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Free Vibration Analysis Of Bolted Lap Joints Under Variable Overlap Length

by
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A THESIS SUBMITTED TO DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR DEGREE OF MASTERS OF SCIENCE IN MECHANICAL SYSTEMS DESIGN AND ENGINEERING

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ABSTRACT

This study presents a comparative analysis of the dynamic behavior of a bolted lap joint with variable lap lengths. The objective of the research is to investigate the effect of overlap length on the stiffness and vibration characteristics of bolted lap joint structures. In the present work, three different overlap lengths, namely 69.5 mm, 55.3 mm, and 39.5 mm, were considered to evaluate the influence of the joint configuration. The study was carried out using three different approaches: numerical simulation using ANSYS, finite element modeling (FEM), and experimental analysis. The natural frequencies obtained from ANSYS and Experimental and FEM from these methods were analyzed and compared to understand the variation of stiffness caused by different overlap lengths. The results indicated that the overlap length considerably affects the structural stiffness and dynamic response of the bolted lap joint. Larger the overlap length greater the contact area between the plates, which results to higher in stiffness and increased natural frequencies, whereas shorter overlap resulted to reduced stiffness. The FEM results show good agreement with the ANSYS simulation whereas there was slight deviation as compared to experimental as per observations, validating the reliability of the developed model. The findings of this study provide useful perception for the design of bolted lap joints in engineering structures subjected to dynamic loading conditions.

Keywords: Natural frequency, Stiffness, Lap joints, Dynamic loading, ANSYS, FEM

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LIST OF ABBREVIATIONS

FOS	Factor Of Safety
FFT	Fast Fourier Transformation
FEM	Finite Element Method
ANSYS	Analytical System
MATLAB	MATrix LABoratory

LIST OF SYMBOLS

K^e	Element stiffness matrix
M^e	Element mass matrix
E	Young's modulus
I	Second moment of area
L_e	Element length
N	Shape Function
f	Frequency
D	Nominal Diameter of Bolt
T	Torque
K	Nut Factor
t	Thickness of plate
M_f	Horizontal Margin
f_u	Ultimate Tensile strength
f_y	Minimum Yield strength
γ_{M2}	Partial safety factor for bolts
A_t	Tensile Stress area
F	Applied Force
ϕ	Eigen Vector
Ω	Angular velocity
$q(t)$	Eigen Vector
$X_{\min}$	Minimum Bolt Spacing
$X_{\max}$	Maximum Bolt Spacing
D_{hole}	Diameter of hole
V_{dsb}	Shear Capacity
V_{dpb}	Bearing Capacity
D_{nominal}	Nominal Diameter

CHAPTER ONE: INTRODUCTION

1.1 Background

Bolted lap joints are one of the most widely used mechanical fastening techniques in structural, aerospace, automotive, and civil engineering applications. Unlike welded joints, bolted connections provide ease of assembly, disassembly, maintenance, and replacement. Because of these advantages, bolted lap joints are extensively used in aircraft fuselage panels, bridge connections, machine frames, pressure vessels, and structural assemblies.

In real engineering structures, joints significantly influence the overall dynamic behavior. Although the connected members may be stiff, the presence of a bolted lap joint introduces local flexibility, contact nonlinearity, frictional slip, and damping effects. These joint characteristics directly affect the natural frequencies, mode shapes, and dynamic response of the structure.

Free vibration analysis is essential for understanding the inherent dynamic characteristics of mechanical systems. The natural frequencies and corresponding mode shapes determine whether resonance may occur under operational loading. If excitation frequency matches the natural frequency, excessive vibration amplitudes can lead to fatigue failure, loosening of bolts, noise, and structural damage. Therefore, accurate prediction of the dynamic behavior of bolted lap joints is critical for safe and reliable design

1.2 Statement of the problem

Bolted lap joints are widely used in engineering structures such as bridges,Cross member of automobile chassis, machine frames, steel structures, and mechanical assemblies because they allow easy assembly, disassembly, maintenance, and replacement of components.The introduction of a joint discontinuity significantly affects the stiffness and dynamic behavior of the structure. In many practical applications subjected to vibration—such as rotating machinery, vehicle structures, and structural where exist dynamic loading can reduced joint stiffness can lead to decrease in natural frequencies, increased in amplitudes of vibration , fatigue failure, and structural instability so, One of the critical parameters influencing the stiffness of a bolted lap joint is bolted lap joint with variable lap length, which determines the frictional contact area between plates and the effectiveness of load transfer through friction and bolt clamping force. There is very few understanding of how variations in lap length affect the effective stiffness and free vibration characteristics of bolted lap joints. Without proper knowledge of this relationship, Designer may either design joints that are insufficiently stiff, leading to major vibration, or over-design them, by increasing material usage and structural weight. So, a systematic study is required to understand how different lap lengths influence the stiffness and natural frequencies of bolted lap joints. This study to compare the dynamic behavior of bolted lap joints with varying lap lengths through numerical simulation using ANSYS and finite element modeling using MATLAB and Experimental method, In order to identify how lap length affects the stiffness characteristics and vibration response of the structure, ultimately helping to design more reliable and vibration resistant mechanical and structural systems.

1.3 Research questions

How does changing the overlap length affect the natural frequencies of a bolted lap joint under free vibration conditions ?

What is the effect of varying overlap length on natural frequency and dynamic stiffness of a bolted lap joint subjected to fixed boundary conditions?

1.4 Research objectives

1.4.1 Main Objective

To study the Vibration behavior of bolted lap joint by changing overlap length

1.4.2 Specific Objective

(i) To Fabricate Bolted Lap Joint for different lap length.

(ii) To perform Numerical simulation and develop FEM model systems by using finite element method

(iii) Validation of Obtained results from Numerical Simulation and FEM Result by Experimental Source of data

1.5 Significance/Rationale of the study

Bolted lap joints are mostly used in structural and mechanical applications because they are easy to put together, take apart, and maintain. In many applications, joints are very important for figuring out how the whole structure will behave when it is under dynamic loading conditions.The widespread use of bolted connections has not yet fully explored how geometric factors like overlap length affect the dynamic properties of the jointed structure.

Most traditional design methods emphasize on static strength criteria and ignore dynamic performance.But vibration behavior is an important design factor in practical situations like aerospace structures, machinery frames, bridges, and mechanical assemblies.So, it is crucial to know how change in the length of the overlap affects the free vibration attributes in order to design structures that are safe , suitable and extend the life of the structure

By applying the Finite Element Method (FEM) for modal analysis, this research offers a reliable computational approach to forecast vibration behavior of bolted lap joints under fixed boundary conditions.The findings of this study will contribute to the development,of dynamic analysis and optimization of mechanical joints in engineering structures.

1.6 Limitations of the Research

(i) The boundary conditions used in the numerical and finite element models are more exemplary. In real world application, the support conditions may slightly vary, which can influence the actual vibration response of the structure.

(ii) The study considers a streamlined bolted lap joint arrangement. In many practical applications such as bridge structures, gusset plate association are more commonly used, which may have slightly different stiffness characteristics.

(iii) Optimization part of lap length is not covered

(iv) The study is limited to free vibration analysis and does not include fatigue behavior, dynamic loading conditions, or long-term structural performance.

CHAPTER TWO: LITERATURE REVIEW

2.1 Mechanical Joints in Engineering Structures

Mechanical joints like bolted connections are important in engineering structures because they make it easy to assemble together and take apart structural parts. Because they are easy to design maintain, these joints are used more in Mechanical,Aerospace, Automotive, and civil engineering. Most Recent studies have focused on how to make bolted joints work better by looking parameters like the diameter of the bolt, the order in which bolt is tightened, and the bolt preload force. Improving these parameters leads to better load distribution better, It also lowers down on the weight of the structure, and makes joints more reliable and durable. So, research on design and optimizing bolted joints is very crucial for making engineering assemblies stronger and more efficient.(Croccolo et al., 2023).

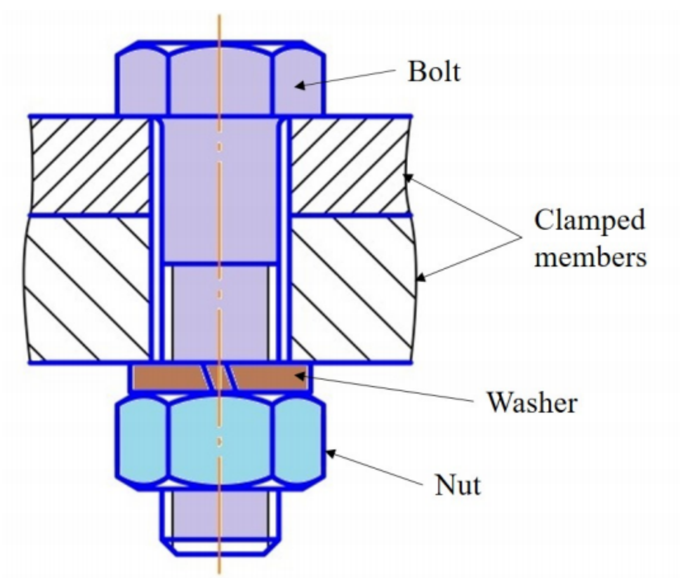

Figure 2.1: Typical Nut Bolt Assembly

Mechanical joints have a huge effect on how engineering structures move and behave. Bolted joints, in particular, add extra damping and stiffness properties that can change how structural systems respond to vibrations. Experimental studies on beam and frame structures have demonstrated that bolted joints can enhance damping characteristics and reduce vibration amplitudes in comparison to monolithic structures. These results show that joints are an important part of controlling the dynamics of a structure and should be carefully thought about when designing and analyzing mechanical assemblies. (Zaman, I. et al., 2013).

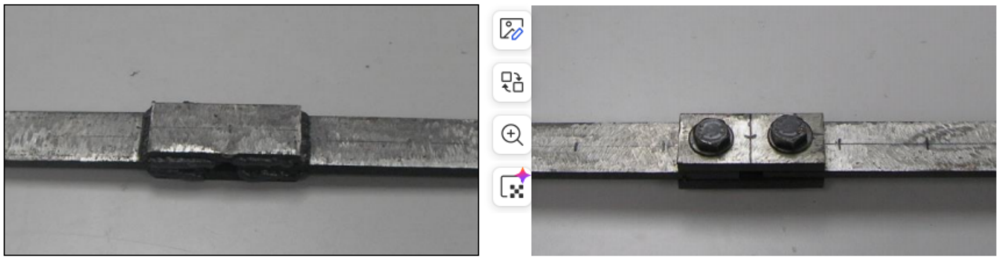

Figure 2.2: Bolted lap joint specimen

Assembled engineering structures like damping characteristics depend a lot on the mechanical joints. Gaul and Becker studied how to predict damping in structures with bolted joints and found that friction at the joint interface plays a important role in the overall structural on damping. The research showed that when material damping is less, the frictional effects in bolted joints had the biggest effect on how the structure moves. The authors also created numerical models that included nonlinear contact and friction behavior in finite element simulations to know how damping would work. The findings indicated that model-reduction techniques can significantly reduce computational expenses while preserving high accuracy in predicting the dynamic response of jointed structures. (Gaul & Becker, 2010).

2.2 Dynamic Behavior of Jointed Structure

(Gaul, Nackenhorst, & Willner, 1994). The dynamic behavior of jointed structures is heavily affected by mechanical joints, like bolted or riveted connections. These joints make the structure's stiffness and damping uneven, which can change how the system vibrates. Studies indicate that friction and micro-slip at the contact interfaces of jointed components facilitate energy dissipation during vibration. Because of this, the dynamic response of structures with joints becomes nonlinear, and it's hard to use simple analytical models to make accurate predictions. (Brake, 2018).

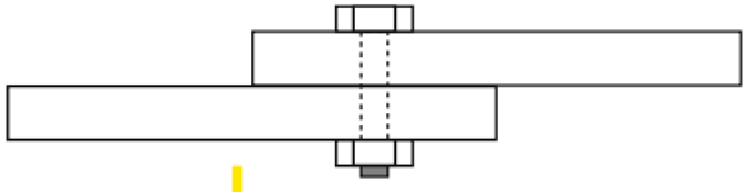

Figure 2.3: Illustration of generic Lap Joint

Mechanical joints have a big effect on how structures respond to dynamic forces because they add nonlinear stiffness and damping mechanisms at the points where two parts are connected. When structures are dynamically loaded, stick-slip motion and frictional contact happen at the joint interface, which causes complex vibrations. Researchers have made analytical and numerical models to better understand these nonlinear interactions and predict how jointed structures will behave in motion. To make engineering assemblies more reliable and better at handling vibrations, you need to know how these systems work.

The research created a model to study how bolted joint structures behave in a nonlinear way when they are excited by a sudden force. The results showed that the vibration response of the structure is greatly affected by the bolt preload, the level of excitation, and the nonlinearity of the interface. Adding cubic stiffness and friction damping models made it easier to predict nonlinear dynamic characteristics. The experimental results confirmed the numerical and finite element models, showing that the responses from the simulations and the measurements were very similar. (Liao, Y. et al., 2016).

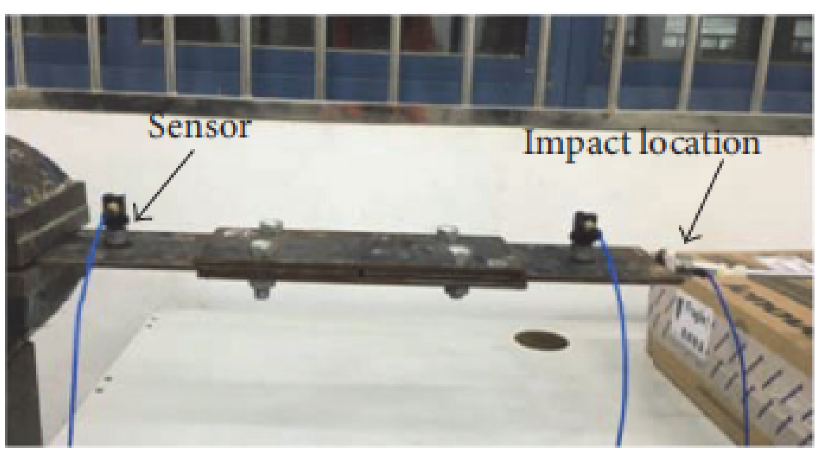

Figure 2.4: Double shear lap beam experimental setup

2.3 Free Vibration of Beam Structure

In structural dynamics, the free vibration behavior of beam structures is an important topic because beams are used a lot as basic parts of engineering structures. When a beam is moved from its stable position and allowed to vibrate without any outside forces, it vibrates freely, which means it has natural frequencies and mode shapes that go with them. The classical Euler-Bernoulli beam theory is often used to explain how slender beams bend and vibrate. The equation of motion that governs this is the one that relates the beam's bending stiffness and mass distribution. You can find out how the beam vibrates by solving the differential equation of motion with the proper boundary conditions.

$$EI\frac{\partial^4 v(x,t)}{\partial x^4} + \rho A\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \tag{2.1}$$

The study examined the free vibration characteristics of Euler–Bernoulli beams subjected to diverse non-classical boundary conditions, including springs, concentrated masses, and torsional restraints. The findings indicated that boundary conditions markedly affect the natural frequencies and mode shapes of the beam. It was noted that the maximum deflection and slope typically occur at the beam ends, and the vibration behavior resembles that of a free–free beam when the stiffness of springs or attached masses approaches zero. The acquired eigenvalues and natural frequencies were found to correlate significantly with findings documented in prior literature. (Silva, J. et al., 2017).

2.1:: Classical Boundary Conditions

Boundary Condition	**Equations**
Free end	$EI\frac{\partial^2 v(x,t)}{\partial x^2} = 0 \quad \text{and} \quad EI\frac{\partial^3 v(x,t)}{\partial x^3} = 0$
Supported end	$v(x,t) = 0 \quad \text{and} \quad EI\frac{\partial^2 v(x,t)}{\partial x^2} = 0$
Clamped end	$v(x,t) = 0 \quad \text{and} \quad EI\frac{\partial v(x,t)}{\partial x} = 0$

Free vibration analysis of beam structures is frequently used for computing the natural frequencies and mode shapes of structural members. The vibration characteristics depend on a number of parameters such as the beam geometry, the material properties, the mass distribution and the boundary conditions. Classical beam theory is often used to analytically derive the frequency equations for beams with different support conditions, such as cantilever, simply supported and fixed beams. Numerical methods, for example, the finite element method, are often used to verify the results from analytical models and improve the accuracy of the vibration predictions

The Euler-Bernoulli beam vibration equation is a fourth order differential equation and so requires four boundary conditions. A cantilever beam is a beam which is fully fixed at one end and has a free end. The fixed end has zero displacement and zero slope, and the free end has zero bending moment and zero shear force. The boundary conditions are applied to the general solution of the beam equation to calculate the natural frequencies and mode shapes of the beam.(Kumar, 2016).

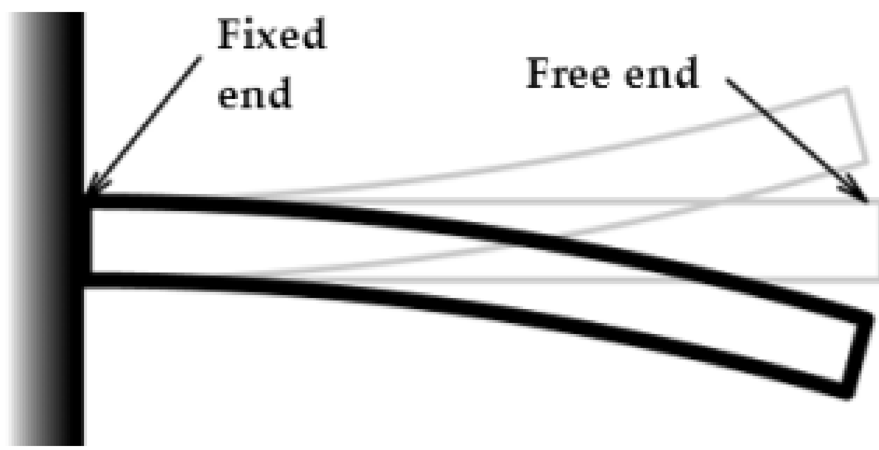

Figure 2.5: Cantilever Beam

In the study of beam vibrations the dynamic behavior is governed by the relationship between bending stiffness and inertia forces acting on the structure. The Euler–Bernoulli beam model posits that cross-sections remain planar and orthogonal to the neutral axis during bending, disregarding shear deformation. When the beam is vibrating freely, its response can be thought of as harmonic motion, where the displacement changes over time and along the beam. Finding the natural frequencies and corresponding vibration modes by solving the governing differential equation is an important part of designing and analyzing structural systems that are subject to dynamic loading. (Gee, 2022).

2.4 Experimental and Numerical Studies on Bolted Lap Joints

Experimental and numerical analyses are frequently employed to comprehend the dynamic behavior of structures featuring bolted lap joints.The research conducted by

Nakalswamy (Nakalswamy, 2010) analyzed that the transient response of structures featuring bolted joints under impact loading through experimental testing and finite element analysis. Accelerometers and Impact hammers was used to measure vibration responses in experimental analysis. The LS-DYNA finite element solver was used to do numerical simulations in ANSYS.

The findings indicated a strong relation between experimental data and numerical forecasts, mostly for lower vibration modes. The research also showed that detailed FEM model with three-dimensional solid elements could accurately capture how bolted structures respond to impact loading.

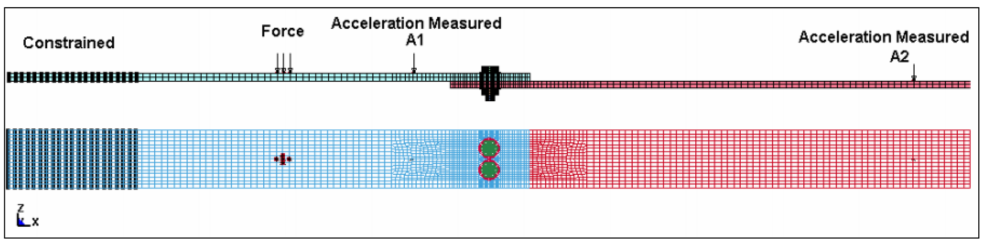

Figure 2.6: Lap Joint Cantilever Beam

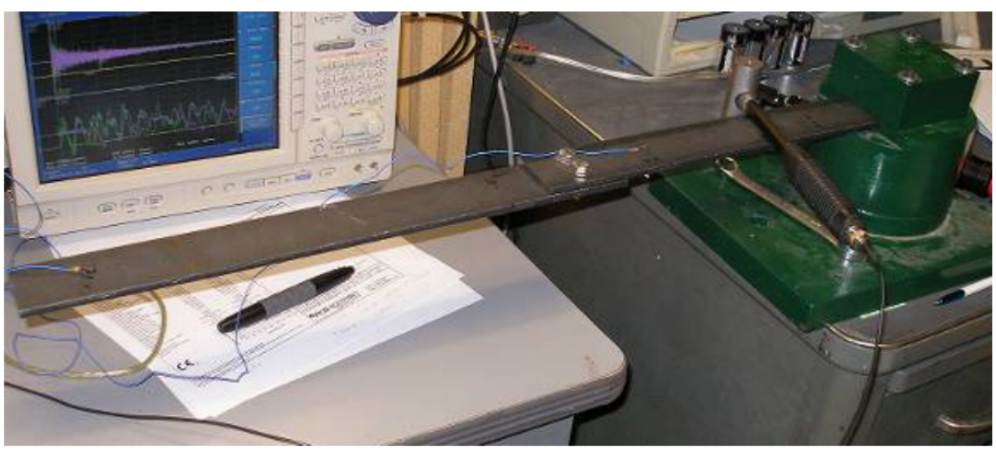

Figure 2.7: Actual Test Setup

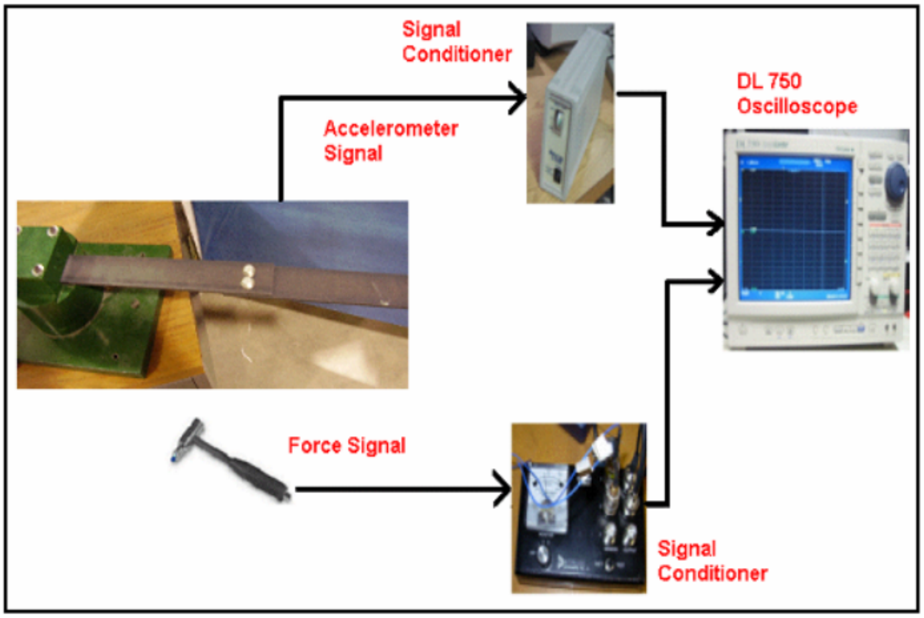

Figure 2.8: Experimentally Data Extraction

More research in the same study looked at how different modeling methods affect the dynamic behavior of bolted joint structures. We compared simplified finite element models that used shell elements to detailed models that had clear bolt representations. The study showed that simplified models predicted higher acceleration responses because they didn't include bolt representation and contact effects. Adding damping parameters, on the other hand, made the numerical and experimental results match up much better. The study also showed that simpler FE models could cut down on computation time while still giving accurate predictions of how shock waves move through bolted joint structures.

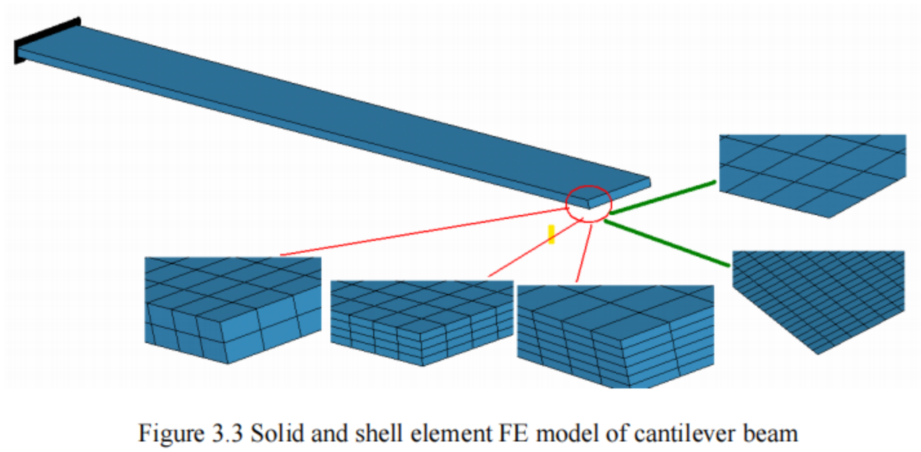

Figure 2.9: Solid and shell element FE model of cantilever beam

Another experimental and numerical study on bolted joints was conducted by Kulkarni and Kulkarni (Kulkarni & Kulkarni, 2017), who studied the effect of bolt pretension on natural frequency of bolted joint structures under low-velocity impact loading. In their experiment, they used an instrumented hammer and accelerometers to hit a cantilever beam with a bolted lap joint and measure its vibration responses. The experimental findings were juxtaposed with pre-stressed modal analysis employing finite element modeling. The research indicated that augmenting bolt preload enhances joint stiffness, leading to elevated natural frequencies of the structure. However, once the preload reaches a certain level, the natural frequency may drop because the effective stiffness at the contact interface between the joint parts goes down.

2.5 Finite Element Modeling of Bolted Joints

A lot of people use finite element modeling to study how structures connected by bolted joints move. In these kinds of analyses, the joint region is often made simpler by using equivalent stiffness or connector elements to show how the connected parts work together.Korolija (Korolija, 2012) created a finite element modeling method for fastener joints in aircraft structures. In ABAQUS, the connected plates were modeled with shell or solid elements, and the bolts were modeled with connector elements. The research showed that this modeling method can accurately show how loads are spread out and how flexible joints are in multi-bolt assemblies. This lets engineers make fairly accurate predictions about how jointed systems will respond structurally. The study also found that the predicted dynamic behavior of the structure is greatly affected by the stiffness values given to the connector elements.

Another study was done byLi et al. who (Li, X. et al., 2020) conducted a study on the dynamic modeling of bolted lap joints through finite element analysis in ANSYS. The researchers created two ways to model the joint interface: a spring-damping element model and a virtual material layer model. To make sure the simulation results were correct, experimental modal testing was done on a lapped beam structure. The comparison showed that the spring-damping element model predicted the natural frequencies of the jointed beam with an error of less than 2.2 percentage This shows that simplified joint models can give accurate results for vibration analysis of bolted structures.

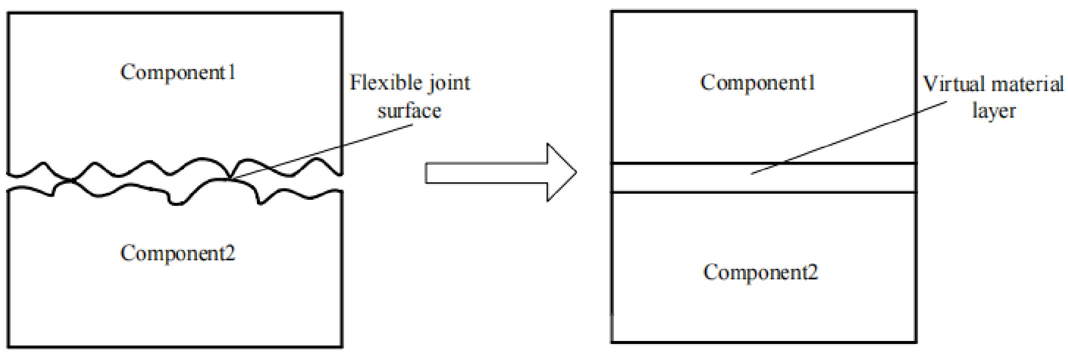

Figure 2.10: Equivalent schematic diagram of virtual material layer

Another method to model bolted joints in dynamic analysis is the use of equivalent material or spring elements to model the joint interface. Zhang et al (Zhang, Y. et al., 2019). conducted modal analysis of bolted structures by establishing equivalent material model to describe the interface between connected components . Their study indicated that the joint interface has a significant influence on the natural frequencies and vibration modes of the assembled structure. The results confirmed that the simplified joint models can accurately model the dynamic behavior of bolted connections and are computationally efficient for finite element simulations.

2.6 Research Gap

(i) Literature shows a strong emphasis in the optimization of bolted joints for improving mechanical strength and static performance with respect to bolt layout, preload, tightening sequence and material choice.

(ii) Some studies have therefore been dedicated to vibration behaviour in adhesive joints or multi-plate bolted systems by finite element and analytical methods

(iii) However, limited research has systematically analyzed and compared the free vibration characteristics of lap joints with varying lap lengths utilizing FEM, ANSYS, and experimental methods.

(iv) Hence, The effect of bolted joint lap length on natural frequencies and mode shape during free vibration is still not well understood. Research gap can be addressed through the free vibration analysis of bolted lap joints with varying lap lengths.

CHAPTER THREE: METHODOLOGY

3.1 Conceptual Framework

The overall flow of the research process is shown in flollowing flow chart :

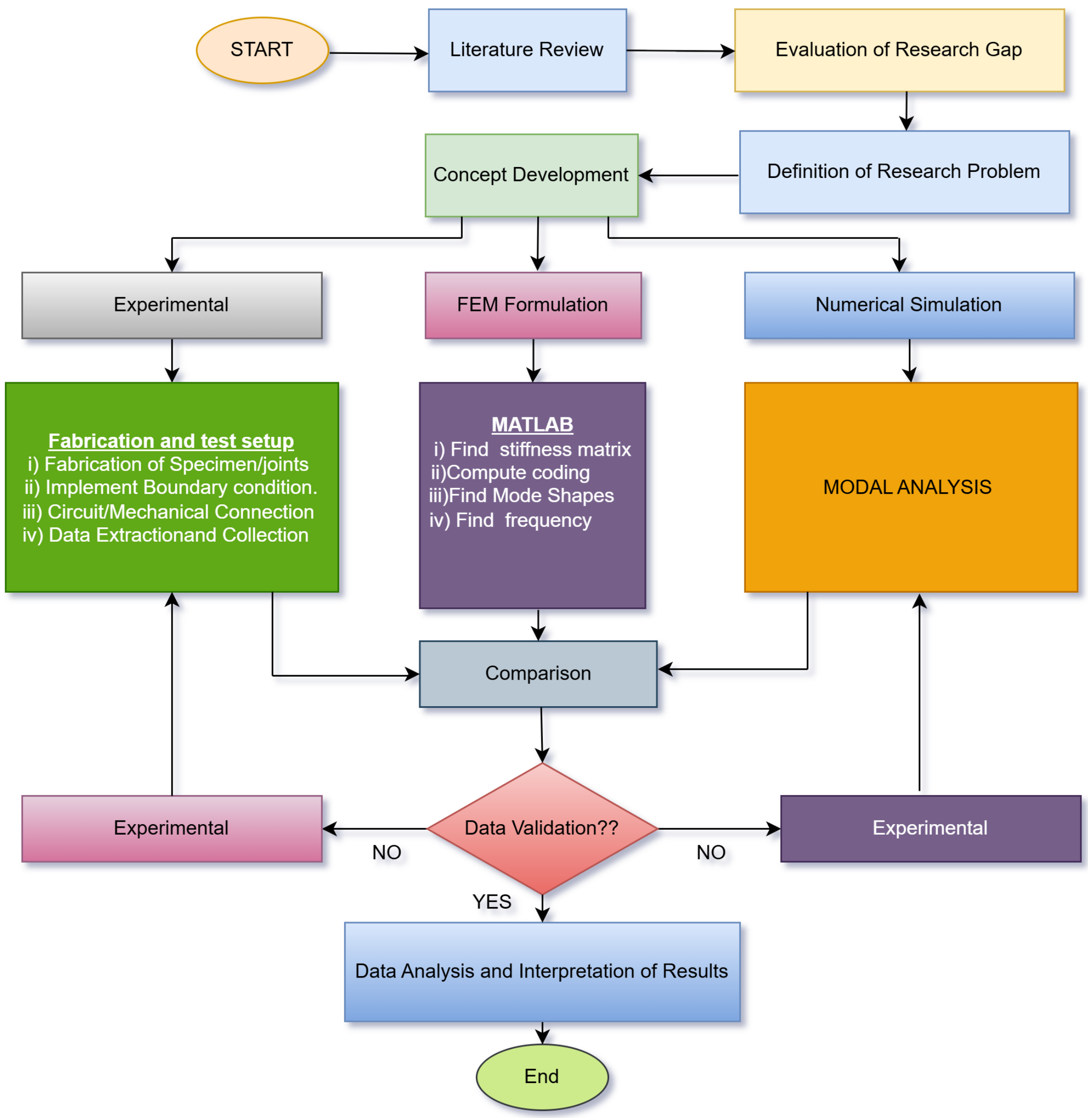

Figure 3.1: Methodology Flow Chart

3.2 Literature Review

This study reviewed existing research related to the topic to understand the current developments and importance of the subject. Previous studies were analyzed based on their methodologies, findings, and limitations to identify research gaps and areas requiring further investigation. By addressing these shortcomings, the present study aims to contribute additional knowledge and provide improved insights into the subject matter.

3.3 Research gap

This study commenced with a review of existing research related to the topic in order to identify the current state of knowledge and the existing research gaps. Previous studies were carefully analyzed to determine the areas that were insufficiently explored or not clearly explained. Based on these observations, the present study was undertaken to address the identified deficiencies and to enhance understanding of the subject through a more detailed and comprehensive investigation.

3.4 Definition of research problem

This study investigated the vibration behavior and stiffness characteristics of bolted lap joints, with particular emphasis on the effect of lap length on their dynamic performance. The research problem was identified through a review of previous studies and practical engineering requirements, which revealed limited understanding regarding the influence of lap length on vibration characteristics. To address this research gap, the free vibration behavior of lap joints was analyzed using both numerical simulations and experimental investigations. This combined approach was adopted to ensure reliable results and to provide a comprehensive understanding of the dynamic behavior and stiffness performance of bolted lap joints.

3.5 Concept Development

In this study, the investigation commenced with the identification of the fundamental concepts and theoretical framework required for analyzing the vibration behavior of bolted lap joints. Key concepts such as beam theory, structural stiffness, natural frequency, and joint interaction mechanisms were studied to establish the basis for the analysis. These theoretical principles were utilized to develop a systematic approach that integrated analytical understanding, numerical simulation, and experimental investigation. The adopted methodology ensured a comprehensive evaluation of the vibration characteristics of bolted lap joints, with the primary objective of examining their dynamic behavior and stiffness performance.

3.6 Experimental Method

The experimental method involved designing and calculating the strength of a bolted lap joint so that it can hold the desired load capacity, which is possible in practice. Here, it means making three different lap joints with different lap lengths: 69.5 mm, 55.3 mm, and 39.5 mm. The length of the lap was measured from straight beams that were 790mm long. The lengths were 9 Percentage, 7 Percentage, and 5 Percentage of the total length, respectively. After the fabrication, the data was taken out with the DAQ system.

3.7 Numerical Simulation

For numerical simulation ANSYS 2021 R2 was use but there were various tools such as ABAQUS,Altair OptiStruct,COMSOL Multiphysics ANSYS 2021 R2 was easy to access and understand.So, a static structural analysis was performed in ANSYS 2021 first to verify the safety of the structure and the static structure result with pre-stress on bolt is extracted for modal analysis to find the natural frequency.

3.8 Finite Element Formulation

Finite element formulation was done starting from strong form of equation and converting them into weak form to solve eigen value problem which finally give natural frequency. The solution was obtained by using MATLAB software.

3.9 Comparison of Data from all analysis

In this study, a comparison of data obtained from FEM, ANSYS and experimental analysis was conducted to determine the accuracy and reliability of the results. The natural frequencies and vibration characteristics of each method were compared systematically to find out the similarities and differences. All methods exhibited similar trends with minor differences attributable to modeling assumptions and experimental conditions. This comparison proved useful in order to validate the numerical models and check that the overall analysis was giving reliable and relevant outputs.

3.10 Data Validation

In this study, the data were verified by comparing the results of FEM, ANSYS and actual tests to ensure the consistency and reliability of the data. We considered results to be good if the error was below 25 percent and all methods showed a similar pattern. Where there were differences we changed things. For the simulation results we double checked and adjusted the settings and assumptions and for the test results we double checked the physical setup. This process ensured that the data used in the final study was reliable and valid. The results of FEM, ANSYS and experimental analysis had to match with each other to ensure the accuracy of the results. The data were verified by FEM, ANSYS and real tests. The results of FEM, ANSYS and experimental analysis should be in agreement.

3.11 Interpretation of result

In this portion we analyzed the results in order to understand the physical behavior of the system and its relation to the quantities of interest. Frequency and stiffness trends with varying lap lengths were studied.Studies was also made for the effect of different factors on the system. Here the comparison of results from FEM, ANSYS and experiments was done to find correct relationship. This allowed us to draw conclusions regarding the performance of the bolted lap and the influence of lap length on the dynamic response. We were looking at the dynamics behavior of the bolted lap joint.

CHAPTER FOUR: FABRICATION AND MODEL BUILDING

4.1 Material Testing of Test Specimen

Different parameters obtained from material testing are as follows :

Parameters	**Sample 1**	**Sample 2**
Thickness (mm)	6.099	6.559
	6.069	6.559
	6.074	6.574
Avg thickness (mm)	6.0806	6.564
Width (mm)	49.12	38.13
	49.02	38.16
	48.94	38.02
Avg width (mm)	49.026	38.103
Yield load (kg)	11200	8600
Ultimate load (kg)	15700	11300
Initial gauge length (cm)	9.5	9
Final gauge length (cm)	12	12

4.1:: Experimental Data for Two Samples

Material testing of mild steel (MS) plates of 6 mm thickness was carried out using a tensile test to find their mechanical properties. Two specimens were prepared, where the average measured thickness was 6.0806 mm for Sample 1 and 6.564 mm for Sample 2, indicating slight variation from the nominal size. During testing, Sample 1 exhibited a yield load of 11200 kg and an ultimate load of 15700 kg, while Sample 2 showed comparatively lower values with a yield load of 8600 kg and an ultimate load of 11300 kg. The initial gauge lengths were 9.5 cm and 9 cm for Sample 1 and Sample 2 respectively, and both samples reached a final gauge length of 12 cm after fracture, demonstrating noticeable elongation and ductile behavior. The results indicate that Sample 1 has higher strength than Sample 2 and both samples have good ductility. This makes MS steel suitable for structural applications where both strength and deformation capacity are important.

4.1.1 Tensile Test Calculations

Cross-Sectional Area

Sample 1:

$$A_1 = t \times w \tag{4.1}$$
$$= 6.0806 \times 49.026 \tag{4.2}$$
$$\approx 298\,\mathrm{mm}^2 = 298 \times 10^{-6}\,\mathrm{m}^2 \tag{4.3}$$

Sample 2:

$$A_2 = 6.564 \times 38.103 \tag{4.4}$$
$$\approx 250\,\mathrm{mm}^2 = 250 \times 10^{-6}\,\mathrm{m}^2 \tag{4.5}$$

Yield Strength

Formula:

$$\sigma_y = \frac{F}{A} \tag{4.6}$$

Sample 1:

$$F = 11200 \times 9.81 = 109872\,\mathrm{N} \tag{4.7}$$
$$\sigma_y = \frac{109872}{298 \times 10^{-6}} \tag{4.8}$$
$$\approx 369\,\mathrm{MPa} \tag{4.9}$$

Sample 2:

$$F = 8600 \times 9.81 = 84366\,\mathrm{N} \tag{4.10}$$
$$\sigma_y = \frac{84366}{250 \times 10^{-6}} \tag{4.11}$$
$$\approx 337\,\mathrm{MPa} \tag{4.12}$$

Ultimate Strength

Sample 1:

$$F = 15700 \times 9.81 = 154017\,\text{N} \tag{4.13}$$

$$\sigma_u = \frac{154017}{298 \times 10^{-6}} \tag{4.14}$$

$$\approx 517\,\text{MPa} \tag{4.15}$$

Sample 2:

$$F = 11300 \times 9.81 = 110853\,\text{N} \tag{4.16}$$

$$\sigma_u = \frac{110853}{250 \times 10^{-6}} \tag{4.17}$$

$$\approx 443\,\text{MPa} \tag{4.18}$$

Engineering Strain

$$\varepsilon = \frac{L_f - L_i}{L_i} \tag{4.19}$$

Sample 1:

$$\varepsilon_1 = \frac{12 - 9.5}{9.5} \tag{4.20}$$

$$\approx 0.263 \tag{4.21}$$

Sample 2:

$$\varepsilon_2 = \frac{12 - 9}{9} \tag{4.22}$$

$$= 0.333 \tag{4.23}$$

Percentage Elongation

$$\%\text{Elongation} = \frac{L_f - L_i}{L_i} \times 100 \tag{4.24}$$

Sample 1:

$$\%E_1 = \frac{2.5}{9.5} \times 100 \tag{4.25}$$

$$\approx 26.32\% \tag{4.26}$$

Sample 2:

$$\%E_2 = \frac{3}{9} \times 100 \tag{4.27}$$
$$= 33.33\% \tag{4.28}$$

True Strain

$$\varepsilon_{true} = \ln\left(\frac{L_f}{L_i}\right) \tag{4.29}$$

Sample 1:

$$\varepsilon_{true,1} = \ln\left(\frac{12}{9.5}\right) \tag{4.30}$$
$$\approx 0.233 \tag{4.31}$$

Sample 2:

$$\varepsilon_{true,2} = \ln\left(\frac{12}{9}\right) \tag{4.32}$$
$$\approx 0.287 \tag{4.33}$$

Summary of Results

Parameter	Sample 1	Sample 2
Yield Strength (MPa)	369	337
Ultimate Strength (MPa)	517	443
Engineering Strain	0.263	0.333
% Elongation	26.32	33.33
True Strain	0.233	0.287
Young's modulus (Gpa)	200	200

Hence the values obtained from material testing was used for load selection criteria and to design bolted structure for its safe limit. The value of young's modulus was found by using simulation software and matlab code to reduce percentage error of result.t

4.2 Experimental Setup

(i) Fabrication of bolted lap joint specimens of variable lap length

(ii) Setup of boundary conditions for the beam structure

(iii) Installation of sensors and data acquisition system.

(iv) Data Extraction By varying lap Length for 69.5mm, 55.3mm, 39.5mm .

4.2.1 Fabrication of Test Specimen as per Design

The workpiece used in this research study consists of a bolted lap joint beam assembly fabricated by two mild steel plates. The upper beam has length of 340 mm and the lower beam has length of 450 mm, both having a uniform cross-section of 50.8 mm × 6 mm. These two beams are assembled in lap joint form by providing different overlap lengths from 69.5 mm, 55.3 mm,up to 39.5 mm in order to study the effect of overlap length on the dynamic characteristics of the jointed structure.

In addition to the lap joint specimens, a straight beam specimen without overlap was also fabricated for comparison. The straight beam had a total length of 790 mm and the same cross-section of 50.8 mm × 6 mm. This specimen represents a monolithic beam without joint discontinuity (i.e full continuous) and is used as a reference beam in the experimental analysis.

During testing, the beam specimen is clamped using a vice fixture. A clamping length of 25 mm is provided at the fixed end in order to ensure a rigid boundary condition to make fixed support, while the remaining length acts as the free vibrating portion of the beam.

Two M8 bolts of grade 4.6 are used to connect the overlapped beams and form the lap joint. The bolts are located at the midpoint of the overlap region for each overlap configuration, namely 69.5 mm, 55.3 mm, and 39.5 mm. The bolts are tightened using a calibrated torque wrench so as to maintain a constant preload all the joint.

Bolt pretension has a significant imapct on the stiffness and dynamic behavior of bolted joints. In the present work, the bolt pretension is taken as 70% of the bolt yield strength. which was made constant so According to this pretension force was taken as

$$F = 6148.4\,\mathrm{N} \tag{4.34}$$

The tightening torque required to produce this bolt preload is calculated using (Nakalswamy, 2010)

$$T = FDK \tag{4.35}$$

where

$$F = \text{bolt pretension force} \tag{4.36}$$

$$D = \text{nominal bolt diameter} \tag{4.37}$$

$$K = \text{nut factor} \tag{4.38}$$

For the present case,

$$F = 6148.4\,\mathrm{N} \tag{4.39}$$

$$D = 8\,\mathrm{mm} = 0.008\,\mathrm{m} \tag{4.40}$$

According to Nakalswamy (2010), the nut factor is taken as

$$K = 0.2 \tag{4.41}$$

Therefore,

$$T = 6148.4 \times 0.008 \times 0.2 \tag{4.42}$$

$$T = 9.83\,\mathrm{Nm} \tag{4.43}$$

Hence, a tightening torque of 9.837 Nm was applied to both the bolts using a calibrated hand torque wrench to maintain a pretension of 6148.4 N in the bolted lap joint. This controlled pretension provides better clamping between the plates and helps maintain consistent experimental conditions during vibration testing.

4.2:: Geometrical specifications of test specimens

Component	**Parameter**	**Value**
Upper Plate	Length	340 mm
	Width	50.8 mm
	Thickness	6 mm
Lower Plate	Length	450 mm
	Width	50.8 mm
	Thickness	6 mm
Straight Beam	Length	790 mm
	Width	50.8 mm
	Thickness	6 mm
Bolt	Type	M8 Hex Bolt
	Grade	4.6
	Pitch	1.25 mm
Bolt Head	Height	5 mm
Nut	Height	7 mm

4.2.2 Geometric Dimension

Clearances for Holes for Fasteners

The bolts may be in standard size oversize short slotted or long slotted holes. In the present work we consider the case of oversized hole as standard. Oversize holes may be used only in slip-resistant connections and hold-down bolted connections, where specified, provided that the oversize holes in the outer ply are covered by a cover plate of sufficient size and thickness and having a hole not larger than the standard clearance hole.

$$D_{hole} = D_{nominal} + 1 \text{ mm clearance} \qquad \text{(For oversized hole)} \tag{4.44}$$

Minimum Spacing

The distance between the centres of fasteners shall not be less than 2.5 times the nominal diameter of the fastener.

Hence the minimum spacing is (Bureau of Indian Standards, 2007).

$$X_{min} = 2.5 \times d \tag{4.45}$$

For $d = 8$ mm

$$X_{min} = 2.5 \times 8 = 20 \text{ mm} \tag{4.46}$$

Hence, the spacing is taken as

$$X_{min} = 22.8 \text{ mm} \tag{4.47}$$

Maximum Spacing

The distance between the centres of any two adjacent fasteners shall not exceed $32t$ or 300 mm, whichever is less, where t is the thickness of the thinner (Bureau of Indian Standards, 2007)..

$$X_{max} = 32t \quad \text{or} \quad 300 \text{ mm} \tag{4.48}$$

For $t = 6$ mm

$$X_{max} = 32 \times 6 = 192 \text{ mm} \tag{4.49}$$

Since $192 < 300$ mm,

$$X_{max} = 192 \text{ mm} \tag{4.50}$$

Margin

The minimum edge and end distances from the centre of any hole to the nearest edge of a plate should not be less than 1.7 times the hole diameter in case of sheared or hand flame cut edges, and 1.5 times the hole diameter in case of rolled or machine flame cut, sawn and planed edges.

The maximum edge distance to the nearest line of fasteners from an edge of any unstiffened part should not exceed $12t$, where t is the thickness of the thinner outer plate.(Bureau of Indian Standards, 2007).

Hence,

$$M_f = 1.5 \times D_{hole} \tag{4.51}$$

For $D_{hole} = 9$ mm

$$M_f = 1.5 \times 9 \tag{4.52}$$

$$M_f = 13.5 \text{ mm} \tag{4.53}$$

Hence, the horizontal margin is taken as

$$M_f = 13.5 \text{ mm} \tag{4.54}$$

4.2.3 Design Calculation

(i) Bolt grade: 4.6

(ii) Pitch: 1.25 mm

(iii) Low carbon steel

(iv) Not heat treated

(v) Ultimate Strength = 400Mpa , Yield Strength = 240 Mpa

For grade 4.6 bolt,

$$f_u = 4 \times 100 \tag{4.55}$$

$$f_u = 400\,\text{MPa} \tag{4.56}$$

Minimum yield strength,

$$f_y = 0.6 \times f_u \tag{4.57}$$

$$f_y = 0.6 \times 400 \tag{4.58}$$

$$f_y = 240\,\text{MPa} \tag{4.59}$$

As per IS 800:2007 (Bureau of Indian Standards, 2007), Recommended factor of safety for different applications

4.3:: Recommended factor of safety for different applications

S.N.	**Application**	**FOS**
1	Simple static joint	1.25 – 2.0
2	Common machinery with variable load	2.0 – 3.0
3	Cyclic or fatigue-sensitive joints	3.0 – 6.0
4	Safety-critical parts	4.0 – 8.0

4.2.4 Design Calculation for Bolted Lap Joint

The design is made to support a tensile load of 6 kN.

$$\mathrm{FOS} = 4 \tag{4.60}$$

$$\text{Yield strength of bolt} = 240\,\mathrm{MPa} \tag{4.61}$$

So,

$$\sigma_{\text{yield}} = \frac{F}{A} \times \mathrm{FOS} \tag{4.62}$$

Therefore,

$$A = \frac{F \times \mathrm{FOS}}{\sigma_{\text{yield}}} \tag{4.63}$$

$$A = \frac{6 \times 10^3}{240} \times 4 \tag{4.64}$$

$$A = 100\,\mathrm{mm}^2 \tag{4.65}$$

This is the required cross-sectional area to carry 6 kN tensile load.
Total number of bolts:

$$n = 2 \tag{4.66}$$

Area of each bolt:

$$A_{\text{bolt}} = \frac{100}{2} \tag{4.67}$$

$$A_{\text{bolt}} = 50\,\mathrm{mm}^2 \tag{4.68}$$

Cross-sectional area of bolt is

$$A = \frac{\pi D^2}{4} \tag{4.69}$$

Hence,

$$D = \sqrt{\frac{4A}{\pi}} \tag{4.70}$$

$$D = \sqrt{\frac{50 \times 4}{\pi}} \tag{4.71}$$

$$D = 7.978\,\text{mm} \tag{4.72}$$

This value represents the root diameter or minor diameter.

Hence, M8 is taken as the nominal diameter.

4.2.5 Design Load Capacity for M8 Bolt

1. Shear Capacity

As per IS 800:2007 (Bureau of Indian Standards, 2007), the design shear capacity of the bolt is given by

$$V_{dsb} = \frac{0.6\, f_u\, A_t}{\gamma_{M2}} \tag{4.73}$$

where,

$$\gamma_{M2} = 1.25 \tag{4.74}$$

$$f_u = 400\,\text{MPa} \tag{4.75}$$

$$A_t = \frac{\pi}{4}(D - 0.9382p)^2 \tag{4.76}$$

For M8 bolt with pitch $p = 1.25$ mm,

$$A_t = \frac{\pi}{4}(8 - 0.9382 \times 1.25)^2 \tag{4.77}$$

$$A_t = 36.6\,\text{mm}^2 \tag{4.78}$$

Therefore,

$$V_{dsb} = \frac{0.6 \times 400 \times 36.6}{1.25} \tag{4.79}$$

$$V_{dsb} = 7027.2\,\text{N} \tag{4.80}$$

$$V_{dsb} \approx 7\,\text{kN} \tag{4.81}$$

2. Bearing Capacity

As per IS 800:2007 (Bureau of Indian Standards, 2007), the design Bearing capacity of the bolt is given by

$$V_{dpb} = \frac{f_u \times D \times t}{\gamma_{M2}} \tag{4.82}$$

where,

$$f_u = 360\,\text{MPa} \tag{4.83}$$

$$D = 8\,\text{mm} \tag{4.84}$$

$$t = 6\,\text{mm} \tag{4.85}$$

$$\gamma_{M2} = 1.25 \tag{4.86}$$

Therefore,

$$V_{dpb} = \frac{360 \times 8 \times 6}{1.25} \tag{4.87}$$

$$V_{dpb} = 13824\,\text{N} \tag{4.88}$$

$$V_{dpb} = 13.824\,\text{kN} \tag{4.89}$$

4.2.6 Equipment used in Experimental setup

Accelerometer

Piezo accelerometer (Brüel & Kjær), also known as a seismic mass, was used to measure the vibration of the structure. Its original sensitivity was 1.004 pc/m/s^2, while the calibrated sensitivity was found to be 0.974 pc/m/s^2.

Programmable DC power Supply

Programmable DC power supply is used to power charged amplified with desired voltage and current .Currently in our text experiment voltage rating is set up to 4.5 volts and current as 0.5 amp

Charged Amplifier

In vibration testing, (Brüel & Kjær) 2635 charge amplifier was used to convert the small electric charge output from a piezoelectric accelerometer into required voltage signal. This conversion is essential because the accelerometer itself produces a high-impedance, low-level picocoulomb charge signal that cannot be directly read by a DAQ system. The charge amplifier conditions this signal by amplifying it, reducing noise using low and high pass filter which is inbuild in amplifier, and matching impedance, allowing accurate data acquisition in systems from LabVIEW to analyzing natural frequency and other dynamic properties

DAQ

The analog signals of the sensors, e.g. the voltage output of a charge amplifier, are collected and converted into digital data by means of a portable DAQ (Data Acquisition) system for the processing in software, e.g. LabVIEW. In vibration testing, it acquires the time-varying signal of the structure's response and allows analysis such as FFT (Fast Fourier Transform) to accurately identify the natural frequency.

4.2.7 Procedure for Experimental Data Collection

(i) Initially fabrication of bolted lap joint was carried out , in total 3 sample were prepared by varying lap length as 69.5mm , 55.3mm and 39.5mm

(ii) Bolt preload condition is set as per the design specification with calibrated torque wrency .

(iii) Providing fixed support at two end points for both the samples with the help of vice.

(iv) Setting the accelerometer at the stiffer portion to reduce noise effect

(v) As per availability, the accelerometer has a sensitivity of 1.004 pC/m/s2. After the calibration of accelerometer new sensitivity was found to be 0.974p C/m/s2 . Which was found using calibration circuit with the help of calibrator

(vi) The output terminal of the accelerometer is connected to the input of the charge amplifier.

(vii) The charge amplifier is powered with a programmable DC supply source with a voltage of 4.5 V and a current of 0.5 A.

(viii) Once the charge amplifier is powered and the sensitivity value is set, the output of the charge amplifier is connected to the DAQ.

(ix) Required Gain is fed into lab view , and with sampling rate as prescribed by Nyquist–Shannon sampling theorem it is kept as 2000 with no of sample 25000

(x) Once the charge amplifier is connected to the DAQ, the output of the DAQ is fed into LabVIEW to obtain to Find Acceleration vs Time in excel file using write to measurement file

4.2.8 Mechanical Circuit connection

The mechanical circuit layout is shown as in the figure below:

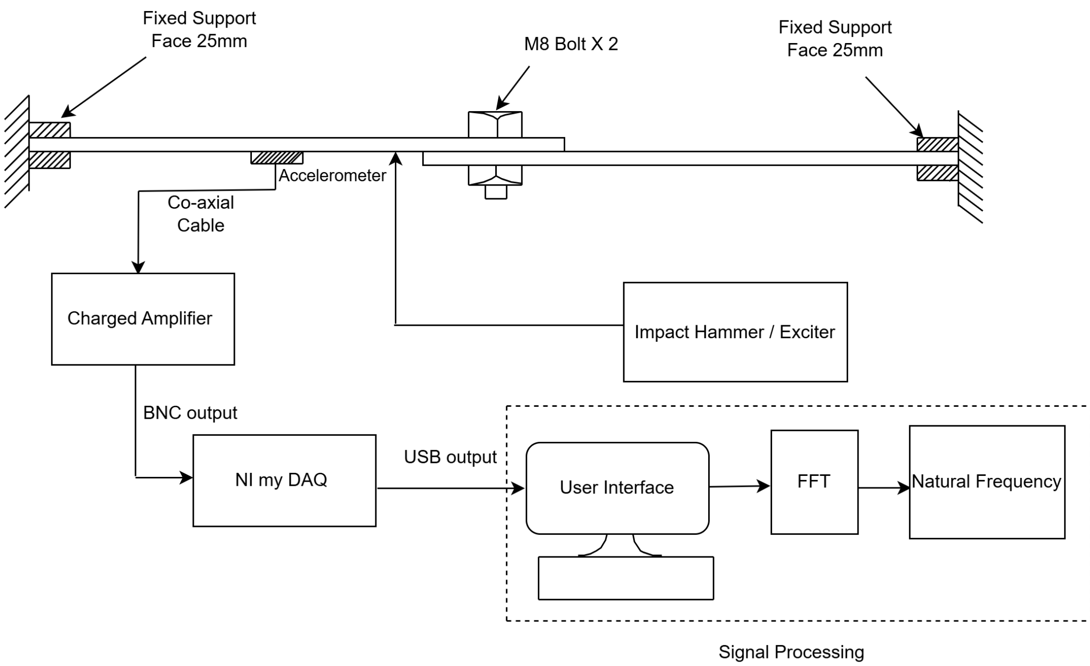

Figure 4.1: Circuit Layout

This setup shows an experimental vibration measurement for a bolted lap joint where the specimen is rigidly fixed at both ends and excited by using an impact hammer (or exciter) to induce vibrations; an accelerometer mounted on the specimen senses the resulting dynamic response and converts acceleration (in m/s^2) into an electrical charge signal based on its sensitivity (1.004 $pC/m/s^2$), which is then passed through a charge amplifier that converts this high-impedance charge signal into a usable voltage signal, maintaining signal quality; this conditioned signal is fed into a data acquisition (DAQ) system (via BNC connection), which digitizes the analog signal and sends it to a laptop through USB, where a user interface processes the time-domain vibration data and applies FFT (Fast Fourier Transform) to convert it into the frequency domain (Done by using lab view software) , allowing identification of natural frequencies and dynamic characteristics of the bolted lap joint for further analysis and validation.

4.2.9 Lab View Circuit Diagram

This LabVIEW block diagram converts vibration data from an accelerometer to acceleration vs time to determine the natural frequency. The DAQ Assistant acquires the raw voltage signals from the accelerometer at the specified rate (sampling frequency) and number of samples, and the stop button controls the data acquisition. Then an offset is subtracted from the raw voltage (bias removal) and the result is divided by the sensitivity of the accelerometer in order to convert voltage into actual acceleration values . The signal is smoothed or analyzed using a mean/filter block and the Set Dynamic Data Attributes block is used to label and format the processed acceleration signal. Finally, the Write to Measurement File block saves the (acceleration vs. time) data into an Excel file, allowing to later perform FFT or other analyses to extract the natural frequency of the test specimen.

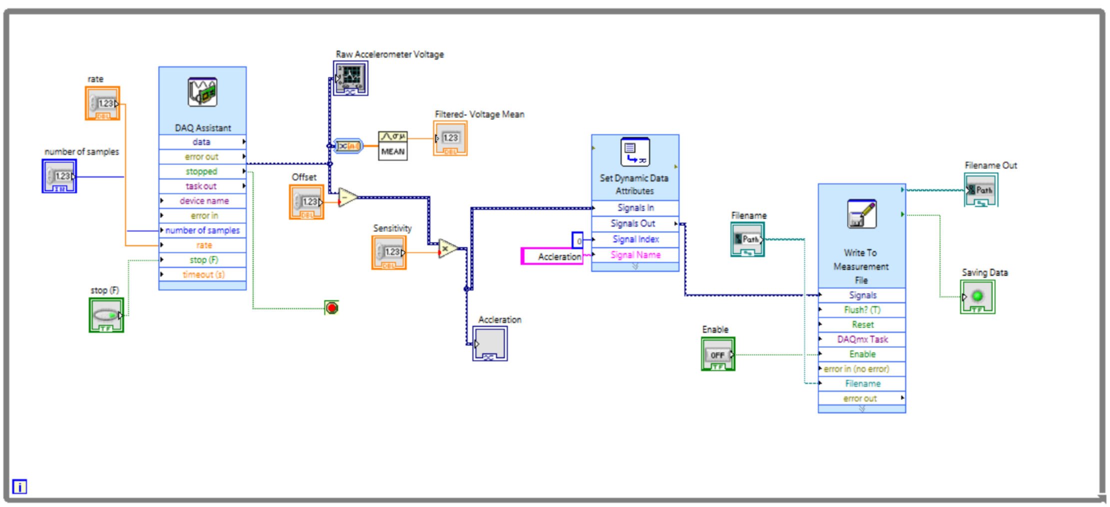

Figure 4.2: Schematic Layout of circuit diagram for lab view

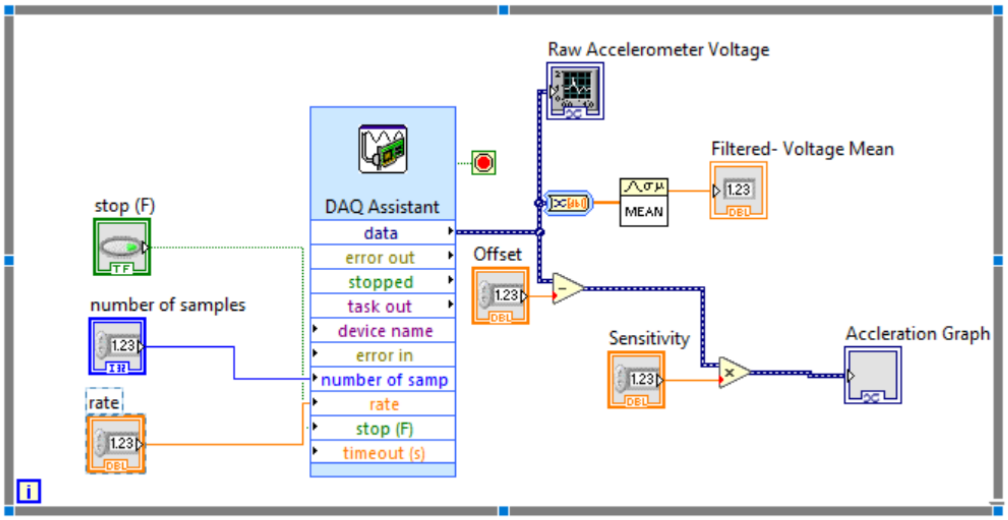

Figure 4.3: Calibration circuit Diagram

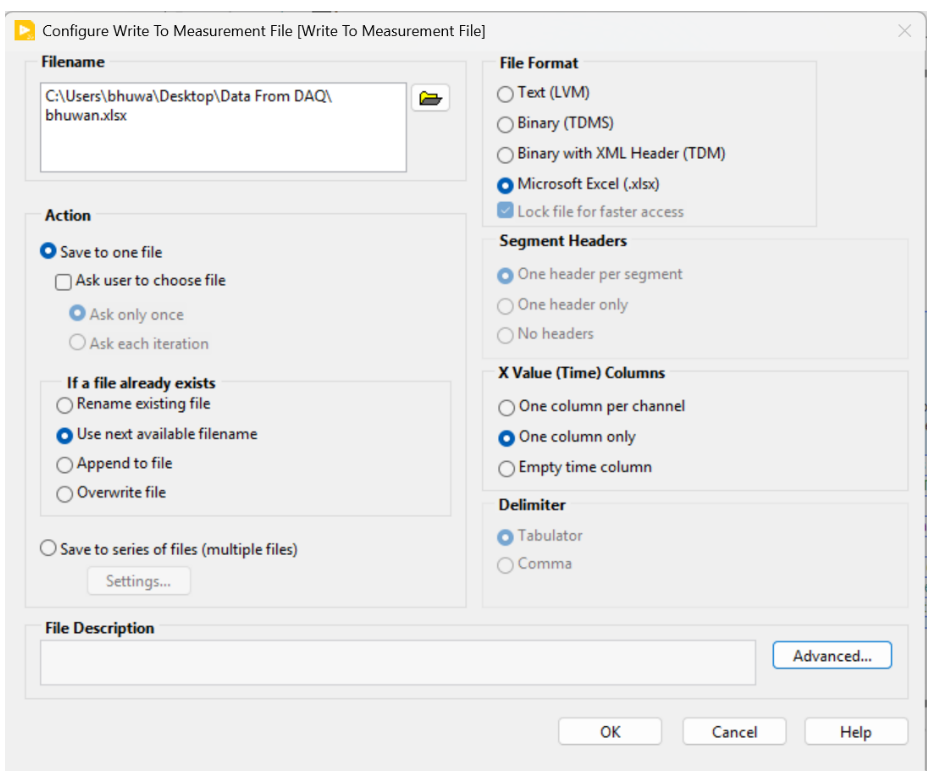

Figure 4.4: Schematic Layout of write to measurement file

Write to measurement Module helps to store the continuous data in terms of excel format, whose interface can be seen in above figure clearly Value of acceleration vs time was extracted.

4.2.10 Interface of lab view

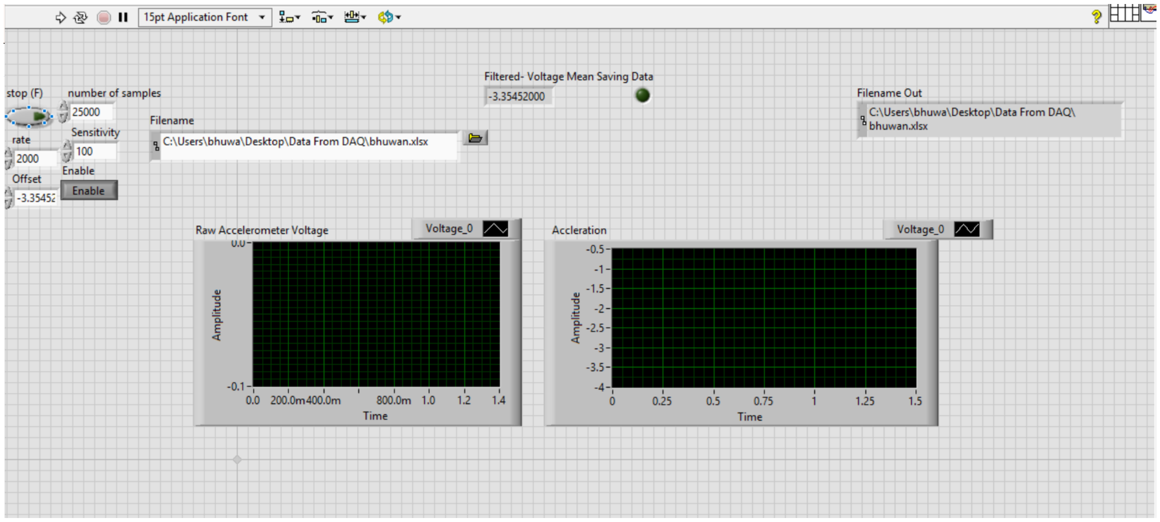

Figure 4.5: Background Interface of Lab View

This LabVIEW front panel is used to acquire and visualize acceleration vs time data from an accelerometer and at the same time save the data to an excel file C:From DAQ.xlsx. The system collects 25,000 samples at a sampling rate of 2000 Hz, which leads to a total recording time of about 12.5 s, which is enough to record the vibration decay for the natural frequency identification. The raw voltage signal from the accelerometer is shown in the graph "Raw Accelerometer Voltage" and then converted into physical acceleration by means of the given sensitivity (100) and offset correction (-3.3545). The processed signal is shown in the "Acceleration" waveform graph (acceleration vs. time) to help you visually evaluate the signal quality, noise level and clarity of the vibration response.The mean / filtered value indicator provides a quick indication of signal bias, and the Enable and Saving Data indicators confirm when data is being written. This setup allows you to store accurate time-history data for later FFT analysis and to check in real time if the waveform is clean enough for reliable extraction of the natural frequency.

4.3 Numerical Simulation

In order to study dynamic characteristics of joint, the numerical simulation of bolted lap joint was carried out by using ANSYS Workbench to do modal analysis. The actual dimensions of the plates and bolt connections was used to develop a 3d model, and the contact region interactions and material properties of mild steel was defined in the overlap region. We discretized the bolted lap joint geometry by using a finite element mesh and applied fixed support boundary conditions to mimic the experimental setup. Then, modal analysis was done to get the natural frequencies and mode shapes of bolted joint.

The results reveal insight into the vibration behavior of the structure and allow comparison with analytical FEM results to verify the numerical model.

4.3.1 ANSYS workbench setup

I used ANSYS 2021 R2 for performing the numerical simulation of bolted lap joint because it is easy to get and gives reliable results for modal analysis. To analyze about the structure's vibration properties, ANSYS made the shapes of a straight beam and a bolted lap joint. The lap joint model was created by changing the overlap lengths to 69.5 mm, 55.3 mm, and 39.5 mm. This was done to see how the length of the overlap affects how the joint moves. To make the bolt modeling easier while keeping its structural effect, the bolt shank was modeled using its tensile stress area, which is 36.6 mm^2. The M8 bolt fit through the bolt holes, which were 9 mm in diameter.

We used the Multizone method to mesh the upper and lower plates for discretization. This method makes structured elements that work well with beam-like shapes. The Hex Dominant method was used to mesh the bolt, and the body size was set to 2 mm. The overall element size for the other parts stayed at 4 mm to keep a good balance between speed and accuracy.

A force of 6148.4 N was used to simulate the tightening effect of the bolt in the joint. The area where the plates overlap was defined as having a friction coefficient of 0.2, and the area where the bolt and nut touch was defined as having a bonded connection to show that it is a strong connection. The Static Structural module was used to apply the bolt's pretension first. Then, the Modal Analysis module was used to look at the assembly's vibration characteristics. Lastly, the clamped areas of the plates were given fixed support boundary conditions to copy the experimental setup and find the natural frequencies and mode shapes of the bolted lap joint system.

4.4:: Contact Region

S.N	Structure	Contact	Remarks
1	340 mm and 450 mm plate	Frictional	$\mu = 0.2$
2	340 mm plate and Bolt shank	Frictional	$\mu = 0.2$
3	450 mm plate and Bolt shank	Frictional	$\mu = 0.2$
4	340 mm plate and Bolt head	Frictional	$\mu = 0.2$
5	450 mm plate and Nut	Frictional	$\mu = 0.2$
6	Bolt shank and Nut	Bonded	X2

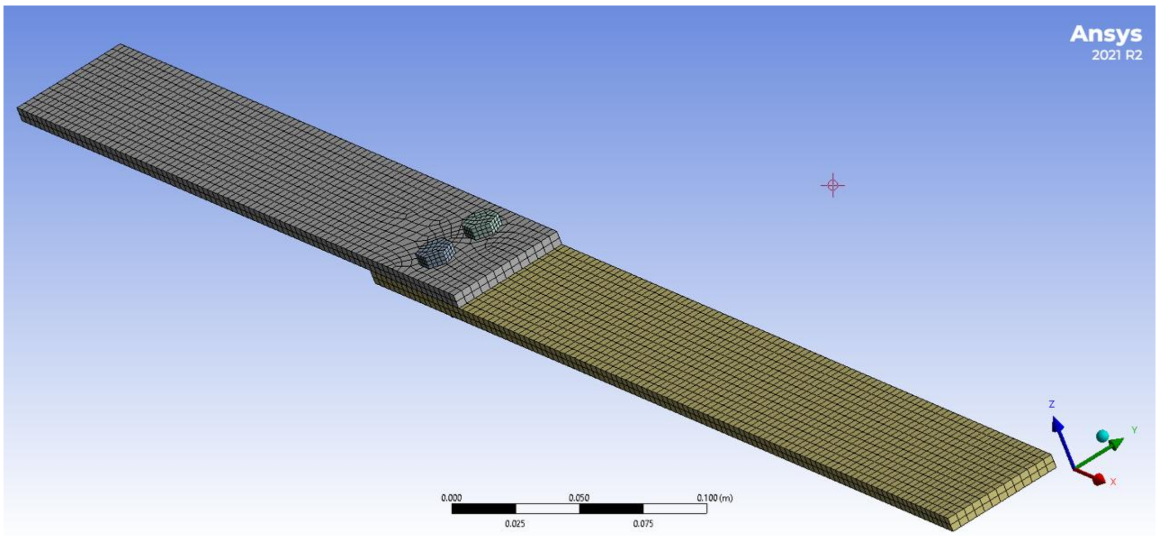

Figure 4.6: Mesh Structure

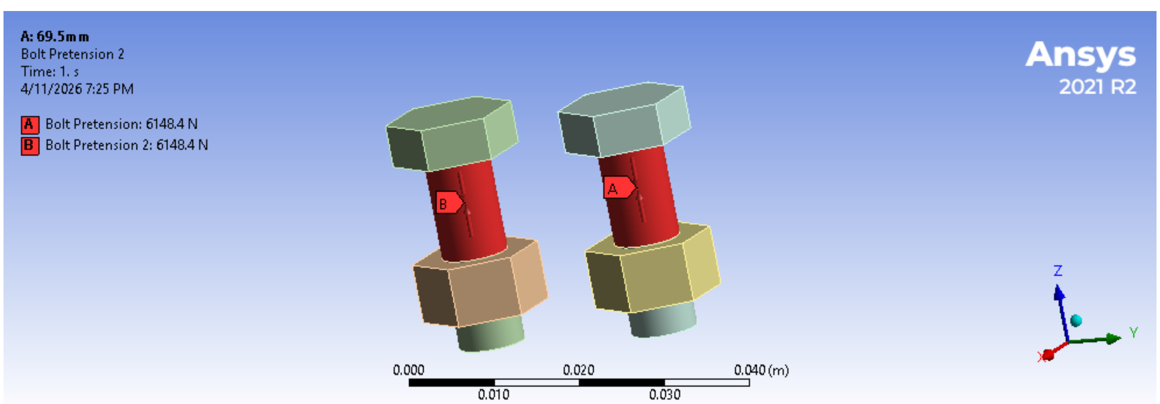

Figure 4.7: Bolt Pretension

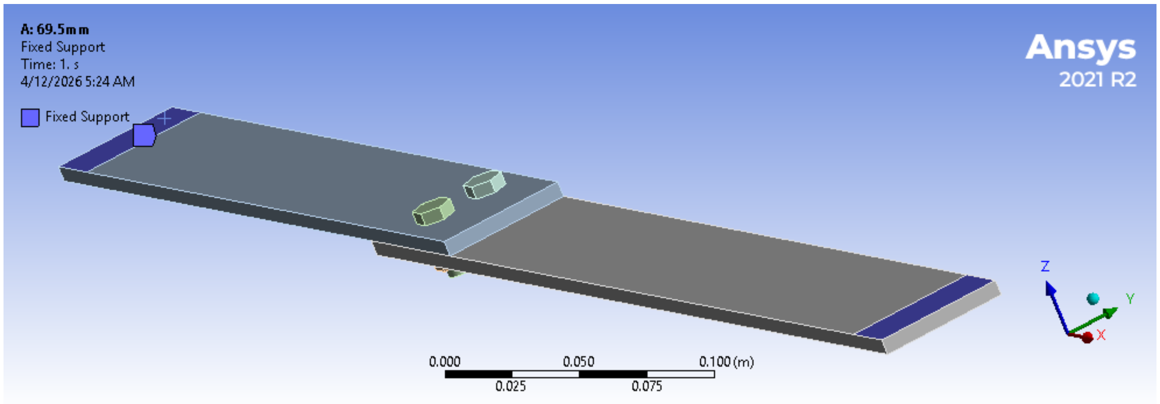

Figure 4.8: Fixed Support

4.3.2 Bolted Joint With overlap length 69.5mm

Basic geometric structure in ansys is shown as in fig below :

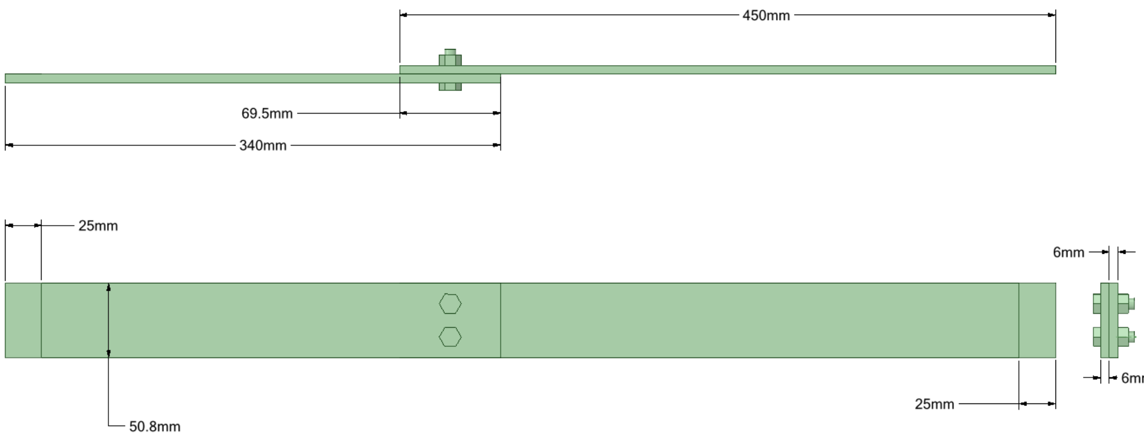

Figure 4.9: Geometric Setup for Bolted joint with 69.5mm

The figure shows the geometrical model of a bolted lap joint in ANSYS for vibration analysis. The assembly consists of two mild steel plates which are joined together by the overlap region using M8 bolts. The purpose of this geometry is to investigate the dynamic behavior and natural frequencies of the bolted lap joint system under clamped boundary conditions.

The upper plate has a length of 340 mm, while the lower plate has a total length of 450 mm. Both plates have the same cross-sectional dimensions with a width of 50.8 mm (2 inches) and a thickness of 6 mm. Both plates are of same material, i.e. mild steel which provides them the structural stiffness and mass for vibration analysis.

The two plates are touching and connected by bolts in an overlap region of 69.5 mm. In this overlap region the plates are fastened together with two M8 bolts. The distance between the two bolts is 22.8 mm, ensuring proper load distribution and structural stability of the joint. The tensile stress area of the M8 bolt shank is 36.6 mm^2 which is the effective area resisting tensile loads in the bolt.

The ANSYS model has fixed supports at both ends of the assembly that are 25 mm long. These fixed supports stop the plates from moving in both directions and rotating, which is like clamped boundary conditions that are often used in experimental vibration studies.

The bolt connections and overlap region make the plates stiff and allow them to interact with each other. The rest of the plates can vibrate freely. This shape lets the ANSYS model see how the plate size, bolt spacing, overlap length, and material properties affect the bolted lap joint structure's natural frequencies and mode shapes.

4.3.3 Bolted Joint With overlap length 55.3mm

Basic geometric structure in ansys is shown as in fig below :

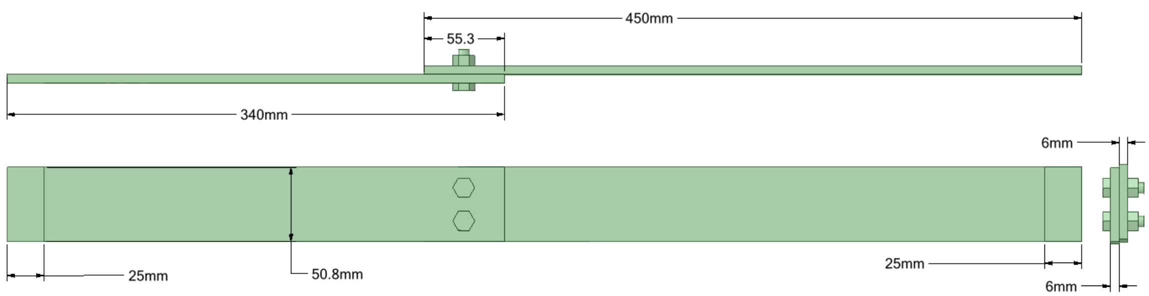

Figure 4.10: Geometric Setup for Bolted joint with 55.3mm

The bolted joint is held together with an overlap length of 55.3 mm, and two bolts are placed in the middle of the overlap length, which is 27.65 mm. The other parameter stays the same.The above picture shows the geometric description of a bolted joint.

4.3.4 Bolted Joint With overlap length 39.5mm

Basic geometric structure in ansys is shown as in fig below

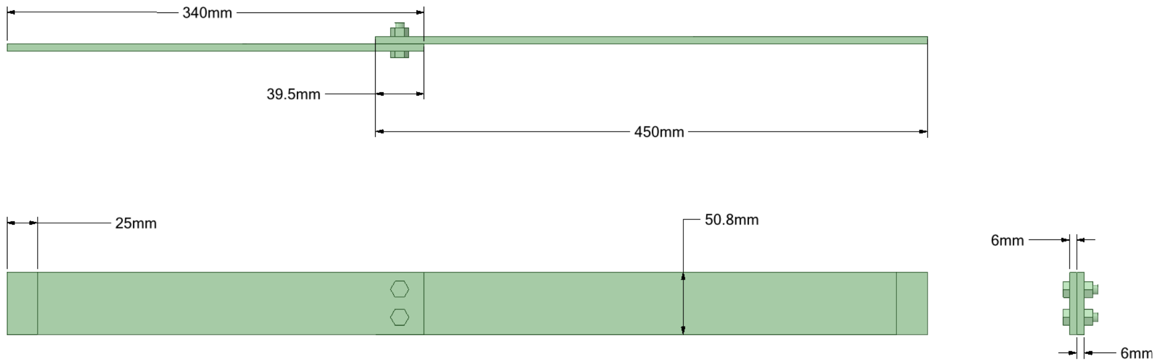

Figure 4.11: Geometric Setup for Bolted joint with 39.5mm

Here the bolted joint are kept with overlap length of 39.5 and two bolt is kept at an mid point of overlap length which is 19.75mm. Other Parameter is keeping same .The Geometric description of bolted joint is shown as above.

4.4 Finite Element Method

The transverse free vibration behaviour of the bolted lap joint was investigated by using the finite element method (FEM). The formulation was based on the Euler-Bernoulli beam theory, which assumes that the plane sections remain plane and perpendicular to the neutral axis during bending. At the ends of the beam, the fixed supports were modeled by imposing the appropriate boundary conditions. The governing differential equation was first transformed into its weak (variational) form to assist the finite element formulation. The beam was then discretized into a number of finite elements and the upper and lower plates in the overlapping region were coupled to represent the joint interaction. The shape functions of the beam element were used to derive the element stiffness matrix and mass matrix, which were then assembled into the global stiffness matrix and global mass matrix of the system.The resulting set of equations was expressed as a generalized eigenvalue problem and solved for the natural frequencies and corresponding modes of transverse vibration of the bolted lap joint. The FEM approach can accurately represent the structural stiffness, mass distribution and coupling effects of the bolted connection, so as to provide reliable predictions of the dynamic characteristics of the jointed beam system.

4.4.1 Finite Element Formulation of Bolted Lap Joint

The bolted lap joint is modeled as two Euler-Bernoulli beams representing the upper and lower plates coupled together. Each plate has width b, thickness t, Young's modulus E, and density ρ. The cross–sectional properties are given as

$$A = bt \tag{4.90}$$

$$I = \frac{bt^3}{12} \tag{4.91}$$

The transverse displacement of the beam is denoted by $w(x, t)$.

4.4.2 Strong Form of Euler–Bernoulli Beam Equation

The governing equation for transverse vibration of an Euler–Bernoulli beam is Silva, J. et al. (2017).

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{4.92}$$

For the lap joint system two coupled beams are considered:

$$EI\frac{\partial^4 w_1}{\partial x^4} + \rho A\frac{\partial^2 w_1}{\partial t^2} = 0 \tag{4.93}$$

$$EI\frac{\partial^4 w_2}{\partial x^4} + \rho A\frac{\partial^2 w_2}{\partial t^2} = 0 \tag{4.94}$$

where $w_1(x, t)$ and $w_2(x, t)$ represent the transverse displacements of the upper and lower beam model respectively.

4.4.3 Boundary Conditions

At the clamped faces the displacement and rotation are zero

$$w = 0 \tag{4.95}$$

$$\frac{\partial w}{\partial x} = 0 \tag{4.96}$$

These conditions replicate the fixed supports applied to the plates.

4.4.4 Weak Formulation

To formulate weak form we Multiply the governing equation by a weighting function $v(x)$ and integrating over the beam length

$$\int_0^L v\left(EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2}\right) dx = 0 \tag{4.97}$$

Expanding the expression

$$\int_0^L EIv\frac{\partial^4 w}{\partial x^4}dx + \int_0^L \rho Av\frac{\partial^2 w}{\partial t^2}dx = 0 \tag{4.98}$$

Integrating the first term by parts twice,

$$\int_0^L EIv''w''dx + \int_0^L \rho Av\ddot{w}dx = 0 \tag{4.99}$$

This equation represents the weak form of the Euler–Bernoulli beam equation.

4.4.5 Finite Element Approximation

Within each finite element the displacement field is approximated using Hermite cubic interpolation Augarde (1998)

$$w^e(x,t) = N_1 w_1 + N_2\theta_1 + N_3 w_2 + N_4\theta_2 \tag{4.100}$$

where

$$\mathbf{q}^e = \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix}^T \tag{4.101}$$

and

$$w^e(x,t) = \mathbf{N}(x)\mathbf{q}^e(t) \tag{4.102}$$

The Hermite shape functions are given by

$$N_1 = 1 - 3\xi^2 + 2\xi^3 \tag{4.103}$$

$$N_2 = L_e(\xi - 2\xi^2 + \xi^3) \tag{4.104}$$

$$N_3 = 3\xi^2 - 2\xi^3 \tag{4.105}$$

$$N_4 = L_e(-\xi^2 + \xi^3) \tag{4.106}$$

where

$$\xi = \frac{x}{L_e} \tag{4.107}$$

4.4.6 Element Stiffness Matrix

The elemental stiffness matrix is formed as

$$\mathbf{K}^e = \int_0^{L_e} EI\mathbf{B}^T\mathbf{B}\,dx \tag{4.108}$$

where

$$\mathbf{B} = \frac{d^2\mathbf{N}}{dx^2} \tag{4.109}$$

Evaluating the integral gives

$$\mathbf{K}^e = \frac{EI}{L_e^3}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \tag{4.110}$$

4.4.7 Element Mass Matrix

The consistent mass matrix is

$$\mathbf{M}^e = \int_0^{L_e} \rho A\mathbf{N}^T\mathbf{N}\,dx \tag{4.111}$$

After evaluation,

$$\mathbf{M}^e = \frac{\rho A L_e}{420}\begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\ 54 & 13L_e & 156 & -22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix} \tag{4.112}$$

4.4.8 Assembly of Global System

The each elemental matrices are assembled to form the global matrices

$$\mathbf{K} = \sum \mathbf{K}^e \tag{4.113}$$

$$\mathbf{M} = \sum \mathbf{M}^e \tag{4.114}$$

For the two beam whose uncoupled matrices are

$$\mathbf{K}_0 = \begin{bmatrix} \mathbf{K}_1 & 0 \\ 0 & \mathbf{K}_2 \end{bmatrix} \tag{4.115}$$

$$\mathbf{M}_0 = \begin{bmatrix} \mathbf{M}_1 & 0 \\ 0 & \mathbf{M}_2 \end{bmatrix} \tag{4.116}$$

4.4.9 Overlap Region Coupling

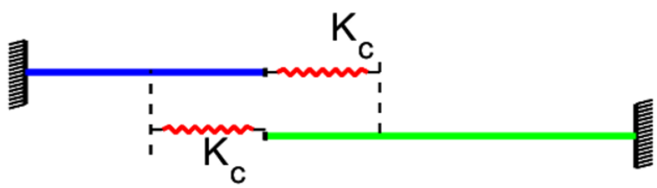

Figure 4.12: Simplified overlap region coupling by distributed spring

The interaction between the two beam in the overlap region is modeled by distributed springs

$$p(x,t) = k_c\,(w_1 - w_2) \tag{4.117}$$

The corresponding spring stiffness matrix is

$$\mathbf{K}_c = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{4.118}$$

This matrix is added between the displacement degrees of freedom of upper and lower.

4.4.10 Bolt Coupling

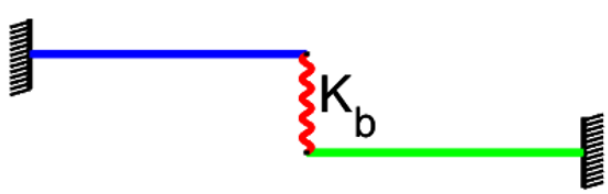

Figure 4.13: Simplified Bolt modeling with contact spring

The bolt is modeled as a discrete spring connecting the two plates at the bolt location, having very high stiffness

$$\mathbf{K}_b = k_b \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{4.119}$$

4.4.11 Global Equation of Motion

The final assembled equation of motion becomes

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{4.120}$$

where

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_c + \mathbf{K}_b \tag{4.121}$$

$$\mathbf{M} = \mathbf{M}_0 \tag{4.122}$$

4.4.12 Eigenvalue Problem

Assuming harmonic motion

$$\mathbf{q}(t) = \boldsymbol{\phi} e^{i\omega t} \tag{4.123}$$

Using the values into the equation of motion gives

$$\mathbf{K}\boldsymbol{\phi} = \omega^2 \mathbf{M}\boldsymbol{\phi} \tag{4.124}$$

The natural frequency is given as

$$f = \frac{\omega}{2\pi} \tag{4.125}$$

The eigenvector $\boldsymbol{\phi}$ denotes the corresponding vibration mode shape of the bolted lap joint structure.

4.4.13 Physical meaning

The consistent mass matrix takes into account the distributed inertia of the beam element through the same interpolation functions as the displacement field. This matrix gives a better accuracy than a lumped mass approximation because the mass is distributed uniformly over the element, especially for the calculation of natural frequencies and mode shapes.

CHAPTER FIVE: RESULTS & DISCUSSION

5.1 Results From Experimental Setup

Result from experimental setup are found for various lap length 69.5mm,55.3mm and 39.5mm which are found as follows

5.1.1 Results From FFT , Overlap length of 69.5 mm

For lap length of 69.5mm , acceleration profile , FFT plot and other logarithmic frequency profile in total of 3 plot were obtained as shown below

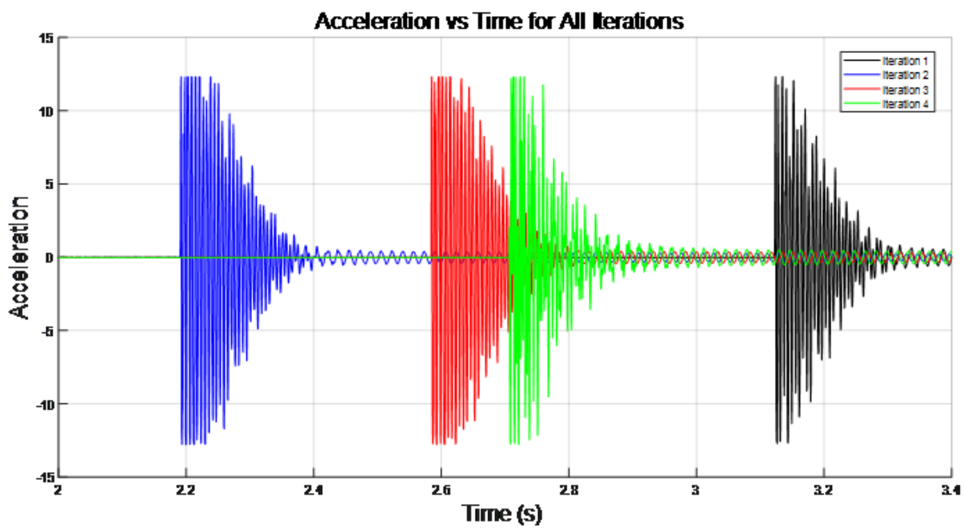

Figure 5.1: Accleration vs Time for over lap length of 69.5mm

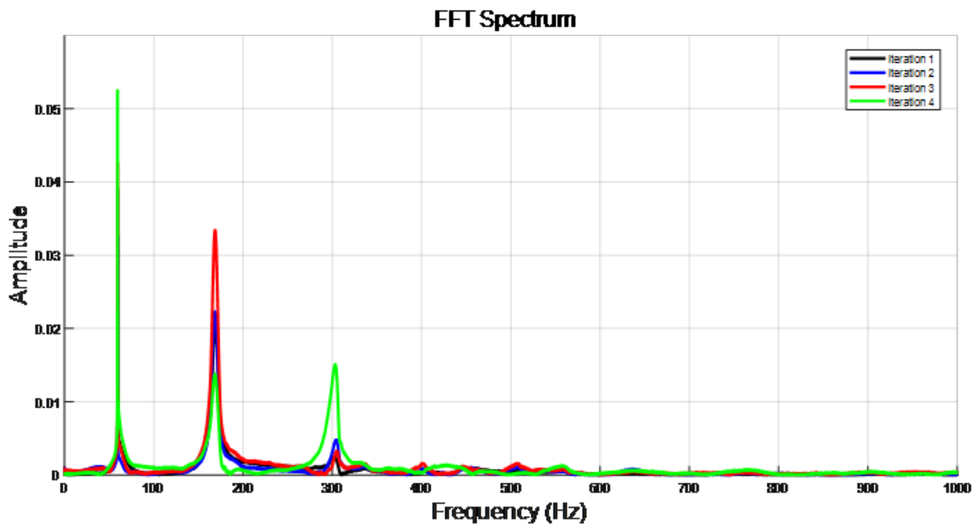

Figure 5.2: FFT plot in normal Scale for 69.5mm

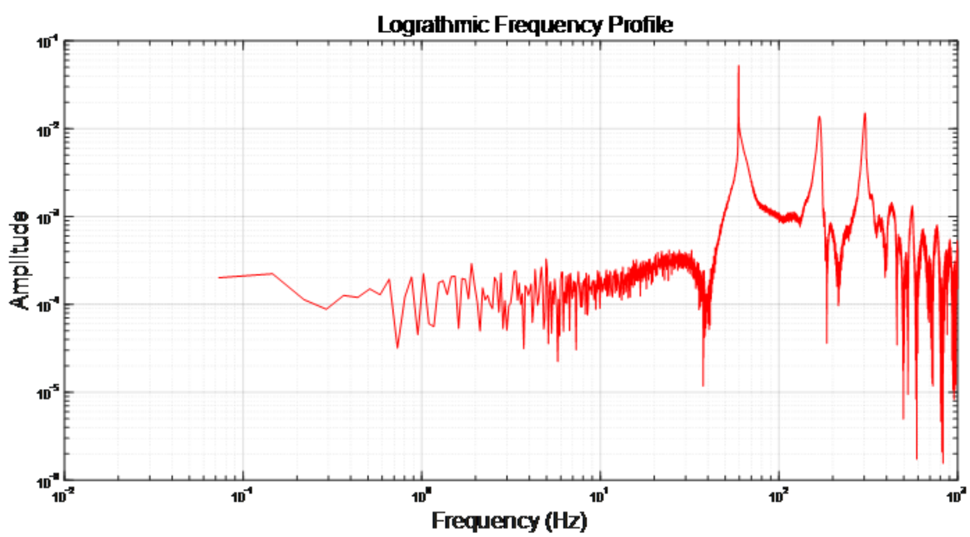

Figure 5.3: Lograthmic Frequency plot for Bolted joint with lap length of 69.5mm

Mode/Iteration	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Iteration 1	59.43	168.27	303.61
Iteration 2	59.55	168.43	304.00
Iteration 3	59.40	168.95	305.04
Iteration 4	59.36	168.45	302.65
Average Natural Frequency	**59.43**	**168.52**	**303.82**

5.1:: Results from Experimental (Only Transverse)

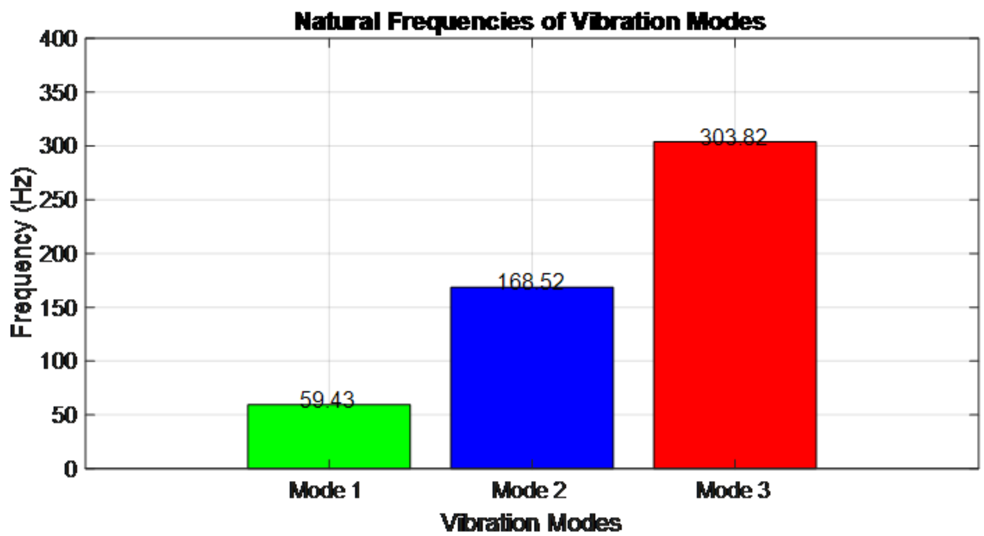

Figure 5.4: Plot showing Natural frequency

5.2:: Overlap with lap length 69.5 mm

S.N.	Mode	Average Natural frequency
1	One	59.43
2	Two	168.52
3	Three	303.82

5.1.2 Results From FFT , Overlap length of 55.3 mm

For lap length of 55.3 mm , acceleration profile , FFT plot and other logarithmic frequency profile in total of 3 plot were obtained as shown below

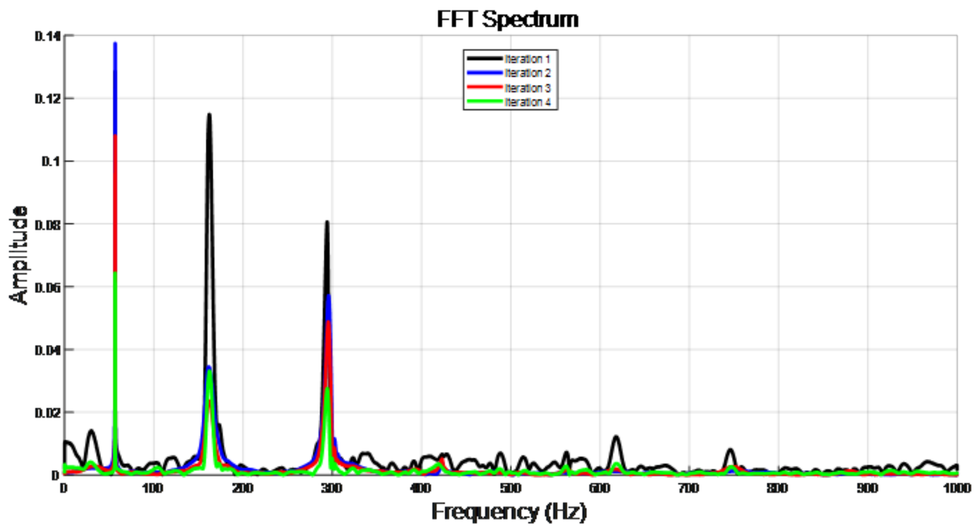

Figure 5.5: FFT plot for bolted joint with over lap length of 55.3mm

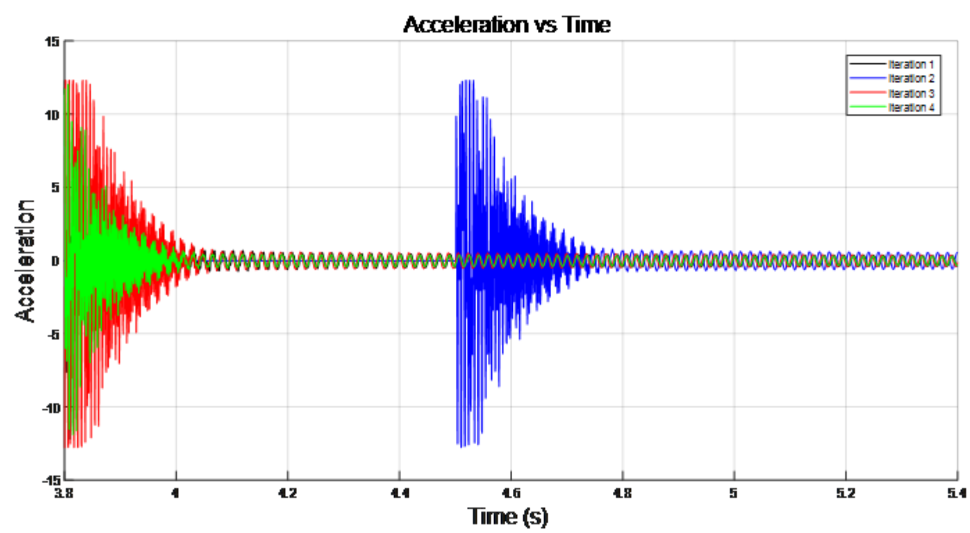

Figure 5.6: Acceleration vs Time plot for bolted joint with over lap length of 55.3

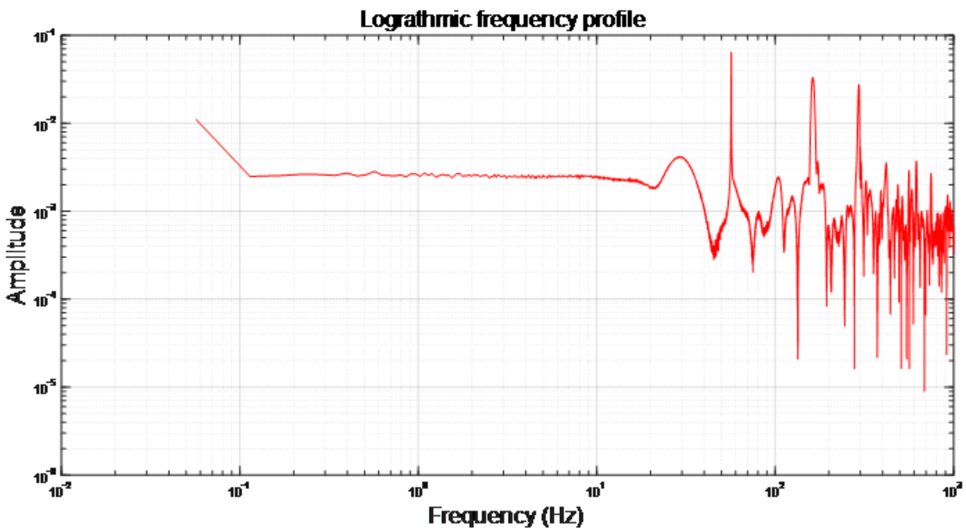

Figure 5.7: Acceleration vs Time plot for bolted joint with over lap length of 55.3

Mode/Iteration	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Iteration 1	56.68	152.15	290.23
Iteration 2	56.65	152.32	289.21
Iteration 3	54.69	152.32	289.21
Iteration 4	56.64	152.00	288.24
Average Natural Frequency	**56.66**	**152.19**	**289.22**

5.3:: Results from Experimental (Only Transverse)

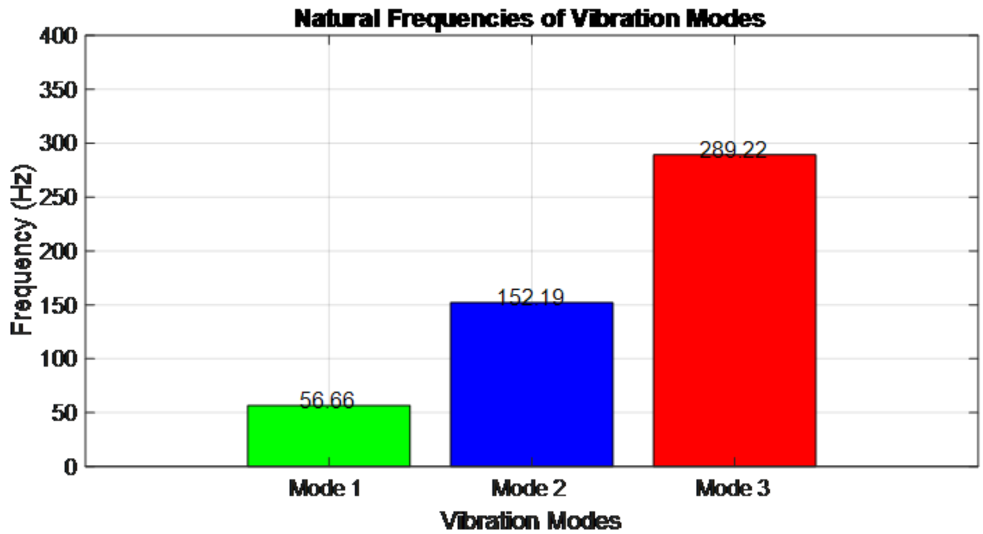

Figure 5.8: Frequency plot for lap length of 55.3mm

5.4:: Overlap with lap length 55.3 mm

S.N.	Mode	Natural frequency(hz)
1	One	56.66
2	Two	152.19
3	Three	289.22

5.1.3 Results From FFT , Overlap length of 39.5 mm

For lap length of 39.5 mm , acceleration profile , FFT plot and other logarithmic frequency profile in total of 3 plot were obtained as shown below

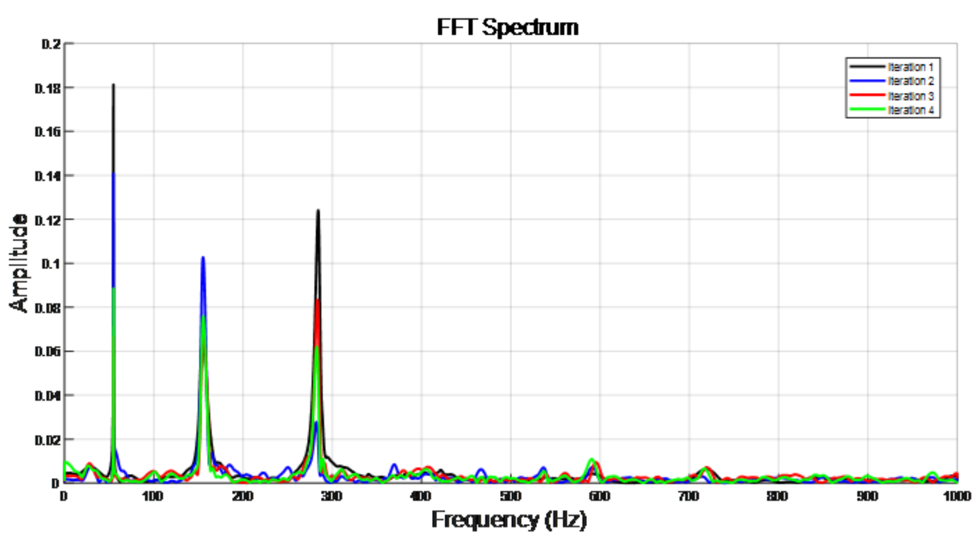

Figure 5.9: FFT plot for bolted joint with over lap length of 39.5mm

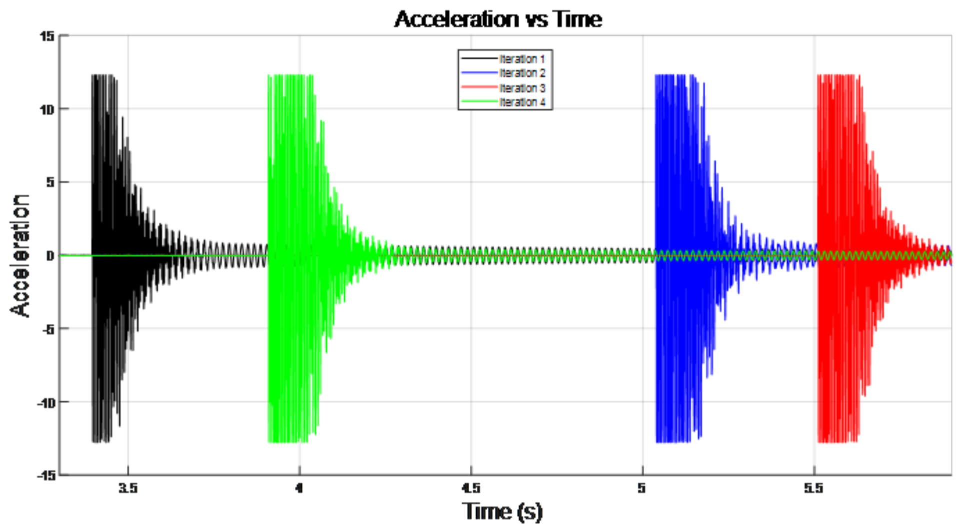

Figure 5.10: Accleration vs Time plot for bolted joint with over lap length of 39.5

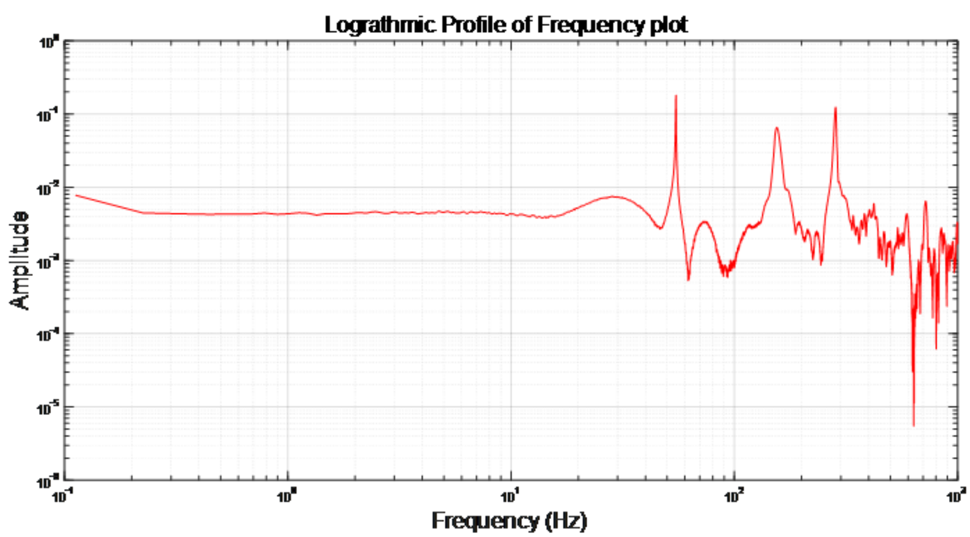

Figure 5.11: Accleration vs Time plot for bolted joint with over lap length of 39.5

Mode/Iteration	Mode 1 (Hz)	Mode 2 (Hz)	Mode 3 (Hz)
Iteration 1	54.63	145.45	294.73
Iteration 2	54.54	145.45	293.97
Iteration 3	54.98	147.23	291.99
Iteration 4	54.92	145.26	293.51
Average Natural Frequency	**54.76**	**145.84**	**293.55**

5.5:: Results from Experimental (Only Transverse)

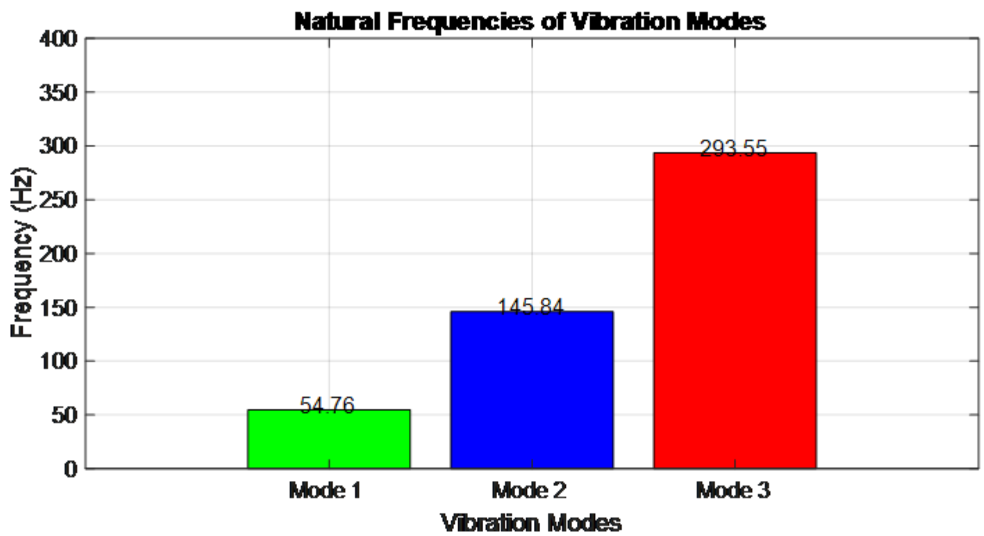

Figure 5.12: Frequency plot for lap length of 39.5mm

5.6:: Overlap with lap length 39.5 mm

S.N.	Mode	Natural frequency(hz)
1	One	54.76
2	Two	145.84
3	Three	293.55

5.2 Result From Numerical Simulation

5.2.1 Result for lap length of 69.5mm

Mode 1

Result for mode 1 is obtained from ansys as:

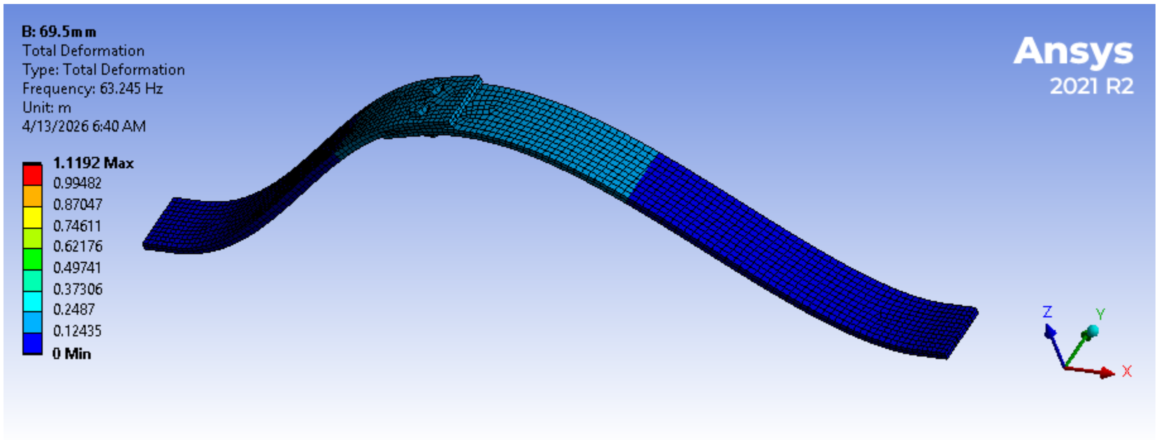

Figure 5.13: Mode 1 for Bolted Joint with overlap length 69.5mm

Figure shows the first vibration mode shape of the bolted lap joint obtained from the ANSYS modal analysis. The natural frequency corresponding to Mode 1 is 63.24 Hz which is the fundamental bending mode of the structure. In this mode, the entire assembly bends smoothly along its length, with maximum deformation occurring near the middle of the beam span while the regions near the fixed supports show little displacement due to the applied clamped boundary conditions. The overlap region and the bolt connections transfer the stiffness between the two plates, and enable the plates to deform as one structure.

Mode 2

Result for mode 2 is obtained from ansys as :

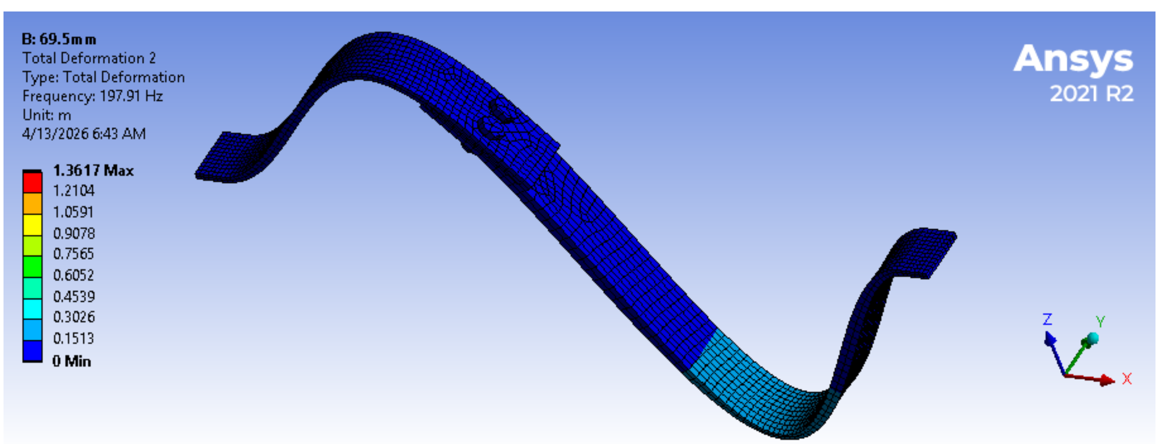

Figure 5.14: Mode 2 for Bolted Joint with overlap length 69.5mm

Fig. 4 shows the second vibration mode obtained from the modal analysis of the bolted lap joint using ANSYS with a natural frequency of 197.91Hz. The structure bends in a more complicated manner than in the first mode. The beam bends in opposite directions from either side of the region of the joint, thus showing the presence of a nodal region where the displacement is minimal. The deformation is localized around the central part of the structure whereas the clamped ends are subject to very little displacement owing to the fixed boundary conditions.

Mode 3

Result for mode 3 is obtained from ansys as :

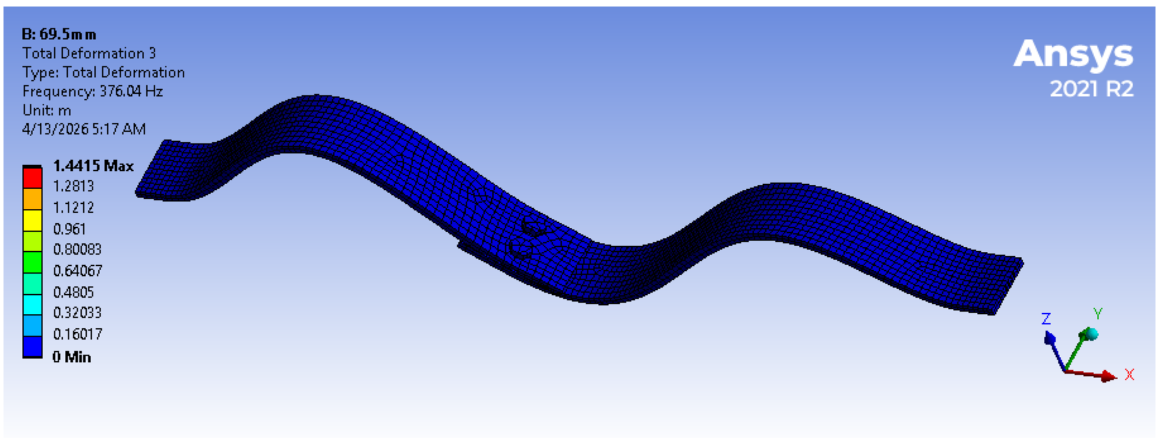

Figure 5.15: Mode 3 for Bolted Joint with overlap length 69.5mm

The third vibration mode, which has a matching natural frequency of 376.04 Hz, is depicted in the figure based on the bolted lap joint's ANSYS modal analysis. The structure displays a higher-order bending pattern with several curvature variations throughout the beam length in this mode. Compared to the first and second modes, the deformation becomes more complicated, exhibiting alternating regions of upward and downward bending. While the clamped ends stay almost stationary because of the provided fixed supports, the overlap region and bolt connection affect the deformation behavior by transmitting stiffness between the two plates.

5.2.2 Result for lap length of 55.9mm

Mode 1

Result for mode 1 is obtained from ansys as :

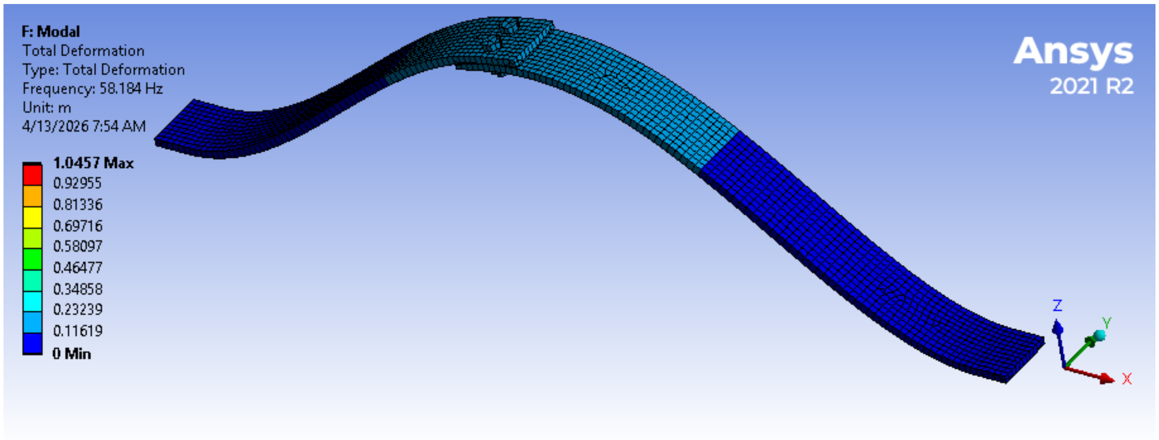

Figure 5.16: Mode 1 for Bolted Joint with overlap length 55.3mm

The initial vibration mode from the ANSYS modal analysis of the bolted lap joint with a 55.3 mm overlap length is displayed in the image. The fundamental bending mode of the structure is represented by Mode 1, which has a matching natural frequency of 58.18 Hz. Because of the clamped boundary conditions, the entire beam assembly bends smoothly along its length in this mode, with greatest deformation happening along to the mid-span and little displacement near the fixed supports. The two plates can vibrate as a single structural system because of the rigidity provided by the overlap region and bolt connection.

Mode 2

Result for mode 2 is obtained from ansys as :

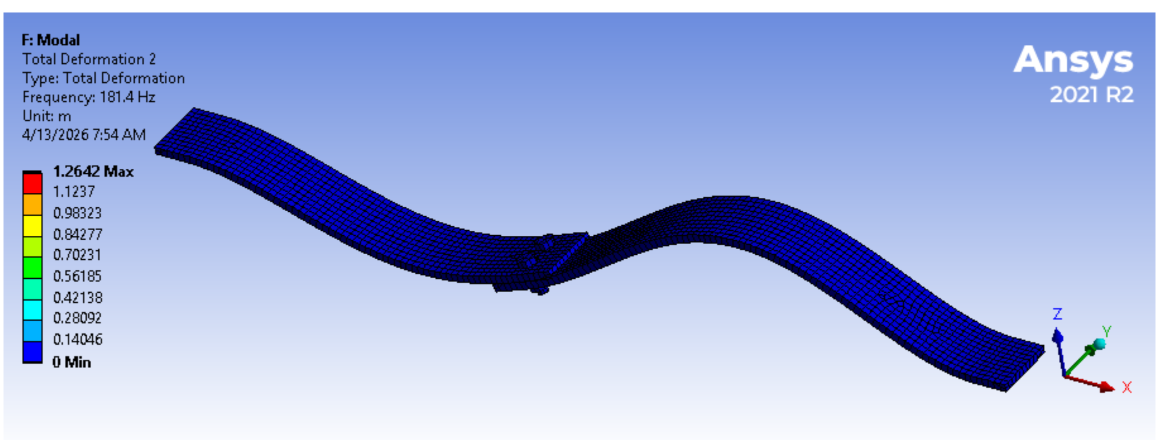

Figure 5.17: Mode 2 for Bolted Joint with overlap length 55.3mm

The second vibration mode from the ANSYS modal analysis of the bolted lap joint with a 55.3 mm overlap length is displayed in the image. For Mode 2, the comparable natural frequency is 181.4 Hz. Compared to the first mode, the structure displays a more intricate bending pattern in this mode, as several beam segments bend in opposing directions to form a nodal region close to the joint area where displacement is minimized. The clamped boundary conditions cause the fixed support regions to move very little, while the deformation is mostly centered around the structure's center.

Mode 3

Result for mode 3 is obtained from ansys as :

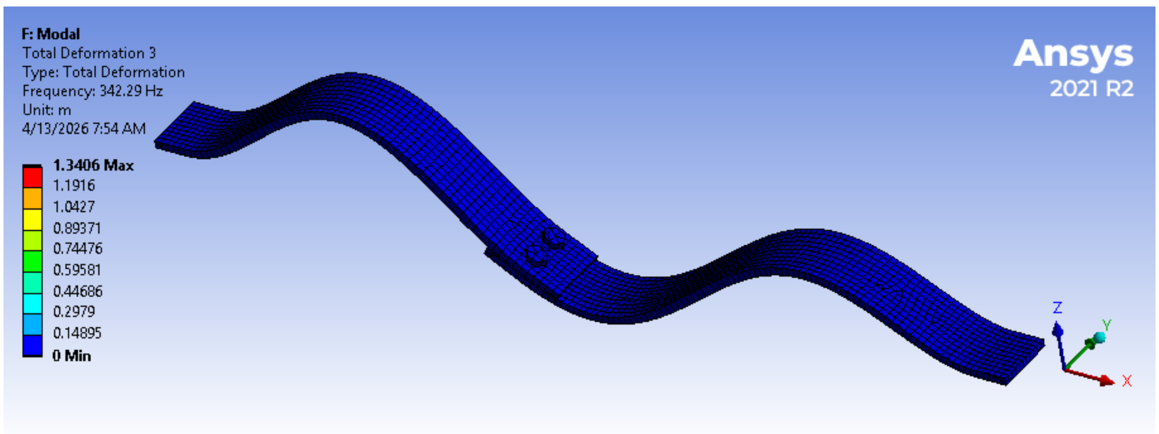

Figure 5.18: Mode 3 for Bolted Joint with overlap length 55.3mm

The figure shows the third vibration mode of the ANSYS modal analysis of the bolted lap joint with an overlap length of 55.3 mm. The natural frequency for Mode 3 is 342.29 Hz. In this mode the structure has a higher order bending pattern with multiple curvature changes along the length of the beam. The deformation is more complex than those of the first and second modes, with alternating regions of upward and downward bending along the beam. The vibration behaviour is influenced by the overlap region and bolt connection transferring stiffness between the two plates, whereas the clamped ends are almost stationary because of the fixed supports applied.

5.2.3 Result for lap length of 39.5mm

Mode 1

Result for mode 1 is obtained from ansys as :

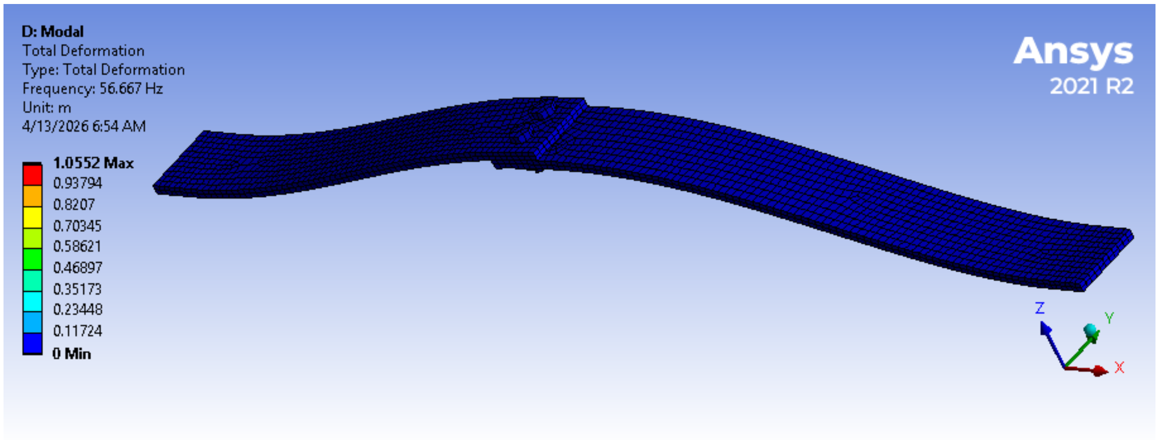

Figure 5.19: Mode 1 for Bolted Joint with overlap length 39.5mm

The first vibration mode from the ANSYS modal analysis of the bolted lap joint with an overlap length of 39.5 mm is shown in the figure. The fundamental bending mode of the structure is 56.66 Hz, which is the natural frequency corresponding to Mode 1. In this mode, the entire beam assembly deflects smoothly along its length with the maximum deformation near the mid-span and the regions near the fixed supports show very little displacement due to the applied clamped boundary conditions. The overlap area and bolt joint give rigidity between two plates so that they can deform together when they vibrate.

Mode 2

Result for mode 1 is obtained from ansys as :

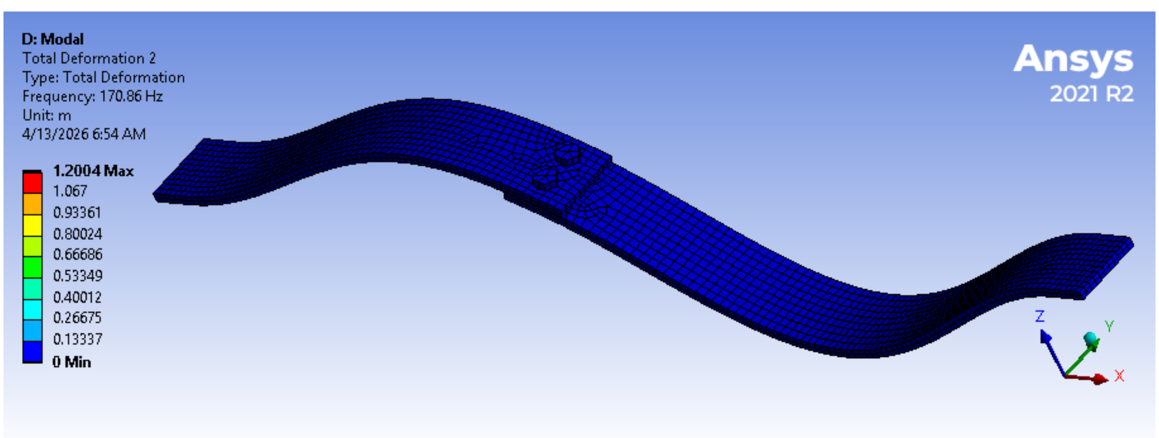

Figure 5.20: Mode 2 for Bolted Joint with overlap length 39.5mm

The second vibration mode obtained from the ANSYS modal analysis of the bolted lap joint with an overlap length of 39.5 mm is shown in the figure. Mode 2 has a natural frequency of 170.86 Hz. In this mode, the structure exhibits an S-shaped bending pattern with different parts of the beam deforming in opposite directions, forming a nodal region near the joint area where the displacement is minimal. The deformation is mainly concentrated around the central portion of the beam while the fixed ends show very small displacement due to the applied clamped boundary conditions.

Mode 3

Result for mode 1 is obtained from ansys as :

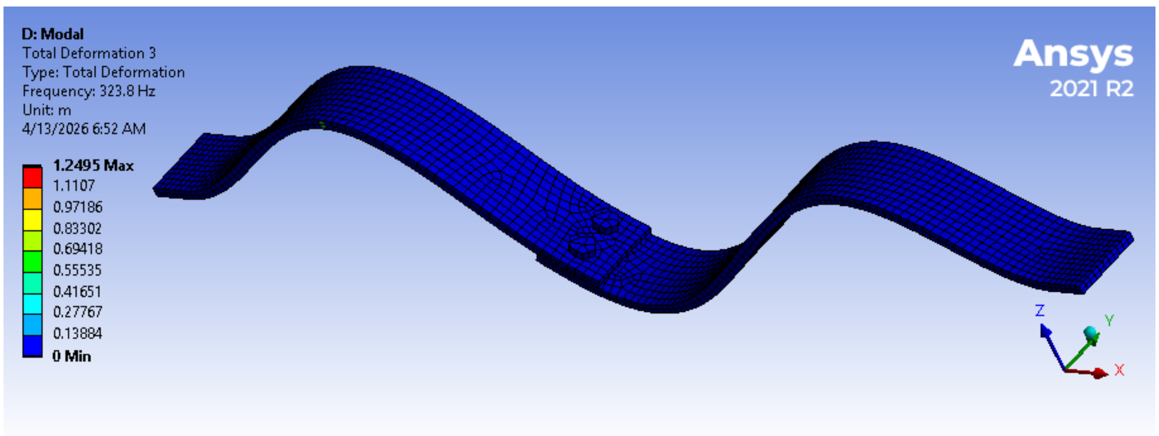

Figure 5.21: Mode 3 for Bolted Joint with overlap length 39.5mm

Figure 3. The third modal vibration of the bolted lap joint by ANSYS modal analysis with overlap length of 39.5mm. The natural frequency for Mode 3 is 323.8 Hz. In this mode, the structure shows a higher order bending pattern with several changes in curvature along the beam length, leading to alternating up and down deformations. The deformation is more complicated than the first and second modes, which shows the vibration energy is higher at higher frequencies. The bolt connection and the overlapping region affect the stiffness of the structure and the transfer of load between the two plates. The clamped ends are fixed in such a way that they remain almost stationary.

5.3 Result From Finite Element Method

5.3.1 MATLAB result for Overlap Length of 69.5mm

Figure 1 shows the first three natural vibration modes obtained by Finite Element Modeling (FEM) for the bolted lap joint with an overlap length of 69.5 mm. The blue curve is for the upper beam (340 mm) and the red curve is for the lower beam (450 mm). The vertical dashed lines show the position of the bolt in the joint and the area of overlap.

Mode 1 – First Natural Frequency (f = 64.30 Hz)

The first mode is the fundamental bending vibration of the bolted lap joint. In this mode the top and bottom beams move in the same direction, resulting in a smooth continuous bending shape along the length of the beam. Gradually the displacement increases from the clamped end to its maximum value near the central part of the structure. The higher contact stiffness of the two plates comes from the longer overlap length (69.5 mm) which makes the structure behave more like one integrated beam and increases the natural frequency slightly.

Mode 2 – Second Natural Frequency (f = 189.19 Hz)

There is a nodal point in the second vibration mode where the displacement is zero. The relative motion between the plates close to the overlap region is indicated by the opposite vibrations of the upper and lower beams in this mode. While permitting differential bending, the bolt and contact stiffness restrict the distance between the plates. Consequently, compared to the first mode, the natural frequency rises and the vibration pattern becomes more intricate.

Mode 3 – Third Natural Frequency (f = 371.78 Hz)

A higher-order bending vibration with several curvature changes throughout the beam length is represented by the third mode. Around the overlap region, both beams exhibit discernible bending as the deformation pattern gets more oscillatory. This vibration pattern is greatly influenced by the relationship between the dispersed contact stiffness and the bolt stiffness. A greater structural stiffness response is indicated by the displacement changing along the beam length more quickly than in the lower modes.

Overall Interpretation

According to the FEM results, a greater contact area between the plates causes the joint region to become more stiff as the overlap length increases to 69.5 mm. Higher natural frequencies result from this increased rigidity as opposed to shorter overlap lengths. Furthermore, as the mode number increases, the vibration modes exhibit more curvature and nodal points throughout the beam length, becoming increasingly more complicated.

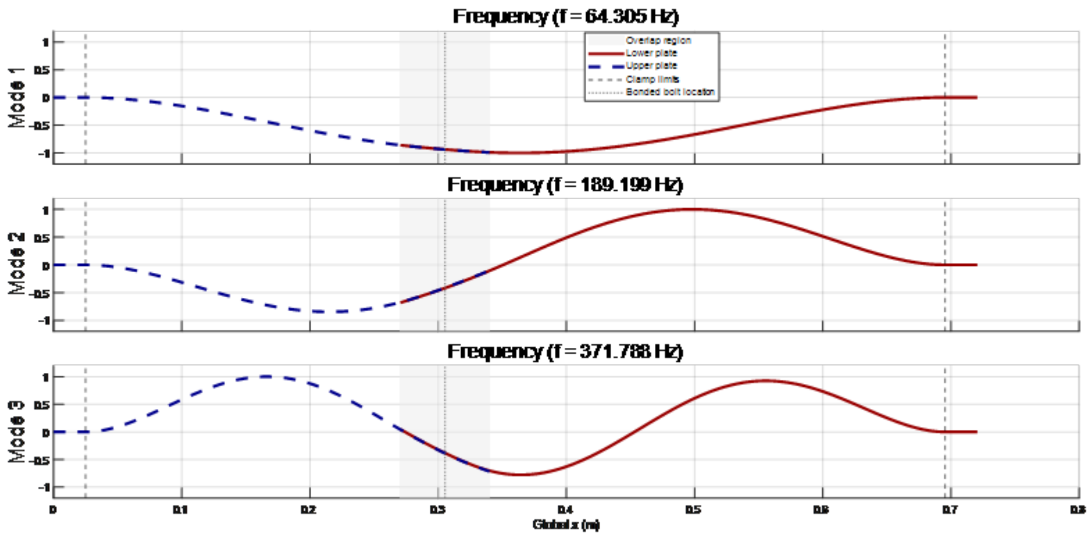

Figure 5.22: Matlab result For bolted joint with lap length of 69.5mm

5.3.2 MATLAB result for Overlap Length of 55.3 mm

In the figure are the first three vibration modes obtained from Finite Element Modelling (FEM) for the bolted lap joint with an overlap length of 55.3 mm. The blue curve is for the upper beam (340 mm) and the red curve is for the lower beam (450 mm). The vertical dashed lines show the overlap region and the location of the bolt in the joint.

Mode 1 – First Natural Frequency (f = 62.64 Hz)

Firstly, there is the basic bending of the bolted lap joint. In this vibration mode, the vibration occurs in the same direction for both upper and lower plates, which shows a steady deformation throughout the length of the beam. The displacement varies progressively, starting from the clamped edge and reaching its peak value around the center of the structure. Due to the overlapping area, the two plates are almost acting together as one single structure.

Mode 2 – Second Natural Frequency (f = 186.20 Hz)

The second mode possesses a node point where there exists zero displacement. This implies that the upper and lower beams vibrate in opposite directions and hence there is motion between the two plates within the overlapping area. However, the stiffness of the bolts as well as the spring contacts restrict the amount of separation between the plates while allowing for differential deformation to occur.

Mode 3 – Third Natural Frequency (f = 359.45 Hz)

The third mode shows more complex deformation, with several changes in curvature along the length of the beam. In this higher mode, you can see that both beams are bending more, and you can see the curve in the area where they meet. The interaction between the bolt stiffness and the contact stiffness in the overlap area has a big effect on the vibration pattern. The deformation becomes more localized and oscillatory than it is in the lower modes.

Overall Interpretation

The FEM results show that as the mode number goes up, the natural frequency goes up too. The vibration shapes also get more complicated with more nodal points. When the plates touch, the larger overlap length (55.3 mm) makes them stiffer, which raises the natural frequencies a little bit compared to smaller overlap configurations. The bolt in the middle holds the structure together while still allowing the plates to move a little bit in relation to each other when the vibration levels are higher.

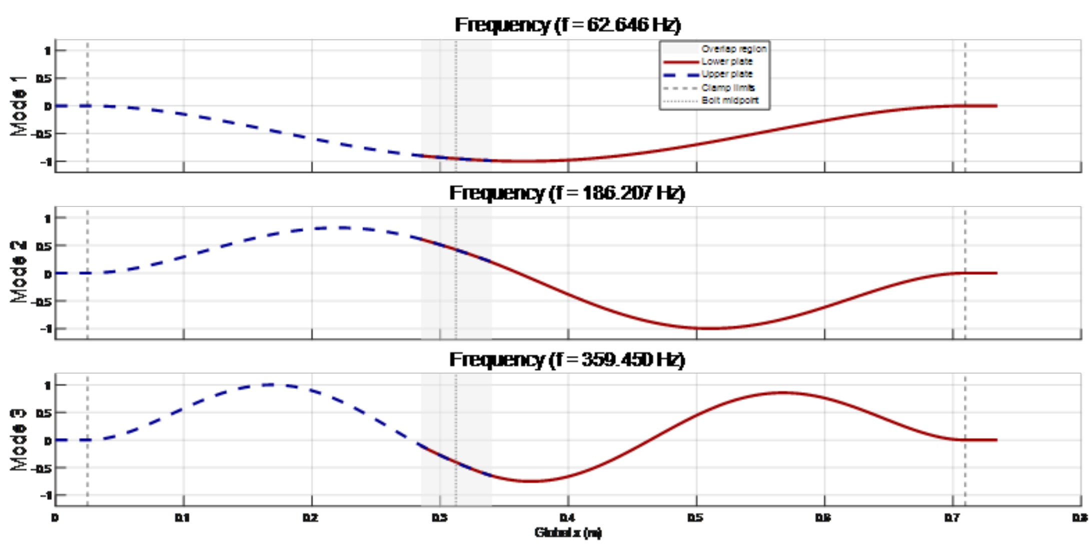

Figure 5.23: Matlab result For bolted joint with overlap 55.3mm

5.3.3 MATLAB result for Overlap Length of 39.5 mm

The figure shows the first three natural vibration modes of the bolted lap joint. This was done using Finite Element Modeling (FEM) for an overlap length of 39.5 mm and a bolt in the middle (19.75 mm). The blue curve shows the top plate (340 mm), and the red curve shows the bottom plate (450 mm). The dashed vertical lines show where the overlap region starts and ends, as well as where the bolt goes.

Mode 1 – First Natural Frequency (f = 60.71 Hz)

The first mode shows how the bolted lap joint bends in the most basic way. The upper and lower plates vibrate in the same direction in this mode, which means that the beam is bending smoothly along its length. The middle of the joint region has the most movement, while the ends that are clamped together don't move at all because of boundary constraints. This mode shows how the whole assembly bends in all directions. The bolt mostly holds the plates together, which move together.

Mode 2 – Second Natural Frequency (f = 179.72 Hz)

The second mode shows one nodal point, which is where the displacement changes from positive to negative. When the upper and lower plates vibrate in this mode, they move in opposite directions. This means that the two plates are moving relative to each other around the joint region. The change is more complicated than it was in the first mode. The plates don't come apart because of the bolt and overlap stiffness, but they do bend a little, which causes this higher frequency vibration pattern.

Mode 3 – Third Natural Frequency (f = 343.56 Hz)

The third mode has two nodal regions and a lot of changes in curvature. This means it is a higher-order bending mode. The vibration shape gets more complicated as the curvature changes more on both plates. The stiffness of the overlap region and the bolt work together to change this mode shape in a big way. This mode shows localized bending behavior, which means that the deformation changes more quickly along the beam's length.

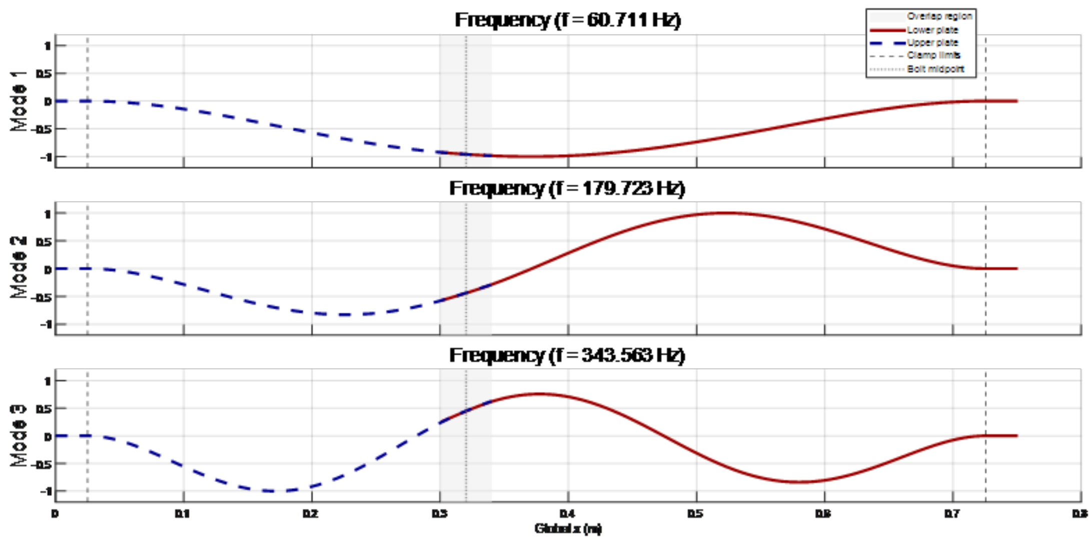

Figure 5.24: Matlab result For bolted joint with overlap 39.5mm

Overall Interpretation

The FEM results show that the natural frequency goes up as the mode number goes up, the mode shapes get more complicated, and the number of nodes along the beam length goes up. The overlap area and the stiffness of the bolts affect how much the two plates can move together. The plates act more like a single structure for the lower modes, but for the higher modes, the joint's flexibility makes it easier to see how the plates move relative to each other.

5.4 Comparison of Natural Frequency for Same Modes with different analysis tool

For lap length of 69.5mm

This bar graph represent comparison of different mode of vibration with different analysis tool for same lap length 69.5mm

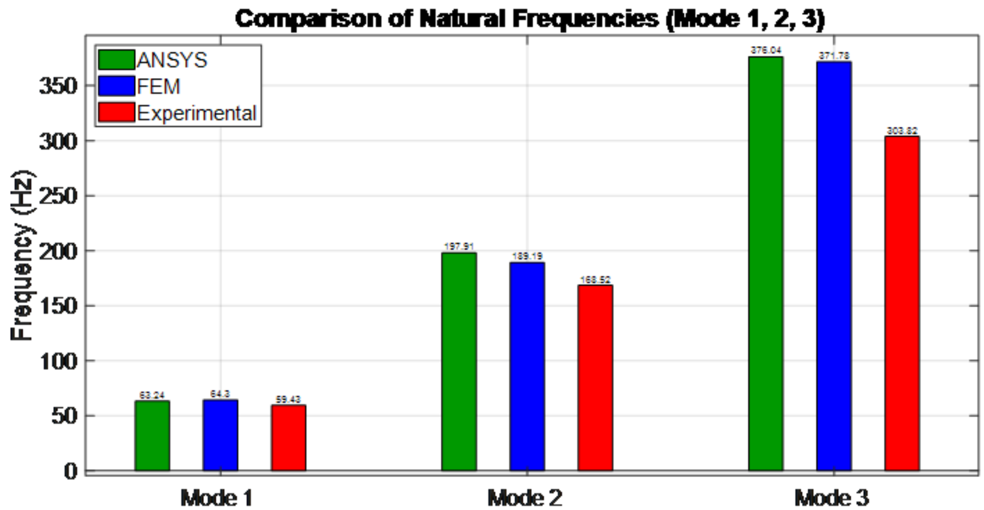

Figure 5.25: 69.5mm comparison

The results of the bolted lap joint with an overlap length of 69.5 mm are consistent with the expected increase in natural frequency from Mode 1 to Mode 3. The ANSYS and MATLAB (FEM) predictions are in very good agreement for all modes, whereas the experimental values are consistently lower. The deviation is small in Mode 1 and increases in Mode 2 and becomes most significant in Mode 3, likely due to practical effects such as damping, bolt looseness and contact nonlinearity. In general the numerical methods are able to capture the vibration behaviour reliably with increasing discrepancy at higher modes.

For lap length of 55.3mm

This bar graph represent comparison of different mode of vibration with different analysis tool for same lap length 55.3mm

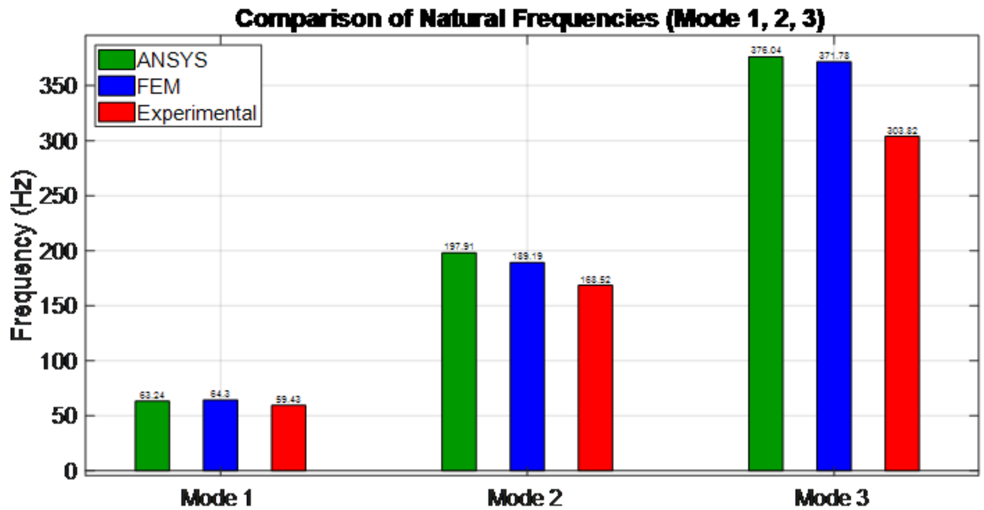

Figure 5.26: 55.3mm comparison

Again, for the bolted lap joint with overlap length 55.3 mm the natural frequencies increase from Mode 1 to Mode 3 for all the methods. The ANSYS and MATLAB (FEM) results remain very close to each other in all modes, indicating strong agreement between numerical approaches. In Mode 1, the difference between numerical and experimental values is small, showing good correlation. In Mode 2 the experimental value starts to deviate more significantly and is lower than both ANSYS and FEM predictions. The mode 3 shows a larger deviation that a much lower experimental frequency, showing the effect of the reduced overlap length, which reduces the stiffness and increases the influence of the factors such as damping, slip, and contact nonlinearity. Overall, the differences are slightly higher for larger overlap lengths because of the reduced joint rigidity.

For lap length of 39.5mm

For this case of the bolted lap joint, the natural frequency increases consistently from Mode 1 to Mode 3 for all methods. The results from ANSYS and MATLAB (FEM) are in good agreement with FEM giving slightly higher values in Mode 2 and Mode 3. In Mode 1, all three results are very close, showing good accuracy of numerical models at lower modes. The experimental value in Mode 2 is much lower than the two numerical predictions. This difference is even more significant in Mode 3, where the experimental result presents a significant drop in comparison with ANSYS and FEM. This increasing deviation at higher modes indicates the influence of practical factors such as lower stiffness, damping and contact effects in the joint, while the numerical methods still follow the overall trend well.

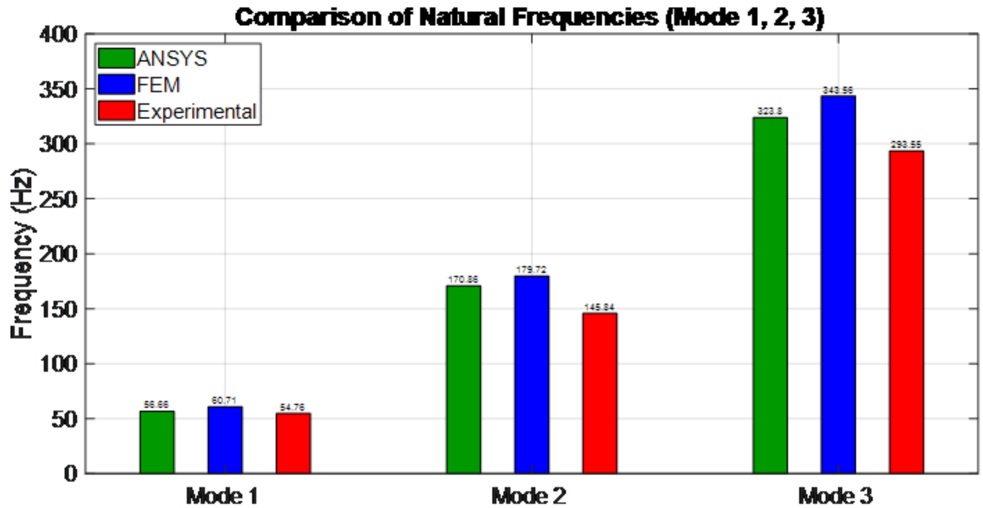

Figure 5.27: 39.5mm comparison

5.4.1 Comparison of Natural frequency for Different mode with same analysis tool

For lap length of 69.5mm

The bar graph indicates that for the same lap length (69.5 mm), the natural frequency increases significantly from Mode 1 to Mode 3 for all three analysis methods, implying that higher vibration modes correspond to higher stiffness response of the structure. The numerical results obtained from FEM and ANSYS are very close for all modes, confirming the consistency and reliability of the numerical methods, while the experimental values are slightly lower, due to real-world effects. Overall, the graph emphasizes that the joint shows higher modes with stronger stiffness behavior and confirms that the numerical predictions follow the actual physical trend closely.

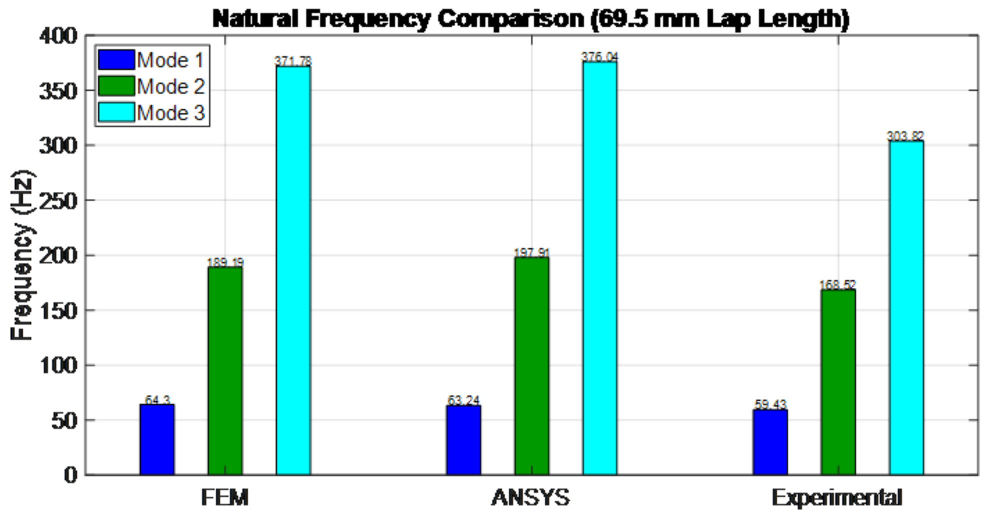

Figure 5.28: 69.5mm comparision

For lap length of 55.3mm

This bar graph shows the natural frequency for the lap length of 39.5 mm increases from Mode 1 to Mode 3 for all analysis methods, but overall the values are lower than higher lap lengths indicating lower stiffness. The results obtained from FEM and ANSYS are in good agreement, which confirms the consistency in numerical predictions. The experimental values are noticeably lower due to real world effects. Visually, it indicates that the structure is less stiff at this shorter lap length and while the trend of increasing frequency with mode is maintained, the joint is more flexible and therefore less resistant to vibration and fatigue.

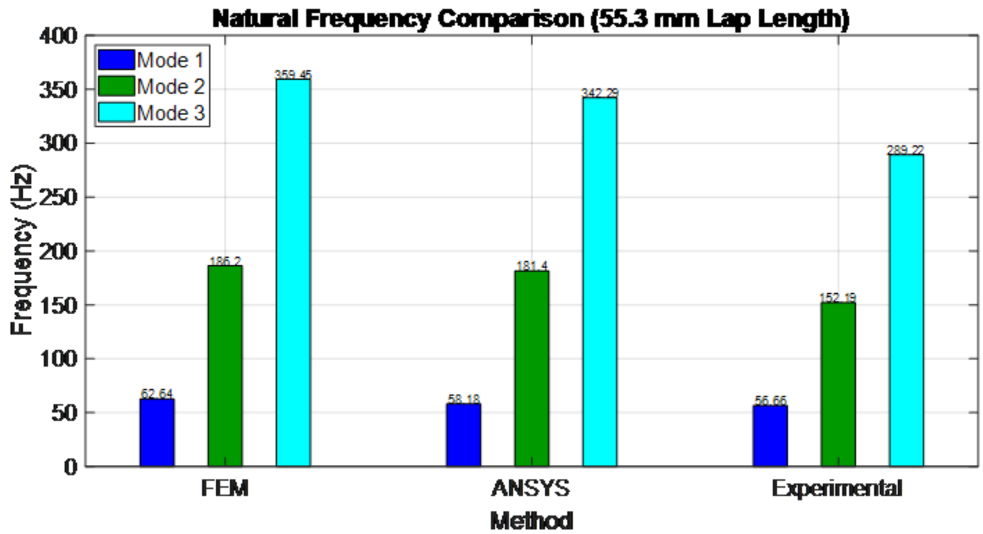

Figure 5.29: 55.3mm comparision

For lap length of 39.5mm

This bar graph shows visually, the natural frequency increases for the lap length of 39.5 mm from Mode 1 to Mode 3 for all methods but the overall values are comparatively lower indicating lower stiffness of the joint. FEM and ANSYS results are in good agreement, implying reliable numerical prediction, while experimental values are lower due to real-world effects. In general, this means that the shorter the lap length, the more flexible the structure, which has less stiffness and less resistance to vibration than longer lap lengths.

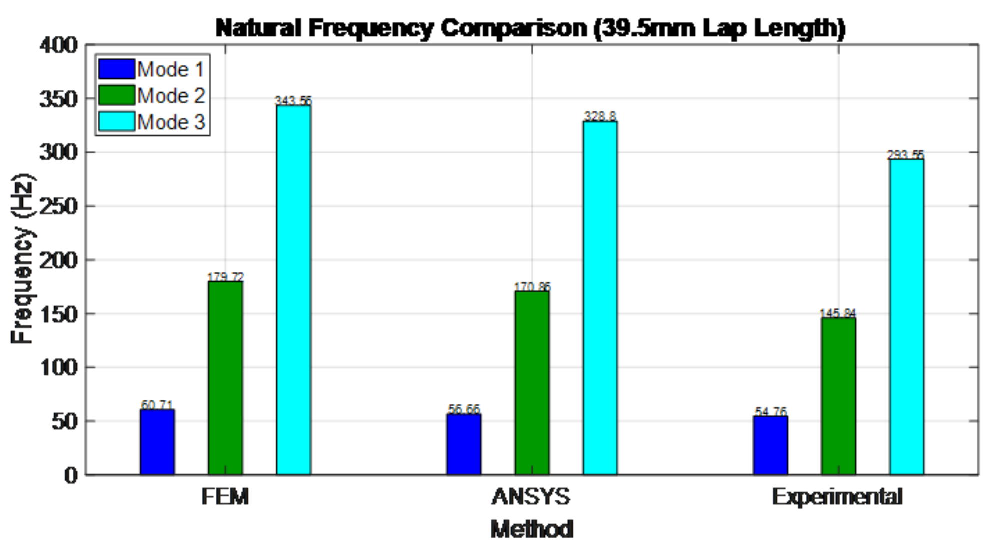

Figure 5.30: 39.5mm comparision

5.4.2 Comparison of same mode of frequency for different lap length

Finite Element Method

Figure shows that natural frequency increases for each mode (Mode 1, 2 and 3) as the lap length increases from 39.5 mm to 69.5 mm for FEM analysis. This clearly shows that higher lap length leads to higher stiffness of the bolted joint as higher stiffness leads to higher natural frequency. This trend is confirmed in all modes, showing that the increase of the overlap length improves the load transfer and the structural rigity, making the joint more resistant to the vibration and fatigue.

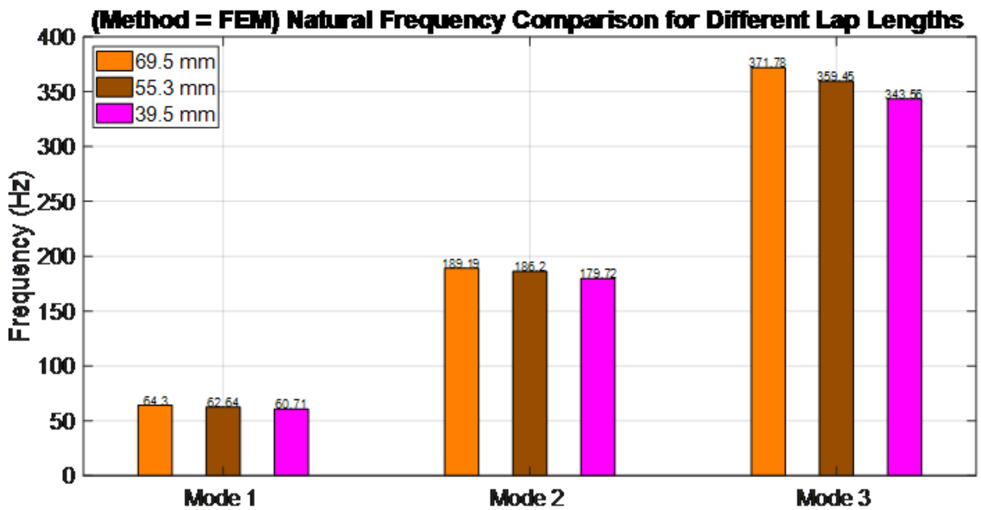

Figure 5.31: FEM comparison, with variable lap length

Numerical Simulation method

This graph clearly illustrates that the 69.5 mm lap length consistently exhibits the highest natural frequency, followed by 55.3 mm and then 39.5 mm across all vibration modes. This trend indicates that stiffness increases with lap length. Additionally, the difference between the bars becomes more pronounced at higher modes, suggesting that lap length has a greater impact on stiffness at higher vibration frequencies. Overall, the graph emphasizes that longer lap joints enhance stiffness and improve dynamic performance.

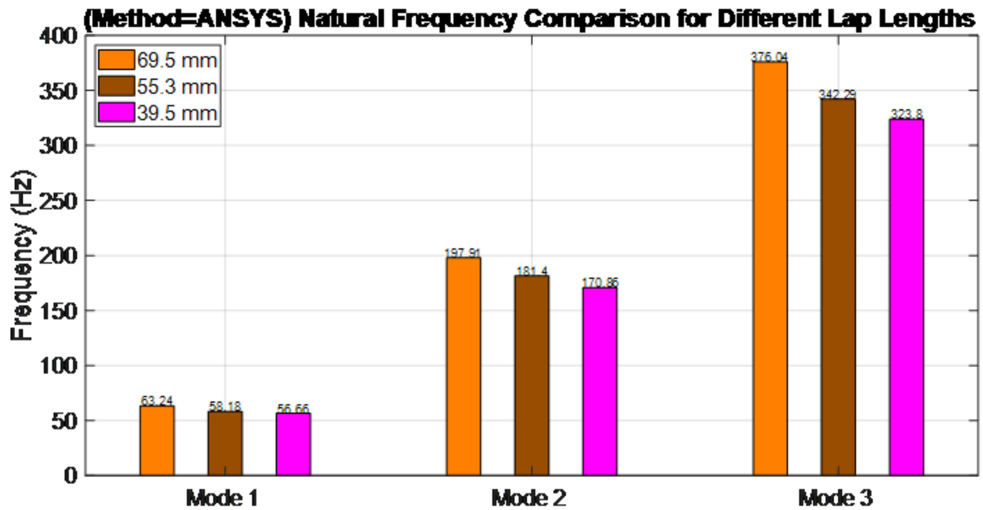

Figure 5.32: ANSYS comparison, with variable lap length

Experimental method

This graph from the experimental method shows that natural frequency increases from Mode 1 to Mode 3 for all lap lengths, with the 69.5 mm lap length still exhibiting the highest frequencies. The differences in the lap lengths (69.5 mm, 55.3 mm and 39.5 mm) are however less pronounced than the numerical results. The presence of real world factors such as friction, bolt looseness and damping reduces the effect of stiffness variation. The graph generally reflects the real structural behavior, where the stiffness increases with the lap length, but not as steeply as predicted by the ideal numerical models.

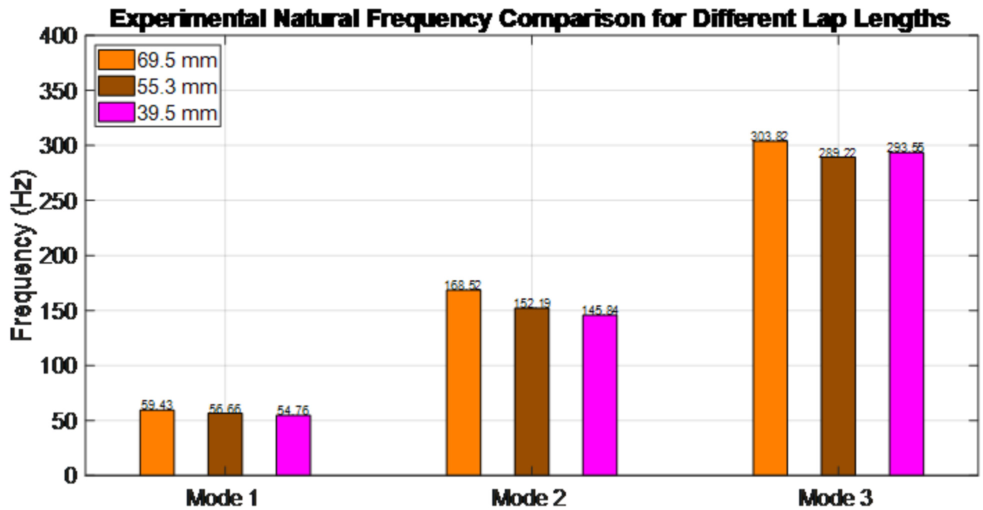

Figure 5.33: Experimental comparison, with variable lap length

5.4.3 Comparison of natural frequency for same lap length for different mode

Finite Element Method

The FEM graph shows that the natural frequency is significantly higher in relation to the increase in the mode from Mode 1 to Mode 3 for each lap length (69.5 mm, 55.3 mm and 39.5 mm), which means that the higher the mode, the higher the structural stiffness. All the lap lengths present this increasing trend; however, the highest frequencies are always obtained for the 69.5 mm specimen followed by the 55.3 mm and then the 39.5 mm. This visually confirms that, while mode shapes drive the frequency increase, the lap length still plays an important role in overall stiffness, with longer lap joints maintaining higher stiffness across all modes.

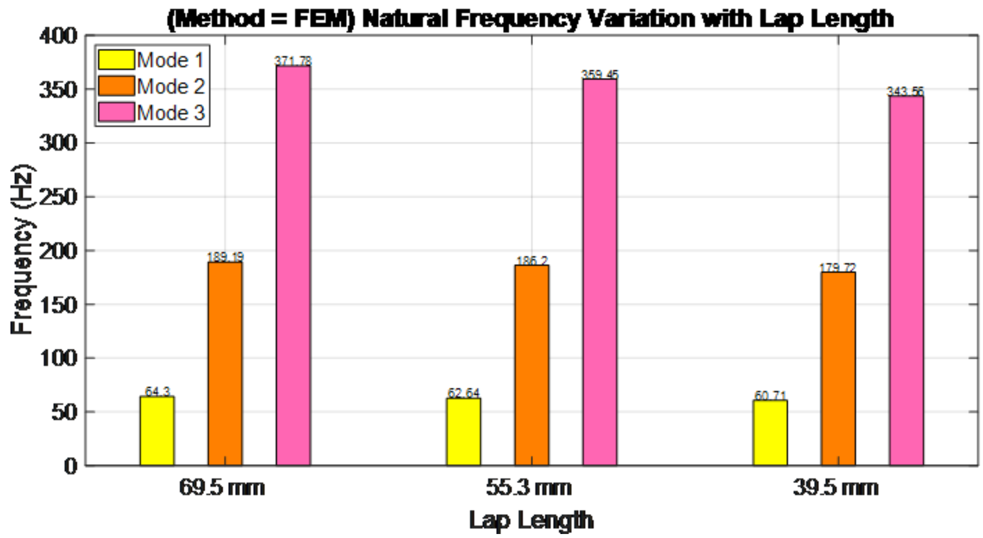

Figure 5.34: FEM comparison, with variable lap length

Numerical Simulation Method

The graph from numerical simulation in ANSYS shows a clear increase in natural frequency from Mode 1 to Mode 3 for all the overlap lengths representing that higher modes correspond to higher stiffness response. The highest frequencies among all lap joint are obtained for 69.5 mm overlap length, followed by 55.3 mm and 39.5 mm, verifying that increasing overlap length increases the stiffness. The smooth and well defined trend shows the greater affect of the mode shape and lap length on the dynamic behavior and expanding the effect of the stiffness for higher modes.

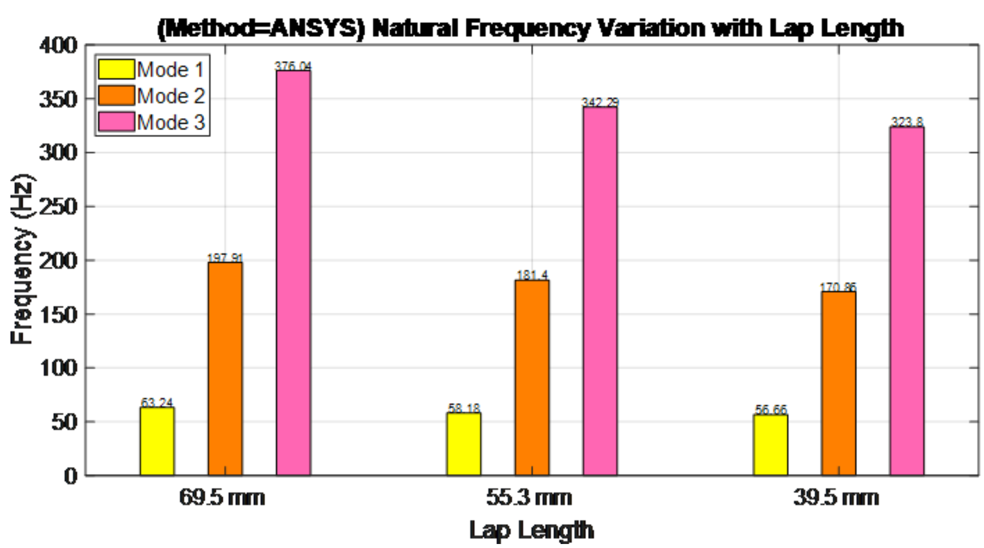

Figure 5.35: ANSYS comparision, with variable lap length

Experimental Method

This experimental graph showed that the natural frequency increase from Mode 1 to Mode 3 for all lap lengths, confirming that higher modes leading to higher dynamic stiffness. However, compared to numerical methods, the increase is more gradual and the differences between lap lengths are less prominent, indicating the influence of real-world factors such as damping, friction, and structural faults. Overall, it showed the actual structural behavior where mode shape governs frequency increase, while lap length still adds to stiffness but with reduced sensitivity.

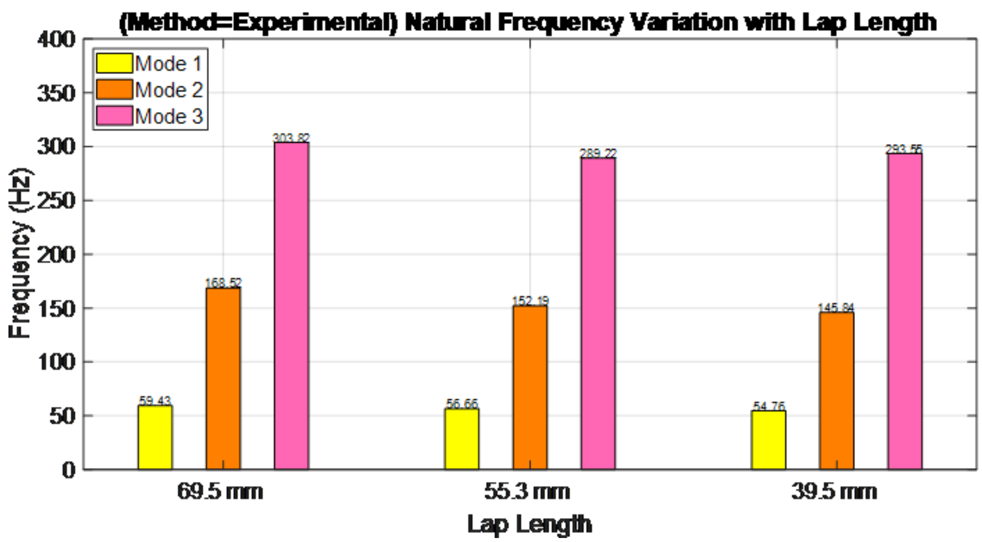

Figure 5.36: Experimental comparison, with variable lap length

5.4.4 Error Comparison

Ansys vs Experimental

Figure 4 shows the error comparison graph between ANSYS and experimental results for the first three transverse modes. The percentage error generally increases with mode number, which indicates that higher modes are more sensitive to modeling assumptions and boundary conditions. The error is relatively low in Mode 1 for all lap lengths, indicating a good agreement in the prediction of the fundamental frequency, while it becomes significantly higher in Mode 2 and peaks or varies in Mode 3. The 69.5 mm specimen shows a steady increase in error, while the 55.3 mm and 39.5 mm specimens show the maximum error at Mode 2 with a slight decrease or variation at Mode 3.In short, ANSYS predicts the lower modes well, but the deviation is higher at higher modes due to contact nonlinearity, damping and joint imperfections which are not fully taken into account in the numerical model.

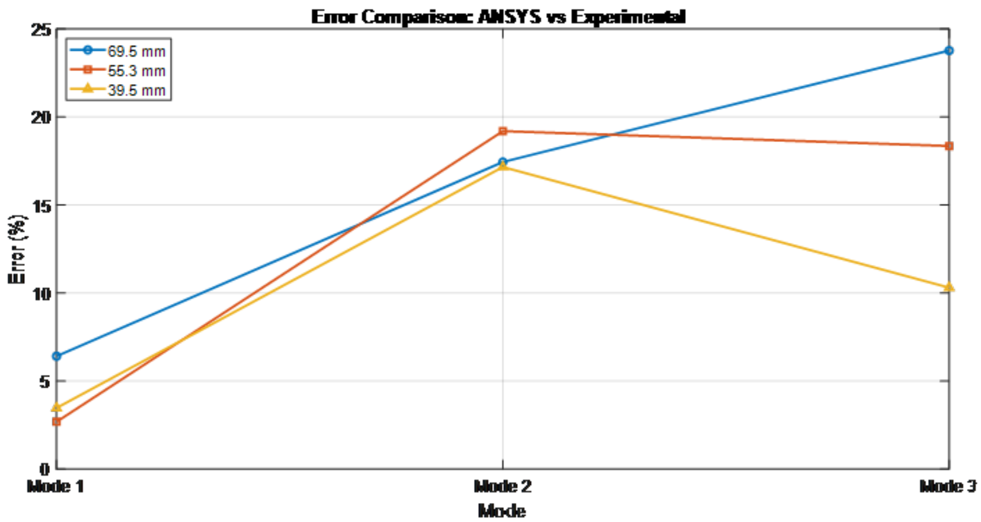

Figure 5.37: Error comparison Ansys vs Experimental

Fem vs Experimental

The error comparison between FEM and experimental results in that the percentage error increases with the mode number for all the lap lengths, which means the higher modes are more difficult to predict accurately. The error is low in Mode 1, showing good agreement for fundamental frequencies, but increases significantly in Mode 2 and Mode 3. For the 69.5 mm specimen, the error grows gradually, while for the 55.3 mm specimen, the highest error is found at Mode 3 and for the 39.5 mm specimen, it is at Mode 2 with a slight decrease afterwards. In general, this suggests that the predictions of the FEM are reliable for lower modes but deviate more for higher modes due to contact behavior, damping, and joint imperfections that are not fully captured by the model.

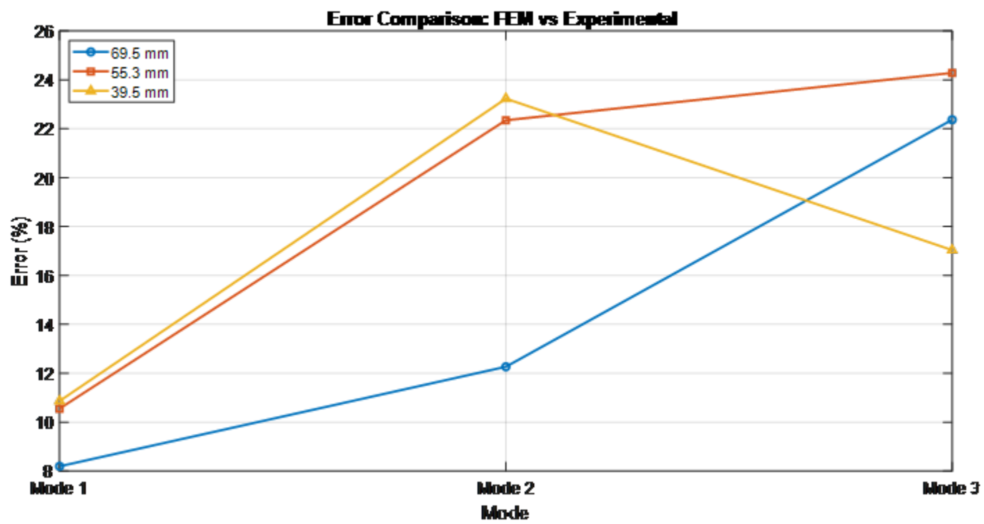

Figure 5.38: Error comparison FEM vs Experimental

Tabular Comparison of Data from different analysis tool with Error percentage

5.7:: Combined Comparison of ANSYS, FEM, and Experimental Results with Error

Lap Length of 69.5 mm							
Mode	**ANSYS (Hz)**	**FEM (Hz)**	**Experimental (Hz)**	**ANSYS Diff. (Hz)**	**ANSYS Error %**	**FEM Diff. (Hz)**	**FEM Error %**
One	63.24	64.30	59.43	3.81	6.4109036	4.87	8.1945146
Two	197.91	189.19	168.52	29.39	17.440066	20.67	12.265606
Three	376.04	371.78	303.82	72.22	23.770654	67.96	22.368508
Lap Length of 55.3 mm							
Mode	**ANSYS (Hz)**	**FEM (Hz)**	**Experimental (Hz)**	**ANSYS Diff. (Hz)**	**ANSYS Error %**	**FEM Diff. (Hz)**	**FEM Error %**
One	58.18	62.64	56.66	1.52	2.6826685	5.98	10.554183
Two	181.40	186.20	152.19	29.21	19.193114	34.01	22.347066
Three	342.29	359.45	289.22	53.07	18.349353	70.23	24.282553
Lap Length of 39.5 mm							
Mode	**ANSYS (Hz)**	**FEM (Hz)**	**Experimental (Hz)**	**ANSYS Diff. (Hz)**	**ANSYS Error %**	**FEM Diff. (Hz)**	**FEM Error %**
One	56.66	60.71	54.76	1.90	3.4696859	5.95	10.865595
Two	170.86	179.72	145.84	25.02	17.155787	33.88	23.230938
Three	323.80	343.56	293.55	30.25	10.304888	50.01	17.036280

The comparison of ANSYS and experimental results indicates that natural frequencies increase with the lap length and mode number, which means the stiffness of the bolted lap joint increases directly. The 69.5 mm specimen always exhibits the highest frequencies, followed by 55.3 mm and 39.5 mm, which confirms that larger overlap improves load transfer and joint stiffness. The higher stiffness reduces the deformation under cyclic loading and so increases the resistance to vibration and the fatigue failure life of the joint. Therefore, increasing the lap length has a positive effect on both the dynamic stiffness and the durability of the bolted structure.

The ANSYS comparison error percentage is relatively low for Mode 1 for all lap lengths, showing good agreement for fundamental frequency prediction. However, the error increases significantly for Mode 2 and Mode 3, especially for the 69.5 mm specimen, which indicates that higher modes are more sensitive to modeling assumptions. These differences are attributed to ideal boundary conditions, absence of damping, and simple contact modeling in ANSYS not fully representing the actual experimental conditions.

Similarly, the FEM and experimental comparison also shows the same trend of increasing natural frequency with the lap length and mode number which confirms that the stiffness increases with the increase of lap length. FEM results closely follow experimental behavior, showing that the numerical model effectively captures stiffness characteristics. The increased stiffness at higher lap lengths is a contributing factor to the reduction in amplitude of vibration as well as better distribution of stresses which is beneficial for extending the fatigue life of the bolted joint.

For FEM comparison, a similar trend is found in error percentage with Mode 1 having lower error and higher modes showing increased deviation. The FEM errors are slightly larger and more variable than ANSYS in some cases especially in Mode 2 and Mode 3 indicating sensitivities to mesh quality, contact definitions and material assumptions. Both methods are generally reliable in predicting lower modes, but the discrepancies increase for higher modes due to the nonlinearities and joint behaviour present in real structures, but not fully modelled in numerical models.

CHAPTER SIX: CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

For the lap joint configurations, the 69.5 mm lap length exhibited the highest natural frequencies, indicating the highest structural stiffness among all the configurations studied. This was followed by the 55.3 mm lap length configuration, while the 39.5 mm lap length showed the lowest natural frequencies due to its comparatively smaller overlap and reduced contact area between the connected plates. The increase in lap length enhances the effective contact surface, resulting in improved load transfer characteristics and greater joint rigidity.

The obtained results clearly demonstrate that the dynamic stiffness of the bolted lap joint increases with an increase in lap length. A larger lap region provides improved contact interaction between the plates, thereby reducing local deformation and enhancing the overall structural stability of the joint. This improvement in stiffness also contributes significantly to enhancing the fatigue life of the bolted connection under cyclic loading conditions.

Higher stiffness in the joint region plays a crucial role in reducing vibration amplitude and minimizing cyclic deformation during dynamic excitation. Since the joint region is generally the most critical location in structures subjected to repeated loading, reducing vibration-induced deformation helps in lowering stress concentration and delaying fatigue damage initiation.

Therefore, among all the configurations analyzed, the 69.5 mm lap length configuration provides the most favorable condition for improving the fatigue life of the bolted joint. The configuration offers superior stiffness and better vibration resistance characteristics, as consistently observed from all the experimental and numerical analyses presented in this study.

6.2 Recommendations

(i) Among the various lap joint configurations, the 69.5 mm overlap lap length showed the highest natural frequencies (highest stiffness), followed by 55.3 mm, while the 39.5 mm lap length exhibited the lowest frequencies due to reduced overlap and contact area and stiffness

(ii) However, the optimum lap length for practical design should be determined considering several design factors such as structural stiffness, material usage, weight, space constraints and manufacturing requirements. Further work can consider the development of optimization models for finding the most efficient lap length based on the trade-off between stiffness improvement and material efficiency.

(iii) Finally, it is recommended to perform experimental validation with better measurement accuracy in order to further verify the numerical results and to support the development of optimized bolted joint designs for real engineering structures.

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APPENDIX A: MATERIAL TESTING AND EXPERIMENTAL SETUP

Fig A1. Material Testing Sample 1

Fig A2. Material Testing Sample 2

Fig A3. Universal Tensile Test

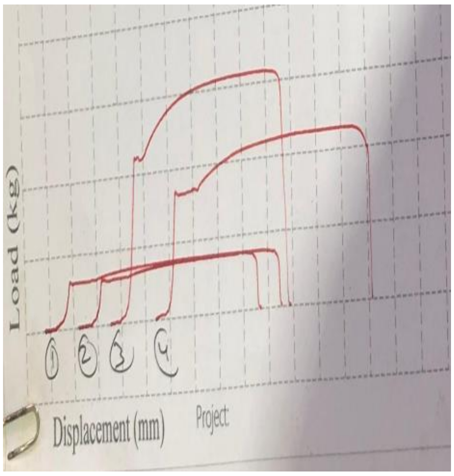

Fig A4. Stress Strain Curve

Fig A5. Programmable Dc Supply

Fig A6. Charged Amplifier

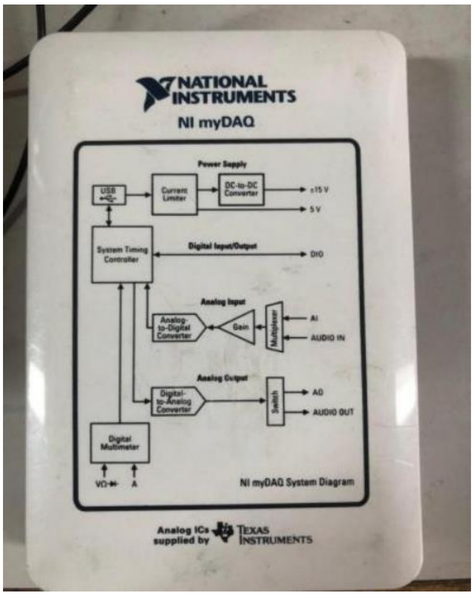

Fig A7. Data Acquisition System

Fig A8. Boundary Condition

Fig A9. Actual Test Setup

Fig A10. Circuit for Hand Calibrator

Fig A11. Fabrication Process

Fig A12. Bolt Pre-Tension With Calibrated Torque Wrench

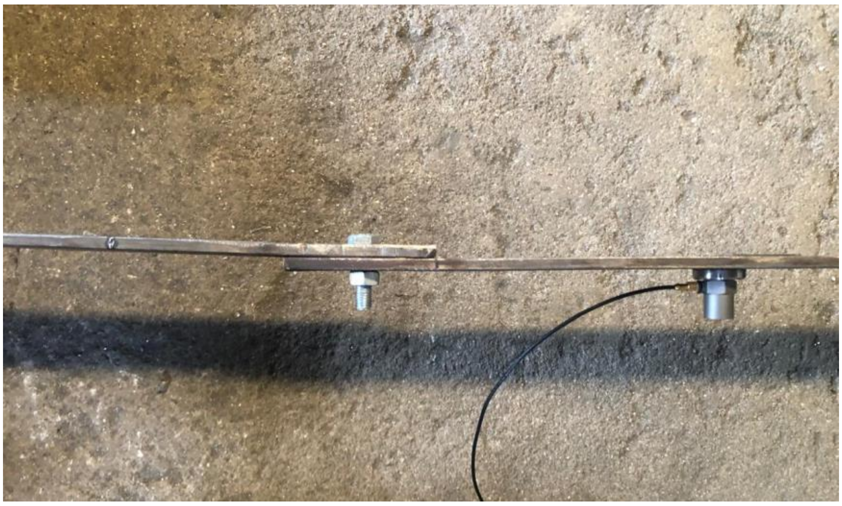

Fig A13. Bolted Joint With Lap Length of 69.5mm

Fig A14. Bolted Joint With Lap length of 55.3mm

Fig A15. Bolted Joint With Lap length of 39.5mm

Fig A16. Welding of Test Specimen

APPENDIX B: MATLAB CODE FOR FEM FORMULATION

```
%% Preamble
clc;
clear;
close all;

%%
============================================================
% FREE VIBRATION ANALYSIS OF BOLTED LAP JOINT USING FEM
% Only Mode 1 to Mode
% MODEL:
% - Two plates modeled as Euler-Bernoulli beam elements
% - Overlap region connected by interface springs
% - Bolt locations treated as attached/tied between plates
% - Bolt mass added at bolt nodes
% - Fixed support over 25 mm at both outer ends
%%
============================================================

%% ---------------- GEOMETRY ----------------
b   = 50.8e-3;       % plate width (m)
t   = 6e-3;          % plate thickness (m)
L1  = 340e-3;        % left plate length (m)
L2  = 450e-3;        % right plate length (m)
Lov = 69.5e-3;       % overlap length (m)
fix_len = 25e-3;     % fixed support length at both ends (m)

x_overlap_start = L1 - Lov;
x_overlap_end   = L1;
L_total = x_overlap_start + L2;

% Bolt positions
x_bolt1 = x_overlap_start + 14e-3;
x_bolt2 = x_bolt1 + 22.8e-3;

%% ---------------- MATERIAL ----------------
E   = 200e9;         % Young's modulus (Pa)
rho = 7850;          % density (kg/m^3)

A = b*t;
I = b*t^3/12;

%% ---------------- BOLT DATA ----------------
d_bolt = 8e-3;       % M8 bolt diameter
r_bolt = d_bolt/2;

% approximate grip length = two plates thickness
L_bolt = 2*t;

% approximate bolt mass (shank only)
A_bolt = pi*r_bolt^2;
m_bolt = rho*A_bolt*L_bolt;

%% ---------------- MESH ----------------
Le = 5e-3;   % element length
```

```
x1 = 0:Le:L1;
if abs(x1(end)-L1) > 1e-12
    x1 = [x1 L1];
end

x2 = x_overlap_start:Le:L_total;
if abs(x2(end)-L_total) > 1e-12
    x2 = [x2 L_total];
end

n1 = length(x1);
n2 = length(x2);

% DOF per node = [w theta]
ndof1 = 2*n1;
ndof2 = 2*n2;
ndof_total = ndof1 + ndof2;

%% ---------------- GLOBAL MATRICES ----------------
K = zeros(ndof_total, ndof_total);
M = zeros(ndof_total, ndof_total);

%% ---------------- BEAM ELEMENT MATRICES ----------------
beamKM = @(E,I,rho,A,L) deal( ...
    E*I/L^3 * [12    6*L   -12    6*L;
              6*L  4*L^2  -6*L  2*L^2;
             -12   -6*L    12   -6*L;
              6*L  2*L^2  -6*L  4*L^2], ...
    rho*A*L/420 * [156    22*L    54    -13*L;
                  22*L   4*L^2   13*L  -3*L^2;
                  54     13*L    156   -22*L;
                 -13*L  -3*L^2  -22*L   4*L^2] );

%% ---------------- ASSEMBLE LEFT PLATE ----------------
for e = 1:n1-1
    Le_local = x1(e+1) - x1(e);
    [Ke, Me] = beamKM(E, I, rho, A, Le_local);

    dofs = [2*e-1, 2*e, 2*(e+1)-1, 2*(e+1)];
    K(dofs,dofs) = K(dofs,dofs) + Ke;
    M(dofs,dofs) = M(dofs,dofs) + Me;
end

%% ---------------- ASSEMBLE RIGHT PLATE ----------------
offset = ndof1;

for e = 1:n2-1
    Le_local = x2(e+1) - x2(e);
    [Ke, Me] = beamKM(E, I, rho, A, Le_local);

    dofs_local = [2*e-1, 2*e, 2*(e+1)-1, 2*(e+1)];
    dofs = dofs_local + offset;

    K(dofs,dofs) = K(dofs,dofs) + Ke;
    M(dofs,dofs) = M(dofs,dofs) + Me;
end

%% ---------------- OVERLAP INTERFACE CONNECTION ----------------
% distributed interface stiffness over overlap
k_interface = 1e8;   % N/m
```

```
findNearest = @(xvec, xval) find(abs(xvec-xval) == min(abs(xvec-xval)), 1);

common_x = intersect(round(x1,10), round(x2,10));

for i = 1:length(common_x)
    xc = common_x(i);

    i1 = findNearest(x1, xc);
    i2 = findNearest(x2, xc);

    dof_w1 = 2*i1 - 1;
    dof_w2 = offset + 2*i2 - 1;

    ks = k_interface * [1 -1; -1 1];
    K([dof_w1 dof_w2],[dof_w1 dof_w2]) = ...
        K([dof_w1 dof_w2],[dof_w1 dof_w2]) + ks;
end

%% ---------------- BOLT ATTACHED TO PLATE ----------------
% Treat bolt locations as strongly tied between two plates
% and add bolt mass at those nodes

k_bolt_tie = 1e12;   % very large stiffness => attached/bonded

bolt_positions = [x_bolt1, x_bolt2];

for xb = bolt_positions
    i1 = findNearest(x1, xb);
    i2 = findNearest(x2, xb);

    % displacement DOF
    dof_w1 = 2*i1 - 1;
    dof_w2 = offset + 2*i2 - 1;

    % rotational DOF
    dof_t1 = 2*i1;
    dof_t2 = offset + 2*i2;

    % tie displacement
    ks_w = k_bolt_tie * [1 -1; -1 1];
    K([dof_w1 dof_w2],[dof_w1 dof_w2]) = ...
        K([dof_w1 dof_w2],[dof_w1 dof_w2]) + ks_w;

    % tie rotation also
    ks_t = k_bolt_tie * 1e-4 * [1 -1; -1 1];
    K([dof_t1 dof_t2],[dof_t1 dof_t2]) = ...
        K([dof_t1 dof_t2],[dof_t1 dof_t2]) + ks_t;

    % add half bolt mass to each connected node displacement DOF
    M(dof_w1,dof_w1) = M(dof_w1,dof_w1) + m_bolt/2;
    M(dof_w2,dof_w2) = M(dof_w2,dof_w2) + m_bolt/2;
end

%% ---------------- BOUNDARY CONDITIONS ----------------
% Left plate fixed for first 25 mm
left_fixed_nodes = find(x1 <= fix_len + 1e-12);

% Right plate fixed for last 25 mm
right_fixed_nodes = find(x2 >= (L_total - fix_len - 1e-12));
```

```
fixed_dofs = [];

for i = 1:length(left_fixed_nodes)
    n = left_fixed_nodes(i);
    fixed_dofs = [fixed_dofs, 2*n-1, 2*n];
end

for i = 1:length(right_fixed_nodes)
    n = right_fixed_nodes(i);
    fixed_dofs = [fixed_dofs, offset + 2*n-1, offset + 2*n];
end

fixed_dofs = unique(fixed_dofs);

all_dofs = 1:ndof_total;
free_dofs = setdiff(all_dofs, fixed_dofs);

Kff = K(free_dofs, free_dofs);
Mff = M(free_dofs, free_dofs);

%% ---------------- EIGENVALUE SOLUTION ----------------
nmodes = 3;

[Phi, D] = eig(Kff, Mff);
lambda = diag(D);

valid = real(lambda) > 1e-6 & abs(imag(lambda)) < 1e-8;
lambda = real(lambda(valid));
Phi = real(Phi(:,valid));

[lambda, idx] = sort(lambda);
Phi = Phi(:,idx);

lambda = lambda(1:nmodes);
Phi = Phi(:,1:nmodes);

omega = sqrt(lambda);
freq = omega/(2*pi);

%% ---------------- EXPAND FULL MODE VECTOR ----------------
Phi_full = zeros(ndof_total, nmodes);
Phi_full(free_dofs,:) = Phi;

%% ---------------- DISPLAY FREQUENCIES ----------------
fprintf('\n=====================================\n');
fprintf(' FIRST 3 NATURAL FREQUENCIES (Hz)\n');
fprintf('=====================================\n');
for i = 1:nmodes
    fprintf('Mode %d : %.4f Hz\n', i, freq(i));
end

%% ---------------- EXTRACT TRANSVERSE DOF ----------------
w1_dofs = 1:2:ndof1;
w2_dofs = offset+1:2:ndof_total;

%% ---------------- PLOT MODE SHAPES ----------------
for mode = 1:nmodes
    phi_mode = Phi_full(:,mode);
```

```
w1 = phi_mode(w1_dofs);
w2 = phi_mode(w2_dofs);

% Make Mode 1 display upward
if mode == 1
    if mean(w1) < 0
        w1 = -w1;
        w2 = -w2;
    end
end

    scale = max([max(abs(w1)), max(abs(w2)), 1e-12]);
    w1 = w1/scale;
    w2 = w2/scale;

    figure;
    hold on;
    grid on;
    box on;

    % undeformed
    plot(x1*1000, zeros(size(x1)), 'k--', 'LineWidth', 1);
    plot(x2*1000, zeros(size(x2)), 'k--', 'LineWidth', 1);

    % deformed
    plot(x1*1000, w1, 'b-', 'LineWidth', 2);
    plot(x2*1000, w2, 'r-', 'LineWidth', 2);

    % bolt positions
    plot(x_bolt1*1000, 0, 'ko', 'MarkerFaceColor', 'g', 'MarkerSize', 8);
    plot(x_bolt2*1000, 0, 'ko', 'MarkerFaceColor', 'g', 'MarkerSize', 8);

    % support limits
    xline(fix_len*1000, 'm--', 'LineWidth', 1.2);
    xline((L_total-fix_len)*1000, 'm--', 'LineWidth', 1.2);

    xlabel('Length (mm)');
    ylabel('Normalized displacement');
    title(['Mode ', num2str(mode), '   Frequency = ', num2str(freq(mode),'%.4f'), ' Hz']);

    legend('Left plate undeformed', ...
           'Right plate undeformed', ...
           'Left plate mode shape', ...
           'Right plate mode shape', ...
           'Bolt 1', 'Bolt 2', ...
           'Left support end', 'Right support start', ...
           'Location', 'best');
end
```

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Letter of Acceptance

Bhuwan Bhandari, Prof. Dr. Mahesh Chandra Luitel, Prof. Dr. Surya Prasad Adhikari

The Editorial Board of Journal of Advanced College of Engineering and Management (jacem) (ISSN No: 2392-4853), is pleased to inform you that your manuscript **"FREE VIBRATION ANALYSIS OF BOLTED LAP JOINT WITH VARIABLE LAP LENGTH"** has been reviewed by the referee and accepted for the publication. Your article will be published in the coming issue of Journal of Advanced College of Engineering and Management, Vol. 13.

We are delighted and thankful for considering this Journal as a venue of your valuable research work.

With Regards

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