VISUALIZATION, FORMULATION AND INTUITIVE EXPLANATION OF ITERATIVE METHODS FOR TRANSIENT ANALYSIS OF RLC CIRCUIT

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राष्ट्रिय युवा परिषद्-नेपाल

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supported by

National Youth Council (NYC)

DEDICATION

To

My mother

Sita Devi Thakur

with

My father

Rameshwar Thakur

STUDENT'S DECLARATION

This thesis entitled "**Visualization, Formulation and Intuitive Explanation of Iterative Methods for Transient Analysis of RLC Circuit**", which has been submitted to the Central Department of Mathematics, Institute of Science and Technology (IOST), Tribhuvan University, Nepal for the partial fulfillment of the Master in Science (M.Sc.) Degree in Mathematics, is a genuine work that I carried out under my supervisor Dr. Jeevan Kafle and that no sources other than those listed in the Bibliography have been used in this work. Moreover, this work has not been published or submitted elsewhere for the requirement of any degree program.

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RECOMMENDATION

This is to recommend that Mr **Bhogendra Kumar Thakur** has prepared this thesis entitled "**Visualization, Formulation and Intuitive Explanation of Iterative Methods for Transient Analysis of RLC Circuit**" for the partial fulfillment of the Master in Science (M.Sc.) in Mathematics under my supervision. To my/our knowledge, this work has not been submitted for any other degree. He has fulfilled all the requirements laid down by the Central Department of Mathematics, Institute of Science and Technology (IOST), Tribhuvan University (TU), Kirtipur for the submission of the thesis for the partial fulfillment of M.Sc. Degree in Mathematics.

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LETTER OF APPROVAL

We certify that the Research Evaluation Committee of the Central Department of Mathematics, Tribhuvan University, Kirtipur approved this research work entitled **"Visualization, Formulation and Intuitive Explanation of Iterative Methods for Transient Analysis of RLC Circuit"** carried out by Mr. **Bhogendra Kumar Thakur** in the scope and generality as a thesis in the partial fulfillment for the requirement of the M.Sc. degree in Mathematics.

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Bhogendra Kumar Thakur

Date: June 25, 2021

ABSTRACT

The time-varying currents and voltages resulting from the sudden application of sources usually due to switching are transients. An RLC circuit is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel. The transient response is dependent on the value of the different characteristics of the damping factor (i.e., overdamped, critically damped, and underdamped). The numerical solutions of first and second-order differential equations with initial value problem (IVP) have been computed by using the Explicit (Forward) Euler method, Implicit (Backward) Euler method, Classical second-order (Heun's or RK2) method, Third-order Runge-Kutta (RK3) method, Fourth order Runge-Kutta method and Butchers fifth-order Runge-Kutta (BRK5) method. The observation compares this numerical solution of ODEs obtained by the above-mentioned methods among them with the necessary visualization and analysis of the error. These iterative methods will be extended and implement to analyze the transient analysis of an RLC circuit. The superiority of these methods over one another has been examined. The Butcher's fifth-order Runge-Kutta (BRK5) method is found to be the best numerical technique to solve the transient analysis due to its high accuracy of approximations. Moreover, we consider the possibility of discussing and analyze above mentioned iterative methods in the cases of different characteristics of damping factors. Different methods are used different iterative methods to analyze the transient analysis of an RLC circuit and compared among them. The consequences of this work lend some limelight to the modern approaches to solving complicated mathematical problems.

LIST OF ACRONYMS AND ABBREVIATIONS

DC: Direct Current

IVP: Initial Value Problem

BVP: Boundary Value Problem

ODE: Ordinary Differential Equation

RLC: Resistor Inductor Capacitor

CFL: Compact Fluorescent Lamp

RK2: Second Order Rungee Kutta

RK3: Third Order Rungee Kutta

RK4: Fourth Order Rungee Kutta

DTM: Differential Transformation Method

BRK5: Butcher Fifth Order Rungee Kutta

EFCAM: Exponentially Fitted Collocation Approximation Method

LIST OF SYMBOLS

i	Current.
v	Voltage.
C	Capacitor.
H	Henry.
F	Farad.
L	Inductor.
R	Resistor.
V_S	Source Voltage.
V_R	Voltage drop across resistor.
V_L	Voltage drop across inductor.
V_C	Voltage drop across capacitor.
I_R	Current through resistor.
I_L	Current through inductor.
I_C	Current through capacitor.
α	Alpha.
ζ	Zeta.
ω	Omega.
Ω	Ohm.
$\approx$	Approximate.
erf	error function.

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Chapter 1

INTRODUCTION

1.1 Background

While regarding the problem of mathematical modelling in the area of engineering, physics, aeronautics, medicine, environmental science, astronomy, chemistry, biology, and many other applied field, the role of a differential equation is momentous [8, 32]. Meanwhile, all of the differential model equation in real life appears to be nonlinear [23]. On contrary, almost all differential equations encountered in course of modelling to be solved analytically is out of the question [8]. The planetary motion under gravity, motion of the simple pendulum, etc. are a typical example of second-order differential equations that won't solve analytically [20, 32]. To wriggle out from such a confounding situation one has to devise a more rigorous alternative approach that would be simple and more pragmatic. One such method is the numerical method [20, 28].

Transients are the momentary fluctuation of energy induced upon the electrical circuit due to the time-varying currents and voltages resulting from the sudden application of sources. It usually occurs at the period of switching [44]. Sensitive components of the electrical circuitry such as millions of inductors, capacitors, resistors, transistors, and diodes embedded inside integrated circuits are most vulnerable to this brief increase in current in voltage [12]. An electrical system under the influence of transient conditions rendered the system in an unsteady state. Thus a transient analysis is to analyze the response of an electrical circuit under these unsteady state conditions [31]. If the variables do not vary with respect to time, then the state of the system is said to be in a steady state otherwise it is said to be in an unsteady state. Performance of any electrical circuit is measured by employing the methodology of transient analysis [21]. Although transients do occur in the most apparent electrical equipment such as charger of an android phone, electric mixture, CFL bulb, iron, etc, it is not at a threatening level because components are adjusted enough to handle such ailments [19]. But transient occurring in the microscopic level inside a microprocessor, motherboard, and memory card could be lethal to the existence of the system.

Mathematical modelling is a state of the art technique to model the real world physical problem into mathematical framework whose theoretical and numerical analysis provides useful outcomes to sort out various problems. Mathematical methods and modeling techniques find their ample applications in different branches of science, such as geophysics [24, 25, 26, 27, 30], environment [37], physics [23, 39], epidemiology and body fluid [3, 4, 5, 6, 38], in particular, in electronics [28, 33, 41].

1.1.1 Series RLC circuit

An RLC circuit constitutes three electrical elements *viz.* a resistor, an inductor, and a capacitor. It is a second-order circuit. These circuits are the most popular as they are applied to construct oscillators and tuners of radio or audio receivers [44]. In the adjacent Fig. 1.1, the electrical elements resistor (R), inductor (L), and capacitor (C) are connected in series with the D.C. source (V_s), which is called the RLC series circuit. A circuit equation is drawn to undertake the analytical solution part of a transient analysis [31]. An ordinary electrical circuitry could contain thousands of components. So any analytical solution in such RLC circuits is virtually impossible. In such a scenario, numerical methods could offer great relief to the solution of the system. Here the possible iterative methods are used to solve the circuit equation under the terms of transient analysis in an RLC circuit and compare different methods. The Kirchhoffs Voltage Law [31] is applied

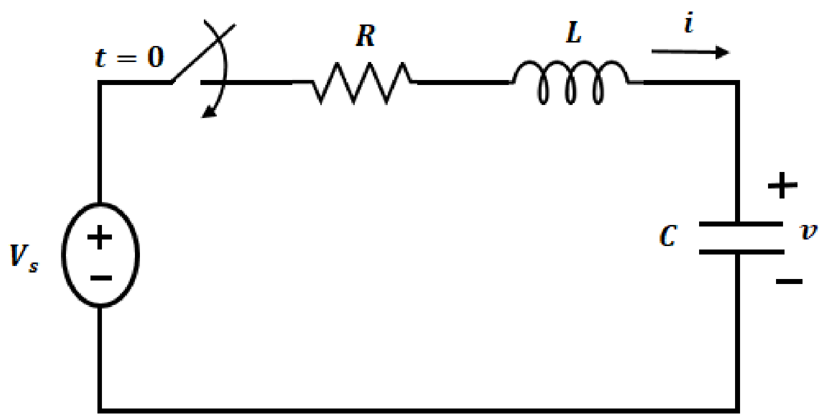

Figure 1.1: Series RLC circuit [19].

around the loop for $t > 0$ in which the voltage source of the circuit in Fig. 1.1 is calculated by the following;

$$V_S = V_R + V_L + V_C \tag{1.1}$$

The differential equations of the RLC in Fig. 1.1 is based on the method of loop currents where the fundamental relationship between the current, and the individual circuit elements (Capacitance, Inductance, Resistance) are given by

$$\begin{aligned} \text{Resistance} \quad &: \quad v(t) = Ri(t), \\ \text{Capacitance} \quad &: \quad i(t) = Cdv/dt, \\ \text{Inductance} \quad &: \quad v(t) = Ldi/dt, \end{aligned}$$

The second-order differential equation of the series RLC circuit with constant coefficients is written as [40]:

$$L\frac{d^2v}{dt^2} + R\frac{dv}{dt} + \frac{1}{C}v = \frac{V_S}{C}; \;\; v(0) = 6V, \;\; i(0) = 6A. \tag{1.2}$$

The series RLC circuit is analyzed in order to determine its transient characteristics once the switch is closed. Equation (1.2) can be solved using different iterative methods [19].

The damping factor is responsible for the amount by which the oscillation of a system gradually decreases with time (t). The transient response is dependent on the value of the damping factor (ζ) [19].

In RLC circuit, the damping factor is given by

$$\zeta = \frac{\alpha}{\omega_o},$$

where $\alpha = \frac{R}{2L}$ (damping coefficient) and $\omega_o = \frac{1}{\sqrt{LC}}$ (resonant frequency). Then

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}. \tag{1.3}$$

The system is (i) overdamped when $\zeta > 1$, (ii) critically damped when $\zeta = 1$, and (iii) underdamped when $\zeta < 1$ [44].

1.1.2 Parallel RLC circuit

In the Fig. 1.2, the electrical elements resistor (R), inductor (L) and capacitor (C) are connected in parallel with the D.C. source (Vs), which is called parallel RLC circuit.

The Kirchhoffs Current Law [31] is applied around the loop for $t > 0$, in which the total current of the circuit in Fig. 1.2 is calculated by the following:

$$I_S = I_R + I_L + I_C. \tag{1.4}$$

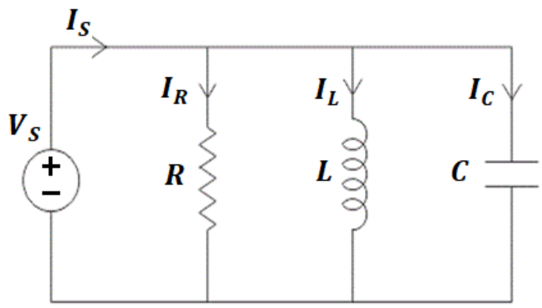

Figure 1.2: Parallel RLC circuit [9].

The differential equations of the parallel RLC in Fig. 1.2 is based on the method of loop currents where the fundamental relationship between the current and the individual circuit elements are given by

$$\begin{aligned} \text{Resistance} &\quad : \quad I_R = \frac{v}{R}, \\ \text{Capacitance} &\quad : \quad I_C = C\frac{dv}{dt}, \\ \text{Inductance} &\quad : \quad I_L = \frac{1}{L}\int vdt, \end{aligned}$$

where v is the voltage drop across each elements *viz.* capacitor, resistor and inductor. Substituting these values in equation (1.4), to get

$$\frac{1}{R}v + \frac{1}{L}\int vdt + C\frac{dv}{dt} = I_S. \tag{1.5}$$

Differentiate equation (1.5) on both sides , we get the second-order differential equation for the parallel RLC circuit with constant coefficients is written as [40]:

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0; \;\; v(0) = 6, \;\; i(0) = 0, \;\; \frac{dv}{dt}(0) = -12. \tag{1.6}$$

The parallel RLC circuit is analyzed in order to determine its transient characteristics once the switch is closed [41]. Equation (1.6) can be solved using different iterative methods [19]. The damping factor is responsible for the amount by which the oscillation of a system gradually decreases with the time (t). The transient response is dependent on the value of the damping factor (ζ) [19]. In parallel RLC circuit, the damping factor is given by

$$\zeta = \frac{\alpha}{\omega_o},$$

where $\alpha = \frac{1}{2RC}$ (damping coefficient) and $\omega_o = \frac{1}{\sqrt{LC}}$ (resonant frequency). Then

$$\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}. \tag{1.7}$$

The system (1.4) is overdamped when $\zeta > 1$, (ii) critically damped when $\zeta = 1$, and (iii) underdamped when $\zeta < 1$ [44].

1.2 Literature Review

Suhag [44] conducted the transient analysis of the second-orderRLC circuit and observed the response of the system by changing the conditions from one steady-state value to another. He concluded RK method was very efficient in solving second-order differential equations. Kee and Ranom [31] studied the transient analysis of series RLC circuits under underdamped, critically damped, and overdamped conditions using RK4 with the different time step size h. The accuracy of the RK4 method is determined gradually as the step size decimates. Thus, the improved RK4 method by decreasing the step size h or increasing the number of steps n was a favorably suited method for solving transient analysis of electric circuits due to its high degree of accuracy and efficiency. Henry et al. [19] undertook the task of transient analysis of a second-order RLC circuit by employing two numerical methods *viz.* Heuns method and Runge-Kutta's method. They observed that Heuns method reaches the stable limit first, thus, converges faster but Runge-Kutta 4^{th} order method proved to be more accurate. Thus, the Runge-Kutta method was recommended for transient analysis of complex electric circuits. Lamichhane [33] studied the internal tank current, phase difference, antiresonance, sensitivity of parallel LCR circuits. He found that the circuit current in the parallel LCR circuit becomes very small in the resonating region, but at the same time, the potential difference across the LC tank becomes very large. The current in the parallel LCR (inductor, capacitor, and resistor) circuit depends not only on the magnitude of the applied electromotive force (emf) but also on its frequency.

Ogbuka et al. [35] in their paper reiterated the need for a sound mathematical background and knowledge of basic laws of circuit analysis before handling the task of circuit analysis. They programmed the state equations of a sample complex circuit in two simulation software to generate the transient responses. Deshpande [15] stated the source DC voltage applied in any circuit (RLC) constituting of energy storage devices gives rise to transients and therefore estimation of the voltages was of significant importance for instrumentation and control purpose. Deshpande carried out the transient analysis to this circuit by employing the Classical and Laplace Transformation methods. In the cases of over-damped, underdamped, and critically damped conditions was evaluated significant for the natural response of the RLC series circuit. The Laplacetransform method had proved to be a better tool with respect to initialization conditions in comparison to the classical method. Ahamad and Charan [8] in order to solve the initial value problem for fourth-order ODEs presented 5^{th} order Runge-Kutta method (RK5). Kamruzzaman and Nath [29] in their paper employed the modified Euler and Runge-Kutta methods to discuss the numerical solution of an ODE with IVP. They carried out the numerical simulation and compared the numerical solution with the exact solution. Kafle et al. [23] studied the thermal diffusivity of heat equation of non-linear differential equation types by using the finite difference method. Faleski [18] undertook the detailed investigation of the transient behavior of the dc-driven underdamped RLC circuit including many interesting features like potential difference across the inductor, capacitor, resistor, and battery by using the mass-spring system to demonstrate the qualitative response of the capacitors potential difference.

Maffezzoni et al. [34] presented a novel approach to the accurate time-domain simulation of non-linear circuits that employs a class of high-order implicit Runge-Kutta (RK) method. The properties of stability and accuracy of these RK methods were briefly reviewed while the implementation in the flow of an analog simulator was described in detail. Alizadeh et al. [9] study the Laplace transformation solution of Caputo-Fabrizio sectional derivative to study the transient response of parallel RLC circuit. They compared the graph obtained for solutions of a different orderof fractional derivatives with the usual solutions. Elton [16] started with the basics concept of the RLC circuit and ended with the concept of circuit design. He analyzed both series and parallel RLC circuits with practical applications using advanced calculus to aid in predetermined results. Maria Selvam and Vignesh [41] established the condition of equilibrium stable and local stable point in a two-dimensional model of a parallel RLC circuit. Different sets of parameter values were used to obtain the phase graphs. Bifurcation of the underdamped parallel RLC circuit was obtained by selective choices of parameter range. Phase graphs and bifurcation simulations exhibit the rich dynamical nature of the system. Falade and Ayodele [17] used the Differential Transformation Method (DTM) and Exponentially Fitted Collocation Approximation Method (EFCAM) to solve the numerical solutions of the second-order electrical circuit. To comparison withthe analytical solution of this method was found to be accurate and compatible. Kafle et al. [28] compared the different iterative methods to analyze the damping conditions of series RLC circuits under the transient situations with DC source and they found the best iterative (BRK5) method to solve the second-order ODE of the RLC circuit. They observed the three damping conditions by using the BRK5 method.

In this work, the RLC circuit is taken with a DC source which can be modeled with a second-order ODE. For solving IVP of ODE and transient analysis we first compared the three iterative methods *viz.* Explicit Euler, Third-order Runge-Kutta (RK3), and Butcher fifth-order Runge-Kutta (BRK5). BRK5 method is observed to be the best method as it gives the least error compared to the other two methods. BRK5 method is then chosen to analyze the three damping conditions of the RLC circuit.

1.3 Objectives

The major objective of this work are listed below:

- to implement the Implicit and Explicit Euler method, Classical second-order Runge-Kutta method (Heun's method or improved Euler or RK2), Third-order Runge-Kutta (RK3) method, Fourth-order Runge-Kutta (RK4) method, Butchers fifth-order Runge-Kutta (BRK5) method for solving IVP for ODEs. To compare these numerical solutionsof ODEs obtained by these methods among them with necessary visualization and also analyze the error and accuracy of these methods.
- to explain the implementation of numerical approximation methods as Explicit Euler method, Third-order Runge-Kutta (RK3) method, and Butcher fifth-order Runge-Kutta (BRK5) method for transient analysis of an RLC circuit with DC source.
- to analyze the current flow of the model equation of series RLC and parallel RLC circuits by variation of DC voltage source (V_s) using iterative methods.
- to discuss and analyze different iterative methods in the cases of three different characteristics of damping factor (i.e., overdamped, critically damped, and under-damped) for both series and parallel RLC circuits.

1.4 Structure of Thesis

The approach of this work is primarily analytical and comparative among the various numerical methods for solving the differential equations. Chapter-2 comprises the theoretical approach and comparison of iterative methods like Euler's (Explicit and Implicit) method and Runge-Kutta method up to 5^{th} order with the necessary visualization and numerical formulation. Chapter-3 depicts the transient analysis of series RLC circuits including both formulations of the second-degree differential equation for the series RLC circuit and a comparison of the above-mentioned method. Chapter-4 describes the transient analysis of parallel RLC circuits including both formulations of the second-degree differential equation for parallel RLC circuit and a comparison of the above-mentioned methods. Chapter-3 and chapter-4 also include the simulated results for both series and parallel RLC circuits. Chapter-5 culminate the summary and findings of this work.

Chapter 2

Numerical Methodologies

In this section, the different iterative methods are discussed to solve initial value problems (IVP) for ordinary differential equations (ODE). First, we introduce Eulers method, the simplest method. Algorithmic development is provided in details for using the Euler method (explicit), Euler method (implicit), Classical second-order Runge-Kutta (RK2) method, Third-order Runge-Kutta (RK3) method, Fourth-order Runge-Kutta (RK4) method, and Butchers fifth-order Runge-Kutta (BRK5) method [13, 14]. These numerical methods are compared with necessary visualization.

For the comparison of numerical methods, an IVP [13] is taken as:

$$y'(t) = ty + t^2, y(0) = 1. \tag{2.1}$$

on the interval $0 \leq t \leq 1$ and with the exact solution.

$$y(t) = \sqrt{\frac{\pi}{2}}\, e^{t^2/2}\, erf\left(\frac{t}{\sqrt{2}}\right) + e^{t^2/2} - t. \tag{2.2}$$

The error function (Gauss error function), often denoted by *erf*, is a complex function of a complex variable defined as [7]:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \tag{2.3}$$

2.1 Euler's (Explicit or Forward) Method

Euler proposed his method for initial value problems (IVP) at 1768. Euler's method is a numerical method that can be used to approximate the solutions to explicit first-order differential equations. It is based on making successive linear approximations to the solution.

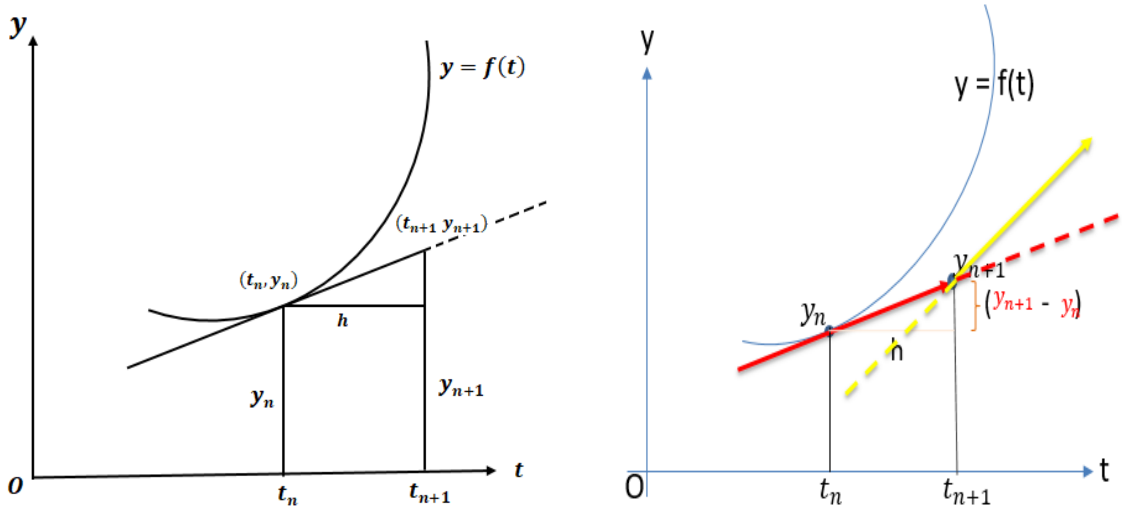

Figure 2.1: Visualization of Euler method **Left:** Explicit and **Right:** Implicit [1].

Consider an initial value problem (IVP) for smooth function $f(t, y(t))$ [1, 11]:

$$y'(t) = f(t, y(t)), \qquad\qquad y(t_0) = y_0. \tag{2.4}$$

Let y_n be the position at time t_n with step size h. By the definition of derivative, we have

$$\frac{dy}{dt} = \lim_{h\to 0} \frac{y(t+h) - y(t)}{h} \Rightarrow \frac{dy}{dt} \approx \frac{y(t+h) - y(t)}{h}.$$

When $h \to 0$, from Fig. 2.1, it is clear that, at that time the slope at that point or the tangential approximation lies completely in the curve. Slope at the point (t_n, y_n) is

$$\frac{y_{n+1} - y_n}{h} \approx \frac{dy}{dt} = f(t_n, y_n).$$

Thus

$$y_{n+1} = y_n + hf(t_n, y_n). \tag{2.5}$$

This formula is called forward Euler's or explicit Euler's method [29].

The simulated result for the IVP (2.1) and exact solution given by equation (2.2) by using the forward Euler method is shown below.

As we understand about the numerical methods, if the step size h is smaller then the numerical method gives the more accurate result i.e., less error. Here, the forward Euler method is more accurate if the step size h is smaller. Table 2.1 below shows the result of Forward Euler method with exact and approximate solutions for the step size $h = 0.1$. The last column in Table 2.1 contains the data of errors which is the modulus of difference between exact and approximate solutions.

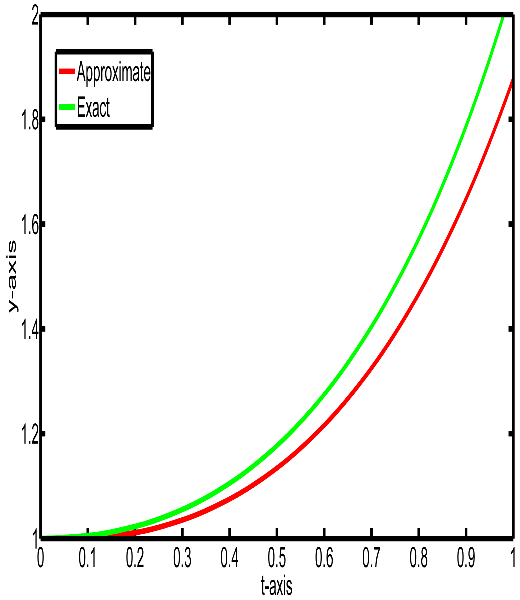

Figure 2.2: Explicit Euler with exact.

t_n	Explicit Euler Method		
	Approximate solution	Exact solution	Error
0.0	0.000000	0.000000	0.000000
0.1	1.000000	1.005347	0.005347
0.2	1.011000	1.022889	0.001189
0.3	1.035220	1.055192	0.019972
0.4	1.075277	1.105319	0.030042
0.5	1.134288	1.176975	0.042687
0.6	1.216002	1.274679	0.058677
0.7	1.324962	1.403988	0.079026
0.8	1.466710	1.571788	0.105078
0.9	1.648046	1.786666	0.138620
1.0	1.877370	2.059407	0.182037

Table 2.1: Explicit Euler with error.

Figure 2.2 shows that the approximate solution curve represented by the red line is approaching the exact solution curve represented by the green line.

2.2 Euler's (Implicit or Backward) Method

The backward Euler method is similar to the forward Euler approach except that it uses the next point t_{n+1} as the point for calculating the derivative [2, 36]. From Fig. 2.1 slope at the point (t_{n+1}, y_{n+1}) is given by

$$\frac{y_{n+1} - y_n}{h} \approx \frac{dy}{dt} = f(t_{n+1}, y_{n+1}),$$

$$\Rightarrow y_{n+1} = y_n + hf(t_{n+1}, y_{n+1}). \tag{2.6}$$

This procedure is then iterated until t_{n+1} converges onto a solution. Which is called implicit (Backward) Euler formula [1].

The simulated result for the IVP (2.1) and exact solution given by equation (2.2) by using the implicit Euler method are shown below.

As we understand about the numerical methods, if the step size h is smaller then the numerical method gives the more accurate result i.e., less error. Here, the implicit Euler method is more accurate if the step size h is smaller. Table 2.2 below shows the result of the implicit Euler method with the exact solution and approximate solutions for the step size $h = 0.1$.

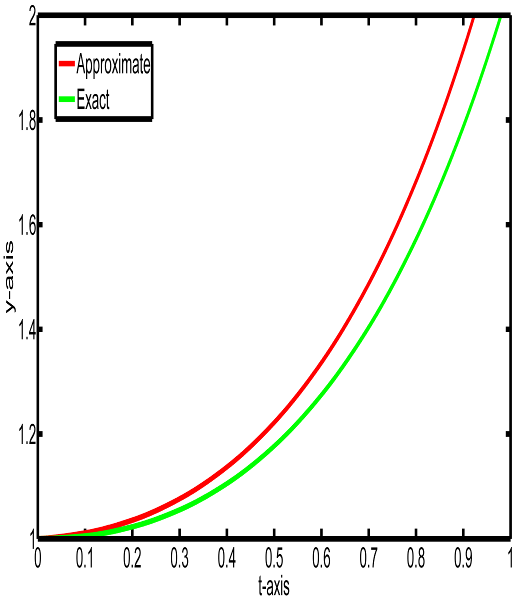

Figure 2.3: Implicit Euler with exact.

t_n	Implicit Euler Method		Error
	Approximate solution	Exact solution	
0.0	0.000000	0.000000	0.000000
0.1	1.011000	1.005347	0.005653
0.2	1.035442	1.022889	0.012553
0.3	1.076247	1.055192	0.021055
0.4	1.136948	1.105319	0.031629
0.5	1.221869	1.176975	0.044894
0.6	1.336347	1.274679	0.061668
0.7	1.487024	1.403988	0.083036
0.8	1.682233	1.571788	0.110445
0.9	1.932506	1.786666	0.145840
1.0	2.251250	2.059407	0.191843

Table 2.2: Implicit Euler with error.

The last column in Table 2.2 contains the data of errors which is the modulus of difference between the exact solution and approximate solutions. Figure 2.3, reveals that the approximate solution curve represented by the red line is approaching the exact solution curve represented by the green line. In this method, the approximate solution curve is converging faster towards the exact solution curve than the explicit Euler method.

2.3 Classical Second-Order Runge-Kutta Method

The Runge-Kutta 2^{nd} order method is a numerical technique used to solve an ordinary differential equation of the form (2.4). Only first-order ordinary differential equations can be solved by using the Runge-Kutta 2^{nd} order method [10, 43]. It is clear from above Fig 2.4 that our error will get minimized if we take an average of the slope at the point (t_n, y_n) represented by the red line and slope at the point (t_{n+1}, y_{n+1}) represented by the yellow line. This concept is used to build the second-order Runge-Kutta method.
Let k_1 be the slope at (t_n, y_n) i.e., $k_1 = f(t_n, y_n)$ and k_2 be the slope at the point (t_{n+1}, y_{n+1}) i.e., $k_2 = f(t_{n+1}, y_{n+1}) = f(t_n + h, y_n + hk_1)$. Thus, the average of k_1 and k_2 is given by

$$k_3 = \frac{k_1 + k_2}{2} = \frac{1}{2}[f(t_n, y_n) + f(t_{n+1}, y_{n+1})],$$

Therefore

$$\Rightarrow y_{n+1} = y_n + hk_3,$$

$$\Rightarrow y_{n+1} = y_n + \frac{h}{2}[f(t_n, y_n) + f(t_{n+1}, y_{n+1})],$$

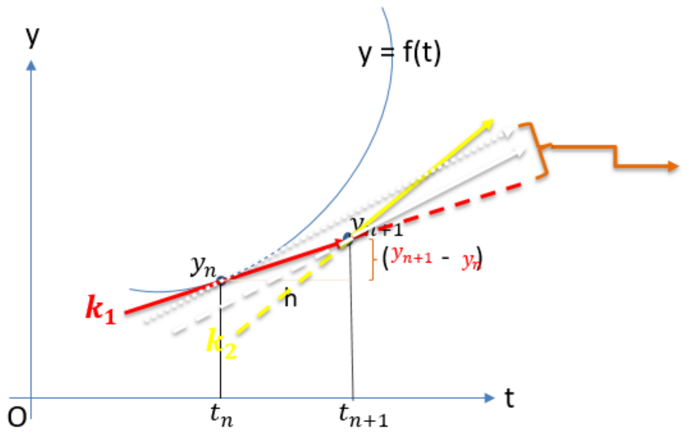

Figure 2.4: Visualization of Heun's method [1].

$$\Rightarrow y_{n+1} = y_n + \frac{h}{2}[f(t_n, y_n) + f(t_n + h, y_n + hk_1)]. \tag{2.7}$$

This is the classical second-order Runge-Kutta method. It is also known as Heun's method or the improved Euler method [2, 43].

The simulated result for the IVP (2.1) and exact solution given by equation (2.2) by using the Classical Second Order Runge-Kutta (Heun's or RK2) method are shown below.

Table 2.3 shows the result of the RK2 method with the exact solution and approximate solutions for the step size $h = 0.1$. The last column in Table 2.3 contains the data of errors which is the modulus of difference between the exact solution and approximate solutions.

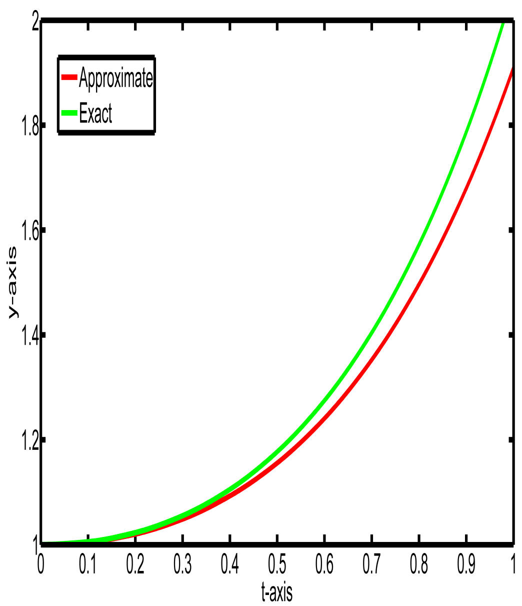

Figure 2.5: Heun's (RK2) with exact.

t_n	RK2 Method		
	Approximate solution	Exact solution	Error
0.0	0.000000	0.000000	0.000000
0.1	1.005500	1.005347	0.000153
0.2	1.023193	1.022889	0.000304
0.3	1.055640	1.055192	0.000448
0.4	1.105901	1.105319	0.000258
0.5	1.177672	1.176975	0.000697
0.6	1.275461	1.274679	0.000782
0.7	1.404804	1.403988	0.000816
0.8	1.572558	1.571788	0.000770
0.9	1.787266	1.786666	0.000600
1.0	2.059649	2.059407	0.000242

Table 2.3: RK2 with error.

Figure 2.5 observe that the approximate solution curve represented by the red line is approaching the exact solution curve represented by the green line. In this method, the approximate solution curve is converging faster towards the exact solution curve than the Explicit and Implicit Euler method.

2.4 Third Order Runge-Kutta (RK3) Method

The third-order Runge-Kutta method (RK3) is broadly used for solving initial value problems (IVP) for ordinary differential equations (ODE). The general formula for the Runge-Kutta third-order (RK3) method is shown below [22, 42]:

$$y_{n+1} = y_n + h\left[\frac{k_1}{6} + \frac{2k_2}{3} + \frac{k_3}{6}\right], \tag{2.8}$$

where

$$k_1 = f(t_n, y_n), \quad k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right), \quad k_3 = f(t_n + h, y_n - hk_1 + 2hk_2).$$

The simulated result for the IVP (2.1) and exact solution given by equation (2.2) using the third-order Runge-Kutta (RK3) method are shown below.

Table 2.4 shows the result of the RK3 method with the exact solution and approximate solution for the step size $h = 0.1$. The last column in Table 2.4 contains the data of errors which is the modulus of difference between exact and approximate solutions. Figure 2.6 observe that the approximate solution curve represented by the red line is approaching the exact solution curve represented by the green line.

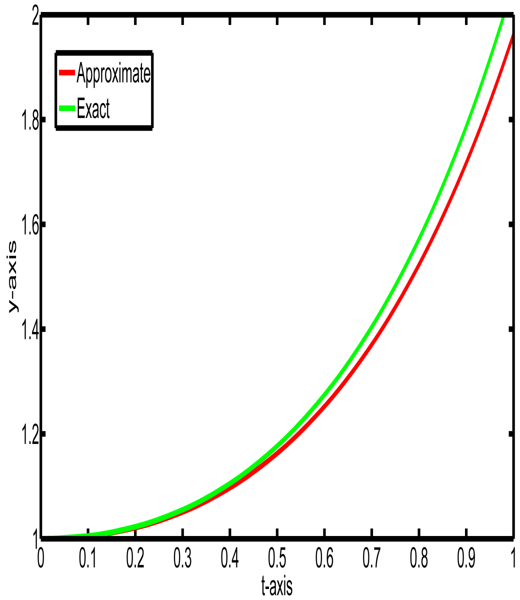

Figure 2.6: RK3 method with exact.

t_n	RK3 Method		Error
	Approximate solution	Exact solution	
0.0	0.000000	0.000000	0.000000
0.1	1.005351	1.005347	0.000004
0.2	1.022899	1.022889	0.000000
0.3	1.055208	1.055192	0.000016
0.4	1.105342	1.105319	0.000023
0.5	1.177006	1.176975	0.000031
0.6	1.274719	1.274679	0.000040
0.7	1.404039	1.403988	0.000051
0.8	1.571848	1.571788	0.000060
0.9	1.786736	1.786666	0.000070
1.0	2.059486	2.059407	0.000079

Table 2.4: RK3 with error.

In this method, the approximate solution curve is converging faster towards the exact solution curve than the explicit Euler method, implicit Euler method, and RK2 method. The error is also less than that of other above-mentioned methods.

2.5 Fourth Order Runge-Kutta (RK4) Method

This method was devised by two German mathematicians, Runge about 1894 A.D., and extended by Kutta a few years later. The Runge-Kutta method is most familiar because it is pretty accurate, steady, and simple to program [2]. This method is notable by their order in the logic that they concur with Taylor's series solution up to terms of h^r where r is the order of the method. The fourth-order Runge-Kutta method (RK4) is broadly used for solving initial value problems (IVP) for ordinary differential equation (ODE) [29]. The general formula for the fourth-order Runge-Kutta method is shown below [14, 22].

$$y_{n+1} = y_n + h\left[\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6}\right], \tag{2.9}$$

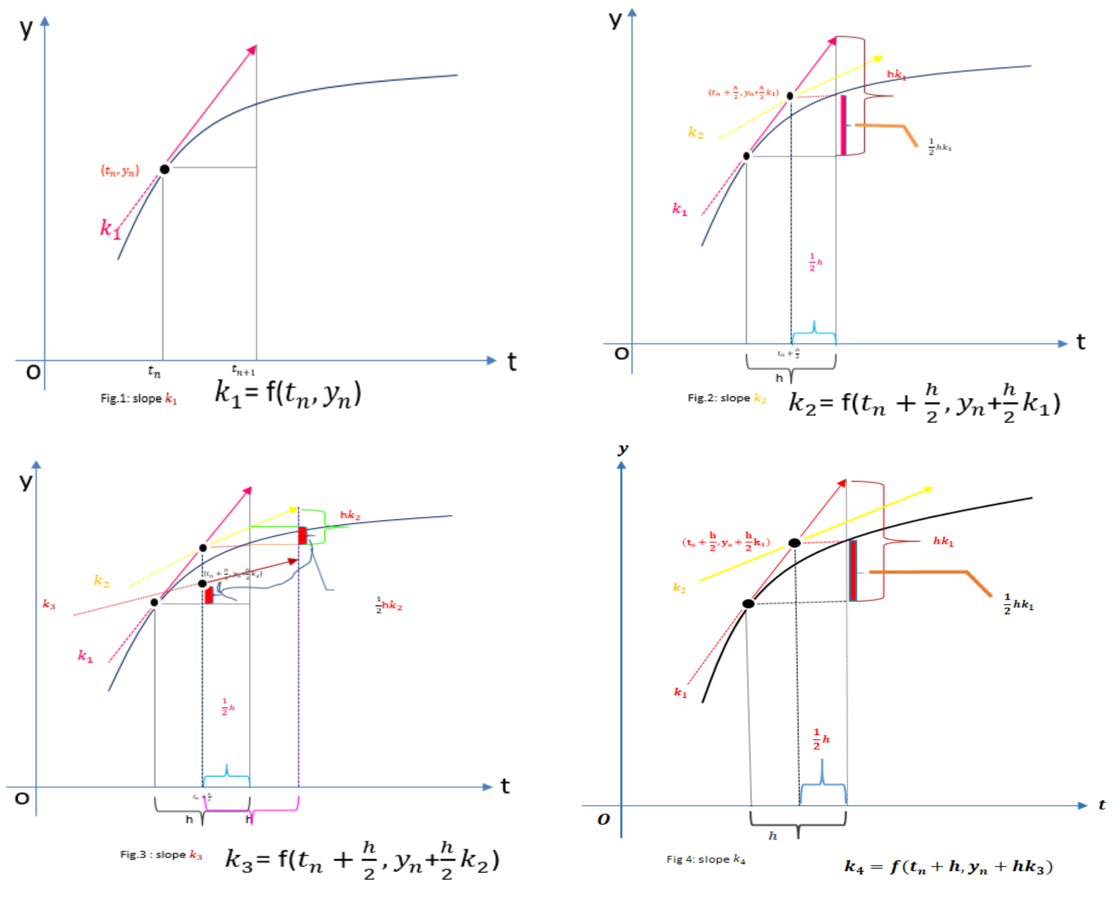

Figure 2.7: Visualization of RK4 method [2, 14].

with

$$k_1 = f(t_n, y_n), k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right), k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right), k_4 = f(t_n+h, y_n+hk_3).$$

The simulated result for the IVP (2.1) and exact solution given by equation (2.2) by using the fourth-order Runge-Kutta (RK4) method are shown below.

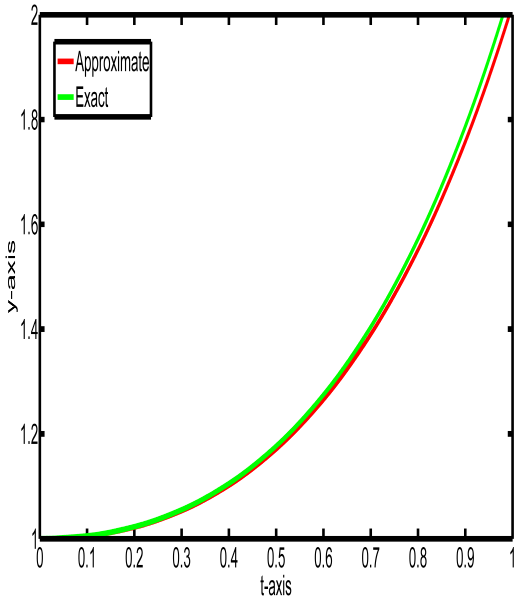

Figure 2.8: RK4 method with exact.

t_n	RK4 Method		Error
	Approximate solution	Exact solution	
0.0	0.000000	0.000000	0.000000
0.1	1.005346	1.005347	0.000001
0.2	1.022889	1.022889	0.000000
0.3	1.055192	1.055192	0.000000
0.4	1.105319	1.105319	0.000000
0.5	1.176975	1.176975	0.000000
0.6	1.274679	1.274679	0.000000
0.7	1.403988	1.403988	0.000000
0.8	1.571787	1.571788	0.000001
0.9	1.786665	1.786666	0.000001
1.0	2.059407	2.059407	0.000000

Table 2.5: RK4 with error.

Table 2.5 shows the result of the RK4 method with the exact solution and approximate solutions for the step size $h = 0.1$. The last column in Table 2.5 contains the data of errors which is the modulus of difference between the exact solution and approximate solutions. Figure 2.8 observe that the approximate solution curve represented by the red line is approaching the exact solution curve represented by the green line. In this method, the approximate solution curve is converging faster towards the exact solution curve than the explicit Euler method, implicit Euler method, RK2 method, and RK3 method. The error is also less than that of other above-mentioned methods.

2.6 Butchers Fifth Order Runge-Kutta (BRK5) Method

This method is distinguished by their order in the sense that agrees with Taylor's series solution up to terms of h^r where r is the order of the method [19]. In this method, consider the IVP (2.4) [14, 21], the general formula for BRK5 method is:

$$y_{n+1} = y_n + \frac{h}{90}\left(7k_1 + 32k_2 + 12k_4 + 32k_5 + 7k_6\right), \tag{2.10}$$

where

$$t_{n+1} = t_n + h, \quad k_1 = f(t_n, y_n), \quad k_2 = f\left(t_n + \frac{h}{4}, y_n + \frac{h}{4}k_1\right),$$

$$k_3 = f\left(t_n + \frac{h}{4}, y_n + \frac{1}{8}hk_1 + \frac{1}{8}hk_2\right), \quad k_4 = f\left(t_n + \frac{h}{2}, y_n - \frac{1}{2}hk_2 + hk_3\right),$$

$$k_5 = f\left(t_n + \frac{3h}{4}, y_n + \frac{3}{16}hk_1 + \frac{9}{16}hk_4\right),$$

$$k_6 = \left(t_n + h, y_n - \frac{3}{7}hk_1 + \frac{2}{7}hk_2 + \frac{12}{7}hk_3 - \frac{12}{7}hk_4 + \frac{8}{7}hk_5\right).$$

The simulated result for the IVP (2.1) and exact solution given by equation (2.2) using the Butcher fifth-order Runge-Kutta (BRK5) method are shown below.

Table 2.6 shows the result of the BRK5 method with the exact solution and approximate solutions for the step size $h = 0.1$. The last column in Table 2.6 contains the data of errors which is the modulus of difference between the exact solution and approximate solutions. Figure 2.9 perceive that the approximate solution curve represented by the red line is approaching the exact solution curve represented by the green line. In this method, the approximate solution curve is converging faster towards the exact solution curve than the explicit Euler method, implicit Euler method, RK2 method, RK3 method, and RK4 method. The error is also less than that of other above-mentioned methods.

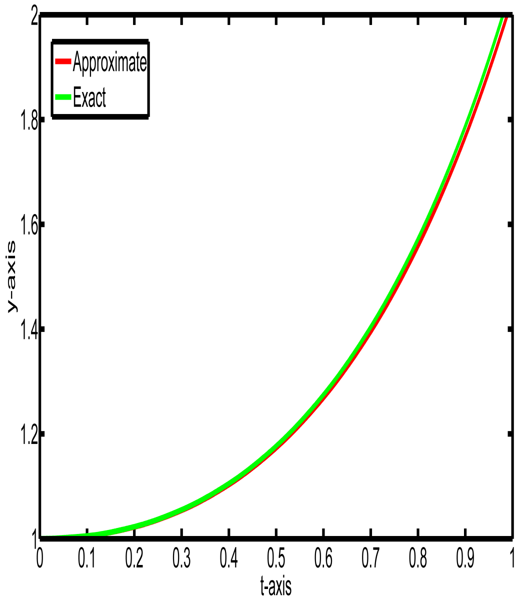

Figure 2.9: BRK5 method with exact.

t_n	BRK5 Method		Error
	Approximate solution	Exact solution	
0.0	0.000000	0.000000	0.000000
0.1	1.005347	1.005347	0.000000
0.2	1.022889	1.022889	0.000000
0.3	1.055192	1.055192	0.000000
0.4	1.105319	1.105319	0.000000
0.5	1.176975	1.176975	0.000000
0.6	1.274679	1.274679	0.000000
0.7	1.403988	1.403988	0.000000
0.8	1.571788	1.571788	0.000000
0.9	1.786666	1.786666	0.000000
1.0	2.059407	2.059407	0.000000

Table 2.6: BRK5 with error.

2.7 Comparison of numerical methods

From the above results, we see that the approximate result converges to the exact solution and if the step size is decreased then the error also decreased. This shows that the small step size gives an improved estimation. The Butcher fifth-order Runge-Kutta method requires five evaluations per step, so it should give more accurate results in comparison with others methods. Other methods were found to be less accurate due to the inaccurate numerical results. From the above study the Butcher fifth-order Runge-Kutta method was found to be more accurate and also the approximate solution converged faster to the exact solution when compared to the other methods. It may be concluded that the Butcher fifth-order Runge-Kutta method is powerful and more efficient in finding numerical solutions of initial value problems (IVP).

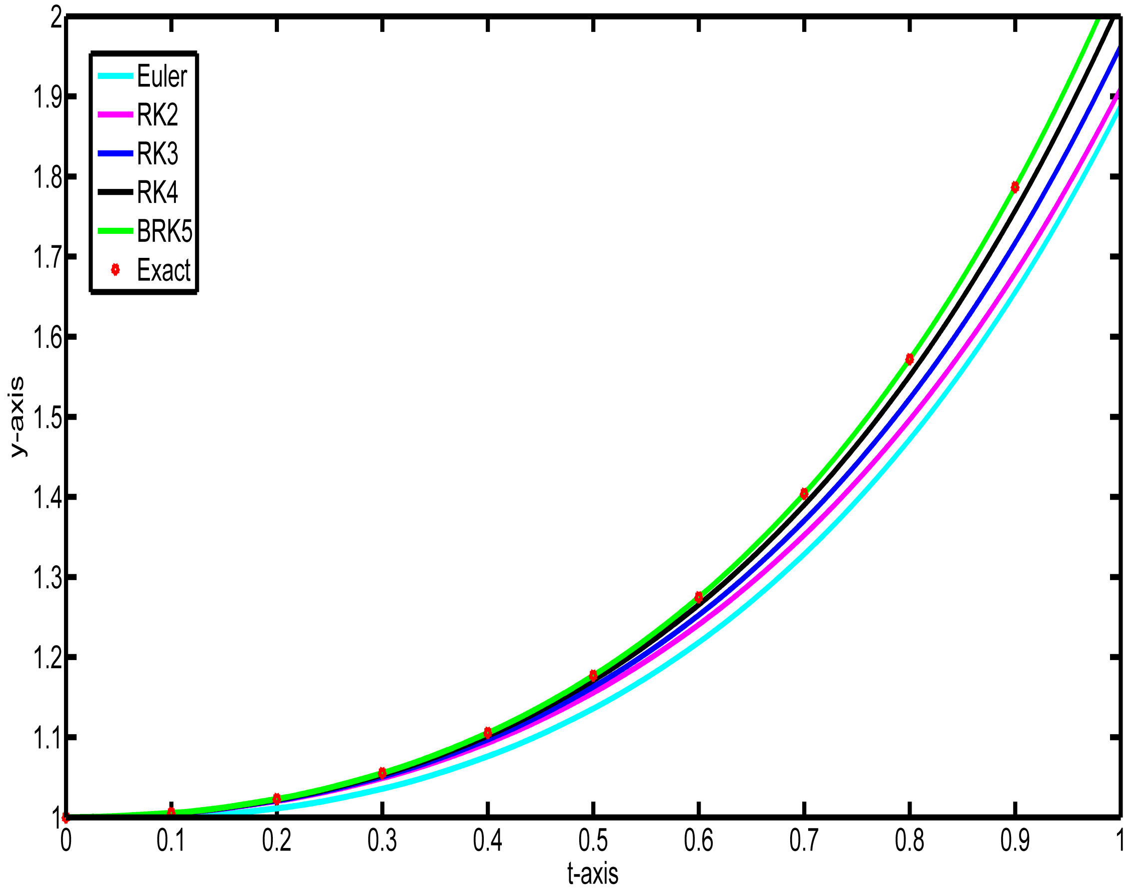

Figure 2.10: Comparing Euler, RK2, RK3, RK4 and BRK5 method.

Chapter 3

Transient analysis of series RLC Circuits

3.1 Series RLC Circuit

The electrical elements resistor (R), inductor (L), and capacitor (C) are connected in series with the DC source (V_s). The second-order differential equation of the series RLC circuit with constant coefficients is given by equation (1.2) [40]:

$$L\frac{d^2v}{dt^2} + R\frac{dv}{dt} + \frac{1}{C}v = \frac{V_s}{C}; \;\; v(0) = 6V, \;\; i(0) = 6A. \tag{3.1}$$

3.2 Numerical Formulation of Series RLC Circuit

Let

$$v = x, \qquad \text{and} \qquad \frac{dv}{dt} = \frac{y}{c} \tag{3.2}$$

then equation (3.1) becomes

$$\frac{dy}{dt} = \frac{V_s - x - R \times y}{L}. \tag{3.3}$$

Hence equation (3.2) and (3.3) form a system of first-order differential equation [19].

Let

$$f = \frac{dv}{dt} = \frac{y}{c}, \qquad \text{and} \qquad g = \frac{dy}{dt} = \frac{V_S - x - R \times y}{L}.$$

3.2.1 Formulating Eulers Method for series RLC Circuit

The system (3.2) and (3.3) can be discretized applying Euler's method as:

$$x_{i+1} = x_i + h\, f(t_i, x_i, y_i),$$

$$y_{i+1} = y_i + h\, g(t_i, x_i, y_i),$$

where h is the time step size.

3.2.2 Formulating RK3 Method for series RLC Circuit

The system (3.2) and (3.3) can be discretized applying RK3 method as:

$$x_{i+1} = x_i + (f_1 + 4f_2 + f_3)/6,$$

$$y_{i+1} = y_i + (g_1 + 4g_2 + g_3)/6,$$

$$\text{where} \quad f_1 = hf(t_i, x_i, y_i), \quad g_1 = hg(t_i, x_i, y_i),$$

$$f_2 = h\,f\left(\left(t_i + \frac{h}{2}\right), \left(x_i + \frac{f_1}{2}\right), \left(y_i + \frac{g_1}{2}\right)\right),\ g_2 = hg\left(\left(t_i + \frac{h}{2}\right), \left(x_i + \frac{f_1}{2}\right), \left(y_i + \frac{g_1}{2}\right)\right),$$

$$f_3 = hf((t_i{+}h), (x_i{-}f_1{+}2f_2), (y_i{-}g_1{+}2g_2)), \quad g_3 = hg((t_i{+}h), (x_i{-}f_1{+}2f_2), (y_i{-}g_1{+}2g_2)).$$

3.2.3 Formulating BRK5 Method for series RLC Circuit

The system (3.2) and (3.3) can be discretized applying BRK5 method as:

$$x_{i+1} = x_i + (7f_1 + 32f_3 + 12f_4 + 32f_5 + 7f_6)/90,$$

$$y_{i+1} = y_i + 7g_1 + 32g_3 + 12g_4 + 32g_5 + 7g_6)/90,$$

$$\text{where} \quad f_1 = hf(t_i, x_i, y_i), \quad g_1 = hg(t_i, x_i, y_i),$$

$$f_2 = hf\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1}{4}\right), \left(y_i + \frac{g_1}{4}\right)\right), g_2 = hg\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1}{4}\right), \left(y_i + \frac{g_1}{4}\right)\right),$$

$$f_3 = hf\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1 + f_2}{8}\right), \left(y_i + \frac{g_1 + g_2}{8}\right)\right),$$

$$g_3 = hg\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1 + f_2}{8}\right), \left(y_i + \frac{g_1 + g_2}{8}\right)\right),$$

$$f_4 = hf\left(\left(t_i + \frac{h}{2}\right), \left(x_i - \frac{f_2}{2} + f_3\right), \left(y_i - \frac{g_2}{2} + g_3\right)\right),$$

$$g_4 = hg\left(\left(t_i + \frac{h}{2}\right), \left(x_i - \frac{f_2}{2} + f_3\right), \left(y_i - \frac{g_2}{2} + g_3\right)\right),$$

$$f_5 = hf\left(\left(t_i + \frac{3h}{4}\right), \left(x_i + \frac{3f_1}{16} + \frac{9f_4}{16}\right), \left(y_i + \frac{3g_1}{16} + \frac{9g_4}{16}\right)\right),$$

$$g_5 = hg\left(\left(t_i + \frac{3h}{4}\right), \left(x_i + \frac{3f_1}{16} + \frac{9f_4}{16}\right), \left(y_i + \frac{3g_1}{16} + \frac{9g_4}{16}\right)\right),$$

$$f_6{=}hf\left((t_i{+}h), \left(x_i{-}\frac{3f_1{-}2f_2{-}12f_3{+}12f_4{-}8f_5}{7}\right), \left(y_i{-}\frac{3g_1{-}2g_2{-}12g_3{+}12g_4{-}8g_5}{7}\right)\right),$$

$$g_6{=}hg\left((t_i{+}h), \left(x_i{-}\frac{3f_1{-}2f_2{-}12f_3{+}12f_4{-}8f_5}{7}\right), \left(y_i{-}\frac{3g_1{-}2g_2{-}12g_3{+}12g_4{-}8g_5}{7}\right)\right).$$

3.3 Simulation Results and Discussion on Series RLC Circuit

In this experiment, the simulation time for all the conditions are from *0 s* to *20 s* with the step size of *h=0.1 s*. Moreover, we consider source voltage $(V_S) = 6V$, resistance $(R) = 1\Omega$, inductance $(L) = 1H$ and capacitance $(C) = 0.25F$ for the simulation result of series RLC circuit [19]. Figure 3.1 describes simulation results of numerical solution of series RLC circuit and exact solution under the above condition. In order to compare the accuracy of the numerical methods, the computed points of the three numerical methods are taken at a specific time. The computed point of each method is compared with the analytic solution presented in Table 3.1. Figure 3.1 shows that the approximate solution curve obtained from the Butcher fifth-order Runge-Kutta (BRK5) method is converging faster towards the analytical solution in comparison with the other two methods. This shows that there is less error between BRK5 and analytical solution in comparison with the other two methods. Table 3.1 contains details of simulated results obtained for iterative methods and exact solutions with errors in different time slices. Hence, it can be concluded that the BRK5 method is a more efficient method to approximate the solution of a series RLC circuit.

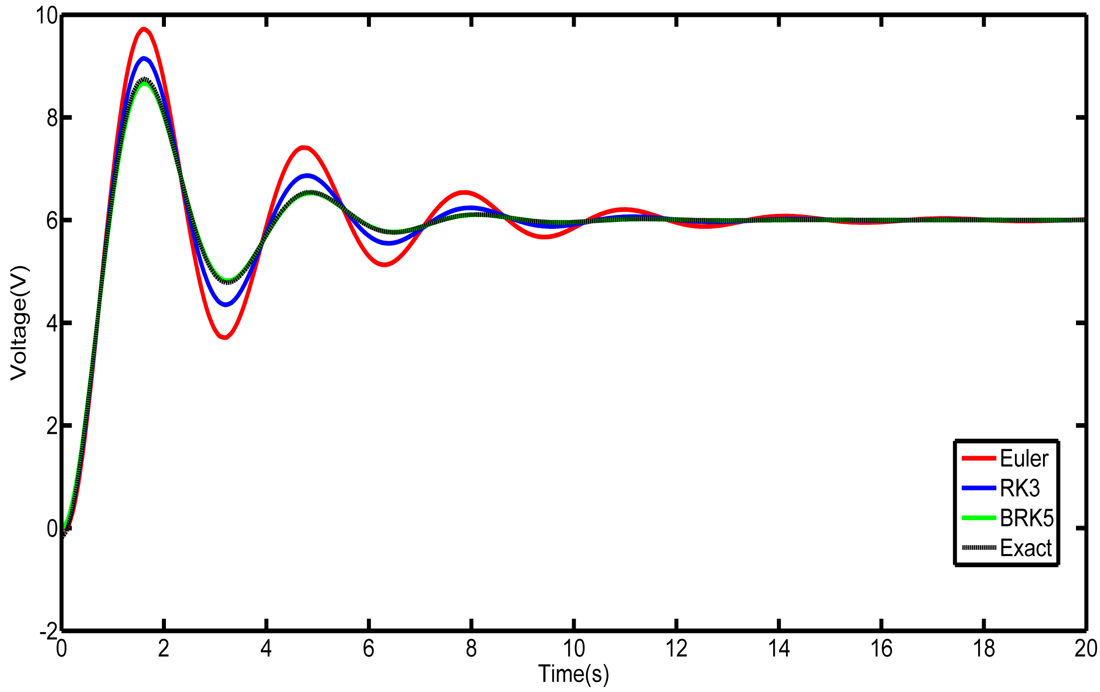

Figure 3.1: Comparison of Euler, RK3, BRK5 methods with analytical solution.

Table 3.1: Simulated results obtained for iterative methods and exact solution with error.

Time(t)	Euler		$RK3$		$BRK5$		Exact
(seconds)	Voltage(V)	$E_R(10^{-3})$	Voltage(V)	$E_R(10^{-3})$	Voltage(V)	$E_R(10^{-3})$	Voltage(V)
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.000000	4.378000	0.116000	1.622000	0.115714	1.336000	0.114378
0.2	0.240000	3.654000	0.444083	0.429000	0.443675	3.000000	0.443654
0.3	0.696000	6.189000	0.951609	0.580000	0.951190	9.990000	0.952189
0.4	1.336800	5.764000	1.602239	0.325000	1.601968	5.960000	1.612564
0.5	2.125680	1.791000	2.357671	8.800000	2.357671	8.800000	2.366471
0.6	3.022200	7.630000	3.179275	0.555000	3.179643	1.870000	3.189830
0.7	3.984041	1.851000	4.029593	4.701000	4.030393	5.501000	4.004892
0.8	4.968810	4.868000	4.873643	0.299000	4.874908	9.034000	4.883942
0.9	5.935740	3.419000	5.680016	2.305000	5.681740	5.810000	5.692321
1.0	6.847225	2.573000	6.421720	2.932000	6.423867	7.850000	6.424652

In the following section, three damping cases (i.e., underdamped, critically damped, and overdamped) are considered to solve ODE by applying Butcher fifth-order Runge-Kutta (BRK5) method. In Fig. 3.2, we simulate the results under-discussed conditions with different time slices.

3.4 Characteristics of Damping Factor by using BRK5 method

The three different damping conditions of the RLC circuit are discussed below:

3.4.1 Condition 1: Underdamped ($\zeta < 1$)

In this condition, the resistance, R must be less than 4Ω in order to achieve an underdamped response. In order to obtain the best numerical method for transient analysis of RLC circuit, three numerical methods are compared with the analytical solution. The three numerical methods mentioned are the Euler method, RK3 method, and BRK5 method. The experiment is conducted by taking 1Ω as the value of the resistor, R for the circuit [19, 31, 44].

3.4.2 Condition 2: Critically damped ($\zeta = 1$)

In critically damped response, the resistance, R must be equal to 4Ω in order to achieve the critically damped condition.

3.4.3 Condition 3: Overdamped ($\zeta > 1$)

In this condition, the resistance, R must be more than 4Ω in order to achieve an overdamped response. The experiment is conducted by taking 6Ω as the value of the resistor, R for the circuit. Table 3.2 contains the data for the conditions of different damping factors. By the variation of the resistance (R), the three different damping conditions can be obtained.

Table 3.2: Data of electrical elements of series RLC circuit for three conditions [31].

Element	Value		
	Condition 1	Condition 2	Condition 3
Resistor	Underdamped	Critically Damped	Overdamped
DC (Vs)	6V	6V	6V
Resistor R	$< 4\Omega$	4Ω	$> 4\Omega$
Inductor L	$1H$	$1H$	$1H$
Capacitor C	$0.25F$	$0.25F$	$0.25F$

Figure 3.2 shows that the underdamped responses are a decaying oscillation decays at a rate determined by the attenuation (α) by using the BRK5 method. The exponential decay describes the envelope of the oscillation. Here the oscillation is sinusoidal with exponentially decaying amplitude.

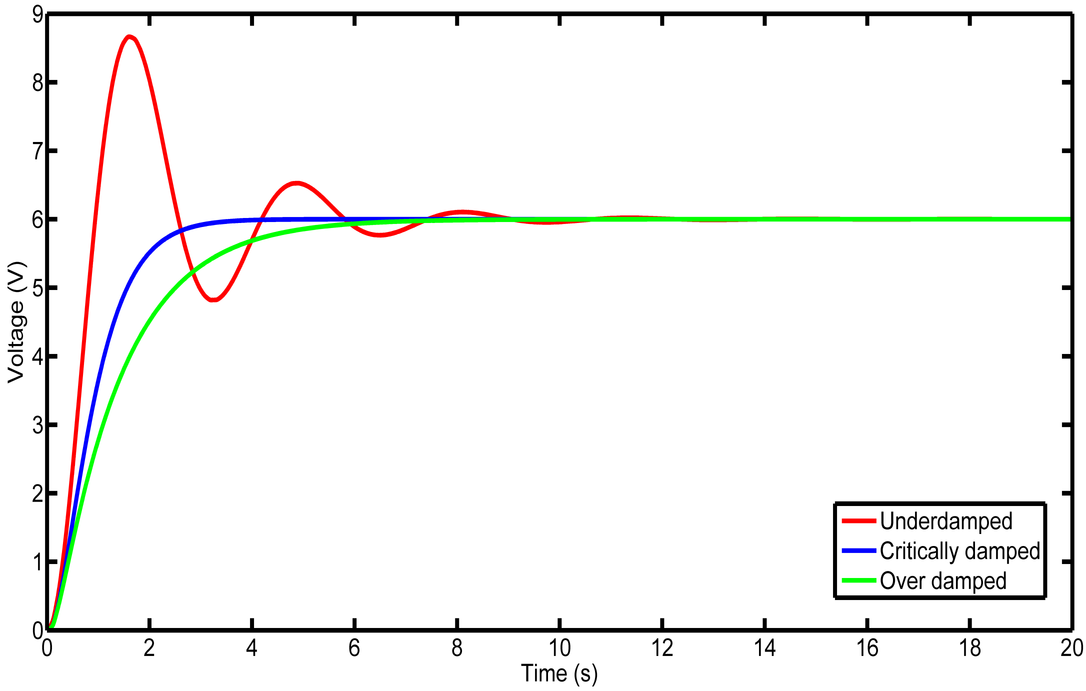

Figure 3.2: Comparison of three different damping conditions by using BRK5 method.

The critically damped response represents the circuit response that decays in the fastest possible time without going into oscillation. This consideration is important in control systems where it is required to reach the desired state as quickly as possible without overshooting. The over-damped response is a decay of the transient current without oscillation, which reaches the stable state slower than the critically damped case. The quantitative description of the numerical solution under three damping conditions by using the BRK5 method is detailed in Table 3.3.

Table 3.3: Values of three damping conditions of BRK5 method.

Time (s)	Value of BRK5 Method		
	Underdamped	Critically Damped	Overdamped
0.0	0.000000	0.000000	0.000000
0.1	0.115714	0.105139	0.098897
0.2	0.443657	0.369312	0.330084
0.3	0.951190	0.731409	0.626944
0.4	1.601968	1.147248	0.950945
0.5	2.357643	1.585448	1.280130
0.6	3.179643	2.024237	1.602290
0.7	4.030393	2.449004	1.910937
0.8	4.874908	2.850415	2.202915
0.9	5.681740	3.222979	2.476992
1.0	6.423867	3.563965	2.733027

3.5 Discussion

The transient analysis of electrical circuits is analyzed using the analytical method, explicit Euler method, RK3 method, and BRK5 method. With the usage of computational software, the process of obtaining results of the transient analysis is done systematically and conveniently. It has been observed that the BRK5 method is very efficient in solving second-order differential equations. Thus, it can be concludedthat by carrying out the transient analysis of a system, the response of the system can be found by changing the conditions from one steady-state value to another. This observation clearly shows that the under-damped decay is oscillatory and exponential. However, the other two are non-oscillatory exponential decay. The decay in the critically damped case is observed faster than in the overdampedcase. This response helps in designing a system, which meets our requirements. we can further optimize the time domain parameters of the system.

Chapter 4

Transient analysis of parallel RLC Circuits

4.1 Parallel RLC Circuit

The electrical elements resistor (R), inductor (L) and capacitor (C) are connected in parallel with the DC source (Vs). The second-order differential equation of the parallel RLC circuit with constant coefficients is given by equation (1.6) [40].

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0; \;\; v(0) = 6, \;\; i(0) = 0, \;\; \frac{dv}{dt}(0) = -12. \tag{4.1}$$

4.2 Numerical Formulation of Parallel RLC Circuit

$$\text{Let} \quad v = x, \quad \text{and} \quad \frac{dv}{dt} = \frac{y}{C}, \tag{4.2}$$

Including damped forced term in equation (4.1) then it turns into

$$\frac{dy}{dt} = \frac{V_s}{LC} - \frac{y}{RC} - \frac{x}{L}. \tag{4.3}$$

Hence equation (4.2) and (4.3) form a system of first order differential equation [19].

$$\text{Let} \quad f = \frac{dv}{dt} = \frac{y}{C}, \quad \text{and} \quad g = \frac{dy}{dt} = \frac{V_S}{LC} - \frac{y}{RC} - \frac{x}{L}.$$

4.2.1 Formulating Euler's Method for Parallel RLC Circuit

The discretization of the system (4.2) and (4.3) applying Euler's method [19] as:

$$x_{i+1} = x_i + h\,f(t_i, x_i, y_i),$$

$$y_{i+1} = y_i + h\, g(t_i, x_i, y_i),$$

where h is the time step size.

4.2.2 Formulating RK3 Method for Parallel RLC Circuit

The discretization of the system (4.2) and (4.3) applying RK3 method [19] as:

$$x_{i+1} = x_i + (f_1 + 4f_2 + f_3)/6,$$

$$y_{i+1} = y_i + (g_1 + 4g_2 + g_3)/6,$$

$$\text{where} \quad f_1 = hf(t_i, x_i, y_i), \quad g_1 = hg(t_i, x_i, y_i),$$

$$f_2 = h\, f\left(\left(t_i + \frac{h}{2}\right), \left(x_i + \frac{f_1}{2}\right), \left(y_i + \frac{g_1}{2}\right)\right), \;\; g_2 = hg\left(\left(t_i + \frac{h}{2}\right), \left(x_i + \frac{f_1}{2}\right), \left(y_i + \frac{g_1}{2}\right)\right),$$

$$f_3 = hf((t_i{+}h), (x_i{-}f_1{+}2f_2), (y_i{-}g_1{+}2g_2)), \quad g_3 = hg((t_i{+}h), (x_i{-}f_1{+}2f_2), (y_i{-}g_1{+}2g_2)).$$

4.2.3 Formulating BRK5 Method for Parallel RLC Circuit

The discretization of the system (4.2) and (4.3) applying BRK5 method [19] as:

$$x_{i+1} = x_i + (7f_1 + 32f_3 + 12f_4 + 32f_5 + 7f_6)/90,$$

$$y_{i+1} = y_i + 7g_1 + 32g_3 + 12g_4 + 32g_5 + 7g_6)/90,$$

$$\text{where} \quad f_1 = hf(t_i, x_i, y_i), g_1 = hg(t_i, x_i, y_i),$$

$$f_2 = hf\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1}{4}\right), \left(y_i + \frac{g_1}{4}\right)\right), g_2 = hg\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1}{4}\right), \left(y_i + \frac{g_1}{4}\right)\right),$$

$$f_3 = hf\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1 + f_2}{8}\right), \left(y_i + \frac{g_1 + g_2}{8}\right)\right),$$

$$g_3 = hg\left(\left(t_i + \frac{h}{4}\right), \left(x_i + \frac{f_1 + f_2}{8}\right), \left(y_i + \frac{g_1 + g_2}{8}\right)\right),$$

$$f_4 = hf\left(\left(t_i + \frac{h}{2}\right), \left(x_i - \frac{f_2}{2} + f_3\right), \left(y_i - \frac{g_2}{2} + g_3\right)\right),$$

$$g_4 = hg\left(\left(t_i + \frac{h}{2}\right), \left(x_i - \frac{f_2}{2} + f_3\right), \left(y_i - \frac{g_2}{2} + g_3\right)\right),$$

$$f_5 = hf\left(\left(t_i + \frac{3h}{4}\right), \left(x_i + \frac{3f_1}{16} + \frac{9f_4}{16}\right), \left(y_i + \frac{3g_1}{16} + \frac{9g_4}{16}\right)\right),$$

$$g_5 = hg\left(\left(t_i + \frac{3h}{4}\right), \left(x_i + \frac{3f_1}{16} + \frac{9f_4}{16}\right), \left(y_i + \frac{3g_1}{16} + \frac{9g_4}{16}\right)\right),$$

$$f_6{=}hf\left((t_i{+}h), \left(x_i{-}\frac{3f_1{-}2f_2{-}12f_3{+}12f_4{-}8f_5}{7}\right), \left(y_i{-}\frac{3g_1{-}2g_2{-}12g_3{+}12g_4{-}8g_5}{7}\right)\right),$$

$$g_6{=}hg\left((t_i{+}h), \left(x_i{-}\frac{3f_1{-}2f_2{-}12f_3{+}12f_4{-}8f_5}{7}\right), \left(y_i{-}\frac{3g_1{-}2g_2{-}12g_3{+}12g_4{-}8g_5}{7}\right)\right).$$

4.3 Simulation Results and Discussion on Parallel RLC Circuit

In the following sub-section, we compare the numerical solutions of parallel RLC circuits by the above discussed numerical methods with the exact solutions and analyze the different characteristics of damping factors by using an iterative method. Here, Table 4.1 contains the data that is used in the simulation of three iterative methods and three damping conditions for the solution of a parallel RLC circuit. Figure 4.1 describes simulation results of numerical solution of parallel RLC circuit and exact solution in overdamped, critically damped, and underdamped conditions from the top, middle, and bottom respectively. In order to compare the accuracy of the numerical methods, the computed points of the three numerical methods are taken at a specific time.

Table 4.1: Data of electrical elements of parallel RLC circuit for three conditions [31].

Element	Value		
	Condition 1	Condition 2	Condition 3
Resistor	Underdamped	Critically Damped	Overdamped
DC (Vs)	6V	6V	6V
Resistor R	$< 4\Omega$	4Ω	$> 4\Omega$
Inductor L	$1H$	$1H$	$1H$
Capacitor C	$0.25F$	$0.25F$	$1H$

The computed point of each method is compared with the analytic solution. We clearly observed in Fig. 4.1 that numerical solutions are more converging with the analytical solution in overdamped conditions and less converging with the analytical solution in underdamped conditions. Therefore, it can be concluded that numerical methods are more suitable in the overdamped condition. Figure 4.1 observe that the approximate solution curve obtained from the Butcher fifth-order Runge-Kutta (BRK5) method is converging faster towards the analytical solution in comparison with the other two methods. This shows that there is less error between BRK5 and analytical solution in comparison with the other two methods. Table 4.2 contains details of simulated results obtained for iterative methods and exact solutions with errors in different time slices. Hence, we concluded that the BRK5 method is a more efficient method to approximate the solution of parallel RLC circuits. So, in the following section, we consider three damping cases (i.e., underdamped, critically damped, and overdamped) for Butcher fifth-order Runge-Kutta (BRK5) method.

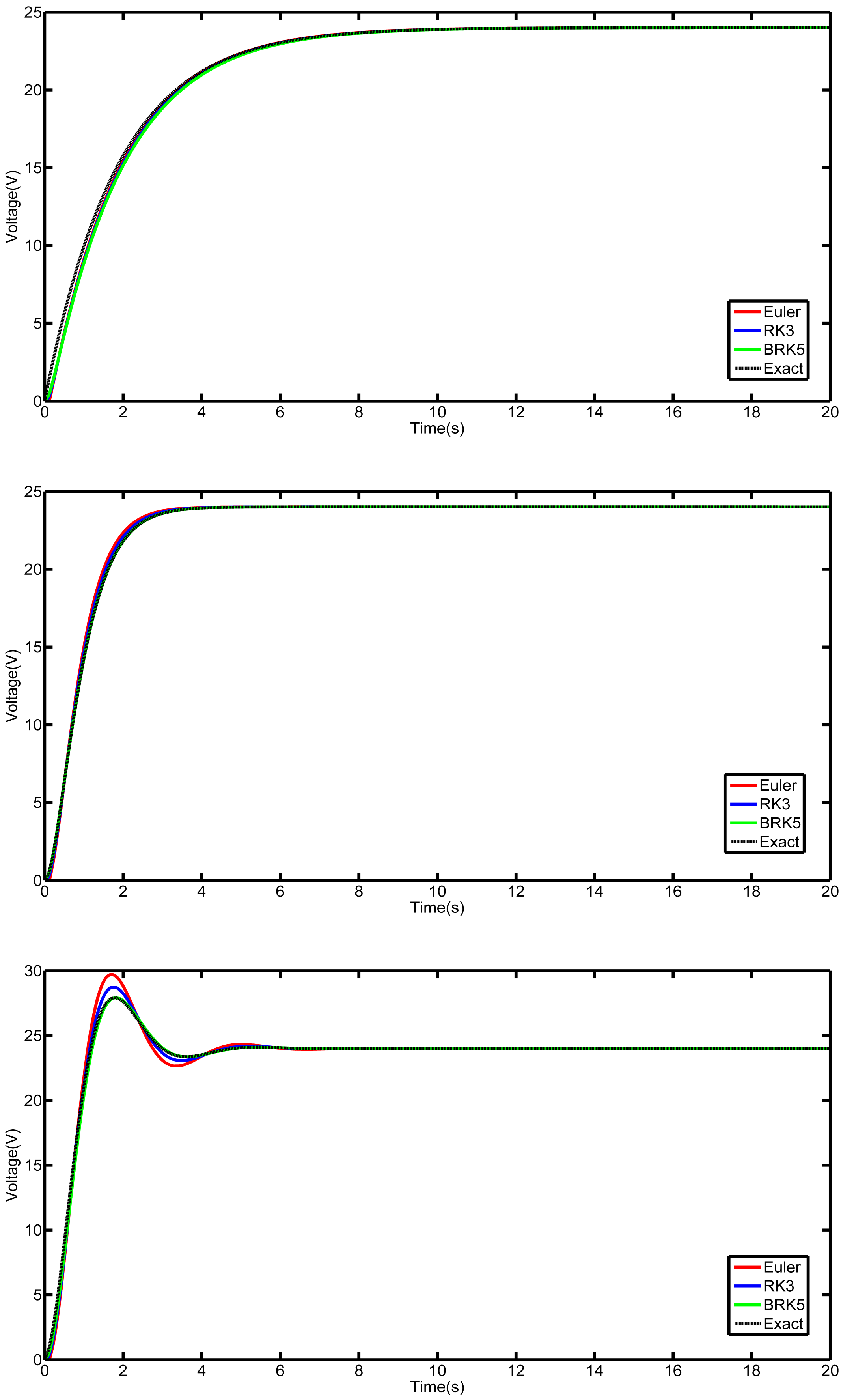

Figure 4.1: Comparison of iterative methods with the analytical solution of overdamped (top), critically damped (middle) and underdamped (bottom).

Table 4.2: Simulated results obtained for iterative methods and exact solution with error.

Time(t) (seconds)	Euler		$RK3$		$BRK5$		Exact
	Voltage(V)	$E_R(10^{-3})$	Voltage(V)	$E_R(10^{-3})$	Voltage(V)	$E_R(10^{-3})$	Voltage(V)
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.1	0.000000	7.550000	0.232000	5.550000	0.448062	9.488000	0.737550
0.2	0.960000	2.195200	1.325157	3.795000	1.665911	6.041000	2.181952
0.3	2.688000	6.488600	3.098752	5.413400	3.470005	2.881000	4.152886
0.4	4.992000	9.169100	5.360915	2.277600	5.688888	4.803000	6.483691
0.5	7.687680	3.723300	7.944409	8.050400	8.167195	7.718000	9.024913
0.6	10.604544	4.204500	10.695937	0.652000	10.768352	8.237000	11.646589
0.7	13.590528	8.875000	13.482075	7.328000	13.376102	3.301000	14.239404
0.8	16.515133	9.770000	16.190937	3.966000	15.895022	9.881000	16.714903
0.9	19.271197	6.292000	18.732741	2.164000	18.250199	4.706000	19.004905
1.0	21.775442	5.153000	21.039441	0.848000	20.386214	4.075000	21.060289

Figure 4.2 observe that the underdamped responses are decaying oscillations that decays at a rate determined by the attenuation (α) by using the BRK5 method. The exponential decay describes the envelope of the oscillation. Here the oscillation is sinusoidal with exponentially decaying amplitude.

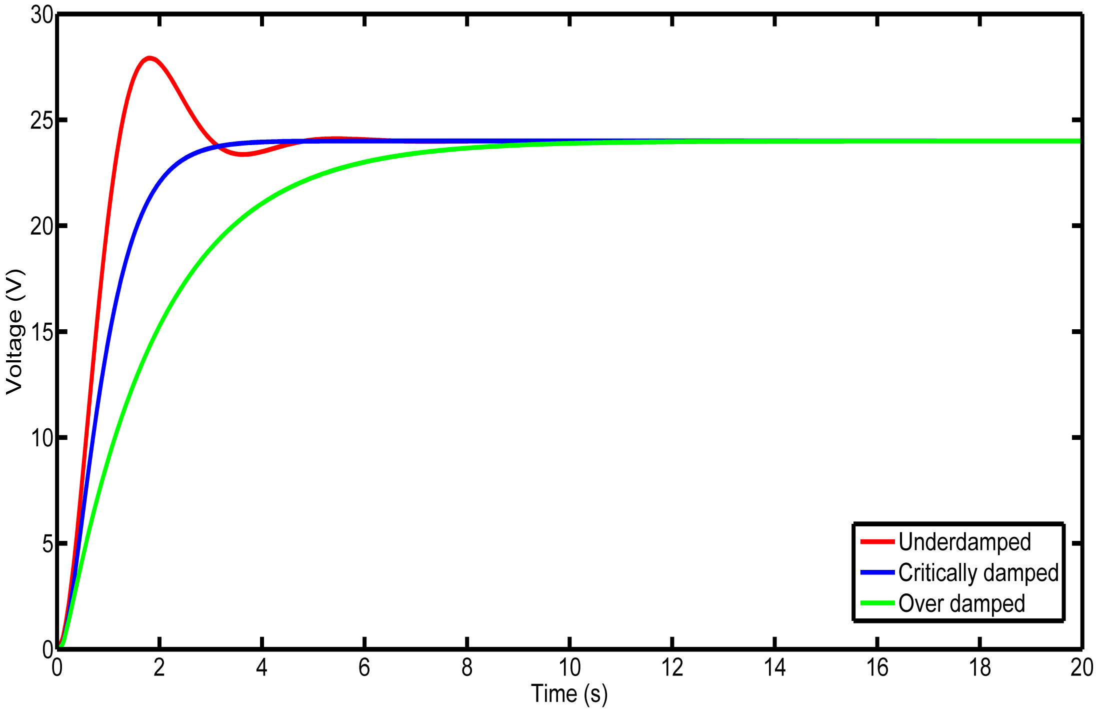

Figure 4.2: Comparison of three different damping conditions by using BRK5.

The critically damped response represents the circuit response that decays in the fastest possible time without going into oscillation. This consideration is important in control systems where it is required to reach the desired state as quickly as possible without overshooting. The overdamped response is a decay of the transient current without oscillation, which reaches the stable state slower than the critically damped case. The quantitative description of the numerical solution for three damping conditions by using the BRK5 method is presented in Table 4.3.

Table 4.3: Values of three damping conditions of BRK5 method in parallel RLC circuit.

Time (s)	Value of BRK5 Method		
	Underdamped	Critically Damped	Overdamped
0.0	0.000000	0.000000	0.000000
0.1	0.448062	0.420556	0.372918
0.2	1.665911	1.477249	1.188828
0.3	3.470005	2.925636	2.181422
0.4	5.688888	4.588992	3.226176
0.5	8.167195	6.341790	4.265703
0.6	10.768352	8.096949	5.274355
0.7	13.376102	9.796018	6.241456
0.8	15.892022	11.401660	7.163347
0.9	18.250199	12.891917	8.039625
1.0	20.386214	14.255861	8.871361

4.4 Discussion

The transient analysis of the electrical circuit is analyzed using the analytical method, Euler (Explicit) Method, RK3 Methods, and BRK5 Method. With the usage of computational software, the process of obtaining results of the transient analysis is done systematically and conveniently. From the simulation results, we conclude that numerical solutions are more appropriate in overdamped conditions than in underdamped and critically damped conditions. It has been observed that the BRK5 method is very efficient in solving second-order differential equations. Thus, it can be concluded that by carrying out the transient analysis of a system, the response of the system can be found by changing the conditions from one steady-state value to another. The observation clearly shows that the underdamped decay is oscillatory and exponential. However, the other two are non-oscillatory exponential decay. The decay in the critically damped case is observed faster than in the overdamped case.

Chapter 5

SUMMARY

Transients are the momentary fluctuation of energy induced upon the electrical circuit due to the time-varying currents and voltages resulting from the sudden application of sources. It usually occurs during the period of switching. An electrical system under the influence of transient conditions rendered the system in an unsteady state. Thus, transient analysis is to analyze the response of an electrical circuit under these unsteady state conditions. The transient response is dependent on the value of the different characteristics of the damping factor (i.e., overdamped, critically damped, and underdamped).

From the above discussion, it is observed that the approximation results converge to the exact solution and if the step size is decreased then the error also be decreased. This shows that the small step size gives an improved estimation. The Butcher fifth-order Runge-Kutta method requires five evaluations per step, so it should give more accurate results in comparison with others methods. Other methods were found to be less accurate due to the inaccurate numerical results. From the above study the Butcher fifth-order Runge-Kutta method was found to be more accurate and also the approximate solution converged faster to the exact solution when compared to the other methods.

It has been observed that the BRK5 method is very efficient in solving second-order differential equations. From the series RLC circuit, it can be concluded that by carrying out the transient analysis of a system. The response of the system can be found changing the conditions from one steady-state value to another. This observation clearly shows that the underdamped decay is oscillatory and exponential. However, the other two are non-oscillatory exponential decay. The decay in the critically damped case is observed faster than in the overdamped case.

From the simulation results of the parallel RLC circuit, it can be concludedthat numerical solutions are more appropriate in overdamped conditions than in underdamped and critically dampedconditions. This observation clearly shows that the underdamped decay is oscillatory and exponential. However, the other two are non-oscillatory exponential decay. The decay in the critically damped case is observed faster than in the overdamped case.

This response helps in designing a system, which meets our requirements, and we can further optimize the time domain parameters of the system. It state that the Butchers 5^{th} order Runge-Kutta method is more appropriate and proficient for finding the numerical solutions of initial value problems (IVP) than 3^{th} order Runge-Kutta method. Hence from this study, it is concluded that to find more accurate results higher order numerical method is appropriate than lower-order methods.

PUBLICATIONS

1. **Published journal in "BIBECHANA"**

   J. Kafle, B. K. Thakur and I. B. Bhandari (2021) Visualization, formulation, and intuitive explanation of iterative methods for transient analysis of series RLC circuit. *BIBECHANA*, *18*(2): 9-17. doi: https://doi.org/10.3126/bibechana.v18i2.31208.

2. **Published journal in "JIST"**

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3. **Journal accepted for publication in "NJMS"**

   J. Kafle, B. K. Thakur and G. Acharya (2021) Formulative Visualization of Numerical Methods for Solving Non-linear Ordinary Differential Equations. *Nepal Journal of Mathematical Sciences*.

PRESENTATIONS/ PARTICIPATION

1. B. K. Thakur (2020) *Visualization, Formulation and Intuitive Explanation of Iterative Methods for Transient Analysis of Series RLC Circuit,* Seminar Cum Workshop on Mathematics and its Applications, August 29 - 30.

2. B. K. Thakur, I. B. Bhandari and J. Kafle (2021) *Visualization, Formulation and Intuitive Explanation of Iterative Methods for Transient Analysis of RLC Circuit,* International Conference on Analysis and its Applications - 2021 (ICAA_NEPAL_2021) April 9-11.

3. 2020 MINDANAWAN MATH - STAT INTERNATIONAL WEBCON on November 4 - 18, 2020. The Department of Mathematics and Statistics of the Mindanao State University-Iligan Institute of Technology, in cooperation with the Department of Mathematics of Central Mindanao University, Mathematical Society of the Philippines-Regions 10, 12 and ARMM Chapter, and the Southern Philippines Society of Theoretical and Applied Statistics.

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