ANALYSIS OF ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSION AT SECONDARY LEVEL

A

THESIS

BY

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This is to certify that **Mrs Samjhana Subedi,** student of the academic year **2073/075** with campus Roll no. 33, Exam Roll No: **7328443,** T.U. Registration No: **9-2-48-1505-2011 and thesis number is 1771** has completed her thesis under the supervision of Mr. **Krishna Prasad Adhikari** during the period prescribed by the rules and regulations of Tribhuvan University, Nepal. The thesis, **Analysis of errors made by learner's in simplifying algebraic expression at secondary level,** has been prepared based on the results of her investigation conducted during the period April 2022 to September 2022 under the Department of Mathematics Education, University Campus, Tribhuvan University, Kirtipur, Kathmandu. I recommend and forward her thesis for evaluation as the partial requirements to award the Degree of Master of Education.

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(Head of Department)

Date:- 14,July,2023

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LETTER OF APPROVAL

This thesis entitled **"Analysis of errors made by learner's in simplifying algebraic expression at secondary level"** submitted by **Miss Samjhana Subedi** in partial fulfillment of the requirement for the Master's Degree in Mathematics Education has been approved.

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RECOMMENDATION FOR ACCEPTANCE

This is to certify that **Mrs Samjhana Subedi** has completed her M.Ed. thesis entitled **" Analysis of errors made by learner's in simplifying algebraic expression at secondary level "** under my supervision during the period prescribed by the rules and regulations of Tribhuvan University, Kirtipur, Kathmandu, Nepal. The study embodies the result of an investigation conducted during the period 2022-2023 under the Department of Mathematics Education, University Campus, Tribhuvan University, Kirtipur, and Kathmandu. I recommend and forward her thesis to the Department of Mathematics Education for the final viva-voice.

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……………….

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Abstract

This study is entitled **"Analysis of errors made by learner's in simplifying algebraic expression at secondary level".** The main aims of the study were to analyze the common errors made by learners simplifying algebraic expressions, to determine learners' errors in simplifying algebraic expressions at grade 9 and to find out the reasons why learners at grade 9 make errors in simplifying algebraic expressions. This is quantitative and descriptive survey design research. The sample of this study consisted of 100 students of Ramkot Secondary School and Rupa Jyoti Secondary School, studying at grade IX.

The paper pencil test paper was prepared with the help of an authentic book of grade IX. After that, the test was administered to the sample of students by the researcher. After the test was administered in the selected sample in-depth interview was taken with students to find out the causes of error.

The study revealed that students had committed several errors in solving the word problems in algebra. The study showed that the maximum error was committed in misapplications of rules and misinterpretation of symbolic notation (15.9%) and Sign error (6.4%). Besides, invalid distribution of brackets, conjoin error, substituting letter by number and lack of basic conceptual knowledge in algebra and language problems. There were many factors such as a lack of understanding on verbal problems and basic conceptual knowledge in algebra, language problems, less practice in word problems, the attitude of students towards algebra, teacher's qualification and performance, lack of practice, mixed-up rules, etc were responsible causes of errors made.

Abbreviations

CDC:	Curriculum Development center
Ce:	Comprehension Error
Ee:	Encoding Error
Pse:	Process Skill Error
T.E;	Transformation Error
T.U.;	Tribhuvan University

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Chapter I
Introduction

Background of the Study

Algebra is an influential mathematics topic in a school curriculum. It is applied in all current branches of mathematics as well as in Science. Algebra is a method of thinking and presents the position that thought, thinking processes and the ability to appreciate mental a spiritual accomplishment is looked upon today as the rightful possession of every individual. MacIntyre (2005) asserts that the success in mathematics substantially hinges on algebraic notions in support of this fact. According to Mensah (2006), algebra makes up a significant chunk of the final matric examination in mathematics. In a similar vein, Christmas and Fay (1990) noted that all fields of mathematics use the basic concepts of algebra to explain and represent a variety of occurrences. All of these perspectives highlight the crucial role that algebra plays in the dissemination and study of mathematics.

The branch of mathematics that deals with general statement of relation, utilizing letters and symbols to represent specific set of numbers, values, vector etc is called algebra. Algebra is a branch of mathematics that uses numbers and letters that represent numbers (Merriam- Webster, 1828). The history of algebra began in ancient Egypt and Babylon, where people learned to solve linear and quadric equation as well as indeterminate equations such as $x^2+y^2=z^2$, where by several unknown variables are involved. The word algebra is derived from the Arabic word Al-Jaber and this comes from the treatise written in the year 830 by medieval Persian mathematician.

Algebra is a barrier for students in school despite playing a crucial part in the learning and development of fundamental mathematics concepts. According to Booth (1988), learning algebra can be confusing for students. Bell (1995) believes that algebra is a common area where students struggle. Understanding mathematical ideas is a challenge for many students. Learners are unable to manipulate algebraic concepts in accordance with conventional guidelines, processes, or methods. This in turn has an impact on their math performance because mastery of algebraic ideas is crucial for success in this subject (Mamba, 2012). The difficulties faced by students are so great that some of them choose to drop out of school or, if they want to stay in school, struggle to advance their education (Wellmann, 2008). Kinney and Purdy (1952: 59) claim that “Algebra has acquired a reputation among teachers, pupils and parents alike, as one of the most difficult and 2 troublesome courses in the secondary curriculum." Kilpatrick and Izsac (2008), in the same line, also regarded algebra as an evil force wreaking havoc across the land and also as a source of difficulty and failure. Reeve (1936) in the America's National Council of Teachers of Mathematics (NCTM, 1936) eleventh yearbook went to an extent of learning difficulties of algebra for students may directly be related to teacher's mathematical knowledge, use of materials, classroom practices, pre-knowledge of student, learning environment , lack of understanding basic concept of the variable in different context , transition from arithmetic to algebra , unable to understanding about the mathematical terminology and poor vocabulary and so on . this study will focus on student's difficulties in simplifying algebraic expressions at grade 9 level. Egodawatte (2011) carried out the students' difficulties comes from misunderstanding and misconception of variables, operation, symbol, expressions notation etc. This different types of misunderstanding and misconception students get difficulties to simplify algebraic expression. Algebra is a buzzword in the landmark of mathematics education. Mathematics is recognized as the "science of patterns" and according to Sawyer "Mathematics is the classification and the study of all possible patterns". And the pattern is the heart of algebra. It shows the unparalleled role of algebra in mathematics. Algebra exists as a fundamental field of modern mathematics. Almost all the development in the field of mathematics is due to the algebraic treatment of the existing structure. It is known from history that algebra ultimately from generalized arithmetic (Acharya, 2017).

In general, why do the students have difficulties in learning algebra? Students struggle to reach procedural and conceptual proficiency in algebra due to a poor understanding of whole numbers, fractions, decimals, and percentages. Additionally, students lack understanding of fundamental algebra concepts such as negativity, variables, equality, and the equal sign. Learning difficulties and errors in mathematics are directly related to the achievement of students in mathematics which is a great challenge to mathematics teachers. Errors in learning algebra for students may directly be related to the teacher's mathematical knowledge, use of materials, classroom practices and school management, pre-knowledge of students, family education, learning environment, and intellectual capacity.

From the above discussion, it seems that more students have been facing number of difficulties in simplifying algebraic expression at grade IX levels. So, Researcher to find out the students' error in simplifying algebraic expression with the propose of explaining some causes of error in simplifying algebraic expression in grade IX levels.

Statement of the Problems

Errors made by learners result in subpar performance in any topic. The majority of these mistakes in mathematics are attributable to weak algebraic abilities. Even worse, algebraic ideas are used to develop practically all mathematical disciplines. As a result, it's important to understand the typical mistakes that algebra students make as well as the reasons why they make them. According to the researcher, one method of doing this is by looking at the mistakes that students make when they try to simplify algebra. The recognition of these faults can assist educators in developing more effective strategies for reducing common errors in algebra as well as in many other fields of mathematics. Hence, the intention of this study is to identify and analyze the common errors among 100(40+60) Grade IX learners from two public secondary schools from Kaski district, Rupa Rular Municipality and find out the root causes of these errors. The objective, after identifying the errors, will come up with better instructional practices for learning algebra.

Objectives of the Study

- To determine and analyze the common errors made by learners simplifying algebraic expressions.
- To find out the reasons why learners at grade IX make errors in simplifying algebraic expressions.

Research Questions

- What errors do students make in simplifying algebraic expressions?
- What are the possible reasons that lead learners to make errors in simplifying algebraic expressions?

Justification of the Study

Learning mathematics starts from when we being to learn how to count. Then, we use mathematics in our everyday lives, sometimes without even realizing. For many years' teachers have been seeking answer of why students have made errors to solve the algebraic expression and what they can do to help them to develop mathematically correct conceptions. In order to provide learning experiences students, have to develop the understanding of importance of algebra. The students in Grade 9 are the main focus of this investigation. It made an effort to solve the algebra-related issues that students experience. It was designed to help students comprehend mathematics by pointing out their mistakes when trying to simplify algebraic formulas. It examined the prevalence of algebraic errors and their root causes.

Algebra is the subject that has the most impact on the development of the majority of mathematical concepts, so learners need assistance in understanding it in order to modify their attitudes about mathematics. The results of this study are intended to inform instructors, curriculum planners and developers, textbook authors, and other stakeholders about the common algebraic errors in the hope that they will be able to identify strategies for either reducing or eliminating these algebraic errors. As a result, students' mathematics performance would be enhanced.

Delimitation of the Study

The study limited in the following boundaries:

- The study conducted on two public schools of Rupa Rural Municipality of Kaski district in Gandaki Province.
- The data collection tools of the study generated through the achievement test and interview guidelines.
- This study delimited to 100 students on achievement test and 10 students and the mathematics teachers were selected for interview guidelines.
- The study delimited to find and analyze the error made by learners to solve algebraic expression of grade IX levels.

Operational Definition of Related Terms

Algebra. Algebra is a branch of mathematics that involves solving equations and inequalities to find an unknown quantity.

Variable. The unknown quantity of an algebraic equation is usually represented by a letter, called a variable. It is the use of variables that makes algebra distinct from regular arithmetic. The techniques used to find the value or ranges of values of the variable(s) are useful in higher levels of mathematics.

Simplifying an algebraic expression. Simplification of an algebraic expression can be defined as the process of writing an expression in the most efficient and compact form without affecting the value of the original expression. The process entails collecting like terms, which implies, adding or subtracting terms in an expression.

Error. The error in solving a mathematical problem is a deviation from the accuracy or rules in solving math problems. This study discusses students' errors in derived algebra material. Errors were analyzed based on students' errors in solving process and interview responds. Errors are diversifying in to four type those are misapplication of rules, conjoining terms, distributive property, misinterpretation of algebraic notation.

Common Error. Errors likes lack of understanding of the equality sign, Incorrect use of inverse operations, including mistakes with sign, conceptual misunderstanding etc are in common error

Misinterpretation. Students are make misinterpretations of terms with invisible coefficients. For example, Students will assume that 0 is in the coefficient of terms with invisible coefficients as there are no numbers before the letters.

Error analysis. Error analysis will be the systematic study and analysis of the error committed by the students.

Misapplication of rules. Students misuse previously learnt procedures and rules in situations where they are not appropriate. For example, when asked to simplify the expression 6n + n, instead of giving 7n as the answer be $6n^2$ as their solution. Students multiplied the terms instead of adding. In this case, it shows learners failed to differentiate n+ n from n× n. This confusion comes about as students try to construct knowledge.

Conjoining terms. Learners made this type of error due to a lack of understanding of the concept of algebraic expression. Learners ignored letters and concentrated on numeric values. They then just added letters in their answers. They also added coefficients and constants and put a letter at the end.

Misinterpretation of algebraic notation. Learners misinterpret the symbolic notation when the ‘invisible’ coefficient of 1 appears in an expression

Conceptual understanding. Conceptual understanding is when a student is able to comprehend mathematical concepts, operations and relation. It is the interrelationship between the basic elements to the larger structure that enables them to function together. Conceptual understanding refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods.

Chapter II

Literature Review

A literature review is a written summary of journal article, books and other document that describes the past and current state of information related to the topic of research study. Algebra in particular has been the subject of several studies on both the teaching and learning of mathematics. Numerous investigations have produced various ideas regarding algebra. They highlight the fact that many students have simplistic theories, preconceived notions, or prejudices that obstruct their understanding of mathematics and cause them to make mistakes when trying to solve issues. This chapter reviews relevant material in an effort to lay the groundwork and ground theory for a systematic investigation of the algebraic errors that students make.

Review of Empirical Literature

Algebra is a branch of mathematics that uses letters and other symbols to describe numbers and situations, according to the Oxford Dictionary of 2003. Students typically lack algebraic understanding. As a result, they are unable to recognize algebra's potential. They lack the knowledge necessary to effectively employ mathematics in a variety of contexts. According to Booth (1988), algebra is a subject that frequently causes perplexity for students (Bell, 1995).

The difficulty with algebra is that the majority of students struggle to grasp its fundamental ideas. Students struggle in mathematics once they don't grasp the fundamentals of algebra. Among these essential components is algebraic expression simplification. Letters, as well as both operation and direction signs, are employed in the generalized form of arithmetic known as algebra. According to Foster (2007), the usage of letters and signs makes it abstract and challenging. This is due to the fact that algebraic concepts are based on abstract notions rather of actual facts or events.

Radatz (1979) distinguished between systematic and unsystematic errors as the two primary categories of errors. The typical mistakes committed over a prolonged period of time by learners are systematic errors. “They are recurring erroneous responses methodically constructed and produced beyond space and time”, (Mamba, 2012: 19). Riccomini (2005)’s views, these errors are symptomatic of a defective cause of thinking. There is misapplication of rules caused by learners’ failure to grasp concepts or rules. Drews (2005) observed that these systematic errors are not only produced by children needing assistance but also able students make incorrect generalizations. Unsystematic errors, as suggested by Riccomini (2005), are non-persistent incorrect responses which learners can easily correct themselves without much intervention from the teacher. They are just random and have no evidence of recurring. Kousathana and Tsaparlis (2002) are of the opinion that these errors could be a result of overloading the working memory, hastiness or recklessness. In their view, learners should be able to correct these errors if given another chance.

Kuchmann (1981) also carried out a study on the 13 to 14 year olds on their errors and misconceptions in algebra. Kuchmann's (1981) study deduced that learners had difficulties in coping with algebraic letters as unknowns or generalized numbers. The study also identified conjoining of terms as one of the most prevalent errors in algebra.

On the other hand, Aremu (1998) explained that; when pupils express lack of interest in the subject, it affects the way they react or listen to the teacher. And when many of the pupils believe that they cannot pass, the teacher is affected. This is because of the negative response from the pupils, he/she as well is already being confronted by many factors (e.g. low income, low status in a society, large teacher-pupils' ratio) and so on. These may cause him or her to resorts to the easiest way of dissemination knowledge that is 'chalk and talk' without the use of instructional materials. He may not also bother to vary his teaching styles to suit individuals: therefore, the cycle goes on.

By supporting above views, Marahatta (2002) studied on "a study on computational errors on fraction by grade VI student in Chitwan district". The descriptive survey method was adopted to find the error of the students on solving mathematical problems of fraction.

Li (2006) noted that mistakes made by students are signs of misunderstanding. According to Muzangwa, Chifamba, and Donald (2012) as well as Donald (2007), an error is a mistake, blunder, miscalculation, or misjudgement. Errors may be the consequence of forgetting, misunderstanding, or a lack of knowledge of important topics. According to Lopez-Valero Fernandez and Clarkson (2008), it is normal for learners to make mistakes and/or errors. According to Mbewe (2013), these mistakes made by students are crucial in letting teachers know what phases their students are in as well as where more instruction or research is needed. They help teachers provide constructive criticism to students.

The majority of pupils in Grades 8 and 9 do have trouble understanding the concepts and techniques of algebra, according to Greens and Rubenstain (2008). The majority of students stop studying mathematics at higher levels for this reason. If students don't stop learning and the misconceptions aren't cleared up, they continue to make blunders even in universities (Gunawardena, 2011). In an effort to pinpoint the causes of algebraic mistakes and misunderstandings, this researcher looked into these issues. The four primary areas of algebra—variables, algebraic expressions, equations, and word problems—were checked for errors and misconceptions. The results showed that learners had widespread misconceptions, most of which were related to algebraic expressions.

A study entitled "An error analysis in solving algebraic problems of six five students" was conducted by Sharma (2009). The goal of this study was to identify students' mistakes in addressing problems involving simplification and algebraic equations as well as to compare their mistakes in terms of knowledge, skill, application, and problem-solving simplification. This study used a descriptive design and was qualitative in nature. All 30 of the grade VI pupils from Surkhet's Janata Primary School served as the study's sample. The primary method used for this study was interview schedules. Major findings included that 75% of errors happened during the comprehension and transmission stage, 12% happened during the process skill stage, 5% happened during the encoding stage, 8% happened because students were careless, and 40%, 34%, and 26% happened during the knowledge level, skill and application level, and problem-solving level, respectively. The study comes to the conclusion that students do poorly in the areas of simplification and equations in terms of knowledge, skill, application, and problem-solving.

Also, Aryal (2017) studied on "Influence of Symbols on Basic Level Students in Solving Algebraic Problem." The objectives of this research were to explore the symbol sense of basic level students in algebra and to identify the students' error in an operating algebraic problem. This study was used mixed method research design. Assessment test and interview schedule were used as tools of the study. The major finding of this study that errors made with these types of symbols can be traced back to misconception in arithmetic. The equal sign has different meanings for students and is not often interpreted to match anticipation, students sometime created new ways of accomplishing their goals that did not follow mathematical rules and student made errors students often worked within one representation type without making connection to any other type. Students often struggle to understand how a symbol could be used to help make sense of symbolic representation of problem. From these reviews of literatures, it is concluded that most of the students makes errors in the area of knowledge and skill levels. Students develop difficulties misunderstanding of variables, bad algebraic thinking and misuse of algebraic relation as well as procedure about algebra.

Shah (2019) studied "Students errors in solving math word problems analysis from the schematic model". The study's major goal was to identify the mistakes made by the students and evaluate the root causes of each mistake at a fundamental level using the Schematic model. The sequential explanatory mixed approach was used to create the study's design. For quantitative data, an achievement exam was employed, and for qualitative data, an interview. 398 children from community and private schools in the Kathmandu district were chosen as the study's sample. This survey found that 3398 errors were produced by pupils, with 1727 errors made by boys and 1671 errors made by girls. The main reasons for dosage errors included a lack of basic concepts, carelessness, a lack of conceptual learning, a lack of desire, exam-focused instruction, and a lack of practice with algebraic word problems.

Chaudhry(2022) conducted research entitled “Student’s error in learning algebra at basic level.” The main objectives of this study ere to find out the student’s errors in learning algebra at basic level and to explore the strategies that reduce errors in learning algebra at basic level, Research based phenomenology research designed used to collect data. The study conducted on Shree Durga Secondary School, Shailung-4, DOlakha with 82 participants for achievement test and 10 purposive students for Focus Group Discussion and 2 teachers for interview. He concluded that 37% of the error were due to misapplication of rules, 22% due to misuse of distributive property, 16% due to conjoin error, 10% due to substituting letters with numeric value, 9% due to sign error and 6 % due to misinterpretation of symbolic notation. According to the teachers 66% of students had no basic knowledge of algebra at basic level in school. The causes behind the error were based on classroom management, size of class, technical problems on test, teacher training, teaching method, motivation, punishment and reward, ignore education, parent consciousness, occupation, anxiety, practice, quality, language problems etc were categorized into 4 factors such as School related factors, Family related factors and students related factors.

A significant arithmetic-algebra divide among learners persists as a root source of learning challenges, as noted by Seng (2010). Learners who have a strong foundation in mathematics are less likely to struggle with algebra. This is so because arithmetic knowledge is the cornerstone upon which algebraic knowledge is built.

Research Gap

Teacher should aware that how they can make their classroom very fruitful and learning algebra become a meaningful which avoids the rote learning and student's errors solving algebraic problems.

Many researchers have conducted the research on the topic analysis of errors made by learner's in simplifying algebraic expression at secondary level but they were based on different factors such as misconception in algebra and sign error, miscalculation and blunder misjudge etc. There is misapplication of rules caused by learners' failure to grasp concepts or rules, Riccomini (2005).

Aryal (2017) studied on "Influence of Symbols on Basic Level Students in Solving Algebraic Problem." The major finding of this study that errors made with these types of symbols can be traced back to misconception in arithmetic. And this research is mixed method design but I studied on quantitative research as well as qualitative research design especially based on survey, this study focused difficulties related to algebra (grade IX) Students. So, this study is differing from Aryal.

This studies fulfilled the gap by analyzing the six types of errors as the common errors made by learners in simplifying algebraic expressions. These errors include misapplication of rules and mis interpretation of symbolic notation, wrong use of sign, misuse of distributive property, substitute letter with value, conjoining error and Misunderstanding the was the most common error in this study.

Review of Theoretical Literature

These errors, as suggested by Olivier (1989), are the inevitable outcomes of students' efforts to build knowledge. Additionally, Labinowicz (1985) saw mistakes made by students as being normal progressions toward comprehension. comprehending learner faults is a method of 15 comprehending learner reasoning, claims Brodie (2014). Errors must therefore be accepted as part of the teaching and learning process.

Numerous studies have examined students' mathematical errors and misunderstandings from various angles (Cline et al., 2020; Muzangwa & Chifamba, 2012). Teachers must create a positive learning atmosphere in the classroom where students are allowed to take chances and try out ideas without fear of ridicule in order to define the functional role of M/Es (Nesher, 1987). To create such a classroom atmosphere, teachers must set a reasonable tone (Hiebert et al., 1997). In order to eliminate M/Es on time, teachers should address them as soon as possible, before they develop into long-term habits among the pupils (Egodawatte, 2011). According to Sisman and Aksu (2015), educators should concentrate on three key areas of concern, including types of misunderstandings and errors, causes of misconceptions and errors, and remedies for misconceptions and errors. Through interiorization of a concept, condensation of the recently learnt concepts with older concepts, and reification of the concept, these misunderstandings and errors may arise at any moment of concept development (Sfard, 1991; Vermeulen & Meyer, 2017).

It can be challenging to understand new concepts, as indicated by Smith, DiSessa, and Roschelle (1993), if fundamental concepts and skills from early learning are not properly grasped. Individual thoughts throughout the learning process are made up of two fundamental alternate procedures, namely assimilation and accommodation (Piaget, 1970). These two procedures explain how a person prepares their mind to accept new information and adapt to new experiences. According to Moodley (2014:11), "Assimilation occurs when a new idea is interpreted in terms of an existing schema." To integrate a new circumstance, one uses an available mental structure that already exists.

Riccomini(2005)'s study entitled "Identification and remediation of systematic error patterns in subtraction." The goal of the current study was to determine whether

90 elementary school instructors could recognize two systematic error patterns in subtraction and then suggest a teaching strategy. Participants were asked to look at each incorrect subtraction issue and explain the mistakes after being given two sets of 20 solved subtraction questions that included basic facts, arithmetic, and word problems. In order to clear up students' misconceptions, participants were then asked which kind of error they would focus on first during arithmetic education. Data research revealed that teachers could identify particular mistake patterns. More than half of the teachers decided to cover fundamental subtraction facts first during instruction regardless of the type of error, but they did not base their instructional focus on the error patterns that were found.

Egodawatte (2011) conducted a study entitled "secondary school students misconception in algebra". This study looked into algebra-related mistakes and misunderstandings made by secondary school pupils in order to identify their nature and root causes and offer teaching recommendations. A mixed-methods research strategy was adopted for the study. An Ontario urban secondary school administered an algebra test to a sample of grade ll pupils after it had been pilot-tested for its validity and reliability. Under each region, the results showed various mistake categories. Misconceptions led to several blunders. The primary cause of misconceptions regarding variables was a lack of comprehension of the fundamental concept of the variable in various settings.

On the other hand, Moodley (2014) describes accommodation as occurring when there is incorporation of new ideas which are not related to the existing schemas. The existing schema may not be enough to assimilate new ideas. Therefore, existing schemas need to be modified or else new schemas have to be created so that the new experience can be taken care of. The process of assimilation can be demonstrated using a situation whereby a learner knows that $a^2 – b^2$ = (a – b) (a + b). When she is asked to evaluate 1012 − 992 without using a calculator, the learner will be able to express it in the same way used for that $a^2 – b^2$. The learner will have 1012 − 992 = (101 -99) (101 + 99). The learner can then simplify what is inside brackets to 2 × 200 = 400. The demonstration shows that the learner has interpreted the new situation in terms of the already known aspect of factorization difference of two squares. The picture of accommodation process can be explained by looking at a situation where learners have to find products of algebraic terms. If a learner has the knowledge that a × b = ab or ba, then when a problem requires the same learner to simplify a × 5 then s/he might have a5 as the answer as the learner will be thinking that a5 is the same as 5a. This means, there is a need for restructuring so that the learner sees when one part of the algebraic term is a number then the number has to be written first. This shows that it is not always possible to connect new ideas to schemas. A new box may be created in the person's mind as a result of their inability to relate new circumstances to their previous schemas. The learner might struggle to connect the information in the box to preexisting schemas, which might drive them to commit the concepts or rules to memory. Some of the rules are only partially retained during the recall phase, which causes confusion and mistakes in the learner.

Ncube (2016) studied "Analysis of errors made by learners in simplifying algebraic expressions at grade IX". The objectives of this study were to determine learners' errors and to find out the causes of errors in simplifying algebraic expression in grade 1A. She used a sequential explanatory design which is a mixed-method approach that is both quantitative and qualitative method. This study is based on the theory of constructivism. She selected 82 students from 300 students randomly grade 9 learners from secondary school in Ga-Sekgopo village, Mopani district in Limpopo Province in South Africa as the sample for this study. In this study, she found six common types of errors in simplifying algebraic expressions. The errors are: Conjoin error (combining unlike terms)- 16%, misapplication of rules-370, misinterpretation of symbolic notation-60, misuse of distributive property-22%, substituting letters by numbers-10%, and sign error-9%. She concluded that learner's mostly misapplied rules when they were simplifying algebraic expressions. They are failing to deal with direction, operation, sign, and algebraic rules. The main reasons behind the errors are poor arithmetic background, Jack of arithmetic skills, lack of awareness and understanding of the meaning of expressions, and lack of conceptual knowledge.

The Australian educator Anne Newman (1977) suggested five significant prompts to help determine where errors may occur in students' attempts to solve written problems. According to Newman's definition, students' errors in solving word problems are classified as follows:

- Reading the words → [Reading error]
- Understanding what they have read → [comprehension]

- Transforming what they have read to be able to form a course of action → [Transformation]
- Following through on procedures → [Process skills]
- Encoding the result of a procedure to answer the question → [Encoding]

Researcher tried to find the errors on the process of simplifying algebraic expression that's why researcher followed the Errors on Processing skills and Encoding Error from the Newman's Suggestion.

On the basis of Newman's suggestion of error analysis, Researcher divided the 6 common errors which were used in Ncube(2016) Studied "Analysis of error made by learners in simplifying algebraic expression at grade IX."

In the Processing Skills, Researcher followed the 3 out of 6 common errors from the Ncube (2016) studied "Analysis of errors made by learners in simplifying algebraic expressions at grade IX".

The errors are:

1. Conjoining Error (Combining Unlike term)

2. Misapplication of rules and Misinterpretation of symbolic notation (Include on Systematic error defined by Riccomini, (2005) also)

3. Invalid Distribution of brackets

4. Sign Error

And in Encoding Error, Researcher followed 1 error from Encube (2016) studies and 1 error from Newman's suggestion.

They are:

5. Substitute letter by numeric value

6. Encoding Error

Conceptual Framework of the Study

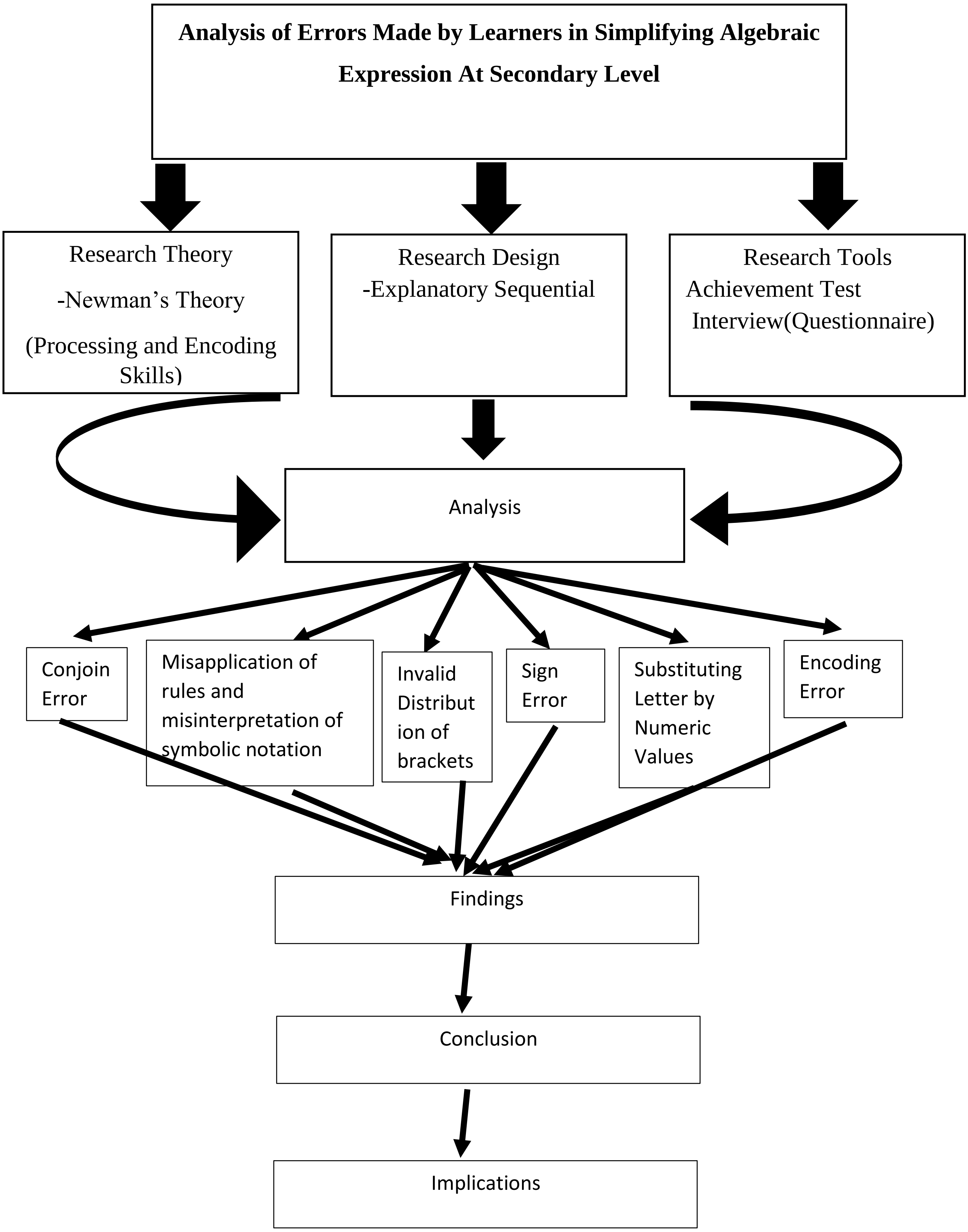


Figure 1: *Conceptual Framework*

Conjoin Error

The concept of 'conjoining errors' in algebra is not attributed to a specific individual or mathematician. It is a general term used to describe errors that occur during the process of combining or simplifying terms in algebraic expression.

Misapplication of Rules and Misinterpretation of Symbolic Notation

The concept of 'Misapplication of rules' in algebra is not attributed to a specific individual or mathematician. It is a general term used to describes errors that occurs the process of commutative rule of addition, multiplication, associative rule of addition and multiplication simplifying terms in algebraic expression.

Misuse of Distributive Property

The concept of the misuse of the distributive property in algebra is not attributed to a specific individual or mathematician. The distributive property is a fundamental property in mathematics that allows us to simplify expressions by distributing a factor to each term inside parentheses.

Substituting Letter with Number Value

The concept of 'Substituting letter by number' in algebra is not attributed to a specific individual or mathematician. It is an error which is produced when learner's responses suggest that the letter has been given numerical value. And it displayed lack of meaning of variable.

Sign Error

The concept of 'sign error' in algebra is not attributed to a specified individual or mathematician either. Sign errors are a common type of mistake that can occur when working with algebraic expressions involving positive (+) and negative (-) signs.

Encoding Error

An error is classified as an encoding error if the students correctly work out the solutions to the problem, but could not express the answer in an acceptable written form.

Chapter III

Methods And Procedures

Research methods and procedures are useful bridges to solve the research problems systematically. It describes the methods and processes applied in the entire aspect of the study. The research method is a plan of study worked out before the commencement of research work by a researcher to gain new knowledge to solve a problem scientifically.

This chapter described the plans and procedures of the study under the separate heading which must be carried out to achieve the objectives of the study, participants of the study, tools for data collection, interview schedule, classroom, observation from, validity, and reliability of tools, data collection procedures, and data analysis procedures.

Research Design

The research design refers to the overall strategy that you choose to integrate the different components of the study in a coherent and logical way, thereby, ensuring you will effectively address the research problem; it constitutes the blueprint for the collection, measurement, and analysis of data.

Researcher used explanatory sequential mixed research design (Quan+Qual). The explanatory-sequential approach is a sequential approach and is used when the researcher is interested in following up the quantitative results with qualitative data. Thus, the qualitative data is used in the subsequent interpretation and clarification of the results from the quantitative data analysis, (Bell, 1995). This study employed a mixed-method research, more specifically sequential explanatory design which is characterized by the collection and analysis of quantitative data followed by the collection and analysis of qualitative data (Creswell, 1998). In many instances, because the Quan design is the emphasis, a generic qualitative design is used in explanatory approaches. This two-phase approach is particularly useful for a researcher interested in explaining the findings from the first phase of the study with the qualitative data collected during Phase 2. However, either the qualitative or quantitative data is the primary focus of the study (see introductory figure) (Aryal, 2017)

Firstly, researcher collected quantitative data through achievement test. The errors obtained by the quantitative part of this research elaborated by qualitative data.

Their underlying concepts revealed through execution of these procedures. In the quantitative phase, a test instrument used to identify and classify errors. The sampling method used in this study was a simple random sample method. This research instrument consists of a set of test questions that focus on the topic of algebraic expressions, namely the Algebraic Expression Diagnostic Test. The test is the main data source before interviews were conducted.

The researcher also used Interview for purposive sample to find the reasons of the " Error made by learning on simplifying algebraic expression in grade IX." committing errors in the qualitative part of the study.

In phase 2, The researcher used interview on proposed sample based on the types of errors made by learners in their achievement test. The researcher asked open ended question related to the error made by them to simplification algebraic expression.

Sampling and Sampling Process

In the study, the population was Grade IX Students. As this was definitely a very large population to handle, the researcher decided to work with a sample of the population. The sample is 100 students from Rupa Jyoti Secondary School and Ramkot Higher Secondary School, Rupa Rural Municipality, Kaski.

Instrument for Data Collection

The study conducted through the primary survey. The data collected through the achievement test from the 100 learners of grade IX levels from Rupa Jyoti Secondary school and Ramkot Higher Secondary School And interview from 10 purposive sampling on the basis of the types of error.

Mathematics Achievement Test

An achievement test is a test of developed skill or knowledge. The most common type of achievement test is a standardized test developed to measure skills and knowledge, ability and application learned in a given grade level, usually through planned instruction, such as training or classroom instruction. To take achievement test, researcher selected 20 item test on simplifying algebraic expressions in 5 group.

In question 1, Researcher selected the question to test the ability to use acquired arithmetic knowledge of basic operations and allow identification of arithmetic-algebra gap. Here were 7 question, among them last 3 questions taken from fraction. The Question are followed the specified grid of grade 9 which provided by Curriculum Development Center of Nepal and the question taken from the knowledge and skills level from Blooms Taxonomy.

In question 2, Researcher selected the question to test the ability of learners to multiply algebraic terms and collect like terms.Here were 9 questions. The Question are followed the specified grid of grade 9 which provided by Curriculum Development Center of Nepal and the question taken from the understanding and application level from Blooms Taxonomy.

Question 3 and 5, Researcher selected the question to test learner's understanding of letters representing numbers. The Question are followed the specified grid of grade 9 which provided by Curriculum Development Center of Nepal and the question taken from the Understanding and Application level from Blooms Taxonomy.

Question 4 has designed to measure the learner's ability to read and understand question and make mathematical sentence and solve the problems. The Question are followed the specified grid of grade 9 which provided by Curriculum Development Center of Nepal and the question taken from the Understand and Higher Ability level from Blooms Taxonomy.

Interview

Data collection is an important part of the study and tools are important factors for data collection. To attain the objectives and get the responses to research questions, the researcher gathered the data by using such tools. There are many tools for qualitative research to get the information from the respondents and the study site. In this study, the researcher intended to fulfill objectives by the find the answers to research questions: What are the possible reasons that lead learners to make errors in simplifying algebraic expressions? For Sequential Explanatory Research Design used Interview on purposive sample as instrument of the data after achievement test.

Data Collection and Processing Procedure

A test administered to the 100 participants. For the explanation of the reasons why learners made errors, the interviewees will purposively select from the sampled 100 participants. The selection of these individuals based on the specific purpose associated with answering the research study questions. The data for the study collected by quantitatively through the structured achievement test. The researcher administered a 20 item test on simplifying algebraic expressions to sample population of Grade IX learners of Rupa Jyoti Secondary and Ramkot Higher Secondary school. The test was of one-hour duration.

The data for the study collected qualitatively through in-depth interview based on quantitative data. I this researcher used interview based on the thematic approach of research design. After taking the achievement test, researcher checked the answer and found the error made by learners. Researcher coding the errors as: code 1 for right answer, 2 for no answer, 3 for conjoining error, 4 for misapplication of rules and misinterpretation of symbolic notation, 5 for misuse of distributive property, 6 for substitute letter by numbers, 7 for sign error and 8 for encoding error. After divided the errors according to the code, Researcher conducted purposive sampling to select sample of 10 students from the population of 100. The interview conducted on the basis of their achievement test result a through thematic approach.

Reliability and Validity of Tools

"Reliability of tools refers to consistency of tools and validity of tools refers to appropriateness of tools Reliability is necessary but not sufficient condition for validity. Reliability is the agreements between two efforts to measures the same trait through maximally similar methods. Validity is represented in the agreement between two attempts to measure the same trait through maximally different methods"Campbell &Fiske (1967; p. 277).

The reliability of the achievement test is determined by the pilot test. The content of the achievement test schedule has been selected based on the grade IX algebra curriculum. For the reliability test, Researcher took 20 participations from the sample by the pilot test in order the Split Half-Method. An individual given scored on the odd-numbered and another given by even-numbered. Therefore, the Split-Half

Method to find reliability for the given data sets is approximately 0.707, indicating a moderate reliable.

The test questions in this study were developed using Grade IX prior exam papers and relied on frequent mistakes the researcher made while teaching and learning. The researcher and supervisor frequently discussed the findings and decided on the interview questions for the follow-up interviews. The researcher was fully aware that prejudice existed. As a result, the researcher saw attitudes, perceptions, and assumptions regarding learners' misconceptions that were simply based on practice. With the aid of an internal supervisor, specialists, the subject matter expert, and other pertinent documents, the test's validity was determined.

Data Analysis Procedure

Researcher analyzed and interpreted the obtained data by using different statistical techniques. After the completion of data collection procedure, researcher started data analysis. For the achievement test, the researcher coded the data in numerical form on the basis theme (6 errors). Researcher were tabulated the code of the error with their frequencies. After these researchers used inferential statistics using SPSS 24.0 and presented the Means percentage and bar diagram to analyze the achievement test.

In second phase, Researcher tried to find out the reasons that led learners to commit the six identified error by thematic approach. For the interview, The researcher conducted the purposive sampling to select the sample of 10 students from population of 100. Researcher was interviewing 10 learners representing each error code. The number of learners representing each code depended on the prevalence of that error in the achievement test.

The questions on interview are selected on the basis of result of their achievement test and based on the error made by learners. The interview participants interviewed by the researcher one by one, translate the answer after interview in English language and record the interviewer answer with reasons why they made representative types of error while solving the problems in given question.

Overall researcher conducted the purposive sampling to collect the data and use thematic approach to analyze the data. Firstly, Researcher record the interview in written form and translate it in English language (familiarization the data), coding and decoding the answer collected by interviewer (Generating Initial code), After coding the answer, researcher kept the interview record and click the picture and attached picture of their answer sheet with types of error and interview record, researcher review it and do correction if any mistake was there(Reviewing theme), Researcher already named the theme in achievement test so no need to naming the theme here and lastly researcher record the interview and find the reasons why learners made error, what are the causes of the errors made by learners on simplifying algebraic expression.

Ethical Consideration

This study conducted for the academic purpose while collecting data, ethical considerations ensured for the primary data privacy. With regarding this study, researcher has clearly informed objectives to respondents. Researcher had not used data for other purpose except this research.

Chapter IV

Analysis and Interpretation of Data

This chapter is completely based on analysis of the data. For this purpose, data were collected through the method of achievement test, class observation form and interview schedule. The researcher administered a 20 item test on simplifying algebraic expressions to sample population of Grade 9 learners of Rupa Jyoti Secondary and Ramkot Higher Secondary school. The results attained from those questions are analyzed according to the purpose of the study. These data were analyzed the solutions presented by the participants and grouped. the participants according to where they had the most errors. In addition, frequencies for each type of error per item were recorded. From the results of the test, the researcher identified the common errors and also recorded the number of learners who committed those errors. Interviews were carried out with 10 learners. These learners were sampled from the group that committed the most prevalent errors. The interviews were held in order to get a clear understanding of how the learners had arrived at particular solutions.

Description of Identified Common Errors

Table 1: Answer codes descriptions

Description	Code
Right Answer	1
No Answer	2
Conjoining Error	3
Misapplication of Rules and Misinterpretation of Symbolic Notation	4
Misuse of Distributive Property	5
Substitute Letter by Numbers	6
Sign Error	7
Encoding Error	8

As the table displays, there were 8 answer codes used. The first code represented correct answer. The second code represented situations where the learner did not attempt to answer the questions. Code 3 was for the error due to conjoin of terms. Code 4 stood the error due to misapplication of rules and error due to misinterpretation of algebraic notation. The error due to wrong application of the distributive property was represented by code 5. Substituting a letter by a numeric value was represented by code 6 followed by wrong use of signs as code 7. Lastly, code 8 represented the encoding in verbal problems. Using the above-mentioned codes, the researcher recorded learners' results and tabled them and kept in the appendix 1.

The table below is an extract of the tabulated results of the code of the error from appendix 1.

Table 2: Number of learners per answer code per item

Code → Item ↓ number	1	2	3	4	5	6	7	8
1.1	62	2	22	0	0	0	14	0
1.2	44	4	32	2	0	0	18	0
1.3	22	8	0	26	20	0	24	0
1.4	16	14	0	36	18	0	16	0
1.5	30	42	2	24	2	0	0	0
1.6	6	38	0	56	0	0	0	0
1.7	10	42	0	48	0	0	0	0
2.1	92	6	0	0	2	0	0	0
2.2	92	2	2	0	2	0	2	0
2.3	76	14	2	2	6	0	0	0
2.4	66	8	2	4	4	2	14	0
2.5	58	14	0	2	4	0	22	0
2.6	64	18	0	12	2	2	2	0
2.7	42	18	6	30	2	0	2	0
2.8	40	32	0	20	6	0	2	0
2.9	18	36	0	42	2	0	2	0
3.1	34	10	0	8	0	48	0	0

3.2	56	10	0	4	0	18	2	10
4	18	32	0	2	10	0	8	30
5	60	20	0	0	4	2	2	12
Total Code:	906	370	68	318	84	72	128	52
Percentage	45.3%	18.5%	3.4%	15.9%	4.2%	3.6%	6.4%	2.6%

Source: Appendix1

Overall, learners provided 45.3% of correctly answered questions, 18.5% of unattended items of questions and 36.1% of incorrect answers. Additionally, "misapplication of rules" and "misinterpretation of symbolic notation" were among the faults found in answers, and "invalid distribution of brackets" was found in 4.2% of the answer codes. Conjoin mistake made up 3.4% of the sample, letter-for-number substitution 3.6%, and sign error 6.4%. and "encoding error" (algebraic concept & improper question reading) 2.6%.

The distribution of errors among the six error codes and the other two non-error codes, one for right answers and the other for missing answers, are shown in the table above. As can be seen, code 4, which represented the misapplication of rules and misinterpretation of symbolic notations, dominated Questions 1 and 2.

to make algebraic expressions simpler. Out of 100 students who completed item 1.2 of the 3pc-4+5cp+8,32 simplification, 32 conjoined words. This indicates that 32% of students combined terms in that item. Their answer was given 12pc. The misuse of rules predominated the other portions of question 1. For question number 2, students had to take the brackets out before simplifying the expression. The misuse of the distributive rules in this question was the most frequent error. Nearly every item in this question applied the distributive property incorrectly. Most students answered Question 3 by replacing the unknown with numerical values. Students struggled to understand verbal problems as well as instruction and operation indications. The learners in Question 4 were unable to transform the verbal issues into mathematical sentences. If they alter the mathematical expression and manage the first phase but not the second, they will fail. Some students chose not to respond to question 5 or, if they did, their responses did not address distributive property.

The overall contribution of each error to this study was also calculated, tabled and illustrated graphically as shown below.

Table 3: Most common errors and their frequencies

Description	Code	Total Item	Percentage
Conjoining Error	3	68	3.4%
Misapplication of Rules and Misinterpretation of Symbolic Notation	4	318	15.9%
Misuse of Distributive Property	5	84	4.2%
Substitute Letters by Numbers	6	72	3.6%
Sign Error	7	128	6.4%
Encoding Error	8	52	2.6%

The contribution of each error to the study was illustrated graphically. The following graph represents the frequencies of each error in the study

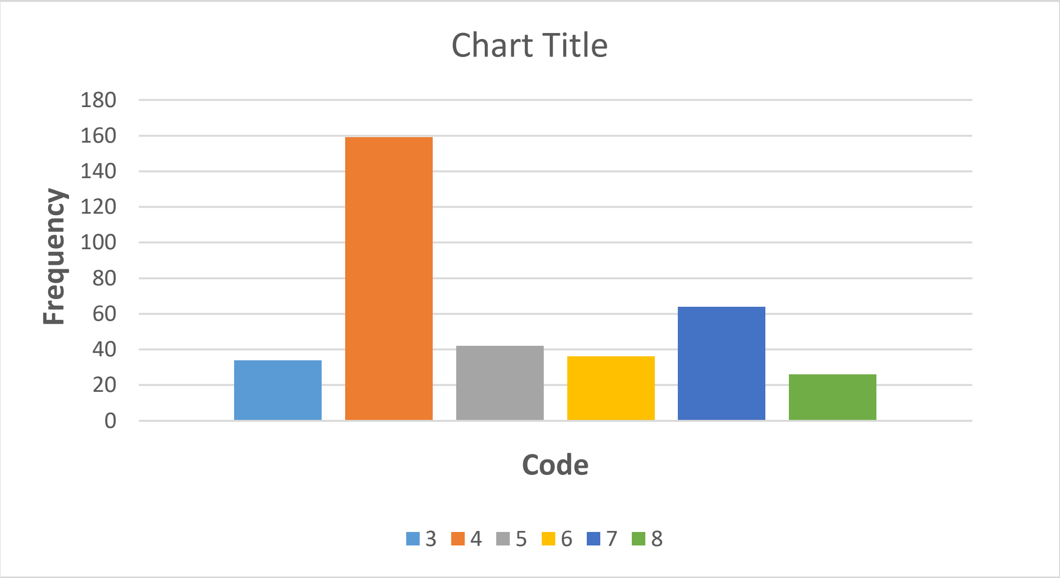


Figure 2: *Frequencies of the most common errors*

It can be clearly seen that the most dominant error was misapplication of rules and misinterpretation of symbolic notations followed by wrong use of sign, misuse of the distributive property, conjoin error, substituting letter by number, and lastly, Misunderstanding of the concept.

Conjoin Error

In order to simplify expressions, students combined terms that were not similar. This was primarily evident in item 1.2, which required students to solve the equation 3pc-4+5cp+8. 4pc was the most often offer 16 common solution by 32% of the students. Some students correctly deleted the brackets from item 2.2 and changed a(5y-3x) to 5ay-3ax. However, they continued to incorrectly simplify 5ay-3ax, arriving at 2axy as the solution. Students did not comprehend the idea of like and unlike concepts. Most of the test questions involved this inaccuracy. Learners were asked to simplify 8k-4k-3y-5y-4x in the first test question, but surprisingly few of them were able to come up with the right response. The most typical response was 2xyk. Instead of adding the terms learners leaving it.In term 2.7, to solve $(x+a)^2$, learners were simplified it by multiplying(x+a)(x+a) by using distributive property, they got x^2+ xa+ax+a^2. After this step learner were leaving here by thought ax and xa are unlike terms. This error was committed by 6% of the learners.

Misapplication of Rules and Misinterpretation of Symbolic Notation

Most of the test questions involved this inaccuracy. In several instances, students applied rules incorrectly. Surprisingly, the majority of students who were asked to simplify (4a)(3a) in the 2.4 item were unable to provide the right response. The most common answer was 7a or 12(2a). Instead of multiplying terms learners add the terms and they multiply constant and adding the variable. Some of the students did $12a^2$and they multiply 12 by 12 and getting answer $144a^2$. Same as this in 2.6,2.7,2.8 and 2.9, most of the learners could not get the correct answer. In item 2.7, students were asked to solve(x+a)(x+a) but they wrote the answer(x^2 +y^2) same as in item 2.9,they were asked to solve $(a-b)^3$, most of the students wrote (a-b)(a^2+ab+b^2) This error was committed by 30% of learners. In item 1.6 and 1.7,in y+1/y, they took y common and wrote(1+1) in numerator and y on denominator, in last step they cancelled the y in both and wrote 2 as answer.

Learners made misinterpretations of terms with invisible coefficients. This error was committed by 64% of the learners. Some students divided each term by m while simplifying ma+mb m+md. The students then cancelled m in each term, leading to the final response of a+b d. They cancelled the m and left nothing for the students in the areas where there was only m.Same as some of the student add ma+mb and got answer 2mab and in denominator 2md and cancelled the 2m and they wrote the answer ab/d,some students were cut the m in the addition form without taking common and wrote a+b/d as answer. They were left with simply d in the denominator as a result. There was evidence that students only understood a portion of the factorization process. Makonye and Nhlanhla (2014) claim that these students depended on unpolished schema, which suggests that their fundamental structures were made up of unstructured information. Although they were unable to effectively retrieve the knowledge, learners probably possessed the right strategies in their long-term memory. The students had views about how regulations should be applied, but they misapplied the rule.

Misuse of Distributive Property

A highly frequent error was the distributive error in bracket expansion, especially in Questions 1.3, 1.4, and 2.2. 19% of the students in the study made this error. Some of the learners just expanded one side of the bracket, ignoring the other. This was evidenced in the expansion of a(5y-3x) where the learners gave 5ay- 3x as their solution.. According to Martz (1980), learners use known rules in appropriate situations but incorrectly adapt the known rules. When expanding the expression Same as this in 2.6,2.7,2.8 and 2.9, most of the learners could not get the correct answer. In item 2.7, students were asked to solve (x+a)(x+a) but they wrote the answer($x+a^2$) same as in item 2.9,they were asked to solve $(a-b)^3$, most of the students wrote ($a-b^3$ x $a-b^3$) that is equal to ab^3. In this instance, as was discovered by Mbewe (2013) in his study on misunderstandings and errors in algebra at the Grade 11 level, students were unable to recall the proper expansion of a binomial. The distributive rule was relatively applied by students. According to the literature, learners' inability to grasp fundamental facts and concepts is what causes incorrect bracket distribution.

Sign Errors

Working with integers and operation signs was difficult for learners. 22% of errors were attributable to the improper usage of signage. Failure to combine operation and direction signs led to the majority of sign errors. When simplifying the expression x(x-6)+7(x-6), learners gave x^2 -6x+7x+42 the answer. It indicated that learner had no concept of the rules to multiply positive number with negative number, there should be negative sign in the product. Learners comprehended what they were intended to accomplish when they were asked to subtract the second expression from the first expression in the question. They successfully typed (a2- 2ab + b2) (a2+ 2ab + b2) but failed to appropriately use the signs when they wrote (a-b) to simplify the equations.2-(a+b)The learners failed to simplify the second term like (a-b)-(a+b), which would have made plus the minus sign instead of writing plus sign in the next stem, where they wrote (a-b+a+b)(a-b-a+b). Too many misconceptions were displayed by learners when using signs.

Substituting Letters by Number

Overall, this error contributed 3.6% to the total errors committed in this study. The substitution of letters by numbers was mostly observed in item 3.1 and 3.2 where 33% of the learners gave wrong answer. In item 3.1, if (a-4)=4 and ab= 2, to find the value of a^2+b^2 the answer was 20 but they found the value of a & b as $\sqrt{2}$ and $\sqrt{2}$ by using both equation and wrote the answer is 4. Same as in 3.2, just they need to multiply 5 with 5 to get answer but they would not read the question properly and found the answer 23 as the value of $(x+1/x)^2$if the value of (x+1/x) was 5.

Encoding Error

Overall, this error contributed 2.6% to the total errors committed in this study. In item number of 4, 30 learners did not understand the verbal problems and did not encode the language in mathematical language what to find and how to find so, 32 students leave it. And 30 students were try to do but they did not reach in answer. If they solve the given area and found the length and breadth of the rectangular ground but in next step they had no idea to get answer by reducing length and breadth by 2, so by this study we conclude that most of the students not understood the concept of problems and making error in encoding the language into code of algebra. Overall the report 52 learners made error in encoding the question.

Analysis of Error

When asked to simplify algebraic expressions for the test, students frequently committed mistakes. The researcher identified six common mistakes with the following percentage distributions: conjoin error, 3.4%; misuse of the distributive property, 4.2%; letters and numbers substituted for each other, 3.6%; sign error, 6.4%; and encoding error, 2.6%. Most rule violations occurred while students were trying to simplify algebraic expressions. The researcher made the decision to look into the causes of the six mistakes that were observed in trainees. The greatest method for learning the reasons why students made those mistakes was thought to be interviewing students. As part of the study's second phase, interviews were conducted with ten students who represented each mistake code. The number of students representing each code varied according to how frequently that error occurred on the test. The distribution of learners, the type of error, and the item number they represented are shown in the table below.

Table 4: Number of learners selected for interviews according to error type

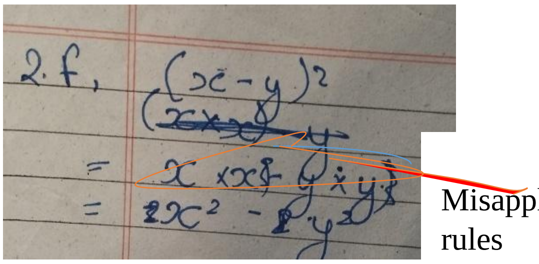
Type of error	% of learners	Number of learners selected	Selected
Conjoin error	3.4	1.2 and 1.1	2
Misapplication of rules and Misinterpretation of symbolic notation	15.9	1.5,1.6,1.7,2.6,2.8	3
Misuse of distributive property	4.2	2.7	1
Substituting letters by numbers	3.1	3.1 and 3.2	2
Sign error	6.4	1.3 and 1.4	1
Encoding error	2.6	4	1

(R Refers researcher and L Refers learners)

Misapplications of rules and misinterpretation of symbolic notation

Learner's :50

Figure no.3



Misapplication of rules

R: Can you go to question 2.6 Read it again and the solution that you provided. Explain to me how you got that solution.

L: In $(x-y)^2$, here is(x-y)(x-y), that means we have to multiply x and x and got x^2 and y and y and got y^2.

So my answer was x^2-y^2.

R: Is your answer right?

L: Yes, My answer was right.

R: Did you know how to multiply 2 terms by two terms?

L: yes, I know, multiply first term by first term and second term by second term.

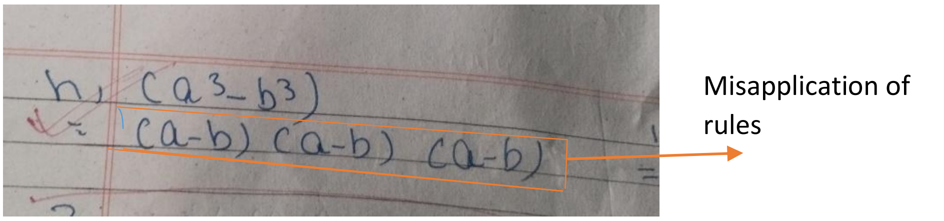
R: Okay thank you.

L : you are welcome

The aforementioned excerpt demonstrates one more that the learner lacked the necessary algebraic facts. Misuse of the distribution property and erroneous application of the rules occurred. The learner need to take the operation signs as instructions to multiply terms one by one like x(x-y)-y(x-y) and add the like terms.

Learner’s 44

Figure no.4



R: : Look at question 2.8. Go through the question and the solution that you provided.

L: yes, I have done

R: How did you come up with your answer?

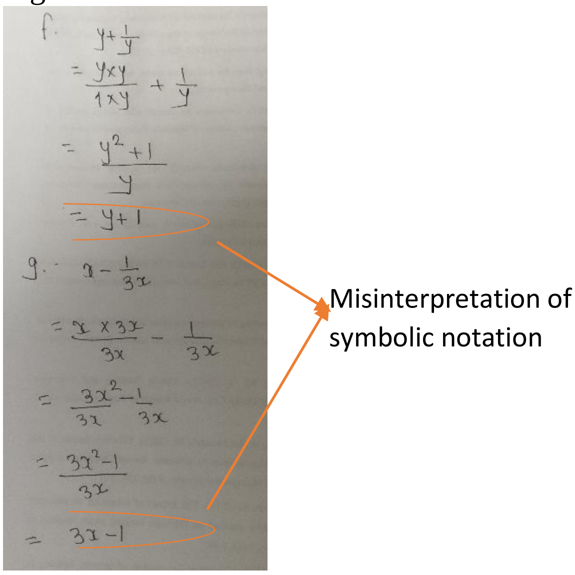
L: In the question, (a^3-b^3), We have to cube means multiply 3 times so, we have to multiply(a-b)(a-b)(a-b). but after this line, I don’t know what is next step so I leave it.

The passage above once more demonstrates the learner's lack of algebraic necessary knowledge. He used $(a-b)^3$ formula to solve a^3-b^3. When I asked how did you think $(a-b)^3$ and (a^3-b^3) same? in this question he asked, “We don’t know ma’am, just I thought.” There was applied misapplication of rules.

Misinterpretation of symbolic notation. This error was seen in situations Fraction form of the algebra with different denominator is given. Learners misinterpreted the binomial terms as monomial terms. They misinterpreted the sign '+ 'and '- '. In question 1.7, learners divided each term of the algebraic fraction by y and were just cancelling the 'y' not thinking what they need to do. The extract below demonstrates how one of the learners cancelled the terms

Learner 16

Figure no.5



R: Read the question 1.6 and 1.7.

L: Yes, I read it.

R: What does the question mean?

L: We have to simplify the question by making denominator same.

R: Did you make the denominator same? By How?

L: Yes ma'am in 1.6 there was, 1+1/y , here we need to make denominator y in both terms. In second term there was already y in denominator so in 1st term only, we need to multiply by y in numerator and denominator both. Answer is y^2+1/y

In next step, answer will be y+1 because in denominator there was y in both term so we can write one y only and in numerator there was y+1 so one y from numerator and denominator is cut.

R: can we cut the y on numerator and denominator which is in adding form.

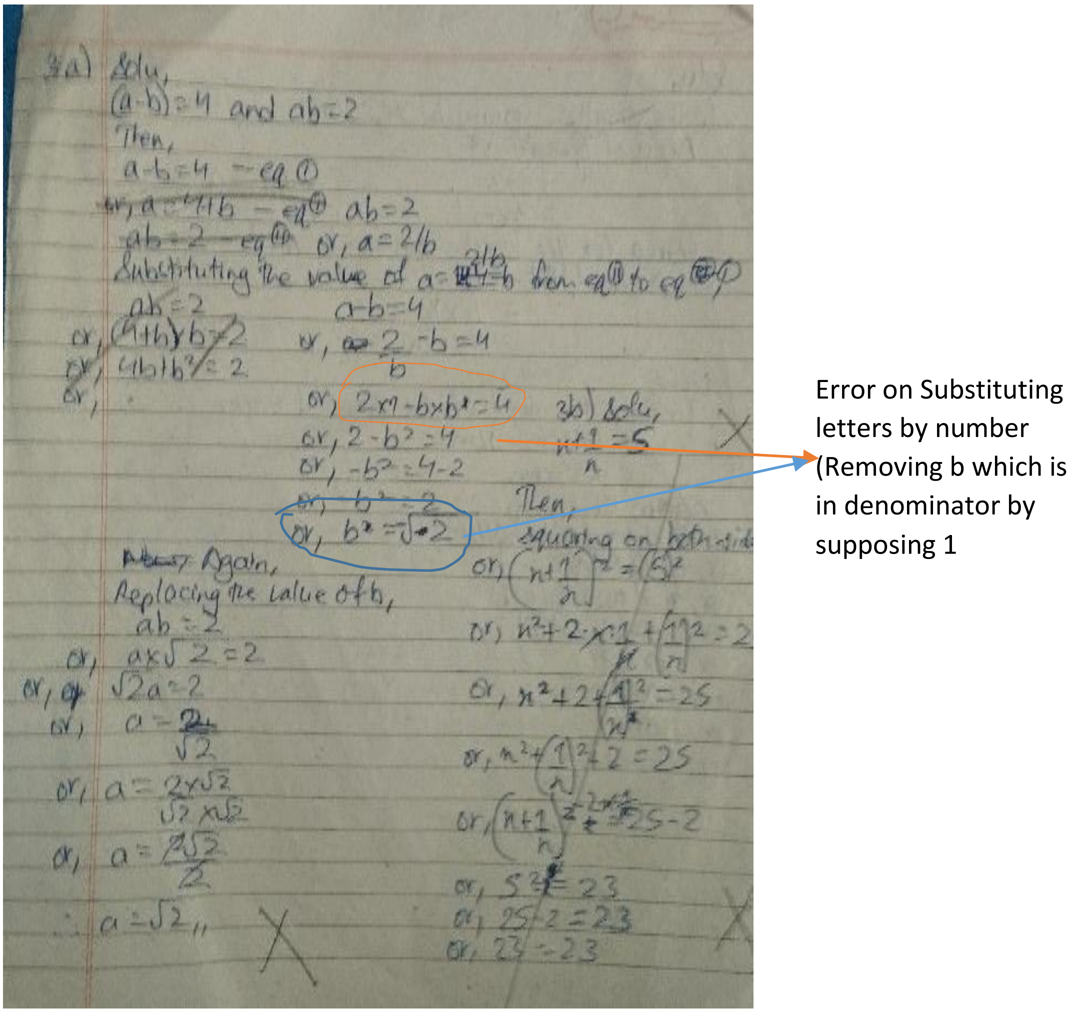
L: yes, ma'am because y and y are like terms.

The learner misunderstood canceling, as seen by the response they provided in response to the sample above. The cancellation implied nothing remained for the learner because something had vanished. The student lacks a fundamental comprehension of algebraic expressions.

Substituting letters by numbers. Variables are not understood by learners. They do not comprehend what a variable means. Question 3.1, which asked students to calculate a2+b2and (a-b)=4 and ab=2 are supplied, made this very evident. It was found that 48% of the students provided 9 incorrect answers. Below is an excerpt from two learners' solutions to item 3.1.

Learner’s:14:

Figure no.6



R: Please read the question no.3.1

L: yes, I have read

R: What does the question mean?

L: Here are the value of (a-b)=4 and ab=2,We have to find the value of a^2+b^2.

R: How do you solve the question?

L: We have to find the value of a and b by using the given vale and find square of a and b and need to add.

R: Explain to me how you came up with this solution.

L: ab=2,so value of a=2/b is the 1st equation.

And substitute the value of a in a-b, it becomes 2/b-b=4

Or, (2 x 1-b xb)=4 (here the learner forget to write denominator.)

Or, $2-b^2=4$

Or, $-b^2=2$

Or, b=√2

Again substituting the value of b on equation 1st and get the value of a=√2

After this step, I forget to find the answer.

R: Is your answer correct?

L: yes, ma’am.

R: Did you think here is next method to find the value?

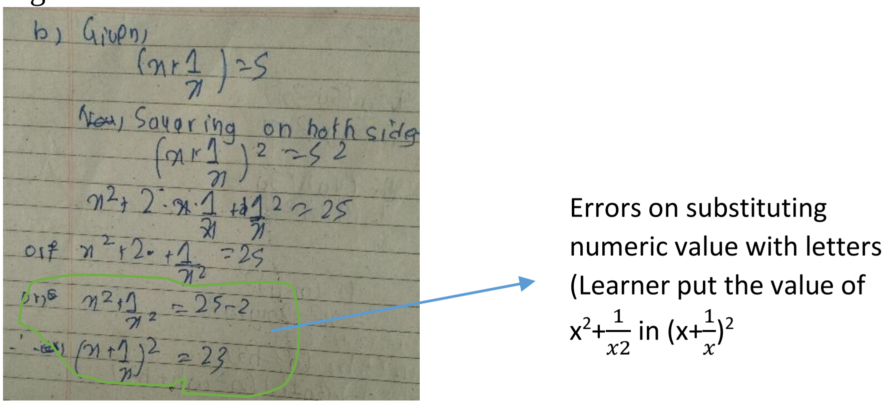
L: No ma’am, I think this method only.

R: okay, thank you.

L: You are welcome ma’am.

Learner 31

Figure no.7



R: Can you please go to question 3. 2. Read the question and the solution that you provided for that question.

L: I am through.

R: Explain to me how you came up with the answer 23.

L: It is because (x+1/x)=5 and need to find the value of $(x+1/x)^2$

R: So what?

L: So In first stage, i am finding the square of both sides it became $(1+1/x)^2=25$

In 2nd stage I factorize $(x+1/x)^2$ as $x^2+2x * 1/x +1/x^2$ and in right side there was 25.

In 3rd stage, x^2+1/x^2 remaining in left side and 25-2 in right side.

R: Why did you factorize$(x+1/x)^2$?

L: We have to find value of x^2+1/x^2

R: I think, you have not read the question properly. You need to find the value of $(x+1/x)^2$ only. No need to find the value of $x^2=1/x^2$.

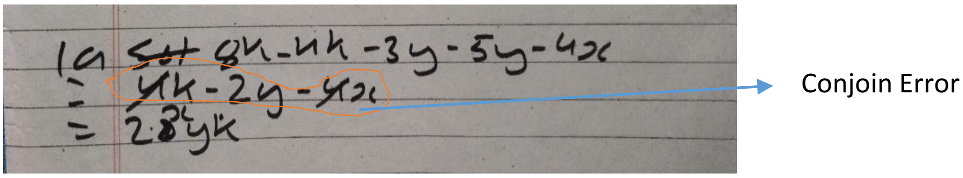
L: okay ma'am but I think $(x+1/x)^2$ is same as x^2+1/x^2.

In the analysis of this response given by learner 14 and learner 31, It is abundantly evident that the variable idea was not fully grasped. Learners appeared to have inherent misconceptions about the variable. They chose to erroneously substitute the specified value of words for that reason.

The conjoin error. According to Tall and Thomas (1991), Learners are not familiar with the fundamentals of algebraic expressions. Students were multiplying coefficients, constants, and coefficients of dissimilar terms. By 26% of learners, the equation 8k-4k-3y-5y-4x was provided as 2xyk. The other expression where learners added unlike terms was when they were asked to expand and simplify a(x+y). While acquiring ax+ay was easy, they ended up giving the response of 2axy. The following excerpts show how learners 7 and 28 also combine incorrect terminology.

Learner 28

Figure no.8



R: : Look at question 1.1. Go through the question and the solution that you provided.

L: I have done.

R: How did you come up with your answer?

L: The question told me to 8k-4k-3y-5y-4x and I did it.

R: How did you do it?

L: I subtract 4k by 8k and get answer 4k, 5y by 3y and get answer 2y and 4x copied from question, and in next step I cut 4 and 4 from 4k and 4x , remained x and k only from 1st and 3rd term and in 2nd term there was 2y so my answer was 2xyk.

R: If you look at the question 8k-4k-3y-5y-4x, there 8k,3y and 4x are like terms?

L: No they are not.

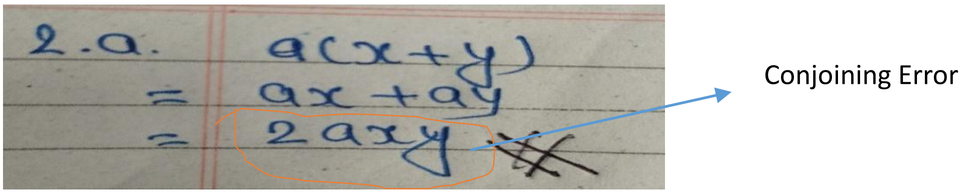
R: So why did you subtract them?

L: The question said simplify that's why I subtracted to get 1 number.

The learner seemed to have taken the subtraction sign as an action verb. This occurred as a result of incorrect question interpretation. For the learner, the word "simplify" had a different meaning. The query seemed to imply that the terms should be condensed into one. The learner in this instance might have agreed with Mamba's (2012) assertion that students may assume algebraic expressions will behave similarly to arithmetic expressions.

Learner 7

Figure no.9



R: Go to question 2.1 and read the question and the solution that you provided. What is your solution?

L: My solution is 2axy.

R: How did you come up with that answer?

L: I removed brackets first and got ax+ay and since the question said I must remove brackets and simplify, I went on to simplify to 2axy.

R: How did you simplify?

L: I added a and a to get and got 2a then I put and my result was 2axy.

R: Is that correct?

L: Yes, it is.

R: If you add x and y what do you get?

L: I cannot add them.

R: Why?

L: They are not like terms.

R: So are ax and ay like terms?

L: No, I think I made a mistake. I was supposed to leave it as ax+ay.

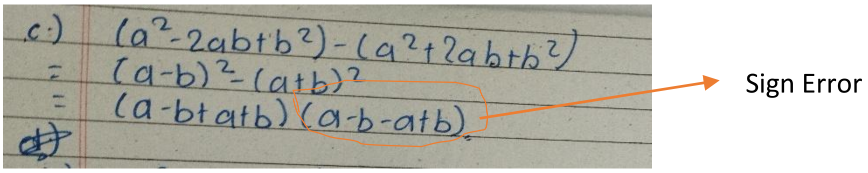
The way the learner answered to the researcher's probing inquiries demonstrates once more that the learner had a different understanding of what the word "simplifies" meant. The learner simply understood the phrase "simplify" to mean "reduce to a single term" and did not evaluate terms that are similar and dissimilar. According to Booth (1998) and Davis (1995), the learner may have struggled to comprehend the absence of closure following expansion. It is clear from the preceding two paragraphs that some learners lack the algebraic facts and concepts that are necessary for success. Learners are perplexed by the idea of similar phrases. When students saw the addition and subtraction sign, they interpreted it as a directive to combine the offered items. Some learners interpret the word "simplify" as requiring them to do away with multiple terms. Because they believed that having an operation sign meant they hadn't simplified, they were unable to consider ax+ay as a potential solution. Because of this, when asked to simplify the phrase ax+ay, the majority of them responded with 2axy.

Sign error. This issue mainly occurred when a negative word was added or subtracted. When signs were applied incorrectly, it had an impact on learners' responses. Both operation and direction indications presented challenges for learners. This blunder was most frequently made when a negative word was added or subtracted..

Taking into consideration the question which instructed learners to subtract $a^2+2ab+b^2$ from $a^2-2ab+b^2$, some learners had their first stages as $(a-b)^2-(a+b)^2$. The second step is (a-b+a+b) (a-b-a-b). The second step already was wrong because of missing brackets. The last two terms in the formula that were to be subtracted from the other were therefore going to operate automatically with the incorrect signs. Some students were aware of the use of brackets in sentences containing more than one term. Even though some of them had correctly written down their second stage as (a-b+a+b)[(a-b)-(a+b)], they were still unable to properly subtract. They experienced issues as a result of ignoring the many indications. In response to this question, 24% of the students gave responses that contained sign-related errors. The researcher realized that learners had problems with combining operation and direction signs. An extract of learner 82 response is provided below to see how learners came up with wrong answers because of failure to deal with both operation and direction signs.

Learner 82

Figure no.10



R: Go to question 1.3 and read the question together with the solution that you provided.

L: I am done.

R: can you please explain how you came up with your answer.

L:At first I used the formula a^2-2ab+b^2 as $(a-b)^2$ and a^2+2ab+b^2 as $(a+b)^2$ In 1st stage the answer became: $(a-b)^2$- $(a+b)^2$. I compare this stage with $a^2-b^2=(a+b)(a-b)$ and 2nd stage It became(a-b+a+b) (a-b-a+b)

R: Explain how did you subtract.

L: In 3rd stage, I subtract a by a and b by b in second term,so answer became 0.

R: While you subtract any term, no need to keep bracket?

L: No, like as plus sign we can use minus sign also.

R: That means you think you have done correct answer, am I right?

L: Yes ma'am.

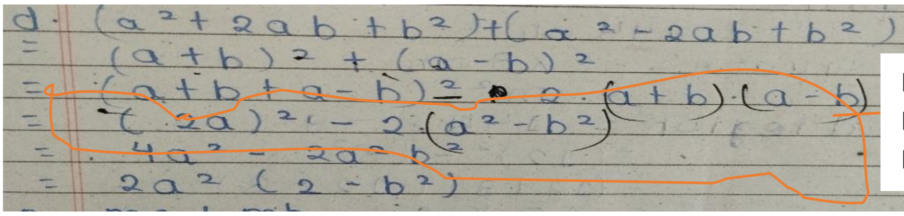
The excerpt shows learners' incompetence in dealing with integers. The error in the learner's response emanated from the learner's failure to deal with signs. It was difficult for the learner to combine operation and direction signs.

Misuse of the distributive property. Many learners struggled to use brackets effectively. They were unsure of the precise phrases that would need to be multiplied by the pre- or post-multiplier. Some students multiplied terms even outside of the brackets, which were meant to be added or subtracted, using the pre-multiplier. After multiplying with the information in the first set of brackets, the other students moved on to the second set of brackets and multiplied using the same pre-multiplier. The three student excerpts that follow show how students abused the distributive property.

The excerpts below for one learners illustrate how learners failed to apply the distributive property.

Learner 24

Figure no.11



Misuse of Distributive Property

R: Go to question 1.4 and go through it together with your solution to it. Now explain to me how you got.

L: I got it by multiplying.

R: How did you multiply?

L: I was using the factorization formula $(a-b)^2$,$(a+b)^2$ and a^2-b^2 get into 3rd stage.

R: In third stage there was $(2a)^2- 2(a^2-b^2)$, Am I right?

L: Yes ma’am

R: In forth stage, what did you do, Can you explain it?

L: Yes ma’am, In forth stage I am taking $2a^2$ common from $4a^2-2a^2b^2$.

And answer is $2a^2(2-b^2)$

R: How did you get $2a^2b^2$ from $2(a^2-b^2)$?

L: I mistakenly write $2(a^2-b^2)$ in the place of $2a^2b^2$ so I have done correction in next stage.

R: If you multiply (a+b)(a-b) , did you get a^2b^2?

L: yes ma’am multiply a with a and b with b and between the bracket there is nothing that means we need to keep multiply sign.

R: No need to multiply a with b in that question?

L: No ma’am we have to multiply 1st term by 1st term and 2nd term by 2nd term.

R: okay, thank you.

This excerpt shows lack of understanding of the distributive rules. The meaning of the invisible multiplier was not understood. The learner could not identify the limits of the bracket multiplication.

Encoding Error. 52 students made error in the verbal problems related to algebraic expression and 32 of the students had not attempted the question because of the encoding problems.

When I asked any 3 students who were not attempting the question no.4 to do.

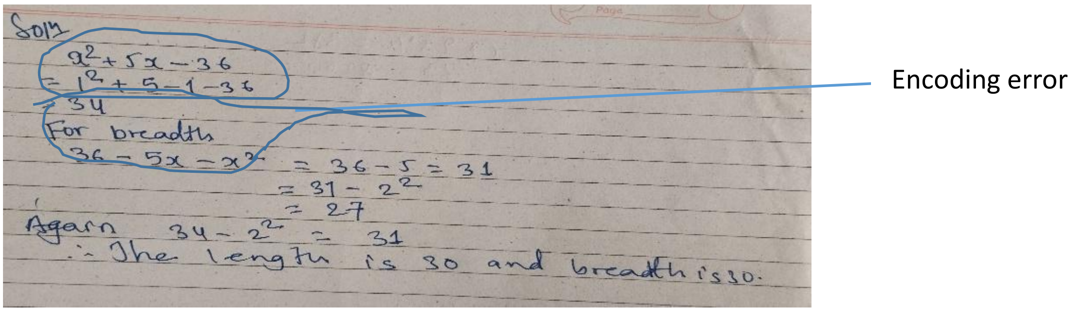
They asked me “We could not understand the question. “

Students give the same answer they don’t know how to write the mathematical sentence of given verbal problems.

This answer shows lake of understanding the verbal problems and difficult to make a mathematical sentence from verbal problems. We can show the excerpt of 1 learner answer of qn.4.

Learner 4

Figure no 12



R: I want you to check question 4 and your solution.

L: I am done.

R: How did you get 30 as answer?

L: Given area is $x^2+5x-36$, we have to find the length and breadth.

R: Explain me about your process in details.

L: I put the value of x as 1 because if there is no value we can put 1 and I solved it for length then got 34 lengths as answer same as I interchange the place of 36 and x^2. After I wrote $36-5x-x^2$ after cancelling x and x there was 36-5 and answer was 31.

By 2nd condition we have to reduce length and breadth 2/2 that means we have to reduce 4 units from the area so I done 34-4= 30 as answer.

R: Is the question asked you to find this type of answer?

L: When I read the question, I thought I have to do that type so I did ma'am.

R: okay thank you.

Above excerpt shows that lake of understanding the verbal problems and difficult to make a mathematical sentence from verbal problems.

Chapter V

Discussion , Conclusion And Recommendation

The main goal of this research study was to identify the most common errors made by learners in simplifying algebraic expressions and also to find out the possible causes of those errors. While trying to simplify algebraic expressions, students made a number of mistakes. This chapter discusses the study's findings, ties them to the theoretical underpinnings and literature review related to the study, draws conclusions based on the findings, and makes suggestions for further research and educational implications.

Discussion of the Findings

A sample of 100 learners participated in the study. A 20-item exam was administered to the subjects. Learners were required to write answers and exhibit their work during the test. The first research question of the study, which sought to identify the typical mistakes made by Grade IX students in simplifying algebraic expressions, was addressed by analyzing the solutions provided by the learners. The researcher evaluated and interpreted the responses given by the students. Following an item analysis, mistakes were coded, as shown in Table 1. The most frequent mistakes from test replies from students were broken down into six categories by the researcher. From highest to lowest frequency, the following six common mistakes were found: misusing the distributive property, misinterpreting algebraic notation, conjoining terms, replacing letters with numbers, sign mistakes, and using the wrong rules to simplify algebraic expressions. Ten students were chosen for interviews after the test scripts were examined in connection to the prevalent faults found. The interviews were intended to address the second research question, which tried to determine the reasons why test takers' attempts to simplify algebraic formulas were unsuccessful.

Misapplications of Rules and Misinterpretation of Symbolic Notation

Misapplications of rules.

The study's conclusions point to rule misapplication as the primary reason for mistakes made when simplifying algebraic statements at the Grade IX level. According to the analysis conducted for this study, students in Grade IX commonly misapplied the rules when trying to simplify algebraic formulas. In this study, incorrect application of the rules was the cause of 15.9% of the errors. Students appeared to have become confused and applied regulations incorrectly. This was consistent with Watson's (2007) discovery that learners become perplexed and incorrectly apply or recall rules for altering phrases. Demby (1997) and Kieran (2007) make the additional claim that many students find the vocabulary and algebraic rules to be meaningless, leading them to memorize the rules with little to no conceptual comprehension. According to Usman (2012), the majority of students just learn how to manipulate rules without considering the meaning of the statement being twisted. As a result, it is challenging for students to follow the rules or use them correctly. According to Erlwanger (1975), in other circumstances, learners make up their own rules that only apply to them. Mbewe (2013) claims learners abuse previously learned procedures and norms in contexts where they are inappropriate. When asked to simplify the phrase (a^3-b^3), most students responded with |(a-b)(a^2+ab+b^2), although others suggested (a^3-3a^2b +3ab^2 –b^3). To solve a^3-b^3, students utilized the formula $(a-b)^3$. This instance demonstrates that students were unable to distinguish between $(a-b)^3$ and (a^3-b^3).This uncertainty results from students' attempts to build knowledge. According to some researchers, when learners attempt to develop the meanings of mathematical information, errors result from misunderstandings from prior knowledge (Luneta and Makonye, 2010). Learners become perplexed while constructing mathematical meanings. Too much meddling from students' peers, professors, and the outside environment, as well as from the students themselves, is the cause of the confusion. Tall and Vinner (1981) contend that errors also come from naive idea pictures that fall short of concept definitions. The constructivists agree that mistakes might result from misunderstandings of prior information. Constructivists believe that learning occurs when existing information is transformed and refined into more complex conceptions, according to Smith et al. (1993). According to Brodie (2014), the emergence of constructivist theories of learning has positioned errors as performances of misunderstandings, conceptual frameworks that learners have created and that make sense to learners in light of their existing knowledge. The pre-knowledge that students bring from prior grades means that they do not enter a new grade as blank slates (Hatano, 1996). Learners, as suggested by Olivier (1989) then use that knowledge to assimilate and adapt new mathematical concepts. The problem is that at times prior knowledge conflicts with new knowledge making it difficult for learners to judge what is correct or not. Because they don't connect new knowledge to prior knowledge, learners go on to make mistakes. The researcher concluded that this inaccuracy was primarily caused by interference with previously learned concepts after taking into account the justifications offered by students during the interview. Looking at the equation $(a-b)^2$, where the most common answer was ab^3 or a^3-b^3, the student said that $(a-b)^2$ meant to multiply a by 2 and b by 2 to create a^2-b^2. The students attempted to incorporate new mathematical concepts into an inadequate, incorrect framework. However, it appeared that students lacked strong math abilities. Because they had no prior knowledge of mathematics, learners made mistakes. This is supported by Norton and Irvin (2007), MacGregor and Stacey (1997) who also indicated that poor arithmetic skills contribute to algebraic errors.

This study has found that students in Grade IX incorrectly apply algebraic rules as a result of interference from other ideas and also because there are many rules in mathematics. Some of the rules are confusing to learners, making it challenging for them to remember them.

***Misinterpretation of symbolic notation*.**

When the 'invisible' coefficient of 1 appears in an equation, learners understand the symbolic notation incorrectly (Seng, 2010). Since there was no number before the letter in the test, students believed that concepts like x and y had no coefficients. Some students believed there was nothing left after dividing a phrase by another term that was equal to it. According to the learners' expectations, nothing would remain after equal terms were cancelled, according to Barcellos (2005). The student who was asked to simplify the algebraic formula 1+1/y made the denominator equal in the first stage, but in the second stage, he cancelled y^2 with y, which is in the numerator in adding form with 1. This means that he multiplied by y in both the numerator and the denominator. Response is y^2 +1/y.

(In next step, answer was y+1 because in denominator there was y in both term so we can write one y only and in numerator there was y+1 so one y from numerator and denominator is cancelled.)

After dividing y by y, the learner added the result. He/she is unaware that in order to cancel any terms that are in the denominator and numerator, we must first take the common element if it exists; otherwise, the term cannot be cancelled. According to Barcellos (2005), learners incorrectly eliminate terms when simplifying algebraic expressions because they do not apply the mathematical principles they have learned for rational numbers to irrational or complex numbers. When the learner who was interviewed's explanation is taken into account, it is clear that the learner did not make the connection between algebra and arithmetic. The student had no trouble providing the right response when asked what would happen if you divide 2 by 2. The student then understood that m divided by m ought to have produced a 1. This is a failure to apply arithmetical meaning, according to Watson (2007). According to Chamundeswari (2014), some errors can be attributed to a lack of basic understanding of mathematical procedures.

Misuse of the Distributive Property

Misusing the distribution property was the second most frequent mistake, accounting for 4.2% of the mistakes made by the students. When attempting to eliminate brackets, students made mistakes. According to Kieran (1992), certain errors were caused by a lack of necessary facts and conceptions. The distributive property was demonstrated by learners as instrumental knowledge. The students became perplexed at this point and were unable to even recognize the brackets' boundaries. Seng (2010) found that the distributive property was misused in a variety of contexts. The situation with this investigation was the same. According to Barcellos' (2005) theories in his suggestions for potential sources of mistakes related to the extension of brackets, errors came from invalid or incomplete distribution. According to Moodley (2014), students don't understand what brackets mean. Moodley (2014) asserts that learners immediately associate brackets with multiplication when they see them. In this study, some students merely factored the pre- or post-multiplier into the first value enclosed in brackets. For instance, they obtained $a^2(2-b^2)$ when asked to simplify $(a^2 -2ab+b^2)+(a^2 +2ab+b^2)$. Other students completed the next step and opened the bracket by comparing the term $(a^2+b^2)=(a+b)^2-2ab$, resulting in the student's writing of (a-b+a+b)-2(a-b)(a+b), after which the student opened the bracket by multiplying a with a and b with b in the second term, but the student did not see any sign between the two brackets so s/he wrote it as $2a^2 b^2$. There is a '-' sign before b in the second and first terms, thus the learner needs to multiply (a-b) by (a) and (a-b) by (b) one at a time. However, they did not do it this way. This indicates that these students understood brackets to have a completely different connotation. Items where students have to take off brackets serve as proof of this. Similar results were also observed by Moodley's (2014) study. Moodley (2014) discovered that students multiply bracket even when a plus or negative sign is present. Learners only partially understood how to expand brackets. The students were using crude schema.

Sign Errors

In the test, this mistake was weighted at 6.4%. Due to their mistakes with integer subtraction, learners erred when simplifying algebraic formulas. Seng (2010) found that students made more mistakes when negative integers were used as coefficients in algebraic formulas in his study on error analysis of Form 2 students. In this study, the negative integer also caused issues for the students, causing them to provide incorrect answers. The signs issue is a sign of a weak foundation in mathematics or an inability to make the connection between mathematics and algebra. According to Norton and Cooper (1999), some learners disregard directed number operating laws and orders of convention. As a result, some of the signs are disregarded by these students. Sometimes students correctly explain the first part of their solutions, but then they make numerous errors when they try to simplify things (Mamba, 2012). This was demonstrated by the students in this study when they were asked to subtract the second expression from the first expression in a question. The students had correctly From that point on, the majority of students struggled with the middle items, where they ultimately scored 3x–7x. Due to this, the students' final response was 3x 2 -4x +2. bra. According to the interviewed learner's explanation, there was already a minus on 7; so, he used that minus to subtract as required by the question. Errors resulting from subtraction of integers prevail because learners have difficulties in operating with negative integers (Seng, 2010). The different uses of the negative sign as suggested by Villasis (2004) are counterintuitive and an obstacle for learners. Learners need to overcome numerous obstacles for them to have few problems in algebra including interpretation of operations (Lee and Messner, 2000). In this study

In situations where there were two sets of brackets and only the first set having a visible pre-multiplier, learners used the visible pre multiplier for both sets of brackets. This was evidenced on the item which had x(a-b)-y(a-b) become xa-xb-ya-yb as an answer. Learner did not check the sign which is in 2nd term before the bracket. The researcher discovered that learners had difficulties in bracket expansion. Learners inappropriately used the distributive property in a variety of ways as shown in the analysis of learners' solutions. Interview responses also showed that learners had many misconceptions about bracket expansion. These misconceptions led them to produce errors.

Conjoin Error

The third-highest number of test mistakes were caused by this issue. 3.4% of the time, this error occurred. This kind of mistake was committed by students since they didn't understand the idea of an algebraic expression. Letters were disregarded, and students focused on numerical values. Then, in their responses, they simply added letters. Additionally, they added constants and coefficients as well as a letter at the end. According to Booth (1999) and Davis (1995), addition is seen as "an action symbol" by students. According to Mamba (2012), the plus sign could have been viewed as a signal to combine terms. This might be the cause of the learners' phrase conjoining. They believed that "+" meant to combine terms, whether they were similar or dissimilar. Brodie (2014) saw errors as resulting from conceptual structures created by students, drawing on a constructivist viewpoint. According to Brodie (2014), although the conceptual structures are not in line with conventional mathematical knowledge, they make sense in light of the learners' prior knowledge. Answers are single-digit numbers in arithmetic, but not in algebra. The hypothesis that learners connect the concept of single-term answers in arithmetic to algebra is supported by Tall and Thomas (1991). For some students, the term "simplify" meant "reduce to a single term.

"Most learners showed that the simplified answers for items 1.1 and 1.2, respectively, were 8k-4k-3y-5y-4x equal to 2kyx and 3pc-4+5cp+8 equivalent to 14pc. Since they held more than one term, they could not accept that 4k-8y-4x or 8pc+4 could be the definitive solutions. Due to the plus and minus signs, students believed that all terms should be combined to form a single term. It was thought that the equation 8pc+4 was not complete. As a result, students chose to finish it by making it a single term.

Failure to recognize similar phrases was the main contributor to this inaccuracy. Most students just inserted terms that contrast. The + sign, which they mistook for a command to combine phrases, deceived them. The word "simplify" originally meant "reduce to a single term" for students. This implies that learners struggle to accept a lack of closure and, as a result, fail to complete or finish expressions. The comments from the students showed that they had completed their expressions.

Substituting Letters by Numbers

3.1% of the test-related errors that were found were attributable to this inaccuracy. It was mostly found in items 3.1 and 3.2, where the majority of students believed that (a-b)= 4 and ab=2 were supplied in order to get the value of $a^2 + b^2$, in which case it would be necessary to first calculate the values of a and b using the given value. The students changed b to a/2 and changed the value of b in the first equation (a-b=4). The learner who was being interrogated assertively stated that he had substituted a/2 for b to obtain the value of a^2+b^2. This meant that instead of utilizing the $(a+b)^2$ form to calculate the value of $a^2 + b^2$, the students believed they needed to calculate the values of a and b. This implies that learners did not take letters to represent unknown numbers.

When the responses of learners imply that a letter has a numerical value, the error of substituting letters for numbers is made. The reason for this, according to Christou, Vosniadou, and Vamvakoussi (2007), is that students frequently draw upon their prior knowledge of numbers when performing arithmetic. Variables are given numbers by learners (Kuchemann, 1978). Additionally, this was found by MacGregor and Stacey (1997) in their study on students who were about the age of 15. According to McIntyre (2005), this letter-to-number substitution is the result of students' poor understanding of the variable.

Learners displayed lack of meaning of a variable. Learners possess little knowledge of a variable because the meaning of a variable is often neglected in the teaching and learning of algebra (Usman, 2012). This results in the learners only knowing algebraic manipulation.

Encoding Error

The answer sheets show misunderstanding and encoding error on meaning of variables, constant and format of factorization relating to verbal problems. This error contributes 2.6% of the errors which were identified in the test. It was mostly identified in item 4. 32 students had not attempted this question to solve because of not understanding the question what to find and how to find the solution. They could not make a schema to find the solution of given problems. When the researcher asked why did you not attempt the question no.4 to solve? 32 student gave same answer, we don’t know what we need to find and how to do, how can we encode the verbal problems into mathematical sentences. When researcher explain the question some of the students were trying to find the solution but they done incompletely and some of the students could not think any solution after explaining also. One interviewed factorized the question by supposing x as 1 like $x^2+5x-36=1-5-36=34$ as length and 36-5=31 as breadth. This implies that learners did not understand the question what to do and how to solve.

Conclusion

The chapter presented a discussion of the six common errors identified in the study. Related literature has also been used to support the findings of the study. The explanations of the origins of the errors have been related to existing literature in a way linking them to broader theoretical views. Having discussed the findings of the study,the next sub point provides conclusions and recommendations based on the study’s findings.

The researcher has been able to identify the most frequent mistakes students make when trying to simplify algebraic formulas thanks to the two phases of the investigation. It also made it possible for the researcher to identify potential root causes of those errors. The researcher identified six categories of faults as the frequent mistakes made by students while they are trying to simplify algebraic statements. These mistakes include incorrect rule application and symbolic notation interpretation, incorrect sign usage, improper use of distributive property, letter substitution with value, conjoining error, and the most prevalent mistake in this study, misunderstanding. The fact that mathematics has so many rules that confuse students is the main reason why students misapplied the rulesLearners frequently learn how to manipulate rules without considering their meaning. According to Mbewe (2013), learners utilize a lot of rules inadvertently since they don't understand the structural aspects in the conceptual space. Then, when using established rules in appropriate circumstances, learners erroneously do so. When asked to simplify y+1/y, the learner came up with the answer y2+1, indicating that he or she had the right concept about where to find the numerator but was unaware that it was the numerator. The student failed to enter the denominator as a result. According to Posamentier (1988), learners' naive assumptions, preconceptions, and misconceptions regarding mathematical concepts obstruct their ability to learn. The notions and beliefs that learners have gained, according to Nesher (1987), are incorrectly applied to a wider domain. Mbewe (2013) also found that learners occasionally utilize poor solution schema due to conceptually superficial similarities rather than formal ones. In addition, according to Fischbein and Barash (1993), learners have deeply ingrained solution schema that, despite intuitive comprehension, they erroneously apply. As a result, students have the right techniques stored in their long term memory. They might have trouble remembering them, which could lead to them applying the rules incorrectly. The incorrect distribution of brackets was the second most frequent issue. This mistake resulted from improper use of the distributive property. The literature primarily blames this blunder on students' inability to master necessary knowledge and ideas. The third most frequent mistake was combining terms. This mistake showed that the students didn't comprehend what like and unlike phrases meant. The error caused by replacing letters with digits came in at number four in terms of frequency. This mistake was caused by the students' poor knowledge of variables. The mistake of misusing signs came right after the one of replacing letters with numerals. There was evidence that the use of direction and operation indicators was challenging for students. The least frequent error involved incorrectly interpreting symbolic notation. Due to the fact that most teachers encourage the usage of notations, this error was not frequently made. The examination of the research data has been the chapter's main topic. The next chapter discusses key issues that were identified from the analysis of test data and interview transcripts.

The discussion of the key issues will be linked to research studies on errors in algebra as well as to the teaching and learning of Mathematics in general.

This chapter presents conclusions of the study based on the analysis and research findings. The chapter starts by stating the aim and objectives of the research, and research questions.

Recommendations for Teaching/Learning

The study's key findings indicated that learners lacked a fundamental understanding of algebra, and as a result, teachers should help students master concepts like grouping like and unlike words and expanding brackets to add and subtract directed algebraic expressions. Understanding the fundamentals of algebra will help you grasp its procedural and conceptual features. Teachers should take the constructivist viewpoint into account and be in a position to give students a solid foundation in mathematics so that they may apply it to algebra.

It is advised for teachers to employ instructional strategies that help students learn both procedural and conceptual material. It should be possible for students to explain their responses using the teaching strategies. Teachers should pay close attention to the explanations given by students, be able to see any misconceptions they may have, and come up with strategies for assisting students in understanding algebraic ideas.

Teachers must foster an environment in the classroom where students can draw their own conclusions from the procedural and conceptual knowledge they are being taught. In order to dispel myths, students should be encouraged to discuss their algebraic challenges and accomplishments. To address the issue of individual variances, students should occasionally receive individualized attention.

A range of algebraic expressions should be provided for learners to simplify. Giving students a range of algebraic expressions, in Falle's opinion, allows them to explore the various ways that algebraic expressions can be spelled down simply. The usage of algebraic manipulation and representation will become second nature to the students.

The researcher advises that there is a need to identify the role of the teacher in the mistakes made by students when simplifying algebraic expressions because the study's findings showed that the methods and approaches used by the teacher to teach algebraic concepts have an effect on how learners grasp the concepts. It is helpful to comprehend how the teacher presents the topics to the students. This will make it possible to determine how much of the students' errors were due to the teacher. The study also advises widening the research by focusing on algebra as a whole rather than only on simplifying algebraic statements. This might increase the research's applicability to the context of teaching and learning. The researcher also suggests the use of a bigger sample including participants from several schools.

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Appendices

Appendix 1

Mathematics Achievement Test question:

Researcher select 20 item test on simplifying algebraic expressions in 5 group.

Criteria to take the question

1. In question 1, Test the ability to use acquired arithmetic knowledge of basic operations and allow identification of arithmetic- algebra gap.

(Here would be 7 question. Among them last 3 questions taken from fraction)

2. In question 2, Test the ability of learners to multiply algebraic terms and collect like terms.

(Here would be 9 questions)

3. Question 3 and question 5 will design by the researcher in a way to test learner's understanding of letters representing numbers.

4. Question 4 will design to measure the learner's ability to read and understand questions and solve the problems.

Topic Algebraic Expression **F.M: 30**

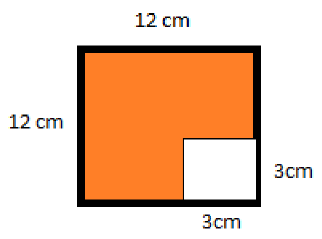
Time: 1 hour

Attempts all question:

1. **Simplify and add like terms where possible.**
   a. $8k - 4k - 3y - 5y - 4x$ 1
   b. $3pc - 4 + 5cp + 8$ 1
   c. $(a^2 - 2ab + b^2) - (a^2 + 2ab + b^2)$ 1
   d. $(a^2 + 2ab + b^2) + (a^2 - 2ab + b^2)$ 1
   e. $\frac{ma+mb}{m+md}$ 1
   f. $y + \frac{1}{y}$ 2
   g. $x - \frac{1}{3x}$ 2
2. **Remove the bracket and multiply. (If there is to determine formula, you have to write it also)** **(1 x 9=9)**
   a. a(x+y)
   b. a(5y – 3x)
   c. (5a)(4b)
   d. (4a)(3a)
   e. x(a-b)-y(a-b)
   f. $(x-y)^2$
   g. (x+a)(x+a)
   h. $(a^3 - b^3)$
   i. $(a - b)^3$
3. a. If (a-b) = 4 and ab=2, find the value of a^2+b^2. 2
   b. If $(x+\frac{1}{x}) = 5$, find the value of $(x+\frac{1}{x})^2$. 2

4. **The area of a rectangular ground is $x^2+5x-36$ unit. If its length and breadth are reduced by 2/2 unit, Find the new area of the ground.** **4**

5. **Find the area of the shaded region using $a^2-b^2= (a-b) (a+b)$** **4**



The End

Appendix 2

Interview Guide

Process	Interview question
1. Read	Please read the question.
2. Comprehension	What does the question mean?
3. Strategy selection	How do you solve the question?
4. Process	Now look at how you solved the question in the test.
5. Explanation	Explain to me how you came up with this solution.
6. Consolidation	What does the answer mean?
7. Verification	Can you check the correctness of your answer
8. Conflict	Ask conflicting questions if there is conflict in Solving

Appendix 3

Item / Learner	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.1	3.2	4	5
L1	7	7	4	2	1	4	4	1	1	1	7	1	1	7	1	5	1	8	7	1
L2	3	7	7	7	4	4	4	5	5	1	1	5	5	5	4	4	6	1	8	8
L3	7	1	2	4	1	4	4	1	1	5	5	1	1	1	7	4	1	8	8	8
L4	7	4	7	2	2	2	2	1	1	5	4	1	4	4	2	2	6	6	5	5
L5	1	1	1	1	1	4	4	1	1	1	1	1	1	1	1	1	1	1	1	1
L6	1	7	7	2	2	2	2	1	1	1	4	7	4	4	4	2	6	2	2	6
L7	1	1	4	4	4	4	4	1	1	2	1	1	1	3	2	2	1	7	8	5
L8	1	1	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	1
L9	1	1	1	1	1	4	4	1	1	1	1	1	1	1	1	1	1	1	1	1
L10	1	1	4	4	2	2	2	1	1	2	3	1	1	3	4	4	4	1	2	8
L11	1	1	5	1	2	1	1	1	1	1	7	5	1	1	1	1	1	1	5	1
L12	1	1	4	4	2	2	2	1	1	2	2	1	1	2	4	4	6	6	2	8
L13	1	1	4	4	2	2	2	1	1	2	2	1	1	2	4	4	6	2	5	2
L14	1	3	1	1	1	4	4	1	1	1	1	7	1	1	1	4	6	6	2	1
L15	7	3	5	5	4	4	4	1	1	1	1	7	1	1	1	4	6	1	2	1
L16	1	1	4	4	1	4	4	1	7	1	7	1	1	1	1	4	1	1	1	1
L17	1	3	1	4	1	1	1	1	1	5	5	7	1	1	1	4	1	6	8	1
L18	1	3	1	2	1	4	1	2	1	1	1	1	7	1	1	4	1	1	2	1
L19	1	3	1	4	1	4	1	1	1	1	1	7	1	1	1	4	1	1	2	1
L20	7	7	4	4	4	4	4	1	1	1	1	2	2	2	2	2	6	6	2	1

L21	1	3	1	1	1	4	2	1	1	2	2	1	2	2	2	2	4	8	1	2
L22	1	3	1	1	1	4	4	1	1	1	1	1	1	1	4	1	6	6	2	1
L23	1	3	4	4	1	4	4	1	1	2	2	1	1	1	1	4	4	1	1	2
L24	1	1	1	5	1	4	4	1	1	1	1	1	4	1	1	4	1	1	1	2
L25	1	1	7	4	3	4	4	1	1	1	1	7	1	1	4	7	6	1	8	1
L26	1	7	4	4	2	2	2	1	1	2	7	7	1	1	2	2	6	6	8	8
L27	1	7	4	4	2	2	2	1	1	3	7	7	1	1	2	2	6	1	2	1
L28	3	3	7	7	4	4	4	2	3	4	6	2	2	4	4	4	6	6	7	7
L29	3	3	4	4	2	4	4	1	1	1	1	7	6	4	4	2	2	1	5	1
L30	7	3	7	7	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2
L31	3	1	7	5	4	4	4	1	1	1	1	1	1	4	1	1	1	6	1	2
L32	1	1	1	7	1	2	2	1	1	1	1	1	4	3	1	4	1	1	4	1
L33	3	3	5	5	2	2	2	1	1	1	1	1	1	1	2	2	2	2	2	2
L34	1	1	5	5	2	2	2	1	1	1	1	1	1	4	2	4	6	1	7	1
L35	1	1	7	5	4	2	2	1	1	1	1	2	2	2	2	2	6	1	8	2
L36	2	7	7	7	2	2	2	1	1	1	7	2	2	2	2	2	6	1	8	8
L37	1	7	2	2	2	4	4	1	1	1	1	1	1	4	2	2	6	1	2	1
L38	3	3	1	1	4	4	4	1	1	1	1	7	1	4	5	4	6	1	8	1
L39	1	1	7	7	2	2	2	1	1	1	1	1	1	4	1	4	6	1	7	1
L40	1	1	4	4	4	4	4	1	1	1	1	1	1	4	4	4	6	2	8	1
L41	1	7	7	4	2	4	2	1	1	1	1	1	1	4	1	2	2	1	8	1
L42	1	1	5	5	4	2	2	1	1	1	1	1	1	1	1	1	1	8	8	1
L43	3	2	2	2	2	4	4	1	1	1	1	1	1	4	2	2	6	1	2	1

L44	7	3	5	7	5	4	4	1	1	1	1	2	2	4	5	4	6	1	8	1
L45	1	3	7	7	2	2	2	1	1	1	1	2	2	2	2	2	6	1	2	1
L46	3	3	4	5	4	4	4	1	1	1	1	7	2	4	5	4	6	1	2	1
L47	3	1	5	4	2	2	2	1	1	1	7	1	4	1	1	1	1	4	1	1
L48	3	1	5	4	2	2	2	1	1	1	1	2	2	2	2	2	2	8	1	2
L49	1	1	5	5	4	4	4	1	1	1	1	1	1	1	1	1	1	1	8	1
L50	3	2	2	2	2	2	2	2	2	1	1	4	4	4	2	2	4	4	8	2
L51	7	7	4	2	1	4	4	1	1	1	7	1	1	7	1	5	1	8	7	1
L52	3	7	7	7	4	4	4	5	5	1	1	5	5	5	4	4	6	1	8	8
L53	7	1	2	4	1	4	4	1	1	5	5	1	1	1	7	4	1	8	8	8
L54	7	4	7	2	2	2	2	1	1	5	4	1	4	4	2	2	6	6	5	5
L55	1	1	1	1	1	4	4	1	1	1	1	1	1	1	1	1	1	1	1	1
L56	1	7	7	2	2	2	2	1	1	1	4	7	4	4	4	2	6	2	2	6
L57	1	1	4	4	4	4	4	1	1	2	1	1	1	3	2	2	1	7	8	5
L58	1	1	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	1
L59	1	1	1	1	1	4	4	1	1	1	1	1	1	1	1	1	1	1	1	1
L60	1	1	4	4	2	2	2	1	1	2	3	1	1	3	4	4	4	1	2	8
L61	1	1	5	1	2	1	1	1	1	1	7	5	1	1	1	1	1	1	5	1
L62	1	1	4	4	2	2	2	1	1	2	2	1	1	2	4	4	6	6	2	8
L63	1	1	4	4	2	2	2	1	1	2	2	1	1	2	4	4	6	2	5	2
L64	1	3	1	1	1	4	4	1	1	1	1	7	1	1	1	4	6	6	2	1
L65	7	3	5	5	4	4	4	1	1	1	1	7	1	1	1	4	6	1	2	1
L66	1	1	4	4	1	4	4	1	7	1	7	1	1	1	1	4	1	1	1	1

L67	1	3	1	4	1	1	1	1	1	5	5	7	1	1	1	4	1	6	8	1
L68	1	3	1	2	1	4	1	2	1	1	1	1	7	1	1	4	1	1	2	1
L69	1	3	1	4	1	4	1	1	1	1	1	7	1	1	1	4	1	1	2	1
L70	7	7	4	4	4	4	4	1	1	1	1	2	2	2	2	2	6	6	2	1
L71	1	3	1	1	1	4	2	1	1	2	2	1	2	2	2	2	4	8	1	2
L72	1	3	1	1	1	4	4	1	1	1	1	1	1	1	4	1	6	6	2	1
L73	1	3	4	4	1	4	4	1	1	2	2	1	1	1	1	4	4	1	1	2
L74	1	1	1	5	1	4	4	1	1	1	1	1	4	1	1	4	1	1	1	2
L75	1	1	7	4	3	4	4	1	1	1	1	7	1	1	4	7	6	1	8	1
L76	1	7	4	4	2	2	2	1	1	2	7	7	1	1	2	2	6	6	8	8
L77	1	7	4	4	2	2	2	1	1	3	7	7	1	1	2	2	6	1	2	1
L78	3	3	7	7	4	4	4	2	3	4	6	2	2	4	4	4	6	6	7	7
L79	3	3	4	4	2	4	4	1	1	1	1	7	6	4	4	2	2	1	5	1
L80	7	3	7	7	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2
L81	3	1	7	5	4	4	4	1	1	1	1	1	1	4	1	1	1	6	1	2
L82	1	1	1	7	1	2	2	1	1	1	1	1	4	3	1	4	1	1	4	1
L83	3	3	5	5	2	2	2	1	1	1	1	1	1	1	2	2	2	2	2	2
L84	1	1	5	5	2	2	2	1	1	1	1	1	1	4	2	4	6	1	7	1
L85	1	1	7	5	4	2	2	1	1	1	1	2	2	2	2	2	6	1	8	2
L86	2	7	7	7	2	2	2	1	1	1	7	2	2	2	2	2	6	1	8	8
L87	1	7	2	2	2	4	4	1	1	1	1	1	1	4	2	2	6	1	2	1
L88	3	3	1	1	4	4	4	1	1	1	1	7	1	4	5	4	6	1	8	1
L89	1	1	7	7	2	2	2	1	1	1	1	1	1	4	1	4	6	1	7	1

L90	1	1	4	4	4	4	4	1	1	1	1	1	1	4	4	4	6	2	8	1
L91	1	7	7	4	2	4	2	1	1	1	1	1	1	4	1	2	2	1	8	1
L92	1	1	5	5	4	2	2	1	1	1	1	1	1	1	1	1	1	8	8	1
L93	3	2	2	2	2	4	4	1	1	1	1	1	1	4	2	2	6	1	2	1
L94	7	3	5	7	5	4	4	1	1	1	1	2	2	4	5	4	6	1	8	1
L95	1	3	7	7	2	2	2	1	1	1	1	2	2	2	2	2	6	1	2	1
L96	3	3	4	5	4	4	4	1	1	1	1	7	2	4	5	4	6	1	2	1
L97	3	1	5	4	2	2	2	1	1	1	7	1	4	1	1	1	1	4	1	1
L98	3	1	5	4	2	2	2	1	1	1	1	2	2	2	2	2	2	8	1	2
L99	1	1	5	5	4	4	4	1	1	1	1	1	1	1	1	1	1	1	8	1
L100	3	2	2	2	2	2	2	2	2	1	1	4	4	4	2	2	4	4	8	2