ERROR ANALYSIS IN PROVING THEOREM BY SECONDARY LEVEL STUDENTS

A

THESIS

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BY

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LETTER OF APPROVAL

Thesis submitted by Jyoti Nath Sharma Entitled ***Error Analysis in Proving theorem by Secondary Level Students*** has been approved in partial fulfillment for the requirement for the degree of Master of Education.

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LETTER OF CERTIFICATE

This is to certify that Jyoti Nath Sharma a student of academic year 2067/2068 B.S. with thesis number 959, exam Roll No. 281402/068 campus Roll No. 2122 and T.U. Reg 9-2-53-714-2006 has completed this thesis under my supervision during the period prescribed by the rules and regulations of T.U. Nepal. The thesis entitled “error analysis in proving theorem by secondary level students” embodies the result of his investigation conducted during the period at the Department of Mathematics Education, University Campus, Tribhuvan University, Kirtipur, and Kathmandu. I recommend and forward this thesis to be submitted for the evaluation to award the degree of Master of Education.

…… ……..

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ABSTRACT

This study was carried out to answer the questions i) What sort of errors do students commit in proving theorem and ii) why do students commit such type of errors? The main objectives of the study were to identify the errors committed by students of secondary level in proving theorem and to explore the causes of error committed by students in proving theorem. Qualitative research design was used and Newman's procedures for analysis and van Hiele theory on levels of thoughts in geometry were considered as theoretical basis of study.

All secondary school students of Baglung district studying in academic year 2070 constituted population of study. Two public schools were selected purposively. 110 students of those schools were taken to constitute sample for written test. A mini-sample of twenty students was selected purposively for interview.

As research tools written test and two interview schedules were used. Written test consist 6 theorems from geometry of grade IX text book. Reliability of test was evaluated by piloting it up to 50 students of grade IX and was found to be 0.94. Validity was constructed by following content points to select question and by judgment of supervisor. Interview schedule (on Newman's procedure) consists five questions. Interview schedule (on van Hiele levels of thinking) consists question from three domains under geometry, corresponding to first four van Hiele levels.

The data were collected from answer sheet and responses to interview. The data collected from answer sheets were classified into five categories as described by Newman's and frequency of each type of error was tabulated. From the interview (on Newman procedures and van Hiele levels of thoughts)

were analyzed qualitatively by interpreting responses to find the causes of committing errors.

The total number of errors committed by the students was 86. These errors were identified by checking out the answer copies of the test and interview with the students. Of all the students more than two third made either comprehension or process skill or transformation level. They were 87% of the total error. Comprehension errors were made because of lack of proper understanding of geometric terminologies and less skill to draw geometric figures representing theorems. Students committed transformation error because they could not transform the language in mathematical term and use appropriate strategy to prove the theorem. Process skill error were made because of insufficient knowledge of definitions, axiom, postulate and already proved theorem and less skill to apply them in unfamiliar situation.

It was found that the causes of committed errors were poor background knowledge on geometry, problem on understanding in teaching new concepts, facts, relation or skill to students and failing student to reprocess geometrical terms definition, axiom, postulations and already proved statement that are needed to prove. Student did not sufficient experiences and prerequisites knowledge at lower levels to encounter formal study of geometry at secondary level was the main causes of committing errors in proving theorem.

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Chapter- I

INTRODUCTION

Background of the Study

Mathematics is regarded as a part of human life. Mathematics education is the study of educational process of mathematics. Today nobody can live without mathematics. Mathematics is intimately involved in every moment of human's life. The study of mathematics is not essential for everyday life rather it has been proved as an indispensable factor in the field of science and technology as well as higher study.

Mathematics is completely an innovative and activity oriented subject. It is deemed as a profound phenomenon in school education system in all parts of the world. Mathematics is the science of numbers and shapes (oxford dictionary). Importance of mathematics is perceived in different ways. For many it is seen in terms of arithmetical skills which are needed for the use at home or in the office or workplace.

According to oxford dictionary geometry is the branch of mathematics that deals with the measurements and relationships of lines, angles, surfaces and solids. It is the studies of the properties of shapes and size of objects. Since the shape of an object is something visible. We begin to acquire geometric knowledge and understanding in early childhood. We learn to distinguish such geometric figures as lines, planes, circles, axioms, postulate and theorem.

Under geometry learning proving theorem is an important activity. It is emphasis teaching some of the techniques of mathematical proof in secondary school mathematics. It is probably not inaccurate to say that the primary reason for teaching geometry in secondary school is to teach students some of the elements of deductive arguments which are used in mathematical proof as well

as everyday discussions. How much time students should spend on proving theorem and the degree of rigor required in proofs can be decided by each mathematics teacher?

According to Hemmerling, (1995) a theorem in geometry is statement or principle that is accepted only after it has been proved by reasoning. Every theorem in geometry consists of two parts: the part which states what is given or known called the given and the part which is to be proved called the conclusion. The formal proving of the theorem consists of five parts. a. statement of the theorem, b. a general figure illustrating the theorem, c. a statement of what is given, d. a statement of what is to be proved and e. a logical series of statements substantiated by accepted definitions, axioms, postulate and previous proved theorem.

It is not any secret that high school geometry with its formal proofs is considered as hard and detached from practical life. Many teachers in public school have tried different teaching methods and programs to make students understand this formal geometry.

Since proving theorem in high school geometry is typically the first time those students who are in high school level. This can obviously present some difficulties it can also lead students to think that two columns geometric proof.

According to Keith Weber one reason that university students find proof so difficult is that their experience with constructing proof is typically limited to high school geometry. To rectify this the NCTM (2000) recommended that proof be introduced early in the mathematics curriculum arguing that reasoning and proof should be a consistent part of students mathematical experience from the pre Kindergarten through grade 12. Further, the NCTM standards and others (1994) argue that proof should not be seen as distinct mathematical topic ,but rather as a way of thinking that can be applied to any mathematical topic. Finally, the NCTM standards argue that by the time students' complete 12^{th}

grade. They should recognize proof as fundamental to mathematics be comfortable with constructs proofs and be able to determine whether a given argument is a proof.

Dimakos and others, (2007) argued geometry is a numerous application in everyday life, holding a central role in many others sciences as well as in the arts. Proofs are probably the most important tool used in geometry. But after all what is proof? In daily life the term proof is linked to conform the products arising from the followed proving practices and to argumentation being accepted by a community of a person. The diverse meaning of proof is identified by the terms explanation, argumentation and demonstration. However, in all circumstance, a common idea exists that of justifying or validating a proposition by proving justifications or arguments.

According to Marthin and Hard (1989), “people think of proof in their everyday life as what persuades them” but Euclid in his Elements makes us believe that proof represents something more powerful for geometry, rational, deductive reasoning based on inalterable axioms, definitions and theorems. Proofs are described as arguments that consist of logically strict deductions of hypothesis.

However, if we pose the question: “What is proof?” to either a student or a teacher it is almost certain that their answers will vary significantly, partly because their interpretation of the question will also be considerably different. In what follows we focus on three factors that justify this phenomenon. Firstly we elaborate on the fact that the term proofs means many different things for students therefore their interpretation of the meaning of proof may be different from that of teachers. At the same time, the interpretation of the meaning of proof may vary from teacher to teacher (Tall 1989). Secondly, we concentrate on the fact that the term proof vary from context to context for instance when referring to daily life issues, empirical sciences or professional mathematics

(Recio and Godino 2001). Thirdly, we focus on the fact that according to the van Hiele model there who are engaged in the teaching learning process. Usually function at different levels of thinking, this phenomenon will be here in after referred as “van Hiele gap”. As a result, proving theorem has been an important goal of the geometry curricula for the secondary schools.

Generally, an error means a simple lapse of cave or concentration which almost everyone makes at least. Occasionally, in mathematics, an error means the deviation from a correct solution of a problem. An error is regarded as a mistake in the process of a solving a mathematical problem algorithmically, procedurally or by any other method. Errors could be found in wrongly answered problems which have flows in the process that generated the answers. (Yong and O’shea, 1981)

Adhikari (2012) argues errors and mistake are taken as synonym in layman’s sense. Technically speaking, all mistakes are not errors". Errors occur as the results of lack of competence whereas mistakes occur due to psychological or physiological reasons or by carelessness. Mistake may be either at competence level or at performance level, mistakes that are committed at competence level are called errors and mistakes that are committed at performance level are called mistakes, slips or lapses. Error occurs because of lack of competence and they tend to occur time and again. So they are said to be systematic. An error is that mistake which occur time and again in a systematic way. So it is better to take into consideration only these deviant an errors which occur regularly in the form of learner’s performance.

It is assumed that, mathematics is difficult than other subjects. Therefore, more errors are committed by students in the field of mathematics. Generally, there may be errors of different types could occur in proving theorem on geometry of mathematics. As we know geometry is a very important topic in secondary level mathematics and for the higher studies in mathematics. The

researcher will take much interest to find the errors committed by students while proving theorem.

Statement of the Problem

Few years ago, when I was a secondary school teacher. I observed many students in my class struggling to cope with learning geometry especially in proving theorem. They had a good algebraic background and they could solve a problem using lengthy algebraic procedures what they came up with themselves but were hesitant to prove theorem. I always tried to teach theorem to motivate them. However, my attempts were not success or rather failed to teach theorem effectively. At the same time I left my teaching and joined a University where I did not have further opportunity to pursue this area. So the problem of this study is to diagnose the errors committed by secondary level students when proving the theorem. Especially this study find answer the following two questions.

- What sort of errors do students commit in proving theorem?
- Why do students commit such type of errors?

Objectives of the Study

The main purposes of this study were as follows:

- To identity the errors committed by students of secondary level in proving theorem.
- To explore the causes of error committed by students in proving theorem.

Significance of the Study

The purposes of error analysis is to identify the patterns of error or mistakes that students make in their work and understand why students make the error. When conducting an error analysis the teacher analyze the student's mathematics problems and categorized the errors. This study has the following significance:

- This study helps teacher educators, mathematics teacher, policy maker, mathematics educationist, curriculum planner, material developers and students.
- It helps teacher to organize his experiences in an appropriate teaching plan, and provide remedies to avoid the errors.
- It provides the students to acquire the power of acquiring knowledge without committing errors.

Delimitation of the Study

The study was limited in the following aspects:

- This study was limited to the students of grade IX who had been completed the chapter geometry.
- It was limit in Baglung district.
- The sample of the school was taken in accordance to the convenience of the researcher, so the result cannot be generalized to the initiative context.
- This study was concerned government school's students only.

Definition of Terms

Error : The first mistake done by the student during the proves of theorem or if the students could not perform the solution for the further process.

Error Analysis: Error analysis refers to the systematic study and analysis of the errors made by students.

Students : Students refers the pupils of grade IX students for the purpose of study from Shree Sanskrit Higher Secondary School and Kalika Kanya Ma.Vi. Baglung.

Proof : The proof in this study means the formal proof which is done by tracing the following stages.

- The statement of the theorem
- The figure
- A statement of what is to be given (Hypothesis)
- A statement of what is to be proved. (Conclusion)
- Construction if any.
- The proof

Figure : In this study the figure means that figures which is drawn by the students on the basis of given information to prove a theorem. It is usually done in the first stage of proof of the theorem.

Chapter II

REVIEW OF RELATED LITERATURES

The researcher has attempted to find out the literature related to error analysis in proving theorem. It helps the researcher to understand the relationship between research problem and the body of knowledge in the area. The most important function of literature review is to ensure the researcher reads widely around the subject areas which intend to conduct research study. This chapter deals with the review of related literature in two section empirical review and theoretical review about error analysis in proving theorem of secondary level students.

Empirical Review

Robert (1968) studies the failure strategies of third graders and identified four types of errors, this includes:

- wrong operation,
- obvious computational errors
- defective algorithm
- Random responses.

Tindal and Marston (1990) suggested that the following list of errors that students commonly make in solving word problem as:

- Difficulty in reading
- Inability to understand the language and vocabulary of the problem
- Difficulty in identifying the relevant and irrelevant information.
- Difficulty in identifying the number of steps required to solve the problem

- Trouble in doing mathematical operations.

Marahatta (2002) studies “A study on computational errors on fraction by grade VI students in Chitwan district". In his study 78 government schools were classified in to two strata as rural located school and urban located school. Three schools are selected from each stratum randomly. 60 students are included for the sample which are selected from each sample school 5 boys and 5 girls randomly. Diagnostic test was administered on grade VI students on each of the schools selected in the sample with the help of the class teacher and research design was qualitative. He found that:

- Students generally commit more error in addition of fraction that in introduction of fraction.
- The mean errors are occurred the addition of fraction and subtraction of fraction were the same?
- The mean error of multiplication of fraction is higher than introduction of fraction.
- The location of school i.e. rural or urban didn't play a significant role in committing errors.

Bhatta (2003) studied on “An error analysis in quadratic equation of grade X”. This study was mainly concerned with identification of errors committed by grade X students in quadratic equations and compares them by using Newman’s Error Analysis technique. In this study the researcher has compared the errors with respect to: gender, location, knowledge, understand, application level and found that

- Students committed more errors in knowledge than understanding of quadratic equation.

- Students generally committed more errors in application of quadratic equation than understanding of quadratic equation.
- There is no effect of location committing equal number of errors in understanding knowledge of solving and application of quadratic equation.
- The role of gender is less important to commit the errors in understanding knowledge of solving and application of quadratic equation.
- Students commit error in simplification process and comprehension.

Pant (2005) studied "Computational errors of grade IV students on operation of fraction in Chitwan district. In this study 50 students were selected from the selected sample school who secured 40% mark in the previous examination. He used three diagnostic tests on introduction addition and subtraction of fraction and used quantitative research design.

The aims of this study were:

- Identifying and classifying the computational errors of grade IV students in operation of fraction.
- Compare the computational errors with respect to gender and types of school.

Main findings were:

- Students generally committing more errors in introduction of traction they aim the addition of fraction.
- Errors were higher in the subtraction of fraction then in the addition of fraction.
- There is no effect of sex and location to committee the errors in areas of operation of fraction considered in this study.

Adhikari (2007) Studied on “An error analysis in menstruation of grade IX students in Kathmandu district.” 80 students out of 120 students were selected by lottery method. Questionnaire made from SLC question banks and used both quantitative and qualitative research design. The aims of this study were:

- Compare the error made by students in knowledge, skill and application and problem solving of menstruation.
- Compare the error with respect to gender
- Analyze the errors made by the students in problem solving.

The main findings of this study were:

- Children’s have committed more errors in area of problem solving
- Student have committed more errors in the area of knowledge than in the area of skill and application
- Girls committed more errors than boys in the area of knowledge
- Students have committed carelessness error

Panti (2009) studied on “An error analysis in equation of grade VII students”. After analysis he found that: 10% errors at reading level. 33.3 % errors at comprehension, 25.5% errors at transformation level , 17% errors at process skill errors and 13.7% error from that she had concluded that student committed more errors of comprehension level while solving verbal problems.

Adhikari (2012) Studied, "An Error analysis in solving verbal problem in probability”. The main objectives of this study were:

- To identify the errors committed by students of secondary level in solving problem of probability.
- To analyze the errors committed by the students in problem solving.

- To analyze the errors with respect to gender

To achieve the objectives two schools were selected from the Kathmandu Valley. Among them one was public school and the other was private school by using purposive sampling procedures. The researcher visited each of the selected school in order to administer the test to the sample of 40 students (20 from each school) of grade X. He concluded that:

- Public school students committed more errors than institutional school students.
- The analysis shows that, 6.86% errors of reading ,29.41% errors at comprehension level, 23.52% errors a t transformation level, 18.14 errors at process skill level, 12.17% errors at encoding level and 9.31% errors at carelessness level from which it can be concluded that students committed more error at comprehension level,
- Students committed more error at skill application level than knowledge and problem solving level.

Chand (2012) studied, "Analysis of errors committed secdondary school student in solving geometric problems". The main objectives of this study were:

- To identify different types of errors in solving geometric problems and analyze them according to Newman analysis of errors.
- To compare errors committed by good performers and poor performers in solving geometric problems.
- To compare some errors in terms of van Hiele model of thinking in geometry.

To achieve the objectives two public schools and two institutional schools were selected from Kavre district by using purposive sampling procedures. He used qualitative research design and concluded that:

Of all the students on the average two third made either comprehension or transformation errors. Comprehension errors were made because of lack of proper understanding of geometric terminologies and less skill to draw geometric figures representing problems/theorems. Transformation errors were made because of insufficient knowledge of definitions, theorems etc and less skill to apply them in unfamiliar situation.

Except better performance of good performance of good performers. Similar patterns of errors were found in short answer type questions and deviation in proof oriented questions. In proof oriented questions errors of poor performers were concentrated on comprehension level and that of good performers were in transformation level. The reasons behind errors were different.

Kafle (2006) studied on "Error analysis of the proof the theorem in geometry in grade X". Twenty students studying a Nagarjun Thalagaun secondary school Jeetpurfedi, Kathmandu were selected. He used qualitative research design. The main findings of this study were:

- To identify the errors
- To classify the errors on the basis of a recognized theory
- To indicate possible causes of errors.

The main findings were:

- The network of logical relations between the properties of geometry was not sufficiently established by the students.

- One fourth of the total errors were concentrated in reading and comprehension skill together.
- Students had weakness in basic for understanding the mathematical problem, language meaning of mathematical terms and understanding of the problem to establish connection of the given condition and the mathematical structure to make a plan for solving the problems.
- Few percentages of errors were committed in comprehending the problems.

Review of Theoretical Literature

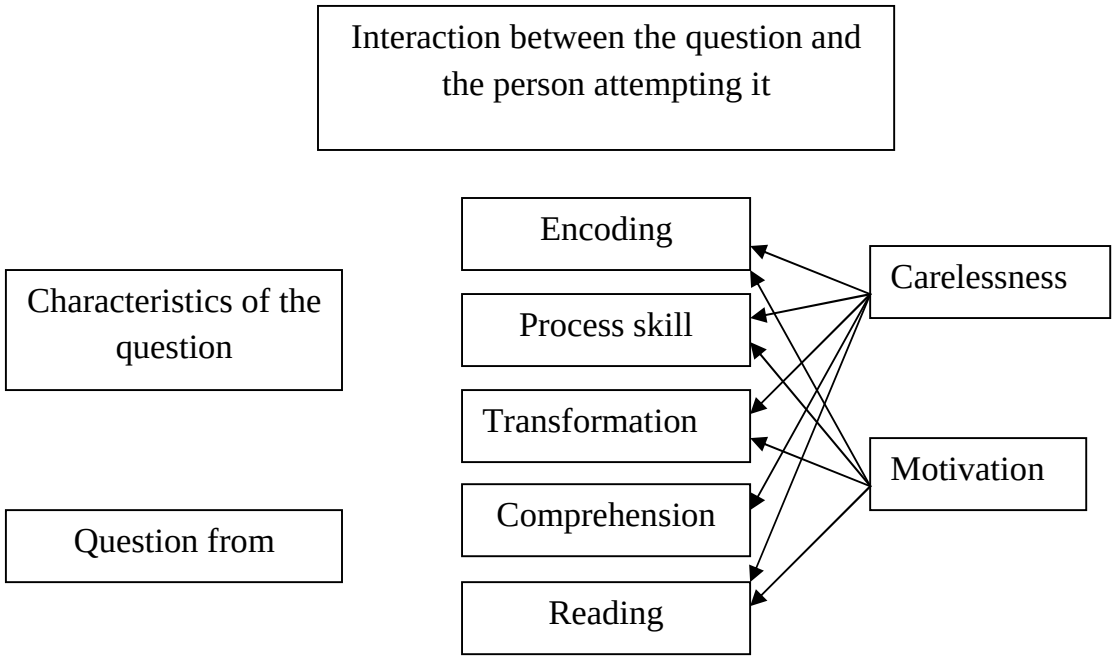
Theoretical basis of study play vital role in the field of research because it helps researcher to describe different concept considered in research and to prepare research design. Some of related theoretical literature is reviewed below.

Newman Error Analysis Procedure

The Australian language educator Eannae Newman (1977) defined five specific literacy and numeric skills as crucial to performance on the mathematical word problems: Reading, comprehension, transformation process skill and encoding. Newman's Error analysis provided a framework for considering the reasons that underlay the difficulties of students experienced with mathematical word problems and a process that assisted teachers to determine where misunderstanding occurred. It also provided directions for where teachers could target effective teaching strategies to overcome them.

Newman (1977) suggested an error classification and some guiding interview questions to analyze mathematical problems. In this classification Newman suggested that error occur in the interaction between the question and the person who is attempting to solve the problem. She classified the sources of

errors into a five elements hierarchy: reading, comprehension, transformation, process and encoding. Other general sources of errors cut of this classification include but not limited to carelessness and lack of motivation. Newman also provided a list of guiding questions pertaining to each stage of the problem solving process.



The Newman Hierarchy for one step Verbal mathematical problems

The importance of Newman's model is that it provides a comprehensive stage wise procedure to analyze mathematical problem solving task. Using this framework, Newman found that 47% of her population of low achievers in grade IX made errors prior to the process stage of which 12% were at the transformation stage. The Newman model was later adopted by Casey (1978) and Clements (1980) found that fewer errors were made at the two lower levels. One quarter of the errors was at the Transformation stage. Clements inferred

that failure in the early stage of problem solving can lead to selection lf incorrect process later.

The unique feature of the Newman model is that it is well suited for word problems although there is no restriction of its use in other context as well. The left block of the diagram represents the difficulties of comprehend or understanding the question which is named as the “question form”. This emphasizes the necessary of providing appropriate questions. Even through the right questions are provided some students may interpret them differently from the implied meaning. The right block represents the five stage of the problem solving process.

Basically, the model depict when a student’s produces an incorrect answer to a question the error resulting in that answer may have occurred at one of several stages in the process of solving that problem. The student may have misread the question (reading error) . Alternatively despite a correct transformation an incorrect method may have been used to solve the problem (process error). Even through all the above steps are correct the answer may have been wrongly encoded (encoding error).

There may also have influences from the student’s lack of academic self concept. This involves a feeling of not having confidence and not recognizing of one’s strengths and weakness. Errors caused by student’s affective altitudes are of different types. Lack of concentration is sometimes caused by over confidence, blockages or forgetfulness.

The Van Hiele Model of the development of Geometrical thought

The model consists of five levels of understanding the level Visualizations, analysis; informal deduction, formal deduction and Rigor describe characteristics of the thinking process. Assisted by appropriate

instructional experiences the model asserts that the learner moves sequent from the initial or basis level (visualization) were space is simply observed he properties of figure are not explicitly recognized through the sequence listed above to the highest level (Rigor) which is concerned with the formal abstract aspects of deduction. Few students are exposed to reach the latter level. A synopsis of the levels is presented below.

Level 0 (Basic Level: Visualization)

At this initial stage, students are aware of space only one something that exists around them. Geometric concepts are viewed as total entities rather than as having components or attributes Geometric figures, for example are recognized by their space as a whole that is their physical appearance not by their parts or properties. A person functioning at this level can learn to reproduce it.

Level 1: Analysis

At this level; an analysis of geometric concept beings for examples through observation and experimentation students begins to discern the characteristics of figures. There emerging properties are then used to conceptualize classes of shapes thus the figures are recognized as having part and are recognized by their parts. After using several example student could make generalization, relationship between properties and figure. However cannot yet be explained by students at this level, interrelationship between properties and figure are still not seen and definitions are not yet understand.

Level 2: Information Deduction

At this level, students can establish the interrelationship of properties both with figures then they can deduce properties of a figure and recognize

classes of figure class inclusion are understood. Definitions are meaningful informal arguments can be followed and given. The students at this level, however does not comprehend the significance of deduction as a whole or the role of axioms. Empirically obtained results are often used in conjunction with deduction techniques. Formal proof can be followed but students do not see how the logical ordered could be altered or do they see how to construct a proof starting from different or unfamiliar premises.

Level 3: Deduction

At this level, the significance of deduction as a way of establishing geometric theory within an axiomatic system is understood. The interrelationship and role of an defined terms axioms, postulates definitions theorems and proofs is seen, A person at this level can construct not just memorize, proofs, the possibility of developing a proof in more than one way is seen the interaction of necessary and sufficient conduction is understood distractions between a statement and its converse can be made.

Level 4: Rigor

At this stage the learner can work in a variety of axiomatic system that is non Euclidean geometric can be studied and different system can be compared. Geometry is seen in the abstract.

This last level is the least developed in original work and has received little attention from researcher. Hiele has acknowledged that he is understood in the first three levels in particular. Since the majority of high school geometry course are thought at level 3. It is not surprise that most research has also concentrated on lower. Perhaps this level will achieve more provinces.

Chapter- III

METHODS AND PROCEDURES

In this chapter the design of study, sampling procedures for the data collection, source of data collection, tools for research and data analysis and interpretation are described.

Research Design

Research design is the conceptual structure within which research is conducted. Qualitative research design was used because it aims to describe the situation addressing the present activities of the students. Newmans procedure for error analysis and Van Hieles theory on level of geometry thinking were considered as theoretical basis of the study.

Population of the Study

Population is any group of individuals that has one or more characteristics in common that are of interest of researcher. This study was concerned with errors committed by secondary schools students in Baglung district. Therefore, all secondary school students of Baglung district were constituted population for this study.

Sample of the Study

A sample is small representative proportion of population that is selected for observation and analysis. For this study out of all secondary school of Baglung district, Shree Sanskrit Higher Secondary School Jaidee and Kalika Kanya Mandir Secondary School, Baglung were selected. As the sample of the study for written test all the students of grade IX were taken from the two sample school. For the purpose of taking interview a small sample consisting 20

students among 110 students (10 from each school were selected so as to include each type of errors categorized by Newman.

Tools of the Study

To get the reliable data the following tools were developed and used.

Written test (subjective test)

This study has concerned with errors committed in proving theorem. Therefore students were asked to write the answer of the questions from geometry section. That's why; paper and pencil test was used for the study. For the purpose of paper pencil test 110 students of grade IX were selected from sample school. The test consisted six theorems of geometry section in grade IX curriculum of mathematics.

Reliability of Test

To evaluate reliability of test, it was piloted to 50 students of grade IX of Shree Bidhya Mandir Higher Secondary School, Baglung. Coefficient of reliability was calculated through Spearman's split half method and found to be 0.94.

Validity of Test

If the question in the test can reflect whole content then test can have content validity. The content validity of this test was established by constructing the questions following content points, objectives and by consulting with supervisor.

In-depth Interview

After the completion of paper pencil test, 20 students among 110 sample students were selected that they made maximum number of error in the test. To identify different types of errors, interview schedule based on Newman's procedure was used to the study. Interview schedule mainly used to find out the

causes of errors that they were found in the written test. Newman's procedures and van Hiele model were also being beneficial to categorize the causes of errors. However, discussions with teachers were used to find out the causes of errors. The interview schedule consists of the field of triangle, parallelogram and circle. Students were asked the questions according to their level.

Procedure of Data collection

To collect the data the selected schools were visited and the administrator was requested for cooperation with the help of mathematics teacher. The test was administered in each school. Answer copies were collected checked and analyzed to categorize different types of errors as categorized by Newman. Small sample of 20 students from the sample of the study (10 from each school) were selected for interview by using purposive sampling procedures. That is the more errors were selected from the first 10. So as to include each type of errors categorized by Newman given below.

a. Please read the question to me If you don't know a word, leave it out (for reading error)

b. Tell me what the question is asking you, draw the appropriate figure and write the given condition of the statement (for comprehension error).

c. Tell me what is to be prove and how you are going to construct? (for transformation error)

d. Show me how you can prove the question explain to me what you are doing as you do it (process skill)

e. Now write down your prove in on acceptable written form to the question (Encoding error)

The samples of study for the interview were selected. Then, the date and time for interview were fixed with their collaboration. The interview data were

taken in the note copy that they were important to the intended data. At the time of interview based on Newman's procedures, some errors were categorized. Similarly causes of errors and some particular responses were noted being based on Newman procedure.

After the completion of interview based on Newman's procedures, the interview based on van Hiele level of geometric thinking was also conducted. At the time of interview, the responses of the students were collected. Then the student's responses were evaluated. Some important responses in different levels were noted.

Data Analysis Procedure

After the completion of the interview, the errors were identifying where the students were given incorrect response as:

- **Reading errors** if students have not seen able to correctly read all the words in the question.
- **Comprehension error** if the students do not draw the appropriate figure and do not write the given condition of the statement.
- **Transformation error** if the students do not write conclusion part and does not use appropriate construction of the given statement.
- **Process skills error** if the student does not know the procedures necessary to carry out these operations accurately.
- **Encoding Errors** if the students cannot express the solution in on acceptable written form.

Data collected from answer sheets were arranged in the form of table. Analysis was made under two separate headings: i) Classification of errors and ii) Causes of error made by students.

After the identification of errors, they were classified into five categories as described by Newman. Analysis was made on the basis of the categorization of these errors committed in the answer copies and identified through interviews.

Data collected from interview were analyzed descriptively by interpreting responses given in the time of interview. This analysis was made to determine causes of different errors in the time of proving theorem.

Chapter- IV

ANALYSIS AND INTERPRETATION OF DATA

In this chapter, the collected data through specific tools and technique has been analyzed and interpreted. In this research the researcher aimed to identify different types of error and to explore the causes of error committed by students in proving theorem by using Newman techniques of error analysis and interpreted the result. Also used van Hiele's model of the development of geometrical thought. The analysis is made under the following headings.

I) Identification of Error

II) Classification of Error

III) Cause of Error made by student

I) Identification of Error

This section deals with the identified errors made by 20 sample students to whom the test was implemented. To identify the errors was tricky job. After the collection of answer sheets, they were checked thoroughly and errors were marked in every problem. The errors are in hierarchical, so if the students did the first error it had not takes other error from the same problem. The incorrect items were counted as errors. The errors which were not visible from the answer copies were later revealed from the interview with those students. Final aggregated errors were categorized relating to the five errors namely: Reading errors, comprehension errors, transformation errors, process skill errors and encoding error according to Newman's' technique.

The error was identified as a reading errors if the students hadn't been able to read the question such type of error were identified mainly from the

interviews with the students. The comprehension error were identified if the students had been able to read all word but had not draw the appropriate figure and had not write the given condition of the statement. If the student couldn't write the condition what is to be proved and appropriate construction to prove such error were counted as transformation error. If the students were unable to prove the theorem with required logical explanations statement and reasons, then such error was identified as process skill errors. At last, if the students unable to use appropriate sign, letters and write proved at last such error identified as encoding error.

The following table shows the frequency of error according to Newman hierarchy or error.

S.N.	**Name of Error**	**Question No**						**Total**	**Percent**
		I	II	III	IV	V	VI		
1	Reading Error	-	-	-	-	2	-	2	2.33
2	Comprehension Error	3	5	5	6	9	3	31	36.04
3	Transformation Error	2	1	5	2	3	6	19	22.09
4	Process Skill Error	3	3	6	7	2	4	25	29.07
5	Encoding Error	2	1	1	1	3	1	9	10.47
	Total							**86**	**100**

Table 1 show that highest percentage of error was concentrated on comprehension error. According to Newman's technique it was found that 36.04% errors were first occurred at the comprehension stage, 29.07% errors were first occurred at the process skill stage, 22.09%. Errors were first occurred

at transformation stage and 10.47% errors were first occurred at the encoding stage. This indicates that most of the student can't comprehend the statement, if comprehend and transformed they can't to prove the theorem with required logical explanations statement and reasons.

ii) Classification of Error

Some errors done by students are listed below:

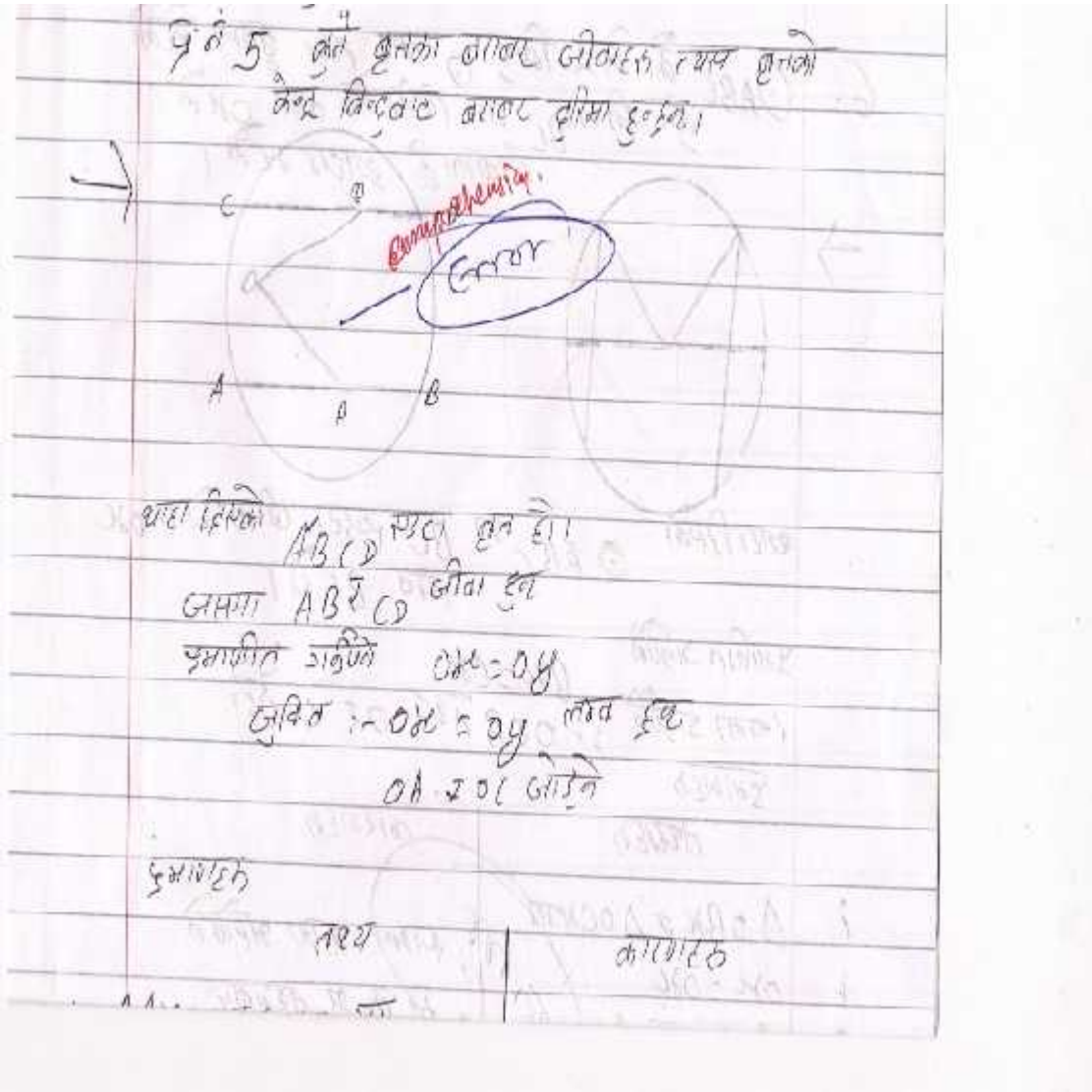
a) Reading Error

This type of error is committed by the students at the time of reading key word or symbol used in written problem. An error is defined as reading error, if the students hadn't been able to read the question that is called reading error. While conducting interview, two students were committed reading error at the time reading question no. 5 (vide Appendix I), which was in mathematical language. Rest of other students managed to read the question easily and meaningfully which were in mother tongue language.

b) Comprehension Error

An error was classified as comprehension error if the students had been able to read all the words in the questions but not grasped the overall meaning of the words and therefore was not able to proceed along an appropriate problem solving path. If the students read the question (statement) but s/he unable to draw the appropriate figure and write the given condition of the statement.

The researcher provided the question to the students i.e. "Prove that in a circle equal chords are equidistant from the centre". Then one student had solved this problem as follows:

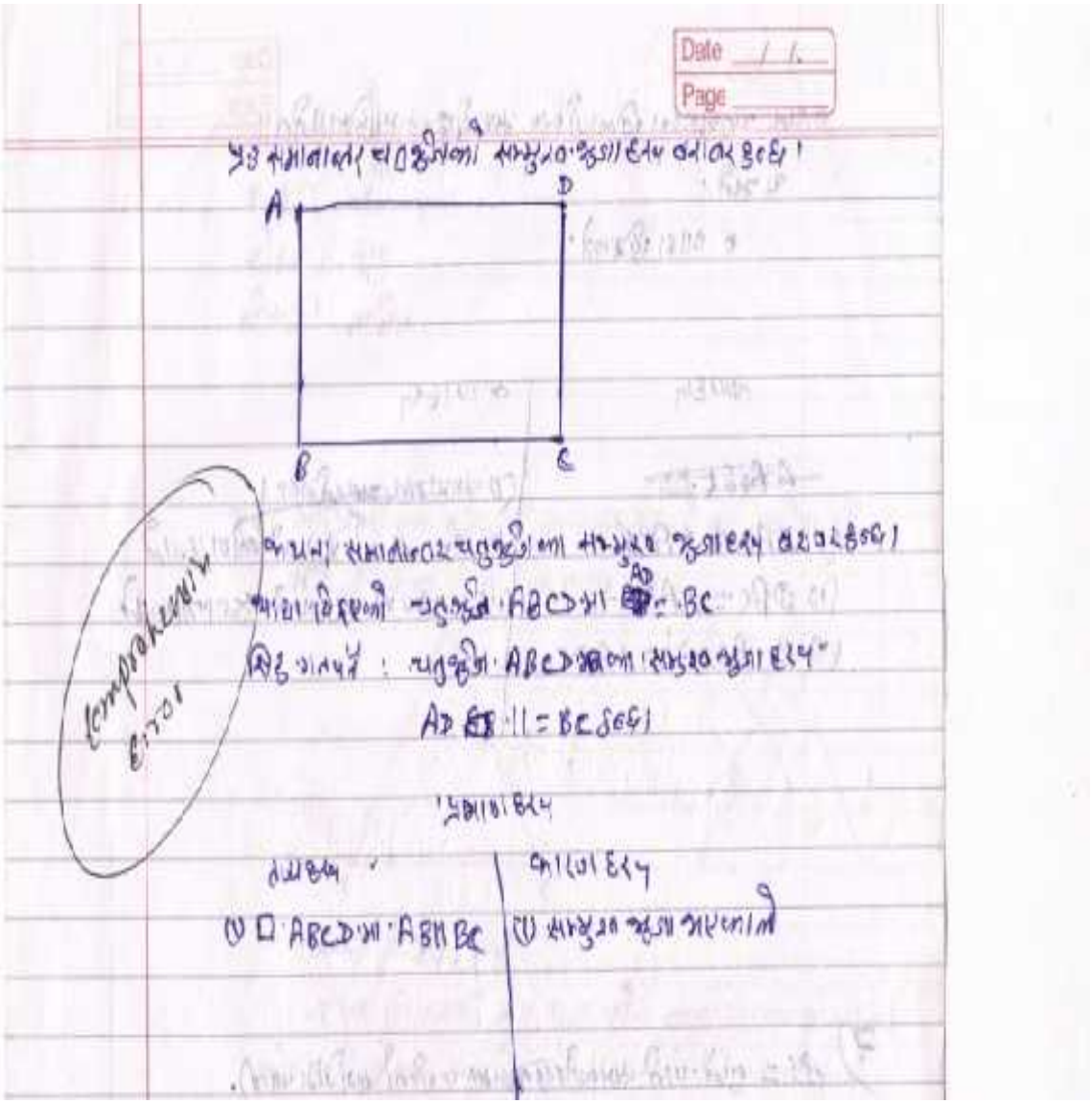


From the above way of solving, the researcher had concluded that students are totally unable to comprehend about the circle.

> *On the other hand, student described ABCD is a circle where AB and CD are chord. When asked to student "What is the question asking you to do? Student replies- the circle has two chords which are equal distance from O. Again the researcher asked to student where is O located? Are your rights now? (By showing the above picture), student replies- yes sir. That means the students unable to comprehend the question. Thus according to Newman it was comprehension error.*

In the above mention solution, student describes ABCD is a circle where AB & CD are two chord which are in equal distant from the centre of its. Here the students Visualize the figure but unable to show the interrelation between figures and properties. Thus according to van Hiele it was the error in analysis level.

Similarly when the researcher gave the question "Prove that the opposite sides of parallelogram are equal". Then one student had solved this problem as follows:



From the above way of solving, the researcher came to know that the student unable to comprehend the term "parallelogram" and failed to prove the theorem correctly.

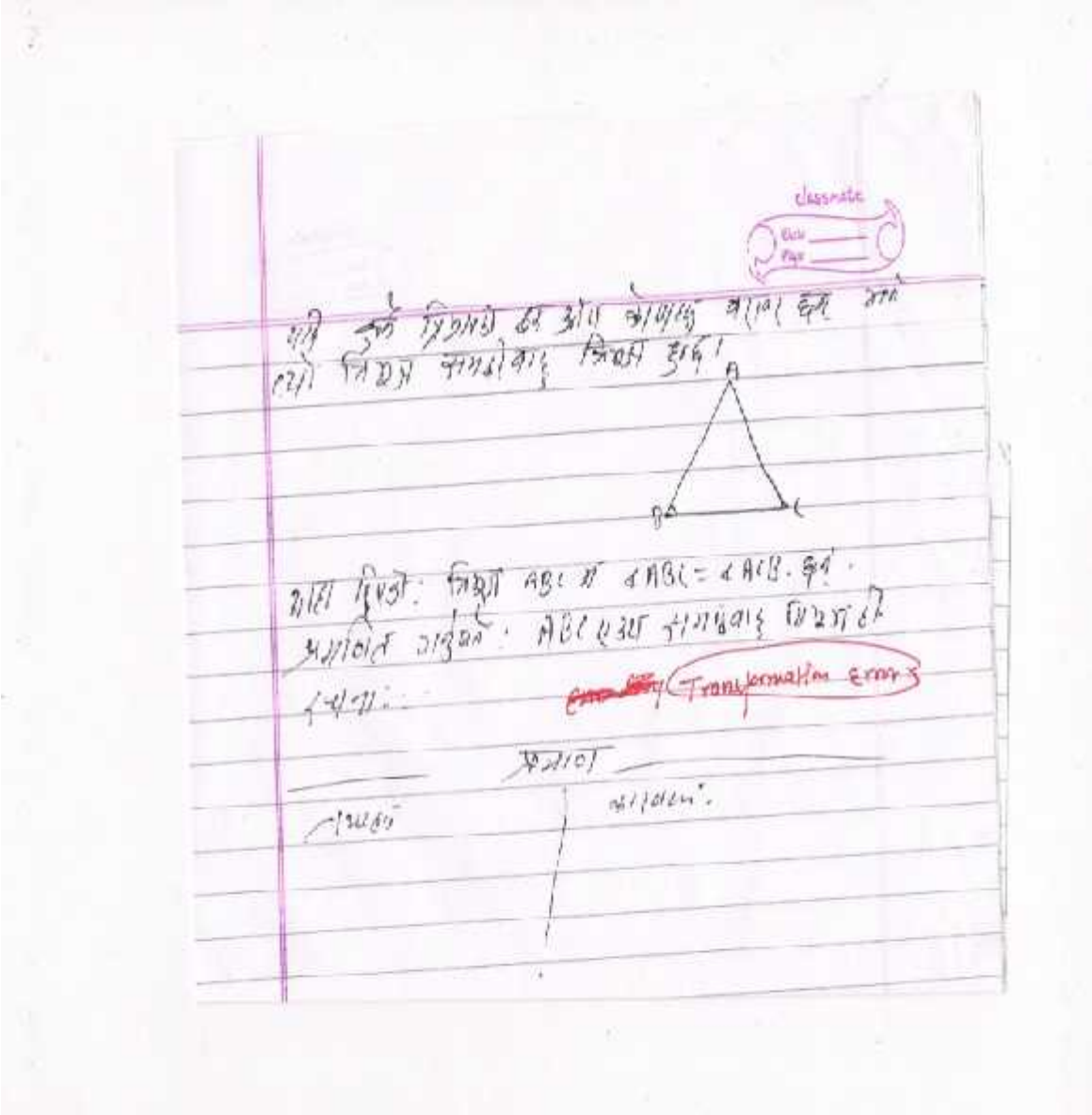
Next student described about parallelogram but when the researcher asked to student, "What was the given condition of the statement? Student replied- in quadrilateral ABCD, AD = BC. Also asked to student (by showing picture), is it parallelogram or quadrilateral? Student replied parallelogram and researcher again asked, why? He also replied that he didn't know. That means the students unable to comprehend the meaning of the parallelogram thus according to Newman it was comprehension error.

Here the students described figures as a collection of its properties which a particular figure satisfies. Students described parallelogram as ‘it has two pair of opposite sides parallel.’ It has two pairs of opposite sides are equal.’ In this way he described figure as a collection of properties but unable to give concise definition of geometric shapes and size of figure. so the student analyzed component part of the figures (opposite sides of parallelograms are parallel and congruent), but interrelationships between figures and properties couldn’t be explained. Thus according to van Hiele it was error in analysis level.

c) Transformation Error

An error is classified as transformation error, if the students understand what the questions wanted him/her to find out but become unable to make appropriate strategy to solve the problem. The researcher has described transformation error, when the student could not write the condition: what is to be proved and appropriate construction to prove of the given statement.

The researcher gave a question "prove that if the base angles of triangles are equal then it is an isosceles triangle." Then one student had solved this problem as follows:



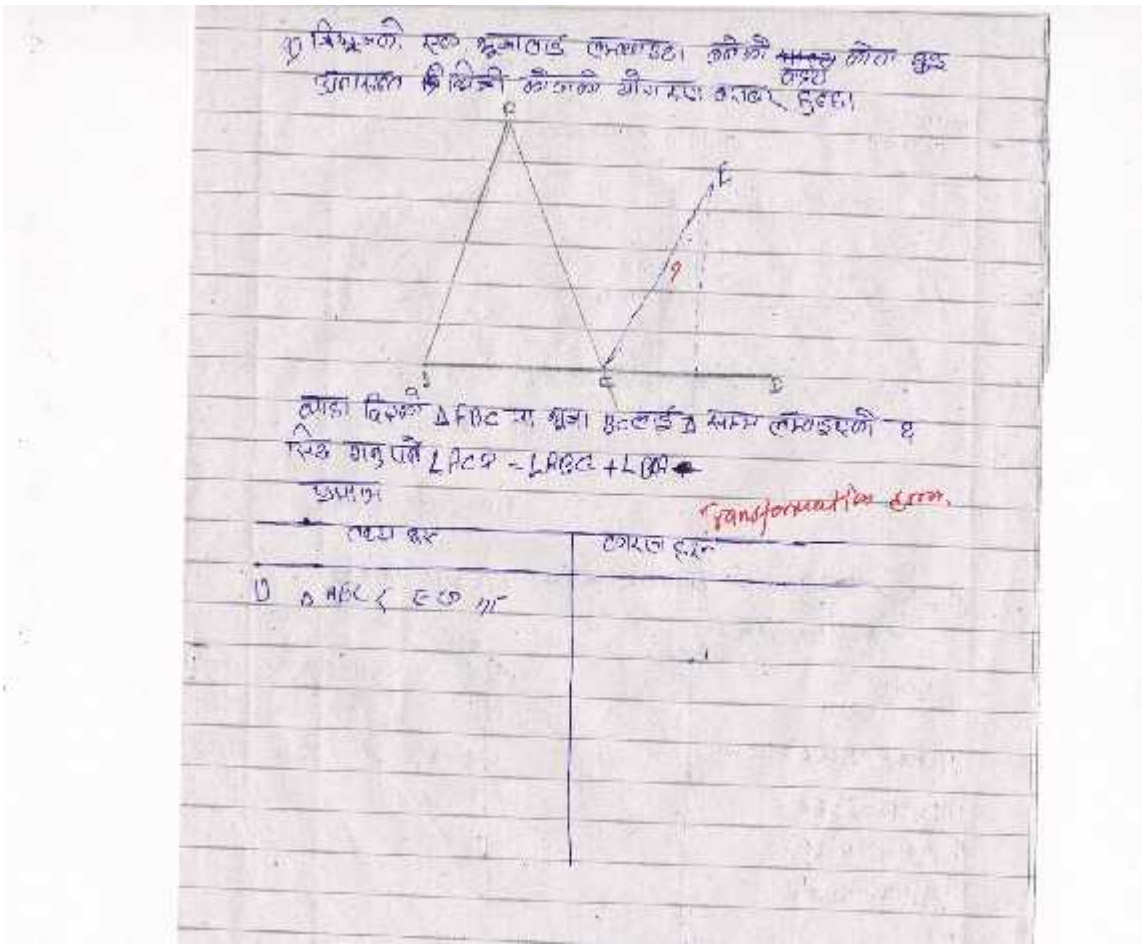
From the above way of solving the researcher came to know that the student had understood the problem, but unable to identify the way of proving.

On the other hand, student wrote the given part of the problem and draws an appropriate figure. When the researcher asked to student,' tell me what the question asking about? Student replied -we have to prove ABC is an isosceles triangle. Researcher also asked to student 'how is it possible? Student replied- I don't know sir. That means student found it difficult to explain what needed to prove. So the student wrote given part in terms of statement and also prepared

figure, but he was unable to make clear what is to be proved and how it can be done. Thus according to Newman it was transformation error.

Student visualized the figure and comprehend its properties and also able to define it, but s/he couldn't made appropriate strategies. Here the students establish the properties within the figure but not comprehend the significance of deduction as a whole. Thus according to van Hiele it was error in informal deduction level.

Similarly the researcher gave a question "Prove that the exterior angle of a triangle is equal to the sum of the opposite two interior angles." Then one student had solved this problem as follows:



From the above way of solving, the researcher came to know that the student had understood the problem, but became unable to succeed in developing appropriate strategy.

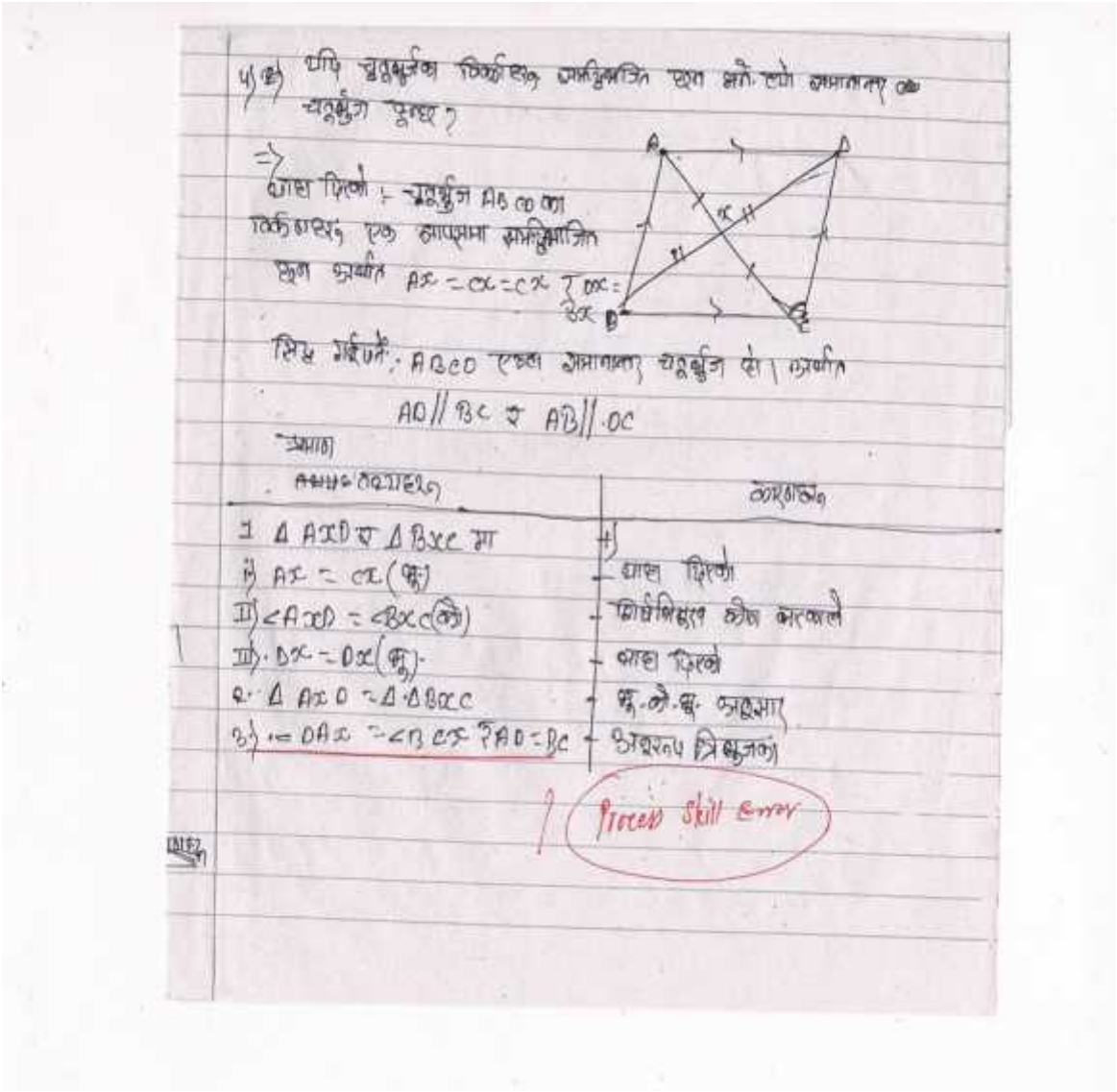
Although the student had understood what the question is asking about but when the researcher asked to student "Tell me what is to be prove on given question?" Student replied-'in triangle ABC ĎACD =Ď ABC+ĎBCA. Again researcher asked to student "which are the opposite angle of exterior angle ĎACD", student replied - ĎABC and ĎBCA. Also researcher asked 'How is it proved'? Student replied- I don't know sir. That means student unable to identify neither opposite angle of exterior angle nor how was possible to prove the theorem. Thus according to Newman, it was transformation error.

Here the student draw the appropriate figure and wrote the given part (hypothesis part), but totally unable to comprehend the conclusion part of the given statement. I.e. the student visualized the figure and comprehend its properties and also able to define it but he couldn't make appropriate strategy. Thus according to van Hiele it was error in informal deduction level.

Process Skill Error

An error is classified as process skill error if the student identifies an appropriate operation or sequence of operations, but does not know the procedure necessary to carry out these operations accurately. The researcher has described process skill error as: if the students were unable to prove the theorem with required logical explanations, statement and reasons or if the student can choose an appropriate operation but cannot complete the prove correctly.

The researcher gave a question "Prove that the diagonals of parallelogram bisect to each other." Then one student had solved this problem as follows:

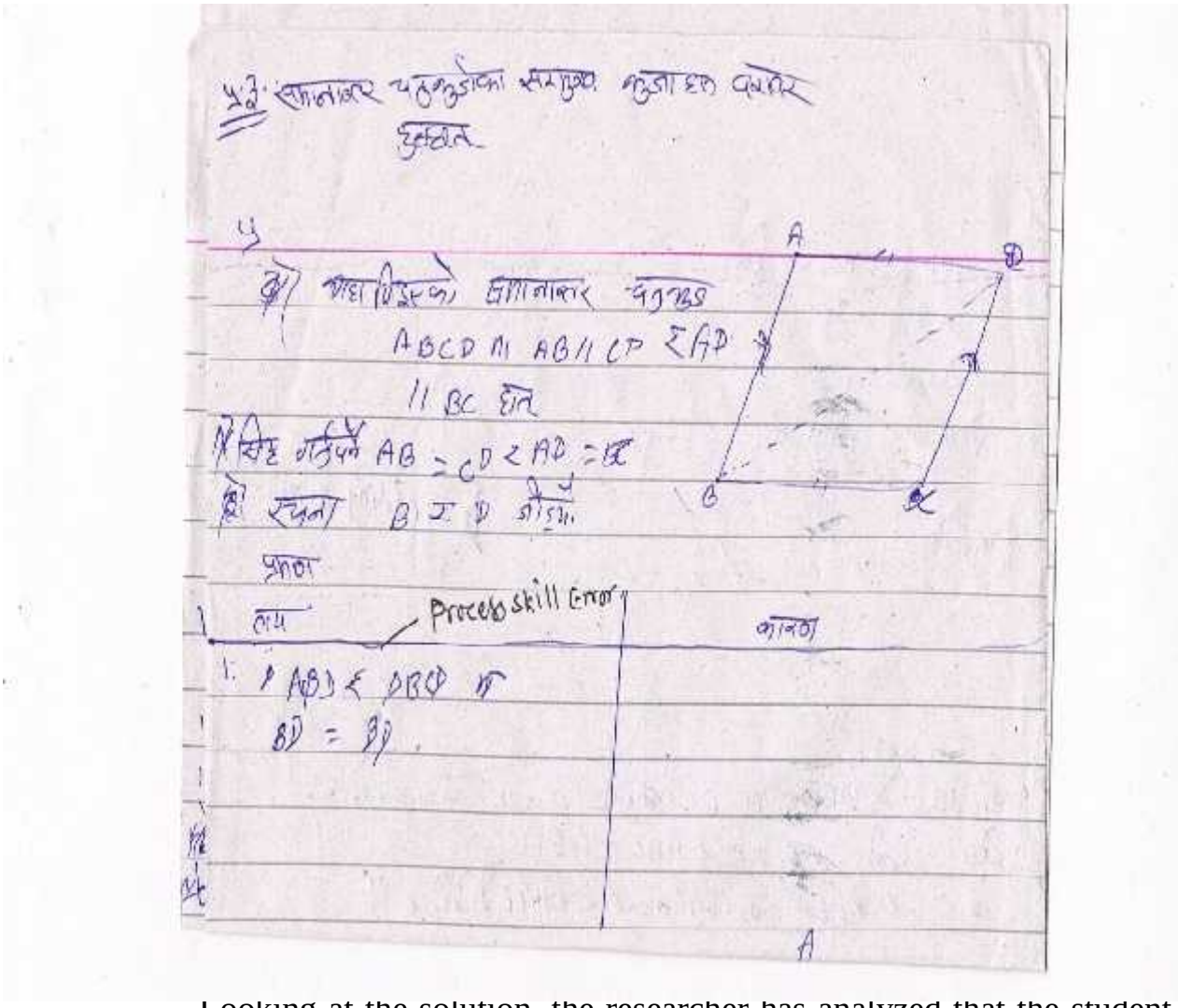


Looking at the solution, the researcher has analyzed that the student had identified the correct way of solving of the given problem but student failed to procedure further step.

On the other hand, the student had familiar about given condition, to prove and what needed to prove of this theorem, when the researcher asked to student 'Explain to me what you are doing as you do it' student replied a*AXD*þ a*BXC then . . . that means student identified the correct way of solving the given problem but he failed to proceed further step. Thus according to Newman it was process skill error.*

The student recognized the figure as its appearance. S/he perceives the relationship between properties and figure, also able to give concise definition of geometric shapes and size of figure. Hence the student visualized and analyzes the given problem but unable to prove theorem by establishing geometric theory within an axiom, undefined terms, and definition. Thus according to van Hiele it was error in deduction level.

Similarly, the researcher gave a question “prove that the opposite sides of parallelogram are equal.” Then one student had solved this problem as follows:



Looking at the solution, the researcher has analyzed that the student had identified the correct way of solving but failed to further procedure.

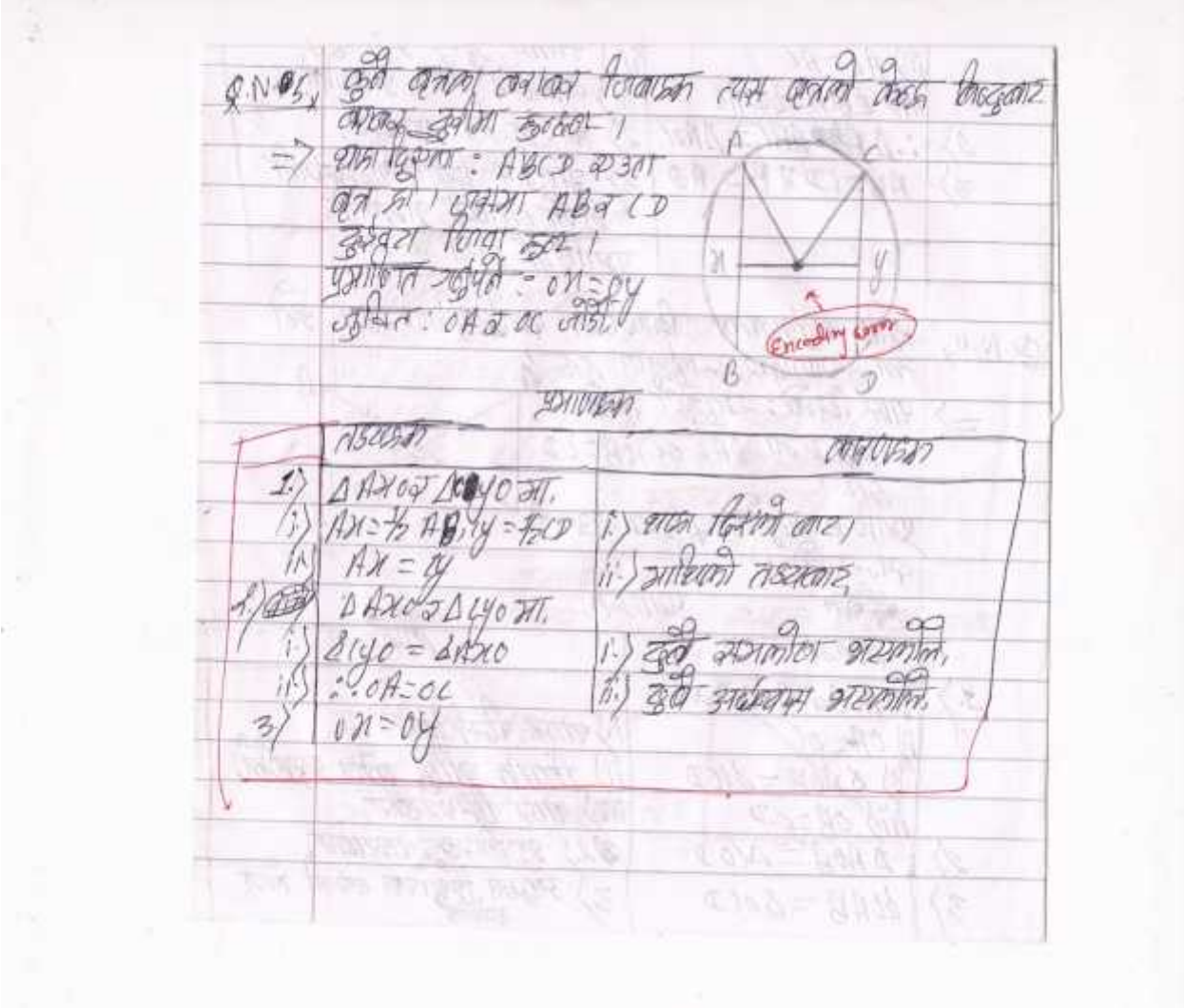
Although the student able to write the given condition, to prove and constructing part of the theorem. But the researcher asked to the student 'Explain to me what you are doing as you do it? Student replied - AB = CD & AD = BC, for this purpose join B&D. Also researcher asked 'how it is possible', but student had not replied. That means the student identified the correct way of solving the given problem but the students failed to further proceed. Thus, according to Newman it was process skill error.

Student draw the appropriate figure, wrote the given part, to prove part and construction part of the given statement but failed further procedure to prove the theorem accurately. That means student recognize the figure, its properties and their interrelationships but unable to show how the logical order could be altered and how to construct a proof starting from different way. Thus according to van Hiele it was error in deduction level.

Encoding Error

The researcher has already described an encoding error as an error if the student can perform the correct operation but failed to write the answer in an acceptable written form i.e. unable to use appropriate sign, letters in appropriate place and do not write proved at last of proof.

The researcher gave a question 'prove that in a circle equal chards are equidistant from the centre." Then one student had solved this problem as follows:



Looking at the solution, the researcher has analyzed that student had perform the correct way but failed to write the answer in an acceptable written form. Although the student able to write the given, to prove construction part and process of the theorem but the student do not write the reason at last statement, not coding the center of circle in figure and not to wrote proved at last. Thus according to Newman, it was encoding error.

The researcher asked to students “can you prove this theorem in other way? Student replied- no sir. Thus according to van Hiele it was error in deduction level.

Causes of error made by students

The interview was administrated to the 20 students selected from sample of 110 students participated in the written test for the interview to find out causes of committing errors in proving theorem. The interview was focused to explore different errors made by students in the written test. The data collected from students interviews presented and analyzed in the subsequent sections.

Reading error was found only 2.33%. The interview was taken 2 students reflected that they made reading error because they were unable to read the symbol ○ included in the written test.

Comprehension errors were found 36% of the total errors. The students could not explain what information was given and are unable to draw diagram representing given problem were more frequent than couldn't explain the meaning of terminologies like: 'isosceles', 'parallelogram', 'circle', 'chord', 'radii', 'parallel', 'congruent' etc. So they were unable to understand overall meaning of problem and hence they committed comprehension error. The subject teacher in their discussion too, stated students could not explain due to lack of concept teaching at lower level.

Students committed transformation errors because neither had they stored sufficient geometric facts and theorems in their mind nor many of them could have ability to recall and apply them in appropriate condition they were 22.09% of the total errors. Therefore they could not transform the language in mathematical term and use appropriate strategy to prove the theorem.

Because of insufficient knowledge of axioms, postulates, previous proved theorem students committed process skill error. They make appropriate strategy but unable to prove the theorem with required logical explanation, statement and reasons as a result students occurred process skill error.

Students committed encoding errors because they have habit of memorizing to prove theorem. This is also appearing because of the carelessness of students.

According to van Hiele level of geometric thinking students at secondary school fall in level three i.e. deduction or formal proofs. In this study only few students could give proof of theorem with no error. Students considered all geometric figures as lookalike manner but they were not given concerned properties of figure. In the interview the students were given the figure of parallelogram. All the participants knew the figure but they could not explain the properties of parallelogram. Some students described figure as collection of properties but unable to give concise definition of geometric shapes and size. The rest of the students were also able to deduct informal proofs not based on axioms and definitions but in totally informal manner.

The researcher found that, teacher use no proper teaching material in teaching learning activity. They always teach geometry at the end of academic session and sometimes they did not teach geometry at all. Students did not get the basic knowledge of geometry since they were hurriedly taught. The teacher faced the problem in proving geometrical theorem due to inability of student reprocess geometrical terms, axioms, postulates and already proved statements and students inability to arrange to justify each step in a proof.

The researcher also found that lack of interaction between guardians and subject matter about the geometrical achievement, lack of approach to analyze difficulty level of geometrical questions asked in exams, interest not shown by school about interaction with guardians on the achievement of students, lack of personal interest on student about the result of geometry with teachers, lack of opportunity for teachers to interact with guardians and students about the achievement of geometry, student’s poor geometrical background, crowded

classroom, student's attitude to memorize theorem/problems without understanding become a problem for effecting learning. That's why students committed higher errors.

Thus, students were not aware of properties of geometric figure so maximum students were unable to prove the theorem by using properties. Since the students possessed ability to list out properties of geometric figure, some students prove the statement by using properties but maximum of them also failed to make logical connection between properties of figures.

Chapter- V

SUMMARY, FINDINGS, CONCLUSIONS AND RECOMMENDATION

After completion of analysis and interpretation of data in chapter IV, this chapter is devoted to summary, findings, conclusion and recommendations of the study.

Summary

This study seeks to answer two questions. What short of errors do students commit in proving theorem? Why do students commit such type of errors? The main objective of this study were to identify the errors committed by students of secondary level in proving theorem and to explore the causes of error committed by students in proving theorem.

The sample in this study consisted of 110 students from two public schools Shree Sanskrit Higher Secondary School and Kalika Kanya Mandir Secondary School of Baglung district were selected for the convenience of the researcher. The descriptive survey design was adopted for this study. Newman's error analysis procedure and theory of van Hiele's level of thought in geometry were considered as a theoretical base of the study. A test consisted of six theorems from grade IX compulsory mathematics text book prescribed by CDC was administered to the sample students to collect the required data. Collection of data was done into two phase. In the first phase error were collected from the answer sheets. The collected answer sheets of the student were checked and errors on those were identified. In the second phase an in-depth interview with each student was done to reveal the causes of the errors.

The identified errors altogether were classified by Newman's technique as well as Van Hiele level of geometrical thinking. Each types of error were

tabulated. The five categories of the errors were reading errors, comprehension error, Transformation error process skill error and encoding error, if the student couldn't read the word or symbol in the question/statement such type of error as reading error. The comprehension error was identified, if the student had been able to read all the word or symbol. But had not draw the appropriate figure and had not write the given condition. If the student could not write the condition of what is to be prove and could not draw the appropriate construction such error were counted as transformation error. If the student were unable to prove the theorem with required logical explanation statement and reasons then such error were identified as process skill error. At last, if the student unable to use appropriate sign, letters and write proved at last such error identified as encoding error.

Findings

The research was conducted to analyze errors committed by secondary school students in proving theorem mainly in the respect of Newman's error analysis procedure. Students problem in geometry in the theory of Van Hiele model of thinking were also assessed to explain students thinking in addition to Newman's error analysis procedure, Therefore the findings based on analysis have been made.

- The total number of errors committed by the students was 86 these errors were identified by checking out the answer copies of the achievement test and interviews with the students.
- This study showed that there were 2 (2.33%) errors at reading level, 31 (36.04%) at comprehension level, 19 (22.09%) at transformation level 25 (29.07%) process skill and 9 (10.47%) errors at encoding level. From which it was concluded that students committed more error at comprehension level.

- Students committed most of the error in comprehension process skill and transformation level. They were 87% of the total. Less error committed in reading and encoding stage 13% error were committed.
- It was found that the causes of errors were poor background knowledge on geometry, problem on understanding in teaching new concepts, facts, relations or skill to student's and failing student to reprocess geometrical terms, definition, axioms, postulates and already proved statement that are needed to prove.
- The students commit errors in the proofs of the theorem in geometry, it was mainly due to lack of enough prerequisite of geometric knowledge at lower level.
- Monotonous in learning, irresponsible administration and lack of appropriate materials are the cause of making higher error.
- Student did not have sufficient experiences and prerequisites knowledge at lower levels to encounter formal study of geometry at secondary level was the main causes of committing errors in proving theorem.

Conclusion

The research was conducted to analyze errors committed by secondary school students in proving theorem based on Newman error analysis procedure and Van Hieles theory on levels of thinking in geometry. On the basis of findings of study following conclusions were made.

The study showed that students committed errors while proving the theorem which might be due to lack of understanding of the language of statement problem to transform in mathematical form. The students committed

errors from the beginning of the solutions to the deduction of the result. The higher percentage of comprehension, process skill and transformation errors were found in the study by using the Newman procedure. Errors were made because of insufficient knowledge of geometric facts, axioms, postulates, already proved theorem and less skill to join them logically.

Relating to the van Hieles' level of geometric thinking, students committed errors only in third level i.e. deduction in proving theorem. The students might have the concept of figures but not the knowledge of properties. The students who had knowledge of properties failed in defining figures.

On the basis of analysis and findings of this study the major causes of the errors in developing the proofs of them in geometry at secondary level are insufficient experiences of geometry at lower level, excessive emphasis on two-column formal proofs, views and beliefs of both students and teacher. Towards geometry, less use of physical objects (models and manipulative) in proofs and problem of language.

Recommendations

This chapter deals with the recommendation given for the educational implication and further study. The researcher intended that the recommendations given for education implication will be used in classroom teaching learning activities and the recommendation for further study will be helpful to the other researcher to carry out other related researcher.

Recommendation for the Educational Implication

On the basis of the above result and conclusion the following recommendations are proposed for the educational implications and improvement in teaching learning.

- Since maximum errors were committed in comprehension and process skill level so while conducting teaching learning activities emphasis should be given in understanding of the problem and representing mathematical concepts, facts, principle skill, already proved theorem, postulate and axioms in mathematical language.

- Geometry course should be taught without excessive emphasis on rigor of axiomatic. Geometry teachers are suggested to avoid treating proofs only as formal proofs.

- Since student which belongs in lower level than they need to have to solve given problems, can't solve the problems, therefore in any domain, of geometry teacher should find out students level of thought and should give sufficient experiences of each level which they must possess before solving the problem.

- Teacher should focus equally to the different steps of the solutions. However mathematics teacher should be aware of the language they use in the classroom they should use simple language and all geometrical concepts should be not only verbally but also symbolically with necessary illustration.

- Efforts should be targeted to develop sound background in geometry at lower levels to study geometry at higher level successfully.

Recommendations for the Further Study

The result and the conclusion of this study generate some other questions, which need to be verified. Some of them are presented here.

- Research can be replicated with wider coverage of sample of students.
- Study of this kind can be conducted at lower secondary as well as primary levels.

- How the result of error analysis be used in classroom teaching learning activities?

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APPENDICES

APPENDIX I

Written test

All the students are requested to give answer in their own words as far as possible. This is because researcher can find actual weaknesses and strengths of students and can give appropriate suggestions to improve their performance in proving theorem.

Attempt all the questions

1. Prove that the exterior angle of a triangle is equal to the sum of the opposite two interior angle.
2. Prove that if the base angles of triangles are equal then it is called an isosceles triangle.
3. Prove that the opposite sides of parallelogram are equal.
4. Prove that the diagonals of parallelogram bisect to each other.
5. Prove that in a circle equal chords are equidistant from the centre.
6. In OABC the perpendicular drawn from O to the chord BC then OX bisects the chord.

APPENDIX II

INTERVIEW SCHEDULE (On Newman's Procedure)

To find causes of errors interviewer is wise to ask some other questions related to each of six subjective questions depending upon the responses.

- Please, read the question.
- Tell me what the question is asking you, draw the appropriate figure and write the given condition of the statement… (corresponding terminologies included in the question)
- Tell me what is to be prove and how you are going to construct?
- Show me how you can prove the question explain to me what you are doing as you do it.
- Write down your prove in on acceptable written form to the question.

APPENDIX III

Interview Schedule (On van Hiele Model of Geometric Thinking)

Interviewer is wise to ask some questions which are not included in schedule if necessary as well as to leave some questions included in the schedule depending upon responses to the questions asked by interviewer.

Triangles

- Which of the figures represent triangle?
- How do you know this is triangle?
- What is triangle?
- Point out interior angles of a triangle in figure.
- How do you know this is an isosceles triangle?
- How can you prove it?
- Some more questions if necessary

Parallelograms

- Which of the figures represent parallelogram?
- How do you know this is parallelogram?
- Define quadrilateral and parallelogram
- What are the properties of quadrilateral and parallelogram?
- Some more questions if necessary

Circles

- Which of the figures represent circle?

- How do you know this is circle?
- Define circle?
- Define chord?
- What are radius/ diameter?
- Some more questions if necessary.