

**Relevance of Knowledge of Real Analysis in Mathematics  
Instruction at Secondary Level**

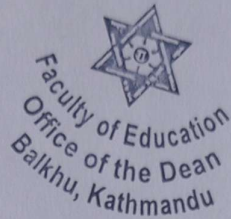
**Submitted By  
Deb Bahadur Chhetri  
Roll No. 12 (Sixth Batch)**

**A Dissertation for the Degree of Doctor of Philosophy (PhD) in  
Mathematics Education**

**Submitted To  
Graduate School of Education  
Office of the Dean, Faculty of Education  
Tribhuvan University  
Kirtipur, Kathmandu, Nepal**

**June 2025**

Relevance of Knowledge of Real Analysis in Mathematics  
Instruction at Secondary Level



Submitted By

Deb Bahadur Chhetri

Roll No. 12 (Sixth Batch)

T.U. Regd. No.: 9-5-28-169-2021

A Dissertation for the Degree of Doctor of Philosophy (PhD) in Mathematics  
Education

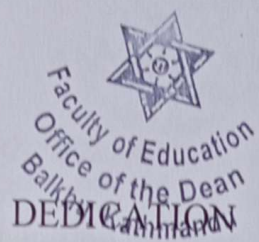
Submitted To

Graduate School of Education

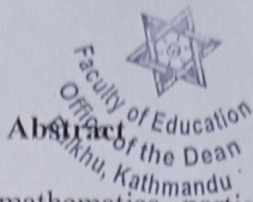
Office of the Dean, Faculty of Education

Tribhuvan University

Kirtipur, Kathmandu, Nepal



This is dedicated to all the mathematics educators and scholars.



The integration of advanced mathematics, particularly real analysis, and its pedagogical rationale in teacher preparation programs has emerged as a critical concern in mathematics education. While real analysis forms a core component of mathematics teacher preparation, its practical relevance for secondary-level mathematics instruction remains underexplored, particularly in Nepal.

In this context, study aimed to assess the relevance of knowledge of real analysis in secondary-level's mathematics instruction. An explanatory sequential mixed-methods design was used. In the first phase, quantitative data were collected from 387 secondary-level mathematics teachers of Gandaki province, selected by using proportionate stratified sampling. Descriptive and inferential statistics were utilized to analyze the data. The results revealed secondary-level mathematics teachers hold positive attitudes toward real analysis in content connection ( $p < .001$ ), pedagogical application ( $p < .001$ ), and mathematical proficiency ( $p < .001$ ), which collectively, predicted teaching efficacy ( $p < .001$ ,  $R^2 = .125$ ). Subsequent analysis showed content connections did not independently predict mathematical proficiency ( $p = 0.25$ ) and teaching efficacy ( $p = .24$ ), similarly, pedagogical applications did not predict teaching efficacy ( $p = .16$ ) independently.

The qualitative phase involved task-based interviews and classroom observations with eight purposively selected secondary-level mathematics teachers from Baglung municipality, along with interviews with three mathematics educators. A comparative analysis of secondary mathematics curricula and real analysis content was also conducted. Thematic analysis was used to analyze the data. The results highlighted real analysis as essential for deepening mathematical understanding, logical reasoning, and mature thinking, all of which contribute to teaching efficacy.

Key secondary-level mathematics topics, functions, limits, continuity, sequences, series aligned closely with real analysis. Additionally, three types of connections between real analysis and secondary-level mathematics were identified: logical, procedural, and alternative representations. These connections enhance conceptual understanding, critical thinking, and sequential learning. Ensuring vertical linkage and applying a bridging strategy strengthens the connection. Problem-solving, abstraction, and concept development are major contexts of applying real analysis knowledge through axioms, theorems, and alternative content representations. Real analysis strengthens teachers' confidence, proficiency, and instructional efficacy. Thus, knowledge of real analysis is relevant for secondary-level mathematics instruction.

This study establishes that content bridging strategies serve as a critical mechanism for connecting advanced mathematical knowledge with classroom practice and teachers need to participate in those programs which enable them to scaffold and transform the abstract mathematical knowledge into classroom practices. The relationship between real analysis knowledge and its dual role in developing teachers' professional competencies and enhancing student learning outcomes remains a critical area for further investigation.

शिक्षक तयारी कार्यक्रममा रियल एनालिसिस (Real analysis) तथा उच्चस्तरीय गणितको उपयोग एउटा महत्त्वपूर्ण सरोकारको विषय बनेको छ। यो माध्यमिक तहका लागि गणित शिक्षक तयारी कार्यक्रमको मूल अङ्ग बनेको भए पनि यसको सान्दर्भिकताको अध्ययन विशेष गरी नेपालको सन्दर्भममा निकै कम छ।


यही क्रममा प्रस्तुत अध्ययनले माध्यमिक तहको गणित शिक्षणका लागि रियल एनालिसिस ज्ञानको सान्दर्भिकता लेखाजोखा गर्ने उद्देश्य राखेको थियो। अनुसन्धानका लागि मिश्रित अनुसन्धान ढाँचाअन्तर्गत व्याख्यानात्मक क्रमिक मिश्रण अनुसन्धान विधि (explanatory sequential mixed-methods research design) उपयोग गरिएको थियो। पहिलो चरणमा आनुपातिक स्तरीकृत नमुना छनौट विधिको उपयोग गरेर परिमाणात्मक तथ्याङ्क सङ्कलन गरिएको थियो। तथ्याङ्कको विश्लेषणका लागि वर्णनात्मक र अनुमानात्मक तथ्याङ्कशास्त्रलाई प्रयोग गरिएको थियो। अध्ययनको नतिजाबाट विषयवस्तुको सम्बन्ध ( $p < 0.001$ ), रियल एनालिसिसमा समावेश ज्ञानको शैक्षणिक उपयोग ( $p < 0.001$ ), गणितीय प्रवीणताको विकास ( $p < 0.001$ ) प्रति शिक्षकहरूको अभिवृत्ति सकारात्मक रहेको पाइयो। गणितीय विषयवस्तुको सम्बन्ध, शैक्षणिक उपयोग, गणितीय प्रवीणताको विकासले संयुक्त रूपमा शिक्षण सक्षमतालाई ( $p < 0.001$ ,  $R^2 = 12.5\%$ ) सार्थक रूपमा अनुमान गर्छ। त्यसै गरी रियल एनालिसिस र माध्यमिक तहका विषयवस्तुबिच रहेको सम्बन्धले शिक्षण सक्षमता र गणितीय प्रवीणतालाई स्वतन्त्र रूपमा अनुमान गर्न नसक्नुका साथै शैक्षणिक उपयोगले शिक्षण सक्षमतालाई सार्थक रूपमा अनुमान गर्न सक्दैन भन्ने कुरा नतिजाले देखाएको छ।

गुणात्मक चरणमा बागलुङ नगरपालिकाबाट उद्देश्यपूर्ण रूपमा चयन गरिएका आठ माध्यमिक गणित शिक्षकहरूसँग कार्य-आधारित अन्तर्वार्ता र कक्षा अवलोकनका साथै तीन गणित शिक्षाविद्हरूसँग अन्तर्वार्ता सम्पन्न गरिएको थियो। यस क्रममा माध्यमिक गणित पाठ्यक्रम र वास्तविक विश्लेषण सामग्रीको तुलनात्मक विश्लेषण पनि गरिएको थियो। तथ्याङ्कको विश्लेषणका लागि विषयगत विश्लेषणलाई प्रयोग गरिएको थियो।

वास्तविक विश्लेषणले गणितीय समझ, युक्तिपूर्ण सोचलाई गहिरो बनाउन महत्त्वपूर्ण भूमिका खेलेको छ । यसले शिक्षण प्रभावकारितालाई सहयोग गर्छ । माध्यमिक तहका प्रमुख गणित विषयहरू: फलन (Functions), सीमान्तमान (limits), निरन्तरता (continuity), अनुक्रम (sequences) र श्रेणी (series) जस्ता विषयवस्तु रियल एनालिसिसका ज्ञानसँग जोडिएका छन् । यसबाहेक, रियल एनालिसिस र माध्यमिक गणितबिच तीन प्रकारका सम्बन्धहरू पहिचान गरिएको छ : तार्किक, प्रक्रियात्मक, र वैकल्पिक प्रस्तुतीकरण । यी सम्बन्धहरूले अवधारणात्मक समझ, समालोचनात्मक सोच, र क्रमिक सिकाइलाई बढावा दिन्छन् ।

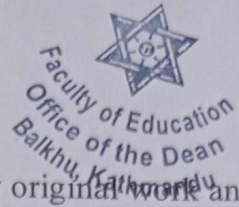
तार्किक, प्रक्रियात्मक, र वैकल्पिक प्रस्तुतीकरणगत सम्बन्धहरूले अवधारणात्मक समझ, समालोचनात्मक सोच, र क्रमिक सिकाइलाई बढावा दिन्छन् । लम्बीय (vertical linkage) र ब्रिजिङ रणनीति (bridging strategy) लागू गरेर यस सम्बन्धलाई अझ बलियो बनाउन सकिन्छ । समस्या समाधान, अमूर्तता (abstraction) र अवधारणा विकास वास्तविक विश्लेषण ज्ञान र वैकल्पिक सामग्री प्रस्तुतीकरणमार्फत रियल एनालिसिसको ज्ञानलाई प्रयोग गर्ने प्रमुख सन्दर्भहरू हुन् । वास्तविक विश्लेषणले शिक्षकहरूको आत्मविश्वास, प्रवीणता, र शिक्षण प्रभावकारितालाई बढाउँछ । यसकारण माध्यमिक तहको गणित शिक्षणका लागि रियल एनालिसिसको ज्ञान सान्दर्भिक छ ।

यस अध्ययनले सामग्री सम्बन्ध (Content Bridging) रणनीतिहरू उच्च गणितीय ज्ञानलाई कक्षाकोठाको अभ्याससँग जोड्ने एक महत्त्वपूर्ण उपकरण हो र शिक्षकहरू अमूर्त गणितीय ज्ञानलाई तार्किक र अर्थपूर्ण तरिकाले विद्यालयस्तरका विद्यार्थीले बोध्ययोग्य हुने गरि कसरी विभाजन गर्ने र अमूर्त गणितलाई सरल तरिकाले कक्षाकोठामा स्थानान्तरण गर्ने भन्नेकुरा सिकाउने पेशागत तालिममा सहभागी हुनु पर्छ । यस्ता कार्यक्रमहरूमा सहभागी हुनुपर्छ भन्ने प्रमाणित गरेको छ । वास्तविक विश्लेषणको ज्ञानले शिक्षकहरूको पेशागत क्षमता र विद्यार्थीहरूको सिकाइ नतिजामा पार्ने द्वैध भूमिकाको सम्बन्ध अझै गहन अनुसन्धानको विषय बनेको छ ।

  
Faculty of Education  
Office of the Dean  
Balkhu, Kathmandu  
© Deb Bahadur Chhetri

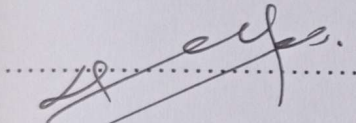
2025

All rights reserved.

**Declaration**

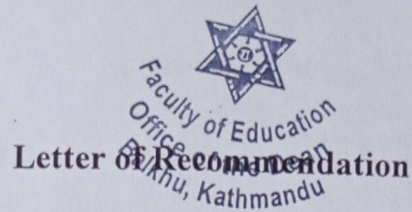
I hereby declare that this dissertation is my original work and has not been submitted for candidature for any other degree.

I understand that my dissertation will be part of the permanent collection of the Tribhuvan University Library. My signature below authorizes the release of my dissertation to any readers upon request.



Deb Bahadur Chhetri

26 June 2025



The undersigned certify that I have read and recommend to the Faculty of Education, Tribhuvan University, for acceptance a thesis entitled **Relevance of Knowledge of Real Analysis in Mathematics Instruction at Secondary-Level** submitted by Deb Bahadur Chhetri in partial fulfillment of requirements for the degree of Doctor of Philosophy in Mathematics Education.

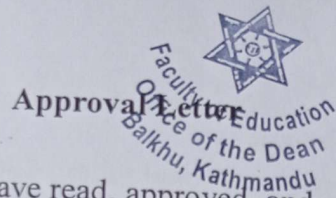
Associate Prof. Bishnu Khanal, PhD

Dissertation Supervisor

Mahendra Ratna Campus, Tahachal

Tribhuvan University, Nepal

26 June 2025



Approval Letter

Faculty of Education  
Office of the Dean  
Balkhu, Kathmandu

The undersigned certify that we have read, approved, and recommended to the Faculty of Education, Tribhuvan University for acceptance, a dissertation entitled **Relevance of Knowledge of Real Analysis in Mathematics Instruction at Secondary-Level** by Deb Bahadur Chhetri in partial fulfillment of requirements for the degree of Doctor of Philosophy in Mathematics Education on 26 June 2025.

Prof. Bed Raj Acyarya, PhD  
Dean, FoE, Tribhuvan University  
and  
Chair, Research Committee

Associate Prof. Bishnu Khanal, PhD  
Dissertation Supervisor  
Tribhuvan University

Prof. Hari Prasad Upadhyay, PhD  
External Examiner  
Tribhuvan University

Associate Prof. Rabindra Siwakoti, PhD  
Research Committee Member  
and  
Acting Director GSE, TU

### Acknowledgments

I wish to convey my heartfelt gratitude to Associate Professor Bishnu Khanal, PhD, for mentoring my research work and providing continuous guidance, constructive feedback, and scholarly insights at every stage of this study. His unwavering academic support has been instrumental in shaping the direction and quality of this research.

I sincerely appreciate Professor Bed Raj Acharya, PhD, Dean of the Faculty of Education, for his invaluable suggestions in framing this study. I am particularly grateful to Professor Hari Prasad Upadhyay, PhD, and Professor Manoj Kumar Sexena, PhD, external examiners. I extend my sincere thanks to Associate Professor Rabindra Siwakoti, PhD, Acting Director of the Graduate School of Education (GSE), for his constructive guidance and consistent support.

I am also deeply indebted to the Research Committee of the Faculty of Education, Professor Basu Dev Kefle, PhD; Professor Bhimsen Devkota, PhD; Professor Peshal Khanal, PhD; Professor Ramesh Adhikari, PhD; Associate Professor Rajendra Kunwar, PhD; and Associate Professor Gopal Prasad Pandey, PhD, for their critical comments and insightful feedback, which significantly contributed to improving the quality of this study and dissertation. I am particularly grateful to the internal examiners, Professor Eka Ratna Acharya, PhD, and Professor Bishnu Dhungana, PhD, for their significant contributions to enhancing the quality of this research.

Likewise, I would like to express my sincere appreciation to Professor Hira Bahadur Maharjan, PhD; Professor Uma Nath Pandey; and Professor Lekhnath Sharma, PhD for their valuable suggestions and continued encouragement. I also extend my gratitude to Lecturer Kamal Sharma, PhD, Lecturer Ashok Sapkota, Janak

Paudel and Mr. Shyam Prasad Sharma for their meticulous review and editing of the language in this dissertation.

I express my sincere gratitude to Professor Ram Prasad Upadhaya, PhD, of Dhawalagiri Campus, Baglung, and Dirgha Raj Joshi, PhD, for their insightful suggestions, from questionnaire preparation to guidance in quantitative data analysis. I am deeply grateful to all respondents and institutions involved in this study for providing the necessary data and information. My appreciation extends to Professor Din Bahadur Thapa, PhD, Associate Professor Sudarshan Silwal, PhD, Lecturer Hiralal Regmi, Lecturer Bhakti Ram Neupane, and all my colleagues at Dhawalagiri Multiple Campus, Baglung, as well as my friends and family members for their continuous encouragement and support throughout this journey.

Finally, I would like to extend my heartfelt gratitude to the dedicated teachers who participated in this study. Their cooperation and valuable time were essential in making this research possible.

## Table of Contents

Declaration .....	vii
Letter of Recommendation .....	viii
Approval Letter .....	ix
Acknowledgments .....	x
Table of Contents .....	xii
List of Tables.....	xix
List of Figures .....	xxi
Abbreviations/ Acronyms .....	xxiii
Chapter 1.....	1
Introduction.....	1
Background of the Study.....	1
Statement of Problem.....	11
Purpose of the Study .....	13
Research Questions.....	14
Rationale of the Study.....	16
Delimitations of the Study.....	17
Definition of Related Terms .....	18
Organization of Dissertation.....	20
Chapter Summary .....	20
Chapter 2.....	21
Literature Review .....	21
Philosophical and Theoretical Perspective on Curriculum Relevance .....	21
Progressivist Approach and the Relevance of Curriculum.....	21
The Constructivist Learning Theory and the Relevance of Curriculum.....	23

Theories on Mathematics Teacher Preparation Program .....	26
Mathematical Knowledge for Teaching .....	28
Habermas Theory of Knowledge Constitutive Interests.....	32
Theoretical Framework.....	33
Model of Curriculum Evaluation.....	35
Social Cognitive Theory .....	39
Application of Theoretical Review .....	39
Dimensions of Knowledge for Mathematics Instruction .....	39
Mathematical Understanding for Secondary Teachers: A Framework .....	41
Forms of Advanced Mathematical Knowledge.....	43
Summary of the MUST Framework and Form of Advanced Mathematics .....	45
Real Analysis as a Source of Mathematical Knowledge for Teaching .....	46
Content Knowledge for Mathematics Instruction .....	47
Advanced Mathematical Knowledge and Teaching Efficacy .....	49
TPACK Framework and the Use of Real Analysis in Instruction.....	50
Relation of Content Knowledge With Proficiency and Learning Outcomes .....	51
Mathematical Content Knowledge to Expertise in Mathematics Instruction.....	52
Connection between School Mathematics and Advanced Mathematics.....	53
Knowledge of Real Analysis for School Teachers.....	55
Place of Real Analysis in Teacher Preparation Programme .....	56
Pedagogical Application of Real Analysis .....	58
Mathematics at the Secondary Level.....	59
Relevance Indicators.....	60
Applications of Thematic Review .....	62
Empirical Studies on Advanced Mathematics in School Instruction.....	63

Relevance of Advanced Mathematics in School Teaching .....	63
Teacher Preparation and Professional Competence .....	64
Role of Mathematical Knowledge in Teaching.....	65
Impact of Advanced Mathematics on Pedagogical Practices .....	66
Applications of Empirical Review.....	68
Conceptual Framework of Relevance of KRA for SLM Instruction.....	69
Mathematical Connection.....	69
Pedagogical Application.....	73
Enhancing Teachers' Proficiency .....	74
Teaching Efficacy.....	76
Research Gap .....	77
Justification of the Study.....	79
Chapter Summary .....	80
Chapter 3.....	82
Research Methodology .....	82
Philosophical Considerations .....	82
Ontology .....	82
Epistemology .....	83
Axiology .....	84
Research Method.....	84
Rhetoric .....	85
Research Design .....	85
Justification of Methodology.....	87
Explanatory Sequential Research Design.....	88
Workflow of Explanatory Sequential Research Design.....	91

Selection of Study Area .....	92
Population and Sample .....	93
Sampling Procedure.....	95
Quantitative Data Collection Tools.....	97
Likert Scale for Attitude Measurement .....	97
Teaching Efficacy Measurement Scale .....	98
Pilot Survey .....	99
Content Validity of Survey Scale .....	100
Construct Validity of Survey Scale .....	101
Reliability of Survey Scale .....	102
Survey Data Collection Process .....	103
Mathematical Connection Framework .....	104
Classroom Observation.....	105
Task-Based Interview .....	107
Interview .....	109
Quality Assurance .....	110
Thrustworthiness .....	110
Data Preparation .....	112
Data Analysis Tools and Techniques .....	114
Quantitative Data Analysis Tools and Techniques.....	114
Qualitative Data Analysis Tools and Techniques.....	116
Data Triangulation.....	118
Methodological Framework .....	118
Report Writing and Language Editing .....	120
Ethical Considerations .....	120

Chapter Summary .....	121
Chapter 4.....	123
Analysis and Interpretation of Data.....	123
Phase i: Analysis and Interpretation of Quantitative Data .....	123
Socio-demographic Characteristics of Respondents .....	123
Teachers' Attitude towards Knowledge of Real Analysis.....	125
Teachers' Teaching Efficacy Level.....	136
Relationship among Attitudes, Proficiency and Teaching Efficacy.....	138
Phase ii: Analysis and Interpretation of Qualitative Data.....	148
Socio-demographic Status of Respondent.....	149
Teachers' Perception of Knowledge of Real Analysis.....	151
Role of Knowledge of Real Analysis in Mathematics Teachers' Preparation.....	175
Key Area of Content Connection between KRA and SLM .....	186
Mapping Real Analysis Content to Secondary Mathematics .....	196
Importance of Link between KRA and SLM .....	197
Techniques of Linking Knowledge of Real Analysis with School Mathematics...	204
Context where Teachers Applying KRA for SLM Instruction .....	209
Teachers' Practices of Knowledge of KRA in School Mathematics Instruction ....	215
Phase iii: Connecting Quantitative Data with Qualitative Explanation .....	227
Teachers' Insight on Knowledge of Real Analysis for Mathematics Instruction...	227
Contribution of Real Analysis to the Development of Teaching Efficacy .....	231
Importance of Knowledge of Real Analysis for Mathematics Instruction .....	234
Connection between Real Analysis and School Mathematics .....	235
Importance of Connecting Real Analysis with the School's Mathematics .....	237
Connection Strengthening Techniques between KRA and SLM.....	238

Teachers' Practices of KRA in School Mathematics Instruction .....	240
Chapter Summary .....	245
Chapter 5 .....	247
Findings and Discussion .....	247
Findings .....	247
Teachers' Attitude towards Mathematical Knowledge of Real Analysis .....	247
Relationship among Attitudes towards CONTN,PALI, MPROF and TE .....	249
Teachers' Perceptions of Knowledge of Real Analysis for Instruction .....	250
Importance of KRA for School Mathematics Instruction .....	251
Content Connection between KRA and SLM.....	252
Necessity of Content Connection .....	254
Techniques of Strengthening Link between KRA and SLM .....	254
Context of the Classroom where Teachers Articulate KRA.....	255
Teachers' Practices of KRA During the Instruction.....	256
Divergent Findings and Instructional Implications .....	256
Discussion .....	257
Teachers' Views on Knowledge of Real Analysis for Instruction.....	257
Relation Among Attitude on Real Analysis and Teaching Efficacy .....	260
Importance of Knowledge of Real Analysis for Instruction.....	263
Content Connection between KRA and SLM.....	265
Linking Real Analysis with Secondary-level Mathematics and Its Role .....	267
Teachers' Use of Knowledge of Real Analysis in Classroom Practice .....	270
Divergence in Relationships Among Key Constructs .....	274
Model for Relevance of KRA for SLM Instruction .....	276
Epistemic Foundation: Mathematical Domain Relevance .....	277

Pedagogical Bridge.....	279
Professional Transformation.....	280
Chapter Summary .....	281
Chapter 6.....	282
Conclusion and Implications.....	282
Conclusions.....	282
Implications for Curriculum Planner.....	288
Implications for Pedagogy and Teacher Education.....	290
Implications for Teachers.....	292
Implications for Further Research.....	292
Chapter Summary .....	294
References .....	296
Appendixes.....	i
Appendix A: Attitude and Efficacy Scale for Mathematics Teachers .....	i
Appendix B: Mathematical Connection Seeking Framework.....	vii
Appendix C: Classroom Observation Protocol.....	viii
Appendix D: Task-Based Interview Protocol .....	ix
Appendix E: Interview Guidelines (for Mathematics Educators) .....	xii
Appendix F: Model Answer of Task Based Interview .....	xiv
Appendix G : Reliability of Survey Tools.....	xviii
Appendix H Construct Validity Index .....	xix
Appendix I: Summary of Category and Theme .....	xx
Appendix J: Consent Form.....	xxi
Appendix K: Toolkit.....	xxiv

## List of Tables

Table 1 Relevance Indicators .....	60
Table 2 Status of Sampling .....	95
Table 3 Nature of Scale .....	99
Table 4 Content Validity Index of Relevance Measurement Scale.....	101
Table 5 Methodological Framework .....	119
Table 6 Demographic Summary of Respondents.....	124
Table 7 Scale of Attitude towards Content Connection between KRA and SLM ....	126
Table 8 Attitude towards Pedagogical Application for School's Mathematics .....	129
Table 9 Scale of Attitude towards KRA in Developing Mathematical Proficiency	131
Table 10 Attitude Test .....	133
Table 11 Attitudes Towards KRA Across Demographic Characteristics .....	134
Table 12 Teaching Efficacy Level of Teacher (Lowest =1 and Highest =5).....	136
Table 13 Measurement Model Fit Criteria.....	139
Table 14 Fornell-Larcker Criterion Value (and HTMT Ratio).....	140
Table 15 Hypothesis Test Results from PLS-SEM.....	143
Table 16 Moderating Effect Test Result.....	146
Table 17 Socio-demographic Status of Teachers Involved in Qualitative .....	150
Table 18 Socio-Demographic Status of Mathematics Educators.....	150
Table 19 Summary Table of Theme on Teacher's Perception.....	152
Table 20 Theme on Why KRA for Teacher Preparation .....	175

Table 21 Category of Mathematical Connection Found in Curriculum and Text Book	187
Table 22 Hierarchical Relationship between MKR and SLM .....	188
Table 23 Axiomatic Connection between MKR and SLM.....	190
Table 24 Procedural Connection between MKR and SLM .....	192
Table 25 Connection between MKR and SLM: Alternative Representation .....	193
Table 26 Connection between MKR and SLM: Generalization .....	194
Table 27 Code Summary of Importance of Link between KRA and SLM .....	197
Table 28 Linking Techniques between KRA and SLM .....	204
Table 29 Contexts of Application of KRA in SLM Instruction.....	210
Table 30 Teachers' Practices of KRA in Mathematics Instruction.....	216
Table 31 Joint Display of Teachers' Attitude and Perception towards Real Analysis.....	228
Table 32 Joint Display of Relationships Among CONTN, PAPLI, MPROF, and TE..	232
Table 33 Joint Display of Connection between Real Analysis and School Mathematic.	236
Table 34 Joint Display of Importance of Knowledge of Real Analysis.....	237
Table 35 Joint Display of Connection Strengthening ways between KRA and SLM	239
Table 36 Joint Display of Teachers' Practices of KRA.....	240

## List of Figures

Figure 1 Area between Two Curves .....	4
Figure 2 Mathematical Knowledge for Teaching .....	29
Figure 3 Conceptualization of School-Related Content Knowledge .....	31
Figure 4 Theoretical Framework of Assing Relevace .....	33
Figure 5 Kirkpatrick Model of Training Evaluation .....	37
Figure 6 Perspective of Mathematical Understanding for Teacher .....	41
Figure 7 Conceptual Framework of Relevance of Real Analysis Knowledge .....	70
Figure 8 Methodological Flowchart of Explanatory Mixed Method.....	92
Figure 9 Measurement Model .....	140
Figure 10 Bootstrap Result of Structural Model .....	144
Figure 11 Bootstrap output of Moderator Analysis.....	147
Figure 12 Content Connection Status .....	152
Figure 13 Logical Connection of Riemann Integral in Area Calculation .....	157
Figure 14 Link Analysis Result of Content Connection.....	159
Figure 15 Status of Pedagogical Application of Real Analysis .....	161
Figure 16 Constructs of Pedagogical Application and their Relationships .....	167
Figure 17 Teachers' Perception Status on MPROF Development by KRA.....	169
Figure 18 Link Analysis Result of Mathematical Proficiency Development.....	174
Figure 19 Role of Real Analysis in Teacher Preparation Program .....	176
Figure 20 A Model of Question Reflecting Mathematical Maturity Check.....	178
Figure 21 Construct Relationships Contributed by KRA .....	185
Figure 22 Exiting Content Connection .....	196
Figure 23 Reasons for Requirement of Link between KRA and SLM.....	198
Figure 24 Teachers' Example of Limit .....	199

Figure 25 Inter-Relationship Among the Importance of Link .....	203
Figure 26 Linking Techniques and its Importance.....	205
Figure 27 Concept of Composite Function Demonstrated by Teacher.....	210
Figure 28 Teachers' Creation of Example of Sequence .....	211
Figure 29 Teacher Practice of Knowledge of Real Analysis in Function Teaching..	213
Figure 30 Example of Strictly Monotonically Increasing Sequence .....	217
Figure 31 Teachers' Alternative Representation of Limit.....	222
Figure 32 Demonstration of Limit by Dividing a Rectangle.....	223
Figure 33 Visualization of existence of Function .....	224
Figure 34 Model for Relevance of KRA for SLM Instruction .....	277

**Abbreviations/ Acronyms**

ANOVA	Analysis of Variance
B.Ed.	Bachelor of Education
CDC	Curriculum Development Centre
CIPP	Context, Input, Process, Product (Evaluation Model)
CK	Content Knowledge
CONTN	Content Connection
EMIS	Education Management Information System
FoE	Faculty of Education
KRA	Knowledge of Real analysis
MKR	Mathematical Knowledge of Real Analysis
MPROF	Mathematical Proficiency
MUST	Mathematical Understanding for Secondary Teachers
PAPLI	Pedagogical Application
PK	Pedagogical Knowledge
PLS-SEM	Partial Least Squares Structural Equation Modeling
RA	Real Analysis
RQ	Research Question
SEM	Structural Equation Modeling
SLM	Secondary-Level Mathematics
SPSS	Statistical Package for the Social Sciences
TE	Teaching Efficacy
TK	Technological Knowledge
TPACK	Technological Pedagogical Content Knowledge
TU	Tribhuvan University

## **Chapter 1**

### **Introduction**

Mathematics teacher preparation programs at the university level cover a broad range of subjects, including mathematics, mathematics education, educational philosophies, learning theories, and aim to equip prospective teachers with knowledge and skills in content, pedagogy, and educational philosophies. This program introduces different branches of pure mathematics and mathematics education. Real analysis is a branch of advanced mathematics that deals with the rigorous study of real numbers, point set topology, sequence, series, function, limit, derivative etc. defined on the set of real numbers. It is typically introduced in undergraduate and graduate courses at the university level and is an important part of the mathematics teacher preparation program. This study identifies the relevance of real analysis for secondary-level mathematics instruction. This chapter includes background, rationale, statements of problem, purposes, research questions, hypothesis, and delimitation of the study.

### **Background of the Study**

A successful mathematics teacher requires a solid foundation in mathematical content as well as a repertoire of pedagogical skills and strategies to deliver instruction effectively and support student learning. The discourse on what mathematics teachers are required to know to be successful mathematics teachers has been at the forefront of the field of research in mathematics education. Secondary-level mathematics teachers need to possess sound knowledge and skills in both content and pedagogy.

In this context, attention has turned to the role of advanced mathematics in mathematics teacher preparation programs. School teacher preparation programs

offered at the university level include advanced mathematics courses such as analysis and algebra which are termed advanced mathematics (Even, 2011). It incorporates various branches of mathematics, including abstract algebra, linear algebra, real analysis, topology, and number theory, forming foundational content knowledge acquired during undergraduate studies (Stockton & Wasserman, 2017; Zazkis & Leikin, 2010). Teachers in mathematics education generally complete advanced mathematics courses, such as algebra, analysis, and topology, as part of their teacher preparation programs at universities or other related academic institutions. Among these courses, real analysis is considered a fundamental subject. However, many teachers question the relevance of such advanced mathematics courses for their future teaching careers (Zazkis & Leikin, 2010). Similarly, it is noted that the amount of advanced mathematics included in teacher preparation programs does not guarantee teaching quality; that is, one's mathematical understanding does not necessarily translate into the ability to enhance others' understanding (Monk, 1994). However, mathematicians and mathematics educators believe that mathematics teachers should know more about mathematics than what they teach (Association of Mathematics Education, 2017; Conference Board of the Mathematical Sciences, 2013; Wasserman & Stockton, 2013). Hence, the inclusion of advanced mathematics remains a central component of mathematics teacher preparation programs. Building on this global perspective, mathematics teacher preparation courses in almost all Universities have introduced advanced mathematics (Schmidt et al., 2013). Different universities in Nepal, such as Tribhuvan University (TU), have introduced advanced mathematics courses such as real analysis in their teacher programs (Panthi & Jha, 2016). These courses are designed to enhance prospective teachers' ability to make connections across mathematical domains, develop mathematical familiarity for problem-solving,

and increase their epistemological awareness of the subject matter (Yan et al., 2022). Besides these necessities of advanced mathematics, relevant questions, such as what kind of mathematical knowledge is required for mathematics teachers and how professional-specific knowledge of mathematics can be characterized, have arisen for mathematics education designing programs (Dreher et al., 2018). In this scenario, this study explores the relevance of knowledge of real analysis in mathematics teacher preparation programs in Nepal.

Real analysis is a branch of mathematics that deals with the study of real numbers and their properties. Abbott (2015) views that real analysis involves examining the characteristics and properties of real numbers in different structures but is not limited to limit, continuity, convergence, differentiation, and integration.

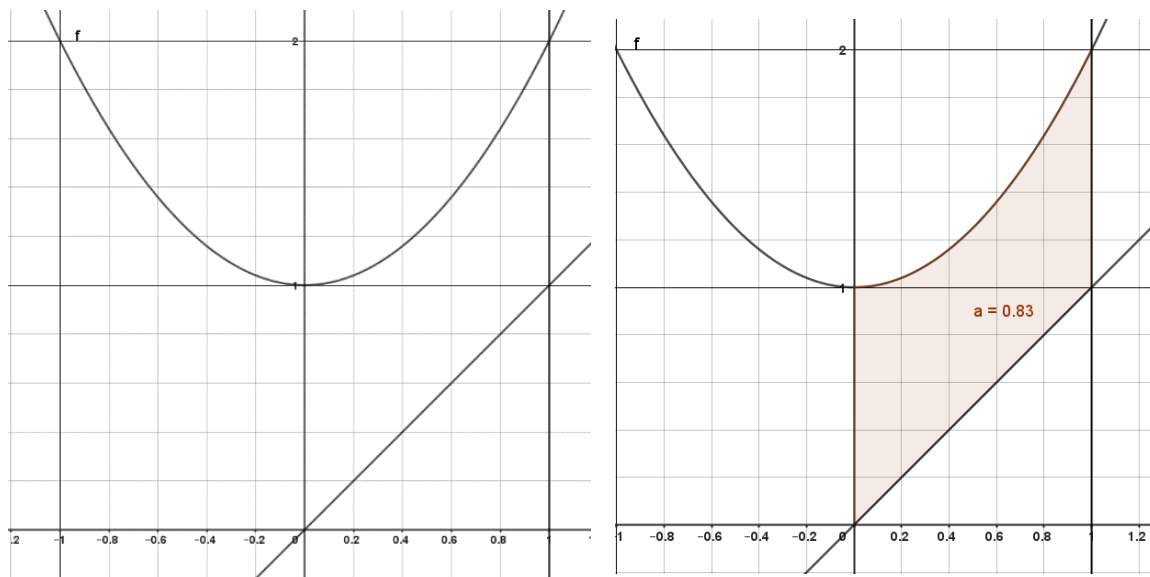
Real analysis is an advanced mathematics course typically included in undergraduate mathematics, science, and engineering programs. However, it is also becoming more common in undergraduate teacher preparation programs as a means of developing a solid foundation of mathematical knowledge for prospective teachers. It is considered an essential component of mathematics education for prospective teachers to develop reasoning abilities (Bhupal, 2021). This highlights the importance of real analysis in mathematics teachers' preparation programs.

Current research trends regarding the significance of advanced mathematics in teacher preparation programs is focused on rationalizing the role of advanced mathematics within teacher preparation programs (e.g., Moreria & Vianna, 2016). Thus, identifying the mathematical knowledge required for school mathematics teachers, its domains, and its relevance is a key focus in current mathematics education research.

In daily teaching practice, teachers frequently rely on fundamental mathematical principles, such as axioms and the properties of real numbers, to guide instruction. For instance, consider the commutative axiom, which asserts that for any two numbers  $a, b \in \mathbf{R}$  the order of multiplication does not affect the result:  $a * b = b * a$ . Another important principle teachers often utilize is the transitive property:  $a = b, b = c$  implies  $a = c$ . Besides these general logical connections, other connections between knowledge of real analysis and school mathematics can be found in mathematical activities. For instance, consider Figure 1, where a problem is considered to find the area of the shaded region. This is a type of problem related to irregular-shaped problem where the parts are counted as shown in the first part of Figure 1. It is related to the Darboux sum and the Riemann integral, which are introduced in B.Ed. (Bachelor of Education) program of TU.

**Figure 1**

*Area between two Curves*



The counting process shows nearly 21 small units, and then its area is  $21 \div 25 = 0.84$  square units because only 25 small units equal one square unit. By using the

formula for calculating integration, shaded area =  $\int_0^1 (x^2 + 1) dx - \int_0^1 x dx = \int_0^1 (x^2 - x + 1) dx = \frac{5}{6} = 0.833$ . The area obtained by both process is identical. These are examples of pedagogical and logical connectivity in real analysis with school mathematics.

Students sometimes ask question, why every open interval  $(a, b)$  contains a rational number. Students at the secondary level know about set theory, rational numbers, irrational numbers etc. Despite this, the answer to the above question is not as easy as they think; the solution itself is subtle. To convince the students with true logic teachers should have proof idea of Archimedean theorem, if  $a, b \in \mathbb{R}$  with  $a > 0$  and  $b > 0$ , then there is a natural number  $n$  such that  $na > b$ . This result is known as the Archimedean theorem.

Archimedean property of real numbers, for any two real numbers  $a > 0$  and  $b$ , there exist a natural number  $n \in \mathbb{N}$  such that  $na > b$ . The following proof illustrate this property using a contradiction argument. Suppose that  $na \leq b$  for every  $n \in \mathbb{N}$ . Let  $S = \{na \mid n \in \mathbb{N}\}$ . Since  $S$  is bounded above by  $b$ , let  $M = \sup S$ . Clearly  $M - a < M$ . By supremum property of  $S$ , there exists a natural number  $n_0$  such that  $n_0 a > M - a$ . But then  $(n_0 + 1)a > M$  where  $(n_0 + 1)a \in S$ . This contradicts the fact that  $M = \sup S$ . Therefore, our supposition is wrong, and it completes the proof of the Archimedean theorem. Based on the theorem, the students' queries can be solved by applying the following proof: every open interval contains a rational number.

**Case i.**  $0 < a < b$ .

By Archimedean theorem, there is a natural number  $k$  such that  $\frac{1}{k} < b - a$ . Let  $A = \{n \in \mathbb{N} \mid \frac{n}{k} > a\}$ . Again, by the Archimedean theorem,  $A \neq \Phi$ . By the well-ordering

principle for  $\mathbb{N}$ ,  $A$  has a first element, say  $n_0$ . Then  $\frac{n_0}{k} > a$  and  $\frac{n_0-1}{k} \leq a$ . Moreover,  $a < \frac{n_0}{k} \leq a + \frac{1}{k} < a + (b - a) = b$ . Thus, the rational number  $\frac{n_0}{k}$  lies in  $(a, b)$ .

**Case ii.**  $a \leq 0 < b$ .

By Archimedean theorem, there is a natural number  $k$  such that  $\frac{1}{k} < b$ . Clearly  $\frac{1}{k} \in (a, b)$ .

**Case iii.**  $a < b \leq 0$ .

Clearly,  $0 \leq -b < -a$ . By Case i. there is a rational  $r \in (-b, -a)$ , and hence the rational number  $-r \in (a, b)$ .

The proof of the Archimedean theorem is natural. However, it delves deeper into the understanding of the supremum of a set in the study of Real Analysis and is the main key to unlocking the knot in the above question. Such examples can be seen while teaching mathematics to secondary-level students. Therefore, knowledge of real analysis is crucial and a deep concern in mathematics instruction at this level.

In teaching of function determining the domain and range is common practices. Considered a function  $f(x) = \sqrt{4 - x^2}$  ? be a function defined in the set of real number, then what is domain and range of this function. To determine its domain, it is required that the expression inside the square root be non-negative, since the square root function is only defined for non-negative values in the real number system. Thus, we solve:  $4 - x^2 \geq 0$

solve the inequality we get

$$x^2 \leq 4 \Rightarrow -2 \leq x \leq 2. \text{ so domain is } [-2, 2].$$

Again, for range, we have to find the possible values that  $f(x) = \sqrt{4 - x^2}$  can take that range. As the result of square root is always positive the minimum value of  $x$  is

zero and the maximum value is 2. When  $x = 0$  then  $f(0) = \sqrt{4 - 0} = 2$ , when  $x = 2$  then  $f(2) = \sqrt{4 - 4} = 0$ , so range is  $[0, 2]$

Though the concept of a domain and range of a function is known to the students of secondary level, finding the domain and range can sometimes be difficult. The key points that matter in identifying the domain and range of a function are the proper knowledge of the given function and the associated algebraic and graphical skills in it. The above problem can be solved both algebraically and graphically by an instructor who has sufficient knowledge of real analysis.

While teaching the concept of open sets on the real line, a common question students raise is: Why does every open interval  $(a, b)$  contain a rational number? Although students at the secondary level are familiar with rational and irrational numbers, set theory, and intervals, they often struggle to grasp this property without a formal explanation. This pedagogical challenge highlights the need for instructors to possess a solid foundation in real analysis, as it equips them to provide rigorous justifications and facilitate deeper conceptual understanding in the classroom.

The concept of nested intervals, a fundamental topic in real analysis, proves invaluable when approximating square roots through iterative methods. The process begins by selecting an interval known to contain the square root of the given number. For instance, in the case of finding the square root of 20, we recognize that it falls within the range of 4 to 5. Hence, our initial interval is set as  $[4, 5]$ . We proceed with the following steps: first, we examine the squares of the endpoints, where  $4^2$  equals 16 (less than 20) and  $5^2 = 25$  (greater than 20). Consequently, we deduce that the square root of 20 lies between 4 and 5. Recognizing this, we refine our interval, creating a new subinterval  $[a_n, b_n]$  such that  $[a_{n+1}, b_{n+1}] \subseteq [a_n, b_n]$  progressively narrowing the range containing  $\sqrt{20}$ . Through this process, we arrive at the approximation that  $\sqrt{20}$

lies within the range of 4.47 to 4.48. The procedure is continuously reiterated to attain even more precise approximations, illustrating the significance of nested intervals in real analysis. In this process, Nested intervals and convergence are applied. The different general problems discussed in school mathematics can be justified using the mathematical knowledge of real analysis.

Scholars (e.g., Grinstein & Lipsey, 2014) have suggested different structures of mathematics courses to make them relevant for secondary mathematics teachers. The traditional and modern approaches can be found in structuring the relevant course for the prospective teacher. Bishop (1992) pointed out three traditions: the pedagogue tradition, the empirical scientist tradition, and the scholars' tradition regarding the mathematics teachers preparation program. The discipline-specific teacher knowledge, including advanced mathematics, was preferred for preparing secondary-level mathematics teachers (Heinze et al., 2016). However, specialized knowledge has received much attention among researchers in mathematics education and is conceptualized as the foundation for advanced mathematical knowledge for teaching. Undergraduate and graduate mathematics teacher preparation programs typically include advanced mathematics. Besides these scenarios, Zazkis and Leikin (2010) raise questions on the relevance of advanced mathematics, such as real analysis, regarding future use: What do mathematics teachers gain from study of real analysis ? Does this course have any connection with secondary-level mathematics courses? These questions have drawn the attention of mathematics educators and researchers.

Besides the aforementioned scholars who have raised questions about the relevance of knowledge of real analysis in mathematics education, I have faced similar issues while teaching real analysis at Tribhuvan University for the past 10

years. Undergraduate students often question the necessity of studying real analysis and its relevance to teaching school-level mathematics.

As an instructor, providing a clear and convincing answer to our students' questions is important. I have tried to explain that studying real analysis helps to develop advanced mathematical ideas, axioms, and properties of the real number system. It provides the foundation for various mathematical concepts, including calculus, a fundamental component of secondary-level mathematics. Furthermore, real analysis enables prospective teachers to develop a deeper conceptual understanding of mathematics, enhancing their ability to teach these concepts more clearly and effectively. However, my students are not entirely convinced. They argue that real analysis has no contribution to teaching school mathematics, which is a valid concern. Therefore, it is important to understand the relevance of mathematical knowledge of real analysis for school mathematics instruction.

The standard for preparing mathematics teachers states that all teachers should possess mathematical, statistical, and pedagogical knowledge necessary for teaching school mathematics (Association of Mathematics Educators, 2017). Single and multivariate calculus, probability, statistics, abstract algebra, real analysis, differential equations, number theory, history of mathematics, etc., are included in the teacher preparation program of the different universities of Nepal (Panthi & Jha, 2016; Kathmandu University School of Education, 2018). Hence, real analysis has been taught in teacher preparation programs at the university level.

The Faculty of Education, Tribhuvan University, Nepal, offers various programs at different academic levels. Among these, the B.Ed. program aims to prepare trained teachers for secondary-level education (Tribhuvan University Faculty of Education, 2023). Therefore, the mathematics program of B.Ed. Tribhuvan

University also develops mathematics teachers required for school mathematics instruction. To achieve this objective, a range of mathematics courses is offered, including real analysis, which is considered a foundational subject.

Mathematicians and mathematics educators agree that subjects like real analysis are important for teachers to deeply understand mathematics and its applications. However, sufficient research is unavailable on the relevance of real analysis for teaching secondary-level mathematics in Nepal.

Researchers (e.g., Wasserman et al., 2017; Patterson, 2019) have found that a solid understanding of real analysis is essential for teaching secondary-level mathematics. In contrast, Zazkis and Leikin (2010) noted, "mathematics teachers at secondary-level were less convinced about the usefulness of more advanced mathematical knowledge for teaching" (p. 310). The researchers (e.g., Wasserman et. al., 2017; Zazkis & Leikin, 2010) further explain that this may be because the mathematical concepts taught at the secondary level are more applied and do not require the same level of theoretical understanding as real analysis. Hill et al. (2005) emphasizes that secondary mathematics teachers benefit most from developing a deep understanding of the specific content they will teach, which directly enhances classroom instruction. In contrast, Tatto (2018) argues for the integration of advanced mathematics coursework into teacher preparation programs to strengthen teachers' overall mathematical understanding and improve their problem-solving skills.

Real analysis is the foundation for developing the concepts necessary for further study in advanced mathematics and enhancing teachers' proficiency in school teaching (Wasserman et al., 2017; Patterson, 2019). Therefore, it is included in mathematics teacher preparation programs in university-level mathematics education curriculum, compared to other branches of advanced mathematics. Mathematicians

and mathematics educators (e.g., Ball et al., 2005; Rowan et al., 2001; Shulman, 1987) believe that real analysis is an essential subject for acquiring mathematical competencies. Therefore, knowledge of real analysis is essential for enhancing mathematical proficiency.

In addition to my active engagement in teaching real analysis, I serve as a university-level instructor for this subject and have frequently encountered challenging questions concerning its relevance and suitability within teacher preparation programs. Moreover, during the literature review, I identified a significant debate surrounding the relevance of real analysis and its possible disconnect from instructional practices at the secondary level. These observations have motivated me to undertake a comprehensive examination of the relevance of real analysis within the framework of school mathematics instruction.

### **Statement of Problem**

Teacher preparation programs offered at the university level aim to develop comprehensive knowledge and skills related to the subject matter among prospective teachers. In mathematics teacher preparation programs, content and pedagogy of mathematics are integral components of the curriculum. Within the content domain, advanced mathematics such as real analysis is included to deepen teachers' conceptual understanding and strengthen their ability to deliver meaningful mathematics instruction (Kumar et al., 2013; Pansell, 2023). However, the appropriateness and relevance of specific advanced mathematical content for secondary school teaching remain a subject of ongoing debate. One such area of discussion is whether knowledge of real analysis typically taught at the undergraduate level holds practical value for secondary-level mathematics instruction.

Real analysis is a foundational course in undergraduate mathematics curricula and is widely included in mathematics teacher education programs (Rolyden & Fitzpatrick, 2010). Mathematics educators such as Hill et al. (2005) argue that secondary school teachers need mathematical knowledge that goes beyond what they are expected to teach, enabling them to reason deeply and respond to diverse student needs. When teachers are able to draw connections between the abstract principles of real analysis and the mathematics taught in school, it demonstrates both advanced content knowledge and pedagogical skill (Wasserman et al., 2017). Despite this theoretical link, research indicates that secondary teachers do not perceive advanced mathematics including real analysis as relevant to their classroom teaching (Leikin & Zazkis, 2010). Changwe (2017) also found that teacher preparation programs fail to equip graduates with the necessary skills for effective mathematics instruction, calling for a re-examination of curriculum content. Additionally, a well-structured mathematical analysis course, which focuses on proof and discovery, significantly contributes to preparing competent future (Naziev & Zemlyakova, 2025). Moreover, Wasserman et al. (2023) highlighted the persistent challenge in mathematics education of aligning university-level mathematics with the instructional demands of school classrooms. Therefore, investigating the relevance of advanced mathematics to school-level instruction remains a significant and contemporary area of research.

This issue is also evident in the context of Nepal, where institutions such as Tribhuvan University (TU) have adopted global standards by including real analysis and other advanced mathematics content in their teacher education programs. The rationale for this inclusion is to improve mathematical proficiency and professional competence in future teachers. Nevertheless, there is a lack of empirical evidence in the Nepalese context about how real analysis knowledge is understood, perceived, or

utilized by in-service mathematics teachers in classroom instruction. The absence of adequate empirical research raises a critical question: why is real analysis taught in teacher preparation programs if its classroom application remains unclear? While advanced mathematics can promote mathematical reasoning and logical thinking both essential for a teacher's professional development the practical impact of such content has not been thoroughly studied in Nepal.

Although international studies such as those by Ball et al. (2008), Patterson (2019), Wasserman and Weber (2018a) and Naziev and Zemlyakova (2025) have highlighted that real analysis helps teachers bridge abstract mathematics with classroom applications and enhances their logical fluency, such findings are limited in their transferability to Nepal due to differences in curriculum design, teacher training practices, and instructional contexts. Moreover, these studies emphasize theoretical value and not classroom-based empirical outcomes. As a result, there remains a significant gap in the literature regarding how, and to what extent, real analysis contributes to instructional effectiveness, content connection, and pedagogical reasoning at the secondary level in Nepal.

Despite its formal inclusion in teacher education curricula, real analysis remains underexplored in terms of its actual relevance to secondary-level mathematics instruction. There is a paucity of empirical research that examines how in-service teachers relate the abstract knowledge of real analysis to their teaching practices, and whether such knowledge enhances their content delivery, pedagogical decisions, or professional competence. This lack of context-specific evidence underscores a critical research gap, which this study seeks to address.

### **Purpose of the Study**

The proposed study is conducted to achieve the following purposes.

1. To assess the teachers' attitude towards the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction.
2. To evaluate the connection between mathematical knowledge of real analysis and secondary-level mathematics, its rationale, and ways to strengthen this linkage.
3. To explore how secondary-level mathematics teachers use knowledge of real analysis in their instructional practices.

### **Research Questions**

The following are the research questions used in the study.

1. What are teachers' attitudes toward mathematical knowledge of real analysis concerning its connection with secondary-level mathematics, its pedagogical applications, and its role in enhancing their mathematical proficiency?
2. What is the relationship among teachers' attitudes towards content connection, pedagogical application, mathematical proficiency, and teaching efficacy?
3. How do teachers perceive the mathematical knowledge of real analysis regarding its content connection, pedagogical applications, and role in enhancing mathematical proficiency?
4. Why is mathematical knowledge of real analysis required to become a mathematics teacher?
5. What are the connections between the mathematical knowledge of real analysis and secondary-level mathematics content?
6. Why is the content connection between mathematical knowledge of real analysis and school mathematics essential?
7. How can the link between mathematical knowledge of real analysis and secondary-level mathematics be strengthened?

8. In what classroom contexts do teachers characterize their mathematical knowledge of real analysis?
9. How do mathematics teachers articulate their mathematical knowledge of real analysis in their classroom practices?

The research is conducted using both quantitative and qualitative inquiry.

Based on the earlier study's findings detailed in the literature (explained further in Chapter 2), the following hypotheses were developed to address research questions 1 and 2.

**H<sub>1</sub>:** Secondary-level mathematics teachers have a positive attitude towards the content connection between real analysis and the content of secondary-level mathematics.

**H<sub>2</sub>:** Secondary-level mathematics teachers have a positive attitude towards applying mathematical knowledge of real analysis for mathematics instruction.

**H<sub>3</sub>:** Secondary-level mathematics teachers have a positive attitude toward contributions of the mathematical knowledge of real analysis for enhancing teachers' proficiency.

**H<sub>4</sub>:** Mathematics teachers' attitudes towards the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction differ significantly across gender, age, qualification, experience, and academic background.

**H<sub>5</sub>:** Teachers' attitudes toward content connection significantly predict that mathematical knowledge of real analysis enhances mathematical proficiency.

**H<sub>6</sub>:** Teachers' attitudes toward content connection significantly predict teaching efficacy.

**H<sub>7</sub>:** Teachers' attitudes toward mathematical knowledge of real analysis as a means to enhance mathematical proficiency significantly predict teaching efficacy.

**H<sub>8</sub>:** Teachers' attitudes toward the pedagogical application of mathematical knowledge of real analysis significantly predict its effectiveness in enhancing mathematical proficiency.

**H<sub>9</sub>:** Teachers' attitudes toward the pedagogical application of mathematical knowledge of real analysis significantly predict teaching efficacy.

### **Rationale of the Study**

Advanced mathematics, like real analysis, is introduced in the teacher preparation program at the university level. Introducing such mathematics courses aims to develop a deep understanding of mathematical knowledge for prospective teachers (Patterson, 2019). The real analysis occurs under the mathematics courses designed for teacher preparation programs. Thus, students who choose a mathematics major takes a real analysis course (Conference Board of the Mathematical Sciences, 2012). The standard topics in real analysis include number systems, point-set topology, functions and limits, one-variable calculus, uniform convergence, and infinite series (Bartle & Sherbert, 2011). These topics have been covered in B.Ed. Major mathematics curriculum of TU. Besides the inclusion of the course, it is viewed that pre-service secondary school mathematics knowledge of real analysis is disconnected from practices (Goulding et al., 2003; Wasserman et al., 2015). As a university teacher, I have noticed a similar sentiment among mathematics education students in Nepal. However, it is essential to investigate whether this perceived disconnection is valid, as there may be explicit links. The general question of pre-service mathematics teachers is Why do we learn real analysis, as there can be explicit links to school mathematics (Wasserman & Weber, 2017a). Real analysis courses can enhance the development of various skills and knowledge necessary for teaching mathematics in secondary education. Teachers may use this knowledge and expertise

in conscious and unconscious ways during their instructional practices. Understanding real analysis can facilitate the enhancement of essential knowledge and abilities for teaching mathematics through developing mathematical fluency, logical fluency, and content connection. The teachers' attitudes towards applying mathematical understanding of real analysis in teaching mathematics, content connection with the school's mathematics, and teacher professional development may be at a particular stage, but have not been measured. A common concern among pre-service mathematics teachers is: Why is real analysis included in our curriculum? How does it help in our instructional practices? very few investigations have been conducted to answer these questions in the context of Nepal. On the other hand, sufficient research is not available in an international context. Therefore, this study is justified because it investigates the relevance of mathematical knowledge of real analysis in terms of its connection to the content of secondary school mathematics, its pedagogical applications, and its role in enhancement of mathematical proficiency.

### **Delimitations of the Study**

The primary objective of this study is to investigate the relevance of mathematical knowledge in real analysis for teaching secondary-level mathematics (Grades 9 and 10). The content connection, pedagogical application, and enhancement of mathematics teachers' proficiency were examined through surveys. Additionally, task-based interviews and classroom observations were conducted with eight secondary-level mathematics teachers, with the qualitative study delimited to Baglung Municipality. This study focused solely on the information provided by these tools. This study focused exclusively on secondary school mathematics teachers in the Gandaki Province of Nepal. Both the compulsory and optional mathematics curricula for grades nine and ten, issued by the Nepal government, and a

recommended curriculum on real analysis for the Bachelor's in Education program under Tribhuvan University were considered for the curriculum analysis. Textbooks for grades nine and ten, published by the government of Nepal, were considered for support during the curriculum analysis to identify content connections.

### **Definition of Related Terms**

*Adaptive Reasoning:* The capacity to think logically and reflectively about mathematical concepts, making sense of problems and justifying solutions in several contexts.

*Attitude:* An individual's disposition, beliefs, and emotional response regarding studying and applying advanced mathematical concepts and principles involved in real analysis.

*Bridging Strategy:* A teaching approach that connects new mathematical concepts with existing knowledge and links new knowledge to previously learned knowledge.

*Content Knowledge (CK):* Content knowledge refers to a teacher's understanding of the subject matter, including the facts, concepts, structures, and theorems within a discipline. This study refers specifically to the teachers' knowledge of real analysis, such as limits, continuity, sequences, and axiomatic properties.

*Conceptual Understanding:* A deeper understanding of a mathematical concept enables differentiation, identity connection, and application in a suitable context.

*Mathematical proficiency:* Knowledge and mathematical skills that enable one to reason, solve problems, and apply mathematical concepts effectively.

*Pedagogical Skill:* Pedagogical skill refers to a teacher's practical ability to apply teaching strategies, adapt instruction, manage classroom interaction, and translate complex concepts into teachable forms. While pedagogy draws on content knowledge, it focuses on how teachers use it to facilitate learning. In this dissertation,

pedagogical skill is conceptualized as applying content knowledge of real analysis in the context of secondary-level instruction.

*Perception:* An individual's interpretation and understanding of a phenomenon.

*Procedural Fluency:* The ability to accurately and efficiently apply mathematical procedures in problem-solving, including recalling facts, performing algorithms, and choosing appropriate methods.

*Productive Disposition:* The tendency to view mathematics as useful, worthwhile, and engaging, accompanied by a belief in one's ability to solve problems and persist through challenges.

*Real analysis:* is a branch of advanced mathematics introduced in the B.Ed. Program at TU.

*Relevance:* Relevance refers to the significance, applicability, and importance of a particular concept, idea, or information in a specific context or situation regarding mathematical knowledge of real analysis.

*Sequential Learning:* The process of acquiring knowledge gradually or in a step-by-step manner, where each concept builds upon previous ones to ensure a more profound understanding.

*Strategic Competence:* The skill to formulate, represent, and solve mathematical problems by selecting and applying appropriate strategies effectively.

*Teaching Efficacy:* Teaching efficacy is a mathematics teacher's belief in their ability to design and implement effective instructional strategies, manage the classroom environment, and actively engage students in meaningful mathematical learning to improve their understanding and achievement.

*Vertical Link:* The connection between concepts taught at different educational levels.

## **Organization of Dissertation**

This dissertation contains six chapters. The first chapter introduces the background of the study, including advanced mathematics for teacher preparation programs, the problem statement, purpose, research questions, hypothesis, and delimitation of the Study. The second chapter presents relevant literature on advanced mathematics, including real analysis and its role and importance for teachers, evaluation models for training programs, the theoretical framework of mathematics teacher preparation, indicators of relevance, the conceptual framework, and the research gap. The adopted research methodology has been explained in chapter three. The fourth chapter comprehensively describes data analysis and interpretation in three phases. Findings and discussion are included in chapter five, whereas the summary, conclusion, and implications of the study are described in chapter six.

## **Chapter Summary**

This chapter elaborates on the background, highlighting the role and significance of advanced mathematics, particularly real analysis, in teacher preparation programs. Additionally, it provides background information on the importance of subject matter knowledge in mathematics education and its impact on teaching effectiveness. This chapter also presents the statement of the problem, explaining the necessity of this research and its rationale. Furthermore, the research questions and related hypotheses focus on assessing the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction. The chapter also includes the delimitations of the study, key terms, and their definitions.

## Chapter 2

### Literature Review

The philosophical perspectives of education and curriculum related theories on mathematics education, particularly those associated with curriculum relevance are reviewed to establish the theoretical guidelines. Additionally, existing literature was reviewed to identify the research gap and guidelines for research. The reviewed literature is organized thematically within this chapter.

#### **Philosophical and Theoretical Perspective on Curriculum Relevance**

This heading discusses the philosophy and theories that make the curriculum relevant and the theories that describe the components and factors required for mathematics teacher preparation programs.

#### ***Progressivist Approach and the Relevance of Curriculum***

Dewey (1920) developed the philosophy of education known as progressivism (Leonall, 2011). This approach emphasized the importance of a flexible, student-centered approach focused on developing critical thinking, problem-solving, and social skills. While Dewey was a major proponent of progressivism, other philosophers also played significant roles in shaping this influential educational movement.

Progressive education emphasizes the active involvement of the learner in the learning process and the relevance of education to the needs and interests of the learner. In curriculum development, it suggests that the curriculum should be relevant, meaningful, and engaging for the learners and focuses on developing problem-solving skills, critical thinking, and creativity (Leonall et al., 2011). In a progressivist approach to curriculum, the curriculum is not a fixed set of knowledge or skills to be transmitted to the learners but rather a dynamic and evolving process shaped by the

learners' interests, experiences, and needs. The curriculum is designed to be flexible and adaptable, allowing for changes and adjustments based on the learners' feedback and input.

Rooted in progressivism, the life-related curriculum underscores the need for education systems to prepare individuals for contemporary realities, enabling learners to navigate new situations through the application of experiential knowledge. Therefore, the curriculum of any subject is relevant when it fulfills the learner's needs. The prospective teacher required subject knowledge and pedagogical skills to be a good teacher. The relevance of undergraduate mathematics education courses lies in their ability to equip students with both content and pedagogical knowledge essential for effective professional teaching practice.

Suppose a teacher perceives that the subject matter they learned in their university courses helped them address the challenges they encounter in their professional life. In that case, it can be considered a relevant curriculum. A curriculum becomes significant when it fulfills the requirement of learners and society and equips them with the necessary knowledge and skills for their personal and professional endeavors. Therefore, if teachers believe their university courses have provided them with relevant subject matter knowledge and pedagogical skills, the curriculum effectively prepares them for their professional roles. Additionally, the undergraduate courses that teachers take during their university studies serve as the foundation for developing the knowledge and skills relevant to school teaching. The undergraduate courses in education programs provide an extensive foundation for aspiring teachers, covering various subjects, skills, and attitudes dynamic for their future profession, indicating the program's relevance (Singh et al., 2020). Therefore, these courses play a crucial role in shaping the teachers' understanding of their subject

matter and teaching methods, ultimately helping them become effective classroom educators.

The relevance of a curriculum lies in its ability to provide practical solutions to real-life problems. This means the curriculum must have practical applications that students can use daily. In the context of teaching and learning, the relevance of a curriculum can be evaluated by its effectiveness in helping teachers deliver effective instruction and solve problems that arise in the classroom. In teaching mathematics, the relevance of real analysis lies in its ability to help teachers solve problems that arise during instruction. For example, suppose a teacher is struggling to explain a particular concept to his/her students. Their mathematical knowledge of real analysis may enable them to approach the problem differently and provide alternative explanations. Similarly, suppose a student struggles to understand a particular concept. In that case, the teacher's mathematical knowledge of real analysis enables his/her to provide additional examples and logical reasoning to clarify the mathematical concept.

In this way, real analysis valuable subject for teachers, enabling them to deliver effective instruction and solve problems that arise in the classroom. However, it is important, the relevance of any subject to teaching and learning ultimately depends on how it is taught and used in practices. Therefore, teachers need to be knowledgeable and skilled in the subjects they teach and continually reflect on their teaching practices to ensure that they provide effective instruction and meet their students' needs.

### ***The Constructivist Learning Theory and the Relevance of Curriculum***

Constructivism is rooted in the work of Piaget (1977) and Bruner (1960) Vygotsky (1978) that further explained constructivism in learning to focus on

knowledge construction (Schunk, 2012). It is a learning theory emphasizing the importance of learners actively constructing their understanding and knowledge of the world rather than simply absorbing information from teachers or textbooks (Roy & Saha, 2021). In the context of teacher preparation programs, constructivism suggests prospective teachers should be encouraged to engage in reflective and collaborative learning processes that allow them to create and develop their understanding of teaching and learning. To support this process, teacher preparation programs should provide opportunities for prospective teachers to engage in inquiry-based learning, problem-solving, and critical reflection. This involves providing prospective teachers with opportunities to observe and analyze classroom practices, collaborate with peers in developing lesson plans, and participate in professional development activities that promote continuous learning and growth.

Constructivism emphasizes the importance of preparing prospective teachers to be active, reflective, and critical learners capable of continually creating and developing their knowledge and understanding of teaching and learning. By providing a supportive and collaborative learning environment, teacher preparation programs can help prospective teachers develop the knowledge, skills, and dispositions they need to be effective educators in today's complex and dynamic classrooms.

Constructivist theory holds that a curriculum is relevant when it supports learners in constructing knowledge and addressing real-life problems across their lifespan. In the case of teacher preparation, the transmission of knowledge from an informed to an uninformed person is not only an education process; learners should have opportunities to evaluate existing knowledge and understanding sufficiently so that they are enhanced to face the problem that occurs during their professional practices (Biswas, 2018). Therefore, it is concluded that the curriculum of the teacher

preparation program becomes relevant if it helps the teacher to create knowledge and solve problems during their professional practice. Additionally, the teacher should get knowledge and skill from a teacher preparation program that enables them to apply their experiences to create an example, solve the problems, able to learn current innovations in a related subject, and facing, develop a research attitude, share knowledge and solve the problem that occurs during the instruction (Kalekar, 2018). These philosophical claims are considered for preparing relevance indicators. Constructivist theory of learning claims that the teachers should be prepared with a curriculum that provides the necessary foundation of knowledge to facilitate their creation of knowledge in the classroom (Cobb & Bowers, 1999). In mathematics education, teacher preparation programs should focus on helping teachers develop the skills to create mathematical examples, define mathematical relations, and solve contextual classroom problems during mathematics instruction (Ernest, 2007). Hence, constructivist philosophy focuses on developing mathematical creations for teaching mathematics in the classroom, indicating the programs' relevance. Real analysis is an important subject in mathematics education that can help teachers develop aforementioned skills.

By studying real analysis, teachers can gain a deeper understanding of mathematical concepts and learn how to apply them in the classroom. For example, they can use real analysis to create mathematical examples relevant to their students' lives and experiences, such as analyzing data from a local sports team or calculating the probability of a particular event occurring. Additionally, real analysis can help teachers develop strategies for addressing contextual classroom problems, such as developing lesson plans considering their students' cultural background and prior knowledge (Sfard & Linchevski, 1994). Thus, teacher preparation programs focus on

providing foundation of knowledge, so that teachers create knowledge in the classroom. Real analysis is relevance and applicable topic when it helps the teachers to develop knowledge and skills required in effective classroom instruction.

### ***Theories on Mathematics Teacher Preparation Program***

Understanding the theoretical foundations of mathematics teacher preparation is fundamental for designing effective programs. The key theories and models related to teacher knowledge, particularly in the context of advanced mathematics, and their role in preparing secondary-level mathematics teachers are explored under this heading.

**Place of Advanced Mathematics in Teacher Preparation.** The starting stage of teacher preparation programs at the university level focuses on content knowledge. The mathematics education program includes subject matter knowledge and pedagogical content knowledge as two broad areas of knowledge. However, the curriculum has strongly emphasized advanced mathematics courses. Such courses are either an integral part of an Education program or requirements for admission to such a program (Tatto et al., 2010; Tatto et al., 2009). This tradition assumed that advanced mathematics studies would enhance teachers' content knowledge, contributing to classroom instruction quality. Besides considering such tradition, the relevance of advanced mathematics courses to the quality of secondary-level mathematics teaching is being debated.

**Models on the Conception of Teacher Knowledge.** The subject of how teachers acquire their mathematical skills to teach secondary-level mathematics is instruction. Under this heading, the theoretical and previously developed models are reviewed. Several models have been developed to describe the domains of knowledge required for teaching. In mathematics education, different frameworks have been

developed to describe the knowledge required for teaching mathematics (Ball et al., 2008; Baumert et al., 2013; Ernest, 1989; O'Meara, 2010).

A growing body of research has focused on identifying what mathematics teachers need to know. Thompson (1984) investigated the relationship between teacher knowledge and classroom teaching from a conception perspective and found that teachers' beliefs, views, and preferences about mathematics and teaching mathematics significantly impacted instruction. He found that teacher' conceptions of mathematics were attributed to the level of integration with other conceptions and the process of reflecting on instruction, mathematics, and their beliefs.

Shulman (1986) proposed a conceptual framework for teacher knowledge, include content knowledge, pedagogical content knowledge, and curriculum knowledge are included in this model as three major categories. Content knowledge was expected for being an expert in mathematics. Ernest (1989) proposed a detailed model for preparing mathematics teachers. Knowledge of the subject matter, teaching mathematics, management of the classroom, specific to mathematics, teaching context of teaching mathematics, and education have been included in Ernest's model. This model is isomorphic to the Shulman model. However, Earnest's model is more specific and detailed. This model focused on beliefs and attitudes by integrating subject matter. Davis and Simmt (2006) provided an approach-based perspective of complexity science, which deals with self-organizing, self-maintaining, and adaptive situations with systems that learn. He partitioned the phenomena of mathematics teachers into categories of knowing and knowledge. The categories of knowing, considered dynamic, consist of the classroom collectively and subjective understanding. Similarly, categories of knowledge often viewed as stable include mathematical objects and curriculum structures.

The entire model consists of content knowledge as a core part of the teacher preparation program model. However, the initial model did not focus on other beliefs and attitudes about mathematics.

### ***Mathematical Knowledge for Teaching***

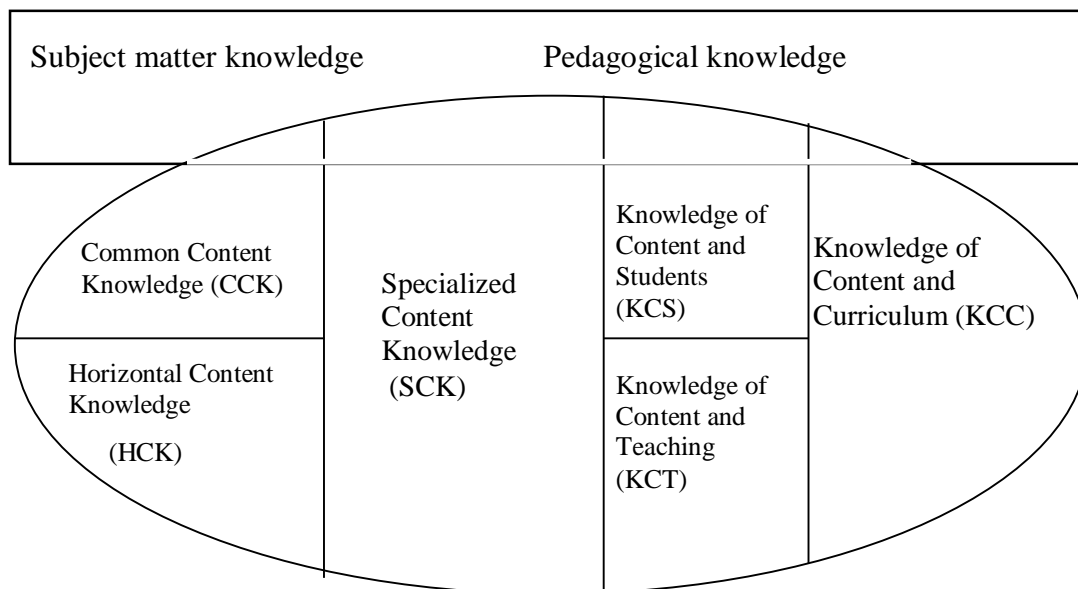
The practice-based theory of mathematics teacher knowledge developed by Ball et al. (2008) and Shulman (1986) served as the foundation for developing the theoretical framework for mathematical knowledge for teaching. Figure 2 illustrates the six types of knowledge needed to teach mathematics (Ball et al., 2008, p. 377). This theory posits that knowledge of subject matter encompasses common content knowledge, horizontal content knowledge, and specialized content knowledge. Similarly, pedagogical knowledge includes knowledge of content and student, knowledge of content and teaching, and knowledge of content and curriculum required for being a mathematics teacher. Common content knowledge (CCK) focuses on a broad knowledge of mathematics. The teacher required broad content knowledge related to mathematical theories, concepts, terminologies, definitions, symbols, rules, etc. All these aspects are important for being a good mathematics teacher. These areas of mathematics are known as common content knowledge (Ball et al., 2008). This structure is utilized in teacher preparation programs. Specialized content knowledge (SCK) is the higher level of mathematical knowledge by which teachers create connections between how mathematics can be represented and it enables teachers, how mathematics can be applied (Jankvist et al., 2015; O'Meara, 2010).

Horizontal content knowledge (HCK) represents the structure of mathematical knowledge, enabling us to explore how concepts are hierarchically related and how these concepts are related to the topic (Koponen et al., 2017). Thus, teachers require

an understanding of the structure and logical connections within mathematical content, particularly from the perspective of content difficulty.

**Figure 2**

*Mathematical Knowledge for Teaching*



Knowledge of content and students (KCS) refers to the requirements for mathematics teachers to know how students learn mathematics and understand whether students are liable to face challenges and learning difficulties (Ball et al., 2008). Thus, it is essential for teachers.

Knowledge of content and teaching (KCT) refers to the skills and knowledge of being a teacher that enable teachers to plan lessons, communicate, and promote interaction in the classroom. Teachers need to be able to change or diversify the teaching method and strategy, and organize learning support for the students to improve their teaching (Ball et al., 2008). Therefore, it enhances the content as well as the pedagogical aspect.

The knowledge of content and curriculum (KCC) is a key dimension of mathematics teacher preparation programs. Mathematics teachers need to possess the

knowledge and skills necessary to select and utilize appropriate teaching materials that align with curricular objectives and mathematical content. All these knowledge features can be encapsulated in terms of knowledge of the content and curriculum required to teach secondary-level mathematics (Jankvist et al., 2015). This focused on the curricular aspect of the content.

The foundation of mathematical knowledge for teaching (MKT) lies within the American context. This framework is based on Shulman's conceptualization (Shulman, 1986a). This framework has spread to all other countries: Finland (Koponen et al., 2017), Malawi (Kazima et al., 2016), Ireland (Delaney et al., 2008), etc. However, MKT is considered a popular framework for describing teacher knowledge. Besides the theoretical claims of the necessity of advanced mathematical knowledge, several questions about what kind of mathematical teaching task is required (Ball et al., 2008) and the relationship between common content knowledge and specific content knowledge still require further inquiry.

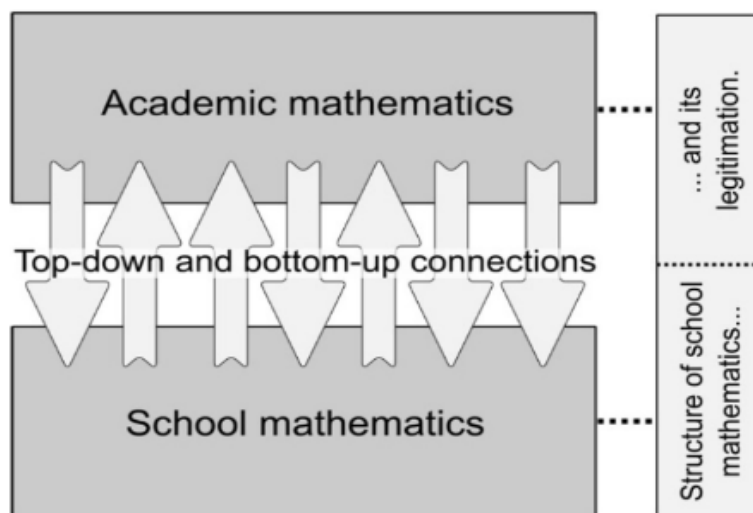
Numbers are defined by their attributes, such as odd, even, composite, and prime numbers, which are considered in real analysis. Real analysis is the source of the knowledge described by the above-mentioned models, allowing us to understand the fundamental principles and characteristics of real number systems.

The School Related Content Knowledge (SRCK), as proposed by Dreher et al. (2018), presents an alternative perspective on mathematical knowledge for teaching (MKT), which was initially formulated by Ball et al. (2008). This novel model strongly emphasizes the interplay between school mathematics and more advanced academic mathematics. It highlights the specific ways advanced mathematics is linked with what is taught in schools, particularly in the context of teacher understanding. The SRCK model, by identifying three primary categories of connections: top-down,

bottom-up, and curricular knowledge, underscores the integral role of educators in shaping the structure of school-related content knowledge, as in Figure 3.

**Figure 3**

*Conceptualization of School-Related Content Knowledge (adopted from Dreher et al., 2018, p.330)*



Top-down knowledge within the School-Related Content Knowledge framework pertains to the connections that originate from the domain of academic mathematics and extend downward to inform the teaching of school-level mathematics. In this context, it represents drawing insights, principles, and advanced mathematical concepts from higher academics and strategically applying them to teaching foundational mathematical concepts in schools. This top-down approach ensures that teachers understand how advanced mathematical concepts relate to and can enhance the comprehension of school mathematics, ultimately enabling them to provide more effective and comprehensive instruction to their students (Drecher et al., 2018). Therefore, top-down model emphasizes transformation of higher mathematics to school-level.

Bottom-up connections are crucial while examining the relationship between school mathematics and academic mathematics. In this context, these connections involve starting with the foundational elements of school mathematics and then considering how they fit into or relate to the broader landscape of academic mathematics (Drecher et al., 2018). This approach is essential for teachers and educators to understand how fundamental concepts taught in schools are building blocks for more advanced mathematical ideas. It allows educators to trace the progression of mathematical concepts from their basic forms in school mathematics to their more complex manifestations in academic mathematics. By making these bottom-up connections, educators can provide a solid and coherent mathematics education that aligns with the gradual development of mathematical knowledge and skills, ensuring that students are well-prepared for advanced mathematical concepts in their academic journey.

Among three categories of connection under the model, curricular knowledge refers to understanding how school mathematics is organized and the rationale behind this structure from the perspective of academic mathematics. For example, teachers use curricular knowledge to determine which mathematical concepts such as infinity are appropriate to introduce at particular grade levels. This knowledge helps ensure that the curriculum progresses logically, building from fundamental ideas toward more complex concepts in a way that supports students' mathematical development.

### ***Habermas Theory of Knowledge Constitutive Interests***

Habermas's (1972) theory of knowledge constitutive interests is applicable to evaluate the relevance of knowledge in an actual classroom. This theory states that human inquiry is guided by three fundamental interests (Habermas, 2005): technical, concerned with control and efficacy, practical, concerned with mutual understanding,

and emancipatory, oriented toward critical reflection and transformation. The transformative learning experience empowers for application (Acharya et al., 2022), which is an emancipatory interest as mentioned by Habermas. This theory provided structure on how knowledge is acquired, and internalized in educational context. The real analysis knowledge is relevant when teachers transform knowledge of real analysis in the classroom.

### ***Theoretical Framework***

Based on the aforementioned theories (Progressive approach and constructivist approach, Mathematical knowledge for teaching, School-related content knowledge, and Habermas), a theoretical framework was prepared to guide the study on the relevance of knowledge of real analysis for secondary-level mathematics instruction.

**Figure 4**

*Theoretical Framework of Assessing Relevance*



The theoretical framework designed to assess the relevance of knowledge of real analysis in secondary-level mathematics instruction synthesizes multiple well-established educational theories to achieve a multidimensional perspective. Ball et al. (2008) model Mathematical Knowledge for Teaching (MKT) emphasizes distinct domains such as Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Horizon Content Knowledge (HCK), Knowledge of Content and curriculum (KCC), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Students (KCS). These domains illustrate how real analysis supports teachers' deep and flexible understanding of the mathematical content required for effective instruction. Complementing this, Habermas' (1972) theory of knowledge-constitutive interests categorizes knowledge into technical, practical, and emancipatory domains. This allows Real Analysis to be examined for its procedural mastery, interpretive understanding, and critical application in classroom contexts.

Further grounding the framework in pedagogical practice, the progressivist (Dewey, 1962) approach to education highlights the importance of real-life application, and student needs, and interest. Constructivist (Bruner, 1960) learning theory focuses on knowledge construction. This perspective supports the idea that Real analysis can contribute to meaningful mathematics instruction that encourages active knowledge construction through reflection and application. Finally, Dreher et al. (2016) propose the Stack model, which integrates both top-down (theory to practice) and bottom-up (practice to theory) processes. This model ensures that the theoretical foundations of real analysis are meaningfully connected to teaching practices, and that classroom experiences also inform and shape theoretical understanding.

### *Model of Curriculum Evaluation*

The curriculum is assessed to identify whether or not the developed curriculum produces the desired result. The relevance of the curriculum is indicated by its application and appropriateness to the needs, interests, aspirations, and expectations of students and society (International Bureau of Education, 2023).

Several models can be found for assessing the developed curriculum. Stufflebeam (2003) suggested four components for curriculum evaluation: context evaluation, input evaluation, process evaluation, and product evaluation. Kirkpatrick (1996) explained the four-level model for assessing curriculum relevance, which includes learners' reactions, learning, behaviour, and results (Alsalamah & Callinan, 2021). Tyler's planning model (2013) suggested evaluating objectives, selecting learning experiences, organizing learning experiences, and evaluating learning experiences (Stone et al., 1990). Bradley (1985) explained 10 indicators to evaluate the effectiveness and relevance of curriculum, including vertical relation, horizontal relation, instruction base of curriculum, curriculum priority, broad involvement, long-range planning, decision-making clarity, positive human relation, theory into practice approach, and planned changes.

Among the aforementioned models, Kirkpatrick's four-level model has focused on evaluating training courses. However, it is applicable to examine the relevance of the curriculum. The participants' reaction is measured in the first stage. The participants' knowledge, attitudes, and skills are assessed during the learning phase. Applying new knowledge to one's employment is evaluated in the third stage, and long-term success and application are evaluated in the final step. Bradley's effectiveness model has provided 10 indicators. Among them, two indicators, vertical

linkage and theory into practice, are applicable to measure the relevance of real analysis.

#### **Application of Kirkpatrick's Model for Measuring Relevance of KRA.**

Kirkpatrick's model of training evaluation, developed by Donald Kirkpatrick in the 1950s, is a widely recognized and utilized framework for assessing the effectiveness of training programs. Over time, the model has undergone refinements and expansions to enhance its applicability and usefulness in evaluating various aspects of training initiatives. The model consists of four distinct levels of evaluation, each emphasizing different dimensions of program effectiveness. These levels provide a comprehensive approach to evaluating programs' outcomes, allowing organizations to gain insights into the impact and value of their training efforts.

The teacher preparation program is also a training program; therefore, this model is suitable for assessing the relevance of the curriculum prepared for teacher preparation. The model provides four levels to evaluate the training program (Paull et al., 2016a). Evaluating the program's relevance is suitable for checking the relevance of the mathematical knowledge of real analysis for secondary schools' mathematics teaching.

Various indicators are needed depending on the context and their use to determine whether a topic is relevant. The Kirkpatrick model is more applicable to identifying the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction.

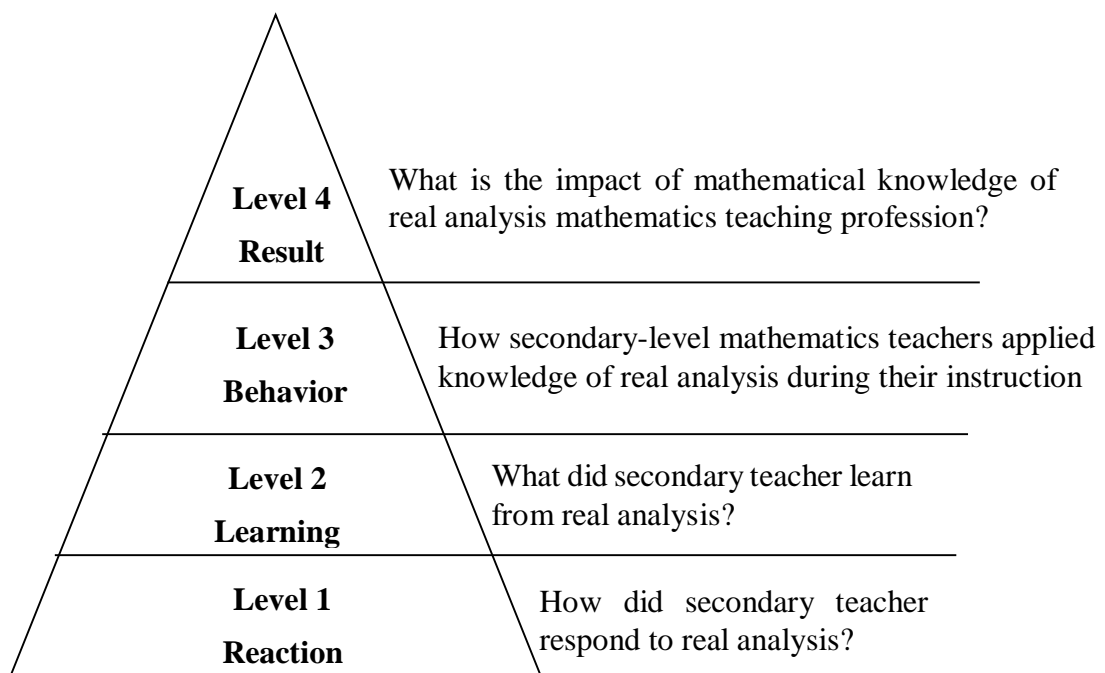
The four levels mentioned by Kirkpatrick in Figure 4 (Kirkpatrick & Kirkpatrick, 2014) can be applied to prepare relevance measurement tools for any subject. Therefore, Kirkpatrick's model was adapted to measure the relevance of mathematical knowledge of real analysis for school-level mathematics instruction.

While retaining Kirkpatrick's four-level model, the indicators were adapted to assess the relevance of knowledge of real analysis by incorporating adjustments based on other theoretical claims.

Trainee mathematics teachers at the secondary level have completed all the stages mentioned by Kirkpatrick and Kirkpatrick (2014). Thus, this model can be used to assess the relevance of the real analysis. This model claims that employee reaction need to be measured. Regarding mathematical knowledge of real analysis, secondary-level mathematics teachers' reactions require to be measured based on their experiences with how mathematics teachers perceive mathematical knowledge of real analysis in terms of their professional activities.

**Figure 5**

*Kirkpatrick Model of Training Evaluation*



Applying the Kirkpatrick model to assess the relevance of mathematical knowledge of real analysis in a teacher preparation program would involve examining the curriculum in light of the students' needs and the program goals.

The first level focuses on assessing experience of trainee. It examines whether the training was enjoyable, helpful, and pertinent to the trainees' roles. Using survey tools, objectives, relevant materials, and knowledgeable facilitators, it is possible to measure trainees' satisfaction, engagement, and thoughts on relevance (Paull et al., 2016b). Therefore, measuring teachers' responses to the mathematical knowledge of real analysis is essential at this level.

In the second level, the evaluation focuses on whether or not the participants have acquired the knowledge, skill, attitude, confidence, and commitment that the training program focuses on. This can be assessed after the program by survey (Paull et al., 2016b). Hence, evaluating teachers' attitudes toward mathematical knowledge of real analysis is crucial to ascertain its relevance.

The third level can be surveyed after some months of training completion. Observation interviews are applicable to measure this level. In the third level, the participant behaviours are measured to identify whether the teachers apply the knowledge of real analysis in their teaching. The fourth level focuses on behavioural change or the impact of the program after the program; it is time-consuming because it focuses on the impact (Liao & Hsu, 2019). Therefore, conducting interviews and observations can serve as valuable tools for assessing the influence of the course, and this approach can be employed to evaluate the relevance of mathematical knowledge of real analysis.

To apply these theoretical ideas to assesses the relevance of knowledge of real analysis, we need to evaluate precisely what the secondary-level mathematics teachers have learned during the learning phase through the training. We should test teachers' attitudes before and after the training to measure their progress and accomplish this efficiently. Using these stage guidelines, achievement from real analysis in the

teaching profession can be measured by checking the teacher's awareness of applying knowledge of real analysis.

### ***Social Cognitive Theory***

Bandura's Social Cognitive Theory emphasizes that individuals' beliefs in their capabilities known as self-efficacy influence their motivation, effort, and performance (Bandura, 1997). In education, teacher efficacy reflects a teacher's belief in their ability to design instruction, manage classrooms, and promote student learning effectively. Bandura (1997) identifies four sources of self-efficacy development: mastery experiences, vicarious experiences, verbal persuasion, and physiological states. These factors shape how teachers approach instructional tasks, solve problems, and respond to classroom challenges. Guided by this theoretical perspective, the present research applies Bandura's self-efficacy construct to prepare a mathematics teacher efficacy scale that captures teachers' confidence in both content knowledge and instructional practices. The scale was designed to assess mathematics teachers' beliefs in their ability to explain concepts, create learning tasks, justify procedures, apply reasoning, and link abstract mathematical knowledge to school-level instruction based on this theory.

### **Application of Theoretical Review**

The study is grounded in several theoretical perspectives that support its objectives and guide its methodology. Progressivist (Dewey, 1962) and constructivist (Bruner, 1960; Vygotsky, 1978) approaches inform the understanding of curriculum relevance and the development of the survey tool and interview protocol. These theories emphasize the application of knowledge to solve real-life problems and the construction of knowledge for practical use.

The Mathematical Knowledge for Teaching (MKT) framework (Ball et al., 2008) was employed to identify the types of knowledge essential for effective mathematics teaching, while Shulman's (1986a) model further reinforced the distinction between content and pedagogical content knowledge. This framework was used to explore the theoretical perspective on the knowledge of mathematics required for teaching.

To explore the connection between school and university-level mathematics, School-Related Content Knowledge (SRCK) (Dreher et al., 2018) was reviewed and applied. This perspective helps clarify how advanced mathematics, such as real analysis, can be made relevant for prospective teachers by aligning it with secondary-level content.

Habermas's theory of knowledge-constitutive interests was used to examine how academic knowledge can be applied in real-life teaching contexts. It supports the study's aim of translating theoretical understanding into practical classroom application.

In addition, Bandura's (1997) Social Cognitive Theory informed the development of a teaching efficacy measurement scale, focusing on teachers' confidence in delivering complex content. The Kirkpatrick model of training evaluation (Kirkpatrick & Kirkpatrick, 2014), though originally developed for training programs, was adapted to assess the relevance of real analysis knowledge in secondary education. These frameworks contributed to the development of research instruments and the validation of findings.

While these theories offer valuable insights into teacher knowledge and curriculum design, their direct application to abstract content, such as real analysis, remains limited. Specifically, existing models do not sufficiently address how such

knowledge can be effectively taught at the secondary level, particularly in contexts like Nepal, where curriculum alignment challenges persist.

### **Dimensions of Knowledge for Mathematics Instruction**

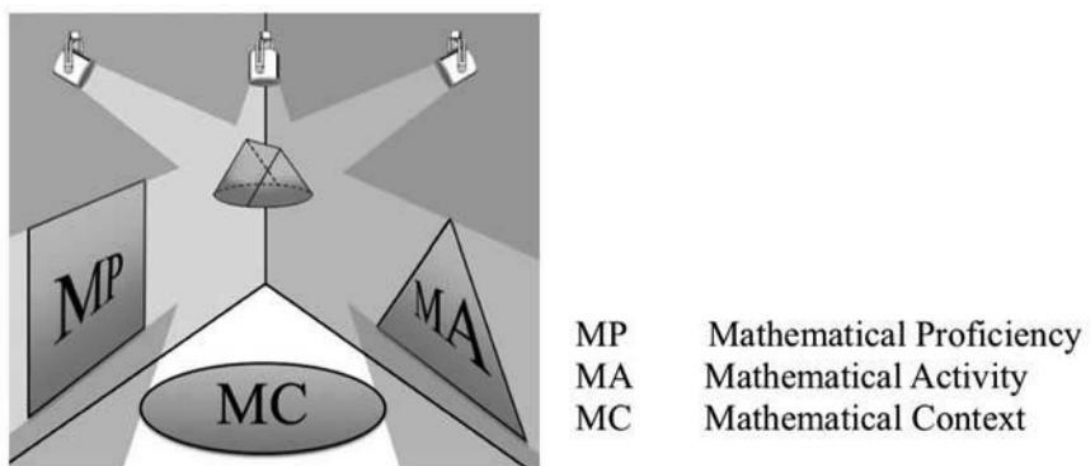
This heading presents key themes related to the use of advanced mathematical knowledge in teaching mathematics.

#### ***Mathematical Understanding for Secondary Teachers: A Framework***

One of the ongoing debates in mathematics education is what kind of mathematical knowledge is required to be a secondary-level mathematics teacher. Different scholars have been researching to provide a framework of knowledge in mathematics required for teaching at the secondary level. Heid et al. (2016) developed a framework of mathematical knowledge for the elementary level, which includes mathematical knowledge and pedagogical knowledge in general.

#### **Figure 6**

*Perspective of Mathematical Understanding for Teacher (Kilpatrick et al., 2015, p. 11)*



Additionally, Ball et al. (2008) described secondary-level mathematics teachers' knowledge, which included common content knowledge, specific content knowledge, knowledge of content and students, and knowledge of content and

curriculum. To represent the mathematical knowledge for secondary-level mathematics instruction, these categories of knowledge were not precise and obvious (Heid & Wilson, 2016). Heid et al. (2016) proposed a framework that described the mathematical knowledge required to be a mathematics teacher at the secondary level. This framework is famously named MUST (mathematical understanding for secondary teachers) and is shown in Figure 6 (Kilpatrick et al., 2015, p. 11). This framework claims that a combination of three components- mathematical proficiency, mathematical activities, and mathematical context required to teach at the secondary level (Heid & Wilson, 2016). These components are explained in coming sections.

**Mathematical Proficiency.** Mathematical proficiency is required for a mathematics teacher at the secondary level. It is capacity for comprehension, application, and definition creation (Krauss et al., 2008). Scholars (e.g., Milgram, 2015; National Research Council, 2001) have explained that conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition are components of mathematical proficiency.

The conceptual understanding refers ability to understand the concept, mathematical operation efficiency in using the concept. Moreover procedure in mathematics comes under procedural fluency. Formulating, representing and solving problem comes included under the domain strategies; reflecting, explaining and justification included in reasoning ability, and disposition includes the attitude and valuing the mathematical knowledge (Heid & Wilson, 2016). Therefore, attitude needs to be measured to assess mathematical proficiency.

**Mathematical Activity.** While teaching and learning mathematics, the activities encompassing pattern-seeking, experimentation, description, critical

thinking, invention, visualization, and conjecture (Cuoco et al., 1996) are integral components of mathematical engagement.

The perspective of mathematical proficiency delineates the overarching domains of mathematical knowledge that a teacher possesses and be proficient in applying. In contrast, the mathematical activity perspective encompasses specific mathematical tasks that a teacher expected to be able to execute effectively (Kilpatrick et al., 2015).

**Mathematical Context.** The mathematical context is another component of the MUST framework. This domain focuses on the context of teaching. The ability of teachers to grasp and understand the curriculum, as well as how to assess students' knowledge, falls under the category of mathematics context. Instructors can use mathematical understanding to improve their students' mathematical comprehension in teaching mathematics. This allows them to combine their content and process knowledge (Kilpatrick et al., 2015). Hence, the mathematical context needs to measure the capacity of applying mathematical knowledge for problem-solving.

### ***Forms of Advanced Mathematical Knowledge***

Research on what kind of mathematical knowledge is required for being a secondary-level mathematics teacher provided a different framework. Additionally, the types of advanced mathematical knowledge that are relevant for preparing secondary-level mathematics teachers have been considered research issues. Leikin and Zazkis (2010) and Stockton and Wasserman (2017) provided a form of advanced mathematics by mapping content relevant to being a mathematics teacher at the secondary level. Peripheral, evolutionary, axiomatic, logical, and inferential knowledge were considered advanced mathematics (Leikin & Zazkis, 2010; Stockton

& Wasserman, 2017). This knowledge has been considered a foundation for preparing mathematics teachers.

**Peripheral Knowledge.** This is a form of advanced mathematics that a teacher may be familiar with, but it is not mathematics that pupil is now learning or will learn in the future learning. Nevertheless, the teacher uses this knowledge for pedagogical purposes (Foster, 2011a). It includes those types of knowledge that study how simple things become complex in mathematics. The learner's upward mathematical trajectory can be supported and moderated by peripheral mathematical information, found anywhere along the vertical axis of difficulty. Even if it is more valuable to the teacher, it does not play a significant role in the learner's mathematical journey (Foster, 2011a). Thus, this knowledge is more complex than its general form.

**Evolutionary Knowledge.** The term evolutionary knowledge describes the expansion of knowledge. Wasserman (2017) refers to understanding the evolution of mathematical concepts as an example of evolutionary knowledge. It corresponds to the definition of mathematical activity given by Heid et al. (2015a). This information enables the teacher to comprehend mathematics from an evolutionary standpoint and persuade pupils why mathematics is important.

**Axiomatic Knowledge.** Mathematical knowledge is rooted in some axiomatic foundation. A mathematics teacher needs to know how mathematical knowledge is rooted. This form of advanced mathematics helps to teach the content of mathematics that demands proof (Lesseig, 2016). Consequently, axiomatic knowledge is represented by understanding mathematical systems' foundations (Patterson, 2021). Therefore, it indicates the basic mathematical system required to prove mathematical theorems.

**Logical Knowledge.** Mathematical logic is central to mathematics and its formal method (Aristidou, 2020). Logical knowledge is concerned with how mathematical reasoning is employed to make inferences. Mathematics teachers need logical knowledge to provide different theorems and facts. Teachers need to have an in-depth understanding of various processes for mathematical proof and the ability to generate their own logically sound explanations, as well as interpret and respond to arguments provided by students (Stockton & Wasserman, 2017). Hence, logical knowledge is required for effective communication in mathematical terms, problem-solving, and using current mathematical processes.

**Inferential Knowledge.** Inferential knowledge is related to the theory of probability and statistics. It provides an understanding of how statistical inference differs from other types of mathematical reasoning. Teachers possess inferential skills to discriminate between mathematical knowledge and statistical aspects (Casey & Wasserman, 2015). Thus, inferential knowledge is required to make informed decisions, provide valuable feedback, and effectively address the dynamic and diverse aspects of teaching and learning.

### ***Summary of the MUST Framework and Form of Advanced Mathematics***

Peripheral knowledge is helpful to secondary mathematics teachers because it enables them to predict future developments more accurately and limits pupils' ability to apply their understanding of a subject or procedure to new situations.

Understanding how mathematical concepts change over time is a necessary skill that enables teachers to defend their subject matter and the advancement of mathematics. That type of skill can be gained from evolutionary knowledge. The axiomatic foundations of the mathematical system are aided by axiomatic understanding. The ability to form arguments and use reasoning in mathematical tasks is a function of the

teacher's logical understanding, which is gained from logical knowledge. Knowledge and abilities to draw conclusions from various experiences are provided by inferential knowledge.

A different form of advanced mathematical knowledge, such as peripheral knowledge, contributes to a more robust understanding of horizontal content knowledge in secondary-level mathematics, as Ball et al. (2008) explained. Advanced mathematical knowledge in different forms supports the three major components: mathematical proficiency, mathematical activity, and mathematical context, as explained by Heid and Wilson (2016). Hence, there are fundamental components of mathematical knowledge for teachers.

The mathematical understanding for secondary teachers (MUST) framework provided by Heid et al. (2015a) and the form of advanced mathematics proposed by Stockton and Wasserman (2017) are used to prepare the research instrumentation and framing methodology and explain the knowledge of real analysis as a source for different types of mathematical knowledge required for secondary school mathematical instruction.

### ***Real Analysis as a Source of Mathematical Knowledge for Teaching***

The MUST framework identifies two fundamental areas of mathematical knowledge for secondary-level mathematics teachers: mathematical activity and mathematical context. Peripheral, evolutionary, axiomatic, logical, and inferential knowledge are the types of advanced mathematics required for secondary-level mathematics teachers (Wasserman et al. 2017). Among these forms of knowledge, real analysis is the source of logical knowledge (Stockton & Wasserman, 2017; Patterson, 2021). Nearly all the number systems, their properties and operations, and related contents of school mathematics are rooted in the axiomatic foundation of the

real number system and its properties (Wasserman et al., 2017; Patterson, 2019). The source of axiomatic knowledge defined by Stockton and Wasserman (2017) is real analysis (Patterson, 2019). Additionally, the content of real analysis provides peripheral and evolutionary knowledge (Wasserman et al., 2017; Patterson, 2019). Hence, real analysis is the source of mathematical knowledge required for being a teacher.

Understanding of limit, continuity, and fundamental concepts of derivative and integration as they relate to calculus and that real analysis is the source of these concepts (Wasserman & Weber, 2017a). The ability to reason is one of the important skills for a teacher. Real analysis is the source of developing and enhancing reasoning ability (Bhupal, 2021). Additionally, reasoning power can be developed by studying real analysis.

The literature above demonstrates how the knowledge of real analysis is essential to a prospective teachers' growth of axiomatic knowledge, logical understanding, and expertise. It serves as a resource for understanding real number systems, their properties, and the way they function in mathematical concepts taught in schools. These literary texts present the idea of a consistent relationship between real analysis and pertinent classroom mathematics practices. This literature identifies methods for determining topic connections and the value of real analysis while teaching secondary-level mathematics.

### ***Content Knowledge for Mathematics Instruction***

Researchers are seriously raising the question of what kind of mathematical knowledge is relevant for being a secondary-level mathematics teacher, and they have developed different models. All the models of different scholars agreed that mathematics teachers require content knowledge of mathematics and pedagogical

content knowledge. The focus point on the types of knowledge of advanced mathematics varies across the model. Among the models, a group viewed common and specialized content knowledge required for mathematics teachers (Shulman, 1987; Rowan et al., 2001; Ball et al., 2005). Another group claimed that common content knowledge, horizontal content knowledge, and specialized content knowledge are required for mathematics teachers (Hill et al., 2008; Baumert et al., 2013; Hurrell, 2013). Nevertheless, the standard view of these models is that they focus on knowledge of advanced mathematics for being a secondary-level mathematics teacher.

The teachers' fundamental knowledge of their subject matter represents the content knowledge that facilitates in-depth understanding.

logic (Shulman, 1987; Mustafa, 2008; Harel et al., 2006). Content knowledge influences pedagogical knowledge (Krauss et al., 2008; Sorto et al., 2009) and real teaching and professional practices (Mustafa, 2008; Arslan, 2017). Knowledge of mathematics beyond what the teacher teaches enables teachers to design classroom activities and mathematical production generated by students and found in teaching resources (Suzuka et al., 2009; Rivas et al., 2012; Markworth et al., 2009). The types of mathematical knowledge have evolved from advanced mathematics, as categorized below.

**Axiomatic Knowledge.** The knowledge that describes the roots of the mathematical system is called axiomatic knowledge of mathematics (Sachan et al., 2017). Mathematics teachers need to gain knowledge of the foundation of mathematical facts.

**Logical Knowledge.** Logical knowledge concerns logical structure and valid rules (Wasserman, 2018b). It has been employed to prove theorems and solve mathematical problems.

**Evolutionary Knowledge.** Mathematical ideas are developed based on some mathematical system. This knowledge helps to understand how mathematical propositions evolved (Wasserman, 2018a). Evolutionary knowledge is similar to mathematical activity, including mathematical reasoning, creating and noticing, axiomatic knowledge, and logical knowledge, similar to the mathematical activities, mathematical proficiency, and mathematical context expressed by Heid and Wilson (2016). Therefore, both types of knowledge are required for a mathematics teacher.

#### ***Advanced Mathematical Knowledge and Teaching Efficacy***

Mathematics teachers' knowledge of advanced mathematics and their teaching efficacy are very close. The knowledge of mathematics influences teachers' teaching efficacy. Mathematical knowledge for teaching (MKT) extends more basic content knowledge of mathematics to the inclusion of pedagogical content knowledge, which enables teachers to explain concepts, address misconceptions, and use multiple representations effectively (Hill et al., 2005; Alshehri & Youssef, 2022). The teachers' mathematical knowledge for teaching significantly contributes to students' learning (Hill, 2005). The teachers' professional development and experience with concepts of advanced mathematical knowledge contribute to effective instruction (Bjerke & Xenofontos, 2024; Fukaya et al., 2023). Hence, advanced mathematics and mathematical proficiency knowledge impact teachers' instructional efficacy. Opportunities for ongoing professional development help teachers experience more confidence in their teaching ability, which improves their methods. Additionally, teaching efficacy impacts motivation for learning and effective instruction (Aksu &

Kul, 2019). Hence, advanced mathematical knowledge and teachers' mathematical efficacy contribute to teaching efficacy.

### ***TPACK Framework and the Use of Real Analysis in Instruction***

The framework, Technological Pedagogical Content Knowledge (TPACK) originally developed by Mishra and Koehler (2006), the concept of pedagogical content knowledge by integrating content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK). It emphasizes the need for teachers to combine subject expertise, effective teaching strategies, and appropriate technological tools to deliver meaningful instruction.

In the context of mathematics instruction, TPACK is particularly relevant for understanding how teachers apply advanced content such as real analysis in school-level instruction. CK is reflected in teachers' knowledge of topics like limits and continuity, PK in their strategies such as linking concepts and using multiple representations, and TK in the potential use of digital tools for visualization and exploration. Although TK was not explicitly addressed in this study, the model provides a useful framework for interpreting how teachers translate advanced mathematics into accessible instruction.

This study observed strong intersections of CK and PK, where teachers used problem-solving, abstraction, and content-bridging techniques rooted in real analysis. These align with the TPACK framework's core idea that effective teaching arises from the integrated application of knowledge domains. TPACK therefore complements models like Mathematical Knowledge for Teaching (Ball et al., 2008) by offering a practice-oriented lens through which the relevance of real analysis in classroom instruction can be better understood. Additionally, Khanal et al. (2024) found that mathematics teachers viewed subject-specific digital tools as essential in enhancing

learners' grasp of mathematical structures and processes. Their reflection confirmed that technology, when aligned with pedagogical intentions and content goals, enhances students' ability to explore and generalize mathematical concepts. Similarly, Joshi et al. (2024) found that developing TPACK competencies improved the clarity and accessibility of mathematics content for diverse learners.

### ***Relation of Content Knowledge With Proficiency and Learning Outcomes***

School teachers are employed according to specified qualifications and selection criteria. A teacher's knowledge and abilities beyond these fundamental standards may impact students' academic success. This section examines teachers' mathematical expertise and how it influences students' academic progress.

Mathematics teachers required to have at least knowledge of pedagogy and content. Mathematics teachers' content knowledge in terms of the level of education influences the students' quality of their performance, grade, and class attainment status of the students (Hill et al., 2004). Moreover, teachers used content knowledge and specific content knowledge on mathematics to help promote the students' achievement and teachers' professional skills (Hill et al., 2004). Additionally, to enhance students' learning outcomes, instructors' professional development programs need to emphasize content understanding, which aid in their mastery of the subject. (Mwinka & Tarmo, 2020; Hill et al., 2005). The prospective teacher's mathematical proficiency and professional expertise are improved by understanding the content knowledge (Tchoshanov, 2011). There is a correlation between teachers' subject-matter expertise and their students' proficiency in a given area of mathematical instruction, including algebra. However, the teachers' knowledge of algebra and real number systems is not correlated with algebraic computation (Begle, 1972). The teachers' mathematical knowledge and attitude toward mathematics positively and

significantly correlate with the students' achievement and motivation toward mathematics learning (Campbell et al., 2014). Additionally, the teachers' professional competence is important in students' success and increasing their achievement and quality of instruction (Yang & Kaiser, 2022). Mathematics teachers have common content knowledge and specific content knowledge under mathematical knowledge for school mathematics instruction, which influences the teacher's mathematical efficacy (Alshehri & Youssef, 2022). In conclusion, teachers' knowledge of advanced mathematics and their understanding significantly influence their mathematical proficiency and teaching efficacy, which consequently impacts students' performance.

This literature emphasizes the importance of teachers' mathematical expertise in student progress, teacher professionalism, and mathematical competence. All of these relationships are beneficial and important. The knowledge of real analysis may have a relatively significant impact on teachers' content knowledge, professional confidence, and students' academic progress because real analysis mathematical knowledge is also a component of mathematical knowledge for school instruction, which is a part of advanced mathematics.

### ***Mathematical Content Knowledge to Expertise in Mathematics Instruction***

Adequate knowledge of the subject matter is required for expertise in mathematics. This statement is considered trivial, but how much knowledge represents adequate content knowledge is unclear. The investigation of teachers' subject-matter expertise and its connection to classroom instruction has increased during the past ten years. The first trend in mathematics teacher preparation approach-based research focused on advanced mathematics to developed expertise in the subject matter (Simmt, 2011). The practice-based approach developed by Even (2011), Kvatinsky and Even (2002), and Ball et al. (2008) define mathematical

knowledge for teaching as mandatory. They view common content knowledge as required in another field. However, specialized content knowledge is helpful in teaching. Mathematics for teaching expected to be a specialized area that differs from mathematics used in other fields (Davis & Simmt, 2006). This approach advocates that advanced mathematics contributes to the development of mastery in mathematics.

Another trend of research focused on mathematics teachers' subject matter knowledge. Research findings claimed that secondary-level mathematics teachers do not have a strong, more profound knowledge of mathematics they need to teach in school (Even, 1990; Knuth, 2002; Ball, 1990). Thus, ordinary university or college mathematics does not adequately support the development of the mathematical knowledge required for school teachers.

### ***Connection between School Mathematics and Advanced Mathematics***

Connection indicates the link between two aspects. The vertical connection between advanced and mathematical knowledge for school-level mathematics instruction can be considered. Making a connection between two things (or ideas) based on similarities, equivalent concepts, or coincidences is known as a logical connection. Skill connection refers to the connection of experience and helps to drive new problems based on previous experience. In the case of mathematics, the skill of using logic at a higher level helps solve problems that occur during school-level mathematics instruction. In this section, the link between advanced mathematics and secondary-level mathematics that has been found in the literature is discussed.

Wasserman (2018a) has identified "the connection between advanced mathematics and secondary-level mathematics and pointed out (1) content connection, (2) disciplinary practice connections, (3) classroom teaching connections, and (4) model instruction connection" (p. 6). The procedural and axiomatic connections are the

silent connections between secondary-level mathematics and advanced mathematics. The connection between advanced mathematics and school-level mathematics indicates the relevance of advanced mathematics for school-level mathematics instruction. The two directions of connections have been discussed: the connections between school mathematics and advanced mathematics. The top-down model takes advanced mathematics as a starting point, and the bottom-up model takes school mathematics as a starting point (Dreher et al., 2016). The transformation from advanced mathematics to school-level mathematics is to make connections between them (Ball et al., 2008). The top-down direction connects advanced and school-level mathematics (Allmendinger, 2016; Klein, 2016). Mathematics teachers at the school-level need to understand the connections between the academic discipline's structures and the courses taught in schools (Dreher et al., 2016). Thus, teachers usually have to deal with textbooks and other learning settings that contain mathematical knowledge that has already been established. It is important that they are able to determine whether the content has been effectively reconstructed.

This literature reveals that mathematicians and mathematics educators adopted the scientific technique to create the relationship between academic mathematics and school mathematics. The top-down and bottom-up connections were used to establish the connection between school mathematics and academic mathematics. The fundamentals of mathematics are introduced in school mathematics, and the content is progressively advanced toward academic mathematics. Following the subject's structure and level of difficulty is one way of making the connection between school mathematics and academic mathematics. The suitable basic definition and axioms are taught in school, and their advanced content and theorems are taught at the university level based on this approach.

University-level courses on foundations and axioms are taught, while general concepts and applications are taught in school mathematics through generalizations rather than theoretical ones. The top-down method is the foundation for this kind of interaction. For instance, basic operations on the real number system are performed during mathematics instruction at the secondary school-level. However, its axioms and theoretical foundations are covered in real analysis courses at the university level. The connectivity of content and logic enables concept conception.

### ***Knowledge of Real Analysis for School Teachers***

Real analysis is a specific branch of mathematics under advanced mathematics (Wasserman et al., 2017). The real number system, function and limit, the topology of real numbers, continuity, sequence and series, derivability, and integration of one variable are included in real analysis (Bartle & Sherbert, 2011). Real analysis is a course that nearly all mathematics majors and some mathematics education majors must take (Conference Board of the Mathematical Sciences, 2012). Under the course of real analysis, the structure of the real number line and its subsets were studied (Bartle & Sherbert, 2011). Mathematical knowledge, peripheral knowledge, evolutionary knowledge, axiomatic knowledge, logical knowledge, and inferential knowledge are required for secondary school teaching (Stockton & Wasserman, 2017). Among these forms of advanced mathematical knowledge, real analysis has the potential to contribute to the conception of axiomatic knowledge (Patterson, 2021). Additionally, real analysis of mathematical knowledge helps conceive unique perspectives on peripheral and evolutionary knowledge (Wasserman et al., 2017). Hence, it is the source of peripheral and evolutionary knowledge.

The MUST (mathematical understanding for a secondary teacher) framework included mathematical proficiency, activity, and context. Among these knowledge

domains, Patterson (2021) found that secondary school teachers used this knowledge via real analysis. This shows that knowledge of real analysis influences secondary-level mathematics instruction. In contrast, the real analysis course is disconnected from the secondary-level mathematics instruction (Goulding et al., 2003). Therefore, pedagogical application of knowledge of real analysis is controversial.

Two approaches can be used to make mathematical knowledge of real analysis relevance to secondary-level mathematics instruction. Wasserman et al. (2017) argue that the traditional method emphasizes advanced mathematics and presupposes that teachers integrate their subject-matter expertise into secondary-level mathematics instruction. This approach indicates the practice of knowledge of real analysis at the undergraduate level is stepping down to school-level instruction. Due to the lack of success of this model, Wasserman et al. (2017) have suggested building on teaching high school mathematics in school practice.

The knowledge of real analysis taught at the university level has contributed to some extent to preparing mathematics teachers for the secondary level with a strong foundation in mathematics. If we can follow the model suggested by Wasserman et al. (2017) then valuable and pedagogical content knowledge can be provided for the prospective teacher.

### ***Place of Real Analysis in Teacher Preparation Programme***

American students who want to teach secondary-level mathematics finish a course in real analysis (Fukawa et al., 2019). Universities around the world also introduce real analysis in their teacher preparation programme (Moreira & Vianna, 2016). Advanced mathematics courses, including real analysis, have been taught to develop problem-solving abilities and boost the consciousness of the subject matter. Different universities in Nepal such as Tribhuvan University, Kathmandu University

introduce real analysis courses in their teacher preparation programme.

The Faculty of Education of Tribhuvan University is implementing the four-year Bachelor of Education programme. Under these programmes, students majoring in mathematics have opportunities to study real analysis. The course includes real numbers, open and closed sets, real sequences, infinite series, functions and limits, continuity of a function, derivability, and Riemann integral (Tribhuvan University Faculty of Education, 2021). In the vague topics, some major concepts are included in the courses. The content, System of real numbers, the algebraic structure of  $\mathbb{R}$ , order axioms and properties of  $\mathbb{R}$ , the absolute value of a real number and its properties., boundedness of subsets of  $\mathbb{R}$ , completeness axioms in  $\mathbb{R}$ , Archimedean property, Dedekind's construction of the set of real numbers, representation of real numbers on a line are included in the chapter of the real number. Open and closed intervals, Neighborhoods, Interior points and interior of a set, open sets, the limit of a set, Bolzano-Weierstrass's theorem, closed sets, covering of a set, compact sets, Cantor sets, connectedness included in the unit, open and closed sets. Unit three real sequence includes convergent sequences, Cauchy sequences, Cauchy's criterion for convergence, non-convergent sequences, Cauchy's first and second theorems on the limit, monotonic sequences, monotone convergence theorem, Cantor's intersection theorem, subsequences, uniform convergence, and summability of sequences. Unit four: Infinite series includes infinite series, partial sums, convergence, different convergence tests, the divergence of series, and series arrangement. Unit five: Function and limit include functions' types with their nature, limits, properties, and types. Continuous function, its type, properties, discontinuous function, inverse function, monotonic function, Lipschitz functions, and related theorems are included in unit six called continuity. Unit seven: Derivability includes a derivative of a

function, Darboux theorem, mean value theorem, and other connected theorems and indeterminate forms. The last unit, Riemann integral, includes fundamental concepts like partition and Darboux sum for starting integration. Additionally, it includes the condition of inerrability and related theorems based on the properties of the Riemann integral and the mean value theorem. All this content is the foundation for teachers and experts in content knowledge.

### ***Pedagogical Application of Real Analysis***

Being a secondary-level mathematics teacher requires having advanced mathematical knowledge. Different courses in advanced mathematics, including real analysis, are included in mathematics teacher preparation programs in Nepal. Knowledge of real analysis is beneficial in enhancing the perspective of teachers' teaching ability, increasing a prospective teacher's competence and confidence (Stockton & Wasserman, 2017). Knowledge of advanced mathematics, including real analysis, is beneficial in developing a more common understanding of disciplinary practices in mathematics concerning proof (Wasserman & Weber, 2017b).

Various types of difficult situations arise when teaching mathematics, such as mathematical logic gaps and student demands for mathematical logic. Knowledge of real analysis offers the basis for mathematical evidence and logic, which is necessary to deal with such situations (Thames et al., 2008). All representations and interpretations formed before systematic instruction of a concept can be regarded as products of the concept's primary intuitive cognition. Advanced mathematical knowledge, including real analysis, improves the intuitive cognition of the prospective teacher, fostering individual independence in systemic instruction and the rigorous process of mathematical logic and its application in mathematical operations (Fischbein, 1999; Roh & Lee, 2017; Edwards & Ward, 2004).

In school mathematics, particularly in teaching the number system and various forms of mathematical induction, the analysis of information expected be grounded in logical reasoning. Real analysis directly supports this process, especially in topics such as sequences, series, and related concepts. The application of reasoning not only enhances understanding but also enables students to solve a wide range of mathematical problems effectively.

The preceding literature focuses on the psychological facet of real analysis. Prospective teachers' attitudes toward real analysis and its application were discovered to be unrelated to teaching mathematics at the secondary level. However, real analysis has made an invaluable contribution to equipping mathematics teachers with a solid foundation in mathematical knowledge. Various mathematical concepts are practised during secondary-level instruction. Professional practices include justifying mathematical concepts and operational procedures, linking them to context, and performing advanced mathematical activities. Real analysis is an authentic source of such knowledge.

In summary, the pedagogical application of knowledge of real analysis contributes to the advancement of mathematical cognition, intuition, and dynamism, all of which are essential for being an effective secondary-level mathematics teacher.

### ***Mathematics at the Secondary Level***

The opportunities for acquiring a mathematics education at the secondary level, which encompasses first two grades (grade nine and ten) are structured to incorporate both mandatory and optional within the broader framework of the general education system, as outlined by the Government of Nepal, Ministry of Education, Science and Technology (2020).

In the compulsory mathematics curriculum, contents such as arithmetic (including interest, commission, tax, home arithmetic, and money exchange), mensuration, algebra (covering sequences and series, quadratic equations, algebraic fractions, and indices), geometry (encompassing triangles, quadrilaterals, construction, and circles), statistics, probability, and trigonometry are included. On the other hand, the optional mathematics curriculum includes algebra, limit and continuity, matrices, coordinate geometry, trigonometry, vectors, transformations, and statistics (Curriculum Development Center, 2022). Therefore, various facets of mathematics are incorporated into the secondary-level mathematics curriculum in Nepal.

The review of the dimensions of mathematical knowledge informed the conceptualization of knowledge for instruction and its pedagogical application. Furthermore, insights derived from the reviewed literature guided the development of the conceptual framework and the construction of survey items.

### **Relevance Indicators**

The reviewed theories and dimensions of mathematical knowledge were utilized to develop the indicators of relevance. The aforementioned theories (e.g., Kirkpatrick and Kirkpatrick, 2014) are the basis for studying the relevance of mathematical theories and the relevance of mathematical knowledge of real analysis for mathematics instruction. The relevance of real analysis in education can be assessed based on several theoretical perspectives. Curriculum development, as proposed by Tyler (1949), need to be responsive to both learner needs and societal expectations, with teachers' perceptions playing a crucial role in its effectiveness (Stone et al., 1990). Thus, it is used to create an indicator for measuring the relevance of real

analysis. If teachers perceive real analysis as irrelevant or disconnected from their instructional needs, its integration into the curriculum could be challenged.

Schwab (1973) supports this notion by arguing that curriculum design needs to be responsive to educators' practical deliberations. Educators evaluate a subject's value based on its applicability in classroom settings. Moreover, Goodlad (1979) suggests that teachers experience the curriculum differently, with their perceptions directly influencing its effectiveness in teaching practice.

Acquiring knowledge of real analysis for teaching aligns with Shulman's pedagogical content knowledge for mathematics teaching. The concept of pedagogical content knowledge (PCK) emphasizes the importance of teachers having in-depth knowledge of their subject matter and understanding how to teach it effectively. Real analysis strengthens this knowledge base, providing teachers with deeper insights into mathematical structures, thus enhancing their teaching competence. Bruner's (1960) spiral curriculum theory further justifies the inclusion of real analysis, as it supports the idea that advanced concepts need to be revisited throughout a teacher's development. Taba (1962) advocates for knowledge structuring in curriculum design, which could be fulfilled by systematically incorporating real analysis to help teachers develop a coherent mathematical understanding.

The application of real analysis in the classroom can be evaluated through Dewey's (1962) experiential learning theory, which stresses the value of learning through real-world experience. Applying real analysis concepts in their teaching can lead to more effective, inquiry-based learning environments for students. Stenhouse (1975) suggests that the curriculum needs to be flexible, allowing teachers to adapt theoretical knowledge to suit diverse educational contexts. Vygotsky's (1978) sociocultural theory also highlights the importance of teachers using mathematical

knowledge to scaffold students' learning, making real analysis a valuable tool for enhancing students' cognitive development.

**Table 1**

*Relevance Indicators*

	Kirkpatrick four levels	Assessing guidelines	Related Research Questions
Relevance indicators	4	What is the impact of knowledge of real analysis on the teaching profession?	Impact of knowledge of real analysis on mathematical efficacy RQ2
	3	How do teachers apply knowledge of real analysis in instruction?	Pedagogical application of mathematical knowledge of real analysis for secondary-level mathematics instruction RQ8 RQ9
	2	What do teachers learn from real analysis?	Knowledge of real analysis for teachers, the existence of a link, and strengthening the link RQ4 RQ5 RQ6 RQ7
	1	How do teachers respond to real analysis?	Mathematics teachers' attitudes towards and mathematical knowledge of real analysis across its content connection, pedagogical application, and enhancement of mathematical proficiency. RQ1 RQ3

All these discussed theoretical guidelines align with Kirkpatrick and Kirkpatrick's training evaluation model (2014); therefore, the relevance indicator, as shown in Table 1, was prepared based on this model and guiding theories.

**Applications of Thematic Review**

The review of dimensions of mathematical knowledge for teaching employed thematic analysis to understand the application of advanced mathematical knowledge to school mathematics instruction and its place. This review examines the various forms of advanced mathematical knowledge (e.g., evolutionary knowledge, axiomatic knowledge) that are essential for secondary school mathematics instruction.

Additionally, it discusses the role of real analysis in secondary-level mathematics instruction.

Overall, while the thematic literature identifies the essential components of mathematical knowledge for teaching (such as mathematical proficiency, activity, and context) and classifies advanced mathematical knowledge into various forms (axiomatic, logical, and evolutionary), the practical application of these categories in the context of teaching secondary-level mathematics remains insufficiently examined. Although scholars recognize that real analysis may support logical reasoning and axiomatic thinking, there is a lack of clarity on how secondary-level teachers can practically employ such knowledge to improve classroom teaching and student understanding. Additionally, frameworks such as the MUST framework (Heid et al., 2015a) outline the knowledge that teachers need to possess. However, they do not provide concrete strategies on how advanced mathematical knowledge, like real analysis, can support teachers' classroom practices. Hence, this study aims to fill this gap by examining how real analysis contributes to the mathematical proficiency, pedagogical application, and content connection required for effective secondary-level instruction, thus applying and extending the existing thematic frameworks.

### **Empirical Studies on Advanced Mathematics in School Instruction**

Several empirical studies have investigated the relevance and role of advanced mathematics, particularly real analysis, in school-level instruction. The reviewed studies are categorized thematically under the following headings.

#### ***Relevance of Advanced Mathematics in School Teaching***

Several studies have investigated whether advanced mathematics, particularly real analysis, is directly relevant to teaching practices in secondary schools. Goulding et al. (2003) explored the significance of undergraduate mathematics experiences for

the preparation of secondary-level mathematics teachers, interpreting these experiences in the secondary level. The results showed that real analysis content does not have a significant connection with instructional practices. Zazkis and Leikin (2010) explored how advanced undergraduate-level mathematical knowledge is applied in teaching careers. Their findings revealed that teachers valued advanced mathematical knowledge because it helped them gain confidence, connect with students, and respond effectively. Even (2011) conducted a qualitative study with Israeli teachers who studied in a special master's program with advanced mathematics courses. The participants responded that advanced mathematics knowledge was relevant to their teaching. In contrast, Wu (2011) reviewed literature and found no direct correlation between university-level course content and classroom instruction.

Suominen (2015) explored the relationship between abstract algebra and secondary-level mathematics, using interviews, observation, and textbook analysis. This study found that abstract algebra has a logical relationship with school-level algebra, establishing a connection between advanced mathematics content and school mathematics instruction. Schmidt et al. (2016) reviewed international perspectives on mathematics teacher preparation programs and found that university mathematics helps prepare top-performing teachers. Teachers with advanced mathematics knowledge are better able to integrate such knowledge into their instruction. This highlights the significance of real analysis knowledge in teacher preparation.

### ***Teacher Preparation and Professional Competence***

Several studies underscore the importance of advanced mathematics in teacher preparation and its impact on professional competence. Pepin et al. (2017) investigated the resources and mobilization strategies that contribute to mathematics teachers' expertise in China. The study found that access to resources, including

advanced mathematics, significantly contributed to teachers' mathematical competence. While this study did not specify real analysis as a source, it suggested that advanced mathematical knowledge, such as real analysis, could serve as a crucial resource for expertise. Koponen et al. (2019) created a framework to identify the domains of mathematical knowledge needed by prospective teachers in Finland. Their study showed that knowledge of content, students, and teaching is interrelated. Knowledge of advanced mathematics, such as real analysis, is linked to stronger teaching practices.

Chua (2021) conducted a meta-analysis of the mathematical knowledge required for teaching and emphasized the importance of advanced mathematical knowledge for teaching. He identified gaps in teachers' understanding of mathematical content and suggested that knowledge of real analysis could help bridge those gaps and improve teachers' mathematical expertise. Schmidt et al. (2020) found that strong foundations in advanced mathematics, such as real analysis, enhance teachers' subject-matter knowledge and teaching capabilities. These studies reinforce the importance of integrating advanced mathematics into teacher preparation programs.

### ***Role of Mathematical Knowledge in Teaching***

Several studies have examined the relationship between advanced mathematics knowledge and teaching effectiveness. Graif (2020) used a mixed-method research design to study mathematical knowledge for teaching proofs in secondary mathematics. The study found that departmental assignments and educational levels did not generate the mathematical expertise required for teaching proof. However, instructors' proof strategies influenced teaching practices. Patterson (2021) investigated how knowledge of real analysis influences the mathematical

proficiency and teaching practices of secondary teachers. The study found that teachers used real analysis knowledge through bottom-up and step-down approaches during instruction, improving their mathematical proficiency and adaptability in the classroom.

Yan et al. (2022) examined mathematicians' views on the value of advanced mathematics for secondary teachers. Their study found that advanced mathematics improves teachers' problem-solving skills and epistemological understanding, supporting the need for real analysis in teacher preparation. Gamboa et al. (2022) explored the relationship between mathematical expertise and the ability to establish mathematical connections in teaching. Their findings recommended including mathematical connections as part of pedagogical content knowledge development, which real analysis knowledge can significantly enhance. Hatisaru (2022) found that teachers with a deeper understanding of mathematics made more connections when teaching functions, showing that advanced mathematics improves teachers' ability to make meaningful mathematical connections.

### ***Impact of Advanced Mathematics on Pedagogical Practices***

Studies also highlight how advanced mathematics shapes teachers' pedagogical practices. Veith et al. (2023) analyzed the use of algebra in instruction and found that advanced mathematical concepts influence teaching practices. This suggests that teacher preparation programs need to emphasize strengthening content knowledge, particularly in advanced mathematics. Wasserman et al. (2023) surveyed university teaching practices for secondary teacher preparation and proposed a model for linking disciplinary practices and engaging in rigorous mathematical work. Their study underlined the need for curriculum reform to ensure that future teachers are adequately prepared with advanced mathematical knowledge. Ko et al. (2024)

examined the relationship between teachers' experience in advanced algebra courses and their teaching proficiency in algebra. The study showed that teachers with more experience in advanced algebra courses demonstrated greater proficiency in teaching algebra, highlighting the significance of subject-specific experience in enhancing instructional quality. Recent empirical research highlights important advances in understanding how secondary mathematics teachers engage with advanced mathematical concepts and pedagogical practices. Shure et al. (2025) conducted a systematic review of professional development programs designed to enhance teachers' abilities to handle typical mathematical tasks effectively. Their findings emphasize that sustained, context-specific professional learning fosters productive teaching practices by equipping teachers with strategies to adapt complex tasks to diverse classroom settings. Complementing this, Cevikbas et al. (2024) explored the experiences of secondary-level teachers in courses focused on mathematical modeling. Through qualitative case studies, they revealed how teachers develop competencies in designing and implementing modelling tasks that connect real-world problems with curricular content, thereby deepening students' conceptual understanding and engagement. Expanding the discourse on advanced mathematical knowledge, Hanke (2024) examined experts' intuitive mathematical discourses about integration in complex analysis, identifying varied interpretative frameworks used by experts. This study contributes to teacher education by elucidating the sophisticated reasoning processes that underpin expert understanding, which can inform instructional approaches that promote deeper mathematical comprehension among secondary students. Collectively, these studies underscore the necessity of integrating both content knowledge and pedagogical strategies in teacher education programs to

enhance secondary mathematics instruction, particularly in advanced mathematics domains.

### **Applications of Empirical Review**

The empirical review of this study highlights the significance of advanced mathematical knowledge for secondary-level mathematics instruction (e.g., Zazkis & Leikin, 2010; Wasserman, 2018). Emphasizing that while advanced mathematics enhances teachers' content knowledge, its direct application in secondary-level instruction often remains unclear about the role of real analysis. Although empirical research acknowledges that real analysis can deepen teachers' understanding of mathematical structures and reasoning, few studies provide practical insights into how this knowledge can be effectively integrated into schools' mathematics instruction. This gap suggests a disconnect between university-level mathematics knowledge and its classroom applicability. Furthermore, the reviewed empirical works focus on general mathematical knowledge for teaching (MKT) and pedagogical content knowledge (PCK), with limited emphasis on context-specific applications of real analysis in secondary education, especially within developing countries like Nepal. This indicates a lack of contextualized evidence on how knowledge of real analysis informs teaching practices aligned with schools' curricula and student needs.

This empirical review indicates a critical need to investigate how knowledge of real analysis can be made relevant and practical for secondary school mathematics teachers. Addressing this gap, the present study contributes to the field by exploring teachers' experiences and strategies in applying knowledge of real analysis in school mathematics, offering context-specific insights that can inform curriculum design and teacher education programs. Additionally, this review guides methodological

procedures and preparing tools focused on the relevance of knowledge of real analysis for secondary-level mathematics instruction.

The literature review has important implications for this research. It included theoretical, thematic, and empirical reviews to provide a comprehensive understanding of the study's context. This review was valuable for understanding the current state of advanced mathematics, particularly real analysis, the theoretical model of mathematical knowledge for teaching, and identifying gaps in existing research. Furthermore, the reviewed literature informed the development of the theoretical framework, relevance indicators, and research tools designed to measure teachers' attitudes toward the relevance of knowledge of real analysis for secondary-level mathematics instruction. In addition, both empirical and theoretical literature were used during the discussion of findings to assess and support the reliability and validity of the study's results.

### **Conceptual Framework of Relevance of KRA for SLM Instruction**

The framework shown in Figure 7 was developed based on a review of prior work on mathematical knowledge and knowledge of real analysis for secondary-level mathematics teachers. It was created to examine the relevance of mathematical knowledge of real analysis in secondary mathematics education.

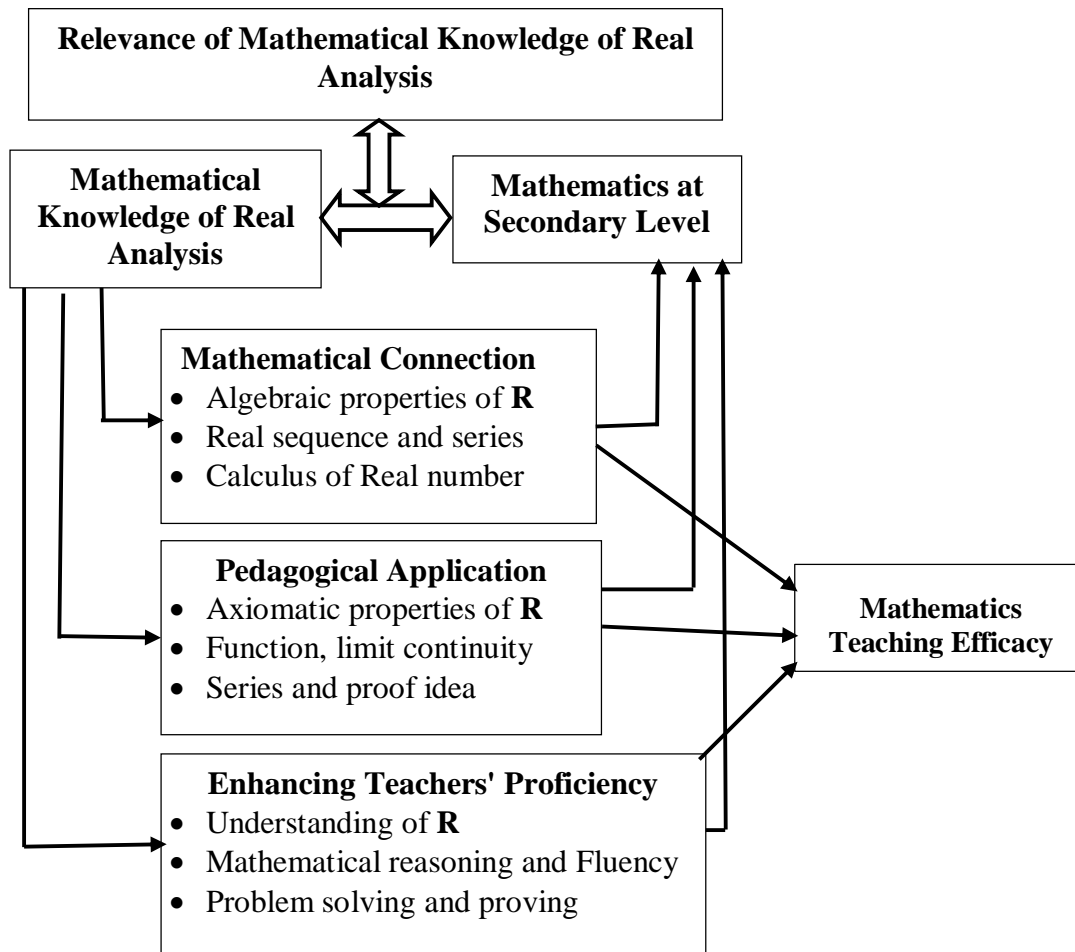
#### ***Mathematical Connection***

Mathematical connection refers to the relationships and interlinkages among mathematical concepts, ideas, and principles. It encompasses various forms such as content connections, axiomatic connections, connections in proofs, and connections in argumentation. These forms of connection enable learners to recognize the overarching structure of mathematics by relating topics like algebra, geometry, and calculus in meaningful ways. Through an understanding of these connections,

students can develop a more comprehensive, integrated, and coherent view of mathematics, enhancing both their conceptual understanding and problem-solving abilities.

**Figure 7**

*Conceptual Framework of Relevance of Real Analysis Knowledge*



Additionally, these connections help students recognize the underlying patterns and relationships across different mathematical areas. This deeper understanding aids in the retention of concepts and enhances problem-solving abilities. Ultimately, fostering mathematical connections supports students in applying their knowledge more flexibly and effectively in both academic and real-world contexts.

**Content Connection.** Content connection refers to the relationship between topics across different academic levels within the same discipline or curricula. The content connection between undergraduate mathematical knowledge and teacher preparation programs indicates the relevance of advanced mathematics for secondary school mathematics teaching. Teachers' awareness of the content connection influences their teaching (Murray et al., 2017). Thus, the content connection between mathematical knowledge of real analysis and secondary-level mathematics justifies the relevance of advanced mathematics for secondary school instruction.

The similarity in content or link between two levels refers to a direct connection in content, such as a polynomial equation in an undergraduate course and a small portion of it in school mathematics content. Curriculum developers use vertical content anchoring to make different mathematics levels relevant (Menon, 2015). The content link between secondary school mathematics and undergraduate mathematics, particularly teacher preparation programs, indicates the relevance of undergraduate mathematics content in instruction. A relatively direct content connection can be used to determine the relevance of real analysis mathematical knowledge for secondary-level mathematics instruction.

**Axiomatic Connection.** Axiomatic connections refer to the relationships between axioms that enable various mathematical tasks in different contexts or situations. During secondary-level instruction, the teacher incorporates some concepts and definitions from their undergraduate course (Patterson, 2019). Axioms taught in undergraduate mathematics, in particular, may connect with secondary-level mathematics (Function, limit, continuity, sequences and series), and teachers may have rationally used such axioms to solve mathematics problems during their

instruction. Thus, using axioms learned in undergraduate mathematics to prove and justify mathematical activities indicates relevance.

**Generalization.** Generalization in mathematics involves extending the application of a particular mathematical concept or rule from one specific instance to a broader category or a related concept without losing its validity. It is the process of recognizing patterns, identifying commonalities, and applying a rule or concept more broadly. The axioms, rules, and theorems may be presented in secondary-level mathematics in their generalized form. This is one indicator of the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction.

**Procedural Connection.** Procedural connection in mathematics involves transferring learned rules, algorithms, and methods from one level or context to another to solve problems and explore concepts. It involves the seamless transfer of procedural knowledge from one context to another. Teachers use their procedural knowledge and skills during their instruction as needed (Wood et al., 2016). The connection between mathematical procedures in real analysis and school mathematics indicates the relevance of real analysis for teaching secondary-level mathematics instruction.

**Alternative Representation.** Alternative representation in mathematics refers to how a particular mathematical concept or object can be depicted. It involves expressing the same mathematical idea through various means, including symbols, formulae, or notations. This versatility allows mathematicians to present concepts from multiple perspectives, offering a richer understanding of the underlying principles.

For instance, the representation of the slope of a line. While the standard form may be  $(y = mx + c)$ , has an alternative representation can take the form

$y - y_1 = m(x - x_1)$ . This flexibility in expression shows mathematical language's adaptability and emphasizes mathematical concepts' interconnectedness. In essence, alternative representation bridges different mathematical domains and enriches the learning experience.

### ***Pedagogical Application***

The pedagogical application of real analysis refers to how mathematics teachers use their mathematical knowledge of real analysis during instruction. The number of mathematical rules is based on the axioms and properties of the real number system. For example, the law of transitivity, trichotomy law, and field axioms are used in school mathematics instruction. Using mathematical knowledge of real analysis, definition, axioms, logic, and argument during instruction demonstrates the pedagogical application of real analysis in secondary-level mathematics instruction (Wasserman & Weber, 2017a). Therefore, relatively moderate pedagogical application of real analysis in secondary-level mathematics can be found.

Thus, the major pedagogical applications of real analysis in teaching secondary school mathematics are the different ways of solving numerical problems that involve the rules of numbers in mathematical practices using the axioms of the real number system (1), reasoning on mathematical operations (2), formulating the verbal problem in mathematical form (3), and so on. The pedagogical application of acquired knowledge demonstrates its relevance for teaching (International Bureau of Education, 2023). This application highlights the importance of applying acquired knowledge to teaching. It suggests that teachers required to possess knowledge in their subject areas and understand how to apply it to teaching.

The pedagogical application of knowledge of real analysis is particularly important in mathematics instruction at the secondary level. Real analysis is a branch

of mathematics that deals with the properties of real numbers and the functions of real variables. It provides a foundation for many other areas of mathematics, including calculus, topology, and complex analysis.

By applying knowledge of real analysis to teaching secondary-level mathematics, teachers can demonstrate to students the relevance and practical applications of this mathematical concept. For example, they may use real analysis concepts to explain function behavior, analyze limits and continuity, or develop mathematical models. Moreover, the contribution of the mathematical knowledge of real analysis for teaching also helps to develop students' critical thinking skills.

By teaching students how to apply mathematical concepts to real-world problems, teachers can help them develop a deeper understanding of mathematical principles and build a strong foundation for future learning. This approach fosters analytical skills, encourages logical reasoning, and enhances problem-solving. Additionally, students are better equipped to understand advanced mathematical topics and their real-life applications. Knowledge of real analysis also strengthens teachers' ability to connect abstract mathematical ideas to concrete examples, making learning more accessible and engaging. Ultimately, this integration of real analysis into secondary-level teaching enriches the overall mathematical experience for students.

### ***Enhancing Teachers' Proficiency***

Teachers need pedagogical, content, and technological proficiency in their related subjects for practical instruction and professional life. By considering real analysis as a source of various knowledge and skills for being a mathematics teacher, only content-based knowledge and skills that enhance the teachers' proficiency are considered.

**Mathematical Proficiency.** Conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition are all components of mathematical proficiency (National Research Council, 2001). Comprehending mathematical concepts, operations, and relations is a form of conceptual understanding; procedural fluency is the ability to carry out procedures flexibly, accurately, efficiently, and appropriately. The ability to formulate, represent, and solve mathematical problems is a key component of strategic competence. Adaptive reasoning encompasses thinking logically, reflecting, explaining, and justifying one's thoughts. A productive disposition includes a habitual tendency to see mathematics as sensible, practical, and worthwhile and faith in one's diligence and efficacy (National Research Council, 2001). These components collectively ensure a well-rounded mathematical ability, enabling learners to understand, apply, and reason through mathematical concepts effectively. They help students develop problem-solving skills, logical thinking, and a positive attitude toward mathematics. Strengthening these areas enhances mathematical learning and application in real-life contexts. Ultimately, mathematical proficiency supports deeper understanding, critical thinking, and long-term academic success.

**Developing Confidence and Professionalism.** Teachers are encouraged to continually enhance their ability to apply their knowledge of subject matter, learning and teaching theories, student characteristics, and workplace contexts, alongside developing their classroom skills. Teachers require confidence in their subjects. The teachers' teaching skills and knowledge of mathematics, which often surpasses what they teach in school, make them confident. Knowledge of real analysis is beneficial for aspiring teachers to develop professional confidence. Real analytical knowledge

enhances a mathematics teacher's worth in the professional community, building the teacher's self-confidence.

Another key element of teacher development is professionalism and self-awareness. The teacher's professional identity and confidence are linked to their professional, pedagogical, and content knowledge (Dalby, 2017). Knowledge of real analysis contributes to the development of content knowledge, as it encompasses core theoretical information on the attributes of the real number system. The teachers' confidence in the content and professional value, gained through knowledge of real analysis, demonstrates the relevance of real analysis knowledge for secondary-level mathematics instruction. The contribution of the mathematical knowledge of real analysis to the development of mathematical proficiency is considered in terms of its relevance for secondary-level mathematics instruction.

### ***Teaching Efficacy***

Teaching efficacy refers to a teacher's belief in their capability to effectively plan, organize, and deliver instruction that leads to meaningful student learning (Bandura, 1997). In mathematics education, this includes the ability to create instructional tasks that foster conceptual understanding (mathematical creation) (Kilpatrick et al., 2015), prepare clear and strategic steps for solving problems (algorithms) (National Research Council, 2005), connect advanced mathematical content with school-level curriculum (content bridging) (Ball et al., 2008), and influence students' thinking through logical reasoning and justification (persuasion). Together, these dimensions reflect the depth and scope of teaching efficacy in supporting effective mathematics instruction.

## Research Gap

The literature review reveals a consistent and central focus in mathematics education: the nature and depth of mathematical knowledge essential for effective teaching at the secondary level. Numerous studies have investigated the foundational mathematical background required for secondary mathematics teachers, aiming to pinpoint the specific domains of knowledge that underpin both deep conceptual understanding and pedagogical proficiency.

The prior research emphasizing the importance of mathematical content knowledge for teaching (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005), the specific role of real analysis in enhancing instructional efficacy at the secondary-level remains underexplored particularly in the context of teacher preparation programs in developing countries such as Nepal. Various research findings explore the importance of advanced mathematics, such as real analysis, in teacher preparation programs (Conference Board of the Mathematical Sciences, 2012; Wasserman et al., 2017). Despite this view, questions persist regarding how much abstract and formal mathematical domains, especially real analysis, translate into practical classroom utility.

The disconnection between theoretical content in teacher preparation programs in university education and pedagogical practices in secondary classrooms has been a long-standing concern (Zazkis & Leikin, 2010). Scholars such as Monk (1994) and Wasserman and Weber (2017a) have drawn attention to the nuanced distinction between knowing mathematics and knowing how to teach mathematics. Moreover, although real analysis is a foundational component of mathematics curricula in mathematics teacher preparation programs (Bartle & Sherbert, 2011), its direct application in secondary-level instruction is not evident to many pre-service and in-

service teachers. This perception fosters a belief that real analysis is esoteric, overly abstract, and removed from the realities of school mathematics.

While evidences of prior research findings from contexts such as the United States (e.g., Patterson, 2019; Wasserman, 2017) and Europe (e.g., Dreher et al., 2018) suggests that real analysis can contribute to the development of pedagogical content knowledge and logical reasoning, there is a scarcity of studies systematically examining this relationship in the South Asian context, particularly Nepal. The existing literature often focuses on general mathematical knowledge for teaching (Ball et al., 2008; Shulman, 1986a), without isolating the contributions of specific domains, such as real analysis.

Furthermore, although frameworks such as Mathematical Knowledge for Teaching (MKT) and the MUST (Mathematical Understanding for Secondary Teachers) model (Kilpatrick et al., 2015) provide theoretical underpinnings for teacher knowledge structures, there remains limited empirical work assessing how knowledge of real analysis contributes to dimensions such as conceptual understanding, procedural fluency, and teaching efficacy within secondary-level contexts.

Teachers' attitudes toward the utility of real analysis, their capacity to create content linkages with secondary school curricula, and their ability to translate this knowledge into meaningful pedagogical practices remain largely undocumented in the Nepalese context. While anecdotal evidence and isolated classroom experiences suggest the latent potential of real analysis for enhancing logical reasoning and abstraction (Bhupal, 2021), no systematic study has examined whether such knowledge substantially improves instructional practices and professional efficacy among secondary-level mathematics teachers in Nepal.

Thus, this study fills a critical gap by evaluating knowledge of real analysis 's perceived and actual relevance in secondary-level mathematics instruction through an explanatory sequential mixed-methods research design. It seeks to determine whether real analysis enhances teachers' content connection, pedagogical application, and teaching efficacy, offering empirical insights into the curriculum's effectiveness. This research addresses a neglected but essential area in mathematics education literature by situating the inquiry within the broader framework of teacher education and curriculum relevance.

### **Justification of the Study**

The integration of pedagogical content knowledge such as knowledge of content and students, knowledge of curriculum, and knowledge of content and teaching alongside mathematical content knowledge, including horizontal, vertical, and specific content knowledge, has been strongly advocated in various theoretical frameworks for secondary-level mathematics instruction (e.g., Ball et al., 2008; Baumert et al., 2013; Ernest, 1989; O'Meara, 2010). These frameworks suggest that advanced mathematics should be included as a component of the specialized subject knowledge necessary to train prospective teachers. However, teacher preparation programs lack a clear and evidence-based justification for the inclusion of advanced mathematics, such as real analysis in curriculum.

Educators widely support the inclusion of advanced mathematics content in teacher education programs. It is essential to equip prospective teachers with a deeper conceptual foundation and opportunities for professional growth. Therefore, a critical examination of the value, rationality, and relevance of each advanced mathematical discipline within the teacher preparation curriculum. It is essential to determine its contribution to effective secondary-level mathematics instruction. Furthermore,

ongoing research is necessary to align the pedagogical approaches used in these programs with the practical needs of prospective teachers, ensuring their preparedness for real classroom challenges.

Among the various branches of advanced mathematics, real analysis is commonly included in university-level mathematics teacher preparation programs. However, the relevance and rational justification for its inclusion, particularly in the context of secondary school teaching, remain underexplored, both globally and within the context of Nepal. This lack of inquiry presents a clear research opportunity to investigate how knowledge of real analysis contributes to the instructional practices, pedagogical reasoning, and professional development of secondary-level mathematics teachers.

Several curriculum evaluation models, such as those proposed by Tyler (1949), Bradley (1985), Stufflebeam (2003), and Kirkpatrick (1996), provide theoretical tools for examining program effectiveness. Among these, Kirkpatrick's four-level model offers a particularly suitable framework for evaluating the applicability and impact of teacher preparation programs and their curricular components. The key questions that emerge from the literature concern the rationale for teaching real analysis in teacher education programs and the extent to which it benefits prospective teachers in their professional practice. Addressing these questions is essential to assess the instructional relevance of real analysis at the secondary level and to inform evidence-based improvements in mathematics teacher education curricula.

### **Chapter Summary**

The literature review examines theories, research papers, dissertations, and conference materials concerning the role of advanced mathematics, particularly real

analysis, in secondary-level mathematics instruction. It includes literature related to mathematical knowledge for teaching, the role of real analysis, and its impact on mathematical proficiency and teaching efficacy to enhance understanding of theoretical aspects. The review discusses educational theories relevant to teacher preparation and curriculum relevance, emphasizing the application of advanced mathematics, especially real analysis, in secondary instruction. Various models of teacher knowledge highlight subject-matter expertise, pedagogical content knowledge, and professional skills. The relevance of real analysis is explained by examining its role in teacher preparation, content knowledge, and pedagogical application, stressing the importance of mathematical proficiency and contextualized activities. Different forms of advanced mathematical knowledge, such as axiomatic, logical, and inferential knowledge, contribute to instruction. The study applies an evaluation model to assess the relevance of real analysis knowledge, focusing on teachers' perceptions, knowledge gained, instructional applicability, and impact on teaching efficacy and student achievement. Previous studies present mixed perspectives some support its role in developing pedagogical skills and problem-solving abilities, while others question its direct application in secondary education. Despite varied views, real analysis remains a core subject in teacher education.

## **Chapter 3**

### **Research Methodology**

This chapter presents the methodological procedures employed in this study, as well as the approach used to address the research questions established in Chapter 1. The research design justifies the chosen methodology and demonstrates that it is suitable to meet the purpose of the study. It also validates the chosen methodology and demonstrates that it is suitable for achieving the study's objectives. It elaborates on the data collection instruments and methods employed, as well as their reliability and validity in capturing relevant information. This chapter also outlines the data analysis tools and techniques. Finally, it presents the ethical considerations.

#### **Philosophical Considerations**

The study aims to investigate the relevance of real analysis for secondary-level mathematics instruction. The relevance of real analysis is connected to the teachers' mental processes, which guide their teaching behavior, the application of gained knowledge, logical fluency, and the ability to provide immediate responses to students' queries, all of which are complex cognitive processes. Therefore, the phenomenon under consideration is complex and a base case of internal and external reality. The pragmatic research paradigm is helpful when a phenomenon is external, internal, and complicated (GHR & Aithal, 2022). Hence, this study was conducted in accordance with the guidelines of the pragmatic paradigm.

The philosophical underpinnings of this study involve key components such as ontology, epistemology, axiology, methodology, and rhetorical stance.

#### ***Ontology***

Ontology is concerned with reality. Pragmatism points multiple realities, subjective and objective. Reality depends on the context to exist and continue

existing, which means that a change in context alters context changes (Maarouf, 2019). In this study, researchers had a positionality that the relevance of mathematical knowledge to real-world analysis reveals two realities. Mathematics teachers' attitudes toward the relevance of mathematical knowledge in real analysis are measurable and, to some extent, can be objectively measured with scientific instruments. It is real and stable to a certain extent. Additionally, the content connection between knowledge of real analysis and school mathematics is objective. However, their practices differ across contexts and individuals. The teachers' practices of applying knowledge of real analysis and their attitude about what they get from real analysis are subjective.

Regarding this study, it is believed that there is a single reality regarding teachers' attitudes toward the relevance of mathematical knowledge in real analysis. However, their perception of its utilization, practices, and essentiality is subjective.

### ***Epistemology***

The epistemological viewpoint of pragmatism is two-sided, encompassing both observable and unobservable knowledge. The knowledge-gaining process is based on its ontological position (Maarouf, 2019). Pragmatic epistemology holds that attitude is shaped by social experience and that knowledge is invariably grounded in experience. Qualitative and quantitative approaches require actionable inquiry to gain proper knowledge (King, 2022). It is expected that a survey tool can be used to examine teachers' attitudes toward the pedagogical application of knowledge of real analysis and mathematical knowledge in secondary mathematics instruction, the content connection, and its contribution to improving teachers' professional development. Additionally, the content connection can be discovered by curriculum analysis, and classroom observation and task-based interviews can reveal the truth of the pedagogical application of real analysis.

Therefore, during the study, it was believed that the reality of teachers' attitudes could be objectively known using survey tools. The teachers' perceptions and practices regarding their knowledge of real analysis during instruction can be observed through classroom observations and task-based interviews.

### ***Axiology***

Axiology concerns the role of values in the research process. In this mixed-methods study, values significantly influenced the evaluation and interpretation of results. The researcher adopted objective and subjective stances, recognizing that each perspective contributes meaningfully to understanding the research questions (Maarouf, 2019). In the quantitative phase, a value-free position was maintained to objectively measure teachers' attitudes toward the relevance of knowledge of real analysis. However, in the qualitative phase, the researcher acknowledged the impossibility of complete detachment from their values and interpretations (King, 2022). Engagement in textbook analysis, classroom observation, task-based interviews, and qualitative data interpretation involved a subjective and contextually informed stance.

While the quantitative phase provided structured data, it offered limited insight into the reality of teachers' lived experiences and instructional practices. The qualitative phase enabled a richer, more nuanced interpretation through the researcher's contextual engagement. Thus, the researcher's positionality during the qualitative inquiry was value-laden, reflecting the interpretive and meaning-making nature of qualitative research.

### **Research Method**

The pragmatic research paradigm permits using both quantitative and qualitative research methods, enabling a comprehensive understanding of the

phenomena studied (Creswell, 2014; Gunasekara, 2015). Researcher used a Likert-type scale for a survey, curriculum analysis, classroom observation, and task-based interviews for qualitative inquiry. Thus, a mixed method was used under the pragmatic research paradigm.

### **Rhetoric**

In research, language plays a crucial role in conveying the research message. Rhetoric refers to the art of language. The use of language in research writing may be formal or informal. It explains how researchers use language in research writing to present, argue, or persuade. This includes formal and informal tones, depending on the research context and design. Formal language is often employed to maintain objectivity, credibility, and scientific rigor, especially when presenting data, discussing methodologies, or drawing conclusions (Creswell, 2015). Moreover, Creswell (2015) emphasizes that while quantitative results are typically presented formally, qualitative data require a more narrative style to convey participants' voices and experiences effectively. Therefore, the research employs the third person in the quantitative phase and the first person (I) in the qualitative phase, with a mix of both in the writing of the findings.

### **Research Design**

The purpose of the study required multiple perspectives to ensure reliable and valid findings. A problem can be studied in-depth using a mixed-methods research approach, which enables the observation of events from multiple angles and integrates and synergizes diverse sources (Creswell, 2015; Maxwell, 2016; Poth & Munce, 2020; Shorten & Smith, 2017). The study investigated how secondary mathematics instruction corresponds to knowledge of real analysis. This was accomplished using the Kirkpatrick approach, in which the primary assessment areas were teachers'

responses to real analysis, what they learned from it, and how they applied what they had learned to their secondary school mathematics education. A mixed-methods research design is appropriate since it involves a complex phenomenon that can be studied using various methods.

Six types of mixed-method research designs are found to be practised. Convergent parallel design, explanatory sequential research design, exploratory sequential design, embedded design, transformative design, and multiphase design are the major types of mixed-methods research designs (Creswell, 2015). Convergent design involves collecting two data streams independently and then comparing and contrasting the results obtained from both sources. This enables a more comprehensive and differentiated understanding of the research problem than is possible with either approach alone. The second type of mixed-methods research design is the explanatory sequential research design. In this design, the researcher first collects quantitative data, analyzes it, and then performs qualitative data collection to explain or extend the quantitative results. The third type is the sequential exploratory research design. The researcher first collects qualitative data, analyzes it, and then collects quantitative data to examine or confirm the qualitative results in the exploratory sequential research design. The fourth type is the simultaneous triangulation design, in which qualitative and quantitative data are analyzed separately and the results are combined. The fifth type is concurrent, which uses one data type to support the other data. Transformative design is one of the six kinds of designs under mixed-method design. In this design, the research is designed to create change in the social context being studied. The explanatory sequential research design aims to create change within the social context being studied. This study was conducted using the explanatory sequential design as part of a mixed methods approach.

## **Justification of Methodology**

This study employed an explanatory sequential mixed-methods design to investigate the relevance of real analysis knowledge in mathematics instruction. The initial quantitative findings revealed both theoretically expected and unexpected results. While teachers expressed a positive attitude toward real analysis knowledge, the hypothesized relationships between content connection and both mathematical proficiency and teaching efficacy, as well as between pedagogical application and teaching efficacy, were not supported. To explore the underlying reasons for these inconsistencies, a qualitative follow-up inquiry was essential. Explanatory mixed methods designs are particularly effective in clarifying unexpected quantitative results by examining participants' experiences and contextual interpretations through qualitative means (Creswell, 2023; Fetters et al., 2013; Plano Clark & Ivankova, 2016). Therefore, the integration of qualitative data enabled a deeper, more contextualized understanding of teacher perspectives and instructional practices.

Once the quantitative research findings are known, the study necessitates a qualitative inquiry to explore how teachers articulate and apply mathematical knowledge of real analysis in their teaching. This phase uncovers deeper insights into what teachers gain from real analysis and how they connect it with secondary-level mathematics. Qualitative methods, such as interviews, enable a more thorough understanding of teachers' experiences and the contextual application of real analysis in their practice. Therefore, a sequential mixed-methods research design is particularly suitable for addressing the study's objectives. The sequential design allows for the first collection and analysis of quantitative data, followed by qualitative data that builds upon and expands the initial findings (Tashakkori & Teddlie, 2010). Adopting a pragmatic research paradigm, the study effectively integrates quantitative

and qualitative methods, focusing on practical outcomes and real-world applications (Johnson & Onwuegbuzie, 2004). This ensures that the research findings are actionable and rooted in the actual experiences of educators, thereby enhancing the relevance and impact of the study (Creswell & Clark, 2018). Hence, the explanatory mixed methods research design is a justifiable methodology for this study.

### ***Explanatory Sequential Research Design***

There are different types of sequential designs under mixed-methods design. Within mixed-methods research design, two types of sequential designs are commonly practiced: exploratory and explanatory sequential research designs (Creswell & Clark, 2018). Explanatory sequential research design is appropriate for studies focused on defining variables to be measured and identifying the phases of the research process (Creswell, 2015). This study identified the relevance of knowledge of real analysis for secondary-level mathematics instruction. Relevance indicators were examined regarding content connection, pedagogical application, enhancement of teacher professional development, and their impacts on teaching efficacy. The variables were identified based on the guidelines of previous studies (e.g., Fukawa et al., 2019; Stockton & Wasserman, 2017; Wasserman et al., 2017; Wasserman & Weber, 2017a; Patterson, 2019; Patterson, 2021). Therefore, the explanatory sequential research design is appropriate for this study.

**Phases of Explanatory Sequential Research Design.** First quantitative survey method was used then qualitative study followed y qualitative study was used. The adopted phases were follows.

**Phase I.** The initial phase of the study involved collecting quantitative data by administering a survey to secondary-level mathematics teachers. The survey aimed to evaluate the teachers' attitudes regarding the relevance of mathematical knowledge of

real analysis for teaching secondary-level mathematics in three main areas: content connection, pedagogical application, and professional development enhancement.

The first phase of the study was quantitative. The survey was conducted in the initial stage. The survey questionnaire was administered to secondary-level mathematics teachers in Gandaki province. The study aimed to identify secondary-level mathematics teachers' attitudes toward accessing the content connection between pedagogical application and real analysis and enhancing mathematical proficiency through mathematical knowledge of real analysis. A survey questionnaire was used to meet this aim.

The content connection refers to the extent to which mathematical knowledge of real analysis is relevant and applicable to the mathematics curriculum taught at the secondary level. Pedagogical application refers to how knowledge of real analysis can improve teaching methods and techniques at the secondary level. Professional development enhancement refers to the degree to which real analysis of mathematical knowledge can enhance the professional development of secondary-level mathematics teachers.

Descriptive and inferential statistics were used to analyze the data collected from the survey questionnaire. In the quantitative data analysis phase, descriptive statistics were computed to summarize the dataset, including frequency, percentage, mean, and standard deviation. Inferential statistical tests, such as the t-test and ANOVA, were conducted to test the research hypotheses. Additionally, structural equation modeling (SEM) was employed to assess the relationships between variables and evaluate the reliability and validity of the measurement model. Statistical analyses were performed using SPSS for descriptive statistics and hypothesis testing, while SmartPLS 4 was utilized for measurement model assessment and structural equation

modeling. The details of these analytical procedures are discussed in the relevant sections below.

**Phase II.** The second phase is qualitative research, following the survey results. The curriculum analysis was conducted using the framework prepared by the researcher (shown in Appendix B). Classroom observations were conducted with mathematics teachers using the classroom observation protocol (Appendix C) to identify how teachers apply mathematical knowledge of real analysis during instruction.

A task-based interview, including follow-up questions based on the task and related questions aligned with research questions, was used by teachers to understand their practices and their application of real analysis knowledge in instruction. That was conducted by using the task-based interview protocol (Appendix D). An interview was conducted with a mathematics educator to understand their views on the relevance of knowledge of real analysis for instruction. For this purpose, interview guidelines (Appendix E) were used.

Thematic analysis is a widely used method in qualitative research that aims to identify, analyze, and report patterns or themes within the data. This approach involves systematically identifying patterns within the dataset and interpreting these patterns to generate meaningful insights.

Conducting a thematic analysis involves several steps. First, the researcher must become familiar with the data by reading and re-reading the material multiple times to identify recurring patterns, ideas, or themes. Then, the researcher codes the data, assigning labels or tags to different data segments that represent different themes or categories. These codes can be created manually or with the help of software such as QDAMinor 2024. Researcher used this software for qualitative data analysis.

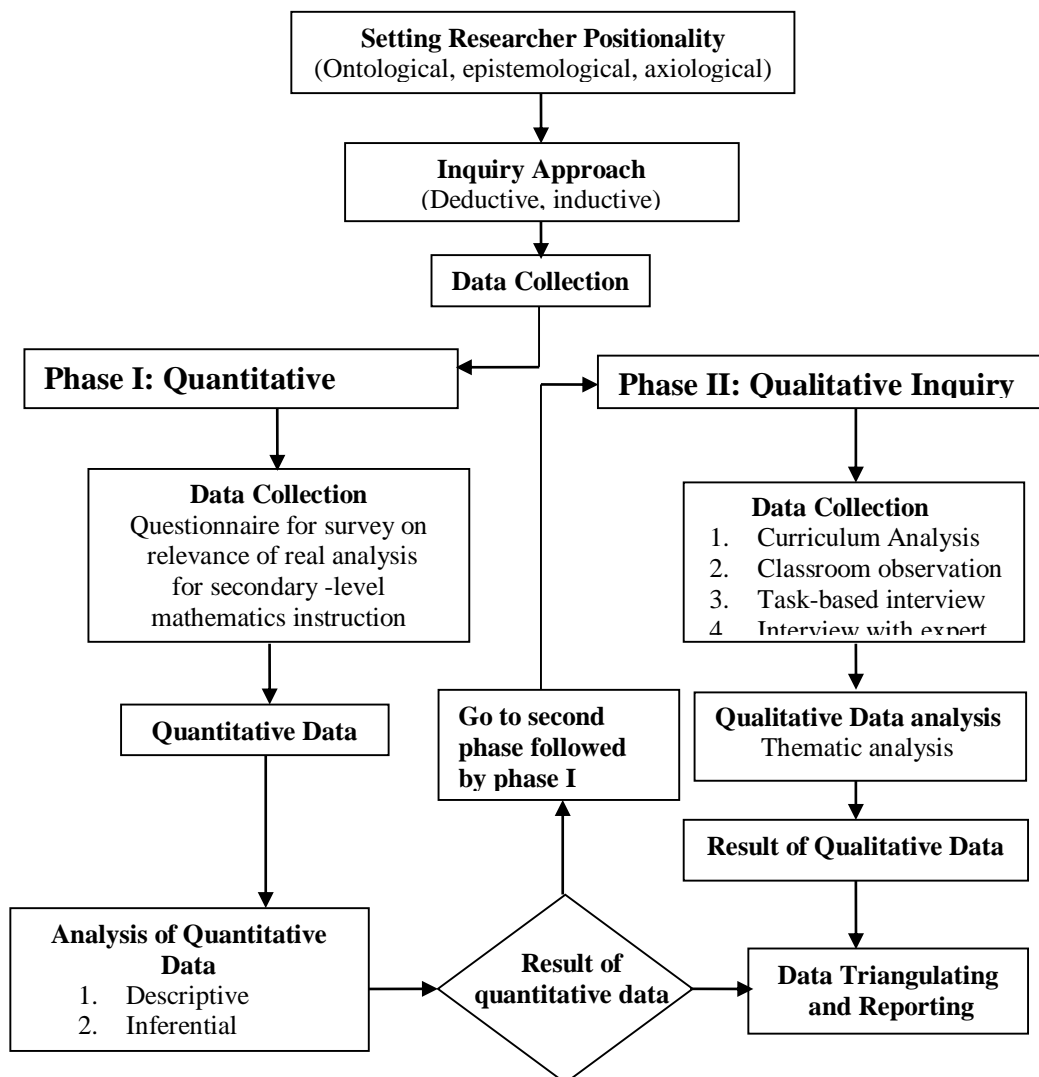
Next, the researcher groups related codes into larger themes that capture the essence of the data. Themes can be identified through inductive or deductive approaches. Inductive coding involves starting with the data and generating themes from the bottom up. In contrast, deductive coding involves starting with a pre-existing theory or framework and testing it against the data.

After identifying the themes, the researcher analyzes and interprets the data to better understand the research question or problem. This interpretation involves synthesizing the themes into a coherent narrative or explanation that captures the essence of the data.

### ***Workflow of Explanatory Sequential Research Design***

The workflow diagram aligns with the overall research design and serves as a strategic roadmap for implementing this study. As illustrated in Figure 8, the workflow outlines the systematic steps involved in conducting explanatory sequential mixed methods research, which was selected to investigate the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction. This research design is particularly well-suited to the study's aims, as it enables the researcher first to quantify teachers' attitudes and perceptions, and then explore those findings in greater depth through qualitative methods.

The workflow diagram thus operationalizes the methodological framework, detailing the phases of sampling, data collection, instrument development, analysis techniques, and integration of results. It serves not only as a visual representation of the research logic but also as a procedural guide that ensured methodological coherence throughout the study. A detailed depiction of this process is presented in Figure 8.

**Figure 8***Methodological Flowchart of Explanatory Mixed Method*

In this study, the quantitative phase preceded the qualitative phase, allowing the initial statistical analysis to inform the development of interview and observation tools. This sequential structure enhances the validity of the findings by explaining the quantitative findings through qualitative findings.

### **Selection of Study Area**

The target respondent group for this study is secondary-level mathematics teachers. Therefore, any geographic area of Nepal with secondary schools can be selected for the study. Gandaki province includes diverse ecological areas such as the

Himali area (Mustang, Manang), the Hilly areas (Baglung, Parbat, Kaski, etc), and the Tarai area (Navalpur) of Nepal. Therefore, Gandaki province was selected to conduct this study. There are eleven districts: Navalpur, Gorkha, Syangja, Tanahun, Lamjung, Kaski, Manang, Parbat, Myagdi, Mustang, and Baglung (Central Bureau of Statistics, 2018), which fall within the study area. The survey was conducted by distributing survey tools to secondary-level mathematics teachers.

Following the completion of the survey, a qualitative inquiry was conducted to further address the research objectives. Baglung Municipality in the Baglung district was selected as the site for this study phase. Given the potential need for frequent interactions with respondents to gain deeper insights, Baglung Municipality was selected due to its accessibility and suitability for ongoing engagement.

### **Population and Sample**

This study selected secondary-level mathematics teachers of Nepal as the universe of this study. The population of this study includes all secondary-level mathematics teachers teaching mathematics at grades 9 and 10 in the Gandaki province of Nepal. There are 1,332 secondary schools in the Gandaki Province of Nepal (Ministry of Education, Science, and Technology EMIS Report, 2019-2020; 2021). Each secondary school has at least one mathematics teacher appointed to teach mathematics. Hence, at least 1332 mathematics teachers are in the secondary schools of Gandaki province of Nepal.

Thus, this study's total population is 1332. William Cochran provided the following formula to determine the sample size (Cochran, 1997).

$$\text{Sample size } n = \frac{N \left\{ \frac{Z^2 \times P(1-P)}{e^2} \right\}}{(N-1) + \left\{ \frac{Z^2 \times P(1-P)}{e^2} \right\}}$$

Where  $N$ = population size,

$Z$ = critical value of normal distribution at a required confidence level,

$P= 0.5$ = proportion of key indicator to be estimated,  $n$  = sample size.

This formula is also embedded in the Raosoft online calculator which gives the sample size from the population. The minimum sample size of 1332 is 299, calculated from the Raosoft online software. Thus, the minimum sample size required for this study is 299, assuming no design effect. However, to enhance statistical precision and account for the design effect associated with proportionate stratified random sampling, the target sample was adjusted. In educational and social science research, a design effect of 1.2 is widely used (Lohr, 2010; Valliant et al., 2018). Therefore, a design effect of 1.2 was applied. Now, design adjusted sample size  $n_{adjusted} = 299 \times 1.2 = 358.8 \approx 359$ . Thus, the design-adjusted sample size was approximately 359. Baruch and Holton (2008) identified a non-response rate of 52.7%, while Sidiropoulou et al. (2022) reported a 34% non-response rate among teacher respondents. To account for non-response and non-contact risks, this study increased the sample size by 50% above the required sample size of 389 (adjusted for design effect), resulting in 539 teachers being surveyed. Among them, 387 valid responses were collected, yielding an actual non-response rate of 28.2%. The details of the sampling structure are shown in Table 2. This adjustment ensured adequate statistical power, improved representativeness, and compensated for the slight increase in variance introduced by the sampling design, thereby supporting the generalizability of the study findings.

There are 146 secondary schools in the Baglung district and 14 in the Baglung municipality (Ministry of Education EMIS Report, 2019-2020; 2021). Ten secondary schools and mathematics teachers were selected using purposive and convenience

sampling, and three mathematics educators were selected (the details are in the sampling procedure) for qualitative inquiry purposes.

### Sampling Procedure

Different sampling methods are available to select the sample from the population. Probability sampling and non-probability sampling are often used. Simple, stratified, cluster, and systematic random sampling can be used when the population is well-defined. Stratified sampling is suitable when strata are well-defined.

**Table 2**

#### *Status of Sampling*

District	Total Schools	Minimum sample size	Sample size covering the Design effect	sample size with design effect and risk coverage estimate	Received number
Gorkaha	136	31	37	55	44
Lamjung	98	22	26	40	31
Tahanun	180	40	49	73	49
Syanja	169	38	46	68	49
Kaski	273	61	74	110	74
Manang	4	1	1	2	1
Mustang	13	3	4	5	4
Myagdi	77	17	21	31	26
Parbat	90	20	24	36	26
Baglung	146	33	39	59	43
Navalpur	146	33	39	59	40
Total	1332	299	359	539	387

In this study, strata are also definable, so stratified random sampling was used. By keeping 11 districts (shown in Table 2) as strata, the proportionate stratified random sampling technique was utilized to meet the sample size because it gives a more reliable and representative sample from the population (Etikan, 2017). Hence, the researcher used proportionate stratified random sampling to select a sample.

Each stratum's sample size was declared using the proportionate stratified random sampling method. First, the total number of schools in Gandaki province was identified, and then a list of secondary schools in each district was prepared.

After that, the sample size for each stratum was declared. Table 2 shows the status of the research sample. The study also covers qualitative inquiry. The different sample sizes have been suggested as a saturated model for qualitative research.

No specific rule is found for sample size in qualitative research. However, a sample size of 2-30 can be taken in qualitative research (Creswell, 2015).

Additionally, Hennink and Kaiser (2022) concluded in their meta-review on sample size in qualitative research that 9-7 participants were needed for an interview and 4-6 participants were needed for a focus group discussion. Small sample sizes and convenience sampling are often employed in educational research involving frequent classroom observations, as they facilitate detailed data collection from readily accessible participants (Gliner et al., 2017). Patton (2002) also supports convenience sampling in qualitative studies, emphasizing its practicality for collecting in-depth insights from a limited number of participants. This study used multiple data collection methods (classroom observation and task-based interview from the same teacher) and frequent visits to the teacher. Therefore, non-probability sampling was used to select the sample for qualitative research. Hence, Baglung municipality was selected purposively and conveniently. Ten mathematics teachers from the municipality were selected purposively and conveniently among 24 secondary schools to ensure frequent visits for classroom observation and task-based interviews, aiming to collect in-depth information. Of the 10, only eight teachers accepted my request to participate in this research.

The study focused on the relevance of mathematical knowledge of real analysis for instruction. Mathematics educators' views were required to assess the relevance of mathematical knowledge of real analysis. For this purpose, mathematics educators engage in teaching real analysis, curriculum development, and publication on real analysis-related work, considering the criteria to select mathematics educators. Five mathematics educators were requested to participate in the interview based on these criteria. However, only three mathematics educators accepted my request, and the researcher finally interviewed three mathematics educators.

### **Quantitative Data Collection Tools**

This study employed a mixed-methods approach, utilizing an explanatory sequential research design. The research objective demanded the integration of both quantitative and qualitative methodologies. Accordingly, tools and techniques from both quantitative and qualitative research paradigms were utilized to ensure comprehensive data collection and analysis.

### ***Likert Scale for Attitude Measurement***

The first phase of this research involves the collection and analysis of quantitative data. The study identified the relevance of mathematical knowledge to real analysis. The first indicator assesses the mathematics teachers' attitudes towards mathematical knowledge of real analysis across content connection and pedagogical application. Another indicator is examining the attitude towards mathematical knowledge of real analysis, which contributes to enhancing mathematical proficiency and teaching efficacy. To meet this requirement, mathematics teachers' attitudes towards mathematical knowledge of real analysis in terms of its connection with secondary level mathematics, its pedagogical application, and its contribution to enhancing mathematical proficiency were assessed. Thus, there are four variables:

teachers' attitude towards the content connection, pedagogical application, and attitude towards the role of knowledge of real analysis for enhancing mathematical proficiency.

The Likert scale and Likert-type survey scale are survey instruments used to measure qualitative or subjective phenomena. This study aims to measure mathematics teachers' attitudes toward the relevance of mathematical knowledge of real analysis for secondary-level mathematics teaching. The Likert scale measures subjective attributes, especially psychological constructs (Beglar & Nemoto, 2014; South et al., 2022). The mathematics teachers' attitude toward the relevance of mathematical knowledge in real analysis is also a psychological construct; therefore, the Likert scale is suitable for measuring this construct.

The self-constructed five-point Likert scale was utilized to measure the attitudes of secondary-level mathematics teachers regarding the relevance of real analysis. The attitude measurement scale consists of the domains: attitudes towards content connection between knowledge of real analysis and secondary-level mathematics, attitude towards pedagogical application of knowledge of real analysis, attitude towards real analysis helps to enhance mathematical proficiency and teaching efficacy. A five-point Likert-type scale was constructed to measure teachers' attitudes across content connection, pedagogical application, and enhancing mathematical proficiency (see Appendix A). The content connection domain consists of 15 items. The pedagogical application consists of 12 items, and enhancing mathematical proficiency consists of 17 items.

### ***Teaching Efficacy Measurement Scale***

The Teaching Efficacy Scale was developed to measure mathematics teachers' teaching efficacy. A self-rating model was used to create the scale comprising 10

items. Teachers can rate their teaching efficacy on a five-point scale, with ratings ranging from one to five. A one rating signifies minimal teaching efficacy, indicating that the ability to teach mathematics effectively is limited. In contrast, a rating of five represents exceptional teaching efficacy, where teachers perceive themselves as highly capable of delivering quality mathematics instruction. The indicators embedded in the scale provide clear definitions for each rating point, helping teachers self-assess with clarity and consistency. The item structure is shown in Table 3.

### ***Pilot Survey***

The reliability and validity of the research instrument are fundamental to the reliability and validity of the research results. The pilot survey was conducted to ensure its reliability and validity, as suggested by different scholars. A sample size of 30 is recommended to test the reliability of the questionnaire (Bujang et al., 2024). Omar et al. (2017) utilized a total sample size of 24; however, Lewis et al. (2012) suggested a sample size of 10-40 for the pilot study. Additionally, 12 samples are suggested by Julious (2005). Hence, 30 respondents participated in the pilot study. However, their responses to the pilot survey were not included in the primary research analysis.

The five-point Likert-type survey scale assessed secondary-level mathematics teachers' attitudes toward the relevance of real analysis mathematical knowledge by clarifying the domains, content connections, pedagogical applications, and knowledge of real analysis to enhance mathematical proficiency. The teachers' mathematics teaching efficacy scale was also developed. The attitude measurement survey tools were crafted based on the literature and theory from the Kirkpatrick and Kirkpatrick (2014) training evaluation model. This model provided a structured framework to assess changes in participants' reactions, learning, behavior, and results.

**Table 3***Nature of Scale*

Category	Domains	Number of items	Response Nature
Attitude-based relevance measuring scale.	Content Connection	15 (1-15)	SA= Strongly agree, A= Agree, N= Neutral, D= Disagree, SD= strongly disagree
	Pedagogical Application	12(1-12)	
	Enhancing mathematical proficiency	17(1-17)	
Efficacy Measurement Scale	Creation	3(1-3)	Self-rating on teaching efficacy based on 1 representing low teaching efficacy and 5 being perfect teaching efficacy by writing a numerical value from 1 to 5
	algorithm	4(4-7)	
	Content bridging	3 (8-10)	

The attitude measurement scale was developed using a five-point Likert-type scale, while the mathematics teaching efficacy scale followed a self-rating scheme ranging from 1 to 5. Table 3 presents the overall structure of the questionnaire used in this study and provides detailed information about the survey tools. Some further explanation or expansion may be required to clarify specific aspects of the instruments used. The draft of the scale items was prepared and subsequently submitted to experts for the face validity check.

Expert suggestions were collected, and corrections were implemented accordingly. This iteration process was repeated five times to modify and improve the items and total scale. The structure and nature of the scale are shown in Table 3.

***Content Validity of Survey Scale***

Content validity refers to the extent to which a research tool accurately represents the entire range of concepts it aims to measure, ensuring that it comprehensively covers all aspects of the construct while excluding irrelevant elements. To ensure content validity, expert judgement, and calculation of the content validity index were employed. Five experts were requested to assess its content validity. The scale was distributed to the expert via Google Forms, asking them to rating their responses, 1= item is not relevant to measure the domain, 2= item is

somewhat relevant to measure the domain, 3= item is quite relevant to measure the domain, and 4 = item is highly relevant to measure the domain. Only two experts responded with their suggestions and evaluation.

**Table 4**

*Content Validity Index of Relevance Measurement Scale*

Part	Domains	Indicators		
		SCVI-Average based on expert	SCVI –Average based on item	S-CVI/UA
Relevance	Content Connection	0.96	0.97	0.93
	Pedagogical Application	1	1	1
	Proficiency development	0.97	0.97	1
Efficacy for teaching	Creation	1	1	1
	Algorithm thought	1	1	1
	Bridging	1	1	1

Two methods were used to ensure the validity of the survey tool. Content validity was ensured by using expert judgment. After collecting the experts' responses, the content validity index (CVI) was computed using the method suggested by Yusoff (2019). The content validity index was calculated to check the extent of content validity. The content validity index value needs to be above 0.80 (Davis, 1992). The computed value crosses the threshold suggested by Davis (200), as shown in Table 4. Hence, content validity is suitable for measuring the target construct.

***Construct Validity of Survey Scale***

Construct validity refers to the measurement tools' ability to accurately measure theoretical constructs. The construct validity of survey tools was established based on the pilot study method.

After the content validation, a pilot study was carried out. Construct validity is another important validity type. It refers to the quality of the questionnaire, which measures those constructs that are supposed to be measured. Calculating the correlation between an item's value and its sum is a typical method of determining

construct validity (e.g., Guloksuz et al., 2020; Khadka et al., 2022). To compute the construct validity indicator value, Pearson's correlation between each item and the item total is calculated (Machuca et al., 2015; Suhartini et al., 2021). The items are valid because Pearson's correlation coefficient is greater than the critical value under a 5% significance level (Machuca et al., 2015; Suhartini et al., 2021).

Thus, Pearson's correlation between items and the item total was calculated to assess construct validity. The result showed that the Pearson's correlation between items and item total was significant at a 5% significance level. The values fall between 0.38 and 0.74 except for the item 'The convergent of series is practiced in school mathematics', which correlates 0.36 (The result of statistical output included in Appendix G). However, it was retained because it measures the importance of real analysis, which is also practiced in school mathematics. This range of significance correlation coefficient satisfies the criteria for construct validity suggested by Schober et al. (2018). Hence, the validity of the survey scale was adequate.

### ***Reliability of Survey Scale***

Cronbach's alpha method was used to compute reliability as suggested Aithal and Aithal's (2020). The alpha value for both the total scale and individual items of both scales falls within the recommended threshold range of 0.70-0.95 (. Hence, the reliability of the scale is good. The scale domains are shown in Table 4, and item details are in Appendix A. A pilot study was conducted to ensure the reliability of the research tools before launching the study, using a pilot sample from the study population. Different views can be found about the sample size for a pilot study. Including 10 to 30 participants is reasonable for the pilot study (Hill, 1998; Julious, 2005; Johanson & Brooks, 2010). Considering these suggestions, 30 participants were

chosen for a pilot study. A pilot study was conducted after the expert judgment of the quality of survey items.

The degree of consistency refers to the reliability; additionally, it indicates that the measurement scale measures the same construct (Huck, 2012; Robinson, 2009). The standard method for estimating the reliability of the Likert scale involves calculating the Cronbach's Alpha coefficient (Taherdoost, 2018). Therefore, Cronbach's Alpha coefficient was calculated in the pilot study. The acceptable value of Cronbach's alpha is 0.70 or higher for the measurement tool to be considered reliable (Kite & Whitley, 2018; Robinson, 2009). Therefore, the items of the survey tool with a Cronbach's Alpha coefficient of 0.70 or higher were retained. The calculation was performed using SPSS software (the result is included in Appendix F).

#### ***Survey Data Collection Process***

Contact information for the selected schools was obtained from District Education Coordination Units, municipality and rural municipality education offices, and colleagues. Subsequently, the contact details of mathematics teachers at the secondary level in these schools were collected. The identified teachers were contacted using the provided contact information, with additional support from the Mathematics Education Council members at the district, municipality, and rural municipality levels to facilitate communication.

During the initial contact, teachers were introduced to the study's purpose and solicited for their participation in the survey. Communication channels, including email addresses, Facebook Messenger contacts, and WhatsApp numbers, were gathered. The research scale was distributed electronically using Google Forms through Messenger, email, or WhatsApp. Besides the online data collection, face-to-face data collection was also conducted by distributing hard copies. These were

delivered in person or through alternative means, such as meetings organized by the Mathematics Education Council, teacher training sessions, or designated contact persons.

Data were collected from Tanahun, Kaski, Parbat, Baglung, and Myagdi face-to-face and online. However, online methods were also used to collect data from other districts under consideration. The survey began on January 20, 2024, and finished on May 5, 2024.

To meet the study's aim, the qualitative method, following the results of the quantitative method, was used. The qualitative data collection methods included the mathematical content connection framework, classroom observation, and task-based interviews. The tools and data collection techniques used are described in the following sections.

### ***Mathematical Connection Framework***

A mathematical connection framework was employed to establish the relationship between mathematical knowledge of real analysis and secondary-level mathematics, thereby achieving the research aim. A mathematical connection is a link between one mathematical concept and another. Additionally, the vertical connection between mathematics content has been introduced at different levels of mathematics and mathematics education programs. A mathematical connection refers to the proper relationship between two mathematical ideas, A and B (Businskas, 2008; Gamboa et al., 2022). Hence, any linkage between two or more mathematical concepts in any manner constitutes a mathematical connection.

This study identified the content connections between real analysis and secondary-level school mathematics, as well as the mathematical connections between mathematical concepts and skills at different curriculum levels (Suominen, 2015).

Therefore, a mathematical connection framework (shown in Appendix B) was employed to identify content connections. The framework encompasses categories such as direct connection, axiomatic connection, alternative representation, and others. It was developed to focus on direct content connection, axiomatic connection, logical connection, procedural connection, and alternative representation.

The course content of real analysis is prescribed in the Bachelor of Mathematics Education program at Tribhuvan University and in the secondary mathematics curriculum, both compulsory and optional, published by the Government of Nepal. It was analyzed using the content connection framework to identify the content connection.

While searching for content connections, the curriculum of real analysis and secondary-level mathematics was analyzed to identify the connection from different lenses of analysis. Additionally, secondary-level mathematics textbooks were used to search for connections. The real analysis curriculum contains the topics of the real number system, open and closed sets, real sequences, infinite series, functions and limits, continuity of a function, derivability, and the Riemann integral. The contents related to these topics were examined within the context of the secondary-level mathematics curriculum. Furthermore, the textbook was utilized to identify whether content generalizations and alternative representations in different forms were incorporated within the text. The analysis focuses on direct content connection, axiomatic connection, logical connection, procedural connection, and alternative representation.

### ***Classroom Observation***

Classroom or lesson observation is a technique for determining teachers' articulation of subject matter (Ingram et al., 2018). This observation has different

objectives, like supporting the teacher. However, the purpose of classroom observation in this study is to explore how secondary-level mathematics teachers articulate the mathematical knowledge of real analysis during their instruction.

The objective of classroom observation in this study was to observe how teachers convey the mathematical concepts of real analysis during classroom instruction. To achieve this, a classroom observation protocol (Appendix C) was developed. This protocol focuses on various concepts within real analysis that may be utilized in secondary-level mathematics instruction.

The classroom observation protocol focused on utilizing different concepts of knowledge of real analysis, such as definitions, examples, properties, and field axioms of real numbers, as well as generalizing abstract concepts of real analysis and deriving proof ideas. Additionally, the observation aimed to observe how teachers demonstrate the connection between content and concepts in real analysis during their instruction.

First, the researcher contacted the teachers from the selected school based on convenience and purposive sampling for classroom observation and task-based interviews. Before conducting classroom observations, the head and mathematics teachers teaching at the secondary level were contacted to obtain their consent and permission for the study. After getting consent from the head teacher, the respective subject teachers were consulted to secure their agreement for classroom observation. Following their approval, dates were fixed, and at least four observations per teacher were conducted during their regular, ongoing classes, as per the agreed schedule. The classroom teaching sessions were observed systematically, with data collected through a classroom observation protocol and video recordings. However, three teachers explicitly requested not to be video-recorded, and their preferences were fully respected. Teachers selected for classroom observation and task-based

interviews also participated in a survey as part of the study. The number of classroom observations for each teacher ranged from a minimum of four to a maximum of six. A total of eight teachers participated in the classroom observations and task-based interviews.

### ***Task-Based Interview***

Researchers occasionally conduct studies to understand better the various mathematical techniques that mathematicians and math teachers perform (Weber, Dawkins, et al., 2020). In certain circumstances, researchers are interested in learning how mathematics teachers or mathematicians successfully solve complex problems to gain knowledge that they can apply to teaching their students. Sometimes, mathematics teachers would like to learn how mathematicians carry out their professional practices to inform the design of classrooms where pupils engage in comparable activities (Mejia-Ramos & Weber, 2020). Semi-structured and task-based interviews are appropriate for exploring how mathematics teachers, educators, and mathematicians display their craft professionally (Grinstein & Lipsey, 2014). This methodology helps to investigate how mathematics teachers and mathematicians engage in various activities during their professional work, such as solving mathematical problems and using algorithms (Carlson & Bloom, 2005; DeFranco, 1996). Therefore, a task-based interview is selected to gather data on how teachers characterize the mathematical knowledge of real analysis in the teaching profession.

A task-based interview requires careful planning and preparation to assess the candidate's skills and knowledge effectively. Before the interview, a task-based interview protocol was developed that outlined the objectives, format, and criteria for evaluating the candidate's performance. The task was designed to focus on real analysis of mathematical knowledge directly relevant to teaching secondary-level

mathematics. This enables the researcher to capture how teachers apply theoretical concepts in practical situations and observe their professional practices during instruction.

To ensure that the candidate fully understands the task, a worksheet was provided that outlined the specific task to be performed. The worksheet included relevant instructions, guidelines, and parameters to be followed during the task. Clear instructions were given to the candidate before the interview to ensure that they understood the expectations and requirements of the task. During the interview, detailed notes were taken from observations to record the candidate's performance, including their approach to the task, strategies, and overall performance. After the task, follow-up questions were asked to allow the participant to explain their thought process and justify their practices.

Consent was taken prior to conducting the task-based interview. With the teachers' consent, the time and place for the visits were arranged based on accessibility and their availability. I conducted the visits at times convenient to the teachers and aligned with their schedules. A detailed description of the purpose and task was made before the task-based interview. Tasks were distributed, each task was explained, and the participants were left to do the work. The sheet and an extra copy were given to the participant for making a record. After the task completion, a follow-up interview based on the task was conducted. During the task-based interview, the researcher asked questions related to the task and connected questions about teachers' activities and strategies they applied during instruction. Especially questions related to connection, pedagogical application, linking techniques, and the necessity of real analysis for teachers beyond the task. Second-time follow-up interviews were conducted with three teachers to provide a more detailed explanation of these

practices related to real analysis. The interviews were recorded with the permission of the participant.

### ***Interview***

An interview is a valuable tool used in research and data collection to gain insight into respondents' perspectives and understand their views (Creswell & Poth, 2023). The interview was conducted with mathematics educator to collect their views on the relevance of knowledge of real analysis for secondary-level mathematics. In the context of this study, interviews were employed as a means to gain a deeper understanding of mathematics educators' viewpoints regarding, "why real analysis is required for being teacher" and techniques of making connection between real analysis and school- mathematics content so that prospective teachers get more benefit from the study of real analysis. An interview protocol was prepared (included in Appendix E) to guide the interview.

To conduct interviews with experts, educators were first contacted and invited to participate. Upon obtaining their consent, permission was sought from the mathematics education department to allocate an appropriate time for the interviews. With their approval, the researcher visited mathematics educators at the scheduled time and recorded the interview sessions. Additionally, educators provided written evidence during the interviews to support their responses.

After a general overview of the quantitative results, qualitative data collection was initiated. It began on June 16, 2024, and concluded in December 2024. However, activities such as member checking and revisiting participants to clarify or verify responses continued until March 4, 2025, ensuring the credibility and trustworthiness of the data.

### **Quality Assurance**

Reliability, validity, and trustworthiness are considered integral parts of research to ensure quality. This study was guided by the explanatory sequential research design under the mixed approach. Therefore, two methods were employed in this study, with different approaches used to ensure reliability and validity.

### ***Trustworthiness***

This research included the qualitative phase in the second phase of data collection and analysis. Trustworthiness is a crucial aspect of qualitative research, referring to the extent to which the study's findings are credible, transferable, dependable, and confirmable. Establishing trustworthiness in qualitative research is important to ensure the results are valid, reliable, and accurate. Trustworthiness in qualitative research is ensured through the principles of credibility, transferability, dependability, and confirmability (Gunawan, 2015). This research followed these guidelines to ensure trustworthiness.

**Credibility.** The congruence of findings and reality indicates credibility (Stahl & King, 2020). For credibility, I used the methods of member checking, peer debriefing, and triangulation (Guba, 1981; Shenton, 2004). To ensure credibility, triangulation was employed by collecting data from multiple sources, including task-based interviews and classroom observations. Member checking was conducted by sharing interview transcripts and the emergent themes with participants to verify the accuracy and authenticity of the interpretations. Additionally, prolonged engagement was maintained through repeated classroom observations and informal discussions, allowing for deeper insight into instructional practices and fostering trust with participants.

**Transferability.** Transferability refers to how the findings can be generalized to other contexts or populations. Qualitative research typically does not aim for generalizability but rather for the transferability of findings to other contexts or individuals. To ensure transferability, researchers need to provide precise descriptions of the study's assumptions and contextual factors, as well as the characteristics of the participants. This can help to establish the applicability of the study's findings to similar settings or individuals. Researchers need to provide rich and thick descriptions of the study context and participants to ensure transformability (Gunawan, 2015). Based on these guidelines, I employed methods that involved providing detailed descriptions of the research settings, including the characteristics of the classroom environments, the instructional practices observed, and the socio-demographic backgrounds of the participants. These contextual descriptions were enriched with direct participant quotations, explanatory accounts, and illustrative scenarios from classroom observations and task-based interviews. This approach enables readers to assess the extent to which the findings apply to similar educational settings or contexts.

**Dependability.** Dependability refers to the consistency of the findings over time and across different researchers. To enhance dependability, researchers required to use clear and systematic data collection and analysis procedures and document the research process in detail (Gunawan, 2015). To ensure dependability, I systematically documented all stages of the research process, including the design, sampling strategy, data collection tools, and analysis procedures. A detailed audit trail consisting of field notes, interview transcripts, observation checklists, coding frameworks, and analytic memos was maintained. I employed a consistent methodological protocol throughout the study to ensure logical coherence and

transparency in the process. Additionally, I engaged in peer debriefing and consulted with my supervisor and colleagues during the coding and interpretation stages to verify consistency in theme development. These steps ensured the findings were stable, consistent, and could be replicated in similar contexts.

**Conformability.** Confirmability refers to the extent to which the findings are shaped by the participants' responses rather than researcher bias, ensuring objectivity and neutrality (Lincon & Guba, 1985). To enhance confirmability, I engaged in reflexivity, maintained an audit trail, and employed peer review. I conducted multiple readings of the transcribed data and carefully compared the emerging themes with participants' original responses. Direct quotations from respondents were included during interpretation to ensure that the findings accurately reflected their views and experiences, rather than my assumptions.

### **Data Preparation**

The data preparation followed a structured approach to ensure quantitative and qualitative data's accuracy, reliability, and validity. This involved multiple steps, including data cleaning, coding, and organization for practical analysis. The collected survey data were examined for missing values and inconsistencies. Cases with substantial missing data were removed (only two respondents' data). For statistical analysis, survey responses, particularly Likert-scale items, were numerically coded (e.g., strongly disagree = 1 to strongly agree = 5). The Likert-type items are combined into a single composite score variable during data analysis (Mumu et al., 2022) which assumes the existence of an underlying continuous variable (Dogan & Demirbolat, 2021). Therefore, the composite score is calculated by averaging the items within the corresponding domain. The variable preparation of each domain and subdomain is guided by the process explained by Hair et al.(2019a).

Initially, the mean was calculated using items from the Likert-type scale to ensure that the research confirmed their representation of the desired domain. These calculations established three subdomains for each domain content connection, pedagogical application, and proficiency enhancement. To achieve this, items 3 and 13 from the original content connection attitude scale and five from the pedagogical application scale were deleted as they violated the initial structural equation modeling assumptions and affected Cronbach's alpha. The corresponding items for each domain, along with the deleted items, are presented in Appendix A (Sections B, C, D, and E). The algebraic properties of real numbers connected with school mathematics content (CONTN1) was calculated as the average of items 1, 2, 4, and 5. The real sequence and series connected with school mathematics content (CONTN2) was calculated as the average of items 7, 8, 9, and 10. The concept of calculus (limit, continuity, and derivatives) connected with the school mathematics curriculum (CONTN3) was calculated as the average of items 6, 11, 12, 14, and 15 from the content connection measurement scale (Appendix A, Section B). For pedagogical application, the pedagogical application of properties of real numbers (PAPLI1) was calculated as the average of items 1, 2, 3, and 4. The pedagogical application of functions, limits, and continuity (PAPLI2) was calculated as the average of items 6, 7, 8, and 9. The pedagogical application of series and proof ideas (PAPLI3) was calculated as the average of items 10, 11, 12, and 13 from the pedagogical application measurement scale (Appendix A, Section C).

Regarding mathematical proficiency, the conceptual understanding of real numbers (MPROF1) was calculated as the average of items 1, 2, 3, and 4. Mathematical reasoning and fluency (MPROF2) was calculated as the average of items 5 to 12. Problem solving and proof construction (MPROF3) was calculated as

the average of items 13 to 17 from the mathematical proficiency measurement scale (Appendix A, Section D).

Mathematical creation for instruction (TEACH1) was calculated as the average of items 1, 2, 5, and 10 for the teaching efficacy domain. Preparing algorithms for problem-solving (TEACH2) was calculated as the average of items 3, 6, and 7. Content bridging and persuasion (TEACH3) was calculated as the average of items 4, 8, and 9 from the teaching efficacy measurement scale (Appendix A, Section E).

Following Braun and Clarke's (2006) framework, thematic analysis was applied to categorize responses into meaningful themes. Data credibility was enhanced by cross-verifying findings from multiple sources, such as curriculum analysis, teacher interviews, and classroom observations. This systematic approach ensured that the data maintained high integrity, allowing for a robust and meaningful analysis.

### **Data Analysis Tools and Techniques**

This study requires two types of inquiry to achieve the research goal. Quantitative data were analyzed in the first stage, and qualitative data were analyzed in the second stage.

#### ***Quantitative Data Analysis Tools and Techniques***

The categorical data were gathered from the Likert scale. These data were recorded using numerical values: strongly disagree, disagree, neutral = 3, agree = 4, and strongly agree = 5. The mean of each item in whole numbers is required to convert the recorded ordinal data of the Likert scale for converting it into scale data, where 3 = neutral represents the average (Morgan, 2016; Spector, 2022). Calculating the mean of Likert scale ordinal data is more reliable than the individual data

(Morgan, 2016). Therefore, the mean was obtained from each scale item to make the scale data. Then, further statistical operations were performed as required.

**Descriptive Statistics.** Means and standard deviation percentages were used to analyze the respondents' demographic characteristics and assess their attitudes toward the relevance of knowledge of real analysis in mathematics instruction.

**Inferential Statistics.** Before using the inferential statistics, the assumptions were established based on the literature guidelines on Skewness and Kurtosis. Regarding the hypothesis testing, inferential statistics were used. Although Likert-type items are ordinal, combining them into composite scores creates approximately interval-level data, allowing the use of parametric tests like t-tests and ANOVA (Carifio & Perla, 2008). This approach is widely accepted when scales are reliable and sample sizes are large enough (Norman, 2010). Parametric tests are robust and often more powerful than nonparametric ones (Norman, 2010). Carifio and Perla (2008) highlight the practical value of this method despite debates. Therefore, this study used t-tests and ANOVA to analyze composite Likert scores, following common research practice. A single mean t-test was used to test whether the teachers' attitudes were significantly positive. The independent sample t-test was used to assess the significance of differences in attitude across gender, training, and whether the students attended private or public schools. Additionally, ANOVA was used to compare the teachers' attitudes toward the relevance of actual analysis knowledge to secondary-level mathematics instruction. The attitude regarding the ethnicity post hoc Tukey test was used for pairwise comparison. This statistical test was performed using data analysis software (SPSS 27).

Moreover, partial least squares structural equation modeling (PLS-SEM) was employed to evaluate the measurement and structural models. It was utilized to test

the impact of attitudes on teaching efficacy. The software SmartPLS 4 was used to check the measurement and structural models. Additionally, the moderating effect was tested. The goal was to ensure the change in dependent variables (teaching efficacy) by attitude relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction, or due to academic qualification and training.

### ***Qualitative Data Analysis Tools and Techniques***

After the quantitative inquiry phase, qualitative data were analyzed. Verbal, textual, and audio data were collected through classroom observations, task-based interviews, and textbook investigations. These data were analyzed using the following techniques.

**Thematic Analysis.** Thematic analysis is a qualitative data analysis technique that includes a detailed examination of the dataset to identify patterns, analyze them, and report them. It is a flexible technique in several contexts related to qualitative data analysis. It helps learn about the experiences and perspectives of study participants and exposes complicated phenomena. Compared with other research methods like focus groups, interviews, or surveys, thematic analysis can offer a more comprehensive understanding of a research subject.

This method aims to explore the hidden meaning of the data by identifying commonalities and differences among the dataset (Braun & Clarke, 2006). Thematic analysis is appropriate when comprehending a group of experiences, thoughts, or behaviours within a data collection (Kiger & Varpio, 2020). Therefore, thematic analysis was used to seek and understand the relevance of mathematical knowledge of real numbers for secondary-level mathematics instruction. Becoming familiar with the data, generating initial codes, searching for themes, reviewing themes, and defining

themes, the write-up is the steps of thematic analysis (Braun & Clarke, 2006; Maguire & Delahunt, 2017). These steps were taken as guidelines for qualitative data analysis.

Based on these guidelines, a systematic thematic analysis was used. The thematic analysis begins with the transcription of audio and video data collected from task-based interviews with secondary-level mathematics teachers and expert interviews. Revisits and comparisons were conducted for the transcription to ensure that verbal data were accurately captured into a written form. The transcribed data were read multiple times to familiarize the reader with them. The transcribed data were imported into QDA Miner 24 to enhance the systematic organization. The imported data has 11 cases (8 cases of task-based interview data and 3 of expert interview data). All cases are read, assigned a code, and grouped under a broader category. The iterative process was used for all cases. The code and category were refined and merged (Demonstrated in Appendix G). Finally, the theme was defined and described with supportive evidence. This process ensured a rigorous and credible way of qualitative data analysis practices (Braun & Clarke, 2006; Maguire & Delahunt, 2017). Therefore, thematic analysis was used to analyze the qualitative data, following the above-mentioned scholarly guidelines. This ensured a systematic and rigorous approach to identifying and interpreting key themes.

**Link Analysis.** Link analysis is useful for uncovering central concepts and structural connections within textual data (Miles et al., 2014). This study employed link analysis to identify meaningful connections between themes. The analysis was conducted using QDA Miner, a qualitative data analysis software that visualizes thematic relationships based on co-occurrence patterns. Following a thematic analysis approach, key themes were identified and categorized. These themes were then examined using co-occurrence analysis, which measures how often two or more

themes appear within the same textual segment (Guest, MacQueen, 2012). Nodes representing the themes were created, and their relationships were mapped according to the strength of co-occurrence. To assess the importance and connectivity of themes, network metrics such as degree centrality, betweenness centrality, and eigenvector centrality were calculated (Dang et al., 2018). The selection of red nodes was based on a combination of these metrics rather than frequency alone.

Thus, link analysis using QDA Miner was conducted to explore the interconnections between constructs concerning the relevance of knowledge of real analysis. This analysis helped answer how teachers perceive real analysis, why it is essential, and how it is utilized in teaching.

### ***Data Triangulation***

Triangulation in qualitative research involves using multiple data sources and methods to gain a comprehensive and accurate understanding of a phenomenon. By comparing evidence from different perspectives, triangulation enhances the credibility, validity, and trustworthiness of research findings (Honorene, 2017). It can take various forms, including theoretical, methodological, time, data, and investigator triangulation (Braun & Clarke, 2006).

In this study, data triangulation was employed to assess the relevance of real analysis knowledge by examining teachers' perceptions, instructional practices, mathematics educators' views, and curriculum connections. Specifically, data from surveys, task-based interviews, educator interviews, and curriculum analysis were combined to strengthen the overall findings.

### **Methodological Framework**

The methodological framework plays a crucial role in any research study. It outlines the systematic approach that researchers use to collect and analyze data.

**Table 5***Methodological Framework*

Purposes	Research Questions	Design	Data collection tools	Data Analysis
1) To assess the teachers' attitude towards the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction.	1. What are teachers' attitudes toward knowledge of real analysis concerning its connection with secondary-level mathematics, its pedagogical applications, and its role in enhancing their mathematical proficiency	Quantitative survey	Likert scale for measuring teachers' attitudes -categorical data	Descriptive analysis: -Statistical tools (Mean, SD, percentage, t-test) PLS-SEM
	2. What is the relationship among teachers' attitudes towards content connection, pedagogical application, mathematical proficiency, and teaching efficacy?			
	3. How do teachers perceive mathematical knowledge of real analysis concerning content connection, pedagogical application, and enhancing mathematics teachers' proficiency?	Qualitative	In-depth interview with the teacher	Thematic Analysis Link analysis
2) To evaluate the connection between real analysis and secondary-level mathematics, its rationale, and ways to strengthen this linkage.	4. Why is mathematical knowledge of real analysis required to become a mathematics teacher?	Qualitative	Task-based interview with teachers, interview with mathematics educators	Thematic Analysis, link analysis
	5. What are the connections between the mathematical knowledge of real analysis and the content of secondary-level mathematics?	Quantitative	Content connection framework, Task-based interview	Descriptive Analysis
	6. Why is content connection between mathematical knowledge of real analysis and school mathematics required?	Qualitative	Task-based interview with teachers, interview with mathematics educators	Thematic Analysis, link analysis
	7. How can strengthening the link between knowledge of real analysis and secondary-level mathematics be achieved?	Qualitative	Task-based interview with teachers, interview with mathematics educators	Thematic analysis, link analysis
3) To explore how secondary-level mathematics teachers use knowledge of real analysis in their instructional practices.	8. In what classroom contexts do teachers characterize their mathematical knowledge of real analysis?	Qualitative	Task-based interview classroom observation	Thematic analysis
9. How do secondary-level mathematics teachers characterize mathematical knowledge of real analysis in their classroom practices?				

This framework is developed based on the research objectives and corresponding research questions, which helps ensure that the data collected is relevant to the research problem and can effectively address the research questions. The

methodological framework used in the study is summarized in Table 5 to provide a clear overview.

This research framework integrates quantitative and qualitative approaches to assess teachers' attitudes, explore the connection between real analysis and secondary mathematics, and examine its instructional applications. The framework typically encompasses the study design, data collection tools, and analysis methods that align with the study's purpose and research questions.

### **Report Writing and Language Editing**

The report was prepared based on the analysis of the data and results. For citation and referencing, the writing process followed APA 7 guidelines. The paid version of Grammarly was used to proofread the language, and English language experts were consulted to refine the language after the draft was prepared.

### **Ethical Considerations**

Ethics in research refers to a set of moral principles that researchers follow to ensure that their studies are conducted responsibly and ethically and that they uphold the rights and dignity of research participants. These principles ensure that the welfare and rights of all participants involved in the research are respected. The ethical principles that govern research involve several key concepts, including informed consent, confidentiality, privacy, and the fair and respectful treatment of all participants. Researchers are required to obtain informed consent from all participants before participating in the study. This involves providing the participants with clear and concise information about the study's purpose, procedures, and potential risks and benefits. Ensuring that participants are prepared and willing to engage voluntarily in the research is essential.

Confidentiality is also a critical ethical principle in research. Researchers must protect the confidentiality of all participants, ensuring that their personal information is not disclosed without their explicit consent. They also take steps to safeguard any data collected during the study, preventing it from being misused or accessed by unauthorized persons. Therefore, the researcher served the ethical consideration of confidentiality.

Privacy is another important ethical principle that researchers are required to respect. Thus, the researcher ensured that participants' privacy was protected during all aspects of the research process, including data collection, analysis, and publication. This means that researchers must take steps to ensure that participants' personal information is not disclosed or misused. To maintain ethical standards, the researcher strictly followed the ethical guidelines set by the Graduate School of Education, Tribhuvan University, Nepal. The researcher took permission to collect data concerning selecting a participant. The researcher requested voluntary participation in research by keeping their rights regarding the response. Probability sampling for quantitative inquiry and convenience sampling for qualitative inquiry were used. The quantitative data were analyzed using SPSS 26 and SmartPLS4 for structural modeling. The data collection tools were validated, and their reliability will be ensured before launching them.

### **Chapter Summary**

The research methodology chapter outlines the philosophical underpinnings, research design, and data collection methods used in the study, which investigates the relevance of real analysis in secondary-level mathematics instruction. This study is grounded on a pragmatic paradigm of research philosophies and accepts multiple realities. It also adopts a mixed-methods approach, integrating qualitative and

quantitative techniques. The research design follows an explanatory sequential mixed methods research design. The initial phase includes a quantitative survey that assesses secondary-level mathematics teachers' attitudes toward knowledge of real analysis, followed by qualitative methods, including curriculum analysis, classroom observation, and task-based interviews, to gain deeper insights into pedagogical applications. A proportionate stratified random sampling method was employed for sample selection, while purposive and convenience sampling strategies guided the collection of qualitative data. The study employs validated Likert-scale surveys to measure teachers' attitudes, ensuring reliability through a pilot study. Descriptive statistics, including mean, standard deviation, and percentage, inferential statistics, t-test, ANOVA, and structural equation modeling, were used to analyze the quantitative data. Thematic analysis supports qualitative insights. The research process adheres to ethical guidelines that ensure compliance with established research norms, including informed consent, confidentiality, and voluntary participation.

## Chapter 4

### Analysis and Interpretation of Data

This study employed an explanatory sequential research design, and the collected data were analyzed and interpreted in three distinct phases. The first phase involved analyzing and interpreting quantitative data; the second phase involved analyzing and interpreting qualitative data. Finally, in the third phase, the findings from both phases were merged and integrated using a joint display to draw a comprehensive conclusion.

#### Phase i: Analysis and Interpretation of Quantitative Data

This chapter presents the results of the analysis of quantitative data, which addresses the first purpose of assessing teachers' attitudes towards knowledge of real analysis and its impact on teaching efficacy. It addresses the first two research questions: *RQ1. What are teachers' attitudes towards knowledge of real analysis concerning its connection with secondary-level mathematics, its pedagogical applications, and its role in enhancing mathematical proficiency?* and *RQ2. What is the relationship among teachers' attitudes towards content connection, pedagogical application, mathematical proficiency, and teaching efficacy?*

#### *Socio-demographic Characteristics of Respondents*

The socio-demographic characteristics of the respondents (n = 387) in the first phase are presented in Table 6. The majority were male (91%), while females accounted for only 9%. Regarding academic qualifications, most respondents held a Master's degree (76.2%), followed by a Bachelor's degree (22.7%), and a small number with an MPhil or PhD (1.0%). The largest ethnic group was Brahmin/Chhetri (74.7%), followed by Janajati (19.9%), Dalit (3.4%), Madheshi (2.1%), and other groups (0.3%). Most respondents were from public schools (83.5%), and a large

proportion (71.6%) had an educational academic background. In terms of teacher training, 74.2% reported receiving professional training in addition to their university education. The largest age group was 30–35 years (31.3%), followed by 25–30 years (24%). Teaching experience varied, with 33.6% having 10–15 years of experience and 19.1% having 20 or more years. The details of socio-demographic characteristics are shown in Table 6.

**Table 6**

*Demographic Summary of Respondent (n=387)*

Socio-demographic Characteristics	Categories	N	Percentage
Gender	Male	352	91
	Female	35	9
Academic qualification	Bachelor	88	22.7
	Master	295	76.2
	MPhil/PhD	4	1.0
Ethnicity	Brahmin/Chhetri	289	74.7
	Janajati	77	19.9
	Dalit	13	3.4
	Madheshi	8	2.1
Types of School	Public	323	83.5
	Private	64	16.5
Academic Background	Education	277	71.6
	Humanities	46	11.9
	Science	64	16.5
Training	Yes	287	74.2
	No	100	25.8
Age group	20-25	53	13.7
	25-30	93	24
	30-35	121	31.3
	35-40	72	18.6
	40 and above	48	12.4
Teaching Experience ( in year)	≤5	35	9
	5-10	58	15
	10-15	130	33.6
	15-20	90	23.3
	≥20	74	19.1

### *Teachers' Attitude towards Knowledge of Real Analysis*

Likert-type scales were used to assess teachers' attitudes towards the content connection between secondary-level mathematics and knowledge of real analysis, the pedagogical application for instruction, and the enhancement of mathematical proficiency. Teachers' attitudes toward content connection, pedagogical application, and mathematical proficiency were measured to gauge their reactions to the application of real analysis in instruction. This measurement represents the first level of relevance assessment, based on the training evaluation model developed by Kirkpatrick and Kirkpatrick (2014).

Item-wise agreement percentages, mean, and standard deviations were calculated. Additionally, sub-domain-wise and domain-wise mean and standard deviations were calculated. A level category was identified using the following formula, which Alharbi (2014) employed to determine the level of response on a Likert-type scale.

$$\text{Level} = \frac{\text{The scale's highest value} - \text{The scale's lowest value}}{\text{Number of levels}} = \frac{5-1}{2} = 2$$

Two levels were required to identify the teachers' attitudes towards the mathematical knowledge of real analysis. The range was defined by adding the lowest value (1) to the range (2). Therefore, the mean score from 1 to 3 represents the negative attitude towards the mathematical knowledge of real analysis, indicating that teachers have a negative reaction. Conversely, if the mean value is greater than 3, teachers have positive attitudes towards the mathematical knowledge of real analysis for school mathematics instruction. Thus, the level was identified and assigned to the response according to the following criteria.

Criteria	Meaning	level
Mean $\leq$ 3	Negative attitude towards mathematical knowledge of real analysis regarding school mathematics instruction.	1
Mean $>$ 3	Positive attitude towards mathematical knowledge of real analysis regarding school mathematics instruction.	2

Based on these criteria, the teachers' attitudes towards mathematical knowledge of real analysis for secondary-level mathematics instruction were categorized and analyzed separately.

**Teachers' Attitude toward Content Connection.** Based on the first research question, teachers' attitudes toward knowledge of real analysis across content connections were analyzed. The response about this domain is arranged in Table 7. Based on the interpretation scale (where a mean greater than 3 indicates a positive attitude), the analysis reveals that teachers generally perceive a strong connection between real analysis and secondary-level mathematics instruction. As shown in Table 7, all items assessing the algebraic properties of real numbers, such as closure, associativity, and commutativity, yielded a mean of 4.1, indicating a positive attitude. This domain showed the highest overall mean ( $M = 4.1$ ,  $SD = 0.74$ ), suggesting that these properties are viewed as highly relevant to school-level content. The domain on real sequences and series received a moderately positive rating ( $M = 3.7$ ,  $SD = 0.77$ ), though individual items, such as the concept of convergent series, showed comparatively lower agreement ( $M = 3.4$ ,  $SD = 1.1$ ). Similarly, the domain on calculus-related concepts ( $M = 3.7$ ,  $SD = 0.67$ ) reflected a positive perception of content connection, particularly regarding limits and continuity.

**Table 7***Scale of Attitude towards Content Connection between KRA and SLM (N=387)*

S.n.	Statements	Percentage of Agreement							
		SD	D	N	A	SA	M	SD	L
1	The closure property of real number (If $a, b \in \mathbf{R}$ then, $a + b \in \mathbf{R}$ & $a \times b \in \mathbf{R}$ ) is strongly connected with content of secondary level mathematics.	4.7	3.9	4.4	55.3	31.8	4.1	1.0	2
2	The associative property of real number [If $a, b, c \in \mathbf{R}$ then, $(a + b) + c = (a + (b + C))$ and $a \times (b \times c) = (a \times b) \times c$ ] is strongly connected with content of secondary-level mathematics.	5.2	5.2	4.4	43.2	42.1	4.1	1.1	2
3	The commutative property of real numbers [ $\forall a, b \in \mathbf{R}$ then $a + b = b + a$ , and $a \times b = b \times a$ ] strongly connected with school mathematics contents	3.4	3.6	4.4	57.9	30.7	4.1	0.9	2
4	The cancellation law (addition and multiplication) is strongly connected with content of school mathematics.	3.1	2.8	6.7	51.4	35.9	4.1	0.9	2
Algebraic Properties of Real Number (CONTN1)							4.1	0.74	2
5	Real sequence studied in real analysis is strongly connected with content of secondary level mathematics.	3.9	9.0	18.9	49.4	18.9	3.7	1.0	2
6	The abstract form of sequence studied in real analysis can be found in a specific form (e.g., sequence in number, diagrammatic form) at secondary-level.	5.2	7.2	11.9	56.1	19.6	3.8	1.0	2
7	The contents of infinite series of real analysis connected with secondary level-mathematic content.	3.4	14.5	16.5	39.8	25.8	3.7	1.1	2
8	The concept of convergent of series is practiced school mathematics.	5.7	19.4	21.4	36.7	16.8	3.4	1.1	2
Real Sequence and Series (CONTN2)							3.7	0.77	2
9	The mathematical concepts of function (especially definition, example, numerical problem) found in real analysis are strongly connected with content of secondary-level mathematics.	5.7	19.4	21.4	36.7	16.8	3.4	1.1	2
10	Limit of a function is expressed in numerical and diagrammatic form at secondary-level mathematics	3.1	9.0	12.9	55.3	19.6	3.8	1.0	2
11	The topic continuity included in real analysis connected with secondary-level mathematics content.	2.6	11.1	14.2	54.5	17.6	3.7	1.0	2
12	The function is interpreted in graphical form at secondary-level mathematics is related to the concept of function studied in real analysis.	2.6	9.0	16.8	52.7	18.9	3.8	0.9	2
13	The concept of interval (open interval, closed interval, semi-closed interval) discussed in real analysis is connected with school mathematics content in verbal form and inequalities.	5.4	14.2	14.5	45.0	20.9	3.6	1.1	2
Calculus of Real number (CONTN3)							3.7	0.67	2
Overall Content Connection							3.83	0.58	2

The overall mean score for content connection ( $M = 3.83$ ,  $SD = 0.58$ ) confirms that mathematics teachers perceive real analysis to be significantly connected to

secondary-level instruction. Among the three domains, algebraic properties of real numbers are seen as most strongly aligned with the school curriculum.

**Teachers' Attitude towards Pedagogical Application of KRA.** The teachers' attitudes towards the pedagogical application of knowledge of real analysis in school mathematics instruction are summarized in Table 8. Using a scale where a mean ( $M$ )  $\leq 3$  indicates a negative attitude (Level 1) and  $M > 3$  indicates a positive attitude (Level 2), the data reveal a generally positive outlook. Specifically, for the application of real number properties, mean scores are high: closure property ( $M = 4.0$ ,  $SD = 0.9$ ), associative property ( $M = 4.2$ ,  $SD = 0.8$ ), and commutative property ( $M = 4.2$ ,  $SD = 0.7$ ). Positive attitudes are also observed for function concepts ( $M = 4.0$ ,  $SD = 0.7$ ), sequences and series ( $M = 4.0$ ,  $SD = 0.8$ ), and limits ( $M = 4.0$ ,  $SD = 0.8$ ). Slightly lower, yet still positive, attitudes appear for infinite series ( $M = 3.7$ ,  $SD = 0.9$ ) and the concept of derivative ( $M = 3.6$ ,  $SD = 1.1$ ).

Composite scores further support these findings: pedagogical application of properties of real numbers (PAPLI1:  $M = 4.1$ ,  $SD = 0.6$ ), application of function, limit, and continuity (PAPLI2:  $M = 4.0$ ,  $SD = 0.6$ ), and application of series and proof ideas (PAPLI3:  $M = 3.7$ ,  $SD = 0.7$ ). Overall, the mean score for pedagogical application is 3.93 ( $SD = 0.49$ ), reflecting a predominantly positive attitude toward integrating real analysis concepts in secondary-level mathematics instruction. Among the three subdomains, the application of properties of real numbers shows the highest mean, indicating that this area is the most consistently and confidently applied by teachers in their classroom practices. This suggests that teachers not only recognize the importance of foundational real number properties but also find them more accessible and relevant to the curriculum, contributing to more effective mathematics instruction.

**Table 8**

*Attitude towards Pedagogical Application for School's Mathematics Instruction*  
(N=387)

S.n.	Statements	Percentage of Agreement							
		SD	D	N	A	SA	M	SD	L
1	I use closure property of real number (If $b \in \mathbf{R}$ then, $a + b \in \mathbf{R}$ and $a \times b \in \mathbf{R}$ ) during mathematics instruction.	3.9	3.9	9.3	56.1	26.9	4.0	0.9	2
2	I use associative property of real numbers [If $a, b, c \in \mathbf{R}$ then, $(a + b) + c = (a + (b + C))$ & $a \times (b \times c) = (a \times b) \times c$ ] during mathematics instruction	2.1	2.3	4.9	56.6	34.1	4.2	0.8	2
3	I use commutative property of real number [ $\forall a, b \in \mathbf{R}$ then $a + b = b + a, a \times b = b \times a$ ] during mathematics instruction	1.0	2.6	5.2	55.0	36.2	4.2	0.7	2
4	I use cancellation laws (for addition, multiplication) discussed in properties of real numbers in numerical calculating process for instruction.	0.8	5.7	13.4	53.2	26.9	4.0	0.8	2
Pedagogical Application of Properties of Real Number (PAPLI1)							4.1	0.6	2
5	I use concepts of function (especially definition, example, numerical problem) found in real analysis for mathematics instruction.	0.5	3.6	12.4	60.2	23.3	4.0	0.7	2
6	I use the concepts of sequence and series studied in real analysis for mathematics instruction	1.3	3.6	10.3	59.4	25.3	4.0	0.8	2
7	I use knowledge of limit and function studied in real analysis to teach school mathematics content like limit, function.	1.8	3.6	8.8	59.4	26.4	4.0	0.8	2
8	I use concept of continuity and discontinuity found in analysis for teaching concept of continuity in school-level.	3.1	4.7	10.6	60.5	21.2	3.9	0.9	2
Application of Function, Limit and Continuity (PAPLI2)							4.0	0.6	2
9	I use knowledge of infinite series found in real analysis for secondary-level mathematics instruction.	2.6	9.6	16.3	54.5	17.1	3.7	0.9	2
10	I use concepts of convergent and divergent found in real analysis for secondary-level mathematics instruction.	6.5	18.3	21.4	40.3	13.4	3.4	1.1	2
11	Mathematical knowledge of real analysis helps to explore the idea of teaching new mathematical content in school.	1.0	4.4	11.4	55.8	27.4	4.0	0.8	2
12	I use the concept of derivative to teach the slope of line in school	4.1	14.7	18.6	45.7	16.8	3.6	1.1	2
Application of Series and proof ideas (PAPLI3)							3.7	0.7	2
Overall Pedagogical Application							3.93	0.49	2

L= level of attitude

**Teachers' Attitude towards KRA for Enhancing Proficiency.** A Likert-type survey scale, including 17 items, was employed to measure the teachers' attitude towards real analysis in developing mathematical proficiency. The key concept included in the scale was how teachers perceive the role of real analysis in enhancing mathematical proficiency. The results are presented in Table 9.

The mean scores for all items range from 3.9 to 4.3, with standard deviations between 0.5 and 0.8, except for item 8 (Real analysis is the root of axioms in the number system applied in school mathematics), which has a mean of 3.9 (SD = 0.8). Three domains were identified through factor analysis.

The first domain, Conceptual Understanding of Real Number System (MPROF1), includes four items (1–4) with a mean of 4.1 (SD = 0.6). This suggests teachers have a strong positive attitude toward the role of real analysis in understanding fundamental mathematical concepts and the axiomatic structure of the real number system. The item means within this domain range from 4.0 to 4.2.

The second domain, Mathematical Reasoning and Fluency (MPROF2), consists of eight items (5–12) with a mean of 4.1 (SD = 0.5). The item ranges from 3.9 to 4.1, and agreement levels generally exceed 80%, except for item 8, which has an agreement level of 78.3%. Teachers recognize real analysis as essential for developing reasoning skills and mathematical fluency.

The third domain, Problem Solving and Proofing (MPROF3), includes five items (13–17) with a mean of 4.1 (SD = 0.5). The item means range from 4.0 to 4.1, with agreement levels above 84%. This reflects teachers' positive attitudes regarding the importance of real analysis in enhancing problem-solving and proof techniques. Such knowledge equips teachers to construct and evaluate mathematical arguments effectively while fostering creativity in developing instructional strategies.

**Table 9***Scale of Attitude towards KRA in Developing Mathematical Proficiency (N=387)*

s.n.	Statements	Percentage of Agreement						SD	L
		SD	D	N	A	SA	M		
1	Mathematical knowledge of real analysis is fruitful to my professional life.	1.6	3.9	9.6	56.1	28.9	4.1	0.8	2
2	Mathematical knowledge of real analysis is required to be a professional teacher.	1.3	3.9	8.3	53.2	33.3	4.1	0.8	2
3	Knowledge of real analysis helps to get fundamental knowledge of real number systems and their properties.	0.5	2.8	6.7	55.3	34.6	4.2	0.7	2
4	Mathematical knowledge of real analysis is a base for understanding the rules of numerical operations.	0.5	2.1	11.9	56.8	28.7	4.1	0.7	2
Conceptual Understanding of Real Number (MPROF1)							4.1	0.6	2
5	Mathematical knowledge of real analysis develops the skill of mathematical reasoning.	0.8	1.6	5.9	55.0	36.7	4.3	0.7	2
6	Mathematical knowledge of real analysis increases the confidence level in content knowledge.	0.8	3.9	11.1	51.9	32.3	4.1	0.8	2
7	Mathematical knowledge of real analysis helps to understand the mathematical structure of real numbers.	1.0	0.8	8.0	56.6	33.6	4.2	0.7	2
8	Mathematical knowledge of real analysis is the root of axioms in number systems applied in school mathematics.	1.0	5.2	15.5	58.4	19.9	3.9	0.8	2
9	Mathematical knowledge of real analysis helps to develop mathematical fluency.	1.3	3.4	11.6	59.7	24.0	4.0	0.8	2
10	Mathematical knowledge is essential to develop the ability which enables us to create ideas to solve mathematical problems during school mathematics instruction.	0.3	1.8	9.3	58.9	29.7	4.2	0.7	2
11	Mathematical knowledge of real analysis is essential to developing mathematical language capacity which enables us to understand new topics in mathematics.	0.8	3.6	12.4	64.3	18.9	4.0	0.7	2
12	Knowledge of real analysis in mathematics enables our mind to understand any new mathematical problem and its basic nature.	0.5	3.6	9.0	63.0	23.8	4.1	0.7	2
Mathematical Reasoning and Fluency (MPROF2)							4.1	0.5	2
13	Mathematical knowledge of real analysis enables us to tackle mathematical problems by improving our mathematical intelligence.	0.8	2.6	9.0	58.7	28.9	4.1	0.7	2
14	Real analysis helps to develop an abstract ability in our brain whereby our brain can suddenly recognize the mathematical relation and solve problems when we see it.	1.6	2.6	11.1	59.4	25.3	4.0	0.8	2
15	Real analysis helps to develop an abstract ability in our brain whereby we recognize the mathematical relationships in a given problem and solve it.	1.3	2.3	10.1	62.5	23.8	4.1	0.7	2
16	Real analysis enables our brains to create mathematical problems and techniques for solving the problems that are needed during instruction.	1.6	3.4	7.8	61.5	25.8	4.1	0.8	2
17	Real analysis enables us to develop techniques of deductive proof of mathematical theorems that are needed in instruction of school mathematics.	1.8	3.4	9.8	60.2	24.8	4.0	0.8	2
Problem Solving and Proofing (MPROF3)							4.1	0.5	2

All domains fall at Level 2 (positive attitude), indicating that secondary-level mathematics teachers highly value real analysis for developing mathematical proficiency.

**Significance of Teachers' Attitude towards KRA.** To answer the research question: *RQ1. What are teachers' attitudes towards knowledge of real analysis concerning its connection with secondary-level mathematics, its pedagogical applications, and its role in enhancing mathematical proficiency?* Three hypotheses were formulated to check whether attitude is significantly more than average.

H<sub>1</sub>: Secondary-level mathematics teachers have significant positive attitudes towards the content connection between real analysis and the content of secondary-level mathematics.

H<sub>2</sub>: Secondary-level mathematics teachers have significant positive attitudes towards applying mathematical knowledge of real analysis to mathematics instruction.

H<sub>3</sub>: Secondary-level mathematics teachers have a significant positive attitude towards mathematical knowledge of real analysis in enhancing mathematical proficiency.

Before using inferential statistics, the basic assumptions were tested. The sample size was 387, and all dependent variables were on a scale. The skewness values ranged from -1.20 to 0.87, while the kurtosis values ranged from 2.1 to 3.04. These values met the normality threshold because the sample size is large, skewness values between -2 and +2, and kurtosis values between -7 and +7 are considered acceptable for t-tests and ANOVA (Hair et al., 2010; Kim, 2013; Damir, 2022). Furthermore, when the sample size is extensive, violations of the normality assumption may not significantly impact the results of parametric tests (Ghasemi & Zahediasl, 2012). Therefore, inferential statistical tools, t-tests, and ANOVA were applied. One sample t-test was conducted to test H<sub>1</sub>, H<sub>2</sub>, and H<sub>3</sub> hypotheses. The test

value equal to 3 was taken because, in the five-point Likert scale, the average is 3. One sample t-test result concerning the three hypotheses is arranged in Table 10. The one-sample t-test result for hypothesis H<sub>1</sub> shows that the mean attitude score toward content connection (M = 3.83, SD = 0.58) is significantly higher than the neutral value of 3,  $t_{(385)} = 28.73$ ,  $p < .001$ , with a very large effect size ( $d = 1.43$ ; Cohen, 1988). This indicates that secondary-level mathematics teachers have positive attitudes toward connecting real analysis with secondary-level mathematics content.

**Table 10**

*Attitude Test Result*

Hypothesis	Variables	Mean	SD	t-value	Sig.	Decision
H <sub>1</sub>	Attitude towards content connection	3.83	0.58	28.73	0.00*	Retain H <sub>1</sub>
H <sub>2</sub>	Attitude towards pedagogical application	3.87	0.47	36.23	0.00*	Retain H <sub>2</sub>
H <sub>3</sub>	Attitude towards enhancing mathematics proficiency	4.10	0.44	48.23	0.00*	Retain H <sub>3</sub>

\* $p < 0.05$  significant

Similarly, the test for hypothesis H<sub>2</sub> yields a significant result,  $t_{(385)} = 36.23$ ,  $p < .001$ , with a very large effect size ( $d = 1.85$ ; Cohen, 1988), indicating that teachers hold positive attitudes toward the pedagogical application of real analysis in secondary-level instruction.

For hypothesis H<sub>3</sub>, the mean attitude score (M = 4.10, SD = 0.44) is also significantly above the test value,  $t_{(385)} = 48.23$ ,  $p < .001$ , with a very large effect size ( $d = 2.50$ ; Cohen, 1988), indicating that secondary-level mathematics teachers perceive real analysis as highly valuable in enhancing students' mathematical proficiency.

**Attitude Across Demographic Characteristics.** Supplementary hypothesis H<sub>4</sub>: Mathematics teachers' attitudes towards real analysis do not differ significantly

across gender, academic background, qualification, types of school teaching, Training, and Ethnicity to answer the first research question. An independent t-test was used to compare two groups (gender, types of schools, and training have two categories).

**Table 11**

*Attitudes Towards KRA Across Demographic Characteristics*

Independent Variable	n	Mean	SD	Test Value	Sig
Gender					
Male	352	3.93	0.35	t = -0.96	0.35
Female	35	4.0	0.40		
Academic Background					
Education	277	3.94	0.40	F = 0.052	0.94
Science	64	3.95	0.32		
Humanities	46	3.94	0.47		
Qualification					
Bachelor	88	3.85	0.37	F = 2.82	0.06
Masters	295	3.97	0.40		
MPhil/PhD	4	3.85	0.39		
School Type					
Government	323	3.93	0.40	t = -1.34	0.22
Private	64	4.00	0.35		
Training					
Trained	287	3.92	0.40	t = 0.67	0.50
Untrained	100	3.95	0.40		
Ethnicity					
Brahmin/Chhetri	289	3.91	0.43	F = 3.98	0.01*
Janajati	77	4.01	0.33		
Dalit	13	3.96	0.36		
Madheshi	8	4.33	0.30		

\* p < 0.05

In comparison, ANOVA was employed for comparisons involving more than two groups (Academic background, qualification, and ethnicity have more than two categories) to test this hypothesis. Table 11 depicts the test results.

An independent samples t-test was employed to examine whether gender significantly affected the attitude towards the relevance of real analysis for secondary-level mathematics instruction. The test result shows that  $t_{(385)} = -0.96$  with  $p = 0.35 > 0.05$ ; hence, there is no significant difference between male ( $M = 3.93$ ,  $SD = 0.35$ )

and female ( $M = 4.00$ ,  $SD = 0.40$ ) participants' attitude towards the relevance of knowledge of real analysis for instruction. A one-way ANOVA was performed to assess the effect of academic background on the attitude towards the relevance of real analysis for secondary-level mathematics instruction. The result shows that  $F_{(2,384)} = 0.052$  with  $p = 0.94 > 0.05$ . Hence, there are no significant differences among participants across the academic background: education ( $M = 3.94$ ,  $SD = 0.40$ ), science ( $M = 3.95$ ,  $SD = 0.32$ ), and humanities ( $M = 3.94$ ,  $SD = 0.47$ ). Additionally, ANOVA was employed to examine the effect of academic qualification on attitudes towards mathematical knowledge of real analysis for secondary-level mathematics instruction. The results indicate no effect,  $F_{(2,384)} = 2.82$ , with  $p = 0.06 > 0.05$  at the 5% significance level. The teachers with a bachelor's degree ( $M = 3.85$ ,  $SD = 0.37$ ), master's degree ( $M = 3.97$ ,  $SD = 0.40$ ), and MPhil/PhD ( $M = 3.85$ ,  $SD = 0.39$ ) exhibited similar attitudes. Thus, more academic qualifications than a bachelor's degree do not significantly influence teachers' attitudes towards the relevance of knowledge of real analysis. An independent samples t-test revealed no significant difference in attitudes between government school teachers ( $M = 3.93$ ,  $SD = 0.40$ ) and private school teachers ( $M = 4.00$ ,  $SD = 0.35$ ),  $t_{(385)} = -1.34$ ,  $p = 0.22$ .

Similarly, training status did not show a significant effect, as trained teachers ( $M = 3.95$ ,  $SD = 0.40$ ) and untrained teachers ( $M = 3.92$ ,  $SD = 0.40$ ) did not differ significantly with  $t_{(385)} = 0.67$ ,  $p = .50 > 0.05$ . However, ethnicity significantly affected the dependent variable with  $F_{(3, 383)} = 3.82$ ,  $p = .01 < 0.05$ . A Post Hoc Tukey test revealed that Madheshi participants ( $M = 4.33$ ,  $SD = 0.30$ ) had significantly higher beliefs than Brahmin/Chhetri participants ( $M = 3.91$ ,  $SD = 0.43$ ),  $p < 0.05$ . No other significant differences were observed among ethnic groups.

### *Teachers' Teaching Efficacy Level*

A survey questionnaire assessed the teachers' efficacy in mathematics instruction. A self-rating based questionnaire was employed. Regarding the efficacy measurement, 1 represents a low level, and 5 represents the highest level of teaching efficacy. Five categories were used to analyze the results. Where 1= low level of teaching efficacy. 2= below average level of teaching efficacy, 3 = average level of teaching efficacy, 4= high level of teaching efficacy, 5 = highest level of teaching efficacy. The results are shown in Table 12.

**Table 12**

*Teaching Efficacy Level of Teacher (Lowest =1 and Highest =5)*

Items		Percentage of response				
How much do you think you can: (rate 1 to 5, where: minimum 1, maximum 5)		1	2	3	4	5
Mathematical Creation for Instruction (M=4.21,SD=0.52)	(1) Create a definition and example of the mathematical concept.	0	0	15.8	75.7	8.5
	(2) Create mathematical problems to teach and evaluate students.	0	0	16.5	46.3	37.2
	(3) Select the appropriate process (algorithm) to solve the mathematical problem.	0	0	19.4	58.1	22.5
	(4) Give reasons for each mathematical process/rule used in schools' mathematics instruction	0	0	20.7	51.2	28.1
	(5) Explain the mathematical procedure and its reasons during instruction.	0	0	17.3	51.2	31.5
Preparing Algorithm (M=4, SD = .44)	(6) Expand and show the axiomatic relation among the structure of mathematical knowledge.	0	0	34.1	46.8	19.1
	(7) Create examples and counterexample of the mathematical concept.	0	0	22.0	52.5	25.5
Content bridging and Persuasion (M=4, SD=0.53)	(8) Give local examples of mathematical content to convince the student.	0	0	14.2	44.2	41.6
	(9) Convince students by providing multiple methods to solve a single problem.	0	0	13.2	52.5	34.3
	(10) Link one concept of mathematics to another concept.	0	0	16.3	50.1	33.6

In the domain of mathematical creation for instruction, respondents rated their efficacy with a mean score of 4.21 and a standard deviation of 0.52, indicating a strong confidence in this area. Specifically, respondents felt very competent in

creating definitions and examples, with 75.7% rating their efficacy as high. Moreover, when creating mathematical problems to teach and evaluate students, 37.2 % rated their efficacy at the highest level, and another 46.3% rated it as high. Efficacy in selecting the appropriate procedure to solve mathematical problems is rated high by a majority (58.1%). Similarly, providing reasons for mathematical processes used in instruction collected high ratings, with 28.1% of respondents having the highest confidence level.

Another domain is preparing algorithms for solving mathematics problems. The mean rating in this domain is 4, with a standard deviation of 0.44. In this domain, explaining mathematical procedures and their reasons demonstrated considerable confidence, with 31.5% of respondents rating their efficacy at the highest level and 51.2% at a high level. However, there was more variability in expanding axiomatic relations, where 34.1% rated their confidence as average and only 19.1% felt at the highest level. Respondents rated the ability to create examples and counterexamples of mathematical concepts as high, with 52.5% rating their efficacy as high and 25.5% at the highest level.

Another domain is content bridging and persuasion; respondents again demonstrated high confidence levels, with a mean rating of 4.00 and standard deviations of 0.53. Respondents were particularly confident in providing local examples to convince students, with 41.6% rating their ability at the highest level and 44.2% at a high level. The ability to provide multiple methods to solve a problem also showed strong efficacy, with 52.5 % rating their efficacy as high and 34.3% at the highest level. Linking one mathematical concept to another was rated highly by 33.6% of respondents and moderately by 50.1 %, reflecting a strong overall ability in this area.

In conclusion, the survey results demonstrate that teachers are highly effective in mathematical creation for instruction, algorithm preparation, and content bridging and persuasion. Teachers are confident in creating mathematical definitions, problems, and examples, explaining procedures, and linking mathematical concepts. However, there is variability in axiomatic relations, with fewer teachers rating their confidence at the highest level. Overall, the findings indicate strong teaching efficacy among mathematics teachers.

### ***Relationship among Attitudes, Proficiency and Teaching Efficacy***

Partial Least Squares Structural Equation Modeling (PLS-SEM) was employed to analyze both the measurement and structural models. To find the relationship among content connection, pedagogical application, mathematical proficiency, and teaching efficacy, related hypotheses were tested using PLS-SEM.

**Measurement Model.** The Partial Least Squares-Structural Equation Modeling (PLS-SEM) was employed for the measurement model, and structural equation modeling was used to examine the impact of teachers' attitudes toward mathematical knowledge of real analysis on their teaching efficacy, indicating the relevance of mathematical knowledge of real analysis for school mathematics instruction. The measurement and structural models were developed using Smart PLS software, which facilitates comprehensive analysis through Partial Least Squares Structural Equation Modeling (PLS-SEM). This approach ensures precise evaluation of relationships between variables and validates the overall model's effectiveness. The models were evaluated following the guidelines provided by Hair et al. (2019). The bootstrapping method with 5,000 resamples was used to analyze the measurement and structural models, as recommended by the same authors. This process helps ensure the validity and reliability of the models, confirming their overall quality. Factor loading

indicates the relationship between an observed variable (indicator) and its corresponding latent construct. The acceptable outer loading ranges between 0.40 and 0.70, with values above 0.70 considered ideal (Hair et al., 2019; Hair et al., 2024).

**Table 13**

*Measurement Model Fit Criteria*

Construct	Items	Loading	P-value	AVE	CA (CR)	VIF
<b>Content Connection</b>				0.63	0.71(0.83)	
Algebraic Properties of real number connected with school mathematics Content	CONTN1	0.837	0.00*			1.475
Real Sequence and Series connected with school mathematics contents	CONTN2	0.647	0.00*			1.295
Concept of Calculus (limit, continuity, derivatives) connected with school mathematics curriculum	CONTN3	0.882	0.00*			1.586
<b>Pedagogical Application</b>				0.72	0.81(0.88)	
Pedagogical Application of Properties of Real Numbers	PAPLI1	0.827	0.00*			1.332
Pedagogical Application of Function, Limit, and Continuity	PAPLI2	0.891	0.00*			1.680
Pedagogical Application of Series and Proof Ideas	PAPLI3	0.834	0.00*			1.489
<b>Enhancing Teachers' proficiency</b>				0.65	0.73(0.84)	
Conceptual Understanding of Real Numbers	MPROF1	0.752	0.00*			1.608
Mathematical Reasoning and Fluency	MPROF2	0.867	0.00*			2.083
Problem Solving and proving statements	MPROF3	0.795	0.00*			1.822
<b>Mathematics Teaching Efficacy</b>				0.71	0.80(0.88)	
Mathematical Creation for Instruction	TEACH1	0.839	0.00*			1.757
Preparing algorithms for problem-solving	TEACH2	0.835	0.00*			1.687
Content bridging and persuasion	TEACH3	0.858	0.00*			1.682

p\* < 0.05

In this study the value of the factor loading of eleven indicators among twelve is more than 0.752 except one (CONTN1) however it is significant predictor. The result of factor loading was shown in Table 13 The factor loading values of all indicators range from 0.647 to 0.891 at a 5% level of significance. Hence, the factor loadings meet the criteria established by Hair et al., (2019a). Cronbach's Alpha (CA), composite reliability (CR), average variance extracted (AVE), the Fornell-Larcker criterion, and the Heterotrait-Monotrait (HTMT) ratio were calculated using Smart

PLS 4 software. The recommended value of quality criteria is  $CA \geq 0.70$ ,  $CR \geq 0.70$ ,  $AVE \geq 0.50$ ,  $HTMT\text{-ratio} \leq 0.85$ , and  $VIF \leq 3$  (Hair et al., 2024). The SmartPLS output results indicate that Cronbach's Alpha (CA)  $> 0.70$ , Composite Reliability (CR)  $> 0.80$ , and Average Variance Extracted (AVE)  $> 0.60$ , as shown in Table 13 The Fornell-Larcker criterion values and Heterotrait-Monotrait (HTMT) ratio values, presented in Table 14 meet the criteria suggested by (Hair et al., 2024). Therefore, all quality criterion for the measurement model are satisfied.

**Table 14**

*Fornell-Larcker Criterion Value (and HTMT Ratio)*

Construct	CONTN	MPROF	PAPLI	TEACH
CONTN	<b>0.80</b>			
MPROF	0.38(0.48)	<b>0.85</b>		
PAPLI	0.53(0.71)	0.66(0.85)	<b>0.81</b>	
TEACH	0.20(0.24)	0.34(0.41)	0.29(0.38)	<b>0.84</b>

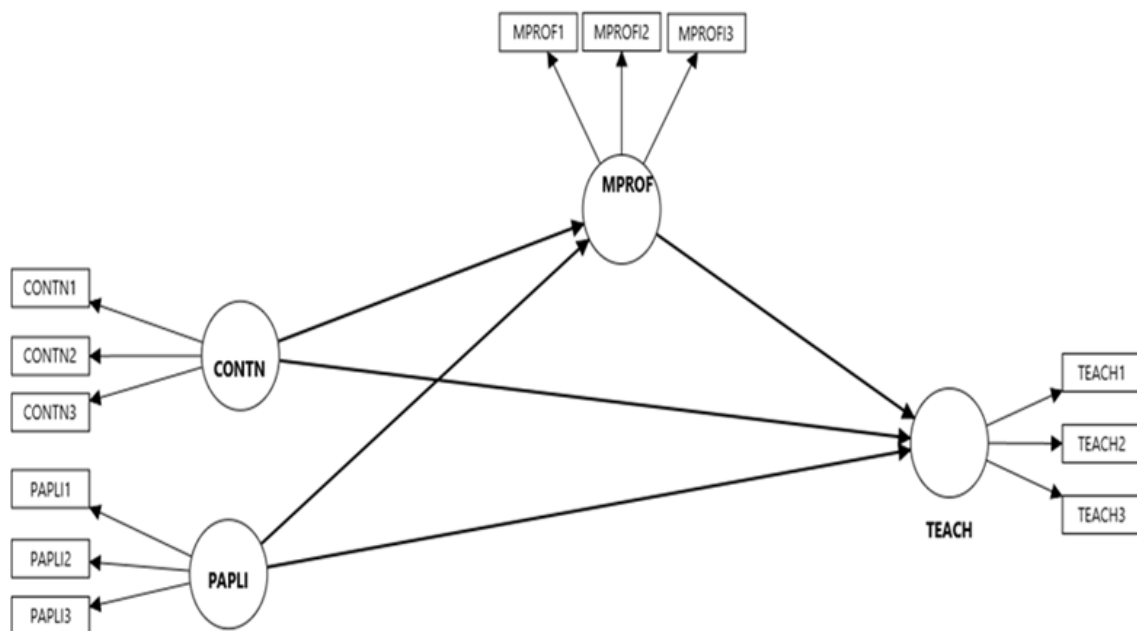
The value of CA, CR AVE, and HTMT ratio resulted from SmartPLS, shown in Table 14, ensures the internal consistency of the measurement and tools. The convergent validity was then examined through AVE analysis with values required to be more than 0.500 (Fornell & Larcker, 1981).

This analysis assesses the extent to which individual indicators reflect their respective constructs while comparing them with other indicators of different constructs. As shown in Table 13, the Average Variance Extracted (AVE) values exceed 0.50, thereby ensuring convergent validity. The values for the Fornell-Larcker test and HTMT ratio shown in Table 14 support the model validity suggested by Fornell and Larcker (1981). Based on these results, the measurement model is sufficiently reliable and consistent. Both convergent and discriminant validity are

established, indicating that the constructs measure what they are intended to. The AVE values exceed the recommended threshold of 0.50, and the HTMT ratios are below 0.90, further confirming construct distinctiveness. These findings validate the measurement model's robustness for subsequent structural model analysis.

**Figure 9**

*Measurement Model*



The measurement model consists of four latent constructs: Content Connection (CONTN), Pedagogical Application (PAPLI), Mathematical Proficiency (MPROF), and Teaching Efficacy (TEACH). Several observed indicators measure these constructs. Content Connection (CONTN) is measured by three indicators: CONTN1, which refers to the algebraic properties of real numbers connected with school mathematics content; CONTN2, which pertains to real sequences and series connected with school mathematics content; and CONTN3, which represents the concept of calculus (limit, continuity, derivatives) connected with the school mathematics curriculum. Pedagogical Application (PAPLI) is measured by three

indicators: PAPLI1, which addresses the pedagogical application of properties of real numbers; PAPLI2, which covers the pedagogical application of functions, limits, and continuity; and PAPLI3, which involves the pedagogical application of series and proof techniques. Mathematical Proficiency (MPROF) is measured by three indicators: MPROF1, which reflects conceptual understanding of real numbers; MPROF2, which assesses mathematical reasoning and fluency; and MPROF3, which focuses on problem-solving and proof techniques. Teaching Efficacy (TEACH) is measured by three indicators: TEACH1, which pertains to mathematical creativity for instruction; TEACH2, which addresses preparing algorithms for problem-solving; and TEACH3, which involves content bridging and persuasion skills. The model was analyzed using Partial Least Squares Structural Equation Modeling (PLS-SEM) to assess the validity and reliability of the constructs.

**Structural Model.** Based on the measurement model, the structural model was also computed using the Bootstrapping algorithm via SmartPLS software. The variance inflation factor (VIF) score for every construct is lower than the offending value of 3.3, as shown in Table 13. Hence, collinearity is not an issue in the model (Diamantopoulos & Siguaw, 2006). Table 15 depicts the path coefficient for each hypothesized relationship.

H<sub>5</sub>: Teachers' attitudes towards content connection significantly predict mathematical proficiency.

H<sub>6</sub>: Teachers' attitude towards content connection significantly predicts teaching efficacy.

H<sub>7</sub>: Teachers' attitudes towards knowledge of real analysis significantly enhance mathematical proficiency and predict teaching efficacy.

H<sub>8</sub>: Teachers' attitudes towards pedagogical application significantly predict mathematical proficiency.

H<sub>9</sub>: Teachers' attitude towards pedagogical application significantly predicts teaching efficacy.

**Table 15**

*Hypothesis Test Results from PLS-SEM*

Hypothesis	Relationship	Standardized Path Coefficient	Standard Error	T - statistic	P values	Decision
H <sub>5</sub>	CONTN -> MPROF	0.053	0.003	1.153	0.25	Not supported
H <sub>6</sub>	CONTN -> TEACH	0.066	0.004	1.165	0.24	Not supported
H <sub>7</sub>	MPROF -> TEACH	0.248	0.013	3.757	0.00	supported
H <sub>8</sub>	PAPLI -> MPROF	0.629	0.032	12.658	0.00	supported
H <sub>9</sub>	PAPLI -> TEACH	0.260	0.013	4.572	0.156	Not supported

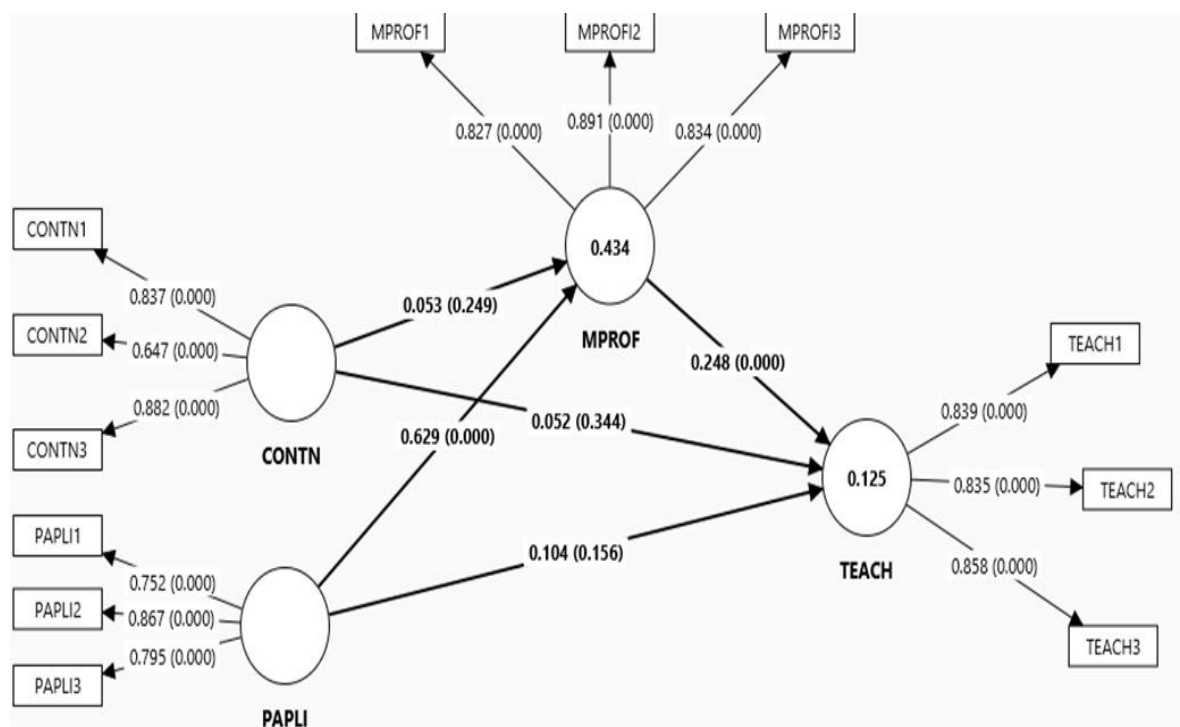
Models	Exogenous Variable	f <sup>2</sup>	Endogenous variables	Q <sup>2</sup>	R <sup>2</sup>	P-values	SRMR
I	CONTNT	0.004	MPROF	0.423	0.434	0.000	0.079
	PAPLI	0.500					
	MPROF	0.04					
II	CONTNT	0.002	TEACH	0.077	0.125	0.000	
	PALI	0.006					

Among the hypothesized relations, H<sub>5</sub>, H<sub>6</sub> and H<sub>9</sub> are not supported. Therefore, mathematics teachers' attitudes towards the content connection between school mathematics and knowledge of real analysis do not predict mathematical proficiency and teaching efficacy. The hypothesized relations H<sub>7</sub> and H<sub>8</sub> are significant at a 5% significance level. Hence, teachers' attitudes towards the pedagogical application of real analysis significantly predict the teachers' attitude towards knowledge of real analysis, enhancing the teachers' mathematical proficiency but not teaching efficacy.

Additionally, teachers' attitude that knowledge of real analysis enhances teachers' mathematical proficiency significantly predicts the teaching efficacy at 5% level of significance.

**Figure 10**

*Bootstrap Result of Structural Model*



The values of f-squared ( $f^2$ ), R-squared ( $R^2$ ), Q-squared ( $Q^2$ ), and standardized root mean square residual (SRMR) are presented in Table 15, which collectively provide insight into the model's explanatory power, predictive relevance, and overall fit. The SRMR value was 0.079, which is below the recommended threshold of 0.080, indicating that the model demonstrates a good fit to the data (Hu & Bentler, 1999).

In the first part of the model, Content Connection (CONTNT) and Pedagogical Application (PAPLI) were specified as exogenous (independent) variables predicting Mathematical proficiency (MPROF). The  $R^2$  value was 0.434, indicating that these two predictors explained 43.4% of the variance in mathematical proficiency.

Hair et al. (2024) identify this as a high level of explanatory power. The corresponding  $Q^2$  value was 0.423, which exceeds the threshold of 0.35, indicating large predictive relevance (Hair et al., 2024). The effect size ( $f^2$ ) for PAPLI was 0.50, representing a large effect (Hair et al., 2019b), while the  $f^2$  value for CONTNT was only 0.004, indicating a negligible effect.

In the second part of the model, Mathematical proficiency (MPROF), Content Connection (CONTNT), and Pedagogical application (PAPLI) were used to predict teaching efficacy (TEACH). The  $R^2$  value for teaching efficacy was 0.125, meaning the model explained only 12.5% of the variance in this outcome. The  $Q^2$  value was 0.007, which is slightly above zero but well below 0.02, indicating very weak predictive relevance (Hair et al., 2024). The individual effect sizes were all small: MPROF ( $f^2 = 0.04$ ), CONTNT ( $f^2 = 0.002$ ), and PAPLI ( $f^2 = 0.006$ ), indicating that none of the predictors had a strong individual influence on teaching efficacy.

This result suggests that pedagogical application of real analysis is a strong and meaningful predictor of mathematical proficiency, both in terms of its effect size and predictive relevance. This highlights the importance of integrating pedagogical knowledge with mathematical content in teacher preparation. On the other hand, content connection, while theoretically relevant, had little predictive value in this context. The model's explanatory and predictive power for teaching efficacy was limited, indicating that other unmeasured factors, such as classroom experience, teacher beliefs, or professional development, play a more significant role. These results highlight the need to expand the model in further studies to capture the complexities of relationship of considered constructs.

**Moderator Effect Analysis.** The literature suggests that academic qualifications and teacher training may act as moderating factors influencing

mathematical proficiency and teaching efficacy. Therefore, academic qualifications (ADQ) and teacher training (TR) are considered moderating variables to assess their impact and ensure they do not significantly influence these factors. Academic qualification (ADQ) does not moderate the relationship between content connection attitude (CONTN) and attitude that knowledge of real analysis is a means for enhancing mathematical proficiency (MPROF).

**Table 16**

*Moderating Effect Test Result*

Exogenous variable → Endogenous variable	F <sup>2</sup>	R <sup>2</sup>	P-value	SRMR
ADQ x CONTN → MPROF	0.395			
ADQ x PAPLI → MPROF	0.976			
TR x PAPLI → MPROF	0.469			
TR x CONTN → MPROF	1.000	0.451	0.00	
ADQ x CONTN → TEACH	0.814			
ADQ x MPROF → TEACH	0.582	0.147	0.00	0.071
ADQ x PAPLI → TEACH	0.829			
TR x PAPLI → TEACH	0.958			
TR x CONTN → TEACH	1.000			

Path direction	Path coefficient	Standard error	T	p-value
ADQ x CONTN → MPROF	-0.068	0.002	1.5563	0.120
ADQ x CONTN → TEACH	-0.039	0.003	0.6489	0.516
ADQ x MPROF → TEACH	-0.094	0.004	1.2344	0.217
ADQ x PAPLI → MPROF	-0.012	0.003	0.2067	0.836
ADQ x PAPLI → TEACH	0.054	0.007	0.6167	0.537
TR x PAPLI → MPROF	-0.215	0.004	1.5667	0.117
TR x PAPLI → TEACH	0.076	0.007	0.2684	0.788
TR x CONTN → MPROF	0.001	0.014	0.0056	0.996
TR x CONTN → TEACH	-0.002	0.006	0.0121	0.990
TR x MPROF → TEACH	-0.190	0.010	0.9702	0.332

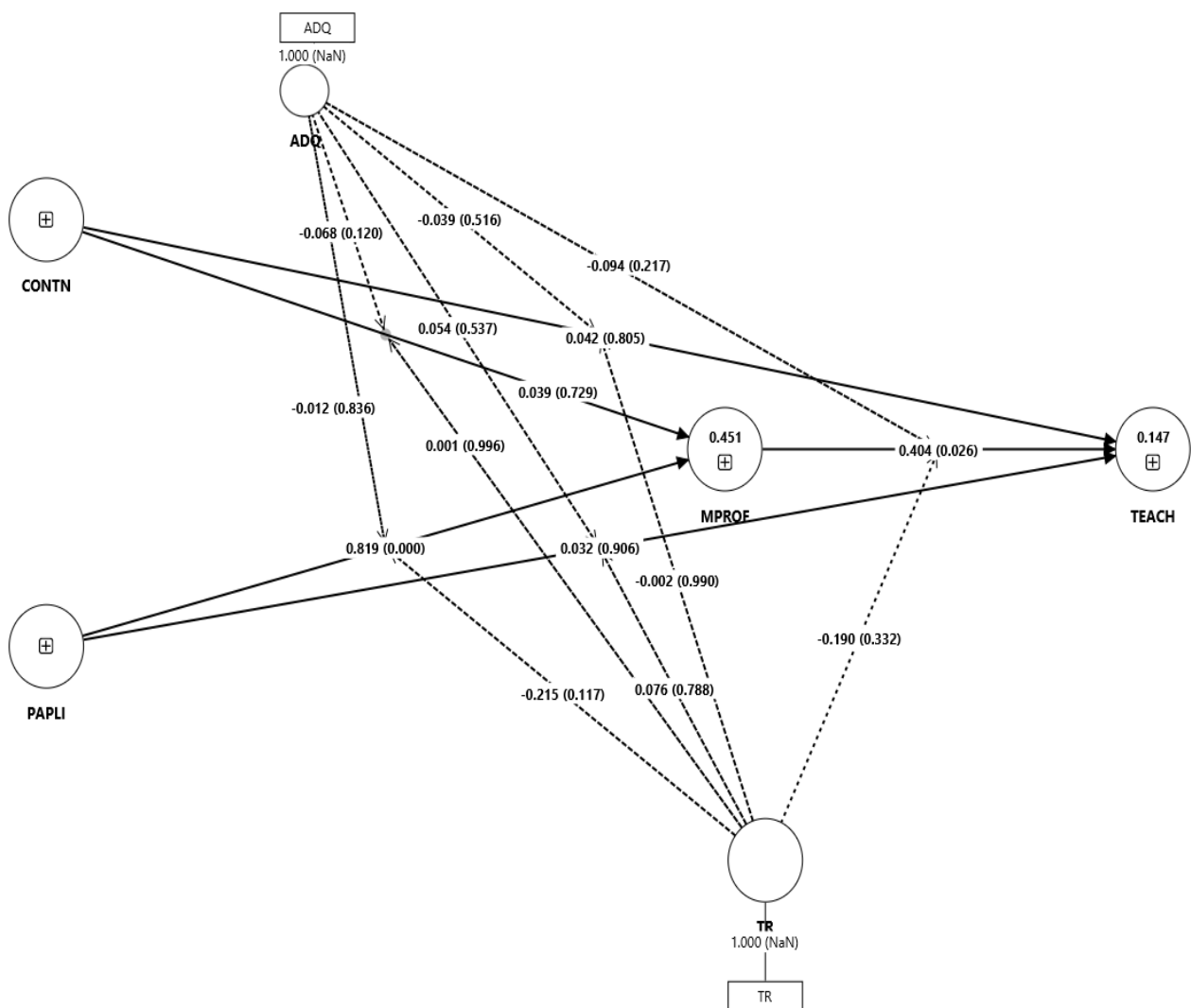
Notably, when examining the path coefficient for the moderation of ADQ on CONTN → MPROF, the following was observed: a coefficient of -0.068, a t-statistic of 1.5563, and a p-value of 0.120, which exceeds the threshold for significance (0.05).

This means that Academic Qualification did not strongly affect the strength of the relationship between Content Connection and Mathematical Proficiency.

Similarly, the interaction effect of academic qualification (ADQ) and content connection attitude (CONTN) on Teaching Efficacy (TEACH) was also insignificant. With a path coefficient of -0.039 and a p-value of 0.516, academic qualification did not significantly affect content connectivity and teaching efficacy. Therefore, the moderator effect is not significant. Figure 11 depicts the results of the moderator analysis output.

**Figure 11**

*Bootstrap output of Moderator Analysis*



The moderation effect of ADQ on the relationship between Pedagogical Application (PAPLI) and Mathematical Proficiency (MPROF) was also found to be insignificant. The path coefficient for  $ADQ \times PAPLI \rightarrow MPROF$  was -0.012, with a p-value of 0.836, indicating that Academic Qualification does not moderate the impact of Pedagogical Application on Mathematical Proficiency.

There was no noticeable moderating influence of Teacher Training (TR) on the connection between Mathematical Proficiency (MPROF) and Pedagogical Application (PAPLI). With a p-value of 0.117 and a path coefficient of -0.215 for  $TR \times PAPLI \rightarrow MPROF$ , this suggests a negative but non-significant moderation effect.

Additionally, with a path coefficient of 0.076 and a p-value of 0.788, Teacher Training (TR) did not significantly moderate the effect of Pedagogical Application (PAPLI) on Teaching Efficacy (TEACH). This demonstrates that the degree of teacher training had no bearing on the impact of pedagogical application on teaching efficacy.

Finally, with a p-value of 0.996 and a path coefficient of almost zero (0.001), it was determined that Teacher Training (TR) had no moderating effect on the association between Content Connection (CONTN) and Mathematical Proficiency (MPROF).

### **Phase ii: Analysis and Interpretation of Qualitative Data**

Regarding the relationship among teachers' attitudes toward content connection, pedagogical application, mathematical proficiency, and teaching efficacy, the quantitative results revealed that mathematical proficiency significantly predicts teaching efficacy ( $p=0.00 < 0.05$ ), and pedagogical application significantly predicts mathematical proficiency ( $p=0.00 < 0.05$ ). The PLS-SEM results further indicated that the combined effect of pedagogical application and content connection on mathematical

proficiency is significant, with large predictive relevance ( $Q^2 = 0.42$ ,  $R^2 = 43.4\%$ ) (Hair et al., 2024). However, content connection does not significantly predict either mathematical proficiency ( $p = 0.24 > 0.05$ ) or teaching efficacy ( $p = 0.16 > 0.05$ ). Despite this, the overall combined effect of content connection, pedagogical application, and mathematical proficiency on teaching efficacy is statistically significant, although it exhibits small predictive relevance ( $Q^2 = 0.07$  and  $R^2 = 12.5\%$ ).

These results did not align with theoretical expectations and raised important questions regarding how teachers perceive and apply their knowledge of real analysis in classroom contexts. For further explanation of these inconsistencies, a positive attitude towards content connection, pedagogical application, and a qualitative inquiry was essential to capture the depth, context, and meaning behind teachers' experiences, interpretations, and instructional practices.

The qualitative inquiry included curriculum analysis, classroom observations, task-based interviews, and interviews with mathematics educators. This section presents the socio-demographic characteristics of the participants involved in the qualitative phase, and the results emerge from curriculum analysis, interviews, and classroom observations.

### ***Socio-demographic Status of Respondent***

Eight teachers participated in classroom observation and task-based interviews. Additionally, three mathematics educators were interviewed to find out why real analysis is required for a teacher preparation program, if a connection is required, and how to establish the link between secondary-level mathematics and real analysis. The socio-demographic status of teachers and their qualification is shown in Table 17. The socio-demographic characteristics of the teachers participating in the qualitative inquiry of this study include age group, academic qualifications, years of

teaching experience, gender, and training status. All sample teachers hold a master's degree, have received professional training, and are male. Among the teachers, two aged 50 or older, while the remaining six fall within the 30–40 age group.

**Table 17**

*Socio-demographic Status of Teachers Involved in Qualitative*

S.n.	Pseudo name of teachers	Age group in year	Qualification	Experience in year	Gender	Training
1	Ram	≥ 50	Master	≥ 20	Male	Trained
2	Shyam	≥ 50	Master	≥ 20	Male	Trained
3	Hari	40-50	Master	18	Male	Trained
4	Rajan	30-40	Master	15	Male	Trained
5	Kapil	30-40	Master	17	Male	Trained
6	Govinda	30-40	Master	10	Male	Trained
7	Milan	30-40	Master	12	Male	Trained
8	Saroj	30-40	Master	10	Male	Trained

Two educators have over 20 years of teaching experience, two have 10 years of experience, one has 12 years of experience, and the rest have between 15 and 18 years of teaching experience. Table 18 shows the status of the participating mathematics educators.

**Table 18**

*Socio-Demographic Status of Mathematics Educators*

S.n.	Gender	Age group in years	Academic Status	Experience in years	Pseudo name
1	Male	≥ 50	Professor	More than 20	Expert 1
2	Male	≥ 50	Professor	More than 20	Expert 2
3	Male	≥ 50	Professor	More than 20	Expert 3

All the mathematics educators interviewed were male professors aged 50 or older with more than 20 years of university teaching experience.

### ***Teachers' Perception of Knowledge of Real Analysis***

In the first phase, results showed that secondary-level mathematics teachers had positive attitudes towards mathematical knowledge of real analysis regarding content connection, pedagogical application, and enhancing their mathematical proficiency. Following this, an inquiry was conducted to assess their perceptions and identify the key focus. To answer the research question RQ3: *How do teachers perceive mathematical knowledge of real analysis concerning content connection, pedagogical application, and enhancing mathematics proficiency?* Task-based interviews were conducted with secondary-level mathematics teachers. The codebook for the thematic analysis of task-based interview data is presented in Table 19.

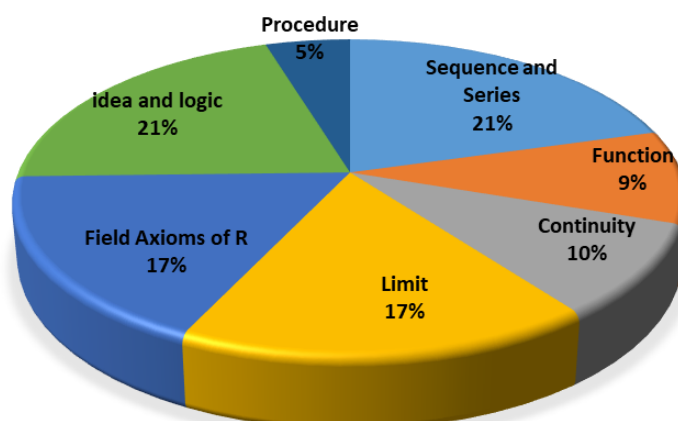
**Perception on Content Connection.** The task-based interview results showed that teachers perceive three types of content connections between mathematical knowledge of real analysis and secondary-level mathematics, as revealed by thematic analysis. Among the themes, the status was as shown in Table 19. The pie chart illustrates that, under the content connection category, teachers perceive a one-fifth (21%) connection between sequence and series in real analysis and school mathematics content. This indicates that teachers find substantial relevance between these topics in university-level real analysis and the concepts taught at the school-level.

Similarly, Idea and Logic also hold nearly one-fifth (21%) connection, suggesting that foundational reasoning and logical structures in real analysis significantly contribute to school mathematics.

**Table 19***Summary Table of Theme on Teacher's Perception*

Category	Code	Theme
Content connection	Sequence and Series	Direct content connection
	Function	
	Continuity	
	Limit	
	Field Axioms of $\mathbf{R}$	
Pedagogical application	idea and logic	Logical connection
	Procedure, process	Procedural connection
	Problem solving, theorem proving,	Mathematical activities
Enhancing mathematical proficiency	Idea creation, thinking of solutions, creating alternative way	Idea creation
	Reasoning in mathematical context, theorem proving, identifying logic behind fact	Mathematical reasoning
	Advancement of knowledge and skill, increasing knowledge, and gaining knowledge	Development of Mathematical Knowledge and Skills
	Know theory and application	Mathematical Reasoning and Identification of Relationships
	Logical reasoning ability	

The Field Axioms of  $\mathbf{R}$  (17%) and Limit (17%) also play crucial roles, implying that the formal properties of real numbers and the concept of limits are moderately integrated into school-level teaching.

**Figure 12***Content Connection Status*

On the other hand, Function (9%) and Continuity (10%) hold comparatively lower connections, indicating that while these topics are essential in real analysis, their direct linkage to school mathematics is perceived to be less pronounced. The Procedure category (5%) is the least represented, suggesting that procedural aspects of real analysis are not a primary concern for teachers when connecting content to school mathematics.

Further analysis of their perceptions regarding these connections is explained in separate topics according to the nature of the connections.

***Direct Content Connection.*** The similarity of topic and introductory context reflects the direct content connections. Teachers showed the direct content connection scenario and evidence during the task-based interview. Some evidence from the interviews is presented below.

Researcher: Here is a series  $1 + \frac{1}{2^2} + \frac{1}{3^3} + \dots$ , what is the result of this series, and do you observe and practice in school mathematics content?

Hari: Such cases are practiced in teaching limits, as they lead to a fixed value or an unknown quantity. The result is fixed and close to 1.

Researcher: Besides this case, what other mathematical concepts have you experienced that are linked with real analysis

Hari: Sequences and series, functions, limits, and continuity are linked with analysis and calculus. The introductory concepts are covered in school, while more advanced contents are taught at the campus level.

A similar response was recorded from another teacher.

Kapil: In the case of functions, order pairs, function types, and inverse functions, these topics are introduced in school mathematics and later revised

in campus-level courses. Similarly, sequences and series, limits, and continuity appear at both levels, though with greater depth in higher education.

The outcomes from the task-based interview highlight the direct content connection between real analysis and secondary-level mathematics. Teachers reported that fundamental concepts, such as sequences and series, limits, continuity, and functions, are introduced in school mathematics and further explored in depth at the university level during teacher preparation programs. Their responses revealed that the conceptual background on sequence, series, function, and limit continuity is introduced in school mathematics content, and this content is further developed in the analysis introduced in the teacher preparation program.

***Axiomatic Connection.*** Axiomatic connection refers to the connection of axioms in the presented mathematical content. The axioms practiced in real analysis guide the real practices in school mathematics content.

Axiomatic connection refers to the relationship between the foundational axioms in advanced fields, such as real analysis, and the schools' mathematics content. Real analysis encompasses a set of axioms that establish the properties of the real number system, including limits, continuity, and other concepts essential for laying the foundation of rigorous mathematical reasoning. While school mathematics does not always explicitly introduce axiomatic systems, many concepts students can learn, such as number properties, operations, and functions, are fundamentally grounded in these axiomatic principles.

During the task-based interview, teachers demonstrated and explained the axiomatic connection between real analysis content and the content of secondary-level mathematics. Milan said:

When we are studying analysis, we call the commutative property  $x+y=y+x$ , and closure, if  $x, y \in \mathbf{R}$ , then  $x, y \in \mathbf{R}$ , for all  $x, y$ . We felt difficult to understand it but now it is in school and practicing in grade 7 and 8. so we know or not little concept connected from lower to higher classes. We teach  $AB = BA$  not hold in matrix; its hidden matter is in real analysis. The case different according to condition and set and operations.

Similarly, Hari said, "Students struggle with applications like  $2x^2 + 6x = 2x(x + 3)$ . Understanding why we take common factors requires a strong grasp of the distributive property, which is rooted in real analysis." Additionally, Govinda said "For example, in real analysis, we study foundational axioms, which are practiced based on numbers. Concepts like the commutative law and distributive law are extensively applied to strengthen understanding through practical applications." Saroj demonstrated that, when  $1+2-4 = -1$ , but behind this fact,

$$\begin{aligned}
 & 1+2- (2+2) \text{ factor 4} \\
 & = 1+2-2-2 \text{ by distributive law} \\
 & = 1+ 0 -2 \quad \text{by using law of identity and inverse} \\
 & = 1 - (1+1) \text{ by factor of 2 in addition law} \\
 & = 1-1-1 = 0-1 = -1, \text{ where the inverse and identity laws are used.}
 \end{aligned}$$

By our memory, we give a direct result, but the fact is found in real analysis and the law provided by it.

In this example, the teacher uses identity law, which is an inverse law, as it comes under the field axiom. Additionally, the teacher uses Piano's axiom to expand 2.

The teachers' statements reveal that they perceive a clear connection between the field axioms mentioned in real analysis and the concepts taught in secondary-level mathematics. This demonstrates that the fundamental rules of real analysis, such as the

commutative and distributive properties, are essential not only for advanced mathematics but also for school-level mathematics. Teachers believe that learning these concepts in higher education helps students make sense of what they study in secondary school. This highlights how real analysis supports and strengthens school mathematics.

**Logical Connection.** In mathematical activities, different logics, such as methods and proving ideas of varying content levels, may be practiced at a higher level. This case refers to the logical connection. Regarding real analysis and school mathematics, the teacher reported evidence of a logical connection between knowledge of real analysis and secondary-level mathematics. In a task-based interview, one question, 'What is the supremum and infimum of the given set value of  $\{\frac{1}{n}\}, n \in N$  ', was asked. In response to this question, Saroj said:

The infimum is 1 in the set of natural numbers, so it is often used in school as a small number or a set member. In grade nine, we teach the concept of limits, where we create increasing and decreasing sequences using the set of natural numbers. For example, we observe terms like 1, 2, 3, and so on in an increasing sequence, which grow infinitely larger. In a decreasing sequence, we see terms like  $1, \frac{1}{2}, \frac{1}{3}$  and so on, which approach zero. In both cases, we discuss where the sequence tends towards infinity or zero.

Similarly, Govinda said "in optional mathematics, many problems require applying mathematical induction to derive the general term of a sequence or prove specific formulas. It's a fundamental method we frequently use to analyze sequences and series systematically." Additionally, Rajan said:

When we are teaching sets in grade nine, there are several problems, especially in verbal and set notation, where we have to teach their maximum and minimum members, without which students cannot properly solve the

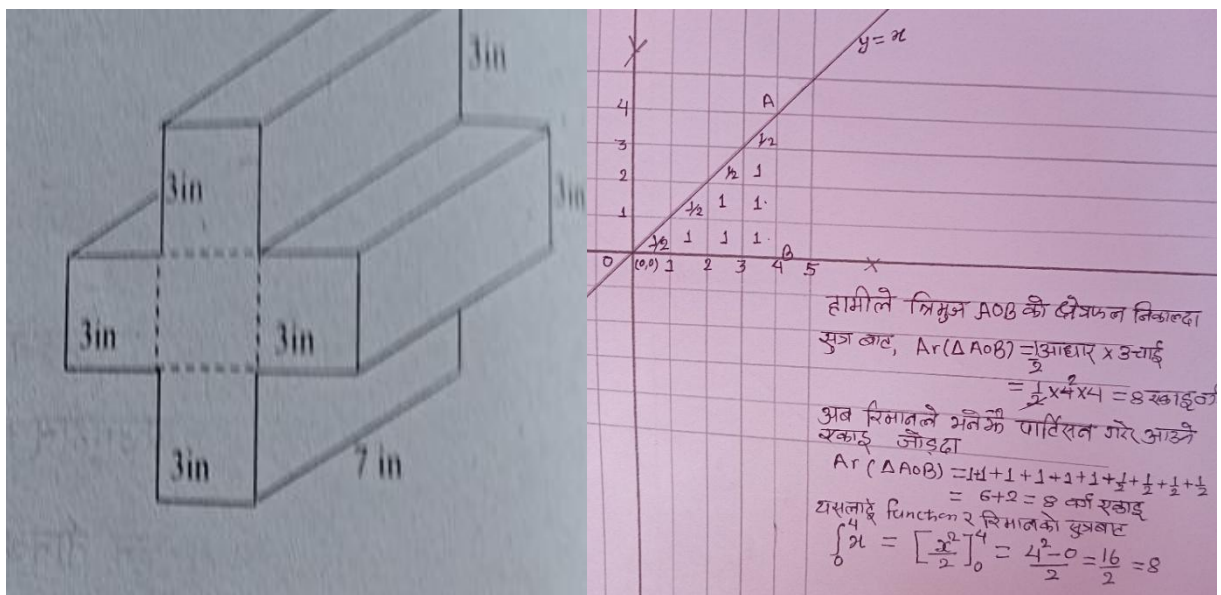
problem. For instance,  $A = \{x: x \text{ is a natural number less than } 10\}$ , in this case, the smallest member of the set is 1, and the largest value of the set  $A$  is 9. This type of concept is coming in different properties of real number like even odd, ...

The aforementioned evidence provided by the participants indicates that they perceived the underlying logic and concepts of real analysis as being relatively connected to school mathematics activities. During the second visit, Milan claimed that "logic used in the Riemann integral is applied in school mathematics." He assured to provide evidence in the third visit. In that visit, he clarified the computation of the area of a cross-section, where partitioning is used to determine the area. Besides this claim, he provided one piece of evidence, which I recorded as shown in Figure 13.

In this evidence, he described how the logic of the Riemann integral, as learned in real analysis, is logically connected to school mathematics. In the first part of Figure 13, he explained the process of partitioning.

**Figure 13**

*Logical Connection of Riemann Integral in Area Calculation*



In the right part of Figure 13, he presented both a curve-based and a partition-based example, demonstrating that they yield the same result, thereby ensuring the logical connection between the concepts. Therefore, a logical connection exists between secondary-level mathematics and the content of real analysis.

***Procedural Connection.*** Among the generated themes under the content connection between real analysis and school mathematics, the procedural connection is one. Procedural connection in mathematics refers to the relationship between different mathematical procedures or algorithms that build on each other across different concepts, topics, or levels of learning. During the task-based interview, the teachers stated the procedural connections. In case of testing existence of limit Ram said, "...exercise is less but, in SEE (Secondary Education Examination), questions have been included that demonstrate the existence of a limit by equating the right-hand limit and left-hand limit." It means teacher used the process of checking the existence of limit which is a part of real analysis. Similarly, Hari said, "The process of checking whether a limit exists follows a structured approach similar to the principles learned in real analysis." The Teacher mentioned that the existence of right-hand limit and left-hand limit is a process of checking the existence of limit practiced in school. Additionally, the teacher induction process was used to find the general term of the series. Kapil expressed that:

We practice the process of induction to find the general term of a sequence, for example, find the general term of the sequence 8, 11, 14, 15..... , In this question, we apply induction as

$$8 = 3 \times 1 + 5$$

$$11 = 3 \times 2 + 5$$

$$14 = 3 \times 3 + 5$$

$$15 = 3 \times 4 + 5$$

.....  
 .....  
 $t_n = 3n+5$

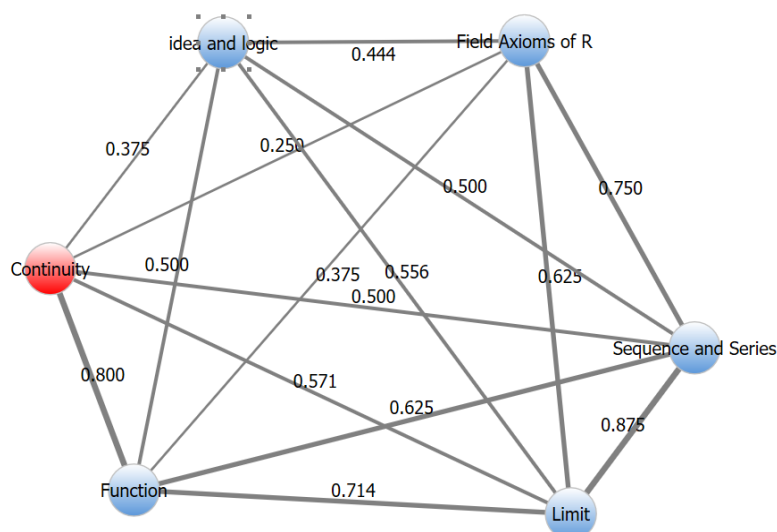
Thus, the process and ideas expressed in real analysis are less or more connected with the problem-solving process of secondary-level mathematics.

Link analysis was performed within the content connection to identify the co-occurrence strength using QDA Minor 2024, classical scaling iteration 500, and tolerance 0.0000001. The result is shown in Figure 14. In the link analysis result, continuity is the target point. The Field Axioms of R were considered the target variable as they form the foundation of mathematical structures essential for school-level mathematics.

The results indicate that Sequence and Series (0.750), Function (0.556), and Limit (0.500) exhibit the strongest co-occurrences with the Field Axioms of R, suggesting that these concepts rely heavily on algebraic properties and arithmetic operations. Continuity (0.375) and Idea and Logic (0.375) exhibit a moderate association, indicating a structural yet less direct connection to the field axioms.

**Figure 14**

*Link Analysis Result of Content Connection*



Procedure (0.125) has the weakest link, indicating that procedural aspects in school mathematics are less directly dependent on axiomatic foundations but are still relevant. These results emphasize the hierarchical nature of mathematical learning, where axiomatic principles underpin higher-level concepts, reinforcing the structured development of mathematical reasoning in educational curricula.

Although teachers demonstrated strong recognition of the content connections between real analysis and secondary-level mathematics, particularly in sequences, limits, and functions, these connections were often framed at a recognition level rather than a transformative or operational level. Teachers could identify where concepts aligned, but did not consistently translate this awareness into deeper instructional actions or self-perceived gains in mathematical proficiency. As supported by classroom observations, many content elements from real analysis were treated as accepted truths (e.g., axioms or algebraic properties) rather than as opportunities for student-centered exploration or reflection. This might explain why the statistical analysis failed to show a significant predictive effect of content connection on mathematical proficiency or teaching efficacy. Recognizing a content link may not be sufficient to enhance proficiency or confidence unless accompanied by robust pedagogical enactment and reflection. Therefore, the pathway from content connection to efficacy appears to be mediated by the depth of pedagogical integration, as was reflected in teachers' successful use of real analysis for reasoning and activity design, which did predict mathematical proficiency.

### ***Perception towards Pedagogical Application***

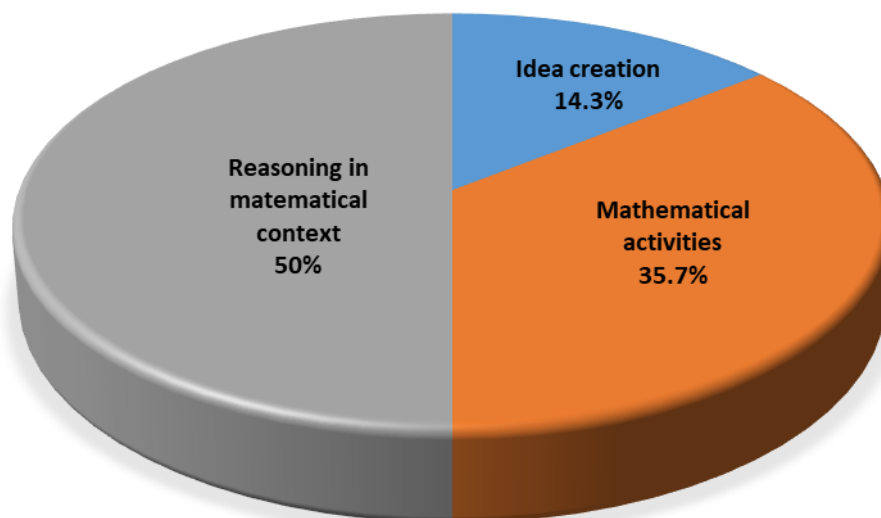
The pedagogical application of knowledge of real analysis refers to instances where teachers incorporate concepts of real analysis into their instructional procedures. The classroom practices of mathematical knowledge of real analysis

indicate the pedagogical application of this knowledge for school mathematical instruction. A positive attitude toward this was observed during the survey phase. This section explores teachers' perceptions through a task-based interview, focusing on specific scenarios. The thematic analysis result indicates three significant aspects of the pedagogical application of knowledge of real analysis, reflecting how teachers perceive its relevance to secondary-level mathematics instruction. The code-based thematic pie chart in Figure 15 demonstrates the status. Among these, the primary application is in mathematical activities (35.7%), indicating that teachers perceive real analysis as significantly contributing to the development and engagement of students in mathematical tasks.

However, reasoning in mathematical contexts (50.0%) holds the highest proportion, emphasizing that teachers view real analysis as a critical foundation for logical reasoning and problem-solving in school mathematics. Meanwhile, idea creation (14.3%) is perceived as a less dominant, yet essential, aspect of applying real analysis in teaching.

**Figure 15**

*Status of Pedagogical Application of Real Analysis*



Teachers' perceptions of the pedagogical application of real analysis are explained based on the identified themes, highlighting how they understand and manage their role in their teaching practices.

*Mathematical Activities.* During the task-based interview, teachers demonstrated that they applied the mathematical knowledge of real analysis during their instruction. Mathematical activities involve problem-solving and applying mathematical theories and ideas to teach mathematics.

Based on teachers' perceptions, the pedagogical application of real analysis in school mathematics instruction plays an important role in various aspects of teaching. Teachers observe the number system as fundamental in structuring mathematical concepts. For instance, Shyam pointed out, "We use the number system for writing different sets and solving probability problems based on sets of numbers." This statement highlights how knowledge of real analysis assists in organizing mathematical reasoning and categorization during lessons. Shyam elaborated that

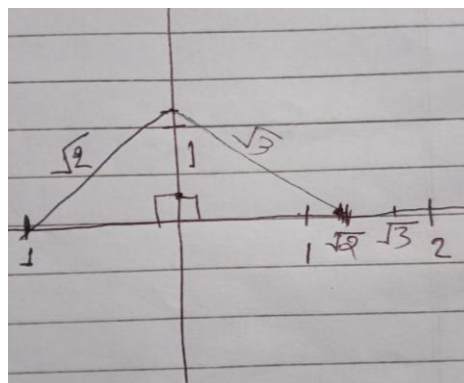
It is has been practicing in teaching set for example  $U = \{x: x \in N, x < 20\}$  this concept is also notational concept without understanding even number are 2, 4, ... and meaning of  $x < 20$  students cannot write the exact elements of  $U$  similarly different problems based on such concept we are teaching in grade six to 10. Therefore, the concept of real numbers and their properties is used to prepare content, answer questions, etc.

Additionally, teachers emphasized the importance of field axioms, such as commutativity, associativity, and distributivity, in algebraic problem-solving. Shyam explained, "Field axioms are used daily when solving algebra problems", while Govinda elaborated, "Especially axioms and theorems are closely linked with teaching activities. For example, in real analysis, we study foundational axioms,

which are based on numbers." This indicates how real analysis concepts are integrated into teaching to reinforce students' procedural fluency and conceptual understanding. Beyond foundational algebra, real analysis is applied in more advanced topics such as linear programming and inequalities. Shyam noted, "In optional mathematics, it is used in linear programming, and the knowledge of inequalities is essential for solving the problems within this chapter." This shows how knowledge of real analysis supports higher-level mathematical reasoning and problem-solving. Teachers also view real analysis as a tool for enhancing students' conceptual understanding through practical applications. Govinda remarked, "Concepts like commutative and distributive laws are extensively applied to strengthen understanding through practical applications." This suggests that teachers incorporate theoretical principles and hands-on activities to deepen students' comprehension of mathematical concepts.

The aforementioned discussion reflects that in their daily activities, teachers apply the knowledge of real analysis to various aspects of mathematical instruction, such as problem-solving, concept teaching, and defining types of numbers (like even and odd numbers). They also use real analysis to create sets, convert verbal problems into mathematical ones, and structure mathematical concepts.

**Idea Creation.** The idea creation refers to the spontaneous mental manner of generating solutions to mathematical problems. It includes thinking critically and creatively, drawing on one's cultivated mathematical thinking skills. This mental



function enables individuals to gain insight into strategies to solve problems effectively. During the task-based interview, teachers shared such scenarios. In this context, Ram elaborated his experience:

When I was teaching optional mathematics, there was a problem showing  $\sqrt{2}$  in a number line, and it was in the book. but students ask me can we show  $\sqrt{3}$ , in this situation I was get confuse it is possible or not, this was I learnt in modern mathematics, then I, suggest them to make  $\sqrt{2}$  in line and spontaneously, I think, after making line of  $\sqrt{2}$  if we make base or perpendicular  $\sqrt{2}$  and another side is 1 unit then its hypotenuse  $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$ , then which was I think during the instruction.

Ram elaborated that he first located  $\sqrt{2}$  on a real line and used it as a base. With the height of 1 unit already in place, he then constructed a new hypotenuse, which, according to the Pythagorean theorem, has a length  $\sqrt{3}$ .

Similarly, Shyam shared his experience as:

During my interview, I was given a rubber band and asked if I could teach the concept of real numbers using it. My response was, "Yes." I stretched the rubber band and used it to explain that there are infinitely many real numbers between any two points (representing real numbers). This idea came to me suddenly, and I believe it was inspired by the deeper understanding I gained during my advanced studies.

During the interview, the teacher shared experiences related to idea creation.

Additionally, Kapil pointed out that "the ideas of real analysis useful in teaching for creating new ideas for solving mathematical problem." Therefore, teacher used the

mathematical knowledge of real analysis for creating ideas that are applicable and suitable for instruction.

Based on the teacher's experience and views, it can be concluded that knowledge of real analysis nurtures the development and maturation of mathematical thinking necessary for effective instruction. It enables learners to cultivate deep insights and apply their knowledge immediately to problem-solving that is connected to real analysis.

***Reasoning in Mathematical Context.*** Providing reasons is essential in immediate settings where individuals engage in mathematical discussions involving proof, disproof, and the justification of why a mathematical statement is true or false. Thus, logical argumentation in a mathematical context defines reasoning in mathematics.

Teachers present different scenarios of reasoning in a mathematical context. They perceive that knowledge of real analysis develops reasoning abilities in various mathematical contexts, such as proving statements, providing examples and counterexamples, and explaining why a statement is true or false.

During the school's mathematical instruction, Govinda observed several contexts where he needed to provide argumentation. The cultivation of argumentation in mathematics is supported by knowledge of real analysis. For example, the sum of two even numbers is always even. Teacher uses different methods with their students. In this scenario, Govinda highlighted inductive and deductive reasoning when exploring the properties of numbers. Govinda explains, "When working the in number system, I use inductive and deductive reasoning." This statement indicated the different types of reasoning used by Govinda in different mathematical explorations. Further, he explained:

For example, consider that the sum of two even numbers is always an even number. While deductive reasoning can be applied to prove this statement by using the definition of even numbers (numbers divisible by 2), inductive reasoning might involve observing multiple instances and generalizing the conclusion. In the case of inductive reasoning, one might begin by testing simple cases such as  $2 + 4 = 6$ ,  $4 + 6 = 10$ , and so on and then hypothesize that the sum of any two even numbers will always result in an even number.

In this way, Govinda highlighted a reasoning technique in school mathematics teaching closely related to knowledge of real analysis. Similarly, Shyam explored that,

I want to share an experience. During an interview, I was given three equal segments and asked, "How would you teach a right-angled triangle using these?" Immediately, I thought that for a triangle to form, the sum of the two sides must be greater than the third side. Based on this, I concluded that these three segments could not form the hypotenuse of a right-angled triangle. I then presented this reasoning during the interview. In this way, the ability to quickly generate mathematical reasoning and problem-solving is a result of the conscious and in-depth study of higher-level mathematics, which I believe is essential.

From the above-mentioned scenario of logical reasoning and its power in mathematical context and reasoning, teachers use mathematical knowledge of real analysis for reasoning during their instruction.

The link analysis was conducted using QDAMiner software to identify the relationship among the constructs in the pedagogical application category. The output is shown in Figure 16. The results indicate that, the practical pedagogical applications

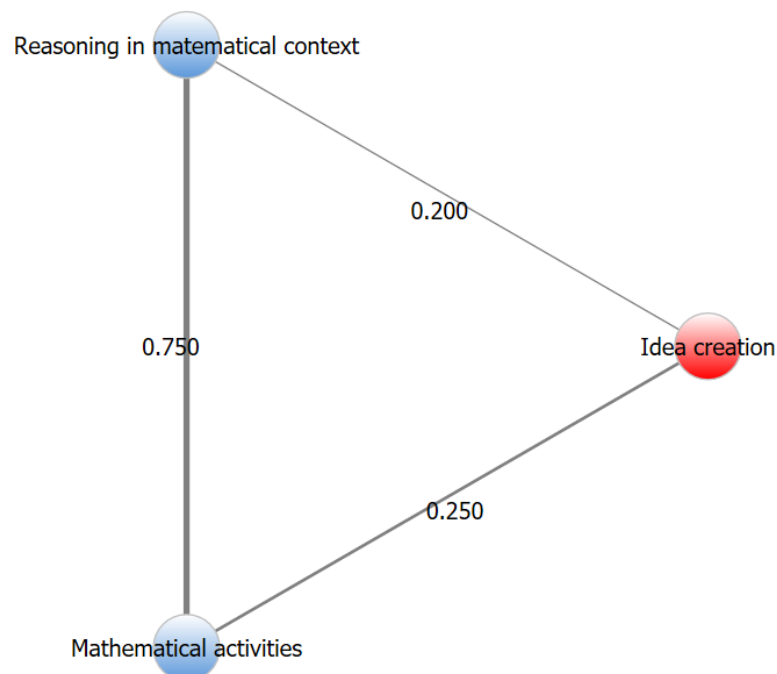
of real analysis knowledge contribute to the development of reasoning skills, thereby enhancing students' engagement in mathematical activities. Fostering logical thinking, problem-solving abilities, and structured exploration offers a strong foundation for promoting active and meaningful participation in mathematics learning.

Additionally, integrating creative problem-solving strategies within mathematical activities can help bridge the gap between reasoning and idea generation, leading to a dynamic effect in learning.

The link analysis presents teachers' perceptions regarding the pedagogical application of real analysis, particularly concerning three key constructs: advancement of knowledge and skill, logical reasoning ability, and identifying mathematical relations.

### Figure 16

*Constructs of Pedagogical Application and their Relationships*



The strength of these interconnections is quantified using coefficient values, reflecting the extent to which these constructs are interdependent. The strongest

connection (0.273) exists between advancing knowledge and skill and identifying mathematical relations, suggesting that teachers recognize a significant relationship between acquiring advanced mathematical knowledge and the ability to discern mathematical structures. This relationship highlights the pedagogical perspective that higher-level mathematical understanding facilitates the ability to establish meaningful connections, which is crucial for problem-solving and theorem application.

A moderate connection (0.182) is observed between identifying mathematical relations and logical reasoning ability, indicating that teachers perceive logical reasoning as instrumental in recognizing mathematical patterns and relationships. This perception aligns with the view that reasoning skills support the identification of structural interrelations within mathematics, which is critical for both secondary and higher-level mathematical learning.

The weakest connection (0.167) is found between advancement of knowledge and skill and logical reasoning ability, implying that while logical reasoning is acknowledged as important, its direct role in the development of mathematical knowledge is seen as somewhat secondary. This may reflect the idea that real analysis strengthens reasoning skills. However, its immediate impact on knowledge acquisition is perceived as more indirect compared to the ability to identify mathematical relations.

The centrality of identifying mathematical relations, indicated by its prominence in the network, suggests that it serves as a crucial bridge between advanced mathematical knowledge and reasoning skills. Teachers appear to consider the ability to establish mathematical relationships as a pivotal component of mathematical proficiency, reinforcing its role in linking theoretical understanding with logical deduction. The comparatively lower coefficients for logical reasoning ability indicate

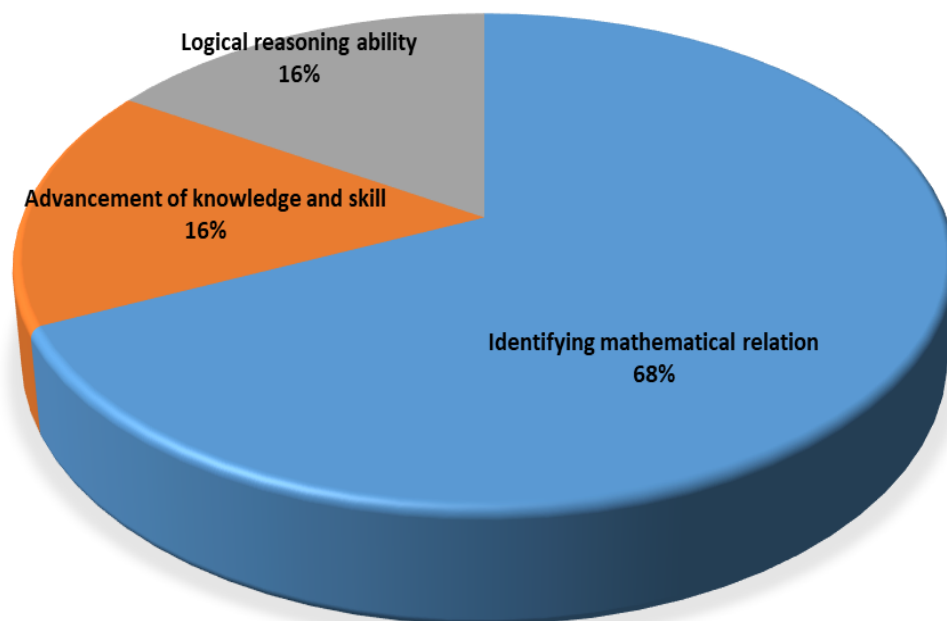
that, while reasoning remains an essential aspect of mathematical competence, it is perceived as less directly influential in advancing knowledge than the capacity to discern and interpret mathematical structures.

***Teachers' Perceptions on KRA Regarding Enhance Mathematical Proficiency***

In the survey phase, teachers' attitudes toward the mathematical knowledge of real analysis were found to be positive in enhancing their mathematical proficiency. This phase examined how teachers perceive the mathematical knowledge of real analysis concerning their professional development. The teachers' perceptions of how real analysis enhanced mathematical proficiency and its primary constructs or areas are identified through task-based interviews. The result of the thematic analysis reflects the teachers' perception of their knowledge of real analysis in enhancing proficiency development. The central area of mathematical proficiency development and teachers' perception status is based on the code was as shown in Figure 17.

**Figure 17**

*Teachers' Perception Status on MPROF Development by KRA*



The results of the thematic analysis, categorized under enhancing mathematical proficiency, reveal three key themes based on teacher perceptions. The findings indicated that teachers believe knowledge of real analysis contributes to the development of skills such as identifying mathematical relations, logical reasoning, and the advancement of knowledge and skills, all of which are important for enhancing mathematical proficiency.

***Identifying Mathematical Relations.*** The knowledge and skills of mathematics teachers that enable them to identify the mathematical relations among different mathematical concepts reflect the mathematical proficiency of mathematics teachers, which is known as identifying mathematical relations.

Teachers view knowledge of real analysis as instrumental in developing the capacity to identify mathematical relationships. Teachers reported that a mathematical knowledge of real analysis helps them identify mathematical relations. Rajan explored that "In my experience, when I learn the absolute value of a real number, then I know that the sum of two numbers is greater than the third, which implies the conditions for a triangle; this relation also holds in metric spaces." In this case, the teacher identified the relations between the existence of a triangle and whether the number represents a triangle or not. Kapil reported that:

In real analysis, this is known as the  $p$ -test. However, at the school level, the concept is often simplified to describe the behavior of a series as tending to a fixed point, which we interpret as convergence in higher education.

Both cases are related to the identification of mathematical relations. They talk about one context linking to another by identifying the relations.

***Advancement of knowledge and Skills.*** During the task-based interview, teachers reported that improving mathematical knowledge and skills is one important

aspect of their professional life. The knowledge of real analysis helps to enhance mathematical knowledge and skill.

Shyam emphasized the importance of an in-depth study of advanced mathematics, stating, "The ability to quickly generate mathematical reasoning and problem-solving is a result of the conscious and in-depth study of higher-level mathematics, which I believe is essential." He pointed out that proficiency in real analysis allows teachers to think critically, construct logical arguments, and approach problem-solving with greater efficiency. For instance, understanding the structure of mathematical proofs in real analysis not only helps in validating results but also improves the way teachers guide students in developing rigorous reasoning skills.

Rajan viewed real analysis as crucial in expanding mathematical knowledge beyond what is directly taught in the classroom. "It helps expand our mathematical knowledge, allowing us to gain more than just what I teach." He explained that it helps us understand exactly what a theory is and how it is applied. This highlights the significance of real analysis in bridging the gap between theoretical understanding and real-world applications. For example, a deeper understanding of measure theory provides valuable insights into probability and statistical models, which are essential for fields such as data science and engineering.

Saroj further reinforced this idea: "It expands our mathematical perspective." He explained that real analysis fosters a broader view of mathematics, encouraging teachers to explore interconnections between branches. Concepts such as uniform convergence, compactness, and completeness are theoretical constructs and fundamental tools in various domains of applied mathematics.

Ram said "Knowledge gained at the campus level helps in acquiring a broader understanding of teaching at lower levels as knowledge increases." This view

emphasizes that university-level mathematics helps to increase the knowledge and skills required to teach at school and is an essential part of proficiency development.

From the above-mentioned teachers' perspective, knowledge of real analysis is beneficial in advancing the knowledge and skills required to teach mathematics in schools.

***Logical Reasoning.*** Mathematics teachers possess the argumentation skills necessary for proving theorems. This type of case is called logical reasoning, which is a part of logical reasoning skills. Real analysis plays a significant role in developing logical reasoning skills among mathematics teachers by providing a foundation for structured argumentation, deep thinking, and problem-solving. Teachers emphasize that engaging with real analysis strengthens their ability to reason systematically and construct mathematical proofs with clarity and precision.

Govinda explained, "We use it for both inductive and deductive reasoning." It means real analysis supports different types of reasoning: inductive, deductive, and practical. He said that inductive reasoning is evident when working with sequences, series, and limits, where patterns emerge through repeated observations before precise mathematical statements are formulated. Deductive reasoning is reinforced through proof-based learning, which requires logical justification step by step.

Milan added "The study of real analysis certainly helps develop logical thinking, deep thinking, the ability to analyze, and the capacity to conclude mathematical results." It means real analysis enhances logical thinking, deep thinking, analytical skills, and the ability to draw mathematical conclusions. Logical thinking is developed through engagement with precise definitions and the need for consistent reasoning. Deep thinking is cultivated through real analysis, which challenges teachers to go beyond surface-level understanding, requiring them to

comprehend abstract concepts such as uniform convergence and the construction of the real number system. Analytical skills improved teachers' skills to learn to break down complex problems, assess the validity of arguments, and interpret mathematical structures systematically.

Furthermore, link analysis was performed to assess the strength of the theme and identify which one contributed based on the teacher's perception. QDA Miner was used to analyze the link. The result is shown in Figure 18. The link analysis presents teachers' perceptions of real analysis to enhance mathematical proficiency, particularly concerning key constructs: Advancement of knowledge and skill, Logical reasoning ability, and identifying mathematical relations. The strength of these relationships is quantified through coefficient values, indicating the extent to which these constructs are interlinked.

The strongest connection (0.273) exists between advancing knowledge and skill and identifying mathematical relationships. This suggests that teachers recognize a substantial relationship between acquiring advanced mathematical knowledge and the ability to separate mathematical relationships.

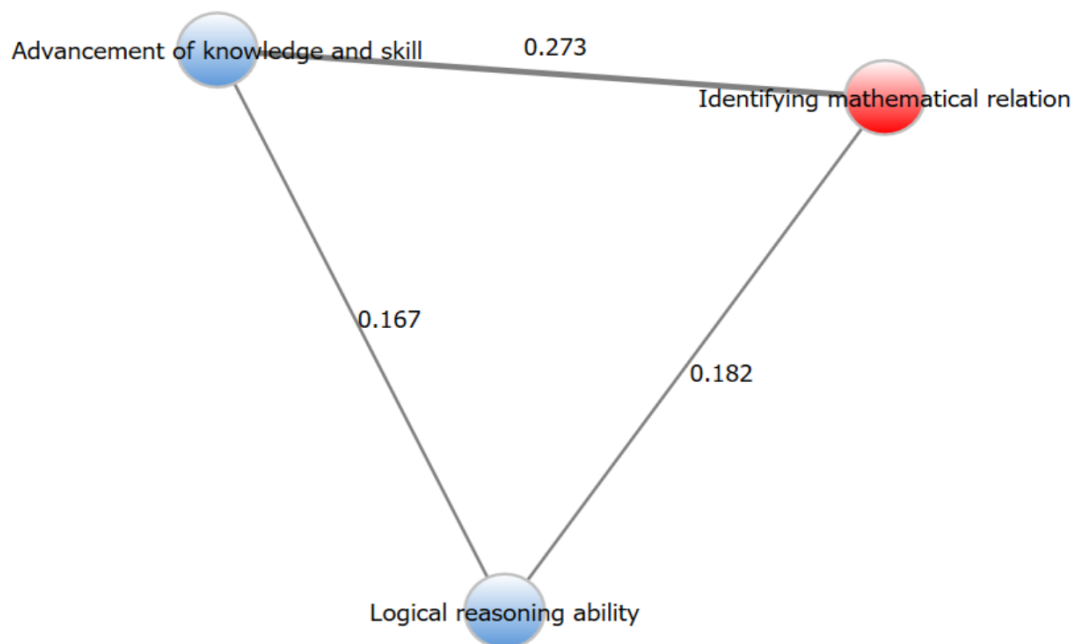
The connection reflects the pedagogical view that higher-order mathematical knowledge enhances the capacity to establish mathematical connections essential for problem-solving and theorem application. The link between the Advancement of knowledge and skill and Logical reasoning ability (0.167) indicates that teachers perceive logical reasoning as an essential aspect of advancing knowledge in mathematics. While this coefficient is relatively weaker than others, it still indicates that logical reasoning is supportive of knowledge acquisition.

This may reflect the idea that while real analysis strengthens reasoning skills, its direct impact on knowledge enhancement is perceived as somewhat limited compared to other factors.

The connection between identifying mathematical relations and Logical reasoning ability (0.182) highlights the perceived role of reasoning in identifying mathematical structures and interrelations. Teachers acknowledge that developing logical reasoning is essential for recognizing mathematical patterns and structures, a skill fundamental to secondary-level mathematics and advanced topics in real analysis. The centrality of identifying mathematical relations, as indicated by its red node, suggests its critical role in linking advanced knowledge acquisition and reasoning skills.

### Figure 18

*Link Analysis Result of Mathematical Proficiency Development*



This central positioning implies that teachers view the ability to identify mathematical relationships as a pivotal skill that bridges theoretical knowledge with logical deduction.

***Role of Knowledge of Real Analysis in Mathematics Teachers' Preparation***

Real analysis has been included in the teacher preparation program. During the survey phase, it was found that secondary-level mathematics teachers hold a positive attitude towards the mathematical knowledge of real analysis as it relates to secondary-level mathematics instruction. To explore its relevance further, interviews were conducted with mathematics teachers and mathematics educators to address the research questions: *RQ4. Why is mathematical knowledge of real analysis required to become a mathematics teacher?* The category, code, and theme under the thematic analysis are shown in Table 20. The thematic analysis results indicate that real analysis plays a crucial role in enhancing mathematical knowledge, fostering maturity in mathematical thinking, developing expertise, and improving mathematical skills, such as generating ideas from context.

**Table 20**

*Theme on Why KRA for Teacher Preparation*

Category	Codes	Themes
Why real analysis	Knowledge, broader understating, knowledge increasing, more knowledge, gain depth of knowledge, clear understating, more exhaustive knowledge	Advancement of knowledge
	Thinking, Mathematical thinking, deep thinking, critical thinking, logical thinking, inductive thinking, deductive thinking	Maturity in mathematical thinking
	Expert, being expert, expertise, acquiring expertise	Exercising
	Acquiring skills, think mathematically, seeking solution, alternative way of solution, reasoning, example, counterexample	Mathematical skill development
	Confidence, self-confidence, acquiring confidence	Being confidence

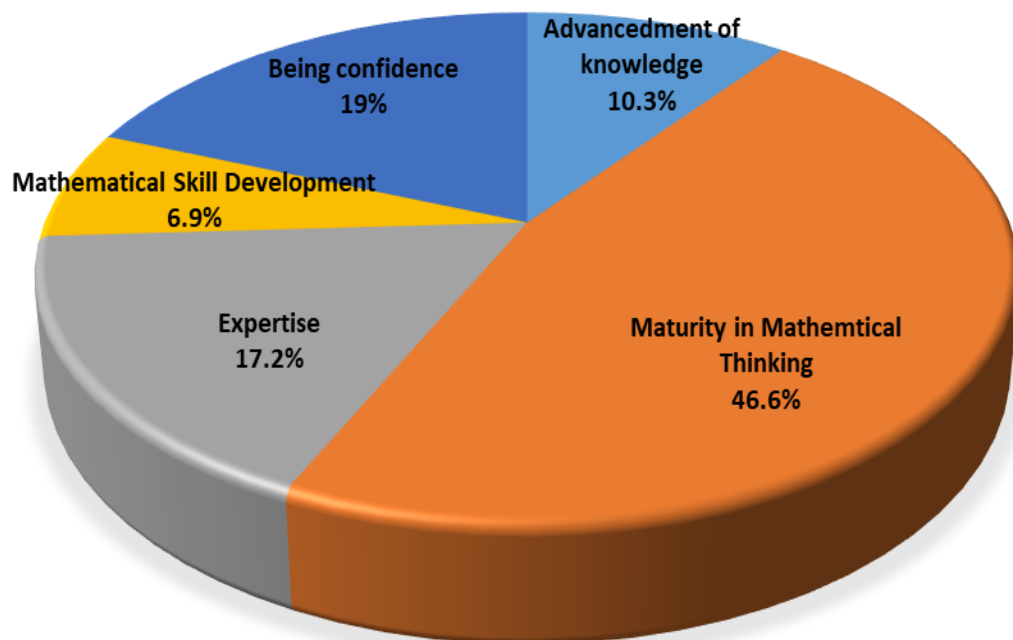
Additionally, it contributes to building confidence in teaching mathematics at the secondary level. The result of the thematic analysis highlights several reasons why real analysis is essential in teacher preparation programs.

The thematic analysis results highlight several reasons why real analysis is essential in teacher preparation programs. These themes are presented in a diagram to illustrate their importance based on the result of thematic analysis.

The pie diagram shows that the significant role of real analysis in teacher preparation is fostering maturity in mathematical thinking (46.6%). This suggests that teachers view real analysis as crucial for fostering deeper mathematical reasoning and abstraction. The second emphasized role is building confidence in teaching mathematics (19%), suggesting that real analysis helps teachers feel more assured in their instructional abilities.

**Figure 19**

*Role of Real Analysis in Teacher Preparation Program*



Following this, developing expertise is another key role, highlighting the perception that real analysis contributes to mastering subject knowledge. However, mathematical skill development is represented at a comparatively lower percentage, indicating that while real analysis supports skill-building, its impact is perceived to be more assertive in conceptual understanding and confidence.

Following this, developing expertise is another key role, highlighting the perception that real analysis contributes to the mastery of subject knowledge. However, mathematical skill development is represented at a comparatively lower percentage, indicating that while real analysis supports skill-building, its impact is perceived to be stronger in conceptual understanding and confidence.

The views of teachers and experts on the role of mathematical knowledge in real analysis for becoming a teacher are described in relation to the developed theme.

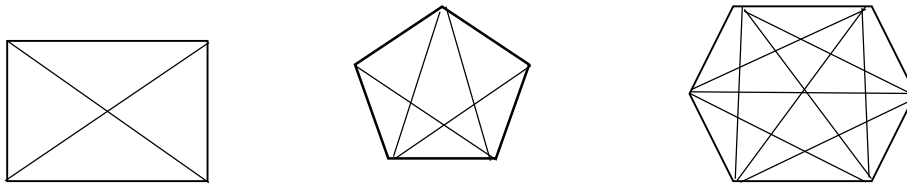
***Maturity in mathematical thinking.*** Maturity in mathematical thinking is considered an essential ability for mathematics teachers. Effective engagement with mathematical situations requires teachers to be proficient in logical reasoning, capable of generating alternative solutions, and skilled in abstract and advanced thinking. In task-based interviews conducted with secondary-level mathematics teachers and mathematics educators, it was reported that a strong foundation in real analysis is crucial for developing mathematical thinking maturity. Such knowledge enables teachers to navigate complex mathematical problems more effectively and equips them with the mathematical expertise needed to foster a deeper understanding of mathematics among their students. Furthermore, mathematical thinking maturity allows teachers to make meaningful connections between mathematical concepts, enhancing their instructional effectiveness. It also helps them identify and address students' misconceptions through logical explanations. A well-developed sense of

mathematical reasoning enables teachers to design engaging and thought-provoking lessons. Additionally, it supports their ability to assess students' problem-solving approaches critically. Ultimately, fostering mathematical thinking maturity in teachers contributes to a more profound and enriching learning experience for students.

During the interview with an expert, Expert 3 illustrated one example, as shown in Figure 20.

**Figure 20**

*A Model of Question Reflecting Mathematical Maturity Check*



Further, he said:

How do you think about this question? How many diagonals can be drawn in a polygon with  $n$  sides? This question is considered a geometry problem, but it requires the ability to think mathematically. Without thinking inductively, it is not easy to find the correct answer. The knowledge of real analysis cultivates this thinking development process because it helps the development of mathematical habits.

The expert elaboration indicates that mathematical thinking is a mathematical behavior that stimulates the teacher to think in a reasoned manner rather than manual thinking or visible-based thought. Expert 1 highlighted that new concepts and ideas develop through analysis, noting that "to advance from basic mathematics to higher-level mathematics, teachers need these foundational concepts." Expert 2 said that

Different operations like  $+$ ,  $-$ ,  $\times$ , and division are used in numerical and algebraic contexts in school mathematics practices. Teachers need to know the

logic and reasons behind mathematical operations executed during instruction. These logics and facts can be known from the study of real analysis. If we find  $1+1 = 2$  is a natural number, because its predecessor is 1,  $2+1 = 3$  is also a natural number, because 2 is a natural number and 1 is the predecessor of 3. It reflects the closure property in a set. When  $1-1 = ?$  Then zero comes, and the concept of a whole number is developed. Similarly, integers, rational numbers, irrational numbers, real numbers, and complex numbers are developed. Teachers need to know the mathematical facts behind what we do. Real analysis helps with this.

Expert 3 further explained that "real analysis plays an essential role in logical reasoning, supporting facts, theorems, and providing arguments in examples."

Teachers expressed that maturity in mathematical thinking cultivates logical, critical, and creative thinking abilities, which are key to effective teaching, especially at the secondary level. As Shyam put it, "I learned during my master's degree are filtered and applied to my teaching activities. Such subjects are essential for developing our minds." This saying emphasizes the mathematical thinking process. All of them agree that knowledge of real analysis helps to develop the thinking that produces maturity in mathematical thinking.

***Being Confident.*** From the perspectives of mathematics educators and teachers, knowledge of real analysis helps develop confidence. Several interviewees mentioned the role of confidence in teaching, Expert 2 said "When teachers have no more knowledge than they need to teach at secondary school, then he/she cannot provide mathematical concepts clearly and confidently." So, knowledge of real analysis is required to develop the teacher's confidence. He emphasizes confidence through gaining more knowledge of real analysis. Teachers like Ram and Kapil

emphasized how self-confidence in teaching is closely tied to a strong foundation in mathematical knowledge.

Ram said "It develops the confidence, which is an important part of teaching." Hari emphasized that mathematical maturity strengthens logical reasoning and enhances a teacher's ability to teach and guide students with confidence. Hari stated that "strengthens our logical reasoning," indicating how this ability boosts confidence. Likewise, Milan also noted that a strong mathematical foundation makes a teacher confidently present concepts to students. The discussion emphasizes the importance of confidence in teaching, which is closely linked to a teacher's depth of knowledge, particularly gained at the university level, such as real analysis. It is argued that a teacher may struggle to present content clearly and confidently without a solid understanding of key mathematical concepts.

A strong foundation in mathematics, especially in areas such as real analysis, not only enhances a teacher's logical reasoning abilities but also boosts their confidence in conveying complex ideas effectively. This increased confidence enables teachers to engage more effectively with students, guiding them through challenging concepts and fostering a deeper understanding of mathematics.

***Expertise.*** A mathematics teacher required to possess expertise in their subject at the level they are appointed to teach. This expertise requires a comprehensive understanding of the curriculum, content, pedagogy, and classroom management. Among these, real analysis plays a crucial role in strengthening content knowledge. Task-based interviews with mathematics teachers and discussions with mathematics educators emphasized the importance of expertise, particularly in deepening content knowledge and conceptual understanding. A solid grasp of real analysis enables

teachers to approach school-level mathematics systematically, ensuring clarity, accuracy, and the ability to present concepts in a structured and meaningful manner.

Experts 1 and 3 emphasized that a teacher's ability to present complex concepts in multiple ways and provide logical explanations is vital. Expert 3 noted, "Qualified teachers are essential for school education, and for that, teachers must have a clear understanding of mathematical concepts, including those related to real analysis." Expert 1 further stated, "Presenting the same concept in multiple ways is also analysis" by highlighting how versatile teaching methods contribute to a deeper understanding. Teachers acknowledged that a deep understanding of foundational concepts, such as those derived from real analysis, is necessary for providing quality education. Ram said, "While it may not be possible to transfer advanced-level knowledge to lower levels directly, its generalization and simplified concepts are extremely useful." Hence, knowledge of real analysis is essential for being an expert. It especially contributes to content expertise.

*Advancement of Knowledge.* The extension of mathematical knowledge with its proper understanding and interpretation capacity. Knowledge of real analysis focuses on the real number system and other further developed forms. During the task-based interview with mathematics teachers and educators, they reported that a solid understanding of real analysis is required to advance mathematical knowledge, as it forms the foundation of school mathematics instruction and other advanced mathematics, such as algebra.

Interviewees agreed that real analysis and mathematical thinking significantly contribute to the advancement of knowledge. For instance, Ram emphasized how advanced learning sharpens knowledge and enhances thinking skills, stating, "It helps us to advance our knowledge, it helps us to sharpen our knowledge, and it helps

develop thinking skills." Similarly, Milan highlighted the importance of continuously gaining mathematical knowledge, stating, "It is required for providing more knowledge of mathematics."

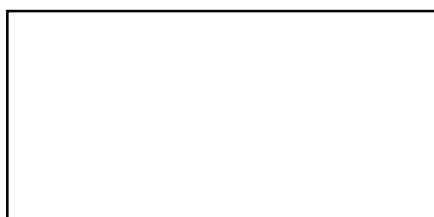
Beyond individual learning, the ability to critically analyze and apply mathematical knowledge was vital for personal and professional development. Real analysis provides the necessary conceptual tools for teachers to approach mathematical concepts systematically, enabling them to present school-level mathematics in a structured and meaningful manner. For example, mathematical maturity, which is cultivated through real analysis, strengthens logical reasoning and enhances a teacher's ability to guide students effectively. Hari pointed out that real analysis "strengthens our logical reasoning", indicating its role in fostering analytical skills essential for teaching. Moreover, a solid grasp of real analysis helps teachers navigate complex mathematical structures and offer multiple representations of the same concept, an ability noted as crucial for effective teaching. Expert 1 mentioned that "presenting the same concept in multiple ways is also analysis", reinforcing the idea that expertise is not just about knowing content but also about adapting and communicating it in various ways to enhance student understanding.

***Mathematical Skill Development.*** Mathematical skills reflect the capacity to demonstrate abstract concepts in visual form, solve problems systematically, and apply logical reasoning to diverse situations. These skills enable individuals to translate theoretical ideas into concrete representations, making complex mathematical structures more accessible. Moreover, pattern recognition is used to construct and validate mathematical arguments.

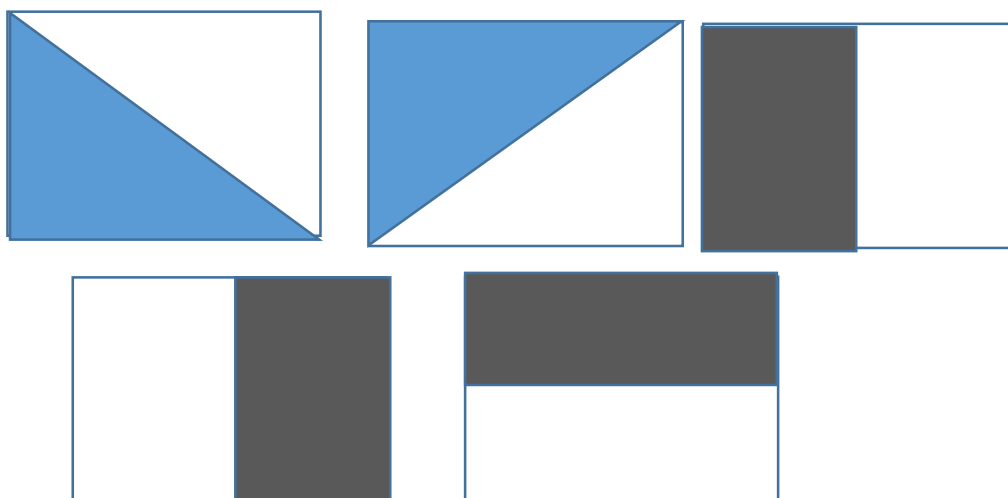
During task-based interviews with teachers and experts, it was found that knowledge of real analysis helps to develop mathematical skills.

It is important that mathematics teachers possess subject-matter expertise appropriate to the level at which they are appointed to teach. This expertise requires a comprehensive understanding of the curriculum, content, pedagogy, and classroom management. Among these, real analysis is crucial in strengthening content knowledge, and providing the logical foundation for advanced mathematical thinking. Experts and teachers recognized that developing problem-solving skills, logical reasoning abilities, and critical analysis through real analysis is essential for teaching and learning mathematics. Expert 2 said:

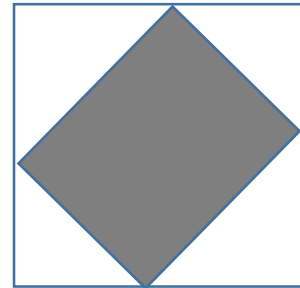
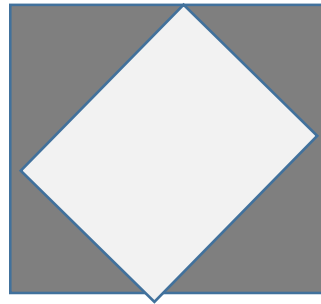
Real analysis is taught to provide knowledge beyond the minimum requirements for teaching at the secondary level. It helps develop fundamental concepts of mathematical processes, such as the development of numbers and their properties. It develops mathematical thinking skills. For example, here is a square; how many ways do you divide it



in general, mathematical thinking may give the dividing process as below



Beside this thinking level, we take half as



All these shaded regions represent half of the square. General-level thinking may not have derived such solutions, but creative and critical thinking help to derive such solutions. Oh, this type of thinking process is cultivated by studying real analysis and other mathematics.

These views highlighted the importance of teachers having skills to think mathematically, logically, and alternatively, and to use visual aids to present complex concepts in simple ways.

Expert 1 noted that "these skills are vital for students' everyday lives, stating that problem-solving and logical reasoning abilities are essential." Expert 3 added that these skills "ensure that teachers and students avoid making incorrect operations" and provide logical reasoning for both accepting and rejecting mathematical arguments. This emphasis on skill development is echoed by Kapil, who said, "Develop mathematical thinking skills." Thus, the one important source of devoting mathematical skills is real analysis.

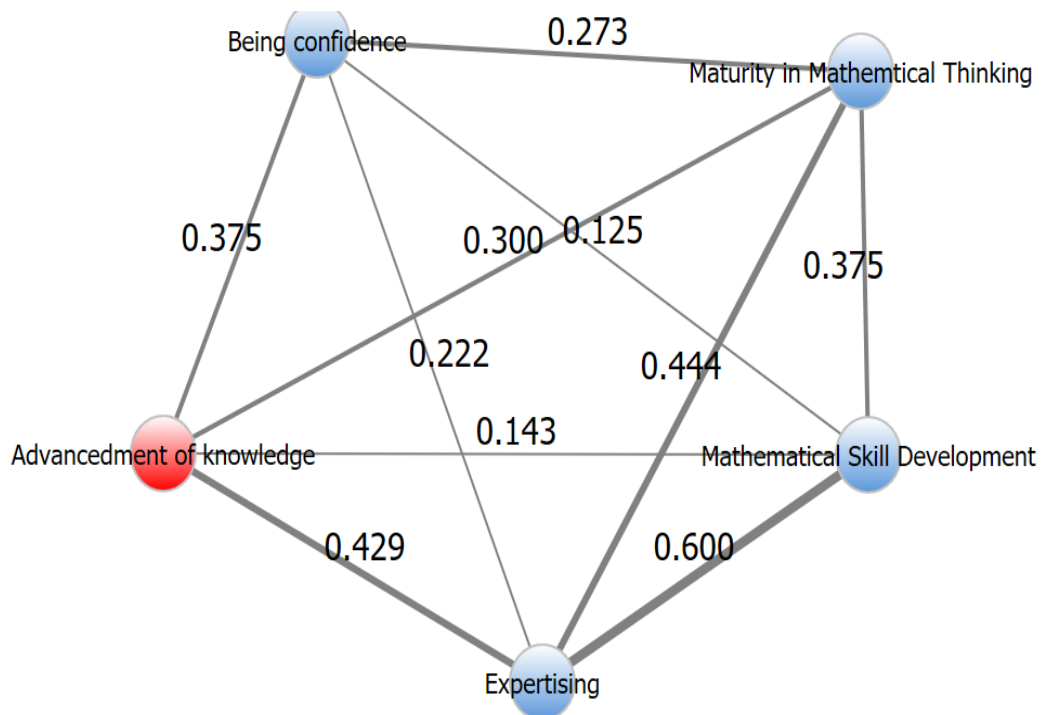
Beside the deification of psychological construct contributed by knowledge of real analysis for enhancing them. The link analysis was conducted to identify the influential themes in the dataset.

The network visualization highlights key concepts based on their connectivity and centrality. This method is employed to gain a deeper understanding of the role these constructs play in shaping the discourse.

Figure 21 illustrates the relationship between idea creation, reasoning in a mathematical context, and mathematical activities as perceived by the teacher in the context of the pedagogical application of knowledge of real analysis. The coefficients between these factors reveal how teachers' interconnection in the teaching process. Maturity in mathematical thinking was considered the target variable due to its higher frequency among the identified themes in the study of real analysis.

**Figure 21**

*Construct Relationships Contributed by KRA*



The link analysis reveals that mathematical skill development (0.375) has the strongest influence on the maturity of mathematical thinking, emphasizing the importance of strengthening problem-solving abilities and computational skills. Confidence (0.273) also plays a significant role, indicating that students who feel assured in their mathematical abilities are more likely to develop mature mathematical thinking. Advancement of knowledge (0.300) contributes to this maturity by deepening conceptual understanding, though its impact is moderate compared to skill development and confidence. Expertise (0.125), while relevant, has the least direct influence, suggesting that while mastery is valuable, it does not independently lead to mathematical thinking maturity. This analysis highlights the need to prioritize skill-building and confidence-enhancing strategies in real analysis education to foster a deeper and more mature mathematical mindset.

***Key Area of Content Connection between KRA and SLM***

The curriculum analysis aimed to identify key areas where secondary-level mathematics and real analysis are connected in various ways, such as direct content connections. A content connection framework was applied to determine the significant links between the real analysis curriculum and secondary-level mathematics. This section examines the relationship between the real analysis curriculum in the Bachelor of Education program at Tribhuvan University and the secondary-level mathematics curriculum and textbooks, addressing Research Question 5: *What are the connections between the mathematical knowledge of real analysis and secondary-level mathematics content?* The primary focus is on identifying major content connections using a connection framework to ensure better alignment with these curricula and textbooks. This is followed by an analysis of the mathematical connections present in both the real analysis curriculum and the

secondary-level mathematics curriculum and textbooks, categorized based on different types of connections. Additionally, some connections may overlap; for example, hierarchical connections can sometimes be interpreted as generalizations, depending on how the contents are presented.

**Mathematical Connection Framework.** A mathematical connection is the inherent relationship between two mathematical ideas or entities. It involves the linkage or association of one mathematical concept with another, highlighting their interdependence and interaction. In essence, a mathematical connection signifies the link existing within the realm of mathematical concepts, fostering a deeper understanding of their relationships and applications. The mathematical connection framework guides the identification of potential links.

The prepared mathematical connection framework was used to identify the mathematical connection between mathematical knowledge of real analysis and secondary-level mathematics.

**Table 21**

*Category of Mathematical Connection Found in Curriculum and Textbook*

Category	Description
Hierarchical relationship	One mathematical concept serves as a foundation or is connected to the mathematical concept of a higher level.
Axiomatic connection	The axioms introduced in real analysis have some form of connection with the content found in secondary-level mathematics.
Procedural Connection	The comprehension and application of interrelated procedures, methods, or algorithms across various mathematical concepts or subjects in real analysis are put into practice in secondary-level mathematics.
Alternative representation	In secondary-level mathematics, one concept of real analysis can be expressed in various ways, including symbolic, algebraic, geometrical, graphical, or manipulative forms.
Generalization	One concept is a generalization of another specific concept.

After reading and analyzing the real analysis curriculum, compulsory and optional secondary-level mathematics curriculum, and textbook based on the prepared mathematical connection framework, I adjusted the mathematical connection framework, which was prepared as a tool. The adjusted mathematical connection framework is given in Table 21. The description of the connection is arranged in the coming sections. In this section, I explore how real analysis and secondary-level mathematics are mathematically connected. This exploration is detailed through the lens of the mathematical connection category, providing a link between mathematical knowledge of real analysis and school-level mathematics.

***Hierarchical Relationship.*** A hierarchical relationship denotes a scenario in which one concept acts as a component or is encompassed within another.

Regarding the mathematical knowledge real analysis and school-level mathematics, this pertains to the connection between the mathematical concepts at these two levels.

**Table 22**

*Hierarchical Relationship between MKR and SLM*

Mathematical concept in real analysis	Mathematics concept at secondary-level
System of Real numbers (Order axioms and properties of Algebraic structure of $\mathbb{R}$ )	Set ideas are linked in mathematical operation
Open and closed set	Functions, Linear inequalities
Real sequence	Sequence, type of sequence, concept of limit by using sequence
Infinite series	series, partial sum of series
Function	Concept of function, types of function, and inverse function
Limit	The concept of limit by using a sequence of numbers
Continuity	Continuity on a set of numbers ( $\mathbb{N}, \mathbb{W}, \mathbb{Q}, \mathbb{R}$ )
Discontinuity	Discontinuity from the number line

The title effectively captures the mathematical content and concepts that involve such a hierarchical link. Table 22 summarizes topics involving those connections found in real analysis and school-level mathematics.

The chapter on 'Real Numbers' in real analysis delves into fundamental concepts related to the real number system and explores their properties. This chapter is crucial for secondary-level mathematics and serves as a basis for structuring mathematical concepts at this level. These concepts are directly linked to constructing problems on sets and performing arithmetic operations.

The content presented in Chapter 2 on 'Open and Closed Sets ' within real analysis is interconnected with creating numerical examples and problems in the context of relations and functions for ninth-grade students. For instance, the textbook provides an example: "Determine the range of each relation  $\{(x, y): y = 2x + 1, 0 \leq x \leq 3, x \in \mathbf{W}\}$ " (CDC, p. 20). Moreover, it introduces another problem: "If  $f(x) = \begin{cases} 4x-1 & \text{if } -3 < x < 0 \\ 1+x & \text{if } 0 \leq x < 2 \\ x^2+9 & \text{if } 4 \leq x < 5 \end{cases}$  is a given function, find  $f(4)$ ,  $f(1)$ , and  $f(-2)$ " (CDC, p. 36). Similar scenarios can be identified in the context of linear inequalities.

Therefore, various concepts related to interval practices in real analysis courses are closely connected to the content taught at the secondary mathematics level.

The topic and concept of real sequence and infinite series of real analysis is connected with the content of secondary-level in the topic "sequence and series" (CDC, 2016, p. 29; CDC, 2016, p. 45). The definitions, examples, and ideas related to these topics are connected to each other. The concept of Function, Limit continuity and discontinuity of real analysis connected with the content of secondary-level mathematics. The concepts "function and its value, searching continuity, discontinuity

" (CDC, 2016, curriculum, p. 104) are included in the topic limit and continuity of secondary-level mathematics.

The mathematical concepts within real analysis, such as real numbers, open and closed sets, functions, limits, and continuity, form a hierarchical connection with the optional mathematics curriculum taught in grades nine and 10. The definitions, examples, and numerical problems presented in the optional mathematics textbooks for these classes are intricately linked to the content covered in real analysis. However, it is worth noting that the depth and complexity of the content are appropriately adjusted to suit the respective academic levels. This alignment underscores the connection between the mathematical knowledge acquired in real analysis and the corresponding concepts introduced in school-level mathematics.

***Axiomatic Connection.*** This connection focuses explicitly on exploring the relationship between the axioms and principles present in real analysis and their link in school-level mathematics. Regarding the axiomatic connection between the MKR and SLM, the axiomatic connection framework, which falls under the mathematical connection framework, was utilized to identify it. Table 23 summarizes the axiomatic connection.

**Table 23**

*Axiomatic Connection between MKR and SLM*

Mathematical concept in real analysis	Mathematics concept at the secondary-level
Field axioms	Number system, arithmetic operations during problem solving
Cancellation Law	Generalization in arithmetic operations is applied in teaching and learning.
Order Axioms	Generalizing in arithmetic operations, inequalities
Law of Trichotomy	Generalization in arithmetic operations

The field axioms, cancellation laws, and order axioms are fundamental concepts in real analysis. These concepts range from basic to more advanced abstract forms in real analysis. These axioms and laws are integral components of secondary-level mathematics curricula. Theoretical axiomatic ideas, including closure, commutativity, associativity, identity, and inverses, are applied in various mathematical activities within the school context.

For instance, students learn about closure under addition and multiplication when exploring the field axioms in real numbers. This means that the sum or product of any two real numbers is also a real number. In the context of cancellation laws, students understand that if  $a + b = a + c$ , then  $b$  must equal  $c$ . However, it is in a specific form. These principles provide a solid foundation for more complex mathematical reasoning and problem-solving in higher-level studies. The order axioms contribute to understanding the relationships between real numbers, such as the law of trichotomy, which states that for any real numbers  $a$  and  $b$ , exactly one of the following holds:  $a < b$ ,  $a = b$ , or  $a > b$ . These concepts and their applications in real analysis form an essential bridge between the basic and more advanced mathematical content taught in secondary-level mathematics.

The following example illustrates the application of law in real analysis, specifically in mathematical procedures and their generalization. Find the value of  $x$ :  $x + 5 = 8$ . Normally, the solution to this problem is  $x = 8 - 5$ , but no reasons are provided to students. The expanded version with rules is as follows:

$$(x + 5) + -5 = (8) + -5 \quad (\text{Additive Equivalence})$$

$$x + (5 + -5) = 8 + -5 \quad (\text{Associativity of addition})$$

$$x + 0 = 8 + -5 \quad (\text{Additive Inverse})$$

$$x = 8 + -5 \quad (\text{Identity Element for addition})$$

$$x = 3 \quad (\text{Closure under addition})$$

The mathematical logic connected with knowledge of real analysis is utilized to complete the solution.

**Procedural Connection.** Procedural connection involves applying the same mathematical processes and algorithms in diverse contexts and difficulty levels while engaged in mathematical problem-solving and creation. This heading highlights the similar process and algorithm used in real analysis and its link with secondary-level mathematics.

The mathematical connection framework's mathematical procedure category was utilized to identify the procedural connections between mathematical knowledge of real analysis and mathematics at the secondary level. Table 24 summarizes the procedural connections of the considered curricula.

Another connection between the mathematical principles of real analysis and secondary-level mathematics exists within the domain of procedural methods. This linkage is vital, as it enables the application of advanced concepts to problem-solving and establishes connections in secondary-level mathematical contexts.

**Table 24**

*Procedural Connection between KRA and SLM*

Procedure in KRA	Mathematics concept at the secondary-level
Fundamental properties of rational numbers (algebraic structure of $\mathbf{Q}$ )	The operational process used in mathematical problem-solving process
Determine the general term ( $t_n$ ) of a series	Determine the general term ( $t_n$ ) of a series
Determine inverse function	Determining domain from range
Determine the value of composite function	Numerical calculation of composite function
Testing the existence of limit	Determine the limiting value of a function

The inverse of the given function practice in real analysis is a procedure. This rule and procedure link with secondary-level mathematics.  $F(x) = y = 2x + 3$

$$y = 2x + 3$$

Interchanging the places of  $x$  and  $y$  we get

$$\text{or, } x = 2y + 3$$

$$\text{or, } x - 3 = 2y$$

$$y = \frac{x-3}{2}$$

Therefore,  $f^{-1}(x) = (x-3)/2$  (CDC, 2076, p. 13). This is a procedure for determining the inverse of a function, but this concept and theoretical concept are included in real analysis. In addition to this case, the procedure used in sequence and series is connected with the content of real analysis.

**Alternative Representation.** The multiple ways of representing the same mathematical concept characterize the alternative representation. The alternative way of conveying the same mathematical concept is used according to the level. Some concepts of mathematics at the secondary level are found to be alternative representations of concepts of mathematics existing in real analysis. The alternative representation category of the mathematical connection framework was used to identify the connection. Table 25 captures the summary of alternative representation.

**Table 25**

*Connection between RA and SLM: Alternative Representation*

Mathematical concept in RA	Mathematics concept at secondary-level
Function	Graphical representation of function and vertical line drawing to test if the given graph is a function or not
Existence of limit	Sequence of numbers in terminating or non-terminating form, sequence of figure
Continuity	Continuity from line and graph
Sequence	Diagrammatic representation
Interval (open and closed interval)	Linear inequalities
Derivative, $\frac{dy}{dx}$	Slope of line (in a line $y = mx+c$ , $m$ ) or $m = \frac{y_2-y_1}{x_2-x_1}$

Various content of real analysis can be found in secondary-level mathematics through diverse presentations. While real analysis delves into theoretical discussions

on functions and limits in a more explicit form, secondary-level mathematics conveys these ideas through graphical representations in a simple form. For example, the curriculum, as outlined in the Curriculum Development Center (CDC) (2016, pp. 102-103), includes teaching methods for representing functions, with an emphasis on the relationship between functions and their graphical representations.

This alternative mode of representation is not only specified in the curriculum but is also reinforced in textbooks. For instance, students are presented with problems such as "Draw the graph of the functions (a)  $y = x + 2$ , (b)  $y = 6$ , (c)  $y = x^2$ " (CDC, 2016, p.5), encouraging a practical application of the theoretical concepts learned.

The sequence of functions connected with real analysis is represented by a diagram found in the textbook of grade nine of optional mathematics (CDC, 2016, pp. 48-56). Additionally, continuity and discontinuity are represented by a line graph. The slope of the line represents the derivative of a linear function.

Therefore, the above-mentioned connective evidence demonstrates the connection between mathematical knowledge of real analysis, as represented in secondary-level mathematics, especially optional mathematics, and its alternative form.

**Generalization.** Using a mathematical notion in various contexts to perform mathematical tasks is called 'generalization' in the context of mathematical connections. This entails expanding the use of a mathematical concept beyond its initial context to facilitate wider application and a deeper understanding of its consequences in various contexts. In mathematics, generalization plays a vital role in identifying patterns, establishing connections across various mathematical domains, and promoting adaptive problem-solving. The generalizations presented in school-level mathematics, when extended through the lens of real analysis, illustrate a

coherent and structured conceptual progression. This progression underscores the mathematical connection between real analysis and secondary-level content, reinforcing both instructional depth and conceptual clarity. As shown in Table 26, this generalized relationship forms the basis for understanding how advanced mathematical knowledge informs effective teaching practices at the secondary level. The findings suggest that when teachers recognize and build on these generalizations, they can more effectively guide students in constructing robust mathematical understandings. This highlights the pedagogical significance of bridging foundational concepts with higher-level reasoning to enhance curriculum coherence and classroom instruction.

**Table 26**

*Connection between RA and SLM: Generalization*

Mathematical concept in RA	Mathematics concept at the secondary-level
Algebraic structure in $\mathbf{R}$	Arithmetic operations are utilized in mathematical problem solving.
Absolute value of real number and its properties	Graph and inequalities
Open and closed set	Solving the problem of inequalities and sets
Convergence of a sequence and series	Limiting the value of a finite series to an infinite series
Limit	Terms of a sequence of numbers
Inverse function	Problems related to the domain of the range
Derivative ( $dy/dx$ )	Slope of lines

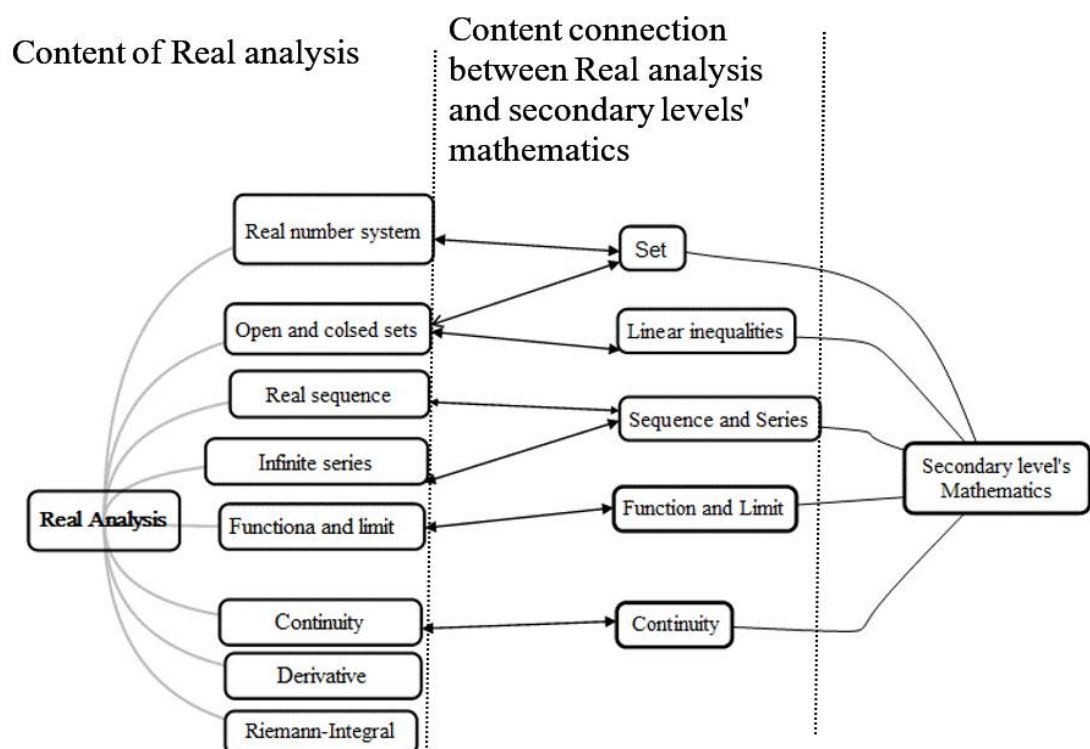
An algebraic structure in  $\mathbf{R}$  is generalized in arithmetic operations in specific mathematical problems. The existence of a limit is generalized when calculating the value of a sequence, which gives the concept of a limit (finite or infinite). The finite implies the existence of a limit and infinite value, like the non-existence of a limit.

### *Mapping Real Analysis Content to Secondary Mathematics*

A direct content connection is considered when there is a direct link between the content taught in real analysis and the content taught in school mathematics. A topic-wise connection was observed during the curriculum analysis to identify commonalities across topics. The content connection framework was used to identify the direct content connection and prepare the mapping. The topic and content of the real number system link to the content discussed in the set during problem formation in the set. During the quantitative phase, the results showed that the mathematics teachers at the secondary level hold a positive attitude toward the mathematical knowledge of real analysis.

**Figure 22**

#### *Existing Content Connection*



The open and closed sets and their concepts link with set formation and problems in linear inequalities in school mathematics. Real sequence, infinite series,

function and limit, and continuity are directly linked with school mathematics content. The facts on the link are expressed in Figure 22. Hence, knowledge of real analysis has a link with school-level mathematics.

### ***Importance of Link between KRA and SLM***

Analysis for school mathematics instruction regarding content connection. Following this result, task-based interviews were conducted with mathematics teachers and educators to answer Research Question 6. *Why is content connection between the mathematical knowledge of real analysis and school mathematics essential?* The thematic analysis was performed based on the interview record. The result under the category is why there is a link between school mathematics and real analysis. Table 27 shows the summary of the category code and theme.

**Table 27**

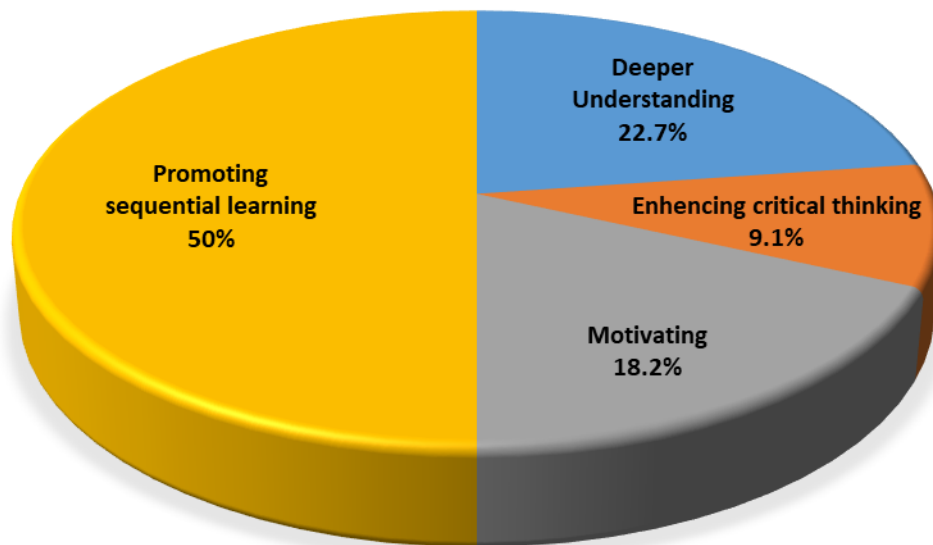
*Code Summary of Importance of Link between KRA and SLM*

Category	Codes	Themes
Why is the link between school mathematics and real analysis essential	Deep understanding, in-depth understanding, understanding link	Deeper understanding
	Critical thinking, thinking alternatively, searching multiple way	Enhancing critical thinking
	Interest, motivation, engage	Motivating
	Learning, linking contents, understanding developmental trends of contents	Promoting sequential learning

The thematic analysis identifies three primary reasons for establishing the link between knowledge of real analysis and secondary-level mathematics: to foster a deeper understanding, enhance critical thinking, motivate, and promote sequential learning. Figure 23 demonstrates the importance of these reasons.

**Figure 23**

*Reasons for Requirement of Link between KRA and SLM*



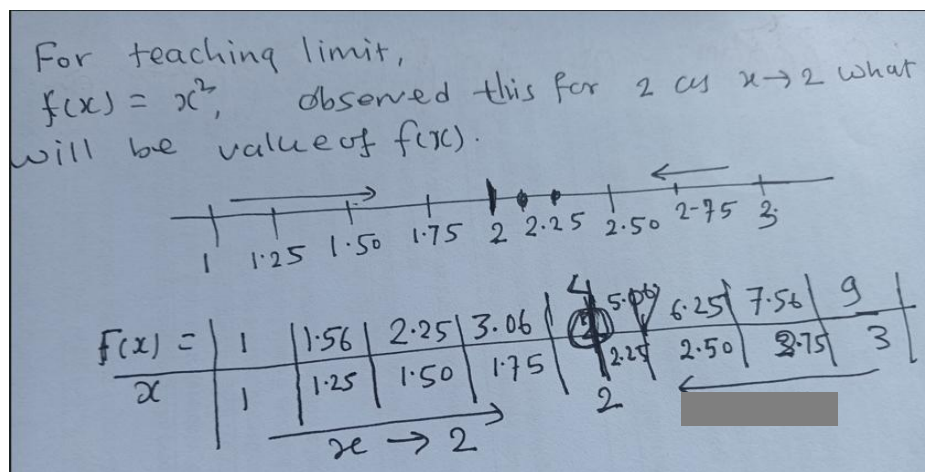
**Deeper Understanding.** In-depth understanding of mathematical concepts refers to a deeper comprehension of the subject. It reflects the ability to justify facts, demonstrate relationships, and create examples and counterexamples while distinguishing between concepts.

A deep understanding of a subject enables individuals to analyze mathematical concepts effectively and distinguish between examples and counterexamples. Establishing conceptual links within the subject matter fosters a more profound comprehension. Expert 2 stated, "Without linking subject matter according to its developmental nature, in-depth understanding is impossible." This perspective highlights the necessity of structuring knowledge in alignment with its natural progression. Similarly, Expert 1 elaborated, "first, we learn number sets, then coordinates, ordered pairs, relations, functions and their types, followed by other areas. Without mastering the fundamentals, how can we progress to higher-level concepts?" This statement underscores the importance of systematically linking mathematical concepts based on their inherent structure to facilitate deeper

understanding. Expert 1 emphasized the importance of linking concepts to facilitate understanding, enabling learners to grasp complex topics by building upon simpler foundational ideas. This structured approach helps learners comprehend intricate concepts. Teachers emphasize the essential link between real analysis and school mathematics in fostering deeper understanding. Rajan stated, "I feel this is another thing, but for an expert, developing the power of understanding mathematics, it is required." Similarly, Milan reflected, "It pushed me to think deeply and search for logical reasoning in other subjects as well." These perspectives highlight how real analysis enhances mathematical maturity and analytical skills, reinforcing its significance in school mathematics." Saroj elaborated on one example, as shown in Figure 24, that illustrates the link and helps with deeper understanding.

**Figure 24**

*Teachers' Example of Limit*



Limits are in classes nine and 10 and linked with real analysis. Knowledge of real analysis helps me teach limits.

I give this example to provide the concept of limit, where the existence of a limit is felt mathematically.

In his example, he uses the existing right-hand and left-hand limits and approaches 4, so limits exist. This example indicates that the link helps a deeper understanding of the subject matter and generalizes it according to the level and context.

**Promoting sequential learning.** Mathematics follows a structured sequence of development, beginning with fundamental assumptions, axioms, or undefined terms, and gradually progressing to more advanced and abstract concepts, such as the definition of groups to fields. These concepts are interrelated, and the vertical link within the subject's structure facilitates step-by-step learning. This step-by-step approach, integral to sequential learning, ensures that mathematical concepts are introduced and built upon in a logical manner. Shyam emphasized the importance of vertical connections according to the knowledge structure for effective learning. Teachers emphasized that linking real analysis with school mathematics fosters deeper understanding by promoting a coherent and structured learning progression.

Rajan stated, "I feel this is another thing, but for an expert, developing the power of understanding mathematics when understanding the link." Milan reflected, "It pushed me to think deeply and search for logical reasoning in other subjects." These perspectives highlight how linking real analysis to school mathematics enhances mathematical maturity and analytical skills, reinforcing its significance in school mathematics.

Expert 2 stated that.

The horizontal link supports learners in gradually understanding mathematics according to its natural progression. For example, Piano's axioms provide foundational ideas about natural numbers and the question of subtraction, such as  $1 - 1 = ?$  Stimulates the discovery of whole numbers. Similarly, the number

system evolves through this process of exploration and extension. In the same way, real analysis presents concepts that are inherently linked to school mathematics. This hierarchical connection facilitates a step-by-step learning process, enabling students to develop a progressive understanding of mathematical concepts from simple to complex.

Therefore, if the course structure establishes a link with knowledge of real analysis, it helps the students learn sequentially and makes it easy to learn analysis.

**Motivation.** The connection between real analysis and secondary-level mathematics enhances learners' motivation by providing deeper insight, reinforcing conceptual understanding, and fostering an appreciation for mathematics. Milan emphasized that linking real analysis to its practical applications increases motivation, stating, "When the link is observed as an application, it motivates the learners. Supporting this view, Expert 1 added, "When the link is established, it motivates the students." This suggests that practical applications of real analysis help cultivate students' interest in mathematics, encouraging them to engage more deeply with the subject.

Expert 2 further highlighted the role of connections in fostering curiosity, stating, "Connection cultivates curiosity in students. This implies that establishing meaningful links between real analysis and school mathematics enhances comprehension and stimulates students' desire to explore mathematical concepts further. Shyam emphasized that such connections inspire students and spark their interest in mathematics. Similarly, Hari noted, "Links focus on application, which increases interest." So, teachers argue that a link is beneficial in motivating students to learn mathematics.

Thus, integrating real analysis with school mathematics in undergraduate courses encourages students to study real analysis more effectively. Emphasizing its applications in teaching contexts is a powerful tool for inspiring and engaging learners, ultimately strengthening their motivation to learn mathematics.

**Enhancing Critical Thinking.** The ability to analyze, evaluate, and synthesize mathematical concepts and problems systematically reflects critical thinking. The study of real analysis strengthens these abilities in students by fostering logical reasoning, analytical thinking, proof justification, problem-solving skills, and mathematical communication. During task-based interviews, teachers emphasized that linking real analysis with secondary-level mathematics is crucial in developing these competencies, ultimately enhancing students' critical thinking in mathematics.

Saroj illustrated this through a real classroom scenario:

When I was teaching grade 10 at the beginning of my career, I explained a problem involving two-digit numbers and stated that it is represented as  $10x + y$ , where  $x$  and  $y$  are digits. A student asked, 'Why do we use 10?' I had no immediate answer, so I assigned another task and reflected on it. Then, I realized the answer was due to place value. The coefficient of  $y$  is 1, and the  $x$  is 10, representing the ones and tens place.

This experience demonstrates critical thinking, particularly in understanding concepts related to the number system. Such instructional challenges require strong mathematical reasoning, which can be significantly supported by knowledge of real analysis.

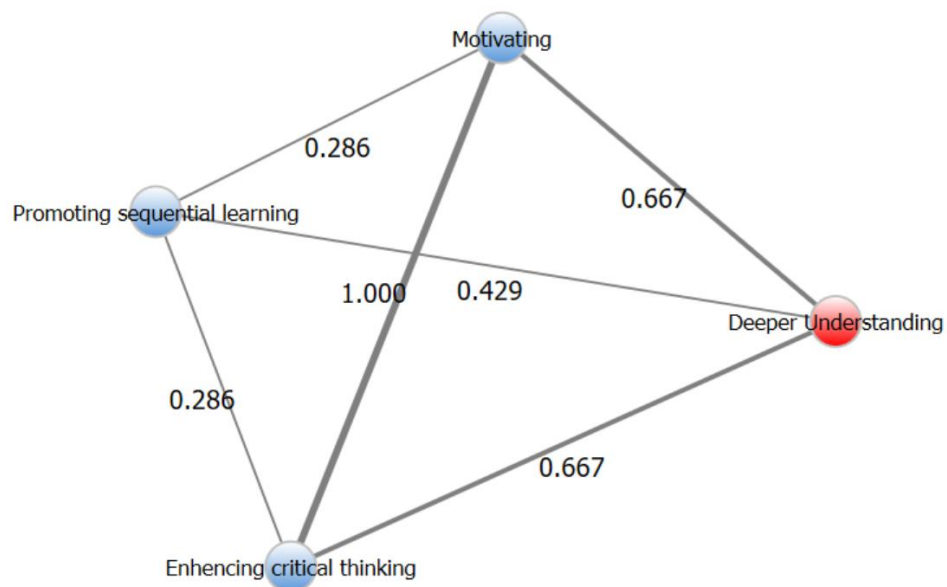
Hari believed that "connection enables mathematical thinking, pattern recognition, and development." This perspective highlights the role of real analysis in cultivating critical thought processes essential for problem-solving. Similarly, Expert

1 emphasized the importance of diverse approaches in teaching, stating, "Teaching should not be one-dimensional or limited to a single method of thought. Presenting the same concept in multiple ways is required, and such thinking is cultivated by real analysis." Hence, the vertical connection between real analysis and school mathematics enhances students' critical thinking by promoting deeper reasoning, flexible problem-solving approaches, and a structured understanding of mathematical concepts.

In addition to the beneficial aspects of the link between knowledge of real analysis and school mathematics identified by the thematic analysis, the relationship among the themes was further explored using the link analysis performed with QDA Minor 2024. The result is shown in Figure 25.

**Figure 25**

*Inter-Relationship Among the Importance of Link*



This link analysis underscores the pivotal role of deeper understanding in fostering mathematical learning, with its centrality highlighted by the red node. Strong associations with motivating (0.667) and enhancing critical thinking (0.667)

suggest that a profound comprehension of mathematical concepts significantly motivates and develops critical thinking skills. Additionally, promoting sequential learning (0.429) maintains a notable connection, indicating that a strong conceptual foundation supports the logical progression of learning. The substantial observed relationship ( $r = 1.000$ ) between enhancing critical thinking and motivation emphasizes the reciprocal reinforcement between these constructs.

A deeper understanding promotes motivation and critical thinking, fostering a cyclical enhancement of learning. While sequential learning plays a role in reinforcing understanding, its influence appears comparatively less pronounced. Strengthening students' conceptual depth is essential for cultivating engagement, logical reasoning, and structured cognitive development in mathematics. This analysis underscores the fundamental importance of deeper understanding as a cornerstone for advancing critical thinking, motivation, and systematic learning progressions in mathematical education.

### ***Techniques of Linking Knowledge of Real Analysis with School Mathematics***

Mathematics teachers at the secondary level hold positive attitudes towards the connection between knowledge of real analysis and schools' mathematics regarding the content connection. Besides this positive attitude and assessing the teachers' perception towards such links, teachers' views and mathematics educators' views were also collected by task-based interviews and interviews to answer the research question RQ7. *How can the link between knowledge of real analysis and secondary-level mathematics be strengthened?* A thematic analysis was performed to analyze the interviewed data. The analysis code and theme are shown in Table 28, which displays the strengthening of the link between KRA and SLM.

### **Table 28**

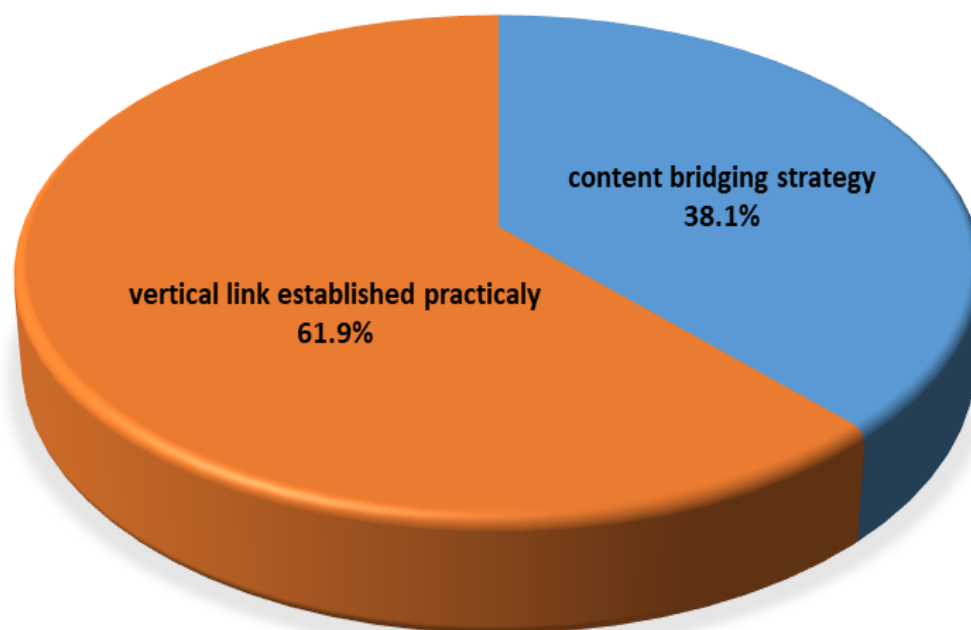
*Linking Techniques between KRA and SLM*

Category	Codes	Themes
How to establish the link	Link contents, link application during teaching, example of schools' mathematics, learning, teaching by connection.	Content bridging strategy
	Upper level and lower level, established link, hierarchical process, application, problem	Established vertical link

The content connection strategy and vertical link establishment were identified by thematic analysis. Figure 26 depicts the importance of these two techniques. It illustrates that establishing the vertical link is the primary method of establishing the link, and the content bridging strategy of instruction serves as a supplement technique.

**Figure 26**

*Linking Techniques and its Importance*



The thematic exploration of linking techniques is explained in the following topics.

**Established vertical link.** The establishment of a vertical link is a scholarly recommended approach to connecting curriculum and content across two educational levels. This method, which links knowledge of real analysis with school mathematics,

is supported by key techniques derived from thematic analysis. In task-based interviews, teachers argue that while the link between real analysis and secondary-level mathematics may already exist, whether consciously recognized or not, it depends on how one perceives and searches for this connection. Beyond the existing link, a clearer connection can be established by incorporating applied examples that are practiced in schools into real analysis. Additionally, including the name of relevant laws and their practical aspects in school curricula can strengthen this link and promote a more effective connection.

Hari highlights the value of using examples from school mathematics to introduce real analysis concepts. Education programs are encouraged to include more examples from school mathematics and illustrate their generalization in real analysis. For instance, when teaching limits, start with intuitive examples like approaching a fixed point, then build toward more formal definitions. This approach helps make abstract concepts more accessible to students by starting with something they already understand and gradually progressing to more formal definitions. While directly transferring advanced-level knowledge to lower levels may not be feasible, generalizing and simplifying complex ideas can make them extremely useful. Ram adds that "While it may not be possible to directly transfer advanced-level knowledge to lower levels, its generalization and simplified concepts are extremely useful." This method helps bridge the gap between secondary-level mathematics and real analysis by adapting advanced concepts in a way that is more manageable for students.

Kapil advocates for incorporating practical examples from school-level teaching, suggesting that these examples, when combined with higher-level concepts, can create a more effective learning experience. He stated that "Incorporating practical and useful examples for school-level teaching, alongside including elements

from higher-level concepts, can create a more effective learning experience." This approach not only reinforces understanding but also makes the higher-level concepts feel more relevant and practical. Milan stresses the importance of explicitly explaining the connection between real analysis and school mathematics to students. He elaborated, "Explicitly making these links helps bridge the gap between levels and fosters a deeper understanding." By clearly demonstrating the connections between these concepts, teachers can help students understand the continuity of mathematical ideas and encourage deeper engagement with the material.

Experts also offer valuable insights into the relationship between these two levels of mathematics. Expert 1 offers a broader view of the connection, noting that analytical thinking is inherently present even in the early grades. He explained that

the forms may differ; we teach these concepts at lower and higher levels. In the early grades, if we mention analysis, students might feel overwhelmed. However, even when teaching addition, multiplication, or division in lower grades, we use analysis, whether through counting on fingers, using beads, sticks, or other objects.

This highlights that even basic operations are rooted in analytical thinking, showing that real analysis is not entirely distant from early mathematical concepts. The foundational skills students develop at lower levels naturally feed into the more formal methods taught in higher-level mathematics.

Expert 2 builds on this idea, emphasizing the need for a deep understanding of mathematical concepts and their progression over time. "First, it is essential to understand the related mathematical concepts and their growth processes." This involves grasping the initial ideas, their progression, and how they interlink with other mathematical topics. Once this understanding is established, the next step is to

elaborate on these interconnected concepts in a way that aligns with the students' level of comprehension. Gradually introducing more complex ideas while building on prior knowledge is essential for creating a strong vertical link between secondary-level mathematics and bachelor-level real analysis.

Finally, establishing a vertical link between real analysis at the bachelor's level and secondary-level mathematics requires a progressive and thoughtful approach. Teachers' perspectives highlight the importance of starting with intuitive, accessible examples and building toward more formal definitions, while experts stress the interconnectedness of mathematical concepts at all levels. By recognizing these connections and making them explicit to students, educators can create a seamless learning experience that empowers students to understand the relevance and progression of mathematical ideas across their academic journey.

**Content Bridging Strategy.** The content bridging strategy emphasizes linking concepts across different levels of learning, ensuring continuity and application. In teaching real analysis at the university level, it is important for educators to explicitly connect advanced mathematical concepts with school-level mathematics. Expert 1 said that, "this strategy fosters a deeper understanding and demonstrates the relevance of higher-level mathematics in foundational learning." Additionally, Ram said that, "University teachers must explain how fundamental real analysis concepts, such as limits, continuity, and sequences, extend school-level mathematics." Moreover, Shyam said, "During lectures, instructors should integrate examples from school curricula, showing how real analysis principles underpin topics like functions, calculus, and number systems." Therefore, a content-bridging strategy is beneficial for understanding the different types of mathematics links at different levels.

Govinda said, "During the university study, if we learn the content by which content works in teaching at the secondary level, it makes it easy in real classroom teaching". Milan and Expert 3 also made similar assertions. Based on this evidence, content bridging is one way of strengthening the link.

The content bridging strategy in real analysis teaching ensures a smooth transition from school to university mathematics. By explicitly linking key concepts, university educators prepare future teachers with a strong conceptual foundation, ultimately improving mathematics education at the school-level.

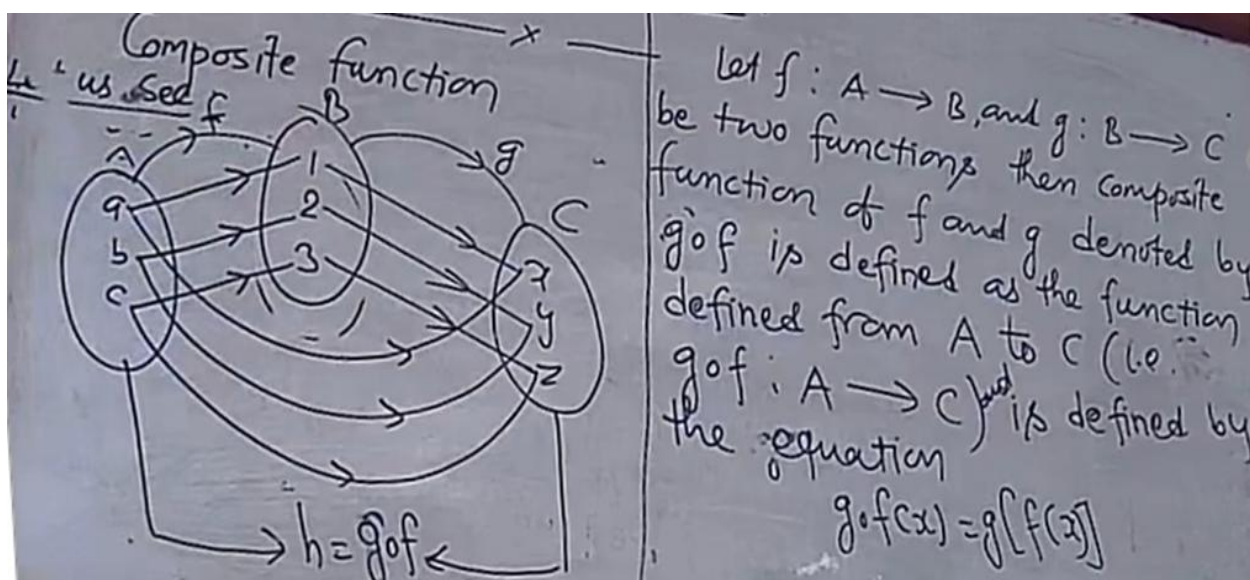
### ***Context where Teachers Applying KRA for SLM Instruction***

The survey results revealed that secondary-level mathematics teachers apply knowledge of real analysis in their instructional practices, as their attitudes toward using real analysis in mathematics were positive. Therefore, qualitative inquiry methods, particularly classroom observations, were conducted in this study phase to identify the key context in which secondary-level mathematics teachers characterize the mathematical knowledge of real analysis. Thematic analysis was conducted using data from classroom observations, observation notes, and transcriptions of key points from recorded sessions to address the research question, *RQ8. In what classroom contexts do teachers characterize their mathematical knowledge of real analysis?* The thematic analysis of the task-based interview and classroom observation note resulted in the following areas of knowledge of real analysis that teachers practice in the school's mathematics instruction. The result of the theme is shown in Table 39. Three major contexts emerged from the thematic analysis: the key concept development context, the problem-solving context, and the abstraction context. These contexts were identified through classroom observations and are described based on common themes that emerged.

**Table 29***Contexts of Application of KRA in SLM Instruction*

Code	Theme	Descriptions
Definition, example, counterexample, terminology, language, notations, define term	Concept development context	The classroom context where teachers and students' activities focus to develop the specific concept of mathematics.
Problem-solving, applying formula, trial and error, algorithm, numerical problem, using formula	Problem-solving context	The classroom context where teachers and students are engaging in problem solving, formula applying activities
Formulating general rules, principles, rules, patterns, symbolizing, explaining properties, expressing in algebraic form, formulation	Abstraction context	The classroom context where teachers and students are engaging in generalization of mathematical concepts and symbolic interpretation of mathematical concept

**Concept Development Context.** During classroom observations, teachers demonstrated their knowledge of real analysis through various instructional activities. They employed definitions, examples, counterexamples, and terminologies associated with real analysis concepts.

**Figure 27***Concept of Composite Function Demonstrated by Teacher*

Specifically, teachers used definitions of limits, functions, continuity, sequences, and series to establish general mathematical concepts.

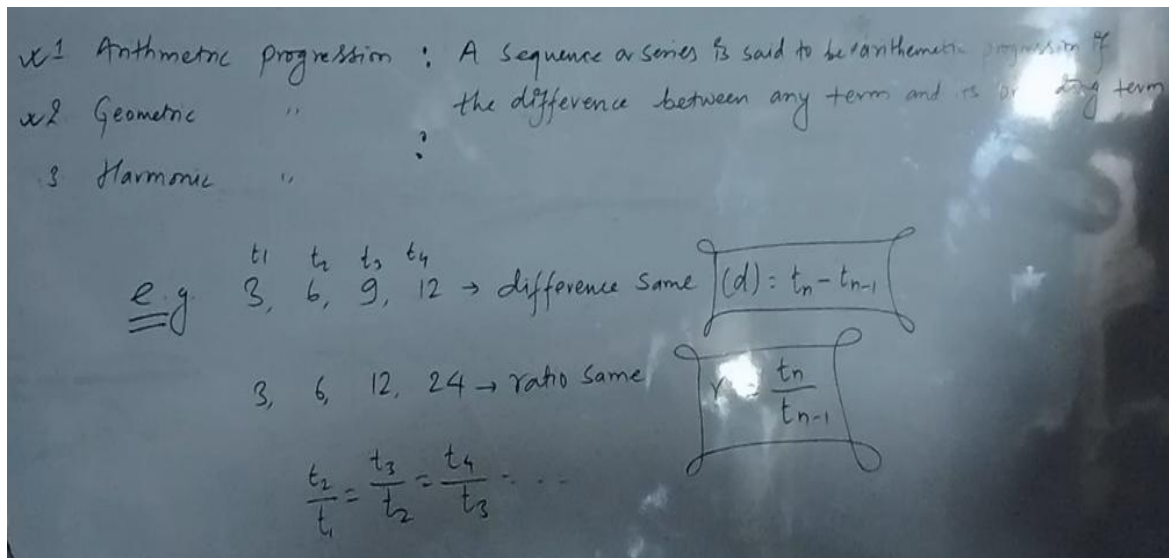
Additionally, examples illustrating these topics were observed during instruction. For instance, Shyam demonstrated the definition of a composite function, as shown in Figure 27.

Similarly, Kapil demonstrated the general example of a sequence. The teacher's expression was recorded, as in Figure 28.

Therefore, the concept development situation is one context where secondary-level mathematics teachers characterize the mathematical knowledge of real analysis. Beyond the previously mentioned example, the classroom observation results reveal that teachers demonstrate varying degrees of knowledge in real analysis, particularly in the concept development of limits, continuity, and series.

### Figure 28

#### *Teachers' Creation of an Example of Sequence*



These topics, which form the foundation of fundamental analysis, were consistently incorporated into classroom discussions, problem-solving activities, and demonstrations. While the depth and clarity of their explanations may differ, the integration of real analysis concepts in these topics was evident across the lessons observed. For instance, when explaining the notion of limits, teachers guided students

through formal definitions, graphical representations, and numerical approximations to establish a comprehensive understanding of convergence. Similarly, the concept of continuity was introduced through intuitive explanations, counterexamples, and rigorous proofs, allowing students to differentiate between continuous and discontinuous functions. In the case of series, teachers emphasized the significance of convergence criteria and the application of series in real-world mathematical modeling.

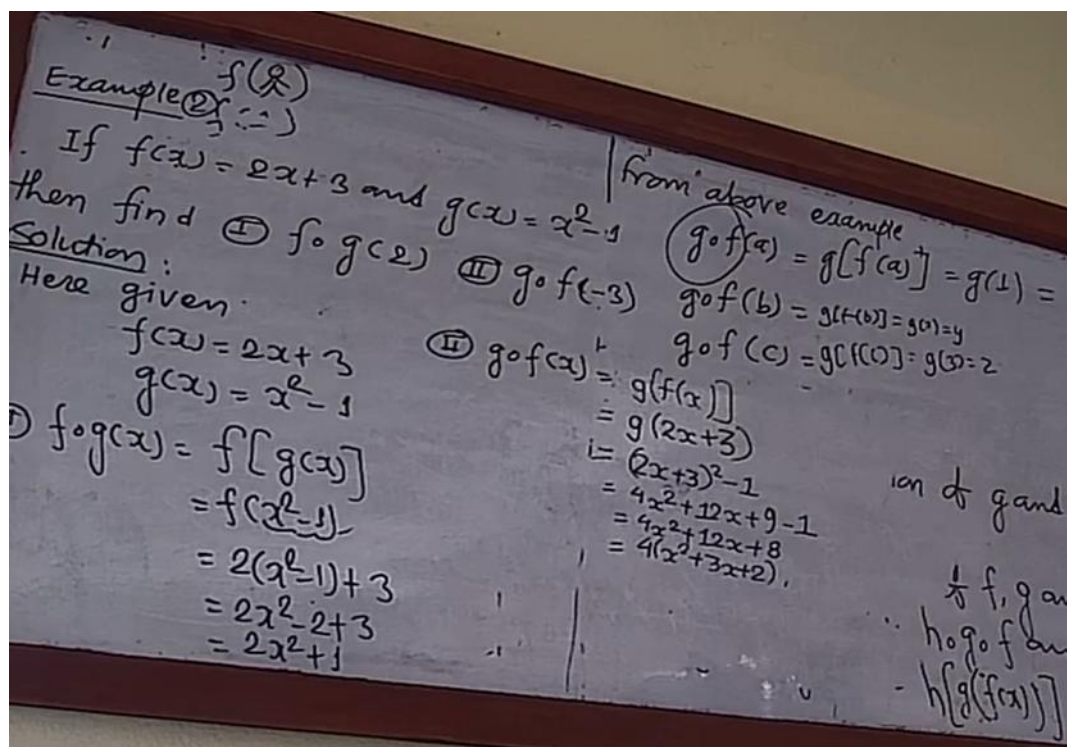
Teachers effectively bridged abstract mathematical theories with practical classroom applications by integrating real analysis principles into their teaching practices. This approach facilitated a structured learning experience, enhancing students' ability to engage with complex mathematical reasoning.

**Problem-Solving Context.** Mathematics teachers at the secondary level demonstrated a high level of knowledge of real analysis in their instructional activities. The logical application of various real analysis concepts was observed during the classroom observations.

When teachers teach, students are engaged in problem-solving contexts related to limits, functions, sequences, series, and continuity. Teachers demonstrate content-related knowledge in these areas. Beyond these topics, teachers also exhibit their knowledge of real analysis, guided by the axiomatic foundation of the real number system and its principles and laws. One direct content-related practice that Ram articulated is shown in Figure 29. Additionally, Milan's activities during equation-solving instruction were noted. His explanation focuses on composite function and utilizing the knowledge of real analysis. This indicates his ability to bridge abstract mathematical concepts with instructional practice, demonstrating a strong connection between subject matter knowledge and pedagogy.

Figure 29

Teacher Practice of Knowledge of Real Analysis in Function Teaching



He used the reverse process of the distributive law. Students often get confused in the initial stages of learning algebra.

$$81x^3 - 9x = 0$$

$$\text{Or, } 9x(x^2 - 9) = 0$$

$$\text{Or, } 9x(x-3)(x+3) = 0$$

Either  $9x = 0$  which gives  $x = 0$

Or  $(x-3) = 0$  gives  $x=3$  and  $(x+3) = 0$  gives  $x = -3$

This indicates that the teacher often uses the field axiom of real numbers. But this is a general and habitual practice. However, the teacher may not explain why this process exists and what operation is performed within the brackets. School mathematics practices the multiple sign near the bracket, but it depends on the

mathematical context; for example, if two functions  $f(x)$  and  $g(x)$  differ from the general multiple.

During the observation, teachers applied the field axioms of real numbers, such as commutative, distributive, inverse, and identity laws, during the problem-solving process, especially in algebra. Rajan demonstrated the utilization concept of the number system during problem-solving in sets. The problem was "Write the following sets using the set builder method: U is the set of natural numbers up to 10, A subset of U is a set if there is a multiple of 3, and B is a set of even numbers." Rajan used the properties of the number system and logic guided by real analysis and demonstrated the solution as  $U = \{x: x \leq 10, x \in \mathbf{N}\}$ ,  $A = \{x: 3x \in \mathbf{N}\}$  and  $B = \{x: x = n^2, n \in \mathbf{N}\}$ .

In addition to this demonstration of knowledge of the real number system and its properties, along with fitting logic and description, a hidden concept was also observed. This suggests the idea of a closed set: the universal set defined here is essentially a closed interval,  $[1, 10]$ , as defined in real analysis, but the members lie within the set of natural numbers.

The above-mentioned teachers' practices reinforce their expertise in the number system and illustrate how abstract mathematical concepts of real analysis, like closed sets, can be effectively integrated into problem-solving contexts in school mathematics instruction.

**Abstraction Context.** The context in which teachers and students engage in generalizing a particular mathematical process, establishing formulas based on logic, and writing about the particular context in mathematical terms is known as an abstraction context. Mathematics teachers characterize knowledge of real analysis in an abstract context. Especially teachers' use of logic and contexts of knowledge in real

analysis. Hari demonstrated the abstraction process during the teaching sequence and solving. The process applied by Hari was recorded as:

Find the  $n$ th term of 2, 5, 8, 11 .....

$$\text{Here, } t_1 = 2 = (3 \times 1 - 1)$$

$$t_2 = 5 = (3 \times 2 - 1)$$

$$t_3 = 8 = (3 \times 3 - 1)$$

$$t_4 = 11 = (3 \times 4 - 1)$$

Therefore,  $t_n = 3n - 1$ . This example explores how teachers use the knowledge of real analysis in an abstract context. The induction is a method of deriving a rule that is part of reasoning and is related to the knowledge of real analysis. Hence, teachers use the knowledge of real analysis in an abstract context.

### ***Teachers' Practices of Knowledge of KRA in School Mathematics Instruction***

Secondary-level mathematics teachers exhibit positive attitudes toward the relationship between mathematical knowledge of real analysis and secondary mathematics content. They accept its pedagogical significance and role in enhancing teachers' mathematical proficiency, as indicated in the survey phase of this study. Classroom observations and task-based interviews were conducted to explore teacher practices of real analysis further to address the research question *RQ9. How do mathematics teachers articulate knowledge of real analysis in their classroom practices?* A classroom observation protocol was used to systematically collect data on teachers' instructional approaches. A thematic analysis was conducted to explain the practices of real analysis concepts within secondary mathematics instruction. The category, code, subtheme, and theme are shown in Table 30. Additionally, teachers' instructional activities and task-based interview records were analyzed regarding real analysis content, focusing on how teachers applied real analysis to explain

mathematical concepts, facilitate problem-solving, and support students' reasoning processes. The emerged themes are shown in Table 30.

**Table 30**

*Teachers' Practices of KRA in Mathematics Instruction*

Category	Codes	Subtheme	Themes
Teachers' practices of knowledge of real analysis	Definition of sequence, series, limit, continuity, example, illustration, explanation of content, applying the theorem	Providing a formal definition Providing concrete examples from real analysis Providing concept related to content of real analysis	Direct content related practices
	Axioms, closure law, commutative law, distributive law, rules, principles	Field axioms, order axiom, Trichotomy law, transitivity law	Practices of axiomatic foundation of real number
	Another way, different, differ, next technique, another term,	Mathematical terminology use, another example, graphical representation	Alternative representation

The analysis is interpreted in light of the emerging theme.

**Direct Content Related Practices.** The direct content connection practices refer to teachers' activities where they use real analysis concepts, such as definitions, examples, and explanations, that directly engage with topics like sequences, series, limits, and continuity. During the classroom observation, it was found that the teachers used knowledge of real analysis with varying degrees of direct connection. Several mathematical topics were observed during the classroom observation. Sequence and series, limit, continuity, function, indices, mensuration, trigonometry, quartile, sets, and related contents within these concepts were observed. During the classroom observation, it was noted that sequence, series, function, limit, and continuity were closely related to real analysis and were observed as content-related

practices. Shyam's definition of function aligns with the definition of function included in real analysis:

If  $A$  and  $B$  are two non-empty sets and  $f$  is a relation in which to each  $x \in A$  there corresponds a unique  $y \in B$ , then  $f$  is called a function. The unique element  $y \in B$  is usually denoted by  $f(x)$ . The value  $f(x)$  is called the image of  $x$  under  $f$ .

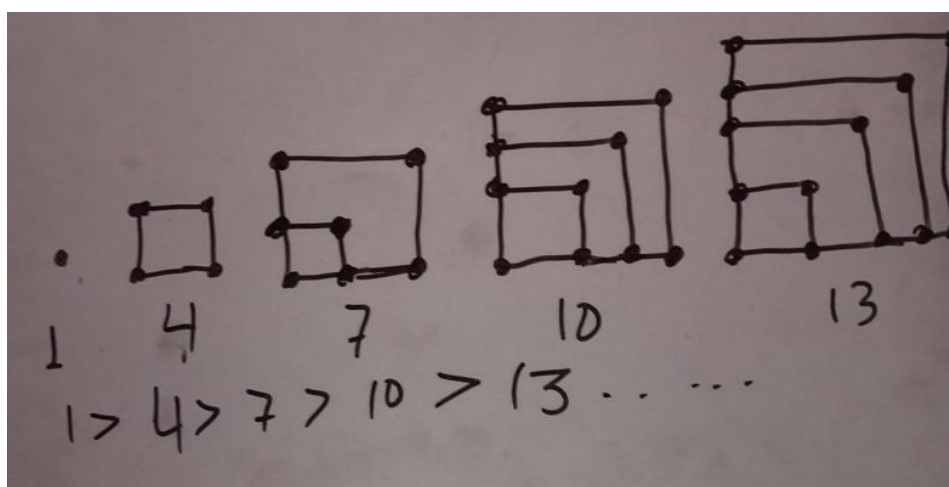
Additionally, he defines function as an example  $f(x) = x + 1$ , where  $x$  is an odd number and the resulting function is an even number.

In addition to this, Shyam explained various types of functions, including onto, into, constant, and composite functions, as introduced in the real analysis course of B.Ed. Second year at Tribhuvan University.

Similarly, Kapil illustrated the concept of finite and infinite sequences, which connects to the introductory review of real sequences in real analysis. Although this level includes topics like convergent and divergent sequences, the explanation remains aligned with the broader concept of sequences. Moreover,

### Figure 30

*Example of Strictly Monotonically Increasing Sequence*



Milan illustrated the arithmetic sequence using a figure, demonstrating it as an increasing sequence, as shown in Figure 30. This example was observed during a classroom observation, where the teacher used it to illustrate common differences in sequences and demonstrate increasing sequences. In real analysis, such sequences are formally studied as strictly monotonic sequences in the review section. These observations demonstrate that teachers apply their knowledge of real analysis directly to various topics, including functions, limits, continuity, sequences, and series.

By aligning their teaching with these central concepts, they support the logical structure and theoretical foundations of real analysis in practical classroom instruction.

**Practices of Axiomatic Foundation of Real Numbers.** The foundation of real analysis begins with the axiomatic system of real numbers, which includes fundamental properties such as field axioms, order axioms, and completeness axioms. These axioms provide a rigorous structure that underpins various mathematical operations and reasoning processes. During classroom observations, teachers articulate these properties in problem-solving and the development of mathematical concepts. During the observations, it was found that teachers apply these axioms, either explicitly or implicitly, to guide students in developing a deeper understanding of algebra, arithmetic, and other mathematical concepts.

During the classroom observation and task-based interview, it was observed that the teacher frequently applied the cancellation law for addition and multiplication, and identified the law in the context of problem-solving. Nevertheless, teachers used them as a habit of mathematical activities rather than lawful steps. For example:

$$2a + 3c = 4b + 3c$$

$$\text{or, } 2a+3c-3c = 4b$$

$$\text{or, } 2a = 4b$$

$$\text{or } a = \frac{4b}{2} = 2b$$

This solution is guided by the properties of real number and its theorem that, if  $a+c = b+c$  then  $a= b$  for all  $a, b, c$  lies in  $\mathbf{R}$ ; if  $ac = bc$  then  $a= c$  for all  $a,b,c$  lies in  $\mathbf{R}$  and  $c$  is not zero and  $a.1 = a$  as multiplicative identity law, and  $a+0 = a$  additive identity law.

During the classroom observation, Rajan demonstrated the order property of  $\mathbf{R}$ . However, he applied it to natural numbers, where he attempted to convince the student to teach the logic of consecutive natural numbers for solving a problem that involves finding three consecutive numbers whose sum is 15 and product is 120. For the solution of this question, one short dialogue was as follows:

Rajan: Suppose three consecutive numbers are  $x-1, x,$  and  $x+1$ .

Student 1: If we assume  $x, y, z,$  or  $a, b, c,$  then what?

Rajan: These consecutive natural numbers have differences of 1, so we suppose them as  $x, y,z,$  or  $a,b,$  or  $c$ . However, we are given only two conditions, so two equations cannot determine three variables. In the case of quadratic equations, we must assume a single variable.

In this example, Saroj said it is difficult to convince students about the nature of the number system, depending on whether Rajan applied discussion to natural numbers or real numbers. He explained the problem logically, considering its origin and the number system used. This demonstrated how Rajan applied the axiomatic foundation of real numbers in practice.

Similarly, an event was observed in Saroj's class where he engaged students in solving a verbal problem on linear equations involving a two-digit number. He wrote

$10x + y$  to represent the number, where  $x$  and  $y$  are its digits. At this point, a student asked why they assumed  $10x + y$  instead of just  $x$ , noting that variables like  $x$  or  $a$  are often used in other cases. This was a thoughtful question about the structure of numbers and the place value system. In response, Kapil explained "the number's structure and the significance of place value in forming and interpreting numbers correctly."

In teaching sets, especially in grade nine, it was found that teachers often used the number system to define sets and identify their members in order to solve set problems presented in set-builder or description methods. Examples such as even numbers, odd numbers, prime numbers, composite numbers, factors, and multiples of particular numbers were commonly used, all deeply aligned with the number system. Without a deep understanding of the number system, identifying the members of these sets becomes quite challenging. Rajan demonstrated an example of set creation. Rajan noted a problem:  $U$  is a set of natural numbers from 1 to 9,  $A$  is a set of composite numbers, and  $B$  is the set of prime numbers. Write these sets and illustrate them in a Venn diagram. In this question, the Ram asked the students to make a list of even and composite numbers, but some got confused. The teacher then demonstrated the number system and prepared the sets using the listing method:  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ;  $A = \{4, 6, 8, 9\}$ ;  $B = \{2, 3, 5, 7\}$ . After this writing, one student raised a question about 1, and the Ram again reminded us of the definition of a composite number as 'a number greater than 1 and having more than two distinct divisors'. This evidence indicates that teachers employ several concepts from the axiomatic foundation of real numbers during instruction.

During the classroom observation, the teacher used the field axiom of real numbers. Milan demonstrated that:

$$81x^3 - 9x = 0$$

$$\text{Or, } 9x(x^2-9) = 0 \quad [\text{step 1}]$$

$$\text{Or, } 9x(x-3)(x+3) = 0 \quad [\text{step 2}]$$

Either  $9x = 0$  which gives  $x = 0$

Or  $(x-3) = 0$  gives  $x=3$  and  $(x+3) = 0$  gives  $x = 3$

In this example, step 1 involves using the reverse of the distributive property, followed by factoring in step 2. After that, the zero-product property is applied. This property is guided by real analysis and algebra, and teachers apply it knowingly or unknowingly.

Teachers frequently use the field axioms of real numbers, such as commutativity, associativity, distributivity, and the existence of additive and multiplicative identities, to facilitate their students' comprehension of number operations and solving algebraic expressions. Hence, the teacher often practices the axiomatic foundation of real numbers.

**Alternative Representation.** Alternative representation is a technique where teachers use different methods or forms to present the same mathematical concept. The thematic analysis of classroom observations and task-based interviews revealed that teachers employed alternative representations of knowledge in real analysis during secondary school mathematics instruction. During the task-based interview, teachers demonstrated some mathematical concepts found in real analysis, which they alternatively applied during their instruction. Teachers used figures to illustrate the concept of a limit. Rajan demonstrated his practice of teaching limits during the task-based interview, as shown in Figure 31.

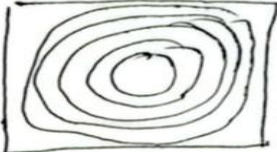
In this figure, the teacher employs an alternative method of defining the limit, rather than the Epsilon-delta definition. He uses a numerical example to show the tending trends, which is also visualized.

Figure 31

Teachers' Alternative Representation of Limit

Limit

$\frac{1}{10} = 0.1$ $\frac{1}{100} = \frac{1}{10^2} = 0.01$ $\frac{1}{1000} = \frac{1}{10^3} = 0.001$	$\frac{1}{10^1} = 100$ $\frac{1}{10^2} = 1000$ $\frac{1}{10^3} = 10000$ $\frac{1}{10^n} = ?$
--	--

How much enlarge the circle to the given Rectangle?

Def<sup>n</sup>: If the value of the function  $f(x)$  approaches to the finite and fixed number  $L$  as approaches to a number 'a' we say that  $L$  is a limit of the function as  $x$  approaches to  $a$ .

i.e  $\lim_{x \rightarrow a} f(x) = L$

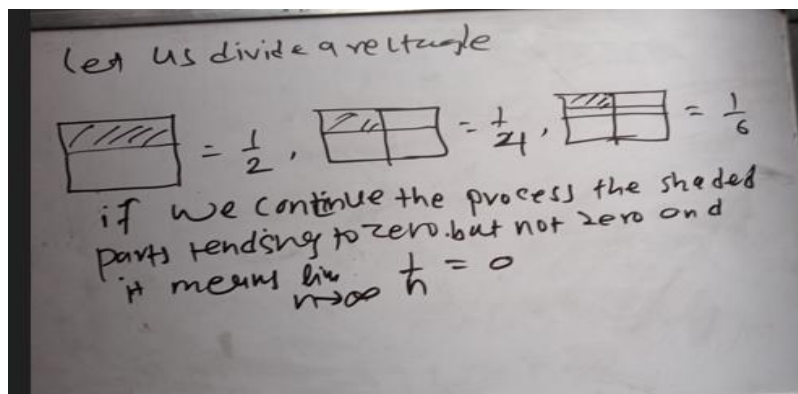
Left hand limit  $\lim_{x \rightarrow a^-} f(x)$        $a$        $\rightarrow$       Right hand limit  $\lim_{x \rightarrow a^+} f(x)$

During the task-based interview, Rajan demonstrated two ways to explain the concept of limits, which he has used in classroom practices. He explained that a function's value approaches a particular number as the input approaches a certain point, termed a limit. Similarly, he used a number line to demonstrate the concept of a limit. During the classroom observation, Saroj demonstrated similar visualization

practices of limit, as shown in Figure 32, where Rajan divided the rectangle successively and represented it in a fraction.

**Figure 32**

*Demonstration of Limit by Dividing a Rectangle*

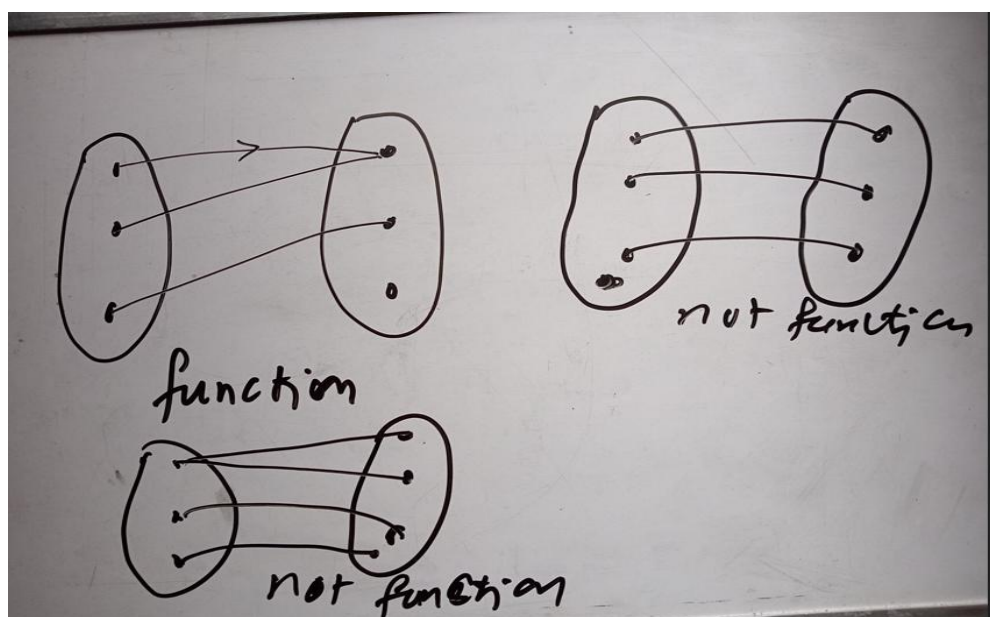


During the task-based interview, Rajan demonstrated two distinct methods for explaining the concept of limits, which he regularly employs in his classroom practices. He described the limit of a function as the value that the function approaches as the input moves increasingly closer to a specific point. He presented multiple approaches to enhance students' conceptual understanding and accommodate diverse learning styles.

During the function instruction, Hari demonstrated a diagram of the function that illustrates its validity, as shown in Figure 33. During the classroom observation, Rajan redefined the concept of a function and illustrated it by presenting examples and counterexamples. He used diagrams to visually represent the relationships between inputs and outputs, clarifying the conditions under which a function exists. Through this approach, he aimed to help students differentiate between functions and non-functions, reinforcing their understanding of the defining characteristics of a function. During the classroom observation, the Rajan defined the concept of a function and illustrated it by presenting both examples and counterexamples.

**Figure 33**

*Visualization of existence of Function*



To clarify the conditions under which a function exists, he used diagrams to represent the relationships between inputs and outputs visually. Through this approach, he aimed to help students differentiate between functions and non-functions, reinforcing their understanding of the defining characteristics of a function.

In teaching the concept of continuity, the teacher relied on graphical representations instead of the formal delta-epsilon definition. By using graphs, he visually demonstrated how a function remains unbroken and smoothly connected, helping students develop an intuitive understanding of continuity. Rather than introducing the formal notation and logical rigor of limit proofs, the Govinda focused on the visual and conceptual aspects, allowing students to grasp the idea of continuity through observation and interpretation of graphs.

Beyond diagrammatic representations, teachers also employ alternative terminologies to simplify abstract mathematical concepts. Govinda, for instance,

replaces the term commutative property of real numbers with the phrase changing the position to make the concept more accessible to students. Similarly, when explaining convergence, they use the phrase goes to a fixed number instead of converges. This approach minimizes cognitive barriers by using familiar language, making mathematical ideas more relatable and easily comprehensible. Shayam Said:

During a mathematics lesson in Class 9, a student posed the question: “If  $\frac{2}{2}=1$  and  $\frac{3}{3}=1$ , then why is  $\frac{0}{0}$  not equal to 1?” This question reflects a common overgeneralization of arithmetic rules, a frequent misconception at the secondary level. To address this, I applied my understanding of real analysis, specifically the notion of indeterminate forms. However, rather than directly using formal limit definitions which can be too abstract for students at this level. I used familiar, concrete scenarios to help students develop a conceptual understanding. First, I presented a case where zero chocolates are to be distributed among three students. The students quickly concluded that each child would receive zero chocolates, thus understanding that  $\frac{0}{3}=0$ . I then offered a second situation: distributing five chocolates among zero students. This prompted confusion, and I explained that since there is no one to receive the chocolates, the division  $\frac{5}{0}$  is undefined. Finally, I asked the students to consider the case where zero chocolates are to be distributed among zero students, representing the expression  $\frac{0}{0}$ . When asked how many chocolates each student would receive, students could not settle on a consistent answer. This scenario created the opportunity to introduce the concept of indeterminate forms. I explained that  $\frac{0}{0}$  does not have a single, well-defined value. In fact, depending on the context, such as the limiting behavior of different functions,

the value can approach 0, 1, infinity, or other outcomes. For instance, the

$$\lim_{x \rightarrow 0} \frac{x}{x} = 1, \lim_{x \rightarrow 0} \frac{x^2}{x} = 0, \text{ and } \lim_{x \rightarrow 0} \frac{x}{x^2} = \infty \text{ all involve the form } \frac{0}{0}, \text{ but yield}$$

different results.

This instructional experience demonstrates how knowledge of real analysis can support meaningful teaching at the school level. By translating abstract concepts into intuitive, real-world analogies, I was able to guide students beyond rote arithmetic and toward a more nuanced understanding of mathematical meaning. This not only corrected the misconception but also laid the groundwork for critical thinking in future mathematical learning.

These alternative teaching strategies demonstrate a shift from purely symbolic and formal mathematical definitions to more intuitive, visual, and linguistic methods. By incorporating diagrams, graphs, and accessible terminology, teachers create a learning environment where students can engage with complex concepts, such as functions, limits, and continuity, in a way that aligns with their cognitive development and understanding. Such instructional strategies highlight the importance of adapting teaching methods to students' levels of comprehension, particularly when introducing abstract mathematical concepts. By integrating multiple representations, visual, symbolic, and verbal, teachers can accommodate diverse learning styles and reinforce conceptual understanding. This multimodal approach not only aids in knowledge retention but also encourages students to think critically about mathematical ideas beyond rote memorization. Furthermore, these methods promote active engagement, allowing students to construct their understanding through exploration and reasoning. A balanced combination of formal definitions and intuitive explanations ultimately enhances mathematical learning by bridging the gap between abstract theory and practical comprehension.

### **Phase iii: Connecting Quantitative Data with Qualitative Explanation**

This study employed an explanatory mixed-methods research design. The quantitative phase used the survey method to assess teachers' attitudes toward real analysis, specifically regarding its connection with secondary-level mathematics, its pedagogical application, and its role in enhancing teachers' mathematical proficiency. Additionally, the impact of teachers' attitudes on their mathematical efficacy was measured in this phase.

In the qualitative phase, teachers' perceptions of their knowledge of real analysis were explored, focusing on content connections, pedagogical applications, and their potential to enhance mathematical proficiency. Furthermore, the study investigated teachers' practices regarding their knowledge of real analysis in instruction, the necessity of such knowledge, and its connection to school mathematics through qualitative inquiry. The results from both phases were combined and analyzed in light of the research questions posed in the study.

#### ***Teachers' Insight on Knowledge of Real Analysis for Mathematics Instruction***

The survey was conducted to answer first research question (RQ1: *what are teachers' attitudes towards knowledge of real analysis concerning its connection with secondary-level mathematics, its pedagogical applications, and its role in enhancing mathematical proficiency?* Additionally, a task-based interview was conducted to address the third research question (RQ3). *How do teachers perceive the mathematical knowledge of real analysis in regarding its content connection, pedagogical applications, and role in enhancing mathematical proficiency?* This aims to meet the first objective OB1. *To assess the teachers' attitude towards the relevance of mathematical knowledge of real analysis for secondary-level mathematics instruction.*

The joint display presented in Table 31 integrates findings from both quantitative and qualitative strands, providing meta-inferences that explain the convergence, divergence, or expansion of results. This integration enables a deeper understanding of how secondary-level mathematics teachers view and apply real analysis knowledge in instructional contexts.

The survey results showed that all the items measuring content connection attitude had a mean score above average (Mean > 3). Additionally, the domain-wise mean is also more than three (Mean of CONT1 = 4.1, Mean of CONT2 = 3.7, and Mean of CONT3 = 3.7). Moreover, the single mean test result showed that  $t_{(385)} = 3.83$  at a 5 % level of significance with  $p = 0.00 < 0.05$  and retained  $H_1$ : Secondary-level mathematics teachers have a positive attitude towards the content connection between real analysis and the content of secondary-level mathematics. Hence, teachers hold positive attitudes toward knowledge of real analysis and its connection to secondary-level mathematics. Two task-based interviews were conducted with secondary-level mathematics teachers to address the research question. The thematic analysis results reveal that teachers perceived three connection types between knowledge of real analysis and secondary-level mathematics. They are direct content connection, logical connection, and procedural connection. In the direct content connection function, limits, sequences, series, and functions were identified, and field axioms, ideas, and logical connections were identified under the logical connection. This finding from thematic analysis supports the result of the survey. Moreover, it was found that teachers easily recognize the direct content connection and accept its relation to the axiomatic foundation of real numbers. However, they did not mention the logical and procedural connections, which only emerged during follow-up questions in the task. Teachers perceive sequence and series as the primary areas of

connection, while limits, field axioms, and functions are viewed as secondary in their perception of content connection. Continuity and procedural connections play a lesser role in their overall perception of content connection.

**Table 31**

*Joint Display of Teachers' Attitude and Perception towards Real Analysis*

Research Focus	Quantitative Findings	Qualitative Findings	Meta-Inferences (Integrated Result)
Content Connection (RQ1 & RQ3)	- Mean scores > 3 for all items measuring content connection.- Domain means: CONT1 = 4.1, CONT2 = 3.7, CONT3 = 3.7.- Single mean test: $t(385) = 3.83$ , $p < 0.05$ , supports positive attitude.	- Teachers perceive three connection types: direct content (limits, sequences, series, functions), logical (field axioms, ideas), and procedural.- A direct connection was easily recognized; logical and procedural elements emerged upon probing.- Sequence and series are seen as primary connections.	Teachers hold positive attitudes toward the connection between real analysis and secondary mathematics content. Qualitative data enriches this by identifying direct, logical, and procedural connections, revealing a nuanced understanding and highlighting areas (logical, procedural) needing further emphasis.
Pedagogical Application (RQ1 & RQ3)	- Mean scores > 3 on all pedagogical items.- Domain means: PALI1 = 4.1, PALI2 = 4.0, PAPI3 = 3.7, overall mean = 3.93.- Single mean test: $t(385) = 36.23$ , $p < 0.05$ , supports positive attitude.	- Real analysis applied mainly in reasoning.- Mathematical activities (problem-solving) important.- Idea creation valued but less emphasized.- Seen as key for conceptual understanding and reasoning development.	Teachers positively perceive pedagogical application of real analysis and actively use it to foster reasoning and mathematical activities. Qualitative findings confirm and specify how real analysis supports teaching practices.
Enhancing Mathematical Proficiency (RQ1 & RQ3)	- Mean scores > 4.0 for proficiency-related items.- Single mean test: $t(385) = 48.23$ , $p < 0.05$ , supports positive attitude.	- Teachers believe real analysis enhances proficiency by improving identification of mathematical relationships, advancing knowledge, and strengthening logical reasoning.- Proficiency is closely linked to real analysis knowledge.	Teachers view real analysis knowledge as essential for improving mathematical proficiency, particularly in logical reasoning and relationship identification. Qualitative findings specify these proficiency domains and deepen understanding of survey results.

Hence, the results from both the attitude and perception assessments show that secondary-level mathematics teachers perceive direct, axiomatic, and procedural connections between knowledge of real analysis and secondary-level mathematics.

Regarding the pedagogical application, the survey result shows that each item has a mean score of more than three on the Likert scale. Additionally, the overall mean and the mean of each domain of pedagogical application (Mean of PALI1 = 4.1, PALI2= 4.0, Mean of PAPI3 = 3.7 and overall mean = 3.93) have mean more than average. Moreover, the single mean test result,  $t_{(385)} = 36.23$ , at a 5% significance level with  $p < 0.05$ , supports the retention of H<sub>2</sub>: Secondary-level mathematics teachers have a positive attitude towards applying mathematical knowledge of real analysis in mathematics instruction. Secondary-level mathematics teachers show a positive attitude toward the pedagogical application of knowledge of real analysis.

The thematic analysis of task-based interviews reveals that teachers primarily apply real analysis to reasoning within the mathematical context, which they consider an important application area. The second important application area is mathematical activities, such as problem-solving tasks and exercises. While idea creation is seen as a valuable tool for generating alternative problem-solving approaches and insights, it is perceived as a minor area compared to reasoning and mathematical activities. These findings suggest that real analysis is key in shaping instructional practices, especially in helping students develop a deeper understanding of mathematical concepts through logical reasoning and engaging activities.

The results of the qualitative inquiry support the findings from the survey. The mixed results indicate that mathematics teachers apply knowledge of real analysis for various purposes during instruction, particularly for reasoning within the mathematical context. Based on the responses from both inquiries, it can be concluded that teachers apply knowledge of real analysis while teaching mathematics at the secondary level.

The attitude towards knowledge of real analysis as a means of enhancing mathematics teachers' proficiency was examined through a survey. The results revealed that the mean scores for all items were more remarkable than three, indicating a positive response. Furthermore, the overall mean score, as well as the means for each domain, MPROF1 ( $M = 4.1$ ), MPROF2 ( $M = 4.1$ ), and MPROF3 ( $M = 4.1$ ), all reflected favorable attitudes. The single mean test result,  $t(385) = 48.23$ ,  $p < 0.05$ , further supports the retention of hypothesis H3: Secondary-level mathematics teachers have a positive attitude toward the contributions of mathematical knowledge from real analysis for enhancing teachers' proficiency. Hence, secondary-level mathematics teachers hold a positive attitude toward knowledge of real analysis, viewing it as essential for developing their mathematical proficiency.

In addition to the survey, task-based interviews were conducted to gain deeper insights into teachers' perceptions of how knowledge of real analysis enhances their mathematical proficiency. The thematic analysis of the interview data revealed that teachers believe mathematical knowledge fosters proficiency by developing skills in identifying mathematical relationships, advancing their mathematical knowledge and skills, and enhancing their logical reasoning abilities.

The qualitative findings support the survey results, as both data sources indicate that teachers have a positive attitude toward the role of knowledge of real analysis in developing their mathematical proficiency.

### ***Contribution of Real Analysis to the Development of Teaching Efficacy***

To examine the relationship among content connection, pedagogical application, mathematical proficiency, and teaching efficacy, Partial Least Squares Structural Equation Modeling (PLS-SEM) was employed.

While content knowledge is valued, its effective classroom application depends on pedagogical orientation, which activates real analysis understanding, enhances teacher proficiency, and reinforces teaching efficacy through reciprocal interaction.

**Table 32**

*Joint Display of Relationships Among CONTN, PAPLI, MPROF, and TE*

Construct/Path	Quantitative Results (PLS-SEM)	Qualitative Results (Themes & Examples)	Meta-Inference
CONTN → TE	Not statistically significant	<ul style="list-style-type: none"> <li>- Teachers noted direct connections (limits, sequences) easily.</li> <li>- Logical/procedural links emerged only after probing.</li> <li>- Teachers viewed content knowledge as useful but passive.</li> </ul>	While valued, content knowledge is not transformed into practice without pedagogical framing. Indicates insufficient pedagogical enactment of content links.
PAPLI → TE	Statistically significant positive relationship ( $\beta = 0.537, p < .001$ )	<ul style="list-style-type: none"> <li>- Teachers apply real analysis to develop reasoning and problem-solving tasks.</li> <li>- Emphasized conceptual clarity, abstraction, and idea creation.</li> <li>- Real analysis used for task design and student engagement.</li> </ul>	Strong alignment. Pedagogical orientation activates real analysis knowledge and directly enhances teaching efficacy.
MPROF → TE	Statistically significant positive relationship ( $\beta = 0.282, p < .001$ )	<ul style="list-style-type: none"> <li>- Teachers linked proficiency with logical reasoning, structural understanding, and identifying relationships.</li> <li>- Teachers with stronger proficiency gave richer classroom explanations.</li> </ul>	Proficiency serves as a bridge between theoretical understanding and classroom instruction. Reinforces model findings.
PAPLI → MPROF	Strong significant path ( $\beta = 0.614, p < .001$ )	<ul style="list-style-type: none"> <li>- Real analysis is seen as a tool for developing students' reasoning.</li> <li>- Teachers engaged in deep explanation, problem modeling, and connecting multiple representations.</li> </ul>	Pedagogical strategies enhance teacher proficiency by activating analytical thinking. Highlights reciprocal reinforcement.

The analysis focused on the extent to which attitudes related to content connection and pedagogical application of real analysis contribute to enhancing mathematical

proficiency, and how, in turn, these attitudes and proficiencies influence teaching efficacy. The results provide insight into the predictive strength of these constructs and how they collectively shape mathematics instruction at the secondary level. To further illustrate how the quantitative and qualitative strands converge in this explanatory sequential research design, the merged results are presented in Table 32 as a joint display. This table summarizes key relationships identified in the structural model, the corresponding qualitative themes derived from task-based interviews, and the resulting meta-inferences that explain how these findings integrate to support a deeper understanding of teaching efficacy among secondary-level mathematics teachers. The findings revealed that the attitude towards content connection did not significantly predict mathematical proficiency or teaching efficacy.

However, the pedagogical application of knowledge of real analysis was found to be a significant predictor of both teaching efficacy and students' mathematical proficiency. Furthermore, mathematical proficiency itself emerged as a strong predictor of teaching efficacy, highlighting the interconnectedness of content knowledge and instructional effectiveness.

The PLS-SEM analysis showed that 43.4% of the variance in attitudes towards knowledge of real analysis for developing mathematical proficiency could be explained by attitudes towards content connection and pedagogical application. Furthermore, attitudes towards knowledge of real analysis accounted for 12.5% of the variance in teaching efficacy. This finding shows that mathematical knowledge has a remarkable role in shaping mathematics teachers' efficacy.

The qualitative findings support the structural model derived from the quantitative phase: attitudes toward pedagogical application are the most powerful predictors of proficiency and efficacy, while content connection alone, though

positively perceived, is not a direct driver unless it is pedagogically enacted. This finding highlights the significance of pedagogical transformation in knowledge for enhancing teaching impact.

### ***Importance of Knowledge of Real Analysis for Mathematics Instruction***

In the survey phase of the research, teachers expressed a positive attitude toward the belief that there is a link between knowledge of real analysis and secondary-level mathematics teaching. This response prompted further investigation to explore the reasons behind this belief. To explore the underlying reasons for the necessity of real analysis knowledge in mathematics teaching, task-based interviews were conducted with secondary-level mathematics teachers, and in-depth interviews were conducted with mathematics educators. It was related to the fourth research question.

Experts and teachers agree that knowledge of real analysis is essential for advancing mathematical understanding. They concurred that mathematics teachers are expected to have mathematical knowledge beyond the level they teach. A broader and more in-depth understanding of the subject matter, along with advanced mathematical knowledge, is crucial. Real analysis is one such area that supports this development. Thematic analysis reveals that gaining maturity in mathematical thinking (46% of the reasons), developing confidence, expertise, and advancing in mathematical thinking are primary reasons for studying real analysis.

Teachers and educators agreed that the knowledge and skills gained from real analysis contribute to developing expertise, another key reason for studying real analysis. This expertise is essential for teachers.

Mathematical thinking skills are also crucial for school teachers. Teachers and educators agreed that knowledge of real analysis helps enhance mathematical skills,

including reasoning, thinking, and creativity. Additionally, real analysis plays a significant role in advancing abstract mathematical knowledge.

These reasons emerged from the thematic analysis of task-based interviews with teachers and interviews with mathematics educators. All themes highlight that knowledge of real analysis is essential for prospective teachers. This finding supports the aptitude test results, indicating that knowledge of real analysis is beneficial for teachers.

### ***Connection between Real Analysis and School Mathematics***

An analysis was conducted to explore the connection between knowledge of real analysis and the secondary-level mathematics curriculum, excluding the task-based interviews with teachers, which specifically aimed to address the research related to the connection between knowledge of real analysis and secondary-level mathematics. The results of the curriculum analysis and the thematic analysis of task-based interviews highlight significant mathematical connections between real analysis and secondary-level mathematics. Using a structured framework, various linkages across different categories were identified. The analysis demonstrates that key concepts from real analysis, such as real numbers and their axioms, functions, limits, and continuity, are directly linked to topics taught in school mathematics, providing a profound understanding of the subject matter.

The hierarchical relationships revealed that foundational concepts in real analysis are the basis for school-level topics. Axiomatic connections illustrate how principles from real analysis, such as field axioms and order axioms, underpin fundamental operations in secondary mathematics. Procedural connections, which use similar algorithms across both levels, emphasize the continuity in mathematical processes. Alternative representations show how the same concepts are presented

differently in real analysis and secondary mathematics. Additionally, the study highlights the importance of generalization, where concepts from real analysis are applied in broader mathematical contexts at the secondary level.

**Table 33**

*Joint Display of the Connection between Real Analysis and School Mathematics*

Data Source	Key Findings	Meta-Inference
Curriculum Analysis	<ul style="list-style-type: none"> <li>- Identified hierarchical connections (e.g., real numbers, sequences, functions)</li> <li>- Foundational real analysis concepts support school-level topics</li> <li>- Revealed axiomatic, procedural, alternative, and generalized links</li> </ul>	Real analysis content forms the structural backbone of secondary mathematics, suggesting deliberate alignment between university-level mathematical foundations and school-level topics. These connections are systematic and scaffolded.
Task-Based Interviews with Teachers	<ul style="list-style-type: none"> <li>- Teachers easily recognized direct content links (e.g., limits, sequences)</li> <li>- Logical/procedural links emerged with probing</li> <li>- Teachers confirmed relevance in problem-solving and reasoning tasks</li> </ul>	Teachers' perceptions align with the curriculum but are more experiential and surface-level unless prompted. Through reflection, they articulate rich conceptual and procedural connections, demonstrating a practical awareness of deeper structures.
Integration of Both Sources	<ul style="list-style-type: none"> <li>- Strong convergence between curriculum structure and teachers' understanding</li> <li>- Curriculum reveals theoretical depth; interviews reflect instructional practice</li> </ul>	There is mutual reinforcement: the curriculum provides theoretical grounding, and the teacher's experiences validate practical relevance. These findings justify the inclusion of real analysis in teacher education and its pedagogical importance.

The meta-inference drawn from the triangulation of multiple data sources is summarized in Table 33. The findings confirm the essential role of real analysis in deepening mathematical understanding at the secondary level by connecting advanced concepts with classroom instruction. While the thematic analysis of task-based interviews and curriculum review was interrelated, the curriculum analysis revealed additional critical aspects that were not fully captured through the interviews. Nevertheless, both sources of data converged to highlight the instructional relevance of real analysis in secondary mathematics. These results underscore its foundational role in providing conceptual, procedural, and axiomatic coherence across topics. Consequently, integrating real analysis into teacher education is vital for

strengthening teachers' mathematical understanding and improving instructional effectiveness.

### ***Importance of Connecting Real Analysis with the School's Mathematics***

The teachers' attitudes toward the connection between real analysis and secondary-level mathematics were positive and statistically significant. This result prompted further investigation into why such a connection is necessary. Additionally, the curriculum analysis revealed a moderate connection between the two areas.

To identify the necessity of content connection between secondary-level mathematics and knowledge of real analysis, task-based interviews were conducted with teachers and mathematics educators. The thematic analysis of the interview results highlighted that this connection is essential for fostering a deeper understanding of mathematics, promoting critical thinking, enhancing learners' comprehension, and supporting a more sequential understanding of mathematical concepts. Table 34 shows the result of the joint display.

**Table 34**

#### *Joint Display of Importance of Knowledge of Real Analysis*

Data Source	Key Findings	Meta-Inference / Interpretation
Quantitative Survey (Teachers' Attitudes)	- Positive, significant attitudes toward the real analysis–secondary math connection.	Teachers recognize the importance of the connection, prompting deeper qualitative exploration.
Curriculum Analysis	- Moderate content connection; direct, logical, procedural, and alternative links identified.	The curriculum structurally supports the foundational role of real analysis in secondary mathematics.
Task-Based Interviews (Teachers)	- Connection is seen as essential for deep understanding, critical thinking, and concept development.	Teacher experiences confirm the applied and pedagogical value of real analysis.
Interviews with Mathematics Educators	- Highlight the role of real analysis in fostering comprehension and reasoning.	Experts reinforce its instructional importance and conceptual coherence in teaching.
Integrated Meta-Inference		Real analysis is both structurally embedded in the curriculum and pedagogically valued by teachers and educators, confirming its essential role in mathematics instruction and justifying its place in teacher education programs.

The positive attitudes of teachers toward the connection between knowledge of real analysis and secondary-level mathematics are supported by curriculum analysis, which revealed multiple layers of connection, including direct content links, logical progressions, procedural relationships, and alternative representations. This alignment suggests that teachers' perceptions are grounded not only in experience but also in the structural design of the curriculum. Additionally, interview data provided further insight by identifying key reasons why this connection is essential, such as fostering deeper mathematical understanding, enhancing critical thinking skills, and supporting a more coherent and sequential learning process for students.

Taken together, these findings demonstrate a strong, mutually reinforcing relationship between real analysis and secondary mathematics education. The theoretical foundations identified in the curriculum validate teachers' positive perceptions, while qualitative insights from interviews provide practical and pedagogical support for this connection. This integrated evidence confirms the importance of incorporating real-world analysis knowledge into teacher education and curriculum development to enhance both teaching effectiveness and student learning outcomes.

### ***Connection Strengthening Techniques between KRA and SLM***

In order to explore ways to strengthen the link between knowledge of real analysis and secondary-level mathematics, task-based interviews were conducted with teachers and mathematics educators. Teachers' attitudes, as reported during the survey phase, indicated that such a link exists and is essential. Therefore, strategies for strengthening the connection between knowledge of real analysis and secondary-level mathematics were explored. The joint display table 35 shows the inferences made from multiple data sources.

**Table 35***Joint Display of Connection Strengthening Ways between KRA and SLM*

Data Source	Key Findings	Meta-Inference / Interpretation
Quantitative Survey (Teachers' Attitudes)	- Teachers reported that a link between real analysis and secondary mathematics exists and is essential.	Teacher recognition of the connection establishes a foundation for exploring strategies to strengthen this link.
Task-Based Interviews with Teachers	- Emphasized the importance of establishing vertical connections between university-level real analysis and school mathematics.- Suggested rethinking university teaching strategies to integrate these connections better.- Advocated for a content-bridging approach demonstrating explicit applications in secondary math.	Teachers desire more explicit, practical teaching that bridges theory and school mathematics, highlighting the need for instructional innovation.
Interviews with Mathematics Educators	- Supported the need for practical examples connecting real analysis concepts to secondary-level content.- Reinforced the importance of strengthening these connections pedagogically.	Educators validate teachers' perspectives and emphasize that university-level mathematics instruction must be aligned with school-level relevance.
Curriculum Analysis	- Confirms the existence of strong content connections that can be leveraged through improved instructional approaches.	Curriculum structure provides a theoretical basis that supports and guides the strengthening of these vertical and bridging links.

A thematic analysis of task-based interviews with teachers and interviews with mathematics educators revealed key strategies for reinforcing the link between real analysis and secondary-level mathematics. The primary strategy involved the practical establishment of vertical connections that explicitly bridge university-level real analysis concepts with school mathematics topics. Teachers emphasized the need to rethink university mathematics teaching approaches, advocating for instruction that moves beyond theoretical exposition to include practical examples demonstrating how real analysis applies to secondary mathematics. This content-bridging approach was frequently suggested as a preferred method to strengthen the connection. Mathematics educators supported these views, underscoring the pedagogical importance of making these links explicit in higher education. Collectively, these perspectives not only confirm the existence of the content connection identified through survey and

curriculum analyses but also highlight practical, instructional strategies favored by stakeholders for enhancing this connection. This integrated evidence provides a comprehensive understanding of both the theoretical and applied dimensions necessary to strengthen the link between advanced mathematical knowledge and school-level mathematics teaching.

### ***Teachers' Practices of KRA in School Mathematics Instruction***

The survey results indicated that teachers utilized their mathematical knowledge of real analysis during secondary-level instruction. After this finding, task-based interviews and classroom observations were conducted to address the research questions: *In what classroom contexts do teachers characterize their mathematical knowledge of real analysis (RQ8)? Moreover, how do mathematics teachers articulate their knowledge of real analysis in their classroom practices (RQ9)?*

**Table 36**

#### *Joint Display of Teachers' Practices of KRA*

Focus Point	Data Source	Key Findings	Meta-Inference / Interpretation
Contexts of real analysis use	Interviews & Observations	Use of definitions, axioms, notation, and examples in explaining limits, sequences, functions, continuity, and problem-solving.	Real analysis concepts form the foundation of classroom instruction and problem-solving activities.
Articulation in teaching practices	Interviews & Observations	Formal explanations with axioms treated as foundational truths; use of alternative representations to aid understanding.	Teachers integrate real analysis knowledge practically but often omit deeper conceptual justification.
Teachers' attitudes	Survey	Positive attitudes toward real analysis content and its pedagogical relevance (mean scores 3.7–4.1).	Favorable attitudes facilitate the integration of real analysis in secondary mathematics teaching.
Data integration	Qualitative + Quantitative	Alignment between teachers' attitudes and classroom practices.	Convergent evidence reinforces the critical importance of real analysis knowledge in secondary math education.

These research questions aimed to identify the key content areas of real analysis that teachers utilize in school mathematics instruction. Additionally, they sought to examine how teachers apply this knowledge to enhance instructional effectiveness and student understanding in the classroom.

The table 36 demonstrates the integrated results from the survey, interviews, classroom observations, and their combined interpretation, providing a comprehensive understanding of teachers' knowledge and classroom practices related to real analysis in secondary mathematics.

**Primary Contexts of Practices.** Teachers use concepts by using definitions, examples, counterexamples, mathematical language, and notation. Observations revealed that teachers frequently explain limits, functions, continuity, and sequences using precise definitions and illustrative examples. For instance, they demonstrated the concept of a composite function and the formulation of sequences, helping students build a foundational understanding of mathematics. Teachers' explanations varied in depth, but real analysis principles were consistently embedded in discussions.

Teachers apply knowledge of real analysis in problem-solving activities involving limits, functions, sequences, series, and continuity. Observations showed that teachers used axioms of the real number system, such as commutativity, associativity, and the distributive law, when guiding students through algebraic manipulations. For example, when solving polynomial equations, teachers implicitly applied field axioms. Additionally, teachers used set notation and logic to structure problems, reinforcing real analysis concepts through structured problem-solving.

Teachers and students generalize mathematical processes, derive formulas from logical reasoning, and symbolically express mathematical relationships. Teachers demonstrated abstraction through mathematical induction and functional rules. For example, Rajan derived the general term of an arithmetic sequence, showcasing real analysis principles in generalization. The process highlighted the importance of reasoning and logical formulation in school mathematics.

The findings from the previously discussed areas highlight the contexts in which teachers applied their knowledge of real analysis. Qualitative inquiry revealed that teachers predominantly applied real analysis concepts in contexts that required an axiomatic understanding of the real number system, functions, sequences and series, limits, and continuity. These conceptual foundations were particularly evident in instructional practices and problem-solving approaches. Additionally, teachers' perceptions of knowledge of real analysis, particularly regarding content connections and pedagogical applications, were positive.

The quantitative results further support these qualitative insights. The attitude test revealed a positive perception of key real analysis concepts in content knowledge and pedagogical applications. Specifically, the mean attitude score for algebraic properties of the real number system was 4.1, while sequences and series, as well as calculus-related content, each had a mean score of 3.7. In the domain of pedagogical applications, the mean scores were 4.1 for teaching the real number system and its properties, 4.0 for functions and limits, and 3.7 for applying series and proof concepts. These results indicate an overall positive attitude toward the pedagogical application of real analysis, as all mean scores exceed 3.

The alignment between qualitative and quantitative findings reinforces the significance of knowledge of real analysis in secondary mathematics teaching. The survey results verify the qualitative insights by demonstrating that teachers engage with real analysis concepts in their teaching and hold favorable attitudes toward their application. This convergence of findings underscores the critical role of real analysis in shaping both mathematical understanding and instructional practices in secondary education. It highlights how content knowledge gained through real analysis not only enhances teachers' conceptual clarity but also informs their pedagogical decisions.

Such alignment between perception and practice strengthens the case for its inclusion in teacher preparation programs.

**Teachers Practices of Knowledge of Real Analysis in Classroom.** The task-based interview and classroom observation explored how secondary-level mathematics teachers' knowledge of real analysis is reflected in their classroom practices. The survey results showed that teachers have positive attitudes toward the relationship between real analysis and secondary mathematics, recognizing its pedagogical significance and role in enhancing mathematical proficiency. Classroom observations and task-based interviews revealed that teachers directly applied real analysis concepts, such as sequences, series, limits, and continuity, through formal definitions, examples, and explanations. For example, Shyam defined a function using real analysis terminology, while Kapil demonstrated finite and infinite sequences in alignment with the content of real analysis.

Teachers also utilized axiomatic foundations of real numbers, including field, order, and completeness axioms, to guide problem-solving and conceptual development. Additionally, teachers applied the axioms and laws of real numbers, such as the inverse law and commutative law, directly in their instruction without offering extensive explanations or reasoning behind these laws. Instead, they treated these laws as established truths and utilized them in problem-solving procedures. This approach was observed in several instances where teachers used these laws to simplify calculations or solve algebraic expressions, assuming that students already understood the foundational reasoning behind them. For example, Milan used the distributive property in solving algebraic equations without explicitly explaining its derivation, presenting it as a given rule. This method of applying laws and axioms without further justification highlights a common teaching practice where certain

mathematical concepts are treated as foundational truths, allowing for efficient problem-solving but potentially limiting deeper exploration and understanding of the underlying principles.

Teachers frequently used alternative representations, such as graphical illustrations and alternative terminology, to make complex concepts more accessible to students. This use of alternative methods reflects a commitment to diverse instructional strategies to support student understanding. Teachers were observed presenting mathematical ideas from different perspectives to encourage broader comprehension, using varied examples and visual aids. Survey findings confirmed teachers' positive attitudes toward content connection and pedagogical application of real analysis, with ratings higher than three on the attitude scale.

These positive attitudes were evident in their instructional practices, where teachers applied real analysis concepts, axiomatic properties, and alternative representations to support students' understanding of the content. The alignment between teachers' attitudes and practices indicates that fostering a positive attitude toward real analysis enhances their teaching methods, benefiting students' mathematical reasoning and comprehension. This integration of knowledge and practice reflects the critical role that teachers' attitudes play in shaping effective teaching and in deepening students' understanding of mathematical concepts.

In conclusion, the integrated findings in the joint display table address the focus of RQ8 and RQ9 by demonstrating that teachers predominantly apply their knowledge of real analysis within key classroom contexts, such as explaining limits, sequences, and continuity through formal definitions, axioms, and problem-solving activities. Furthermore, the alignment between teachers' positive attitudes and their instructional practices shows that they actively articulate real analysis concepts using

precise mathematical language, alternative representations, and structured reasoning. This strong integration of knowledge and practice underscores the crucial role of real analysis in shaping effective secondary mathematics instruction and deepening students' conceptual understanding.

### **Chapter Summary**

This chapter presents the analysis and interpretation of data collected through an explanatory mixed-methods research design. The study was conducted in two phases: the first phase was quantitative, and the second phase was qualitative. The data were analyzed in three phases: the first phase involved analyzing quantitative data, the second phase focused on qualitative data, and the final phase integrated the quantitative and qualitative results. Quantitative analysis assessed teachers' attitudes toward knowledge of real analysis and its implications for teaching efficacy. The results of the Likert-scale responses indicate a positive attitude toward knowledge of real analysis, with higher mean scores indicating favorable perceptions across content connection, pedagogical application, and enhancement of mathematical proficiency. Statistical analysis, including one-sample t-tests, confirmed the significance of these attitudes, supporting the hypothesis that teachers identify the importance of real analysis in mathematics instruction. Furthermore, PLS-SEM results demonstrated a strong relationship between pedagogical application and mathematical proficiency, which in turn influenced teaching efficacy, while the direct impact of content connection was statistically insignificant.

Qualitative findings, derived from curriculum analysis, classroom observations, and task-based interviews, provided an in-depth understanding of teachers' perceptions. Thematic analysis discovered distinct content connections between real analysis and secondary mathematics, categorized into direct, axiomatic,

logical, and procedural connections. Participants highlighted the alignment of real analysis topics such as sequences, series, limits, and continuity with school mathematics while also highlighting the axiomatic structure underpinning number properties. The pedagogical application of knowledge in real analysis was observed in reasoning processes, problem-solving strategies, and the design of mathematical activities, facilitating instructional effectiveness. Additionally, the role of real analysis in developing mathematical proficiency was noted in three key areas: identifying mathematical relationships, logical reasoning, and advancing mathematical knowledge and skills.

The integration of qualitative and quantitative findings highlighted the relevance of knowledge of real analysis for mathematics instruction. Teachers' positive attitudes toward real analysis, as found in quantitative research, were supported by their classroom practices, where axiomatic foundations were frequently applied during instruction. Furthermore, it was discovered that qualitative insights shed light on how teachers' attitudes were reflected in practice, whereas quantitative results offered a more comprehensive understanding of their attitudes. The study highlights that knowledge of real analysis is important for teacher preparation programs.

## Chapter 5

### Findings and Discussion

This study investigates the significance of knowledge of real analysis in secondary-level mathematics education. To achieve this objective, nine research questions were developed. A mixed-methods research design was employed for the study. The chapter presents findings and discussion. The findings are organized under each heading corresponding to the research question, while the discussion blends findings from two or more research questions that share similar attributes.

#### Findings

The research findings based on the research questions are summarized under the following headings.

##### *Teachers' Attitude towards Mathematical Knowledge of Real Analysis*

The survey results indicated that teachers hold positive attitudes toward the content connection between real analysis and secondary-level mathematics. All items measuring content connection had mean scores above the average ( $M > 3$ ), with domain-specific means of CONT1 ( $M = 4.1$ ), CONT2 ( $M = 3.7$ ), and CONT3 ( $M = 3.7$ ). Statistical analysis results,  $t_{(385)} = 3.83$ ,  $p = 0.00 < 0.05$ , further confirmed this positive attitude. Teachers primarily recognized the direct content connection, especially regarding sequences, series, and functions, which were seen as directly related to secondary mathematics content. They also acknowledged the axiomatic foundations, such as field axioms, although logical and procedural connections only emerged during follow-up questions. These findings suggest that teachers perceive direct, axiomatic, and procedural connections between real analysis and secondary-level mathematics.

Teachers also exhibited positive attitudes toward the pedagogical application of knowledge of real analysis. The survey results showed that mean scores for each item related to pedagogical application exceeded the average Likert scale score. The overall mean score was 3.93, with domain means of PALI1 ( $M = 4.1$ ), PALI2 ( $M = 4.0$ ), and PAPI3 ( $M = 3.7$ ). The t-test result  $t_{(385)} = 36.23$ ,  $p = 0.00 < 0.05$  confirmed the positive attitudes. Teachers primarily apply knowledge of real analysis in reasoning within the mathematical context, considering this as an important application area. Meanwhile, Problem-solving tasks and exercises are followed as secondary applications. Though idea creation was considered valuable, it was less significant than reasoning and mathematical activities. These findings indicate that knowledge of real analysis plays a key role in shaping instructional practices, particularly in fostering deeper students' understanding of mathematical concepts.

In terms of contributing to mathematical proficiency, teachers also demonstrated positive attitudes toward the role of knowledge of real analysis in enhancing their mathematical skills. The survey results showed that all items related to this aspect had mean scores of more than three, with domain-specific means for MPROF1 ( $M = 4.1$ ), MPROF2 ( $M = 4.1$ ), and MPROF3 ( $M = 4.1$ ), indicating favorable attitudes. The t-test output,  $t_{(385)} = 48.23$ ,  $p < 0.05$ , supported this view. Teachers perceived knowledge of real analysis as essential for developing their mathematical proficiency. They believed it enhanced their ability to identify mathematical relationships, expanded their mathematical knowledge, and improved their logical reasoning abilities.

The Partial Least Squares Structural Equation Modeling (PLS-SEM) results indicated that teachers' attitudes towards content connection do not predict how knowledge of real analysis enhances mathematical proficiency and teaching efficacy.

However, teachers' attitudes towards pedagogical application (PALI) significantly predict how knowledge of real analysis contributes to the development of mathematical proficiency (MPROF) and teaching efficacy (TEACH).

***Relationship among Attitudes towards CONTN, PALI, MPROF and TE***

The structural model analysis showed that 43.4% of the variance in teachers' attitudes towards knowledge of real analysis enhancing mathematical proficiency can be explained by content connection and pedagogical application. Additionally, 12.5% of the variation in teaching efficacy is explained by content connection, pedagogical application, and mathematical proficiency.

A moderating analysis considered academic qualifications and training as potential moderators. However, these variables did not significantly impact the original model's relationships between the exogenous and endogenous variables. Hence, knowledge of real analysis predicts teaching efficacy and helps enhance mathematical proficiency.

The qualitative data strongly support the result from partial structural equation modeling gathered through classroom observations and task-based interviews. During classroom observations, teachers across diverse demographic profiles were noted to engage with mathematical content in similar ways. For instance, whether a teacher was relatively new or highly experienced, they demonstrated comparable skill in integrating concepts of real analysis into their lesson delivery. Teachers consistently employed strategies such as linking abstract axiomatic properties to concrete examples, scaffolding complex ideas through stepwise explanations, and encouraging students to reason rigorously about mathematical definitions and proofs. No observable differences emerged in the ways teachers approached these instructional practices that could be attributed to their background characteristics.

Similarly, task-based interviews provided further insight into the cognitive processes underpinning teachers' instructional application of real analysis knowledge. When encouraged with real analysis tasks or hypothetical teaching scenarios, participants from all demographic groups showed an equivalent depth of understanding and capacity to articulate the relevance of advanced mathematical concepts in the secondary school curriculum. Their explanations revealed a firm conceptual grasp and the ability to translate abstract real analysis principles into accessible learning experiences for students. This consistent pattern across interviews reinforces the quantitative finding that demographic variables do not moderate the link between attitudes toward real analysis knowledge and teaching efficacy.

#### ***Teachers' Perceptions of Knowledge of Real Analysis for Instruction***

The task-based interview was conducted to explore teachers' perceptions of the content connection between knowledge of real analysis and secondary-level mathematics content. The findings from the thematic analysis revealed that teachers perceive real analysis as corresponding to the content of secondary-level mathematics. Particularly, topics like functions, limits, continuity, sequences, and series are linked areas. Additionally, they recognize that knowledge of real analysis is applied in mathematics instruction at the secondary level. Mainly, it is used for problem-solving, idea generation, and mathematical reasoning in their teaching practices.

Moreover, teachers consider knowledge of real analysis to be essential for developing mathematical proficiency. They believe that it facilitates the advancement of academic mathematics by providing theoretical insights and increasing students' capacity to identify and understand mathematical relationships.

Finally, the analysis of the link between real analysis concepts and secondary mathematics shows that knowledge of field axioms, sequences, series, limits, and

functions forms the primary connection points. The remaining concepts also exhibit significant relationships with one another, contributing to a deeper understanding of mathematics.

The link analysis result shows that teachers perceive identifying mathematical relations as a key factor in mathematical proficiency, playing a crucial role in both acquiring advanced knowledge and enhancing logical reasoning. The relatively lower coefficients for Logical reasoning ability suggest that while reasoning is important, its direct role in knowledge advancement is perceived to be secondary to the ability to establish mathematical connections.

Additionally, the link analysis results show that the Advancement of Knowledge and Skill is a driver for identifying mathematical relationships and, to a slightly lesser degree, building logical reasoning. Although all three constructs are viewed as important, the analysis highlights the notion that teachers perceive real analysis powerfully as a vehicle for broadening mathematical knowledge and skills, which in turn enables educators to recognize patterns and connections throughout mathematics.

### ***Importance of KRA for School Mathematics Instruction***

The thematic analysis of task-based interviews with teachers and interviews with mathematics educators revealed several key insights regarding the importance of knowledge of real analysis for prospective mathematics teachers. A key theme that emerged out of it was the role of real analysis in developing maturity in mathematical thinking. Teachers and educators emphasized that real analysis enables teachers to cultivate deeper, more abstract thinking skills, which are essential for effectively teaching complex mathematical concepts. This was a major reason real analysis is considered vital in teacher preparation.

Knowledge of real analysis was also key to building confidence and improving pedagogical skills. Mathematics teachers and educators agree that real analysis is remarkable in cultivating mathematical thinking skills. Teachers focused on how real analysis boosts the critical thinking, reasoning, and problem-solving skills necessary for teaching and learning mathematics at higher levels. Teachers additionally highlighted that if a teacher has a strong foundation in real analysis, they handle complex mathematical problems and present concepts clearly and logically. This expertise was crucial for delivering high-quality instruction and helping students understand advanced topics. Additionally, participants noted that real analysis deepens their understanding of mathematical concepts, extending beyond the typical secondary-level curriculum.

Knowledge of real analysis helps develop the ability to generalize and prove mathematical concepts. These findings highlight that knowledge of real analysis is fundamental for cultivating the advanced thinking, confidence, and expertise for effective mathematics teaching.

The link analysis reveals that knowledge advancement serves as the backbone of real analysis learning, influencing expertise, confidence, and skill development. Strengthening students' foundational understanding of real analysis concepts can facilitate expertise and maturity in mathematical thinking, ultimately enhancing problem-solving abilities and confidence in handling abstract mathematical structures.

### ***Content Connection between KRA and SLM***

The analysis of the content connection between real analysis and secondary-level mathematics reveals several direct links between concepts taught at these two levels. The key connection between knowledge of real analysis and secondary-level mathematics is founded in the fundamentals of real number systems and their

axiomatic properties. Teachers use these concepts when instructing set problems and solving different mathematical problems. This connection illustrates how the foundation laid in real analysis supports the teaching of set theory and mathematical operations in secondary-level mathematics.

The concepts of open and closed sets in real analysis and linear inequality problems in school mathematics share another important content link. Open and closed sets are crucial to understanding increasingly complex subjects in real analysis, and secondary mathematics utilizes these concepts to formulate problems and investigate solutions involving linear inequalities. This link demonstrates the practical application of theoretical ideas in real analysis to secondary-level problem-solving.

The topics of real sequences, infinite series, functions, limits, and continuity highlight strong content connections between real analysis and secondary mathematics. Real sequences and infinite series, for example, are covered in both curricula. While secondary-level mathematics focuses on applying these concepts to solve specific problems, real analysis offers a more in-depth, theoretical understanding of principles such as convergence and divergence. Similarly, functions, limits, and continuity are studied at both levels. Secondary-level mathematics presents these ideas in a more tangible and comprehensible way, whereas real analysis offers a formal and abstract framework.

These direct connections between real analysis and secondary mathematics are evident in essential concepts such as the real number system, open and closed sets, sequences and series, and functions. These connections help bridge the gap between the theoretical foundations established in real analysis and their practical applications in secondary mathematics.

### ***Necessity of Content Connection***

The result highlighted the necessity of linking real analysis with school mathematics for adequate mathematical understanding. Making this connection enables learners to distinguish between examples and counterexamples of particular mathematical concepts and identify relationships, all contributing to a better understanding. Additionally, it enhances critical thinking by improving their capacity for logical reasoning, problem-solving, and mathematical communication, while motivating students to approach issues from multiple perspectives. Furthermore, the link serves as a motivational factor, as recognizing real analysis applications increases students' curiosity and engagement in mathematics. Moreover, promoting sequential learning through real analysis ensures a structured progression from fundamental to advanced concepts, facilitating a coherent and logical learning process. The link analysis highlights deeper understanding as a central factor in mathematical learning, strongly linked to motivation and critical thinking, indicating their mutual reinforcement. The connection between enhancing critical thinking and motivation is robust, emphasizing their role in fostering engagement and intellectual growth. Promoting sequential learning also contributes, but plays a comparatively smaller role. Strengthening conceptual depth is essential for fostering engagement, logical reasoning, and structured cognitive development, making deeper understanding a key driver of effective mathematical learning.

### ***Techniques of Strengthening Link between KRA and SLM***

Thematic analysis identified two key strategies for strengthening this link: the content bridging strategy and establishing a vertical link. The vertical link was recognized as the primary method, facilitating a structured progression of mathematical concepts across educational levels. Teachers emphasized the

importance of incorporating school-level examples into real analysis instruction, making abstract concepts more accessible. Experts highlighted that analytical thinking is inherently present even in early mathematics education, reinforcing the need for a progressive approach to curriculum integration. Strengthening this connection requires explicitly demonstrating the links between secondary and higher-level mathematics, promoting logical progression, and ensuring that students recognize the continuity of mathematical concepts.

### ***Context of the Classroom where Teachers Articulate KRA***

The thematic analysis of classroom observations revealed that secondary-level mathematics teachers apply their knowledge of real analysis in three primary instructional contexts: concept development, problem-solving, and abstraction. In the context of concept development, teachers engage students in defining mathematical terms, using examples and counterexamples, and introducing mathematical notations and terminology. During the observation, it was found that teachers used definitions with elaboration related to function, limit, continuity, sequence, and series, which helped develop a clear mathematical concept in the presented topic.

Students gain a deeper comprehension of the subject matter by making this connection because it helps them to grasp relationships, defend ideas, and distinguish between examples and counterexamples. Additionally, it improves their critical thinking abilities, bolstering their capacity for logical reasoning, problem-solving, and mathematical communication, all of which motivate students to approach problems from various angles. These abstraction context covers teachers' engagement in generalizing mathematical processes, framing principles, and symbolizing mathematical expressions. Observations reveal that teachers guide students through reasoning processes such as induction, pattern recognition, and logical derivation to

establish formulas and mathematical rules. Teachers effectively bridge abstract mathematical theories with school mathematics by integrating knowledge of real analysis into their instructional practices, enhancing students' conceptual understanding and problem-solving skills.

### ***Teachers' Practices of KRA During the Instruction***

The findings from classroom observations and task-based interviews indicate that secondary-level mathematics teachers actively apply their knowledge of real analysis in their instructional practices. Teachers demonstrated real analysis concepts such as sequences, series, limits, and continuity through formal definitions, examples, and problem-solving approaches. They also incorporated the axiomatic foundations of real numbers, including field, order, and completeness axioms, though often without explicitly explaining their derivations. Instead, these axioms and laws, such as the inverse and commutative laws, were presented as established truths to facilitate problem-solving. Additionally, teachers employed alternative representations, including graphical illustrations and varied terminology, to enhance student comprehension.

### ***Divergent Findings and Instructional Implications***

The PLS-SEM results showed that attitudes toward content connection did not significantly predict either mathematical proficiency ( $\beta = 0.053, p > .05$ ) or teaching efficacy ( $\beta = 0.066, p > .05$ ). In contrast, pedagogical application significantly predicted both mathematical proficiency ( $\beta = 0.614, p < .001$ ) and teaching efficacy ( $\beta = 0.537, p < .001$ ). Mathematical proficiency also significantly influenced teaching efficacy ( $\beta = 0.282, p < .001$ ).

Qualitative data revealed that while teachers recognized content connections, such as limits and sequences. These were often treated as static, rather than dynamic,

knowledge. In contrast, they described using real analysis pedagogically, especially for reasoning and task design. Creative use (e.g., idea generation) was less emphasized. Mathematical proficiency was mainly linked to reasoning, though some teachers equated it with procedural fluency.

The study's findings indicate that teachers are aware of the content connections between real analysis and secondary-level mathematics. However, this knowledge alone does not enhance their proficiency or teaching efficacy unless it is actively applied in pedagogical contexts. The strong predictive role of pedagogical application indicates that instructional use, particularly for reasoning and task design, is the key mechanism by which real analysis knowledge becomes instructionally meaningful. The divergence between data strands highlights the gap between content recognition and pedagogical enactment, underscoring the need for teacher preparation programs to support not only content understanding but also its effective integration in the classroom.

## **Discussion**

The findings from both the quantitative and qualitative phases were integrated and organized under the previously described thematic headings. The detailed discussion of these findings is explained under the respective headings.

### ***Teachers' Views on Knowledge of Real Analysis for Instruction***

A survey and task-based interview were conducted to assess teachers' attitudes and perceptions regarding the role of knowledge of real analysis in instruction. The study examined three key aspects: attitudes toward content connections, attitudes toward pedagogical applications, and attitudes toward enhancing mathematical proficiency through knowledge of real analysis.

The study's results showed that teachers hold positive attitudes towards knowledge of real analysis, particularly regarding its connection with secondary-level mathematics, its pedagogical applications, and its role in enhancing mathematical proficiency among teachers. These findings align with the finding that real analysis is not only the foundation of higher mathematics but also the structure and logic underlying school-level mathematical concepts, such as functions, limits, continuity, and number systems (Tall, 2013). The teachers' attitude and perception indicate that they recognize the pedagogical application and role of content connection in developing mathematical proficiency. This recognition of connection reflects an awareness of how deep conceptual understanding supports effective instruction, and it justifies the theoretical claim that teachers require not only content knowledge but also specialized content knowledge that enables them to understand and explain the mathematical ideas that make them accessible to students (Ball et al., 2008). Hence, knowledge of real analysis that emphasizes rigor and logical progression equips teachers with this form of knowledge, enhancing their instructional clarity and responsiveness to students' misconceptions.

Furthermore, the results showed that teachers perceive real analysis as having pedagogical applications, which is consistent with Shulman's concept of pedagogical content knowledge, which bridges the gap between learning mathematics and learning how to teach it effectively. When teachers understand real analysis, they can effectively scaffold abstract ideas for secondary-level students, resulting in a more meaningful learning experience. Additionally, teachers perceive that real analysis is a means to develop mathematical proficiency with their positive attitude towards it. This finding is linked with the five standards of proficiency outlined by the National Research Council (2001): conceptual understanding, procedural fluency, strategic

competence, adaptive reasoning, and productive disposition. Real analysis fosters all five standards, especially adaptive reasoning and conceptual understanding, which are fundamental to mathematical thinking.

Moreover, the finding indicates that teachers perceive a positive link between knowledge of real analysis and school mathematics. The findings of this study support prior research that highlights the direct connections between knowledge of real analysis and school mathematics, emphasizing both axiomatic and procedural aspects (Fukawa et al., 2019; Patterson, 2021). Teachers also exhibit a positive attitude toward the pedagogical application of knowledge of real analysis for secondary-level mathematics instruction, and they perceive that knowledge of real analysis has been applied to reasoning in mathematical contexts for problem-solving. This finding is linked to the finding that knowledge of real analysis plays a key role in shaping instructional practices, particularly in fostering a deeper understanding of mathematical context (Wasserman & Weber, 2017a; Patterson, 2021; Wasserman & Stockton, 2013). Hence, knowledge of real analysis is applied in mathematics instruction at the secondary level.

Despite the prevailing positive attitude and perception towards the knowledge of real analysis regarding its relevance for instruction in terms of content connection, pedagogical application, and its contribution to enhancing mathematical proficiency, there is literature that contradicts this finding. The findings of this study contradict those that suggest teachers perceive real analysis as too theoretical or disconnected from practical classroom needs (Biza et al., 2007). Additionally, the findings real analysis is too abstract and not directly useful for secondary level mathematics instruction (Fried, 2001) and advanced mathematical content like real analysis may not improve unless paired with pedagogical strategies (Tatto, 2013) More over Zazkis

and Leikin (2010) found that teachers perceive utilization of advanced mathematics and its application not well for school mathematics instruction beside its importance. These prior research findings contradict the results of this study. The curriculum of secondary-level mathematics varies from country to country and changes over time; therefore, such a temporal and contextual gap may cause contradictory results.

The research questions (RQ1 and RQ3) and their corresponding answers capture the teachers' initial perceptions of real analysis and their attitudes towards its relevance for their teaching. The findings indicate that teachers' initial attitudes and perceptions towards knowledge of real analysis are positive. Teachers demonstrate insightful engagement and a readiness to incorporate real-world analysis into their teaching methods. This positive attitude aligns with the theoretical claim that when teachers have a positive perception of a subject and find it beneficial, they are more likely to integrate it into their teaching practices (Kirkpatrick & Kirkpatrick, 2014). Therefore, the first relevance indicator is supported by the findings of this research.

#### ***Relation Among Attitude on Real Analysis and Teaching Efficacy***

This research examines the impact of knowledge of real analysis on enhancing mathematical proficiency and teaching efficacy by assessing teachers' attitudes toward its connection with secondary-level mathematics, its pedagogical applications, and its role in improving mathematical proficiency and teaching efficacy. The PLS-SEM results indicate that content connection attitude and pedagogical application attitude have significant impacts on developing mathematical proficiency, and these attitudes also affect mathematics teachers' teaching efficacy.

These findings align with prior research highlighting the importance of advanced mathematical understanding in teaching. For instance, Bjerke and Xenofontos (2023) highlighted that deep content knowledge enables teachers to make

meaningful connections between abstract concepts and classroom instruction.

Similarly, Fukawa et al. (2019) demonstrated that familiarity with advanced mathematics boosts teachers' pedagogical confidence and adaptability. In this study, teachers reported that mathematics teachers with robust backgrounds in real analysis exhibited skills in guiding and scaffolding the complex mathematical knowledge into simple pieces, which enables students to grasp the concept.

Additionally, the results of this study show that teachers and educators agreed that knowledge of real analysis contributes to enhancing mathematical proficiency and instructional practices. In particular, teachers frequently highlighted the ability to engage in mathematical justification and proof-based instruction as a key factor in developing their confidence and efficacy in classroom settings. For example, teachers with stronger backgrounds in real analysis demonstrated a remarkable ability to guide students through multi-step reasoning processes when solving algebraic and geometric problems, reinforcing their pedagogical effectiveness. This finding supports prior research, such as Ottmar et al. (2011), who demonstrated that teachers' understanding of algebraic patterns, particularly their perceptual and conceptual similarities, plays a crucial role in students' learning outcomes. Similarly, Mustafa (2008) highlighted that teachers' perspectives on mathematical proof significantly influence classroom practices, suggesting that a deep comprehension of proof structures enhances instructional quality. Arslan (2017) further corroborated this by illustrating that teachers' mathematical thinking directly impacts their pedagogical approaches and classroom effectiveness.

The findings of this study support the theoretical framework proposed by Ball et al. (2008), who argued that advanced mathematical knowledge contributes to common and specialized content knowledge, enhancing mathematical proficiency and

instructional practices. Their work emphasizes that a robust understanding of advanced mathematics is essential for effective teaching, as it enables educators to connect complex concepts with classroom instruction in a meaningful way. This justifies knowledge of real analysis contributions in mathematical proficiency and teaching efficacy.

Besides this supportive literature, one notable contradiction with this study's findings comes from the study by Zakis and Leikin (2018), which found that secondary mathematics teachers often perceive real analysis as having limited application to their professional teaching practices. Additionally, Chand (2021) argues that teachers perceive real analysis as difficult due to their limited ability to translate knowledge of real analysis into their teaching practices. Behind this discrepancy, prior research covered a limited area of real analysis rather than its overall application.

The research question (RQ2) and hypotheses (H<sub>5</sub>, H<sub>6</sub>, H<sub>7</sub>, H<sub>8</sub>, H<sub>9</sub>), along with their corresponding answers and results, emphasize the contribution of knowledge of real analysis to enhancing teachers' mathematical proficiency and teaching efficacy. This finding is supported by the theoretical framework of mathematical understanding for secondary teachers (Kirkpatrick & Kirkpatrick, 2014) and the mathematical knowledge for teaching (Ball et al., 2008). For example, teachers with formal exposure to real analysis demonstrated a stronger ability to construct mathematical explanations based on logical reasoning rather than relying solely on procedural methods in their teaching practices. This supports the argument that deeper subject knowledge leads to more dynamic and effective teaching methods. Therefore, the fourth relevance indicator is supported by the findings of this study.

While teachers expressed strong appreciation for the content connection between real analysis and school mathematics, this did not directly predict teaching efficacy or mathematical proficiency. The qualitative findings revealed that many of these connections were recognized in principle but not always operationalized in practice. Without pedagogical mediation, content awareness alone may not suffice to impact efficacy.

### ***Importance of Knowledge of Real Analysis for Instruction***

The thematic analysis of task-based interviews with in-service mathematics teachers and educators highlights the crucial role of knowledge of real analysis in developing key aspects of professional mathematical competence. Specifically, the findings demonstrate that real analysis significantly contributes to the development of mature mathematical thinking, rigorous problem-solving skills, deep conceptual understanding, and enhanced confidence in teaching mathematics effectively.

One of the core insights of this study is that teachers, through their engagement with real analysis, acquire a sophisticated understanding of the axiomatic system underlying the real number structure. This leads to the expansion of their advanced mathematical knowledge and reinforces their capacity for formal reasoning. Mathematics educators participating in this study emphasized that the advanced nature of real analysis fosters not only mathematical abstraction but also a systemic understanding of mathematical principles, supporting claims made in prior scholars' studies (Ball et al., 2008; Biza et al., 2007; Weber et al., 2020). Hence, knowledge of real analysis is fruitful to expand teachers' mathematical knowledge for schools' mathematics instruction.

The findings reveal that from the study of real analysis, teachers develop a clear understanding of the axiomatic structure of the real number system, thereby

enriching their advanced mathematical knowledge. Mathematics educators affirm that real analysis is a critical avenue for acquiring higher-level mathematical concepts, and teachers' perspectives are largely aligned with this view. This convergence of perspectives reinforces the notion of peripheral knowledge advanced mathematical understanding that, while not always explicitly taught in school curricula, is essential for effective mathematics instruction and professional expertise (Leikin & Zazkis, 2010; Stockton & Wasserman, 2017). Additionally, research findings show that a solid understanding of real analysis enhances the maturity of mathematical thinking and related skills. Moreover, the findings of this study indicate that real analysis is a fundamental source for learning the axiomatic system of the real number system, which plays a fundamental role in enhancing mathematical proficiency and conducting mathematical activities. This finding is aligned with the theoretical framework and mathematical understanding of school teachers proposed by Kirkpatrick and Kirpatrick (2014). The framework emphasizes that a deep understanding of foundational mathematical structures, such as those encountered in real analysis, is essential for developing the conceptual clarity, reasoning skills, and instructional confidence required for effective teaching.

The findings of this study dissents with with the findings of Wasserman and Weber (2017a), argue that the depth of understanding required in real analysis might not align with the immediate instructional needs of secondary educators, leading to a disconnect between advanced mathematical knowledge and classroom application, however, it is in the context of America and structure of secondary-level curriculum of American context.

The finding of this study aligns with the finding that mathematical proficiency, mathematical activity, and mathematical content can be gained from the

knowledge of real analysis (Patterson, 2021). The study results highlight that knowledge of real analysis is essential for building confidence and expertise in the subject matter. This finding is supported by the theoretical framework proposed by Shulman (1987), who argued that content knowledge is essential for becoming an expert. Hence, the prospective teachers acquired maturity in mathematical thinking, expertise, confidence, and advanced knowledge through the study of real analysis, which are essential for being an effective teacher. The link analysis reveals that teachers consider mathematical skill development a primary reason for the importance of real analysis, as it is strongly linked to expertise and the development of mature mathematical reasoning. The interconnections in the diagram suggest that real analysis supports knowledge advancement, expertise, and confidence, all of which are crucial for effective mathematics instruction at the secondary level.

Research question (RQ4) and its corresponding answers highlight the importance of knowledge of real analysis for becoming a teacher. The answer to this question fulfills the second relevance indicator. It supports the second level of the training evaluation model (Kirkpatrick & Kirkpatrick, 2014), which focuses on what teachers gain from the training.

### ***Content Connection between KRA and SLM***

The connection between knowledge of real analysis and secondary-level mathematics represents a critical dimension of mathematics education. It not only establishes a solid theoretical foundation but also enables practical applications in classroom instruction.

The theoretical framework of Mathematical Knowledge for Teaching (MKT), horizontal content knowledge is categorized under content knowledge, a fundamental component of teacher preparation programs (Ball et al., 2008). The findings of this

study indicate that real analysis is directly connected to secondary-level mathematics through key mathematical concepts, including functions, limits, sequences, and series. These connections are substantiated through logical structures and alternative representations and verified through empirical survey data. The results are consistent with the MKT framework articulated by Ball et al. (2008).

Furthermore, the study highlights that while many mathematical concepts are applied in secondary-level classrooms, their theoretical underpinnings are often rooted in more advanced areas such as real analysis. The vertical linkage between foundational and applied mathematics is further supported by Dreher et al. (2016), who emphasized the importance of connecting higher-order theoretical knowledge to instructional practices.

The study also underscores the hierarchical relationship between real analysis and elementary mathematical topics. Foundational constructs in real analysis—such as the axiomatic properties of the real number system, the definitions of limits and continuity, and the behavior of sequences and series correspond directly to secondary-level curricular content. A similar alignment was reported in prior research involving textbook analyses that mapped real analysis concepts onto the Common Core State Standards.

At a more structural level, real analysis reinforces the completeness of fundamental topics. For example, the understanding of field and order axioms central to real analysis provides clarity and coherence to the numerical operations taught at the secondary level. Procedurally, these theoretical concepts inform both instructional strategies and the use of varied representations that bridge abstract and concrete understandings. This alignment is supported by the findings of Patterson (2021) and

Wasserman et al. (2017), who both emphasize the pedagogical importance of connecting formal mathematical structures to school-level content.

Concerning Research Question 5 (RQ5), which examined the link between knowledge of real analysis and secondary mathematics, the findings reaffirm the applicability of both the frameworks proposed by Ball et al. (2008) and Dreher et al. (2016).

Additionally, they substantiate the second relevance indicator in this study: the effective utilization of acquired mathematical knowledge in actual classroom settings.

### ***Linking Real Analysis with Secondary-level Mathematics and Its Role***

Regarding the inquiry into content connection and how to strengthen it, the study's results highlight that a connection is necessary for sequential learning, which enhances critical thinking, fosters deeper understanding, and increases motivation in mathematics. Additionally, findings highlight two key techniques: establishing a vertical link and implementing pedagogical practices through a bridging strategy to reinforce the connection between knowledge of real analysis and secondary-level mathematics.

The findings of this study indicate that linking knowledge of real analysis with secondary-level mathematics promotes conceptual understanding and helps students grasp knowledge by identifying patterns and relevance between mathematical concepts. This finding aligns with Sfard (2008), who argued that understanding mathematics involves both procedural and conceptual knowledge. Additionally, Foster (2011a) and Wasserman and Weber (2017) support the study finding that conceptual understanding enhances students' ability to make meaningful connections between mathematical concepts according to the structure of knowledge. Therefore, the knowledge of real analysis and its connection with school mathematics promote an in-depth understanding of mathematical concepts. Wasserman et al. (2023)

highlight that linking real analysis concepts with school mathematics deepens teachers' content knowledge and pedagogical skills, enlightening student understanding. Furthermore, Chhetri (2023) found that the link enhances student motivation for learning and critical thinking, making abstract concepts more accessible and relevant. Another key finding of this study is that linking real analysis with school mathematics fosters sequential learning, ensuring the structured progression of concepts. This finding aligns with Skemp's (1996) claim of a need for a coherent mathematical structure and is further supported by the finding that integrating realistic mathematics education improves students' problem-solving skills (Ventistab & Tsani, 2024). The study highlights that content connections motivate learners and enhance critical thinking, as Schoenfeld (2016) found that subject relevance and connections increase student engagement. The link between real analysis and school mathematics also enables students to become mathematical thinkers capable of solving problems, aligning with (2008), who argue that problem-solving and conceptual linkages enhance critical thinking. Additionally, Tyler's (2013) planning model supports the need for organizing learning experiences to ensure relevance, reinforcing the significance of establishing strong connections between real analysis and secondary-level mathematics.

The establishment of a vertical link between knowledge of real analysis and secondary-level mathematics is the primary idea for strengthening the connection, and this finding aligns with both the top-down and bottom-up linking approaches, as outlined in the theoretical frameworks of Ball et al. (2008) and Dreher et al. (2016). The top-down approach believes that knowledge of real analysis shapes secondary-level mathematics instruction, and this link helps the teacher present the mathematical concept in a more straightforward, structured, and coherent way, emphasizing

foundational theories that support secondary-level mathematics. Dreher et al. (2016) explain that this approach facilitates the incremental development of mathematical skills, with secondary-level mathematics serving as a foundation for more advanced topics such as real analysis. The findings of this study support both ideas.

The findings highlight another technique for establishing the connection between knowledge of real analysis and secondary-level mathematics: pedagogical practices that emphasize the link between real analysis and school-level mathematics. It is essential to elaborate on which aspects of real analysis correspond to specific school mathematics content and why this connection is necessary during instruction. Additionally, explaining similarities and alternative approaches to the same concept is an important aspect of teaching real analysis. This technique is termed the bridging strategy.

This finding aligns with Chhetri's (2023) findings, who emphasized that teacher preparation programs should incorporate content bridging strategies to enhance teachers' quality.

The research questions (RQ5, RQ6) and their corresponding answers examine teachers' understanding of connecting knowledge of real analysis to secondary-level mathematics and the techniques for strengthening this connection. These answers assess what teachers learn from real analysis and whether it aligns with the framework of Knowledge for Mathematics Teaching. The findings highlight the benefits of linking concepts. They are consistent with the theoretical framework proposed by Ball et al. (2008) and the top-down and bottom-up linking approaches outlined by Dreher et al. (2016). Additionally, these findings support the second relevance indicator.

### *Teachers' Use of Knowledge of Real Analysis in Classroom Practice*

The findings of this study indicate that teachers apply their knowledge of real analysis in problem-solving, reasoning during proof-based instruction, concept development, and the abstraction of specific mathematical knowledge. These areas represent key aspects of classroom instruction where teachers integrate knowledge of real analysis into their teaching practices.

To further illuminate how knowledge of real analysis directly assists secondary-level mathematics teachers in their instructional practice, the following classroom vignette illustrates a real-world scenario where such knowledge plays a crucial role in addressing a common student misconception. A common misconception among secondary-level students is that the equation  $x^2 - 4 = 0$  has only one solution  $x = 2$ , neglecting the negative solution  $x = -2$ . This misunderstanding can be effectively addressed through the lens of real analysis, which emphasizes rigorous reasoning, function behavior, and the structure of the real number system. From an algebraic standpoint, real analysis builds on the field axiom, particularly the zero-product property. The expression  $x^2 - 4$  can be factored as  $(x - 2)(x + 2)$ . According to the zero-product property, if the product of two real numbers is zero, then at least one of the numbers must be zero. Therefore, the solution of given equation is  $x = 2$ ,  $x = -2$ . This logical structure helps students understand that equation involving even power generally two real roots, not one.

Further, real analysis provides a precise definition of square root function. While  $\sqrt{4} = 2$  denotes the principal (positive) square root, solving the equation  $x^2 = 4$  requires finding all real solutions, which include both  $x = 2$  and  $x = -2$ . Real analysis reinforces that if  $x^2 = a$ , then  $x = \pm\sqrt{a}$  for any  $a \in R$  where  $a > 0$  (?).

Graphical analysis, another technique rooted in real analysis, helps correct misconceptions visually. When plotting the function  $y = x^2$  and the horizontal line  $y = 4$ , their intersection points clearly occurs at  $x = 2$  and  $x = -2$ , visually conforming the two solution. This approach supports students who benefit from visual learning and helps bridge intuition with formal reasoning.

Finally, real analysis emphasizes that the function  $f(x) = x^2$  is not injective over  $\mathbb{R}$ , as  $f(x) = f(-x)$ . Recognizing that the squaring function is even clarifies why both positive and negative values of  $x$  yield the same output. Thus, knowledge of real analysis equips teachers to address this miscomputation not only through correct answers but through rigorous explanation and multiple representations.

The finding is supported by the theoretical frameworks of Ball et al. (2008) and Shulman (1987), which emphasize the role of both standard and specialized content knowledge in effective mathematics instruction. In this study, teachers' instructional practices align with these frameworks, demonstrating how content knowledge informs teaching practices. Furthermore, the findings correspond with the conceptualization of school-related knowledge proposed by Dreher et al. (2016), which argues that academic mathematics knowledge extends downward to inform the teaching of school-level mathematics.

The study also reveals that teachers incorporate fundamental properties of real numbers and their applications in problem-solving, a finding consistent with research by Wasserman et al. (2017) and Patterson (2021). Additionally, using knowledge of real analysis for mathematical reasoning aligns with Patterson's findings. At the same time, the observed teaching practices correspond with the theoretical framework of Ball et al. (2008). These findings diverge from those of Goulding et al. (2003), who found that secondary-level mathematics teachers believe that knowledge of real

analysis does not have a notable connection. Additionally, Wu (2011) found no direct connection between the school-level mathematics taught at the university level. This discrepancy may be attributed to prior studies that have focused primarily on teachers' perspectives rather than classroom observations and curriculum analysis.

The findings of this study indicate that teachers apply knowledge of real analysis in a direct content-related manner, such as in sequences, series, and functions. This finding aligns with Patterson's (2021) research, which suggests that the content of real analysis is linked to secondary-level mathematics in various ways, such as direct content connections, and teachers utilize it in various ways, including graphical representation. However, it contrasts with the findings of Zazkis and Leikin (2010), who found that teachers do not incorporate advanced mathematical knowledge, such as real analysis, into their instruction. This discrepancy may be attributed to methodological and contextual differences between the studies.

Additionally, the findings reveal that teachers employ alternative methods to integrate knowledge of real analysis into their school mathematics instruction. This finding is consistent with Patterson (2021), who similarly observed such practices. Moreover, the study shows that teachers use knowledge of real analysis as a logical framework, particularly in reasoning and proof contexts. This aligns with Bhupal (2021), who argued that teachers apply knowledge of real analysis in these contexts. These findings are further supported by Wasserman (2018a), who noted that the disciplinary practices of advanced mathematics are closely related to school-level mathematics instruction.

The relevance of knowledge of real analysis (KRA) in mathematics instruction can be better understood using Habermas's (1972) theory of knowledge-constitutive interests: technical, practical, and emancipatory. These three domains reflect how

teachers apply and internalize real analysis. The technical interest explains how teachers utilize KRA to enhance their teaching techniques, such as field axioms, thereby developing accurate and effective instruction. The practical interest supports how teachers create mutual understanding by connecting abstract concepts to classroom practice, for example, explaining domain and range or limits using everyday examples. The emancipatory interest highlights teachers' transformation through reflection, where real analysis strengthens logical thinking and shifts their professional identity toward more critical and conceptually driven teaching (Habermas, 1972; Acharya et al., 2022). This theoretical lens confirms that KRA contributes to procedural skills and deeper reflective and transformative teaching practices.

Teachers' integration of content knowledge and pedagogical strategies in the classroom aligns with Mishra and Koehler's (2006) TPACK framework, which combines content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK) for effective teaching. In this study, teachers effectively connected CK and PK using advanced real analysis concepts like limits and continuity to create meaningful tasks and multiple representations linking secondary and university-level math. Although technology (TK) was not emphasized, the TPACK model highlights future opportunities to enhance learning through digital tools. Supporting studies (Khanal et al., 2024; Joshi et al., 2024) demonstrate that utilizing digital resources and developing TPACK skills enhance the clarity and engagement of math instruction. Thus, TPACK validates the integration of content and pedagogy, pointing to the benefits of expanding technological fluency in teaching real analysis.

The findings on how teachers articulate mathematical knowledge of real analysis during the instruction highlight that knowledge of real analysis serves not

only as a foundational pillar for advanced mathematical reasoning but also as a vital pedagogical resource that informs and enriches secondary-level instruction particularly in conceptual development, problem-solving, and proof-based teaching despite some contrasting perspectives in prior research.

### ***Divergence in Relationships Among Key Constructs***

The integration of quantitative and qualitative findings revealed meaningful divergences that deepen the understanding of how teachers conceptualize and use knowledge of real analysis (KRA) in secondary-level instruction. While survey results showed that teachers held positive attitudes toward the content connection between real analysis and school mathematics, this construct did not significantly predict mathematical proficiency or teaching efficacy in the structural model. In contrast, qualitative findings provided evidence that teachers recognized direct content links, such as limits, sequences, and series, but often treated these links as fixed content rather than as a basis for instructional transformation. This suggests that content knowledge, when not actively translated into pedagogical practice, may remain inactive. This aligns with Ball et al.'s (2008) theory of Mathematical Knowledge for Teaching (MKT), which distinguishes between possessing content knowledge and having the specialized content knowledge required to make mathematical ideas understandable to students. Furthermore, the lack of predictive strength in quantitative models may also reflect the nature of axiomatic content, which was often presented by teachers as unquestioned truths rather than reasoning tools, a finding echoed in their tendency to use axioms without explaining their logical foundations.

By contrast, the pedagogical application of real analysis emerged as a significant and consistent predictor of both mathematical proficiency and teaching efficacy, as evidenced in both PLS-SEM results and qualitative interviews. Teachers

described using KRA in problem-solving contexts, particularly to structure logical reasoning and design tasks that enhanced students' conceptual understanding. These results support Shulman's (1986) construct of Pedagogical Content Knowledge (PCK), emphasizing the teacher's ability to represent and adapt content to learners' needs. However, the qualitative data revealed that idea generation, a more creative and exploratory component of pedagogy, was less emphasized by teachers, suggesting a reliance on familiar procedural reasoning rather than innovative instructional design. This nuance is important, as the literature suggests that pedagogical application requires not just transfer but transformation of knowledge (Wasserman & Stockton, 2013; Ko et al., 2024), a shift that may not yet be fully realized among participants. This divergence is further shaped by Habermas's (1972) theory of knowledge-constitutive interests, which categorizes knowledge into technical, practical, and emancipatory forms. While teachers may approach real analysis with a technical interest (e.g., mastering procedures), translating this knowledge into effective teaching demands a practical interest in applying concepts contextually to interpret and facilitate learning. Additionally, the National Research Council's (2001) five strands of mathematical proficiency, particularly adaptive reasoning and strategic competence, are better fostered when teachers leverage real analysis in instructional decision-making rather than merely grasping its theoretical foundations.

In light of these findings, the limited predictive role of content connection may be explained by its lack of pedagogical activation. Teachers understand the relevance of real analysis but do not consistently operationalize that understanding into strategies that foster student learning. This insight is corroborated by empirical studies (e.g., Wasserman, 2018a; Patterson, 2021), which emphasize that while advanced mathematics enhances content knowledge, its instructional utility remains

underdeveloped without structured professional learning and context-aware integration. Therefore, teacher preparation programs need to address not only what teachers know but also emphasize how and why that knowledge is enacted in the classroom, thereby closing the gap between theory and practice.

### **Model for Relevance of KRA for SLM Instruction**

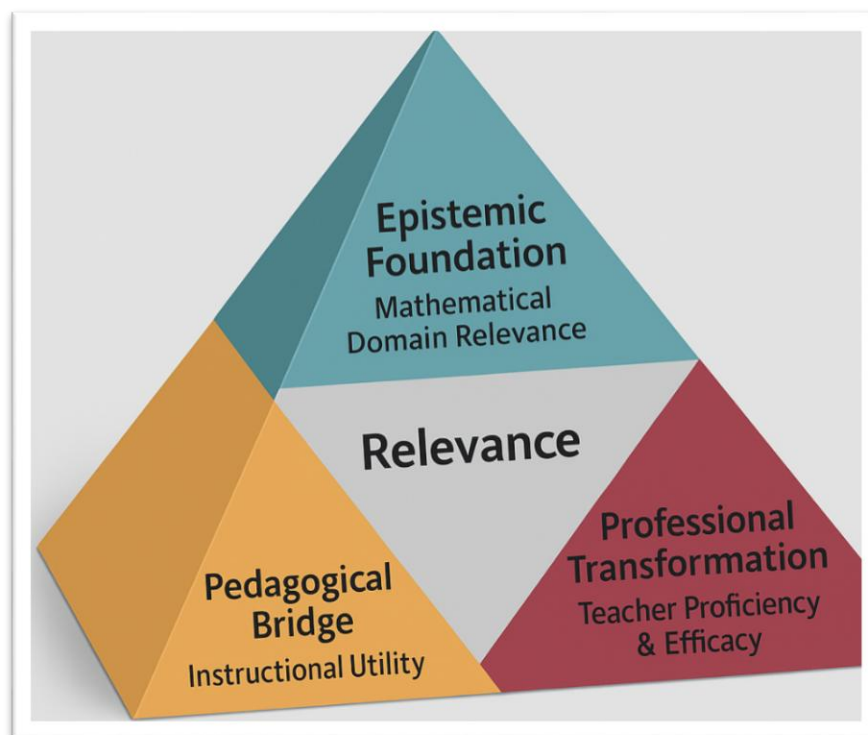
The findings of this study suggest that real analysis is fundamental for secondary-level mathematics instruction. It shapes teachers' mathematical proficiency and pedagogical effectiveness, both of which are essential for effective mathematics teaching and learning. This study hypothesized that real analysis serves as an essential cognitive and instructional scaffold, enabling teachers to develop a deeper understanding of the mathematical structure. This understanding enables teachers to provide meaningful and coherent mathematics instruction. Mathematical proficiency is a combination of knowledge and skills that reflects the teacher's capacity to understand mathematical facts, concepts, principles, and procedures, and utilize these tools to solve problems and perform mathematical activities. It is an essential skill for teachers and is cultivated through knowledge of real analysis. It equipped the teacher in such a way that they can consult logical connections through abstraction and applied mathematics.

This study argues that knowledge of real analysis fosters mathematical maturity, equipping teachers with the ability to generalize mathematical properties, construct rigorous proofs, and present alternative representations of mathematical concepts in the classroom setting. Knowledge of real analysis enables teachers to translate advanced mathematical reasoning into accessible learning experiences. The study theorizes that real analysis contributed to teachers' instructional repertoire by

reinforcing content, connections, fascinating structured explanations, and cultivating students.

**Figure 34**

*Model for Relevance of KRA for SLM Instruction*



Logical reasoning abilities. This study conceptualizes that teachers with clear knowledge of real analysis enhance the teacher's capacity to bridge secondary-level mathematics and advanced mathematics by establishing a connection between real analysis and secondary-level mathematics, strengthening students' competencies, and fostering progressive learning that aligns with mathematical cognition. The insight model from the study is shown in Figure 34.

***Epistemic Foundation: Mathematical Domain Relevance***

The epistemic foundation for demonstrating the relevance of real analysis to school mathematics instruction lies in the systematic arrangement of knowledge of real analysis on the content taught at the secondary level. A well-structured

arrangement aims to establish vertical coherence by integrating both bottom-up and top-down approaches, as proposed by Dreher et al. (2016). Through this dual approach, foundational concepts from school mathematics can be revisited and deepened via real analysis. In contrast, advanced concepts from real analysis can be traced back to their pedagogical roots in school-level content. Furthermore, the knowledge of real analysis contributes meaningfully to the domains of Mathematical Knowledge for Teaching (MKT) outlined by Ball et al. (2008), including common content knowledge, specialized content knowledge, and knowledge of content and teaching. Therefore, the epistemic dimension of the proposed model extends and reinforces the theoretical grounding of prior frameworks by establishing a dynamic and reciprocal relationship between advanced mathematical knowledge and instructional practice.

This dimension represents the content knowledge gained through real analysis. It encompasses axiomatic structures, logical reasoning, and foundational concepts such as continuity, limits, convergence, and properties of real numbers. These elements contribute to a deeper understanding of mathematics and serve as the base for justifying and interpreting secondary-level mathematical content. As mentioned, the content of real analysis needs to be linked with school mathematics using Bruner's (1960) spiral curriculum model.

Knowledge of real analysis is a foundational subject that enables teachers to establish content continuity between higher-level mathematics and secondary-level concepts. Additionally, knowledge of real analysis enhances teachers' logical fluency, problem-solving skills, and conceptual depth, which are essential for explaining all mathematical activities, justifying each mathematical process as mathematically true, and ensuring the mathematical validity of each activity. The epistemic foundation,

with its relevant features mentioned above, helps make real analysis relevant to secondary-level mathematics instruction.

Ensuring a solid epistemic foundation promotes sequential learning, enables teachers to abstract mathematical knowledge, scaffold theoretical knowledge for practice, and justify mathematical activities during secondary-level mathematics instruction.

### ***Pedagogical Bridge***

The second dimension focuses on how abstract knowledge from real analysis can be adapted and transformed into pedagogically actionable strategies. This dimension encompasses processes such as generalization, procedural linkage, visualization, and alternative representations, all of which enable teachers to present school mathematics in a more meaningful and coherent manner. Within the context of real analysis instruction, it becomes essential for instructors to explicitly justify the connections between advanced mathematical concepts and corresponding elements of school-level mathematics, demonstrating the interrelatedness of these ideas wherever possible. The study reveals a critical need for a bridging strategy in teaching real analysis that equips prospective teachers with the capacity to translate abstract theoretical constructs into instructional tools. Such a strategy fosters cognitive readiness in which pedagogically relevant ideas emerge naturally as teachers engage with mathematical tasks in the classroom.

This theoretical claim can be operationalized through pedagogical application by strategically guiding prospective teachers in understanding and teaching the concept of limits. Within the framework of the pedagogical bridge, it is important that knowledge of real analysis, particularly the formal and abstract definition of the limit, is introduced at the university level. Subsequently, teacher educators can support

prospective teachers in exploring its general conceptual interpretation and how it underpins secondary-level mathematics instruction. By explicitly modeling how abstract reasoning is translated into classroom practice, instructors help prospective teachers internalize the idea that abstraction is not an endpoint, but a resource for meaningful instruction. This approach encourages them to view their advanced mathematical training as a foundation for constructing accessible, school-level mathematical ideas. The suggested model for implementing this instructional bridge, including sample activities and representations, is presented in Appendix K.

### ***Professional Transformation***

The third dimension focuses on how real analysis contributes to the personal and professional growth of mathematics teachers. It enhances mathematical maturity, critical thinking, and confidence in instructional decision-making. Furthermore, it strengthens teaching efficacy by equipping teachers with the skills to analyze, reflect on, and improve their instructional practices.

At the core of this three-dimensional model is a construct that emerges only when all three dimensions interact effectively. The model suggests that relevance is not a static attribute of content but a construct developed through the interplay of foundational knowledge, pedagogical application, and professional development. As such, this theory extends existing models of mathematical knowledge for teaching (Ball et al., 2008; Shulman, 1986a) by emphasizing how advanced mathematics, such as real analysis, becomes relevant in classroom instruction when supported by structured curricular bridges and reflective practice.

This theoretical framework can guide curriculum designers, teacher educators, and researchers in understanding and enhancing the role of advanced mathematics in teacher preparation programs. It provides a comprehensive lens through which the

value of real analysis can be assessed, taught, and applied in meaningful ways across educational levels.

### **Chapter Summary**

This chapter presents the findings of quantitative and qualitative analyses, as well as the discussion. The discussion section compares findings with existing research and theoretical perspectives. The study discloses that teachers hold a positive attitude toward real analysis and perceive it as relevant to secondary-level mathematics instruction. Real analysis plays a crucial role in advancing mathematical knowledge, contributing to the development and maturity of mathematical thinking. While teachers initially perceived a limited connection between real analysis and its application in the classroom, their real-world experiences led to a shift in attitude and perception. The findings are supported by previous literature, further supporting the implication of real analysis in mathematics education.

## Chapter 6

### Conclusion and Implications

This chapter includes a conclusion and implications of the study on the 'Relevance of Knowledge of Real Analysis in Mathematics Instruction at the Secondary Level'.

#### Conclusions

Real analysis serves not merely as a subject in teacher preparation programs but as a foundational framework that enhances teachers' mathematical reasoning, cognitive maturity, and instructional decision-making. In Nepal, however, the pedagogical value of advanced mathematics, particularly real analysis, is insufficiently explored in research and underutilized in practice. Prevailing perceptions of real analysis are often governed by misconceptions, leading to its disconnection from secondary-level mathematics instruction. A deeper understanding of real analysis contributes significantly to teachers' professional identity, pedagogical adaptability, and content mastery, ultimately enriching classroom practice.

The findings do not suggest a uniform relevance of real analysis. Instead, they reveal a context-dependent relationship between advanced mathematical knowledge and its application in secondary-level instruction. Teachers in this study highlighted the value of real analysis for its role in advancing mathematical proficiency, strengthening logical reasoning, fostering cognitive maturity, and bolstering teaching confidence. However, a divergence emerges when compared with existing literature, which often questions the classroom applicability of such abstract content. Mathematical proficiency is fostered through real analysis, where present is a predictor of teaching efficacy. It has direct integration into classroom content. However, its influence manifests more in teachers' reasoning and decision-making

processes than in explicit topic coverage. Logical habits and abstract reasoning are cultivated through real analysis and often guide instructional decisions, framing mathematical arguments and problem-solving strategies.

Notably, KRA has an implicit link with secondary mathematics through alternative representations, logical structures, and generalized applications of abstract theory, even when not explicitly illustrated in the curriculum. Qualitative data provided additional explanatory depth to the quantitative findings. Classroom observations and interviews revealed that teachers who had internalized KRA demonstrated greater flexibility, employed diverse representations (visual, logical, procedural), and promoted deeper reasoning. The knowledge of real analysis helped shift their instructional practice from a textbook-centric delivery to a reasoning-oriented approach, thereby cultivating mathematical thinking in their students. This dual-phase inquiry confirms that real analysis supports both as foundational content knowledge and as a cognitive-pedagogical resource. Teachers used it for abstraction, justification, and inductive reasoning, translating formal structures into meaningful instructional moments. The axiomatic foundation of real numbers supported scaffolding and transformation of concepts in daily teaching practice. The relevance of KRA can be further enhanced through curriculum alignment and reflective pedagogy.

Finally, the study identified logical, procedural, and representational dimensions of the connection between real analysis and secondary mathematics. These dimensions provide a useful typology for teacher education, curriculum design, and future research by illustrating how abstract knowledge can be transformed into clear, effective instruction.

Besides the connection between knowledge of real analysis and secondary-level mathematics, this linkage facilitates hierarchical learning and enables the development of mathematical thinking. This study further explores that strengthening the connection can be achieved through strategies such as emphasizing bridging techniques, alternative representations, and axiomatic perspectives in teacher education programs. Theoretically, this view aligns with constructivism, which posits that knowledge is constructed through reflection and inquiry (Bruner, 1960), and progressivism's emphasis on applying knowledge to real-world instructional problems (Dewey, 1962). Providing prospective and in-service teachers with opportunities to connect advanced and secondary-level mathematics explicitly, reflect on these connections, and practice applying them in instructional scenarios can help consolidate this linkage. The study highlights the necessity for teacher education programs to create spaces within their curricula where future and in-service teachers can grapple with advanced mathematical concepts, reflect upon their structures, and learn strategies for applying these structures in their classroom practice.

This inquiry demonstrates that the inclusion of real analysis in teacher preparation programs plays a vital role in shaping proficient, flexible, and insightful secondary mathematics educators. It highlights how knowledge of real analysis enhances teachers' ability to reason mathematically, design rigorous instruction, and connect abstract concepts with classroom applications, ultimately contributing to more effective mathematics teaching at the secondary level. Theoretically, this view highlights Habermas's interest theory's technical and practical dimensions of knowledge, where advanced mathematical content provides a powerful toolset for instructional practice, and Shulman's specialized content knowledge underscores the unique knowledge base required for effective teaching. Furthermore, from a

progressivist perspective, this knowledge prepares future educators to respond to instructional problems in context and to foster a deep, conceptual understanding of mathematics in their students.

This study revealed that secondary-level mathematics teachers who possess deep knowledge of real analysis exhibit greater mathematical maturity in their pedagogy. Specifically, their practice is marked by an ability to connect abstract mathematical structures to school content, employ rigorous explanations, and select appropriate representations to illuminate challenging concepts. Furthermore, this depth of knowledge facilitates instruction, which increases their confidence and efficacy in delivering coherent, conceptually rich lessons. Theoretically, this underscores Shulman's view that a strong understanding of content knowledge underpins instructional expertise, and it resonates with progressivism's view that education must empower practitioners to address instructional challenges through deep content knowledge and reflective inquiry.

Developing mathematical maturity through real analysis directly strengthens instructional practice and enhances teacher efficacy. It fosters deeper conceptual understanding, sharpens analytical reasoning, and equips educators with the tools needed for effective teaching in secondary mathematics classrooms. Specifically, when secondary mathematics teachers possess this maturity, their pedagogy reflects a deep understanding of mathematical structures, their explanations become more coherent and rigorous, and their ability to respond to student questions and misconceptions is enhanced. Furthermore, this maturity is a key component in developing specialized content knowledge and mathematical proficient thinking, which, in turn, underpins the ability to connect advanced and school mathematics in a coherent framework. Theoretically, this highlights the view that specialized content

knowledge comprises not just knowledge of mathematical facts and procedures but also the knowledge processes that connect content to pedagogy and foster student understanding.

Strengthening the linkage between real analysis and school mathematics is not a passive process; it requires deliberate cultivation through targeted instructional strategies, a thoughtfully designed teacher education curriculum, and ongoing professional development. Theoretically, this underscores constructivism's perspective that knowledge is actively constructed through reflection and inquiry, alongside progressivism's view that education empowers practitioners to solve problems within their specific contexts. Providing future and in-service teachers with opportunities to reflect upon advanced mathematical structures, practice applying them in instructional scenarios, and connect this knowledge to their pedagogy helps consolidate these linkages. Furthermore, employing strategies such as alternative representation, bridging techniques, and axiomatic perspectives can aid in this process.

Real analysis, when explicitly integrated into teacher education and instructional practice, offers substantial benefits to the discipline of mathematics education. It enhances teachers' conceptual understanding, deepens their mathematical reasoning, and equips them to foster more meaningful learning experiences for students. This study highlights that this specialized knowledge does not remain an isolated discipline within the curriculum but permeates pedagogy and instructional practice. Theoretical perspectives from Shulman, progressivism, constructivism, and Habermas collectively underscore this view that advanced mathematical knowledge is not an optional addition; it is a prerequisite for developing proficient, flexible, and

insightful mathematics educators who can connect theory to practice coherently and purposefully.

Therefore, this study highlights that a solid mathematical foundation in real analysis is crucial for secondary-level mathematics instruction and should be a core component of teacher education programs. It plays a central role in developing specialized content knowledge, mathematical maturity, instructional expertise, and pedagogy, all of which collectively enable secondary mathematics teachers to become more effective, proficient, and insightful practitioners. Furthermore, this knowledge strengthens their ability to connect advanced and school content, respond to instructional problems, reflect on their practice, and foster deep mathematical understanding in their students. Importantly, this view highlights the need for education programs to integrate real analysis into their curricula and pedagogy in a purposeful and systematic manner.

A significant scholarly contribution of this study lies in its application and extension of established theoretical frameworks to the context of teacher preparation in Nepal. By integrating Ball et al.'s (2008) MKT model, the research confirms that specialized content knowledge developed through real analysis plays a vital role in shaping effective instructional practice, particularly in reasoning, justifying, and interpreting mathematical ideas. The findings support the model's claim that deep content knowledge strengthens pedagogical performance, not only by enhancing accuracy but by cultivating a teacher's flexibility and responsiveness in classroom settings.

Moreover, this study adds conceptual depth to the Habermasian theory of knowledge interests. Teachers who engaged reflectively with real analysis demonstrated technical interest (using procedures), practical interest (understanding

meanings in context), and emancipatory interest (transforming their instructional beliefs). In this sense, real analysis becomes not just a content domain but a transformative one, activating critical awareness about the epistemic structure of mathematics and its role in teaching.

The study also aligns with the SRCK framework by Dreher et al. (2018), demonstrating how top-down, bottom-up, and curricular connections between university-level and school-level mathematics can be articulated and mapped. Teachers were found to utilize bottom-up strategies when linking students' prior knowledge with advanced ideas. In contrast, top-down strategies emerged when teachers drew upon theorems or abstract reasoning to clarify school-level concepts such as limits or sequences. This contributes to the ongoing theoretical work in mathematics education by emphasizing how abstract knowledge can be transformed into instructional value.

### **Implications for Curriculum Planner**

This study highlights the crucial need for curriculum planners in Nepalese teacher education to establish a systematic alignment between university-level advanced mathematics and secondary school curricula. The findings necessitate collaborative policy action between Tribhuvan University and the Curriculum Development Centre to create a national framework that explicitly maps how core mathematical concepts in real analysis (including functions, limits, and continuity) pedagogically scaffold secondary-level instruction. Such alignment requires carefully designed advanced mathematics courses that train pre-service teachers in transforming abstract content into teachable school mathematics through three identified connection types: logical structures, procedural approaches, and alternative representations. This study focuses on the importance of designing subject-specific

pedagogy courses that integrate mathematical content and pedagogical approaches. Teacher education programs should use a hybrid model that combines bottom-up and top-down perspectives to illuminate how advanced mathematical structures inform and guide school-level instructional practices. Specifically, a hybrid approach can help prospective and in-service teachers appreciate both the axiomatic properties and algebraic structures of real numbers and how these translate into their instructional decisions in the classroom. Furthermore, this approach can help educators connect abstract mathematical knowledge to school content through alternative representations, logical structures, and the justification of mathematical operations.

The study provides a focused examination of the need to reassess assessment models in teacher preparation programs. Rather than focusing predominantly on formal proof skills, assessments should reward pedagogical reasoning, representational fluency, and the ability to connect advanced mathematical knowledge to instructional practice. This change in emphasis would foster a more coherent and applicable mathematical understanding that directly supports effective classroom teaching.

For in-service teacher education, the study advocates for ongoing professional development programs designed by the Faculty of Education at Tribhuvan University in collaboration with the CDC. Such programs should aim to help practicing secondary-level mathematics teachers apply advanced mathematical knowledge to their instructional practice. Importantly, this form of professional development would enable in-service teachers to reflect upon their pedagogy, deepen their mathematical maturity, and respond more effectively to instructional problems in their classrooms.

This study reveals an urgent need for transformative reform in mathematics instructional materials, demonstrating that textbook design must systematically bridge

real analysis concepts with school mathematics pedagogy. Specifically, curriculum planners should mandate textbooks that: (1) explicitly demonstrate how axiomatic structures of real numbers (e.g., field axioms, completeness) underpin secondary mathematical operations, (2) directly address common student misconceptions (e.g.,  $0/0=1$  or  $\infty/\infty=1$ ) through formal analytical reasoning, and (3) provide preservice and in-service teachers with concrete examples of transforming real analysis knowledge into classroom-appropriate explanations. The findings particularly emphasize the necessity for real analysis textbooks tailored to teachers, not mathematicians, that model this pedagogical translation through side-by-side comparisons of abstract theorems (e.g., limit definitions) and their corresponding instructional approaches in school mathematics. Such materials would prevent the persistent disconnect between advanced mathematical knowledge and classroom practice identified in this study, where teachers often default to procedural rather than conceptual teaching due to inadequate training resources.

### **Implications for Pedagogy and Teacher Education**

Educational institutions should prioritize professional development initiatives that encourage teachers to approach mathematics from a conceptual and logical perspective, rather than merely following procedural steps. Specifically, institutions should offer targeted workshops, collaborative learning platforms, and ongoing professional development opportunities to enable both school and university teachers to deepen their subject knowledge, refine instructional strategies, and effectively connect real analysis content with school-level mathematics. Such programs will empower teachers to demonstrate and justify the applicability of real analysis in enhancing the quality of secondary mathematics instruction.

Moreover, institutions should support mathematics educators within teacher preparation programs by promoting bridging teaching strategies that emphasize the relevance of real analysis knowledge for school mathematics. This study suggests that real analysis should not be presented as an isolated formalism but rather as a conceptual toolkit to enrich secondary-level teaching, necessitating a shift in both curriculum content and instructional approaches.

Firstly, teacher educators need to explicitly model how real analysis concepts can be contextualized for school instruction. For instance, while teaching the epsilon-delta definition of limits, instructors should simultaneously illustrate how this abstract framework helps diagnose student misconceptions about continuity and function behavior at the secondary level. Similarly, concepts such as nested intervals or the supremum property should be integrated alongside practical problems involving approximation or inequality reasoning relevant to school mathematics.

Secondly, curriculum designers are encouraged to integrate bridging modules within B.Ed. and M.Ed. programs that explicitly focus on translating advanced mathematics into teachable content. These modules would provide pre-service teachers with structured opportunities to practice scaling down complex concepts without compromising mathematical rigor.

Thirdly, professional reflection emerges as a critical component of effective mathematics teaching. Teachers who have benefited most from the knowledge of real analysis (KRA) are actively engaged in reflective practices, considering how their university learning informs their instructional decisions. Hence, teacher education programs should incorporate opportunities for structured reflection, peer collaboration, and school-based practicum aligned with real analysis concepts to consolidate this linkage.

Collectively, these pedagogical implications underscore the necessity of embedding real analysis meaningfully within teacher education curricula and professional development programs to cultivate proficient, reflective, and conceptually grounded mathematics educators.

### **Implications for Teachers**

Knowledge of real analysis provides the foundation for mathematical operations used during instruction at the secondary level. It offers the logic behind each mathematical operation. Therefore, teachers should study real analysis, especially the axiomatic system of real numbers, as it provides the rationale for each mathematical operation. Additionally, school teachers should transition from teaching mathematics in an applied mode to an abstract mode, thereby enhancing students' mathematical thinking abilities. Mathematical knowledge of real analysis serves as the foundation for learning other advanced mathematics. Therefore, teachers can utilize knowledge of real analysis during instruction to improve mathematical thinking and motivate students to study advanced mathematics.

Knowledge of real analysis is meaningful for enhancing students' mathematical thinking and problem-solving ability. Teachers should present complex mathematical concepts in various ways, enabling the students to grasp the concept. Teachers should also focus on nurturing students' critical thinking by encouraging them to explore mathematical relationships and build logical reasoning skills. Additionally, teachers should actively seek professional development opportunities to enhance their understanding of real analysis and improve their teaching methods.

### **Implications for Further Research**

This study underscores the necessity for further research to deepen understanding of how mathematical knowledge of real analysis contributes to

instructional practice and student outcomes in secondary education. Future research should be conducted to explore the impact of knowledge of real analysis on teachers' performance in the classroom. Furthermore, future investigations could aim to determine how teachers' understanding of real analysis influences their students' mathematical thinking and problem-solving abilities. Future researchers may also employ both qualitative and quantitative approaches to examine how mathematics teachers utilize the axiomatic properties of real numbers in their instructional practices and how these properties facilitate the conceptualization of mathematical content.

The effectiveness of various instructional strategies in incorporating real analysis into secondary mathematics classrooms could be another valuable area of inquiry. Comparative studies might be conducted to investigate different approaches to linking real analysis knowledge to school mathematics instruction at both the lower secondary and upper secondary levels. Furthermore, future research needs to explore obstacles to integrating real analysis into classroom practice and identify strategies to help teachers connect advanced mathematical knowledge to their pedagogy purposefully and coherently.

Additionally, it is expected that future research incorporates variables such as classroom management, time management, feedback, and student performance to provide a more comprehensive view of instructional practices. These constructs might be useful illuminate their roles in mediating or moderating the relationship between knowledge of real analysis and effective classroom teaching. Furthermore, a mixed-methods approach can be used to explore the relevance of knowledge of real analysis for teaching at the secondary level, employing both qualitative and quantitative methods with a larger number of participants.

Meanwhile, longitudinal studies could track how pre-service teachers' understanding and use of real analysis evolve, particularly as they transition to full-time teaching. Comparative studies could explore how real analysis is perceived and used in teacher education programs across different universities in Nepal and South Asia. Furthermore, theoretical work is needed to develop new pedagogical models that show how abstract mathematical knowledge can be transformed into instructional knowledge. Research into student outcomes may assess whether teachers with a firm grounding in real analysis have a positive impact on student learning in topics such as functions, limits, or mathematical reasoning. By pursuing these directions, the field can continue to refine its understanding of how to bridge theory and practice in mathematics education.

### **Chapter Summary**

This chapter summarizes the study's findings, conclusions, and implications regarding the relevance of knowledge of real analysis in secondary mathematics instruction. The study examined teachers' attitudes toward real analysis in terms of content connection, pedagogical application, and its role in enhancing mathematical proficiency and teaching efficacy. In Gandaki province, Nepal, the study employed an explanatory sequential mixed-methods research design, involving 387 survey participants and eight teachers in the qualitative phase.

Findings indicate that teachers perceive a strong connection between real analysis and secondary mathematics, particularly in the areas of sequences, series, functions, limits, and continuity. They recognize that real analysis supports problem-solving, theorem proving, and reasoning, enhancing their mathematical proficiency by enhancing logical reasoning and abstraction skills.

Real analysis is essential for effective mathematics instruction, as it provides a solid theoretical foundation for mathematical operations and supports the development of sequential learning. Its integration into teacher preparation enhances instructional clarity and strengthens the logical structure of mathematics teaching. Teachers' positive attitudes suggest real analysis supports their instructional practices, improving their confidence and expertise.

Implications include recommendations for curriculum planners to integrate real analysis into teacher preparation programs through a bridging strategy. Professional development programs focused on real analysis are essential at the institutional level, while teachers are responsible for actively applying this knowledge to improve students' problem-solving abilities. Limitations of the Study

Significantly, few females (9%) participated in the survey, and no female participants participated in the qualitative inquiry. The survey was conducted among secondary-level mathematics teachers in Gandaki province, so the findings of this research may not be generalizable to other levels, such as the basic level. Additionally, fewer teachers participated in the qualitative inquiry compared to the survey, with only one district and one municipality represented. Additionally, the non-response rate is 28.20% (with good coverage of the minimum sample size). Therefore, the findings may not be generalizable to different cultural and geographical settings. Only three mathematics educators participated in the study, so future research on the same issue could involve more mathematics educators. Furthermore, the study included only secondary-level mathematics teachers. Therefore, it does not explain scenarios of other levels.

## References

- Abbott, S. (2015). *Understanding analysis* (2nd ed.). Springer. [http://ndl.ethernet.edu.et/bitstream/123456789/88631/1/2015\\_Book\\_UnderstandingAnalysis.pdf](http://ndl.ethernet.edu.et/bitstream/123456789/88631/1/2015_Book_UnderstandingAnalysis.pdf)
- Acharya, B. R., Belbase, S., Panthi, R. K., Khanal, B., Kshetree, M. P., & Dawadi, S. D. (2022). Critical conscience for construction of knowledge in mathematics education. *International Journal of Education in Mathematics, Science and Technology (IJEMST)*, 10(3), 670–688. <https://doi.org/10.46328/ijemst.2203>
- Aithal, A., & Aithal, P. S. (2020). Development and validation of survey questionnaire & experimental data - A systematical review-based statistical approach. *International Journal of Management, Technology and Social Sciences*, 5(2), 233–251. <https://doi.org/10.47992/IJMTS.2581.6012.0116>
- Aksu, Z., & Kul, U. (2019). The mediating role of mathematics teaching efficacy on the relationships between pedagogical content knowledge and mathematics teaching anxiety. *SAGE Open*, 9(3). <https://doi.org/10.1177/2158244019871049>
- Alharbi, E. (2014). *A study on the use of ICT in teaching in secondary schools in Kuwait* [Doctoral thesis, Cardiff Metropolitan University, Cardiff School of Education]. [https://figshare.cardiffmet.ac.uk/articles/thesis/A\\_Study\\_on\\_the\\_Use\\_of\\_ICT\\_in\\_Teaching\\_in\\_Secondary\\_Schools\\_in\\_Kuwait/20456799](https://figshare.cardiffmet.ac.uk/articles/thesis/A_Study_on_the_Use_of_ICT_in_Teaching_in_Secondary_Schools_in_Kuwait/20456799)
- Allmendinger, H. (2016). Die Didaktik in Felix Kleins "Elementarmathematik vom höheren Standpunkte aus". *Journal Für Mathematik-Didaktik*, 37(1), 209–237. <https://doi.org/10.1007/S13138-016-0089-1>
- Alsalamah, A., & Callinan, C. (2021). Adaptation of Kirkpatrick’s four-level model of training criteria to evaluate training programmes for head teachers. *Education Sciences*, 11(3). <https://doi.org/10.3390/educsci11030116>

- Alshehri, K. A., & Youssef, N. H. (2022). The influence of mathematical knowledge for teaching towards elementary teachers' mathematical self-efficacy. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(6).  
<https://doi.org/10.29333/ejmste/12086>
- Aristidou, M. (2020). Is mathematical logic really necessary in teaching mathematical proofs? *Athens Journal of Education*, 7(1), 99–122.  
<https://doi.org/10.30958/aje.7-1-5>
- Arslan, S. (2017). Effects of mathematics content knowledge on mathematics pedagogical content knowledge. *Journal of Teacher Education and Educators*, 6(1), 53–68. <https://files.eric.ed.gov/fulltext/EJ1214990.pdf>
- Association of Mathematics Teacher Educators (2017). *Standards for preparing teachers of mathematics*. <https://amte.net/standards>
- Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21(2), 132–144.  
<https://doi.org/10.2307/749140>
- Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching. *American Educator*, Fall, 14–22.
- Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.  
<https://doi.org/10.1177/0022487108324554>
- Bandura, A. (1997). *Self-efficacy: The exercise of control*. W.H. Freeman.
- Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis*. Wiley.
- Baruch, Y., & Holtom, B. C. (2008). Survey response rate levels and trends in organizational research. *Human Relations*, 61(8), 1139–1160.  
<https://doi.org/10.1177/0018726708094863>

- Baumert, J., Kunter, M., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (2013). *Cognitive activation in the mathematics classroom and professional competence of teachers: Results from the COACTIV project, September*.  
<https://doi.org/10.1007/978-1-4614-5149-5>
- Beglar, D., & Nemoto, T. (2014). Developing Likert-scale questionnaires. In N. Sonda & A. Krause (Eds.) *JALT2013 Conference proceedings*, (pp.1–8). JALT.  
[https://jalt-publications.org/sites/default/files/pdf-article/jalt2013\\_001.pdf](https://jalt-publications.org/sites/default/files/pdf-article/jalt2013_001.pdf)
- Begle, E. (1972). *Teacher knowledge and student achievement in algebra* (School Mathematics Study Group Reports Number 9). Stanford University  
<https://files.eric.ed.gov/fulltext/ED064175.pdf>
- Bhupal, R. S. M. (2021). *Ability of reasoning and learning real analysis*.  
*International Journal of Mathematics and Statistics*, 514(3), 3–8. [https://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume10/volume10-issue3\(7\)/1.pdf](https://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume10/volume10-issue3(7)/1.pdf)
- Bishop, A. J. (1992). International perspectives on research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 710–723). Macmillan.
- Biswas, S. (2018). Constructivism in teacher education. *International Journal of Creative Research Thoughts*, 6(1), 57–80.
- Biza, I., Nardi, E., & Zachariades, T. (2007). Using tasks to explore teacher knowledge in situation-specific contexts. *Journal of Mathematics Teacher Education*, 10(4), 301–309.
- Bjerke, A. H., & Xenofontos, C. (2023). Teachers' self-efficacy in teaching mathematics: tracing possible changes from teacher education to professional practice. *Teachers and Teaching: Theory and Practice*, 30(1), 1–15.  
<https://doi.org/10.1080/13540602.2023.2219982>

- Bradley, L. H. (1985). *Curriculum leadership and development handbook*. Printice Hall.
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101.  
<https://doi.org/10.1191/1478088706qp063oa>
- Bruner, J. S. (1960). *The process of education*. Harvard University Press.  
[http://edci770.pbworks.com/w/file/attach/45494576/Bruner\\_Processes\\_of\\_Education.pdf](http://edci770.pbworks.com/w/file/attach/45494576/Bruner_Processes_of_Education.pdf)
- Bujang, M. A., Omar, E. D., Foo, D. H. P., & Hon, Y. K. (2024). Sample size determination for conducting a pilot study to assess reliability of a questionnaire. *Restorative Dentistry and Endodontics, 49*(1), 1–8.  
<https://doi.org/10.5395/rde.2024.49.e3>
- Businskas, A. M. (2008). *Conversation about connections: How secondary mathematics teachers conceptualize and contend with mathematical connections* (Master's thesis, Simon Fraser University). Simon Fraser University.  
<http://summit.sfu.ca/item/9245>
- Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., DePiper, J. N., Frank, T. J., Griffin, M. G., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419–459. <https://doi.org/10.5951/jresmetheduc.45.4.0419>
- Carifio, J., & Perla, R. (2008). Resolving the 50-year debate around using and misusing Likert scales. *Medical Education, 42*(12), 1150–1152.  
<https://doi.org/10.1111/j.1365-2923.2008.03172.x>
- Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An

- emergent multidimensional problem-solving framework. *Educational Studies in Mathematics*, 58(1), 45–75. <https://doi.org/10.1007/s10649-005-0808-x>
- Casey, S. A., & Wasserman, N. H. (2015). Teachers' knowledge about informal line of best fit. *Statistics Education Research Journal*, 14(1), 8–35. <https://doi.org/10.52041/serj.v14i1.267>
- Curriculum Development Center. (2022). *Secondary education curriculum 2078: Compulsory mathematics, grade 9 and 10*. Ministry of Education, Science and Technology, Government of Nepal. <https://moecdc.gov.np/storage/gallery/1672286726.pdf>
- Central Bureau of Statistics. (2018). *Sthaniya taha (Local level)*. Author. <https://cbs.gov.np>
- Cevikbaş, M., Kaiser, G., & Schukajlow, S. (2024). Teacher education for mathematical modelling: Exploring the experiences of secondary school teachers in two courses. *ZDM – Mathematics Education*, 56(7), 1109–1122. <https://doi.org/10.1007/s11858-024-01609-4>
- Chand, H. B. (2021). Difficulties experienced by undergraduate students in proving theorems of real analysis. *Scholars' Journal*, 4(1), 149–163. <https://doi.org/10.3126/scholars.v4i1.42475>
- Changwe, R. (2017). *Effectiveness of the mathematics teacher education curriculum at the University of Zambia in preparing secondary school teachers of mathematics* (Doctoral dissertation, The University of Zambia). <https://dspace.unza.zm/server/api/core/bitstreams/ae3a08dd-e0d5-48f5-937b-34a360d46e3a/content>
- Charanji, K. S., & Marappan, P. (2020). A review of research on the importance of higher order thinking skills (HOTs) in teaching english language. *Journal of*

- Critical Reviews*, 7(8). <https://doi.org/10.31838/jcr.07.08.161>
- Chhetri, D. B. (2023). Teaching real analysis in teacher preparation program through content bridging strategy. *Janabhawana Research Journal*, 2(1), 50–58. <https://doi.org/10.3126/jrj.v2i1.57488>
- Chua, V. C. G. (2021). A meta-synthesis of studies on deficiencies and affordances in mathematical knowledge for teaching. *Araneta Research Journal (Indagatio)*, 43(1), 15–21. <https://ejournals.ph/article.php?id=17751>
- Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher*, 28(2), 4–15. <https://doi.org/10.2307/1177185>
- Cochran, W. G. (1997). Sampling techniques. In *John Wiley and Sons, Inc.* (3rd). John Wiley & Sons.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Routledge.
- Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II*. American Mathematical Society. <https://www.cbmsweb.org/the-mathematical-education-of-teachers-ii-2012/papers3://publication/uuid/0A48BDE7-6D25-42EA-9079-52AFC3475AB5>
- Creswell, J. W. (2015). *Educational Research* (4th ed.). Pearsons. <http://repository.unmas.ac.id/medias/journal/EBK-00121.pdf>
- Creswell, J. W., & Clark, V. L. P. (2018). *Mixed method research* (3rd ed.). SAGE Publications. <https://www.pdfdrive.com/designing-and-conducting-mixed-methods-research-d196908993.html>
- Creswell, J. W., & Poth, C. N. (2023). *Qualitative inquiry and research design: Choosing among five approaches* (5th ed.). SAGE Publications.

- Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior*, *15*(4), 375–402. [https://doi.org/10.1016/S0732-3123\(96\)90023-1](https://doi.org/10.1016/S0732-3123(96)90023-1)
- Dalby, D. (2017). The professional identity of mathematics teachers in further education. *Adults Learning Mathematics: An International Journal*, *12*(1), 7–16.
- Damir, S. (2022). Comparison of normality tests in terms of sample sizes under different skewness and kurtosis coefficients. *International Journal of Assessment Tools in Education*, *9*(2), 397–409. <https://doi.org/10.21449/ijate.1101295>
- Dang, T. T. C., Nguyen, H. V., & Tran, P. T. T. (2018). The SAGE handbook of qualitative data collection. In *Applied linguistics and language education research methods: Fundamentals and innovations* (Chapter 4). IGI Global. <https://doi.org/10.4018/979-8-3693-2603-9.ch004>
- Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics*, *61*(3), 293–319. <https://doi.org/10.1007/s10649-006-2372-4>
- Davis, L. L. (1992). Instrument review: Getting the most from a panel of experts. *Applied Nursing Research*, *5*(4), 194–197. [https://doi.org/https://doi.org/10.1016/S0897-1897\(05\)80008-4](https://doi.org/https://doi.org/10.1016/S0897-1897(05)80008-4)
- Dawkins, L., Bauld, L., Ford, A., Robson, D., Hajek, P., Parrott, S., Best, C., Li, J., Tyler, A., & Uny, I. (2020). A cluster feasibility trial to explore the uptake and use of e-cigarettes versus usual care offered to smokers attending homeless centres in Great Britain. *PLOS ONE*, *15*(10), e0240968. <https://doi.org/10.1371/journal.pone.0240968>
- Delaney, S., Ball, D. L., Hill, H. C., Schilling, S. G., & Zopf, D. (2008). “Mathematical knowledge for teaching”: Adapting U.S. measures for use in

- Ireland. *Journal of Mathematics Teacher Education*, 11(3), 171–197.  
<https://doi.org/10.1007/S10857-008-9072-1>
- Dewey, J. (1962). *Reconstruction in philosophy*. Henry Holt and Company.
- Diamantopoulos, A., & Siguaw, J. A. (2006). Formative versus reflective indicators in organizational measure development: A comparison and empirical illustration. *British Journal of Management*, 17(4), 263–282. <https://doi.org/10.1111/j.1467-8551.2006.00500.x>
- Dogan, E., & Demirbolat, A. O. (2021). Data-driven decision-making in schools scale: A study of validity and reliability. *International Journal of Curriculum and Instruction*, 13(1), 507–523.  
<https://files.eric.ed.gov/fulltext/EJ1285547.pdf>
- Dreher, A., Lindmeier, A., & Heinze, A. (2016). Conceptualizing professional content knowledge of secondary teachers taking into account the gap between academic and school mathematics. In C. Csíkós, A. Rausch, & J. Sztányi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 219–226). PME.
- Dreher, A., Lindmeier, A., Heinze, A., & Niemand, C. (2018). What kind of content knowledge do secondary mathematics teachers need? A conceptualization taking into account academic and school mathematics. *Journal für Mathematik-Didaktik*, 39(2), 319–341. <https://doi.org/10.1007/s13138-018-0127-2>
- Edwards, B. S., & Ward, M. B. (2004). Surprises from mathematics education research: Student (mis)use of mathematical definitions. *American Mathematical Monthly*, 111(5), 411–424. <https://doi.org/10.2307/4145268>
- Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, 15(1), 13–33.

<https://doi.org/10.1080/0260747890150102>

- Ernest, P. (2007). Mathematics education and philosophy: an internal perspective. *Journal of Mathematics Teacher Education*, 10(3), 155–165.
- Etikan, I. (2017). Sampling and sampling methods. *Biometrics & Biostatistics International Journal*, 5(6). <https://doi.org/10.15406/bbij.2017.05.00149>
- Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics*, 21(6), 521–544.  
<https://doi.org/10.1007/BF00315943>
- Even, R. (2011). The relevance of advanced mathematics studies to expertise in secondary school mathematics teaching: Practitioners' views. *ZDM - International Journal on Mathematics Education*, 43(6), 941–950.  
<https://doi.org/10.1007/s11858-011-0346-1>
- Fischbein, E. (1999). Intuitions and schemata in mathematical reasoning. *Educational Studies in Mathematics*, 38, 11–50. <https://doi.org/10.1023/A:1003488222875>
- Fetters, M. D., Curry, L. A., & Creswell, J. W. (2013). Achieving integration in mixed methods designs: Principles and practices. *Health Services Research*, 48(6pt2), 2134–2156. <https://doi.org/10.1111/1475-6773.12117>
- Fornell, C., & Larcker, D. (1981). Evaluating structural equation models with unobservable variables and measurement error. *Journal of Marketing Research*, 18(1), 39–50.
- Foster, C. (2011a). Peripheral mathematical knowledge. *For the Learning of Mathematics*, 31(3), 24–26. <http://www.jstor.org/stable/41319604>
- Foster, C. (2011b). Productive ambiguity in the learning of mathematics. *For the Learning of Mathematics*, 31(2), 3–7. <https://doi.org/10.2307/41319555>
- Fried, M. N. (2001). Can mathematics education and history of mathematics coexist?

*Science and Education*, 10(4), 391–408.

<https://doi.org/10.1023/A:1011205014608>

Fukawa-Connelly, T., Mejia-Ramos, J. P., Wasserman, N. H., & Weber, K. (2019).

An evaluation of ULTRA; an Experimental real analysis course built on a transformative theoretical model. *International Journal of Research in Undergraduate Mathematics Education* 2019 6:2, 6(2), 159–185.

<https://doi.org/10.1007/S40753-019-00102-8>

Fukaya, T., Fukuda, M., & Suzuki, M. (2023). Relationship between mathematical

pedagogical content knowledge, beliefs, and motivation of elementary school teachers. *Frontiers in Education*, 8(January), 1–11.

<https://doi.org/10.3389/feduc.2023.1276439>

Gamboa, G. De, Caviedes, S., & Badillo, E. (2022). Mathematical connections and

the mathematics Teacher's specialised knowledge.  $\Sigma$  *Mathematics*, 10(21), 4010.

<https://doi.org/10.3390/math10214010>

Ghasemi, A., & Zahediasl, S. (2012). Normality tests for statistical analysis: A guide

for non-statisticians. *International Journal of Endocrinology and Metabolism*, 10(2), 486–489. <https://doi.org/10.5812/ijem.3505>

GHR, G., & Aithal, P. S. (2022). Why is it called Doctor of Philosophy and why

choosing appropriate research philosophical paradigm is indispensable during Ph.D. program in India? *International Journal of Philosophy and Languages (IJPL)*, 1(1), 42–58. <https://doi.org/10.2139/ssrn.4263406>

Gliner, Jeffrey, A., Morgan, G. A., & Leech, Nancy, L. (2017). Research methods in applied setting. An integrated approach to design and analysis (3 rd.). Routledge.

Goodlad, J. I. (1979). *Curriculum inquiry: The study of curriculum practice*.

McGraw-Hill. <https://archive.org/details/curriculuminquir0000unse>

- Goulding, M., Hatch, G., & Rodd, M. (2003). Undergraduate mathematics experience: Its significance in secondary mathematics teacher preparation. *Journal of Mathematics Teacher Education* 2003 6:4, 6(4), 361–393. <https://doi.org/10.1023/A:1026362813351>
- Graif, F. (2020). *Mathematical knowledge for teaching proof in secondary mathematics teacher preparation* [Doctoral dissertation, University of Minnesota]. University Digital Conservancy. <https://hdl.handle.net/11299/216393>
- Grinstein, L., & Lipsey, S. I. (2014). Encyclopedia of mathematics education. In I. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer. <https://doi.org/10.1007/978-94-007-4978-8>
- Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Communication and Technology*, 19 (2) (75–91). <https://www.jstor.org/stable/30219811>
- Guest, MacQueen, N. (2012). *Applied thematic analysis*. Sage
- Guloksuz, S., Pries, L. K., Ten Have, M., de Graaf, R., van Dorsselaer, S., Klingenberg, B., ... & van Os, J. (2020). Association of preceding psychosis risk states and non-psychotic mental disorders with incidence of clinical psychosis in the general population: A prospective study in the NEMESIS-2 cohort. *World Psychiatry*, 19(2), 199-205. <https://doi.org/10.1002/wps.20755>
- Gunasekare, U. L. T. P. (2015). Mixed research method as the third research paradigm: A literature review. *International Journal of Science and Research*, 4(8), 361–367. <https://www.ijsr.net/archive/v4i8/SUB157170.pdf>
- Gunawan, J. (2015). Ensuring trustworthiness in qualitative research. *Belitung Nursing Journal*, 1(1), 10–11. <http://belitungraya.org/BRP/index.php/bnj/>

- Habermas, J. (2005). Knowledge and human interests: A general perspective. In *The international encyclopedia of communication* (pp. 310–321). Wiley.  
<https://doi.org/10.1002/9780470755501.ch23>
- Hair, J. F., Black, W. C., Babin, B. J., & Anderson, R. E. (2019a). *Multivariate data analysis* (7th ed.). Pearsons.
- Hair, J. F., Risher, J. J., Sarstedt, M., & Ringle, C. M. (2019b). When to use and how to report the results of PLS-SEM. *European Business Review*, *31*(1), 2–24.  
<https://doi.org/10.1108/EBR-11-2018-0203>
- Hair, J. F., Sarstedt, M., Ringle, C. M., Sharma, P. N., & Liengaard, B. D. (2024). Going beyond the untold facts in PLS-SEM and moving forward. *European Journal of Marketing*, *58*(13), 81–106. <https://doi.org/10.1108/EJM-08-2023-0645>
- Hanke, E. (2024). Experts' intuitive mathematical discourses about integration in complex analysis. *ZDM – Mathematics Education*, *56*(7), 1403–1416.  
<https://doi.org/10.1007/s11858-024-01610-x>
- Harel, G., Selden, A., & Selden, J. (2006). Advanced mathematical thinking. *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future, May 2014*, 147–173.
- Hatisaru, V. (2022). Mathematical connections established in the teaching of function. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *41*(4) 1–21. <https://doi.org/https://doi.org/10.1093/teamat/hrac013>
- Heid, M. K., & Wilson, P. S. (2016). Framework for mathematical understanding for secondary teaching: A mathematical activity perspective. *15Th Annual Conference on Research in Undergraduate Mathematics Education*, 512–516.
- Heid, M. K., Wilson, P. S., & Blume, G. W. (Eds.). (2015a). *Mathematical*

*understanding for secondary teaching: A framework and classroom-based situations.* IAP

Heid, M. K., Wilson, P. S., & Blume, G. W. (2015b). Mathematical understanding for secondary teaching: A Framework and classroom. In G. W. Blume (Ed.), *Mathematical Understanding for Secondary Teaching: A framework and Classroom-based situation.* IAP.

[https://books.google.com.np/books?hl=en&lr=&id=kgYoDwAAQBAJ&oi=fnd&pg=PA41&dq=Heid,+M.+K.,+Wilson,+P.+S.,+%26+Blume,+G.+W.+\(2015\).+Mathematical+understanding+for+secondary+teaching:+A+framework+and+classroombased+situations.+National+Council+of+Teachers+o](https://books.google.com.np/books?hl=en&lr=&id=kgYoDwAAQBAJ&oi=fnd&pg=PA41&dq=Heid,+M.+K.,+Wilson,+P.+S.,+%26+Blume,+G.+W.+(2015).+Mathematical+understanding+for+secondary+teaching:+A+framework+and+classroombased+situations.+National+Council+of+Teachers+o)

Heinze, A., Dreher, A., Lindmeier, A., & Niemand, C. (2016). Akademisches versus schulbezogenes Fachwissen – ein differenzierteres Modell des fachspezifischen Professionswissens von angehenden Mathematiklehrkräften der Sekundarstufe. *Zeitschrift Für Erziehungswissenschaft 2016 19:2, 19(2), 329–349.*

<https://doi.org/10.1007/S11618-016-0674-6>

Hennink, M., & Kaiser, B. N. (2022). Sample sizes for saturation in qualitative research: A systematic review of empirical tests. *Social Science and Medicine, 292, 114523.* <https://doi.org/10.1016/j.socscimed.2021.114523>

Hill, H.C., Rowan, B., & Ball, D. L. (2005). Effect of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42 (2) 3711–406.* <https://doi.org/10.3102/00028312042002371>

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39(4), 372–400.* <https://doi.org/10.5951/jresmetheduc.39.4.0372>

- Hill, H. C., Rowan, B., & Ball, D. L. (2004). Effects of teachers' mathematical knowledge on student achievement. Paper presented at the Annual Meeting of the American Educational Research Association, San Diego, CA.
- Hill, H. c., Rowan, B., & Ball, D. l. (2005). Effect of teachers' mathematical knowledge for teaching on student achievement. *American Education Research Journal*, 2(42), 371–406.  
<https://doi.org/https://doi.org/10.3102/00028312042002371>
- Hill, R. (1998). What sample size is “enough” in internet survey research. *Interpersonal Computing and Technology: An Electronic Journal for the 21st Century*, 6(3–4), 1–12. [https://d1wqtxts1xzle7.cloudfront.net/51136900/IPCT-J\\_Vol\\_6\\_Robin\\_hill\\_SampleSize-libre.pdf](https://d1wqtxts1xzle7.cloudfront.net/51136900/IPCT-J_Vol_6_Robin_hill_SampleSize-libre.pdf)
- Honorene, J. (2017). Understanding the role of triangulation in research. *Scholarly research journal for interdisciplinary studies*, 4(31), 91-95.
- Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, 6(1), 1–55. <https://doi.org/10.1080/10705519909540118>
- Huck, S. W. (2012). *Reading statistics and research* (3rd ed.). Pearson.
- Hurrell, D. P. (2013). What teachers need to know to teach mathematics: An argument for a reconceptualised model. *Australian Journal of Teacher Education*, 38(11), 54–64. <https://doi.org/10.14221/ajte.2013v38n11.3>
- Ingram, J., Sammons, P., & Lindorff, A. (2018). *Observing effective mathematics teaching: A review of the literature*. Education Development Trust.  
<https://www.educationdevelopmenttrust.com>
- International Bureau of Education.(2023). *Curriculum relevance*.  
<http://www.ibe.unesco.org/en/glossary-curriculum-terminology/c/curriculum->

relevance

- Jankvist, U. T., Mosvold, R., Fauskanger, J., & Jakobsen, A. (2015). Analysing the use of history of mathematics through MKT. *International Journal of Mathematical Education in Science and Technology*, 46(4), 495–507.  
<https://doi.org/10.1080/0020739X.2014.990528>
- Johanson, G. A., & Brooks, G. P. (2010). Initial scale development: Sample size for pilot studies. *Educational and Psychological Measurement*, 70(3), 394–400.  
<https://doi.org/10.1177/0013164409355692>
- Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, 33(7), 14–26.  
<https://doi.org/10.3102/0013189X033007014>
- Joshi, D. R., Khadka, J., Adhikari, K. P., Khanal, B., & Belbase, S. (2024). Exploring the effects of learning challenges on mathematics achievement. *International Journal of Instruction*, 17(4), 78–98. <https://e-iji.net/ats/index.php/pub/article/view/643>
- Julious, S. A. (2005a). Sample size of 12 per group rule of thumb for a pilot study. *Pharmaceutical Statistics*, 4(4), 287–291. <https://doi.org/10.1002/pst.185>
- Julious, S. A. (2005b). Sample size of 12 per group rule of thumb for a pilot study. *Pharmaceutical Statistics*, 4(4), 287–291. <https://doi.org/10.1002/pst.185>
- Kalekar, S. (2018). Approaches for implementing constructivism in teacher education: Hurdles and means. *Scholarly Research Journal for Interdisciplinary Studies*, 5(6), 48–55. <http://www.srjis.com/pages/pdfFiles/14694366777.Pr.Kalekar.pdf>
- Kazima, M., Jakobsen, A., & Kasoka, D. N. (2016). Use of mathematical tasks in teaching and the corresponding LMT measures in the Malawi context. *Mathematics Enthusiast*, 13(1–2), 171–186. <https://doi.org/10.54870/1551->

3440.1371

- Kathmandu University, School of Education. (2018). *Master of education (mathematics education)*. Kathmandu University School of Education. <http://www.ku.edu.np/school-of-education>
- Khanal, B., Joshi, D. R., & Sharma Chapai, K. P. (2024). Mathematics teachers' reflection on essential digital skills and techniques for effective mathematics instruction. *Education and Development*, 32(1), 79–92. <https://doi.org/10.3126/ed.v32i1.61571>
- Khadka, R., Pokharel, B., & Rimal, A. (2022). Validation of a Nepali version of the Fear of COVID-19 Scale in the general population. *PLOS ONE*, 17(1), e0262120. <https://doi.org/10.1371/journal.pone.0262120>
- Kiger, M. E., & Varpio, L. (2020). Thematic analysis of qualitative data: AMEE Guide. *Medical Teacher*, 42(1), 1–9. <https://doi.org/10.1080/0142159X.2020.1755030>
- Kilpatrick, J., Blume, G., Heid, M. K., Wilson, J., Wilson, P., & Rose, M. (2015). Mathematical understanding for secondary teaching: A framework. In M. K. Heid, S. P. Wilson, & G. W. Blume (Eds.), *Mathematical understanding for secondary teaching: A framework and classroom-based situations* (pp. 9–30). Information Age Publishing.
- Kim, H.-Y. (2013). Statistical notes for clinical researchers: Assessing normal distribution (2) using skewness and kurtosis. *Restorative Dentistry & Endodontics*, 38(1), 52. <https://doi.org/10.5395/rde.2013.38.1.52>
- King, R. (2022). The utility of pragmatism in educational research. *Creative Education*, 13(10), 3153–3161. <https://doi.org/10.4236/ce.2022.1310199>
- Kirkpatrick, D. L., & Kirkpatrick, J. D. (2006). *Evaluating training programs*. Berrett-Koehler Publishers. <https://doi.org/10.1021/cen-v034n010.p1083>

- Kirkpatrick, J., & Kirkpatrick, W. (2014). The Kirkpatrick four level: a fresh look after 55 years. In *Kirkpatrick Partners*.  
[http://www.kirkpatrickpartners.com/Portals/0/Resources/White Papers/Introduction to the Kirkpatrick New World Model.pdf](http://www.kirkpatrickpartners.com/Portals/0/Resources/WhitePapers/Introduction%20to%20the%20Kirkpatrick%20New%20World%20Model.pdf)
- Kite, M. E., & Whitley, B. E. (2018). Principles of research in behavioral science. In *Routledge* (4th ed.). Routledge. <https://doi.org/10.4324/9781315450087>
- Klein, F. (2016). Elementary mathematics from a higher standpoint: Volume I: Arithmetic, algebra, analysis. *Elementary Mathematics from a Higher Standpoint: Volume I: Arithmetic, Algebra, Analysis, I*.  
<https://doi.org/10.1007/978-3-662-49442-4/COVER>
- Knuth, E. J. (2002). Secondary school mathematics teachers' conceptions of proof. *Journal for Research in Mathematics Education*, 33(5), 379–405.  
<https://doi.org/10.5951/jresmetheduc.33.5.0379>
- Ko, I., Mesa, V., Duranczyk, I., Herbst, P., & Kohli, N. (2024). Understanding the characteristics of mathematical content knowledge for teaching algebra in high schools and community colleges. *International Journal of Mathematical Education in Science and Technology*, 55(5), 590–614.  
<https://doi.org/10.1080/0020739X.2021.2006348>
- Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2017). How education affects mathematics teachers' knowledge: Unpacking selected aspects of teacher knowledge. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(6), 1943–1980. <https://doi.org/10.12973/eurasia.2017.01209a>
- Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2019). Using network analysis methods to investigate how future teachers conceptualize the links between the domains of teacher knowledge. *Teaching and Teacher*

*Education*, 79, 137–152. <https://doi.org/10.1016/j.tate.2018.12.010>

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology*, 100(3), 716–725. <https://doi.org/10.1037/0022-0663.100.3.716>

Kumar, R. S., Dewan, H., & Subramaniam, K. (2013). *The preparation and professional development of mathematics teachers: Context and background*. Publisher

Kvatinsky, T., & Even, R. (2002). Framework for teacher knowledge and understanding about probability. *Developing a Statistically Literate Society. Proceedings of the Sixth International Conference on Teaching Statistics (ICOTS 6)*, 1–6.

Leikin, R., & Zazkis, R. (2010). On the content-dependence of prospective teachers' knowledge: A case of exemplifying definitions. *International Journal of Mathematical Education in Science and Technology*, 41(4), 451–466. <https://doi.org/10.1080/00207391003605189>

Leonall, H., Advisor, F., & Kelleher, R. (2011). Progressivism in the 21st century elementary school: Is John Dewey relevant today? *International Journal of Art & Sciences*, 4(16), 196–283.

Lesseig, K. (2016). Investigating mathematical knowledge for teaching proof in professional development. *International Journal of Research in Education and Science*, 2(2), 253–270. <https://doi.org/10.21890/ijres.13913>

Lewis, G., Jones, B., & Baker, C. (2012). Translanguaging: Developing its conceptualisation and contextualisation. *Educational Research and Evaluation*, 18(7), 655–670. <https://doi.org/10.1080/13803611.2012.718490>

- Liao, S.-C., & Hsu, S.-Y. (2019). Evaluating a continuing medical education program: New world Kirkpatrick model approach. *International Journal of Management, Economics and Social Sciences*, 8(4), 266–279.  
<https://doi.org/10.32327/ijmess/8.4.2019.17>
- Lincon, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. SAGE Publications.
- Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.  
<https://doi.org/10.1177/0022487108324554>
- Lohr, S. L. (2010). *Sampling: Design and analysis* (2nd ed.). Brooks/Cole.  
<https://www.cengage.com/c/sampling-design-and-analysis-2e-lohr/9780495105275/>
- Maarouf, H. (2019). Pragmatism as a supportive paradigm for the mixed research approach: Conceptualizing the ontological, epistemological, and axiological stances of pragmatism. *International Business Research*, 12(9), 1–11.  
<https://doi.org/10.5539/ibr.v12n9p1>
- Machuca, C., Baker, R. S., Sufi, F., Mason, S., Barlow, P. S. A., & Robinson, P. G. (2015). 10 - Derivation of a short form of the dentine hypersensitivity questionnaire. In *Dentine hypersensitivity* (pp. 155–164).  
<https://doi.org/10.1016/B978-0-12-801631-2.00010-5>
- Maguire, M., & Delahunt, B. (2017). Doing a thematic analysis: A practical, step-by-step guide for learning and teaching scholars. *All Ireland Journal of Teaching and Learning in Higher Education*, 8(3), 3351–3362.  
<http://ojs.aishe.org/index.php/aishe-j/article/view/335>
- Markworth, K., Goodwin, T., & Glisson, K. (2009). The development of mathematical knowledge for teaching in the student teaching practicum. In M.

- Mewborn & S. Lee (Eds.), *AMTE monograph, 6. Scholarly practices and inquiry in the preparation of mathematics teachers* (pp. 67–83). Association of Mathematics Teacher Educators.  
<https://search.proquest.com/docview/1651837224?accountid=12347>
- Maxwell, J. A. (2016). Expanding the history and range of mixed methods research. *Journal of Mixed Methods Research, 10*(1), 12–27.  
<https://doi.org/10.1177/1558689815571132>
- Mejía-Ramos, J. P., & Weber, K. (2020). Using task-based interviews to generate hypotheses about mathematical practice: mathematics education research on mathematicians' use of examples in proof-related activities. *ZDM - Mathematics Education, 52*(6), 1099–1112. <https://doi.org/10.1007/s11858-020-01170-w>
- Menon, U. (2015). *Mathematisation-vertical and horizontal*. May.
- Miles, M. B., Michael, H. A., & Saldana, J. (2014). *Qualitative data analysis a methods sourcebook* (3 rd). SAGE Publications Inc.
- Milgram, R. J., & Mathematical Sciences Research Institute. (2007). What is mathematical proficiency? In A. H. Schoenfeld (Ed.), *Assessing mathematical proficiency* (pp. 31–58). Cambridge University Press.  
<https://scispace.com/pdf/what-is-mathematical-proficiency-1ueu03m1i5.pdf>
- Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017–1054. <https://doi.org/10.1111/j.1467-9620.2006.00684.x>
- Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), 125–145. [https://doi.org/10.1016/0272-7757\(94\)90003-5](https://doi.org/10.1016/0272-7757(94)90003-5)
- Moreira, P. C. B., & Vianna, C. R. D. (2016). Why real analysis in mathematics

- teacher education? A comparison between mathematics educators' and mathematicians' views. *Bolema: Mathematics Education Bulletin*, 30(55), 515–534. <https://doi.org/10.1590/1980-4415v30n55a14>
- Morgan, L. D. (2016). Which method should I use to present the mean of a 5-point Likert scale?. <https://www.researchgate.net/post/Which-method-should-I-use-to-present-the-Mean-of-a-5-point-Likert-scale>
- Mumu, J., Tanujaya, B., Charitas, R., & Prahmana, I. (2022). Likert scale in social sciences research: Problems and difficulties. *FWU Journal of Social Sciences*, 16(4), 89–101. <https://doi.org/10.51709/19951272/Winter2022/7>
- Murray, E., Baldinger, E., Wasserman, N., Broderick, S., & White, D. (2017). *Exploring connections between advanced and secondary mathematics*. Montclair State University. <https://digitalcommons.montclair.edu/cgi/viewcontent.cgi?article=1024&context=mathsci-facpubs>
- Mustafa, Z. (2008). *Teachers' levels of use in the adoption of task-based language teaching in the classrooms* (Unpublished Ed.D. thesis). Universiti Sains Malaysia.
- Mwinka, E. J., & Tarmo, A. P. (2020). Influence of teachers' mathematical knowledge for teaching on students' academic achievement in secondary mathematics in Tanzania. *Education and Development*, 38(2), 178–199.
- National Research Council. (2001). *Helping children learn mathematics*. National Academies Press. <https://doi.org/10.17226/10434>
- Naziev, A., & Zemlyakova, I. (2025). Teaching proof to future mathematics teachers in mathematical analysis classes. *arXiv*. <https://arxiv.org/abs/2503.14507>

Norman, G. (2010). Likert scales, levels of measurement and the “laws” of statistics.

*Advances in Health Sciences Education*, 15(5), 625–632.

<https://doi.org/10.1007/s10459-010-9222-y>

O’Meara, N. (2010). *Improving mathematics teaching at second level through the design of a model of teacher knowledge and an intervention aimed at developing teachers’ knowledge* (Doctoral dissertation, University of Limerick). University of Limerick.

[https://ulir.ul.ie/bitstream/handle/10344/3254/OMeara\\_2011\\_teaching.pdf](https://ulir.ul.ie/bitstream/handle/10344/3254/OMeara_2011_teaching.pdf)

Omar, M., Klawonn, F., Brand, S., Stiesch, M., Krettek, C., & Eberhard, J. (2017).

Transcriptome-wide high-density microarray analysis reveals differential gene transcription in periprosthetic tissue from hips with chronic periprosthetic joint infection vs aseptic loosening. *The Journal of Arthroplasty*, 32(1), 234–240.

<https://doi.org/10.1016/j.arth.2016.06.036>

Ottmar, E. R., Rimm-Kaufman, S. E., Larsen, R., & Merritt, E. G. (2011). *Relations between mathematical knowledge for teaching, mathematics instructional quality, and student achievement in the context of the* [Conference paper].

Society for Research on Educational Effectiveness.

<https://files.eric.ed.gov/fulltext/ED528798.pdf>

Özden, M. (2008). The effect of content knowledge on pedagogical content

knowledge: The case of teaching phases of matters. *Kuram ve Uygulamada*

*Eğitim Bilimleri*, 8(2), 633–645. <https://eric.ed.gov/?id=EJ831172>

Pansell, A. (2023). Mathematical knowledge for teaching as a didactic praxeology.

*Frontiers in Education*, 8, Article 1165977.

<https://doi.org/10.3389/feduc.2023.1165977>

Panthi, D., & Jha, K. (2016). A study on teaching applicable mathematics in the

- universities of Nepal. *Kathmandu University Journal of Science, Engineering and Technology*, 1(5), 10–12. <https://doi.org/10.3126/kuset.v4i1.2888>
- Patterson, A. (2019). *Real analysis mathematical knowledge for teaching: An investigation* [NC State University].
- Patterson, B. (2021). Real analysis mathematical knowledge for teaching: An investigation. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1(Content Knowledge), 1–10.  
<https://files.eric.ed.gov/fulltext/EJ1290170.pdf>
- Patton, M. Q. (2002). *Qualitative research and evaluation methods* (3rd ed.). Sage Publications.
- Paull, M., Whitsed, C., & Girardi, A. (2016a). Applying the Kirkpatrick model: Evaluating an Interaction for Learning Framework curriculum intervention. *Issues in Educational Research*, 26(3), 490–507.
- Paull, M., Whitsed, C., & Girardi, A. (2016b). Evaluating an interaction for learning framework curriculum intervention: Applying the Kirkpatrick Model. *Education + Training*, 58(3), 490–502. <https://doi.org/10.1108/ET-06-2015-0046>.
- Pepin, B., Xu, B., Trouche, L., & Wang, C. (2017). Developing a deeper understanding of mathematics teaching expertise: An examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics*, 94(3), 257–274.  
<https://doi.org/10.1007/s10649-016-9727-2>
- Plano Clark, V. L., & Ivankova, N. V. (2016). *Mixed methods research: A guide to the field*. SAGE Publications.
- Poth, C., & Munce, S. E. (2020). Commentary: Preparing today's research for a yet unknown tomorrow: Promising practices for a synergistic and sustainable

mentoring approach to mixed research learning. *International Journal of Multiple Research Approaches*, 12(1), 56–64.

<https://doi.org/10.29034/ijmra.v12n1commentary>

Rivas, M. A., Godino, J. D., & Castro, W. F. (2012). Desarrollo del conocimiento para la enseñanza de la proporcionalidad en futuros profesores de primaria.

*Bolema: Boletim de Educação Matemática*, 26(42b), 559–588.

<https://doi.org/10.1590/s0103-636x2012000200008>

Robinson, W. S. (2009). Ecological correlations and the behavior of individuals.

*International Journal of Epidemiology*, 38(2), 337-341.

<https://doi.org/https://doi.org/10.1093/ije/dyn357>

Roh, K. H., & Lee, Y. H. (2017). Designing tasks of introductory real analysis to

bridge a gap between students' intuition and mathematical rigor: The case of the convergence of a sequence. *International Journal of Research in Undergraduate*

*Mathematics Education*, 3(1), 34–68. <https://doi.org/10.1007/s40753-016-0039-9>

Rowan, B., Schilling, S., & Ball, D. (2001). Measuring teachers' pedagogical content

knowledge in surveys: An exploratory study. *Study of Instructional*

*Improvement*. [http://www.sii.soe.umich.edu/newsite\\_temp/documents/pck final report revised BR100901.pdf](http://www.sii.soe.umich.edu/newsite_temp/documents/pck_final_report_revised_BR100901.pdf)

Roy, S., & Saha, B. (2021). Essence of social constructivist approach in teaching-

learning scenario and revolving questions on its relevance in technological era.

*IAR Journal of Humanities and Social Science*, 2(5), 72–76.

<https://www.iarconsortium.org/article/download/1187/>

Sachan, M., Dubey, A., & Xing, E. P. (2017). From textbooks to knowledge: A case

study in harvesting axiomatic knowledge from textbooks to solve geometry

problems. *EMNLP 2017 - Conference on Empirical Methods in Natural*

*Language Processing, Proceedings*, 773–784. <https://doi.org/10.18653/v1/d17-1081>

Schmidt, W., Burroughs, N., & Cogan, L. (2013). *World class standards for preparing teachers of mathematics*. Center for the Study of Curriculum, Michigan State University.

Schmidt, W. H., Burroughs, N. A., Cogan, L. S., & Houang, R. T. (2016). The role of subject-matter content in teacher preparation: An international perspective for mathematics. *Journal of Curriculum Studies*, 49(2), 111–131. <https://doi.org/10.1080/00220272.2016.1153153>

Schmidt, W. H., Burroughs, N. A., Houang, R. T., & Cogan, L. S. (2020). The role of content knowledge in mathematics teacher preparation: A study of traditional and alternative teacher preparation in Texas. *Journal of Teacher Education*, 71(1), 1–12. <https://doi.org/10.1177/0022487119886290>

Schober, P., Boer, C., & Schwarte, L. A. (2018). Correlation coefficients: appropriate use and interpretation. *Anesthesia & Analgesia*, 126(5), 1763–1768. <https://doi.org/https://doi.org/10.1213/ANE.0000000000002864>

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, 196(2), 1–38. <https://doi.org/10.1177/002205741619600202>

Schunk, D. H. (2012). *Learning theories: An educational perspective* (6th ed.). Pearson. <https://doi.org/10.1007/BF00751323>

Schwab, J. J. (1973). The practical 3: Translation into curriculum. *The School Review*, 81(4), 501–522. <https://doi.org/10.1086/443100>

Sfard, A. (2008). Introduction to thinking as communication. *The Mathematics Enthusiast*, 5(2–3), 429–436. <https://doi.org/10.54870/1551-3440.1121>

- Sfard, A., & Linchevski, L. (1994). The gains and the pitfalls of reification: The case of algebra. *Educational Studies in Mathematics*, 26(2–3), 191–228.  
<https://doi.org/10.1007/BF01273663>
- Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, 22(2), 63–75. <https://doi.org/10.3233/EFI-2004-22201>
- Shorten, A., & Smith, J. (2017). Mixed methods research: Expanding the evidence base. *Evidence-Based Nursing*, 20(3), 74–75. <https://doi.org/10.1136/eb-2017-102699>
- Shulman, L. S. (1986a). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14. <http://www.jstor.org/stable/1175860>
- Shulman, L. S. (1986b). Those who understand knowledge. *Educational Researcher*, 15(2), 4–14.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1–22.  
<http://people.ucsc.edu/~ktellez/shulman.pdf>
- Shure, V., Lehmann, M., Friesen, M., Roesken-Winter, B., & Prediger, S. (2025). Professional development research on promoting productive practices for teachers' handling of typical tasks of mathematics teaching: A systematic review. *ZDM – Mathematics Education*. Advance online publication.  
<https://doi.org/10.1007/s11858-025-01659-2>
- Sidiropoulou, M., Nikoloudakis, I. A., & Xenos, M. (2022). Online survey design for teachers' attitudes toward technology integration in Greek secondary schools. *Computers*, 11(9), 127. <https://doi.org/10.3390/computers11090127>
- Simmt, E. (2011). Teacher expertise explored as mathematics for teaching. *Expertise*

*in Mathematics Instruction: An International Perspective*, 151–164.

[https://doi.org/10.1007/978-1-4419-7707-6\\_8/COVER](https://doi.org/10.1007/978-1-4419-7707-6_8/COVER)

Skemp, R. R. (1996). Relational understanding and instrumental understanding.

*Mathematics Teaching in the Middle School*, 77(1), 20–26.

<https://doi.org/10.5951/mtms.12.2.0088>

Singh, C. K. S., Singh, T. S. M., Ja'afar, H., Tek, O. E., Kaur, H., Moastafa, N. A., &

Yunus, M. (2020). Teaching strategies to develop higher-order thinking skills in

English literature. *International Journal of Innovation, Creativity and*

*Change*, 11(80), 211-231.

[https://www.ijicc.net/images/vol11iss8/11816\\_Singh\\_2020\\_E\\_R.pdf](https://www.ijicc.net/images/vol11iss8/11816_Singh_2020_E_R.pdf)

Sorto, M. A., Marshall, J. H., Luschei, T. F., & Carnoy, M. (2009). Teacher

knowledge and teaching in Panama and Costa Rica: A comparative study in

primary and secondary education. *Revista Latinoamericana de Investigación en*

*Matemática Educativa*, 12(2), 251–290.

South, L., Saffo, D., Vitek, O., Dunne, C., & Borkin, M. A. (2022). Effective use of

Likert scales in visualization evaluations: A systematic review. *Computer*

*Graphics Forum*, 41(3), 43–55. <https://doi.org/10.1111/cgf.14521>

Spector, P. E. (2022). *Job satisfaction: From assessment to intervention*. Routledge.

Stahl, A. N., & King, J. R. (2020). Expanding approaches for research: Understanding

and using trustworthiness in qualitative research. *Journal of Developmental*

*Education*, 44(1), 1–28. <https://files.eric.ed.gov/fulltext/EJ1320570.pdf>

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating

productive mathematical discussions: Five practices for helping teachers move

beyond show and tell. *Mathematical Thinking and Learning*, 10(4), 313–340.

<https://doi.org/10.1080/10986060802229675>

- Stenhouse, L. (1975). *An introduction to curriculum research and development*. Heinemann. <https://archive.org/details/introductiontocu0000sten>
- Stockton, J. C., & Wasserman, N. H. (2017). Forms of knowledge of advanced mathematics for teaching. *Mathematics Enthusiast*, *14*(1–3), 575–606. <https://doi.org/10.54870/1551-3440.1412>
- Stone, J. C., Madaus, G. F., Stufflebeam, D., & Tyler, R. W. (1990). Educational evaluation: Classic works of Ralph W. Tyler. *Educational Evaluation and Policy Analysis*, *12*(1), 102. <https://doi.org/10.2307/1163590>
- Stufflebeam, D. L. (2003). The CIPP model for evaluation. In *Annual Conference of the Oregon Program Evaluators Network* (pp. 31–62). [https://doi.org/10.1007/978-94-010-0309-4\\_4](https://doi.org/10.1007/978-94-010-0309-4_4)
- Suhartini, R., Ekohariadi, L. N., Wahyuningsih, U., Yulistiana, & Prihatina, Y. I. (2021). Validity, reliability, intra-rater instrument parameter teaching factory and learning outcomes of industrial clothing. *Advances in Social Science, Education and Humanities Research*, *11*(8), 211–226. <https://doi.org/10.2991/assehr.k.211223.214>
- Suominen, A. L. (2015). *Abstract algebra and secondary school mathematics: Identifying and classifying mathematical connections* (Doctoral dissertation, University of Georgia). University of Georgia Libraries. [https://getd.libs.uga.edu/pdfs/suominen\\_ashley\\_1\\_201505\\_phd.pdf](https://getd.libs.uga.edu/pdfs/suominen_ashley_1_201505_phd.pdf)
- Suzuka, K., Laurie, S., Loewenberg, B., Hyman, B., Jennifer, M., & Thames, H. (2009). Designing and using tasks to teach mathematical knowledge for teaching. In D. S. Mewborn & H. S. Lee (Eds.), *Scholarly practices and inquiry in the preparation of mathematics teachers* (pp. 7–23). Association of Mathematics Teacher Educators. <https://www.amazon.com/Scholarly-Practices-Preparation->

Mathematics-Monograph/dp/1623969514

Taherdoost, H. (2018). Validity and reliability of the research instrument; How to test the validation of a questionnaire/survey in a research. *SSRN Electronic Journal*, 5(3), 28–36. <https://doi.org/10.2139/ssrn.3205040>

Tall, D. (2013). *How humans learn to think mathematically: Exploring the three worlds of mathematics*. Cambridge University Press.

<https://doi.org/https://doi.org/10.1017/CBO9781139565202>

Tashakkori, A., & Teddlie, C. (2010). *SAGE handbook of mixed methods in social & behavioral research* (2nd ed.). SAGE Publications. <https://us.sagepub.com/en-us/nam/sage-handbook-of-mixed-methods-in-social-behavioral-research/book233405>

Tatto, M. T. (2013). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries* [Technical report]. In *The Teacher Education and Development Study in Mathematics (TEDS-M)* (Issue January).

<http://proxy.library.vcu.edu/login?url=http://search.proquest.com/docview/1651827260?accountid=14780>

Tatto, M. T., Ingvarson, L., Schwille, J., Peck, R., Senk, S. L., & Rowley, G. (2009).

Overview of teacher education systems around the world. In R. Even & D. L. Ball (Eds.), *The 15th ICMI study (pp. 1–26), the professional education and development of mathematics teachers*. Springer.

<http://eric.ed.gov/?id=ED542390>

Tatto, M. T., Lerman, S., & Novotna, J. (2010). The organization of teachers' mathematics preparation and development: A report from the ICMI Study 15.

*Journal of Mathematics Teacher Education*, 13(4), 313–324.

<https://doi.org/10.1007/s10857-009-9139-7>

- Tatto, M. T. (2018). The mathematical education of secondary teachers. In M. T. Tatto, M. C. Rodríguez, W. M. Smith, M. D. Reckase, & K. Bankov (Eds.), *Exploring the mathematics education of teachers using TEDS-M data* (pp. 409–450). Springer. [https://doi.org/10.1007/978-3-319-92144-0\\_14](https://doi.org/10.1007/978-3-319-92144-0_14)
- Tchoshanov, M. A. (2011). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics*, 76(2), 141–164. <https://doi.org/10.1007/s10649-010-9269-y>
- Thames, M. H., Sleep, L., Bass, H., & Ball, D. L. (2008). Mathematical knowledge for teaching (K-8): Empirical, theoretical, and practical foundations. *Icme 11, Tsg 27*.
- Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15(2), 105–127.
- Tribhuvan University, Faculty of Education.(2023). *The objectives*. <https://tufoe.edu.np/index.php/about-foe/introduction>  
<https://link.springer.com/article/10.1007/BF00305892>
- Tribhuvan University, Faculty of Education. (2021). *TU-Syllabus: Bed II year* (Vol. 4, Issue 1). <https://sukuna.edu.np/syllabus/>
- Tyler, R. W. (2013). Basic principles of curriculum and instruction. In *Curriculum Studies Reader E2* (pp. 60–68). <https://www.taylorfrancis.com/chapters/edit/10.4324/9780203017609-14/basic-principles-curriculum-instruction-ralph-tyler>
- Valliant, R., Dever, J. A., & Kreuter, F. (2018). *Practical tools for designing and weighting survey samples* (2nd ed.). Springer.

[https://books.google.com.np/books?id=0R0\\_AAAAQBAJ&printsec=frontcover&redir\\_esc=y#v=onepage&q&f=false](https://books.google.com.np/books?id=0R0_AAAAQBAJ&printsec=frontcover&redir_esc=y#v=onepage&q&f=false)

Veith, J. M., Beste, M. L., Kindervater, M., Krause, M., Straulino, M., Greinert, F., & Bitzenbauer, P. (2023). Mathematics education research on algebra over the last two decades: quo vadis? *Frontiers in Education*, 8.

<https://doi.org/10.3389/feduc.2023.1211920>

Ventistas, G., Ventista, O. M., & Tsani, P. (2024). The impact of realistic mathematics education on secondary school students' problem-solving skills: A comparative evaluation study. *Research in Mathematics Education*, 2024, 1–25.

<https://doi.org/10.1080/14794802.2024.2306633>

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds. & Trans.). Harvard University Press.

<https://home.fau.edu/musgrove/web/vygotsky1978.pdf>

Wasserman, N. H. (2018a). *Connecting abstract algebra to secondary mathematics, for secondary mathematics teachers*. <https://doi.org/10.1007/978-3-319-99214-3>

Wasserman, N. H. (2018b). Exploring advanced mathematics courses and content for secondary mathematics teachers. In N. H. Wasserman (Ed.), *Connecting abstract algebra to secondary mathematics, for secondary mathematics teachers* (pp. 1–15). Springer Nature Switzerland AG. <https://doi.org/10.1007/978-3-319-99214-3>

Wasserman, N. H., Buchbinder, O., & Buchholtz, N. (2023). Making university mathematics matter for secondary teacher preparation. *ZDM - Mathematics Education*, 55(4), 719–736. <https://doi.org/10.1007/s11858-023-01484-5>

Wasserman, N. H., Fukawa-Connelly, T., Villanueva, M., Mejia-Ramos, J. P., &

- Weber, K. (2017). Making real analysis relevant to secondary teachers: Building up from and stepping down to practice.  
*Http://Dx.Doi.Org/10.1080/10511970.2016.1225874*, 27(6), 559–578.  
<https://doi.org/10.1080/10511970.2016.1225874>
- Wasserman, N. H., & Stockton, J. (2013). Horizon content knowledge in the work of teaching: A focus on planning. *For the Learning of Mathematics*, 33(3), 20–22.  
[https://digitalcommons.sacredheart.edu/math\\_fac](https://digitalcommons.sacredheart.edu/math_fac)
- Wasserman, N. H., & Weber, K. (2017a). Pedagogical applications from real analysis for secondary mathematics teachers. *For the Learning of Mathematics*, 37(3), 14–18. <https://www.jstor.org/stable/26548463>
- Wasserman, N. H., & Weber, K. (2017b). *Pedagogical applications from real analysis for secondary mathematics teachers*. FLM Publishing Association.  
<https://www.jstor.org/stable/26548463>
- Wasserman, N., Villanueva, M., Mejía-Román, J. P., & Weber, K. (2015). Secondary mathematics teachers' perception of real analysis in relation to their teaching practice. *Annual Conference on Research in Undergraduate Mathematics Education (RUME)*, 1052–1055.
- Weber, K., Mejía-Ramos, J. P., Fukawa-Connelly, T., & Wasserman, N. (2020). Connecting the learning of advanced mathematics with the teaching of secondary mathematics: Inverse functions, domain restrictions, and the arcsine function. *Journal of Mathematical Behavior*, 57.  
<https://doi.org/10.1016/j.jmathb.2019.100752>
- Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (2016). Assessing mathematical argumentation through automated conversation. *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the*

*Psychology of Mathematics Education*, 1471–1478.

Wu, H. (2011). The mis-education of mathematics teachers. *Notices of the American Mathematical Society*, 58(3), 372–384.

<https://www.ams.org/notices/201103/rtx110300372p.pdf>

Yan, X., Marmur, O., & Zazkis, R. (2022). Advanced mathematics for secondary school teachers: Mathematicians' perspective. *International Journal of Science and Mathematics Education*, 20(3), 553–573. <https://doi.org/10.1007/S10763-020-10146-X>

Yang, X., & Kaiser, G. (2022). The impact of mathematics teachers' professional competence on instructional quality and students' mathematics learning outcomes. *Current Opinion in Behavioral Sciences*, 48, 101225.

<https://doi.org/10.1016/j.cobeha.2022.101225>

Yusoff, M. S. B. (2019). ABC of content validation and content validity index calculation. *Education in Medicine Journal*, 11(2), 49–54.

<https://doi.org/10.21315/eimj2019.11.2.6>

Zazkis, R., & Leikin, R. (2010). Advanced mathematical knowledge in teaching practice: Perceptions of secondary mathematics teachers. *Mathematical Thinking and Learning*, 12(4), 263–281. <https://doi.org/10.1080/10986061003786349>

## Appendixes

### Appendix A: Attitude and Efficacy Scale for Mathematics Teachers

#### Respected Teachers

Here are some statements intended to measure teachers' attitudes toward the relevance of mathematical knowledge of Real Analysis for secondary school mathematics instruction. Please express your rate in the given statement.

Your involvement in this study is optional, and you can decline participation. The procedure entails completing a survey that is expected to take approximately 20 to 30 minutes of your time. The information in your survey responses will be treated with confidentiality (Participants' responses will be kept secret to preserve privacy). As a result, the survey excludes questionnaires containing your name and other personal details.

Researcher

Deb Bahadur Chhetri

PhD Scholar

Tribhuvan University, Nepal

#### Section A

Please express your socio-demographic status in the following questions.

1. Your Gender:             Male         Female         Transgender
2. Your Age        .....
3. Academic qualification:    Bachelor    Master    MPhil/ PhD
4. Your ethnicity:  
 Bramin/Chhetri    Janajati    Dalit    Madheshi    Other  
Specify of any \_\_\_\_\_
5. Working experience in year \_\_\_\_\_
6. Educational Background    Education    Humanities    Science
7. Working District \_\_\_\_\_



## Section B

The following statements are related to the content connection between mathematical knowledge of Real analysis and the content of school mathematics. You are requested to rate the following statements.

**Table A1**

*Content connection Measurement Scale*

<i>To what extent do you agree or disagree with the content connection between mathematical knowledge of Real Analysis and the content of school mathematics?</i>		<i>Domain</i>
<i>SD= Strongly disagree, D= Disagree, N= Neutral, A= agree, SA = Strongly agree</i>		
s.n.	Statements	SD D N A SA
<b>Items Related to Content Connection</b>		
	यो भागमा <b>Real Analysis</b> का विषयवस्तुहरु विद्यालयको (कक्षा ९ र १०) अनिवार्य गणित र अतिरिक्त गणितमा प्रत्यक्ष वा फरक स्वरुपबाट के कस्तो सम्बन्ध छ भन्ने कुरामा आधारित कथनहरु छन । राम्रो सँग अध्ययन गरि अभिमत दिनुहोस् । अभिमत राख्दा ✓ चिन्ह लगाउनुहोस्	
1	The closure property of real numbers (If $b \in \mathbf{R}$ then, $a + b \in \mathbf{R}$ & $a \times b \in \mathbf{R}$ ) is strongly connected with contents of secondary- level mathematics.	CONTN1
2	The associative property of real number [ <b>If <math>a, b, c \in \mathbf{R}</math> then, <math>(a + b) + c = (a + (b + C)</math> and <math>a \times (b \times c) = (a \times b) \times c</math></b> ] is strongly connected with content of secondary-level mathematics.	CONTN1
3	Absolute value of real numbers [E.g., cases: $ a  +  b  \geq  a + b $ , $ a - b  \geq  a  -  b  \forall a, b \in \mathbf{R}$ ] is strongly connected with secondary-level mathematics.	Deleted
4	The commutative property of real numbers [ <b><math>\forall a, b \in \mathbf{R}</math> then <math>a + b = b + a</math>, and <math>a \times b = b \times a</math></b> ] strongly connected with school mathematics contents	CONTN1
5	The cancellation law (addition and multiplication) is strongly connected with content of school mathematics.	CONTN1
6	The mathematical concepts of function (especially definition, example, numerical problem) found in real analysis are strongly connected with content of secondary-level mathematics.	CONTN3
7	Real sequence studied in real analysis is strongly connected with content of secondary level mathematics.	CONTN2
8	The abstract form of sequence studied in real analysis can be found in a specific form (e.g., sequence in number, diagrammatic form) at secondary-level.	CONTN2
9	The contents of an infinite series of real analyses are connected with secondary-level mathematical content.	CONTN2
10	The concept of convergent of series is practiced school mathematics.	CONTN2
11	Limit of a function is expressed in numerical and diagrammatic form at secondary-level mathematics	CONTN3
12	The topic continuity included in real analysis connected with secondary-level mathematics content.	CONTN3
13	The concept of derivative $\frac{dy}{dx}$ practiced in real analysis is studied in slope of line in different form (e.g. slope of line m) in secondary-level mathematics.	Deleted
14	The function is interpreted in graphical form at secondary-level mathematics is related to the concept of function studied in real analysis.	CONTN3
15	The concept of interval (open interval, closed interval, semi-closed interval) discussed in real analysis is connected with school mathematics content in verbal form and inequalities.	CONTN3

Note: CONTN1 = Algebraic Properties of real number, CONTN2 = Real Sequence and Series, CONTN3 = Calculus of real number

## Section C

**Table A2***Pedagogical Application Measurement Scale*

	SD	D	N	A	SA	
Application of Real Analysis in Teaching School Mathematics						
<i>SD= Strongly disagree, D= Disagree, N= Neutral, A= agree, SA = Strongly agree</i>						
यो भागमा <b>Real Analysis</b> का विषयवस्तुहरूलाई यसमा समावेश तर्कहरूलाई माध्यमिक तहको (कक्षा ९ र १०) अनिवार्य गणित र अतिरिक्त गणित शिक्षण गर्दा प्रयोग हुन सक्ने अवस्थामा आधारित कथनहरू छन् । राम्रो सँग अध्ययन गरि आफ्नो अभिमत राखिदिनुहोला । अभिमत राख्दा ✓ चिन्ह लगाउनुहोस्						Domain
1	I use closure property of real number (If $b \in \mathbf{R}$ then, $a + b \in \mathbf{R}$ and $a \times b \in \mathbf{R}$ ) during mathematics instruction.					PAPLI1
2	I use associative property of real numbers [ <i>If <math>a, b, c \in \mathbf{R}</math> then, <math>(a + b) + c = (a + (b + c))</math> &amp; <math>a \times (b \times c) = (a \times b) \times c</math></i> ]during mathematics instruction					PAPLI1
3	I use commutative property of real number [ $\forall a, b \in \mathbf{R}$ then $a + b = b + a, a \times b = b \times a$ ] during mathematics instruction					PAPLI1
4	I use cancellation laws (for addition, multiplication) discussed in properties of real numbers in numerical calculating process for instruction.					PAPLI1
5	I use absolute value of real numbers [E.g. cases: $ a  +  b  \geq  a + b ,  a - b  \geq  a  -  b  \forall a, b \in \mathbf{R}$ ] during mathematics instruction.					Deleted
6	I use concepts of function (especially definition, example, numerical problem) found in real analysis for mathematics instruction.					PAPLI2
7	I use the concepts of sequence and series studied in real analysis for mathematics instruction					PAPLI2
8	I use knowledge of infinite series found in real analysis for secondary-level mathematics instruction.					PAPLI2
9	I use concepts of convergent and divergent found in real analysis for secondary-level mathematics instruction.					PAPLI2
10	Mathematical knowledge of real analysis helps to explore the idea of teaching new mathematical content in school.					PAPLI3
11	I use knowledge of limit and function studied in real analysis to teach school mathematics content like limit, function.					PAPLI3
12	I use the concept of derivative to teach the slope of line in school					PAPLI3
13	I use the concept of continuity and discontinuity found in analysis for teaching the concept of continuity at the school-level.					PAPLI3

Note: PAPLI1 = Pedagogical Application of properties of Real Number

PAPLI2 = Application of Function, Limit, and Continuity

PAPLI3 = Application of Series and proof ideas

## Section D

Table A3

*Mathematical Proficiency Attitude Measurement Scale*

<b>Enhancing Mathematical proficiency</b>		SD	D	N	A	SA	Domain
<i>To what extent do you agree or disagree on the relevance of knowledge of real analysis for enhancing teachers' mathematical proficiency.</i>							
<i>(SD= Strongly disagree, D= Disagree, N= Neutral, A= agree, SA = Strongly agree)</i>							
Real analysis को ज्ञानले पेशागत विकासमा प्रत्यक्ष वा अप्रत्यक्ष रूपमा पुन्यउन सक्ने योगदानमा आधारित कथन समावेश छन् अध्ययन गरि आफ्नो अभिमत राखिदिनुहोला। अभिमत राख्दा ✓ चिन्ह लगाउनुहोस्							
1	Mathematical knowledge of Real analysis is fruitful to my professional life.						MPROF1
2	Mathematical knowledge of Real analysis is required to be a professional teacher.						MPROF1
3	Knowledge of real analysis helps to get fundamental knowledge of real number systems and their properties.						MPROF1
4	Mathematical knowledge of real analysis is a base for understanding the rules of numerical operations.						MPROF1
5	Mathematical knowledge of real analysis develops the skill of mathematical reasoning.						MPROF2
6	Mathematical knowledge of real analysis increases the confidence level in content knowledge.						MPROF2
7	Mathematical knowledge of real analysis helps to understand the mathematical structure of real numbers.						MPROF2
8	Mathematical knowledge of real analysis is the root of axioms in number systems applied in school mathematics.						MPROF2
9	Mathematical knowledge of real analysis helps to develop mathematical fluency.						MPROF2
10	Mathematical knowledge is essential to develop the ability which enables us to create ideas to solve mathematical problems during school mathematics instruction.						MPROF2
11	Mathematical knowledge of real analysis is essential to developing mathematical language capacity which enables us to understand new topics in mathematics.						MPROF2
12	Knowledge of real analysis in mathematics enables our mind to understand any new mathematical problem and its basic nature.						MPROF2
13	Mathematical knowledge of real analysis enables us to tackle mathematical problems by improving our mathematical intelligence.						MPROF3
14	Real analysis helps to develop an abstract ability in our brain whereby our brain can suddenly recognize the mathematical relation and solve problems when we see it.						MPROF3
15	Real analysis helps to develop an abstract ability in our brain whereby we recognize the mathematical relationships in a given problem and solve it.						MPROF3
16	Real analysis enables our brains to create mathematical problems and techniques for solving the problems that are needed during instruction.						MPROF3
17	Real analysis enables us to develop techniques of deductive proof of mathematical theorems that are needed in instruction of school mathematics.						MPROF3

## Section E

**Table A4**  
*Mathematical Efficacy Measurement Scale*

### Mathematical Efficacy for Teaching

<b>How much do you think you are able to: (rate 1 to 5 where: minimum 1 maximum 5)</b>		<b>Domain</b>
तपाइले तलका सक्षमतामा आफुलाई १ देखि ५ सम्मको अंक प्रदान गर्न सक्नु हुन्छ		
1	create a definition and example of the mathematical concept.	D1
2	create mathematical problems to teach and evaluate students.	D1
3	explain the mathematical procedure and its reasons during instruction.	D2
4	give local examples of mathematical content to convince the student.	D3
5	select the appropriate process (algorithm) to solve the mathematical problem.	D1
6	expand and show the axiomatic relation among the structure of mathematical knowledge.	D2
7	create examples and counterexamples of the mathematical concept.	D2
8	convince students by providing multiple methods to solve a single problem.	D3
9	link one concept of mathematics to another concept.	D3
10	give reasons for each mathematical process/rule used in schools' mathematics instruction	D1

Note: D1 = Mathematical Creation for instruction

D2= Preparing Algorithm

D3= Content bridging and Persuasion

## Appendix B: Mathematical Connection Seeking Framework

Required Sources of observing content connection

1. \_\_\_\_\_ 2. \_\_\_\_\_ 3. \_\_\_\_\_

**Table B**

### *Mathematical Connection Seeking Framework*

<b>Categories/perspectives</b>	<b>Concept of mathematical knowledge of Real analysis</b>	<b>Content of Secondary-level Mathematics</b>
Content Connection	Real number system Open and Closed Sets Real Sequence Infinite Series Functions and Limit Continuity of a function Derivability Riemann Integral	
Axiomatic connection	Real number system Open and Closed Sets Real Sequence Infinite Series Functions and Limit Continuity of a function Derivability Riemann Integral	
Generalization	Real number system Open and Closed Sets Real Sequence Infinite Series Functions and Limit Continuity of a function Derivability Riemann Integral	
Procedural connection	Real number system Open and Closed Sets Real Sequence Infinite Series Functions and Limit Continuity of a function Derivability Riemann Integral	
Alternative representation	Real number system Open and Closed Sets Real Sequence Infinite Series Functions and Limit Continuity of a function Derivability Riemann Integral	

### Appendix C: Classroom Observation Protocol

Observation no. \_\_\_\_\_ Date of Observation \_\_\_\_\_

Name of School \_\_\_\_\_ Address \_\_\_\_\_

Teacher's Name \_\_\_\_\_ Class \_\_\_\_\_

Teaching Topic \_\_\_\_\_ Observation time \_\_\_\_\_

#### Table C

##### *Classroom Observation Protocol*

Possible category related to knowledge of real analysis	Is the teacher using these categories during instruction (Yes/No)	Particular practice (record)
Definition		
Example		
Algorithm		
Using properties of real number		
Using concept of closed set and open set		
Using concept/content/example/logic of sequence		
Using concept/content/example/logic of series		
Using concept/content/example/logic of function.		
Using concept/content/example/logic of limit		
Using concept/content/example/logic of continuity		
Using concept/content/example/logic of derivability		
Using concept/content/example/logic of Riemann integral		

#### Particular events that are used by teacher (why he/she

## Appendix D: Task-Based Interview Protocol

Dear Teachers

Greeting

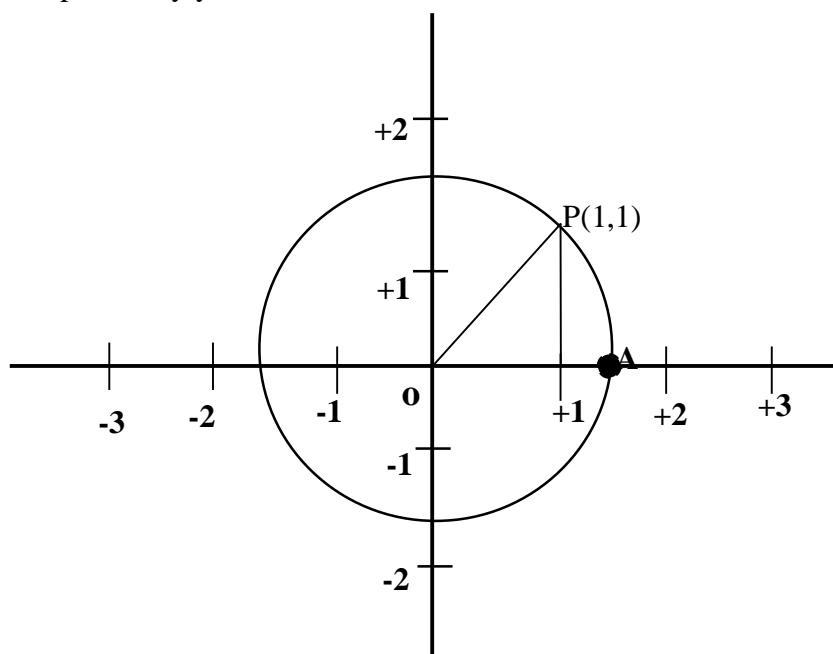
My name is Deb Bahadur Chhetri. I am a PhD student at the Graduate School of Education. I am researching the relevance of mathematical knowledge in real analysis for secondary-level mathematics instruction. I kindly invite you to participate in this task-based interview voluntarily. Your insights will be valuable for this study. This task-based interview is designed for a context of real analysis content. Please read the context and write your practices within it.

1. Here is a graphic showing the number line. Observe the character, choose the solution, and write your experiences in the questions.

a) What is the coordinate of A, select from the given options?

- i)  $(2,0)$
- ii)  $(\sqrt{2}, 0)$
- iii)  $(1.5,0)$

b) Can you explain why your choice with reasons



2. If  $x, y$  be any two real number then we can write  $x + y = y + x$ ,  $xy = yx$  in numerical

$$2 + 3 = 3 + 2, \quad 2 \times 3 = 3 \times 2 \text{ in our practices}$$

- a) Select which is your answer i This writing is accurate ii) this writing is false
- b) Can you explain the reasons for this writing?
- c) If you have reasons, what are its sources
- d) What could have been done in your real analysis course to help you incorporate this connection into your teaching?
3. The series  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$  can be written in the n-term  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ , In the series  $\sum \frac{1}{n^p}, p > 1$  then it is convergent ( has fixed value). The solution of given series  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$  is convergent
- a) Can you explain the solution of this series?
- b) Do you use this type of case in teaching school mathematics?
- c) Do you use sequence and series and their concepts in school mathematics teaching?
4. In a set  $S = \left\{ \frac{1}{n} : n \in \mathbf{N} \right\} = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \dots \right\}$ , the supremum is 1 and infimum is 0.
- a) If your students ask you what the largest value of this set will be, how do you answer them? Can you explain your solution?
- b) Do you use this case in teaching school mathematics?
- 5) suppose that your students question to you that  $1+2 = 3, 1+2+3 = 6$ , how I can find  $1+2 + \dots + 100$ .
- a) What will be your answer? How do you show it?
- b) Do you use this type of case in school mathematics instruction? Can you give an example?
- 6) A set  $A$  of  $x \in \mathbf{R}$  such that  $|2x + 3| < 7$ .
- a) Can you explain, how you suggest your students write the members of these sets?
- b) Do you use these types of cases during school mathematics instruction?

Based on your experience and task, let us discuss the following questions.

1. Could you please give me examples of pedagogical applications of real analysis in secondary-level mathematics instruction?
  - a) Could you please give me an example or scenario of its application?
  - b) Can you give me specific examples of how real analysis is used in the classroom?
  - c) What are some benefits of using real analysis in the classroom?
2. Could you please give your view on what knowledge we get from real analysis that is useful in our professional lives?
3. Can we generalize the mathematical knowledge of real analysis in our instructional practices?
4. Could you please mention the relevance of knowledge of Real analysis for enhancing our professionalism?
5. Could you please mention the ideas that help make the connection between mathematical knowledge of real analysis and secondary-level mathematics?

## Appendix E: Interview Guidelines (for Mathematics Educators)

Dear Sir/Madam,

Greetings

My name is Deb Bahadur Chhetri. I am a PhD student at the Graduate School of Education. I am researching the relevance of mathematical knowledge in real analysis for secondary-level mathematics instruction.

I kindly invite you to participate in this interview voluntarily. Your insights will be valuable for this study.

I appreciate your time and participation in this interview. Below are some fundamental questions regarding real analysis and its connection to secondary-level mathematics. Your insights will be valuable in exploring these topics. Rest assured that your responses will remain confidential. I look forward to discussing each question in detail.

Best regards,

Researcher

DEB Bahadur Chhetri

PhD Students, GSE, TU

### A. (Introduction)

Please express your socio-demographic status in the following questions.

1. Your Gender:             Male         Female         Transgender
2. Your Age group:         20-29         30-39         40-49     50  
and above.
3. Working experience in year \_\_\_\_\_
4. Faculty/ institute Studied  Education  Humanities  Science

**B. Warmup Questions**

1. How did you become interested in teaching mathematics, and what motivated you to pursue real analysis?
2. Can you briefly describe your teaching experience with real analysis, including the grade levels or courses you have taught?

**C. Relevance of Mathematical Knowledge**

1. Could you explain why we teach real analysis in the teacher preparation program?
2. Based on your experience, could you give suggestions on how to establish the link between mathematical knowledge of real analysis and mathematics at the secondary level?
3. Is a connection between mathematical knowledge of real analysis and secondary-level mathematics required?
4. Could you give suggestions/ ideas on connecting the mathematical knowledge of real analysis with secondary-level mathematics?

Thank You for your Contribution

**Appendix F: Model Answer of Task Based Interview**

Here is a graphic showing the number line. Observe the character, choose the solution, and write your experiences in the questions.

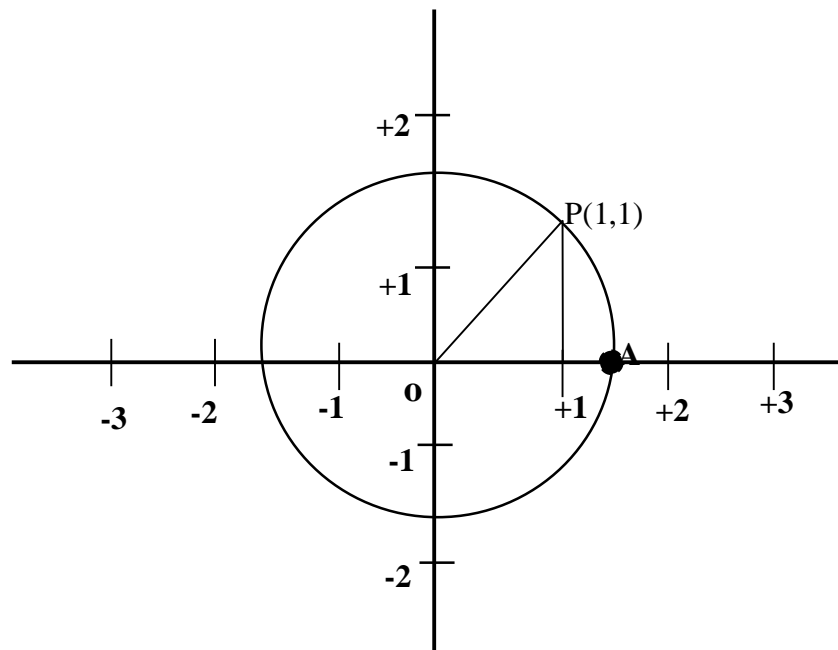
c) What is the coordinate of A, select from the given options?

iv) (2,0)

v)  $(\sqrt{2}, 0)$

vi) (1.5,0)

d) Can you explain why your choice with reasons



$$P = (1, 1)$$
$$b = 1$$
$$OP = \sqrt{1^2 + 1^2} = \sqrt{2}$$

a) Select which is your answer

i) This writing is true

ii) this writing is false

b) Can you explain the reasons of this writing

Commutative property of  $\mathbb{R}$  has been applied for this reason.

c) If you have reasons what is its sources

This fact is found in textbook but its true explanation is in analysis and algebra studied in higher level.

d) What could have been done in your real analysis course to help you incorporate this connection into your teaching?

We can connect the theoretical knowledge to the practical practices doing at school level.

3. The series  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$  can be written in the n-term  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ . In the series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$ ,  $p > 1$  then it is convergent (has fixed value). The solution of given series  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$  is convergent

a) Can you explain the solution of this series

It is related to p-Test. In which when  $p > 1$ , and  $n \rightarrow \infty$  then the given series converges to zero. In practical, if denominator increased keeping numerator constant then it converges to zero.

b) Do you use this type of case in school mathematic teaching?

Even direct connection is not found in school level but this concept can be seen in limit, Continuity, Sequence and Series.

c) Do you use the sequence and series and its concept in school mathematics teaching?

Yes, which is explained in (b).

4. In a set  $S = \{\frac{1}{n} : n \in \mathbb{N}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$ , the supremum is 1 and infimum is 0.

a) If your students ask to you what will be the largest value of this set then how do you give answer to your students? Can you explain with solution?

In a sequence, if the denominator increases infinitely, then it tends to 0. So for the given set the max value is 1 and the minimum value is 0.

b) Do you use this case in school mathematics teaching?

Yes. In different units of algebra like, limit & continuity, sequence and series this concept has been used.

5) suppose that your students question to you that  $1+2=3$ ,  $1+2+3=6$ , how I can find

$1+2+\dots+100$ .

a) What will be your answer? How do you show it?

Two alternative ways can be used.

i. By formula.  $S_n = \frac{n}{2} [2a + (n-1)d] = \frac{n(n+1)}{2}$

ii. Induction teaching technique.

c) Do you use this types of case in school mathematics instruction, can you

give example. This type of concept is using for teaching sum of ~~Sequence~~ <sup>sets</sup> of natural numbers, series.

6) A set  $A$  of  $x \in \mathbb{R}$  such that  $|2x+3| < 7$ .

a) Can you explain, how do you suggest your students to write the member of this sets?

Given that  $|2x+3| < 7$

$$\Rightarrow -7 < (2x+3) < 7$$

$$\Rightarrow -7 - 3 < 2x < 7 - 3$$

$$\Rightarrow -10 < 2x < 4$$

$$\Rightarrow -5 < x < 2$$

The members of this set lies between -5 to 2.

b) Do you use these types of cases during school mathematics instruction?

This type of problems are not directly practicing but such type of concept are highly applied, specially in graph.

1. Could you please give me pedagogical applications of real analysis in secondary-level-mathematics instruction?

Properties of real number, its axioms are used for teaching mathematics at school level.

a) could you please give me an example or scenario of its application?

for solving equation. e.g.  $2x + 3 = 5$   
 $\Rightarrow 2x + 3 - 3 = 5 - 3$  [Additive inverse]  
 $\Rightarrow \frac{2x}{2} = \frac{2}{2}$  [multiplicative inverse]  
 $\Rightarrow x = 1$

b) Can you give me some specific examples of how real analysis is used in the classroom?

As expressed in (a).

c) What are some of the benefits of using real analysis in the classroom?

In depth understanding of logic, mathematical process and operation are really practical connect with real analysis in the classroom.

2. Could you please give your view on what types of knowledge we get from real analysis that is useful in our professional life?

Theoretical knowledge, skills, that are applying in school mathematics.

4. Can we generalize the mathematical knowledge of real analysis in our instructional practices?

Yes, generalization practice exists in school but its form is simple and different than in analysis.

## Appendix G: Reliability of Survey Tools

Scale	Domains	Cronbach's Alpha	Range of adequacy	Interpretation
Relevance of Mathematical Knowledge	Content Connection	0.826	0.70-0.95	Reliability is adequate
	Pedagogical Application	0.845		
	Enhancing Proficiency	0.892		
	Total item	0.943		
Mathematics teaching efficacy	Creation	0.740	0.70-0.95	Reliability is adequate
	Procedural fluency	0.792		
	Reasoning	0.710		
	Total item	0.897		

Total Item Statistics of Relevance Measurement Scale								
Item	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted	Item	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted	Item	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
1	0.619	0.941	16	0.373	0.943	31	0.510	0.942
2	0.480	0.942	17	0.326	0.943	32	0.559	0.942
3	0.307	0.943	18	0.653	0.941	33	0.500	0.942
4	0.674	0.941	19	0.427	0.942	34	0.522	0.942
5	0.354	0.943	20	0.597	0.941	35	0.632	0.942
6	0.565	0.942	21	0.564	0.942	36	0.512	0.942
7	0.593	0.941	22	0.502	0.942	37	0.538	0.942
8	0.379	0.943	23	0.645	0.941	38	0.504	0.942
9	0.640	0.941	24	0.504	0.942	39	0.414	0.943
10	0.354	0.943	25	0.488	0.942	40	0.605	0.942
11	0.407	0.943	26	0.670	0.941	41	0.556	0.942
12	0.440	0.942	27	0.399	0.943	42	0.623	0.941
13	0.608	0.941	28	0.424	0.943	43	0.633	0.941
14	0.388	0.943	29	0.500	0.942	44	0.568	0.942
15	0.402	0.943	30	0.508	0.942	45	0.467	0.942

## Total Item Statistics Teaching Efficacy Measurement Scale

Item	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
1	0.764	0.878
2	0.733	0.881
3	0.521	0.894
4	0.658	0.886
5	0.679	0.885
6	0.584	0.891
7	0.543	0.893
8	0.695	0.884
9	0.601	0.890
10	0.676	0.885

































## Appendix H Construct Validity Index

Attitude measurement Scale				Efficacy measurement scale	
Item	r	items	r	Items	r
1	.645 <sup>1</sup>	25	.518**	1	.822**
2	.532**	26	.657**	2	.790**
3	.369*	27	.484**	3	.599**
4	.692**	28	.464**	4	.730**
5	.374*	29	.520**	5	.760**
6	.607**	30	.538**	6	.678**
7	.617**	31	.520**	7	.632**
8	.381*	32	.575**	8	.775**
9	.664**	33	.541**	9	.671**
10	0.357	34	.560**	10	.748**
11	.424*	35	.640**		
12	.421*	36	.568**		
13	.610**	37	.574**		
14	.428*	38	.537**		
15	.427*	39	.439*		
16	.415*	40	.636**		
17	.370*	41	.560**		
18	.667**	42	.653**		
19	.468**	43	.654**		
20	.636**	44	.583**		
21	.567**	45	.484**		
22	.544**				
23	.652**				
24	.571**				

\* p < 0.05

\*\*p < 0.01

## Appendix I: Summary of Category and Theme

	Count	% Codes	Cases	% Cases
 Content Connection				
 Sequence and Series	14	5.5%	8	72.7%
 Function	6	2.4%	5	45.5%
 Continuity	6	2.4%	4	36.4%
 Limit	11	4.3%	7	63.6%
 Logical connection				
 Field Axioms of R	11	4.3%	6	54.5%
 idea and logic	13	5.1%	7	63.6%
 Procedural connection				
 Procedure	3	1.2%	3	27.3%
 Pedagogical application				
 Idea creation	2	0.8%	2	18.2%
 Mathematical activities	5	2.0%	3	27.3%
 Reasoning in mathematical context	7	2.8%	4	36.4%
 Professional development				
 Identifying mathematical relation	17	6.7%	10	90.9%
 Advancement of knowledge and skill	4	1.6%	4	36.4%
 Logical reasoning ability	4	1.6%	3	27.3%
 Linking technique				
 content bridging strategy	8	3.2%	7	63.6%
 vertical link established practicaly	13	5.1%	5	45.5%
 Why link is required				
 Deeper Understanding	5	2.0%	3	27.3%
 Enhencing critical thinking	2	0.8%	2	18.2%
 Motivating	4	1.6%	2	18.2%
 Promoting sequential learning	11	4.3%	7	63.6%
 Real Analysis knowlede(Why)				
 Advancement of knowledge	6	2.4%	5	45.5%
 Maturity in Mathematical Thinking	27	10.7%	8	72.7%
 Expertise	10	4.0%	5	45.5%
 Mathematical Skill Development	4	1.6%	3	27.3%
 Being confidence	11	4.3%	6	54.5%

**Appendix J: Consent Form****Title of Research** Relevance of Knowledge of Real Analysis in Mathematics

Instruction at Secondary-Level

Dear Teacher,

Namaste!

I am conducting a research study to explore the relevance of Knowledge of real analysis in teaching secondary-level mathematics, as part of my PhD study at Tribhuvan University. Since you are an experienced secondary-level mathematics teacher, your valuable insight, experiences, and opinions are crucial for the success of this study.

Therefore, I kindly request your voluntary participation in this research. Your honest responses will contribute to improving the mathematics teacher preparation programs and help to establish a strong connection between advanced mathematics and school mathematics teaching.

I assure you that all the information you share will be kept strictly confidential and used only for academic purposes. Your cooperation will be highly appreciated.

**Researcher**

Deb Bahadur Chhetri

PhD Scholar, Mathematics Education

Graduate School of Education, TU, Nepal

**Supervisor**

Assoc. prof. Bishnu Khanal, PhD

**Purpose of the Study**

This research aims to explore and analyze the relevance of Knowledge of real analysis in teaching secondary-level mathematics. The study focuses on teachers'

attitudes, perceptions, and classroom practices regarding the application of Knowledge of real analysis.

### **Participation**

You are invited to participate in this research because you are a secondary-level mathematics teacher. Your participation will involve responding to a questionnaire and/or participating in interviews, classroom observations, or task-based interviews.

### **Confidentiality**

All information you provide will be kept strictly confidential. Your name, school name, or any identifying details will not be mentioned in the research report. The data will be used only for academic purposes and stored securely.

### **Voluntary Participation**

Your participation is completely voluntary. You may choose not to participate or withdraw from the study at any time without any penalty or negative consequences.

### **Risks and Benefits**

There are no known risks associated with participating in this study. The study may benefit you and other teachers by providing insights into the importance and practical application of advanced mathematical knowledge like Real Analysis in secondary-level mathematics teaching.

### **Contact Information**

If you have any questions about the research, you may contact:

Deb Bahadur Chhetri

Email: devchhetri010@gmail.com

Phone:

**Consent Statement:**

I have read and understood the information provided above. I understand that my participation is voluntary and that I may withdraw at any time. I consent to participate in this research and allow the researcher to use the information I provide for academic purposes.

Participant's Full Name .....

Signature .....

Date .....

Researcher's Name & Signature .....

Date of Consent Received .....

## Appendix K: Toolkit

This toolkit is designed to support secondary-level mathematics educators in teaching the foundational concept of limit to Class 9 students by applying selected principles from real analysis. It reflects a bridging strategy, recommended by this study, that connects the abstract definitions from university-level mathematics with intuitive teaching methods appropriate for school learners.

The toolkit incorporates story-based reasoning, graphical visualization, symbolic expression, and guided reflection, aligned with real analysis concepts as outlined in Pandey (2024). It illustrates how advanced mathematical knowledge can be transformed into practical instructional strategies.

### Link to Real Analysis Definition

In *The Real Numbers and Real Analysis*, Pandey (2024) defines the limit of a sequence as:

“A sequence  $(a_n)$  has a limit  $L$  if for every  $\varepsilon > 0$ , there exists a natural number  $N$  such that for all  $n \geq N$ ,  $|a_n - L| < \varepsilon$ .” (p. 2)

While this formal  $\varepsilon$ - $N$  definition is rigorous, it is inaccessible to Class 9 students.

Therefore, this toolkit uses a contextual problem and inductive reasoning to model the behavior of converging sequences in an intuitive manner. This approach allows students to develop a conceptual foundation that can later support formal understanding.

### Learning Objectives

Upon completion of this lesson sequence, students will be able to:

1. Understand the idea of approaching a value without necessarily reaching it.
2. Construct tables and graphs to observe converging sequences.

3. Use verbal and symbolic representations to describe limit behavior.
4. Reflect on mathematical processes involving infinity and approximation.

### Overview of Toolkit Modules

Module	Title	Description
1	Journey Toward a Limit	Introduces the concept of limit using a story about a man walking 100 km by halving the remaining distance each day.
2	Numerical Table and Graph	Students construct a table of distances and plot the cumulative sum to visualize convergence towards 100 km.
3	Symbolic Formulation	Introduces limit notation: $\lim_{n \rightarrow \infty} S_n = 100$ , where $S_n = \sum_{k=1}^n \frac{100}{2^k}$ .
4	Computational Bridging	Classroom discussions often introduce real analysis concepts, such as convergence and boundedness.
5	Reflection and Application	Students reflect on the mathematical meaning of "approaching a value" and apply the concept in new contexts.

#### Module 1: Journey Toward a Limit – The 100 km Traveler

Instructional Prompt (Story): A man plans to walk 100 km. On the first day, he walks 50 km. The next day, he walks half the remaining distance (25 km), and continues this pattern every day always walking half the remaining distance.

Ask students: “Will he ever reach 100 km?”

Purpose: To illustrate the concept of a limit through inductive reasoning and real-life analogy.

Expected Reasoning: Students will observe that the man never actually reaches 100 km but gets closer and closer. This intuitively introduces the concept of a converging sequence.

#### Module 2: Table and Graph Construction

Activity: Students fill in a table showing distance traveled each day and the total distance so far (partial sums):

Day	Distance Covered	Total Distance
-----	------------------	----------------

1	50 km	50 km
2	25 km	75 km
3	12.5 km	87.5 km
4	6.25 km	93.75 km
...	...	...

Students then plot these totals to visualize the approach toward 100 km.

### **Module 3: Symbolic Formulation**

Teacher Explanation: Introduce the limit symbolically using

Explain how this infinite process models the man's journey. This illustrates the idea of a limit of a converging geometric series, a core topic in real analysis.

### **Module 4: Conceptual Bridging to Real Analysis**

Discussion Prompts:

1. How is this different from finishing something exactly?
2. Why does it still make sense to say the total distance is 100 km?
3. How would mathematicians describe what's happening here?

Concepts Introduced (in simplified form):

1. Converging sequence
2. Infinite process
3. Boundedness
4. Limit as the value approached but never exactly reached

### **Module 5: Reflection and Extension**

Student Writing Prompt:

Write a short explanation of what a limit means using your own words and the 100 km example. What does it mean to say that a number is being approached?

Extension Ideas:

1. Explore similar problems using sequences like
2. Visualize function limits using graphing tools or GeoGebra
3. Challenge students to find a situation in real life where limits occur

#### Toolkit Materials

1. Story prompt sheet (“The 100 km Traveler”)
2. Student worksheet (table template, reflection questions)
3. Graph paper or plotting software
4. Teacher discussion guide
5. Simplified explanation of limit notation

#### Teacher Implementation Tips

1. Use storytelling to spark curiosity
2. Allow students to complete the table themselves before graphing
3. Connect the activity to real-world ideas like speed, distance, or finance
4. Avoid formal  $\epsilon$ - $N$  language; focus on conceptual clarity
5. Emphasize that limits describe what a value gets closer to, not necessarily what it reaches

#### Conclusion

This toolkit serves as a model for how concepts from real analysis like convergence and limits can be transformed into meaningful, age-appropriate classroom instruction.

It demonstrates a practical application of theoretical knowledge, aligning with this study’s recommendation to develop bridging frameworks that connect higher mathematics to school-level pedagogy.

#### References

Pandey U.N. (2024). *Real Analysis*. Vidharthi Publication.