

**CULTURAL METAPHOR FOR MATHEMATICAL
UNDERSTANDING IN NEPALESE CONTEXT**

Jaya Bishnu Pradhan

A Dissertation

Submitted to

Tribhuvan University, Faculty of Education

Kathmandu, Nepal

for the Degree of

Doctor of Philosophy in Mathematics Education

2019

LETTER OF RECOMMENDATION

This is to certify that Mr. Jaya Bishnu Pradhan, a student of academic year 2014-2019 with T.U. Registration Number 8915-90 has completed his Ph. D. dissertation under our supervision for the period prescribed by the rules and regulations of Tribhuvan University. The dissertation entitled **Cultural Metaphor for Mathematical Understanding in Nepalese Context** embodies the results of his research conducted during the period of 2014-2019 A. D. under the Faculty of Education, Tribhuvan University. We recommend and forward his dissertation to be submitted for evaluation and awarding the degree of Doctor of Philosophy in Mathematics Education.



(Prof. Hari Prasad Upadhyay, Ph. D)

Research Supervisor



(Prof. Bidya Nath Koirala, Ph. D.)

Research Co-Supervisor

Date: 19 April, 2019

ABSTRACT

An abstract of the dissertation of Jaya Bishnu Pradhan for the degree of
Doctor of Philosophy in Mathematics Education presented on 19 April, 2019.

Title: Cultural Metaphor for Mathematical Understanding in Nepalese Context

Abstract approved by:



 (Prof. Hari Prasad Upadhyay, Ph.D)

Research Supervisor



 (Prof. Bidya Nath Koirala, Ph.D)

Research Co-supervisor

The main intention of my study was to explore the mathematical knowledge and ideas embedded in everyday activities in out-of-school culture, and to assess the effectiveness of community knowledge in the process of teaching and learning of school mathematics. I adopted the multimethod qualitative research to answer the research questions based on the emerged themes as mathematical ideas and concepts observed in everyday activities of different groups of people facilitated to teaching and learning of abstract concepts. The combination of ethnography and action research methodology was adopted to carry out the research.

With the help of ethnography, the exploration of cultural metaphors regarding mathematics practice in out-of-school environment was studied using multiple methods such as participant observation, in-depth interview, focus group discussion, study of cultural artefacts, and observation of everyday activities of different group of

people. With the help of action research, I examined the contribution of cultural metaphors in the process of teaching abstract concepts of mathematics at school. The very fundamental concepts of ratio and proportion, estimation and approximation, measurement, algebraic concepts of like terms, unlike terms, algebraic expressions, and the equations with one and two unknown variables were identified in the household activities. Highly sophisticated mathematical ideas were also observed in the professional activities such as weaving of a mat, stone and wood carving, carpentry and metal works. Similarly, the analysis of different cultural artefacts also exhibited the mathematical ideas embedded in it. Both teachers and students indicated that the emic knowledge of the group of people in out-of-school community and the craft model approach helped them in knowledge generation and distribution of mathematical knowledge with ease.

The findings of my study showed that the cultural metaphor was a mediated tool that connects the students' out-of-school context to the school mathematics. The familiar cultural contexts of students were found to be the source for the teaching and learning of difficult and abstract concepts of mathematics. The students and participant teachers reported that they enjoyed teaching and learning through cultural metaphors and found it to be meaningful and interesting. Finally, from this study, I contend that cultural metaphors are the most prominent mediated tools for teaching and learning of mathematics. The findings of my study could be significant to the stakeholders of mathematics education to incorporate cultural metaphors embedded in out-of-school cultural activities in order to develop mathematical understanding.



 (Jaya Bishnu Pradhan)

Ph. D. Scholar

Copy Right

Jaya Bishnu Pradhan

2019

All Right Reserved

DECLARATION

I hereby declare that this Ph. D. dissertation entitled **Cultural Metaphor for Mathematical Understanding in Nepalese Context** submitted to the office of the Dean, Faculty of Education, Tribhuvan University is an entirely original work. I have made due acknowledgements to all ideas and information borrowed from different sources in the course of writing this dissertation. The results presented in this dissertation have not been presented or submitted anywhere else for the award of any degree or for any other reason. No part of the content of this dissertation has ever been published in any form before. I shall be solely responsible if any evidence is found against my dissertation.

I understand that my dissertation will become a part of permanent collection of Tribhuvan University Library. My signature below authorizes the release of my dissertation to the interested readers upon their request.



.....
(Jaya Bishnu Pradhan)

Ph. D. Scholar

19 April, 2019

ACCEPTANCE AND RECOMMENDATION

The undersigned certify that we have read, approved, and recommended to the Faculty of Education, Tribhuvan University for acceptance, a dissertation entitled CULTURAL METAPHOR FOR MATHEAMTICAL UNDERSTANDING IN NEPALESE CONTEXT submitted by Jaya Bishnu Pradhan for the degree of DOCTOR OF PHILOSOPHY IN MATHEMATICS EDUCATION.

Date of Approval: 19 April, 2019

Prof. Hari Prasad Upadhyay, Ph. D
Research Supervisor
Faculty of Education, TU Nepal



Prof. Madan Man Shrestha, Ph. D
External Examiner
Tribhuvan University, Nepal



Prof. Ganesh Bahadur Singh, Ph. D
Member, Research Committee
Faculty of Education, TU, Nepal



Prof. Krishna Prasad Gautam, Ph. D
Dean, Faculty of Education
Tribhuvan University, Nepal




DEDICATION

This dissertation is dedicated to my ancestors, family members of late Jarna Singh Shrestha, my live god-goddess father Mr. Babu Lal Pradhan and mother Mrs. Aashika Pradhan for their immense love, support and blessings.

ACKNOWLEDGEMENTS

The successful completion of this work was made possible with the support and encouragement of a number of individuals. First and foremost, I would like to express my heartfelt gratitude to Prof. Dr. Hari Prasad Upadhyay who supervised the research work and provided scholarly guidance at all stages of the study and without his generous support and encouragement this study would not be in present form. I am equally indebted to express my sincere appreciation to Prof. Dr. Bidya Nath Koirala, Co-Supervisor, for his scholarly guidance, feedback and generous support throughout the study.

I would also like to express the deepest appreciation to Prof. Dr. Siddhi Prasad Koirala and Prof. Dr. Arbinda Lal Bhomi, the internal examiners, for their contributions to improve the quality of this research and valuable advice in designing the study. I owe profound gratitude to the members of research committee, Faculty of Education for providing critical comments and suggestions in the designing of the study and on the draft report of the study for its further improvement.

My sincere gratitude goes to Prof. Dr. Madan Man Shrestha, Prof. Dr. Hira Bahadur Maharjan, Prof. Dr. Bashu Dev Kafle, Prof. Dr. Tirtha Raj Parajuli, Prof. Dr. Ramjee Prasad Pandit, Prof. Dr. Min Bahadur Shrestha, Prof. Dr. Abdul Quaiyum, Prof. Dr. Lekha Nath Sharma, Prof. Dr. Shiva Ram Neupane, Prof. Uma Nath Pandey, Prof. Dr. Indra Kumari Bajracharya, Associate Professor Bhola Nath Bhattarai and all my gurus for their sensible advice and encouraging remarks. I am equally grateful to Prof. Dr. Chandreshwor Mishra for his patience in going through my writing and editing the language of my dissertation. I would like to extend my sincere thanks to Dr. Deepak Khadka, Director of Research Division, UGC Nepal for his

encouragement and support. I am extremely indebted to University Grants Commission for providing scholarship to carry out this study.

I am forever indebted to my research participants for their support to carry out this study by providing necessary information. Moreover, I would like to thank Mr Natikaji Maharjan, the head teacher of Gyanodaya Secondary school, Bafal, Kalimati, Kathmandu for providing me a place to carry out my study. I also like to thank Mr. Arjun Shrestha, Assistant head teacher of Saraswati Secondary School, Tokha, Kathmandu for his support during the data collection. I would like to thank researchers and scholars whose work I used in my dissertation. Furthermore, I greatly appreciate to the authors of the books, articles and conference paper that I drew on in this work mentioned in the reference section.

I am thankful to Associate Professor Keshab Adhikari, Department of Nepali Education, Mahendra Ratna Campus, Tahachal and Dr. Ganga Bahadur Gurung for their encouragement, motivation and support to complete my study. My genuine appreciations also goes to my wife Kamala, sons Jenish and Jebish for their understanding and support during my study. I have a long list of friends who supported me in different stages of my study, whose names are inadvertently remained to come here, my sincere apology are due to them.



.....
(Jaya Bishnu Pradhan)

Ph. D. Scholar

ACRONYMS

CAR	Collaborative Action Research
CBS	Central Bureau of Statistics
CDC	Curriculum Development Center
CERID	Research Center for Educational Innovation and Development
CMT	Conceptual Metaphor Theory
CPBL	Cultural Project Based Learning
ERO	Education Review Office
FGD	Focus Group Discussion
FOE	Faculty of Education
FOK	Funds of Knowledge
HOD	Head of the Department
MOE	Ministry of Education
NASA	National Assessment of Students Achievement
NCF	National Curriculum Framework
Ph. D	Doctor of Philosophy
RME	Realistic Mathematics Education
SAP	Students Awareness Program
SLC	School Leaving Certificate
TFP	Teacher Facilitation Program
TIMSS	Trends in International Mathematics and Science Study
TU	Tribhuvan University
UNESCO	United Nations Educational Scientific and Cultural Organization
VDC	Village Development Committee
ZPD	Zone of Proximal Development

TABLE OF CONTENTS

LETTER OF RECOMMENDATION	ii
ABSTRACT	iii
COPY RIGHT	v
DECLARATION	vi
ACCEPTANCE AND RECOMMENDATION	vii
DEDICATION	viii
ACKNOWLEDGEMENTS	ix
ACRONYMS	xi
TABLE OF CONTENTS	ix
LIST OF TABLES	xxii
LIST OF IMAGES	xxiii
LIST OF FIGURES	xxvi
CHAPTER I INTRODUCTION	1
Motivation Towards My Research Journey	1
Philosophical Underpinning of Mathematics	5
Understanding of Mathematics Education	7
Statement of the Problem	11
Objectives of the Study	14
Research Questions	15
Significance of the Study	15
Operational Definition of Key Terms	17
Structure of the Thesis	23
CHAPTER II LITERATURE REVIEW	26
Etymology of Cultural Metaphors	26

Metaphor and Teaching-Learning Mathematics	33
Gestural Metaphor and Communicating Mathematical Ideas	40
Mnemonics: The Tool of Memory Aid	42
Cultural Games and Mathematical Pedagogy	47
Ethnomathematics and its Pedagogical Roles	50
Cultural Artefacts and Mathematical Pedagogy	56
Cultural Relevant Pedagogy and Mathematics Curriculum	57
Out-of-School Mathematics and Academic Mathematics	60
Gap in Literature and My Attempt	63
Summary of the Chapter	67
CHAPTER III THEORETICAL UNDERSTANDING AND	69
CONCEPTUAL FRAMEWORK OF THE STUDY	
Constructivism as the Theoretical Basis of Study	69
Constructivist Framework for Knowledge Construction	71
Vygotsky's Sociocultural Theory	75
Funds of Knowledge Theory	82
Conceptual Framework of My Study	86
Summary of the Chapter	88
CHAPTER IV RESEARCH METHODOLOGY	89
Philosophical Underpinnings in My Research Journey	90
Research Design: Qualitative Worldview	92
Multimethod Research Model: Ethnography and Action Research	93
Ethnography and its Choice in My Study	94
Choice of Action Research in My Study	96
Selection of Study Location	101

Selection of Research Participants	102
Procedures for Ethnographic Data Collection	103
Procedures of Data Collection for Action Research	108
Tools of Data Collection for Action Research	112
Procedural Framework for Data Collection	116
Data Recording Procedures	117
Data Analysis and Interpretation	118
Data Authentication	120
Ethical Considerations of My Study	121
Delimitations of the Study	122
Summary of the Chapter	123
CHAPTER V EXPLORATION OF CULTURAL METAPHORS IN MATHEMATICS	124
Knowledge and Practice of Community People on Measurement	124
Mathematics in Weaving	131
Traditional Houses and Exploration in Geometrical Ideas	139
Carving and Exploration of Geometry	142
Monuments and Mathematical Exploration	145
Geometrical Exploration in Cultural Arts and Artefacts	150
Mathematical Ideas in Cultural Games	156
Summary of the Chapter	168
CHAPTER VI CULTURAL METAPHORS IN TEACHING AND LEARNING OF SCHOOL MATHEMATICS	170
Selection of Cooperating School	170
Meeting with HOD of Mathematics	173
Conducting Teacher Facilitation Program	173

Reflection of Teachers' on TFP and Possibilities of Use of Cultural Metaphor	175
Selection of Teacher Participants for CAR Project	183
Collection of Cultural Metaphors	185
Instructional Design through Cultural Metaphors	188
Gestural Metaphors for Writing Numerals	189
Metaphors for Understanding Place Value	194
Metaphors for Teaching Rounding off Numbers	200
Metaphors for Teaching and Learning of Prime Numbers	216
Metaphors for Conceptualizing Integers	227
Metaphors for Teaching Operations on Integers	236
Metaphors for Developing Algebraic Concepts	245
Selection of Spot for CPBL	252
Selection of Students for Participation in CPBL	254
Student Awareness Program for Field Visit	254
Lesson Design for the Field Visit	256
Teachers' Reflections on CPBL	271
Summary of the Chapter	274
CHAPTER VII FINDINGS AND DISCUSSIONS	275
Findings Related to Cultural Metaphor Practices in Everyday Mathematical Activities in Out-of-School Context	275
Findings Related to Possibilities of Use of Cultural Metaphor in Teaching and Learning of Mathematics at Basic Level of Education	288
Findings Related to Contribution of Cultural Metaphor in Teaching and Learning of School Mathematics	294

Summary of the Chapter	313
CHAPTER VIII CONCLUSIONS AND IMPLICATIONS	314
Conclusion of the Study	314
Implications of My Study	318
Researcher's Reflections	324
Researcher's Final Words	329
REFERENCES	332
APPENDICES	
Appendix I: Interview Protocol for Parents	358
Appendix II: Observation Checklist: Collection of Cultural Metaphors	359
Appendix III: Interview Guidelines for Mathematics Teachers	360
Appendix IV: Interview Guidelines for Students	361
Appendix V: Protocol for FGD of Teachers	362
Appendix VI: Protocols for FGD of Students	363
Appendix VII: Development of Field Project	364
Appendix VIII: Students Participations and Group Formation for Field Visit	372
Appendix IX: Student Awareness Program for Field Visit	373
Appendix X: TFP: Pedagogical Knowledge	375
Appendix XI: TFP: Content Knowledge	382
Appendix XII: Consent Forms	392
Appendix XIII: Some Images from the Field Study	397

LIST OF TABLES

Table 4.1	Tools and Respondents for Ethnographic Data Collection	104
Table 4.2	Identification of Contents for Classroom Teaching	110
Table 4.3	Tools and Respondents of Data Collection for Action Research	113
Table 5.1	Land Measurement Units and its Conversion	128
Table 5.2	Knowledge and Practices of Community members on Area Measurement	129
Table 5.3	Conversion of Local Units of Weight Measurement with Standard Units	130
Table 5.4	Exploration of Algebraic Concepts in Cultural Game	166
Table 5.5	Frequency Distribution of Score in Game	167
Table 6.1	Metaphors Capturing from Out-of-School Culture	185
Table 6.2	Metaphors Capturing from the Literature and Classroom	187
Table 6.3	Gesture of Devanagari Numerals and Word Pronunciation	190
Table 6.4	Content of Rounding Off Numbers at Basic Level	201
Table 6.5	Content of Integers at Basic Level	228
Table 6.6	Balance as a Conceptual Metaphor for Zero	230
Table 6.7	Balance as a Conceptual Metaphor for Positive Integers	231
Table 6.8	Balance as a Conceptual Metaphor for Negative Integers	231
Table 6.9	Integers is Money Metaphor	234
Table 6.10	Number as a Conceptual Metaphor for Integers Teaching	235
Table 6.11	Developing Algebraic Concepts: Connecting to Real Life Context	247
Table 6.12	Image Metaphor for Conceptualizing Algebraic Concepts	248
Table 6.13	Conceptual Metaphor for Like and Unlike Terms: A Cultural Activities	250
Table 6.14	Conceptual Metaphors for Like and Unlike Terms	251
Table 8.1	Potential Cultural Metaphor for Mathematics Instruction	322

LIST OF IMAGES

Image 2.1	The Primary Colors	38
Image 2.2	Gesture Representing the Curve $y = x$	40
Image 5.1	A <i>Haat</i> as Length Measurement Unit	125
Image 5.2	<i>Amal</i> , <i>Kuret</i> and <i>Bitta</i> : Units used in Length Measurement	126
Image 5.3	Frame for Mat and Preparing to Weave	131
Image 5.4	Measuring Diagonals of Mat's Frame	133
Image 5.5	Rolling a Mat: An Exclusive Example of Circular Cylinder	136
Image 5.6	Sitting Mat: An Exclusive Example of Circular Shapes	137
Image 5.7	Constructing Frame for Textiles Weaving	138
Image 5.8	Geometrical Patterns in Textiles Weaving	139
Image 5.9	Gable Roof in the House of Cultural Village	139
Image 5.10	Patterns and Symmetry	143
Image 5.11	Temple: A Monument	145
Image 5.12	Doorways Surmounted by Semi-circle	147
Image 5.13	Bell and Exploration of Geometrical Ideas	149
Image 5.14	Hyperbolic Plane	149
Image 5.15	<i>Rangauli</i> : Unity of Aesthetics and Geometrical Patterns	151
Image 5.16	Swastik found in Textiles	153
Image 5.17	Board of <i>Bagha Chal</i> Game	157
Image 5.18	<i>Chungi</i> : A Bunch of Rubber	164
Image 5.19	Students are Playing <i>Chungi</i> Game	164
Image 6.1	Cooperating School for Action Research	171
Image 6.2	Researcher Delivering in TFP	174

Image 6.3	Teachers' Participation in TFP	174
Image 6.4	Bima's Reflection on TFP	176
Image 6.5	Reflection of Sonu on TFP	178
Image 6.6	Shyam's Reflection on TFP	179
Image 6.7	Shyam's Reflection about the use of Cultural Metaphor	180
Image 6.8	Participants in CAR Project	184
Image 6.9	Image Taken on Special Request of the Student Participants	193
Image 6.10	Nepalese Currency of Different Denominations	196
Image 6.11	Money as a Metaphor for Place Value	197
Image 6.12	Base-ten Blocks as a Metaphors for Place Value	199
Image 6.13	Analog Watch as a Metaphor for Rounding-off Numbers	202
Image 6.14	Number Line Metaphor for Teaching Rounding-off Numbers	207
Image 6.15	Colors in Venn Diagram	218
Image 6.16	Displaying the Composite Colors	218
Image 6.17	Students Participations in the Learning Process	219
Images 6.18	Colorful Rainbow Factors in Teaching of Prime and Composite Numbers	221
Image 6.19	Artefacts Showing the Rectangular Area	223
Image 6.20	Students are Exploring Mathematical Relations	225
Image 6.21	Gestural Metaphor and Students Participation in Learning	232
Image 6.22	Integers and Number Line Metaphor	233
Image 6.23	During Integers Lecture	234
Image 6.24	Student Participations in Problem Solving	244
Image 6.25	Activity Based Instruction in Teaching Algebraic Expression	246
Image 6.26	Visualization of Like and Unlike Terms	250

Image 6.27	Old Guheshwory Temple	253
Image 6.28	Researcher Delivering in SAP	255
Image 6.29	Planning to go in the Field Visit	255
Image 6.30	On the Way to Destination	255
Image 6.31	Teacher Assign the Task to his Group	256
Image 6.32	Sources of Geometrical Objects in the Surrounding of Temple	258
Image 6.33	Mandalas and Geometrical Shapes	259
Image 6.34	Searching for Task: Researcher with Cooperating Teachers	260
Image 6.35	Students Report of the Task Given to Group I	264
Image 6.36	Students' Engagement in Task	264
Image 6.37	Students Report of the Task given to Group II	267
Image 6.38	Students are Exploring the Symmetry in the Artefacts	269
Image 6.39	Students Views on Learning Mathematics in Out-of-School Context	270

LIST OF FIGURES

Figure 1.1	Different Forms of Cultural Metaphors	22
Figure 2.1	Gaps in Literature and My Attempt	66
Figure 3.1	Constructivist Framework for Knowledge Construction	73
Figure 3.2	Conceptual Framework Connecting School Mathematics through Cultural Metaphors	87
Figure 4.1	Cycle of Action Research: P-A-O-R Model	99
Figure 4.2	Selection of Research Participants: A Tree Diagram	102
Figure 4.3	Procedural Framework for Data Collection	116
Figure 4.4	Procedures for Data Analysis and Interpretation	119
Figure 5.1	X-test in Rectangular Figure	134
Figure 5.2	Constructing Rectangular Shape	135
Figure 5.3	Process for Construction of Mandala	152
Figure 5.4	<i>Swastik</i> Curve	155
Figure 5.5	Constructing Board of <i>Bagha Chal</i> Game	158
Figure 5.6	Exploring Geometrical Shape in Board of <i>Bagha Chal</i>	159
Figure 5.7	Exploring Centroid of Triangle	159
Figure 5.8	Infinite Series in Board of <i>Bagha Chal</i>	160
Figure 5.9	Movement in <i>Bagha Chal</i> Game	160
Figure 5.10	Mid-point theorem in Board of <i>Bagha Chal</i> Game	161
Figure 5.11	Pythagorean Theorem in <i>Bagha Chal</i> Game	162
Figure 6.1	Rounding-off Numbers as a Path Metaphor	210
Figure 6.2	Deepa's Path Metaphor and Rounding-off Numbers	214
Figure 6.3	Integers as Number Line Metaphor	236
Figure 6.4	Mnemonics and Sign Rule in Multiplication	240

Figure 6.5	Rules of Multiplication (Translated)	240
Figure 6.6	Mnemonics for Sign Rules of Multiplication	241
Figure 6.7	Source Domain and Target Domain of Arrow Metaphor	244
Figure 7.1	Weavers' Mathematics and School Mathematics: The two Cultures	279
Figure 7.2	Dichotomy about the Possibilities of Using Cultural Metaphor	291
Figure 8.1	The Cultural Conflict?: A Poem	330

CHAPTER I

INTRODUCTION

This chapter primarily deals with the motivating factors that contribute to conduct ongoing research. There has been a serious complain that the students' cultural context is largely ignored in the classroom teaching of mathematics. The learners' familiar context and its connection with the educational process enhance the teaching and learning of mathematics. The cultural metaphors are the tools that map students' familiar context to another target domain as abstract concepts of mathematics.

The objective of my study was to explore and document the cultural metaphors for the teaching and learning of school mathematics and to analyze the contribution of those metaphors as the pedagogical tools in the educational process. The present chapter also discusses the significance of my study. The definition of technical terms that I have used explicitly in my report has been explained in a separate section. The structure of this research report has been explained at the end of this chapter.

Motivation towards My Research Journey

On a certain day, I was reading an article in which the author cited a book by George Lakoff and Rafael Nunez entitled "Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being". I searched e-sources but there were very few pages of the book available. Then, I bought this book and read thoroughly. In this book, the authors discussed the creation and understanding of fundamental mathematical concepts and ideas from the perspective of cognitive science. The cognitive scientists believe that mathematics is limited and structured by the human brain and mental capacities. In this connection, Lakoff and Nunez (2000)

viewed that “mathematics as we know it and teach it can only be humanly created and humanly conceptualized mathematics” (p.2). The human mathematics makes fundamental use of conceptual metaphor in characterizing mathematical concepts (Lakoff & Nunez, 2000). The conceptual metaphor is the mechanism that helps to conceptualize abstract mathematical concepts in concrete terms.

Aforesaid readings reminded me at the statement by Morish Klien (1962) who said, “Mathematicians produce formulas, no formula produces mathematicians” (p.26). Morish’s idea tried to establish the fact that mathematical knowledge is not only the cognitive process of the human kind but also the cultural product. This indication was supported by a number of humanist mathematicians such as Reuben Hersh (1997) who viewed that “mathematics as a socio-cultural-historic activity” (p. 48). The author further said “Mathematics is part of human culture and history, which is rooted in our biological nature and our physical and biological surroundings. Our mathematical ideas, in general, match our world for the same reason that our lungs match earth’s atmosphere” (p. 17). Thus, mathematics is inseparable entities from the human culture and society. The present-day advanced mathematical concepts and knowledge are the human creation on the basis of everyday socio-cultural activities. In his world-renowned book “What is mathematics really?” Reuben Hersh (1997) uses the analogy of restaurant to explain the nature of mathematics. In the words of the author,

“the front and back of mathematics are not physical locations like dining room and kitchen. They’re its public and private aspects. The front is open to outsiders; the back is restricted to insiders. The front is mathematics in finished form- lectures, textbooks, journals. The back is mathematics among working mathematicians, told in offices or at cafe tables” (p.36).

The front stage of mathematics is the “mathematics with capital M” and that can be taken as the mathematics of the global village or the mathematics that has been practised by academic mathematicians and professors at universities. Thus, Mathematics is an internationalized discipline (Clements & Ellerton, 1996) and has influenced all over the world. It is so called formal mathematical system and schools everywhere have practised it, developed it, and they all are along with the mainstream of the Mathematics.

Mathematics is basic to every culture and their everyday activities. The mathematical ideas of counting, locating, measuring, designing, explaining, and playing are embedded in the environmental activities of culture of each group of people (Bishop, 1988). Every cultural group has its own mathematical ideas and knowledge to perform their everyday activities. The mathematical ideas and thinking that are neglected by the formal school mathematics curriculum is identified as small m mathematics. The different group of people has develop particular mathematical concepts and ideas in out-of-school environment.

However, it is impossible to understand the front side of mathematics while ignoring the backstage of mathematics (Hersh, 1997). He further viewed that to become professional, one must move from front stage to backstage. One can understand a restaurant meal if s/he is aware of kitchen. The backstage of mathematics is a system that enables people to count, calculate, measure and the various knowledge related to fulfill their daily lives. It is the mathematical system of the groups of people practices in out-of-school culture. The recognition and acknowledgement of backstage of mathematics enables to understand the front or formal mathematics that are practices in school and universities.

Anyway, student and teacher consider that mathematics is one of the difficult subjects in school and may experience significant learning problems (Chinn & Ashcroft, 1998). One of the reasons of taking mathematics a difficult subject is ignoring the out-of-school mathematical knowledge and ideas in formal mathematics curriculum. The students' cultural environment practices many mathematical ideas to perform their everyday activities. The cultural activities enable to communicate the spatial relations and geometrical ideas. However, our school mathematics curriculum largely ignores the cultural mathematical ideas in the process of teaching and learning of school mathematics. Learners' experiences and knowledge regarding mathematics can help to teaching and learning of school mathematics.

In this connection, Rosa and Orey (2016) argue that it is important to acknowledge contributions that members from diverse cultural groups make to mathematical understanding, their cognition and identification of diverse mathematical practices in varied contexts, and the link between academic mathematics and student experiences should all become central ingredients to a complete study of mathematics. I argue that the mathematical ideas and knowledge of out-of-school environment can enable to communicate abstract concepts of school mathematics. The tools for allowing us to connect out-of-school mathematical ideas to formal mathematics is the metaphors. The metaphors are the mechanism used for visualization of abstract concepts of mathematics through familiar and concrete objects. The mathematical ideas and concepts embedded in out-of-school environment of the students can play important role to communicate formal mathematics concepts. These are the tools for visualization or ease of understanding of school mathematics.

The students' everyday experiences, out of school knowledge and ideas are powerful tool to understand the abstract knowledge of mathematics. The cultural

friendly environments and activities that facilitate to understand mathematics is the cultural metaphors. Lakoff and Nunez (2000) pose a question: How do we understand basic concepts of mathematics using our everyday conceptual apparatus? Are everyday conceptual apparatuses the metaphors for the understanding of mathematics? This is my central question which motivated me to explore more about what are such everyday conceptual apparatus and how they contribute to the understanding of abstract concepts of mathematics via conceptual metaphors.

Philosophical Underpinning of Mathematics

What is mathematics? The answers to this question are given by different mathematicians and philosopher differently. As different people has their own philosophical bases to see the world, in the same way, different people has different ways of understanding and defining the term mathematics. Carl Frederich Gauss defined it as the queen of the sciences and Alfred North Whitehead suggests mathematics as the most original creation of the human spirit (Boyer, 1956, p. 402). More prosaically George Polya (1954) claims as many students feel it as a set of rigid rules, some of which they should learn by heart before the final examinations, and all of which you should forget afterwards. Most of the students believe that mathematics is full of meaningless symbols unrelated to daily life. These views about mathematics are in the line of the absolutist philosophers that mathematics is an objective, absolute, inhuman entities (Hersh, 1997). They believe that mathematical reality is out there that can be discovered. It assumes that mathematics is independent of human knowledge and culture.

On the other hand, the fallibilist philosophers see that mathematics is fallible, it is a human construct that has its origins in social and physical interaction. From this perspective mathematical truth is fallible and it is firmly the human and cultural

enterprises (Hersh, 1997). Further, Fasheh (1997) believes that “one equals one is mathematical fact, but its description and interpretation and application differ from one situation to another and from one culture to another” (p. 281). For example, a fresh and delicious apple is not equal to a rotten apple. So, one equals one does not have true in every context. For the cognitive scientist, mathematics is abstract domain and it does not reside outside of human cognition.

Furthermore, the cognitivist theorist believes that the mathematics is highly technical domain and characterised by the entities that constitute idealise mental abstractions. They suggest that most of the idealised abstract technical entities in mathematics are created via a human cognitive mechanism that extends the structure of bodily experience while preserving the inferential organisation of these domains of bodily experience. In cognitive science, the complexities of abstract, literal and nonliteral phenomena have been studied through a mechanism such as conceptual metaphors, conceptual blends, conceptual metonymy, fictive motion and dynamic schemas, and aspectual schemas (Nunez, 2006).

Thus, mathematics gets different meanings to the different philosophers and schools of thought. However, it is true that the present day mathematics is the combination of the human practice and experience in their everyday activities. People have own way of understanding mathematical phenomenon and they frequently practice the mathematical knowledge in daily life knowingly and/or unknowingly. The teaching and learning of mathematical concepts, ideas and knowledge will be meaningful if the mapping between the already known domain to abstract and unknown domain is established. The abstract mathematical concepts can be explained in terms of students experiences, everyday cultural activities, artefacts, games they are playing and others activities. As we know through cultural metaphors, we can easily

conceptualise different abstract mathematical ideas. It is obvious to say that every human activity involves mathematical knowledge, ideas and concepts implicitly or explicitly.

Understanding of Mathematics Education

Mathematics education is a term that refers both to the practice of teaching and learning of mathematics, as well as to a field of scholarly research on this practice. Researchers in mathematics education are in the first instance concerned with the tools, methods and approaches that facilitate practice and/or the study of practice. Thus the mathematics education is little more than that of the mathematical contents. It is not only concerned with mathematical contents to be taught and learnt but also all the issues and problems related to what, how, when, why, where etc., of the mathematical knowledge and skills to all levels and age groups. In this regard Clements and Ellerton (1996) say:

Mathematics education is concerned with the development and implementation of appropriate mathematics curricula, and with all issues associated with the teaching and learning of mathematics. In keeping with the concept of lifelong learning, mathematics education covers learners of all ages and at all levels- from early childhood to adult (p.6).

Thus the mathematics education is concerned with all the issues and problems in the field of teaching and learning of mathematics within and beyond the school. Mathematics education also concerns to the social and cultural issues in general and of the specific group of people in particular. For some mathematicians, the obvious purpose of mathematics education is to produce more mathematicians and also scientists, engineers, and others who will use substantial technical mathematics in their work (Greer & Mukhopadhyay, 2003). The domain of mathematics education is

wide and huge and its aim is to develop the overall aspects of society. Mathematics education has the more responsibility to uplift and upgrade all the issues and problems of the society as a whole.

Teaching method in mathematics is to be based on the objectives of the mathematical contents. In our context, mathematics teaching and learning is far from the recent developed approaches. We are still believed that mathematics is best learned by drill and practice and viewed as a hierarchy of mental habits or connections that must be carefully sequenced, explicitly taught and then practiced with much repetition in order to learning to occur. This method of teaching mathematical results, definitions and concepts through repetition and memorization is explicitly deny the ability of students to reason about mathematical concepts (Ellis & Berry, 2005). Further L. Tom as cited in Gupta (1989) said:

Mathematics education in third world countries suffers because most of the good mathematicians, who are concentrated in the universities and who have a high prestige with the government and the public, do not take sufficient interest in mathematics education. They themselves get poor quality students, produce poor quality teachers and the vicious circle continues. Sustained efforts by some university teachers with initiative and drive can transform the situation (p. 46).

With references to the above saying, the development of the situation on the mathematics education needs to be revamped in our context through the change in the education policy. Since mathematics is considered as the science of all sciences, there has to be national commitment to quality of mathematics education.

Mathematics education is fundamental not only for mathematics itself but for education in all fields of science and technology. As mathematics education is solely

concerned with the issues and problems of society, it is more technical than medical science. The mathematics educationist must be more responsible than medical practitioner because the mathematics educationists practice on the huge mass of the population in the society at a single time but medical doctor is not. That is to say, if the processes and system is faulty in medical laboratory test or diagnosis of the patients can suffer single person but the whole society suffers when there is fault in education system. For improvement of mathematics education, there should be large scale retraining of teachers, improve students' evaluation system, reform in the curriculum as the needs of the society. Further, the different group of people use their own ways of problem solving techniques while dealing with the situations in their everyday activities. The incorporation of the culture friendly pedagogy in the process of teaching and learning of school mathematics enhance the level of understanding of the students.

The mathematical history has the ancient roots of the different civilizations and their experiences and practices. The cumulative knowledge of the particular cultures forms the structure of the present day's formal mathematics known as so-called western mathematics. Although formal mathematics is heavily based on the cumulative knowledge and practices in different civilization but fail to acknowledge to the knowledge of the indigenous culture, their cultural practices, experiences, use and applications to the daily lives. If we wish to make mathematics relevant and meaningful for all students, it is required to embrace a wide variety of real-world applications as we know diverse cultures provide rich and interesting contexts in which students can experience and explore mathematics. There are various mathematical activities found in every culture that they have their own ways of understanding, presenting and explaining the phenomenon. The teaching of

indigenous students' needs not only to take account of indigenous culture but also to support the manifestation of this culture in education. With this regards, Barton (2008) opinions that

The problem is that, especially in indigenous peoples' education, not paying attention to the nature of mathematics may have devastating educational effects. The relative alienation of indigenous children to mathematics (and science) can be explained at least partially because they are less likely to accept mathematics as relevant to their lives. ... Because we do not teach mathematics as culturally dependent ... we generate alienation and thereby under-achievement by communities of indigenous children. Individual children or communities often interpret reduced success as personal or group failure rather than systemic failure (pp 168-169).

The practical reality is that every indigenous people's context is different with respect to their knowledge, the political situation in their own country, their aims for education, their access to resources and more. However, some features of the many indigenous contexts are generally similar. Most indigenous people are relatively culturally homogenous and have some identity within the wider society of the country in which they live. In many cases, the children are deeply immersed in their indigenous cultural world. Their home language and world-view are predominant in their lives, although they are subordinate in the wider community (Barton, 2008). Thus, successful educational performance is closely linked to a healthy sense of indigenous identity.

Children have been learning mathematical concepts through different ways at home, in the playground, while working in a team, interacting with their colleagues and so on. They have been using various mathematical concepts while playing

different games, interaction with peers, observing and imitating the cultural artefacts, and the other cultural activities. Mathematics teaching can be made more effective and interesting by connecting to different cultural traditions, rituals, activities, games and mathematical ideas and concepts that can be related to the cultural situations. But there are scarcely such practices carried out in our schooling.

Hence, this present study is intended to explore the metaphors practising in the various sectors of lives in out-of-school context of learners' community and to observe the mathematical relations, concepts, ideas and knowledge embedded in these activities. Such study can help to develop the mathematical understanding and acknowledge through the ethnomathematical practices of the groups of people in out-of-school environment. The learners of mathematics feel harmonious with the school mathematics curriculum when acknowledges and regards of their culture activities, experiences, knowledge and hence give due regards to them. Thus, it is necessary to carry out the research in the Nepalese context so as to explore the cultural metaphors that help to the conceptual understanding of mathematics at the basic level of education.

Statement of the Problem

Mathematics is believed to be the key for all other subjects but it is surprising that most students fail in it and but pass in other subjects. Current School Leaving Certificate (SLC) results showed that about half of the students were unable to pass the exam. Most of the unsuccessful cases were due to the failure in mathematics. Such conditions are repeated since long. In this regard, mathematics achievement at grade eight observed by National Assessment in Students Achievement (NASA) in 2013 was found eight percent lower than the previous 2011 assessment (ERO, 2015). The reasons behind such decline in students' achievement demand further research to

get the root of the fact (ERO, 2015). However, there are various causes behind the lower achievement in mathematics. Some of the reasons might be the students' ineffective problem-solving strategies, lack of computational fluency, cognitive deficits, negative attitudes towards the subject, belief that mathematics is hard subject and another hidden aspect of the students' lives.

Cajete (1994) and Smith (1994) opine that one of the reasons for the poor performance in mathematics is the teaching and learning styles in school which is bereft of indigenous culture and environmental context. Further accusing figures have also been pointed that the way of mathematics teaching in school and lack of relevance of mathematics content to the students' real life experience (Ezeif, 2002). Gay and Cole (1967) did a study of the learning difficulties in the mathematics of children from the *Kpelle* community (Liberia). They concluded that there do not exist any inherent difficulties. What happened in the classroom was that the transplanted contents and methods did not make sense from the point of view of *Kpelle* culture. The *Kpelle* children experienced mathematics as something alien to their culture, which they thought it was not their cup of tea and took it as a strange white men's creativity and ability. Furthermore, it turned out that to the majority of children of the third world countries, mathematics appears rather useless. It is taken as one of the subjects that have hardly any implications in their personal life and they study it merely to pass the exams without understanding the concepts, ideas and knowledge. Every human culture and their environmental activities of counting, locating, measuring, designing, playing, and explaining exist in all culture and societies (Bishop, 1991; Orton, 2004). Children have been learning mathematical concepts through different ways at home, in the playground, while working in a team, interacting with their colleagues and so on. They have been using various

mathematical concepts while playing different games, interaction with peers, observing and imitating the cultural artefacts, and the other cultural activities.

Mathematics teaching can be made more effective and interesting if it is based on the students' experience and their everyday activities. The students' household activities including their different cultural traditions, rituals, games etc. provide rich sources to understand mathematical ideas and concepts. Home and school are two different worlds where the children live and learn simultaneously. It is believed that the teaching and learning mathematics is enriched if home and school knowledge interact each with other. But there are scarcely such practices carried out in our schooling. In this regards, Nunes, Schliemann and Carraher (1993) cited the views of Bruner (1966) as

“Bruner argues that school learning is distanced from reality, it is carried out in the context of general representations of situations. It is this distance of school learning from reality that makes it more abstract and consequently more general than learning in context. Vygotsky also argues that school learning leads to more general and abstract thinking because it is mediated by language rather than stemming from representations of reality” (p.50).

I agree with the view that the school mathematics itself is not a difficult subject; it is compelled to take as a difficult subject. The difficult thing is due to inability to connect school mathematical concepts with the real-life situations of the learner and vice versa. Various study showed that the several groups of people use their indigenous counting systems to solve arithmetic problems through counting, decomposition, and regrouping (Gay & Cole, 1967; Ginsburg, 1982). Connections are advocated between mathematical content and the home cultures of learners, as well as between different branches of mathematics, various disciplines in which mathematics

is used, historical roots of mathematical content, and connections with the real world and the world of work (Civil 1995). “To unlock and capitalize on the knowledge students already possess” (Gonzalez, Moll & Amanti, 2005, p. x) regarding the mathematical ideas provides some opportunities to the students.

ERO (2015) presented that the average achievement of Nepalese students in mathematics is the below average achievement of Trends in International Mathematics and Science Study (TIMSS). This indicates that the Nepalese students are weak in mathematics as compared their achievement with the international standard. It is the matter of concern and serious problem not only for students and parents but also for the mathematics teachers, school head teacher, school management, mathematics educationist, curriculum developers, textbook writer and all the stakeholders of mathematics education. To overcome this problem, various studies have been done to examine why the students have been performing so low in mathematics for long. The learning and achievement of the children might depend upon the cognitive and affective domain of the children. However, these are not only the major aspects for the achievement of the students in mathematics. As stated above, the social and cultural aspects of the children also play a significant role in learning and achievement of the students. But the researches on the role of social and cultural aspects are scarce.

Objectives of the Study

The main objective of the present study was to explore the contributions/ implications of cultural metaphors regarding mathematical ideas and knowledge embedded in the everyday activities for teaching and learning school mathematics at the basic level of education. This can be elaborated into following:

1. To explore cultural metaphor practices in everyday mathematical activities in out-of-school environment.
2. To analyze the contribution of cultural metaphors in the process of teaching and learning mathematics at basic level of education.
3. To assess the possibilities of using cultural metaphors as an effective approach in teaching and learning of mathematics at basic level of education.

Research Questions

To fulfil my research objectives, the following research questions are formulated:

1. How are the knowledge and ideas in relation to cultural metaphors embedded in everyday mathematical activities in out-of-school culture? To what extent do people understand and practice cultural metaphors to communicate different domains of mathematics in their daily life?
2. How do teachers use cultural metaphors during mathematics teaching at the basic level of education? And what effects do they have on students' understanding of the subject matter?
3. Does cultural metaphors contribute as an effective approach in teaching and learning of school mathematics? What are possible metaphors for the teaching and learning of different domain of mathematics at basic level of education?

Significance of the Study

Every culture has developed mathematics practices consistency with their needs and interest for everyday activities. Research on cultural metaphors for the understanding of mathematics in Nepalese context is voided and the study in the area of indigenous knowledge system is scarce. Only a few studies have been carried out about indigenous mathematics and their ways of teaching and learning. However,

these studies did not provide an integrated model of mathematical understanding addressing the cultural metaphors and cultural pedagogy. The research studies investigating cultural metaphors for mathematical understanding have significant importance to solve the everyday problems of indigenous culture. The cultural metaphors and its contribution to the teaching and learning mathematics have not been researched in our context.

In learning mathematics and children psychology need to be connected along with students' culture. It is believed that effective learning mathematics is impossible without connecting mathematics to students' cultural context. Students should be provided with appropriate learning opportunities so that they can develop a sound mathematics understanding. My study will be significant for three reasons. First, it has the potential to contribute to the literature on the understanding of basic mathematical concepts by addressing omissions of cultural metaphors noted in my rather exhaustive literature search. Much of students' understandings of formal mathematics emerges from interactions and activities in their cultural settings. Many indigenous cultures have developed counting, measuring, locating, playing, explaining, and designing (Bishop, 1996) in their cultural environment. They have made use of arts and design that were rich in symmetry, transformation, proportions and many other activities that employ mathematics concepts. My research contributes to mathematics education by closely examining the cultural metaphors to understand the mathematics as student and teacher activity in the classroom in moments of mathematical learning.

The next contribution of my study will be a new method of teaching and learning of mathematics through the use of cultural metaphors. This seems important because traditional methods of teaching mathematics cannot help students develop a

rich conceptual understanding nor can overcome their learning difficulties. This study will explore the culture friendly instructional strategies using their cultural metaphors to learn and teach different mathematical concepts, knowledge and skills. I believe that the methodology put forward in this study can be one possible method of teaching mathematics in indigenous culture in Nepal.

Finally, my study suggests the best model of mathematics teaching unit that can be used to develop curriculum from the indigenous perspective and it will be promising for students' understanding based on their culture friendly pedagogy. It integrates the cultural metaphors in such a way that students themselves can take *aha* feelings in the process of learning the mathematics of using cultural metaphors as tools in their concept development. Therefore, this study does have potential to reveal important educational implications to improve mathematics education in Nepal.

Operational Definition of Key Terms

I have defined some key terms that frequently appear in this study. These operational definitions of different terms are define operationally:

Culture

The ways to looking of the world, beliefs, attitudes and practices embedded in their everyday activities and their professions of the particular group of people are the culture. In my study, I took the meaning of culture beyond the classical anthropological definition that deals with the beliefs, values, attitudes, customs, social relationships, art and literature that define an ethnic group of people (Abidi, 1996; Banks, 2008). The different group of people has their own set of practices and beliefs system. In my study, the set of practices and experiences outside the school environmental activities is termed as a culture. The teachers culture, students' culture, classroom culture, carvers' culture, masons' culture are some names.

School Culture

Culture can be described as the shared way of living of a group of people. It is used to encompass commonly experienced aspects of the community, such as shared knowledge, backgrounds, values, beliefs, forms of expression and behaviours that may impact classroom interactions (Bishop, 2002; Bourdieu, 1984). The culture is the practice and experience embedded in their circumstances. I believe that the learners bring rich cultural experiences and a variety of cultural resources in the school. Schools could help students learn about their culture as well as the culture of others through learning activities that demonstrate the relationship between culture and mathematics. In my study the school culture or the classroom culture is the set of practices that are based on routine and structural framework of the everyday activities according to the formal mathematics curriculum.

Out-of-School Culture

Learners come in school from different community and cultural groups. Those children bring some sort of ideas, knowledge and practices from their home culture. In my study, the beliefs, values, attitudes, customs, social relationships, art and literature (Abidi, 1996; Banks, 2008) that define students' home environment was known as an out-of-school culture. Thus, the out-of-school culture is set of practices that are embedded in the everyday activities of the different group of people in different community. In my study, the out-of-school culture is set of experiences and the cultural resources of the learners regarding the mathematical ideas and knowledge.

School Mathematics

School mathematics is formal body of knowledge shaped in the written form. The mathematics taught and learnt in school on the basis of curriculum developed by Curriculum Development Center (CDC) of Nepal is known as school mathematics.

School mathematics has developed with the purpose to achieve some predetermined objectives. In school mathematics, one does not usually have a freedom of making a choice of using alternate techniques other than those taught in the classrooms. In schools, mathematical activities are based on symbols which get detached from any meaningful context (Resnick, 1987). School mathematics is aimed at improving individuals' performances and skills. For Ginsburg (1982) school mathematics is systematic knowledge that included written symbols, algorithms, and explicitly stated principles generally taught and learnt in school system.

Out-of-School Mathematics

Out-of-school mathematical knowledge of the learners has been studied extensively with the pioneering work of “Everyday Mathematics” of Nunes, Carraher, and Schliemann (1985), Ethnomathematical ideas of Ubiratan D’Ambrosio (1985), and other authors in the Latin American and African mathematicians and educators. The out-of-school mathematics is the form of mathematics that people use in everyday activities engaging in contextually embedded practices (Bose, 2014). The different cultural group of people practices and uses mathematical concepts in the everyday activities viz. work place activities, household activities, shopping, playing games and so on. The mathematical ideas, knowledge that are practices in the home environment of the learner is known as out-of-school mathematics. In my study, out-of-school mathematics entails non-written and informal mathematics which involve oral techniques in calculations (Nunes, Carraher & Schliemann, 1985; Saxe, 1988). In out-of-school mathematics, the doer has a continuous engagement with the objects and the situations and she does not burden herself with the extra effort to remember the algorithms, calculation-techniques and the reasoning used. They acquire a

mathematical knowledge generation, and distribution from generation to generation from craft model approaches.

Ethnomathematics

The Brazilian educator and mathematician D'Ambrosio introduced the term ethnomathematics in 1977 during a presentation for the American Association for the Advancement of Science. According to him, "Ethnomathematics can be defined as the mathematics which is practiced among identifiable cultural groups such as national-tribe societies, labor groups, children of certain age brackets and professional classes and so on..." (p. 44).

The ethnomathematics is the study of the relationship between mathematics and culture. Its aim is to contribute both to the understanding of culture and the understanding of mathematics. Eglash (1997) viewed how ethnomathematics is the study of mathematical concepts embedded in the out-of-school environmental activities. Every cultural group has its own knowledge and skills to perform their work even though they are unable to play with the global mathematical knowledge. Thus, ethnomathematics is the pedagogical tool that seeks to study how students have come to understand, comprehend, articulate, process and ultimately use mathematical ideas, concepts and practices to solve the problems related to their everyday activities (Rosa & Orey, 2010).

The ethnomathematical ideas of the group of people have generally been excluded from discussions of formal and academic mathematics. Rosa and Gavarrete (2017) also viewed that their mathematical knowledge and learning approaches are not taken into consideration in the formal school mathematics curricula. In my study, ethnomathematics is the mathematical ideas from the group of working class people, their ways of knowledge generation and transmission to create and integrate school

mathematics and possible to apply these strategies in teaching and learning mathematics.

Mathematical Understanding

Understanding is a psychological process. If a person understand about the phenomenon, they are able to think and relate the abstract object with that phenomenon. With this connection, Bereiter (2002) mentions understanding is a psychological process related to an abstract object whereby one is able to think it and use concepts to deal with that object. Skemp (1976) define two types of understanding: Relational and Instrumental Understanding. According to Skemp, relational understanding is “knowing what to do and why,” whereas instrumental understanding is “rules without reasons” (p. 152). Thus, the instrumental understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships. In my study, the mathematical understanding is the ability of learner to use and apply mathematical knowledge and ideas in the variety of context encountered in everyday life.

Cultural Metaphors

Daniel Pink (2008) presents a most powerful statement “A picture is worth a thousand of words, but the right metaphor is worth a thousand pictures” (Cited in Wormeli, 2009). Metaphors provide as a powerful tool to understand one domain of knowledge in terms of another. The conceptual metaphors help to deal with relatively unfamiliar and more abstract domains of experience in a familiar and tangible way. Metaphors not only allow for the expression of ideas and knowledge through language but also through thought and action (Hong-mei, 2010; Geary, 2011).

The mathematical ideas and knowledge that are practicing in the out-of-school environment of children everyday activities can facilitate to develop school mathematics. The use of cultural metaphors as a pedagogical tool could help to mathematical knowledge transformation and distribution in classroom teaching. Thus the cultural metaphor connects different target concepts through their relationships in the common source domain.

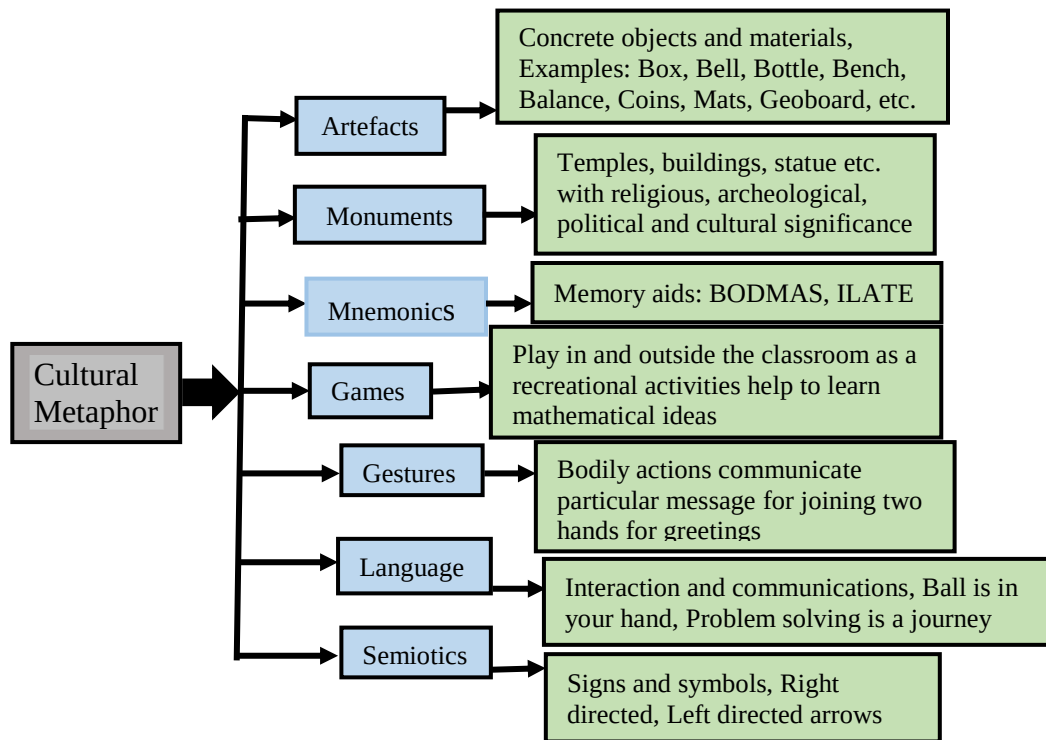


Figure 1.1. *Different forms of Cultural Metaphor*

In my study, the cultural metaphors are the tools whose source domains are the out-of-school mathematical ideas embedded in artefacts, monuments, arts, language and literature, games, and their everyday activities and the school mathematics as the target domain. The use of the cultural artifacts in mathematics classroom activities can be articulated in various stages of teaching and learning of content objectives. The use of some artifacts (bottle and labels, the weather forecast presented in a newspaper, ...), enable the teacher to pose a great deal of possible questions, remarks, culturally and scientifically interesting inquiries in the process of instruction. I take the term

cultural artefacts to refer to the materials cultural tools that can be mediated to the teaching and learning of school mathematics.

The monuments have religious, archeological, political and cultural significance. It has also educative significance and source of inspiration as the mathematical knowledge generation, transformation, distribution and diffusion to the students and teachers of mathematics. The ethnomathematical ideas embedded in games and other cultural activities can serve as most powerful metaphor in the process of teaching and learning of formal mathematics. In my study, the cultural metaphors are tools which connects students' out-of-school environmental activities to understand school mathematics.

Structure of the Thesis

This thesis consists of eight chapters. After this introductory chapter, the second one presents a review of the literature on different themes which provide a basis for the whole research study. The chapter presents the literature review on metaphors theory and focuses on how metaphors can concretize abstract concept of mathematics in a visual mode and how metaphors are used in a pedagogical setting as a mediational tool for scientific discourse. This chapter ends with the evaluation of research gap in the area of cultural metaphors and developed the framework that guides this study.

The third chapter presents the theoretical underpinnings of the study. This starts with a brief description of constructivism and continues with Vygotsky's sociocultural theory, Bourdieu cultural capital, Funds of Knowledge theory and its pedagogical implications, and ends with conceptual framework of the study. The conceptual framework for the understanding of mathematical ideas based on cultural metaphors have been described. The conceptual framework is based on the modified

version of the ethnomathematical curriculum model (Lipka, 1994, Adam, 2004) showing the connection of ethnomathematical ideas, cultural metaphors and out-of-school culture to school mathematics at the basic level of education. Under this model, a lesson is designed based on the students' cultural environment and the connections among them is presented together with the important aspects of that model which drove the design and data analysis.

The fourth chapter presents the methodology of the study. This includes brief description of the research design. Since this study was conducted in two different phases including ethnography and collaborative action research, the data collection procedures were described extensively for each methodology. This chapter also provides details of the research instruments and methods of data analysis.

The fifth chapter presents data analysis and interpretation based on my first research question. This chapter primarily deals with mathematical ideas and concepts embedded in the everyday activities in out-of-school environment. The ethnomathematical ideas of the group of people and how ethnomathematics are considered as potential metaphors that could be employed for the pedagogical purpose was discussed. The different arts, artefacts and design of the students' home environment and mathematical ideas associated on it were described.

The sixth chapter attempts to answer my second research question about the use of cultural metaphors and its effects as a pedagogical tool for mathematics teaching and learning at the basic level of mathematics education. Results leading to the answers of the research questions are summarised at the end of each respective section. This chapter also included the data analysis and results regarding the mathematical ideas developed beyond the four walls of the classroom. The mathematical task that embedded in the cultural arts, artefacts and design were given

to different group of students and individual interviews were taken during the study. The FGD of students and teachers was conducted after the implementation of CPBL approaches. The students and teachers' reflections about the teaching and learning mathematical ideas outside the classroom and possibility to incorporate as a pedagogical tool were also presented in this chapter.

The seventh chapter of this thesis presents the findings and discussions based on the analysis of data. The answers to the research questions are provided in separate sections. These are followed by a discussion regarding the effectiveness of the cultural metaphors for teaching and learning of mathematics at the basic level of education. The thesis ends with the chapter includes the conclusions, my reflections during the whole process of the study and implications for different areas of study.

CHAPTER II

LITERATURE REVIEW

This chapter begins with the extensive search of etymology of cultural metaphor. I have explained several definitions of metaphor and its categorizations found in literature. Review on various studies based on the role of metaphors in teaching and learning of different domain in mathematics education have been made. The literature regarding the role of gestural metaphor, mnemonics, cultural games, and ethnomathematical ideas embedded in cultural artefacts as a pedagogical tool for the development of mathematical understanding have also been reviewed.

The research carried out in the field of everyday mathematics, informal mathematics practiced in out-of-school environment have also been examined. Following the review, I have presented the research gap in the domain of cultural metaphors for the teaching and learning of school mathematics and finally have developed a figurative framework to carry out this study. Throughout the chapter, I have also noted relevant questions, issues and concerns raised within each topic for the methodology of the study.

Etymology of Cultural Metaphors

The term metaphor is derived from the Greek word *metaphora*, which means to carry meaning from one word, image or idea to another. Thus, a metaphor is a figure of speech in which connection is made between two, unlike things that actually have something important in common. Initially, the metaphor was studied only with linguistic point of view, so most of the early work on the topic concerns how metaphor is generated through language. Searle (1979) argues that metaphor is primarily a linguistic phenomenon, processed by the brain as language and that the processing of metaphorical or figurative language takes more time than the processing

of literal language. With this connection, Blasko (1999) viewed that metaphor was mostly regarded as a poetic device, something that could be used to make language pretty.

The metaphor has been a central idea within cognitive linguistics. Cognitive linguistics is the study of language in its cognitive function where cognitively refers to the crucial role of intermediate informational structures in relation to our encounters with the world. It assumes that our interaction with the world is mediated through informational structures in the mind (Geeraerts & Cuyckens, 2007). Cognitive linguistics is hardly the first area of scholarship to treat metaphor as an object of study. Thus, the study of metaphor has traditionally been a part of linguistics, literary theory and rhetoric (Grady, 2007). However, the metaphor is not merely a linguistic phenomenon, it is a fundamental principle of thought and action (Johnson, 1981 cited in Aubusson, Harrison & Ritchie (2006). However, with the advent of linguistic and dialogical models of communication, researchers began to consider metaphor in a new way, as a subject in its own right.

In the early 1980s, George Lakoff and Mark Johnson published a radical new model of metaphor that challenged the established thinking on the subject (Lakoff and Johnson, 1980). They introduced the concept that metaphor might not only be concerned with language but also about the way we think. Indeed, they suggested that making metaphors is a cognitive process, and is something that the human brain does naturally. According to Lakoff and Johnson, our thoughts are shaped or framed by metaphor. Our brains work by relating new knowledge to old, and we are constantly looking at things as if they were something else. The model that Lakoff and Johnson proposed, suggested that the whole way we understand and relate to the world is metaphorical and that linguistic metaphor is just a surface reflection of the deeper

level of cross-domain linking that happens in our minds. Lakoff and Johnson's model then is as much to do with psychology, cognitive science and communication theory as with linguistics.

Lakoff's work caused a sea change in the field of metaphor research. Prior to the publication of the book titled "Metaphors we live by", which he co-authored with Mark Johnson in 1980, the metaphor was studied primarily as a linguistic phenomenon. After the publication of Lakoff and Johnson's contemporary theory of metaphor, there was a change of focus away from the linguistic study of metaphor and toward the notion that metaphorical thought is a cognitive process that pervades our everyday cognition (Stern, 2000; Blasko, 1999). The principle feature of Lakoff and Johnson's theory of metaphor is that it identifies metaphor as being a transfer between two conceptual domains, or subject areas: the source and the target. This has become known as the "two domain mapping model" of metaphor. This model is widely acknowledged as being the first move away from exclusively linguistic studies of metaphor. However, in recent years the model has been adapted and expanded to reflect new thinking in the area of conceptual metaphors.

Conceptual Metaphors

Lakoff and Nunez (2000) define metaphor as a conceptual mapping from one domain to the other. Regarding the metaphorical thought, Lakoff and Nunez (2000, p.5) explain: "...human beings conceptualize abstract concepts in concrete terms, using ideas and modes of reasoning grounded in the sensory-motor system. The mechanism by which the abstract is comprehended in terms of the concrete is called conceptual metaphor". Thus, the conceptual metaphors play a major role for the understanding of abstract mathematical ideas. In this regards, Lakoff and Nunez (2000) created three types of conceptual metaphors and they are known as grounding

metaphors, linking metaphors and extraneous metaphors. The grounding metaphors allow us to ground our understanding of mathematical ideas in based on everyday experience. For instance, the abstract mathematical idea of addition as putting things into piles/ bag or adding things to a collection, subtraction as taking things away from a collection, sets as containers, members of a set as objects in a container. Linking metaphors allow us to link one branch of mathematics to other branches of mathematics. For example, the metaphor “Numbers is a point on a Line” links arithmetic to geometry. Thus the linking metaphors allows to conceptualize the ideas of arithmetic in terms of geometry.

Lakoff and Nunez (2000) discussed another type of metaphor called extraneous metaphors. According to them “...extraneous metaphor or metaphors that have nothing whatever to do with either the grounding of mathematics or the structure of mathematics itself. Unfortunately, the term metaphor when applied to mathematics has mostly referred to such extraneous metaphors” (p. 53). The staircase is an example of extraneous metaphor of the “step function” because when graphed the step function can look similar to a staircase. Likewise, “Function is a machine” is a metaphor. The source domain of this metaphor is “machine” and the target domain is the mathematical concept of “functions”. The image of staircase has nothing whatever to do with either the inherent content of the grounding of the mathematics, although the visual is a helpful reminder of how this function would look when graphed (Lakoff & Nunez, 2000, Bearden, 2012). Extraneous metaphors can be eliminated without any substantive change in the conceptual structure of mathematics, whereas eliminating grounding or linking metaphors would make much of the conceptual content of mathematics disappear.

A metaphor is a description of an object or event, real or imagined, using concepts that cannot be applied to the object or event in a conventional way. The object or the event being described is called the target, and the concepts that cannot be applied conventionally are called the source. The source does not just include the concepts mentioned in the description that can be applied conventionally to the target, but other related concepts as well. The source of a metaphor is always supplied by the description (the text). The target may be explicitly hinted at by the description may be determined by the context, or may not be provided at all. Whenever the target is provided, context plays a more dominant role in determining it than the description.

Lakoff and Nunez (2000) claim that “metaphors are an essential part of mathematical thought, not just auxiliary mechanisms used for visualization or ease of understanding” (p.6). The metaphors have two important tasks. First one is that, it contributes in the development of mathematical content knowledge. Secondly, it acts as the tool for the understanding of one thing in terms of another. Thus, metaphor contributes as the pedagogical knowledge for the understanding of abstract concepts of mathematics. Consider the metaphors that “Numbers are points on line”. The conceptions of numbers are no geometrical. The number line is one of the fundamental concepts in different branches of mathematics. Fyhn (2007) viewed that patterns in snow are examples of linking metaphors. Fyhn further maintained that

Some Sami mittens have the pattern grouse footprints on their edges. These repeating patterns show a connection between embodied experiences and ornamentation. Ski trails as well as animal footprints perform patterns in the snow, patterns with different symmetry properties (p. 246).

Barton (2005, p. 100) points out that “Rather than thinking of the mathematics which is known world over through formal education, we need to expand our vision

to include any form of quantitative, relational or spatial systems”. Barton uses boats as a metaphor for mathematics. He claims that different boats can be used for different purposes, the fishing boat can go to rocky places where the ferry cannot navigate and the ferry can travel under conditions too hard for the fishing boat. “It is the same world, but it is a different understanding. Neither is the truth” (p. 100). From this views, metaphors can be any thing that conveys the meaning of abstract thing in terms of more familiar, concrete and everyday experience.

Pedagogical Meaning of Metaphors

Metaphors provide rich evidence about the ways in which some aspects of our experience are associated with others, for reasons that reflect basic aspects of perception, thought, and possibly neurological organisation. Within cognitive linguistics the term metaphor is understood to refer to a pattern of conceptual association, rather than to an individual metaphorical usage or a linguistic convention. Lakoff and Johnson (1980, p. 5) describe metaphor as: ‘The essence of metaphor is understanding and experiencing one kind of thing in terms of another’. The emphasis within cognitive linguistics on this conceptual dimension suggests a view in which metaphor is not inherently a linguistic phenomenon.

Everyone uses metaphors in their everyday interaction with others. People may use metaphors without even noticing it. Some common everyday metaphors include: “*Jeevan euta yatra ho*” (Translate: Life is a Journey). People usually use this phrase to describe the essence of life. Every journey may or may not have destination. While there may arise obstacles during the journey, sometime there may come joyful moments as well. Thus the meaning of life can be taken as a journey. Moreover, there are many metaphors to represents same thing. People also say “*Jeevan nadi ho*” (Translate: Life is a River) metaphor to convey the meaning of life as the river which

continuously flows in its path. Other metaphor to convey life is “*Jeevan Ek Sangharsha ho*” (Translate: Life is Struggle) that explains the life as ups and downs in different situations. And in the same way, “*Jeevan euta Khel ho*” (Translate: Life is a Game) is another metaphor that expresses life as the combination of loss and win in the result of the game. Here, the term “Life” is target domain to be explained in terms of various sources domain that are more familiar.

People go through without noticing the use of metaphors, but metaphors will always be used. In the course of the teaching and learning of mathematics, different linguistics metaphors are used. For instance, the “Addition is Deposit in a Bank”, and “Addition is Collection of Objects in a Bag” are the linguistic metaphors to communicate the ideas of addition. From the view of conceptual metaphor, the abstract concept of addition is explained with the help of everyday experiences and activities as the term deposit and collection of objects are familiar and concrete to everyone.

Metaphor simply means the representation of abstract entities through very simple, familiar and meaningful objects. It can be taken as a mapping that the abstract ideas map into concrete, strong and meaningful images that were developed in different social and cultural context for a different purpose. Thus, metaphors help to understand abstract ideas by mapping them into strong, meaningful images that were originally developed in a different context. The primary function of metaphor is to communicate the meaning of abstract thing in terms of relatively familiar context. Thus, metaphors are the powerful tool for the communication of one thing in terms of other. Teacher uses conceptual metaphors to make meaning of abstract concepts of mathematics and to explore hidden intellectual avenues contained in a metaphor’s

frame (Bearden, 2012). Metaphor is the matter of teaching because it provides insight into how metaphors assist a teacher in making meaning of his or her teaching.

English (1997) sees metaphors as tools for creating formal concepts out of image schemas, and of restructuring these concepts in complex ways. Lakoff and Nunez (2000) also argue that the conceptual metaphor's "primary function is to allow us to reason about relatively abstract domains using the inferential structure of relatively concrete domains," (p. 42) with structures of image schemas preserved by this mapping. Further, Petrie and Oshlag (1993, p. 580) discussed metaphors facilitated through three aspects.

1. Metaphors work by transferring chunks of experience from well-known to less well-known contexts.
2. Metaphors permit and impress a more memorable learning due to the greater imagery or concreteness or vividness of the full-blooded experience conjured up by the metaphorical vehicle.
3. Certain aspects of natural experience are never encoded in language and that metaphors carry with them extra meanings never encoded in language.

Thus, metaphor plays unique educational role in helping students to acquire new knowledge because it enables one to transfer learning and understanding from what is well known to what is less well known in a vivid and memorable way.

Metaphor and Teaching-Learning Mathematics

In previous chapter I mentioned the meaning, definition and different forms of metaphor that gives fundamental role in the development of mathematical understanding. I start this part by presenting the role of metaphors in the development of mathematical understanding found in literature, and reviewing categorizations and analysis of metaphors done by different researchers. However, many of the

researchers who studied metaphors argue that all of our thinking is fundamentally metaphorical. With this view Lakoff and Johnson (1980) speak “the way we think, what we experience, and what we do every day is very much a matter of metaphor” (p. 3) even though metaphor may remain largely unnoticed. Similar to the Lokaff and Nunez, Pimm (1987) talks about extra-mathematical and structural metaphors:

There are two main sources of metaphor, which may be of interest in mathematics education. The first consists of what I term extra-mathematical metaphors. These attempts to explain or interpret mathematical ideas and processes in terms of real world events, and such metaphors can involve everyday objects and processes... The second main source, which I call structural metaphors... involves a metaphoric extension of ideas from within mathematics itself (p. 95).

Another takes on roles of metaphor in learning comes from frameworks involving reification, in which a mathematical process becomes reified as a mathematical object, so that it can be used as a unit in a further process at a higher level of abstraction, until this process in turn becomes reified as a mathematical object, and so on (Presmeg, 1997, p. 275).

Metaphors may play a role in transitions between informal and formal understanding. English (1997) sees metaphors as tools for creating formal concepts out of image schemas, and of restructuring these concepts in complex ways. In this context, I investigate this role of cultural metaphor for the understanding of mathematical ideas, concepts and knowledge, where metaphor can be considered as a mechanism for connecting informal and formal layers of knowing. The concrete material and everyday experiences help to conceptualize abstract mathematical ideas.

The defining theme of the interaction perspective (Ortony, 1993) is that metaphors play a constitutive role in creating new knowing. Metaphors are not just connecting old ideas in new ways, but changing both source and target ideas. Sometime as time passed the source domain may detach from the target domain. For instance, 'kite' was concrete device to conceptualize the geometrical quadrilateral but it has now become the name of this quadrilateral. This poses the question of what happens when a metaphor becomes a mathematical concept (Parzysz, Pesci, & Bergsten, 2005).

Droujkova (2004) observed another pedagogical possibility comes from the data on using "the language of metaphor" to communicate with learners. In her dissertation on teacher interventions, she speaks "when I was suggesting a multiplicative relation to Annie, I used the terms "a new kind of sharing pattern" which came from her micro-world. Such use of the language of metaphor as a tool (Vygotsky & Kozulin, 1986), is a powerful pedagogical intervention. In her study, learners could choose their representations, even though some of them were influenced by common computer entities such as buttons, and all of them were influenced by mathematical culture at large.

Droujkova (2004) observed the role of metaphor in the growth of mathematical understanding in the area of proportionality. A conceptual framework for the study was based on constructivist perspective. In addition, she used ideas from the areas of linguistic, ethnography, and commercial software development. The software development process of the study was adapted from agile processes methodologies (Droujkova & Droujkov, 2003). It is characterized by active learners' input and participation, supporting learner-centered principles of software-related research in education (Clements & Battista, 2000). In turn, the fact that learners have

active roles supported by the enactivist idea of co-construction of meaning the idea needed for integration of different theories. Data analysis was based on the study of sources and targets of metaphors, as represented in learners' actions.

The Pirie-Kieren model for the growth of mathematical understanding, in combination with a model for proportional reasoning development that was created for the study, were used to map learners' knowing actions. Micro worlds created by learners supported metaphoric systems, which in turn helped to coordinate the process of knowledge development in proportionality domain. The model used for mapping this process included the notions of equivalence class, relation and invariance. Findings indicate that each of these notions may be developed, during local and context-specific growth of understanding, through actions in additive, multiplicative and qualitative analogy worlds. Image making and folding back to image making from levels beyond are different, from the point of view of metaphor. Having separated the source and the target, learners coordinate their image making activities within this two-part system through analogical reasoning. Pedagogical implications of the analysis of roles of metaphor in creation of new knowledge include the stress that should be placed on image making activities. Image making is also the level to which learners frequently fold back from property noticing, refining the target of their metaphor through working on the source. Droujkova's findings underlie the importance of image making activities in formation of new concepts and conceptual structures. Image making is of especially great significance in the area of proportionality that requires complex multi-concept structures involving coordination of additive, multiplicative and analogical reasoning.

Kilhamn (2011) did a research to investigate how students make sense of negative numbers through metaphorical reasoning. The physical versions, known as

line segment, as metaphors can conceptualize the abstract concepts of numbers. The metaphors seem to break down, making a transition from intuitive to formal mathematics necessary. The study was based on assumptions about mathematics as both a social and an abstract science and of metaphor as an important link between the social and the cognitive. Conceptual metaphor theory was used to analyze teaching and learning about negative numbers. She suggested that conceptualizing the number line as a continuous rather than discrete representation of the number system that evolves from the notion of a repeated unit was less important than carrying out actions on the number line. This emphasize caused ambiguity in the way teachers referred to the number line and restricted understanding amongst the children that focused upon the ordering of numbers and the actions that could be associated with this ordering. However, the number line is limited used as a tool because teachers, and consequently children display little of them. Different metaphors give different meanings to statements such as finding the difference between two numbers, and result in incoherent mappings onto mathematical symbols. This study highlights the importance of metaphors to make sense of negative numbers to students. Students with a highly developed number sense for positive numbers seemed to incorporate negatives more easily than students with a poorly developed numbers sense, implying that more time should be spent on number sense issues in the earlier years, particularly with respect to subtraction and to the number zero.

Everyone loves the magic of color and especially children like to play with colors, the stuffs of art and beauty. Children can have experience the magic of mixing three primary colors (red, yellow, and blue) each other to watch a third color that appears. Children will see that a secondary color is made by mixing two primary colors. It helps students in understanding the concepts of prime factorization and thus

generating different composite numbers (Upadhyay, 2015). The concept of prime and composite number is an

abstract idea to be taught. The

prime numbers are primary

colors and the composite

numbers can be seen as the

secondary colors. Mixing two

different colors gives third

color is same as the generation



Image 2.1. Primary Colors

of composite numbers as the product of two primes. For instant mixing two primary

colors red and yellow generates orange (secondary) color. There are the primary

colors as source domain which is better known concepts and the secondary colors is

target domain with the more abstract or less innate concepts. Hence metaphors,

ranging from verbal to artistic ones, become a privileged form of communication,

exactly because they are able to communicate with people in depth. This is

remarkably interesting in the educational context and in particular in relation to

mathematics, a discipline which often shapes individuals' personal history in difficult

ways. In this sense it was already pointed out (Pesci, 2003) that talking about

mathematics to students through symbolic images, actions, gestures, not typical of the

world of mathematics but rather of everyday life could promote communication on

such a subject. In the art world, traditional colour theory states that there are three

types of primary colours. These are the colours of red, yellow and blue which cannot

be created by any combination of other colours. They are the colour equivalents of

mathematical prime numbers. Like prime numbers, the basic three colours can't be

reduced to more fundamental colours. They form the building blocks of all other colours.

From the study of literature on metaphors, it was observed that many of the researchers who studied metaphors argue that all of our thinking is fundamentally metaphorical: “the way we think, what we experience and what we do every day is very much a matter of metaphor” (Lakoff & Johnson, 1980, p. 3) even though metaphor may remain largely unnoticed. Metaphors are used not only for communication purposes, but also can be considered as thinking devices intended for teaching and learning abstract mathematical ideas. Lakoff and Nunez (2000) categorizes two types of metaphors: grounding metaphors and linking metaphors. Grounding metaphors ground mathematical ideas in everyday experience and the linking metaphors allow us to link one branch of mathematics to another.

In this study I took Lakoff and Nunez (2000) conceptual metaphors as the working definition of metaphors which enable to support mathematics teaching and learning. From the available literature I reviewed, there is no study have been done in the area of cultural metaphors. I assumed that all metaphors are primarily cultural. So in my study the metaphors which can carry mathematical meaning from the everyday activities of the students’ culture are taken as the cultural metaphors. My study focused on how students and teachers used cultural metaphors in the process of teaching and learning mathematics. By analyzing how people use mathematical ideas and knowledge metaphorically in their everyday activities; interviews with the teachers, students and community members; and observations of mathematics classroom further help me to explore the cultural metaphors and to analyze the contributions of such metaphors in the process of teaching and learning of school mathematics.

Gestural Metaphor and Communicating Mathematical Ideas

A gesture is a form of communication in which visible bodily actions communicate particular messages. It includes movement of the hands, face, or other parts of the body. Gestures are a crucial part of everyday conversation such as chatting, describing a route, negotiating prices on a market; they are ubiquitous. Gestures allow individuals to communicate a variety of feelings and thoughts often together with body language in addition to words when they speak. Gestures play a major role in many aspects of human life. Gesturing is probably universal; there has been no report of a community that does not gesture. These are conventional, culture-specific gestures that can be used as replacement for words, such as the joining hands used for respecting and greetings. A single emblematic gesture can have a very different significance in different cultural contexts, ranging from complimentary to highly offensive. These gestures are closely coordinated with speech.

Metaphoric gestures have typically been conceived of bodily activities or movements that represent the source domain of a metaphor. The psychologist Wilhelm Wundt was the first scholar to recognize that gestures may be used metaphorically (Cienki & Muller, 2008). The gestures that transfer concepts from one domain to another “symbolic gestures,” and offered as an example the use of spatial gestures to represent temporal concepts (Wundt 1922). It attests to the metaphor passing from something concrete to the physical representation of something abstract. Calbris (1990) note the following context of gesture use in which a metaphor is expressed in the gesture.



Image 2.2. Gesture Representing the Curve $y = x$

When someone is uttering a word with an abstract meaning, one sometimes produces a gesture which reflects the concrete origins of the word. Gestures play an important role in teaching and learning mathematics. One can teach the abstract concepts of mathematics through proper body language. For instant, one can use different positions of hands to represent the functions of different types. The image 2.2 representing the identity function $y = x$.

Edwards (2005) did the research in the area of physical gesture and language and viewed that the human gestures form an integral part of language, thought and communication. She viewed McNeill's three kind of gestures such as deictic gestures (pointing to an object), metaphoric gestures (the content represents an abstract idea that has no physical form), and iconic gestures (bearing a relation of resemblance to the semantic content). The teachers frequently use such gestures without noticing it to represents the abstract concepts of mathematics. The second type of gesture found in the data were metaphorics (Edwards, 2005). Metaphorics are gestures where "the pictorial content presents an abstract idea rather than a concrete object or event" (McNeill, 1992, p.14). Metaphorics were found referring to a great variety of mathematical abstractions, including comparisons of numbers, equality or similarity, generalized actions, and generalized mathematical entities like statistics, ratios, and formulas. The gestures associated with ideas of "more" and "less" showed an interesting contrast. There were two cases in which students talked about something more or additional, and in both cases, the gesture consisted of a single tap or touch of the table, followed by another tap or touch to the right of the first one.

The final type of gesture found in the corpus of student gestures were deictics (Edwards, 2005). A deictic is a "pointing movement that selects a part of the gesture space" (McNeill, 1992, p. 80). Sometimes diectic gestures point to actual objects near

the speakers, or to directions in the real world (north, south, front, back, etc.).

However, deictic gestures can also indicate imaginary objects, people, or elements of a “space” that has already been constructed through previous gestures and speech.

Hence, the gesture would enact or depict the concrete grounds of abstract concept.

Mnemonics: The Tools of Memory Aid

A mnemonic is defined as a device designed to aid memory. Mnemonic instruction gives students tools they can use to better encode information so that it is easier to recall later on. Higbee (2001) defines mnemonics as memory aids that assist one in remembering specific information by using a process, strategies or techniques that enables a person to improve memory. In this study, I observed the different mnemonics that the students and teachers use in the process of teaching and learning of mathematics. I believe that the mnemonics process help students to retain information and connect the concepts in mathematics.

There are at least three distinct methods for teaching mnemonics: keyword, pegword, and letter strategies. The keyword strategy is based on linking new information to keywords that are already encoded to memory. According to Scruggs and Mastropieri (1989), the keyword strategy works best when the information to be learned is new to students. Wang and Thomas (2000) have shown that the initial benefits of using the key word mnemonic are not very durable over time when retention interval is manipulated as a between-participants' variable. Pegword strategy uses rhyming words to represent numbers or order. The rhyming words or “pegwords” provide visual images that can be associated with facts or events and can help students associate the events with the number that rhymes with the pegword. Pegword mnemonics has proven useful in teaching students to remember ordered or numbered information (Scruggs & Mastropieri, 1989). The pegword example that I

used in my classroom was for an improper fraction. My students were confused between a mixed numerals and an improper fraction. So, I presented a picture of an ice cream cone and told them that the upper part of a snow cone is bigger than the base of the cone, thus representing an improper fraction. This eliminated the confusion between the two terms.

Letter strategy involves the use of acronyms and acrostics. Acronyms are words whose individual letters can represent elements in lists of information. Acrostics are sentences whose first letters represent to-be-remembered information. Teachers can use these letter strategies to help students remember lists of information (Scruggs & Mastropieri 1989). Being able to make those connections by using letter strategies gives students another tool in order to retain the new key concepts they have learned. Most of the teacher use the BODMAS letter strategy or acronym in mathematics classroom for the correct order of operations in simplification.

In my classroom observation in the elementary trigonometry course, the teacher used “Some People Had Curley Brown Hair Turned Permanently Black”. In this mnemonics, the first alphabet of the consecutive words stands for trigonometric function with their ratios such as $\text{Sin} = \frac{P}{H}$, $\text{Cos} = \frac{B}{H}$ and $\text{Tan} = \frac{P}{B}$ where the underlines alphabets S, P, H, C, B, H, T, P, B comes in a regular pattern as in above mentioned equations. In the class many students write their own mnemonics (*saen pen harayo, casle bhetyo hola, tanson paryo baalai*) related to the same concepts that was more meaningful to them than the mnemonics that the teacher had created for them. I got astonished when so many other students shared their own brilliant mnemonics which were easier for them to remember.

Mnemonic instruction as a strategy that provides a visual or verbal prompt for students who may have difficulty retaining information. It is a memory enhancing

instructional strategy that involved teaching students to link new information that is taught to information they already know (Delashmutter, 2007). He argues that mnemonics works to address problems related to students' poor retention and useful tool in mathematics classroom. Mnemonic strategies that use imagery and visual cues to facilitate memory recall are commonly used in the classroom. Hayes (2009) study showed that when a student with learning disabilities is taught using mnemonic strategies, their ability to retrieve information is greatly enhanced.

Sanders (2007) did a Quasi- experimental study on embedded strategies in mathematics vocabulary instruction. The purpose of the study was to explore the use of the keyword mnemonics strategy in mathematics vocabulary instruction. The quasi-experimental study was attempted to observe the significant difference between scores on vocabulary assessments for third grade students taught mathematics vocabulary by direct instruction without keyword mnemonics and those taught by direct instruction with keyword mnemonics. Recent accountability requirements increase the emphasis on mathematics achievement for all students, including low-performing students and students with learning disabilities. However, these students progress very slowly, with weaknesses evident in computation and problem solving. These limitations affect their success in mathematics at lower grades and make earning a high school diploma difficult.

It is imperative to isolate components of mathematics comprehension and identify methods to teach them in order to help these lower performing students be successful. Research indicates a strong relationship between vocabulary and mathematical comprehension, identifying vocabulary understanding as a key component in understanding the subject. Vocabulary instruction incorporating mnemonic strategies has consistently resulted in substantial increases in learning and

retention for students with disabilities as well as nondisabled peers when compared with other approaches.

Mathematics vocabulary is a significant factor in mathematics comprehension. Students often have trouble remembering the definitions of difficult mathematics terms, causing them to miss problems that are dependent upon those terms. Keyword mnemonics was shown by this research to be an effective way to teach mathematics vocabulary words to third grade students. Therefore, it is important that teachers receive training in this strategy and that administrators place emphasis on using such strategies to enable students, especially minimally competent ones, to learn important mathematical vocabulary terms accurately and effectively.

The keyword strategy is a memory strategy, not a concept development technique. As shown in these examples, keyword mnemonics retrieve the vocabulary term at whatever level of conceptual understanding the student has at that time, accurate or inaccurate. Research has been done in reading that stresses the importance of teaching conceptual development along with the definition. However, more research in this area is warranted with mathematical concepts. Also, more research is needed about the interplay of name confusion and conceptual misunderstandings and their role in students' levels of understanding. This information is critical to assist teachers in effective mathematics vocabulary instruction.

The research reported here focused on the use of keyword mnemonics in mathematics vocabulary instruction. This mixed model multi-strand study with a quasi-experimental quantitative design tested for significant differences between groups on a vocabulary assessment and conducted a repeated measures analysis of variance was on two levels of instruction (direct instruction versus keyword mnemonic instruction) and across three measures (pretest, posttest, and follow-up).

Although both groups did show significant improvements, the students who participated in the keyword mnemonic classes outperformed the students in the direct instruction classes as measured on the both the posttest and the follow-up test of the vocabulary assessment. There was no significant difference between the groups on overall mathematics achievement as measured by Curriculum-Based Measurement probes or on attitudes toward mathematics. The qualitative data identified a relationship between the use of elaboration techniques, level of performance, and conceptual misunderstandings. However, as always, the effective use of any retrieval technique is dependent on the accuracy of the information encoded.

Low achievement is a significant problem in mathematics education. It has been an ongoing cause for concern and a major factor in mathematics education reform. One possible cause of underachievement in mathematics is the student's negative attitude towards the subject, math anxiety or fear of math (Balbuena & Buayan, 2015). This attitude is attributed to the psychological nature of students and typically to the pedagogical practices of schools. Emenaloa (1984) cited that among the root causes of mathematics phobia in Nigerian schools are lack of effective teaching aids, a shortage of qualified math teachers, lack of adequate in-service training programs and proper incentives for math teachers, and an inherent fear of mathematics. Wong (1999) stated that an overdependence of the chalk-and-talk method has resulted in unsatisfactory performance in and poor attitude towards school mathematics for many students. Math anxiety may have something to do with the classroom experience. Yara (2009) mentioned that students' outlook can be directly influenced by the teachers' outlook and teaching approaches.

Sousa (2001) said that some children develop a fear of mathematics (or math phobia) because of negative experiences in their previous schooling or a simple lack

of self-confidence in dealing with numbers. The negative attitude towards mathematics is also a product of students' inadequate knowledge and skills in basic mathematics. Sousa (2001) recognized that students need to have mastered a certain number of skills before they can understand and apply the principles of more complex mathematical operations. Children experiencing these difficulties are suspected of possessing certain types of mathematical disorders. Students with mathematical disorders have difficulties solving simple and complex arithmetic problems. Their difficulties stem mainly from deficits in both numerical procedures and working memory. Students with memory disorders have difficulty recalling number facts, have a high error rate when they do retrieve arithmetic facts, and retrieve incorrect facts that are associated with correct facts (Sousa, 2001). To name a few, Scruggs and Mastropieri (1999) conducted a research on mnemonic instruction with world history classes where the analysis of strategy use data revealed that students employed appropriate strategies, and observational data confirmed that student time on task was higher in the mnemonic condition. Survey data revealed general overall satisfaction with mnemonic strategies on the part of teachers and students.

A review of the literature indicates that memory and recall improve when lyric and song are integrated as a learning strategy. The literature also reveals that musical mnemonics are particularly effective as a memory aid for the learning disabled and mildly handicapped student. They further claimed that mnemonic methods may also be helpful for computation and recommended to teachers the development and use of letter strategies to help students remember lists of information.

Cultural Games and Mathematical Pedagogy

Mathematical games can be played in whole-class, small group or paired settings. They are a resource which is usually highly motivating to children and,

consequently, encourages greater levels of concentration and engagement with mathematics. Games can be used in different ways to consolidate learning, practice skills, explore mathematical relationships and develop problem-solving strategies. Thus, the games play in and outside the classroom is an approach that the teacher can use to provide a source of strong motivation for student engagement in learning and to encourage students' social, emotional and cognitive development (Kamii & DeVries, 1980). The benefits of transforming classroom activities of ostensibly difficult or tedious subjects into opportunities for learning through playing cannot be underestimated. In fact, the use of card games was proven to enrich science teaching (Perlota, Reyes, Rivera, Sato, Solis, & Vega, 2008).

In this study, I proposed the cultural games, played in the community, as a metaphor for the development of mathematical concepts and ideas. The use of games as metaphor is also common in natural language. Some of the common word metaphor regarding games are “play your cards right”, “the ball is in your side”, “life is a game”. In mathematics metaphors are used everywhere (Lakoff & Nunez, 2000), as in probability (Lakoff & Johnson, 1999), even though most students and teachers are unaware of their use. This study combines the potentials of mnemonics and games in improving effectiveness for teaching operation of integers. Araya (2007) identified a game used as a metaphor for some statistics and probability concepts is proposed. It is a pattern discovery game where players guess what is inside boxes. Random variables are considered box features, the classification variable is viewed as the box content, and methods to find the most discriminating variables are viewed as concrete ways of arranging boxes. Based on findings about the source of difficulties with probabilities the game promotes a strategy that uses ecologically valid formats. The verbalization of detected patterns as explicit rules expressed with algebraic language

is also encouraged. Teaching is presented as hints to play better: how to store and graph information, and how to describe patterns. Evidence of student's deep understanding of the targeted mathematical concepts is presented. Metaphors can allow large chunks of information to be transferred from one domain of knowledge to a new one.

Black (1979) suggested that metaphors can help generate new knowledge when the known domain and the new domain interact in the mind of the learner so that each is enriched by the other. While many mathematical games are designed as competitive games, they can often turn into co-operative games where pupils support each other to obtain the greatest success. Carefully planned, these types of games can provide opportunities for developing skills related to mathematical thinking – predicting, generalizing, justifying and explaining. For Ainley (1988, p. 243), the main value of mathematical games lies in the linking together of mathematical problems which are 'real' to the children, the use of such process skills as listed above, and the need such activities present for children to think in a mathematical way.

Pradhan (2015) presented the results based on cultural games and the mathematical knowledge, ideas and concepts involved in it. With the analysis of the various cultural games, varieties of geometric ideas, the concepts of probability and statistics were observed. Different cultural games provide a variety of skills such as developing strategies, organizing information, making quick decision, formulating and testing hypothesis, logic and reasoning which are the foundations for mathematical understanding. It was also observed that cultural games develop children's good mental coordination, their concentration levels, memory and their ability to communicate and use mathematical language greatly improves, providing an

atmosphere for children to share ideas with their peers. My study (Pradhan, 2015) revealed that the mathematical ideas embedded in the various cultural games can be used effectively in the teaching and learning of school mathematics. It was also found that the games can enhance attitudes towards mathematics by creating fun and enjoyment for the students.

The literature review on the area of cultural games observed that the games play significant role for the development of mathematical ideas. The problems with mathematics achievement, math anxiety, and mathematical disorders are the problems in the students around the world and the use of mathematical games in instruction are solutions to address them. My study identifies the potentials of games of the students' community to develop the mathematical ideas in the basic level of education.

Ethnomathematics and its Pedagogical Roles

Ethnomathematics is the study of the relationship between mathematics and culture. It refers to a broad cluster of ideas ranging from distinct numerical and mathematical systems to multicultural mathematics education. The goal of ethnomathematics is to contribute both to the understanding of culture and the understanding of mathematics, but mainly to appreciate the connections between the two. D'Ambrosio (2006) has interpreted the notion of ethnomathematics by considering its three conceptual elements: 'Ethno' refers to people; *mathema* to understanding; while *tics* refer to techniques as well as to art. Thus, ethno-mathematics refer to culturally embedded ways of understanding of mathematical techniques by the group of people practiced different mathematical concepts in their own ways. It must be noted that the notion of *mathema* is broader than mathematics as we normally consider it; that 'ethno' has to be understood as people/culture, and that it does not include any reference to 'ethnicity' understood as a racial category.

Ethnomathematics can be described as the way people from a particular culture have common systems for dealing with quantitative, relational, and spatial aspects of their lives (Barton 1996). It follows what Knijnik (1997) describes as an ethnomathematical approach, one that is characterized by the investigation of the traditions, practices, and mathematical concepts of a subordinated social group and the pedagogical work, which was developed in order for the group to be able to interpret and decode its knowledge (as cited in Alangui, 2017).

The term ethnomathematics is coined with the different perspectives. Some saw it is the new area of mathematics as Ascher and Ascher (1997) defined ethnomathematics is the study of the mathematical ideas of non-literate peoples. Other believed that it is the methodological postures in the learning process of mathematics (Ferrerira, 1989). In D' Ambrosio (2006) view ethnomathematics is a research program about the history and philosophy of mathematics and it is also the program of the way in which cultural groups understand, articulate and use the concepts and practices, which we describe as mathematical, whether or not the cultural group has a concept of mathematics. Thus, ethnomathematics is relatively new as a field of research. It may be described as the study of mathematical ideas and activities as embedded in their cultural context. Further D' Ambrosio defined ethnomathematics as "the mathematics which is practiced among identifiable cultural groups such as national-tribe societies, labor groups, children of certain age brackets and professional classes and so on..." (p. 44).

Bishop (1991) believes that mathematics is a cultural product, which has developed as a result of various activities, and that counting, locating, measuring, designing, playing, and explaining are all part of that cultural

product. Everyday life is impregnated in the knowledge and practices of a culture. At all times, individuals are comparing, classifying, quantifying, measuring, explaining, inferring, generalizing, and evaluating, using material and intellectual instruments that belong to their culture (D'Ambrosio, 2006). The cultural activities of the human being invented some kind of the mathematical knowledge and concepts.

The study of Barton (1996) study provides a new conceptualization of ethnomathematics which avoids some of the difficulties which emerge in the literature. A definition is proposed for ethnomathematics as the study of mathematical practices within context. Four types of ethnomathematical activity are identified: descriptive, archaeological, mathematizing, and analytical activity. The definition also gives rise to a categorization of ethnomathematical work along three dimensions: the closeness to conventional mathematics; the historical time; and the type of host culture. The mechanisms of interaction between mathematical practices are identified, and the imperialistic growth of mathematics is explained. Particular features of ethnomathematical theory are brought out in a four examples. By admitting the legitimacy of other viewpoints, ethnomathematics opens mathematics to new creative forces. Within education, ethnomathematics provides new choices, and turns cultural conflict into a useful tool for teaching. Mathematical activity exists in a variety of contexts. Learning mathematics involves being aware and integrating of diverse concepts. Ethnomathematics expands mathematical horizons, so that cultural diversity becomes a richer contributor to the cultural structures which humans use to understand their world.

The ethnomathematics exist in different cultural practices of Chitwan's Tharus and such practices has supported in preserving their cultural identity. There is discontinuity between traditional and modern measurement system but similar practices were found still among the illiterate group (Paudel, 2008). Tharus ethnomathematical practice is needed to preserve and to be linked in formal education of the community connecting to the "local corpora of knowledge with the global corpora of knowledge. With this view Luitel (2003, as cited in Paudel, 2008) argued that Nepalese students have confronted with gunny pig destiny being tested with prescribed curriculum where the curriculum developer are unaware of the socio cultural particularity. Similar result is also found in the literature of ethnomathematical studies done in the Nepalese context. Measurement system, numerical system, geometrical application and other mathematical concept preserved by *Gopali* people, situated in *Chitlang* village of *Makawanpur* district, were different from the school mathematics (Dahal, 2007). The children before school age have some mathematical concept developed in their mind but after the enrollment in school, they have to learn totally different concepts from which *Gopali* children certainly face the problem. He concluded that the ethnomathematics found in *Gopali* culture is more applicable and can be used easily in the practical life because ethnomathematics is emerged from cultural practices. Primary level school mathematics education can be related to the ethnomathematics which was the major concluding remarks of the study.

UNESCO (2008) was set up to develop culturally contextualized mathematics curriculum resource materials for the lower secondary schools of Nepal in order to foster a culturally pluralistic society. This study found that numerous local practices

were identified as being linked to formal mathematics at the lower secondary level. In particular, practices of farming, local business, household activities, children's games, cultural practices, artifacts and social events were found to be strongly linked to school mathematics concepts. These contexts were incorporated into the curriculum resource materials in order to enhance children's learning of school mathematics, especially girls whose lived experiences and family social roles will be legitimated in formal education by such contextualization. Analysis of the lower secondary school mathematics curriculum of Nepal revealed formal geometric, arithmetic, algebraic and set theory concepts that could be linked directly to local practices and day-to-day life activities documented in the fieldwork data. The field study indicated how parents and elders can help their children's learning of mathematics at home by engaging them consciously in local activities linked directly to school mathematics, such as wages, expenses and cost calculation and estimations. Given the fact that children cannot escape from household work, a possibility observed in the fieldwork was that children can enrich their mathematical understanding by linking their classroom learning with their daily household activities.

Pradhan (2012) did a study to explore the knowledge generation of *Chundara* and the compare and contrast the ways that they learn and teach in their everyday activities from the school pedagogy. The main finding of the study was that *Chundaras* teaching and learning activity involved participatory and cooperative approaches in which they learn with the help of their parents as Vygotsky seeing learning as an activity in which shared mathematical meanings are constructed socially. It was also observed that indigenous knowledge could act as a powerful tool in a learning environment to teach students. But my experience in the *Chundara* community says that conventional curricula and achievement tests, however, do not

support students' learning based on their indigenous knowledge. I found that the prescribed school pedagogy is alien and indifferent to the local knowledge of the *Chundara*. However, they are using implicit mathematical knowledge in their everyday activities.

Millroy (1992) found that the group of carpenters used the concept of tacit knowledge of mathematical ideas. This concept of tacit knowledge implies that most people know more than they can tell. Tacit knowledge manifests itself through human activities, and is not necessarily expressed or expressible in written or spoken form. According to Millroy (1992) "the physical act of designing and building furniture would involve tacit mathematical knowledge" (p. 13). The studies reveal that all cultures have basic counting, sorting and deciphering methods, and that these have arisen independently in different places around the world. This can be used to argue that these mathematical concepts are being discovered rather than created. However, others emphasize that the usefulness of mathematics is what tends to conceal its cultural constructs. Naturally, it is not surprising that extremely practical concepts such as numbers and counting have arisen in all cultures.

The review of literature on ethnomathematics provides a new conceptualization of ethnomathematics which avoids some of the difficulties which emerge in the literature. There is no consistent view of ethnomathematics in the literature. The relationship with mathematics itself has been ignored, and the philosophical and theoretical background is missing. The literature also reveals the ethnocentricity implied by ethnomathematics as a field of study based in a culture which has mathematics as a knowledge category. In particular, this work has been studied the mathematical ideas embedded in the everyday activities of indigenous

people of Nepal. The need of incorporation ethnomathematical ideas of other people has been highly emphasized in school curriculum.

Cultural Artefacts and Mathematical Pedagogy

The use of artefacts such as concrete materials to support mathematics learning is commonplace in primary education, though less common in upper secondary and tertiary education. Most research studies on the use of concrete materials in mathematics education have focused on the effect on learning outcomes, often by experimental design comparing a treatment group and a control group (Sowell, 1989). Investigations of how students interact with such materials are rarer (Bergsten & Fransson, 2005). An artefact may be considered generally as any human creation, such as physical tools, production schemes, language or skills. Artefacts used for supporting learning, such as concrete materials designed for educational use, are ‘secondary’ as compared to ‘primary’ artefacts used directly in the production (Wartofsky, 1979). To make an artefact an ‘instrument’, for example for learning, it is necessary for the user to develop ‘utilization schemes’, i.e. ways to use the artefact (Strasser, 2004).

The physical materials or concrete materials used in school are common tools or educational materials (Szendrei, 1996). Everyday tools used in society for different purposes for different peoples. The everyday apparatuses for the member of group of people has their own significant. For example, the matches or money of different denominations has some sort of purpose for some community members but in educational context to be used as an aid for teaching and learning of different concepts of mathematics. There is, in general, three types of artefacts such as static, dynamic and responsive artefacts in terms of the character of the artefacts itself. The artefacts that can be manipulated but do not change or give any feedback to the user is

known as static artefacts. The Cuisinaire rods, geoboards, match boxes are the examples of static artefacts. An artefact is responsive if it has a mechanism to produce an output to the user's deliberate input request, e.g. a calculator. In a dynamic artefact, by a series of input responses a dynamic sequence evolves under the guidance of the user, as for example constructions in a dynamic geometry software by the use of 'drag mode' (Bergsten & Fransson, 2005). In an overview of research about the 'effectiveness' of concrete materials in mathematics education, Sowell (1989) concluded that such materials may have a positive effect on learning and attitudes towards mathematics through long term use, provided that their use is properly handled by knowledgeable teachers (Thompson & Lambdin, 1994; Hall, 1998).

Metaphors are used not only for communication purposes, but also can be considered as thinking devices intended for teaching and learning abstract mathematical ideas. In this study, I took artefacts as the metaphor which enable to support mathematics learning. With this regard, the research about the effectiveness of the concrete materials showed positive effect on learning and attitude towards mathematics (Sowell, 1989). The use of artefacts such as concrete materials to support mathematics learning is commonplace in mathematics education. My study focused on how students and teachers interact with concrete materials used in teaching mathematics.

Cultural Relevant Pedagogy and Mathematics Curriculum

Realistic Mathematics Education (RME) is a learning theory formulated in the early 1990's by respected Dutch mathematician, Freudenthal, who believed mathematics to be a, "human activity that combines learning and problem solving as a simultaneous activity" (Zawojewski, Magiera & Lesh, 2013). In particular, RME outlines that the progression of knowledge begins in a contextual, concrete scenario

and then develops toward a more informal and abstract understanding, i.e. discovering connections between ideas, creating strategies to solve problems different ways, finding shortcuts. While this learning pattern may not be a “one size fits all” for students, using Vygotsky’s classic “zone of proximal development” can be helpful in creating goals and scaffolding to help students engage with the content (Zawojewski, Magiera & Lesh, 2013). Researcher Michelle Stephan (2009) lists three points that define RME:

- RME suggests that instruction start with an experientially real context for students, meaning that students do not have to have actually experienced the situation but have to be able to imagine themselves in it.
- The work should build students’ reasoning gradually from the concrete to the abstract. Teachers should use manipulatives, pictures, tools, and other items to reinforce students’ reasoning with imagery, not just hope that students will think in the abstract by magic. Instruction should be intentionally designed so that students reorganize their thinking progressively toward more abstract ideas.
- Students should be encouraged to create models of their concrete activity. These models should become reasoning devices for more abstract thinking.

The National Curriculum Framework (NCF) was prepared in 2005. It emphasized on inclusive curriculum for all students in respective of gender, ethnicity, cultures, religion, caste, ethnicity, abilities and disabilities, socio-economic conditions and geographical conditions. Similarly, key guiding principles of national curriculum framework were: curriculum should address diverse curricular concerns through integration, curriculum should follow child-centered developmental approach, curriculum must recognize mother language in teaching and learning, curriculum

should be inclusive, curriculum should provide opportunity for local need-based learning, curriculum should recognize *Sanskrit* education as the foundation for eastern knowledge and philosophy, curriculum should provide opportunity for information communication and technology (ICT) education, curriculum development should follow collaborative approach, and curriculum should recognize alternative learning related to multicultural education and teaching in multicultural classroom directly or indirectly.

Acharya (2015) observed that most of the contents of primary mathematics curricular materials were irrelevant with culture of diverse learners. The curricular materials were reported to be interrelated with each other but they were not prepared from the multicultural perspectives. From the findings of the Acharya (2015), it can be concluded that the mathematics education and its curricular materials are not culturally relevant in Nepalese context. One of the factor that the students feel mathematics more difficult is the disparity of the teaching-learning approach of the students' culture and school pedagogy.

Acharya (2015) observed that “teachers used self-focused pedagogical process and ignored students' subjectivity” (p. 347). As mathematics is a cultural phenomenon, the mathematical ideas and knowledge are embedded in the students' home cultural activities. The cultural traditions of the people manifested in their development of settlement, construction of buildings, temples and its architectural structures, craftsmanship to work displaying the implicit mathematical ideas. One of the objective of my study was to explore the possibilities of using cultural artefacts, monuments, games that the students play as mathematics learning metaphors. The main concern of present study was to link students' cultural activities to school

mathematics and make school mathematics pedagogy more relevant from cultural perspective.

Out-of-School Mathematics and Academic Mathematics

Children learn mathematics implicitly in out-of-school setting while they are involved in different cultural activities. The research carried out in the field of everyday mathematics (Carraher & Schliemann, 2002) has repeatedly produced evidence that children learn mathematics outside of school settings. Children are participating in the interaction involving different mathematical ideas and knowledge in specific cultural activities such counting, measuring, playing locating and so on. Such activities promote the development of mathematical ideas in out-of-school culture. Nunes, Carraher and Schliemann (1993) mentioned that their research shows that groups with restricted schooling master arithmetical operations, properties of integers and of the decimal system, and proportional relations. Besides arithmetic, concepts and procedures related to measurement, geometry and probability are used in everyday activities. The data collection and handling, and the concepts of probability are practicing in the course of playing different cultural games (Pradhan, 2015). However, they practicing those mathematical concepts without knowing the meaning and essence of it. Mosvold (2007) viewed that the mathematics that the children practices in their everyday life should be incorporated in the overall teaching of mathematics. However, most of the teachers are not able to connect the out-of-school mathematical ideas with school mathematics. In this connection, Mosvold (2007) viewed that “The connection between mathematics and everyday life, which is strongly emphasized in the Norwegian national curriculum of 1997, appears to be far from trivial, and the transfer of knowledge from one context to another is often considered to be problematic” (p. 390).

With regard to the ways to connect the two worlds of mathematical ideas and knowledge, Owens (2015) cited the Matang's ways of incorporating the children's out-of-school mathematical ideas to formal mathematics. He argues that there are two ways in which learners' culture is connected to school mathematics. One way is that the definition of mathematical knowledge is somewhat implicit. The mathematics which practices in the sociocultural context is quite implicit. The advocates of this view argue that "the incorporation of representations and procedures by children into their cognitive systems is a process that occurs in the context of socially constructed activities" (Owens, 2015, p. 13). The second way suggested by Matang is that the analysis of cultural influences on mathematical knowledge can demonstrate both the differences and invariance in mathematical knowledge across cultures (Bishop, 1988; Kimball, 1974; Nunes, 1992 cited in Owens, 2015). She also focuses that there are often rich metaphors in cultures that can be used in mathematics. The cultural artefacts of the slope of a roof is a good introduction to work on angles.

Children are learning mathematical ideas and concepts in out-of-school environment via the interaction of the members of the group of people. Pradhan (2017) also observed that Chundara children were enjoyed the learning process at home but exhibits their resentment about the school pedagogy. The students' home culture and their traditionally handed down teaching and learning approaches make mathematics education more relevant to the students. The connection between mathematics and everyday activities of people has received much attention in international curriculum reforms in the last decades (Mosvold, 2007). However, there does not appear to be a consensus about how this connection is supposed to be carried out. Mathematization and contextualization are also proposed as important practices to create a bridge between everyday activities of the group of people and practices in

mathematics, but familiar contexts do not always make life easier for the pupils (Arcavi, 2002). I believe that the people are using mathematical ideas in their everyday activities tacitly. The children perform these mathematical ideas in out-of-school environment, but the connection should be made to formal classroom mathematics.

Resnick's (1986) work exhibited that the informal mathematics and the mathematics of the school are more closely related. However, the way they both often did through distinctly. The group of people outside the formal school mathematics had to respect many of same basic properties of school arithmetic such as the associative law, distributive law, commutative law. With this regards, the street vender might take away 100, then 30, then 5 when subtracting 135 from 200. This strategy relies on the decomposition of 200 into $100 + (70 + 20 + 10)$; likewise, 135 is implicitly treated as the sum, $100 + 30 + 5$. Although the street vendors did not know how to explicitly express the associativity of addition they revealed their implicit use of the property through the transformations they made on the values given (Schliemann & Carraher, 2002).

The ethnomathematical ideas of group of people have performed strong mathematical ideas. Every cultural group has its own knowledge and skills to perform their work even though they are unable to play with the global mathematical knowledge. Thus, ethnomathematics is the pedagogical tool that seeks to study how students have come to understand, comprehend, articulate, process and ultimately use mathematical ideas, concepts and practices to solve the problems related to their everyday activities (Rosa & Orey, 2010). This implicit mathematical knowledge is embedded in their activities tacitly. Mosimege and Lebeta (2000) also reported that the indigenous people use different mathematical concepts like estimation,

tessellations, and symmetry in the construction of the traditional artifacts and cultural activities. The Chundara use lots of mathematical concepts and knowledge to perform their wooden artefacts. They use highly sophisticated mathematical ideas and knowledge and this out-of-school mathematical knowledge is sufficient for performance that is consistent with rules, even though the person might not be aware of the rules (Pradhan, 2017). The connection between the mathematical ideas practicing in out-of-school context and the formal mathematics supposed to be carried out. In present study, my objective is to develop the concrete strategies to connect the learners' out-of-school ideas and knowledge to teach some particular content of school mathematics at the basic level of Nepalese mathematics curriculum.

It is believed that the mathematics has the origin in a different culture to fulfil the everyday needs of human life. They are using mathematics in their own ways implicitly via metaphors. However, mathematics is considered as a difficult subject to learn and understand due to its abstract nature. Thus, the metaphors help us to understand and comprehend the subject meaningfully. It helps the people who are trying to learn mathematics, understand the material better. In this study, my job is to identify the cultural metaphors that enable learning school mathematics through the cultural mindset of the mathematical concepts. As a generation of a cultural metaphor follows an inductive, structured reasoning process based on the grounded theory approach of Glaser and Strauss (1967), I was equally interested to analyze the contribution of cultural metaphors in the process of teaching and learning of school mathematics.

Gap in Literature and My Attempt

From the review of empirical study, I observed that metaphors help to understand abstract mathematical ideas. Lakoff and Nunez (2000) explained three

types of conceptual metaphors: grounding metaphor, linking metaphor and extraneous metaphor. The grounding metaphors allow us to ground our understanding of mathematical ideas in based on everyday activities and experience. For example, “Temperature is Integers”, and “Primary colors are Prime Numbers” are grounding metaphors. The linking metaphors allow us to link one branch of mathematics to other branches of mathematics. The metaphor “Number is a point on a line” and “Square root is a side of a square” links arithmetic to geometry. Although grounding metaphor and linking metaphor are essential for the understanding of abstract concept of mathematics, Lakoff and Nunez (2000) discussed about third kind of metaphor known as extraneous metaphor that is mostly used to visualize the mathematical content.

Bishop (1991) identified that counting, locating, measuring, designing, playing and explaining are environmental activities that exists in all culture. These activities are in all cultures in which mathematics is embedded. The abstract mathematical ideas of integers as temperature (Kilhamn, 2011), colors as the metaphor for prime and composite numbers (Upadhyay, 2015), animal footprints perform pattern in snow as metaphor for symmetrical properties (Fyhn, 2007), gestures form an integral part of thought and communication of abstract mathematical concepts of fraction (Edwards, 2005), games used as a metaphor for some statistics and probability concepts (Pradhan, 2015; Araya, 2007; Mosimege & Lebeta, 2000) and out-of-school mathematical ideas grounded in the children’s home culture allows to understand the abstract mathematical ideas. I observed that the various artefacts, monuments, games playing in each culture, and the everyday activities embedded mathematical concepts, ideas and knowledge facilitated to connect abstract and complex mathematical concepts in concrete form.

The cultural metaphors provide powerful tool to communicate mathematical ideas and knowledge practicing in the students' community to classroom mathematics teaching. In this connection, Owens (2015) viewed that many metaphors are used in mathematics and teachers need to explain the metaphor used in everyday talk in the out-of-school environmental context rather than using western colloquial metaphors. Thus, literature review done in metaphors and incorporation in mathematics teaching and learning different context need to be redefined in Nepalese context.

In my visit of the available literature, there is a dearth of recent literature regarding study on metaphor for teaching and learning of mathematics in our context. On the basis of reviewed literature that I did in this chapter, I identified that most of the research on the areas of metaphors were done in the western context. It is necessary to contextualize the metaphors capturing in the literature in our context. In my study, the main objective was to explore the cultural metaphors regarding mathematical ideas and analyze its contribution in the process of teaching and learning of school mathematics. In the following figure 2.1, the main task of this study was to capture the cultural metaphors and observe its contribution in mathematics classroom. They were depicted within the larger rectangle and connected by the bold arrow. After reviewing the literature, I have developed the figure that presented my attempt to be carried out and which are connected by dashed arrow. Viewing the importance of the areas in the field of mathematics education I captured the conceptual metaphors grounded in the different context that facilitates to development of abstract mathematical ideas. At the same time, it was needed to identify the areas of mathematics that most of the students have fail to develop conceptual understanding and also figure out the difficult areas of mathematics to teach.

From the literature I reviewed, it was identified that cultural artefacts can be used as motivating stepping-stones to launch new mathematical knowledge (Bonotto, 2004). Keeping this view, I trace out the cultural artefacts and ethnomathematical ideas practices in out-of-school environment. The different group of people in out-of-school context have their own ways to understand mathematical phenomena. The ethnographic study regarding cultural artifacts offered the opportunity of making connections between the mathematics incorporated in real-life situations and school mathematics. In the same vein, my attempt also was to observe the cultural games as they are part of their real-life experience and observe the possibilities to incorporate in the classroom teaching. The figure 2.1 displays my ways of addressing the gaps that I identified in literatures through this empirical study.

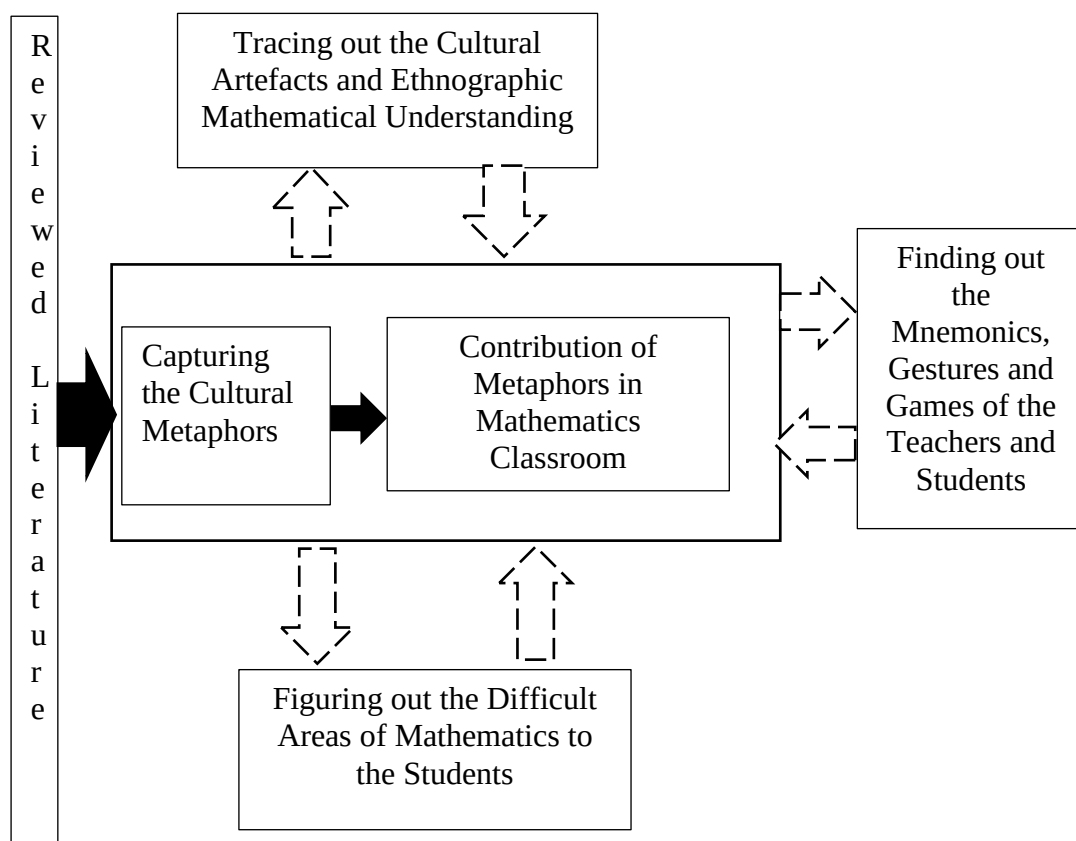


Figure 2.1. Gaps in Literature and My Attempt

The research identified that the metaphors used by students and teachers help in the process of teaching and learning of school mathematics. Viewing the importance of the areas in the field of mathematics education in Nepalese context, it is necessary to explore the cultural metaphors that help to connect the school mathematics. In the review of the available research, it was arguing that the ethnomathematical ideas of the groups of people and the educative framework under this area enhance the students understanding of school mathematics. However, very few of them provided the pedagogy that connect the out-of-school mathematics to formal mathematics. Thus, it is necessary to carry out the research that helps to connect the learners' home culture to the teaching and learning of formal mathematics in Nepalese context.

Most of the research done in this area focused on the mathematical ideas embedded in the indigenous culture. There is also an absence of research about the incorporation of children ethnomathematical ideas in formal mathematics teaching and its effects in the understanding of school mathematics. More research needs to be done in this complex area. In an extensive search, no studies appeared to examine use of a cultural metaphors as tool that supported for the development mathematical concepts which could be used to underpin the present study.

Summary of the Chapter

This chapter began with the defining the meaning of metaphor. Metaphor is one of the powerful tool to communicate the abstract ideas of mathematics. Not only the literature, metaphor is using in every area as a communicative tool. The linguistic and pedagogical meaning of metaphor have been described. The conceptual metaphors theory of Lakoff and Nunez have been described extensively. The cultural

metaphors are the connecting tools of out-of-school mathematical ideas to the school mathematics.

The available literature has been reviewed thematically with critical consciousness. The empirical study done in the area of metaphors in teaching and learning of mathematics has been reviewed. The research done by Edwards on gestural metaphors and visual spatial reasoning of Kay Owens that the indigenous community used to conceptualize the geometrical ideas have been reviewed. The teacher and students are using mnemonics as memory aids that assist one in remembering specific information by using a process, strategies or techniques that enables a person to improve memory. The mathematical ideas embedded in the learners' context, the cultural games they play in the context, the mathematics embedded in their out-of-school context need to be connected with the teaching and learning of school mathematics.

CHAPTER III

THEORETICAL UNDERPINNING AND CONCEPTUAL FRAMEWORK OF THE STUDY

This chapter begins with brief discussion of constructivist learning theory. It includes brief history and characteristics of the constructivism. In my study, I focused how the constructivism is used to develop mathematical ideas in the classroom using cultural metaphors. Here, I have discussed about the Sociocultural Theory of Vygotsky, and Funds of Knowledge (FOK) Theory of Velez-Ibanez and Greenberg. As my study is intended to explore the cultural metaphors for the understanding of school mathematics, the metaphor practices regarding mathematical ideas in students' culture are primarily focused. The conceptual framework of the study explains the interconnectedness of cultural metaphor, ethnomathematical ideas and learners' out-of-school culture which supports to carry out my study.

Constructivism as the Theoretical Basis of Study

There is no all-accepted single definition of learning. For some, learning is observable changes in behaviour, other say it means acquiring new knowledge. Psychologists have made different philosophic assumptions about the nature of the learning process. The behaviourist believes that learning is determined by the forming connections between the stimuli and responses. The cognitive psychologists emphasise the inner mechanisms of human thought and mental activities. Whereas constructivist believes that people create their own new understanding based upon the interaction of their prior knowledge and surroundings (Richardson, 1994). In this section, I reviewed the characteristics of constructivist learning theory in the context of theme of my study.

The constructivism is latest catchword in educational circle, which applied both to learning theory and to epistemology. Epistemology is a detailed study of knowledge. From the philosophical point of view, constructivist epistemology seeks to investigate the scope, structure, and the nature of human knowledge (Pritchard & Woollard, 2010). From the pedagogical view, constructivism concerns about how to people learn. The constructivist believes that learning is the product of the interplay between the internal cognitive processes and the environmental conditions (Brooks & Brooks, 1993). That is learners actively construct their own meaning and knowledge by interacting with their environment.

The term constructivism is often used to describe as a theory that human knowledge is constructed by individuals and within social communities (Phillips, 1995). Constructivism has a strong voice in the current dialogue on mathematics education. The term constructivism refers to the idea that learners construct knowledge for themselves. It signifies that students need to construct their own understanding of each mathematical concept so that the primary role of teaching is not to lecture, explain, or otherwise attempt to transfer mathematical knowledge but to create situations for students that will foster their making the necessary mental constructions. The main proposition of constructivism is that learning is an active, creative process, which involves individuals interacting with their physical environment and with other learners. Bransford, Brown and Cocking (2000) describe constructivism as a theory of learning which holds that humans construct meaning from current knowledge structures. These arguments about the nature of human learning guide constructivist learning theories and teaching methods or strategies. Shepard (1989) describes the process of creating understanding as learners'

constructions of their own cognitive maps of the interconnections among concepts and facts.

Learning, in constructivist terms, is the process and result of questioning, interpreting and analysing information, but furthermore to be able to use the information to develop and alter meaning and understanding of concepts and ideas (Marlowe & Page, 2005). The constructivist framework was followed during the different stages in my study. The students were given rich learning environment and allow learners to create their own meaning by providing the different task during the teaching and learning inside and beyond the four walls of classroom. As my theoretical orientation about the knowledge generation is based on the premises of the constructivist philosophy, the children construct mathematical knowledge as a result of active participation in their social context. The learner construct mathematical knowledge based on their experience and interaction with peers. In every stage of data collection from the students' participants, I created the environment favourable to them for construct mathematical knowledge.

Constructivist Framework for Knowledge Construction

Teaching and learning are the complex human phenomenon. We use metaphors while something is difficult to explain during the teaching and learning. The metaphors that teachers and students use in the process of teaching and learning mathematical concepts are contextual and cultural based on everyday experiences. It is believed that people construct mathematical meaning about the phenomena on the bases of personal experiences and environment. The learner construct meaning based on their experiences and beliefs system within their particular culture. They are acquired, internalise and understand the mathematical language, symbol system from context that they insisted. The prior knowledge and experiences of children in their

culture help to establish the relationship with the new objects. The social constructivist theory emphasises the importance of culture and context in the process of knowledge construction. Students bring their own cultural experiences and knowledge into the classroom. Being multicultural classroom, there is distinctive ideas and knowledge to understand the mathematical concepts among the students as well as a teacher.

The five E's (Engage, Explore, Explain, Elaborate and Evaluate) learning cycle is a model of learning that is built in the constructivism theory of knowledge. The five E's is an instructional model based on the constructivist approach to learning, which says that learners construct new ideas on the basis of past experiences and ideas. In the five-pointed star, each point represents the cyclic process of knowledge construction. The mathematical knowledge is situated at the centre of the model diagram. Each of the five E's describes a phase of learning, and each phase begins with the letter "E": Engage, Explore, Explain, Elaborate, and Evaluate. The following figure 3.1 represents the how five E's allows students and teachers to experience common activities, to use and build on prior knowledge and experience, to construct meaning, and to continually assess their understanding of a concept.

Engage: This is the first phase in the process of constructivist knowledge construction. In this stage, the students first encounter and identify the instructional task. Here they make connections between past and present learning experiences, lay the organizational groundwork for the activities ahead and stimulate their involvement in the anticipation of these activities (Barufaldi, 2002). Asking a question, defining a problem, showing a surprising event and acting out a problematic situation are all ways to engage the students and focus them on the instructional tasks. If we are going to teach Prime and Composite numbers, first teachers ask few questions related to the

topic. For example, how many types of numbers are there? List the even and odd numbers, positive and negative numbers, fractions, counting numbers. Series of further questions arise. What is prime number? Give the examples and non-examples of prime numbers. How can you identify prime numbers? Then let student divides into small group of 4-5 members to work each other with this question. The teacher will provide the instructional materials and facilitate them to explore.

Explore: This is second phase of five E's design. In the exploration stage, the students have the opportunity to get directly involved with phenomena and materials.

Involving themselves in these activities they develop a grounding of experience with the phenomenon (Barufaldi, 2002). As they work together in groups, students build a base of common experience which assists them in the process of sharing and communicating. The teacher acts as a facilitator, providing materials and guiding the students' focus. The students' inquiry process drives the instruction during an exploration.

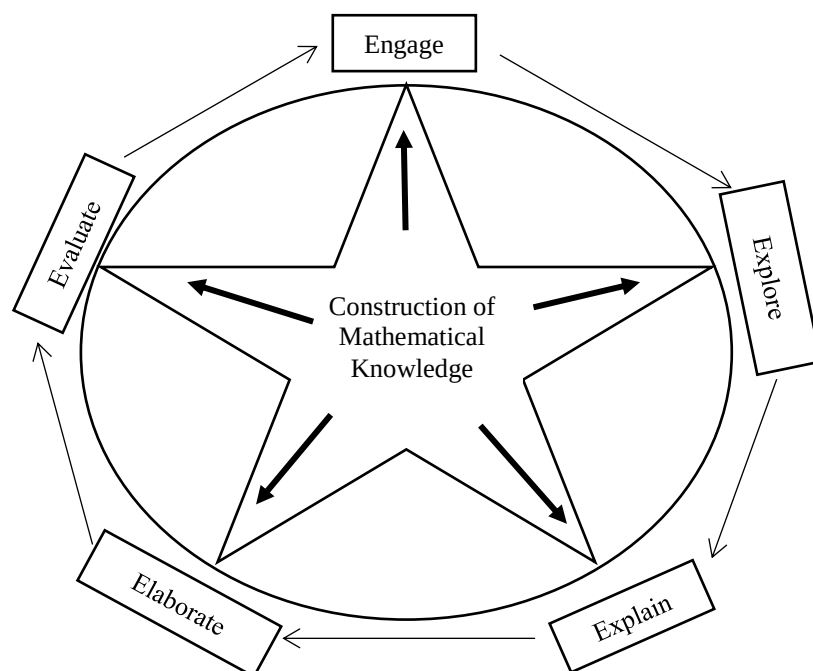


Figure 3.1. *Constructivist Framework for Knowledge Construction*

Explain: The third stage of this instructional design is explained. This phase of the five E's helps students explain the concepts they have been exploring. They have

opportunities to verbalize their conceptual understanding or to demonstrate new skills or behaviours. This phase also provides opportunities for teachers to introduce formal terms, definitions, and explanations for concepts, processes, skills, or behaviours (Barufaldi, 2002). For example, a child, through her exploration, may state they have noticed that the prime number can have only two rectangular form in geoboard. The facilitator, in her discussion with the child, might at this stage introduce terminology referring to prime numbers and the area representing in rectangular form.

Elaborate: This phase of the five E's design extends students' conceptual understanding and allows them to practice skills and behaviours. Through new experiences, the learners develop deeper and broader understanding of major concepts and apply their understandings to the world around them (Barufaldi, 2002). For example, student can elaborate that the prime numbers exist as bigger and smaller numbers. The smallest prime number is two. The largest prime number is at infinity. There is only one even prime number that is two. These connections of one type of mathematical concept often lead to further inquiry and understandings of other type of knowledge.

Evaluate: This phase of the five E's design encourages learners to assess their understanding and abilities. It is an on-going diagnostic process that allows the teacher to determine if the learner has attained understanding of concepts and knowledge. Evaluation and assessment can occur at all points along the continuum of the instructional process. Some of the tools that assist in this diagnostic process are rubrics (quantified and prioritized outcome expectations) determined hand-in-hand with the lesson design, teacher observation structured by checklists, student interviews, portfolios designed with specific purposes, project and problem-based learning products, and embedded assessments (Barufaldi, 2002). These evidences of

learning services guide the teacher in further lesson planning and may signal the need for modification and change of direction. For example, if a teacher perceives clear evidence of misconception, then he/she can revisit the concept to enhance clearer understanding. If the students show profound interest in a branching direction of inquiry, the teacher can consider refocusing the investigation to take advantage of this high level of interest. Viewing the evaluation process as a continuous one gives the constructivist instructional design a kind of cyclical structure. The learning process is open-ended and open to change. There is an ongoing loop where questions lead to answers but more questions and instruction is driven by both predetermined lesson design and the inquiry process.

Vygotsky's Sociocultural Theory

The central role of the social and cultural contexts in learning was articulated in a social constructivist theory developed by Vygotsky, a Russian who lived between 1896- 1934 (Hausfather, 1996). Vygotsky's theory states that human cognitive development is a socially mediated process. According to Vygotsky, social interactions, especially cooperative dialogues with more knowledgeable others, are necessary for children to acquire ways of thinking and behaving that make up a community's culture.

Vygotsky (1978) argued that an understanding of how knowledge develops requires an understanding of the social and historical origins of knowledge and of changes in that knowledge. He also proposed a central role for language in cognitive development. Vygotsky argued that human knowledge originates in socially meaningful activity and is shaped by language. Processes that originate in the social world are transferred to the inner mental world and children develop cognitive

understanding of the minds of others. In this regards Borich and Tombari (1997) explains:

The theme of Vygotsky's work is that a learner's cognitive development takes place in a social context. Throughout their lives, learners are surrounded by the parents, siblings, relatives, friends, teachers, and fellow students. They communicate with one another, stimulate one another. Parents and teachers, in particular, are more knowledgeable and skilled than learners and...Learners acquire knowledge about their culture and history from their encounters with adults, peers and the media. This cultural knowledge includes shared beliefs, ways of viewing the world, patterns of interacting with people and language (pp.53-54).

Thus the cognitive development is a child's increasing mastery over the culturally determined developmental tasks imposed by social agents. Therefore, part of the development of children's thinking requires apprenticeship into culturally specific cognitive and social practices. According to Vygotsky, cognitive development does not happen just in the head of the child. Rather, it is a process of learning to operate with physical, symbolic, and cognitive tools in ways that in themselves change cognitive processes. The difference between a child's individual performance and that child's performance when guided by experts is metaphorically described by Vygotsky's Zone of Proximal Development (ZPD). In this ZPD a learner, although not capable of solving problems independently, can do so with help of others. The ZPD was described by Vygotsky (1978) as

When it was first shown that the capability of children with equal levels of mental development to learn under a teacher's guidance varied to a high degree, it became apparent that those children were not mentally the same age

and that the subsequent course of their learning would obviously be different. This difference between twelve and eight or between nine and eight is what we call the zone of proximal development. It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (p. 86).

He opined that the ZPD is the current or actual level of development of the learner and the next level attainable through the use of mediating semiotic and environmental tools and capable adult or peer facilitation. In other words, it is what a child can do alone at a particular point in time. The second, potential development is defined as that which a child can achieve if given the benefit of support during the task. It is the ability to solve problems under adult guidance or in collaboration with more capable peers. This notion of an enhanced level of mental functioning when an expert guides an apprentice has been influential in education and in the study of learning disability. He conceives of the ZPD as central to instructional enhancement and classroom change in mathematics. The ZPD implies that a certain stage in development, learners can solve a certain range of problems only when they interact with teachers and cooperate with peers.

Vygotsky (1978) seeing learning as an activity in which shared mathematical meanings are constructed socially. He emphasized the social aspect of learning and the interplay of speech and action in children’s learning activities. In the same way, Borich and Tombari (1997) explain the Vygotsky’s term of ZPD in the following lines:

This zone encompasses a range of skills or abilities bounded on one side by what the learner can do independently and on the other side by the skills that

the learner needs adult assistance to perform. For any learners, there are as many zone of proximal development as there are development task to master (pp. 54- 55).

Thus, the principal theme of Vygotsky's theory is that a learner's cognitive development takes place in a social context. Socio-cultural theory emphasizes that the most advantageous learning environment is one where a dynamic interaction between teachers, learners and tasks provides an opportunity for learners to create their own understanding during interaction with others (Atkinson, Derry, Renkl & Wortham, 2000). Atherton (2005) emphasizes that in a socio-cultural classroom, learners are active makers of meanings and the role of the teacher is to guide learners to gain meaningful understanding of the learning material.

The concept of scaffolding is closely related to the ZPD, although Vygotsky himself never mentioned the term; instead, scaffolding was developed by other socio-cultural theorists applying Vygotsky's ZPD to educational contexts. Scaffolding is a process through which a teacher or more competent peer gives aid to the student in her/his ZPD as necessary. According to Balaban (1995), "scaffolding refers to the way the adult guides the child's learning via focused questions and positive interactions"(p. 52). Scaffolding is the process in which the student masters a skill under the guidance of an expert. There are four basic steps in the process (Balaban, 1995). The student observes the teacher modelling an activity. The student tries the activity under the guidance of the teacher. The teacher prompts with cues only when needed. The student is free to practice the skill independently.

The ZPD theory has significant implications in teaching and learning in the mathematics classroom. Regarding the use of Vygotsky's theory in the mathematics classroom, he recommends two approaches. First aspect is the use of assisted

instruction which is quite common for teachers to give students a lot of mathematical tasks as homework, without considering the sort of assistance that is available to the learner. According to Vygotsky (1978), problem-solving should be under the guidance of a competent or capable peer. Mathematics could be an enjoyable subject if there is expert intervention and is timorously rendered.

Vygotsky considers that collaboration is another crucial aspect for mathematics instruction. For this, teachers need to identify capable students in the class. Those who have low capacity for mathematical mastery should be assisted by the capable peers. Teachers should be convinced that the capable peers possess the mathematics ability before they are allowed to assist others. The more expert partner should provide support at the moments where maturing functions are inadequate. It is important to understand that a learner is able to perform a certain task alone, while in collaboration, is able to perform a greater number of tasks.

The teacher of mathematics considers individual zones of proximal development of learner. Teaching each learner mathematics according to his/her ZPD has two major advantages: the tasks are made simpler for the learner and the learner's intellectual ability to deal with the task is considered. Teaching within the ZPD implies providing mathematical activities that are of appropriate difficulty as measured by the mistakes that students make while trying to come through in a process that should be higher than zero for a student to learn. However, the tasks assigned to the learners in mathematics, sometimes fall outside the ZPD that the learner can already do, or tasks that the learner would not be able to do even with help. The theory implies that the challenge for all teachers is to pose problems that most students would perform with some assistance. One way to support this development is through scaffolding, which involves structuring the ideas to be

understood in an order that is likely to lead children to develop further and faster than they would on their own (Bruner, 1996). In mathematics learning, for example, a student may know a formula but may fail to apply it. With the assistance of the knowledgeable teacher or capable peers, the student learns to apply the formula.

Roosevelt (2008) posits that the main goal of education from Vygotskian perspective is to keep learners in their own ZPDs as often as possible by giving them interesting and culturally meaningful learning and problem- solving tasks that are slightly more difficult than what they do alone, such that they will need to work together either with another, more competent peer or with a teacher or adult to finish the task. The tasks given should be challenging to such an extent that mediation by the knowledgeable teacher or peer is needed. After the student had attained mastery of concept with the assistance from others, he/she should be able to do the task independently. If the student accomplishes the task individually through that process, then the learner's ZPD for that particular task will have been raised. This process is then repeated at the higher level of task difficulty that the learner's new ZPD requires.

According to Vygotsky (1978), the learning process should be based on the student's engagement in an activity where the teacher is the director of the social environment in the classroom, the governor and guide of the interactions between the educational process and the student (Vygotsky 1997). The teacher's job in teaching mathematics is to push the child into his or her zone of proximal development, coach for success with a task slightly more complex than the child can manage alone, and thus, push forward the area of independence. It is through repetition of such cycles that learners grasp new ideas, master new skills and become increasingly independent thinkers and problem solvers in mathematics (Daniels, 2001).

In a mathematics class, for example, the teacher may scaffold a multiplication problem by relating the problem to an activity that is familiar to the students, by reviewing skills needed to solve the problem, by providing tools for students to work with and by offering support while allowing the students to find their own solutions (Brown, Collins, & Duguid, 1989). Scaffolding is an interactive process of assessing and assisting and being sensitive to the needs and the readiness of the learner.

A big challenge for a classroom teacher is that they teach learners who all have different zones of proximal development. This suggests that scaffolding should be flexibly designed to meet the needs of diverse students. It should provide that extra support learners need to successfully complete a just out of- reach task in mathematics (Savery, 1998). In spite of the obvious limitations of scaffolding within the notion of the ZPD, scaffolding remains increasingly popular among educators, researchers and practitioners. The ZPD plays a critical role in mathematical instruction because it allows the teacher to find an entry point for each student's learning needs, so that his or her learning potential can be developed in mathematics.

Social constructivism encourages the learner to arrive at his or her own version of understanding, influenced by his or her background, culture or embedded worldview. Historical developments and symbol systems, such as language, logic, and mathematical systems, are inherited by the learner as a member of a particular culture and these are learned throughout the learner's life. The learner's cultural background can provide the necessary context for effective mediation (Nieman & Monyai, 2006). This also stresses the importance of the nature of the learner's social interaction with knowledgeable members of the society. Without the social interaction with more knowledgeable people, it is impossible to acquire the social meaning of important symbol systems and learn how to utilise them. From the social constructivist

viewpoint, it is thus important to take into account the background and culture of the learner throughout the learning process, as this background also helps to shape the knowledge and truth that the learner creates, discovers and attains in the learning process (Wertsch, 1997). This study examines the use of cultural metaphors in the process of teaching and learning that occurs through shared practices that emerge in the out-of-school community. As a community of practice, the teachers developed a shared set of instructional practices and pedagogical knowledge on the basis of cultural practices and mathematics teaching, and learning comes through experience and interaction within the context of their community of practice.

Funds of Knowledge Theory

Cultural capital is the central ideas of French philosopher Pierre Bourdieu's theory of cultural reproduction. Cultural capital means the cultural resources that involve attitudes, knowledge, behaviour, modes of appreciation that can be used to obtain access to other valued social resources (Pavic & Dukic, 2016). For them, these cultural resources indicate familiarity with the dominant culture. Bourdieu (1986) differentiates between three forms of cultural capital which are embodied, institutionalised, and objectified. Embodied cultural capital references cultural knowledge, attitudes, and behaviours that are inscribed in a person's habitus. Institutionalised cultural capital is manifested in educational credentials. Finally, objectified cultural capital refers to cultural artefacts.

Bourdieu believes that the possession of cultural capital into economic and social advancement because of the convertibility of cultural capital into economic capital. Children of the "dominant class" are crucially advantaged over the children of subordinate classes in that they enter the educational system already well prepared to succeed within it. In their case, a clear continuity exists between the culture of the

home and that of the school. These children will share a common mode of speech, style of social interaction, and aesthetic orientation with their teachers. For those children, neither the content of what they are taught (syllabus) nor the manner in which they are taught (pedagogy) is likely to appear strange to them. In contrast, for children from the working class, the school will represent an alien and indeed a holistic environment.

As human is social organism, the children are often bound in social, cultural relationship. The broad features of children's learning are developing in the various social and cultural institutions. Children's home as well as their surroundings contain lots of resources of knowledge and helps to connect to the teaching and learning school curricula. That concept of knowledge was first introduced by anthropologists Velez-Ibanez and Greenberg (1992) termed as Funds of Knowledge (FOK). The concepts of FOK theory is based on the premises "No learner is a blank slate". That means "the people are competent, they have knowledge, and their life experience have given them that knowledge" (Gonzalez, Moll & Amanti, 2005, p. ix-x). They described FOK as an array of knowledge and skills that are of strategic importance to working-class households living (Velez-Ibanez and Greenberg, 1992). Further, the term FOK is defined by researchers Luis Moll, Cathy Amanti, Deborah Neff, and Norma Gonzalez (1992) and they offered the definition of FOK "to refer to the historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being" (p. 133).

The term FOK is coined to describe the prior knowledge that students bring into the classroom from their cultural and experiential backgrounds. The concept of funds of knowledge is based on premise that the people are competent, they have the knowledge, and their life experiences have given them that knowledge (Gonzalez,

Moll & Amanti, 2005). With this new knowledge, they can begin to see that the households of their students contain rich cultural and cognitive resources and that these resources can and should be used in their classroom in order to provide culturally responsive and meaningful lessons that tap students' prior knowledge. From the cognitive viewpoint, FOK is extremely useful classroom tool because they base new learning on prior knowledge, building a deep understanding of big ideas within the variable domain of mathematics. Information that teachers learn about their students in this process is considered the student's funds of knowledge. The theoretical concepts of FOK facilitate a systematic and powerful way to represent communities in terms of resources, the wherewithal they possess, and how to harness these resources for classroom teaching. Gonzalez (1995) writes,

The basic premise of Funds of Knowledge for teaching is that classroom learning can be greatly enhanced when teachers learn more not just about their students' culture in an abstract sense but about their particular students and their students' households. ... The teachers then draw upon that knowledge to develop curricula and teaching innovations that have roots in the experience and forms of knowledge of the students and of the community. (p. 3)

FOK for teaching is grounded on the theory that household and community knowledge can provide strategic resources for classroom practice. It is assumed that all households have historically accumulated and culturally developed bodies of knowledge and skills on which they draw for daily survival and wellbeing (Moll & Greenberg, 1990). As Gonzalez (1996) writes, "this assumption is critical in terms of reconceptualizing households, not as the source of barriers to educational attainment, but as repositories of resources that can be strategically tapped" (p. 3).

In order for teachers to gain this kind of knowledge about the households and social networks of their students, teachers must be willing to go into the homes and communities of their students to observe and learn not simply about, but also from and with their students and the families of their students. In order to become “researchers” about students’ funds of knowledge teachers must be willing not only to talk more to their students but to be willing to travel to the homes of their students to meet and visit with the family members of the child’s household. Most of the literature on funds of knowledge involves teachers collaborating with ethnographers and conducting ethnographic fieldwork. These teachers must learn ethnographic methods as well as reflexivity (Moll et al., 1992; Gonzalez et al., 1993; 2005).

It is important to highlight a critical component embedded in the definitions provided by Velez-Ibanez and Greenberg (1992) and Moll and Gonzalez (1994): The notion of funds of knowledge appears to share some commonalities with other types of capital, including the human, social, cultural, linguistic and symbolic capital. For example, similar to Bourdieu’s (1986) definition of social capital, the concept of funds of knowledge contemplates the fact that belonging to a cluster of households can translate into benefits for its members (Velez-Ibanez and Greenberg, 1992). Furthermore, there seems to be a link between the notion of practices, embedded in the concept of funds of knowledge, and Bourdieu’s (1977) notion of habitus, defined as a set of dispositions through which the world is perceived, understood and evaluated. The connection between these two concepts stems from a common understanding of culture, not as a static and normative notion, but as something dynamic that is experienced in everyday life. The empirical study by Fasheh (1991), formally trained in mathematics, wrote about his illiterate mother’s mathematics embedded in her practice of sewing,

My mother's math was so deeply embedded in the culture that it was invisible through eyes trained by formal education... The math she was using was beyond my comprehension. ... It was a shock to me to realise the complexity and richness of my mother's relationship to mathematics. Mathematics was integrated into her world as it never was into mine' (p. 59).

Similarly, Millroy (1992) writes about what she considers to be a paradoxical situation in ethnomathematics work, 'how can anyone who is schooled in conventional Western mathematics 'see' any form of mathematics other than that which resembles the conventional mathematics with which she is familiar?' (p.11). Socio-cultural studies in mathematics and science education have argued that cultural resources and funds of knowledge (Gonzalez, Andrade, Civil & Moll, 2001) of people from non-dominant and underprivileged backgrounds are often not leveraged (Barton & Tan, 2009) in school teaching and learning practices. Neither is their knowledge from everyday life experience valorized (Abreu, 2008) and built upon in the classrooms nor is their identity acknowledged.

Conceptual Framework of My Study

The primary goal of my research objective was to connect the learners' out-of-school mathematical ideas and knowledge with the school mathematics. The conceptual framework of my study is based on the theoretical framework of the Bourdieu's cultural capital and the funds of knowledge theory of Velez-Ibanez and Greenberg and sociocultural theory of Vygotsky. The framework developed in this research study shows how the characteristics of the implementation design can be described based on a constructivist learning environment.

As my study was intended to facilitate the teaching and learning of school mathematics through the exploration of cultural metaphors, this framework puts the

school mathematics at the centre of the research environment. The mathematical understanding is important for everyone and they are using mathematical ideas and knowledge to perform everyday work. The mathematical knowledge and ideas are implicitly uses in out-of-school culture. The learners culture and everyday activities regarding ethnomathematical ideas is an integral part of education in general and learning mathematics in particular. It is believing that classrooms and other learning environments cannot be isolated from the communities in which they are embedded, and students come to school bringing with them values, norms and concepts they have acquired from their culture and environment. The students' home culture and out-of-school mathematical ideas and knowledge provides opportunities to connect to the formal mathematics.

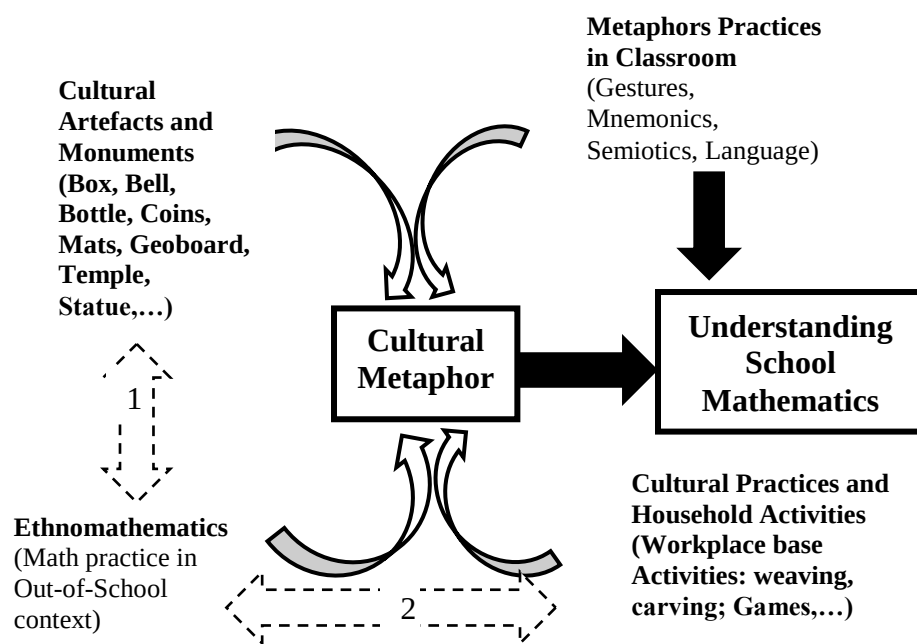


Figure 3.2. Conceptual Framework: Connecting School Mathematics through Cultural Metaphors

Figure 3.2 shows a conceptual framework to connect the students' out-of-school knowledge, embedded mathematical ideas in their cultural context and the learners' cultural metaphors to school mathematics. This framework allows for a wider set of out-of-school culture environment has possibility in the discussion of

school mathematics. Cultural practices and cultural artefacts may not always be mathematical as Millory (1992) viewed that mathematics is not everywhere. Thus, the connection (1) from between cultural artefacts and monuments, and Out-of-school mathematics (Ethnomathematics), as well as the connection (2) from cultural practices and Ethnomathematics, are no longer straightforward (represented here by the dashed arrows). However, cultural artefacts and monuments, ethnomathematical ideas in out-of-school context, cultural practices and household activities in learners' home culture, and the metaphors practices in classroom are highly connected with the cultural metaphors. That is to say, it is possible to connect metaphors practices in classroom and the cultural metaphors to school mathematics is straightforward. This suggests that it is possible to connect the school mathematics through the cultural metaphor, ethnomathematical ideas and the out-of-school context of the learners.

Summary of the Chapter

I have reviewed the sociocultural theory of Vygotsky, Funds of Knowledge theory of Velez-Ibanez and Greenberg and constructivist theory of knowledge construction and transmission. The funds of knowledge theory are the principal theory of my study. FOK is grounded on the theory that household and community knowledge can provide strategic resources for classroom practice. Children bring some sort of mathematical knowledge and ideas while entering to classroom which contributed to the learning of school mathematics. FOK is based on the premises "No learner is a blank slate" as opposed to the philosophical ideas "tabula rasa" of John Locke. The mathematical understanding is important for everyone and they are using mathematical ideas and knowledge to perform everyday work. The mathematical knowledge and ideas are implicitly uses in out-of-school culture.

CHAPTER IV

RESEARCH METHODOLOGY

This chapter discusses philosophical dimensions of qualitative research design adopted in my study. My study intended to explore the cultural metaphors practiced in out-of-school culture to understand mathematical knowledge and ideas and then analyze the contribution of cultural metaphors in the process of teaching and learning of school mathematics. The exploration of students' everyday activities regarding mathematical ideas and knowledge outside the school environment play paramount importance in the process of data collection. The observations of different activities performed by the members of the diverse groups of people from out-of-school community regarding mathematical ideas and knowledge were the main source of data. The metaphors used by the teachers and students being based on their experiences and everyday activities were also the source of my data.

In my research, I have chosen qualitative research design as I wanted to make sense of the complex world of the mathematical knowledge embedded in the out-of-school context of the students and teachers' understanding in teaching learning of mathematics. It would not be possible for me to quantify such ideas, perceptions and knowledge in numbers. The particular paradigm I adopted to conduct my research was multimethod, a blend of two different qualitative methodologies: ethnography and action research.

I chose ethnographic methodology to explore the cultural metaphors regarding mathematical knowledge, ideas and concepts embedded in the out-of-school culture of learners. For the same, I used observation, in-depth interviews, and study of artefacts as my principal method for the study. The contribution of cultural metaphors in the process of teaching and learning of school mathematics was viewed through the

collaborative action research. It aimed to observe the effective change in the practices of teaching and learning in school and improving students understanding of the concepts. The teacher facilitation program was conducted in the sample school in order to acquaint cultural metaphors in education process. This chapter discusses procedures of collection, recording, analyzing of the qualitative research data gathered from the field. The ethical issues were also addressed during the data collection, analysis and interpretation. This chapter finally sums up with the ways of data authentication and delimitations of the study.

Philosophical Underpinnings in My Research Journey

Everyone has own perspective to understand the world. In other words, individual's way of doing, thinking and the activities are based on their own philosophical ideas. The same things occur in the case of doing research, although philosophical ideas remain largely hidden in research (Slife & Williams, 1995). They still influence the practice of research and need to be identified (Creswell, 2009). In this process I have used the qualitative research for exploring and understanding the meaning of the complex world of the social and cultural behavior of the different group of people's out-of-school environment about mathematics.

The ontological issue relates to the nature of reality and its characteristics (Creswell, 2007, p.16). My ontological understanding in present study is that there are multiple ways to understand the mathematical ideas, concepts and knowledge in the out-of-school culture. It is also assumed that the mathematical knowledge is subjective. Bishop (1991) identified counting, locating, measuring, designing, playing and explaining exists in all culture and societies and they have different ways to perceive the reality.

The particular paradigm I have adopted to conduct my research is interpretative which Denzin and Lincoln (2005) refer to a set of approaches where the central research interest is the meaning that humans give to their experiences and social interactions. Interpretative research involves long-term participation in a field setting, and careful recording and collecting of data. This is followed by reflection and writing using rich descriptions, narrative descriptions and direct quotations (Erickson, 1986). My epistemological assumption seeks to observe about what the relationship between the researcher and that being researched is (Creswell, 2007). In my study, I have tried to get as close as possible to the research participant that provide easy access to observe their everyday activities and capture necessary information from their culture.

My sources to carry out this study is cultural metaphors to communicate everyday mathematical activities embedded in out-of- school environment practiced by different group of people. I believe that informal mathematical knowledge is generated and handed down from generation to generation in their own ways and it has to recognize and pay high regards. In this study, I equally like to admit the stories and the narrative of the research participants and includes my interpretation and presentation (Creswell, 2007).

I have applied qualitative study procedure as by Creswell (2007) who said that “the procedures of qualitative research methodology are characterized as inductive, emerging, and shaped by the researcher’s experience in collecting and analyzing the data” (p.19). In this sense, my study is a bottom up approach to research. It is especially useful for generating and constructing new ideas and theories. In this study I have adopted inductive method that follows three steps: observe the world, search for a pattern in what is observed, and make a generalization about what is occurring.

Research Design: Qualitative Worldview

The purpose of this study was to explore cultural metaphors practiced in out-of-school environment related to the mathematical ideas, concepts and knowledge. Also this study analyzed the contribution of cultural metaphors in the process of teaching and learning mathematics at the basic level of education. To achieve these objectives, this study demands multiple methods in its own right. Particularly this study adopts sequential qualitative research methods of ethnography and action research. The everyday mathematical ideas and knowledge practiced in the out-of-school environment were observed and documented through the methods of qualitative research design. The contribution of cultural metaphors in the process of teaching and learning of mathematics also demands quantitative method. However, in this study I used collaborative action research to measure the effect of cultural metaphors in the teaching and learning of school mathematics.

I am aware that qualitative research relies primarily on the collection of qualitative data. Qualitative research is a field of inquiry that crosscuts disciplines and subject matters (Denzin & Lincoln, 2005). Qualitative research also claims to describe life-worlds from the inside out, from the point of view of the people who participate (Flick, Kardorff & Steinke, 2004). So as a qualitative researcher, I have aimed to acquire an in-depth understanding of human behavior and the reasons that govern human behavior. Qualitative research relies on reasons behind various aspects of behavior and their daily lives. In my research, I chose qualitative research method as I want to make sense of the complex world of the cultural practices, knowledge and understanding for the mathematical ideas embedded in everyday activities in out-of-school culture. The descriptive data in the form of words and observations were obtained from the natural setting of the participants. The ideas, perceptions, beliefs

and understanding regarding the mathematical ideas and concepts would not be possible for me to quantify in figures and numbers.

Following Glesne and Peshkin (1992) I have been a qualitative researcher. So my role was confined to a meaning maker who draws on own experiences, knowledge and theoretical outlooks, to collect data and to present their understanding to the world. Unlike other research approaches, I have attempted to answer my research questions holistically i.e., contextually. The setting, its people, their activities, their interactions, and their points of view were all taken into account. As qualitative researcher I have interviewed participants, spent time in the setting in order to understand the context in which behaviors occur, and reviewed documents related to the focus of the research in order to construct the meaning a particular situation (Bogdan & Biklen, 2007). In line with Creswell (2007) I have chosen ethnographic research design to gain knowledge and understanding of a particular culture-sharing group.

Multimethod Research Model: Ethnography and Action Research

Collier and Elman (2008) viewed the idea of multimethod research as encompassing three different meanings: the heterogeneity of qualitative methods, the interconnections between qualitative and quantitative research procedures, and the relationship with interpretative and constructivist methods. In my research, I adopted the multimethod, which is viewed as the heterogeneity of qualitative methods to answer my research questions. This study was blended with two different qualitative methodologies: ethnography and action research.

The ethnography was used as a methodology in which I studied everyday activities of out-of-school cultural group regarding the mathematical ideas and knowledge in a natural setting over a prolonged period of time (Creswell, 2009) by

collecting through participant observation and in-depth interview as well as the document analysis of the cultural group. The action research was conducted at the second stage of the study. I chose action research because I was trying to answer the research questions: “How do teachers use cultural metaphors during mathematics teaching at the basic level of education? And what effects do they have on students’ understanding of the subject matter?” Both are of mutual interest and benefit (Naughton & Hughes, 2009) to my collaborative partners. The detail of the methodologies that I adopted in my process of inquiry, the rationale to choice in my study have been described in the consequent sections.

Ethnography and its Choice in My Study

One of my objectives of this study was to collect and document the cultural metaphors embedded in the out-of-school context. With this objective in mind I realized that mathematics is pan-cultural phenomena (Bishop, 1988 as cited in Clements and Ellortien, 1996). It is used in everyday activities of all human culture. The mathematical ideas and concepts are embedded in the different sectors of lives. However, previous study exhibited that our school mathematics is alien from the out of school mathematical concepts and ideas.

As my study was intended to investigate the contribution of cultural metaphors for the understanding mathematics, I explored the cultural metaphors regarding the mathematical ideas, concepts and knowledge practiced in out-of-school environment of the learners’ community. To overcome my research objective, I resolutely situated myself as a qualitative researcher. Choosing the most appropriate methodology that reflective process led me to select ethnography. I chose ethnography as the methodology because it looks at mathematical ideas practices in the everyday activities of group of people as a sociocultural process (Taylor & Bogdan, 1998). It is

inward looking to uncover the tacit knowledge through investigation. That is, the study seeks to uncover and describe the beliefs, values, perceptions, understanding and attitudes towards the cultural metaphors regarding mathematical ideas in their everyday activities (Merriam, 2009).

I realized that ethnography is a primary methodology for collecting empirical data from out-of-school culture regarding the mathematical ideas, knowledge, and practices in their everyday activities. The process of collecting field notes in ethnographic research has the ability to inform several methods of observations, in-depth interview, document analysis and study of artefacts. Therefore, ethnography reveals human complexity that may not otherwise be available through other methodologies. The culture is how people make meaning from their experiences, and it is not bounded by a single place or uniform behaviour. I used ethnography to describe, interpret, and reveal the meaning of cultural activities regarding ethnomathematical ideas embedded in my informant's everyday lives.

As defined by Lecompte and Schensul (1999), an ethnography promises to appropriately aid in the discovery of insight into the questions of interest. They write:

Ethnography is an approach to learning about the social and cultural life of communities, institutions, and other settings that is scientific, is investigative, uses the researcher as the primary tool of data collection, uses rigorous research methods and data collection techniques to avoid bias and ensure accuracy of data, emphasizes and builds on the perspectives of the people in the research setting, and is inductive, building theories for testing and adapting them for use both locally and elsewhere. (p. 1)

While conducting this research, I continuously addressed the questions of interest involving cultural metaphors embedded in everyday activities regarding

mathematics. The metaphors used by teachers and its contribution for the development of mathematical concepts in the students motivated for positioning myself to engage in the research techniques as described within an ethnographic paradigm. It was my intent to gain insight into the social and cultural life of the classroom, use rigours in my observations, and attempt to frame the perspectives of the people in the research through sustained interaction.

The process of data collection in ethnography is extensive. Ethnography emphasized the importance of listening and asking appropriate questions, observing the day-to-day lives of the group people in natural setting. Although the main data gathering is accomplished through participant observation, interviews are also an important part of the research, with the opportunities for interviews many times occurring as a natural event, not having to be scheduled (Flick, 2006). In this process, I studied Spradley (1979) who describes that ethnographic interviews are “friendly conversation into which the researcher slowly introduces new elements to assist informants to respond as informants” (p. 166). But regarding the ethnographic data recording, I chose from several classical and contemporary positions and recorded behavior and activities of the people in natural setting through field notes (Angrosino & Rosenberg, 2011). These field notes include the traditional paper and pencil as well as more contemporary media, such as photographs, video or audio recorders, which can capture the lived experiences of the person or group within the context of the social setting. Here, I found that ethnographic research is an inductive process in which events occur within a culture that influence the research purpose and questions.

Choice of Action Research in My Study

The term action research was introduced by social psychologist, Kurt Lewin in his paper Action Research and Minority Problems (Kemmis & McTaggart, 2005).

Action research is a form of research which is intended to solve classroom problem and immediate improvement of the teacher practice. In this connection, I followed Kemmis and McTaggart (2005) who described “classroom action research typically involves the use of qualitative interpretative modes of inquiry and data collection by teachers often with the help from academics with a view to teachers making judgements about how to improve their own practices” (p. 561). Through this research, I have gathered information on the problem they faced during the classroom teaching and tried out to make some other changes in practices.

My research has been focused on change in teachers’ classroom practices related to the incorporation of cultural metaphors into mathematics teaching and learning. Thus, my action research generated the concerns of people in an immediate problematic situation. Allan Feldman (2002) provides a definition of action research. In his words:

Action research happens when people research their own practice in order to improve it and to come to a better understanding of their practice situations. It is action because they act within the systems that they are trying to improve and understand. It is research because it is systematic, critical inquiry made public. (p. 240)

He also explains that action research can help to develop a professional community, illuminate the power of relationships in educational situations, and help one recognize their own expertise. Further, Mills (2003) explains,

Action research is any systematic inquiry conducted by teacher researchers to gather information about the ways that their particular school operates how they teach, and how well their students learn. The information is gathered with the goals of gaining insight, developing reflective practice, effecting

positive changes in the school environment and on educational practices in general, and improving student outcomes. (p.4)

Keeping both the definitions in my mind, I came to conclude that action research is obviously an effective means to make positive changes in favour of student by improvement of teachers and administrators in their practices. It is based on the premise that the teachers are fully involved as voluntary participants (Clements & Ellerton, 1996) to improve their own teaching practice to overcome a problem encounter in the process of teaching and learning. Faced with the problem, the action researcher goes through a series of phases (plan, action, observe, reflect) called the action research cycle to systematically tackle the problem.

The definitions and the importance of action research made me interested to use it as a method of my study. As a teacher and researcher, I tried to capture the same concept, some have more steps and articulate them more fully, but the essence of learning through planning, acting, observing and reflecting on the outcomes, and reworking as needed exist throughout. Usually, I observed that “teachers do plan their lessons, enact their plans in their classrooms, observe the effect of their actions and reflect on what they might do differently next time” (Clements & Ellerton, 1996, p. 115). However, the process might not be as neat as this spiral of self-contained cycles of planning, acting, observing, and reflecting (Kemmis & McTaggart, 2005). With this realization in mind, I made a common image of action research that it is spiral and one cycle of planning, acting, observing and reflecting leads to another in the light of learning from experience. In each cycle, I reflected and used my participants as researcher and change makers.

The collaborative action research (CAR) is the joint research between two or more teachers or between universities and teachers. They collaborate and influence in

changing the curricular approach, and their focus is on practical problems of individual teachers or schools (Raymond, 2004). Regarding CAR, Sumekh (2006) viewed that

“Action research is conducted by a collaborative partnership of participants and researchers, whose roles and relationships are sufficiently fluid to maximize mutual support and sufficiently differentiated to allow individuals to make appropriate contributions given existing constraints”. (p.7)

Fasheh (1997) viewed that the mathematics teaching and learning cannot be taught effectively and meaningfully without relating it to culture or to the individual students. All the stakeholders of mathematics education presume that out-of-school context would be the powerful source and funds of knowledge for mathematics teaching and learning. However, connecting mathematics teaching with the students’ out-of-school culture is really great problem for the teachers as well. There was common interest and mutual understanding between the mathematics teachers of sample school about the incorporation of cultural metaphors in teaching and learning of school mathematics. My research based on CAR aimed to connect out-of-school context as cultural metaphors to improve the classroom practices of the teachers. This methodology enabled me to find

out more appropriate ways of teaching aimed at acquisition of basic mathematical concepts through the cycles of collaborative planning, acting, observing and reflecting (P-A-O-R) model.

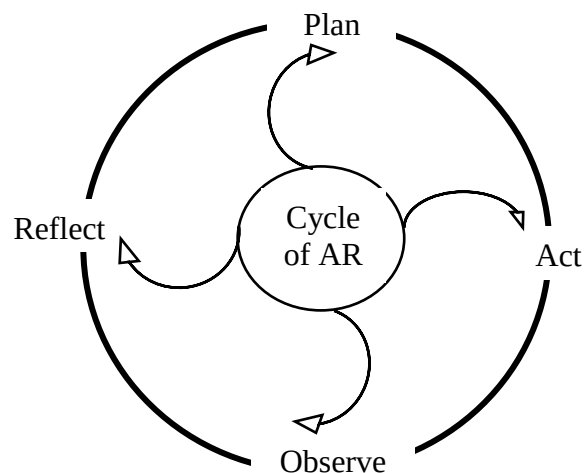


Figure 4.1. Cycle of Action Research: P-A-O-R Model

As my study is intended to analyse the contribution of cultural metaphors in the process of teaching and learning of school mathematics, it is important to incorporate students' out-of-school mathematical ideas and concepts in the classroom. Regarding the incorporation of cultural metaphors of the students', Paulo Friere (1970) suggested that "children's cultural capital—the knowledge children bring to school from their home and cultural environment—should be welcomed and utilized in school for teaching and knowledge building purposes" (as cited in Stringer, Christensen & Baldwin, 2010, p.24). In this way, students' out-of-school knowledge embedded in their culture is celebrated and utilized as the cultural metaphor in an appropriate way in the classroom teaching and learning process.

The framework of my action research is modified version of the framework given by Stringer, Christensen, and Baldwin (2010) in their book "Integrating Teaching, Learning, and Action Research: Enhancing Instruction in the K-12 Classroom". The meaningful mathematics teaching and learning provide the means for teachers to reveal within each student—what they know and can do, and to use them to move into the unknown—what they must learn (Stringer, Christensen, & Baldwin, 2010). This framework is based on the theory of conceptual metaphor that indicates the connection between source domain and the target domain for meaningful mathematics teaching and learning. The source domain is the students' cultural capital regarding mathematical ideas, concepts and knowledge embedded in their out-of-school context. The connection of these concepts and ideas in the teaching and learning to enhance school mathematics is target domain (learning objectives) of the conceptual metaphor.

In this framework for collaborative action research, we, the research group, identified students' familiar context and mathematical knowledge embedded in work

places activities, household activities, shopping and other everyday activities, and playing games in out-of-classroom context. The contextualization of school mathematics enables them to successfully accomplish the rewarding task of facilitating their students' learning. The understanding of learners' culture and their out-of-school activities not only enables teachers to gain a greater understanding of their students, but also provides rich material to incorporate into classroom learning activities (Stringer, Christensen, & Baldwin, 2010). It also provides an opportunity for teachers to understand their students in a richly meaningful way, and in the process, develop relationships that enable them to accomplish their work more effectively. In the subsequent sections, I have described the selection of study location, selection of research participants, procedures for ethnographic data collection, procedures of data collection for action research, data recording procedures, data analysis and interpretation, data authentication, ethical considerations and delimitations of the study.

Selection of Study Location

My study was intended to explore the cultural metaphors regarding the mathematical ideas, concepts and knowledge of the group of people and connecting it to the teaching and learning of school mathematics. I selected Kathmandu as the study location as it is the center of social, cultural and political activities. The location of the study was a community which had been dispersed in household based workshops, small scale business, manufacturing, trade and service of Kathmandu district.

The school that I selected is the government funded community school. Most of the students of this school were from multilingual, multi-religious, multiethnic community basically migrated from different parts of the country. The community of the students were largely engaged in the small-scale business, household activities

including food cooking, weaving, carving stone and wood, workplace activities, garment stitching, carpentry, leather goods, pottery and other household activities. The low-income settlement is economically active with diverse workplaces, activities and their practices create varied opportunities for school going children (Bose, 2014) and it also provides the funds of knowledge for teaching and learning of school mathematics. Considering this, I focused my study in those settlement of Kathmandu.

Selection of Research Participants

As my study is qualitative in nature, it does not seek for representative characteristics of the large population (Gay, 1987). However, it demands three most common sampling methods: purposive sampling, quota sampling, and snowball sampling (Mack & Woodson, 2005). Among them, I chose purposive sampling. Purposive sampling is one of the most common sampling strategies in qualitative research and group participant according to pre-selected criteria relevant to a particular research question. Sample sizes, which may or may not be fixed prior to data collection, depend on the resources and time available, as well as the study's

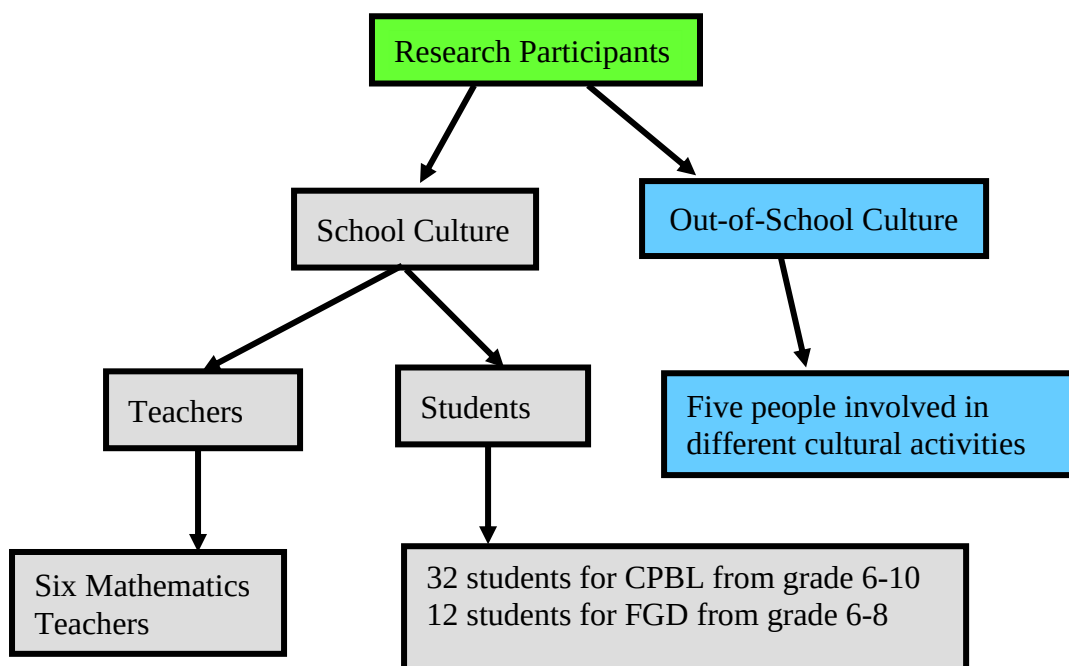


Figure 4.2. Selection of Research Participants: A Tree Diagram

objectives. Its sample sizes are often determined on the basis of theoretical saturation. It is non random sampling used in ethnographic research (Neuman, 2008).

Keeping these characteristics of the purposive sampling, I purposively selected five people involving in different income generating cultural activities. The cultural metaphors regarding mathematical ideas embedded in out-of-school environment were captured by using ethnographic tools. To achieve my second research question, six school mathematics teachers teaching at basic level of education were selected. In the same, twelve students were selected for FGD from grade six to eight and 32 students from grade 6 to 10 were selected for Cultural Project Based Learning (CPBL). The details of the students' participations in this study are presented in Appendix VIII.

Procedural Framework for Data Collection

The collection of data in a qualitative research study is ongoing process and it do not terminate until the report have been submitted. The data collection steps

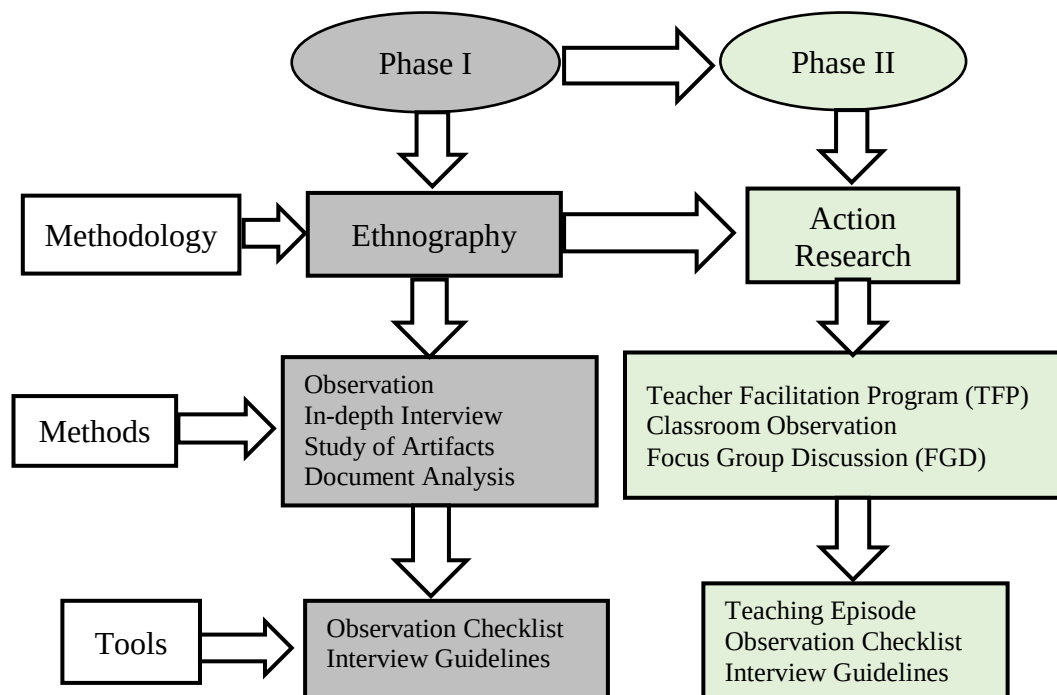


Figure 4.3. *Procedural Framework for Data Collection*

include the setting the boundaries for the study, collecting information through the observations, interviews, document analysis and visual materials including images, audio-video materials as well as establishing the protocol for data recording procedures (Creswell, 2014). The figure 4.3 shows the procedural framework for data collection to achieve my research objectives.

As my research design is based on the multimethod model, obviously there are multiple ways of data collection procedures to answer the different research questions. My study was conducted in two phases: the ethnography (Phase I) and the collaborative action research (Phase II). The ethnographic data of group of people and their mathematical ideas and concepts embedded in their everyday activities was explored through the observations, interviews, documents analysis and study of artefacts. While the contribution of out-of-school mathematical ideas and concepts in the process of teaching and learning of school mathematics was observed through the P-A-O-R model of collaborative action research. The data collection tool and process of CAR are presented in Chapter VI in sufficient detail.

Procedures for Ethnographic Data Collection

The data collection tools and techniques are important to gather the data from the field. Following Creswell (2014), I gathered multiple forms of data through interviews, observations, and documents. For this, I developed interview guidelines, observation protocols, protocols for study of cultural artefacts and format for document analysis before entering to field.

Initial Field Visit

I visited the some community of Tokha municipality of Kathmandu district and observed the scenario of everyday life of the group of the people. I also spent one week in the community. I prepared the field notes for everyday. The major themes I

observed during this initial field visit were the community members engaging in workplace activities, metal works, small scale business, garment stitching, textile weaving, wood carving, household activities, restaurant, food cooking and so on. I also observed cultural arts, traditions, artefacts, monuments and other cultural activities. I talked with some of the people who were involved in their everyday income generating work. The experience from my initial field visit helped me to develop more reliable tools for my study.

Development of Tools for Ethnographic Study

The community was rich in different cultural activities regarding mathematical knowledge and ideas. After the initial field visit, I explained the details of my experience with my research supervisors. I also distributed the developed tools in a small group of teacher educators and teacher trainers to establish the validity of the tools. Considering the suggestion, I reviewed the observation protocols and interview guidelines and then developed final tools to gather information from the field. In this way, the validity of the tools was established by the help of expert's judgement.

Table 4.1

Tools and Respondents for Ethnographic Data Collection

Tools	For whom	For what
Observations Protocols	Parents (5)	To collect metaphors from their everyday activities.
		To observe the mathematical ideas embedded in monuments and cultural artefacts.
Interview Guidelines	Parents (5)	To explore the mathematical ideas embedded in their activities.

Participant Observation

Participant observation is a qualitative method with roots in traditional ethnographic research, whose objective is to help researchers learn the perspectives

held by study populations. Erickson (1985) uses “the term interpretive refers to the whole family of approaches to participant observational research” (p.119), one of the most common types of qualitative methods. Regarding the participant observation, Fraenkel and Wallen (1993) opine:

In participant observation studies, researchers actually participate in the situation or setting they are observing. Participant observation can be overt, in that the researcher is easily identified and the subjects know that they are being observed, or covert, in which case the researcher disguises his/her identity and acts just like any of the other participants. (p. 390)

Participant observation always takes place in community settings, in locations believed to have some relevance to the research questions. While in these community settings, I as a researcher did careful observation and wrote field notes about what they see. I also have to make specific schedules according to the need of my research participants. When using participant observation as an ethnographic research method, I entered the world of the people to study (Wolcott, 2008). I developed observation checklist, which is presented in Appendix II.

During my study, I observed the activities of research participants where they were engaged in day-to-day activities. I observed the household activities of the children, their participation in playground, in the street, interaction with peers, in their homeroom, in their school classes, at lunch, and leaving the school with their permission. I spent round the clock with them. I sat for the meals and talked about the knowledge whenever they felt like talking with no hesitation. I followed them, with their permission, wherever they moved about. But less structured, unscheduled participant observation also occurred anytime I moved around their community and interacting with the people. I talked with people on the streets, at water taps, buffalo

shed, at marketplace, workplaces etc. I jotted down such things in my notebook so that I could take advantage of spontaneous opportunities without relying completely on memory.

While I was observing, I took extensive field notes. My field notes included two types of information: descriptive and reflective. The descriptive part of the field notes captured the detail of physical setting, the people involved in the interactions observed, accounts of the interactions observed, the reconstruction of any dialogue, and the behaviours of the participants in the setting (Taylor & Bogdan, 1998). As noted, in addition to descriptive material, my field notes contained reflective information, the subjective part of the observation experience. The emphasis of the reflective part of the field notes was “on speculation, feelings, problems, ideas, hunches, impressions, and prejudices” (Bogdan & Biklen, 2007) relative to the student’s inclusion in high school general education classes. They were interspersed with the descriptive notes.

In-depth Interview

Interviews are most common method used to collect the data in qualitative research. The type of interview conducted by the researcher is determined by the type of information required to carry out the research problem. There are three types of interview mostly used in qualitative research and they are structural interview, semi-structural interview, and unstructured or in-depth interview (Seidman, 2006; Taylor & Bogdan, 1998).

The in-depth interview is a technique designed to elicit a vivid picture of the participant’s perspective on the research topic. During in-depth interviews, I considered the person being interviewed as an expert and me, the researcher as a student. I engaged with participants by posing questions in a neutral manner, listening

attentively to participants' responses, and asking follow-up questions and probes based on those responses. I did not lead participants according to my preconceived notions, nor did I encourage them to provide particular answers by expressing approval or disapproval of what they say for "greater breadth" of the information (Fontana & Frey, 2005).

In my study, I used this technique of in-depth interview to gather the data from the community members engaged in different cultural activities regarding the mathematical ideas. I listened carefully and recorded the cultural metaphors to practice mathematical ideas during their everyday activities. The paper and pencil was used to make field notes and their voice was recorded using voice recorder. The interview was conducted based on interview guidelines for parents presented in Appendix I.

Documents Analysis

Document analysis is just what its name implies-the analysis of the written or visual contents of a document (Fraenkel & Wallen, 1993, p. 389). Documents can be categorized in different ways and sometimes these categories overlap. Personal documents, public records, and official documents are common types of documents (Merriam, 2009). Personal documents can be broadly defined as any first-person narrative that describes an individual's actions, experiences, and beliefs which are found in diaries, letters, and scrap books. Official documents include internal communications that are circulated within an organization. Such documents often allow only limited access. The student records, personnel files, newsletters, curricular materials and so on (Merriam, 2009; Bogdan & Biklen, 2007) were analyzed.

The documents I analyzed were primarily public records and official documents i.e. school mathematics curriculum, mathematics textbook including

interschool memos, the student's class schedule, sample worksheets from the student's classes and notes. There I figured out whether these documents were incorporated with out-of-school cultural aspect or not.

Study of Cultural Artefacts

A cultural artifact is an object that derives from and illuminates the history of a particular community. It can be supermarket bills, bottle or can labels, railway schedules, a cover of a ring binder, a weekly TV guide (Bonotto, 2007). Geoboard, classroom materials, and other materials that the children typically meet in their real-life situations can also be the cultural artifacts. Photographs, music, clothing, and dance that can provide a lot of information about the group of people and their cultural beliefs. But, in my study, I figured out as the physical objects surrounding in the students' culture that can be brought in the mathematics classroom for teaching and learning.

Procedures of Data Collection for Action Research

The main objective of my study was to analyze the contribution and possibilities of incorporation of cultural metaphors in teaching and learning of school mathematics. To do this, I visited a public school in the community and talked to the Head-teacher. I explained him my objectives of the study in detail. As my study was intended to connect the out-of-school mathematical ideas in the process of teaching and learning of school mathematics, I developed detailed plan to carry out my research work in collaborative action research framework. The three different stages: pre-action research, actual action research and post-action research were carried out to collect the data.

Pre-Action Research Stage

In this phase, Teacher Facilitation Program (TFP) was conducted in order to equip mathematics teachers through cultural metaphors and constructivist framework of teaching mathematics. Along with this, some teachers were selected as participants for action research, TFP stage II was conducted and the abstract contents of mathematics at the basic level of education was selected. In order to orient teachers about cultural metaphors and analyze its contribution in the process of teaching and learning of school mathematics, the following programs were conducted in pre-action research stage.

Teacher facilitation program (TFP). After the meeting with HOD of mathematics and Head teacher, I had developed detailed plan (Appendix X) to conduct Teacher Facilitation Program (TFP) to acquaint about the knowledge of cultural metaphors and connection of out-of-school mathematical ideas and knowledge for the teaching and learning of school mathematics. The general orientation about the cultural metaphors to the teachers of mathematics of cooperating school was conducted in 21-22, July 2017. Twenty mathematics teachers were participated in the program.

Selection of teacher participants. After conducting two day's teacher facilitation program, I provided the open-ended questions and blank sheet to write the perceptions and their reflections on TFP. Only 15 participants provided me their written reflections about the TFP. I collected the reflection notes of fifteen participants and analyzed them with the collaboration of my research supervisors. I took oral interview with fifteen teachers who participated in my project based on collaborative action research. Six teachers, who had keen interest on cultural metaphors and its use in mathematics classroom, were selected.

Selection of difficult contents. After the selection of teacher participants, the contents of mathematics at basic level which they felt difficult in classroom teaching was discussed. They were requested to write the topic and the difficult area regarding those topics. From the teachers' observation, the content and difficult areas were identified and presented in table 4.2.

Table 4.2
Identification of Contents for Classroom Teaching

S. N.	Contents	Specific Difficult Areas
1	Numbers	Writing Devanagari Numeral
2	Place Value	Identifying Place value, Face value and Total value
3	Rounding off Numbers	Rounding off middle value to its nearest 10, 100, and 1000.
4	Prime and Composite Numbers	Definition and identification of Prime and Composite Numbers
5	Integers	Concept of Integers Subtraction of Negative Integers
6	Basic Algebraic Concepts	Variables, Terms, Algebraic expressions
7	Transformation Geometry	Symmetry, Reflection

The selected contents were also presented in the teachers group, and discussed with some mathematics educationist and teacher trainer if they are really difficult to teach in basic level of mathematics classroom.

Teacher facilitation program (Stage II). Six teachers participated in second stage TFP which was conducted for five days. I prepared model lesson plan in view of cultural metaphors in constructivist teaching and learning framework and demonstration classes were conducted by researcher (Appendix X). The cultural

metaphors were collected and selected for the teaching and learning of identified difficult areas of mathematics. Each teacher had prepared their lesson plans with the help of researcher in view of cultural metaphor. The simulation classes were done by each teacher participants before going to actual classroom (Appendix XI). The developed teaching episode were also discussed among the teachers group.

Action Research Stage

In this stage, the lessons were taught in the real classroom with the view of cultural metaphors in constructivist teaching and learning framework. The action research methodology enabled me to adopt more appropriate ways of teaching through the cycles of planning, acting, observing and reflecting (P-A-O-R) model. In this cycle, teachers made plan to teach above mentioned contents and identified the cultural metaphors, intervened them into the classroom, made observation of the effect of the metaphors on students' achievement and made decision for next action.

Plan for action research. Teachers who were participated in collaborative action research, developed a teaching episode to deliver in classroom with the view of cultural metaphor. The lessons were selected and designed based on TFP stage II. In this phase, teachers selected the cultural metaphors (grounding metaphors, linking metaphors and extraneous metaphors) of different levels to teach particular contents.

Classroom intervention. In this stage, teacher used cultural metaphors to teach abstract concepts of mathematics. Teachers intervened the cultural metaphor in classroom to facilitate the understanding of abstract concepts of mathematics. Series of interventions (First, Second, Third,...) may require to teach students if the hierarchy of difficulty in content increases.

Observation of classroom teaching. After the intervention of the classroom, teacher observed the effectiveness of the cultural metaphor for the teaching and learning of particular contents of school mathematics.

Reflection of the teachers. Reflection is a continuous process throughout each phase and all cycles of the action research process. It provides to evaluate the effectiveness of the treatment or the intervention of researchers. In this regard, Naughton and Hughes (2009) opines that “ To reflect on our practice is to be curious about what we’re doing, why we’re doing it and what its effects are; and it also involves trying to think openly about our past and present practices in order to improve on them” (p.98). Thus, this step helped to make a decision about research problem and identify next possible actions whether the metaphors did well to conceptualize abstract ideas of mathematics or not.

Post Action Research Stage

This stage was conducted immediately after finishing each class. After each class, post class interview was taken with the students’ and teachers. The main objective of this stage was to analyze the effectiveness of cultural metaphors in the process of teaching and learning of mathematics. It was measured by observing the students’ and teachers’ psychological and behavioral changes after the intervention of cultural metaphors.

The behavioral aspects of the students and teachers were measured in terms of their activeness in classroom participation, cooperation to their peers, engagement in the classroom task, and other behavioral changes. In the same way, the psychological aspects of the children and the teachers were evaluated in terms of reaction, feelings, emotions, facial expressions, and other psychological traits. As my study was

qualitative multimethod, cognitive aspects of the students were measured by observing class work and home work copies.

Tools for Data Collection in Action Research

The data collection tools and techniques are important to gather the data from the field. Following Creswell (2014), I gathered multiple forms of data through interviews, observations, and documents. For this, I developed interview guidelines, observation protocols, classroom observation checklist and lesson design before entering to field.

Table 4.3

Tools and Respondents of Data Collection for Action Research

Tools	For whom	For what
Interview Guidelines	Students (12) Teachers (6)	To analyze the contribution of cultural metaphors in the process of teaching and learning of mathematics
FGD Guidelines	Teachers (6)	To analyze the possibilities of use of cultural metaphors in classroom teaching
	Students (12)	To examine the contribution of CPBL approach in mathematics learning
Observation Checklist	Classroom observation	To observe the effectiveness of cultural metaphors in the process of teaching and learning of mathematics
Workshop Materials	Teachers (20) teaching mathematics from K-12	To collect teachers perceptions on the use of cultural metaphors To select the teachers participants in the CAR
Teaching Episodes	Teachers (6)	To facilitate teachers in view of cultural metaphors

The table 4.3 presented the different tools which were developed for different participants to collect the data to overcome my research questions. The tools that I

used to collect the data have been put in the appendices of this report. In the following section, I have discussed about the data collection procedures: interviewing, FGD and photographs recording for action research. I have also mentioned the ways in which I used each data collection tools during my field visit.

Semi-Structured Interview

In my study, semi-structured interview was used to gather the information from the students and teachers. The semi-structured interview contains a mix of more- and less-structured questions. The interview was guided by interest in a particular topic and subsequent subtopics but the exact wording of the questions and the order in which the questions were asked were not determined ahead of time (Seidman, 2006). An interview guideline is a list of questions one intends to ask in an interview (Merriam, 2009; Seidman, 2006). I used semi-structured interview to communicate with teachers and students with the help of interview guidelines. Similar questions were asked in order to provide a comparison among the research participants. The validity of interview guidelines were established through expert judgement. The constructed interview guidelines for teachers and students are presented in Appendix III and IV respectively.

Focus Group Discussion

Focus Group Discussion (FGD) is a qualitative data gathering method in which people are informally interviewed in a group discussion setting (Neuman, 2008). There I initiated loosely structured discussion with my homogenous participants. I asked very specific questions (Fontana & Frey, 2005) for cross-flow of our communication (Darlington & Scott, 2002). In my study, I used FGD on the issue of using the possibilities of incorporation of cultural metaphors that are practiced in out-of-school context in classroom mathematics teaching and learning.

The FGD was conducted with the mathematics teachers and students separately along with my technical supporter. In that session, my role was facilitating the discussion about the issues I raised. I had hired a technical supporter during my data collection phases. The video, audio and images of several behaviors of interest occurring rapidly in different moments were taken with their consents (Fraenkel & Wallen, 1993). In the same vein, my technical supporter ran a video recorder and took snap during different session which was permanent records of behaviors for comparison with later and different samples. The FGD of the teachers (Appendix V) and students (Appendix VI) included six and twelve members respectively, and each discussion lasted for about 45 minutes. The data gathered from the FGD helped me to compare the ideas, knowledge and their beliefs being interviewed of individual teacher and their versions in the group discussion that also helped to produce valid results.

Photography as Data Sources

Harper (2004) says that a text is never equivalent to the image, but images by themselves do not communicate fully. Thus, photography itself is a source of my data and at the same time it eases my analysis and interpretation because I have used both images and words to give meaning to my findings. About photography as visual data, Atkinson and Delamont (2005) state:

During recent years, the development of small digital camcorders and the development of digital photography have created an enormous range of possibilities for ethnographers in the field. Consequently, visual anthropology and sociology should not be treated as separate genres or specialties. There are many aspects of culture that are intrinsically visual...there are many

phenomena that can and should be analyzed in terms of their appearances and performances that may be captured in visual terms. (p. 825)

Therefore, visual data should not be overlooked or ignored. This is because many cultural domains and artifacts can be grasped only through their visual representations and the structured properties of their visual codes (Balls & Smith, 2001). These data eased me to capture the social realities of the groups and their culture which might not be possible through other means. Visual data also provided an opportunity for my participants to share their reality directly (Creswell, 2014). For capturing the visual data of the different moments in out-of-school environment and everyday activities of students' home culture, I acknowledged their cultural identity and maintained ethical considerations.

Data Recording Procedures

It is necessary to make a plan their approach to data recording before entering to the field. To overcome my research objectives, I used in-depth interviews which are mostly known as open ended interviews or ethnographic interviews. I prepared an interview guideline for parents (Appendix I), teachers (Appendix III) and students (Appendix IV) so that it would be easier for me to generate the data in the fields. I prepared unstructured questions for the same as qualitative research does not opt for the structured questionnaire. I availed the situation so that my research participants would feel comfortable to express what they felt regarding mathematical understanding in their everyday activities.

I used participant observation to generate the data to support my study. I carefully recorded all the possible observational data with the help of the video camera and take field notes as much as I could. I collected the data from the multiple observation sources during the course of my study and used an observational protocol

(Appendix II) for recording information while observing (Creswell, 2009). I collected data through examining documents, observing behavior and interviewing participants. Thus I reviewed all of the data gathered from the multiple source of data (Creswell, 2014) and then organize it into categories or themes that cut across all of the data sources. At the same time, I converted the conversations and interviews into manuscripts so that I could easily analyze and interpret them.

My approach to research is heavily influenced by an ethnographic tradition. In that sense, I spent considerable time in the community building rapport with its members, just being there. Also, whenever possible, I interacted with the students, their parents, and their teachers. That is, I argue that to gain an understanding of students' experiences with in-school and out-of-school mathematics, we need to talk to the various people that are likely to play a role in these students' experiences.

Data Analysis and Interpretation

The main task in the research is to analyze and interpret the data collected from the field. Data collection and interpretation process is simultaneous process in qualitative research. According to Creswell (2014), qualitative data analysis is conducted with gathering data, making interpretations and writing reports. During data analysis, the data was organized categorically and chronologically, then reviewed repeatedly, and coded and decoded continually. I also took the idea of Creswell (2007) who divided data analysis into five parts: 1) data managing, 2) coding and developing themes, 3) describing, 4) interpreting, and 5) representing. In each part, I analyzed the data in spiral form rather than fixed linear approach.

I collected voluminous data by using different research methods such as participant observation, in-depth interview, classroom observation, semi-structured interview, FGD, study of cultural artefacts, documents analysis and various informal

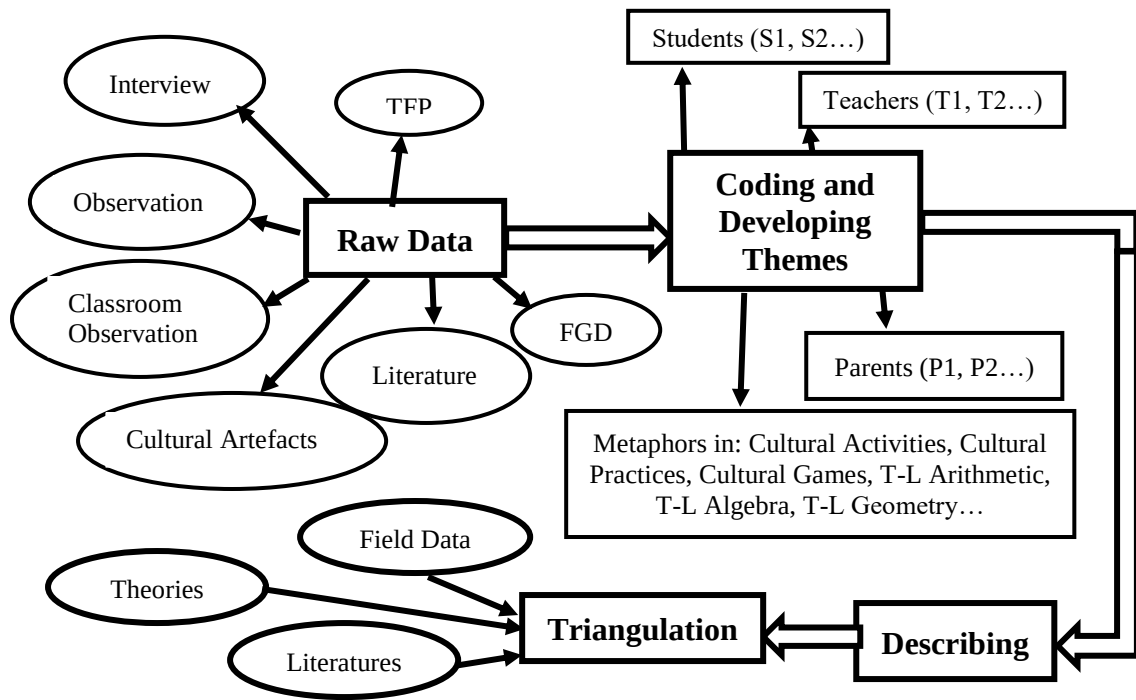


Figure 4.4. *Procedures for Data Analysis and Interpretation*

conversation. I was aware that raw data would not help me to understand the mathematical ideas embedded in sociocultural settings and contribution of cultural metaphor in classroom teaching which is being studied, or the way the participants view it, unless the data were systematically analyzed to illuminate the existent situation (Merriam, 2009). So, I began coding the data during analysis to organize and make sense of textual data gathered from different sources. Codes or categories are tags or labels for allocating units of meaning to the text compiled during a study. Codes usually are attached to chunks of text of varying sizes: words, phrases, sentences or even whole paragraphs. Codes or categories came from the professional reading, or are the words and phrases used by my informants (Miles & Huberman, 1994).

I read transcripts of interviews, field notes, classroom observation, study of cultural artefacts and documents to make sense of the whole before examining unique and individual pieces of data. I then read the data for a second time and I developed themes using short phrases, ideas, or key concepts related to the research questions.

Thematizing the data helped me to remove unnecessary information and concentrate precisely at the research questions. The themes are further described without dismantling the essence of the research participants and ethical considerations. I used the thematic approach for data analysis based on the themes developed according to the research questions. The data were analyzed and interpreted with consideration of the literature and theories discussed in previous chapters II and III. I tried to establish the relation of the gathered data from multiple sources and triangulated with the theories and literatures.

Data Authentication

There are four types of triangulation: the use of multiple methods, multiple data sources, multiple investigators, or multiple theories to confirm emerging findings (Merriam, 2009). This study uses multiple methods of data collection and multiple sources of data: interviews, observations, study of cultural artefacts and document review. One important point about the data collection is to identify whether the collected data are sufficient to answer the research questions. In the process of data collection, I checked the information that I gathered from my interviews against both my observations and my notes from my document review. I looked for consistent pattern across the data. I began to recognize pattern in the data, collected from the classroom observation and interview of teachers and students. I did not find any significant differences among these sources. The data gathered from in-depth interview and FGD were transcribed without dismantling the essences and themes of the research participants and were coded as many categories as possible. Once the transcript had been checked for accuracy, I listened the tape again and again to get the real sense of the text. Throughout the data analysis process, field notes and diaries were categorized according to the themes and were reviewed regularly.

The analysis of the data from FGD presents both challenges and opportunities in comparison to the data obtained from face to face interview and observations. The researcher should have careful recording of individual's opinion and observe their shared understandings or the common views about the topic of focus. In my study, the cultural metaphors regarding the mathematical activities in their different sectors of lives and their ways of understanding the natural phenomena, and their mathematical knowledge were analyzed with the notions of pluralism. In this study, the analysis of the research was validated and made more reliable by triangulating the statements among the research participants, their ways of presenting the text in the several times of data collection periods. The interpretations of the context and their reports should not be biased to the same degree.

Ethical Considerations of My Study

Ethical issues relevant to qualitative research is intended to provide a context for discussion in subsequent modules of procedures for safeguarding research participants' interests (Naughton & Hughes, 2009). Research ethics deals primarily with the interaction between researchers and the people they study. Agreed-upon standards for research ethics help ensure that as researchers we explicitly consider the needs and concerns of the people we study, that appropriate oversight for the conduct of research takes place, and that a basis for trust is established between researchers and study participants. Whenever we conduct research on people, the well-being of research participants must be our top priority.

The first objective of my research was to collect the cultural metaphors regarding mathematical ideas and knowledge in different group of people embedded in their everyday activities. It was necessary to visit their community and observe their everyday activities. First of all, I made sure to the participants that my research

was just for the fulfillment of academic degree and not for any other purpose. I acquainted my research participants about the purpose of this research and made them very clear that this research would not bring any immediate change in their lives. I got the consent from the research participants to have in-depth interviews and got the due permission to live with them and follow them to their work places. I also provided the detail of the information sheet which explain the purpose, nature and possible effect of my study. I made clear to the participants that I respected their self-termination and autonomy for providing any information on all aspects of research. I even let my research participants access the versions they made and asked if they wanted any change. I acquired the permission from them to use their names, addresses, photos and videos for the purpose of this study (Appendix XII: Consent Forms). In addition, I have used unbiased language or words against gender, racial group, and ethnic group in writing report. Similarly, I have cited the works of different scholars, researchers that I referred to during my study.

Delimitations of the Study

Delimitations describe the boundaries set for the study. My study aimed to document and explore the cultural metaphors regarding mathematical ideas in out-of-school environment of the students and analyse the contribution of those metaphors in the process of teaching and learning of school mathematics.

- This study was delimited to the household activities, workplace activities, cultural activities, cultural arts and artifacts of the community surrounding of one coeducational secondary school of Kathmandu district.
- In this study the different group of people were selected not on the basis of their ethnic, religious, sex, and language group. The cultural group in this study was categorized on the basis of their everyday activities.

- The action research was conducted in one of the government funded secondary school of Kathmandu district.
- This was a descriptive and analytical investigation based on multimethod qualitative research study. So, the findings, conclusions and reflections were based on qualitative evidence, literature, field data and my personal experiences. Therefore, the findings of this study are not generalizable over the country. However, it can be expected that it will be applicable in similar cases in similar contexts.

Summary of the Chapter

My study was intended to explore the cultural metaphors regarding mathematical ideas and knowledge and also analyse the effectiveness of those metaphor in the process of teaching and learning of school mathematics. In order to address the research questions that had been developed on the basis of objectives, I did multimethod research design which integrated ethnography and action research. Initially, I tried to capture out-of-school mathematical ideas embedded in the everyday activities of different group of people. To understand the field reality, I also used the participant observation, interviews, document analysis and study of cultural artefacts. I brought both field data and literature together to interpret my field information. I used different sociocultural theories and literature to analyse and interpret the field data based on my research questions.

CHAPTER V

EXPLORATION OF CULTURAL METAPHORS IN MATHEMATICS

In this chapter, I collected and documented the mathematical ideas practiced in everyday activities outside the school environment. The mathematical concepts and ideas hidden in the different artefacts and embedded in the everyday activities were explained. This chapter attempts to answer first research question and its sub-question in relation to cultural artefacts and mathematical understanding of different group of people.

The mathematical ideas in the environmental activities involved in counting, measuring, locating, designing, playing and explaining of members of any cultural group. This chapter is devoted to an exploration of the different mathematical ideas embedded in cultural arts, artefacts and activities of a group of people found in the out-of-school environment.

Knowledge and Practice of Community People on Measurement

The major disciplines in mathematics first arose from the need to perform calculations in commerce, to understand the relationships between numbers, to measure land, and to predict astronomical events. Each discipline used different kinds of tools. For calculations, the Chinese abacus, Egyptian clay tablets, Quipu knotted strings, and Napier's bones were used. In the 17th century, as technology changed, mechanical calculators came into existence, which was followed by electronic calculators. Different units are used to measure physical features such as area, volume, weight, or time. Important aspects of measurement include the use of compatible units for the attributes being measured, estimating measurements, selecting appropriate units and scales that determine the accuracy of the measurements, and using measurements for solving problems.

Distance Measurement

The first units of length in almost every culture were based on body parts. Over the centuries, many systems of measurement were developed and refined as people found the need for consistent measures in commerce. Today, the most common standard measuring system is the metric system which uses the meter as its standard unit of length. Some countries, however, still use the imperial system, with the foot as the basic unit of length (based on the size of the foot). The foot (12 inches) was originally the length of a human foot, though it has evolved and now is longer than the length of an average person's foot.

Most of the young people are knowledgeable about modern measurement system. To measure the length of any things, they use their hands as a unit, the distance between the elbow and the tip of middle finger is called *Haat*. It is found that the people from rural village community use the traditional units of measurements to measure the length of their house, cow shed, clothes, fields etc. Among these units, *Haat* is mostly used as a unit of length. *Haat* is commonly used as a measuring unit to measure the plot of house-hold land even today.

For the shorter length, they measure the distance between the tip of the thumb and the tip of the little finger, which is called *Bitta*. To measure even shorter distance, they use finger width which is called *Amal*.

Kureet is another measuring unit of mat weavers which is the distance between the tip of the thumbs and the tip of the index finger with stretched palm. They also use the length

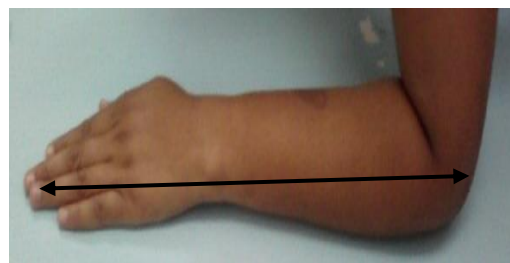


Image 5.1. A *Haat* as Length Measurement Unit

of their foot which is called *Paila*. Some of the major measuring units are shown in the figures given below.

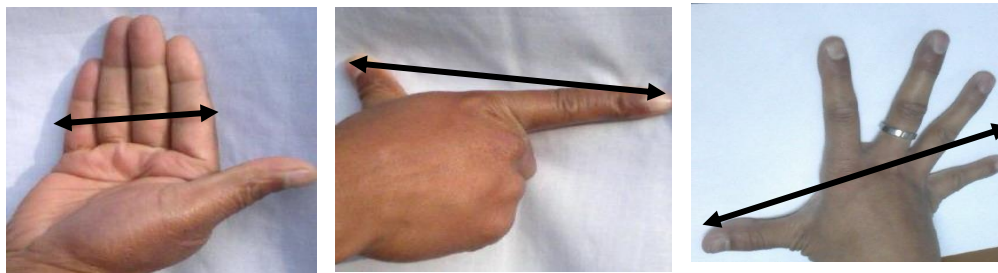


Image 5.2. *Amal, Kuret and Bitta*: Units used in Length Measurement

To estimate the long distance especially the distance between two places, they use '*Kosh*'. *Kosh* is the walking distance of a man until a wet cloth dries up. Officially one *kosh* is approximately equal to 2 miles or 3.2 km. Many people interviewed were aware of the new measuring system of the length centimetre, meter and kilometre, but they do not feel comfortable with using those units. They use their own measurement system among themselves within the community. They use the modern system of measurement when they go to buy and sell the required things in the market. They measure the length and distance to meet their own requirements rather than for business transaction. So, possible errors in measurements are not of much count.

The methods that the community people use to measure the length and distance suit their environment and cultural requirement. The whole community still agrees on those measuring units and the measuring system. Generally, everyone use distance in terms of time without any mathematical reasoning involved in it. When I asked one of my research participants: how long/far is your house? He immediately replied that it takes only five minutes. It is customary to get the answer involving the distance between two locations.

Area Measurement

The measurement of area is based on a utilitarian concept. The small area like weaving mats, *Dhaka-Pattasi* and the estimation of land for a household is measured

with bodily measurement units. The utilitarian concept of area measurement is assessed on its productivity, utility and seed-holding capacity (CERID, 1990). This concept has strongly rooted in the village people. However, the new generation and the school children are using modern measurement units to measure the area. The *Haat* is also common measuring units to measure the plot of land in household purpose in a rural area too.

The big plot of farm land is usually measured in three ways: (i) in terms of the seed quantity required, (ii) in terms of ploughing day, and (iii) in terms of grain yield. The description of each, in brief, are presented consequently. The seed of grain is generally measured in the local measuring unit of *mana* and the *pathi*. The *pathi* is a bigger unit and its conversion having $8 \text{ mana} = 1 \text{ pathi}$. This unit is fairly accurate as all people sow seeds almost in the same way and at the same space intervals. One *mana* of paddy seed is sown in an approximately land area of 53 square meters (CERID, 1990).

Ploughing with the help of oxen was very common in a cultural village in Nepal. In present days, the ploughing with the help of oxen is seen rarely even in the rural part of Nepal. The modern ploughing machine, tractors are available instead of oxen helped system. However, the rural villagers are still using the traditional ways of measuring sizes of the farm land. They estimate the size of land that could be ploughed in one day and called “*Ek halo ko melo*”. One *halo ko melo* is the area ploughed in one day at normal speed and is approximately 1500 square meters (appro. 3 *ropani*). The *ropani* is the official land measurement unit in the hilly region of Nepal. A *ropani* is a land area covering about 508.7 square meters. The smaller unit of *ropani* is *Dam*. The conversion of *ropani* and standard unit of measurement are presented in table 5.1.

Table 5.1
Land Measurement Units and its Conversion

Units of Measurement	Conversion
4 <i>Dam</i>	1 <i>Paisa</i>
4 <i>Paisa</i>	1 <i>Aana</i>
16 <i>Aana</i>	1 <i>Ropani</i>
1 <i>Ropani</i>	508.7 Sq. m.

The another ways of measurement of farm land is *muri mato* (quantity of soil). This is a very interesting way how the people connect the area of land in terms of the volume of soil. It is commonly believed that one *ropani* of field land have 4 *muri mato*. Regarding the measurement of land in terms of volume of soil, I asked the parents of the village, “How much farm land do you have?” One of my research participant says “I have no more farm land. We six people are living if we do hard work in our farm land”. He further says, “I have just farm land of 20 *muri mato* and about 20 *muris* of grain would yield”.

The production of grain is varying according to the quality of land. In the hilly region, there are two types of farmland: *Tari* and *Gairi* in terms of facility of the irrigation system and the quality of the soil. The *Gairi* is a land which has regular irrigation facility and more fertile soil as well than in a *Tari*. Thus the less grain would yield in *Tari* land than in *Gairi* land though the land area is equal. A grain yield measured in *muris* is sometimes used to express the size of the farmland. But, this unit does not give the exact area measurement because it inseparably utilises both land area and quality.

The production of grains is based on the quality of soil, facility of irrigation and other components. The quantity of seeds required are different according to the

quality land and the types of grain. More seeds are required for *tari khet* land than the *gairi khet*. In the same way, more paddy seeds are required than maize seeds. It was observed that the community people have beautiful ideas of relating the area of land

Table 5.2

Knowledge and Practices of Community Members on Area Measurement

Community People Practice	Local unit of Measurements	Metric System	Remarks
<i>Ek hal ko melo</i> (Ploughed in one day)	3 <i>Ropani</i>	1500 sq. m.	Area measure in terms of time
<i>Ek mana biu lagne melo</i> (Land require seed grains of one <i>mana</i>)	About 2 <i>aana</i>	53 sq. m.	Area measure in terms of quantity of seed grains
<i>Ek muri mato</i> (Land contains soil of one <i>Muri</i>)	About 8 <i>aana</i>	250 sq. m.	Area measure in terms of volume

in terms of volume of seed grains, soil and the time to plough the land. The community people and classical Nepali units of measurement are different than the rest of the world. We have units that are unique to Nepal and they are still used popularly. They are also used for official purposes. The *Dam*, *paisa*, *Aana*, *Ropani* in hilly region and *Dhur*, *Kattha*, *Bigaha* in Terai are still popular in measuring land. These are more popular units when compared to FPS and MKS system of measurement. The following table presents the conversion of traditional units of measurement to different international standards.

Weight Measurement

The balance is a common measurement tool to measure weight. The traditional community in Nepal use Tulo as a measurement tool. The *Sher*, *Bishauli*, and *Dharni* are the measuring unit of weight in rural part of Nepal. The Tulo also can be the

powerful metaphor for the understanding of concept of moment. The various mathematical concepts and ideas can be connected with the Tulo.

The people of rural part still practicing these traditional unit of measurement and the *Laal*, *Masa*, *Tola* are still use to measure the weight like gold and silver in present days. The *Laal*, *Masa*, and *Tola* are still popular in measuring weight. These units of measurement are still practice in the market to measure the gold and silver. These are more popular units when compared to FPS and MKS system of measurement in Nepalese context. Table 5.3 presents the conversion of traditional units of measurement to different international standards.

Table 5.3

Conversion of Local Units of Weight Measurement with Standard Units

Local Units of Measurement		Standard Units of Measurements
1 <i>Laal</i>		0.1166 grams
100 <i>Laal</i>	1 <i>tola</i>	11.66 grams
1 <i>Chhatak</i>	4.5 <i>tola</i>	52.47 grams (50 grams in practice)
1 <i>Pao</i>		194.4 grams (200 / 250 gms in practice in different parts of country)
1 <i>Dharni</i>	12 <i>pau</i>	2.34 kg

The traditional units prevalent in Nepal have not entirely disappeared from the world of commerce and industry. The buying and selling of gold and silver uses traditional units of *Laal*, *Masa* and *Tola*. However, the textbook and mathematics curriculum adopted the MKS system of measurement and omitted the interconversion with the western system of measuring units.

Mathematics in Weaving

There is a lack of evidence or the exact date that the people of the world started to weave the items. For thousands of years, people have weaved baskets, hats, mats and other items. Different cultures have carried out weaving using different materials like grass, leafs, clothes, feathers, paper and other local materials.

Mat Weaving

Weaving mat is one of the common activities in the rural part of the different community in Nepal. The weaving of sitting mat and sleeping mat reflects the cultural values and identities. Handloom (*hataso*) is a major tool to weave a mat. It is rectangular parallelepiped in shape and approximately $2\frac{1}{2}$ *haat* length, 3 *amal* breadth and 4 *amal* height. There are odd numbers of holes in handloom (*hataso*) and maximum numbers of holes contain 15. The distance between the holes is approximately 4 *amal*. They fix a frame for the mat with the length of 6 *haat* and the breadth of the mat is flexible as the number of whole used in handloom (*hataso*). The mat weavers make a mat with different size. The size (breadth) of the mat (*Gundri*) is called '5 *Ghare*', '7 *Ghare*', '9 *Ghare*', '11 *Ghare*' etc. according to as 5 holes, 7 holes, 9 holes, 11 holes used in handloom (*hataso*). The breadth of the mat depends upon the length of the paddy bushes available and the length is determined on the basis of purpose of the mat weavers.



Image 5.3. Frame for Mat and Preparing to Weave

The best time to weave the mat is winter season. This time is just after the harvest of paddy and they feel free from busy days of farming and other households job. The women of the rural community engaged in weaving mat because it is considered that weaving a mat is a female activity and responsibility. The weaving a mat is a multi-step process. First of all, they make the frame in a rectangular shape with four poles. The two poles (close to each other) are connected by the beam and is formed and the opposite sides of the rectangle. The mat weavers follow the certain pattern and the ways to weave through a straw. The strips of straw are started to weave from left to right and then from right to left parallel. Each time a strip is completely woven into the mat, it is pushed up against the previous weft strip. The pattern is continued with each strip until all of the strips are used and then resulting mat is tight and firm.

The various mathematical ideas and thinking are inherited in the process of constructions of the mats. Indigenous people have implicit mathematical knowledge with the ability to perform a task but without the ability to explain their deeds (Pradhan, 2017). This implicit mathematical knowledge is embedded tacitly. Mosimege and Lebeta (2000) also reported that the indigenous people use different mathematical concepts like estimation, and tessellations, and symmetry in the construction of the traditions artefacts and cultural activities. Chundara people use lots of mathematical concepts and knowledge in their everyday activities. Their knowledge without sufficient justification for the rule they have been using are consistent (Pradhan, 2010). In recent years, most of the cultural activities are degrading. It is because of the rapid development of science and technology and globalisation. The artefacts that the community members developed for their everyday

use and for survival is redundant. The modernization replaces the cultural art and artefacts.

Geometry in Mat Weaving

Weaving is a common activity with some women in the rural community of Nepal. The mat weaver starts by taking four poles and two beams. The two poles are to be laid on the ground at a distance just longer than the length of handloom (*hataso*). And these two poles are tied together with the beam. The third pole is laid as per the length of mat required to be weaved and the fourth pole is moved to form the close of a quadrilateral. The distance between the first and third pole is measured with the help of a rope. The fourth pole is further adjusted until the distance measured from the second pole become equal and the third pole and fourth pole are tied together with the beam. The design and making of the mat present opportunities to work with



Image 5.4. Measuring Diagonals of Mat's Frame

various geometrical concepts like measurements and pattern concepts. The knowledge they practised in the process of construction of a net of the sleeping mat is equivalent to the theorems in Euclidean geometry. The process of constructing rectangle is adjusting four corners and is termed as making *Chaukon* as the vertices. In the construction of frame to weave a mat, the mat weavers formulate the rectangle axiom:

A₁: A parallelogram with equal diagonals is a rectangle.

A₂: A quadrilateral with congruent diagonals that intersects at their midpoint is a rectangle.

A₃: Opposite sides of a rectangle is equal and parallel.

In quadrilateral ABCD, if $AC = BD$ then it is a rectangle. The activities across the different cultural villages are largely the same as most of the indigenous people are culturally homogenous (Barton, 2008). In this

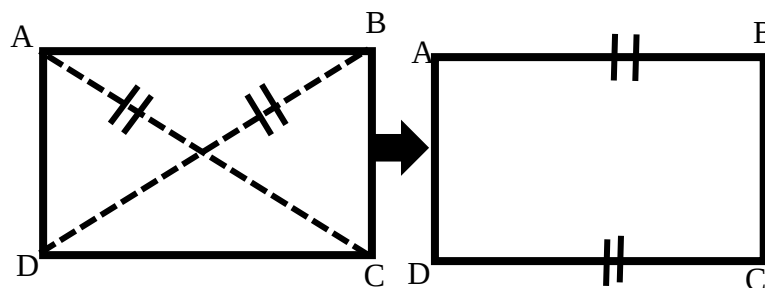


Figure 5.1: *X-test in Rectangular Figure*

regards, Gerdes (1999) mentioned that the traditional Mozambican house builders also used the similar knowledge as used by the mat weavers.

The use of x-test is to find the four points of the rectangle where they can make their pole stand for the swing. When they finish step three, they take one rope and draw a diagonal measurement of the rectangle. Whatever the length is, they mark that length and use it from the next corner. When they measure the length AC, they use the same length from B to D. Now they have, A, C and B and need to fix D as they consider three points (A, B, C) as fixed points and just to determine the last point (D). When they do it, the problem is solved. At this time, the x-test is applied. The builders do not really care about the vertices: A, B and C. They just want to fix the vertex D. They already have taken $AB=CD$, now just they have to make sure they x-test ($AC=BD$) is there. Once AC becomes equal BD, they determine the four corner of the swing. Asking students about why and how does it work? The way students try to explore mathematical reasons is a reach environment for developing mathematical creativity and problem-solving skills.

Gerdes (1999) mentioned that the most of the Mozambican peasantry use two methods for the construction of rectangular bases of traditional houses. The first method presents the house builders start by laying down on the floor two long

bamboo sticks of equal length (figure a). Then these first sticks are combined with two other sticks also of equal length, but normally shorter than the first ones (figure b). Now the sticks are moved to form the closure of a quadrilateral (figure c). The figure is further adjusted until the diagonals – measured with a rope- became equal (figure d). Then, from where the sticks

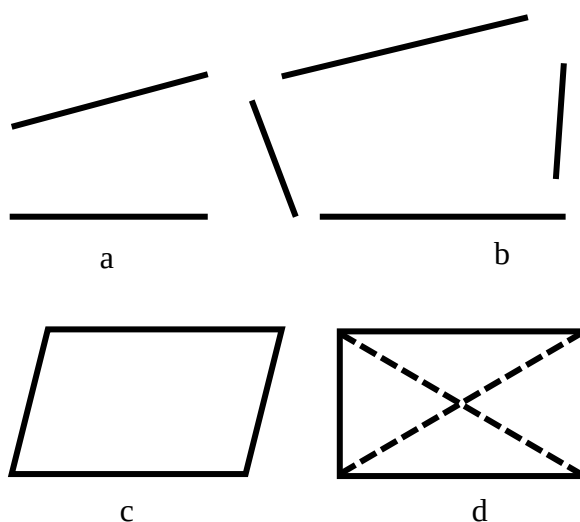


Figure 5.2: *Constructing Rectangular Shape*

are now lying on the floor, lines are drawn and the house builders can start.

The mathematical thinking and methods are quite different in different cultures (Gerdes, 2001). The mathematical ideas hidden in the process of constructing the frame to weave a mat in the rural community in Nepal is mostly equivalent to the methods that the Mozambican peasantry used methods of constructing the rectangular bases of houses (Gerdes, 1999). Further, Gerdes (1997) explored that the construction techniques of the house builders facilitate to formulate the rectangle axiom. From the observation of the different process of mat weaving, it was found that it includes a lot of mathematical ideas in the mat weaving activities. It is argued that it is possible to use indigenous mat weaving in the teaching and learning of mathematics at different levels from primary to a higher level. In a cultural village of Nepal, a rectangular form of sitting or sleeping mat may be rolled up when it is not in use. Some people use sleeping mats to larder for the purpose of reserve paddy. It then takes approximately the form of a cylinder. From the observation of the situations, this cultural experience provides an opportunity for students to obtain an idea of calculation of the volume of

a cylinder. The finished product is analysed to assess the embedded mathematical ideas in the mat and other cultural artefacts.

We have a mat ABCD of length L and breadth H . Let us transform the mat ABCD into a cylinder by rolling up such that AC and BD are just in contact.

Here, Area of initial mat ABCD = Surface area of the cylinder = $L \cdot H$

It is evident that the circumference of the base of cylinder (C) = L

This implies that $2\pi R = L$ gives $R = \frac{L}{2\pi}$. Let A be the area of the cross-section area of the circular cylinder, $A = \pi \left(\frac{L}{2\pi}\right)^2$

Also $V = A \times H = \frac{L^2}{4\pi} \times H$. It gives, $V = \pi R^2 H$

Gerdes (1999) points out that by taking the thickness of the initial mat as a unit of linear measurement, the volume of the mat is equal to the length (L) times width (W). After rolling up, we have the top area A equal to L . Therefore, the



Image 5.5. Rolling a Mat: An Exclusive Example of Cylinder

volume of cylinder is approximately given by

$V = L \cdot W = A \cdot H$, where H represents the height of the cylinder which is the same as the width W of the mat. Advancing with the limit, the volume of a cylinder is found to equal to $\pi R^2 H$ (p. 94).

Different groups of people are performing various activities so as to sustain their life since long. The people that are weaving mats are not using mathematics; they are doing it traditionally (Gerdes, 1997) through craft model approach. From the eyes of ethnomathematics, it was observed that the powerful mathematical ideas are embedded in their cultural activities. The investigation of the mathematical ideas

embedded in mat weaving activities is beneficial in order to connect them with school mathematics. In this regard, the Tanzanian curriculum specialist Mmari stresses that “...there are traditional mathematical methods still being used in Tanzania.... A good teacher can utilize this situation to underline the universal truths of the mathematical concepts” (Gerdes, 1997, p. 226). Thus, the ethnomathematical ideas embedded in the process of mat weaving help to contextualize the mathematical problems.

Geometry in Sitting Mat

Weaving Mat is one of the common activities in the rural part of the different community in Nepal. The weaving of sitting mat reflects the cultural values and identities. The sitting mat (*Chakati*) generally weave with a dry leaf of maize and with dry rice plants. There are two types of sitting mat according to the shape and size. Most common sitting mat is in circular shape. Another type of sitting mat is rectangular shape. It is longer



Image 5.6. Sitting Mat: An Exclusive Example of Circular Shapes

in size in comparison to sleeping mat. The sitting mats are in small breadth than the sleeping mat. It is sometimes very long and different ways to weave than the sleeping mats. These mats are made from peddy bushes and maize carts. It is common production in a rural community.

The mats are woven in symmetrical patterns. To waive a mat from bush of rice plant, two groups of strips that are woven perpendicular to each other. The strips can be of different colors and sizes. The vertical strips and the horizontal strips are knoted together. The knot would be the center of the circular mat. Another strip is knoted at the same point and weaving is begun by connecting the strips. The strips can add to the rotated strips according to the need of the diameter of mat. Once the first rotating

strips has been done, more strips are simply added as needed. Such a mat weaving activities are common activities in different cultural group of peoples.

Students from the different community are familiar from those cultural artefacts. The shape of sitting mats exhibited the exclusive examples circular shape and the examples of concentric circles. The household activities regarding the construction of such artefacts facilitate to understanding of geometrical concepts. The geometrical pattern observed in the sitting mat also mediated to conceptualize the symmetry.

Geometrical Patterns in Textiles Weaving

Weaving practices in Newar culture have been using ideas from mathematical concepts. In order to create patterns on clothes which are to be knitted, many mathematical techniques and tools are used. Diagonal shapes, rectangular shapes, circles and other different geometrical shapes are created by passing the threads which are laid parallel with each other. These shapes are made by passing the weft thread over two or more warp threads then repeating that patterns one wrap thread over another. The process of weaving is linked to the innovative ideas of mathematics. People know how to make shapes even if they haven't gained formal mathematical knowledge.



Image 5.7. Constructing Frame for Textiles Weaving

Since symmetry in general and bilateral symmetry in particular have been permanent, and very visible, ingredients of man's culture from the dawn of human history, they are expected to permeate to some extent all kinds of literary works. In order to establish this, we will look for examples in a variety of works of literary art. Attention will be directed especially towards fiction. The cultural activities of textiles

weaving involves implicit mathematical ideas. The observation of the activities is devoted to an investigation of the mathematical principles present in the patterns found specifically in the *hakku patasi* textiles. The fundamental aim is to elucidate and present explanations of the



Image 5.8. Geometrical Patterns in Textiles Weaving

presence of symmetry in the patterns in these textiles. By doing this, an acknowledgment and enrichment of this aspect of Newari culture uncovered the mathematical ideas embedded in thier cultural artefacts.

The design and pattern in the home made *hakku patasi* involves some mathematical aspects and ideas incorporated in the patterns invented by textiles weavers and to support further initiatives that may contribute to a fuller realization of the mathematical potential of the rural crafts made by men and women in their community. Symmetry in its various forms is one of the most fundamental features of the designs on the arts and artefacts constructing from the different group of people.

Traditional Houses and Exploration of Geometrical Ideas

The house in the cultural village of Nepal mostly use roof with zinc sheet. The roof is easy and economical to construct because it requires less numbers in wooden



Image 5.9. Gable Roof in the House of Cultural Village

Source: <http://himaland.com/en/nepal-village-tour-home-stay.html>

beams and related materials. There are two design in common: one roofer, and two roofers according to the interest of house owner.

The house builders use a lot of mathematical ideas in the process of construction of buildings. They are using traditional measurement unit of *haat* and *bitta* to measure the base area of the house. Most of the house in the village be constructed by using stone and mud. The foundation of two/three *haat* be dug out and then the walls construct. The house builders uses sophisticated mathematical concepts implicitly in the process of the construction. Regarding the ethnomathematical ideas and thinking of the house builder, one of my research participant Arjun viewed that

I have passed my Bachelor in Education with major mathematics. I learnt most of the mathematical ideas just for securing marks in the examination. I have good knowledge of mathematical concepts but without knowing its real life implication. I was surprised to observe house builders using Pythagorean Theorem without knowing the name of the Pythagorean Theorem. I do teach my students in the same way as I was taught. But now I have become aware of practical knowledge of mathematics.

Here I observed a dichotomy of two types of people in the same community, the people who have formal knowledge do not have practical knowledge and vice versa. Who is the real owner of the mathematical ideas? It is believed that mathematics is a cultural product. The people before Pythagoras also had constructed house, pyramid and other such artefacts in their own society. The house builders though lacks formal knowledge, they live and sustain their lives. They use their emic knowledge in their everyday job. The mason use tools in the process of the constructing wall. The *buttom* and *bell* are the tools they used to form in each new

row of the wall. I asked one of mason names *Maila* (53): why do you use these tools in the construction of each new row of the wall? He says

The wall should be in vertical position. The bell that we used helps to every consequent step of stone/ bricks in a straight line in both horizontally and vertically. The walls must be in a vertical position to keep the wall strong.

Different group of people uses their mathematics to tackle everyday problems. Their mathematical thinking and ideas might be different from the formal ways of thinking. Their mathematical knowledge is experiential as opposed to classroom bookish knowledge. Students participate in two cultures that is the culture of home and the culture of the school. The contextualizing and making connections are powerful processes in the development of school mathematical understanding. In view of importance of connecting out-of-school cultural activities to school mathematics, my research participant Arjun viewed that

I think the connection between out-of-school culture and school mathematics can improve the understanding of the concepts. The children can get a lot more interested if the mathematics teaching is based on rich situations from their culture. The mathematical concepts can be tied to their cultural activities- anything just from simple games they play, kitchen and household activities, observations of the artefacts in the surrounding. All these cultural activities embeds a lot of mathematical ideas and problem solving skills.

The indigenous people have their own ways teaching and learning process in social phenomena. The cultural activities of the *Chundara* and the knowledge generation process involved participatory and cooperative approaches in which they learn with the help of their parents as Vygotsky seeing learning as an activity in which shared mathematical meanings are constructed socially (Pradhan, 2012). I also

observed that they are using implicit mathematical knowledge in their everyday activities. According to Millroy (1992) “the physical act of designing and building furniture would involve tacit mathematical knowledge” (p. 13). The house builder *Maila* mason used the concept of tacit knowledge of mathematical ideas in the process of constructing wall. The incorporation of cultural activities and ways of teaching and learning in out-of-school environment facilitates the development of mathematical ideas.

Carving and Exploration of Geometry

The term symmetry has been derived from the Greek word *symmetros*, which means “measured together”. Symmetry is most familiar to us as a description of the similarity between the two halves of any form separated by a dividing line or plane known as the axis of symmetry. The concept is not only limited to science but also a mainstay of aesthetics from the beginning. Thus we speak of the symmetry of the human face, we can all understand what it is and how it looks. Its influence in the arts is pervasive. In music, we hear the symmetry of exposition and recapitulation in the so-called sonata form, where virtually the same music is played and played again.

The notion of symmetry has an important role in the development of mathematical concepts and science. In everyday life, the most common case of symmetry is associated with a mirror reflection. In this vein, Capra (1975) defines symmetric figure is that which one can draw a line through it and thereby divide the figure into two parts which are exact mirror images of each other. There are reasons why researchers into African art and Ethnomathematicians consider the property of symmetry in the analysis of culturally produced patterns and designs on material culture. By understanding the role of symmetry in the visual recognition process, we can better recognise its frequency throughout a number of cultural domains.

The mirror symmetry, or bilateral symmetry, is the symmetry with respect to reflection. A figure, which is indistinguishable from its mirror image, is called mirror symmetric. A way to think about this symmetry is to fold the image in the middle and check if the two halves are identical (Sparavigna,2000). The rotational symmetry is different because it is quite natural for us to follow the image in its rotation in the space. This symmetry produces a feeling of motion and evolution. A breaking of symmetry, in this case, enhances the idea of interpenetration or contrast. Let us consider for instance a well-known symbol, the Yin-Yang symbol of Taoism. The image actually intends to be a representation of the complementary need for male and female concepts. The rotational symmetry of the shape plus the breaking of color symmetry catches our attention with a feeling of development in the frame of complementary actions, not of mere recurrence.

Symmetry in Stone Carving

Patharkatta, a stone carver, constructed different images that possesses to symmetry. One of stone carver Biru says that the image he treasured to construct to make the stuff more beautiful. The image will be more beautiful, and eye pleasing if the images are in symmetric in the pattern.

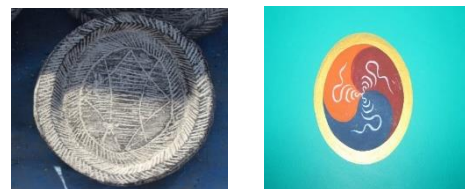


Image 5.10. Patterns and Symmetry

It exists not only in art and design, but it also exerts its strong presence in mathematics, in the physical world of science, in history, and even in story plots and character development. They construct different geometrical pattern in their works. They use rectangular coordinate axes while start to draw a pattern. The symmetries are found in their activities.

In the village of Nepal symmetrical images are painted on either wall of the door in a house. The wonderful mathematical ideas and concepts which are practised

in the construction of different patterns in stone carving. The stone carver use emic view of the mathematical knowledge in the process. The mathematical knowledge they practised since long complimented to their everyday lives and developed across generations through verbally. So, it is impossible for a researcher to go through the emic approach because they all have preconceived perspectives, ideas and the theories. In this regards, Fetterman (2010) viewed that “most ethnographers start collecting data from the emic perspective and then try to make sense of what they have collected in terms of both the native’s view and their own scientific analysis” (p. 22). Further, Lett (1996) mentioned that an etic approach seeks to compare and examine cultural practices of the members of certain cultural groups by using standardised methods (as cited in Orey and Rosa, 2015, p. 376).

This implicit mathematical knowledge is embedded in their activities tacitly. Mosimege and Lebeta (2000) also reported that the indigenous people use different mathematical concepts like estimation, and tessellations, and symmetry in the construction of the traditions artefacts and cultural activities. The stone carver use lots of mathematical concepts and knowledge. This knowledge is sufficient for performance that is consistent with rules, even though the person might not be aware of the rules. The field data also supported the findings of Millroy (1992) that the stone carver use tacit mathematical knowledge in the process of constructing different stuffs as Millroy (1992) observed that the carpenters’ physical act of designing and building furniture involved tacit mathematical knowledge. In this situations, they practice the mathematical ideas of the circular shape, centre and radius of the circle; the coordinate axes, reflection and symmetry of different geometric patterns.

The indigenous language pattern of the speakers reflects mathematical concepts and ideas (Owens, 2010) who observed how the most indigenous languages

of Papua New Guinea (PNG) possess mathematical concepts like volume, mass, area, and length. The children from indigenous communities are deeply immersed in their own cultural world. Their home language reflects the mathematical thinking of the people in own ways (Barton, 2008; Owens, 2010) and I also observed that stone carver have tacit mathematical knowledge of circle, identification of centre, diameter in the various stage of construction of geometrical pattern and in their conversation.

Monuments and Mathematical Exploration

The logs of woods are shaped in a beautiful and appropriate way with axe and knife. They do not use nails to build the temple. Roofs are slanted to an appropriate angle in order to keep safe of different calamities and leakage of water. No thatch or tin sheets are used on the roofs. Planks of woods are used to make roofs. No professional training has been given for the architectures to bring such notion of architects. Here, we do encounter the geometry embedded in the temple. We can see the different varieties of woods cut in proportionately for different purposes. Different colour are put in woods of different pictures of deities in Hindu culture. What D' Ambrosio (1985) says is to maximise the gold mine of cultures for different purposes and identify the mathematical applications. Here we can see the beautiful mathematical artefacts in the architecture. The rectangular pyramidal walls which add



Image 5.11. Temple: A Monument

more beauties to the buildings. The rectangularly shaped walls are added one upon the other in diminishing order which tends to a point if it is extended to infinity.

Image 5.11 shows the typical temple of Nepalese culture. The temples were built with traditional architecture preserving the cultures of the particular community (Newari community). But it is interesting that Nepalese temples do have the particular design of windows and roofs and there is the difference between the normal type of house and the religious temple. My one research participants opined and related the architecture of the temple with the religious beliefs. But, what is the significance of a peculiar type of windows? The windows of temples are distinct from the normal windows so as to preserve the culture and tradition of the community. The temple resemblances not only the cultural and religious traditions but also exhibited a different type of mathematical designs.

From the observation of the temple, it was noticed that windows are put at an equal distance from one to another. The special design put on the windows (depicted below) give the special type of pattern of the number system. The roof of the temple that we could see is built in the shape of a rectangular based pyramid. Most of the temples do preserve this peculiarity. The roof of the temple has been supported with wooden poles inclined at an angle of 45 degrees. It has got the scientific reason for this particular angle. It is assumed that a large amount of load can be borne, with a small amount of force put at an angle of 45 degrees.

Doorways and Mathematical Ideas

It is obvious that mathematics is embedded in every culture in every step of work. But we must be able to trace out for this. The mathematics that is found in every aspect of daily life is called ethnomathematics. Ethnomathematics include history, philosophy and pedagogy. For a particular type of artefact one knows its

originality, essence of that in the culture and mathematics, and how it can be associated with the different application. In the observation of the temple, the doorways is surmounted by the semi-circle. The arts in woodcarving has rich sources of symmetry and reflection.



Image 5.12. Doorways Surmounted by Semi-circle

To encounter such aspect of mathematics the sole content knowledge may not be sufficient for the students. As such students must be given some project based on mathematical artefacts so that they can encounter such knowledge in different cultures and broaden their knowledge. These days the cultural root of each mathematical knowledge is very important and students should be allowed to explore to the utmost so that they can relate the local knowledge with the global corpora. In this regard, Orey (2000) had suggested the following: “Students of today must be able to use mathematics to better their lives; it is the overall quality of a successful mathematics education that depends on the following characteristics that enable them to become full participants in their society. Because we all live in a globalized society, students: Must be able to reason logically; Must be able to use their own creativity; Must strengthen their own unique onboard ability to quickly adapt and learn in new and innovative situations; Must be able to become active knowledge gatherers and disseminators; Must develop a sense of responsibility towards the environment, including the preservation of its rich and diverse cultures; Must be able to develop initiative for resolving problems; Must develop processes for daily professional life”.

Further, Orey (2000) believes that through the use of interdisciplinary activities, students learn to see mathematics as linked to other areas by using research

tools that analyse problems and develop mathematical models. However, the selection of content alone is not enough to guarantee their meeting of mathematical objectives. The way in which subjects are organised by the classroom teacher (pedagogy) as equally important since individual or personal responsibility, involvement and connection to the disciplines are important to the success of each new aspect of the mathematics learners find for themselves. This enables the formation of flexible, curious teachers and learners who readily engage in the basic research and critical exploration needed to collect the mathematical data and tools necessary for life in a globalised information-rich society.

In particular, Nepal is a multi-cultural country consisting of 126 ethnic groups. Each ethnic group has a unique culture and different mathematical knowledge is embedded in each culture. As such, the inclusion of ethnomathematics in our mathematics curriculum is indispensable so as to incorporate the ideas of different groups of students. Our classroom pedagogy should allow the acculturation with thorough discussion with collaborative learning.

Bell and Exploration of Geometry

The term geometry may be evolved as a tool to measure the land. It is thus known as a measurement of “Geo”. However, our school geometry really does not measure the associated problem regarding measurement of the distance between different two places. It also insufficient to measure the land area of the different landscape. Thus, Euclidean geometry is the geometry of plane.

Nepal is a secular country. The main religion of Nepali is Hindu. They worship Brahma (Shiva), Bishnu, and Maheshwar and they are taken as the Trimurti (त्रिमूर्ति) in Sanskrit. The three forms are the supreme divinity in Hinduism. They are considered as the cosmic functions of creation, maintenance, and destruction.

Particularly, they are leveled as Brahma the creator, Bishnu the preserver, and Shiva the destroyer or regenerator. Hindu people are often visit temples to worship god and goddess. We did encounter the bells to enchant the prayers and to symbolize the presence of the devotee in



Image 5.13. Bell and Exploration of Geometrical Ideas

the temple to worship the god. In this vein, while observing the temple bell, it is possible to describe the distinctive features of hyperbolic geometry and was the metaphor to contextualised the notion of it. The different type of triangles can be drawn on the surface of the bell.

From the above hyperbolic triangle, it is seen that the sum of the angles of a triangle is always less than 180 degrees, which contradicts the notion that the sum of the angles of the triangle in Euclidean plane is always equal to 180 degrees. Bell is one of the interesting artefacts that has lots of the properties of non-

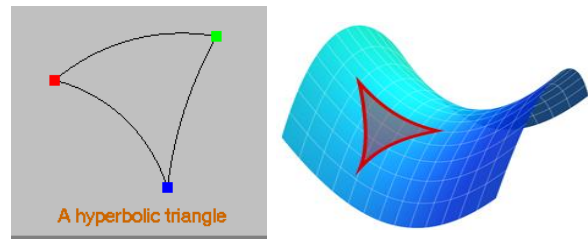


Image 5.14. Hyperbolic Plane

Euclidean geometry. The various property of hyperbolic geometry can be found in the bell. The parallel lines meet at a point, the sum of the internal angles of a quadrilateral is less than 360 degree and other related properties also can be found from the observation of bell. To distinguish the different types of plane, we would like to draw three different planes which may be helpful to generalise the different types of geometrical figures of Euclidean plane, Hyperbolic plane and Elliptic plane. Luintel and Taylor (2007) exhibited that the *shanai* and pseudo-sphere reveal that two parallel lines meet in a plane asymptotically which helps to contextualise the hyperbolic geometry.

Geometrical Exploration in Cultural Arts and Artefacts

Mathematical anthropology uses mathematical modeling in historic, ethnographic, and material culture studies to describe material and cognitive patterns of certain group of people (Eglash, 2001). In this vein, Rosa and Orey (2006) viewed that mathematical modeling is methodological tool that may be used in the ethnomathematical program. Ethnomathematics can reshape our greater cultural identity in positive way by requiring the inclusion of a greater representation of the true mathematical practices and problems of a students' own community (D' Ambrosio, 1998; Zaslavsky, 1996). The people of the different culture around the world prevalence of symmetry in designing textiles, sand painting, wall painting, pottery, arts, and artefacts.

The western mathematical way of thinking spread and grew along with the mathematical practices of a particular group of people in their everyday activities. The mathematical ideas practising in students environmental context may be used as cultural vehicles for the teaching and learning of school mathematics. The informal mathematical ideas and knowledge that practices and uses in the everyday activities are the source domain for the understanding of abstract mathematical ideas. Thus, the ethnomathematical system of the cultural group can enable a better understanding of school mathematics.

Rangauli: Connecting Geometric Pattern and Aesthetics

Nepal has rich cultural traditions and peculiar rituals. This is mandala of *rangauli*. It is constructed in the occasion of *Dipawali*. The Rangauli is drawn by the rice flower including seven different colors and seen on the floors of buildings and in front of worship room (*Puja Kotha*). This drawing requires different geometrical patterns and spatial reasoning. The number of concentric circles, number of

symmetries, vertical and horizontal reflections and rotations etc are found in the *Rangauli*. The different colorings are also associated with this drawing. Regarding the construction of *Rangauli* mandala, I asked to one of my research participant Sushila, how to draw a Mandala pattern. She replies:

The mental image of the pattern helps to draw a *Rangauli mandala*. The cognitive map of the *mandala* is important things for that. We start from a point and draw some concentric circles. Anyone can draw different shapes inside the regions in concentric circles. Some may draw an eight-pointed star or eight petals of a lotus as per their choices.

From the interview with Sushila, I have come to know that the construction of mandala is a mental construction. The different geometrical pattern found in their construction is largely hidden from the participants. And the ways of drawing of such cultural arts and artefacts is craft model approach. The image 5.15 was the mandala constructed by Sushila herself in front of main doorways of house. I also asked her

that how to draw the eight-pointed star. She had already done this before I meet to her but said that she would work it out. She started with a rough diagram. The process of her drawing was that first, she draw concentric circles, and then she first divide the circle in four equal parts by



Image 5.15. Rangauli: Sources of Geometrical Patterns

drawing two mutually intersecting lines. Further the four parts be divided in to another halves and making isosceles triangles whose vertex at the ends of each of eight lines.

From the observation of rough sketch of the ways to drawing an eight-pointed star, I have come to know that they are practising sophisticated mathematical ideas in

the process of constructing mandala. I asked another research participants Kamala about the knowledge they learnt to construct such beautiful artefacts and her mathematical ideas associated with it. She replies:

I learnt to make this artefact by observing the ways of drawings of my mother as in the same of how these children observing patiently to my activities here. First I draw a sitting place for goddess Laxmi at the centre as we are going to worship the goddess. Then I drew some larger circle and eight petals of lotus as we believe that she likes to lye over the lotus flower.

Rangauli is a symbol of good fortune. Rice powder used for *Rangauli* drawings provides nourishments for smaller animals, such as birds and ants. This is a symbolic act of showing the necessity of sharing and maintaining proper relationships with others. It also reinforces the respect for nature and relationship with other human beings and animals. This art form is almost exclusively practiced by Hindu women. Rangauli has been constructing as cultural product from generation to generation in a process of craft model learning approach.

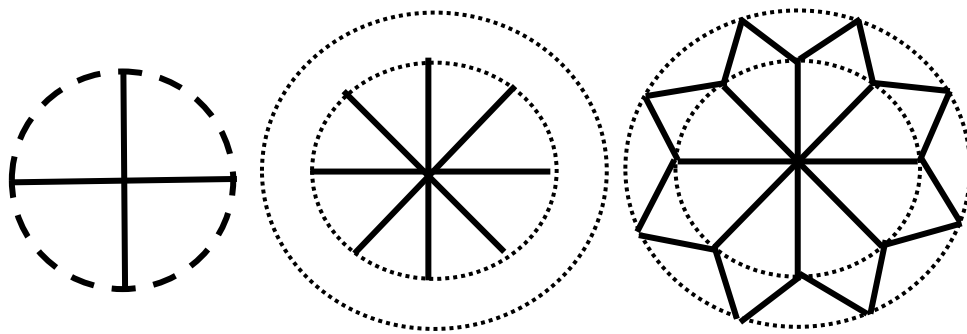


Figure 5.3. *Process for Construction of Mandala*

The *astamatrika* or eight-pointed star has great significance and has religious meaning in Hindu culture. The creation and perception of the pattern have a logical as well as aesthetic component (Ascher, 1991). The Rangauli drawing involves complex mathematical activity and strategic understanding of spatial reasoning. It is important to explore the hidden mathematical ideas in the process of constructing such cultural

artefacts. For that, I have conducted interviews with some people who are constructing such artefacts as their cultural tradition. They possess sophisticated mathematical ideas in construction having very little knowledge about academic mathematics.

Teaching mathematics becomes more interesting if the contents are related to the students' familiar environments. Students' everyday interaction with the cultural environment provides more opportunities to interact and conceptualise mathematical ideas. The cultural environment activities and artefacts of the students involve a lot of mathematical knowledge and ideas. The mathematical ideas embedded in the students' cultural environment which is familiar to the students provide a good environment to learn. This environment plays important tool to communicate mathematical ideas.

Swastika Symbol and Mathematical Exploration

The word Swastika symbol is deriving from Sanskrit. Swastika takes the meaning of good fortune or well-being and *Subha Labha* in Nepali. It is believed first to have been used in Neolithic Eurasia to represent the movement of the sun through the sky. Even today a similar hooked cross is a sacred symbol in Hinduism, Buddhism, Jainism and Odinism and is commonly found in temples and houses. It is common that the Swastika symbol is found in main entrance gate of the house and the money collection box in the shop. This symbol is commonly printed in the collection box in the shop and the sticker of this symbol is hanging in the shop. Regarding the values of Swastika symbol, I ask shopkeeper. Why are the people using this symbol? The meaning of Swastika symbol in Nepali is Shubha Labha. It is



Image 5.16. Swastik found in Textiles

believing that more collection of money be made if this symbol is printed in the collection box. This finite symbol, most famously known as a swastika, is a common symbol found in multiple cultures around the world.

This image was used in Native American rugs. There have been variations of this symbol throughout human history and it has been used by the Hindus, Buddhists, Ancient Greeks, Romans, Chinese, Christians, and most recently (and famously) by Nazi Germany. One of the more common places to find this symbol is in Native American pieces of art. Other Native American folklore has stories about one of their own travelling down the river and using the swastika as a symbol of good luck. The symbol as has the meaning of "well-being." Within the last sixty years, the symbol has gained the negative connotation of "death" and "fear".

As a group, we see a lot of mathematics behind this symbol. A very obvious connection is the reverse symmetry. If we were to diagonally cut the swastika and then reflect it over a mirror, the image we see would be reversed of the actual symbol. Another mathematical feature is the 90-degree angle that separates the four arms. We see the pattern as a rotation of an arm around its axis at 90-degree intervals. The symmetry in its various forms is one of the most fundamental features of the symbol. *Abas (2001)* pointed out that the beautiful Islamic geometric patterns were excellent materials to teach the idea of symmetry at all levels, from properties of simple geometric figures to tessellation, transformation, and to providing “a visual gateway for the teaching of abstract notions of Group Theory at the university level”. The swastika symbol mediated to conceptualize the symmetry from basic level to the university level as symmetry is fundamental concepts of mathematics.

One of the major objectives of the ethnomathematical research is to look for possibilities of incorporating the mathematical ideas embedding into the cultural

context of students and teachers for improving the teaching and learning of school mathematics. The connections between art and mathematics are well established and are firmly rooted in the history of art. Art historians often use the characteristics of symmetry and the rules of the golden mean to assess the formal qualities of a work of art. In further establishing the interrelationship between art and mathematics, Hardy (1940) compared a mathematician to a painter or a poet and a maker of patterns made with ideas. He wrote:

The mathematician's patterns, like the painter or poet's, must be beautiful. The ideas, like the colours or the words, must fit together in a harmonious way.

Beauty is the first test: there is no permanent place in the world for ugly mathematics.

There are reasons why researchers into African art and ethnomathematicians consider the property of symmetry in the analysis of culturally produced patterns and designs on material culture. By understanding the role of symmetry in the visual recognition process, we can better recognise its frequency throughout a number of cultural domains. Crowe (1988) has shown that patterns can be described by their symmetries and that repeated designs occur frequently on many types of objects.

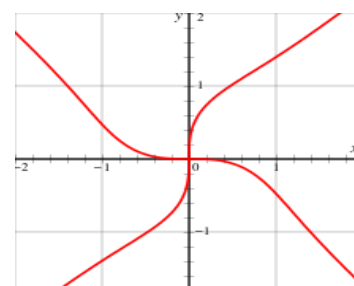


Figure 5.4: Swastik Curve

The swastika curve is the name given by Cundy and Rollett (1989) to the quartic plane curve with the Cartesian equation $y^4 - x^4 = xy$ or, equivalently, the polar equation $r^2 = -1/2 (\tan 2\theta)$. The curve looks similar to the right-handed swastika. It can be inverted with respect to a unit circle to resemble a left-handed swastika. The Cartesian equation then becomes $x^4 - y^4 = xy$.

Mathematical Ideas in Cultural Games

The different game provides rich opportunities for all children to build and extend mathematical ideas and strategic thinking. In fact, through the game, children not only build mathematical ideas but also interact with aspects of their culture. Games are helpful to develop a social attitude and explore important social behaviours. Moreover, introducing this game, either in or out of school, can help children understand that humans encode their mathematical ideas in diverse cultural products, including architecture, art, games, music, written texts, and so on. Games help students to develop quick decision making, logic and reasoning to every step in games which are the foundations of mathematics learning.

The different cultural games are embedded mathematical knowledge, ideas and concepts. In the above games that I discussed have a lot of geometrical ideas such as line, point, triangle, quadrilateral, parallel lines, perpendicular lines, and mathematical concepts like counting, measurement of time, the basic concept of arithmetical operations, relation, function, ratio, proportion and so on. So these games are helping to contextualise mathematics teaching and learning.

***Bagha Chal* Game and Mathematical Exploration**

Bagha Chal is a strategic, two-player board game that originated in Nepal (Karki, 2012). There are twenty goats and four tigers. One player controls four tigers and the other player controls up to twenty goats. The Tigers hunt the goats while the goats attempt to block the tigers' movements. Pieces are positioned at the intersection of the lines and not inside the areas delimited by them. Directions of valid movement between these points are connected by lines. At the start of the game, all four tigers are placed on the four corners of the grid, facing the centre. All goats start off the

board. The pieces must be put at the intersections of the board lines and moves follow these lines.

The player controlling the goats moves first by placing a goat onto a free intersection on the board. Then it is the Tigers' turn. One tiger is then moved to an adjacent position along the lines that indicate the valid moves. Moves are alternate between players. Tigers capture goats by jumping over them to an adjacent free position. Goats cannot

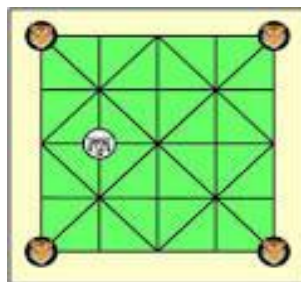


Image 5.17. Board of Bagha Chal Game

move until all twenty goats have been put on the board. The Tigers win once they have captured five goats. Goats try to avoid being captured (jumped over) and they win by blocking the Tigers' moves till they are unable to move. Sometimes the game can fall into a repetitive cycle of positions. Goats especially may use this resort to defend themselves against being captured. To avoid this situation, an additional rule has been established: when all the goats have been placed, no move may return the board to a situation that has already occurred during the game.

Although no historical proof exists, *Bagha Chal* is said to have been played first by goatherds. They were on constant lookout for tigers and other predatory animals and played it to pass their time as goats grazed around in the forest. Times have changed now. The *Bagha Chal* boards can be found in bronze, wood and metal. However, this can be used as a didactical game which supports teaching and learning certain mathematical concepts and knowledge. The analysis of the *Bagha Chal* reveals a number of mathematical concepts. It is important to indicate that the list of mathematical concepts specified below is not exhaustive. My study identified the following mathematical concepts in the analysis of board of *Bagha Chal* game.

Exploration of geometrical shape. The analysis of the *Bagha Chal* reveals a number of mathematical concepts. It is important to indicate that the list of mathematical concepts specified below is not exhaustive. Identification of varieties of geometric figures in the construction of the board for *Bagha Chal* game such as squares, rectangles, triangles, parallel lines, transversal line and associated angles. The board of the *Bagha Chal* also reveals the properties of various geometrical figures viz. the sum of the internal angles of the triangles of the square/ rectangle; vertically opposite angles, the sum of the adjacent angles, corresponding angles, alternate angles and so on.

Bagha Chal is a board game and the board is square in size. While drawing a net of *Bagha Chal* game, a player practising lot of geometrical knowledge. First of all, a square is drawn. The different player can make a net of *Bagha Chal* board differently. I observed the ways of making board of three different players and noticed two different ways. The ways of drawing the net of the board of each player exhibited the geometrical thinking of each player. The styles of different players to draw the net are the outcomes of their connection of geometrical knowledge they have.

Player I: This player divides each side of the square into four equal parts and then they can join and parallel lines are drawn (Red Pen and Brown Pen). Then they get 16 small squares. After that, the two diagonals were drawn (Black Pen).

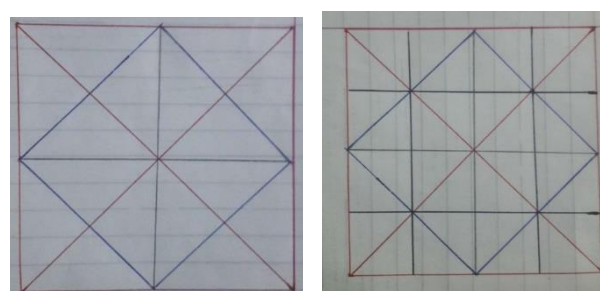


Figure 5.5. Constructing Board of *Bagha Chal* Game

Player II: the second player draws a square at first and then draw two diagonals. The point of intersection of two diagonals is the centre of the board (Red Pen). Two

perpendicular lines were drawn (Brown Pen). The point meets the sides of the square are the midpoints of the sides of the square. Then draw the lines that join the midpoints of the side of the square (Blue Pen). The line joining the midpoints of the sides of a square also forms a square.

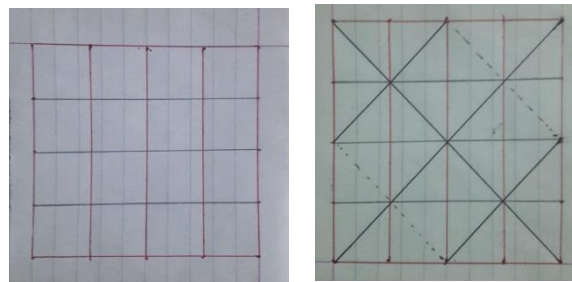


Figure 5.6. Exploring Geometrical Shape in Board of *Bagha Chal*

Centroid of triangle. The board of the *Bagha Chal* game has exhibited rich mathematical contents. The centroid of a triangle is the point of the intersection of the medians. It is a point which divides the ratio of 2:1. In the observation of the board of the *Bagha Chal* game, E and F are the midpoints of the sides AB and AC of triangle ABC. The quadrilateral BCFE is a trapezoid and the diagonals BF and CE of Tpz BCFE intersect at a point G. here the line BF and CE are the medians of triangle ABC and hence G is the centroid of triangle ABC. I observed the

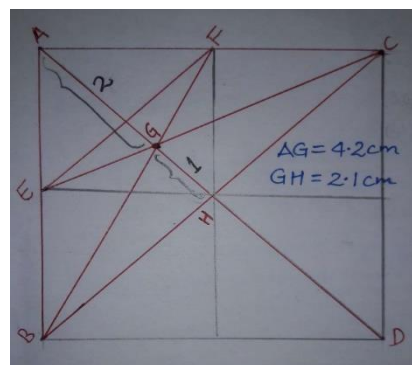


Figure 5.7. Exploring Centroid of Triangle

ways of making board of three different players and noticed two different ways. The ways of drawing the net of the board of each player exhibited the geometrical thinking of each player. The styles of different players to draw the net are the outcomes of their connection of geometrical knowledge they have.

Infinite series. Identification of varieties of geometric figures in the construction of the board for *Bagha Chal* game such as squares, rectangles, triangles, parallel lines, transversal line and associated angles. The board of the *Bagha Chal* also reveals the properties of various geometrical figures viz. the sum of the internal

angles of the triangles, of the square/ rectangle; vertically opposite angles, sum of the adjacent angles, corresponding angles, alternate angles and so on. The concept of symmetry was also found on the board of *Bagha Chaal*. The *Bagha Chal* board can be obtained by folding the square plane lamina in possible symmetric shape in square and triangle. It also helps to conceptualize the infinite geometric sequences by joining the middle points of the squares. If 'a' is the length of a side of the square, the second square which is obtained by joining the midpoints of the sides of the square, the length of the side

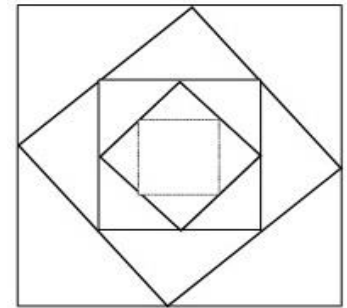


Figure 5.8. Infinite Series in Board of Bagha Chal

of the second square will be $a/\sqrt{2}$ and the side of the third square is $a/2$ and so on and we get the infinite number of squares. The sides of the length of the squares possess an infinite geometric sequences with common ratio $1/\sqrt{2}$.

Isometric transformation. The concepts of transformation geometry can be observed by the movement of the tiger according to the rule of the game. When the tiger is at the centre of the board and has the opportunity to catch the goat in any direction, the tiger can jump from the position of the goat to the empty slot. Here, the position of the goat is the point of reflection and the line through the goat is a line of reflection. When $P(x, y)$ represents the position of the tiger and P' represent the position after the jump of a tiger, it can conceptualize the reflection about the various axes.

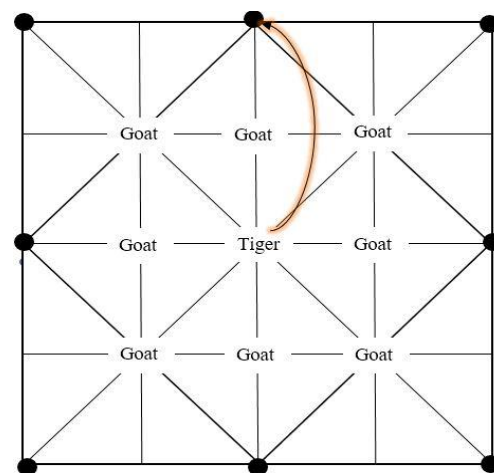


Figure 5.9. Movement in *Bagh Chal* Game

a) When the tiger moves left and right to catch the goat, it can be conceptualized as the reflection of the point (tiger's position) in Y-axis and the point of reflection of $P(x, y)$ is $P'(\pm x', y)$.

b) When the tiger moves below and above to catch the goat, it is obvious of reflection in X-axis and the point of reflection of $P(x, y)$ is $P'(x, \pm y')$.

c) When the tiger moves along the diagonal line or the line parallel to it, at that situations the tiger's positions can be conceptualized as the reflection in the line $y = \pm x$ and the point of reflection of $P(x, y)$ is $P'(y', x')$ or $P'(-y', -x')$.

Mid-point theorem of plane geometry. The board of *Bagha Chal* is square in size. Each side of the square is divided into four parts and then the larger square will be divided into 16 small squares as shown in the diagram. Further, two diagonals are drawn so that the large square be partitioned into different triangles. The various

geometric theorems related to the triangles and parallelogram can be illustrated with the net of the *Bagha Chal* board.

The midpoint theorem states “the line joining the

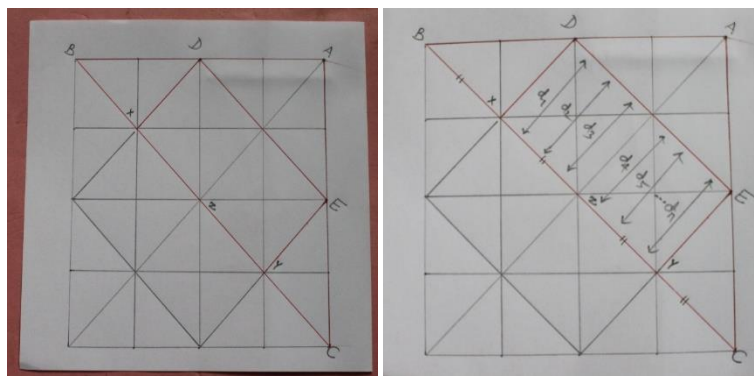


Figure 5.10. Mid-point Theorem in Board of *Bagh Chal* Game

midpoints of any two sides of a triangle is parallel and half of the third side”. In the given diagram, any one of the diagonal be taken as the base of the triangle and two of the sides of the square are other two sides of the triangle.

Let us suppose the diagonal BC is the base of triangle ABC. Here the point D and E are the midpoints of the sides AB and AC. The diagonals DX, EY of the small square are evidently equal and hence $d_1, d_2, d_3, d_4, \dots, d_n$ are all equal. Hence the line

joining the midpoints D and E of two sides of the triangle ABC are parallel to the third side BC. Also, side $BC = AX + XZ + ZY + YC = 4 ZX = 2 DE$. Thus, the line joining the midpoint of any two sides of the triangle is half of the third side.

As there was the historical evidence that the have been played first by goatherds because they were on constant lookout for tigers and other predatory animals as goats grazed around in the forest. The goatherds draw the board of *Bagha Chal* on the plane slate of stone and the wooden plank. The layman who is familiar with this board game has such wonderful mathematical knowledge that they practised since long.

Pythagorean Theorem. The board of *Bagha Chal* is square in size. Each side of the square is divided into four parts and then the larger square will be divided into 16 small squares as shown in the diagram.

Further, It is also seen that each small square is divided into two right-angled triangles. The concepts of right-angled triangle and verification of Pythagorean theorem can be conceptualized through the board of *Bagha Chal*. In the adjoining

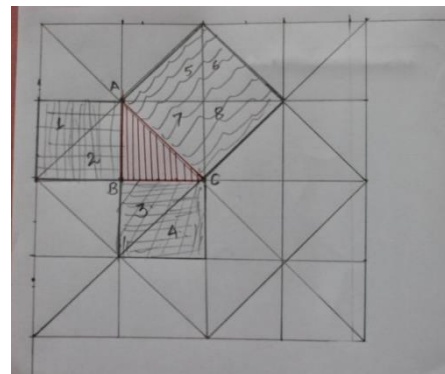


Figure 5.11. Pythagorean Theorem in *Baah Chal* Game

figure, ABC is a right angled triangle. The side AB, BC and CA are the altitudes, the base and the hypotenuse of the triangle ABC. Now,

$$(AB)^2 = \text{Area of a square whose one side is AB}$$

$$= \text{Area of triangle 1} + \text{Area of triangle 2}$$

$$= \text{One small square}$$

$$(BC)^2 = \text{Area of a square whose one side is BC}$$

$$= \text{Area of triangle 3} + \text{Area of triangle 4}$$

$$= \text{One small square}$$

$(AC)^2$ = Area of a square whose one side is AC

= Area of $\Delta 5$ + Area of $\Delta 6$ + Area of $\Delta 7$ + Area of $\Delta 8$

= Two small square

The people construct the board of *Bagha Chal* game use implicit mathematical knowledge. The way that they construct is usual activities but possesses high mathematical concepts from the eyes of ethnomathematics. Ethnomathematics is the study of the relation between mathematics and culture. D' Ambrosio (1997) explains ethnomathematics as the mathematics practiced among identifiable cultural groups. Using ethnomathematics as the pedagogical tools in the mathematics classroom help students learn not only mathematical concepts but also cultural elements.

The *Bagha Chal* game can provide opportunities for developing skills related to mathematical thinking – predicting, generalizing, justifying and explaining. For Ainley (1988, p. 243), the main value of mathematical games lies in the linking together of mathematical problems which are ‘real’ to the children, the use of such process skills as listed above, and the need such activities present for children to think in a mathematical way. The board of *Bagha Chal* game can also be the metaphor to communicate the abstract ideas of mathematics. It mediated for the understanding of different geometrical concepts, centroid of triangle, geometric series, the concept of isometric transformation, verification of mid-point theorem and Pythagorean theorem.

***Chungi* Game and Exploration of Mathematical Ideas**

A *chungi* might be just a bunch of rubber bands tied together but it looks like an aster, a sunflower and a daisy. It's a kicker's game, rooted in the soil of Nepal. For children, *chungi* is a prized possession played whenever school or time permits.

The craze for *Chungi*, however, disappears with adulthood. It becomes a string of dirty rubbers crawling with bacteria. Most forsake the game and prefer to slump in

their couches. But children play it with the same fervour as the outgrown adults once did. They are the real envoys of this popular game. *Chungi* is noticeably seen during winters when parents encourage their young ones to go out in the sun. To become the first participant of



Image 5.18. Chungi: A Bunch of Rubber

the game, one has to kick the *Chungi* in the air without putting one's foot down for the longest time. Then the number of winning the game is decided, most often 50 or 100 kicks, played in the pattern of kicking the *chungi* and putting one's feet down.

The winner in a game of *chungi* is the one who can make the biggest score first. It requires concentration and stamina. One can play it with either leg, and every game ends with a back kick. Your back kick should land the *chungi* in the distance decided between the players, some 10 or 20 steps away from the player. In due course of time, rubbers replaced feathers, and its shape took a symmetrical round form. Perhaps, the low cost of a *chungi*, its accessibility and simplicity are the reasons behind its wide acceptance and presence. *Chungi* is very interesting and popular game for the children of all ages. It is a very useful game both mentally and physically.

Chungi game played on normally in empty plots, playground, or in a big room where a player has to move during the game. The



Image 5.19. Students are Playing *Chunai* Game

player should be at least two and play with *Chungi*.

During my data collection period, I discussed with a mathematics teacher if the games played by students could be linked up with the learning of school mathematics. The methods involved during the game play made me to think that there

could be numerous such concepts that could be linked up with the classroom teaching. Therefore, I asked him if we could observe the students playing the game. The teacher assisted me to the ground and we started observing the game play of the students.

After the session of game play completed, I asked two playing students to meet us.

Then I started talking with them about the game they had been playing earlier. They were playing *Chungi* and I had observed them competing to win. Then I asked them about the game.

Researcher: Who was the first player of the game today?

Boy A: Sir, me!

Researcher: What was the score you scored in the game?

Boy A: 74, sir.

Researcher: [Pointing out the other boy] then what was the score you made?

Boy B: 52. Sir but I did not lost the game. In fact, no one won the game as the bell had rung before the completion of the game.

Researcher: It was too unfortunate. How much more score was required for you to have the score leveled with your friend?

Boy B: [After few seconds] 22 more. In addition, to win the game, I was just 23 score back.

Then after a few descent conversations, I thanked the students and told the students to move towards their respective classroom. The teacher enthusiastically asked what I was looking forward to see in the game. My concern about the game was whether the game could incorporate the teaching learning of mathematics in classroom. I told him if any such mathematical concepts could be associated with such methods of game play.

Teacher: There could be many such ideas for school teaching such as counting of numbers during score counting, measurement strategies required in the game and others.

Researcher: (That was true but not so limited. I was thinking that there could be some other concepts, which are generally supposed to be vague but exist in the out of classroom surrounding itself). What if any such concepts regarding algebra could be synchronized with the game play of the students?

Teacher: (He started to think too.) That would be exquisite. However, which concept of algebra could be linked with the game then?

Researcher: We observed the whole process of game in the natural setting and interviewed with the players too. On this basis, how would it be if we draw the following algebraic concepts?

Table 5.4

Exploration of Algebraic Concepts in Cultural Game

Activities in Play	Mathematical Language
Initial Stage: Score of player A and player B are equal.	Score of A = Score of B Mathematical condition of equality.
The game begins. After the completion of game, player A scores 74 and player B scores 56.	Counting Grouping: Group of 10
For player B to equalize the score of Player A, player B further requires the score “74- 52”	For the calculation of the required score, we introduce a algebraic variable, ‘x’, $x + 52 = 74$ $\text{i.e. } x = 74 - 52$ This is the algebraic equation.
For player B to beat the score of Player A, player B further requires the score “>(74- 52)”	For the calculation of the required score, we introduce a algebraic variable, ‘y’, $y + 52 > 74$ $\text{i.e. } y > 74 - 52$ This is the algebraic inequality.

Researcher: What do you think of this idea. Is this viable for its incorporation in teaching the algebraic concepts in classroom?

Teacher: This can be a wonderful idea, sir.

Teaching mathematics becomes more interesting if the contents are related to the students' familiar environments. Students' everyday interaction with the cultural environment provides more opportunities to interact and conceptualise mathematical ideas. The swing game can be extraneous metaphor (Lakoff & Nunez, 2000) for the understanding of the concepts of rotation and angle of rotation. The *Chungi* game possesses various mathematical concepts and ideas. The major mathematical idea found in the analysis of this game is counting concepts. This game helps to develop the addition, subtraction, multiplication and division of the numbers. At the same time comparison of the numbers (greater and smaller) also imply in this game. The winner of the game is those who have to kick the *chungi* in large numbers of times in the air without putting one's foot down. Generally, 100 kicks with a back kick 10 steps away from the player are decided to win the game. To aid their memory the player can make a tally bar table where they would write the names of players and record against each name when the player hits 100 kicks with at least 10 steps away from the player to the landing point of the *chungi*. The tally bar table for four players is shown in the following:

Table 5.5
Frequency Distribution of Score in Game

Player's Name	P ₁	P ₂	P ₃	P ₄
Games won				
Frequency	4	3	6	2

Even though the player did not write their score on paper or elsewhere but kept in their mind. Same results have been displayed in the study of Ismael (2002) and

he revealed that the basket weavers used implicit mathematical knowledge. In *Chungi* game players would conceptualise the tallying system by using their mental recording system. The idea of frequency can develop as they count the number of tallies for each player to find the number of games won by each player. The player would learn the concept of mode as the player who won the highest number of games. The above table reveals P_3 won the game six times and evidently, the mode is 6.

The cultural activities like games can use to provide a source of strong motivation for student engagement in learning and to encourage students' social, emotional and cognitive development (Kamii & DeVries, 1980). In this regards, Araya (2007) viewed that the cultural games can be a powerful metaphor in the process of teaching and learning of school mathematics. Black (1979) suggested that metaphors can help generate new knowledge when the known domain and the new domain interact in the mind of the learner so that each is enriched by the other. For Ainley (1988, p. 243), the main value of mathematical games lies in the linking together of mathematical problems which are 'real' to the children, the use of such process skills as listed above, and the need such activities present for children to think in a mathematical way. The different game provides rich opportunities for all children to build and extend mathematical ideas and strategic thinking. The different cultural games are embedded mathematical knowledge, ideas and concepts which help to contextualizing mathematics teaching and learning.

Summary of the Chapter

Mathematical thinking and ideas are inherited in the environmental activities of different group of people. The counting, measuring, locating, designing, explaining and playing are mathematical ideas embedded in every human culture and use implicitly. Mathematics teaching and learning cannot be isolated from the

environments of the student communities as classrooms are part of a community with defined cultural practices. Students bring values, norms, and concepts they have acquired in their community to the school. Some of these are mathematical (Bishop, 1990). An exploration of the different mathematical ideas embedded in cultural arts, artefacts and activities of a group of people found in the out-of-school environment is one of the major task in this chapter.

The counting is most basic and fundamental mathematical ideas embedded in different group of people. The numbers and cultural significant in Nepalese context have been explained. The traditional measurement system of community people and its importance in present time also have been explored. This chapter is also devoted to explore the mathematical ideas embedded in the different cultural activities and cultural artefacts. The geometrical shapes, properties of different shapes, the symmetry, transformation, are found in the cultural arts and artefacts. The exploration of mathematical ideas embedded in out-of-school environment are paramount importance in the field of mathematics education.

The cultural game provides rich opportunities for all children to build and extend mathematical ideas and strategic thinking. The different cultural games are embedded mathematical knowledge, ideas and concepts. In this chapter, I discussed some of the games that involve a lot of geometrical ideas. The mathematical ideas embedded in the cultural games facilitates to the development of mathematical understanding.

CHAPTER VI

CULTURAL METAPHORS IN TEACHING AND LEARNING OF SCHOOL MATHEMATICS

This chapter deals with the cultural metaphors as a tool for teaching and learning of mathematics at the basic level of education. The teacher support program was conducted in order to equip the cooperating teachers in view of cultural metaphors. The lessons were developed in the framework of collaborative action research and delivered in the classroom in view of cultural metaphors. The cultural artefacts, household-based knowledge and practices were considered as the metaphor for the teaching and learning of abstract mathematical concept.

These artefacts resemble the lots of geometrical ideas. I developed a project-based pedagogy related to the study of monument and games that the children play in the different cultural settings. In this chapter, I prepared a teaching module to teach the different mathematical and geometrical ideas based on the above scenes. The effectiveness of cultural metaphor was measured in terms of the critical pedagogy used by the teachers and the amount of students' elation observed.

Selection of Cooperating School

Teachers are one of the main components of the educative process. They are the agent to deliver the mathematical knowledge and content to the students. The teacher's content knowledge and the pedagogical knowledge is important vehicle for the teaching and learning of mathematics. Giving high regards to the teachers' knowledge on culture friendly pedagogy, in the beginning of collaborative action research (CAR), I had developed the survey form for mathematics teachers regarding the perceptions on cultural metaphors. I had taken the survey study regarding the perceptions of mathematics teachers on cultural metaphors with the objective to locate

the sample schools and selection of mathematics teachers' participants in my study. The survey form was developed by researcher himself. There were thirty-seven Likert type statements regarding the perceptions of mathematics teachers on cultural metaphors. The form was piloted on thirty mathematics teachers from Bara and Tanahun district.



Image 6.1. Cooperating School for Action Research

The public school of Kathmandu valley was the study area of my research. Being the capital city of Nepal, the people from outside the valley migrate and get involved in different occupations here in Kathmandu. As my visit to the school in the valley in order to fill up the survey form regarding the perceptions of mathematics teachers on cultural metaphors, I underwent innumerable bitter experiences. Most of the teachers did not want to participate even to fill up the survey form. Some of the teachers asked me if they could fill the form after finishing their classes, which was really considerable. Some of them asked me if they could fill the form at their house and to bring it the next day. This was also considerable. Nevertheless, I found that some of them did not return the form, and some escaped saying that the form was lost. Time and again, I visited them every day for about a week to get back those survey

forms. Thus, the selection of school for any research study is not only in the hands of researcher. The volunteer contribution of the participants and their unbiased perceptions about every issues in the project plays a vital role not only in the process of research but also in the findings of the study. Thus, the random sampling methods for selection of sample school might not work well because of the varied interest of the school administration and the teacher participants.

In the meantime, I found that Gyanodaya Secondary School had no such indifferent behaviours, but the teachers were so excited about my research project and I felt as if I could get the full support from this school. The Gyanodaya Secondary School is one of the reputed public schools in terms of having a number of students, students' representation from different parts of the country, academic excellence, well developed physical infrastructures and so on. Finally, I decided to choose the school as my sample school to accomplish my research project with full enthusiasm.

Then, I explained the full detail about my research project to the Head teacher. I found him as one of the very few among the Head teacher who could give that opportunity for conducting such project in his school. As my study was based on collaborative action research (CAR), the volunteer support and active participation of mathematics teachers was exquisitely important. I requested him to support me in every aspect required for me to conduct the CAR project. He asked me for the tentative time required for their involvement in my project. He somehow felt uncomfortable when I urged for some about four months of time with his school. He asked me to have a detail conversation of my research project regarding every requirement of my research with the HOD of the Mathematics Department of the school. Then, he called the HOD of mathematics and told her to provide necessary support in the process of my CAR project. They asked me to feel free to express

everything about my research. They looked appreciably satisfied regarding the conducting of my research in their school.

Meeting with HOD of Mathematics

After getting the access to conduct my project in Gyanodaya School, I met with the HOD of mathematics and I explained full detail of my research project. As my study was intended to connect the out-of-school mathematical ideas in school mathematics curriculum, she was excited to participate in this CAR project. I developed detailed plan to carry out this project in collaborative action research framework. She requested me to explain in full detail to the mathematics teachers in a meeting organized in the school. Ultimately, we arrived at the conclusion that the teacher facilitation program must be conducted for the betterment of the CAR project. There were twenty teachers teaching mathematics from kindergarten through grade twelve.

After the completion of meeting, I provided the information sheet about the detail of my research project that contains the purpose, nature and possible effects of the study to the teachers. I also gave the framework of the consent form to all the teacher participants and requested to make necessary change in the form. With the discussion about the ethical issues concerning in my study, they provided me to include their name, excerpts without dismantling the essence in my final research report. The sample of the commitment and consent of the teacher participants were presented in the Appendix XII.

Conducting Teacher Facilitation Program

After the meeting with HOD of mathematics and Head teacher, I had developed detailed plan (Appendix X) to conduct Teacher Facilitation Program (TFP) to acquaint about the knowledge of cultural metaphors and connection of out-of-

school mathematical ideas and knowledge for the teaching and learning of school mathematics. Thus, my intention to conduct the TFP was to provide the answer of questions of the teachers' participants based on what are the cultural metaphors. How cultural metaphors plays the important role to communicate abstract mathematical ideas? How the cultural metaphors incorporate to in the process of teaching and learning of school mathematics at the basic level of education? To answer these questions, I had prepared powerpoint slide to present about the cultural metaphors to the teachers of cooperating school with the help of my research supervisors.



Image 6.2. Researcher Delivering in TFP



Image 6.3. Teachers' Participation in TFP

The general orientation about the cultural metaphors to the teachers of mathematics of cooperating school was conducted in 21 July 2017. The research supervisor also participated as the chairperson of the opening ceremony of the teacher facilitation program and discussed about the role of metaphors for the construction of mathematical knowledge. The first session of second day of the teacher facilitation program was about the mathematical ideas embedded in the out-of-school culture. The next session of the second day was devoted to the constructivist approach of teaching and learning mathematics. I prepared model lesson in view of cultural metaphors for the teaching and learning of school mathematics with the collaboration of my research supervisors which are presented in the Appendix X.

Reflection of Teachers on TFP and Possibilities of Use of Cultural Metaphor

In this section, I observed the teachers' reflections about the cultural metaphors and classroom practices of mathematics teachers. Most of the research participants reported that mathematics originated from the cultural activities of different society, from different racial groups around the world. Different professional group of people are using mathematical concepts and ideas without knowing the meaning of mathematics. Their implicit mathematics knowledge of the group of people provide an opportunity to their children to learn mathematics in the classroom. Teachers who have children's cultural background and funds of knowledge can link in the classroom. I ask the teachers participants about the cultural metaphors and its possibilities in the teaching and learning of school mathematics. One of my research participant Bima speaks:

I believe that mathematics is evolve from the culture. The different group of people are using mathematical ideas implicitly in their profession. Their children are also doing same work what their parents do. Children are also using same kind of mathematics. But the real problem is to connect their mathematics to school mathematics.

I observed that the teachers were indifferent or did not feel necessary to apply the knowledge what they learnt in the facilitation program, workshop and training. It is because of teachers do not have collaborative traditions on their peers. With this respect, Bima further says:

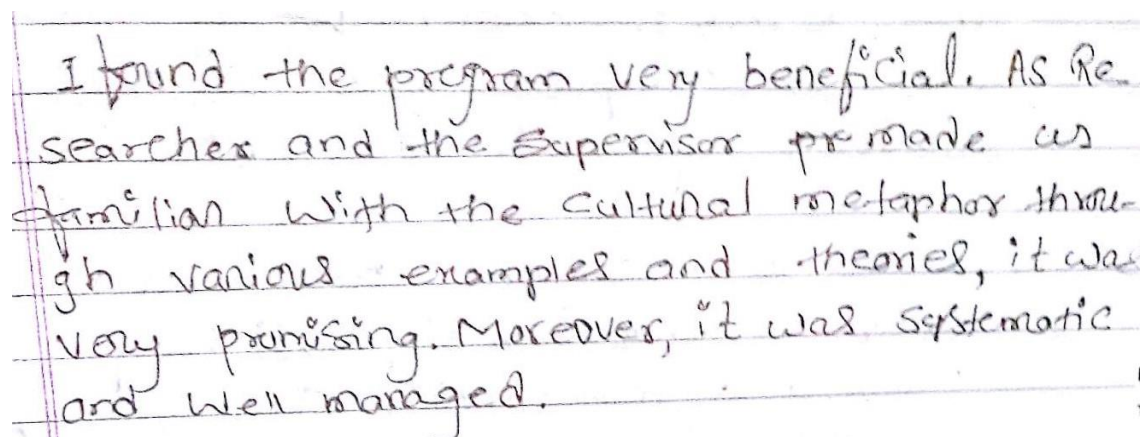
Teachers are demotivated to use culture friendly pedagogy. There are several reasons. One of the reason is school itself because school administration expects very excellent results in the national examination rather than the conceptual development of the subject matter. If teacher wants to use students'

culture friendly pedagogy, the course cannot be finished on time. For teacher there is great dilemma whether to use the culture friendly pedagogy or focus textbook-based examination instructional design.

My other research participants agreed with the views of Bima. The culture friendly teaching-learning approach creates environment for conceptual understanding of the students. Further, she speaks:

Teachers have no time to manage instructional design from the cultural friendly pedagogy. Teacher have to take five and six periods a day, how can teacher think about using cultural metaphors in their teaching of particular piece of mathematical content. At least teacher needs time to think about this. So first thing is time and second thing is the attitude and eagerness of the teacher to shift their own tradition to new one. The teaching-learning of mathematics is disconnected with the research work. There is no rewards and punishment in the teaching and learning of school mathematics.

In the first phase, I had made two days' TFP on connecting cultural metaphors and school mathematics and constructivist teaching and learning framework. The program manual was developed, tried out, improved and validated in collaboration with my research supervisors. It was unduly to state that the teacher support program

A photograph of a handwritten note on lined paper. The text is written in cursive and reads: "I found the program very beneficial. AS Researcher and the Supervisor pre made us familiar with the Cultural metaphor through various examples and theories, it was very promising. Moreover, it was systematic and well managed."

I found the program very beneficial. AS Researcher and the Supervisor pre made us familiar with the Cultural metaphor through various examples and theories, it was very promising. Moreover, it was systematic and well managed.

Image 6.4. Bima's Reflection on TFP

which was conducted by myself changes in teachers' practices in their classroom mathematics teaching. I am not a such person to make drastic change after giving some sort of intervention to the teachers. Despite this, teachers were very much grateful to the program. Twenty mathematics teachers from kindergarten level to secondary level participated in two-day facilitation program. After the program, I asked to write the reflection of their own about the program. One of my research participants Bima mentioned that the TFP opens the new avenue to the teachers of mathematics. She wrote:

Most of the teachers participating in the program reflected that the content presented in the TFP were more useful to connect our daily activities with the school mathematics. The question regarding the use of cultural metaphors in the teaching and learning of school mathematics, most of the teacher participants viewed that their teaching is based on the textbook. My one of the research participant viewed that

I never felt that mathematical ideas can be found in the out of school culture. However, mathematics is the cultural product. How school mathematics can be relating to the students' home culture is really problematic.

Other research participant opined that:

We teachers do not have time to think about to make mathematics teaching more culture friendly because of the overloaded curricula and over duties in classroom teaching.

Connecting with same question, other research participants questioned that how teacher can think his lesson more culture friendly if they do not have sufficient time to manage. But, other teacher gives serious limitations of the teachers to make their class more culture friendly. Time is not the problem at all to use cultural metaphors in the teaching and learning of school mathematics. Attitude and

knowledge of the teachers about the cultural metaphors for teaching school mathematics is more problematic. But the problem is that the lack of ideas on the selection of cultural metaphors for the teaching of particular content of school mathematics. The teacher of mathematics need not to think about the cultural metaphors for teaching and learning. They need more time and unnecessary effort to think about the teaching of lesson. Bima further wrote:

Obviously, such programs are fruitful to invest the different aspect of our classroom teaching. It is widely understood that if we connect different aspect of our daily life practice with mathematics, we can make mathematics more useful. Moreover, in classroom we can explain the usefulness of various topics in our daily life.

The next participant in the TFP, Sonu speaks that

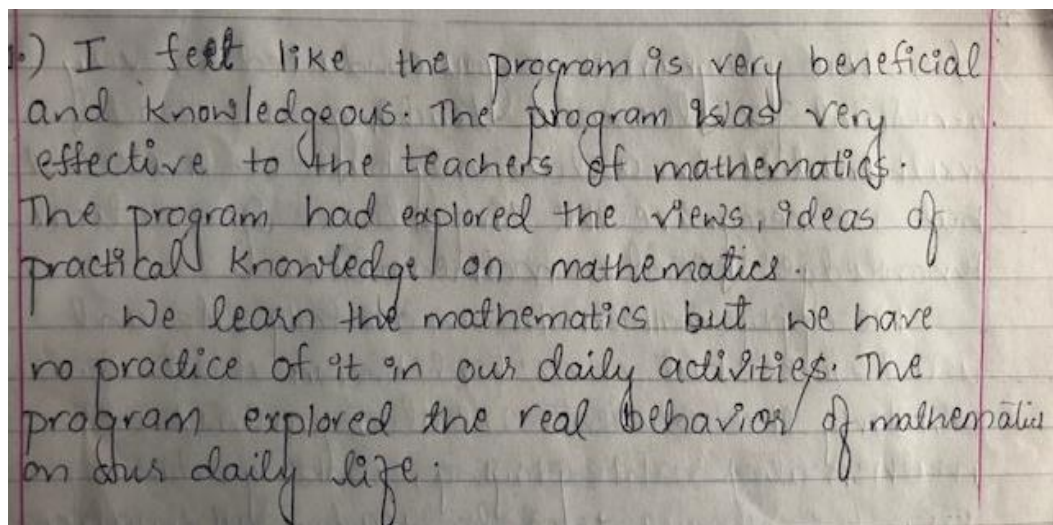


Image 6.5. Reflection of Sonu on TFP

All my participant were agreed with Sonu. They felt the program was very effective to the teachers of mathematics. In this regards, my another research participant viewed that the students' household activities could be the funds of knowledge for the teaching and learning of school mathematics. The program was effective for teachers of mathematics. This program explored the existence of

mathematical concepts in out of textbook culture. After this program, I came in the situations that the mathematics can be found in the everyday activities of different professional peoples either literate or illiterate. The school mathematics if relate to the students home cultural environment would be more effective and interesting.

My one research participant Shyam pointed out the importance of the cultural metaphors in the process of teaching and learning of school mathematics. With the view of his writing, it was observed that the mathematics is originated from the culture and everyday activities of group of different people to fulfil their needs. Mathematics is taken as abstract subject as the teachers are unable to teach

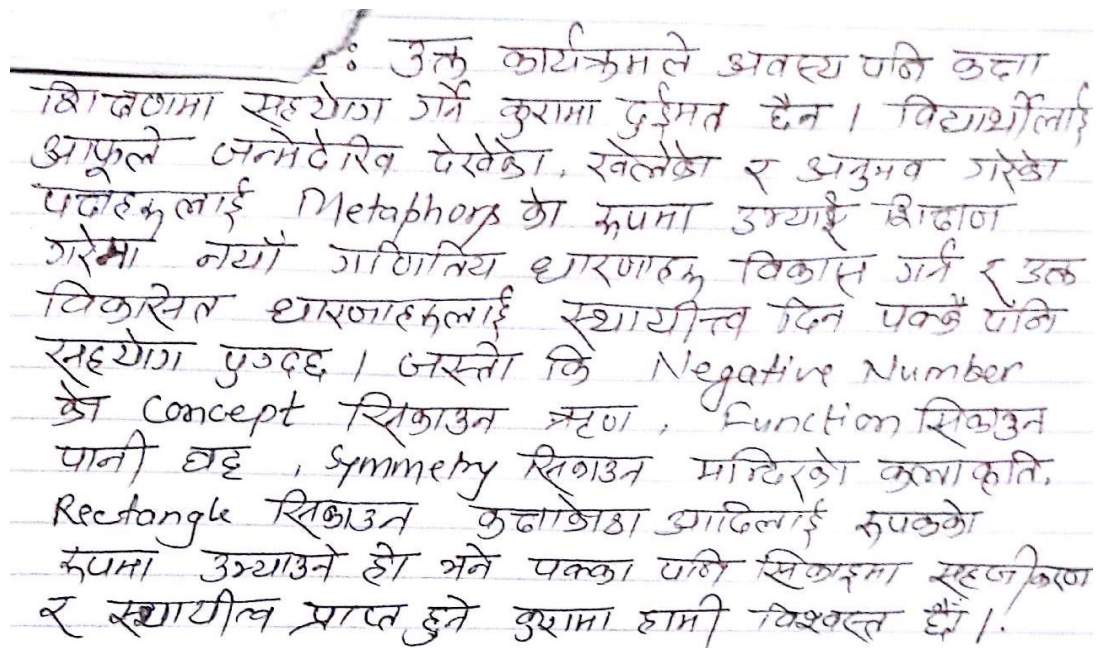
Ans of Q.N.1: सञ्चालित उक्त कार्यक्रम एकदमै महत्वपूर्ण भएछो महसुस भएछो छ। गणित समाजवाहे उत्पत्ति भएछो भएता पनि हामी शिक्षकहरूले समाजमै बिहित कुराहरूले गणितलाई जोडेर पढाउन नसक्दा विद्यार्थीहरूलाई गणित अगुत प्रायः बन्दै आएको छ। तसर्थ गणित शिक्षणको लागि Cultural Metaphors आते महत्वपूर्ण छ है भन्ने आशय भएछो उक्त कार्यक्रम हाम्रो लागि फलदायी भएछो महसुस गरेका छौं।

Image 6.6. Shyam's Reflection on TFP

mathematics through connecting cultural activities and knowledge. The teacher of mathematics should have ideas on funds of knowledge of the children culture and their everyday activities. The TFP really facilitate to explore the mathematical ideas embedded in the out-of-school culture. Relating cultural metaphors for teaching and learning of abstract mathematical ideas will certainly make mathematics teaching effective, interesting and meaningful.

Shyam further viewed that the students' funds of knowledge if we bring as a metaphor into classroom teaching of school mathematics certainly enhances the level

of mathematical understanding of abstract concepts. The TFP was seems to be benefitted to the teachers of mathematics. The teacher training and workshop



इस उक्त कार्यक्रम ले अवश्य पनि कक्षा शिक्षणमा सहयोग गर्ने कुरामा पुईमत हैन । विद्यार्थीलाई आफूले जन्मेदेखि देखेको, खेलेको र अनुभव गरेको पदहरूलाई Metaphors को रूपमा उभ्याए शिक्षण गरेमा नयाँ गणितीय धारणाहरू विकास गर्ने र उक्त विकसित धारणाहरूलाई स्थायीत्व दिन पक्कै पनि सहयोग पुग्दछ । जस्तै कि Negative Number को Concept सिक्नुन ऋण, Function सिक्नुन पानी छद्, Symmetry सिक्नुन मन्दिरको कलाकृति, Rectangle सिक्नुन कुडाछेडा आदिलाई रूपको रूपमा उभ्याउने हो भने पक्का पनि शिक्षण सहजीकरण र स्थायीत्व प्राप्त हुने कुरामा हामी विश्वस्त हौं ।

Image 6.7. Shyam's Reflection about the use of Cultural Metaphor regarding culture friendly pedagogy in teaching and learning of school mathematics opens new avenue for them. Most of the teachers use market-oriented pedagogy for mathematics teaching which focuses on the final output of the results. The market-oriented pedagogy focuses mostly on the procedural understanding rather than conceptual understanding of the content. The long-term memory of the content requires the procedural understanding of the abstract concept of mathematics. The funds of knowledge of the students' cultural activities are the metaphors for the teaching and learning of school mathematics.

There was controversy about the use of cultural metaphors in the teaching and learning of mathematics. Most of the participants viewed that the culture friendly pedagogy and the funds of knowledge for teaching and learning of school mathematics is effective for the teaching mathematics. But, they do not like to use cultural metaphors in their classroom teaching. The time factor is one of the most

effective factor not to use culture friendly pedagogy in mathematics teaching. The overloaded curricula are also the hindrance for the use of cultural metaphors in the classroom. With regarding to the use of cultural metaphors for the teaching of school mathematics, one of my research participant Bima opine that:

The use of cultural metaphor in the mathematics teaching is evident. But we are unable to use it in our classroom teaching. How can a teacher who have responsibility of taking five six periods in a day can manage their own class from culture friendly pedagogy? How to prepare class with the view of cultural metaphor for certain mathematical content? If a teacher has one or two periods of responsibility, then s/he can think about which cultural metaphor fitted most appropriately.

Most of my research participants agreed with the view of Bima. They all accept the importance of cultural metaphors for the conceptual understanding of the mathematical concepts. The incorporation of cultural metaphors in the process of teaching and learning of mathematics enhance mathematical understanding to the students. The process and product teaching, both are important in the educative process. Product teaching focuses on the final output or the results of mathematical problems. Most of the teacher use market-oriented pedagogy for mathematics teaching. Such pedagogy focuses on the final output of the results. The market-oriented pedagogy emphasizes mostly on the procedural understanding rather than a conceptual understanding of the content. Our current tendency is to focus on achievement of the students in a final examination rather than how knowledge and skills are achieved by the students. Regarding this, one of my research participants Bima speaks:

Teachers are demotivated to use culture friendly pedagogy. There are several reasons. One of the reason is the school itself because school administration expects excellent results in the national examination rather than the conceptual development of the subject matter.

Most of the teacher participants in my study supported the views of Bima. Regarding this issue, my another participant Shyam partly accepted to the views in above. He speaks: The use of cultural metaphor in teaching mathematics is not a great deal. The connection of students' cultural context to the teaching of mathematics facilitates the development of conceptual understanding. But, if a teacher wants to use students' culture friendly pedagogy, the course does not finish on time. There is a great dilemma whether to use the culture friendly pedagogy or instead focus on textbook-based examination oriented instructional design.

From the interaction with teachers and the ways of their teaching of mathematics, I observed that the school teacher did not want to change their way of teaching. Using culture friendly pedagogy requires some extra planning for the teaching of mathematical content. Considering the views of the teachers, they cannot manage extra time other than ten to four for making the use of such pedagogy which if used, exceeds the teaching hour from the design in completing the course.

Regarding this, my another participant Ranadhir viewed;

We teachers generally cannot give time to manage instructional design from the cultural friendly pedagogy. A teacher has to take five to six periods a day, how can a teacher think about using cultural metaphors in their teaching of a particular piece of mathematical content. At least teacher needs some time to think about this but our system gives no such actions probably due to the overloaded curriculum.

Overloaded curricula did not provide sufficient time to think about the cultural friendly pedagogy. My research participant Shyam viewed that using cultural metaphor in teaching and learning mathematics is based on the choice of the teacher. He speaks:

So first thing is time, and the second thing is the attitude and eagerness of the teacher to shift their own tradition to the new one. The teaching-learning of mathematics is disconnected with the research work. There is no rewards and punishment in the teaching and learning of school mathematics.

Teachers do not have collaborative traditions in the teaching and learning of mathematics. The weekly meeting of the Department of Mathematics mainly focuses on the completion of courses taught in the different sections. The another most occurring agenda in the meeting was the construction of test items for the monthly and terminal examinations. The meeting has some space for the discussion of the achievement of the students in a mathematics test. Twenty school mathematics teachers participated in two days' general orientation program. After this program, I had distributed the open-ended questions regarding the perceptions and beliefs of participants in TFP and requested to write their own views about the program. On the basis of their perceptions, beliefs, knowledge and eagerness to volunteer participation, the six mathematics teachers were selected as research participants in the CAR.

Selection of Teacher Participants for CAR Project

After conducting two day's teacher facilitation program, I provided the open-ended questions and blank sheet to write the perceptions and their reflections on TFP. Only 15 participants provided me their written reflection about the TFP. About seven teacher participants were not interested to write their views and they did not provide me the written sheets. I had collected the reflection notes of fifteen participants of two

day's teacher facilitation program and analyzed their written documents with the collaboration of my research supervisors. I had taking oral interview to fifteen teachers to participate in my project based on collaborative action research.



Image 6.8. Participants in CAR Project

I had provided the information sheet and explained orally about the purpose, nature and possible effect of the study to fifteen participants to gain access for collaborative action research. The nature of the data collection instrument and their rights and responsibilities as participants have been explained to them, individual consent was obtained from six participants included in the study. The researcher convinced them to safeguard the confidentiality of collected data and the privacy of each participant as well.

I was interested to participate at least eight teachers in the project from those fifteen on the basis of analyzing their views on cultural metaphors and possibilities of incorporation in mathematics classroom teaching. The teachers who had keen interest on cultural metaphors and practices on mathematics classroom was selected. The only six teachers were ready to participate and committed for the collaborative action research. Their participation and cooperation in my project was presented in the appendix section of my research report on the title of “Commitment and Consent of School Teachers”.

Collection of Cultural Metaphors

Metaphors are mapping between conceptually separate domains of knowledge. The attributes related to one domain are used to understand another domain. The domain, which is concrete and familiar that mediated to convey the meaning of another (target) domain, is known as source domain. The source domain must be from the students' cultural environment. There are different sources of metaphor. Some of them were collected from the out-of-school culture based on the everyday activities of different group of people and cultural artefacts embedded in the surrounding of students community. Other source of metaphors were collected from the observation of classroom and literature that I reviewed. In this section, the matrix of metaphors and its possible contribution in the process of teaching and learning of mathematics were briefly presented.

Metaphors Capturing from the Out-of-School Culture

I have collected the metaphors from the different sources. One of the major sources of metaphor was the out-of-school culture. From the ethnographic observation of the household activities of students' home culture, I have collected the some of the metaphors regarding the mathematical ideas and knowledge. The following table presents the brief about the metaphors and the possible contribution to the particular areas of mathematics.

Table 6.1

Metaphors Capturing from Out-of-School Culture

Metaphor	Areas to Mathematics	Remarkable Contribution
Balance	Arithmetic	Integers {0, positive integers, negative integers} Principle of the moment of forces
Money	Arithmetic	Place Value Integers

Games	Statistics and Probability	Data Collection, Frequency Distribution, Measure of Central Tendency
	Geometry	Basic geometric shape and their Properties
	Algebra	Basic concept of algebra, Algebraic equation and inequation.
Kitchen Mathematics	Arithmetic and Algebra	The matheamtical ideas embedded in the kitchen related to everyday activities can related to the different concepts of arithmetic and the fundamental concepts of algebra.
Weaving	Geometry	Geometric Shapes, Different properties of rectangle
Carving	Geometry	Symmetry, Coordinate Axes, Transformation geometry
Plumbing	Geometry	Basic concepts of geometry, parallelism and perpendicularity
Artefacts	Geometry	Symmetry, Geometric shapes and its properties

Hiebert and Carpenter (1992) demonstrated that children have a considerable amount of mathematical knowledge of addition and subtraction in their everyday activities involving game play. They can solve a wide range of addition and subtraction problems in their own ways without understanding the symbolic representation of addition and subtraction. The use of students experience based on the cultural metaphor as “Addition is collecting objects in a bag” and “Subtracting is taking thing away from a bag” can mediated to understand the formal operations of addition and subtraction. Everyday activities of different group of people possesses mathematical activities of counting, measuring, locating, explaining and playing. The ethnomathematical ideas of students’ home community mediated to understand

formal mathematics. Table 6.1 displayed the list of the conceptual metaphors that facilitated to communicate the abstract mathematical ideas of different areas.

Metaphors Capturing from the Literature and Classroom

Another source of metaphors that I collected was from the literature and the classroom practices of teachers. Lai (2013) mentioned that the teachers treated different artefacts and familiar model to develop abstract concepts of mathematics. In this regards, the teachers used images in photographs to construct the mathematical concepts of similarity. Further, Danesi (2007) viewed that the teaching of abstract concepts of mathematics “by developing them through explanatory and ancillary devices that allow their referential structure to be shown in some concrete way” (p. 227). The abstract concepts can be comprehended with the use of concrete vehicles, for instance, the charts, diagram, geoboard and commercially produced materials.

The conceptual metaphor allows us to understand one domain (target domain) in terms of another (source domain). As the source domain of conceptual metaphor is familiar and contextualized class of concrete vehicles, this generally involves the use of mathematical ideas embedded in the everyday activities of different group of peoples. In my study, the team of CAR project used the cultural metaphors to communicate the abstract ideas of mathematics. These metaphors were captured from the literature that I draw and the classroom practices of the teachers that help to developed abstract mathematical concepts. The table 6.2 presented the brief about the metaphors and the possible contribution to the particular areas of mathematics.

Table 6.2

Metaphors Capturing from the Literature and Classroom

Metaphor	Areas to Mathematics	Remarkable Contribution
Color	Arithmetic	Prime and composite numbers

Gestures	Numeral writing	Seems more effective for the correction of Error in numeral writing
Clock	Numbers	Ways of perceiving the time of morning, day and night to conceptualize “rounding off numbers”
	Circle	Minute hand of clock is radius. Circumference of circle is a path of minute hand passed through a revolution (Lai, 2013).
Area	Prime and Composite Numbers	The area presented in Geoboard reflected for ease to conceptualize the prime and composite numbers
Arrow	Integers	Addition and subtraction of negative integers from integers (Upadhyay, 2010).
Number Line	Integers	Conceptualize the integers, Addition and subtraction of integers
Path/ Fictive motion	Integers	Rounding-off numbers
Mnemonics	Arithmetic, Algebra	Memory aids to some concepts and rules

Instructional Design through Cultural Metaphors

This was an exploratory research study, and the lesson design could not be tested ahead of time. On the basis of curricular objectives, I have developed lesson design with the sufficient feedback of from my research supervisors and the cooperating teachers of the sample school. The lessons that I developed were a little different from the conventional design. It was based on constructivist design and

students were encouraged to investigate the mathematical ideas embedded in the cultural activities. The metaphor is a communication tool and hence provides the opportunity to teach abstract mathematical ideas and concepts by using concrete apparatus. Conceptual metaphors are the mapping between two domains. The vehicle terms are familiar and the abstract and difficult concepts of mathematics are conceptualized through already known concepts.

Gestural Metaphors for Writing Numerals

The writing numeral is difficult task for most of the children at the first grade of schooling. It is obvious that most of the children commit error while writing the numerals. Rasmita is a grade teacher for Grade I. When the class was going on, I was watching the class from the window. The teaching method that she uses and the reaction from students was my subject of interest at that time. So, I was just watching the class from outside and watched her teaching, the *Devanagari* numerals. The structure of some numbers like १, २, ३, ४, etc. probably made some of the children difficult to make off in their mind as the teacher was repeating a same number walking around the students. Then after the end of period;

Researcher: Miss Rasmita, how are you?

Rasmita: I am fine sir. Nice to see you!

Researcher: Thank you. And miss, how are the classes going on?

Rasmita: Sir, the classes are running well but what to say! Ummm... The students really take some time to get the numbers in their mind and they make serious errors during the learning of the numerals.

Problem Identification: During the teaching of writing numerals, the students get confused and commit errors like structural error and writing mirror shapes of numerals. They could not grasp the perfect structure of the numerals.

Making Plan: For teaching it, she planned to teach the numerals by using number charts.

Intervention I: During the teaching, she taught the numerals by reading all the numbers pointing them from the number-chart. She made the students write the numbers along with her. Some of the students were still unable to write, to whom she hold their hands and got them to write those numerals.

Observation: During our observation in the classroom, some students were still making mistakes while writing those numerals.

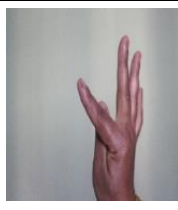
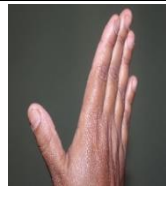
Researcher: I think they are not being able to visualize those numbers, so they are making such errors.

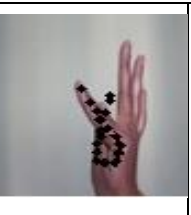
Rasmita: Perhaps the thing is the same sir. But what can be done to overcome such errors?

In making the teaching easy, adding of gestures in the regular teaching could be fruitful. Use of hands and fingers to represent numeral values during the class could make the students visualize those numbers. For each numeral, we can show gestures as given in Table 6.3, to visualize the abstract concepts of numbers. So, I mediated her to use gestural metaphor in the classroom.

Table 6.3

Gesture of Devanagari Numerals and Word Pronunciation

Hindu Arabic Numbers	1 (One)	2 (Two)	3 (Three)	4 (Four)	5 (Five)
Gestures					

Gestures with Devnagari Numeral					
Devanagari Numeral Symbol	१	२	३	४	५
Pronunciation in Devanagari	एक (Ek)	दुई (Dui)	तीन (Tiin)	चार (Chaar)	पाँच (Paanch)

Intervention II: Previously, using of number chart during the teaching of numerals could not help all the students in the class. So she used the gestural metaphors in her classroom as second intervention. She used both gestures and their pronunciation simultaneously in teaching it.

Observation: The students' error in writing numerals was highly minimized. The students felt so easy to get the writing of those numerals.

Reflection: After some days, I met to the Miss Rasmita. I really wanted to know whether the gestural metaphors did well to remove the confusion that the students committed. I asked to Miss Rasmita about the effectiveness of gestural metaphors to remove the general error that students did in the past. She replies:

“Mathematics teaching is difficult because of the teachers. The idea that you gave me was not so big and difficult to incorporate. Nevertheless, I never thought those ideas to put ever in my teaching activities. I also use gestures to teach the numerals but my gestures did not work well to that of yours. The gestures that you advised me to incorporate in teaching numerals are powerful. By using it, teachers can facilitate to teach and help students to learn in a better way.”

From her views, it was found that the gestural metaphors can be the powerful tools to represent the visual representation of numerals. She started teaching the numerals by using gestures of fingers and hand to visualize the symbols of numerals. From the discussion with Rasmita, the selection of metaphor is difficult task. My other research participants also agreed with the view of Rasmita. They viewed that the overloaded curricula is main problem to incorporate cultural metaphors in teaching and learning of mathematics. She further says:

“The students are feeling happy to participate in the learning process of telling and writing the numerals. The active participation of children in the learning process encourage them to learn. The use of gesture created a lovely and entertaining environment during teaching of numerals”.

The gesture of fingers and hand are effective tools to connect the learning with the value and symbol of the numerals. Her gestures and body language regarding the gestural metaphors and its incorporation in the classroom teaching seems to be more effective and meaningful to the students. The different gestural metaphors help to improve students' errors and changes the conventional way of teaching.



Image 6.9. Image Taken on Special Request of the Student Participants

As my study was framed within multimethod qualitative research, the development in understanding of the content knowledge and improvement of the

students' cognitive aspect was assessed through the observation of their psychological and behavioural changes. The cognitive development of the children was assessed through the observation of their class copy. The class copy was observed to see if the improvement was made. I observed that the students who had committed error in writing numerals were improved after the incorporation of gestural metaphors in the classroom teaching.

The psychological and behavioral activities of the children were also assessed through the classroom observations. The children were happy to read and write the numerals by copying what their teacher did. Children were interested to capture photographs and requested me to take one (Image 6.8). Their classroom engagement during the teaching and learning of numerals seems to be effective and interesting. The indicatory gestures help children to develop cognitive map about the numerals. The incorporation of symbolic representational activity is full of indicatory gestures (Vygotsky, 1987) is the powerful tool to teach the numerals.

Metaphors for Understanding Place Value

Learning mathematics in primary grades begin with counting numbers. Most probably, children started counting of numbers before entering the school. But the counting might not be effective. Someday, I was wondering here and there in my cooperating school. I was attracted towards the class of Dipika, the grade teacher of grade I. Her qualification was intermediate with major mathematics. In her class, a student was counting the numbers pointing by the stick on the number chart and other students were chanting loudly after him. Some students were in head down posture and some students did not have any interest to speak. However, they were compelled to follow the students who had the stick. I asked with Miss Dipika for a consent to observe her class. She agreed and permitted me into the class the next day.

Next day, I went to the staff room and met with Miss Dipika. As per our plan, she asked me to come along with her to her class in Grade 1. She had planned to teach the Counting of Numbers from 1- 100. We both entered into the classroom. All the students stood out to greet her. The teacher gave my introduction to the children and explained briefly about the purpose of the class observation. Students greeted me as well. I thanked them and requested them if I could observe the class and sit behind one of the student in the last bench. All the students accepted my request and the formal class begun.

Dipika: Children! We have learned about the counting of numbers. Can everybody write what I had taught yesterday?

Students: Yes Madam!

Dipika: Then all of you start writing the numbers from 1- 50.

(After all the students finished their classwork.)

Dipika: Okay, then Jayanti come here, please.

Jayanti: Okay, Miss.

(Dipika, sticks 28 number on the chart-board.)

Dipika: So tell me, which number is this?

Jayanti: Madam, the number is; Two Eight: Twenty-eight.

She called all of the students one by one. Everyone could speak out the numbers shown by the teacher. The student sitting behind me was so amazingly counting all the numbers from 1- 50 continuously even having difficulties in his continuous breathing. Then I started the interaction with that child just behind me.

Researcher: You were so amazing.

Student: Thank you, sir!

Researcher: Okay, for instance, show me by writing a number; let it be Twenty-eight without viewing in the copy or the board.

(The student wrote the number 28 in a blank sheet of his copy.)

Researcher: Wonderful, then answer one question. In this 28, which digit is greater? 2 or 8?

Student: (Instantly!) Eight sir.

Researcher: How can that be true?

Student: Because my madam taught me the counting in which 2 comes before 8 sir. So, probably 8 is greater than 2.

Problem Identification: The students were not clear between the face value, place value and total value of the digit. For eg: In number 28, for digit 2, its face value is Two and the place value is tens with the total value of twenty. I was not satisfied with such outcome. I got to know that place value was not yet taught to the class. My idea was that basic concept of place value can make the counting of numerals more meaningful.

Making Plan: The lesson of place value was at back of the course and bringing the concept of place value in this counting of numbers was a little challenging. Students at least knew the different denomination of money. So, we planned to use money metaphor.

First Intervention: Money Metaphor

The cultural artefacts can be used as motivating stepping-stones to launch new mathematical knowledge (Bonotto, 2007). He further explained that cultural artefacts offered the opportunity of making connections between the mathematics incorporated in real-life situations and school mathematics. These artefacts are relevant to children;

they are meaningful because they are part of their real-life experience, offering significance references to concrete situations.



Image 6.10. Nepalese Currency of Different Denominations

Our number system is a place-value system. This means that each place in a number is assigned with a value. The value of each place in the base-ten system is ten times the value of the place to its right. And the use of money for developing the concept of place value can give it an actual significance. When a student is asked about the values of each digit in a number, for example, be 28, the student cannot easily answer the questions like how many tens are there in 28 without some intellectual thinking. But when you give them the denomination of 2 tens and 8 ones then they easily calculate that they have Rs. 28 in total. Then when they are asked how many tens are in that Rs.28, they easily answer that there are 2 tens in that amount of Rs.28 and in bonus even may reply as there are 8 ones.

Observation: The use of place value during the counting of numbers seemed to be really appealing with positive response from both teacher and the students. They gained the value of each digits from the numbers.

Reflection: About the use of metaphors in teaching and learning of place value at the primary level, one of my research participant Dipika speaks:

“After I joined the collaborative action research, I got wonderful ideas and opportunities to teach place value using money metaphors. As money is familiar to each student, its connection to the mathematical ideas of place value seem to be very fruitful. To teach the place value at primary grades, I used the note of 1 rupee to represent the ones units, the 10 rupees to represent the tens units and the hundred rupees to represent the hundreds units.”

My other research participant agreed with the views of Dipika. Further, he speaks, “Money is a powerful metaphor as students seem to like talking about and playing with money. So I started using the money metaphor to teach the concepts of place value at basic level of education. So I took the ones, 10 ones for a ten, 10 tens for a hundred and 10 hundred for a thousand and so on”.



Image 6.11. Money as a Metaphor for Place Value

The place value is very fundamental concept in arithmetic. It is presented at the very basic level of education, however, the students of the higher grade also made error in different problem-solving situations. The procedural error of the students caused by the lack of conceptual understanding of the concepts. It is to say that place value is one of the most important concepts in learning arithmetic.

Mathematics lesson can be made interesting and effective from the students' point of views if it is relating to real-life context of the students. If we relate a

mathematics lesson to real life of the students, the attitude of students towards mathematics is positive and student interest increases about the learning of mathematics. Money is one of the real-life example to teach different concepts of mathematics. It has strong connection to all culture and it has given high valued. Every one, however illiterate, recognizes the values of the different currency. How do people recognize the values of the different notes? In this regards, Hutchens (2015) viewed that the beauty of money with place value is that it gives place value a real meaning. Further, the place value is very fundamental concepts which helps to teach the mathematical operations of addition, subtraction, multiplication and division. So, the concept of place value could be the linking metaphor for the teaching and learning of other concepts of arithmetic.

A study conducted by Bose and Subramaniam (2011) found that the range of number knowledge varied among the students. They found that 21 students out of 23 had difficulty in writing the numbers dictated to them correctly, making place value errors, especially for numbers bigger than 100. They wrote the numerals reflecting the number-names, i.e. 1001 for 'one hundred one', 10010 for 'one hundred ten', 10051 for 'one hundred fifty-one', 20060 for 'two hundred sixty', 10001 for 'one thousand one', etc. Numbers that were multiples of hundred or thousand like 5000 for 'five thousand' were written correctly. However, irrespective of the place-value errors that students made while writing the numbers in figures, they seemed to understand the numbers through their names. This knowledge probably is rooted in their regular use of money.

Second Intervention: Cultural Artefacts

Base-Ten Block is one of the important teaching tool for place value and arithmetical operations. Base Ten Blocks is a virtual math manipulative playground where students explore and discover place value concepts and also learn addition and

subtraction with regrouping. Our Base Ten Blocks allow students to experience and develop math concepts through interaction and discovery. Another research participant Shyam viewed that the Base-Ten Block could be the effective metaphor for teaching place value.

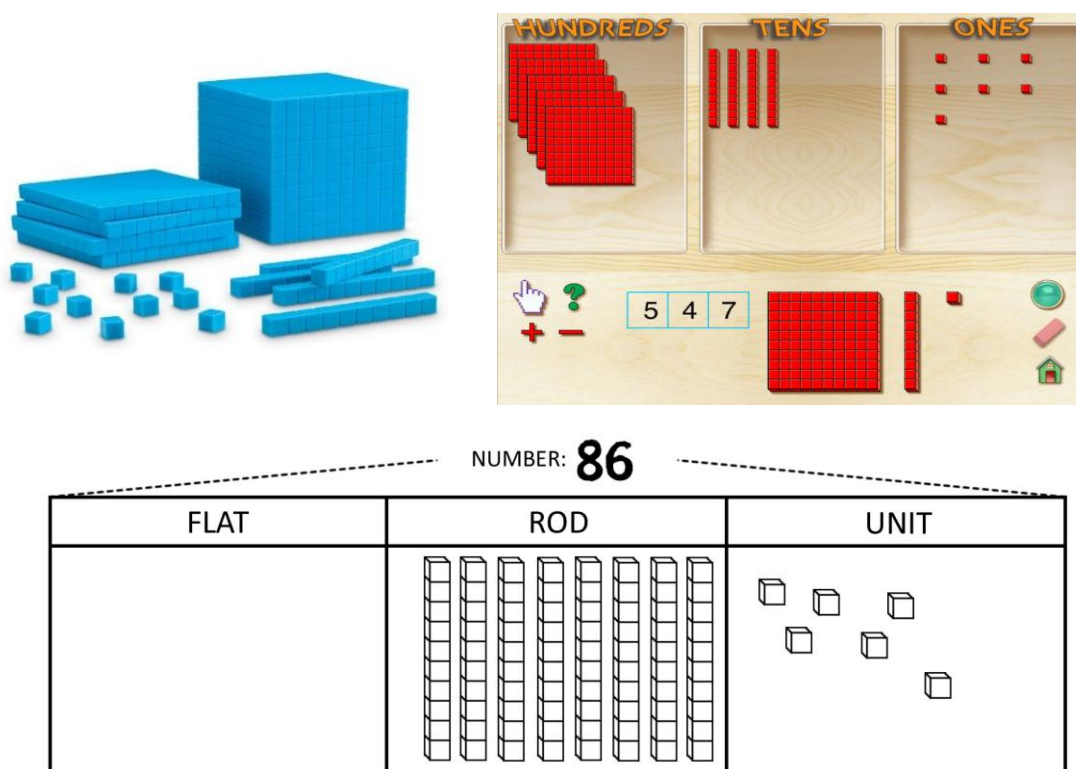


Image 6.12. Base-ten Blocks as a Metaphor for Place Value

They are a great concrete model for helping students to understand the relationship between ones, tens, hundreds and thousands. Students can count the blocks and set them side by side to see that ten ones is the same as one hundred or that one hundred tens is equal to one thousand. I also think it is important to use other examples for students to find meaning in the base ten system. A lot of teachers have students bring in objects to count on the hundredth day of school and sort them into groups of ten to see the relationship between ones, tens, and hundreds. For years I used blocks to teach and review base ten from ones through one thousands and then showed the students the patterns on the place value chart.

The National Curriculum Framework (NCF) argues that the way the students understand mathematics is by allowing the students to construct their own meanings by linking new information and concepts to what they already know (CDC, 2005). The household ideas and knowledge, students experience in their everyday activities in their familiar context provides the opportunities to learn and teach the school mathematics. The students' familiar context facilitates a systematic and powerful tool to represent the abstract concepts of mathematics. In this connection, I always advocate for Cummins (1996) "No learner is a blank slate" cited in Gonzalez, Moll and Amanti (2005, p. ix). It is believing that the students bring some sort of mathematical ideas and knowledge in the classroom. The ideas, knowledge that the children already possess help to unlock the difficult and abstract concepts of mathematics.

Metaphors for Teaching of Rounding off Numbers

Rounding off numbers is an important skill in every sector of human lives in general and particular in mathematics. This skill allows us to estimate numbers and make them easier to understand. Rounding off numbers to the nearest multiple of 10, 100, 1000, or million has great significance. In Nepal, the Central Bureau of Statistics (CBS) conducted a widespread national census in 2011. The CBS determined that the population of the country was 2,64,94,504. That number is hard to remember and if we say the population of Nepal is 30 million it is a good estimation because it doesn't make any real difference with what the exact number is. Thus, rounding off numbers and estimation are important skills in mathematics and everyday life. Estimation skill gives an idea about possible answer without doing a lot of work. In daily life, estimation is great for giving intelligent guesses about the possible amounts. Our school curriculum

gave due importance in rounding of numbers from grade three. The contents of rounding off numbers in different grade level are presented in the following table 6.4.

Table 6.4
Content of Rounding off Numbers at Basic Level

Grades	Units	Contents
3	Rounding Numbers	off Rounding off numbers to the nearest ten
4	Rounding Numbers	off Rounding off numbers to the nearest hundred
5	Rounding Numbers	off Rounding off numbers to the nearest thousand

There are different metaphors to communicate the abstract concepts of rounding off numbers. The common metaphors used in the discussion of the contents were time metaphor, number line metaphor, path and fictive motion metaphor. The incorporation of these metaphor in the classroom teaching and its effectiveness was evaluated on the basis of teachers and students behavioural, cognitive and psychomotor perspective. The metaphors that contributed in the process of teaching and learning of the concepts Rounding-off numbers were discussed in the following subheadings.

First Intervention: Clock Metaphor

The concept of rounding off is used in everyday activities and situations of morning, day and night. We usually conceptualize the time in these three state of time. We eventually say that its morning, it is already day and it is night and so on. Which time is belonging to the morning, day and night? That is not a great deal here. The important thing is that our everyday interaction involves the rounding-off the time as

morning, day and night. Anyone who says, its morning! It is not difficult to understand that he is rounding the interval of time at morning.

If we ask a person with a watch about what the time it is in there, then the person probably answers the time to be 10 o'clock even when the exact time in the clock would have been 10:03 or 9:56 or any nearest digits. The watch is grounding metaphors for the teaching and learning of the concept "Rounding off Numbers". How do we conceptualize the concept for the rounding off the numbers implicitly in course of viewing the time in an analogue watch? For example, let us take a watch



Image 6.13. Analog Watch as a Metaphor for Rounding-off Numbers

and at a certain instance, let it shows a time at 8:32. Here, in general, we take this time rounded off to 8:30 but without the theoretical concept of the rounding off numbers. And hereby, a question arises if an analogue watch can be used as a metaphor for the teaching and learning of the content Rounding off Numbers.

Second Intervention: Linking Metaphors

After a weeklong TFP, we had made a lesson note of different grade level in view of cultural metaphors based on constructivist teaching framework. I asked a teacher to observe her class as my first class observation. Deepa was excited and gave me the permission for the observation of her class. She requested me to come along her in the grade three. She had planned to teach the lesson of Rounding off the Numbers in that day. We both entered in the classroom. All the students stood out to greet the teacher. She gave my introduction to the children and explained briefly about the purpose of the class observation. I thanked them and requested them if I

could observe the class and sit behind one of the student in the last bench. I had my technical supporter with me too. So, I made them alert not to take it otherwise about the photo shoot in the class which also was already notified to the teacher. I asked them to take the class regularly. I even said them to learn at the best level and ask to the teacher if anything is not understood. All of the students accepted my request and the formal class begun.

Deepa: Children! Do you know what are we going to study today?

Students: Yes, Madam, you had told us yesterday about the today's lecture.

Deepa: Oh nice! So, Dipshikha what is the lesson for today?

Dipshikha: Madam, the lesson you asked us to see at home was Rounding off the Numbers.

Deepa: Excellent. So, students we hereby start our chapter for today

“Rounding off the numbers”.

I was happy to see the bonding between the teacher and the students. I further observed the class.

Deepa: So, what time do you wake up Kashish?

Kashish: Madam, I wake up 6 in the morning.

Deepa: Good, and Karuna what time do you walk to the school?

Karuna: Madam, I walk to the school at about 9 in the morning.

Deepa: See students. This is what we are to learn today. In this both cases, the action though has been performed at two exact time of 6 and 9 but we define both the time condition as morning. Is not it? Likewise, we Round off the number to bind up its original value with the nearest ones as for the easiest remembrance and for the foremost estimation in our daily life. We can round

off the number to the nearest 10, 100, or even to the greater extent. But in our level, we learn to round off the numbers to nearest 10.

Then, Miss Deepa noted a number 13 in the board and asked the student if they remember the concept of place value that she herself had taught them last week. And all the student loudly shouted, “Yes, Mam”. And, she continued.

Deepa: See, for the rounding off a number to the nearest ten, we must see the value at the ones place. For example, if we take this number 23 then, first of all, we see the value in the unit place. And what is the value at unit place?

Students: Mam, three (3).

Deepa: Good observation. Then the main note to be remembered is that, if the value of the unit place is less than five (5) then the number is rounded off to the lower nearest 10th value and that is? What is that value? Can you answer it Mansun?

Mansun: Madam, is that number 20?

Deepa: Wait. Let the other friend try too. Bikash, what do you think the value would be?

Bikash: Madam, is not that 20?

Deepa: Great! The correct answer is 20. Did not everyone make the same value?

Then can you round off the number 78 to the nearest 10? All of you, try this.
(After few second)

Students: Madam, the value would be 80. Is not it?

Deepa: Yes, excellent my children. You all are so good. So, if the value of the unit place is greater than five (5) then the number is rounded off to the upper nearest 10th value. So, quickly round off 45 to the nearest 10.

(All the students were finishing up their work and bell suddenly rung.)

Deepa: See, in this case, we always see the upper 10^{th} value. This is the trait we are following which is probably the correct way of rounding for such numbers at the centre.

Okay students, turn the page no. 45 and try solving Exercise 10 at home. If any confusion arises, please ask me in the next class Okay!

(All the students start to solve the exercise and the period ended to the lunchtime.)

After the class, I also walked along Miss Deepa toward the staff room and asked her if my presence in the class created any disturbance in today's lecture. She said that she did not feel any uncomfortable situation due to my presence but she further asked about her class if it was good one or not. I answered her that it was great interactive class with maximum participation of almost all the students.

Third Intervention: Number Line Metaphor

Then as my second observation class the same day, I walked along with Deepa to her class. She had the same topic to teach as she taught in grade 3, in grade 4 too, The Rounding off Numbers but in nearest 100. So, we both entered in the classroom.

Deepa: Good afternoon, class!

Students: Good afternoon Madam.

Deepa: Today we have a guest observer in this class. Sir, I request you to share about you today.

Researcher: Good afternoon children. Today I am present here for the purpose of a research that I am performing for my doctorate level studies. So, I request you all if I could observe the class. Can I sit behind one of the students in the last bench?

Students: Yes, sir. Why not.

Researcher: I have my technical supporter with me. So do not take it otherwise about the photo shoot and video shoot that can occur at any time during the class. Take the class as the regular one. Learn to the best level and ask the teacher if a thing is not understood.

Students: It's all right sir. We will cooperate your delightful purpose.

All of the students accepted my request and the formal class begun.

Deepa: Children! Do you know what are we going to study today?

Students: Yes, Madam you had told us yesterday about the today's lecture.

Deepa: Oh nice! So, Leela what is the lesson for today?

Leela: Madam, the lesson you asked us to see at home was Rounding-off Numbers.

Deepa: Excellent. So, students we hereby start our chapter for today

“Rounding off the numbers”.

I really found the bonding between the teacher and the students real and appealing. I further went on observing the class.

Deepa: Do you all remember about the similar lesson that we had learnt in the Grade 3 only rounding off a number to the 10th place.

Students: We do madam.

Deepa: In this grade, we are to learn about the rounding off the number to the nearest hundred. We round off the number to bind up its original value with the nearest ones as for the easiest remembrance and for the foremost estimation in our daily life. We can round off the number to the nearest 10, 100, 1000 or even to the greater extent. But here as I said we learn to round off the numbers to nearest 100. For this, we see the extent of value at the tens place. Okay, children!

Then, Miss Deepa noted a number 613 in the board. And all the student loudly shouted, “Yes, Mam”. And, she continued.



Image 6.14. Number Line Metaphor for Teaching Rounding off Numbers

Deepa: See, for the rounding off a number to the nearest hundred, we must see the value at the tens place. For example, if we take this number 613 then, first of all, we see the value in the tens place. And what is the value at tens place?

Student: Mam, one (1)

Deepa: Good observation. Then Jeevan you answer me what is the value after you round off to the nearest hundred?

Jeevan: Madam, 613 if rounded off to the nearest hundred then the value becomes 600.

Deepa: Excellent Jeevan. So, can you say why the value becomes 600?

Jeevan: The main note is that, if the value of the tens place is less than five (5) then the number is rounded off to the lower nearest hundred value.

Deepa: Absolutely. Then probably all of you can round off 880 to its nearest hundred.

Students: Madam, 900 is the answer.

Deepa: Wonderful. Now answer me for rounding off 450 to its nearest hundred.

(All the students finish up their work)

Deepa: So, what is the value? Dijesh, you answer me.

Dijesh: Mam the value should be 400.

Bipul: Mam, he is wrong probably. The correct value is 500.

Deepa: Yes. The correct value is 500. So, if the value of the unit place is even equal to five (5) then the number is rounded off to the upper nearest 10^{th} value only.

Dijesh: But why mam? Why cannot it be 400? It also has the same difference with 450 as 500 has with 450.

Deepa: See, in this case, we always see the upper 10^{th} value. This is the trend we are following which is probably the correct way of rounding for such numbers at the centre. Is that fine?

Dijesh: (Softly)... Yes, miss!

Deepa: Okay students, turn the page no. 40 and start solving the given exercises. Try to complete it here within class. And the remaining will be the homework. If any confusion arises, please ask me in the next class Okay!

After the class, I also walked along Miss Deepa toward the staff room. I was pleased seeing that she could make a great interactive class with maximum participation of almost all the students. But I mentioned her about some faces in the classroom which were probably not satisfied with some confusing answers given to them. I further mentioned about the rounding off the number which is at equal interval from both the upper and lower nearest 10^{th} place whose answer could not satisfy the

answer of Dijesh. She further told that the information mentioned in the text was also the same and no further information was mentioned.

The number line is one of the most central concepts in all of the mathematics (Lakoff and Nunez, 2000, p. 6). It is one of the powerful tools to explain the abstract concepts of numbers. The number line is taken one of the best metaphors for the teaching and learning of the concepts Rounding-off-Numbers. It is the linking metaphor which enables to teach the arithmetical concepts through number line. The Curriculum Development Center (CDC) introduced the concept of Rounding off Numbers from grade three. The prescribed textbook of CDC explained this concept through the number line. However, the text does not explain sufficiently about the rounding off the middle numbers (the text is found in the Appendix--). For example, 45 is a number lying between 40 and 50 on the number line. So 50 is the number rounding off 45 to the nearest ten. One of my research participants asked me and why not 40 is the rounding number of 45 since 45 is just between 40 and 50.

I observed a classroom teaching of the unit of Rounding-off Numbers. My research participants taught in her class as explained in the textbook. Almost Nepalese mathematics teachers considered textbook as the main source of teaching and learning of mathematics. The mathematics textbook is not presented in view of cultural metaphors. The examples given in the textbook is not culture friendly. Then, in the staffroom, I asked some of the teachers about teaching of the same issue and they too simply agreed to Deepa's way of teaching. This was not the thing I was searching for as the same thing was not able to make that student understand it with full satisfaction. So, I thought how would it be if path metaphor could be introduced in the understanding in this case. And I explained about all the procedures.

In this “Rounding off Numbers”, we can use the familiar examples regarding the everyday activities of moving from one place to another by students themselves.

Let a person has to travel from A to B and let C be the centre of AB. The point B is the destination and A is the initial point of the journey. If a person is on the way to B and let the road is blocked due to any circumstances at a time, then there arise three cases.

Situation I: When a person is near to A or at a distance less than AC, then s/he better moves back to the initial point A because of the uncertainty of the disclosing of the blockage.

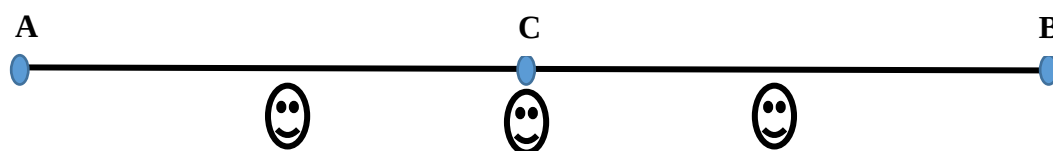


Figure 6.1. Rounding off Numbers as a Path Metaphor

Situation II: If a person gets to the point C i.e. at the centre of the journey, then s/he must go towards B because the main target for which the journey began was only to achieve the aim. Since every journey has a purpose, it is necessary just for its accomplishment, which signifies that the distance, however, must be covered even if s/he returns to the initial point which is to waste the time and everything else. So as s/he gets to half of the way to it, it's better moving towards the target rather than returning to the initial place.

Situation III: For a person beyond the midpoint, near to the end, it is always better to move to the final point only and not to the initials as s/he she has gained a distance near to the aim.

Then, the following conversation took place:

Deepa: Sir, this can be so true to make that confusion clear.

Researcher: Do you too think so?

Deepa: Undoubtedly sir. Why do not we try this in my next class? I have same class of rounding off the numbers in the nearest 100 in class 5 after 5th period.

May I try the same pattern for teaching in that class?

Researcher: Why not! But, can I get the permission to be in that class too?

Deepa: Sir, why not. I would welcome you in that class sir.

Third Intervention: Path Metaphor

With the same lesson plan, I along with the teacher moved towards the classroom of Grade 5. We both entered in the classroom.

Deepa: Good afternoon, class!

Students: Good afternoon Madam.

As usual, she started giving my introduction and purpose. I shared my some of the objective of the class observation and explained about the actions me and my technical supporter will perform in the class. I requested them to take the class normally, learn to the best level and ask the teacher if a thing is not understood. All of the students accepted to cooperate with me, and the formal class begun.

Deepa: Children! Do you know what are we going to study today?

Students: Yes, Madam.

Deepa: Oh nice! So, Kushal what are we studying today?

Kushal: Madam, the lesson you asked us to see at home was Rounding-off Numbers.

Deepa: Excellent. So, students we hereby start our chapter for today

“Rounding off the Numbers”.

Her bonding with the students was truly pleasing and her style of teaching was really appreciable. I further went on observing the class.

Deepa: Do you all remember about the similar lesson that we had learnt in the previous grades. Rounding off a number to the 10th place or 100th place.

Students: We do madam.

Deepa: And as you are growing, the course too is taking its height. In this grade, we are to learn about the rounding off the number to the nearest thousand. We round off the number to bind up its original value with the nearest ones as for the easiest remembrance and for the foremost estimation in our daily life. We can round off the number to the nearest 10, 100, 1000 or even to the greater extent. And here as I said we learn to round off the numbers to nearest 1000. For this, we see the extent of value at the hundreds place. Okay, children!

Then, Miss Deepa noted a number 6110 in the board. And all the student loudly shouted, “Yes, Mam”. And, she continued.

Deepa: See, for the rounding off a number to the nearest thousand, we must see the value at the hundreds place. For example, if we take this number 6110 then, first of all, we see the value in the hundreds place. And what is the value at hundreds place?

Student: Mam, one(1)

Deepa: Good observation. Then Krity you answer me what is the value after you round off to the nearest thousand?

Krity: Madam, 6110 if rounded off to the nearest thousand then the value becomes 6000.

Deepa: Excellent Krity. So, can you say why the value becomes 6000?

Krity: The main note is that, if the value of the hundreds place is less than five (5) then the number is rounded off to the lower nearest thousand value.

Deepa: Absolutely. Then probably all of you can round off 8800 to its nearest thousand.

Students: Madam, 9000 is the answer.

Deepa: Wonderful. Now answer me for rounding off 4500 to its nearest thousand.

(All the students finish up their work)

Deepa: Have you done students? Himal, can you give the answer?

Himal: Mam, the answer is 5000.

Deepa: Good, did everyone get the thing?

(After some time.)

Bishal: Miss, I have a query.

Deepa: Feel free Bishal.

Bishal: Why 450 is rounded off to 500? Cannot it be rounded off to 400?

Deepa: Bishal, you have raised one critical question. Students, what might be the reason?

Do anyone in the class can give me some logic?

(No one answered)

Deepa: Can anyone... You Jyoti, you can make an effort!

Jyoti: No idea madam.

Deepa: Can no one answer me?

(There was a pin drop silence in the classroom.)

Deepa: Then listen carefully. Suppose you are in Kathmandu. Then you have a schedule of travel to Pokhara. You get on to your car and start the journey. And suddenly there is an uncertain blockage of your way. Let us note that Kurintar, way to Manakamana temple, is the centre of the total distance of the journey. Then Kamal, answer some of my questions. Will you?

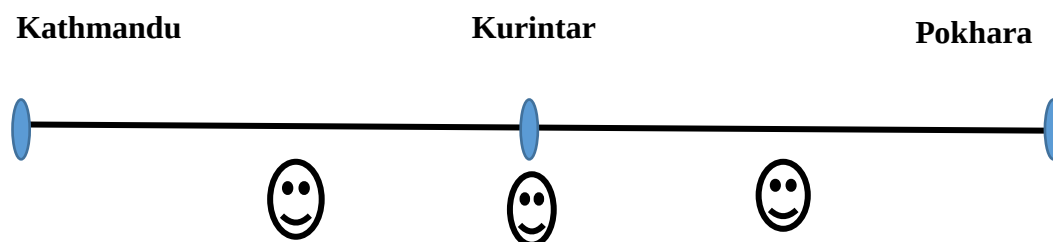


Figure 6.2. Deepa's Path Metaphor for Rounding off Numbers

Kamal: I will madam.

Deepa: Would you go on or return to Kathmandu if you have even not reached to Kurintar, the halfway of your destination.

Kamal: Madam, I would better back.

Deepa: But if you have reached to the halfway?

Kamal: Madam, If I have already travelled half of the way then I would probably chose to wait for the way to unlock.

Deepa: Exactly. The same thing can be compared with the special case of rounding off 450 to the nearest 100. Likewise, 450 is at the midpoint of 400 and 500. And like the same way as said by Kamal, we can prefer moving to 500 from 450 rather than towards 400.

Students: [Totally amazed] Madam, wow.

Deepa: Yes. Then you all are clear now.

Students: Yes, Madam.

Deepa: Okay students, now turn the page no. 22 and start solving Exercise 2.3.

Try to complete it here within class. And the remaining will be the homework.

If any confusion arises, ask me in the next class Okay!

The classroom teaching and learning is meaningful if it is connected to everyday activities and familiar context of students. From the observation of different lesson regarding rounding the middle numbers into nearest 10, 100, 1000 and so on can be made meaningful by contextualizing the mathematical ideas. The mathematical activities in different cultures and their mathematical thinking can be facilitated to develop mathematical achievement and to provide rich background knowledge to the students. By motivating students as mathematics, a cultural enterprise will develop a positive attitude towards the learning of mathematics. Regarding the teaching and learning of “Rounding off Numbers” at grade 3 students, in this regards, my next research participants opined that:

Mathematics is an important tool to solve the everyday need of the people. They use mathematics in the diverse field of their activities without explaining and knowing mathematically.

The mathematical thinking of the people depends on the context, cultural traditions, beliefs, attitudes and political situations. The members of the cultural group practising their own mathematical knowledge to perform their activities. They are using their emic view of the knowledge they developed across the generations. The mathematical ideas that the diverse cultural groups use in their everyday activities and its integration into mathematics curriculum enhance the students’ mathematical ideas. In this vein, Gerdes (1997) viewed that the incorporation of mathematical knowledge from different cultures into mathematics curriculum enhance everyone’s confidence and learning to respect all ethnic groups and culture.

Metaphors for Teaching and Learning of Prime Numbers

Numbers are very fundamentals of mathematics. The number is an abstract concept. The abstract concepts can be made more meaningful by connecting learners' real-life situations. In this vein, Bonotto (2007) viewed that the extensive use of cultural artefacts makes school mathematics more meaningful. For him, the cultural artefacts that introduced into mathematics classroom are concrete materials which children typically meet in real life situations. Those concrete materials can be the suitable tools to transfer from one domain to another domain of knowledge. The prime and composite numbers can be conceptualized through the different cultural artefacts.

During the teacher training cum workshop programme, in our CAR programme, we had listed out some mathematical concepts that were comparatively difficult for the teachers to teach in the classroom. Among the teacher fellows, Pooja and Deepa both asked about the way of teaching prime and composite number in an easier method as they have to teach the concept to the next day. Then for the teaching of prime and composite number, we discussed about the measures of teaching it to the students in the group of many teachers.

Researcher: Miss, I think you both have a great experience in teaching since a long time.

Deepa: I have been teaching for about 10 years.

Pooja: And I have been teaching for 5 years.

Researcher: This is a great experience. Till date, how have you been teaching the concept of the chapter?

Deepa: I, during the class, used to give definitions of prime and composite numbers. (numbers having more than two factors are called composite numbers and remaining numbers are known as prime numbers).

Researcher: And Miss Pooja, how did you teach?

Pooja: I do follow the same pattern as by Deepa.

Then I put this issue in the group of CAR project. Is definition method only permissible in the teaching of prime and composite number? If not true, what other ways there may exist in course of teaching this concept? And as result, we arrived at a conclusion that different metaphors can be used in the teaching to make it simpler and effective. And the metaphors could be Color Metaphor, “Sieve of Eratosthenes”, Area Metaphor, Cultural artefacts and so on. Then, Deepa mentioned that she would use the area metaphor and Pooja would use “Sieve of Eratosthenes” in the future class of Prime and composite numbers.

First Intervention: Color Metaphor

It should be clear that the traditional colour theory states that there are three types of primary colours. These are the colours of red, yellow and blue. The primary colour is that colour which cannot be created by any combination of other colours. They are the colour equivalents of mathematical prime numbers. Like prime numbers, the basic three colours cannot be reduced to more fundamental colours. They form the building blocks of all other colours. The main objective of this class will help each student to understand the concept of prime and composite numbers. Color is one of the common metaphor that can help to conceptualize the concepts of Prime numbers and composite numbers. Then we together developed the lesson plan according to constructivist teaching framework. Pooja had prepared a chart board as shown in the figure. The figure represents a Venn-diagram showing the intersections of three

primary color sets; Red, Yellow, and Blue. The chart was put in the flannel board and explained about the formation of different colors when the primary colors were mixed with each other. Then the whole class was divided into four small interactive groups each of eight members. Each group was provided with a color pad with three primary colors; Red (R), Blue (B) and Yellow (Y) and a blank painting sheet. They were asked for mixing the colors $R+B$, $R+Y$, $B+Y$ and $R+B+Y$.



Image 6.15. Colors in Venn diagram



Image 6.16. Displaying the Composite Colors

After few minute for the task, she asked each group to share their result with the whole class. Then, Pooja asked the students if all of them got the same outcome as per discussed before. All the students had the reply of having the same outcome as per her teaching of the formation of those colors. Then she concluded that the color mixing can be compared with the prime and composite numbers. She further described the prime numbers as those primary colors which are not a mixture of any other colors, and the colors like orange, grey, green and violet formed as a mixture of more than a color were compared to the composite number. After this discussion, the teacher then encouraged to write the definition of prime and composite numbers.

After some time, the teacher gave an opportunity to each group leader to share their definition with the whole class. Encompassing all the concepts, prime number can be defined as a number which has just a factor, itself other than the factor 1. For



Image 6. 17. Students Participations in the Learning Process

further clarification, the prime number can be considered as a primary color, a unique item. For instance, if we take yellow color, then its mixing item, if considered as the factor, is just single color i.e. yellow is an independent item with a single factor concluding that yellow is a prime color. Likewise, composite number can be defined as a number which has two or more factors other than the factor 1. For further illustration, composite color can be directly connected with the secondary colors composed of two or more color items. For instance, if we take orange color, then its mixing items if considered as the factors are of 2 colors i.e. two factor concluding that orange is a composite color. At the end of the classroom, I asked the students near to me:

Researcher: Students, what did you find about today's class? Was it interesting?

Students: Yes. Today's class was the most interesting class till date.

Researcher: What made you feel that interesting about today's class?

Students: Madam, brought so many interesting items like colors and charts.

This really made us feel fantastic about the class today.

Researcher: Did not she brought those items before in the class?

Students: Actually, not sir. Mam, always used to teach textbook directly and solve the problems from the exercise.

Researcher: Did she made you to solve some of those problems or not?

Students: Yes. She made us to do that.

Seeing that keen participation of the students in today's class, I could find that the teacher had never used this way of teaching approach till now. She always used chalk and talk methods during her teaching.

Researcher: Did you enjoy in learning the concepts?

Student: The class was the really enjoyable and interesting today. We learnt those bookish concepts with a simultaneous playing as if those were games. This class will be really memorable for us.

Researcher: That's good. Thank you for your compliments.

Yes, the students were true from their side. Some teachers generally lack in the understanding of children's psychology. Every teacher to gain a successful career must take care of the thing that is the knowledge one is providing being effective to the learning of the class or not. The teacher must use child-centered pedagogy for applying those bulks of theories in the teaching. This can really uplift the level of understanding of students to a great extent. As the teacher used special approaches while she taught prime and composite numbers today, the students were really pleased as they could understand those concepts so easily. She had never used this way of teaching approach till now. She always used chalk and talk method only inside the classroom. But after this special approach i.e. the use of metaphors in teaching today, I could see the active participation of the students and constructivist learning framework flourishing in the classroom environment. The children were really pleased.

Second Intervention: Colorful Rainbow Factor Metaphor

For teaching prime and composite numbers, she used the color metaphor by making a rainbow design for the better understanding. She further mentioned that; For a prime number, there would be only two factors “I and Me factor metaphor” which means 1 and itself. For a composite number, she created another beautiful colorful rainbow factor metaphor “I, Me and My factor metaphor” suggesting 1, itself and all other factors of the number in a colored rainbow pattern. For example, in case of a number 36, it has the factors 1,2,3,4,6,9,12,18 and 36. Then she gave some examples to check whether the numbers were prime or composite. Then the bell rang and we left of the classroom. On the way to staffroom, I asked her how she felt about the class today. She replied:



Images 6. 18. Colorful Rainbow Factors in Teaching of Prime and Composite Numbers

“In course of my long term teaching career, I always had some feeling like could I make the students understand about the concept of prime and composite numbers or not. I mean, this chapter has no such way for making its understanding in a simple way but a general pattern of topic and definition was only permissible. I knew that I was not being able to explain the core concept with a path of ease. I just used to teach the chapter and ended the class by giving them the work to check if a number is prime or composite. And to be frank, I used to enlist all the prime and composite number in two sides of

the board and ask them to rote them until they are memorized. But after today's teaching design, I think I rather could give them the work as homework instead as the classwork. And I can feel the difference in the student from last batch and the students of today's class. They were so jolly to perform such use of colors and other cultural artifacts, which make them very keen in the studies. I have started feeling really gratified during the teaching of this lesson."

With the interaction with Pooja, I found that there is a need of providing workshop and training to the teachers of mathematics. She was heartily convinced about the contribution of the metaphors in the teaching and learning of abstract concept of mathematics. Mathematics is not so abstract concepts. It can be made concrete by connecting it to the familiar contexts of the students. The connection of students' familiar context to the school mathematics can make the regular mathematics class very interesting, and meaningful. There are different metaphor for teaching and learning of abstract concepts of mathematics. But the teachers of matheamtics should know the students household activities to contextualize the school mathematics.

Third Intervention: Area Metaphor

Area is one of the common metaphor that can help to conceptualize the concepts of Prime numbers and composite numbers. The area can be visualized as the small square unit in the geoboard. It is one of the common visual instructional material for the teaching and learning of mathematical concepts. It is equally important to teach prime and composite numbers. Then we together developed the lesson plan according to constructivist teaching framework. The whole class was

divided into five small interact groups of members six. Each group was provided with a geoboard and rubber bands of different colours. Each groups were assigned with two numbers, one prime and other non-prime(Composite). For example; the numbers 5 and 6 are given to group A, the numbers 7 and 8 are given to another group B and so on. Each group has requested to form possible rectangular plot with the help of rubber bands in the geoboard. After few minutes to discuss within the group, she asked each group's reporter to share their answer with the whole class.

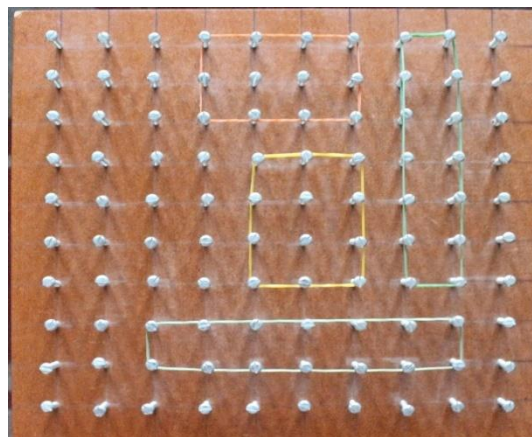


Image 6.19. Artefacts Showing the Rectangular Area

Deepa: Are you finished with your task?

Reporter of each group replied yes madam.

Deepa: Good, my children. If so, please tell your answer to the class turn by turn.

Students: Okay madam.

Deepa: At first, who is the reporter of Group A?

Reporter of Group A: Yes, madam, me Rohan.

Deepa: Its okay. Please tell me your numbers in task.

Rohan: We have two numbers 5 and 6, madam.

Deepa: Then now tell me how many possible rectangles did you get for the number 5 and 6?

Rohan: Madam, for 5 we could make only 2 rectangular plots but for 6 we could make 4 rectangular plots.

Deepa: What were the sizes of rectangle you could prepare?

Rohan: We could prepare 1×5 and 5×1 in case of 5 and 1×6 , 2×3 , 3×2 and 6×1 in case of 6 Madam.

Deepa: Okay sit down! Can other group tell me if any other sizes of rectangular be made for 5 and 6?

Groups: Madam, Rohan is probably correct.

Deepa: That's good. Now one of you from group B, stand up!

Reporter of Group B: Yes Miss.

Deepa: So, Sharwin. Talk about the same in your task.

Sharwin: Yes, Madam. We had 7 and 12 as two numbers given to us and we found that there were only 2 for 7 and 6 rectangular plots for 12.

Deepa: And Group C and D. Which numbers did you get with just 2 rectangular plots?

Reporter from Group C: Madam, among 3 and 8, 3 had just two rectangular plots.

Reporter from Group D: And Miss, out of 2 and 9, 2 had just two rectangular plots.

Then, Deepa enlisted those numbers having only 2 rectangular plots in board as said by her students. Then she asked if there were other numbers that could have similar result. And she concluded that these numbers can be called as prime numbers and the numbers that can have more than 2 rectangular plots can be called as composite numbers. Then she asked her students if they could write the definition of prime numbers and composite numbers. Then comprising the answers from her students she modified it and wrote it as: A prime number is a natural number that is

greater than 1 and has only two factors i.e. 1 and itself. Then she gave some Examples to check whether the numbers were prime or composite.



Image 6.20. Students are Exploring Mathematical Relations

Then the bell rang and we jointly came out of the class. Then I asked her about how she felt about today's class. She replied the class to be very interesting, participatory and as a whole meaningful. Regarding the effectiveness of incorporation of cultural artefacts in the classroom teaching, she speaks:

I have been teaching for 10 years, and I always had to think about the class to be a successful class during the time of teaching prime and composite numbers as I really felt difficult to express its essence. I directly used to give definition to the students and instantly jumped to the exercises. This was rather easier for me to end up the class but still I did not found any interest both in me to teach this and in the face of students to learn it. But today after the use of area metaphor in teaching of prime and composite numbers I felt really gratified during the teaching of this Prime and Composite numbers in fact after 10 years of my teaching.

From the interaction to her, metaphor plays significant role in the process of knowledge construction. The cultural artefacts facilitates students to develop the mathematical relation and helps to understand the concepts of prime and composite

number. Metaphors really contribute to the teaching and learning of abstract concept of mathematics. The connection of students' familiar context made to teach the school mathematics can make the regular mathematics class very interesting, and meaningful.

After the class, I asked about the effectiveness of cultural artefacts in the process of teaching and learning with the group member of each group. The reporter of group A viewed that

This class seems very interesting. We are working collaboratively. This is providing us the opportunity in learning together. The collaborative learning situations and group discussions enhance the level of confidence.

From the above view of student participants, I have come to conclude that the constructivist learning situations encourages in learning mathematics. The collaborative work provides ample opportunities for social interaction and self-expression. They also make their students develop a sense of their active roles as producers not only consumers of knowledge. The learning pedagogy that was used denied the rote and memorization of the definition of prime and composite numbers. The member of another group views that:

We are having a real fun today. We are learning mathematical concepts just like by playing games. We feel this learning environment to be perhaps student friendly. Our teacher is providing us a great opportunity to work in a group. In fact, the cultural artefacts provides us to understand more mathematical concepts.

From the conversation, I have come to realize that mathematics teacher need to provide opportunity to students for active participation in learning process. Playing games is common to everyone. Games has some sort of rules, procedures and structure,

as in the mathematics. The incorporation of games and puzzles play important role to achieve aesthetic spirit of mathematics and to increase mathematical creativity and curiosity. The above mentioned conversation is good evidence that the cultural artefacts mediated for the construction of mathematical ideas and enhance the level of students understanding.

Metaphors for Conceptualizing Integers

Peled, Mukhopadhyay and Resnick (1989) observed that the third and fifth graders did not have knowledge about the integers. They found that the student did not performed appropriate answer to the subtraction problem like $5-7$. Students either inverted the subtraction or treated as $7-5$ or they stated that $5-7$ yielded zero. Kilhamn (2011) further noted that “since these students did not receive any formal instruction about negative numbers before sixth grade, they construct mental models that includes negative numbers before school instruction drawing on their models for positive numbers to do this” (p.38). Further, Kilhamn (2011) advocated that the procedural errors that are caused by students’ lack of conceptual understanding of the negative integers. The students who could perform the operations also did not have conceptual knowledge regarding the negative integers. The CDC introduced the concept of integers from sixth grade in Nepalese mathematics curriculum. Even though, most of the secondary level students around the world have misconceptions in tackling the problems regarding the negative integers. The table 6.5 depicted the contents of integers in mathematics curriculum at the basic level of education in Nepal.

Table 6.5
Content of Integers at Basic Level

Grades	Units	Contents	Teaching Hours
6	Integers	Introduction to integers	2

7	Integers	Four Fundamental Laws on Integers	5
8	Integers	Four Fundamental Laws on Integers Simplification with Brackets	4

Number itself is an abstract concept. What is number? Can it be perceived through our five sensory organs? What is Zero? How does student perceive negative numbers/ integers? How the students have perceived the quantity infinity? The answer of all these questions are really problematic for basic level students. However, students of this grade level are frequently practicing all these concepts in their everyday lives. Students can estimate the height of a house, tree, electrical pole, the depth of the well and so on. They are looking for measuring and estimating the height by taking any references as they have assessable. In the same way, students can be conceptualised the negative integers through the depth of well, the level of electrical pole below ground level, the debt, and the background of the house below the ground level.

For making the concept of integers more easily understandable, there can be the use of many metaphors which are familiar to the students' out-of-school context. However, it becomes a difficult task for the teacher to figure out which metaphor would suit its use in the classroom teaching to give them the conceptual understanding of the ideas.

First Intervention: Balance Metaphor

Today, I had decided to observe the class about introduction to the integers in Grade VI. For this, I had to observe the class by Chhaya. She had developed a lesson note for teaching integers in view of cultural metaphors based on constructivist teaching framework. I asked Miss Chhaya for her first class observation. She gave me

the permission for the observation of her class. We went toward the classroom. All the students stood out to greet us. The teacher gave my brief introduction and purpose of the class observation. I had my technical supporter with me too. So I made them alert not to take it otherwise about the photoshoot in the class which also was already notified to the teacher. All of the students accepted my request. Then I went back to the last bench of the class. The formal class begun.

Chhaya: Today, I am going to start a new lesson. We are reading about integers today.

Do you know about this term?

Students: We have heard of it but we do not know its exact meaning.

Chhaya: Cool! Then let us start our class. I guess everyone knows about balance meter.

Students: Yes Mam we have seen them.

Chhaya: Good. Have you noticed the level of the meter when no any load is kept in the bucket of the balance?

Table 6.6
Balance as a Conceptual Metaphor for Zero

Source Domain	Target Domain
 Figure (a): Equilibrium	Zero (0)

One of the students shouted

Student: Yes! Mam. The level of the meter are balanced at a same level.

Chhaya: Wonderful. Then note that the level means 0 deviation in the buckets of the balance. So, this is generalized as 0 value.

Chhaya used equilibrium condition of balance as a source domain to represent the value 0 as a target domain.

One of the student asked.

Student: Hmmm... Mam, but how can this represent 0.

Chhaya: I appreciate. But, “*Samaya se Phele aur Kismat se Jyaada, Kisiko Kuch Nahi Milta: Balak*”. First let me complete my saying.

Students: (She along his friends smiled hearing to their teacher.)

Chhaya: Okay, lets continue. And you probably know what happens when a load is put onto a bucket of the balance which exceeds the weight in the next bucket.

Students: All watching to the teacher very keenly.

Chhaya: Then the value comes in the consideration is a positive value as the bucket with the load exceed the bucket with the weight object in mass.

Chhaya used overloaded condition of balance as a source domain to represent the positive numbers as a target domain.

Table 6.7
Balance as a Conceptual Metaphor for Positive Integers

Source Domain	Target Domain
	Positive Integers $\{+1, +2, +3, \dots\}$

Figure (b): Over loaded

Students: Oh yes.

Chhaya: Everyone is getting right?

Students: Yes, Mam.

Chhaya: Then what can we consider for the condition in which the weight object exceeds the load in mass?

All students got it correct!

Students: Miss, perhaps negative numbers.

Chhaya: Correct. Then now you probably got the answer of your questions right?

Chhaya used low loaded condition of balance as a source domain for the negative numbers as a target domain.

Table 6.8

Balance as a Conceptual Metaphor for Negative Integers

Source Domain	Target Domain
	Negative Integers $\{-1, -2, -3, -4, -5, \dots\}$

Figure (c): Low Weight

The teacher used balance meter as the metaphors for the teaching of integers.

Balance metaphor can represent positive integers, negative integers and zero as per situations seen in the balance. The equal weights on both sides of balance creates equilibrium as in the illustrated below in figure (a) above. This views represents no difference or zero difference in weights on both left and right sides of balance. The figure (b) depicted the right of balance has overloaded than the left side balance. The overload means some weight are more in right than the left side of the balance. The more weight or more quantity represents the positive quantity and hence positive integers. In the same vein, the figure (c) represents the left hand balance has low

weight in right side balance than the left sided balance. The lower weight or less quantity represents the negative quantity and hence negative integers.

Second Intervention: Gestural Metaphor

My cooperating teachers of the sample school used varieties of metaphors to communicate the ideas of integers to the sixth graders. The teachers were using multiple metaphors with different sources for a single target (Chiu, 2000). The idea of balance metaphor probably did not help some students to understand the concept of integer. So, she further used gestural metaphor in her teaching of integers.



Image 6.21. Gestural Metaphor and Students Participation in Learning

Up-Down and Above-Below is one of the best way to teach the concept of integers. The teacher used this concept to support her balance metaphor in the teaching of integers. She used her hands to show the up-down and above-below directions in front of students. The use of such gestural metaphors gave clear understanding to the students. The use of child-centered pedagogy could really create huge participation of the children in exciting and enjoyable manner.

Third Intervention: Money Metaphor

When analyzing discussion during a workshop in a teacher support program in my sample school, different metaphors were proposed for the teaching and learning of integers. One of my research participants gave the money is integer as a metaphor to conceptualize integers. Many researches have been done in the area of integers. How

can students of basic level be conceptualized the notion of negative integers thereby students developed conceptual understanding about the subject. While teachers are going to deliver new abstract mathematical concepts, the metaphors are the effective tools to communicate the concepts. However, the mathematics teachers are not always aware of the metaphors they use (Font, Bolite, and Acevedo, 2010).

In the classroom observation of one of my research participant, I found that the teacher used several metaphors in the process of teaching of the concept integers. But she was unaware about the question of what type of metaphor did she used.

Most of the teachers used metaphors without noticing it as they were the metaphors. I

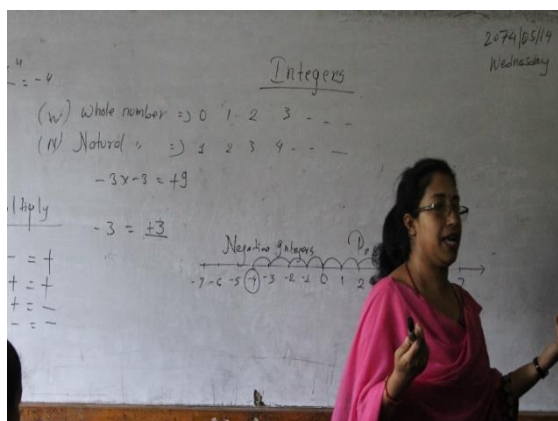


Image 6.22. Integers and Number Line Metaphor

analyzed that the metaphors she used in their study of teaching integers at grade six was MONEY IS INTEGER metaphor. She told me that she was taking some familiar examples that students encounter in their everyday lives. I had proposed several metaphors to conceptualize the notion of integers.

Table 6.9
Integers is Money Metaphor

Source Domain	Target Domain
Neither loose nor won money	Zero
You won Money	Positive Integer
You lose Money	Negative Integer

Money, as a metaphor, can be used as a very efficient tool in the teaching of integers as money holds a great impact in this physical world. There is probably no child who lags the idea of earning and spending money. And we hereby can use these situations to provide a contextual support for solving problems regarding integers. The assets and liabilities can also be the metaphor for the understanding of positive and negative integers. Assets add positive income each period and liabilities (debt) require payment to service it. As per my observation in Deepa's class about integers, I found the connection of money with the teaching of integers very graceful. She had connected the earning, spending and owing of money directly with the concepts of integers. The winning of money as positive integers, the losing of money as negative integers and the neither loss nor gain situation as zero was linked mesmerizingly.

Fourth Intervention: Number Line Metaphor

The number line is one of the common metaphor to conceptualize integers in the basic level of mathematics classroom. Bruno and Martinon (1999) found that the number line became an indispensable tool when solving problems where the identification of addition with subtraction was a primary focus. A mental number



Image 6.23: During Integers Lecture

line is an important feature in the conceptualization of negative numbers (Kilhamn, 2011, p.39). In her study, students implicitly used a mental number line, scale, thermometer or other representation when dealing with negative integers. She further pointed that people in general associate large magnitudes with right side of space and small magnitudes with left side of space (Fisher & Rottmann, 2005). Gleaser (1981,

cited in Kilhamn, 2011) identified metaphorical meaning of zero as absolute. It is understood as the bottom, below which there is nothing. The another metaphorical conception of zero is taken as origin or initial point. The initial point represents an arbitrary point on the axis of orientation from which there are two directions (Kilhamn, 2011). The number line extended to the left of zero and every number on the right is greater than the left. Thus the number left to zero is smaller than zero and hence they are negative numbers.

Table 6.10

Number Line as a Conceptual Metaphor for Integers Teaching

Source Domain	Target Domain
Initial Point/ Origin Point	Number Zero
Bottom Point	Number Zero
Right to Initial Point	Positive Integers
Left to Initial Point	Negative Integers

Teachers used different kind of metaphor to communicate the concepts of integers at the basic level of education. Number line is one of the powerful metaphor to communicate the abstract concept of integers. Deepa used several metaphors to teach the concept of integers at grade six. She used different gesture to communicate the ideas of positive integers, zero, and the negative integers. She used the head or the

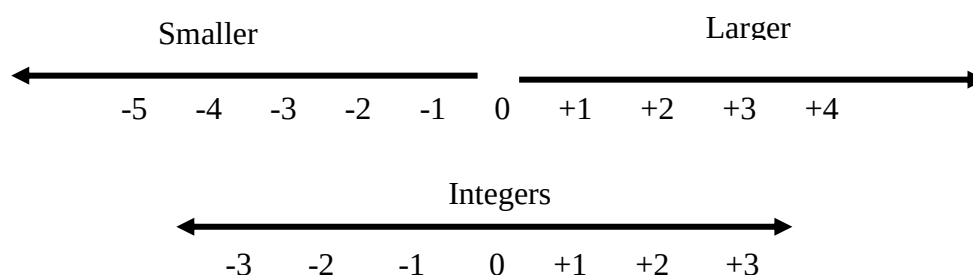


Figure 6.3. Integers as Number Line Metaphor

central part of her body as initial point to represent the number zero. The numbers towards right side for positive integers and the negative integers towards left was used.

The subject of integers is a very important part of school mathematics curriculum at basic level and it symbolizes a move from concrete to abstract thinking (Lamb & Thanheiser, 2006). Different teachers use several metaphors to assist student learning and encourage them to understand abstract ideas and concepts of integer. The metaphors used by the teachers in the mathematics classroom were Balance, Money, Gesture of hands and body parts, Number line as the source to communicate the abstract concepts of integers. With this vein, Chiu (2000) viewed that the students used multiple metaphors with different sources to understand a single target. The connection of two domains between sources and target be effective and interesting if the sources are familiar to them. The students' familiar out-of-school context and their funds of knowledge enhance to understand the concepts easily.

Metaphors for Teaching Operations on Integers

The rules for fundamental operations with integers have been formulated in many cultures. They were treated as a rule without producing mathematical proofs for them (Kilham, 2011). Mumford (2010) mentioned that the *Brahma-sphuta-siddhanta* includes two chapters on mathematics which are a compendium of the mathematical concepts and techniques which had been developed over the previous centuries. The rules of performing arithmetical operations with integers were explained in the following lines.

[The sum] of two positives is positive; of two negatives, negative; of a positive and a negative [the sum] is their difference; if they are equal, it is zero. The sum of a negative and zero is negative, of a positive and zero positive, of two

zeros, zero. [If] a smaller [positive] is to be subtracted from a larger positive, [the result] is positive; [if] a smaller negative from a larger negative, [the result] is negative; [if] a larger from a smaller, their difference is reversed – negative becomes positive and positive negative.

The product of a negative and a positive is negative, of two negatives positive, and of positives positive; the product of zero and a negative, of zero and a positive, or of two zeros is zero. A positive divided by a positive and negative divided by a negative is positive; a zero divided by a zero is zero; a positive divided by a negative is negative; a negative divided by a positive is negative (Mumford, 2010, p.123).

The rule teaching is sometimes important than the concepts teaching if the explanation of the concepts becomes difficult or time consuming. In our mathematics classroom, the teacher and the students use different types of mnemonics as a memory tool. By using the mnemonic in mathematics classroom, students are provided a better encode information so that it is easier to recall later on. The symbolic mnemonics as a source domain help to connect metaphorically to understand the target domain. With this connection, one of my research participants Deepa used symbolic mnemonics to teach the operations on integers.

First Intervention: Mnemonics

The subtraction of negative numbers from an integer, one of my research participant Dipa explained symbolic mnemonics. She explained to subtract negative (-5) from -3 to the children by imaging that one of the minus rotates itself ninety degrees and moves on to the other (-) to form a (+) plus (Chiu, 2000). She also explained that if there are two same operations in consequent order yield the positive

sign (+). Thus the subtraction of minus -5 from -3 is same as adding 5 in -3 . i.e., $(-3) - (-5) = (-3) + 5 = 2$. But the students did not understand the concepts.

Then she used another technique to make her students understand: Number Line Metaphor. And the class continued:

Deepa: Students, how can we transform the above expression into a number line?

Students: (No response from students!)

Deepa: (Drawing a number line) See here, where do (-3) lies in the number line.

Students: Miss, it lies three steps towards the left side from 0.

Deepa: Then what operation do we have to perform here? (Showing the minus sign)

Students: Subtraction miss.

Deepa: What will be the answer of this problem?

Students: (Students were making bizarre face with no answer at all)

Deepa: (Pointing to the number line) First move three units left from 0. Then, instead of going five units left from (-3) , we move five units in opposite direction from (-3) . And in this manner, we finally reach at $(+2)$, which is our answer.

Student: Madam we did not understand.

(one student; Ramita stood up to ask her queries)

Ramita: Miss, but subtraction means taking out of some items from a group of it. Is not so? Then, how can we show about taking (-5) away from (-3) ? Can that be really possible miss?

Deepa: See. In case of today' learning, we are not discussing numbers as the concrete objects like marbles, apples etc. as in the lower grades but here we are considering the subtraction of numbers from its position. Are you clear?

Students: Miss, we still did not understand. Would you mind repeating it once more?

Deepa: Okay. I will repeat it again. (Teacher explained more loudly than before) For (-3) we shall move three steps left from 0. After this, if there was $-(+5)$ only then we would still move 5 steps left from (-3) but as there is $-(-5)$ we move in reverse direction to $-(+5)$ i.e. we move 5 steps to the right from (-3) and we reach to $(+2)$. Did you understand now?

Students: (They still were not satisfied) Yes Mam.

Deepa: Okay then, now solve the problems which I have enlisted in the board. You must write the answer in the same pattern that I have taught you.

Students: (Small Sound) Okay, Miss!

Many students completed the problems as per said by their teacher. But I could still visualize those unsatisfied faces inside the classroom. After some time the bell rang and we came out of the class. Then I asked Miss Deepa how did she feel about today's class. Then Deepa frankly admitted that she could not give her best today. She further asked me how could this be made clearer to be understood. After we reached the staff room. We, along with some teacher participants of my CAR project, discussed about the problem of today's class and making it more effective. One of my CAR project members, Shyam, put in front his idea to make the students clear about the multiplication of two negative integers at first. He even claimed that the students should get this rule in their mind. Further, next participant arose and told that there was a mnemonic that could make the use of the rule more simplified.

This table which explains the rule of sign for multiplication of integers shows a real disciplinary act about a person who undergo the output by the corresponding actions they perform. This could be a really meaningful mnemonic that may help the

चिन्हको गूणन सम्बन्धी नियम		
सम्मो गरे (+) × (+)	=	सम्मो हुन्छ (+) हुन्छ
सम्मो नगरे (+) × (-)	=	नसम्मो हुन्छ (-) हुन्छ
नसम्मो गरे (-) × (+)	=	नसम्मो हुन्छ (-) हुन्छ
नसम्मो नगरे (-) × (-)	=	सम्मो हुन्छ (+) हुन्छ

Figure 6.4. Mnemonics and Sign Rule in Multiplication

students in both memorizing the rule and even recalling the moral values. This can really assist the students to memorize those rule of sign easily, which halts for the difficulties to be created in the solving of problems related to the operations in integers. The translation of above was presented in the following figure 6.5.

Rule of Sign in Multiplication			
Do	Good	=	results Good
+	+		+
Do not	Good	=	results Bad
-	+		-
Do	Bad	=	results Bad
+	-		-
Do not	Bad	=	results Good
-	-		+

Figure 6.5. Rules of Multiplication (Translated)

Shyam further explained that when students get clear about the rules of multiplication of integers, basically, $(-) \times (-) = (+)$ then the students can easily

solve the problem $(-3) - (-5)$. Here $-(-5)$ can be understood as $(+5)$ resulting the expression to be $(-3) + (+5)$. Then this can easily be carried for the further solution from number line itself. He also provided a mnemonic that he used in the teaching of the integer number during his class period. The mnemonic he provided was:

“Two like signs make a positive sign; Two, unlike signs, make a negative sign”

Like Signs				Unlike Signs			
+	+	=	+	+	-	=	-
-	-	=	+	-	+	=	-

Figure 6.6. Mnemonics for Sign Rules of Multiplication

This mnemonic readily explained about the rule of multiplication of integers at a level of ease. This even could help the student to recall those rule of sign which might create various difficulties in solving the mathematical problems related to the operations in integers. Mnemonics process help students to retain information and connect the concepts in mathematics. It is a device designed to aid memory for particular concepts of mathematics. Mnemonic instruction gives students tools they can use to better encode information so that it is easier to recall later on. Higbee (2001) defines mnemonics as memory aids that assist one in remembering specific information by using a process, strategies or techniques that enables a person to improve memory. In this study, I observed the different mnemonics that the students and teachers use in the process of teaching and learning of multiplication of integers.

Still, Miss Deepa appealed me if there was some other method for teaching those concepts regarding integers. She further told that the concepts that were arising in there were just about the rules and laws but not about the conceptual ideas. So, she further insisted to gain some conceptual thinking for taking the understanding regarding

the ideas of integers to the level of simplicity. After getting such a good reflection from a teacher, I really wanted to appreciate her, not with the words but with those conceptual ideas that she was searching for. Then I decided to use arrow metaphor from the constructivist framework. Finally, I put off my concepts in front of my CAR project participants:

Researcher: (Showing different types of arrows: $\rightarrow \leftarrow$) I introduce the meaning of these arrows I have drawn. The arrow $\rightarrow$ (pointed towards right) represents $(+1)$ and $\leftarrow$ (pointed towards left) represents (-1) . Similarly $\rightarrow \rightarrow$ represents $(+2)$. Then probably $\leftarrow \leftarrow \leftarrow$ represents (-3) . Then answer me what does this operation results $[(+3) + (+5) = ?]$?

Participants: (Drawing arrows: $\{\rightarrow \rightarrow \rightarrow\}\{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow\}$) Eight Sir!

Researcher: What does this operation results $[(+5) - (+3) = ?]$?

Participants: (Drawing arrows: $\{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow\}\{\leftarrow \leftarrow \leftarrow\}$) Two sir!

Researcher: Then what types of arrows can be shown to depict the number 5?

Participant: $\{\rightarrow \rightarrow \rightarrow\}\{\rightarrow \rightarrow\}$; $\{\rightarrow \rightarrow\}\{\rightarrow \rightarrow \rightarrow\}$; $\{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow\}\{\leftarrow\}$; $\{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow\}\{\leftarrow \leftarrow \leftarrow\}$ etc. . There are many ways sir.

Researcher: Fine, what about the arrows that can depict the number 0? Are there some?

Participant: For 0! Ummm... There, too, can be many options sir:

$\{\rightarrow\}\{\leftarrow\}$; $\{\rightarrow \rightarrow\}\{\leftarrow \leftarrow\}$; $\{\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow\}\{\leftarrow \leftarrow \leftarrow \leftarrow \leftarrow\}$... etc.

Researcher: Cool! Then if 0 can be represented by many arrow combination how can (-3) be represented?

Participant: There are so many ways sir:

1st way: $\{\leftarrow \leftarrow \leftarrow\}$

2nd way: $\{\leftarrow \leftarrow \leftarrow\}\{\rightarrow \leftarrow\}$

3rd way: {← ← ←}{→→←←}

4th way: {← ← ←}{→→→←←←}

5th way: {← ← ←}{→→→→→←←←←←} etc...

Researcher: Then when (-5) is subtracted from the 5th way, which you have used to represent (-3) , what arrow combination shall we get?

Participant: {← ← ←}{→→→→→←←←←←} - {←←←←←}

= {← ← ←}{→→→→→}

= {→→}

(The participants were amazed to see the result!)

Researcher: And what is the output?

Participant: Two!!! Brilliantly done sir. Wow!!

Researcher: Do you think that the students can understand this teaching technique.

Participant: Why not. They will easily understand this technique.

Deepa: Sir, I believe that all of them can understand easily. And I would use this technique in my tomorrow's lesson. Sir, if you are free tomorrow, I invite you to be in my class tomorrow. I ensure you that you can feel a great change in tomorrow's class compared to today's.

Researcher: Why not! I would enjoy to be in there.

Second Intervention: Arrow Metaphor

Next day, I went to the school to observe Deepa's lesson. Before entering inside the classroom, I shortly made her recall about the technique that we had used the previous day. She explained that the right directed arrow denoted positive integers and left directed arrow denoted negative integers and equal number of right and left directed arrow represented zero as shown in figure 6.7.

Source Domain	Target Domain
Right Directed Arrow $\{\rightarrow\}; \{\rightarrow\rightarrow\}; \{\rightarrow\rightarrow\rightarrow\}; \dots$	Positive Integers $+1, +2, +3, \dots$
Left Directed Arrow $\{\leftarrow\}; \{\leftarrow\leftarrow\}; \{\leftarrow\leftarrow\leftarrow\}; \dots$	Negative Integers $-1, -2, -3, \dots$
Equal number of Right and Left Directed Arrow $\{\rightarrow\leftarrow\}; \{\rightarrow\rightarrow\leftarrow\leftarrow\}; \{\rightarrow\rightarrow\rightarrow\leftarrow\leftarrow\leftarrow\}; \dots$	Zero 0

Figure 6.7. Source Domain and Target Domain of Arrow Metaphor

After she gave the concepts regarding integers by using arrow metaphor, the students seemed to be very satisfied and relaxed. During the class, I especially noticed Ramita who seemed very pleased and was smiling. I felt really happy observing the class. Deepa enlisted some problems for classwork at the board. I also collect the copy of some of the students. The image herewith was one of them. And after the class observation, I asked to Rasmita and other students about class. They were really satisfied today's teaching. Bell rang and we came out from the class together. I asked Deepa about the lesson. She was pleased and she replied that the level of understanding had really gained its height after the use of arrow metaphor for teaching of operations on integers.

Handwritten student work for the problem $5 - (-3)$. The student uses the arrow metaphor, representing 5 as five right arrows and $-(-3)$ as three left arrows. The work shows the process of adding the three left arrows to the five right arrows, resulting in eight right arrows, which is equal to +8.

$$\begin{aligned}
 &5 - (-3) \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \{\leftarrow\leftarrow\leftarrow\rightarrow\rightarrow\} \\
 &\quad \text{Taking away three left directed} \\
 &\quad \text{= arrow,} \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \{\rightarrow\rightarrow\rightarrow\} \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \\
 &= +8
 \end{aligned}$$

Handwritten student work for the problem $5 - (-3)$. The student uses the arrow metaphor, representing 5 as five right arrows and $-(-3)$ as three left arrows. The work shows the process of adding the three left arrows to the five right arrows, resulting in eight right arrows, which is equal to +8.

$$\begin{aligned}
 &5 - (-3) \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \{\leftarrow\leftarrow\leftarrow\rightarrow\rightarrow\} \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \{\leftarrow\leftarrow\leftarrow\rightarrow\rightarrow\} - \{\leftarrow\leftarrow\leftarrow\} \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \{\rightarrow\rightarrow\rightarrow\} \\
 &= \{\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\} \\
 &= +8
 \end{aligned}$$

Image 6.24. Student's Participation in Problem Solving

The students' participation in the task, their facial expressions during the classroom teaching was entertaining and happy. They were actively participated in the classroom activities. After the classroom discussion, teachers listed some problems for their classroom task. All of them were engaged in the given task. Most of them were able to solve the given problems. I observed that the arrow metaphors really contributed to understand the abstract concepts of addition and subtraction of integers.

Metaphors for Developing Algebraic Concepts

Algebra is a generalized form of arithmetic. It is one of the important fields of mathematics. We use it in our everyday lives without even realizing it. The algebraic concept is needed to solve everyday problems. Algebra is an abstract part of mathematics introduced in the very beginning of schooling. Probably, most of the students say that algebra is one of the hardest subject to be studied. In this connection, Leitzel (1989) the concept of a variable is more fundamental ideas in algebra and it frequently becomes a barrier to a student's understanding of algebraic ideas. Thus, the understanding the concept of a variable provides the basis for the transition from arithmetic to algebra and is necessary for the meaningful use of all advanced mathematics (Schoenfeld & Arcavi, 1988).

The history of algebra is as old as the history of arithmetic and geometry. The Greeks first introduced algebra in the third century and it was traced to the early Babylonians. The Babylonians were the ones who created formulas and equations that we still use to solve situations until today. Eventually, Diophantus was considered as the father of algebra. A fourteenth century and Viète is considered as the golden era in view of the development of syncopated algebra. Algebra is actually needed in our everyday life. We are required to find unknown values all the time in our lives. We

just do not realize that we are doing algebraic calculations, but feel they are merely arithmetic calculations.

Metaphor for Teaching and Learning of Variables

Number and equations are actually used in almost anywhere in the everyday interactions. But, our teaching of algebraic concepts is far from the connection of household activities and real-

world problems. Most of the students of higher level as well

are not able to see the

connection of algebra to the

real-life situations. Here, I

developed a teaching episode of

fundamental concepts of algebra

with the view of culture friendly pedagogy. The students' funds of knowledge and their activity based learning environment could provide the good opportunity to explore algebraic concepts.

Plan: After a weeklong teacher facilitation program, I along with CAR members had also made a discussion about how the algebra could be linked with our daily life.

Pooja, one of my CAR project members designed a plan to link the students' household activities and basic concept of algebra. To do so, she developed a project based on the sociocultural context of the students. Before starting the regular class, she assigned a project to the whole class. She encouraged the students to bring different types of beans available in their own house in a small amount.

First intervention: She, divided the whole class into six groups containing six students in each. Each group was requested to collect the beans, categorize them



Image 6.25. Activity Based Instruction in Teaching Algebraic Concepts

according to their type and count those beans which are of similar type separately. Then she asked the students to suppose each type of the beans available as variables x , y , z ... simultaneously. Then she finally asked students to write the total number of beans in terms of the variables used. This easily gave the fundamental concepts of algebra i.e. about constants and variables to the students. The table 6.11 presents the grounding metaphor for the development of some basic concepts of algebra.

Table 6.11

Developing Algebraic Concepts: Connecting to Real Life Context

Source Domain	Target Domain
Different types of Beans (For eg. Soybeans, green beans, corn, other some pea)	Variables (Let them be represented by x , y , z etc.)
Green Bean	→ Variable x
Soybean	→ Variable y
Number of Green Bean	→ ax (a is constant)
Number of Soybeans	→ by (b is constant)

I observed the class patiently and post class interview was taken. Then I asked a student to define what variable was. The students were unable to answer the question and stood speechless. I thought to suggest the teacher to use another activities that could make them easier to understand the concepts of variables and constant.

Second Intervention: Image Metaphor

Plan: The connection of students' familiar context in the process of teaching and learning of mathematics facilitated to understand the abstract concepts. So, I intervened the teacher to use image/ extraneous metaphor.

For easiness in understanding of variables, image metaphor can be used. For it, few male students were asked to stand in the class and their names were asked respectively which were written in the board.

Table 6.12

Image Metaphor for Conceptualizing Algebraic Concepts

Ramesh	is a boy.	Concrete
Suresh	is a boy.	
Dinesh	is a boy.	
	is a boy.	Semi-concrete
X	is a boy.	Abstract

Then teacher drew a male sign below the names in the board and asked the name of the image. He further clarified that the image could represent any of the names. As an image can represent more than one names, it is the variable. He mentioned a letter “X” below all which represented all boys which acted as the image.

Observation and Reflection: It was observed that the students were actively engaged and collaborative participation was found in the present lesson. The use of image metaphor made the students understand the concept of variable easily. The effect of cultural metaphors for the understanding of teaching and learning of algebraic concept be measured by evaluating students’ and teachers’ psychological and behavioral aspects. Their reaction, facial expression, active participation, and engagement in the classroom showed the effectiveness of the metaphor used in the classroom.

Teacher used sequence of concrete, semi-concrete and abstract model in the process of teaching of algebraic concept of variables. The concept was first modeled with visual and concrete object. Students were provided many opportunities to

practice and play with those concrete objects. After that, students represent their concrete understanding by drawing simple picture as an image metaphor that replicate the concrete objects previously used. The practice and mastery at representational (semi-concrete) stage supported students to develop conceptual understanding of abstract concepts of mathematics.

Metaphors for Teaching Like and Unlike Terms

Algebra can be found in various field of our everyday living. Algebraic concepts can also be found in the process of cultural game play as presented in chapter V. Students are practices the ideas of algebraic equation and inequality in the process of game play. However, the algebraic concepts they practiced are largely hidden in their discussion. The concepts of algebra are of great use in the systematic and well-functioning of a general living of a person.

First Intervention: The classroom activity was based on the constructivist pedagogy and was considered as the conceptual metaphors for the teaching and learning of algebraic concepts. Pooja asked her students:

Pooja: Students, do you know any other algebraic concepts around us?

Students: Around us! Ummmmm... (The class was silent)

Pooja: Think it. Just around your environment. It may be your home, school, shops anywhere you go!

Students: No idea miss!

Pooja: Then do remember your kitchen. Can you remember a kitchen rack in the kitchen where there are kept with a lot of utensils?

Students: Yes, Miss!

Pooja: And have you noticed the arrangement of those utensils in the rack?

Plates in a row, spoons at a side, glasses at one corner, bowls at the other!

Could you visualize what I said?

Students: Ummmm... Yes, miss!

Pooja: And have you noticed that a similar item are kept at some certain place in the shops?

Student: Oh Yes!

Pooja: Cool! So answer me how are those items arranged at a certain place?



Image 6.26. Visualization of Like and Unlike Terms

Students: Miss, probably it could

make easy to take the items if kept in a certain specified area.

Pooja: Yes, Good! So here we are using the algebraic concepts implicitly in our daily life.

She further explained that the utensils in same type in a kitchen rack, varieties of stuffs in shop like noodles, biscuit, chocolates, soap, etc reflected tacit knowledge of like and unlike terms. At the same time Pooja also use same types of beans to conceptualize the like and unlike terms.

Table 6.13

Conceptual Metaphor for Like and Unlike Terms: A Cultural Activities

Source Domain	Target Domain
Same type of Beans →	Like Terms
Different type of Beans →	Unlike Terms
ax →	Algebraic Terms
Total number of Green Bean and Soya Bean →	Algebraic Expression ($ax + by$)

Students were able to identify the like terms and unlike terms from the examples taken from their culture and activities. Everyday activities of students and their familiar context facilitate to understand the concepts. From the post class interview, I observed that most of the students were unable to symbolize their understanding into algebraic form. Teachers asked students to identify the like terms and unlike terms. $2x$, y , $5y$, x^2 , $3x$, x^3 , y^3 and so on. However, most of the students did not response correctly. After the class, I asked to some of the students near to me. They told me that “ $2x$ and $5y$ are unlike terms as same as corns and beans are different, but $2x$, and x^2 are like terms because of both contained same variable x ”.

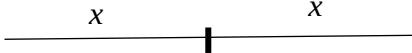
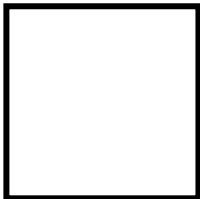
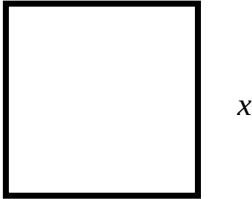
Second Intervention: From this observation, teacher developed another lesson to conceptualize the like and unlike terms. She asked, are $2x$ and x^2 same?

One of the students replied: yes mam. Why and how? Can you tell me?

Yes mam. Both terms contained the same variable x .

Table 6.14

Conceptual Metaphor for Like and Unlike Terms

Source Domain	Target Domain
	$2x$
	Square
	x^2

Pooja: yes, you are partly correct! There is some different between x^2 and $2x$.
Who can differentiate it? No one pronounce a single word.

Let's see x^2 is the product of two x ($x \cdot x$) and $2x$ is the sum of two x ($x + x$).
Still some of the students did not understand the concepts clearly. She further explained the concepts through the use of extraneous metaphors.

The term $2x$ gives distance between two points. Whereas x^2 gives the area of square plot having a side x . From the table 6.14, it was observed that the source domain “line” and “square” for the terms $2x$ and x^2 facilitate to understand the terms. The image metaphors facilitated the conceptual understanding of like and unlike terms.

Selection of Spot for CPBL

Mathematics can be taught beyond the boundaries of four walls of the classroom. The study of monuments and the ethnomathematical ideas embedded in it are the sources mathematics teaching and learning. The selection of study area is really difficult for us. The Headmaster suggested me to select one monument out of three. One of my research participants and me together visited all possible location and observed for the possibilities of connecting monuments and cultural artefacts to school mathematics. We visited this temple before select the study area and search for possibilities of incorporation in mathematics lesson. Bonotto (2007) viewed that “the connection between real world and classroom mathematics is not easy because the two contexts differ significantly” (p. 187). However, the mathematics teaching and learning would be more interesting and effective if the appropriate connection is made between out-of-school context to the school mathematics. Ultimately we decided Old Guheshwory temple as the study location for field visit with the aim to connect

monuments and cultural artefacts as the metaphor for teaching and learning of school mathematics.



Image 6.27. Old Guheshwory Temple

First, I explained briefly about the history, myth, religious belief of the Old Guheshwory temple. The temple of Guhyeshwori located in Goldhunga VDC of Kathmandu is known as old (purano) Guhyeshwori, while the Guhyeshwori of Pashupati Mrigadavan is known as new Guhyeshwori. Guheshwory temple erected near the Shakti Peetha near to the Pashupatinath Temple. It is said that this temple is the Shakti chair of Pashupatinath Temple. The temple houses an idol of Parvati as Guhyeshwori surrounded by idols of Ganesh, Buddha, Surya, and Kumara. As the new temple was built by King Pratap Malla in the 17th century, it is retrospective nominalization that the Old Guheshwory temple was erected before 17th century.

Gay and Cole (1967) assumed the need of incorporation of ethnomathematical ideas of indigenous students in achieving better results. It is considered that the inclusion of ethnomathematical perspectives in school mathematics valued students cultural background and experiences. The observation of mathematical ideas in the surrounding of the temple would motivate students to take mathematics has relevant to their lives outside the classroom (Adam, 2004). The field observation of the temple

provides to learn mathematical ideas embedded in out-of-school context and connect to the school mathematics lesson.

Selection of Students for Participation in CPBL

The selection of sample students from each grade level was felt more difficult because of the enthusiasm of participation in field visit. As my objective of the study was intended to examine the effectiveness of cultural metaphors at the basic level of mathematics education, my interest was to select the students from that level only. But, the teacher participation and head teacher as well suggested me not to take the students from smaller grades due to various constraints and difficulties. It was also felt that the inclusion of secondary grades students in the group would provide better opportunity for collaborative learning. So we decided to select thirty-two students from the grade six to ten randomly. Out of them, selected eight students from each grade of 6 to 8 and 4 students from grade 9 and 10 each. Divided the students in four different groups. Each group included 8 members from five different grades, 2 students from grade 6 to 8 each and one from 9 and 10. The group thus formed would be heterogeneous and provide an opportunity for collaborative learning. The list of students participants were presented in Appendix VIII.

Student Awareness Program for Field Visit

I had designed Student Awareness Program (SAP) connecting to the mathematical ideas in the monuments (Appendix IX). The main objective of this project was to provide an opportunity to connect school mathematical concepts and knowledge with the cultural activities and assets. Thirty-two students from grades six to ten were selected randomly. Before visited to the monuments, the researcher was presented the mathematical ideas embedded in out-of-school context and encourage them to observe and connect the real-life situations and the school mathematics. The

SAP was conducted before going to field visit and lasted about one hour. The thirty-two students and four mathematics teachers were participated in this program.

After the orientation in SAP, the thirty-two students were divided into four groups. Each group had eight members. The group was formed in such a way that they represent the students of all grade level with the aim to provide collaborative learning environment. The list of the students in each group was presented in the Appendix VIII.



Image 6.28. Researcher Delivering in SAP

Four teachers were assigned to facilitate to each group. We visited this temple before select the study area and search for possibilities of incorporation in mathematics classroom. They were encouraged to explore more about the geometrical, algebraic and arithmetical ideas with the measurement of different parts and observation of the surrounding of the temple. They were provided measuring tape, paper, pencil and other instruments by which they able to explore and verify the different geometrical properties. Students were also requested to write their feeling and perceptions of field visit with respect to the mathematical ideas learnt and the ways of gaining knowledge through this approach of teaching and learning school mathematics.



Image 6.29. Planning to go in Field Visit



Image 6.30. On the Way to Destination

Lesson Design for the Field Visit

For further data collection, I decided to take the students to the field visit. And, in course of finding a proper field place, I along with a few of my research participants. And the place we decided as the place for the field visit was the Old Guheshwory Temple. We, at the same time, had almost visualized the probabilities of the arts and the surrounding objects of the temple that could be examined as the mathematics. I was keen to take the students to the field regarding the outcome they would provide about linking the school mathematics with the surrounding mathematics that they could see in the temple premises. Finally, the day came to take the students to field along with teacher fellows.

After reaching to the field place, the students were first provided the common task. All the students were requested to observe the temple and its surroundings and note down the geometrical objects and the probable mathematical concept that they could identify. I along with the teachers had to identify the tasks for the four groups of students after viewing the scenario in the temple premises. Finally, four major tasks were



Image 6.31. Teacher Assign the Task to his Group

identified. The identified tasks were to identify the centre of the circle, to identify the type of quadrilateral by measuring its sides, to explore the concept of symmetry and identify the axis of reflection, and to identify the reason of getting error while measuring the sides of the triangle. Then the designed tasks were given one to each group.

Task I: To Explore the geometrical objects and Probable Mathematical Concepts

The first task given to each group was a common task which was to collect the different geometrical objects found in the temple and its surroundings. There are many geometrical objects found in the surrounding of the monuments. After we reached to the temple premises, we let the students to observe the temple, monuments and its surroundings. I asked the participant students to note down the geometrical objects and the probable mathematical concept that they could identify in and around the temple. Each student was assorted within four groups. All the students were actively participating as a group and performing the task very efficiently. Some students in a group were measuring the stuffs where some were sorting the place for the successful accomplishment of the project and some holding a pen and a copy with such a keen face of them. I was very happy to watch such willingly performed efforts by the students in learning the similar concepts that they used to learn sitting inside the class. They were getting true essence from the visualization of those mathematical concepts. They observed the temple and noted down the shapes and the mathematical contents around the temple in their copies.

After some time of observation, each group was requested to present their observations and findings. Each group involved in the common task serially presented their findings. The common observation made by the students were the 2D shapes of the objects like triangle, square, quadrilateral, circle, oval, octagon, trapezium, parallelogram and rectangle. They also observed 3D shapes like hemisphere, cone, frustum, sphere, cube, cuboid, cylinder, prism and pyramid. One of the major part of the observation was the mathematical concepts that the students could find around them. They sorted out the concepts of concentric circles, transformation, reflection, rotation, symmetry, pattern, and tessellations.

The common task of observing the temple and its surroundings and noting down the geometrical objects and the probable mathematical concept that they could identify was provided to each of the students participating in the field visit. There are two mandalas found in the temple, one mandala which is erected by carving stone is found in front of main entrance gate and another metallic mandala is observed inside the gate of the temple. The mandala consisted of two concentric circles. Image also contains the twelve petals of the lotus flower each petal representing the place for the 12 deities. This can arise another question of why are they carved in a same style and what might be the significance of this style of carving the similar item.



Images 6.32. Sources of Geometrical Objects in the Surrounding of Temple

The Chaitya are also found in the surrounding in the temple. The different images of gods and goddess are found in the art in stone carving. The Chaitya is observed as the combination of different geometrical objects. The regular octagons are found in the base of Chaitya and octagonal prism on it. The conjugate hemisphere is put in the top of the Chaitya.

The temple Old Guheshwory is a prehistoric temple established centuries ago. There are so many monuments and artefacts present in and around the temple. Each monuments and artefacts were so precisely cut for resulting the design of the artefacts. There was an artefact with the cuboid overlapped by other cuboid but of smaller size continuing till the top point was formed. It gave a shape like that of pyramid. There,

we also could see that each sides of the consecutive cuboid were in a same proportion in length and area covered. Also, there was artefact with octagon shape fulfilling

every criteria to be an octagon, all the sides were equal.

Concepts of concentric circles used to form such impressive artefacts



Image 6.33: *Mandalas and Geometrical Shapes*

also give us a hint how the mathematical ideas were created at that time being far of today's formal studies based on bookish knowledge. So, we can see that the abstract mathematical knowledge was used in such creations. For observation, there are many geometrical objects found in the surrounding of the monuments.

Task II: Identify the Center of the Circular Surface of the Chaitya.

I had designed a lesson for the exploration of mathematical ideas outside the classroom with the collaboration of members of CAR projects. I had made an assumption of existence of the mathematical ideas beyond the four walls of classroom and the students' mathematical competencies would be enhanced with the help of out-of-school mathematics. The observation of the cultural artefacts and the mathematical ideas embedded in those artefacts motivate and encourage students to explore more mathematics. It also develops positive attitude of the students towards school mathematics. Students were encouraged to explore the different mathematical ideas and provide an opportunity to learn in collaborative framework. This image reflects the observation of different geometrical objects surrounding of the temple for the selection of possible tasks for the students participating on the field visit.

We, for the first group of students, decided to assign a task to identify the centre of the circular surface physically. We took the students near a Chaitya. Chaitya possessed different geometrical shapes along with a circular face. The excitement in students was getting higher. They just knew they were to a task assigned. And probably, they were trying to figure out about the mathematics



Image 6.34. Searching for Task: Researcher with Cooperating Teachers

present around the Chaitya itself as they were making an angle to angle view of it. Finally, my teacher-fellow provided the task “So, are you seeing this large circular surface? You have to find out the centre of this surface”. And the student replied, “Why not sir, we shall give a try”. Required physical materials like measuring tapes, ropes, markers etc. was provided to the group. They were left for the accomplishment of the task for about 15 minutes.

Finally, they came out to the teacher:

Student: Sir, we found out the centre sir.

Teacher: Wow, what did you do children?

Student: Sir, first of all, we used the measuring tape to find the circumference of the circle

Teacher: Hmmm...

Student: Sir, then we calculated the radius of the circular surface by using the working formula

$$\text{Circumference of a circle} = 2\pi \times \text{radius of the circle}(r)$$

And by calculating the radius (r), we calculated the diameter of the circle as we have: $Diameter(d) = 2 \times r$

By using the diameter, we located the longest chord in the circular surface and finally, we marked its mid-point. Sir, this perhaps is the centre of the circular surface.

[We did it, friends!!! 😊 😊 😊]

Teacher: Students, ummmmm... This is possibly a good idea. What are there in the circular surface?

Students: Sir, we see that there are concentric circles in there.

Teacher: So, once you think. How old is this artefact after it has been made?

Students: Sir, probably centuries of years!

Teacher: And how do you think were those concentric circles made?

Students: Sir, probably by using a huge compass from the centre of the first circle.

(Everyone laughs!)

Teacher: Did our ancestors used those same formulas to consider the centre of the face?

Student: Ummm... We have no idea about it, sir. Probably they used some other knowledge behind those formulas, I mean the knowledge like used by the wood carvers.

Teacher: Possibly it's true. The similar type of knowledge should have been used. Though we may not get that exact concept of what they have used, we perhaps may get some similar ideas? Can you determine those ideas?

Student: Hmmm... Are there some more? Let us think.

(They start their group discussion and perform various activities with that chaitya!)

Student: Sir we got the idea that possibly was used in the past.

Teacher: Wow, can you share that with me?

Students: Sir first we took a point in the circumference of the circle and put one end of the measuring tape in that point. Then, we loosely hold other side of the measuring tape and moved the tape along the circumference until we found the tightest point of the measuring tape. This probably was the longest chord, the diameter. Hence we marked the diameter and calculated the position of its midpoint. This was the centre of that circular side.

Teacher: Wonderful! This was a great procedure, my children. I am really happy with you!

Student: Thank you, sir.

As a researcher, I heard their presentation to their teacher facilitator. I observed their facial expressions which seemed to have shining face, enjoying at each step of their work. I also observed their behaviours. They were showing an active participation and involvement in the task. All the students were involved as a group. Then I had a brief conversation with the students:

Researcher: Students, I appreciate your achievement!

Students: Thanks, sir!

Researcher: What did you learn today?

Students: Sir, we really enjoyed today's practical work. We got that opportunity to work out the way that is not even in our textbook.

Researcher: For example.

Student: Sir we figured out the way of determining the centre of a given circle in the way that is not mentioned in the textbook. You probably have noticed that in our presentation.

Researcher: Yes! Then what do you think about this project? Would it contribute in the teaching of those textbook materials?

Student: No doubt about that. In fact, we internalized the mathematical concepts. We touched those mathematical ideas by our senses. We are never forgetting this learned items all our life.

Researcher: Thank you for your wonderful compliments.

Students: (Job, well done ☺ ☺ ☺) Especial thanks goes to you sir as you made all this provision.

I was really happy to see these students doing their task so eagerly. I could feel their excitement and enhanced facial expression. This task made the students to think for some more probabilities for a certain item. We had formed a heterogeneous group of members involving from grade six to ten. The involvement of different grade level students provides an opportunity to learn mathematics concepts, even of the higher level, in collaborative learning framework. The image 6.34 depicted the brief report presented by the group I. Their report shows that they had got empirical knowledge to solve the mathematical problem of finding the centre of the circle in the case of non-given other dimensions of a circle.

In my view, the project-based learning is more effective than the method which have been used in the classroom. One of the student participants says: “When I was in class, I could have handled the problem given by my teachers, the challenges of the problems, even if would be unable to attain correct solution. But when I was in the field, the task given by my teachers made me really shocked.” This happened in

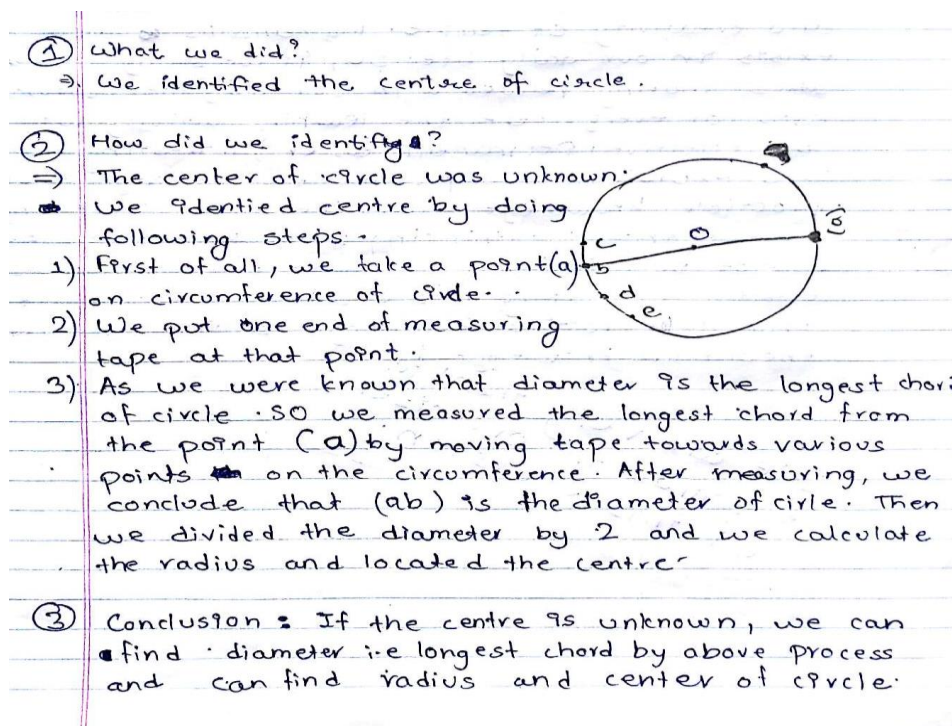


Image 6.35. Students Report of the Task Given to Group I

part because “doing mathematics in school” took a different meaning and the thing she performed in the field was off-task work. “But now I have developed confidence in facing the new tasks and handling the new situations that may arise.” This off-task work they performed shows the step-up of the level in the understanding of mathematical concepts.

Task III: To identify the type of quadrilateral by measuring its sides.

In course of assigning the task to the groups, the II group of students were led to a terrace. The excitement in the students was going on increasing; what are we going to be assigned? they were very interested about the platform they were taken to. And probably, they were trying to figure out about the mathematics present



Image 6.36. Students' Engagement in Task

around them. Finally, the teacher provided the task “Cannot we find what shape this terrace is off?” And the student replied, “Why not sir, we shall probably get the correct one.” Each and every required physical device like measuring tapes, makers etc. were provided to them. They were left for the accomplishment of the task for about 15 minutes.

Finally, they came out to the teacher:

Student: Sir, we got it.

Teacher: What did we get the outcome children?

Student: Sir, we got that the shape of the terrace was a quadrilateral.

Teacher: Are you, sure children. How did you get that?

Student: Sir, first we measured the length of each and every side of the terrace.

We got that all sides of terrace were not equal. And, that probably could be a quadrilateral.

[We did it, friends!!! 😊 😊 😊]

Teacher: Students, ummmmm... Can that only be defined as a quadrilateral?

Aren't there some other possibilities?

Student: Hmmm... Are there some more? Let us think.

(One of the group member raises a point.)

Student: Yes! though the quadrilateral doesn't have equal sides but if any two opposite sides comes to be parallel then the shape is said to be a trapezium.

Teacher: And have my students tested for that purpose?

Students: We really did not think about that sir. Let us to the terrace and we will come faster with the result sir.

Teacher: Take your time! Find those things out.

Students: Thank you, sir.

[Let's go, friend]

(After some time)

Students: Oh sir! The shape of the terrace came to be a trapezium.

Teacher: See, so think and give it a final touch.

As a researcher, I heard their conversation and then I had a brief conversation with the students:

Researcher: Students, then you found out the terrace to have a shape of trapezium. Right?

Students: Yes, sir!

Researcher: What way did you do to get that output?

Students: Sir, as you have heard, we initially found out the terrace to be a quadrilateral as we sorted out that the sides of the terrace were not equal to each other.

Researcher: Yes!

Student: Then after a discussion with our supervisor, we were made to think for further shapes that a quadrilateral shape may claim.

Researcher: Oh! Then what did you do?

Student: Then my friend remembered that if any two sides of the plane was parallel then the terrace would claim the shape of a trapezium. So, we figured out our observation to see if any two sides were parallel or not. We noticed that, among four sides of the plane, two opposite sides measured 6.22m and 6.30m, but the next opposite sides measured a common length of 6.44m. And two diagonals in the plane was also of equal length. So, the previous sides could be considered as parallel sides. And as the two opposite sides of the

quadrilateral were verified to be a parallel side, we got that the terrace had the shape not only of a quadrilateral but also the shape of trapezium.

Researcher: Your findings are quite appreciable! Good job guys!

Students: We also thank you for providing such an opportunity, sir.

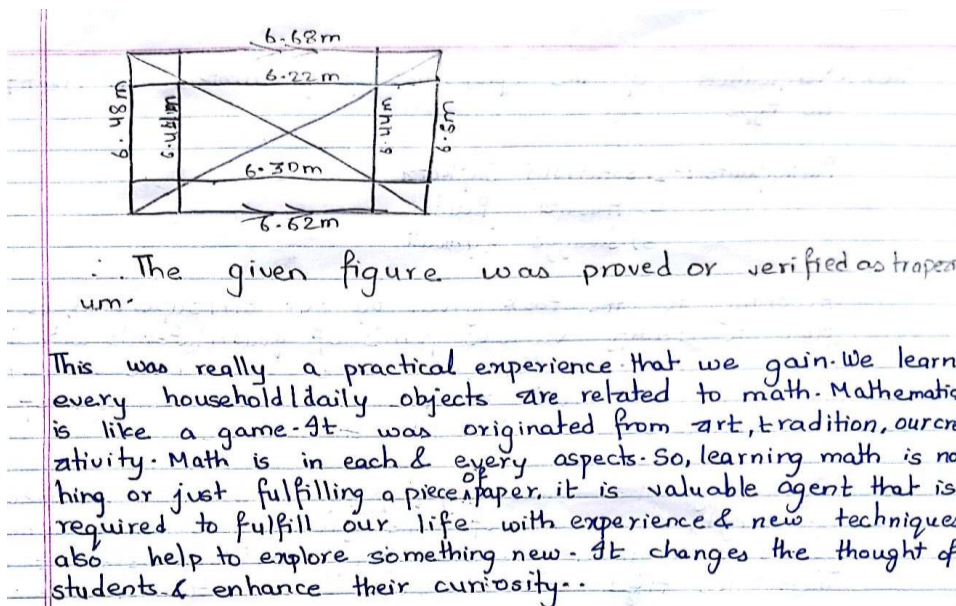


Image 6.37. Students Report of the Task given to Group II

I was really happy to see these students doing their task so eagerly. I could feel their excitement and enhanced facial expression. They showed keen interest in the completion of their task. Their strong enthusiasm even provided me a power for my completion of this research. This task made the students to think for more probabilities to solve a single problem. The junior students were so critically analyzing each terms that they got to hear for the first time by their teacher and senior student participants. And, there was the transfer of all that higher level knowledge by the seniors to the junior participants which was a thing to be mesmerized.

Task IV: Examining the Symmetry and Identify the Axes of Reflection

For the further exploration of mathematical ideas embedded in the monuments, I, along with the collaboration of members of CAR projects, had developed some more ideas that could be explored with the view of the students. This

would motivate and encourage students to explore more mathematical concepts and develop positive attitude in the students towards school mathematics. We, for the III group of students, decided to assign a task to examine the symmetry and identify the axes of reflection in the field. The excitement in students was getting really higher. Finally, my teacher-fellow provided the task “Today we have observed various artefacts around and in this temple. So, we have decided to provide you with a task to determine the concept of symmetry and identify the axes of symmetry in different objects”. And the student replied “Okay Sir! We’ll do that.”. All the required materials like measuring tapes, ropes, markers etc. were provided to the group. They were left for the accomplishment of the task for some time.

(After sometimes)

Teacher: Are you getting some of the objectives of today’s task?

Students: Yes, sir we are getting some.

Teacher: What objects have you chosen for observing symmetry?

Students: Where could we find symmetry, sir?

Teacher: Do you really know the concept o symmetry or not?

Students: (The students get confused)

Teacher: First of all, you should become clear about the concept of the thing that you are searching for. See at your friend’s face! (smilingly), how beautiful she is! Why is she so pretty?

Student: Hmmm... (With a thinking face) Is that due to the symmetry in her face sir?

Teacher: Exactly, you came to the point. Then, where can you find symmetry around us?

Student: Sir, anywhere but where exactly?

Teacher: Everywhere! Look at the leaf of this *Peepal tree*, in the carvings in stones, in the carving of doors, windows, and even in the *mandala*. See, everywhere!

Student: Oh, Yes sir! We will do now!

Teacher: Do it and you can! Go on...

Then the students observed different artefacts inside the temple premises.

They compared the halves of those artefacts and determined if they were symmetrical or not. They even distinguished the symmetric artefacts as having bilateral symmetry and rotational symmetry. They

measured the parts of the artefacts to check if it was symmetrical. Some of them were taking the measured data, while some were comparing



Image 6.38. Students are exploring the Symmetry in the Artefacts

those data to determine if the artefact of that corresponding data was symmetrical or not. They finally reported that the images that were carved in the doors, ceilings, windows and other places were completely symmetrical. In fact, one part of the artefacts which possess bilateral symmetry exactly seemed to have the axis of reflection which acted as a mirror giving the same image as compared to that part which formed the complete view of those artefacts. The carving of different objects, in woods and stones were so systematically carved. If observed with reference to any line, we get each part of the object has exactly same reflected image. This is to say that, the artists at that time possessed highly sophisticated mathematical ideas in their job.

The student participants of this project reflected that the object found in the premises of the temple has both bilateral symmetry and rotational symmetry. They found bilateral symmetry in the images carved on the doors, windows, and ceilings of gods and goddess, heavenly creatures, arts and designs. Similarly, they observed rotational symmetry in the artefacts like circular mandala which further possessed twelve petal designs constantly at a same interval and with a same shape signifying whole artefact as a flower of lotus and also in the eyes carving in the doorways of the temple. They got the concepts of symmetry in innumerable artefacts and objects associated directly or indirectly with the religious aspects of the temple. They figured out the concepts of tessellations on the floor floored with objects of same design perhaps symmetrical objects and on the walls of the temple. The idea of symmetry in the premise was probably infinitely present.

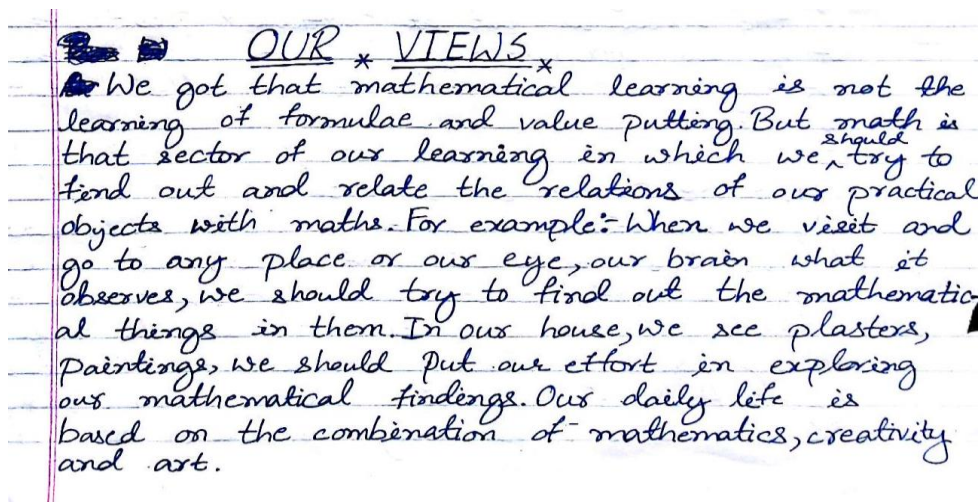


Image 6.39. Students Views on Learning Mathematics in Out-of-School

Every artefact that were considered to be a part of the symmetrical design were really attractive. The way that they were formed and the portraying of those artefacts really needed great thinking and perhaps a lot of physical and mental workouts. In fact, the design should probably be the result of deep mental concentration in getting positive feedback from the viewers. This might have resulted the implicit knowledge of symmetricity in those beautiful minds. By which we may

really conclude that all those beautiful creations were based on the combination of mathematics, arts and culture as said by my research participants.

Teacher's Reflections on CPBL

Teacher is a one of the main agent in the transformative process in an education system. Teacher has significant role for the adoption of appropriate teaching pedagogy in classroom. In my study, teacher has key role to connect student's household activities, and informal mathematical ideas to teach formal school mathematics. As my study was intended to connect the out-of-school mathematical ideas and school mathematics, I, with my cooperating teachers, decided to explore the mathematical ideas embedded in the monuments. In doing so, we had visited the Guheshwory temple located in Balaju, Goldhungga. After the field visit, I asked one of my cooperating teachers about their view regarding the teaching and learning of school mathematics through the study of cultural artefacts, and monuments.

Researcher: How did you feel about today's field visit?

Teacher: Sir, I am really delighted to have a chance taking part in this type of educational field visit. I really appreciate your efforts in this regard.

Researcher: Oh! It's my pleasure. I guess you have observed the temple very closely. What were the different geometrical shapes that you observed in the temple?

Teacher: Of course, there are different geometrical shapes like triangles, rectangles, parallel lines, semi-circles with a rectangular base that can be helpful to contextualized geometrical shapes. Viewing the shape of the floor made up of the rectangular shape of stones we can see the mathematical structure which has a peculiar shape. Each brick consists of rectangles kept in horizontal and vertical position. Such pattern resemblance the different

geometric knowledge. The window is in the shape of a rectangle. Each window consists of various squares which resemble the puzzle box. In fact, such pattern of the window does represent the pattern of numbers, but this is a matter of research. The woods and stone carving exhibited the sophisticated mathematical ideas of symmetry and transformation geometry.

Researcher: Have you ever been taught school geometry via field visit of this kind?

Teacher: Oh... no sir, you know how we the teacher can manage this project due to overloaded curricula.

Researcher: Would you enjoy to teach and learn school geometrical concepts through the field visit approach?

Teacher: Really sir. I think it is one of the best approaches to conceptualize abstract knowledge of mathematics. We can teach ever difficult ideas of mathematics in a simple way and student-centred approach. I think our students were enjoying in learning various geometrical concepts through the observation of the temple.

Researcher: If it is a good method of teaching geometrical ideas, why should not we adopt this approach in teaching?

Teacher: You know; we have to teach at least six periods a day. There are at least 40 students in each class. The classroom teaching, homework checking, unit test, terminal examinations, test papers checking and other responsibilities of the school are not favorable to think and adopt this method. It takes so many time to make a plan and conduct a field study. More teachers and mentors are required to manage such field study even for taking students from a single class.

Researcher: Do you have any good memories in learning geometry in your school day?

Teacher: Um... no as such. But I did good in geometry in my examination.

Researcher: How did you do good in geometry though you did not have good memory during learning geometry?

Teacher: Hahahaha... (long laughing). It's our method.

Researcher: Which method?

Teacher: Parroting... Drill and Practice....

The mathematics we are teaching at school can be contextualized to our everyday activities and surroundings. Our school mathematical pedagogy must be changed so as to enrich our students with the overall knowledge of mathematics. Our pedagogy should be student-friendly, in a sense we must be able to build the rapport of different cultures of different societies in our mathematics curriculum. This reveals that our mathematics curriculum should be dynamic one which incorporates the overall knowledge of students and values the knowledge of different cultures which makes different students access to the knowledge of different cultures.

The interview with the cooperating teachers exhibited that our school teaching is conventional. Teachers' approaches are similar to the method that they learnt in the school. However, teachers were convinced and assured that the classroom teaching should be student-centred. The mathematical ideas can be taught in better way out of the four walls of the classroom. It was a serious problem that the teacher did not speak out the mathematics out of the prescribed curricula. Teachers teaching activities were mostly textbook driven. The mathematics teaching and learning can be effective and meaningful if the school mathematics is connected with the out-of-school environment.

The teacher is feeling good in the CPBL approaches to teaching and learning of mathematics. The teacher participants in my study agreed that the cultural metaphors enhanced the improvement of teaching and learning of abstract mathematical concepts. From the interaction with the teachers, I found that the teachers have a positive attitude towards the CPBL approach to teaching and learning of mathematics. Teachers themselves enjoy the approaches to teaching and learning through CPBL, but they do not have sufficient time to manage and develop the teaching plan for culture friendly. The CPBL approach to teaching and learning requires more time which was the obstacles to use in mathematics teaching.

Summary of the Chapter

The metaphor is communicative tool for teaching and learning of abstract concepts of mathematics. It maps familiar and concrete domain of knowledge embedded in the students' home and cultural setting to the target domain as the abstract concepts of mathematics. The cultural artefacts, household-based knowledge, everyday practices, monuments, game play that involves ethnomathematical ideas were considered source domain of cultural metaphor for the teaching and learning of school mathematics.

My study was intended to examine the contribution of cultural metaphor for the understanding of abstract mathematical ideas at the basic level of education. The cultural metaphor were captured from the different sources and teachers were equipped with views of cultural metaphor through TFP. The mathematics lessons on numerals writing, prime and composite numbers, rounding off numbers, place value, integers, algebraic expression, and geometric concepts at the basic level were developed with the view of cultural metaphor in constructivist framework. The CPBL was developed to the study of geometric concepts in monument and cultural artefacts.

CHAPTER VII

FINDINGS AND DISCUSSIONS

During the analysis stage of my research in the previous two chapters, I have answered three questions about my study; what are the cultural metaphors practiced in the out-of-school context regarding mathematical ideas and knowledge? Further, It was also discussed that how the cultural metaphors contribute teachers and students in course of teaching and learning of school mathematics. Finally, I have discussed about the possibilities of use of cultural metaphor in the process of teaching and learning of school mathematics at basic level.

While doing so, I have presented thematic findings from the data and interpreted them. It is now essential to put these findings in light of the theoretical framework of my study to see triangulation with theories, literature and to go beyond the data to generate new knowledge from this study. For this purpose, I did three works simultaneously: I revisited the literature that I studied, then I consulted the information drawn from the previous two chapters (Chapter V and VI) with my field experience, and finally I evaluated them in light of my methodologies and theories. In this process, I have listed some major findings in substance first, and then linked them to the theories and discussed them.

Findings Related to Cultural Metaphor Practices in Everyday Mathematical Activities in Out-of-School Context

Mathematics is an abstract domain of knowledge. Familiar and concrete source related to the mathematical ideas and knowledge can effectively comprehend the abstract concept of mathematics. The mathematical ideas practiced in students' environmental context played a significant role of cultural metaphor for the teaching and learning of school mathematics. The informal mathematical ideas and knowledge

that are practiced and used in everyday activities are the source domains for the understanding of abstract mathematical ideas. Thus, the ethnomathematical system of the cultural group can contribute for a better understanding of school mathematics.

Mathematics Embedded in Cultural Practices

Different groups of people practice different mathematical ideas in the process of performing their everyday activities. They perform a wide range of activities like measurement and estimation of length, volume and time, geometrical ideas, and mathematical reasoning. Saxe and Moylan (1982), as cited in Nunes, Schliemann, and Carraher (1993), investigated transitive inferences based on measurements taken with non-Western measuring systems.

The activities involving mathematical ideas of the people in the rural area of Nepal use local measurement unit for measuring distance, area and volume. The non-standard and local units of *haat* and *bitta* were used to measure the length, *Kosha* for the measuring the distance, *Pathee*, *Maana*, *Chauthai*, *Chakhanti*, *Chimti* to measure the volume; and working time of ploughing, in terms of seeds require and production of crops are the bases to calculate and estimate the area of land. The *Sher*, *Bishauli*, and *Dharni* are the measuring unit of weight in a rural part of Nepal. The unit of measurement of *Laal*, *Masa*, *Tola* are still used to measure the weight like gold and silver in present days. The *Tulo* (Balance) is a tool to measure the weight of the stuffs and it was found to be a powerful metaphor for the understanding of the concept of moment of forces, integers and so on. The various mathematical concepts and ideas can be connected with the *Tulo*.

Mathematics Embedded in Cultural Activities

One of my objectives was to explore the cultural metaphor embedded in the out-of-school environment regarding mathematical ideas. A metaphor is the powerful

tool that assists to communicate the abstract mathematical ideas through familiar and meaningful objects embedded in everyday activities. The students' cultural environment, everyday household activities, and games that they play are the funds of knowledge that can help them to learn school mathematics. In my study, I gathered information from the students' out-of-school environment embedded mathematical ideas, concepts, and knowledge. I also gathered information of the families and their extensive knowledge on the workplace activities, household management, carpentry, farming, small-scale business, and everyday activities regarding mathematical ideas and knowledge. It was observed that children actively engage in their household activities. They are resourceful and creative problem-solvers in the out-of-school environment as they performed their household activities beautifully. In this regard, the children perform their task involving mathematical ideas in their everyday work in the out-of-school environment though they are characterized as poor and low-achievers in the school community.

From the interaction with my research participants, there are a lot of informal mathematical ideas observed in our surroundings. Those mathematical ideas are the effective metaphors to communicate the abstract mathematical ideas. I found that the teachers were using ethnomathematical ideas of different groups of people as a metaphor in the process of teaching and learning of classroom teaching. In the following sections, I have discussed the informal mathematics practiced by different groups of people in their everyday activities.

Mathematical ideas in weaving. Weaving is one of the common cultural activities practiced in different communities in Nepal. The weaving of textiles, sitting mat, sleeping mat, basket, and other stuff reflect the cultural values and identities. Various mathematical ideas and thinking are inherited in the process of construction

of mat. Indigenous people have implicit mathematical knowledge with the ability to perform a task but without the ability to explain their deeds (Pradhan, 2017). This implicit mathematical knowledge is embedded tacitly in their activities. Mosimege and Lebeta (2000) also reported that the indigenous people use different mathematical concepts like estimation, tessellations, and symmetry in the construction of the traditional artifacts and in various other cultural activities. I observed that the mat weavers exhibited sophisticated geometrical ideas in the process of construction of a net of the mat. The knowledge they practiced in the process of construction of a net of the sleeping mat used the theorems of Euclidean geometry implicitly.

Mat weavers practice mathematical ideas and knowledge implicitly in the process of construction of mat. They use sophisticated geometrical ideas while constructing rectangular frames. Two poles are fixed in accordance to the length of mat required and the third pole is adjusted as per the breadth of the mat. To adjust the

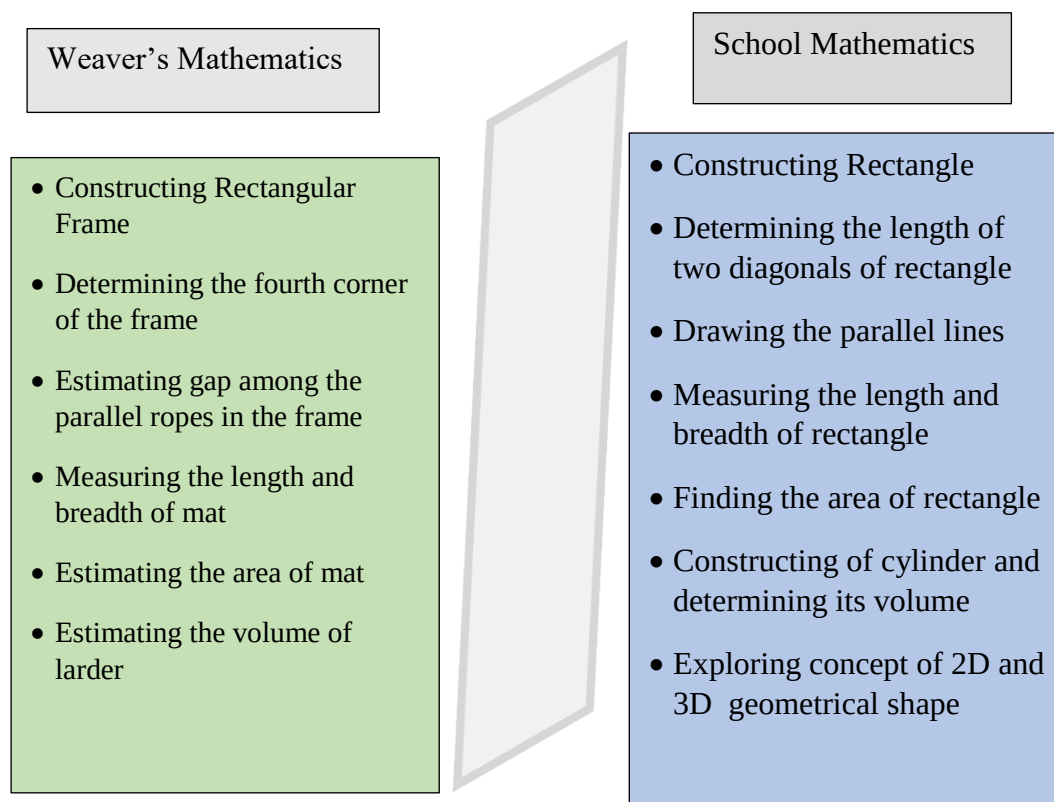


Figure 7.1. Weavers' Mathematics and School Mathematics: The two cultures

fourth pole, they use X-test that is the process of constructing the frame of the mat involving tacit geometrical ideas that two diagonals of a rectangle is equal in length. Figure 7.1 displays the mathematical worlds of two cultures of professional mathematics and academic mathematics.

In a cultural village of Nepal, a rectangular form of sitting or sleeping mat may be rolled up when it is not in use. Some people use sleeping mats as larder for the purpose of storing paddy. It then takes approximately the form of a cylinder. From the observation of the situations, this cultural experience provides an opportunity for students to obtain the idea of calculation of the volume of a cylinder and the study of different geometrical properties inherited in it. It is argued that it is possible to use indigenous mat weaving as a metaphor for the teaching and learning of mathematics at different levels from primary to a higher level. In the same way, the design and pattern in the homemade *hakku patasi* involve some mathematical aspects of symmetry.

The use of ethnomathematical practices of mat weavers as a pedagogical tool shows the importance of the contextualization of problems (Rosa & Orey, 2012). The mathematical thinking and methods are quite different in various cultures (Gerdes, 2001). The mathematical ideas hidden in the process of constructing the frame to weave a mat in the rural community in Nepal is mostly equivalent to the methods that the Mozambican peasantry used for constructing rectangular bases of houses (Gerdes, 1999). Further, Gerdes (1997) explored that the construction techniques of the house builders facilitate to formulate the rectangle axiom. From the observation of different process of mat weaving, it was found that it includes a lot of mathematical ideas, implicitly and tacitly embedded in it.

Mathematics in carving. The connections between art and mathematics are well established and firmly rooted in the history of art. Art historians often use the characteristics of symmetry and the rules of the golden mean to assess the formal qualities of a work of art. In further establishing the interrelationship between art and mathematics, Hardy (1978) compared a mathematician to a painter or a poet and a maker of patterns made with ideas. The use of mathematics in art and design makes things pretty and aesthetic. Every beautiful thing in the world possess mathematical properties, and the images possess the golden ratio. I patiently observed the activities of stone carver, where many mathematical ideas of ratio and proportion, symmetry, tessellations, reflection were found being used. However, the designers and workers did not have formal mathematical knowledge. In this context, one of the stone carvers Biru says that the image that he carved possess mathematical properties of symmetry tacitly in order to make the stuff more beautiful. The carver had implicit and tacit knowledge of symmetry that help to make the images more beautiful and eye-pleasing. They construct different geometrical patterns and design in their works. From the observation of workplace of carver, it was found that the coordinate axes, symmetry, reflection are the mathematical ideas embedded in their drawing a pattern.

In the villages of Nepal, symmetrical images are painted on either wall of the door in a house. Symmetry in its various forms were found in the designs on the arts and artifacts constructed in different community of Nepal. Rosa and Orey (2006) viewed that one of the major objectives of the ethnomathematical research is to look for possibilities of incorporating the mathematical ideas embedded into the cultural context of students and teachers for improving the teaching and learning of school mathematics. There is prevalence of symmetry in designing textiles, sand painting, wall painting, pottery, arts, and artifacts of the people in different cultures.

Synchronizing Small m and Capital M Mathematics

Different groups of people applied a wide range of mathematical ideas and thinking in their everyday activities. In case of mat weavers, highly sophisticated geometrical ideas were used. In the similar vein, the masons used their own way to construct the rectangular shape of the foundation of the house. They implicitly used 3-4-5 method in the construction of rectangular form. They practiced the mathematical ideas in order to perform their everyday job. From the field data, it was observed that there were some sort of conflict between the mathematics practiced in out-of-school cultural activities, and teaching and learning in formal schooling. In this regard, one of my teacher participants Arjun viewed that learning of mathematics is intended just to secure good marks in the examination. Even the good marks securer might not have conceptual understanding in mathematics and be unable to link school mathematical knowledge in real life situations.

From the views of Arjun, I realized that our school pedagogy is alien to the everyday activities of students' home culture. Mathematics is considered as one of the compulsory subjects studied just for passing the exam. I believe that lack of connection of mathematics to the everyday life of people is one of the main reason to consider mathematics as a difficult and dry subject. After I visited my research participant, I got an opportunity to make a talk with *Maila Dai* (brother after the eldest brother), a mason. He was on his duty. I was observing his way of performing his task patiently. The bottom and bell are the tools they use to form each new row of the wall. I had asked *Maila Dai* (53): why these tools were used in the construction of each new row of the wall. They use these tools to keep the wall in straight vertical position, which keeps the building strong.

From the interaction with Arjun and *Maila Dai*, I came to know that, different people have their own different ways of understanding the mathematical ideas and using them for their purpose tacitly. The people who have formal knowledge might not have practical knowledge and those who can apply all those theories in practical life might not know its western mathematical knowledge. Who is the real owner of the mathematical ideas? It is believed that mathematics is a cultural product. The people before Pythagoras had also constructed house, pyramid and other such artifacts in their own society. The house builders though lacked western mathematical knowledge; they used all those theoretical concepts implicitly. In fact, they possessed emic knowledge in performing their everyday job, which had contributed to live and sustain their lives.

Similar types of mathematical ideas were practiced in two different worlds. But, the ways of teaching and learning, and purpose were different. The small ‘m’ mathematics was practiced in the world of mason and the school practiced the capital ‘M’ mathematics (Bishop, 1996). From the field data, mason practice lot of mathematical ideas in their everyday life. The estimation of cement and sand to make mixtures for wall construction exhibited high level of mathematical thinking and knowledge. The estimation of cements and sands, its mixtures reflected their ideas in ratio, knowledge of fraction and other mathematical ideas. The dissemination of mathematical knowledge in out-of-school context is extremely context bound.

On the other hand, Arjun taught textual mathematics without connecting to the real world situations of the children. With this regards, Bishop (1994, p. 16) pointed out that “many young people in the world are experiencing a dissonance between the cultural tradition represented out-of-school environment and that represented inside the school”. From the interview of my research participants, Arjun often experience

conflicts in terms of contradictory understandings generated through his participation in two different mathematics cultures: mason's mathematics based on their everyday practices and the school mathematics. Bishop (1994) believes that all the formal mathematics education is a process of cultural interaction and every child experiences some degree of cultural conflict in that process. He argued that "schools are different social institutions from others such as homes" (Bishop, 1994, p. 16). The conflicting situations concerning learning and teaching of mathematics at out-of-school environment and the formal schooling are especially language, geometrical concepts, calculation procedures, symbolic representations, logical reasoning, attitudes, goals and cognitive preferences, values and beliefs. In my study, I found that the incorporation of ethnomathematical perspective enhance the understanding of abstract concepts of school mathematics. Synchronizing the small m mathematics and the capital M mathematics would facilitate the teaching and learning of school mathematics and give due regard to students cultural backgrounds and experiences.

Mathematical Ideas in Cultural Arts and Artefacts

Mathematical ideas and thinking are embedded in every environmental activity of a group of people. However, the mathematical ideas they possess in order to perform their everyday job remains largely hidden. Ethnomathematics can reshape our greater cultural identity in a positive way by requiring the inclusion of a greater representation of the true mathematical practices and problems for a student's own community (D' Ambrosio, 1998; Zaslavsky, 1996).

Mathematics in Rangauli. Nepal has rich cultural traditions and peculiar rituals. The mandala of *rangauli* is constructed on the occasion of *Dipawali*. The *Rangauli* is drawn by the rice flour with different colors seen on the floors of buildings and in front of *Puja Kotha* (prayer room) during the festival, *Dipawali*. The

drawing of *Rangauli* requires different geometrical patterns and spatial reasoning. It is observed that the number of concentric circles, number of symmetries, vertical and horizontal reflections, rotations and so on were found in the *Rangauli*. However, the mathematical concepts embedded in *Rangauli* mandala were hidden for the constructor. From the observation of field data, it was found that one of my research participants Sushila constructed the *Rangauli* mandala having implicit mathematical knowledge. The tacit mathematical knowledge is used in the artefacts. The common practice of creating mandala is based on the concept of both bilateral and rotational symmetry. Different colorings are also associated with drawing. The *astamatrika* or eight-pointed star has great significance and has religious meaning in Hindu culture. The creation and perception of the pattern have a logical as well as aesthetic component (Ascher, 1991).

The *Rangauli* drawing involves complex mathematical activity and strategic understanding of spatial reasoning. It is important to explore the hidden mathematical ideas in the process of constructing such cultural artifacts. Based on an interview with some people and observations of ways to construct as their cultural tradition, I have come to know that it possess sophisticated mathematical ideas of concentric circles, coordinate axes, and symmetry in the process of construction of the mandala.

Mathematics in Swastik. The word Swastik symbol derives from Sanskrit. Swastika takes the meaning of good fortune or well-being and *Subha Labha* in Nepali. Swastik symbol is a sacred symbol for most of the culture and is commonly found in temples, houses, departmental store, and shop. From the interaction of my research participants, I have come to know that people pasted striker of swastika symbol in the shop and on the money collection box with the belief to attract more money in the days. The symbol reflects a lot of mathematical ideas and concepts.

The most obvious connection found in the *Swastik* symbol is the reverse symmetry. If we were to diagonally cut the swastika and then reflect it over a mirror, the image we see would be reversed of the actual symbol. Another mathematical feature in this symbol is the 90-degree angles that separate the four arms. We see the pattern in the symbol as rotation of an arm around its axis at 90-degree intervals. Crowe (1988) has shown that patterns can be described by their symmetries and that repeated designs occur frequently on many types of objects. *Abas (2001)* pointed out that the beautiful Islamic geometric patterns were excellent materials to teach the idea of symmetry at all levels, from properties of simple geometric figures to tessellation, transformation, and to providing “a visual gateway for the teaching of abstract notions of Group Theory at the university level”. The *Swastik* symbol mediated to conceptualize the symmetry from basic level to the university level as symmetry is fundamental concepts of mathematics.

Mathematical ideas in Bell. It is one of the common cultural artefacts encountered in the temple. It enchant the prayers and symbolize the presence of the devotee in the temple. In addition, bell mediates the development of different geometric ideas. During the observation of the temple and its surrounding, students encountered the bell and asked what geometric shape did it had. One of the teacher participants described that the shape was possibly a cone. It is the canonical example of extraneous metaphor to describe the distinctive features of hyperbolic geometry and the study of surface. Different types of triangles can be drawn on the surface of the bell which resembles the distinctive features of hyperbolic geometry. In this context, Luintel and Taylor (2007) exhibited that the *shanai* and pseudo-sphere reveal that two parallel lines meet in a plane asymptotically which helps to contextualise the hyperbolic geometry. The bell can be described metaphorically as a bell-shaped curve

to normal distribution. The bell was found to be effective tool to conceptualize the notion of hyperbolic geometry at higher level than at basic level.

Mathematical Ideas in Cultural Games

Games are recreational part of every society and common environmental activity. They help to develop a social attitude and explore important social behaviors. Playing games is common to everyone. Games have some sort of rules, procedures, and structure, as in the mathematics. The incorporation of games and puzzles play an important role to achieve the aesthetic spirit of mathematics and to increase mathematical creativity and curiosity. Moreover, introducing the games, either in or out of school can help children understand that humans encode their mathematical ideas in diverse cultural products, including architecture, art, games, music, written texts, and so on. From the observation of game, it was found that the game help the student to develop quick decision making, logic, and reasoning to every step in games which are the foundations of mathematics learning.

I disclosed a lot of geometrical ideas in the cultural games, such as line, point, triangle, quadrilateral, parallel lines, perpendicular lines, and mathematical concepts like counting, measurement of time, the basic concept of arithmetical operations, relation, function, ration, proportion and so on. The cultural games help to contextualize the abstract mathematical concepts. Some of the games and embedded mathematical ideas are discussed in the subsequent headings.

Mathematics in *Bagha Chal* game. *Bagha Chal* is a board game and the board is square in shape. The analysis of the *Bagha Chal* game reveals a number of mathematical concepts. It was observed that varieties of geometric shapes in the construction of the board for *Bagha Chal* game such as squares, rectangles, triangles, parallel lines, transversal line and associated angles were identified. The board of the

Bagha Chal game as a metaphor provides an opportunity to verify the properties of various geometrical figures viz. the sum of the internal angles of the triangles of the square/ rectangle; vertically opposite angles, the sum of the adjacent angles, corresponding angles, alternate angles and so on.

The mathematical concepts of the centroid of a triangle, infinite series, reflection and rotation, and the properties of mid-point theorem, the Pythagorean Theorems were also observed and verified in the board of the games. From the observation of the board game and the analysis of board of *Bagha Chal*, I noticed that games contribute to develop mathematical thinking and explore mathematical ideas and concepts. Different pattern and structure found in the board are the metaphors that mediated for the development of mathematical reasoning.

Mathematics in Chungi game. It is popular game for both boys and girls of different age groups. During the game, it was found that children understand the rules of the game, make strategies, calculate their scores, compare the scores, develop strategy to win the game, and use various mathematical concepts without using the technical term. In this connection, Zaslavsky's (1989) opinions that students have lot of ideas of mathematics though they lack the terminology. This also shows that the mathematical ideas and concepts are rooted in the cultural games. In a single attempt, not everyone may complete the cycle of the game. The player may fail to hit the *Chungi* (bunch of rubber) by securing some points and continuing the game from the already secured points in their own turn. To aid their memory the children would learn the tallying system to record the level attained by each player. In the observation of the children game, I found that the player kept record against players in their mind rather than make a tally chart on paper. Through the steps, they secured and the game won, children practice the concepts of the frequency distribution in

mind itself.

From the interview of players and the observation of the *Chungi* game, it was found that the various algebraic concepts such as equations and inequations were practiced implicitly during the process. My cooperating teacher also found that, ethnomathematical ideas embedded in the games were beneficial to teach algebraic equation at the basic level of school mathematics. The mathematical games are valuable for stimulating and encouraging mathematical discussion between groups of children and between pupil and teacher (Ernest, 1986). Enjoyment and fun play an important role in learning mathematics and are some indicators of attitudes (Ernest, 1986). From the observation of the field I found that, playing of games acts as a source of strong motivation for student engagement in learning situations and encourage them to social, emotional and cognitive development (Kamii & DeVries, 1980). My study revealed that the mathematical ideas embedded in various cultural games mediated for the teaching and learning of school mathematics. The cultural games help to develop positive attitudes towards mathematics by creating fun and enjoyment for the students. Thus, different games provide rich opportunities for all children to build and extend mathematical ideas and strategic thinking.

Findings Related to Possibilities of Use of Cultural Metaphor in Teaching and Learning of Mathematics at Basic Level of Education

In view of conceptual metaphor theory of Lakoff and Nunez (2000), learners connect their experiences and knowledge to construct the new mathematical knowledge. They can connect the relatively abstract and difficult ideas of mathematics with familiar context based on students' everyday activities. As cultural metaphor provides opportunities to understand relatively abstract concept through

familiar and concrete context, it has to do more on mathematics due to its abstract nature.

Students construct mathematical knowledge with the help of their prior knowledge, experience, active participation in their environmental activities and everyday household knowledge. The household activities of the children involved many implicit mathematical ideas. However, our school mathematics curriculum and the teacher training is largely ignoring the learners' cultural activities and their ethnomathematical ideas (Pradhan, 2017, Ezeif, 2002). Rosa and Gavarrete (2017) also viewed that children's ethnomathematical knowledge and learning approaches are not taken into consideration in the formal school mathematics curricula. The mathematical ideas from the group of working-class people culture, the acknowledgment of their ways of knowledge generation and transmission, and students' experiences should be blended with formal mathematics in the classrooms (Pradhan, 2017).

The mathematical ideas embedded in the everyday household activities of children provide an opportunity to learn school mathematics. In my observation in the field, metaphors play significant role in the teaching and learning of mathematics. The connection of students' familiar context in the process of teaching mathematical content provides rich opportunity to the learners. From the FGD of teacher participant, I marked some dichotomy in teachers regarding the use of cultural metaphors in the process of teaching and learning of school mathematics.

Teachers' Dichotomy on the Use of Cultural Metaphors

While viewing through participants' perspective, there was controversy about the use of cultural metaphor in the teaching and learning of mathematics. Most of the participants viewed that the culture friendly pedagogy and the funds of knowledge for

teaching and learning of school mathematics are effective for teaching mathematics.

Regarding the role of cultural metaphor in the teaching of school mathematics, one of my research participant Bima viewed the cultural metaphor as a powerful tool for the development of mathematical concepts. Almost all of my research participants agreed that the cultural metaphor is potentially a very powerful tool for teaching learning of the mathematical concepts. The incorporation of cultural metaphor in the process of teaching and learning of mathematics enhances mathematical understanding of the students and eases to transfer abstract mathematical ideas to teachers.

My other research participant Deepa exhibited how cultural metaphor was powerful tool for the teaching and learning of abstract mathematical concepts. Deepa viewed that the use of cultural metaphor facilitate the teaching and learning of abstract concepts of mathematics. The ethnomathematical ideas and concepts embedded in the out-of-school context and its incorporation in formal classroom eases students to learn. The use of students' cultural knowledge in school mathematics also develop positive attitude towards the subject. From the personal interview of teacher participants and FGD, it was found that all most all of them agreed about the importance of cultural metaphors in teaching and learning of abstract concepts of school mathematics.

There are many things in the out-of-school environment that can be connected to the teaching and learning of school mathematics. The historically accumulated and culturally developed bodies, and knowledge and skills of the learners essential for household functioning and well-being (Moll, Amanti, Neff & Gonzalez, 2005) refer to the funds of knowledge. The funds of knowledge of children and their everyday activities, life experiences regarding mathematical ideas provide powerful tools in the process of teaching and learning of school mathematics. In this vein, Vygotsky (1987)

argued that the cultural practices and resources mediate children in the process of development of thinking and help to learn school mathematics. Thus, the cultural metaphor helps to connect cultural capital of learners and teaching and learning of school mathematics.

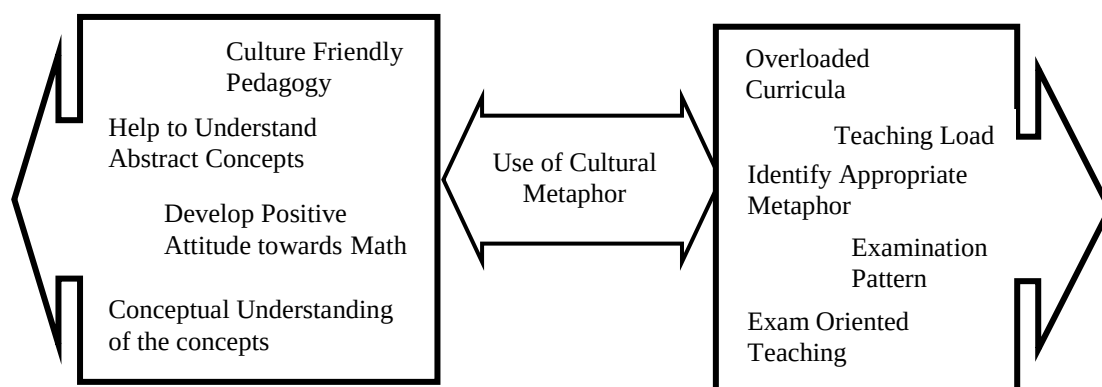


Figure 7.2 Dichotomy about the Possibilities of Using Cultural Metaphor

Nevertheless, they do not like to use cultural metaphor in their classroom teaching. The time factor hinders the use of cultural metaphor and culture friendly pedagogy in the mathematics classroom. The overloaded curricula is also the hindrance for the use of cultural metaphor in the classroom. Revealingly, most suggested that the main difficulty they faced was the selection of appropriate metaphor practiced in out-of-school environment for particular mathematical content.

Teachers are main agents to deliver formal mathematical content knowledge to their students. Their attitudes, beliefs and values determine how classroom teachers demonstrate alignment for the most effective and meaningful teaching practice (Bearden, 2012). From the field data, I found two types of teacher participants. My research participant Deepa was very intrinsically motivated towards her job and seriously wanted to understand cultural metaphors to teach abstract concept of mathematics. She had her inclination towards the collection and usage of different metaphors in her classroom teaching. In the other hand, Bima and Rana hold the

belief that the collection of appropriate metaphor from out-of-school culture was an overload to the teachers. They viewed that the collection of household activities regarding mathematics is difficult task for teachers. From the observation of the teachers' classroom activity and interview with them, some of them were determined to keep the status-quo. I found that have positive attitude towards the use of cultural metaphors and its incorporation in classroom teaching. They themselves enjoy the approaches to teaching and learning through cultural friendly pedagogy but they do not have sufficient time to manage and develop the teaching in view of cultural metaphors.

From the interview and FGD with the teachers, they viewed that "if a teacher wants to use students' culture friendly pedagogy, the course do not finish on time. There is great dilemma whether to use culture friendly pedagogy or instead focus on text-book based examination oriented instructional design". So, they loved to use their habitual method and did not want to entertain new approach in classroom teaching and some dichotomy on the use of cultural metaphors were found. However, other research participants viewed that the connection of out-of-school context in the process of teaching and learning mathematics has significant importance. The incorporation of ethnomathematical approach in school mathematics classroom encourage students to learn and develop positive attitude towards mathematics.

Mathematics beyond the Four Walls: Students' Reflection

The mathematical ideas embedded in out-of-school context and the pedagogy they used in their cultural setting could be the powerful metaphors for the teaching and learning of school mathematics. The cultural friendly pedagogy provide students to explore mathematical ideas embedded in different cultural arts and artifacts. From the FGD of the students, I found that the students have come to agree with the view

that the mathematics is a cultural phenomenon. The mathematical ideas embedded in the arts and artifacts showed that the artisans, centuries ago, had implicit mathematical knowledge. On view of a student- “when we visit and go to any place, our eye, our brain should try to find out the mathematical things in them” (Image, 6.38), one can find many mathematical ideas in every artefacts if tried to perceive from the eyes of mathematics. Moreover, the embedded mathematical ideas can be the cultural metaphor, which facilitates in teaching and learning of abstract mathematical ideas.

CPBL is one of the collaborative and constructivist approaches for teaching and learning of mathematics. Regarding the students’ views on the teaching and learning approach of CPBL, one of the student participants says: “It is our first trip of this kind. I had never imagined that mathematics could be learnt without textbook and worksheets beyond the classroom. We learned a lot of mathematical ideas and became able to explore mathematical ideas embedded in the cultural artifacts”. The CPBL approach for teaching and learning of mathematics changed the meaning of traditional ways of mathematics instruction. They believed that the mathematics learning is developing of skill and ability to solve the problems given in the textbook. From the observation in the field and the FGD of the students, I found that the CPBL approach is one of the effective approaches for the teaching and learning of mathematics. The CPBL approach for teaching mathematics provided an opportunity to learn mathematical concepts embedded in the cultural arts and artifacts beyond the four walls of classroom. This approach created an environment for the students for the construction of knowledge in their own ways.

In my observation, in every stage of data collection from the student participants, constructivist learning created a favourable environment to construct

mathematical knowledge. Most of the student participants were happy in the process of teaching and learning when teacher used metaphor in the teaching of mathematical concepts. The constructivist framework was followed during different stages in my study. The students were given rich learning environment and allowed to create their own meaning by providing different task during the teaching and learning through the use of cultural metaphors. As my theoretical orientation about the knowledge generation is based on the premises of the constructivist philosophy, the children construct mathematical knowledge as a result of active participation in their social context. The learner constructs mathematical knowledge based on their experience and interaction with peers.

Findings Related to Contribution of Cultural Metaphor in Teaching and Learning of School Mathematics

Metaphors are tool that helps to understand abstract domain of knowledge in terms of another. The domain of knowledge that mediate to understand new knowledge are termed as the source domain. The source domain of metaphor are concrete and familiar to the learner's context. My study identified two different sources of metaphor that mediate the teaching and learning of abstract concept of mathematics. The metaphors were collected from the out-of-school culture based on the everyday activities of different group of people, observation of classroom and literature that I reviewed. Following were my findings that analyze the contribution of cultural metaphor in the process of teaching and learning of different areas of school mathematics.

Synchronizing Gestures, Words and Symbols for Numerals Writing

The writing numeral is a difficult task for most of the children at the first grade of schooling. It is obvious that most of the children commit an error to write the

numerals. From the classroom observation in the cooperating school, I found that the errors were sufficiently minimized after the use of gestural metaphor. After the interaction with the class teacher Ms. Rasmita, it was found that the gestural metaphor helps to remove the errors that students did in course of writing numerals. Teachers use different gestural metaphors to communicate abstract mathematical ideas without noticing it. My research participants also used gestures to teach the numerals but her gestural metaphor did not work well to develop students understanding in numeral writing. The selection of appropriate metaphor for the teaching of particular concept is important. From the classroom observation and the interview with Rasmita, it was found that the gestural metaphor that I suggested her to teach numerals writing mediated to be an effective tool. The gestures of hands and fingers to represent the shape and value of the Devanagari numerals were found to be effective and interesting.

In view of gestural metaphor, Vygotsky (1987) viewed that gestures write in the air and there is a link between gesture and writing the numerals. The figurative gestures with fingers and hand indicate the model for writing the numerals. The metaphor is common to everyone in their everyday communication. Even the child use metaphors while they play games without noticing them. From this point of view, gestures are powerful metaphors to represent the abstract concept of mathematics. My findings support the line of Edwards (2005) who viewed that human gestures form an integral part of the language, thought, and communication. From the observation of classroom environment, I found that children became very happy and participated actively in the teaching and learning process of numerals. The effectiveness of role of gestures in teaching numerals and improvement of the students' cognitive aspect was assessed through the observation of class worksheets. Synchronizing gestures, words

and symbols help to improve students' errors and change the conventional ways of teaching and learning of numeral writing in the first year of primary school.

Metaphors for Teaching and Learning of Place Value

The study of Bose and Subramaniam (2011) identified that students make place value error while writing the numbers dictated to them especially for the numbers bigger than 100. The students made error in different problem solving situations caused by the lack of conceptual understanding of the concepts.

Base-Ten block: An artefacts. It is one of the important teaching tool for place value and arithmetical operations. Base-Ten blocks is a concrete manipulative through which students explore and discover place value concepts. My research participant Shyam viewed that the Base-Ten Block could be an effective metaphor for teaching place value. It is a great concrete model for helping students to understand the relationship between ones, tens, hundreds and thousands.

Money metaphor. Money has a strong connection with all culture and it has been highly valued in this physical world. Every one, however illiterate, recognize the denominations of different currency. Money is the real-life example to teach different concepts in mathematics. The cultural artifacts can be used as motivating stepping-stones to launch new mathematical knowledge (Bonotto, 2004). The cultural artifacts offered the opportunity of making connections between the mathematics incorporated in real-life situations and school mathematics. These artifacts are relevant to children; they are meaningful because they are part of their real-life experience, offering significance references to concrete situations. About the use of metaphors in teaching and learning of place value at the primary level, one of my research participant Dipika used money metaphor to teach place value. As money is familiar to each student, its connection to the mathematical ideas of place value seems to be very fruitful. To

teach the place value at primary grades, she used the note of rupee one to represent the one's units, the 10 rupees to represent the tens unit and the hundred rupees to represent the hundred of unit.

Realizing this view, it was found that the use of money metaphor to teach the concepts of place value at the basic level of education really made a difference in the learning process. From the observation of class, it was found that the concepts of ones, tens, hundreds, thousands through money mediated to conceptualize the ideas of place value for ones, 10 ones for a ten, 10 tens for a hundred and 10 hundred for a thousand and so on. The classroom activities of teacher and the interaction supported the lines of Hutchens (2015) who viewed that the beauty of money with place value is that it gives place value a real meaning. Thus, it is concluded that money is a powerful metaphor to teach and learn one of the most important ideas of arithmetic because the concept of place value is very fundamental which help to teach the mathematical operations of addition, subtraction, multiplication and division.

The household ideas and knowledge, which students experience in their everyday activities in their familiar context provides the opportunities to learn the school mathematics. The students' familiar context facilitates a systematic and powerful tool to represent the abstract concepts of mathematics. In this connection, I always advocate for Cummins (1996) "No learner is a blank slate" cited in Gonzalez, Moll and Amanti (2005, p. ix). It is believed that the students bring some sort of mathematical ideas and knowledge in the classroom. The ideas, knowledge that the children already possess help to unlock difficult and abstract concepts of mathematics.

Metaphors for Teaching and Learning of Rounding off Numbers

Rounding off numbers is an important skill in every sector of human lives in general and particular in mathematics. Rounding off numbers and estimation are

important skills in mathematics and everyday life. Estimation skill gives an idea about possible answer without doing a lot of work. In daily life, estimation is great for giving intelligent guesses about the possible amounts. There are different metaphors to communicate the abstract concepts of rounding off numbers. The common metaphors used in the discussion of the contents were clock metaphor, number line metaphor, path and fictive motion metaphor.

Clock metaphor. The concept of rounding off is used in everyday activities and basically used to describe situations like morning, day and night. We usually conceptualize the time in these three state of time. We eventually say that its morning, it's already a day and its night and so on. From the observation of classroom, I found that the teacher used an analog watch to conceptualize the ideas of Rounding off numbers. One of my research participants explained to the class “If we ask a person with a watch about what time it is, the person will probably answer the time to be 10 o'clock even when the exact time in the clock would be 10:03 or 9:56 or any nearest digits”. Hereby, we generally take this time rounded off to 10:00 but without the theoretical concept of the rounding off numbers. They use such concepts tacitly. From the discussion, I found that the watch is a grounding metaphor for the teaching and learning of the concept Rounding-off numbers.

Number line metaphor. Number line is one of the most central concepts in every branch of mathematics. Analytic geometry would not exist without it nor would trigonometry. Thus, the number line is one of the common metaphor to mediate different concepts of mathematics. From the observation of the classroom, I found that the teacher used number line metaphor to round off the numbers to the nearest 10, 100, and 1000 and their multiples. The findings of this study are in line with the claim made by Lakoff and Nunez (2000) as they argued the number line is one of the most

central concepts in all of the mathematics. I found that the number line is taken as one of the best metaphor for the teaching and learning of Rounding-off numbers.

Path metaphor. There are different metaphors used in the teaching and learning of Rounding-off numbers. In the classroom teaching, one of my research participants taught the concept of rounding to the nearest 10. For example, 45 is a number lies in between 40 and 50 on the number line. Therefore, 50 is the number after rounding off 45 to the nearest ten. One of my research participants asked me why 40 was not the rounded number of 45 since 45 is just between 40 and 50. It was found that the path metaphor found to be meaningful and effective to conceptualize the rounding-off the middle number to the nearest 10, 100, 1000 and so on.

The path metaphor used by the research participants was described as “Suppose a person begins his journey from initial point A to destination B. If a person gets to the point C i.e. at the center of the journey, then s/he must go towards B because the main target for which the journey began was only to achieve the aim. Since every journey has a purpose, it is necessary just for its accomplishment, which signifies that the distance, however, must be covered even if s/he returns to the initial point, which is to waste the time and everything else. So as s/he gets to half of the way to it, it’s better moving towards the target rather than returning to the initial place”.

I observed a classroom teaching of the unit of Rounding off Numbers. My research participants taught in her class as explained in the textbook. Almost all the Nepalese mathematics teachers considered textbook as the main source of teaching and learning of mathematics. The mathematics textbook is not presented adequately to address the rounding off the middle of the numbers. However, the path metaphor is one of the effective tools to facilitate the learning of rounding-off numbers at all level.

Metaphors for Teaching and Learning of Prime and Composite Number

The concepts of prime and composite numbers can be taught and learnt with the use of different metaphors. From the observation of classroom, my teacher participant used series of intervention of different metaphors.

Color metaphor. Color is one of the common metaphor that can help to conceptualize the concepts of Prime numbers and composite numbers. From the observation of classroom activities, teacher participants used color metaphor as first intervention to conceptualize the prime numbers and composite numbers. It was found that the prime numbers are as those primary colors which are not a mixture of any other colors, and the colors like orange, grey, green, violet, and so on formed as a mixture of more than a color were compared to be the composite number.

The connection of students' familiar context to the school mathematics can make the regular mathematics class very interesting, and meaningful. The familiar context of the learners and physical apparatus enables students to get a sense of mathematical ideas (Delaney 2001). With this connection, it was found that the ideas of prime and composite numbers could be mediated by manipulating different colors.

Area metaphor. Area is one of the common metaphor that can help to conceptualize the concepts of prime numbers and composite numbers. The area can be visualized as the small square unit in the geoboard. Each group has requested to form possible numbers of rectangular plot from the given numbers with the help of rubber bands in the geoboard. The numbers that have only two rectangular plot in a board is labeled as prime numbers and the numbers that can have more than two rectangular plots can be called as composite numbers.

The connection of students' familiar context to teach the school mathematics can make the regular mathematics class very interesting, and meaningful. In this

connection, Edwards (1998) viewed that teaching and learning of mathematics can be mediated by images and physical apparatus (as cited in Delaney, 2001). In the context of teaching and learning of prime and composite numbers through the use of physical apparatus (the geoboard) provides students with a rich environment to develop a sense of some aspect of mathematics. Enactive mode, as described by Jerome Bruner, representing the mathematical ideas of prime and composite numbers by means of manipulating physical apparatus and their actions (Delaney, 2001).

Colorful rainbow factor metaphor. Teacher participant created and used grounding metaphors of “I and Me factor metaphor” for prime numbers and “I, Me and My factors metaphor” for composite numbers to teach the concepts as the third intervention. From the classroom observation, it was found that teacher used “colorful rainbow design” artefact for different prime and composite numbers to teach the concepts. The “I and Me factor metaphor” depicted for prime numbers, which has only two factors 1 (I) and itself (Me). Another “I, Me and My factor metaphor” for composite numbers, which has more than two factors 1 (I), itself (Me) and other factors (My). These metaphors were found to be powerful metaphor in the process of teaching and learning of prime and composite numbers. There were several other metaphors used by the teachers in teaching abstract concepts of mathematics.

The use of different metaphors in constructivist framework facilitated students to develop the concepts of prime and composite numbers. After the class, I asked about the effectiveness of cultural artifacts in the process of learning the concept of prime and composite numbers with the reporter of each group. The reporter of almost all group viewed that the class was very interesting and learning was fun. From the post class interview with student participants, I came to know that the constructivist learning situations encourage in learning mathematics. The incorporation of cultural

metaphor in school pedagogy denied the rote and memorization of the definition of prime and composite numbers. The conversation with student participants was exhibited good evidence that the cultural artifacts mediated for the construction of mathematical ideas and enhance the level of students understanding. Finally, I have come to know that cultural metaphor mediates to conceptualize the mathematical ideas of prime and composite numbers.

Metaphors for Teaching and Learning of Integers

Integers are one of the fundamental concepts in mathematics. Kilhamn (2011) advocated that the procedural errors are caused by students' lack of conceptual understanding of the negative integers. Some students though could perform the operations, did not have conceptual knowledge regarding the integers. The abstract ideas of integers can be taught using Balance metaphor, Money metaphors, Temperature metaphors, and other examples that the students encounter in their daily life like the depth of well, the level of electrical pole below ground level, the debt, and the background of the house below the ground level and so on. Following were some of the metaphors practiced in the process of teaching and learning of integers discussed in the consequent subheadings.

Balance metaphor. Balance meter is one of the common tools to measure the weight. It can serve as a metaphor for the teaching and learning of integers. Balance metaphor can represent positive integers, negative integers and zero as per situations are seen in the balance. The equal weights on both sides of balance create equilibrium and there is a condition of no difference or zero difference in weights on both left and right sides of the balance. The more weight or more quantity represents the positive quantity and hence positive integers and the less weight or less quantity was used to represent the negative integers. From the classroom observation and interview of the

students, the balance was found to be an effective metaphor for conceptualizing integers.

Gestural metaphor. Gesture is a common metaphor that the teachers use frequently in their teaching without noticing it. The gestures of Up-Down and Above-Below also mediate to teach the concept of integers. In the classroom teaching, I observed that teacher used gesture to conceptualize the positive integers, negative integers, and zero according to the positions of the hands. Keeping their activities in mind, I found that the use of such gestural metaphors contributed to grasp a clear understanding of the concepts by the students.

Money metaphor. Money, as a metaphor, can be used as a very effective tool in the teaching of integers as it money holds a great impact in this physical world. There is probably no child who lags the idea of earning and spending money. The assets and liabilities can also be the metaphor for the understanding of positive and negative integers. Assets add positive income each period and liabilities (debt) require payment to service it. I found that money metaphor was equally significant in course of teaching integers. Teachers use money metaphor as the basis of integer that was compared with having or not having the money. The winning of money signified positive integers, the loss of money signified negative integers and the situation of neither a loss nor gain signified zero. So, money metaphor could be the grounding metaphor for the teaching and learning of integers.

Number line metaphor. The subject of integers is an important part of school mathematics curriculum at the basic level and it symbolizes a move from concrete to abstract thinking (Lamb & Thanheiser, 2006). From the classroom observation, my research participants used several metaphors to assist student learning and encourage them to understand abstract ideas and concepts of an integer. I found that the teacher

used number line to conceptualize positive and negative integers. My study supported the findings of Kilhamn (2011) who viewed that physical version, known as a line segment, as metaphor can conceptualize the abstract concept of integers.

Metaphors for Teaching and Learning of Operations on Integers

Students of the basic level were found to be difficult during addition and subtractions of integers. To illuminate students' difficulty and ease the teaching and learning of operations on integers, my teacher participants used series of intervention regarding different forms of cultural metaphors. Some metaphors used in the process of teaching and learning of operations on integers are discussed in the consequent subheadings.

Mnemonics as metaphor. The rule teaching is sometimes important than the concept teaching if the explanation of the concepts becomes difficult or time consuming. In our mathematics classroom, the teacher and the students use different types of mnemonics as a memory tool. By using mnemonics in mathematics classroom, students are provided a better encode information so that it is easier to recall later on.

Higbee (2001) defines mnemonics as memory aids that assist one in remembering specific information by using a process, strategies or some techniques that enable him/her to improve memory. This could be a meaningful metaphor that may help the students in both memorizing the rule and even recalling the moral values. In the observation of classroom and interview with the teachers, I found that teachers used different mnemonics regarding the sign rules of multiplication. The rule of multiplication of two negative numbers gives positive, was described by the teacher using mnemonics "Don't Do Bad, results Good". My research participant Shyam opines that "when students get clear about the rules of multiplication of

integers, basically, $(-) \times (-) = (+)$ then the students can easily solve the problem $(-3) - (-5)$. Here $-(-5)$ can be understood as $(+5)$ resulting the expression to be $(-3) + (+5)$ ".

Another research participants used other mnemonics "Two like signs make a positive sign; Two unlike signs, make a negative sign" to teach the sign rules of multiplication. Delashmutt (2007) viewed that mnemonic is a memory-enhancing instructional strategy that involved teaching the students to link new information that is taught to information they already know. My study identified that mnemonics work to address problems related to students' poor retention and is a useful tool in solving mathematical problems.

Number line metaphor. The number line is one of the common metaphor to conceptualize integers at the basic level of mathematics classroom. Bruno and Martinon (1999) found that the number line became an indispensable tool when solving problems where the identification of addition with subtraction was a primary focus. I found that the teacher frequently use number line in the process of solving problems regarding integers. From the classroom observation, the subtraction of negative integers was felt quite difficult than the addition of two integers.

Arrow metaphor. There were different metaphors used in the teaching and learning of integers at the basic level of education. The arrow metaphor seemed to be the most effective one for the teaching of addition and subtraction of integers. From the classroom observation, one of my research participants used arrow metaphor for the teaching of subtraction of negative integers. By observing the classroom activities, I found that the arrow metaphor contribute in the teaching and learning of abstract concepts of subtraction of negative integers from negative integers.

The teacher and students were pleased while using the arrow metaphor in the process of teaching and learning of the concepts. The students' active participation, facial expressions, emotions, feelings, cooperating behaviours and engagement in the teaching and learning process was observed and found to be effective. The effectiveness of arrow metaphor and students cognitive changes were assessed through the observation of worksheets.

Metaphors for Teaching Algebraic Concepts

Algebra is an abstract part of mathematics introduced in the very beginning of schooling. Probably, most of the students say that algebra is one of the hardest subject to be studied. In this connection, Leitzel (1989) mentions that the concept of a variable is a fundamental ideas in algebra and it frequently becomes a barrier to student's understanding of algebraic ideas. Thus, understanding the concept of variable provides a base for the transition from arithmetic to algebra and is necessary for the meaningful use of all advanced mathematics (Schoenfeld & Arcavi, 1988). The algebraic concepts of constant, variables, like and unlike terms, algebraic terms, algebraic expressions and the equations can be linked with the students' household activities and locally available materials. The student centered project based on the sociocultural context of the students helps to connect algebraic concepts. The collection of different types of beans available in students house, and categorizing them according to their type, the numbers of a particular type of beans and total number of beans provides the opportunity to learn different algebraic concepts.

The out-of-school environment has lot of mathematical knowledge and ideas. From the observation, it was found that the connection of household activities of the students in the process of teaching and learning help to understand the algebraic concept in a better way. The incorporation of students' familiar context helps them to

think critically and enhance the problem-solving skills and activities. In this vein, D' Ambrosio (2006) claims that the ethnomathematics program contributes to restore culture and offers the intellectual tools for the exercise of citizenship. The classroom practices need to govern the meaningful connections to the mathematical ideas embedded in the out-of-school context. My teacher participants connected ethnomathematical ideas embedded in the kitchen work to teach the concepts of like and unlike terms. A number of image metaphors were found to be an effective tool for developing the basic algebraic concepts.

CPBL Approach for Teaching and Learning of Mathematics

The mathematical ideas and concepts embedded in the out-of-school environment could have great opportunities and assert for the teaching and learning of school mathematics. The connection between students' cultural knowledge and ideas in the process of teaching and learning of school mathematics helps to develop a positive attitude towards the subject. The incorporation of culturally contextualized mathematical ideas in the formal mathematics curriculum recognizes the indigenous knowledge that reinforces cultural self-respect and offers a broad view of humanity.

From the observation of the students in the field visit at the cultural heritage and monuments regarding the possibilities of connecting out-of-school context and mathematics, it was observed that the temple and its surroundings were contented with a lot of mathematical ideas. The students were given to identify four major tasks. The tasks were to explore the mathematical ideas embedded in the surrounding of the temple, to identify the center of the circular surface, to identify the type of quadrilateral by measuring its sides, and finally to explore the concept of symmetry and identify the axis of reflection. Following four subheadings present the findings of the groups of students who participated in the field visit.

Mathematical ideas embedded in the surrounding of the temple. The exploration of mathematical ideas embedded in the surroundings of the temple was one of the common task given to each group. This task of observing the temple and its surroundings and noting down the geometrical objects and the probable mathematical concept that they could identify was provided to each of the students participating in the field visit. The findings in the observation made by the students were 2D shaped objects like triangle, square, quadrilateral, circle, concentric circles, oval, octagon, trapezium, parallelogram, and rectangle. They observed 3D shapes like hemisphere, cone, frustum, sphere, cube, cuboid, cylinder, prism and pyramid. They also sorted out the concepts of concentric circles, transformation, reflection, rotation, symmetry, pattern, and tessellations.

There were two mandalas found in the temple, one erected by carving stone was found in front of the main entrance gate and another metallic mandala was observed inside the gate of the temple. The mandala consisted of two concentric circles. The image also had a design of the twelve petals of the lotus flower each representing the place of 12 deities. The *Chaitya* is also found in the surrounding of the temple. Many images of gods and goddess were found in the art of stone carving. The *Chaitya* was observed as the combination of different geometrical objects. The regular octagons are found in the base of *Chaitya* and octagonal prism on it. The conjugate hemisphere is put at the top of the *Chaitya* (Image 6.33).

Considering the student's interaction and their report, they identified the highly sophisticated mathematical ideas embedded in the monument and artifacts of the temple. However, the artifacts were created centuries ago, different arts and artifacts found in the surrounding of the temple were exhibited full of mathematical ideas. They identified that the *Chaitya* and other artifacts of the temple embedded beautiful

creation as a combination of mathematics and creativity. The consecutive cuboid in a proportion forms the pyramidal shape. Concepts of concentric circles in stone carving form impressive artifacts, which also give us a hint about how the mathematical ideas were created at the beginning of time being so far from today's formal studies based on bookish knowledge.

The students' observation of the cultural artifacts and monuments and their findings confirmed the argument made by Mosimege and Lebeta (2000) who reported that the indigenous people use different mathematical concepts like estimation, tessellations, and symmetry in the construction of the traditional artifacts and cultural activities. The ethnomathematical ideas of a group of people have performed strong mathematical ideas. From the observation of different artifacts present in surrounding of the temple, students came to know that mathematics was a cultural product of different group of people. The mathematical ideas found in the artifacts developed a positive attitude in the children towards the learning of mathematics.

Identify the center of the circular surface. A task to identify the center of the circular surface found in the *Chaitya* and mandala was given to the group I. This was collaborative work and students from grade six to ten were engaged in the given task. From the discussion of the group, they came out with a result based on the formula *Cicumference of a circle* $= 2\pi \times \text{radius of the circle}(r)$ to find the radius. Then they identified the midpoint of diameter as a center of the circular surface. This shows that the students linked the formal mathematical ideas to understand the cultural mathematics. Their findings are in line with the claim made by Winter, Andrews, Greenhough, Hughes, Salway, and Yee (2009) as they found many examples of children doing mathematics at home which seemed much more similar to the mathematics they were doing in school (p.22).

The artifacts were crafted centuries ago and the mathematical thinking of people in their artisanship might have been different, as indigenous people have their own ways of doing and understanding the mathematical phenomenon (Pradhan, 2017). In view of this, students were motivated to explore the ideas for the identification of the center of that circular surface crafted by the people at that time. After the discussion among the members of a group, the following was the procedures they constructed to solve the problems they encountered. Their report shows that they had got the empirical knowledge to solve the mathematical problem of finding the center of the circle in case other dimensions of the circle were unknown. From the observation of the students' engagement in the task, I found that they construct the mathematical knowledge based on a constructivist approach to learning. They adopt the cyclic process of engaging, exploring, explaining, elaborating and evaluating in view of construction of the mathematical knowledge (Barufaldi, 2002) in this given task.

According to the underlying tenet of constructivism, the teacher acted as a guide, suggesting paths that may be profitable (Delaney, 2001). In the context of cultural metaphor, the constructivist teacher's role would be to facilitate students to explore mathematical concepts and relations. With this regard, we had formed a heterogeneous group of members involving students from grade six to ten that could provide an opportunity to learn from more knowledgeable peers (Vygotsky, 1978). From the CPBL in this study, I found that the involvement of different grade level students provides an opportunity to learn mathematics concepts, even of the higher level, in collaborative learning framework.

Identifying the types of quadrilateral. One of the major objectives of CPBL was to search for the possibility of connecting out-of-school mathematical ideas in the

process of teaching and learning of formal mathematics. The task given to the second group had an aim to explore the mathematical properties of the shape of terrace of the *sattal* present in the premises of the temple. The constructivist learning situations were created to solve the problem given to them. Students were engaged in a task with measuring tape, marker and some other were engaged with the paper and pencil, marker. My research participants facilitated students' activity to make a connection between past and present learning experiences and organize students' thinking towards the learning outcomes through current activities.

From the students' report on their task, I found that the project provided good opportunity to explore the properties of the quadrilateral. First, they identified that the given surface was a quadrilateral on the basis of the result obtained; the length of each side was unequal. Evidently, their findings did not get its final form. They discussed in their group and came to the conclusion that their finding was incomplete. Further, they observed that the diagonals of the given surface are equal and two of the sides are parallel. Finally, they identified that the given surface was trapezium. Though the task given to the students was simple, the learning opportunity provided encouraged them to explore different mathematical ideas from students centered approaches.

While interviewing the student participants, they reacted positively and enjoyed this CPBL approach to explore mathematical ideas and knowledge embedded in the monuments and cultural artifacts. In view of Nunes (1992), the concept of non-school mathematics facilitates the understanding of school mathematical concepts. The incorporation of out-of-school mathematical ideas and concepts play a vital role in successful learning of school mathematics.

Symmetry and reflection in artefacts. The exploration of symmetry and axis of reflection in objects of different artifacts was given to the heterogeneous group of

students from grade six to ten. On the basis of the observation of different objects inside the temple premises, students compared the halves of those artifacts and determined if objects were symmetrical or not. Some of the students actively participated in the task given to them. The wood carvings in doors, windows, ceilings, and other parts of the temple were observed. Some of them were taking the data, while some were comparing those data to determine if the artifact of that corresponding data was symmetrical or not.

The student participants of this project reflected that the object found in the premises of the temple has both bilateral symmetry and rotational symmetry. They found bilateral symmetry in the images of gods and goddess, heavenly creatures, arts, and designs carved on the doors, windows, and ceilings. Similarly, they observed rotational symmetry in the artifacts like circular mandala, which possessed twelve petal designs as in a flower of lotus, constantly at the same interval and with the same shape and in the art of eyes carved in the doorway of the temple. They got the concepts of symmetry in innumerable artifacts and objects associated directly or indirectly with the religious aspects of the temple.

The data of my study supports the idea of term Zone of Proximal Development (ZPD) supplied by Vygotsky (1978) in the context of CPBL. He described that the distance between actual developmental level is determined by independent problem solving and the level of potential development through problem-solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978). This notion of an enhanced level of mental functioning when an expert guides a learner has been influential in teaching and learning. From the observation in my field study, I found that the lower grade students learn different mathematical concepts involved in participatory activities and cooperative approaches in which they

learn with the help of their senior graders as Vygotsky see learning as an activity in which shared mathematical meanings are constructed socially.

Summary of the Chapter

Metaphors are common to everyone in their everyday communication. Even a child uses metaphors in their gameplay without noticing it. Metaphor is one of the powerful mediated tools that gives a real meaning of objects in view of observers. In my study, the ethnomathematical ideas that are practiced in everyday activities of different group of people, the mathematics embedded in different arts and artifacts in out-of-school environment, the cultural games, and gestural activities that contributed to conceptualizing the abstract mathematical ideas were considered as metaphors.

The culturally relevant pedagogy bridges students' home with school culture in the process of teaching and learning of mathematics. Further, Bartell (2011) viewed that culturally relevant pedagogy builds on students' home culture as a means to foster success in learning of school mathematics. The students' familiar context facilitates to understand school mathematics. In my study, different cultural heritages, monuments, and artifacts provide an opportunity to explore the mathematical ideas embedded in the cultural context and helps to connect school mathematics. The culture friendly pedagogy facilitates incorporation of students' cultural values, experiences and perspectives into the curriculum (Gay, 2002). From the FGD of students, I found that students view mathematics as useful and relevant subject when mathematical ideas are connected to the cultural artefacts. I also found that the CPBL is one of the activity based student friendly method to explore mathematical ideas in out-of-school environment.

CHAPTER VIII

CONCLUSIONS AND IMPLICATIONS

The purposes of my study were to explore the cultural metaphors in relation to mathematical ideas and knowledge practiced in out-of-school culture, the analysis of contribution of cultural metaphors in teaching and learning of school mathematics and assess the possibilities of use of cultural metaphor in the teaching and learning at basic level of education. In the previous chapter VII, the findings of the study based on the data analysis were discussed in detail.

This chapter presents conclusion of the study followed by the discussion of the themes, grounded in the findings of the study and triangulated with literature and theory delineated in chapters II and III. Finally, the chapter discussed the implications for teachers, curriculum materials developers, and propose area for further researches based on the limitations regarding methods of data collection, analysis techniques, and the findings of the study. I have also expressed my reflections during the process of this study and end with the researcher's final words about the study.

Conclusion of the Study

Mathematics is everywhere. It is embedded in physical environment as well as different cultural activities. The petals of the different flowers possesses the Fibonacci numbers, self-similarity in cauliflowers, concavity of curves is observed in the ups and downs in the geographic landscape and so on. Different mathematical ideas and concepts are embedded in the everyday activities of human culture. Chatpatte wala uses the ratio and proportion, makes correct estimation of the integrant, surface area and volume of a cone etc. implicitly. The ethnomathematical ideas practicing in the out-of-school environment can facilitate the understanding of school mathematics. However, the connection of students' out-of-school mathematical ideas to classroom

teaching is difficult task for most of the school teachers. The incorporation of ethnomathematical ideas of children and cultural metaphor in the mathematics classroom was common interest of the mathematics teachers of my cooperating school.

The people outside the classroom using sophisticated mathematical ideas to perform their everyday activities. For example, mason uses 3-4-5 method to construct the rectangular shape without knowing the properties of right angle triangle and the Pythagorean Theorem. The roof of the houses and temples are inclined in forty-five degree without noticing the formal mathematical ideas. Mat weavers use implicit knowledge of Euclidean geometry while they construct the frame for mat. Their indigenous knowledge and skills are sufficient to survive them. The craft model approach is the ways to knowledge generation and distribution in out-of-school culture. The ethnomathematical ideas embedded in the everyday activities of different group of people can also be linked while teaching in classroom to make class lively.

The ideas of estimation and approximation are fundamental and critical processes in the development of mathematical concepts. The major characteristics of home mathematics is the estimation and approximation. The people in out-of-school environmental context use mathematical ideas implicitly since long. Their mathematical ideas and knowledge are practical validity which is neither acknowledge nor utilized in school mathematics curriculum (Burton, 1996). The measuring system of carpenter, estimation of Chundara in the process of constructing wooden stuffs, the ratio of different integrant in the preparation of tea and the different mathematical ideas embedded in the kitchen involves the estimation of approximation. The different group of people use mathematical ideas implicitly in their everyday activities. The mat weavers, textiles weaver, stone carver, wood carver,

small-scale businessperson and different group of people possesses highly sophisticated mathematical ideas in their everyday activities. This help me conclude that mathematics can be taught at work place more effectively than in the classroom.

The connection of two domains between sources and target be effective and interesting if the sources are familiar to them. The student's familiar out-of-school context and their funds of knowledge enhance to understand the concepts easily. The mathematical ideas embedded in the cultural arts and artefacts, everyday activities of different groups of people reflected highly sophistication of the world of mathematics. The traditional measurement system practices in out-of-school environment still exist. But, our school mathematics teachers hardly use the socio cultural plurality (Poudel, 2008) to teach mathematics. Instead they unknowingly follow the process of demathematization that makes mathematics a difficult subject. This led me to conclude that ethnomathematical practices are need to preserve and to link cultural as well as in school mathematics curriculum with the help of the innovative pedagogy.

The out-of-school environment identified numerous mathematical practices in farming, local business, household activities, children games, cultural arts, artefacts and social events. The ways of teaching and learning of mathematics practices in formal school and out-of-school context is different to each other. There is difficulty to connect out-of-school mathematical ideas in the teaching and learning of school mathematics. The cultural metaphor is a tool to connect the students' out-of-school context and the school mathematics. The students bring some sort cultural experiences and knowledge in classroom. The familiar context of the students are the sources for the teaching and learning of unfamiliar and abstract concepts of mathematics. It enabled me to conclude that "No children are blank slate" as opposed to the philosophical ideas of "the tabula rasa" of John Locke.

The incorporation of cultural metaphor in the process of teaching and learning of mathematics was found more effective and meaningful. In my study, the cultural metaphors enhanced the teaching and learning of school mathematics. It provides the mediated tools to communicate abstract concepts of mathematics. The cultural metaphors mediated to frame the cultural responsive pedagogy in mathematics. The students' out-of-school cultural and their household mathematical activities help to learn mathematics. From the interaction to teachers and students, it was concluded that the cultural metaphors substantially contribute in the process of teaching and learning of mathematics. For instance, the area metaphor, and colors metaphor really contributed to teaching and learning of prime and composite numbers. In the same way, the number line metaphor, arrow metaphor, balance metaphors, gestural metaphors and mnemonics facilitate to conceptualize integers and operations on it. The gestures of fingers and hands really contribute to develop students' level of confidence and level of understanding in numerals writing. This enabled me to conclude that the teachers need to figure out the cultural metaphor and use them to teach and help students learn abstract concepts of mathematics.

This study on connecting out-of-school environment in the teaching and learning of school mathematics supports Zaslavsky's (1990) and Uy (1996) who claim that 1) students appreciate the culture that are different from their own and 2) linking the study of mathematics with other disciplines and cultures provides more meaning to the mathematics studied. Evidence was provided by student questionnaires and in interviews. When asked why they enjoyed a CPBL approach, most of the students answered that they saw use of applications of mathematics outside the classroom and other cultures that they had not encountered in their previous mathematics classes. Interview comments from the interview like "It was fun

learning about mathematics and another culture at the same time”...supported these claims. The students participated in the CPBL approach indicated that they appreciated the ways to teaching and learning of mathematics because they saw a direct and human way of applying the topic. Based on evidence from field observation and the FGD enabled me to conclude that the incorporation of cultural metaphors mediated a powerful tool in the process of teaching and learning of abstract concepts of mathematics.

While viewing through participants’ perspective, there was controversy about the use of cultural metaphor in the teaching and learning of mathematics. Most of the participants viewed that the culture friendly pedagogy and the funds of knowledge for teaching and learning of school mathematics are effective for teaching mathematics. They agreed that the use of cultural metaphors and its incorporation in classroom teaching creates positive attitude towards mathematics. They themselves enjoyed the approach of teaching and learning through cultural friendly pedagogy but they lacked sufficient time to manage and develop the teaching in view of cultural metaphors. From the interview and FGD with the teachers, they viewed that “if a teacher wants to use students’ culture friendly pedagogy, the course would not finish on time. There arises a great dilemma whether to use culture friendly pedagogy or instead focus on text-book based examination oriented instructional design”. Concisely, they loved to use their habitual method and did not want to entertain new approach in classroom teaching and some dichotomy on the use of cultural metaphors were found.

Implications of My Study

I am not going to claim that my study opens up the floor gate in the field of cultural metaphors for understanding of mathematics. However, my study probably opens the new avenue in the field of research in mathematics education in Nepalese

context. As my study is limited on some particular cultural activities, no longer implications and generalization can be made. From the field data, the following implications in the diverse fields can be established.

Implications for Policy Makers

The three words- contextualization, ruralization and glocalization, can easily be practiced in mathematics through cultural metaphor. This implies that the curriculum and material developer need to transfer these buzzwords into action through textbooks and materials. The specific implication would be

- Teacher training and workshop can be conducted in order to equip mathematics teachers in view of cultural metaphors for teaching and learning of school mathematics.
- Mathematics was originated in order to solve practical problems of everyday activities. So the teaching and learning of mathematics should be based more on practical activities.
- From my field observation, I got to know that, teachers generally do not go into the topics related to the practical problems since they are not asked in the examination. As examination and testing device is the major tool to assess the students' achievement, the system should focus on practical problems related to learners' household activities.
- Same curriculum and pedagogy are used throughout the nation, which largely ignore the local knowledge. This reminded me that our education, the curriculum we design and the pedagogy we adopt should be corresponding to the needs of the indigenous knowledge. I, thus, realize that the concerned bodies need to give due respect to the approaches to indigenous knowledge generation and education needs to be life long and practical.

Implications for Textbook Writer

Textbook is one of the common resources in educative process. A good textbook can be an excellent teaching aid for both teachers and students. From the field study, it was observed that the teachers teaching activities were mostly textbook driven. On the basis of my study, the following suggestions were made:

- Textbook is one of the important tools in the classroom teaching and learning process. The textbook writer must be aware of cultural plurality and incorporation of everyday cultural activities of learners.
- The incorporation of everyday activities and culture of learners in mathematics textbook encourage them to learn, develop self-respect towards their culture.
- Mathematics textbook itself is a resource material in the process. The mathematics teaching and learning can be effective and meaningful if the problems given in mathematics textbook is connected with the out-of-school environment.

Implications for Mathematics Teachers

The classroom observation and interview with the cooperating teachers exhibited that our school teaching is conventional. Teachers' approaches are similar to the method that s/he had learnt in the school. However, teachers were assured that the classroom teaching should be student-centred. They were convinced that mathematical ideas could be taught in better way beyond the four walls of the classroom. From the field study, I realized that mathematics teaching could be more meaningful if following points are considered from the teachers' side.

- Teacher need to recognize the learner's out-of-school activities regarding to the mathematical ideas and knowledge and connect to school mathematics.

- They need to identify cultural metaphors for the teaching and learning of some of the mathematical concepts at the different level of education.
- They require to design collaborative work and the knowledge sharing sessions among the faculty members to address difficult concepts of mathematics.
- In a classroom representing multicultural students, it is better for mathematics teachers to know about the culture of different students of classroom. The incorporation of cultural activities belonging to such diverse groups in a mathematics classroom would facilitate the students to take each other's culture as a subject of pride and self-respect creating a harmonious environment in the classroom.
- Students take mathematics as a subject studied to secure passing marks and teacher, as a subject to make their students pass. This has been a trend that is discharging the creativeness of teachers and potential of the students. If the teaching and learning gets connected with the out-of-school culture, this would pacify the opposed feeling towards the subject.
- Teachers can apply CPBL approach to connect the students out-of-school context to school mathematics thereby develop positive attitude of students towards mathematics.
- Finally, they can enable students to make school mathematics as "My mathematics- Our mathematics" rather than author's mathematics and teacher's mathematics.

Implications to Use of Cultural Metaphors in Mathematics Instruction

Primarily, metaphors has communication power to teach new concepts through familiar and well-known ideas and knowledge. Most of the learning theorist believe that learners' prior knowledge is essential for successful learning. Learners'

prior knowledge includes knowledge acquired from their everyday activities, home culture, rituals, varieties of games, interaction with seniors and peers, language, storytelling, myths, etc. All of these can be seen as thesis of that culture and can be considered as cultural capital (Upadhyay, 2015). During my field visit, I collected some cultural metaphors from out-of-school environment, as described in Chapter V, and some of them from classroom observation and the literature reviewed. I observed that some of the metaphors thus collected contributed in the process of teaching and learning of mathematics at basic level education. Some of the metaphors that I collected were not intervened in the classroom. I suggest that the cultural metaphors presented in the following matrix can have potential contribution to develop different abstract concepts of mathematics at various level of education.

Table 8.1

Potential Cultural Metaphors for Mathematics Instruction

Cultural Metaphors	Areas to Mathematics	Contents
Weaving (Mat weaving, Textile Weaving)	Geometry	Geometrical Shape (2D and 3D), Different properties of rectangle, Symmetry
Carving (Stone and wood)	Geometry	Coordinate axes, Symmetry, Reflection
Games (<i>Bagha Chal</i> , <i>Chungi</i> , Swing)	Geometry	Geometric shapes and their properties
	Algebra	Basic ideas of algebra, variables, Equations and its roots, inequality
	Statistics	Data collection, frequency distribution, Central tendency
	Transformation	Concept of rotation, Angle of rotation, Axis of rotation
Building constructors (Mason)	Arithmetic	Counting, Measuring, Estimation, Ratio
	Geometry	

		Perpendicular, parallel, different geometrical shape, properties of right angled triangle (Pythagorean Theorem)
Cultural Arts and Artefacts (<i>Rangauli</i> mandala, <i>Swastik</i> symbol)	Geometry	Coordinate axes, geometric shape, Symmetry, rotation, perpendicularity, parallelism

Areas for Further Researches

Every path leads to another path and every journey begins with a single step. Thus, my study does not claim as a complete and final in this area. Evidently, no this study is the final, must be continuing to dig out the effectiveness of cultural metaphors in the process of teaching and learning of mathematics. As I mentioned the limitations and delimitations in chapter IV as a researcher and the research process, it demands the need for further research on different domain of cultural metaphors and mathematics instruction. On the basis of the findings of this study, the following areas have been proposed for further research.

- My study was delimited to the ethnomathematical ideas regarding the everyday household activities, workplace base knowledge, cultural artefacts and monuments of Kathmandu district. For the wider application of the research findings, similar study with different cultural group from different ecological zone can be conducted.
- The connection of out-of-school mathematics of different cultural group provides learner with rich opportunity to learn other's culture, gain self-respect and motivate themselves towards the subject. The similar study can be conducted in different indigenous community with different sociocultural contexts.

- Teachers are one of the main agents in the educative process. The perceptions of teachers on the use of cultural metaphors in the teaching of mathematics play vital role for the understanding of school mathematics. The further study might go in this direction.
- The comparison of the perceptions of pre-service and in-service teachers on the use of cultural metaphors in the teaching of mathematics might be the domain for further research.
- Further study needs to be directed towards the study of ethnomathematical ideas and its incorporation in school mathematics curriculum. It is necessary to explore the mathematical ideas embedded in the different occupations and workplaces.

Researcher's Reflections

The particular paradigm that I adopted was interpretative and it involves long-term participation in a field setting. The role of participants is very important in social research work and their experiences and social interaction is central interest in the qualitative research. Without their genuine involvement and contribution in the research process, no work would be completed. During the different stage of my study, I have candid reflection over the selection of the sample school, research participants, teaching and learning strategies in school, field visit approach and overall process of my study.

Reflection about the Selection of Cooperating School

In the beginning of my selection of participants for the research, I found the selection to be rather difficult. The role of cooperating school and the teacher participants have important role in the process of data collection. To locate the suitable school and the teachers who have keen about the cultural metaphors was very

essential component in my research work. During the process of the data collection in the beginning, I found out that the teachers to be involved in my research were not as supportive as I thought they would be. They would hardly take interest on filling up the data form and some would even make excuses to get away from it.

As I found out, there were various reasons for them not participating in my research. There arose different questions for them to participate in my job, e.g., why is it necessary to voluntarily participate in other's task? What incentives do they get for paying their time investment? Teachers seemed to be more professional rather than academic. The grim way to persuade the teachers to make them involved in my research led me to find a perfect school to work on. Then I got to work with the Gyanodaya School's teachers. Firstly, I explained my purpose and then shared what I needed to get my results with the head teacher of the school. He was not comfortable in the beginning and suggested me to share my ideas with the HOD of the mathematics department with other teachers.

Then I met the HOD and she called the teachers meeting at the end of class on the same day. After I shared my ideas, I found them to be different than the teachers from previous schools I tried to work with. They were not like other money minded teachers who valued time with money, rather they believed in academic knowledge than professional way the teaching institutions are turning into. Then I got my green signal, started to work with them, interviewed with some of them during the piloting the survey form and other data tools. I worked in that school for my data collection about three months. I found the school management and teachers more supportive and cooperative. From this study, I have come to know that the selection of sample school is difficult since random selection may not include self-volunteering participants. Hereby, the qualitative researcher adopts purposive sampling for the study. s

Reflection on Selection of Research Participations

Regarding the selection of sample school and research participants, from the field visit, I experienced that the selection of school for any research study is not only in the hands of the researcher. In search of teacher participants for my research, I faced many teachers who simply did not have any interest for volunteering contribution for the research work. I even got that they neglected the volunteering interest thinking that their time would get wasted. The volunteer contribution of the participants and their unbiased perceptions about every issues in the project play vital role not only in the process of research but also in the findings of the study. Thus, the random sampling methods for selection of sample school might not work well because of the varied interests of the school administration and the teacher participants.

After selecting the cooperating school for my study, I had equipped mathematics teachers with the views of cultural metaphor and culture friendly pedagogy. I had conducted a week long TFP with twenty-two participants. But in that time, I even found that all the teachers did not want to give their time to express their perceptions regarding the program as well. By assessing their reflections about the program, they viewed that there is need to incorporate of out-of-school mathematical ideas in classroom teaching through cultural friendly pedagogy. I found that there was dichotomy about the use of cultural metaphor in the teaching and learning of mathematics.

Most of the participants viewed that the culture friendly pedagogy and the funds of knowledge for teaching and learning of school mathematics is effective tool in the process of classroom teaching. But, they do not like to use cultural metaphor in their classroom teaching. The teaching load is one of the most effective factor that do

not let them use culture friendly pedagogy in mathematics teaching. The overloaded curricula are also the hindrance to the use of cultural metaphor in the classroom.

Reflection about the School Pedagogy

It is believed that the teaching and research are two separate disciplines. Teachers do not have serious commitment to their professions. There is need to differentiate between job and profession. Most of the teachers are taking teaching as a job. Their job starts from 10 am up to 4 pm. The good teachers must have enthusiasm about the innovation about teaching and learning approaches developed around the world. But, most of the teachers do not think about what is the effective methods to teach particular content of mathematics. It is lack of traditions of collaborative work in members in mathematics department. The meeting of the members of department is limited to the preparation of test items in terminal examinations. There is very hard to find the traditions of interacting about the ways of teaching and learning of mathematics.

Teachers generally do not accept new ways of teaching and learning approaches than what they followed since long. However, the textbook encourages to the teachers to bring the local knowledge of the students. They mostly use the daily teaching materials in their regular teaching. The teachers were largely unaware of the cultural artefacts, household activities, children cultural knowledge, monuments and its connection to the mathematical contents.

No any research can be all exhaustive and comprehensive. So, this study is not the final. There must be continuity to dig out the effectiveness of cultural metaphors in the process of teaching and learning of school mathematics.

Gestural metaphors used by the teachers seems to be more effective. Teachers frequently use gestural metaphors during the explanation of the concepts of

mathematics. It was observed that the metaphors help to change the teaching practices of the teachers. However, it was due to the effects of a week-long teacher training and workshop to orientate connection of school mathematics through the cultural metaphors. The teachers reflected that the use of cultural metaphors facilitates in the process of teaching and learning of school mathematics. It also enhances the students' level of understanding of the concepts.

Reflection about the CPBL Approach

It was really memorable event for me also in the sense that the experience that I gathered from the involvement of the students in the field visit allowed them to explore the mathematical knowledge from beyond the four walls of the classroom environment. Students from different levels were selected for the field visit which gave them the opportunity for collaborative learning that boosted their happiness since it was their first collaborative study. The response from the students during the field visit while working on the various tasks set up by me with the help of other teachers was phenomenal.

The students were given various tasks which they accomplished by team work and their enthusiasm to find out the results. They were keen to know how they could use mathematical concepts to find out the center of the circular object found on the temple premises which was common tasks set up for the group of students that we divided into. The use of measuring tapes in determining whether the doors in the temple were symmetrical or not was also taken interest by the students during the visit. Also, the artifacts in the temple were source of mathematical knowledge that was developed in the students while performing the tasks. The teachers who were participants in the event were very supportive and played the role of facilitators in answering the problems students arose while performing the tasks. Moreover, the

cultural artefacts connect the teaching and learning of formal mathematical ideas and concepts. The field visit provides the opportunities to learn and practices mathematical ideas in collaborative manner, as they were heterogeneous groups of students from different grade level.

Researcher's Final Words

From the study of literature and the field visit, I identified the cultural artefacts, mnemonics, gestures, and the ethnomathematical ideas of group of people embedded in their activities are the metaphors for the teaching and learning of school mathematics. The ethnographic study identified that the lot of mathematical ideas and knowledge were embedded in everyday activities. The activities of different groups of professional people use implicit mathematical ideas in order to perform their everyday job. The highly sophisticated mathematical ideas were found in the weaving of mat, stone and wood carving, carpentry, mason's work, cultural arts and artefacts and different cultural activities. Those mathematical ideas are being practiced since long while performing their everyday works which helped them to sustain their lives.

From the action research, it was found that the cultural metaphors that were found in the out-of-school context, and constructed by the teachers during the teaching and learning of mathematics facilitated to communicate abstract mathematical ideas. I had designed a lesson for the exploration of mathematical ideas outside the classroom with the collaboration of members of AR projects. I had made an assumption that the existence of the mathematical ideas embedded in out-of-school culture and the students' mathematical competencies would be enhanced with the help cultural metaphors. However, our school curriculum, textbook, examination system, and the school pedagogy are not culture friendly. My teacher participants have some

sort of dichotomy regarding the use of cultural metaphors. The following lines on “The Cultural Conflict: A Poem” reflects what I learnt from my study.

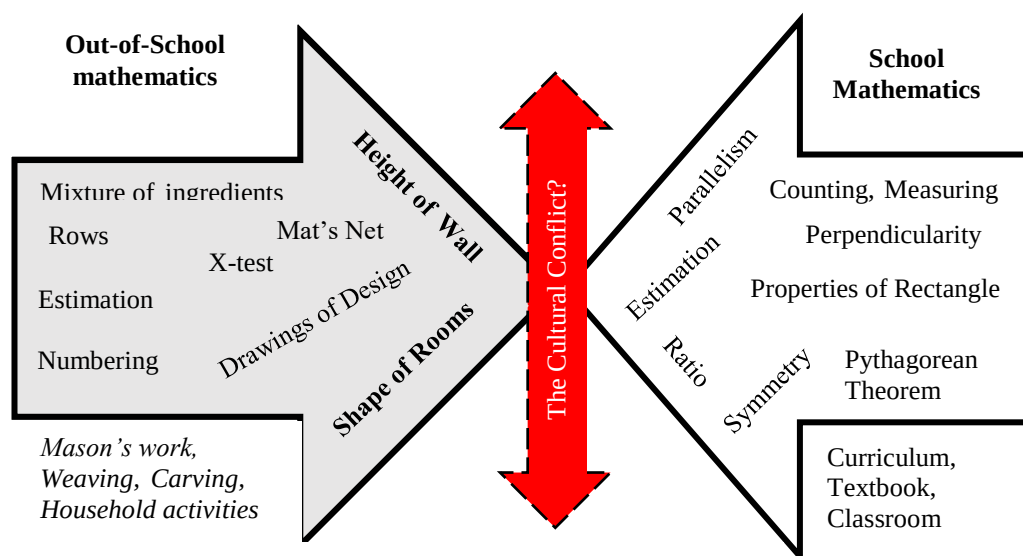


Figure 8.1. The Cultural Conflict?

Object is one, but their names are different

Yeah, they are same

but perceived as different

You say it is a mathematics,

and you see it as mathematics

but, I say it is my doing,

my way of living

because it's my everything

my existence and being.

Yes, I believe I am quite far,

The world where I belong

might be more bizarre,

but the stuffs that I perform

are something like those shining stars!

Infact I just don't see mathematics

I feel it

I just don't solve the mathematics

I carve it

But probably, you might not understand my way and I might not understand yours

There it is, where the conflict glows

The conflict of culture, the life which we blow

The conflict of school and home

Oh no!

We are the same

Together, lets congeal

Yeah, Its my humble appeal

Though we act as two different folds

Believe me, we make this world a shining gold,

A Shining Gold.

Finally, from this study, I envisioned the ideas about cultural metaphors as most prominent mediated tool for mathematical understanding. I observed that the incorporation of cultural metaphors brought change in teaching and learning of mathematics. My attitude in the teaching and learning field was also changed after doing this research. My further direction will be towards the implication of my findings to make school mathematics pedagogy more culture friendly. Due to the limitation of this study, many things are to be figured out about the cultural metaphors that help for the conceptual development of students. In my further study as an independent researcher, my study will be oriented to address the problems and issues that bracketed as the limitations of this study.

REFERENCES

- Abidi, A. H. S. (1996). *Editorial in Makarere University Newsletter*, 26, 1-2.
Makarere, Uganda: Makarere University.
- Acharya, B. R. (2015). *Relevancy of primary level mathematics education in Nepal: A cultural perspective*. Unpublished Doctoral Dissertation, Faculty of Education, Tribhuvan University, Kathmandu, Nepal.
- Adam, S. (2004). Ethnomathematical ideas in the curriculum. *Mathematics Education Research Journal*, 16 (2), 49-68.
- Alangui, W. A. (2017). Ethnomathematics and culturally relevant mathematics education in the Philippines. In M. Rosa, L. Shirley, M. E. Gavarrete & W. V. Alangui (Eds.), *Ethnomathematics and its Diverse Approaches for Mathematics Education* (183-208). Cham, Switzerland: Springer.
- Angrosino, M. & Rosenberg, J. (2011). Observations on observation: Continuities and challenges. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE Handbook of Qualitative Research* (467-478). Thousand Oaks, CA: Sage.
- Araya, R. (2007). Guess what is inside this box: Look at these opened boxes for clues. *CERME*, 5, 101-110
- Arcavi, A. (2002). The everyday and the academic in mathematics. *Journal for Research in Mathematics Education*. Monograph 11, 12-29.
- Ascher, M. & Ascher, R. (1997). Ethnomathematics. In Powel A.B. and Frankenstein, M. (Eds.). *Ethnomathematics: Challenging Eurocentrism in Mathematics Education*. New York, NY: State University of New York.
- Ascher, M. (1991). *Ethnomathematics: A multicultural view of mathematical ideas*. Pacific Grove, California: Brooks/ Cole Publishing Company.

- Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures*. Princeton, New Jersey, NJ: Princeton University Press.
- Atherton, J. S. (2005). *Learning and teaching: Constructivism in learning*. Retrieved from <http://www.learningandteaching.info/learning/constructivism.htm>.
- Atkinson, P. & Delamont, S. (2005). Analytic perspectives. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of Qualitative Research*, 2nd ed. (pp. 821-840). Thousand Oaks, CA: Sage.
- Atkinson, R. K., Derry, S. J., Renkl, A. & Wortham, D. W. (2000). Learning from examples: Instructional principles from the worked examples research. *Review of Educational Research*, 70(2), 181-214.
- Aubusson, P. J., Harrison, A. G., & Ritchie, S. M. (Eds) (2006). *Metaphor and analogy in science education*. Dordrecht, Netherlands: Springer.
- Balaban, N. (1995). Seeing the child, knowing the person. In Ayers, W. *To Become a Teacher: Making a Difference in Children's Lives* (pp. 49-57). New York, NY: Teachers College Press.
- Balbuena, S. E., & Buayan, M. C. (2015). Mnemonics and gaming: Scaffolding learning of integers. *Asia Pacific Journal of Education, Arts and Sciences*, 2 (1), 14-18.
- Balls, M. & Smith, G. (2001). *Analyzing visual data*. Newbury Park, CA: Sage.
- Banks, J. (2008). *An introduction to multicultural education* (4th ed.). Boston, MA: Pearson Education Inc.
- Bartell, T. G. (2011). Caring, race, culture and power: A research synthesis toward supporting mathematics teaching in carrying with awareness. *Journal of Urban Mathematics Education*, 4 (1), 50-74.

- Barton, A. C., & Tan, E. (2009). Funds of knowledge and discourses and hybrid space. *Journal of Research in Science Teaching*, 46(1), 50–73.
- Barton, B. (1996). Making sense of ethnomathematics: Ethnomathematics is making sense. *Educational Studies in Mathematics*, 31, pp.201-233.
- Barton, B. (2005). Mathematics and language: Divergence or convergence? In C. Bergsten & B. Grevholm (Eds.), *Conceptions of mathematics*. Proceedings of NORMA 01. The 3rd Nordic Conference on Mathematics Education, (pp. 96–104). Linköping: SMDF.
- Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. New York, NY: Springer.
- Barton, W. D. (1996). *Ethnomathematics: Exploring cultural diversity in mathematics*. Unpublished Ph. D. Dissertation, The University of Auckland, Auckland, New Zealand.
- Barufaldi, J. (2002). The 5e model of instruction. *Eisenhower Science Collaborative Conference*. Austin, Texas.
- Bearden, L. O. (2012). An examination of the relationship between high school mathematics teachers' dispositions and their metaphors for teaching. *Dissertations, Thesis and Capstone Projects*, Kennesaw State University, Kennesaw, GA.
- Bereiter, C. (2002). *Education and mind in the knowledge age*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Bergsten, C. & Fransson, T. (2005). Students interacting with an artefact designed to visualise three-dimensional analytic geometry. *CERME*, 4, 71- 81.
- Bishop, A. (1988). *Mathematical enculturation*. London: Kluwer Academic Publishers.

- Bishop, A. (1991). Mathematics education in its cultural context. In M. Harris (Ed.), *Schools, mathematics and work* (pp. 29-41). London: Falmer Press.
- Black, M. (1993). More about metaphor. In A. Ortony (Ed.), *Metaphor and Thought*, (pp. 19-43). Cambridge, MA: Cambridge University Press.
- Blasko, D. G. (1999). Only the tip of the iceberg: Who understands what about metaphor? *J. Pragmat.* 31, 1675–1683.
- Bogdan, R., & Biklen, S. (2003). *Qualitative research for education: An introduction to theory and methods* (4th Ed.). Boston: Allyn & Bacon.
- Bonotto, C. (2007). How to replace word problems with activities of realistic mathematical modelling. In W. Blum, P. L. Galbraith, H. Henn & M. Niss (Eds.), *Modeling and Applications in Mathematics Education: The 14th ICMI Study* (185-192). New York, NY: Springer.
- Borich, G. D. & Tombari, M. L. (1997). *Educational psychology: A contemporary approach*. New York, NY: Addison – Wesley Educational Publishers Inc.
- Bose, A. & Subramaniam, K. (2011). Exploring school children's out of school mathematics. In Ubuz, B. (Ed.). *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 177-184. Ankara, Turkey: PME.
- Bose, A. (2014). *Work, knowledge and identity: Implications for school learning of out-of-school mathematical knowledge*. Retrieved on 10 January, 2016 from <http://www.hbcse.tifr.res.in/research-development/ph.d.-theses/bose-phd-synopsis.pdf>
- Bourdieu, P. & Passeron, J. (1990). *Reproduction in education, society and culture* (2nd Ed). London: Sage.

- Bourdieu, P. (1977). *The school as a conservative force: scholastic and cultural inequalities*. Chicago, IL: The University of Chicago Press.
- Boyer, C. B. (1956). *A history of mathematics*. New York, NY: John Wiley and Sons.
- Brooks, J. G. & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classroom*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Brown, J., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, 18(1), 32-42.
- Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.
- Bruner, J. S. (1996). *The culture of education*. Cambridge, MA: Harvard University Press.
- Bruno, A. & Martinon, A. (1999). The teaching of numerical extensions: The case of negative numbers. *International Journal of Mathematics Education in Science and Technology*, 30 (6), 789-809.
- Cajete, G. A. (1994). *Look to the mountain: An ecology of indigenous education*. Skylad, NC: Kivaki Press.
- Calbris, G. (1990). *The semiotics of French gestures*. Bloomington: Indiana University Press.
- Capra, F. (1975). *The Tao of physics: An exploration of the parallels between modern physics and eastern mysticism*. Boulder, Colorado: Shambala Publications.
- Carraher, D.W. & Schliemann, A.D. (2002). Is everyday mathematics truly relevant to mathematics education? In J. Moshkovich & M. Brenner (Eds.) *Everyday and Academic Mathematics in the Classroom*. Monographs of the Journal for Research in Mathematics Education. Reston, VA: NCTM.

- CDC (2005). *National curriculum framework for school education (Pre-primary- 12) in Nepal*. Sanothimi, Bhaktapur: Author.
- CERID (1990). *Elementary process of learning mathematical concepts in Nepal*. Kathmandu, Nepal: Author.
- Chinn, S. J. & Ashcroft, J. R. (1998). *Mathematics for dyslexics: A teaching handbook*. London: Whurr.
- Chiu, M. M. (2000). Metaphorical reasoning: Origins, uses, development and interactions in mathematics. *Education Journal*. The Chinese University of Hong Kong, 28(1).
- Cienki, A. & Muller, C. (2008). Metaphors, gesture, and thought. In Raymond W. Gibbs, Jr. (Editor), *The Cambridge Handbook of Metaphors and Thought*, (pp. 483-501). Cambridge, UK: Cambridge University Press.
- Civil, M. (2002). Culture and mathematics: A community approach. *Journal of Intercultural Studies*, 23(2), 133-148.
- Civil, M. (2007). Building on community knowledge: an avenue to equity in mathematics education. In Nasir, N. S. & Cobb, P. (2007). *Improving access to mathematics: Diversity and equity in the classroom* (pp.105-117). New York: Teachers College Press.
- Clements, D., & Battista, M. (2000). Designing effective software. In R. L. A. Kelly (Ed), *Handbook of Research Design in Mathematics and Science Education* (pp. 761-776). Mahwah, NJ: Lawrence Erlbaum.
- Clements, M. A. & Ellerton, N. F. (1996). *Mathematics education research: Past, present and future*. Bangkok: UNESCO Principal Regional Office for Asia and the Pacific.

- Collier, D. & Elman, C. (2008). Qualitative and multi-method research: Organizations, publications, and reflections on integration. In J. B. Box-Steffensmeier, E. Henry, D. Collier (Eds.), *The Oxford Handbook of Political Methodology* (pp. 779-795). Oxford, UK: Oxford University Press.
- Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches (Third Edition)*. Thousand Oaks, CA: Sage.
- Creswell, J. W. (2009). *Research design: Qualitative, quantitative and mixed methods approaches (Third Edition)*. New Delhi, India: Sage.
- Crowe, M. J. (1975). Ten laws concerning patterns change in the history of mathematics. *Historia Mathematica*, 2 (2), 161-166.
- Cummins, J. (1996). Foreword. In Book, *Affirming diversity: The sociopolitical context of multicultural education (xiv-xviii)*. Boston, MA: Allyn & Bacon.
- Cundy, H. & Rollett, A. (1989). *Mathematical Models (3rd Ed)*. Stradbroke, England: Tarquin Publication.
- D'Ambrosio, U. (1994). Ethnomathematics, the nature of mathematics and mathematics education. In P. Ernest (Ed), *Mathematics, education and philosophy: an international perspective* (pp 230- 242). Bristol, PA: The Falmer Press.
- D'Ambrosio, U. (1998). *Ethnomathematics: The art or technique of explaining and knowing*. Las Cruces: ISGEm.
- D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. In *For the Learning of mathematics*. 5(1): 41-48
- D'Ambrosio, U. (2006). *Ethnomathematics: Link between traditions and modernity*. Sao Paulo: UNICAMP.

- Dahal, R. C. (2007). *Indigenous mathematical concepts of Gopali community from their cultural perspective*. Unpublished M. Phil. Thesis, Tribhuvan University, Faculty of Education, Kathmandu.
- Danesi, M. (2007). A conceptual metaphor framework for the teaching of mathematics. *Studies in Philosophy and Education*, 26 (3), 225-236.
- Daniels, H. (2001). *Vygotsky and pedagogy*. New York, NY: Routledge/Falmer.
- Darlington, Y. & Scott, D. (2002). *Qualitative research in practice: Stories from the field*. Crows Nest, Australia: Allen & Unwin.
- Delaney, K. (2001). Teaching mathematics resourcefully. In P. Gates (Ed.), *Issues in Mathematics Teaching* (pp.123–146). New York, NY: Routledge.
- Denzin, N. K. & Lincoln, Y. S. (2005). Introduction: The discipline and practice of qualitative research. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research*, (pp. 1-32). Thousand Oaks, CA: Sage.
- Droujkova, M. A. & Droujkov, D. (2003). *Using XP with children for learning mathematics*. Paper presented at the 4 th International conference on eXtreme Programming, Genova, Italy.
- Droujkova, M. A. (2004). *Roles of metaphor in the growth of mathematical understanding*. Unpublished Ph. D. Dissertation, North Carolina State University, Raleigh, North Carolina, USA.
- Edwards, L. D. (2005). Metaphors and gestures in fraction talk. The role of metaphors and image in the learning and understanding of mathematics. *CERME*, 4, 92-101.
- Eglash, R. (1997). When Math Worlds Collide: Intention and Invention in Ethnomathematics. *Science, Technology and Human Values*, 22(1), pp. 79-97.
- Eglash, R. (2001). Rethinking symmetry in ethnomathematics. *Symmetry: Culture*

- and Science*, 12 (1-2), 159-166.
- Ellis, M. W. & Berry, R. Q. (2005). The paradigm shifts in mathematics education: Explanations and implications of reforming conceptions of teaching and learning. *The Mathematics Educator*, 15 (1), 7- 17.
- Emenaloa, S. I. (1984). Mathematics education in Nigeria. *International Journal of Mathematical Education in Science and Technology*, 15.
- English, L. D. (1997). Children's reasoning processes in classifying and solving computational word problems. In L. D. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 117-147). Mahwah, NJ: Lawrence Erlbaum Associates.
- Erickson, F. (1986). Qualitative methods in research on teaching. In M.C. Wittrock (Ed.), *Handbook of research teaching* (pp. 119-161). New York: Macmillan.
- ERO (2015). *National assessment of student achievement 2013 (Grade 8: Mathematics, Nepali and Science)*. Sanathimi, Bhaktapur: Author.
- Ezeif, A. N. (2002). Mathematics and cultural nexus: The interactions of culture and mathematics in an aboriginal classroom. *International Education Journal*, 3 (3), 176–187.
- Fasheh, M. (1991). Mathematics in a social context: Math within education as praxis versus math within education as hegemony. In M. Harris (Ed.), *Schools, Mathematics and Work* (pp. 57-61). New York, NY: Falmer.
- Fasheh, M (1997). Mathematics, culture and authority. In Powel A. B. & Frankenstein, M. (Eds.). *Ethnomathematics: Challenging eurocentrism in mathematics education* (pp. 273-290). Albany, NY: State University of New York Press.
- Fetterman, D. M. (2010). *Ethnography: Step by step*. Thousand Oaks, CA: Sage.

- Fischer, M., & Rottmann, J. (2005). Do negative numbers have a place on the number line? *Psychology Science*, 47(1), 22-32.
- Flick, U. (2006). *An introduction to qualitative research*. London: Sage Publications.
- Flick, U., Kardorff, E. V., & Steinke, I. (2004): *A companion to qualitative research* (eds.). London: Sage Publications.
- Font, V., Bolite, J., & Acevedo, J. (2010). Metaphors in mathematics classrooms: Analyzing the dynamic process of teaching and learning of graph functions. *Educational Studies in Mathematics*, 75 (2), 131-152.
- Fontana, A. & Frey, J. H. (2005). The interview: From neutral stance to political involvement. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 695-727). Thousand Oaks, CA: Sage.
- Fraenkel, J. R. & Wallen, N. E. (1993): *How to design and evaluate research in education*. New York, NY: Mc Graw-Hill.
- Freire, P. (1970). *Pedagogy of the oppressed*. New York, NY: Seabury Press.
- Fyhn, A. B. (2007). Sami culture as basis for mathematics teaching. In C. Bergsten, B. Grevholm, H. S. Masøval, & F. Rønning (Eds.), *Relating Practice and Research in Mathematics Education*. Proceedings of NORMA 05, Fourth Nordic Conference on Mathematics Education (pp. 245–256). Trondheim: Tapir Academic Press.
- Gajardo, A. & Dasen, P. (2006). Des ethnomathématiques à l'école ? Enjeux politiques et propositions pédagogiques. *Format et pratique d'enseignement en questions*, 4, 121-138.
- Gannon, M. J. (2004). *Understanding global cultures: Metaphorical journeys through 23 nations (Third Edition)*. Thousand Oaks, CA: Sage

- Gay, G. (2010). *Culturally responsive teaching: Theory, research and practice* (Second Edition). New York, NY: Teachers College Press.
- Gay, J., & Cole, M. (1967). *The new mathematics and an old culture: A study of learning among the Kpelle of Liberia*. New York, NY: Holt, Rinehart and Winston.
- Geary, J. (2011). *I as another: The secret life of metaphor and how it shapes the way we see the world*. New York, NY: Harper Collins Publishers.
- Geeraerts, D. & Cuyckens, H. (2007) (Eds). *The oxford handbook of cognitive linguistics*. New York, NY: Oxford University Press.
- Gerdes, P. (1997). On culture, geometrical thinking and mathematics education. In A. B. Powell & M. Frankenstein (Eds.), *Ethnomathematics: Challenging Eurocentrism in Mathematics Education* (pp. 223-247). Albany, New York, NY: State University of New York.
- Gerdes, P. (1999). *Geometry from Africa: Mathematical and educational explorations*. USA: The Mathematical Association of America.
- Gerdes, P. (2001). Symmetry aspects of Mavuku baskets among the Makhuwa (Mozambique). *Symmetry: Culture and Science*, 12 (1–2), 87–114.
- Ginsburg, H. P. (1982). The development of addition in contexts of culture, social class, and race. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 99-116). Hillsdale, NJ: Erlbaum.
- Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory*. Chicago, IL: Aldine.
- Glesne, C. & Peshkin, A. (1992). *Becoming qualitative researchers: An introduction*. New York, NY: Longman.

- Gonzalez, N. (1995) (Ed.). Educational innovation: Learning from households. *Practicing Anthropology*, 17(3), 3-24.
- Gonzalez, N. (2005). Beyond culture: The hybridity of funds of knowledge. In N. Gonzalez, L. C. Moll, & C. Amanti (Eds.), *Funds of knowledge: Theorizing practice in households, communities, and classrooms* (pp. 29–46). Mahwah, NJ: Erlbaum.
- Gonzalez, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging Funds of Distributed Knowledge: Creating Zones of Practices in Mathematics. *Journal of Education for Students Placed at Risk (JESPAR)*, 6(1-2), 115-132.
- Gonzalez, N., Moll, L. C. & Amanti, C. (2005). *Funds of knowledge: Theorizing practices in households, communities and classrooms*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.
- Grady, J. E. (2007). Metaphor. In Dirk Geeraerts & Hubert Cuyckens (Eds), *The oxford handbook of cognitive linguistics*. New York, NY: Oxford University Press. (Pp. 188-213).
- Greer, B. & Mukhopadhyay, S. (2003). What is mathematics education for? *The Mathematics Educator*, 13 (2), 2- 6.
- Hall, N. (1998). Concrete representations and procedural analogy theory. *Journal of Mathematical Behavior*, 17, 33-51.
- Hardy, G. H. (1978). *Ramanujan: Twelve lectures on subjects suggested by his life and work*. New York, NY: Chelsea Publishing Co.
- Harper, D. (2004). Photography as social science data. In U. Flick, E. V. Kardorff & I. Steinke (Eds.). *A Companion to Qualitative Research*, (pp. 231-236). London: Sage.

- Hausfather, S. J. (1996). Vygotsky and schooling: Creating a social context for learning. *Action in Teacher Education*, (18), 1-10.
- Hayes, D. (2012). *Foundations of primary teaching* (Fifth Edition). Abingdon, London: Routledge.
- Hersh, R. (1997). *What is mathematics really?* New York, NY: Oxford University Press.
- Higbee, K. L. (2001). *Your memory: How it works and how to improve it*. Boston, Massachusetts: Da Capo Press.
- Hong-mei, S. (2010). The cognitive study of metaphor and its application in English language teaching. *Canadian Social Science*. 6(4), 175-179.
- Hutchens, M. (2015). *Using dollar bills to teach place value*. Retrieved from <http://surfingtosuccess.org/2015/08/using-dollar-bills-to-teach-place-value.html>
- Jayaram, V. (2016). Symbolic significance of numbers in Hinduism. Retrieved from <http://www.hinduwebsite.com/numbers.asp>
- Kamii, C. & DeVries, R. (1980). *Group games in early education: Implications of Piaget's theory*. Washington, DC: National Association for the Education of Young Children.
- Kamii, C., & DeVries, R. (1993). *Physical knowledge in preschool education: Implications of Piaget's theory*. New York, NY: Teachers College Press.
- Kapoor, J. N. (1989). *Fascinating world of mathematical sciences* (Vol. 4). New Delhi: Mathematical Science Trust Society.
- Karki, D. (2012). *Know your Nepali games*. Retrieved from http://archives.myrepublica.com/2012/portal/?action=news_details&news_id=39913

- Kemmis, S. & McTaggart, R. (2005). Participatory action research: Communicative action and the public sphere. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 559- 603). Thousand Oaks, CA: Sage.
- Kilhamn, C. (2011). *Making sense of negative numbers*. Unpublished Ph. D. Dissertation, University of Gothenburg, Gothenburg, Sweden.
- Kimball, M. M. (1994). The worlds we live in: Gender similarities and differences. *Canadian Psychology*, 35(4), 388-404.
- Kline, M. (1972). *Mathematical thought from ancient to modern times*. New York, NY: Oxford University Press.
- Knijnik, G. (1997). An ethnomathematical approach in mathematical education: A matter of political power. In A. B. Powell & M. Frankenstein (Eds.), *Ethnomathematics: Challenging Eurocentrism in mathematics education* (pp. 403–410). Albany, NY: State University of New York Press.
- Lai, M. Y. (2013). Constructing meanings of mathematical registers using metaphorical reasoning and models. *Mathematics Teacher Education and Development*, 15 (1), 29-47.
- Lakoff, G. & Johnson, M. (1980). *Metaphors we live by*. Chicago, IL: University of Chicago
- Lakoff, G. & Johnson, M. (1999). *Philosophy in the flesh: The embodied mind and its challenge to western thought*. New York, NY: Basic Books.
- Lakoff, G. & Nunez, R. E. (1997). The metaphorical structure of mathematics: Sketching out cognitive foundations for a mind-based mathematics. In L. English (Ed.) *Mathematical Reasoning: Analogies, metaphors and images* (pp. 21-89). Hillsdale, NJ: Lawrence Erlbaum Associates.

- Lakoff, G. & Nunez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books.
- Lamb, L. C., & Thanheiser, E. (2006). Understanding integers: Using balloons and weights software. *Algebraic Thinking*, 2, 163-164.
- Lareau, A. (2000). *Home advantage: Social class and parental intervention in elementary education* (2nd Edition). Lanham, MD: Rowman and Littlefield Publishers.
- Lave, J. (1996). Teaching as learning in practice. *Mind, Culture, and Activity* (3)3: 149-164.
- LeCompte, M. D., & Schensul, J. J. (1999). *Designing and conducting ethnographic research*. Walnut Creek, CA: AltaMira Press.
- Leitzel, J. (1989). Critical considerations for the future of algebra instruction. In S. Wagner and C. Kieran (Eds.), *Research Issues in the Learning and Teaching of Algebra*. Reston, VA: NCTM.
- Lett, J. (1996). Emic-etic distinctions. In D. Levinson & M. Ember (Eds.), *Encyclopedia of cultural anthropology* (pp. 382–383). New York, NY: Henry Holt and Company.
- Lipka, J. (1994). Culturally negotiated schooling: Toward a Yup'ik mathematics. *Journal of American Indian Education*, 33(3), 14-30.
- Luitel, B. C. & Taylor, P. C. (2007). The shanai, the pseudosphere and other imaginings: Envisioning culturally contextualized mathematics education. *Cultural Studies of Science Education*, 2 (3), 621-638.
- Mack, N. & Woodsong, C. (2005). *Qualitative research methods: A data collector's field guide*. North Carolina, NC: Family Health International.

- Marlowe, B. A. & Page, M. L. (2005). *Creating and sustaining the constructivist classroom* (Second Ed.). Thousand Oaks, CA: Sage.
- Masingila, J. O., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics*, 31, 175-200.
- McDaniel, M. & Pressley, M. (1987) (Eds.). *Imagery and related mnemonic process: Theories, individual differences and applications*. New York, NY: Springer.
- McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. Chicago, IL: University of Chicago Press.
- Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.
- Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco, CA: Jossey-Bass.
- Miles, M. B. & Huberman, A. M. (1994). *An expanded sourcebook: Qualitative data analysis (Second Edition)*. Thousand Oaks, CA: Sage.
- Millroy, W. L. (1992). An ethnographic study of the mathematics of a group of carpenters. *Journal for Research in Mathematics Education Monograph No. 5*. Reston, VA: National Council of Teachers of Mathematics.
- Mills, G. E. (2003). *Action research: A guide for the teacher researcher*. Upper Saddle River, NJ: Merrill/Prentice Hall.
- Moll, L. C., & Greenberg, J. (1990). Creating zones of possibilities: combining social contexts for instruction. In L. C. Moll (Ed.), *Vygotsky and Education* (pp. 319-348). Cambridge, UK: Cambridge University Press.
- Moll, L., Amanti, C., Neff, D., & Gonzalez, N. (2005). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. In N.

- Gonzalez, L. C. Moll, & C. Amanti (Eds.), *Funds of knowledge: Theorizing practice in households, communities, and classrooms* (pp. 71–86). Mahwah, NJ: Erlbaum.
- Mosimege, M. & Lebeta, V. (2000). An ethnographic study of mathematical concepts at the Basotho cultural village. In: S. Mahlomaholo, M. Nkoane and K. Smit (Eds) *Proceedings of the 8th Annual Conference of the Southern African Association for Research in Mathematics and Science Education*. University of Port Elizabeth.
- Mosvold, R. (2007). Teaching mathematics in everyday life. In C. Bergsten, B. Grevholm, H. S. Masøval, & F. Rønning (Eds.), *Relating Practice and Research in Mathematics Education*. Proceedings of NORMA 05, Fourth Nordic Conference on Mathematics Education (pp. 389–399). Trondheim: Tapir Academic Press.
- Mumford, D. (2010). What’s so baffling about negative numbers? A cross-cultural comparison. In C. S. Seshadri (Ed.), *Studies in the history of Indian mathematics*. New Delhi, India: Hindustan Book Agency.
- Naughton, G. M. & Hughes, P. (2009). *Doing action research in early childhood studies: A step by step guide*. New York, NY: Open University Press.
- Neuman, W. L. (2008). *Social research methods: Qualitative and quantitative approaches (Sixth edition)*. New Delhi, India: Dorling Kindersley.
- Nieman, M. M., & Monyai, R. B. (2006). *The educator as a mediator of learning*. Pretoria, South Africa: Van Schaik.
- Nunes, T. (1992). Ethnomathematics and everyday cognition. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: McMillan.

- Nunes, T., Schliemann, A. D. & Carraher, D. W. (1993). *Street mathematics and school mathematics*. New York, NY: Cambridge University Press.
- Nunez, R. E. (2006). Do real numbers really move? Language, thought, and gesture: The embodied cognitive foundations of mathematics. In R. Hersh (edt), *18 unconventional essays on the nature of mathematics*. New York, NY: Springer.
- Orey, D. C. & Rosa, M. (2006). Ethnomathematics: Cultural assertions and challenges towards pedagogical actions. *The Journal of Mathematics and Culture*, VI (1), 57-78.
- Orey, D. C. (2000). The ethnomathematics of the Sioux tipi and cone. In H. Selin (Ed.), *Mathematics across culture: The history of non-western mathematics* (pp. 239–252). Dordrecht, The Netherlands: Kulwer Academic Publishers.
- Orey, D. C., & Rosa, M. (2010). Ethnomodelling: A pedagogical action for uncovering ethnomathematical practices. *Journal of Mathematical Modelling and Application*, 1(3), 58– 67.
- Orey, D. C., & Rosa, M. (2015). Three approaches in the research field of ethnomodelling: Emic (local), etic (global) and dialogical (glocal). *Revista Latinoamericana de Ethnomathematica*, 8 (2), 364–380.
- Ortony, A. (1993). *Metaphor and thought* (2 nd ed.). New York, NY: Cambridge University Press.
- Owens, K. (2015). *Visuospatial reasoning: An ecocultural perspective for space, geometry and measurement education*. New York, NY: Springer.
- Parzysz, B., Pesci, A. & Bergsten, C. (2005). The role of metaphors and images in the learning and understanding of mathematics. *CERME*, 4, 67-71.

- Paudel, K. P. (2008). *Tharu culture: An ethnomathematical perspective*. Unpublished Master Degree Thesis. Kathmandu University, Lalitpur, Nepal.
- Pavic, Z. & Dukic, M. (2016). Cultural capital and educational outcomes in Croatia: A contextual approach. *Sociologia*, 48 (6), 601-621.
- Paznokas, L. (2003). Teaching mathematics through cultural quilting. *Teaching Children Mathematics*, 9 (5), 250-256.
- Peled, I., Mukhopadhyay, S., & Resnick, L. B. (1989). Formal and informal sources of mental models for negative numbers. In G. Vergnaud, J. Rogalski & M. Artigue (Eds.), *The 13 th International Conference for the Psychology of Mathematics Education* (Vol. 3, pp. 108-112), Paris, France: PME.
- Perlota, G. C., Reyes, R. G., Rivera, J. D., Sato, R. A., Solis, M. T., & Vega, A. B. (2008). *The effectiveness of scientific games in teaching chemistry concept*. Undergraduate Thesis, Eulogio “Amang” Rodriguez Institute of Science and Technology, Manila.
- Petrie, H. G. & Oshlag, R. S. (1993). Metaphor and learning. In A. Ortony (Ed.), *Metaphor and Thought* (p. 579-609). Cambridge, MA: Cambridge University Press.
- Phillips D. C. (1995) The good, the bad, and the ugly: The many faces of constructivism. *Educational Researcher*, 24 (7), 5–12.
- Pimm, D. (1981). Metaphor and analogy in mathematics. *For the Learning of Mathematics*, 1(3), 47-50.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York, NY: Routledge.
- Polya, G. (1954). *Mathematics and plausible reasoning: Patterns of plausible inference* (Vol. 2). Princeton, NJ: Princeton University Press.

- Pradhan, J. B. (2010). *Uncovering frozen knowledge of Chundara: An ethnomathematical perspectives*. An unpublished M. Phil. Thesis, FOE, Tribhuvan University, Kathmandu, Nepal.
- Pradhan, J. B. (2012). *Mathematical knowledge generation and distribution of Chundaras: Communalities and disparities from school pedagogy*. University Grants Commission, Bhaktapur, Nepal.
- Pradhan, J. B. (2015). *Ethnomathematical studies on games: Pedagogical implications for classroom teaching*. An unpublished Research, Research Division, Tribhuvan University, Kathmandu, Nepal.
- Pradhan, J. B. (2017). Mathematical ideas in Chundara Culture: Unfolding a Nepalese teaching and learning system. In M. Rosa, L. Shirley, M. E. Gavarrete & W. V. Alanguí (Eds.), *Ethnomathematics and its Diverse Approaches for Mathematics Education* (pp.125-152). Cham, Switzerland: Springer.
- Presmeg, N. (1997). Reasoning with metaphors and metonymies in mathematical learning. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors and images* (pp.267-280). Mahwah, NJ: Lawrence Erlbaum.
- Presmeg, N. (2007). The role of culture in teaching and learning mathematics. In F. Lester, Jr. (Ed.), *Second Handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age Publishing.
- Pritchard, A. & Woollard, J. (2010). *Psychology for the classroom: Constructivism and social learning*. New York, NY: Routledge.
- Resnick, L. B. & Omansson, S. (1987). Learning to understand arithmetic. In R. Glaser (ed.), *Advances in Instructional Psychology* (Vol 3), Erlbaum, NJ: Hillsdale.

- Resnick, L. B. (1987). Learning in school and out. *Educational Researcher*, 16 (9), 13-20.
- Richardson, V. (1994). Conducting research on practice. *Educational Researcher*, 23 (5), 5-10.
- Robson, E., & Stedall, J. (2009). *The oxford handbook of the history of mathematics*. New York, NY: Oxford University Press.
- Rosa, M., & Gavarrete , M. E. (2017). An ethnomathematics overview: An introduction. In M. Rosa, L. Shirley, M. E. Gavarrete & W. V. Alangui (Eds.), *Ethnomathematics and its Diverse Approaches for Mathematics Education* (3-19). Cham, Switzerland: Springer.
- Rosa, M., & Orey, D. C. (2010). Ethnomodeling as a pedagogical tool for the ethnomathematics program. *Revista Latinoamericana de Ethnomatheamtica*, 3(2), 14–23.
- Rosa, M., & Orey, D. C. (2011). Ethnomodeling: The pedagogical action of ethnomathematics as a program. *XIII CIAEM-IACME*, Recife, Brasil.
- Rosa, M., & Orey, D. C. (2012). The field of research in ethnomodeling: Emic, etic, and dialectical approaches. *Educação e Pesquisa*, 38 (4), 865–879.
- Rosa, M., & Orey, D. C. (2013). Culturally relevant pedagogy as an ethnomathematical approach. *Journal of Mathematics and Culture*, 7(1), 74-97.
- Savery, J. R. (1998). Fostering ownership for learning with computer-supported collaborative writing in an undergraduate business communication course. In C. J. Bonk and K. S. King (Edts.), *Electronic collaborators: Learner-centered technologies for literacy, apprenticeship, and discourse* (pp. 103–127). Mahwah, NJ: Lawrence Erlbaum.

- Saxe, G. B. (1989). Transfer of learning across cultural practices. *Cognition and Instruction*, 6 (4), 325-330.
- Schliemann, A. D. & Carraher, D. W. (2002). The evolution of mathematical understanding: Everyday versus idealized reasoning. Invited paper. *Developmental Review*, 22 (2), 242-266.
- Schoenfeld, A. & Arcavi, A. (1988). On the meaning of variable. *Mathematics Teacher*, 81, 420-442.
- Scruggs, T. and Mastropieri, M.A. (1999). Mnemonic strategy instruction in inclusive secondary social studies classes. Retrieved September 11, 2010, from <http://rse.sagepub.com/content/28/6/345.refs>
- Searle, J. (1979). *Expression and meaning: Studies in the theory of speech acts*. Cambridge, England: Cambridge University.
- Seidman, I. (2006). *Interviewing as qualitative research: A guide for researchers in education and the social sciences* (Third Edition). New York, NY: Teachers College Press.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.
- Skemp, R. R. (1987). *The psychology of learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Smith, R. M. (1994). Social Research. *Social Research: An International Quarterly*. 61 (3), 631-670.
- Somekh, B. (2006). *Action research: A methodology for change and development*. New York, NY: Open University Press.
- Sousa, D. A. (2001). *How the special needs brain learns*. California: Corwin Press.

Sowell, E. (1989). Effects of manipulative materials in mathematics instruction.

Journal for Research in Mathematics Education, 20, 498-505.

Spradley, J. (1979). *The ethnographic interview*. New York, NY: Holt, Rinehart &

Winston.

Stern, J. (2000). *Metaphor in context*. Cambridge, MA: MIT Press.

Strasser, R. (2004). Artefacts – instruments – computers. In C. Bergsten et al. (eds.),

Mathematics and Language (212-218). Proceedings of Madif 4, Malmo,

Sweden.

Stringer, E. T., Christensen, L. M. & Baldwin, S. C. (2010). *Integrating teaching,*

learning, and action research: Enhancing instruction in the K-12 classroom.

Thousand Oaks, CA: Sage.

Szendrei, J. (1996). Concrete materials in the classroom. In A. Bishop et al. (eds.),

International Handbook of Mathematics Education (411-435), Dordrecht,

Kluwer.

Taylor, M. G. (1996). The development of children's beliefs about social and

biological aspects of gender differences. *Child Development*, 67 (4), 1555-

1571.

Taylor, S. J., & Bogdan, R. (1998). *Introduction to qualitative research methods: A*

guidebook and resources (3rd ed.). New York, NY: John Wiley & Sons.

Thompson, P. & Lambdin, D. (1994). Research into practice: Concrete materials and

teaching for mathematical understanding, *Arithmetic Teacher*, 41, 556-558.

UNESCO (2008). *Developing culturally contextualized mathematics resource*

materials: Capturing local practices of Tamang and Gopali communities.

Kathmandu: United Nations Educational, Scientific and Cultural Organization

Office.

- Upadhyay, H. P. (2001). *Effect of constructivism on mathematics achievement of grade V students in Nepal*. Unpublished Ph. D. Dissertation, Punjab University, Chandigarh, India.
- Upadhyay, H. P. (2013). A Dialogue: Mathematics as an umbrella concept unifying all the disciplines. *Mathematics Education Forum*, II (34), 1-6.
- Upadhyay, H. P. (2015). Metaphor as a reification of abstract concepts in mathematics. *Proceeding on Second National Conference on Mathematics Education* (1-7). Council for Mathematics Education. Kathmandu, Nepal.
- Upadhyay, H. P., Pradhan, J. B., & Dhakal, B. P. (2012). *Trends of mathematics education*. Kathmandu, Nepal: Balbalika Publication.
- Velez-Ibanez, C. G., & Greenberg, J. B. (1992). Formation and transformation of funds of knowledge among U.S.-Mexican households. *Anthropology and Education Quarterly*, 23 (4), 313-335.
- Vygotsky, L. S. & Kozulin, A. (1986). *Thought and language*. Cambridge, MA: MIT Press.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Vygotsky, L. S. (1997). *Educational Psychology* (R. Silverman, Trans. Original 1926). Boca Raton, FL: St. Lucie Press.
- Wartofsky, M. W. (1979). *Models: Representation and the scientific understanding*, Dordrecht: Reidel.
- Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. New York, NY: Cambridge University Press.
- Wenger, E., McDermott, R., & Snyder, W. M. (2002). *Cultivating communities of practice*. Cambridge, MA: Harvard Business School Press.

- Wertsch J. V. (1997). *Vygotsky and the formation of the mind*. Cambridge, MA: Cambridge University Press.
- Winkle-Wagner, R. (2010). *Cultural capital: The promises and pitfalls in education research*. San Francisco, CA: Jossey-Bass.
- Winter, J., Andrews, J., Greenhough, P., Hughes, M., Salway, L., & Yee, W. C. (2009). *Improving primary mathematics: Linking home and school*. New York, NY: Routledge.
- Wolcott, H. F. (1994). *Transforming qualitative data: Description, analysis and interpretation*. Thousand Oaks, CA: Sage.
- Wolcott, H. F. (2002). Writing up qualitative research better. *Qual Health Res*, 12 (1), 91-103.
- Wolcott, H. F. (2008). *Ethnography: A way of seeing*. Plymouth, United Kingdom: AltaMira Press.
- Wong, K. Y. (1999). Multi-modal strategy. Retrieved from <http://math.nie.edu.sg/kywong/Multimodal%2520SEACME%2520paper%2520with%2520added%2520diagram.pdf>
- Wormeli, R. (2009). *Metaphors and analogies: Power tools for teaching any subject*. Portland, ME: Stenhouse Publishers.
- Yara, P. O. (2009). Relationship between teachers' attitude and students' academic achievement in mathematics in some selected senior secondary schools in south-western Nigeria. *European Journal of Social Sciences*, 11(3), 364-369.
- Yee, S. P. (2012). *Students' metaphors for mathematical problem solving*. Unpublished Ph. D. Theses. College and Graduate School of Education, Kent State University, Kent, Ohio.

Zaslavsky, C. (1996). *The multicultural mathematics classroom: Bringing in the world*. Portsmouth, NH: Heinemann.

Zaslavsky, C. (1998). *Math games and activities from around the world*. Chicago, IL: Chicago University Press.

Zavojewski, J., & Lesh, R. (2003). A models and modeling perspective on problem solving. In M. D. R. Lesh (Eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Teaching, Learning and Problem Solving*. Mahwah, NJ: Lawrence Erlbaum.

Appendix I

Interview protocol for Parents

For the data collection, I prepared some interview guidelines before going to the field. For this, I recorded information from interviews by making handwritten notes, by videotaping, by audiotaping, a digital camera and other required materials to explore mathematical ideas, concepts and knowledge embedded in out-of-school community.

Name of Research Participants/ ID No:

Age:

Place:

Date:

Time of Interview:

Some Sample Questions for in-depth Interview:

- How many family members do you have?
- What is your profession? Other than this?
- How much land do you have?
- How many crops do you produce?
- How do you practice the counting, measuring, locating, designing, playing, and explaining the phenomena in their everyday activities?
- What different artefacts do you make?
- How do you learn to make different objects and artefacts?
- How you create things of different shapes and sizes?
- How do you calculate the measurement?
- How do you measure the area of land? Carpet? ...
- How do you calculate the capacity contains in vessels?
- What do you mean by “a land of a pair of oxen”?
- How do you authenticate your measurement approaches?
- How do you learn the knowledge? And transfer to younger generation?
- How do you teach your children to perform the daily activities?
- Are your children able to assist everyday work?
- Do your children ask the mathematics lesson being learnt at school?
- Do your children feel comfortable to learn your approach to teaching mathematical concepts than the approach learnt in the school?
- How is the knowledge being transferred to the younger generations?
- Do they (children) enjoy in your style of teaching?

And the questions that arise while talking to them.

Appendix II

Observation Checklist: Collection of Cultural Metaphors

Before going to the site, I developed some observation checklist to record my data. I observed the artefacts, everyday activities of the members of the society, their communication with the groups of peoples regarding mathematical concepts, ideas and knowledge, and the way the teaching takes place in the real situation. The cultural metaphors they practising to communicate mathematical concepts and knowledge implicitly was my concerned. I also observed the way that the knowledge generation, practice, drilling, working in a group, working individually and working professionally of their community in general and mathematics classroom in particular. To record the observational data, I engaged in multiple observations during the course of my study on the basis of following:

Demographic Information			Descriptive Notes			Reflective Notes
Date	Time	Place	Participants/ Source	Physical Setting	Account of Particular event or activities	Researcher's Personal note

Appendix III

Interview Guidelines for Mathematics Teachers

Interviewer Name:

Age:

Teaching Experience:

Gender:

Level of Teaching:

Question Format: Semi-structure

Guiding Questions for Interview

1. Is mathematics taught at basic level an abstract entity?
2. Do you use cultural metaphors while teaching and learning of abstract mathematical concepts?
3. How do cultural metaphors help to conceptualize the abstract ideas of mathematics? Could you exhibit the cultural metaphors for the teaching and learning of particular content of school mathematics?
4. How can you manage your classroom teaching in view of incorporating the out of school knowledge in teaching and learning mathematics?
5. Are you believing that the cultural metaphors enhance the learning abstract mathematical ideas to the students?
6. Could you notice any remarkable changes in your classroom practices after taking Teacher Facilitation Program about cultural metaphors and Constructivist Teaching Approaches?
7. Did you see the students' cognitive, emotional and behavioral changes in the process of learning mathematics while you teach mathematics by using cultural metaphors?

Appendix IV

Interview Guidelines for Students

Name:

Age:

Grade:

Gender:

School:

Question Format: Semi-structure

Guiding Questions for Interview

1. In which grade do you study?
2. Which subject do you like the most? What about mathematics?
3. Do you feel mathematics difficult?
4. Which content do you feel difficult? Why do you feel so?
5. Do your parents help you to learn mathematics at home?
6. What is the occupation of your parents?
7. Do you find any mathematics in their job?
8. Do you find any mathematics in everyday household activities?
9. Do you help to solve household mathematical problems at home?
10. Can you recall any mathematics in your kitchen?
11. Does your teacher connect out-of-school knowledge while teaching school mathematics?
12. Does your teacher use cultural artefacts while teaching school mathematics?
13. Have you seen/recognized any mathematical idea in the game play?
14. Do you find any mathematical ideas in the monuments?

Appendix V

Protocol for Focus Group Discussion of Teachers

Date: 6 September 2017

Time of Discussion: 1Hr 30 Minutes

Place/ Location: Gyanodaya Secondary School, Bafal, Kathmandu

Interviewer Group Members: 6 School Mathematics Teachers

Question Format: Semi-structure

Issue: Effectiveness of Incorporation of Cultural Metaphors in Teaching and Learning of Mathematics

Guiding Questions for Group Discussion

1. Selection of metaphors to teach particular content of mathematics.
2. Classroom management and number of students in mathematics classroom.
3. Overloaded mathematics curricula and teaching load of the teachers.
4. Examination system and exam oriented teaching and learning system.
5. Diversity of classroom and teacher should have mathematical knowledge and ideas embedded in students' culture and everyday practices.

Appendix VI

Protocol for Focus Group Discussion of Students

Time of Discussion:

Date:

Place: Gyanodaya Secondary School

Interviewer Group Members: 10 students of grade 6-8

Question Format: Semi-structure

Issue: Effectiveness of Incorporation of Cultural Metaphors in Learning of Mathematics

Guiding Questions for Group Discussion

1. Which subject do you like the most to study? And why?
2. Why is mathematics most interesting (non-interesting) subject for you?
3. Do you find the mathematical ideas in your household activities? What mathematical ideas can you connect to the school mathematics?
4. Do your teachers use the household activities while teaching mathematics?
5. How do you feel when your mathematics teacher give examples from your everyday activities?

Appendix VII
Development of Field Project

Project 1: Exploring Mathematics in Monuments (*Guheshwari* Temple)

Project 2: Cultural Games and Teaching-Learning Mathematics

Project 3: Exploring Mathematical Ideas in Weaving

**Project 4: Student Activity Based Learning: Exploring Mathematical
Ideas**

Project: 1

Exploring Mathematics in Monuments (*Guheshwari Temple*)

Topic: Geometric Shapes and Measurement

Objectives:

4. To explore the different geometrical concepts: congruence and similarity while measuring artefacts.
5. To analyze the contribution of out of school mathematical ideas in the process of teaching and learning formal mathematics at basic level of education.

Location: Purano Gueheshwari Temple, Gol Dhunga, Kathmandu

Special Equipment Needed: Paper, pen/pencil, Measuring Tape, String/ Rope, Scissors

Motivation: Ask the students whether they have ever been visited the Guheshwori temple. Ask the students if anyone knows the temple. After hearing from students, tell them a little religious history of this temple, then, researcher describes the purpose of the field visit to students.

Methods: Select 32 students from each grade of 6 to 10 randomly. Out of them, select eight students from each grade of 6 to 8 and 4 students from grade 9 and 10 each. Divide the students in four different groups. Each group includes 8 members from five different grades, 2 students from grade 6 to 8 each and one from 9 and 10. The group thus formed would be heterogeneous and provide an opportunity for collaborative learning. Assign each group with different task and supply needed equipment.

Provide each students with a paper and a pen and ask them to write geometrical shapes, number of doors, windows, the shapes and arts weave in the walls of temple, and other mathematical ideas found in the surrounding of temple. The mathematics teacher facilitates students to explore more mathematical ideas found outside the classroom.

Role of Researcher: Researcher himself observe the numbers and geometrical pattern found in the temple and its surrounding. Explore the symmetry in the temple and observe the special number relation in the construction of different pattern found in the temple. It would be more interesting to search for golden ratio in the different artefacts.

Student Based Activities: The following activities were given to the students.

1. Count the number of stairs to enter the temple.
2. Measure the length, breadth, and height of each stair.
3. Decide whether each of the stairs have same dimensions.
4. Measure the length and breadth of the temple.
5. How was the height of the temple?
6. Count the number of bell in the temple and its surrounding.
7. Find the base area of the temple.
8. Count the number of door and window in the temple.
9. Measure the dimension of the door and window.
10. Find the area of each of the door and window.
11. Observe the shape of the paving stone in the ground of the temple.
12. Are the shapes of the stone similar?
13. Find the dimension of the stone.
14. Find the area of the ground with temple/ without temple.
15. Trace the location of temple from your school.
16. Draw a tentative map of the temple with reference to your school.

17. Estimate approximate distance between your school and the temple.

Incorporation of Cultural Artefacts in Mathematics Curriculum

After the field visit, researcher asked to the mathematics teachers about the possibilities of incorporation of cultural artefacts for the teaching and learning of the contents of school mathematics at different level.

Grades	Areas	Units	Contents
4	Geometry Algebra Arithmetic		
5	Geometry Algebra Arithmetic		
6	Geometry Mensuration Transformation		
7	Geometry Mensuration Transformation		
8	Geometry Mensuration Transformation		

Reflective Notes of the Students/ Teachers

After the field study, students and teachers were requested to write their reflection about the different themes and issues given in the separate sheet.

1. Mathematical ideas embedded:
2. Feelings of learning mathematics outside the classroom without textbook:
3. Compare the methods of Teaching Learning mathematics: Inside/outside the Classroom

Project: 2

Cultural Games and Teaching-Learning Mathematics

Topic: Geometric Shapes and Algebra

Objectives

1. To explore the mathematical concepts and ideas embedded in cultural games like Chungi, Bagha Chal.
2. To find the pedagogical implications of these games in teaching and learning of mathematical concepts and ideas in the classroom.

Location: Play Ground, Street, Home compound, Village

Special Equipment Needed: Paper, Pen/pencil, Measuring Tape, String/ Rope, Chalk

Motivation:

The purpose of selection of this cultural games played in the various cultural groups in Nepal is to uncover the hidden mathematical ideas embedded in this game and their implication in mathematics teaching. My intention is to see how we can consolidate such cultural games into day to day teaching learning activities in mathematics so as to enhance performance of students in mathematics that will ensure the students to learn mathematical concepts, ideas and knowledge with utmost ease.

Methods:

Ask the common game that the students play. Ask them the rules of the game. Provide them with an opportunity to play game. Observe the whole process of the game play and then give them a little idea of mathematical knowledge embedded in this game. Then, researcher encourages to uncover hidden mathematical ideas associated with the games.

Role of Researcher

The mathematics teacher facilitate to explore more mathematical ideas embedded in the games that students play.

After the observation of the game that students play, researcher request to write the reflection of students and teachers about the cultural games. Encourage teacher to observe its possibility to incorporate in the process of teaching and learning of school mathematics.

Student Activities: The following questions will be asked to the students.

1. Which one is most favorable game to you?
2. How many players do this game needed?
3. Draw a net of games in a ground.
4. Who can start the game?
5. Is the game team or individual?
6. Do you know the rule of the game that you play?
7. Have you known the rule to solve any mathematical problems?
8. Have you seen any relation between games and mathematical concepts you learnt in the school?
9. Do you find any mathematical concepts during the process of the game that you played?

Incorporation of Games in Mathematics Curriculum

After the field visit, researcher asked to the mathematics teachers about the possibilities of incorporation of particular game for the teaching and learning of the contents of school mathematics at different level.

Grades	Areas	Units	Contents
4	Geometry	Geometrical Shapes	Observation the process of game The net of the game
	Algebra	Equations and its solution	Two teams of game [=] at initial stage. Eg. if the team A got 5 points, and the B get 2 point at some interval, how much point do B require to equalize? $\{x + 2 = 5\}$
5	Geometry Algebra		
6	Geometry Mensuration Transformation		
7	Geometry Mensuration Transformation		
8	Geometry Mensuration Transformation		

Reflective Notes of the Students

1. Mathematical Ideas Found in the Cultural Games:
2. Possibilities of Teaching and Learning Mathematics through Games:
3. Which environment do you love to learn mathematics: text book based mathematics in the classroom or Learning through playing games?
4. Has your teacher ever taught mathematics concepts outside the classroom environment?
5. Do you think that the mathematics concepts could be understood better if your teachers adopt the games play way method?

Project: 3 Exploring Mathematical Ideas in Weaving

Topic: Geometric Shapes and its Properties

Objectives

1. To explore the mathematical concepts and ideas embedded in the construction of mat.
2. To find the pedagogical implications of activities regarding the construction of mats and its model in teaching and learning of mathematical concepts in the classroom.

Location: Play Ground, Street, Home compound, Village

Special Equipment Needed: Paper, Pen/pencil, Measuring Tape, String/ Rope, Chalk

Motivation: The purpose of selection of this cultural activities is to uncover the hidden mathematical ideas embedded in the various activities in the students' culture and their implication in mathematics teaching. My intention is to see how we can consolidate such cultural activities into day to day teaching learning activities in mathematics so as to enhance performance of students in mathematics that will ensure that students learn mathematical concepts, ideas and knowledge with utmost ease.

Methods: Divide the students in three different groups. Each group consisting of 5-6 members. Each group have given different assignment with needed equipment. Paper and pen/ pencil are provided to each students and ask them to write geometrical shapes, and mathematical ideas observed in the process of construction of mats.

Ask the common game that the students played. Ask them to explain the rules of the game. After students' explanation, I will tell them the ideas of mathematical knowledge embedded in the game. Then, researcher encourages students to uncover hidden mathematical ideas associated with the games.

Role of Researcher

The mathematics teacher will facilitate students to explore more mathematical ideas outside the classroom.

After the field study, students and teachers were requested to write their reflection about the issues given in the separate sheet.

Student Based Activities: The following task will be given to the students:

1. Measure the dimension of the net of mat
2. Sketch the relation with mathematics do you observe in the construction process?

Incorporation of Cultural Activities in Teaching and Learning of Mathematics

Grades	Areas	Units	Contents
4	Geometry		
5	Geometry		
6	Geometry Mensuration Transformation		
7	Geometry Mensuration Transformation		
8	Geometry		

	Mensuration Transformation		

Reflective Notes of the Students

1. Is there any possibility of learning mathematical ideas easily through the help in the field of games? Give your opinion.
2. Do you ever think that the cultural activities involve mathematical ideas related to our school mathematics?

Project: 4**Student Activity Based Learning: Exploring Mathematical Ideas****Topic:** Algebra**Objectives:**

- To explore the mathematical ideas embedded in students' cultural activities.
- To ensure the possibilities of incorporation of cultural activities and their practices in the teaching and learning of algebraic concepts at basic level of education.

Location: Classroom**Special Equipment Needed**

Paper, Pen/pencil, different type of beans, corn seed, Balance

Motivation

Algebra is symbolic form of arithmetic. The basic algebraic concepts can be found in everyday activities. Students are encourage to bring different beans available in their kitchen. Provide an opportunity to explore different algebraic concepts like variables, algebraic terms, like and unlike terms, algebraic expressions and so on.

Methods

Divide the students in three different groups. Each group consisting of 5-6 members. Each group have given different assignment to bring the different beans available in their own house. Students are requested to write the beans they bring in school. Ask them to write names and amount of beans they bring in their own. Represent any type of bean by a symbol x ; count the number of that bean and write ax and so on

Role of Researcher

The mathematics teacher was facilitating to explore more mathematical ideas outside the classroom. After the field study, students and teachers were requested to write their reflection about the issues given in the separate sheet

Student Based Activities

The following activities were encouraged to the students.

1. Ask the name of the beans that the students bring into classroom
2. Let them to encourage to symbolize per item of each bean.
3. How many of any type of bean (i.e. $7x$)?
4. How many different types of bean be taken by any student?
5. Like and unlike terms ($7x$ and $5y$ are unlike)

Reflective Notes of the Students

1. Is there any possibility of learning and teaching algebraic concepts with the help of such activities?
2. Do you ever think that the students' activities involve mathematical ideas related to our school mathematics?

Appendix VIII

Student Participations and Group Formation for Field Visit

Location: Old Gueheshwory Temple, Gol Dhunga, Kathmandu

Date: 2074-05-13 (29 August, 2017)

Time: 11 AM- 2 PM

Group	Name of Students	Grade	Section	Gender	Remarks
A	Sarajya Gyawali	10	A	F	
	Alisha Poudel	9	B	F	
	Samridhi Sapkota	8	C	F	
	Santosh Thapa	8	A	M	
	Bishal Magrati	7	A	M	
	Aprila Maharjan	7	C	F	
	Samundra Kuwar	6	D	M	
	Krishna Hari Poudel	6	C	M	
B	Sabina Khadki	10	B	F	
	Ranjana Gyawali	9	C	F	
	Dikshya Yadav	8	D	F	
	Sudeep Thapa	8	B	M	
	Rabin Devkota	7	B	M	
	Kopila Pudasaini	7	D	F	
	Anjila Poudel	6	C	F	
	Birendra Yadav	6	A	M	
C	Sandesh Panthi	10	C	M	
	Kusha Rawal	9	D	M	
	Asmita Poudel	8	D	F	
	Dinesh Dhakal	8	A	M	
	Saugat Shrestha	7	B	M	
	Santosh Thapa	7	D	M	
	Mahima Sah	6	B	F	
	Priya Syawa	6	A	F	
D	Bijaya Kingring Magar	10	D	M	
	Jenish Gautam	9	A	M	
	Sushil Adhikari	8	C	M	
	Ritik Panta	8	B	M	
	Sara KC	7	A	F	
	Riya Pariwar	7	B	F	
	Nabina KC	6	B	F	
	Suman Thapa magar	6	D	M	

Appendix IX

Student Awareness Program for Field Visit

Objective of Field Visit: The main objective of field visit is to explore mathematical ideas embedded in the monuments. The students and the mathematics teachers will be able to connect the mathematical ideas in the teaching and learning of school mathematics.

Location: Purano Gueheshwari Temple, Gol Dhunga, Kathmandu

Historical and Archeological Important of Studying Monuments: Ask the students whether they have ever been visited the Guheshwori temple. Ask the students if anyone knows the temple. After hearing from students, tell them a little religious history of this temple. Then, researcher describes the purpose of the field visit to students.

Special Equipment Needed: Paper, pen/pencil, Measuring Tape, String/ Rope, Scissors, Cameras (Still and Video)

Role of Teachers: Select 32 students from different grade level of 6 to 10 randomly. Divide the students in four different groups. Each group consisting of 8 members from five different grades as mentioned in Appendix XI. Each group have given different assignment with needed equipment. I believe that mixing the students of different grade level will provide the opportunity to learn collaboratively.

Paper and pen/ pencil are provided to each students and ask them to write geometrical shapes, number of doors, windows, the shapes and arts weave in the walls of temple, and other mathematical ideas found in the surrounding of temple. The mathematics teacher facilitates students to explore more mathematical ideas found outside the classroom.

Role of Researcher: Researcher himself observe the numbers and geometrical pattern found in the temple and its surrounding. Explore the symmetry in the temple and observe the special number relation in the construction of different pattern found in the temple. It would be more interesting to search for golden ratio in the different artefacts.

Role of Student: The students are encouraged to observe the surrounding of the temple and engage in their assign task:

Task I:

Task II:

Task III:

Task IV:

As given by their teacher.

Searching for Incorporation of Cultural Artefacts in School Mathematics

After the field visit, researcher asked to the mathematics teachers about the possibilities of incorporation of cultural artefacts for the teaching and learning of the contents of school mathematics at different level.

Grades	Areas	Units	Contents
4	Geometry Algebra Arithmetic		
5	Geometry		

	Algebra Arithmetic		
6	Geometry Mensuration Transformation		
7	Geometry Mensuration Transformation		
8	Geometry Mensuration Transformation		

Reflective Notes of the Students/ Teachers

After the field study, students and teachers were requested to write their reflection about the different themes and issues given in the separate sheet.

1. Mathematical ideas embedded:
2. Feelings of learning mathematics outside the classroom without textbook:
3. Compare the methods of Teaching Learning mathematics: Inside/outside the Classroom

Appendix X
Teacher Facilitation Program: Pedagogical Knowledge

Session 1: Exploring Mathematical Ideas in Out-of-School Environment

Session 2: Overview and Orientation of Cultural Metaphors

Session 3: Teaching and Learning Mathematics: Constructivism Approach

Session 4: Demonstration and Self-Assessment (Sample Lesson)

Session 1: Exploring Mathematical Ideas in Out-of-School Environment

Time: 1. 30 hrs.

Objective

At the end of the session, the participants will be able to explore mathematical ideas embedded in an out-of-school environment.

Tools and Instruments

- Handouts (Ethnomathematics/ Out-of- school mathematics)
- Papers (Blank sheets)

Warm Up

- Ask about the development of history of mathematical knowledge.
- Ask about the relation between the mathematics and culture.

Activities

- Facilitator will raise the issues regarding the mathematical ideas embedded in the everyday activities of different group of people.
- Put the participants into two groups and distribute handouts.
- Ask the participants read the given handouts and request them to share their views with the whole group.
- Facilitator will ask the participants to list the mathematical ideas embedded in their environment regarding the different areas of mathematics.

Reflection/ Self-Evaluation

Participants are requested to write their feelings and understanding about the relation between the mathematics and culture. Also, encourage to state their views on the incorporation of out of school mathematical ideas in school mathematics curriculum.

Session 2: Overview and Orientation on Cultural Metaphors

Time: 1.30 hrs.

Objective

At the end of the session, the participants will be able to identify the conceptual metaphors and use it to teach the abstract concepts of mathematics.

Tools and Instruments

- Handouts (PPT)
- Papers (Blank sheets);

Warm Up

- How do we understand about the phenomenon like children (Children are blank slate)
- “We establish the relation between the objects familiar with us to the concepts to be known”. Comment about this observation.

Activities

Activity I: The facilitator will answer the following questions regarding cultural metaphors on the basis of power point slides.

- What is metaphor?
- Where do Metaphors come from?
- Why metaphors are important in teaching and learning of mathematics?

Activity II: The researcher will discuss about the cultural metaphors for the teaching and learning of school mathematics. The researcher will give questionnaire to the participants and let them to collect the metaphors that are familiar in their cultural setting.

Reflection/ Self-Evaluation

- Participants are requested to write down their feeling and understanding about the relation between the mathematics and culture and also their views on the incorporation of out of school mathematical ideas in school mathematics curriculum.

Session 3: Teaching and Learning Mathematics: Constructivist Approach

Time: 1.30 hrs.

Objectives

At the end of the session, the participants will be able

- to define learning theories in teaching mathematics.
- to use constructivist method for teaching and learning mathematics

Tools and Instruments

- Handouts (PPT)
- Papers (Blank sheets)

Warm Up

- Ask about the different situation where the different learning theories in mathematics are best suited
- Ask participants' opinion about the best methods of teaching of mathematics they perform.

Activities

Activity 1: Divide the participants into two/three groups. The facilitator writes the following questions on the board and ask the participants to discuss among their group members.

- a. What methods of teaching mathematics we are using?
- b. Differentiate between teacher centred and student centred methods of teaching?
- c. What are the characteristics of each method of teaching mathematics?
- d. Which methods do you follow most in the process of teaching mathematics?

After 10 minutes, ask each group to present their thoughts for question (a) and facilitator will write all the points on the board. For this question (a) there will be some very specific answers

Activity 2: The facilitator will distribute the handouts regarding the constructivist teaching method and ask them their experiences.

Reflection/ Self-Evaluation

- Participants are requested to write their feeling and understanding about the relation between the mathematics and culture. Also their views on the incorporation of out of school mathematical ideas in school mathematics curriculum.

Session 4: Demonstration of Sample Lesson and Self-Assessment

Time: 1.30 hrs.

Objectives

At the end of the session, the participants will be able

- to prepare lesson plan with the view of cultural metaphors in teaching mathematics.
- to encourage to use constructivist teaching and learning environment in the classroom teaching.

Tools and Instruments

- Handouts (PPT)
- Papers (Blank sheets)

Warm Up

- Ask about the importance of lesson plan before entering to the classroom.
- Ask about the components of lesson plan.

Activities

Activity I: The facilitator presented a model lesson in view of cultural metaphors. The participants are encouraged to evaluate the teaching lesson.

Sample Lesson Plan

Teacher's Name:

Subject: Mathematics

Content: Prime and Composite

Numbers

Grade: 4/5

Number of Students:

Objectives: At the end of this lesson, students will be able to

- Define prime and composite numbers
- Recognise the prime and composite numbers up to 100

Instructional Materials

Colours, Chart Paper, Color sign pen, Geoboard, Rubber Band

Warm Up

The teacher will ask following questions to the students.

- How many types of numbers do you know?
- Ask the students to list different types of numbers that they know. (In general students may list Natural numbers, Whole numbers, Integers, Fractions, Even and Odd, ...)

- Ask about the factors of the given numbers for e.g., 12, 13, 14, 15, 17, 18 etc.
- By the use of geoboard, it is indicated that the area enclosed by the four nearest pins, which form a smallest square, represents 1 sq. unit.
- Six pins taken in a rectangular form represents the area of 2 sq. units, where eight pins taken in the same manner represents the area of 3 sq. units and so on.

Teaching Activities

Activity I: Area Metaphor

- Divide the students into 5 groups each consisting of 7 members. Provide a geoboard and rubber band to each group.
- Students of each group are asked to enclose the area in rectangular form of numbers 3, 5, and 7.
- How many different rectangular forms can be made for the above numbers?
- The teacher will provide the opportunity to each group to share their answers. Take a few minutes to discuss within the groups and come with the conclusion.
- Students of each group are asked to enclose the area in rectangular form of the numbers 4, 6, 8, and 9.
- How many different rectangular forms can be obtained for the above numbers?
- The teacher will provide the opportunity to each group to share. Take a few minutes to discuss within the groups and come with the conclusion.

Prime numbers have only two ways of representing the rectangular form.

Composite numbers have more than two ways of representing the numbers in rectangular form.

Activity II: Color Metaphors for Prime and Composite Numbers

The teacher will ask about the primary colours. It should be clear that the traditional colour theory states that there are three types of primary colours. These are the colours of red, yellow and blue. The primary colour is that colour which cannot be created by any combination of other colours. They are the colour equivalent to mathematical prime numbers. Like prime numbers, the basic three colours cannot be reduced into other two colours. These primary colors in different combinations form the building blocks of many other secondary colours. The main objective of this class will be help each student to understand the concept of prime and composite numbers. After this discussion, the teacher then encourages to write the definition of prime and composite numbers.

After some time, the teacher will give an opportunity to each group's reporter (team leader) to share their answer with the whole class.



Definition 1: A number which has only two factors itself and one is called prime number.

Definition 2: A prime number is a natural number that is greater than 1, no positive divisor other than 1 and itself.

Activity III: The sieve Metaphor

The sieve of Eratosthenes's is the most intuitive method to generate prime numbers. Under this method, write down all the numbers and check the numbers one by one and mark the numbers that are divisible by 2, and then 3, and 4 and then 5 and so on. The number thus is not marked; it is prime numbers.

Evaluation:

Ask the meaning of prime and composite numbers.

Examples and Non-examples

Students are encouraged to find the factors of 12 and 13.

The factors of 12 are 1×12 , 2×6 , 3×4 . So, 12 is not prime.

The factor of 13 is 1×13 . So 13 is a prime number.

Check whether prime or composite: 15, 16, 17.

Students are asking to find the prime numbers less than 100.

Reflection/ Self-Evaluation

Participants are requested to write their feeling and understanding about the relation between the mathematics and culture. Also their views on the incorporation of out of school mathematical ideas in school mathematics curriculum.

Appendix XI
Teacher Facilitation Program: Content Knowledge

Session 1: Rounding Off Numbers

Session 2: Prime and Composite Numbers

Session 3: Introduction to Integers

Session 4: Multiplication of Integers

Session 5: Algebraic Expressions

Session 6: Algebraic Equations

Session 7: Symmetry

Lesson Plan 1

Topic: Numbers

Lesson: Rounding Off Numbers

Grade: 3/4/5

Estimated Time: 45 minutes

Expected Outcomes

- To define rounding off numbers.
- To round numbers into nearest 10, 100, and 1000.

Tools and Instruments

Models, Flannel Board, Table watch,

Warm Up

- Students are asked to tell the concept of estimating the length, weight, time, money and so on
- Students are asked to tell the time how the people conceptualised the round-off. (morning, day, evening and night) on the basis of shadow of sun.
- Students are used to draw the number line.

Teaching Learning Activities:**Activity I: Clock Metaphor**

How do the students conceptualise the concept for the rounding off the numbers implicitly in course of viewing the time in an analogue watch? Let us present the watch with adjusted time to the students and ask the questions by showing the nearest 5, 15, 30, 45, 60 minutes and so on.

**Activity II: Number Line Metaphor**

Draw a number line on the black/ white board and ask the questions regarding round off numbers to the nearest 10, nearest 100 and nearest 1000.

Activity III: Path Metaphor

Use path metaphors to conceptualise why the middle number be rounded to the right handed number.

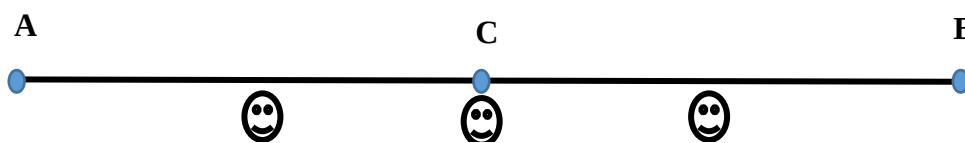


Figure 3: Round Off Numbers as Path Metaphor

Reflections: The participants are encouraged to state their views on the incorporation of other possible metaphors for the teaching of this concept.

Lesson Plan 2

Topic: Whole Numbers

Lesson: Prime and Composite Numbers

Grade: 4 & 5

Estimated Time: 45 minutes

Expected Outcomes:

1. To define Prime and Composite numbers.
2. To identify the prime and composite numbers.

Tools and Instruments:

- Colour box, paper sheet
- Models, Flannel Board, Geoboard, Rubber Bands

Warm Up:

- Ask some prime numbers to the students.
- Ask the definition of prime numbers.
- Students are asked to write the factors of certain numbers (for eg. 12, 15, 17, 18 etc.)

Teaching Learning Activities:

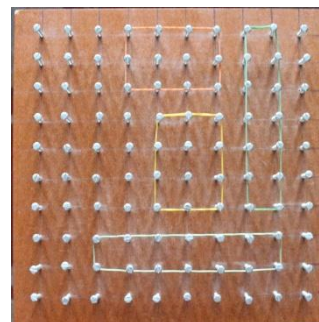
Activity I: Conceptual Metaphors

- Primary colour is PRIME NUMBERS
- Mixing colours is COMPOSITE NUMBERS

Source Domain	Target Domain
Primary colors	Primary numbers
Mixing colors	Taking product of prime numbers
Getting new colour	Composite numbers

Activity II: Geo-board: Area Metaphors

- Use examples and non-examples of prime numbers. For this, let us take an example that the area of certain rectangular garden is (i) 5 sq. m. (ii) 6 sq. m. Students are requested to list all possible rectangular plot covering given area of the garden can have.



Activity III: The sieve of Eratosthenes's is the most intuitive method to generate prime numbers. Under this method, write down all the numbers and check the numbers one by one and mark the numbers that are divisible by 2, and then 3, and 4 and then 5 and so on. The number thus is not marked; it is prime numbers.

Reflections:

The participants are encouraged to state their views on the incorporation of other possible metaphors for the teaching of this concepts.

They are asking to state best metaphors for the teaching and learning of prime and composite numbers.

Lesson Plan 3

Topic: Integers

Lesson: Introduction to Integers

Grade: 6

Estimated Time: 45 minutes

Expected Outcomes

1. To define integers.

Tools and Instruments

- Models, Flannel Board, Balance

Warm Up

- Ask about the natural numbers to the students.
- Let ask them to perform the operations $5-5=?$ $5-8=?$ and other similar questions.
- What does a number zero mean?
- Have you ever been practiced such numbers in your everyday activities?
- Create some examples and related to everyday experiences of students regarding negative integers.

**Teaching Learning Activities****Activity I: Balance Metaphors**

Source Domain	Target Domain
Equilibrium on both sides	Number Zero
Overloaded	Positive Integers
Low	Negative Integers

Activity II: Up- Down and Left- Right: A Gestural Metaphor

Up-Down and Above-Below are activities that can be helpful to conceptualised integers.

Activity III: Number Line is Integers Metaphor

Source Domain	Target Domain
Initial Point/ Origin Point	Number Zero
Right to Initial Point	Positive Integers
Left to Initial Point	Negative Integers

Activity IV: Money is Integer Metaphor

Source Domain	Target Domain
Neither loose nor won money	Zero

You Won Money	Positive Integer
You Lose Money	Negative Integer

Activity V: Temperature is Integers Metaphor

Source Domain	Target Domain
0°C	Number 0
Temperature above 0°C	Positive Integers
Temperature below 0°C	Negative Integers

Reflections:

Students are asked to state best metaphors for the teaching and learning of integers.

Lesson Plan 4

Topic: Integers

Lesson: Multiplication of Integers

Grade: 7

Estimated Time: 45 minutes

Expected Outcomes

1. To define integers.
2. To perform multiplication operation between two negative integers.

Tools and Instruments

- Models, Flannel Board, Number Line

Warm Up

- Tell about the skip counting of numbers
- Ask students to draw number lines in their work-sheet.
- Have you ever practiced multiplication of two negative numbers in your everyday activities?
- Create some examples and related to everyday experiences of students regarding the multiplication of two negative integers.

Teaching Learning Activities**Activity I:** Mnemonics as a Metaphors

चिन्हको गुणन सम्बन्धी नियम			
राम्रो गरे (+) × (+)	=	राम्रो हुन्छ (+) हुन्छ	
राम्रो नगरे (+) × (-)	=	नराम्रो हुन्छ (-) हुन्छ	
नराम्रो गरे (-) × (+)	=	नराम्रो हुन्छ (-) हुन्छ	
नराम्रो नगरे (-) × (-)	=	राम्रो हुन्छ (+) हुन्छ	

Figure : Rule of sign in Multiplication through the Rule of Discipline

Activity II: Mnemonics: Two Like and Unlike Signs

Two like signs make a positive sign

Two, unlike signs, make a negative sign

Like Signs				Unlike Signs			
+	+	=	+	+	-	=	-
-	-	=	+	-	+	=	-

Activity III: Number Line Metaphors

- Discuss how the multiplication of two negatives numbers give positive number through the use of number lines.

Reflections

Students are asked to state best metaphors for the teaching and learning of operations on integers.

Lesson Plan 5

Topic: Algebra

Lesson: Algebraic Expressions

Grade: 4/5

Estimated Time: 45 minutes

Expected Outcomes

1. To define variables.
2. To identify like and unlike terms and relate to their everyday activities.
3. To write algebraic expressions

Tools and Instruments

- Models, Flannel Board, Balance

Warm Up:

- Have you ever been practices mathematics in your everyday activities?
- Create some examples and related to everyday experiences of students regarding the algebraic concepts.

Teaching Learning Activities**Activity I:** Developing Algebraic Concepts: Connecting to Real Life Context

Source Domain	Target Domain
Different types of Beans (For example, corn, Soybeans, green beans, other some pea) →	Variables (Let them be represented by x, y, z etc.)
Green Bean →	Variable x
Soybean →	Variable y
Number of Green Bean →	ax (a is constant)
Number of Soybeans →	$b x$ (b is constant)
Same type of Beans →	Like Terms
Different type of Beans →	Unlike Terms
ax →	Algebraic Terms
Total number of Beans →	Algebraic Expression ($ax + by$)

Reflections:

They are asking to state best metaphors for the teaching and learning of prime and composite numbers.

Lesson Plan 6

Topic: Algebra

Lesson: Algebraic Equations

Grade: 5/6

Estimated Time: 45 minutes

Expected Outcomes:

1. To solve algebraic equations involving single variable.

Tools and Instruments:

- Models, Flannel Board, Balance

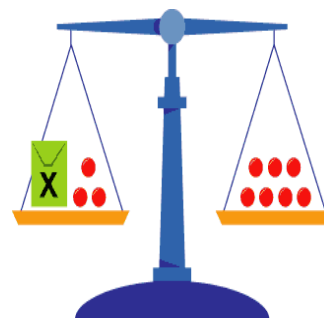
Warm Up:

- Ask the equality relations to the students.
- Create some examples and related to everyday experiences of students regarding the equation involving single variable.

Teaching Learning Activities:**Activity I: Teaching Algebraic Equations: Balance Metaphor**

To find out what the block weighs, the following equality axioms are use:

- Add the same amount to both sides (either X 's or ones)
- Subtract the same amount from both sides (either X 's or ones)
- Multiply both sides by the same number (but not by zero)
- Divide both sides by the same number (but not by zero)

**Reflections:**

They are asking to state best metaphors for the teaching and learning of prime and composite numbers.

Lesson Plan 7

Topic: Geometry

Lesson: Symmetry

Grade: 7

Estimated Time: 45 minutes

Expected Outcomes:

- To define line symmetry
- To find different symmetries in their environment.

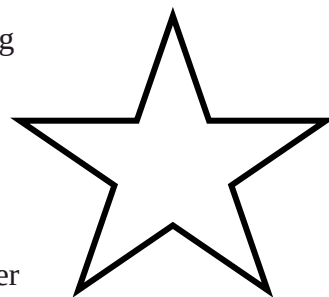
Tools and Instruments:**Brainstorming**

- Could you tell me about the Transformation of an Object?
- What types of transformation do you know? And what are they?
- Can you give me some examples of rotation and reflection of object?
- Have you seen such objects in your home culture?

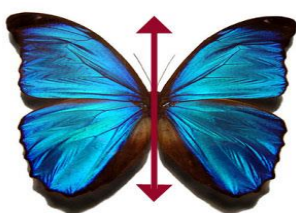
Teaching Learning Activities

Activity I: Activity Based Learning: Great idea for teaching Symmetry

- Students are requested to put their own handkerchief on the desk. Then, ask the questions: Can you see any symmetry in it? How many lines could you draw that would reveal symmetry?
- Look at the bedsheet cover, Cushion cover, and other textiles. Do these textiles show any symmetry?
- Have you ever visualized the artists using symmetry in their paintings? Do symmetries provide a pleasing image?
- Did you find symmetry in the five pointed star? How many different line of symmetry can be drawn here?

**Activity II:** Symmetry in Nature

- Observe and find the different object that shows symmetry in your environment.



Reflections: They are asking to state best metaphors for the teaching and learning of prime and composite numbers.

Appendix: XII
Consent Forms

Commitment and Consent of the School Head Teachers

Commitment and Consent of the School Teachers

Commitment and Consent of the Community Members

Commitment and Consent of Students

Appendix A

Consent of the Parents

Researcher: Jaya Bishnu Pradhan

Title of the Research: Cultural Metaphors for Mathematical Understanding in
Nepalese Context

I have read the information sheet provided by the researcher. He also explained orally about the purpose, nature and possible effects of the study. I agree to cooperate and participate in the discussion about the cultural activities of our community. I give permission for the researchers to include my name attached to selected quotations and the images of mine without dismantling the essence and cultural identity in the final research report.

I understand that the researchers will maintain our identity, confidentiality and any information, I supply to the researchers will be used only for the purpose of the research.

Name: -----

Signature: _____ Date: _____

Address: -----

Appendix B

Commitment and Consent of School Head Teachers

Researcher: Jaya Bishnu Pradhan

Title of the Research: Cultural Metaphors for Mathematical Understanding in
Nepalese Context

I have read the information sheet provided by the researcher. He also explained orally about the purpose, nature and possible effects of the study. With my full understanding, I give permission to the researcher to conduct collaborative action research in our school. I gave him permission to observe the class and conduct the focus group discussions with the students and teachers team of collaborative action research. I understand that the researcher includes my name and my excerpts without dismantling the essence in the final research report.

I understand that the researchers will maintain my identity, confidentiality and any information I supply to the researchers will be used only for the purpose of the research.

Name: _____
(Head Teacher)

Signature: _____ Date: _____

School Name: Gyanodaya Secondary School, Purano Kalimati, Bafal, Kathmandu.

Appendix C

Commitment and Consent of School Teachers

Researcher: Jaya Bishnu Pradhan

Title of the Research: Cultural Metaphors for Mathematical Understanding in
Nepalese Context

I have read the information sheet provided by the researcher. He also explained orally about the purpose, nature and possible effects of the study. With my full understanding and I agree to participate as a critical friend in the collaborative action research that has been designed by the researcher and the participants. I willingly let the researchers to include my name and my excerpts without dismantling the essence in the final research report.

I believe that the researchers will give me the opportunity to review the draft to see how and where my name is used in the report and to resolve any concerns I may have about the use of my name. I understand that the researchers will maintain my identity, confidentiality and any information I supply for the purpose of the research.

Name: -----

Signature: _____ Date: _____

School Name: Gyanodaya Secondary School, Purano Kalimati, Bafal, Kathmandu.

Appendix D
Commitment and Consent of Students
(Focus Group Discussion)

Researcher: Jaya Bishnu Pradhan

Title of the Research: Cultural Metaphors for Mathematical Understanding in
 Nepalese Context

We, the following students of Gyanodaya Secondary School, Purano Kalimati, Bafal, have read the information sheet provided by the researcher. He also explained orally about the purpose, nature and possible effects of the study. With our full understanding, we agree to participate in the field visit project that has been designed by the researcher and our mathematics teachers. We give permission to the researcher to include our name and excerpts without dismantling our essence in the final research report. We understand that the researcher will maintain our identity, confidentiality and any information we supply to the researcher will be used only for the purpose of the research.

S. No.	Name of Participants	Grade	Signature
1			
2			
3			
4			
5			
6			
7			
8			

Appendix XIII

Some Images from the Field Study

1. Meeting to Head teacher for Gaining Access



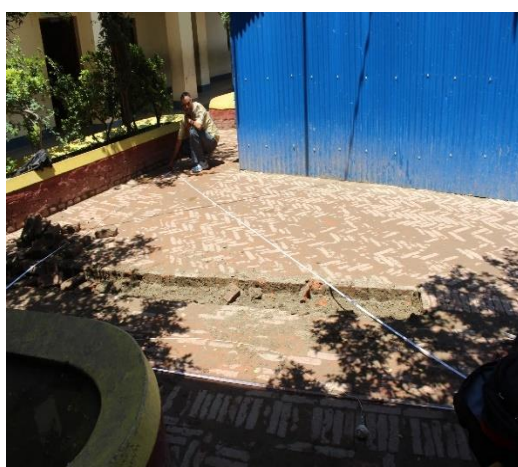
2. Meeting with HOD of



3. Opening Ceremony of TFP



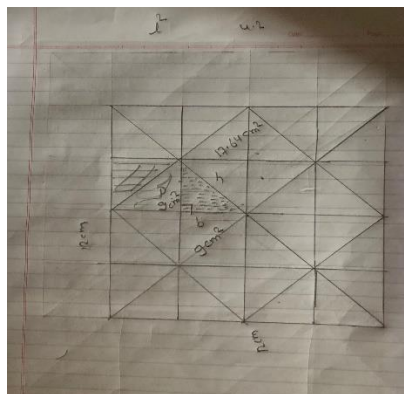
4. 3-4-5 method to determine the Rectangular shape



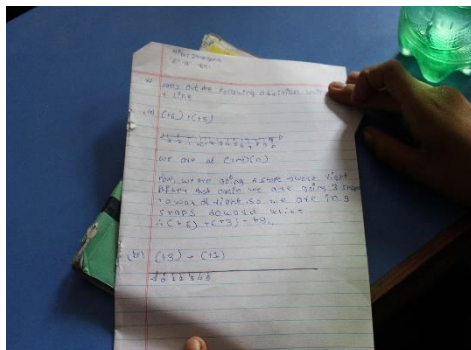
5. Students' are in Peer Learning



6. Construction of Board of Bagha Chal Game and Exploration of Mathematical Ideas



7. Assessing Contribution of Cultural Metaphor in Learning of Mathematics



8. Teachers Facilitation Program in Cooperating School

