

**INVESTIGATING STUDENTS' MATHEMATICAL DIFFICULTIES IN
QUADRATIC EQUATIONS AT GRADE IX**

**A
THESIS
BY
TEK RAJ JOSHI**

**IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE
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Letter of Approval

This thesis entitled “**Investigating Students’ Mathematical Difficulties in Quadratic Equations at Grade IX**” submitted by Mr. Tek Raj Joshi in partial fulfillment for requirement of Degree of Masters in Mathematics Education has been approved.

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Recommendation for Acceptance

This is to certify that **Mr. Tek Raj Joshi**, has completed his M.Ed. thesis entitled
**“Investigating Students’ Mathematical Difficulties in Quadratic Equations at Grade
IX”** under my supervision during the period prescribed by the rules and regulation of
Tribhuvan University, Kirtipur, Kathmandu, Nepal. I recommend and forward his thesis to
the Department of Mathematics Education to organize final viva-voce.

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Krishna Prashad Bhatt

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Defense Date:

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Dedication

Dedicated

To

My parents Mr. Jayanand Joshi, Devaki Devi Joshi, my brothers' Bhakta Raj
Joshi, Shiv Shankar Joshi, my sister in-law Basanti Joshi, my wife Sabina
Lamichhane Joshi and my Nephew Dikshit Joshi whose inspiration, encouragement
and support lead me where
I am today.

Declaration

This thesis does not contain any other work which is offensive and beyond the copyright norms. To the best of my knowledge and belief this research is truly based on my effort and it does not match with any researches that were published earlier in this university as well as other. I take all the ethical and legal responsibility for submitting this thesis.

.....

Tek Raj Joshi

Date:

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.....

Tek Raj Joshi

Abstract

This was a case study related to investigating students' mathematical difficulties with quadratic equation at grade IX. The main purpose of this study was to explore the students' difficulties while learning quadratic equations and to find out the causes of difficulties while learning quadratic equations at grade IX. This study was based on the qualitative nature. The researcher selected the one school from Kanchanpur district by purposive sampling, which was Baijnath Secondary School, Gailnadi. The case for this study was the students of the grade IX. This study was conducted in 24 students at grade IX. The data were collected from students' written test and in-depth interview with students at grade IX. Also in-depth interview was taken with mathematics teacher. Total ten questions related to quadratic equations were used in the written test which was constructed on the basis of research objectives. The collected information's were analyzed at five different levels (Conceptual, Factorization, Completing Square, Quadratic Formula and solving Process Difficulties). The data were analyzed with the help of related literatures. On the basis of test result five respondents were selected for the interview. The collected information through the interview were coded, transcribe and making themes then findings are drawn. The data were analyzed with the help of related literatures.

The findings of this study shows that, the students have conceptual and procedural difficulties in solving quadratic equations at grade IX students such as; unable to factorize, unable to apply factorization principle appropriately, inability to use completing square for solving quadratic equation, students always emphasis on product only rather than process, students remembered the quadratic formula incorrectly, had difficulty in determining the factors through cross multiplication and completing square principles. The causes of difficulties were lack of pre- requisite

knowledge, lack of the basic knowledge about quadratic equation, irregular in school, not sufficient interaction on teacher and student's carelessness in study and students did not have sufficient practices of mathematics.

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Chapter-I

Introduction

Background of the Study

Mathematical learning includes thinking, communicating and expressing mathematically (Akpinar, 2004 as cited in Gunar, 2017). One of the mathematical expression types is algebra. Algebra is an important part of mathematics and a subject to be understood (Chazan, 1996). Although there are many different perspectives related to the definition of algebra, the common points of the definitions are solving equations, finding unknowns and using symbols. In general terms, algebra is considered as generalized arithmetic, a study of procedures for solving problems, the study of relationships among quantities and the study of structure (Usiskin, 1988). It is situated in every domain of life.

Learning of algebra is needed in terms of students due to the fact that algebra establishes a connection between subdomains of mathematics and the other branches of science in terms of theoretical and conceptual learning through its abstract thinking structure, algebra teaching is an important issue (Erbas & Ersoy, 2009). Students have difficulty in understanding algebra and they usually tend to engage in algebra without realizing the real purposes and thinking context (Chazan & Kieran, 1992). According to Kaput (1999) students do not like algebra since it is taught based on the rules and independent from the other domains in mathematics.

In general, students feel the necessity of algebra in terms of accomplishing their objectives such as passing exam, entering a good high school or university (Usiskin, 1988 as cited in Gunar, 2017). However, indeed they do not believe the importance of it due to their thought about uselessness of algebra in daily life. Thus learning algebra must be made worthwhile. On the other hand, students learn

arithmetic thinking and work with numbers at the beginning. As time goes by it replaces algebraic thinking since the development of algebra proceed from concrete to abstract (Didis, 2018). Therefore, this transition is not easy for students and algebra is considered abstract and meaningless. All these difficulties present the greatness of negative situation encountered in algebra teaching (Dede, Yalin & Argun, 2002).

Many students of our schools are unable to understand the mathematical concepts, facts and skills and feel uneasy while solving mathematical problems. Jena, (2013) states that there are many children and adults who experience difficulties with mathematics. A mathematical difficulty refers to children or adults who struggle or fail to cope with some of the aspects of algebra which are necessary for educational or practical purposes. Centeno, (1998) points out that a difficulty is something that inhibits the student in accomplishing correctly or in understanding quickly a given item. Difficulty may be due to several causes: related to concept that is being learned to the teaching method used by the teacher, to the students' previous knowledge, or to his ability.

The history of quadratic equation began in ancient Egypt and Babylon. Quadratic equations have rich historical development process. The past documents showed that the origin of the quadratic equations was based on the concepts of rectangle which had length and width. Depending on known quantities, the unknown such as length, width or area could be calculated. These easy problems led to constitution of quadratic equations and in time, they were transformed into complex structural form (Gunar & Uygen, 2016).

For the first time, a solution of quadratic equation appeared in Berlin Papyrus (Ca.2160-1700 Bc) in Egypt (Smith, 1953 as cited in Gunar, 2016). However, the type

of the examples in this period was pure quadratic equations (Gandz, 1940 as cited in Gunar, 2016).

Quadratic equation is one of the important topics in algebra and quadratic equations have been a fundamental topic, not only in secondary mathematics curricula around the world but also in the historical development of algebra. Various approaches for solving quadratic equations were used at different stages in this historical development, algebraic or symbolic and visual or geometric (Didis & Erbas, 2015). From a contemporary perspective, quadratic equations are considered important in school mathematics curricula because they serve as a bridge between mathematical topics such as linear equation, functions and polynomials (Saglam & Alacaci, 2012).

Furthermore, like linear equation, quadratic equations are powerful representations used in other disciplines, such as physics, engineering and design due to their usefulness in solving many kinds of word problems and modeling realistic or real- life situations (Didis & Erbas, 2015). There are many methods that are normally taught in schools for solving quadratic equations (i) factorization, (ii) completing square and (iii) the quadratic formula. Many researches related to mathematics education show that students have difficulty in quadratic equations and they comprehend quadratic equations as to make a calculation, focus on only symbols in order to solve equation and they are not aware of the essential concepts in quadratic equations (Vaiyavutjamai & Clements,2006).

Therefore, there are many difficulties performed by the students particularly in solving quadratic equations (Zakaria & Maat, 2010). Taylor and Mittag (2001) suggest, the factorization technique is only symbolic in its nature. Since students simply memorize the procedures and formulas to solve quadratic equations, they have little understanding of the meaning of quadratic equations and do not understand what

to do and why. Some researchers have conducted studies on learning quadratic equation and noted that it is an important and interesting research object. Some of those studies indicate that there are many students have difficulties in solving the quadratic equation. The difficulty, for instances, around inability to uncover the relationship between variables & roots, difficulties in solving quadratic equation by factorization, completing square and quadratic formula.

The focus of this study is to investigating students' mathematical difficulties with quadratic equation. Student with learning difficulties is a very general term used widely and without much precision. Usually, the term is applied to those students whose learning problems in school are not directly related to any physical, sensory or intellectual impairment. Through this research I attempted to find learning difficulties of students in grade IX in quadratic equation which consequently affected their learning strategies.

Statement of Problem

Formulating a research problem is the first and most important step in the research process. A research problem identifies our destination. It should tell us our research supervisor & our readers what we intend to research (Kumar, 2011).

Students' difficulties give rise to poor performance in any subject. In mathematics most of these difficulties are attributed to poor algebraic skill and lack of the instrumental and relational understanding of algebraic skill. To make matters worse, almost all topics in mathematics are developed using algebraic concepts. Understanding of algebra in school mathematics is one of the most important goals for mathematics education. On the other hand, algebra has been an obstacle and a challenge for many students, yet we know that many students fail it in the ninth grade. Not only are these students unprepared for algebra, they also come to fear and dislike

it (have a negative attitude towards mathematics) translation problem, carelessness, difficulties stemming from a lack of basic concept and to think of themselves as mathematically weak. Algebra is frequently taught as though it is unrelated to any prior mathematics that students have experienced. The major reasons for students either continuing in mathematics or avoiding mathematics was their own perception about how good they are at mathematics. Therefore, there is a need to recognize difficulties made by students in quadratic equation as well as causes of those difficulties. The researcher is of the opinion that analyzing difficulties encountered by students in solving quadratic equation is one way of achieving this.

It is our experience that no matter how much we teach students to factorize, quadratic formula, completing square they still cannot seem to get the concept and understand it fully. Factorizing in school mathematics is a key technique serving as one of the pillars required to process mathematics.

When I review the many literatures related to difficulties in solving quadratic equation problem in algebra then I have found that students difficulties in factorization, completing square affecting the process of problem solving. Many research shows that there are many causes of difficulties in solving quadratic equation like: lack of conceptual understanding, lack of appropriate formula, reasoning, process of solving, inability to factorize, lack of basic concept of completing square and students always emphasis on product only rather than process and I had also same difficulties to solve the quadratic equation in mathematics. It was very difficulties to solve the quadratic equation when I was in secondary level. Therefore, I would like to study on “Investigating students’ mathematical difficulties with quadratic equations at grade IX”. In this study I want to identify the actual causes of difficulties on solving quadratic equations.

The concern of this study is investigating students' difficulties with quadratic equation at grade IX. Hence the intention of this study was to identify and explore the common difficulties among grade IX student from one secondary school and find out the root causes of these difficulties. The following questions state my research problem.

- What are the common difficulties faced by students' in solving quadratic equations?
- What are the causes of difficulties in learning quadratic equations?

Objectives of the Study

The objectives of this study were as follows;

- To explore the students difficulties while learning quadratic equations.
- To find the causes of difficulties while learning quadratic equations.

Significance of the Study

Different studies and researches are important for their own field. This study provides information about students' difficulties in learning quadratic equation and cause of difficulties in learning quadratic equations at grade IX. This study is significant for the students, teachers and planner who are to know students' difficulties in learning quadratic equations. Research is also beneficial for those who are directly or indirectly involved in the teaching activities of secondary level mathematics. Hence this study has following significance.

- This study helps teacher to find out area of difficulties in learning school algebra and helps to choose appropriate teaching method and strategies, which helps to improve students' understanding and learning.

- This study helps students to dig out own mistakes and helps to minimize mistakes in dealing with algebraic equation in solving quadratic equations.
- This study supports for policy makers, educators, curriculum planners, textbook writers and related persons in finding students difficulties in learning and solving quadratic equation and notify them to select content in basis of students' knowledge.

Delimitations of the Study

The study was limited in the following ways.

- The study was focused only on grade IX of secondary level.
- The study was centered with quadratic equation of algebra at grade IX.
- The study was delimited to find out students' difficulties and causes of difficulties in learning quadratic equation.
- The study was based on qualitative data.
- The study was conducted at grade IX of Kanchanpur district.
- This result cannot be generalized by students of other school because different context affected the result.

Operational Definitions of the Related Terms

Quadratic equation. In this study quadratic equation means the equation of second degree. It consists at least one term that is squared. The standard form is $ax^2+bx+c=0$, $a \neq 0$ with a, b and c being constants, or numerical coefficients and x is an unknown variable.

Difficulties. In this study the meaning of difficulties is as the thing or situation that cause problem faced by students during solving quadratic equation, factorization, applying formula, completing the square.

Factorization. In this study factorization is a method of writing quadratic equation into two linear factors. It is a process of finding factors of a number or an expression or an equation. The factors are defined as the integer or expression which when multiplied together result is original number or expression.

Completing Square. In this study completing square is a method of converting a quadratic equation which is not a perfect square into the sum or difference of a perfect square and a constant by adding or subtracting the suitable constant terms.

Misconception. In this study the meaning of misconception is the misapplication of a rule, an over- or under- generalization, misbelief about quadratic equation.

Conceptual Difficulties. A mathematical concept is a collection of meaning that one associates with quadratic equation used in algebra of mathematics, in this study students are able to understanding and what to find and how to find quadratic solutions has no conceptual difficulties.

Procedural Understanding. In this study procedural understanding is the knowledge exercised in the performance of quadratic equation. It can involve more sense, such as problem solving of quadratic equation, solution process, steps of process.

Quadratic Formula. In this study quadratic formula is a method of solving quadratic equation when the equation is not factorized. The formula is as $x =$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a, b, c \text{ belongs to } \mathbb{R} \text{ and } a \neq 0$$

Chapter-II

Review of Related Literature

Literature review is the process of locating, obtaining, reading and evaluating the interest and also called summary form of related articles and journals allied to the topics, which is going to be studied. It gives an overview of what has been said, who the key writer are, what are the prevailing theories and hypothesis, what question are being asked and what methods and methodologies are appropriate and useful as such, it is not in itself primary research but rather is report on other finding. Literature review provides knowledge to find out the different facts in research for further study of task. In this section books, reports and related studies that are reviewed in order to explain the present problem of the study.

It helps to find research gap, conduct the research program and make the better ideas in the research process. The literature is classified into the groups such as empirical literature, theoretical literature and conceptual framework. These will be discussed in the following paragraph.

Empirical Literature

The review of the Empirical literature pertains the systematic summary of scientific researches and true investigations including their topics, the reasons why this study has to have conducted, methods of the study, data collection tools and methods of ensuring their validity and reliability and key findings in the related field.

Didis and Erbas (2015) studied on “Performance and Difficulties of Students in Formulating and Solving Quadratic Equations with One Unknown”. The main purpose of this study was to investigate the performance & difficulties of 10th grade

students in solving quadratic equations with one unknown using symbolic equation and word problem representations. The study used mixed method. The case respondents were selected by convenient sampling. The participants were 217 10th grade students, from different public high schools.

Data was collected through an open- ended questionnaire comprising 8 symbolic equation and 4 word problems; furthermore, semi-structured interviews were conducted with 16 students. The collected data were qualitatively analyzed.

The finding revealed that although students have difficulties in solving both symbolic quadratic equation and quadratic word problem, they performed better in the context of symbolic equations compared with words problems. Student difficulties in solving symbolic problems were mainly associated with arithmetic and algebraic manipulation error. In the word problem, however, students had difficulties comprehending the context and were therefore unable to formulate the equation to solved. Thus, it is concluded that difference in the structural properties of the symbolic equations and word problems representations affected students' performance in the formulating and solving quadratic equation with one unknown.

Also, O'Connor & Norton (2016) did a case study on "Learning difficulties in quadratic equation". The main purpose of this study was to investigate students' difficulties in learning quadratic equation. The researcher used qualitative research design. The case respondents were class XI of 25 students. To collect data researcher used written test to examine their understanding of concepts and procedures associated with this topic. The collected data were analyzed through descriptive analysis.

The findings revealed that students' success was inhibited by errors that arose from a lack of procedural understanding regarding fractions, algebraic processes, and conceptual understanding regarding algebraic conventions. The researcher found and categorized students' difficulties into four patterns: difficulties in fractional reasoning, difficulties in conceptual understanding regarding, algebraic misconceptions regarding and lack of procedural understanding and inefficient techniques, without this pre-requisite knowledge working with and understanding the nature of quadratics was hindered.

Similarly, Usman (2015) worked on "Analysis of Problem Solving Difficulties with Quadratic Equations Among Senior Secondary School Students" in Zaria, Nigeria. The main aim of this study were to analyze the problem solving difficulties in quadratic equation, to determine the extent to which individual students ability affects their performance in solving problems involving quadratic equation. The Study used descriptive survey research design. Jackson- Ashmore model was used for theoretical base. A total of 126 senior secondary 2 mathematics students randomly selected from three private schools in Zaria with a mean age of 17 constituted the sample size for the study.

The mathematics achievement tests (MAT), mathematics competence test (MCT), and problem-solving test in quadratic equation were used for the study. The students were classified as high achievers and achievers using categorization test score designed by the investigator. Data were analyzed using facility values (FV), mean, simple percentage (%) and chi-square statistics.

The findings from the study showed that the students performed poorly in mathematical problems involving quadratic equations. The students had difficulties

not only in defining a problem goal and recall of appropriate mathematical information, but also with the correct strategies required to reason through and solve the problem.

Zakaria & Maat (2010) did a case study on “Analysis of Students’ Error in Learning of Quadratic Equations”. The purpose of the study was to determine the students’ error in learning quadratic equation. The study was case study that used survey method. The samples were 30 form three students from a secondary school in Jambi, Indonesia. Diagnostic test was used as the instrument of this study that included three components: factorization, completing the square and quadratic formula. Diagnostic interview was also used to identify at which level students’ error occur in solving problems. The type of error was based on Newman Error Hierarchy model that includes reading type error, comprehension, transformation, process skill and encoding error. Data was analyzed using descriptive statistics: percentage and frequency.

The findings showed that most students make error in transformation and process skill in solving quadratic equations. There was no error found in reading. The number of students who made encoding error and carelessness was small. The students’ error in solving quadratic equation was due to their weakness in mastering topics such as algebra, fractions, negative numbers and algebraic expansions.

Similarly, Makgakga (2016) entitled “Errors and Misconceptions in Solving Quadratic Equations by completing a square”. The main aim of this study was to diagnose errors learners made in solving quadratic equations by completing a square and found the reasons why those occurred. The study used qualitative study design

that was conducted in five South African schools of Limpopo province in Capricorn district.

The results revealed that learners in those schools experienced problems in solving quadratic equations by completing a square. Findings revealed that teaching approaches used by teachers contributed towards learners' errors and misconceptions they possessed in solving quadratic equation by completing square.

Further, findings showed that learners were not given an opportunity to discuss the concepts and they did not have enough time to practice their work. Other reasons found in the five schools are- learners are afraid to be mocked at by their peers during the lessons, learners themselves not committed to their school work, some have parental role to play at home, and some teachers are not committed as they come to class late and not controlling their works.

Also, Limbu (2018) conducted research on "Students' Difficulties in Learning Algebra at grade VIII". This was a case study on lower secondary school students' difficulties in learning algebra with view to explore the students' difficulties in learning algebra at grade VIII and to find the causes of difficulties in learning algebra at grade VIII. The study used mixed method research design. The study selected two schools from Kathmandu district by using convenience sampling. Written test, face to face interview and class observation were used as the tools to collect the data.

The result indicated number of difficulties categories under each area; under variable, students were unable to differentiate like and unlike term and their simplification. The abstract structure of algebraic expressions posed many problems to students such as distributive errors in bracket expansion, difficulties in operating with negative integers. Inadequate understanding of the uses of equal sign and its

properties were major difficulties in solving equation, students were confused not only lack of technical words and vocabulary but also translating algebraic form with proper sign. Under transition from arithmetic to algebra students were unable to substitute the value in expression and equation.

The causes of difficulties in algebra were due to the lack of pre requisite knowledge, solving ways and rules, traditional teaching method and did not use teaching materials.

Khanal (2018) conducted research on “Difficulties in Solving Word Problem in Algebra”. This was a case study. The study used qualitative research design. The aim of this study was to explore and analyze the difficulties in solving word problem of algebra at grade IX. He selected one school from Kathmandu district by purposive sampling. He took written test and face to face interview with students and teacher. The collected information was analyzed five different levels (Reading and language, conceptual, comprehension, transformation and solving process difficulties). The data were analyzed with the help of theories and related literature.

He concluded that many students were unable to understand the mathematical concepts, facts and skills. The major difficulties of students were deficiency of comprehension difficulties, transformation difficulties and solving process difficulties. The finding of this study showed that there were many difficulties in solving algebra word problem at grade IX students such as: unable to give the meaning of mathematical term properly and unable to choose appropriate operation to solving the problem, inability to use alternative method for solving the problem, students always emphasis on product only rather than process, lack of pre-requisite knowledge, lack of the basic knowledge about algebra, unable to translate into correct

mathematical form, traditional trend of problem solving style, students did not have the sufficient practices of mathematics.

Similarly, Bhandari (2017) did a case study entitled on “Difficulties in Learning Group Theory”. The purpose of this study was to explore the learning difficulties and to analyze the cause of these difficulties in learning group theory. He used written test, in-depth interview for the research tools. He used APOS Theory for theoretical framework. He found that the most prominent difficulty was to understand the group theoretic concepts and lack of link between pre-learned concepts and new perception towards group theory. Reasoning was another difficulty for learning group theory because students were unable to used inductive reasoning due to their weak cognitive structure.

The result showed that the students had difficulty in proving process of abstract ideas. They were unable to used proper symbols as well as quantifiers. Moreover, they were unable to provided sequential logic to reach in up to conclusions. Most of the higher secondary level students neglected to solve the group theoretic problem due to having less interest and motivation toward solving group theoretic problems. They were interested to solve numerical instead.

Also, Pangali (2012) conducted research on “Difficulties in Learning Arithmetic at grade IX”. The study used qualitative research design. The main purpose of this study was to identify the difficulties and its causes in learning arithmetic in grade IX students. The researcher used purposive sampling method to collect data from field. He used written test, face to face interview and observation as research tools. The collected data were analyzed by cross match approaches.

He found that there were many difficulties in learning arithmetic content grade IX students such as: students were not clear handwriting and slow to write, difficulties in languages of arithmetic, they were out of attention in mathematics class, also many organizational difficulties in arithmetic.

The students had some causes of difficulties which were student participation in learning arithmetic, teacher-students interaction interpersonal relation and so on. There was no proper interaction between teacher and students .Most of the students were not sufficiently practice a home, because they were not clear concept of arithmetic, also their parents were illiterate and they were not commanding to their child for learning mathematics.

Reflection

Quadratic equations difficulties defined in this study is presenting students difficulties consisting of one unknown variables with symbolic equation. Quadratic equations are integral part & important in school mathematics curricula because they serve as a bridge between mathematical topics.

Review of the articles, the journals and thesis of my related topic on Investigating students' mathematical difficulties with quadratic equation shows that learning difficulties faced by the students are particularly related to conceptual aspect of algebra, confusion about solution, solving process, and how to make plan to solve, difficulties stemming from a lack of knowledge of basic quadratic equation, concept and covered on previous course, carelessness, lack of comprehension are some of the common difficulties encountered by respondents in solving quadratic equation. Similarly it is due to negative feeling of the students towards algebra of quadratic problem. Thus it is concluded that students feel easy to learn algebra if the above mentioned aspects are addressed.

Theoretical Literature

This chapter deals with the review of related literature. The review of related literature is an essential aspect of document the research findings drawn from the different researchers related to present study. Theoretical literature describes learning theories in mathematics. It supports to construct the framework to achieve the objectives of this study. This chapter also deals with the review of other related literature about facing problems concerning with curriculum, activities, observation and disabled students characteristics etc.

Various textbooks, journals, magazines which are related to the research topic were reviewed. There are many theories which can be used to understand the learning difficulties in mathematics. The researcher used constructivist theory and theory of disability.

Constructivist Learning Theory

This study is informed by constructivist theory of learning. The constructivist theory of learning imply that learners come to new grade not as empty vessels but they come with some pre knowledge acquired in the previous grades, which knowledge they use to assimilate and adapts incoming mathematical concepts (Skemp, 1976). So the knowledge they learn interacts with their prior knowledge and learners try to find the balance; to equilibrate between what they know already and what they are learning now. The process of finding the balance between prior and current knowledge may lead to difficulty and errors. This occurs, for example when a learner uses an unmodified earlier acquired schema to mediate new knowledge when in fact that schema is not appropriate for the extended domain.

This theory assumes that concepts are not taken unchanged directly from experience, but that a person's ability to learn and what they learn depends on the

quality of the ideas that they bring to new experience. Knowledge is not entirely derived from experience but is regarded as a combination of experience as well as ones' current knowledge structures. It is common knowledge that a persons' mind is active most of the time and on each occasion that the mind receive information, the next step is that of processing that information so that it makes sense or is usable. Oliver (1989) asserts that it cannot be assumed that knowledge can be transferred intact from one person to another, where the learner is viewed as a simple recipient to take it as is. This view is also shared by Luneta & Makonye (2012) who suggested that learners come into the classroom with relatively homogeneous important views of mathematics, capable of assimilating and valuing mathematical understanding when the subject is being taught. The assertion that learners do not come into the classroom as "empty vessels" that need to be filled with knowledge is quite realistic during the introduction of new topic. If a teacher throws some probing questions to the learners as a way of assessing what they already know, they will find out that learners will respond to most of the questions, although the responses might not be necessarily correct. The fact that learners provide some responses to the questions given to them confirms that they have some idea of their own when they come into the classroom.

The constructivist theory of learning regards the learner as an active participant in the construction of his/her own knowledge and the learners exiting ideas and ones interact, the new ideas being interpreted and understood in the light of the learners present knowledge with its foundations on the previous experience(Hatano, 1996; Cobb, 1994 as cited in Luneta and Makonye,2012). The learning activity according to the constructivist perspective is not a matter of adding or of stockpiling new concepts to the ones that the learners already have but rather leads to changes in our schema.Siegler (1995) argued that there are two very important aspects that need

to be understood namely assimilation and accommodation which are defined as follows: assimilation is incorporation of some new recognizably familiar encountered idea into an existing schema (interrelated idea in the mind) and accommodation is when the new idea is quite different from the existing schema and cannot be assimilated; then reconstruction and re-organization take place so that a new schema is formed. Thus to the constructivist, learning leads to changes in our schema according to Cobb (1994).

From a constructivist point of view difficulties play an important role in teaching and learning. Since they are the way that learners interact with new mathematical concepts to make sense of them (Oliver, 1989). If the learners mechanically use old schema without adapting them to new situations they make difficulties due to instrumental rather than relational understanding (Skemp, 1976). Constructivism describes 'how learning happens' and 'learners construct the new knowledge'. It is a description of human cognition which approaches to promote the slogan 'learning by doing'.

Vygotsky's Social Constructivism

According to social constructivism, knowledge is best constructed when learners collaborate together. Vygotsky states "Every function's in the child's cultural development appears twice, first on the social level and later on the individual level: first between people (inter-psychological), and then inside the child (intra-psychological). This applies equally to voluntary attention, to logical memory and to the formulation of concepts. All the higher functions originate as actual relations between human individuals."

In social constructivism, there are three main things which are presented in below:

Making meaning:

Learners built up any things meaning from the society. So, there is great role of society in learning.

Tools of cognitive development:

Culture, language and ancestors are the tools of knowledge from them a learner can get a lot of knowledge in learning their mobilization and their nature towards education affects the learning.

The zone of proximal development (ZPD):

ZPD means the difference between capacity of solving problem by learners themselves and solving problem from the supports of others. The full development of the ZPD depends upon social interaction, in that the range of skill can be developed with social guidance or collaboration, thus it often exceeds what can be attained alone. To ensure development in the ZPD, the assistance given must have certain features and they are inter- subjectivity, scaffolding and guided participant (Acharya, 2015).

Reflection

The main view of constructivism learning is that students construct their own understanding of the world, and in turn their own knowledge. Learners come to new grade not as empty vessels but they come with some pre knowledge acquired in the previous grades. Classroom teaching practice is likely to be more effective when it is formed by and understanding of how students learn. Allowing students to write about their understanding of mathematical concepts invites all students, including those with learning difficulties into the mathematical conversation. There is a need for students to be able to communicate mathematical thinking. It allows them to engage in construction of mathematical knowledge.

Mathematical ideas are conceptualized by constructing mathematical concepts. During peer interaction, the ideas taught are either reconstructed or reinforced to enable correction and emphasized to ensure mastery and students' confidence. Thus students easily learn to mathematics. Also students learn with collaboration from society.

Theory of Disability

Disability is the lack of ability to relative to a personal or group standard or norm. It is the functional consequence of an impairment or change in the functioning. The global concept of learning disability includes problems in listening, concentrating, speaking, thinking, memory, reading, writing, arithmetic and social skills and learning disabilities are specific, not global impairments (psychologytoday.com). The term learning disability is used to refer to a range of neurological conditions that affect one or more of the ways that a person takes in, stores or uses information. Learning disabled are those whose function at average or above average intellectual level but have significant specific academic problems coupled with an executive processing deficit (Jeane, S. 2017 as cited in Limbu, 2018).

Jena(2013) learning difficulties is a classification including several areas of functioning in which a person has difficulty learning in typical manner, usually caused by unknown factor or factors. Given the 'difficulty learning typical manner' this does not exclude the ability to learn in a different manner. Therefore, some people can be more accurately described as having a 'Learning difference', thus avoiding any misconception of being disabled with a lack of ability to learn and possible negative stereotyping. It means a disorder in one or more of the basic psychological processes involved in understanding or in using language spoken or written, which may

manifest itself in an imperfect ability to listen, think, speak, read, write, spelling, or to do mathematical calculations.

NJCLD (National Joint Committee on Learning Disability, 1980) defines learning disability as: A heterogeneous group of disorders manifested by significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning or mathematical abilities. These disorders are intrinsic to the individual and presumed to be due to central nervous system dysfunction and may occur concomitantly with other handicapping conditions. It is not the direct result of those conditions or influences. Studies revealed that apparently there is no single cause for all learning disabilities. Learning disability arises due to minimal brain dysfunction which occurs in the central nervous system (Jena, 2013). Learning disabilities tend to run in families and they think that heredity play a role, un-stimulating environment at home may develop learning disability. A child may inherit a learning disability or it can also result from lack of early learning experience that stimulates mental growth and development. They are neither caused by cultural or linguistic differences nor by poor instruction. Statistics reveal that learning disorders are more common in densely populated areas of large towns than in more affluent sections of the society (Khanal, 2017).

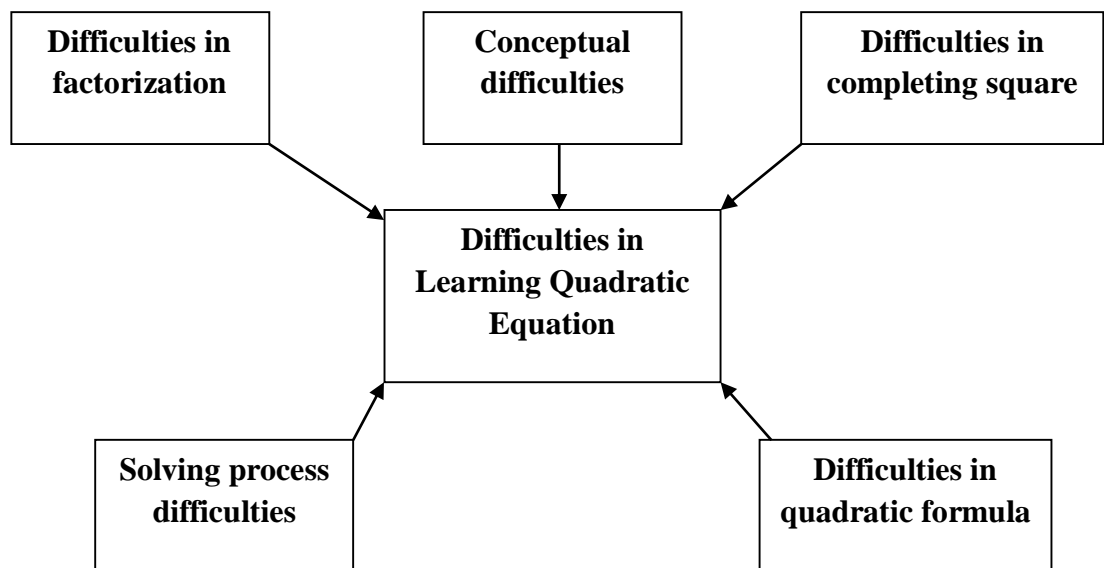
Reflection

Mathematics learning difficulties are common significant and worthy of serious instructional attention in classes. Students may respond to repeated failure with withdrawal of effort, lowered self- esteem, and avoidance behaviors. Math learning problems range from mild to severe and manifest themselves in a variety of ways. Most common are difficulties with efficient recall of basic algebraic facts and reliability in written computation. When these problems are accompanied by a strong

conceptual grasp of mathematical and spatial relations it is important not to bog the student down by focusing only on remediating computation. While important to work on, such efforts should not deny a full math education to otherwise capable students. Finally, learning disability is a burning problem of modern area, one in secondary school mathematics students is likely to suffer this condition. Teacher and parents are becoming more and more aware of the condition. Similarly, the state has also shown sensitivity to the reality of learning disability and has created certain provisions in the policy and practice to help students with learning disabilities.

Conceptual Framework

Conceptual framework refers to the mental picture of the things in consideration. When we think of something an image is created in our mind. The type of mental structure is known as conceptual framework. This study attempted to investigate the students' mathematical difficulties with quadratic equation at grade IX students. These have been described in the different empirical and theoretical literatures. The purpose of the conceptual framework is to identify the difficulties of quadratic equation at grade IX in mathematics. I have constructed the conceptual framework on the basis of the research studies carried out by; Didis & Erbas (2015), O'Connor & Norton(2016), Usman (2015), Zakaria & Maat(2010), Makgakga (2016), Limbu (2018), khanal (2018), Bhandari (2017), Pangali (2012). In this study the researcher used constructivist and disability theory as theoretical framework. From the above empirical & theoretical literature the researcher constructed following framework.



Source: Khanal (2018)

For this study the conceptual framework devised through the literatures studies facilitated to attain research objectives, to get answer of the research questions and carry out the research work as a whole smoothly. Analyzing various literatures in relation to investigate difficulties in quadratic equation, researcher apply constructivism theory. Constructivism claims that knowledge must be actively constructed by learners as they are already “knowing beings” who brings previous knowledge and experience to any learning events. Learning depends on the way the learners interact with situation, beliefs, attitude and previous experiences (Biggs & Moore, 1993 as cited in Acharya, 2015). Constructivism views learning as a process in which the learner actively constructs or builds new ideas or concepts based upon current and past knowledge or experience. The above conceptual framework can be described as below.

Conceptual Difficulties

Mathematics emphasizes the need to build a deep understanding of concepts. This involves making connection among mathematical ideas, facts, skills and reflection upon refining ones' own understanding. In algebra, it is necessary to understand quadratic equation first, what is quadratic equation? How many solutions? , Various forms, null factor law, techniques of solving (O' Connor & Norton,2016). If the students are able to understanding and what to find these problems no conceptual difficulties they are able to solve conceptual problems. Student should have deep understanding of algebraic processes, like terms and variables.

Difficulties in Factorization

Factorizing a quadratic equation required learners to take into consideration all other three terms at the same time. The addition or subtraction of the factors of the first term and last term must give the middle term in the equation. However, the students neglected some of these terms or confused the rules such as multiplication or addition of the number pairs. This kind of difficulties based on their incorrect guesses and no attempt to control the accuracy of linear factors while using the cross-multiplication method. Besides, this failure probably resulted from their lack of conceptual knowledge regarding factorization method. The students confused the concept of factorizing. It is understand that they knew some rules but they could not apply correctly. The students confused how to use the principle of factorization. They do not know how to factorize. If students are able to factorize the given problem it will be considered as no difficulties in factorization.

Difficulties in Completing Square

Solving the quadratic equation from completing square is challenging for many students. This method of solving quadratic equations by completing a square is

helpful as it was appropriately applied in finding the solution to the equations; learners were alerted to use this method appropriately to provide them with the correct answers. Students do not know how to make a square & unable to complete square. Learners should ensure that the coefficient of x^2 is 1 and if it greater or less than 1, they should divide by that coefficient. To solve the quadratic equation by completing the square method encountered different challenges that led them to failure. They have difficulties in forming square, adding correct number or lack of addition of number. If students are able to complete square and find solution of the problem it will be considered as a no difficulties in completing square.

Difficulties in Quadratic Formula

Quadratic formula is another method to solve the quadratic equation. The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The students encountered different challenges while applying the quadratic formula to find the roots of the quadratic equations. Students are tend to memorize quadratic formula, when they cannot gain a procedural and conceptual understanding of quadratic equations. Students either computed the discriminant incorrectly because of calculation difficulty or could not compute it at all, students computed the discriminant correctly, but applied the quadratic formula incorrectly since they had misremembered it. Student confused the concepts of discriminant and root. They also ignore the sign operation & solution of the quadratic equation. If the students are able to apply the quadratic formula and find the correct solution then it will be considered as no difficulties in quadratic formula.

Solving Process Difficulties

Process is more important for problem solving skill. Can students solve quadratic equation? Can students perform the mathematical calculation? In quadratic problem there is difficulties in solving quadratic equation, student solve the quadratic

equation using factorization, completing square, quadratic formula method. The difficulties that occur when students completing misapplication of the mathematical rule in solving process some students fail to even to get first step correct, that is failing to notice the different forms of equations can take, therefore, struggling to further solve the problem. If students are able to correct mathematical operation it will be as no solving process difficulties. If students are unable to do correct mathematical operation, it will be consider as solving process difficulties.

To fulfill second objective of this study, researcher mentions the following categorists.

Pre- Knowledge

Prior knowledge acts as a lens through which we view and absorb new knowledge. Students learn and remember new information best when it is linked to relevant prior knowledge. Teacher who link classroom activities and instruction to prior knowledge and build on their students background knowledge they understand the lesson easily.

A common problem faced by instructor in education is that students lack important prior knowledge and skills needed when they enter the more courses in their subjects. Students who had a more integrated prior knowledge base and were able to operate on higher levels of procedural prior knowledge at the beginning of the course were more likely to successful.

Participation in Learning

Different aspects are interrelated in learning. Social status, educational background of family, learning environment, qualification of teacher etc. are the different factors which affect the learning. If the students participate and show interest in learning, they progress in learning. Students' participation can be defined through

the factor; irregular in school and class, extra classes and tuition classes in addition to class by peer groups and teacher.

Teaching Learning Strategies

Effective teachers are always on the prowl for new and exciting teaching strategies that would keep their students motivated and engaged. They follow methods or strategies such as cooperative learning, inquiry based learning, different instrument use, utilizing technology in classroom.

Mathematics is not limited to learning from textbook, lessons, or testing strategies, students have different learning styles and need to have lessons that help improve all styles to learning to get the best results. When the teacher teach to moving beyond the simple concepts to abstract and in corporate time to test that review the previous class or several classes.

Teacher Students Relation

Instruction is social activity; instruction may be with in person or in group. Personal instruction refers to the mental activity with his/ her mind and soul. It depends upon the person's intellectual capacity. Inter- individual interaction refers to the sharing, adjustment and co-operation. Interaction brings maturity in learning. The students are away from teacher because they do not have time to talk to them as they are hurry to go to another school or class. They also did not want to interest with their teacher. So this is also cause for difficulties in learning quadratic equation.

Chapter-III

Methods and Procedures

This chapter contains the methods and procedures which are helpful to achieve the objectives of the study and to get answers of on the same. An extremely important feature of research is the use of appropriate methods. The selection of an appropriate research design is crucial in enabling you to arrive at valid findings, comparisons and conclusions (Kumar, 2011). When selecting a research design it is important to ensure that it is valid, workable and manageable.

This chapter presents the procedures of the study to achieve the objectives of this study and to get the answer of the statement of problem. The present research was focus on the difficulties in solving quadratic equation at grade IX. It also describes the design of the study, site selection, selection of respondents, data collection tools, quality standard, data collection procedure, data analysis and interpretation procedure.

Design of the Study

This was a case study relating to difficulties and causes of difficulties of grade IX student in solving quadratic equation. Kumar (2011) states that; the case study method is an approach to studying a social phenomenon through an analysis of an individual case. (Creswell, 2009) define case study as “researcher explores in depth a program, an event, an activity, a process, or one or more individual”. The case study can be either a single case or a case bounded by time and place. (Creswell, 2009), suggest the structure of a case study should be the problem, the context, the issues, and the lessons learned. The data collection for a case study is extensive and draws from multiple sources as direct or participant observation, interviews, archival records or documents, physical artifacts, and audiovisual materials. The researcher must

spend time on- site interacting with the people studied. The report would include lessons learned or patterns found that connect with theories.

In this research students ' difficulties in learning quadratic equation in algebra grade IX is a case study because it provides an opportunity for the intensive analysis of many specific details often overlooked by other methods. The approach has a potential to deal with the simple through complex situations. This design of the research based on description qualitative on which meanings derive from the total picture, logic and reasoning.

Selection of School

There are many types of schools in Kanchanpur district such as government, private and community base schools. An objective of research is to explore students' difficulties so selection of school not affected the result. The researcher had taken only one government school of Kanchanpur district. The main respondents of this study were grade IX students of Baijnath secondary school, Gailnadi. This school was selected using purposive sampling method because researcher was familiar with this institution, teachers, and hence its' very easy to gather the in depth information from familiar institution.

Selection of Respondents

The respondents of this research were ninth grade students and teacher for teaching quadratic equation of algebra. There were total 24 students in grade IX for the purpose of this study. Researcher took exam/ test of all these students in their school. Ten open-ended question related to quadratic equations took place in examination and researcher checked the every answer sheet and selected highly difficulties occurred answer sheet. After checked the answer sheet and selected the

researcher selected the five students for interview. The respondents were selected in such a way so that objectives of the study would fulfilled.

Research Tools

This is a case study which is qualitative in nature so that the researcher collected the information only by the primary sources and analyzed with the help of literatures. In this study the following tools were used:

Written Test

Test is a kind of assessment which is given to the students. It is widely used to evaluate the students formally. It helps to examine the knowledge of students on the particular subject matter. The researcher was constructed ten open- ended questions related to solving quadratic equation. Questions were prepared on the basis of research objectives (Appendix-A). In the process of preparation of the questions 4 questions were related to factorization, 3 questions were related to quadratic formula and 3 questions were related to completing the square method. After making the question, the researcher administered it with grade IX students. After getting the response of students the researcher discussed on the students results and the researcher found out difficulties and analyzed them in detail. The written test question is given in Appendix-A.

Interview

Kumar (2011) states that; any person to person interaction between two or more individual with a specific purpose in mind is called an interview. Interview is very flexible, when the interviewer has the freedom to formulate questions as they come to mind around the issue being investigated and on the hand, it can be inflexible, when the investigator has to keep strictly to the questions decided beforehand.

The researcher took the interview on the basis of the objective. The interview was designed on the basis of conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in quadratic formula and solving process difficulties. The researcher took interview with the highly difficulties in written test with selected five students and mathematics teacher. The interview guidelines for students are given in Appendix-B for teacher is given in Appendix-C.

Quality Standard

After completing the construction of the research tools, it is necessary to maintain quality standard of the research, To maintain the quality standards Guba and Lincoln (1998) suggests the following criteria so, I followed these criteria to maintain the quality standard in my research.

Credibility

This concept replaces the idea of *internal validity*; by which researcher seek to establish confidence in the truth of their findings. Guba and Lincoln (1998) recommend several techniques inquirers may use to enhance the credibility of their research: persistent observation, triangulation, peer debriefing, negative case analysis, progressive subjectivity checks and member checking. To maintain credibility of my research I tried to spend as much as time for case respondents and selected school. The data were collected from multiple sources and got several information on related with research problem. After getting information I again met participants to make result realistic. I also asked question to other people and tried to find real difficulties from those information.

Transferability

Transferability replaces the concept of *external validity*. This criterion refers to the applicability of findings in one context (where the research is done) to other

contexts or settings (where the interpretation might be transferred). To maintain transferability I had included photos of taking examination, interview and related person of my research. I had tried to capture most of scenario by using thick description of interviews and my meaning making.

Dependability

This concept replaces the ideas of reliability. This is the third standard for judging qualitative standards and refers to the stability or consistency of the inquiry processes used over time. To maintain it I had taken written test and logic used for selecting participant, interview and include in the study. I would try to maintain credibility and transferability to ensure dependability standard.

Conformability

A fourth standard is conformability, which refers to the quality of the results produced by an inquiry in terms of how well they are supported by informants (members) who are involved in the study and by events that are independent of the inquirer. This is sometimes referred to as audit trail (a record of how decisions were made throughout the study). I am also a part of mathematics students. So, to maintain conformability before concluding information I reviewed those information myself several times and sometimes I conform those information to my other students/ friends before concluding information as well.

Data Collection Procedures

Data collection is the process of gathering and measuring information on variables of interest, in an established systematic fashion that enables one to answer the research objectives. This is a case study so it is qualitative research. Researcher collected data only through primary sources visiting the school site and meeting head

teacher and secondary level mathematics teacher of the school allowing permission for the administration of the test and interview with the students of grade IX.

Test was conducted with 24 students of class IX where ten open-ended questions were from quadratic equation. The researcher marked each students answer sheet and found out different difficulties (Conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in quadratic formula, solving process difficulties). After selected the five students with interview highly difficulties on the test, researcher conducted the face to face interview with the selected these students related the highly difficulties problem and also interview with the mathematics teacher related problem.

Data Analysis and Interpretation Procedures

Data analysis is described as a systematic search for meaning such that the qualitative and /or quantitative data observed may be communicated to others in understandable ways. Analyzing qualitative data requires understanding make sense of text and images so that can form answers to your research questions (Creswell, 2009). This research qualitative research is mostly associated with words, language and experience rather than statistics. The collected information from the primary source such as test and interview were analyzed and interpreted. The researchers collect the data only through the primary sources.

In this study, researcher used the test and interview with student and mathematics teacher to find out the different difficulties, while the collected the data student was provided with test to find out the difficulties in their answer sheet. Interview was conducted with selected each student who was done in maximum difficulties in test and interview was conducted with mathematics teacher to know the view or experience about difficulties made by his students.

The collected data was classified into: conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in quadratic formula, solving process difficulties. The researcher has checked every answer sheet on the basis of above categories. The collected data from selected students and mathematics teacher was analyzed first these all interview from information transcribe and transcribe these information are coding and build description upon themes. These themes were considered as a code and similar code version of respondents students were collected together and explained in their perspectives. At last the main themes were analyzed with the help of the above reviewing literature.

Ethical Considerations

Ethical issues arise in discussions about codes of professional conduct for researchers and in commentaries about ethical dilemmas and their potential solutions (Punch, as cited in Creswell, 2009). In the process of my research, initially I explained the purpose of my study to the participants to convince them. I convinced them that I would not put them at risk. In addition, I used unbiased language or words against gender or age in writing this research. Moreover, I was very honestly explaining the objectives of my research. I respected self-termination and autonomy of the researcher and provide information on all the aspects of the research and its possible result. I never tried to ask about their private information to make them feel embarrassed or uneasy. When analyzing the data obtained from the site, I tried to protect the privacy of the participants. I presented and interpreted the participants understanding, ideas and perceptions as they were expressed and shared. I have cited the literatures that I referred to during my study and references have been maintained. It is considered as the ethic of my research. Being a qualitative researcher I don't claim objective realities.

Chapter-IV

Analysis and Interpretation of Data

Different themes were developed in the process of data analysis with reference to learning difficulties in quadratic equation in algebra. Learning in quadratic equation problem has seen an obstacle and a challenge for many students. Students could not understand the problem. It seems that they just try to solve without understanding. This indicates that due to understanding in quadratic equation in algebra students are not able to solve and are poor in algebra quadratic equation as well as learning and achieving high grades in mathematics. The final themes are conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in quadratic formula and solving process difficulties. The analysis and interpretation of data based on these themes are presented in the chapter. Many students could not understand the conceptual meaning of factorization, completing square problem, some students were careless in their study and examination. In algebra quadratic problem, the researcher found so many difficult areas; conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in quadratic formula and solving process difficulties.

There were 24 students in grade IX. First the researcher took the exam/ test to all students in their school. And researcher checked the answer sheets and selected highly difficulties occurred in the answer sheet. From these answer sheets the researcher selected five students for the interview. They were selected on the basis of mistakes they committed in their answer sheets.

Difficulties in Learning Quadratic Equation Problem of Grade IX students

Quadratic equation problem has been an obstacle and challenge for many students. Lack of many mathematical skills caused difficulties in solving quadratic equation problem. Yet we know that many students fail in the mathematics. In this research, found that the students were poor in quadratic problem in learning mathematics.

Many students did not understand the conceptual meaning of the problem. Some students were careless in their study. They do not know the conceptual meaning of quadratic equation. In quadratic problem, the researcher found so many difficult areas;

Conceptual Understanding Difficulties

A mathematical concept is a construct in mind that has collection of meanings and is associated with a set of symbolic equation used in mathematics. Cangelosi (1996) states that a concept is a category students' mentally construct by creating a class of specific possessing a common set of characteristics. The process, by which creation qualities of actual objects or events are internalized as concepts while other qualities are ignored, is called abstraction. Thus a concept can be described as an abstract meaning, mental picture or idea that an individual has about something. Conceptual knowledge consists of connected relationships between idea and concept internalized by individuals.

Researcher analyzed the conceptual level difficulties on the basis of two categories. First, if the students can understand the problem their own it is considered as no conceptual difficulties. Second, if the students are unable to what they are trying to find out it is considered as conceptual difficulties.

The problem in this level was, solve the quadratic equation by using factorization method: $x^2 + 9x + 20 = 0$. The answer sheet of a student is given below for example;

2. $x^2 + 9x + 20 = 0$

$x^2 + 9x = -20$

$x^2 \times x = -20 - 9$

~~Ques 1~~: $x^2 \times x = -11$

$x^3 = -11$

conceptual + factorization difficulties

Above solution shows the conceptual difficulties level of students. Based on their work and talking about the process, it is the level of students that they did not try to make solution of quadratic equation. They started to solve without understanding the factorization, student does not understand to factorize, rather applies linear algebra thinking to solve for one possible x . This is a lack of conceptual understanding regarding quadratics. Student takes 9 from both sides and also changes the addition to a multiplication sign indicating lack of conceptual understanding regarding basic algebraic processes. Student incorrectly applies integer procedures as $x^2 \cdot x = -20 - 9$ implies $x^2 \cdot x = -11$. This was incorrect. Student now correctly applies index laws on this mistake as $x^3 = -11$. The student has assumed that x^3 is equal to -11 , but cannot proceed. This dead end is a result of conceptual understanding regarding quadratics and algebraic processes and factorization. Thus this problem is related to conceptual difficulties as mentioned by O'Connor and Norton (2016) research the conceptual

level difficulties are describe as the above mentioned problem. In their research also the same condition was found, so we can categories it as the students' conceptual difficulties. In this problem students donot know about coefficient and variable and even how to solve these type of problem in quadratic topic. Learning involves constructing one's own knowledge from one's own experiences. Constructivism learning therefore, is a very personal endeavor, whereby internalized concepts, rules and general principles may consequently be applied in a practical real world context. Child builds new concepts on what they already knows. Constructivism focused on how humans make meaning in relation to the interaction between their experiences and ideas. To understand more about the concern on student conceptual difficulties researcher took the face to face interview with student asking the oral question as mentioned in the following dialogue box.

I: Interviewer, S: Student

I: What is given in this question?

S: in this question, quadratic equation is given sir.

I: What is quadratic equation?

S: the equation whose degree is 2, but I donot know about degree our teacher told so.

I: How many solutions does quadratic equation have?

S: I think quadratic equation have 3 solutions sir.

I: Can you say how to solve this problem?

S: I will try, $x^2 + 9x = -20$. I donot know how to solve this problem, how to factorize and what to find I confused.

I: Why did you feel hard this question?

S: I did not know what was meant by question and did not understand factorize.

In this interview mentioned above with, it was reflected that the student has difficulties in conceptual because student unable to understand problem and unfamiliar with factorization reasoning. O'Connor & Norton (2016) shows that when student unable to understand factorization reasoning because lack of basic number facts, quadratics and its various forms.

After that, researcher interviewed with mathematics teacher relating to this problem. The researcher asked, "What are the causes of conceptual understanding?" He replied that, "*Many students are lacking of pre- knowledge of quadratic equations. The students of this school are belonging from poor family background, they donot learn in home rather they work with his/her family member. They have poor basic knowledge background with quadratic equation.*" Researcher asked, "How can it be done to teach students so that they can understand easily?" He told that, "*First, teacher should teach based on the previous knowledge, teacher will explain what the question said and teacher will also explain that what to find out*".

O'Connor and Norton (2016) have also mentioned if the students are unable to understand the problem it is considered as conceptual difficulties level. This research shows that the main causes of difficulty was conceptual difficulties as students' lack of pre- knowledge of quadratics. Students are poor to perform the connection among the mathematical symbol and operation, do not apply null factor law, techniques of solving, carelessness in the mathematics class and lack of knowledge of quadratic equation and its various forms.

In another problem students' solution was analyzed through their answer sheet. The problem in this level was also linked with conceptual understanding. The

question; solve the quadratic equation by completing square; $x^2 - 7x + 12 = 0$. The sample answer of a student is given below as an example.

5. $x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 2x \cdot \frac{7}{2} + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 = 0$

$\Rightarrow x^2 - \left(\frac{7}{2}\right)^2 = 0$

$\Rightarrow x - \frac{7}{2} = 0$

$\therefore x = \frac{7}{2}$

Conceptual + difficulties in Completing Square

The solution shows the conceptual difficulties related to the problem. In this problem, student adds $(7/2)^2$ to complete square and same number subtract and neglected constant term 12. This was incorrect. Student directly started to solve the quadratic equation by writing x^2 minus $2 \cdot x \cdot 7/2$ add $(7/2)^2$ and subtract $(7/2)^2$. Then after next step student writes x^2 multiply $(7/2)^2$ is equal to zero. Students solved the problem without understanding. Students should know the concept of square and how to complete square. Students should know $(a + b)^2 = a^2 + 2ab + b^2$ and apply this formula to complete square. Students do not know how to complete square. The student could not solve correctly which is in the above example. Thus, this problem is related to conceptual difficulties.

To understand more about the concern on student conceptual difficulties, researcher took the face to face interview with a student asking the oral question as mentioned in the following dialogue box.

I: In this question, what is given?

S: In question, quadratic equation, $x^2 - 7x + 12 = 0$ is given sir.

I: How do you solve this question?

S: By completing square sir

I: What is completing square?

S: Completing Square means addition of Number Square.

I: Why did you feel hard to solve this question?

S: I did not understand, in case of completing square. I did not know how to solve this question and I confused.

After that, I interviewed with mathematics teacher relating to this problem. I asked, “What is the cause of conceptual difficulties?” He replied that, “*Student was confused and donot know how to complete square because of their pre knowledge & carelessness in the class & lack of basic knowledge about integers*”. Again I asked “How to teach to students so that they can understand it easily?” He told that “*Teacher should teach this problem through group discussion and teach student centered teaching method, problem solving method and teaches through basic knowledge*”.

In the same line, O’Connor & Norton (2016) also shows that when the students cannot understand the concept of problem and unable to making the connection among mathematical ideas, facts and reflection upon refining one’s own understanding such as the conceptual difficulties. In the above interview and written solution, the causes of conceptual difficulties are student could not understand the quadratic equation, coefficient and variable, unable to understand its various forms, solving techniques and execute the solution process. Therefore, it shows the difficulty

of student lack of pre knowledge about quadratic equation, could not understand solving techniques of quadratic equation.

Difficulties in Factorization

Factorizing a quadratic equation requires student to take into consideration all other three terms at the same time. For example, the factors of the first term and the last term in the equation added together or sometime subtracted must work out to the middle term. Many students have challenges in factorization; the students have difficulty with basic multiplication table fact retrieval. Difficulty retrieving multiplication facts directly influences students' ability to engage effectively in factorization of quadratics. Since factorization is a process of finding products within the multiplication table. Finally students also find it challenging to recognize and understand varied representation of the same quadratic relationship. Many students encounter difficulties, recalling main multiplication facts, which directly influences their ability to engage in quadratics and since the factorization techniques of solving quadratic equations requires students to be able to rapidly find factors, factoring simple quadratic (i.e. $x^2 + b x + c = 0$, where $b, c \in \mathbb{R}$) become a quite challenge. Researcher analyzed factorization difficulties on the basis of two categories. First, if the students able to factorize quadratics it is considered as no factorization difficulties. If the students are unable to factorize the quadratic equation then it is considered as factorization difficulties.

Almost students have difficulties in factorization. The problem: solve the quadratic equation by using factorization method; $4x^2 - 3 = 4x$. This was not in standard form. The answer sheet of a student is given below.

Q no. 9 $4n^2 - 3 = 4n$
 Soln $4n^2 - 3 = 4n$
 $\Rightarrow 4n^2 - 3 + 4n = 0$
 $\Rightarrow 4n^2 - 4n - 3 = 0$
 $\Rightarrow (2n)^2 - 3 \cdot 2n - 3 = 0 \rightarrow$ difficulties in factorization
 $\Rightarrow 2n^2 - 2n(n-3) + 1(2n-3) = 0$
 $\Rightarrow (2n-3)(n+1) = 0$

 $\therefore (2n-3) = 0 \quad | \quad (n+1) = 0$
 $2n = 3 \quad | \quad n = -1$
 $n = \frac{3}{2}$

The solution shows the factorization as one of the difficulties of students. The students tried to solve the quadratics without understanding the factorization although the equation was factorable, the student factorized this incorrectly. First, student writes this quadratics in standard form as: $4x^2 - 3 = 4x$ implies $4x^2 - 4x - 3 = 0$. Then apply square in the first term $(2x)^2$, the student confused the concept of factorizing. It is understand that they knew some rules but they could not apply correctly. Then the students write $(2x)^2 - 3x - 1x - 3 = 0$ which is wrong, student takes x common to both as $x(x-3) + 1(2x-3) = 0$ which also shows that students had a difficulty to take common factor. Then again write $(2x-3)(x+1) = 0$ and $x = 3/2$, $x = -1$ is the solution of quadratic equation $4x^2 - 3 = 4x$ which is completely incorrect. This level of difficulties matches with the definition of factorization difficulties mentioned by the researcher Didis & Erbas (2015). That is when these solution describe we come to decide that through understand the factorization method but lack of pre knowledge related factorization confirmed that students have factorization difficulty.

Moreover, for justification of students' factorization difficulties in this problem, the researcher took face to face interview with the students.

I: In this question which methods do you used?

S: Factorization sir.

I: How could you solve this question by factorization?

S: First, take the quadratic equation then multiply first term & second term sir.

I: Do you know principle of factorization?

S: I know but I confused.

I: Why did you feel difficulties while solving this problem?

S: I could not understand the factorization, I do not know basic number facts, multiplication of numbers & don't know how to factorize quadratic equation.

From this fact of the interview we came to know that students have difficulties in factorization. After, that researcher interviewed with mathematics teacher relating to this problem. I asked the mathematics teacher, “How to teach to students so that they can factorize easily?” He told that, “*Teacher should teach this problem through student centered and discussion method. The teacher should give pre knowledge about factorization and principle of factorization*”. Again researcher asked “Why do students have factorization difficulties and what its causes? He told that, “*Many students are confused in basic number facts; even they do not know multiplication of numbers, addition and subtraction. They do not know how to factorize because of lack of basic knowledge, carelessness in the classroom and do not do homework in their house.*”

Didis & Erbas (2015) shows that, students face difficulties in factorization because students are unable to find factors, false guess of factors, attempting to factorize non- factorable quadratic equations. The study concluded that the

factorization difficulties is determined if the students are unable to factorize the quadratic equation.

Difficulties in Completing Square

Completing square difficulties are those difficulties which student cannot solve to complete square form as $(a + b)^2 = a^2 + 2 ab + b^2$. Many of the students have challenges for completing square. Students were alerted to use square form.

Researcher described the completing square difficulties in two categories. If the students are able to complete square for solving quadratic equation then it considered as no completing square difficulties. If the students are unable to completing square for solving quadratic equation, it is categorized as completing square difficulties.

The problem: solve the quadratic equation by using completing square; $x^2 + 10x - 39 = 0$. In this problem almost students committed such type of the difficulties. The answer sheet of a student is given below for example.

Solve the quadratic equation by using Completing Square

6. $x^2 + 10x - 39 = 0$

or, $x^2 + 10x = 39$

or, $x^2 + 2 \cdot x \cdot 5 + 5 = 39 + 25$ *difficulties in*

or, $(x+5)^2 = 64$ *Completing Square*

+ चिन्ह लिखें		- चिन्ह लिखें
$(x+5)^2 = 64$		$(x+5)^2 = 64$
$x+25 = 64$		$x+25 = -64$
$x = 64 - 25$		$x = -64 - 25$
$x = 39$		$x = 89$

Above solution shows the completing square difficulties. Student tried to complete square but cannot succeed. Student writes question $x^2 + 10x = 39$ instead of $x^2 + 10x - 39 = 0$. Student takes constant term to right side as $x^2 + 10x = 39$. Then

student tried to complete square as $x^2 + 2.x. 5 + 5 = 35-45$. In first term and second term are right but last term 5 forget to put square as 5^2 and in constant term in right student subtract 45 which is wrong. Then after student form square $(x+5)^2 = \pm 40$. That is student subtract $35-45$ get result 40 and take square root ± 40 which is also wrong. Students' do not know how to complete square and what should be added and subtracted to each side. From the above solution students had difficulties in completing square. Didis and Erbas (2015) shows that when students cannot able to form squared and can't solve problem from completing square then such type of difficulties is known as completing square difficulties.

Moreover, for clear justification of student completing square difficulties face to face interview was taken by researcher with that students asking oral question to him stated as follows:

I: What does the question ask you to do?

S: Solve quadratic equation by completing square that we have to find the value of x.

I: How would you apply completing square?

S: I remember, first I form square then take square root as $x^2 + 10x - 35$ implies $x^2 + 2.x. 5 + 5 = 0$, in this way sir.

I: Can you explain me about solution of question $x^2 + 10x - 39 = 0$?

S: I tried but I confused. Our teacher told so.

I: Why did you feel difficulties while solving this problem?

S: I don't know how to complete square. Then I don't know what number should add on each side and difficulty in taking square root.

From the above interview the researcher came to the conclusion that students had problem in completing square. I asked the question in face to face interview with

mathematics teacher, “Why do students have completing square difficulties?” Then teacher said that, *“Because in attempt to make thing easy for students, teacher has give complete explanation by focusing students concepts about quadratic equation, because of laziness, carelessness.”* Again I asked mathematics teacher “How the students make completing square difficulties?” He told that, *“Teachers teach the quadratic problem with explanation and ask the general and regular question like, understood? Most of the question is answered in group, yes sir but rarely do they think about concept of completing square. So the students’ commits completing square difficulties.”*

Makgaka (2016) research shows that, student difficulties to completing square due to misconception and errors, unable to form squared, not given an opportunity to discuss the concepts and other describe as the above mentioned problem. The same condition was found in their research also. So we can categories it as the students completing square difficulties.

In the interview mentioned above it was found that the student has difficulties in completing square because the students could not form square, lack of basic knowledge about completing square, carelessness. So the researcher concluded that the student has difficulties.

The result out of interview and written solution shows that the similar fact. The main reason of completing square difficulties is students’ were not given an opportunity to discuss the concepts and they did not have enough time to practice their work. Students are afraid to be mocked at by their peers during the lessons, carelessness.

Difficulties in Quadratic Formula

Quadratic formula difficulty is defined as the students who do not apply correct formula. Students are tend to memorize the quadratic formula, when they cannot gain a procedural and conceptual understanding of quadratic equations. The students feel difficulties in quadratic formula since they remembered and applied the quadratic formula incorrectly. Some students computed the discriminant incorrectly because of calculation difficulty and some students computed the discriminant correctly but could not use the quadratic formula correctly since they misremembered it. Researcher described the quadratic formula difficulties in two categories. If the students are able to solve by use of quadratic formula, it is considered as no quadratic formula difficulties. If the students are unable to solve by using quadratic formula, it is considered as quadratic formula difficulties.

The question; solve the quadratic equation by using quadratic formula; $x^2 - 5x - 24 = 0$. In this question student challenges so many difficulties. The answer sheet of a student is given below for example.

The image shows a student's handwritten work on lined paper. The work is as follows:

- Equation: $x^2 - 5x - 24 = 0$
- Quadratic formula: $x = \frac{-b \pm \sqrt{2ab^2 + 4ac}}{2}$
- Substitution: $x = \frac{-24 \pm \sqrt{(5x)^2 + 4 \cdot ac}}{2}$. To the right of this line, the text "difficulties in quadratic formula" is written in red.
- Correction: $x = \frac{-24 \pm \sqrt{25 + 4}}{2}$. The "5x" in the previous line is crossed out and replaced with "25".
- Further correction: $x = \frac{-24 \pm \sqrt{29}}{2}$. The "25" is crossed out and replaced with "29".
- Final result: $x = \frac{-24}{7}$. The entire previous line is crossed out.

Above solution shows the quadratic formula difficulties of student. Many students have challenges to quadratic formula. The students assumed that the formula $x = \frac{b \pm \sqrt{2ab^2 + 4ac}}{2}$ instead of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Students don't know the formula and write incorrect formula then after students apply this form to find root of the quadratic equation as x^2 minus 24 square root $(5x)^2 + 4ac$ divide 2 which is also wrong. This type of difficulty was considered as quadratic formula difficulties. The students apply incorrect formula and not compare the coefficient with standard quadratics form. Didis & Erbas (2015) shows that when the student unable to use the formula to solve quadratic problem then such type of difficulties is known as quadratic formula difficulties. Further they are unable to calculate and process the quadratics to find roots of equations.

Moreover, for clear justification of student quadratic formula difficulties in this problem, face to face interview was taken by researcher with that students asking oral question to him stated as follows.

I: What does the question asks to do?

S: Solve the quadratic equation by using quadratic formula.

I: Can you tell me what is the formula/

S: umm... I tried $x^2 = \frac{b \pm \sqrt{2ab^2 + 4ac}}{2}$ sir.

I: is it correct formula?

S: May be... I confused and misremembered it.

I: Why did you feel difficulties in quadratic formula?

S: I don't know what is quadratic formula? I don't like mathematics subject because it bored me too much. I don't have pre knowledge about it.

I asked the question in ace to face interview to mathematics teacher, "Why students commit the quadratic formula difficulties?" The teacher replied that,

“Students focuses on the calculation part, they do not focus on rules, formula to the mathematics subject, and lack of pre-requisite knowledge of their previous lesson”.

Again the researcher asked “What are the main causes of quadratic formula difficulties?” He told that, *“Because of lack of mathematical concept, such as square root, multiplication; sign and common factor to the quadratic equation. They do not learn in house rather they play, do not learn quadratic formula.”*

Didis & Erbas (2015) research the difficulties in quadratic formula are described as the above mentioned problem. The difficulties in quadratic formula are due to lack of mathematical concept, unable to learn quadratic formula, do not know how to apply the formula. The result out of interview and written solution shows that the similar facts. Thus many students have calculation difficulty while solving the quadratic equation.

Solving Process Difficulties

A solving process difficulty is defined as the carrying out difficulties such as using in appropriate method, inappropriate formula and calculation error in which students become unable to solve the problem correctly. The difficulty in solving process is a skill difficulty when a students' been able to identify the difficult operation but did not know the procedures to carry out these operations correctly. Looking at the solution researcher has analyzed that the students had identified the correct way of solving problem but failed to carry out the operation correctly.

This kind of difficulty was made in the process of making operations in order to reach the solution and find the possible values of the unknown. The question: solve the quadratic equation by using factorization; $2x^2 + 3x - 5 = 0$. Many of the students perform such type of difficulties. It was observed that most of the students were poor in the solving process. A sample answer of a student is given here for example.

⑧. $2x^2 + 8x - 5 = 0$
 $2x^2 + (5-2)x - (5)^2 = 0$
 $2x^2 + 5x - 2x - 5 = 0$
 $x(2x+5) - 1(2x-5) = 0$ solving process difficulties
 ~~$x(x-1)(2x+5) = 0$~~
 Either:
 $x-1=0$
 $x=-1$

$2x+5=0$
$2x=-5$
$x=-\frac{5}{2}$

The above solution shows that the skill requires in solving process is challenging because of the difficulties of students in computational skills. In this solution although the quadratic equation was factorized correctly, the student put the incorrect sign in front of the number 5 while taking common factor. Then after equating both parts of the equation by leaving x alone on one side, namely student transferred the number to the right side of equation without changing the sign of it. Therefore, students found -1 instead of 1. When this solution process was examined carefully, it was realized that the student continued solution carelessly. These types of difficulties can be considered as solving process difficulties. This shows that students usually face difficulties in the process skill. In this problem students able to find factor of quadratic equation but unable to perform correct operation/ process (Didis&Erbas, 2015).

Moreover, for clear justification of students' solving process difficulties in this problem, face to face interview was taken by researcher with students asking oral question to him stated as follows:

I: What does the question asks to do?

S: Solve the quadratic equation sir.

I: Can you go over each step of your work?

S: yes, firstly, I factorize the quadratic equation as $2x^2 - 5x - 10x^2$ and get two factor such that addition or subtraction get middle term of equation, $5x$ minus $2x$ is equal to $3x$. Then after I take common and solve it.

I: Explain how you solve it?

S: First, I take common factor as $x(2x + 5) - 1(2x - 5) = 0$ then common $(x-1)(2x+5) = 0$ and I assumed $x-1 = 0$ and $2x + 5 = 0$ implies $x = -1$ and $x = -5/2$ and I got the answer $x = -1$ & $x = -5/2$.

From this interview the researcher came to realization that students had problem in solving process. Again I took face to face interview with the mathematics teacher related to this problem, first I asked, “*What are the causes of process difficulties?*” He told that, “*students are careless in mathematics class, donot attend in learned and students do not practice at home*”. I asked, “*Why the students make a process skill?*” The teacher replied, “*Because the students cannot present in regularity class, then they do not pay attention on the class.*”

O’Connor & Norton (2016) mentioned that students’ difficulties related to solving process because of the computation difficulties, incorrect operation, and lack of mathematical basic concept. The difficulties in the problem can be linked with computation difficulties and sign difficulties. Interview with students and mathematics teacher relating to the problem found that the students difficulties in solving problem because of the students inability to perform computational operation, lack of the knowledge of multiplication,, lack of practice mathematics at home,

irregularity in mathematics class and students focus on the product is not more important than the process.

In second problem, students' written solution was analyzed through their answer sheet. The question: solve the quadratic equation by using quadratic formula $x^2 - 5x - 24 = 0$. The answer sheet of a student is given below for example.

8. $x^2 - 5x - 24 = 0$
 द्विघात समीकरण $ax^2 + bx + c = 0$ का सूत्रानुसार हल करें

$a = 1$
 $b = -5$
 $c = -24$

We know that
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

8. $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot (-24)}}{2 \cdot 1}$ *conceptual + solving process difficulties*

9. $x = \frac{25 + \sqrt{25 - 96}}{2}$

10. $x = \frac{25 + \sqrt{121}}{2}$

11. $x = \frac{25 + 11}{2}$

+ चिह्न लिए हैं	- चिह्न लिए हैं
$x = \frac{25 + 11}{2}$	$x = \frac{25 - 11}{2}$
9. $x = \frac{36 + 18}{2}$	10. $x = \frac{14}{2}$
$\therefore x = 18$	$\therefore x = 7$

$\therefore x = 18 \text{ or } 7 \neq$

The above solution shows that the student has solving process difficulties. In this problem student has compare the given quadratic equation with standard quadratic form $ax^2 + bx + c = 0$, then $a=1$, $b=5$ $c=24$. Then next step student apply quadratic formula, and put the values of coefficient in the formula. Inside square root student multiply $-4 \cdot 1 \cdot (-24) = +96$. In next step, student ignores sign. Student get calculation and simplification difficulty in this question and got wrong solution, it is considered as solving process difficulties. O'Connor and Norton (2016) shows that students apply correct formula but not able to perform correct calculation & simplification such type of difficulties is solving process difficulties.

Moreover, for clear justification of student solving process difficulties in this problem, face to face interview was taken by researcher with the student asking the oral question to him stated as follows.

I: can you go over each step of your work?

S: Yes, firstly I compared this question $ax^2 + bx + c = 0$ then apply quadratic formula sir.

I: explain how you solve it?

S: Firstly, I put value of a, b, c in quadratic formula then simplifies to find solution of quadratic equation.

Again, I took face to face interview with the mathematics teacher related to this problem. First, I asked, “what are the main causes of solving process difficulties?” Then the teacher replied that, “*students always focus that the product is more important than the process.*” O'Connor and Norton (2016) also showed that if students are able to apply correct formula but could not attempt with an correct calculation then that is known as solving process difficulties. The evidence shows that main reason of facing process difficulties is linked with students' inability to perform

correct computational operation, lack of mathematical basic concept, lack of knowledge of multiplication and student unable to correct mathematical operation such difficulties lies on solving process difficulties.

Chapter-V

Findings, Conclusions and Recommendations

This section deals with the summary, findings, conclusions and recommendations concerning the student difficulties in solving quadratic equation at grade IX. This chapter is divided into three sections: findings, conclusion, and recommendations for the further study.

Summary of the Study

The topic of study was ‘difficulties solving quadratic equation at grade IX.’ The objectives of this study were to explore the difficulties in quadratic equation and find the main causes of difficulties in solving quadratic equation. Design of this study was case study based on qualitative method. This study was conducted in Baijnath Secondary School Gailnadi, Kanchanpur. The researcher was taken exam to grade IX students after checking the answer sheet and highly / maximum difficulties in given problem to the students, the researcher selected the five students for interview, To achieve the objectives of the study, data and information were collected through the test and interview with student and mathematics teacher. The researcher analyzed the test result and interviewed on the basis of above reviewed literature. Many students were responds to incorrect answer in test and only few students respond correctly. Lack of conceptual understanding of the factorization principle, students always emphasis on product only rather than process, student don’t know how to factorize, how to complete square and even they don’t know how many solutions does quadratic equation has? Difficulties in quadratic formula are the major obstacles in solving quadratic equation.

Findings and Discussions

The main concern of this study was to explore the difficulties in solving quadratic problem in algebra and main causes of difficulties of solving quadratic problem in algebra. In this study researcher included the five difficulties (Conceptual difficulties, Difficulties in factorization, difficulties in completing square, difficulties in quadratic formula and solving process difficulties). The result of data analysis showed that the felt difficulties are in following.

Conceptual Difficulties

The conceptual knowledge consists of connected relationship between idea and concept in internalized by individuals. The students can understand the problem in their own it is considered as no conceptual difficulties but if the students are unable to understand the question and what trying to find out it is considered as conceptual difficulties. It is found that students could not understand the problem. For example, solving quadratic equation by factorization as $x^2 + 9x + 20 = 0$ students proceed this problem as $x^2 + 9x = -20$ implies $x^2 \cdot x = -20 - 9$ implies that $x^3 = -11$, which is incorrect. This is a lack of conceptual understanding regarding quadratics, algebraic processes and factorization. In conceptual difficulties student face difficulties in addition, subtraction, multiplication etc. They could not understand the problem, it is considered as conceptual difficulties. The main causes of conceptual difficulties are lack of pre knowledge, lack of basic concept about quadratics, various form, solution, students practice.

Difficulties in Factorization

The factoring plays a vital role in mathematics as well as in several other fields. Factoring or factorization is the process of finding factors of a number or an expression or an equation. The factors are defined as the integer or expression which

when multiplied together result in original number or expression. Factoring quadratics finds the roots of quadratic equations in standard form, can often be accomplished by finding two numbers that add to give b and multiply to give ac . Many students have challenges with factorization. The students have difficulty with basic multiplication table. Factoring simple quadratic equation become quite challenges because of that the students could not find two correct linear factors, they could not find the root accurately.

The above study indicated that student have difficulty in factorization. Factorizing a quadratic equation required to consider the first, middle and last term at the same time. For example, the addition of the factors of the first term and last term must give the middle term in the equation. However, the students neglected some of these terms or confused the rules such as multiplication or addition of the pairs. This kind of difficulty based on their incorrect guesses and no attempt to control the accuracy of linear factors while using the cross multiplication. Besides, this probably resulted from their lack of conceptual knowledge regarding factorization. In conclusion, difficulty in multiplication, basic number facts related to factorization difficulties. The factorization difficulties occurred due to unable to understand factorization principle, lack of pre knowledge, lack of basic number facts, lack of multiplication table, and lack of attention to the teacher's explanation.

Difficulties in Completing Square

The method of converting a quadratic equation which is not a perfect square into the sum or difference of a perfect square and a constant by adding or subtracting the suitable constant terms. Many students have challenges to completing square. Completing square to the quadratic equation is become quite challenge. In this research student don't know how to perform square and what number should be

added or subtracted to each side, then after students confused in taking square root.

For example, solving $x^2 + 10x - 39 = 0$ by completing square as perform this way $x^2 + 2 \cdot x \cdot 5 + 5^2 - 5^2 = 39$ then it form complete square but student could not do so.

Student did such problem as $x^2 + 2 \cdot x \cdot 5 + 5 = 39 - 45$ which is incorrect. In above examples students have difficulties in completing square.

In conclusion, completing square difficulties occurred due to unable to understand the problem, unable to apply square formula, unable to clarifying problem, lack of choose appropriate operation. The main causes of difficulties in completing square are lack of general concept, rule, sign, misinterpretation of question, carelessness.

Difficulties in Quadratic Formula

Quadratic formula is a method of solving quadratic equation when the equation is not factorized. The formula is as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ which has two solutions but students do not know how many solution this formula gives. The data showed that students encountered the following challenges while applying the quadratic to find the roots of the quadratic equations: (i) students either computed the discriminant incorrectly because of calculation difficulty, or could not compute it at all; (ii) students computed the discriminant correctly, but applied the quadratic formula incorrectly, since they had misremembered it; (iii) students computed the discriminant incorrectly but they applied the quadratic formula correctly. The main causes of quadratic formula difficulties are lack of mathematical concept, lack of learning formula, carelessness, lack of practice at home.

In conclusion, calculation and simplification difficulties are related to quadratic formula, lack of concept of square root, skill & more emphasis on calculation part on mathematics that challenges many students.

Solving Process Difficulties

Problem solving is seen as high level skill. A boarder definition of problem solving is a goal directed actively that entails interpreting a situation mathematically. Many students understand the meaning of the quadratic problem and ability to factorize the quadratic equation but they were unable to perform correct operation and correct procedures. For example: student solves $2x^2 + 3x - 5 = 0$ using factorization, they find correct factor as $5x + 2x$ but in process they take $2x^2 - 5x + 2x - 5 = 0$ implies $x(2x - 5) + 1(2x - 5) = 0$. They take common $(2x - 5)(x + 1)$. For solving x , $x = 1$ & $x = -5/2$, which is incorrect. Another problem using quadratic formula student calculation & simplification as $x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 24}}{2 \cdot 1}$ which is also incorrect. The main causes of solving difficulties is lack of mathematical knowledge, lack of procedural knowledge, less practicing quadratic equation, lack of multiplication table, basic number facts, carelessness in solving quadratic problem. Teacher should be minimizing the solving process difficulties should care above points and must force the students to solve the problem.

In conclusion, the study shows that the students had conceptual and procedural difficulties regarding quadratic equations, difficulty in multiplication, various forms of quadratic equation, solving techniques. Solving quadratic equation is one of the most conceptually challenging aspects in school level. Student may have blindly memorized rules and applied them mechanically without enough thought. Students do not realize what finding the roots of quadratic equation means. On the other hand students memorize the rules, formulas and algebraic procedures to solve quadratic equations without understanding the meaning, they could not transfer these rules, formulas and procedures to quadratic equation. They also have a tendency to forget

the formula after some time has passed since they learnt it, so students had conceptual and procedural difficulties in solving quadratic equation.

Causes of Difficulties

This was a case study related to investigating students' mathematical difficulties with quadratic equations at grade IX. From written test and interview data, it showed that students' have difficulties in solving quadratic equation. Hence the causes of difficulties were as follows.

- Lack of pre- knowledge about quadratic equation.
- Lack of pre- requisite background knowledge of basic number operation.
- Lack of multiplication fact, table, and lack of conceptual knowledge related to quadratic and its various forms & algebraic processes.
- Students always emphasis on product rather than process.
- Traditional teaching method.
- There was no proper interaction between students and teacher.
- Students were irregular in the classroom and did not pay attention while learning mathematics.
- Students did not interest in mathematics. They felt bored while learning mathematics.
- Lack of sufficient practice at home and carelessness in learning mathematics.

Conclusions

In this research, the researcher found many difficulties the students face in solving quadratic equation problem in algebra grade IX. The objective of the study was to investigate students' mathematical difficulties with quadratic equation at grade IX. After analysis and interpretation of data it is found that students committed the

maximum difficulties at factorization, completing square and solving process. Most of student do not solve question, they don't know how to solve these question. Students see completing square is more challenging than factorization and using quadratic formula because in completing square requires thinking more mathematically than others. In completing square the square provides the algebraic manipulation of a quadratic equation for rendering it more suitable form. However, students do not prefer it since they have lack of knowledge regarding basic properties square root such as $\sqrt{x^2} = |x|$ & if $x^2 = a$ then $x = \pm\sqrt{a}$. it reveals the need for more time and attention of teaching these basic properties.

This study showed that the students having difficulties in solving quadratic equation categories in five level (Conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in quadratic formula and solving process difficulties). These difficulties are due to lack of basic knowledge, lacking of pre requisite background knowledge of basic number operation, lack of multiplication fact, lack of conceptual knowledge related quadratic & its various forms and algebraic processes, students always emphasis on product only rather than process, traditional teaching method. Similarly there was no proper interaction between students and teacher as a result students had difficulties in learning. Moreover, students did not have sufficient practice of mathematics, students were not given an opportunity to discuss the concepts and they did not have enough time to practice their work, on the other side, learners are afraid to be mocked at by their peers during the lessons, learners themselves not committed to their school work. Some have parental role to play at home and not pay attention in classroom, carelessness, some teachers are not aware of the importance of linking algebra learning to students previous experiences. Children learn mathematics effectively when they are given opportunity to grapple

with problem. So students provide conceptual and procedural knowledge of quadratic problem.

Educational Implication

- Teacher should teach quadratic problem of algebra by applying the basic concepts.
- Before starting the chapter, the teacher has to give the fundamental knowledge about the topic, conceptual teachings are most needed in secondary level.
- Teacher should encourage his/ her students to focus on process rather than product.
- The teacher should try to find out the reason about committing the difficulties.
- Teacher should be diagnostic test and must identify the area of difficulty and must use remedial teaching to avoid the difficulties.
- It is important for students to make clear connections between symbolic representations relating to quadratic problem.
- Teacher should be aware that solving quadratic equations is not based only on procedures and rules and attempt to seek out alternative way of teaching quadratic equations.
- Students should also be given opportunities to explore a range of situations in which they are required to construct, interpret and then solve quadratic equation.

Recommendation for Further Study

The researcher had tried to make some suggestion for further study in this field. The following suggestions could be beneficial for further research study:

- This type of study can be extended to private school as well.
- The study area of this research should be extended like other development region and other parts of country.
- Due to lack of sufficient time, resources and economic problem, the researcher conducted only one government school. Therefore, further study can be carried out sampling various schools from different parts of Nepal.
- Further study can be done on “Error Analysis of Students’ in Solving Quadratic Equation.”

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Appendix-B

Interview guideline for students

Name of Students: Gender:

Class: Roll No:

The interview with mathematics students will be taken on the basis of following topics:

1. Conceptual Difficulties
 - Coefficient and variable.
 - Quadratic equation.
 - Various forms
 - Techniques of solving
2. Difficulties in Factorization
 - Basic Number Facts
 - Multiplication, addition and subtraction
 - Common factor
 - Principle of factorization
3. Difficulties in Completing Square
 - Addition of Number & Subtraction of number
 - Square root
 - Solution & Sign operation
4. Difficulties in Quadratic Formula
 - Formula
 - Simplification
 - Calculation
 - Rule operation
5. Solving Process Difficulties
 - Knowledge for understanding problem
 - Strategic to approach problem solve take to the task
 - Correct mathematical operation or procedures

Appendix- C

Interview Guidelines for Mathematics Teacher

Name of teacher:

Class:

Teaching Experience:

Gender:

The interview with mathematics teacher will be taken on the basis of following main topics:

1. Pre- knowledge of quadratic equation of algebra
 - Operation and Symbol
 - Pre knowledge of arithmetic skill for algebra
 - Solving technique
2. Teaching Learning Strategies:
 - Using learning material
 - Student centered teaching method
 - Co- operating learning
 - Problem solving.
 - E- learning
3. Participation in Learning
 - Students are regular or irregular
 - Engagement in mathematics class
4. Students' Teacher Interaction
 - Sufficiently interaction between teacher and students
 - Students are openly asked any question with teacher.

Appendix- A

Question Paper for Written Test

Class: 9

Times: 45 Min.

Dear all,

This is a partial work of my master's thesis entitled **Investigating Students' Mathematical Difficulties with Quadratic Equation at Grade IX**. I would be grateful by your appreciation of this questionnaire.

Student's Demographic Information

Name:

Age:

Gender:

School's Name:

Questions

Solve the quadratic equation by using factorization method

1. $x^2 - 2x - 48 = 0$

2. $x^2 + 9x + 20 = 0$

3. $2x^2 + 3x - 5 = 0$

4. $4x^2 - 3 = 4x$

Solve the quadratic equation by using competing square method

5. $x^2 - 7x + 12 = 0$

6. $x^2 + 10x - 39 = 0$

7. $x^2 - 2x + \frac{3}{4} = 0$

Solve the quadratic equation by using quadratic formula method.

8. $x^2 - 5x - 24 = 0$

9. $x^2 + 3x = 28$

10. $2x^2 + 11x + 12 = 0$

Thank you for your participation.

