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By

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Ordered Successive Interference Cancellation and Sphere Detection Technique for MIMO

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A midterm report of thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Information and Communication Engineering

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Recommendation

The undersigned certify that they have read and recommended to the Department of Electronics and Computer Engineering for acceptance, a thesis entitled **"Ordered Successive Interference Cancellation and Sphere Detection Technique for MIMO"**, submitted by **Prabhat Adhikari** in partial fulfillment of the requirement for the award of the degree of **"Master of Science in Information and Communication Engineering"**.

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ABSTRACT

The advent of multiple inputs, multiple output (MIMO) system has had a major influence on how the current capacity limits of a single antenna system could be increased without the need of overwhelming computational complexity. MIMO systems may be broadly classified into two categories: those designed to increase the transmission rate through spatial multiplexing and those designed to increase reliability using diversity techniques. Most of the current researches are concerned with the spatial multiplexing design scheme as it offers very high increase in data rate. The detection scheme for spatial multiplexing techniques includes linear receivers like Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) and non-linear receivers like Successive Interference Cancellation (SIC), Ordered Successive Interference Cancellation (OSIC) and Sphere Detection (SD).

This thesis reviews briefly the MIMO system and the spatial multiplexing technique and compares various detection schemes used in MIMO system. Based on the BER analysis, the performance of various detections schemes in Rayleigh Channel is conducted. Different aspects have been considered in the evaluation of performance namely signal to noise ratio (SNR), modulation order and the number of transmitter and receiver antennas. The Maximum Likelihood (ML) detection method has shown optimum detection for MIMO system however the complexity involved is very high which leads to use of low complexity detection schemes. The performance of linear detection scheme like ZF, MMSE and non-linear schemes like SIC and the OSIC are first reviewed and then a quite new SD algorithm is analyzed where the SD provides near ML solution with much less computational complexity. The obtained results are inter-compared and discussed from different performance aspects.

Keywords**:** *Multiple Input Multiple Output (MIMO), Zero Forcing (ZF), Minimum Mean Square Error (MMSE), Successive Interference Cancellation (SIC), Sphere Decoder (SD), Maximum Likelihood (ML).*

LIST OF SYMBOLS

h_{ij}	Channel coefficient between i-th receiving and j-th transmitting antenna
E_b	Average received energy per bit at each receiver antenna
E_s	Average received energy per symbol at each receiver antennas
$\hat{\boldsymbol{x}}$	Estimate of transmit symbol vector $\boldsymbol{x}$
σ^2	Noise variance
Δ	Step size
$\boldsymbol{H}^{+}$	Moore-Penrose pseudoinverse of $\boldsymbol{H}$
C	Channel capacity
$E\{.\}$	Expectation operator
M	Number of transmit antennas
N	Number of receive antennas
$\boldsymbol{H}$	Channel matrix
$\boldsymbol{R}$	Upper triangular matrix
$\boldsymbol{W}$	Weight matrix
$\boldsymbol{n}$	Noise vector
$\boldsymbol{x}$	Transmit symbol vector
$\boldsymbol{y}$	Received symbol vector
$\mathcal{Q}(.)$	Quantizer or Slicer
α	Real part of h_{ij}
β	Imaginary part of h_{ij}
$(.)^H$	Hermitian transpose

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ABBREVIATIONS

3GPP	3rd Generation Partnership Project
AWGN	Additive White Gaussian Noise
BER	Block Error Rate
HSPA+	Evolved High-Speed Packet Access
LOS	Line of Sight
LTE	Long Term Evolution
MIMO	Multiple-Input Multiple-Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
MMSE-OSIC	MMSE- Ordered Successive Interference Cancellation
MMSE-SIC	MMSE -Successive Interference Cancellation
OSIC	Ordered Successive Interference Cancellation
SD	Sphere Decoder
SIC	Successive Interference Cancellation
SISO	Single Input Single Output
WiMAX	Worldwide Interoperability for Microwave Access
ZF	Zero Forcing
ZF-SIC	ZF-Successive Interference Cancellation
ZF-OSIC	ZF- Ordered Successive Interference Cancellation

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CHAPTER 1: INTRODUCTION

1.1 Background

Wireless system designers are facing a number of challenges. These include the limited availability of the radio frequency spectrum and a complex space–time varying wireless environment. In addition, there is an increasing demand for higher data rates, better quality of service, and higher network capacity [1]. The mobile data applications has increased the demand for wireless communication systems offering high throughput, wide coverage, and improved reliability. The main challenges in the design of wireless communication systems are the limited resources, such as constrained transmission power, scarce frequency bandwidth, and limited implementation complexity—and the impairments of the wireless channels, including noise, interference, and fading effects. Multiple Input Multiple Output (MIMO) communication has been shown to be one of the most promising emerging wireless technologies that can efficiently boost the data transmission rate, improve system coverage, and enhance link reliability. By employing multiple antennas at transmitter and receiver sides, MIMO system enable a new dimension – the spatial dimension – that can be utilized in different ways to combat the impairments of wireless channels [2].

As opposed to traditional wireless systems, in which there is one transmitting and one receiving antenna, MIMO systems use arrays of multiple antennas at both transmitter and receiver, all operating at the same frequency at the same time. This introduces spatial diversity into the system, which can be used to tackle the problem of multipath. In wireless communications system, such as point to point radio links, radio waves do not simply propagate from the transmit antenna to the receive antenna. Rather they bounce and scatter off objects, this effect is known as multipath. This effect is regarded as hindrance to the accurate transmission of data in traditional wireless links. MIMO systems exploit multipath by using the rich scattering environment to increase the spectral efficiency and/or reliability of the wireless system. There are different ways of exploiting multiple antennas at both ends of the MIMO communication link. For example, to achieve the best transmission reliability possible, the transmit antennas should be used such that transmit diversity is achieved. The transmission rate achieved in this case is normally comparable to that achieved in single-input single-output (SISO) systems. That is, all the degrees of freedom of the MIMO channel are used for improving the transmission reliability and not the transmission rate. An alternative is to use the transmit antennas to maximize the transmission rate. In this case, independent signals are transmitted from the different transmit antennas, i.e., there is no correlation among the transmitted signals from different antennas. While this approach increases the transmission rate, the corresponding reliability is poor. A combination of these two approaches is also possible, that is, one could trade rate for reliability or vice versa. The use of MIMO systems and associated signaling approaches are critical for the future of high data rate, extremely reliable wireless communications [3].

1.2 Problem Statement

It is commonly known that at the receiver side the best performance can be achieved when a full maximum likelihood (ML) search is performed over the complete dimensions. It is obvious that the complexity of such a receiver grows exponentially with the size of the symbol constellation and/or antenna numbers [4]. So, the receiver scheme with less complexity is needed without much performance loss. In general less complex receiver schemes result in a performance loss. The fundamental problem of MIMO systems is the mapping operation at the transmitter and the corresponding inversion at the receiver to optimize the overall performance of the wireless system.

1.3 Objectives

The use of MIMO technology improves the overall performance of wireless communication system. The work focuses on the advantages of using the MIMO system in communication link and compares the performance of various receivers namely ZF, MMSE, ML, OSIC, and SD detection scheme and their variants employed with MIMO system. The goal is to have a receiver with a manageable complexity that performs closely to the maximum likelihood receiver. It includes the following:

- To review and study the single antenna (SISO) system and multiple antennas (MIMO) system
- To study the linear detection method such as ZF and MMSE detection algorithm
- To study the detection algorithm such as ML, Successive Interference Cancellation (SIC) and Ordered Successive Interference Cancellation (OSIC) for ZF and MMSE detection schemes.
- To study the Sphere Detection algorithm for low complexity ML receiver.
- To compare the performance of the abovementioned detection schemes.

1.4 Scope of Thesis

MIMO offers significant increases in data throughput and link range without additional bandwidth or transmit power. It achieves this by higher spectral efficiency and link reliability and/or diversity [5]. Because of these three properties, MIMO is an important part of modern wireless communication and is incorporated in 3GPP, WiMAX, LTE, HSPA+.

The scope of the work presented in this thesis primarily centers on the performance comparison of detection techniques, for Rayleigh flat fading MIMO channel used for spatial multiplexing MIMO system. The thesis contributes to the analysis of performance of different types of detection scheme like ZF, MMSE, ZF-OSIC, MMSE-OSIC, ML and SD.

1.5 Organization of Thesis

In Chapter 2, a brief description of MIMO system is given. This chapter includes description on SISO and MIMO channel capacity. The spatial multiplexing MIMO system is for achieving higher data rate is in section 2.4. Chapter 3 includes the description symbol detection technique in spatial multiplexing MIMO system. The symbol detection algorithm namely ZF, MMSE, ZF/MMSE – OSIC and SD are described in this section. Chapter 4 provides the methodology for and the symbol detection problem in MIMO system. The simulation setup and results is presented in Chapter 5. Finally, discussions and future enhancement of this thesis work is given in Chapter 6 and 7 respectively.

CHAPTER 2: LITERATURE REVIEW

2.1 Multiple Input Multiple Output (MIMO) System

Multiple Input Multiple Output (MIMO) communication is born out of the presence of multiple antenna elements at both the transmission and reception sides of a communication channel. The channel between the transmit antennas and the receive antennas takes the form of a matrix. The signal paths between each transmit and receive antenna pair is a channel coefficient in the matrix.

This technology was a result of the move towards the use of multiple antennas in communications. The number of antennas was increased to exploit a newer form of diversity called spatial diversity. Diversity refers to the reception of multiple independent copies of the same information which allows a more reliable determination of the original transmitted information. Spatial diversity comes from the separation between the antennas transmitting signals and/or due to the separation of the antennas at the receiving side as well. The reduced probability of all paths present being lost of deep fading leads to a reduced loss of information and improved capability.

Initially, techniques like smart antennas provided multiple antennas at either the transmit or the receive side, these techniques initially provided for only receive or transmit diversity using methods like Maximum Ratio Combining or Space Time Block Coding. These methods were successful in providing diversity gain which improved the reliability of the communication channel and reduced the BER. Diversity gain is the ratio of the SNR with diversity to SNR without diversity. It is also defined as the slope of the BER curve in a log-scale. For the above multiple antenna cases, it is seen that if there are N independent paths for a given signal (i.e. there are N antennas), the maximum diversity gain is N. this is because, in theory, the average error rate decays as $1/(SNR)^N$ with N antennas a opposed to 1/SNR for the single antenna case. At a fundamental level, these techniques used diversity to combat fading.

When multiple antennas were used both at the receiving and transmitting side it was determined that Spatial Multiplexing Gain is also obtainable. Spatial Multiplexing Gain implies for M transmit antennas and N receive antennas, it is possible to recover min (M,N) different signals. It must be stressed that the fundamental assumption for the above gains to be achieved is the independence of the fading between all transmit-receive antenna pairs. In this case, fading turns from foe to a friend because of uncorrelated fading leads to a channel matrix with good condition which results, in turn, in the independence and separatability of the incoming data streams [6].

2.2 MIMO System Model

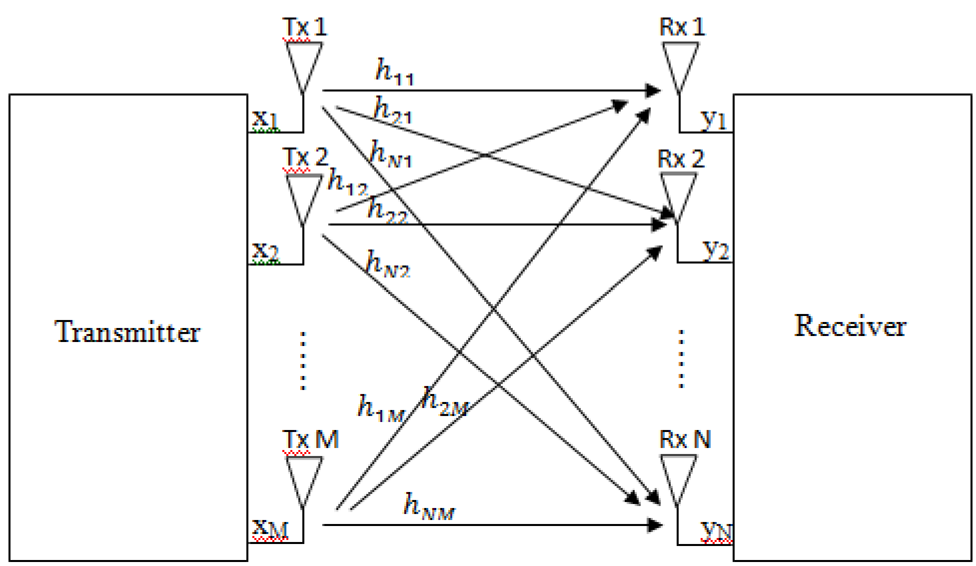

Figure 2.1: MIMO System with M transmit and N receive antenna

In a single user MIMO system with *M* transmit antennas and *N* receive antennas, the received signal are:

$$y_1 = h_{11}x_1 + h_{12}x_2 + \cdots + h_{1M}x_M$$

$$y_2 = h_{21}x_1 + h_{22}x_2 + \cdots + h_{2M}x_M$$
$$\vdots$$
$$y_N = h_{N1}x_1 + h_{N2}x_2 + \cdots + h_{NM}x_M \quad (2.1)$$

In the matrix form,

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & & h_{2M} \\ \vdots & & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \quad (2.2)$$

Thus input/output relations of a narrowband, single-user MIMO link by the complex baseband vector notation is represented as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \quad (2.3)$$

where $\boldsymbol{x}$ is the (M×1) transmit vector, $\boldsymbol{y}$ is the (N×1) receive vector, $\boldsymbol{H}$ is the (N×M) channel matrix with zero mean and unit variance, and $\boldsymbol{n}$ is the (N × 1) additive white Gaussian noise (AWGN) vector at a given instant in time with zero mean and variance σ^2. It is assumed that the channel matrix is random and that the receiver has perfect channel knowledge. It is also assumed that the channel is memory less, i.e., for each use of the channel an independent realization of $\boldsymbol{H}$ is drawn. A general entry of the channel matrix is denoted by $\{h_{ij}\}$. This represents the complex gain of the channel between the jth transmitter and the ith receiver. For a MIMO system consisting of M transmits antennas and N receive antennas, the channel matrix is written as,

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & & h_{2M} \\ \vdots & & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix} \quad (2.4)$$

where,

$$h_{ij} = \alpha + j\beta \quad (2.5)$$

where both α and β are zero mean Gaussian random variable In a rich scattering environment with no line-of-sight (LOS), the channel gains $|h_{ij}|$ are usually Rayleigh distributed. If α and β are independent and normal distributed random variables, then $|h_{ij}|$ is a Rayleigh distributed random variable [7].

In this thesis, we are primarily concerned with detection at the receiver of the transmit vector **x** based on knowledge of $\boldsymbol{y}$, $\boldsymbol{H}$. The parameters of $\boldsymbol{H}$ can be learned at the receiver via techniques collectively known as *training* and *tracking*, in which $\boldsymbol{H}$ is estimated by sending vectors jointly known to the transmitter and receiver across the channel. If the channel changes with time, then the estimate of $\boldsymbol{H}$ can be updated using the detection decisions. Sometimes it is also useful to periodically perform training in case tracking becomes unsuccessful. It is assumed in this work that $\boldsymbol{H}$ is explicitly known at the receiver.

2.3 Channel Capacity

A measure of how much information that can be transmitted and received with a negligible probability of error is called the channel capacity. It is common to represent the channel capacity within a unit bandwidth of the channel. The channel capacity is then measured in bits/s/Hz. [7]

Representing the input and output of a memoryless wireless channel with the random variable X and Y the information capacity of the Gaussian channel with power constraint P is given by

$$C = \max_{p(x)} I(X;Y) \tag{2.6}$$

Where $I(X;Y)$ represents the mutual information between X and Y. Mutual information is a measure of the amount of information that one random variable contains about another variable.

The mutual information between X and Y can also be written as

$$I(X;Y) = H(Y) - H(Y|X) \tag{2.7}$$

where, $H(Y|X)$ represents the conditional entropy between the random variables X and Y. The entropy of a random variable can be described as a measure of the amount of information required on average to describe the random variable. It can also be described as a measure of the uncertainty of the random variable. Note that the mutual information between X and Y depends on the properties of the channel (through a channel matrix **H**) and the properties of X (through the probability distribution of X)

2.3.1 SISO channel capacity

The ergodic (mean) capacity of a random channel with $M = N = 1$ and an average transmit power constraint P_T can be expressed as

$$C = E_H\left\{\max_{p(x):P\leq P_T} I(X;Y)\right\} \tag{2.8}$$

where P is the average power of a single channel codeword transmitted over the channel and E_H denotes the expectation over all channel realizations. Compared to the definition in equation 2.7, the capacity of the channel is now defined as the maximum of the mutual information between the input and the output over all statistical distributions on the input that satisfy the power constraint. If each channel symbol at the transmitter is denoted by s, the average power constraint can be expressed as

$$P = E[|s|^2] \leq P_T \tag{2.9}$$

Using equation 2.9, the ergodic (mean) capacity of a SISO system (N =M = 1) with a random complex channel gain h_{11} is given by

$$C = E_H\{log_2(1+\rho|h_{11}|^2)\} \text{ (bits/s/Hz)} \qquad (2.10)$$

where ρ is the average signal-to-noise (SNR) ratio at the receiver branch.

2.3.2 MIMO channel capacity

The capacity of a random MIMO channel with power constraint P_T can be expressed as,

$$C = E_H\left\{max_{p(x):tr(\phi)\leq P_T} I(\boldsymbol{x};\boldsymbol{y})\right\} \qquad (2.11)$$

Where $\phi = E\{xx^H\}$ [2] is the covariance matrix of the transmit signal vector $\boldsymbol{x}$. The total transmit power is limited to P_T, irrespective of the number of transmit antennas. In [6] it is shown that equation 2.11 can be expressed as

$$C = E_H\left\{log_2\left[det\left(\boldsymbol{I}_N + \frac{P_T}{\sigma^2 N}\boldsymbol{H}\boldsymbol{H}^H\right)\right]\right\} \text{ (bits/s/Hz)} \qquad (2.12)$$

This can also be expressed as:

$$C = E_H\left\{log_2\left[det\left(\boldsymbol{I}_N + \frac{\rho}{N}\boldsymbol{H}\boldsymbol{H}^H\right)\right]\right\} \text{ (bits/s/Hz)} \qquad (2.13)$$

where, $\rho = \frac{P_T}{\sigma^2}$ is the average SNR ratio at each receiver branch. By the law of large numbers, the term $\frac{1}{M}\boldsymbol{H}\boldsymbol{H}^H \rightarrow \boldsymbol{I}_N$ as M gets large and N is fixed. Thus the capacity in the limit of large M is

$$C = E_H\{N.\log_2(1+\rho)\} \text{ (bits/s/Hz)} \qquad (2.14)$$

The expression $\log_2(1+\rho)$ is the capacity limit of SISO channel i.e. the capacity of MIMO channel is scaled by N times than that of the SISO channel without the need of any additional increase in transmit power or bandwidth. Thus it is intuitive to express the capacity limit of MIMO channel as

$$C=\min(N,M).\ \log_2(1+\rho) \text{ bits/sec/Hz} \tag{2.15}$$

2.4 Spatial Multiplexing

Spatial Multiplexing (SM) seems to be the ultimate solution to increase the system capacity without the need to additional spectral resources [8]. However, spatial demultiplexing or signal detection at the receiver side is a challenging task for SM MIMO system. This thesis work is primarily concerned with the study various detection techniques employed for the SM MIMO system. The basic block diagram of Spatially Multiplexed MIMO (SM MIMO) system with M transmit and N receive antennas is shown in Fig. 2.2. The channel matrix is represented by $\boldsymbol{H}$ with its channel coefficients h_{ij} for the channel gain between the j^{th} transmit and i^{th} receive antenna, $i=1, 2,\dots, M$ and $j=1,2,\dots,N$. The spatially multiplexed user data and the corresponding received signals are represented by $\boldsymbol{x}=[x_1,x_2,\dots,x_M]^T$ and $\boldsymbol{y}=[y_1,y_2,\dots,y_N]^T$ respectively, where x_i and y_j denote the transmit signal from the i-th transmit antenna and j-th receive antenna respectively. The elements of transmitted signal vector $\boldsymbol{x}$, i.e. x_i are drawn independently from a complex modulation alphabet set $\mathcal{A}$, e.g. Quadrature Amplitude Modulation(QAM). The (M,N) MIMO system is represented as:

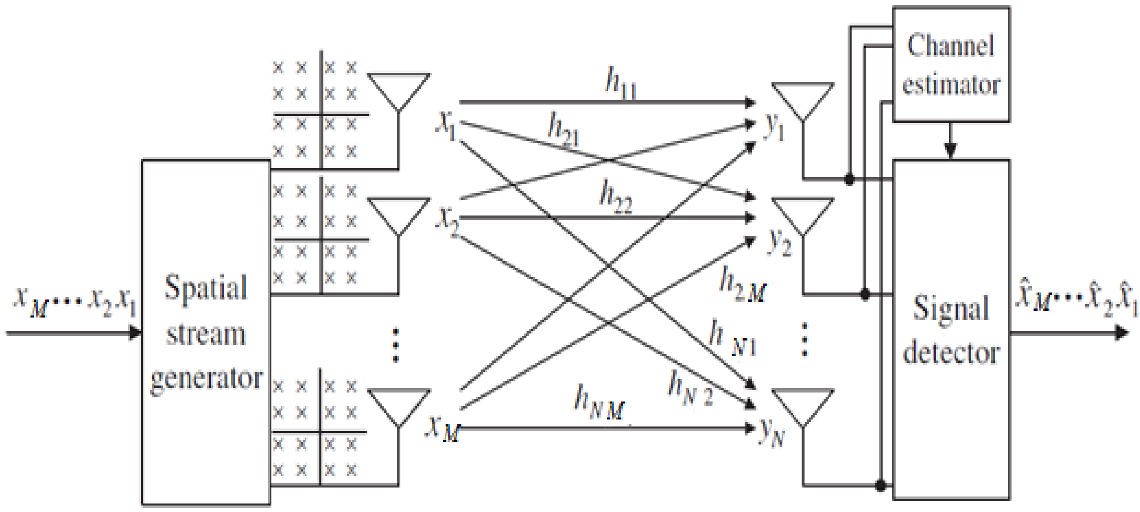

Figure 2.2: Spatial Multiplexing MIMO system [9]

CHAPTER 3: SYMBOL DETECTION IN SPATIAL MULTIPLEXING MIMO SYSTEM

The symbol detection problem considered in this thesis is the problem of estimating the MIMO channel input vector $\boldsymbol{x}$ given the received vector y assuming that the receiver has perfect knowledge of channel ($\boldsymbol{H}$). During transmission, the actual value of the transmitted signal constellation is shifted by noise. The aim of the receiver is to estimate as close a value to the transmitted symbol as possible. In essence, the receiver would need to decide on which value of the signal constellation is closest to the actual value of the transmitted signal. This decision is made on a symbol by symbol basis without taking into account any statistical dependencies that may be present in the sequence of vectors $\boldsymbol{x}$. In other words, we exclude coding across the time dimension and consider only the modulation-demodulation problem as depicted in Fig. 3.1. The goal is to minimize the probability of decision error.

$$P_e = Pr\{\hat{\boldsymbol{x}} \neq \boldsymbol{x}\} \tag{3.1}$$

where $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, \ldots\ldots, \hat{x}_M)^T$ is the demodulator's estimate of $\boldsymbol{x}$.

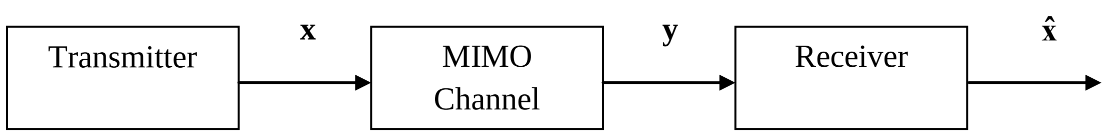

Figure 3.1: Transmission and decision in MIMO wireless systems.

3.1 Linear Detection

Linear detection is relatively simple to implement and has less computational complexity which in turn reduces the processing time in the receiver. However these receivers have intermediate performance. Linear detectors are the class of detectors for which the symbol estimate $\hat{\boldsymbol{x}}$ is given by a transformation of the received vector **y** of the form,

$$\hat{\boldsymbol{x}} = Q(\boldsymbol{W}\boldsymbol{y}) \tag{3.2}$$

where, $\boldsymbol{W}$ is a weighting matrix that may depend on $\boldsymbol{H}$ and Q is a quantizer (slicer) that maps its argument to the nearest signal point.

3.1.1 Zero Forcing (ZF) receiver

Zero Forcing is linear detection scheme which forces the interference to zero.
However it may result in increase in noise level. ZF output is given by

$$\hat{\boldsymbol{x}} = Q(\boldsymbol{W}_{ZF}\boldsymbol{y}) \tag{3.3}$$

where,

$$\boldsymbol{W}_{ZF} = (\boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H})^{-\boldsymbol{1}}\boldsymbol{H} = \boldsymbol{H}^{+} \tag{3.4}$$

and $\boldsymbol{H}^{+}$ denotes Moore-Penrose pseudoinverse of $\boldsymbol{H}$, which is generalized inverse that exists even when $\boldsymbol{H}$ is rank-deficient. The ZF receiver performs well in the high SNR regime, whereas in the low SNR regime there is significant noise enhancement [10].

3.1.2 Minimum Mean Square Error (MMSE) receiver

Unlike ZF, MMSE attempts to minimize both interference and noise at the same time. This results in better performance than ZF but with little additional complexity. The output of MMSE receiver is given by

$$\hat{\boldsymbol{x}} = Q(\boldsymbol{W}_{MMSE}\boldsymbol{y}) \tag{3.5}$$

where, $\boldsymbol{W}_{MMSE}$ is chosen to minimize $E\{\|\boldsymbol{W}_{MMSE}\boldsymbol{y}-\boldsymbol{x}\|^2\}$.

The MMSE estimator matrix is given by [11]

$$\boldsymbol{W}_{MMSE}=\frac{\rho}{M}\boldsymbol{H}^{\boldsymbol{H}}\left(\frac{\rho}{M}\boldsymbol{H}\boldsymbol{H}^{\boldsymbol{H}}+\sigma^2\boldsymbol{I}_{\boldsymbol{N}}\right)^{-\mathbf{1}} \quad (3.6)$$

3.2 Maximum Likelihood (ML) Receiver

Maximum Likelihood is the optimum detection scheme considered here. It has high computational complexity hence time consuming for decoding. It has non-linear procedure and basically consists of a exhaustive search through all possible signal vectors. The complexity increases with increase in the number of antennas or the size of constellation.

The output of ML receiver is given by

$$\widehat{\boldsymbol{x}}=arg\ min_{\boldsymbol{x}\in\mathcal{A}^M}\{\|\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x}\|^{\mathbf{2}}\} \quad (3.7)$$

Thus, the ML detector chooses the message $\hat{\boldsymbol{x}}$ which yields the smallest distance between the received vector **y** and the hypothesized message $\hat{\boldsymbol{x}}$. Note that the search for the smallest distance for ML detection increases the complexity with increase in constellation size and/or number of transmits antennas. For QAM constellation alphabet $\mathcal{A}$ with M transmit antennas, the receiver has to perform search over a set of size $|\mathcal{A}|^M$. So for higher-order modulation such as 16-QAM this complexity can become prohibitive for a small number of transmit antennas. For example, for 16-QAM and number of transmitter, $M=4$ the receiver has to search over 65,536 different vectors. ML detection is not feasible for larger number of transmit antennas or higher modulation schemes [10].

3.3 Successive Interference Cancelation

3.3.1 Zero Forcing with Successive Interference Cancellation (ZF-SIC)

The linear detection schemes are just a bank of separate filters to estimate the data streams. However, the result of one of the filters could be used to aid the operation of the others. This is the idea of the successive cancellation strategy: once a data stream is successfully recovered, subtract it off from the received vector and reduce the burden on the receivers of the remaining data streams.

With this motivation, the receiver structure is modified as shown in Fig. 3.2

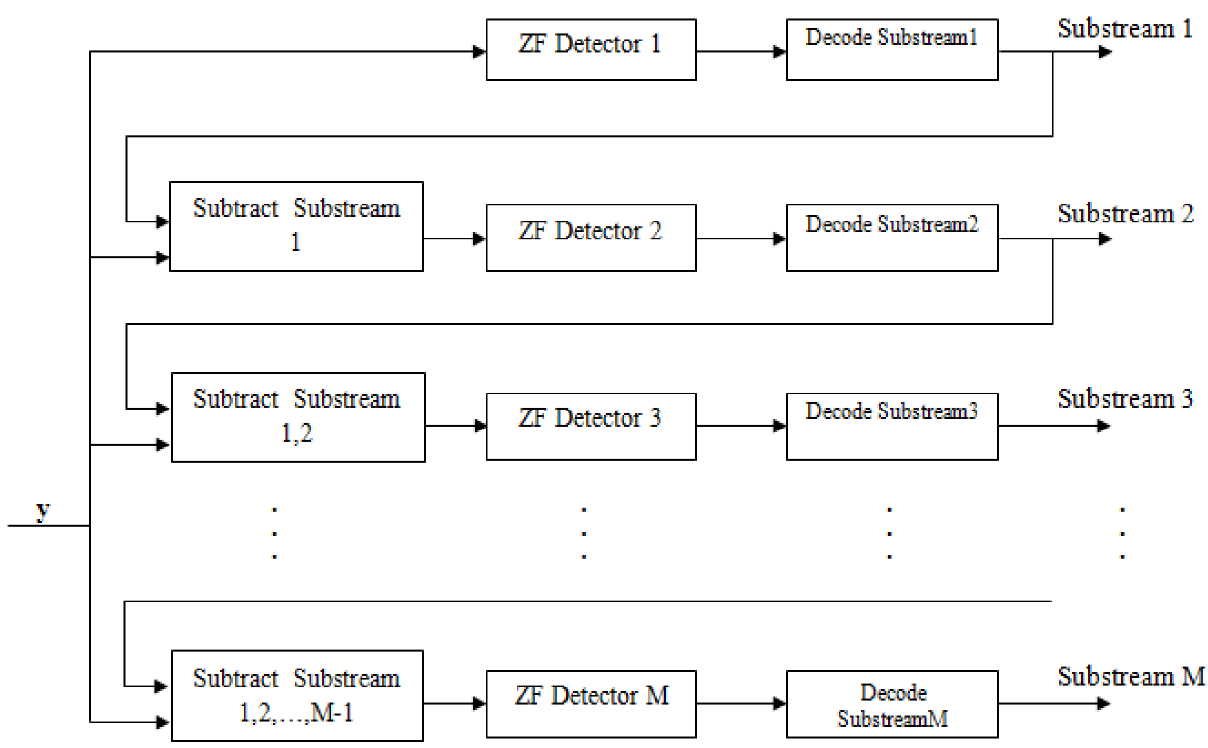

Figure 3.2: Zero Forcing with Successive Interference Cancellation (ZF-SIC) [12]

The Zero-Forcing with Successive Interference Cancellation detector architecture (ZF-SIC) is illustrated in Fig. 3.2. After the nulling operation, assuming that $\hat{x} = x_i$, cancel x_i from y_i and generate modified received signal vector $\boldsymbol{y_{i+1}} = \boldsymbol{y_i} - \boldsymbol{\hat{x}w_i^H}$ for $i = 1, 2, \ldots, M$.

Assuming the first stream is successfully decoded, and then the Zero-Forcing Detector (ZF Detector 2) only needs to deal with the streams of $x_3, x_4, \ldots, x_M$ as interference, since x_1 has been correctly subtracted off. Thus, the ZF Detector 2 projects onto a subspace which is orthogonal to that spanned by $h_3, h_4, \ldots, h_M$. This process is continued until the final ZF detector does not have to deal with any interference from the other data streams (assuming successful subtraction in each preceding stage).

One problem with this SIC receiver structure is *error propagation*: an error in decoding the k^{th} data stream means that the subtracted signal is incorrect and this error propagates to all the streams further down, $k+1, \ldots, M$. To minimize the error and ordering strategy can be employed which is described in later section.

3.3.2 Minimum Mean Square Error with Successive Interference Cancellation (MMSE-SIC)

Minimum Mean Square Error Successive Interference Cancellation (MMSE-SIC) is signal detection method where weight matrix is used same as linear detector MMSE In Fig. 3.2, banks of separate filters were considered to estimate the data streams. However, we can use the result of one filter to help the operation of other filters similar to ZF-SIC detectors. Once a data stream is successfully recovered, we can subtract it from the received vector and reduce the burden on the receiver of the remaining data streams.

3.4 Ordered Successive Interference Cancelation (OSIC)

3.4.1 ZF Successive Interference Cancelation with ordering (ZF-OSIC)

Consider the problem of the error propagation we discussed in Section 3.3.1. In a real communication system, it is impossible to cancel the streams perfectly, thus the error propagation is unavoidable. Furthermore, the error propagation has a negative impact on the error rate performance of the receiver. The goal is trying to make this error propagation as small as possible.

In order to combat the error propagation, it is necessary to utilize an ordering strategy in the system. Ordering is a method of easing the error propagation by sorting the order of the streams being detected. Streams with higher post detection SNRs may have lower error probabilities, thus processing these streams at earlier stages lead to less errors. Each signal is an interferer to the rest of the signals. The best signal stream in terms of SNR is selected for detection and is also removed from the remaining signals. This way, the remaining signals have one interferer less. Optimal combining is repeated until all signals are detected.

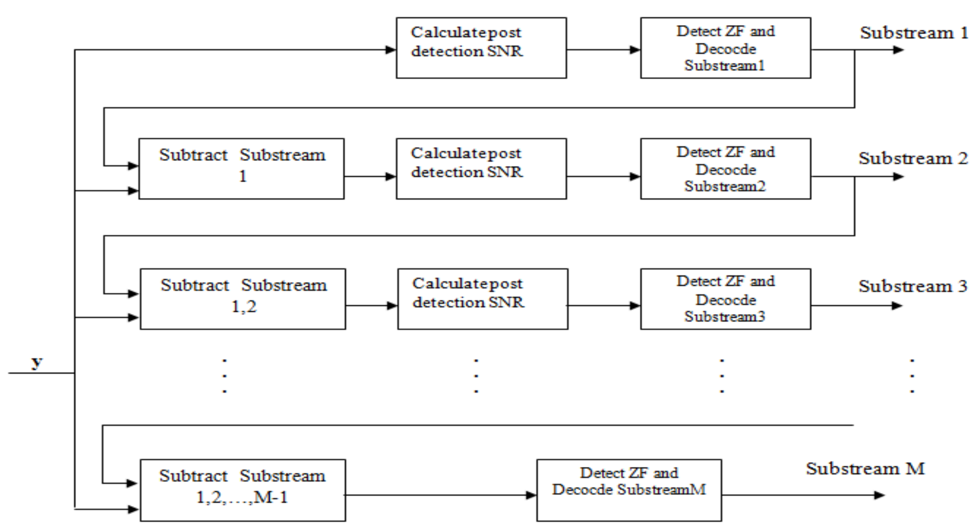

Figure 3.3: ZF Successive Interference Cancelation with ordering (ZF-OSIC)

Three consecutive phases take place for SIC ordering Zero-Forcing in [12] and [13]:

- Linear interference suppression
- Interference cancellation of the substreams detected
- Reordering of the detection process through SNR post-detection

This operation is realized in three steps:

$$w_{k_i} = (G_i)_{k_i} \tag{3.8}$$

$$z_{k_i} = w_{k_i}^T . y_i \tag{3.9}$$

$$\hat{x}_{k_i} = Q(z_{k_i}) \tag{3.10}$$

The first step consists on finding the zero-forcing vector w_{k_i} that satisfies:

$$w_i^T . (H)_j = \begin{cases} 0 & j > i \\ 1 & j = i \end{cases} \tag{3.11}$$

In above equations, $(H)_j$ represents the j^th column of H which is a vector composed of all the channel coefficients produced from transmit antenna j. It turns out that the condition of above equation is realized by taking w_i to be the equivalent j-th column from the Moore-Penrose pseudoinverse. Hence, the algorithm finds the zero-forcing vector w_{k_i} through consecutively calculating the Moore-Penrose pseudo-inverse. Also, i is the iteration number, k_i is the index of the transmit antenna whose substream is detected during iteration i and G_i is the Moore-Penrose pseudo-inverse at iteration stage i. With w_{k_i} , the statistic decision z_{k_i} is obtained from the received vector y_i at stage i, and is quantized (represented by the operator $Q(.)$ in order to obtain the k_i^{th} detected substream.

The interference of the detected substream is cancelled in the second step of the algorithm by removing its interference from the received signal:

$$y_{i+1} = y_i - \hat{x}_{k_i}.(H)_{k_i} \tag{3.12}$$

$$G_{i+1} = H_{k_i}^{\pm} \tag{3.13}$$

A new received vector $\boldsymbol{y_{i+1}}$ is formed by removing y_i from the estimated transmit signal of the detected substream. The Moore-Penrose pseudo-inverse (represented by the + sign as superscript) is recalculated from the previous stage transfer matrix after annulling its k_i^{th} row. The operations realized in Equation 3.9 can be seen as redefining a new system where the antenna that used to transmit the previous detected substream is eliminated. This is achieved by removing the signal produced by the detected substream and by annulling the channel coefficients related to the estimated substream's antenna.

The last step of the process is to select the optimum order for decoding. In other words, the layers are rearranged in the process in order to minimize the probability of error. This means finding an index k_i of stage i. The method proposed in the algorithm is to select the antenna with the best post-detection SNR. The post-detection SNR of each transmit antenna j is the absolute value of the j^{th} column of the Moore-Penrose pseudo-inverse G_i at stage i. If S is the optimum ordering and $S = \{k_1, k_2, \ldots, k_M\}$, the search for the best SNR post- detection at stage i is performed for the set $\{k_1, k_2, \ldots, k_M\}$, since the substreams transmitted from antennas k_1 to k_i already have been detected in previous stages. The best post-detection SNR is the minimum absolute value calculated:

$$k_i = arg \;\; min_{J \in S, j \notin \{k_1, k_2, \ldots, k_{i-1}\}} \left\| (G_i)_j \right\|^2 \tag{3.14}$$

The generalized algorithm is composed of two phases, an initialization and a recursive phase. The initialization phase consists of calculating the Moore-Penrose pseudo-inverse:

Step 1: Initialization

$$G_1 = W_{ZF} = H^+$$

$$i = 1$$

Step 2: The recursive phase is composed of the previously described steps as:

$$k_i = arg \ \ min_{J \in S, j \notin \{k_1, k_2, \ldots, k_{i-1}\}} \left\| (G_i)_j \right\|^2$$

$$w_{k_i} = (G_i)_{k_i}$$

$$z_{k_i} = w_{k_i}^T . y_i$$

$$\hat{x}_{k_i} = Q(z_{k_i})$$

$$y_{i+1} = y_i - \hat{x}_{k_i}(H)_{k_i}$$

$$G_{i+1} = H_{k_i}^{\pm}$$

$$i = i + 1$$

So, instead of processing the substreams according to the order in Fig. 3.2, the post detection SNR of each substream was calculated; the substreams in order from the highest SNR to the lowest SNR were ordered. Then selection of the substream with the highest post detection SNR as the substream 1 was done. This stream was applied with its corresponding Zero-Forcing detector, decoded and subtracted from the received vector. After subtraction, we now have a new MIMO system with $M - 1$ transmitting substreams and N receiving antennas. Now we calculated the post detection SNR for the new $(M - 1)\ x\ N$ system, selected the substream with the highest SNR as the substream 2, continued the above steps until all the transmitted substreams were decoded. An illustration of the SIC ordering ZF detector is shown in Fig. 3.3.

3.4.2 MMSE Successive Interference Cancelation with ordering (MMSE-OSIC)

When the cancellation of the detected substream is employed, the order in which the components of $\boldsymbol{x}$ are detected is important to the overall system performance. In the ordering strategy, we choose the data substream with the highest post detection SNR every time; this makes sure that the number of the errors being propagated to the next stage is the smallest.

Instead of processing the substreams according to the order as described in section 3.3.2, we calculate the post detection SNR of each substream. Then we put the substreams in order from the highest SNR to the lowest SNR and apply this substream with its corresponding MMSE detector, decode and subtract it from the received vector similar to the zero-forcing algorithm. An illustration of the SIC ordering MMSE detector is shown in Fig. 3.4.

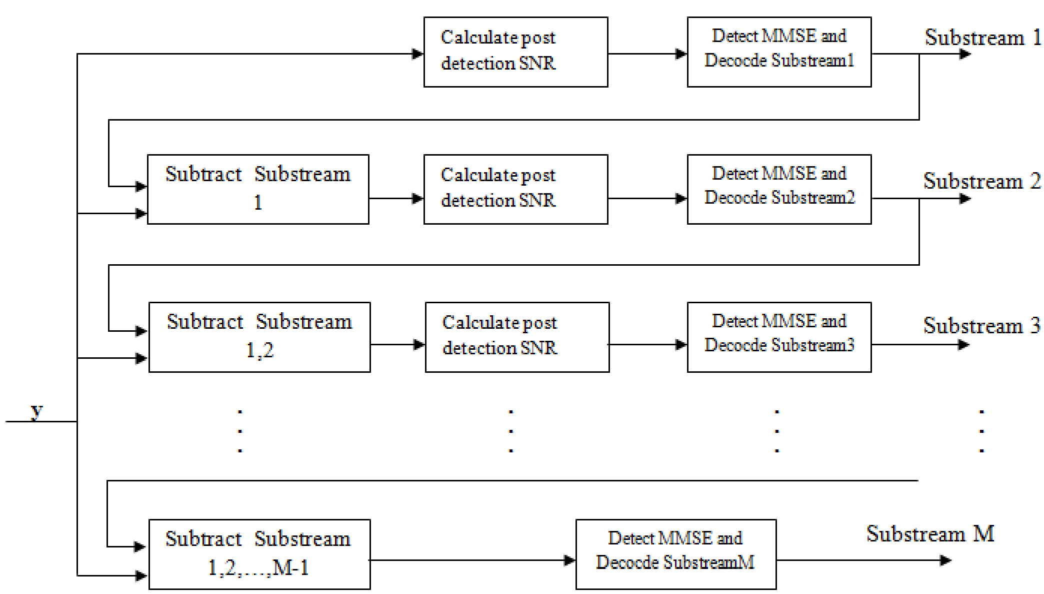

Figure 3.4 MMSE Successive Interference Cancelation with ordering (MMSE-OSIC)

The mathematical description for the minimum mean square error detection algorithm which is a recursive procedure with ordering strategy is shown as:

Step 1: Initialization

$$G_1 = W_{MMSE}$$

$$i = 1$$

Step 2: Recursion

$$k_i = arg\ \ min_{J \in S, j \notin \{k_1, k_2, \ldots, k_{i-1}\}} \left\| (G_i)_j \right\|^2$$

$$w_{k_i} = (G_i)_{k_i}$$

$$z_{k_i} = w_{k_i}^T . y_i$$

$$\hat{x}_{k_i} = Q(z_{k_i})$$

$$y_{i+1} = y_i - \hat{x}_{k_i} (H)_{k_i}$$

$$G_{i+1} = H_{k_i}^{\pm}$$

$$i = i + 1$$

3.5 Sphere Detection

The output of ML receiver as described in equation 3.7 is given by

$$\hat{\boldsymbol{x}} = arg\ \ min_{\boldsymbol{x} \in \mathcal{A}^M} \{ \| \boldsymbol{y} - \boldsymbol{H}\boldsymbol{x} \|^{\mathbf{2}} \} \qquad (3.15)$$

It can be seen clearly that the ML detector attempts to retrieve the desired constellation point, $\hat{\boldsymbol{x}}$ that would minimize $\| \boldsymbol{y} - \boldsymbol{H}\boldsymbol{x} \|^{\mathbf{2}}$. In essence, an ML detector needs to calculate $\| \boldsymbol{y} - \boldsymbol{H}\boldsymbol{x} \|^{\mathbf{2}}$ for every transmitted constellation point before deciding on which symbol was sent. Although the ML estimate is a very accurate means of estimating the transmitted symbols, it is clear that for higher order systems where *M* and/or constellation size is high, the computational complexity of ML would become very significant. The high computational complexity necessary for ML detector operation led to research into other detection algorithms which offer much lower complexity though they might be sub-optimal in nature.

The basic idea of Sphere Decoder (SD) is to limit search only to the lattice points $\boldsymbol{Hx}$ that lie in a sphere of radius *R* around the given vector y and in this way save on computations. It is clear that the closest point inside the sphere is also the closest point in the lattice. Therefore, there is no need to make an exhaustive search over all lattice points. Moreover, if the radius of sphere is properly chosen one can limit the number of operations used in order to find the desired point in sphere. The sphere radius constraint can be included in the in the ML detection rule as follows

$$\hat{\boldsymbol{x}} = arg\ min_{\boldsymbol{x}\in\mathcal{A}^M}\{\|\boldsymbol{y}-\boldsymbol{Hx}\|^2 \leq R^2\} \qquad (3.16)$$

Where, *R* is the radius of the sphere.

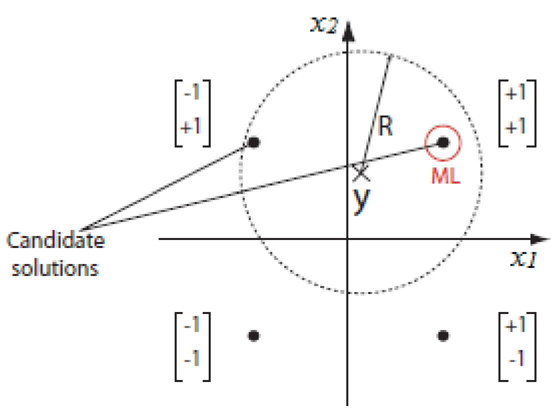

Figure 3.5: Decoding sphere of radius R for limiting the candidate lattice points in a 2x2 MIMO system using a BPSK constellstion [14]

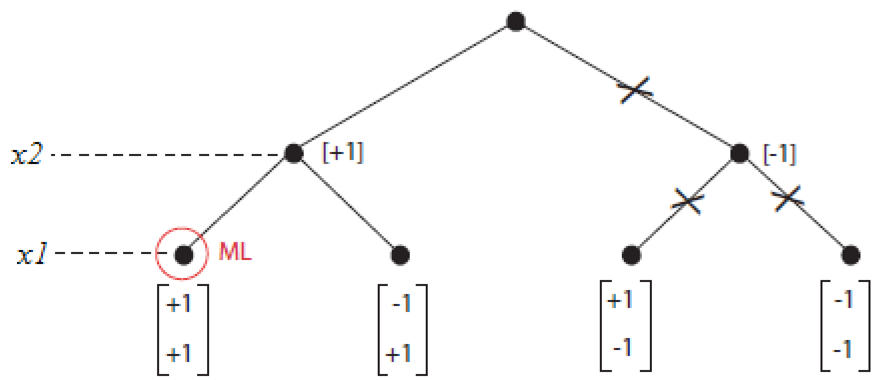

Figure 3.6: Decoding tree associated to the decoding of sphere of Figure 3.5.

A widely recognized manner of interpreting the search characteristics for sphere decoder can be visualized as a tree search algorithm. Different tree search strategies have been proposed, but they can be classified into two main types of tree search: Depth- First and Breadth-First.

In the Depth-First algorithms the tree is explored beginning from the root descending to the leaf nodes. In the Breadth-First algorithms the tree is explored descending level by level up to the leaf nodes, every child in the same level has to be visited before starting to visit the following level. In this thesis, we are only concerned with depth first search algorithm.

3.5.1 Depth-first algorithm

The real domain system is expressed as

$$\begin{bmatrix}\mathrm{Re}(\boldsymbol{y})\\ \mathrm{Im}(\boldsymbol{y})\end{bmatrix} = \begin{bmatrix}\mathrm{Re}(\boldsymbol{H}) & -\mathrm{Im}(\boldsymbol{H})\\ \mathrm{Im}(\boldsymbol{H}) & \mathrm{Re}(\boldsymbol{H})\end{bmatrix}\begin{bmatrix}\mathrm{Re}(\boldsymbol{x})\\ \mathrm{Im}(\boldsymbol{x})\end{bmatrix} + \begin{bmatrix}\mathrm{Re}(\boldsymbol{n})\\ \mathrm{Im}(\boldsymbol{n})\end{bmatrix} \tag{3.17}$$

Equivalently,

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{3.18}$$

Note that by transformation to real domain the MIMO system is now transformed from *N* dimensional to *2N* dimensional but is computationally efficient. The vector $\boldsymbol{x}$ is chosen from the set of real entries in the constellation $\mathcal{A}$, for example $x_i \in \{-3, -1, +1, +3\}$ in the case of 16-QAM [15].

The channel matrix H can be decomposed using QR decomposition as

$$\boldsymbol{H} = \boldsymbol{QR} \tag{3.19}$$

Where, $\boldsymbol{Q}$ is 2*Nx2N* and orthogonal matrix and $\boldsymbol{R}$ is 2*Nx2M* upper triangular matrix. For simplicity, we assume *M*=*N*, in rest of the section.

Since $\boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{I}$, multiplying both sides of equation 3.12 by $\boldsymbol{Q}^T$, we have

$$\boldsymbol{y}' = \boldsymbol{Rx} + \boldsymbol{Q}^T\boldsymbol{n} \tag{3.20}$$

Where,

$$\boldsymbol{y}' = \boldsymbol{Q}^T\boldsymbol{y} \tag{3.21}$$

After the complex to real transformation performed by equation 3.17 a new channel matrix $\boldsymbol{H}$ is gained and the QR decomposition is performed on it. The transformed ML solution is

$$\hat{\mathbf{x}} = \arg\ \min_{\mathbf{x}\in\chi^{2M}}\{\|\boldsymbol{y}' - \mathbf{Rx}\|^{\mathbf{2}}\} \tag{3.22}$$

Due to upper triangular $\boldsymbol{R}$ matrix, $\|\boldsymbol{y}' - \mathbf{Rx}\|^{\mathbf{2}}$ has a structure enabling the progressive search as

$$\left\|\begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_{2M}' \end{bmatrix} - \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,2M} \\ 0 & r_{2,2} & & r_{2,2M} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & r_{2M,2M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2M} \end{bmatrix}\right\|^2 \tag{3.23}$$

The tree based search algorithm, namely the sphere decoder is based on this modified cost function as:

$$R^2(\boldsymbol{x}) = \|\boldsymbol{y}' - \boldsymbol{Rx}\|^2$$

$$= \sum_{i=1}^{2M} \left| y_i' - \sum_{j=i}^{2M} r_{i,j} x_j \right|^{\mathbf{2}}$$

$$= \left| y_1' - \sum_{j=1}^{2M} r_{1,j} x_j \right|^{\mathbf{2}} + \left| y_2' - \sum_{j=2}^{2M} r_{2,j} x_j \right|^{\mathbf{2}} + \cdots + \left| y_{2M}' - r_{2M,2M} x_{2M} \right|^{\mathbf{2}} \quad (3.23)$$

Thus the distance $R^2(\boldsymbol{x}) = \|\boldsymbol{y'} - \boldsymbol{Rx}\|^2$can also be computed recursively as follows [16]:

$$T_i\left(\boldsymbol{x}^{(i)}\right) = \quad T_{i+1}(\boldsymbol{x}^{(i+1)}) + \left| y_i' - \sum_{j=i+1}^{2M} r_{i,j} x_j - r_{i,i} x_i \right|^2 \text{ where } i = 2M, \ldots, 1$$

$$= T_{i+1}\left(\boldsymbol{x}^{(i+1)}\right) + \left| b_{i+1}\left(\boldsymbol{x}^{(i+1)}\right) - r_{i,i} x_i \right|^{\mathbf{2}} \quad (3.24)$$

And $T_i(\boldsymbol{x}^{\boldsymbol{i}}) = 0$ if $i = 2M + 1$

Here, $\boldsymbol{x}^{(\boldsymbol{i})} = [x_i, x_{i+1}, \ldots, x_{2M}]$ with $i = 1, \ldots, 2M$. And, the term $\sum_{j=i+1}^{2M} r_{i,j} x_j = 0$ for $i = 2M$, then after 2M steps, the distance $R^2(\boldsymbol{x})$is obtained as $R^2(\boldsymbol{x}) = T_1(\boldsymbol{x}\,)$. Also, the distance increments are non-negative; it follows immediately that whenever the partial weight of a node violates the sphere constraint then all of its children will also violate the sphere constrained. This approach effectively reduces the number of transmit vector symbols i.e. leaves of the tree to be checked. Also the radius is progressively reduced every time a leaf is reached at a distance that is smaller than current radius.

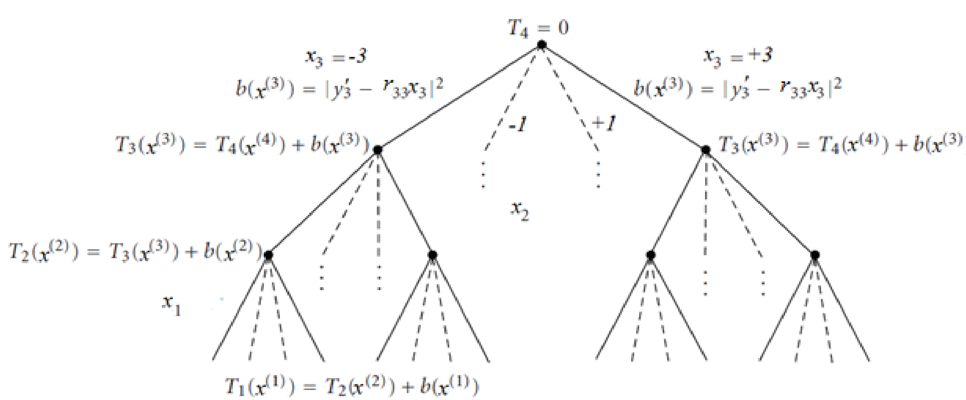

Figure 3.7: Tree organization for sphere decoder for constellation {-3,-1,+1,+3} [16]

Several algorithms have been studied in order to make the tree traversal efficient. First algorithm, proposed by Fincke and Pohst [17], needs to chose explicitly an initial radius. This resulted in two major problems: if the sphere radius is initialized to too small value then, all the branches may be pruned before finding a ML solution. And if the sphere radius is too large, then many constellation points has to enumerated before reaching the ML solution, which increases the complexity of the detection algorithm. Also the, searching order children nodes of FP SD is based left to right which can be reduced if the candidates are enumerated based on their branch metric. A more efficient solution was proposed by Schnorr and Euchner(SE) [18]. In this case, the initial radius can be set to infinity. Obviously, in this way the event of declaring an empty sphere never occurs. Also, the first point In this case, corresponds to the Babai point and a "depth and best first" traversal of the tree is performed[16].

With radius reduction, it is desirable to find candidate solutions that lie close to the ML solution as early as possible in order to shrink the sphere as fast as possible and hence speed up the tree pruning. A scheme proposed by Schnorr and Euchner [19] and modified for the finite lattice case in [20] traverses the members of the admissible sets in ascending order of their PEDs. In the case of real-valued lattice constellations, given a starting point and an initial direction, this ordering is predefined. The starting point is given by:

$$x_i = Q((y_i' - \sum_{j=i}^{2M} r_{i,j} x_j)/r_{i,i}) \tag{3.25}$$

The decoder starts with the center of the admissible interval and proceeds to the boundaries in a zigzag fashion as depicted in Fig. 3.7. Figure 3.8 shows the initial searching strategy employed with SE enumeration.

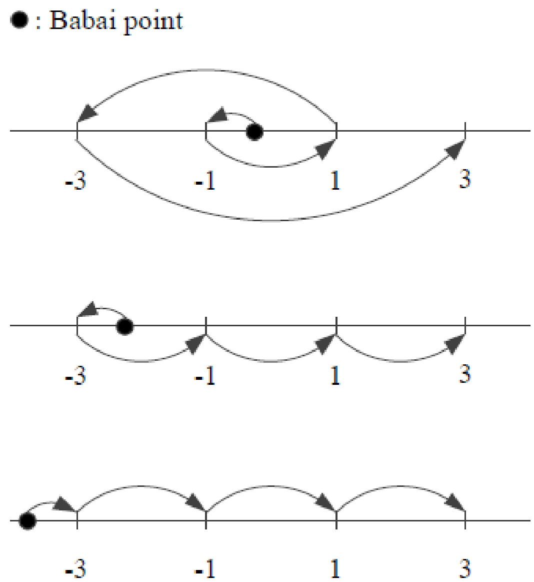

Figure 3.7: SE Enumeration [15]

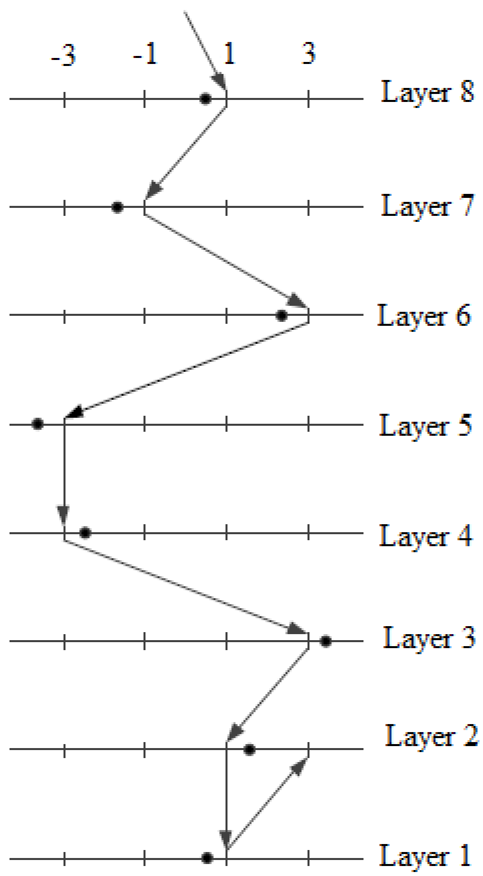

Figure 3.8: Depth first searching strategy with SE Enumeration [15]

CHAPTER 4: METHODOLOGY

4.1. Symbol Detection

The symbol detection problem considered in this thesis is the problem of estimating the MIMO channel input vector **x** given the received vector **y** assuming that the receiver has perfect knowledge of channel (H). During transmission, the actual value of the transmitted signal constellation is shifted by noise. The aim of the receiver is to estimate as close a value to the transmitted symbol as possible. In essence, the receiver would need to decide on which value of the signal constellation is closest to the actual value of the transmitted signal. This decision is made on a symbol by symbol basis without taking into account any statistical dependencies that may be present in the sequence of vectors **x**. In other words, we exclude coding across the time dimension and consider only the modulation-demodulation problem as depicted in Fig. 4.1. The goal is to minimize the probability of decision error.

$$P_e = Pr\{\hat{\boldsymbol{x}} \neq \boldsymbol{x}\} \qquad (4.1)$$

Where $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots\ldots, \hat{x}_M)^T$ is the demodulator's estimate of **x**.

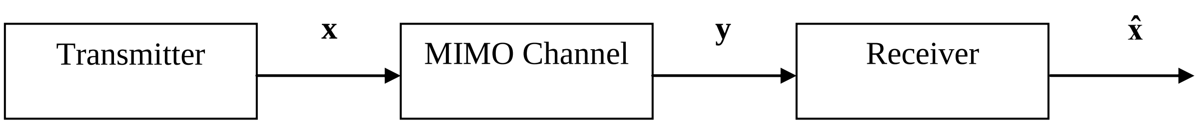

Figure 4.1: Transmission and decision in MIMO wireless systems.

The following assumptions are considered to address the detection problem on the input vector **x** [22].

i. Each element x_i of input vector **x** belongs to common modulation alphabet $\mathcal{A}$, i.e. $x_i \epsilon \mathcal{A}$, $i = 1,2\ldots.,M$.

For example the modulation alphabet, $\mathcal{A}$ = { -1-j -1+j 1-j 1+j } for 4-QAM modulation scheme.

ii. Each element in $\mathcal{A}$ have equal a priori probabilitites.

iii. The vector **x** is a random vector over $\mathcal{A}^M$ such that

$$E\{\boldsymbol{x}\boldsymbol{x}^{\boldsymbol{H}}\} = \frac{\rho}{M}\boldsymbol{I}_M \tag{4.2}$$

where ρ is a constant, $\boldsymbol{I}_M$ is the identity matrix of size M, $E\{\ .\ \}$ is the expectation operator and $\boldsymbol{x}^{\boldsymbol{H}}$ denotes Hermitian transpose of **x.** Assumption (iii) implies that the elements of $\boldsymbol{x}$ are uncorrelated and each has energy

$$E\{|x_i|^{\mathbf{2}}\} = \frac{\rho}{M} \tag{4.3}$$

yielding a total average transmitted energy of ρ per symbol, combined over all antennas. The parameter ρ also has the significance of being the average received energy per symbol E_s at each receiver antennas i.e.

$$E_s = \rho \tag{4.4}$$

Using above equation, the average received energy per bit at each receiver antenna may be computed as

$$E_b = \frac{E_s}{\log_2|\mathcal{A}|} = \frac{\rho}{\log_2|\mathcal{A}|} \tag{4.5}$$

and receiver signal to noise ratio (SNR) is defined as

$$SNR = \frac{E_b}{N_0} = \frac{\rho}{N_0\log_2|\mathcal{A}|} \tag{4.6}$$

While designing a receiver structure for this MIMO system, two main considerations that should be taken into account are the error performance and the implementation complexity. The aim of this thesis work is to study various receiver structures in terms of error performance.

To analyze the performance of MIMO detection system, BER and SNR plots are developed and observed under various conditions: number of transmit and receive antennas ((2,2) and (4,4) MIMO system), modulation order of 16QAM.

In digital transmission, the number of bit errors is the number of received bits of a data stream over communication channel that has been altered due to noise, interference, distortion or bit synchronization error. The Block Error Rate (BER) represents the quality of digital link, is calculated from the number of blocks received in error divided by the number of blocks transmitted.

BER = (Blocks in Error) / (Total Blocks Received)

4.2 Development of Algorithm

The general algorithms for various detection algorithms are as follows:

1. Specify the number of transmit and receive antennas.
2. Specify the SNR value
3. Generate the transmit signal vector using appropriate constellation set (16QAM)
4. Generate the channel matrix
5. Generate noise and compute the received signal
6. Specify one of the detection scheme and find the estimate of transmit signal vector
7. Compare the estimated signal with the transmit signal to find error.
8. Simulations are done for BER vs SNR on different detection schemes.

The algorithm of various detection scheme are described in the following section.

4.2.1 Zero Forcing (ZF) receiver

It uses the weight matrix as $\boldsymbol{W}_{ZF} = (\boldsymbol{H}^{\boldsymbol{H}}\boldsymbol{H})^{-\mathbf{1}}\boldsymbol{H} = \boldsymbol{H}^{+}$ in order to estimate the transmitted symbol.

4.2.2 Minimum Mean Square Error (MMSE) receiver

It uses the weight matrix as $\boldsymbol{W}_{\boldsymbol{MMSE}} = \frac{\boldsymbol{\rho}}{\boldsymbol{M}}\boldsymbol{H}^{\boldsymbol{H}}\left(\frac{\boldsymbol{\rho}}{\boldsymbol{M}}\boldsymbol{H}\boldsymbol{H}^{\boldsymbol{H}} + \boldsymbol{N}_{\mathbf{0}}\boldsymbol{I}_{\boldsymbol{N}}\right)^{-\mathbf{1}}$ in order to estimate the transmitted symbol.

4.2.3 ZF/MMSE-SIC

The ZF/MMSE Successive Interference Cancellation uses liner combinational nulling and symbol cancellation to successively compute the $\boldsymbol{z}_{\boldsymbol{k}_{\boldsymbol{i}}}$, proceeding as follows:

Step 1: Using nulling vector w_{k_1}, form a linear combination of the components of $\boldsymbol{y}_{\mathbf{1}}$ to yield z_{k_1}

$$z_{k_1} = w_{k_1}^T.\boldsymbol{y}_{\mathbf{1}}$$

Step 2: Slice z_{k_1} to obtain $\hat{x}_{k_1}$

$$\hat{x}_{k_1} = \mathcal{Q}(z_{k_1})$$

Step 3: Assuming that $\hat{x}_{k_1} = x_{k_1}$, cancel from the received vector $\boldsymbol{y}_{\mathbf{1}}$, cancel x_{k_1} from the received vector $\boldsymbol{y}_{\mathbf{1}}$, resulting in modified received vector $\boldsymbol{y}_{\mathbf{2}}$

$$\boldsymbol{y}_{\mathbf{2}} = \boldsymbol{y}_{\mathbf{1}} - \hat{x}_{k_1}.(\boldsymbol{H})_{k_1}$$

Where $(\boldsymbol{H})_{k_1}$ denotes k_1th column of $\boldsymbol{H}$.

The steps 1-3 are then performed for components k_2,…, k_M by operating in turn on the progression of modified vectors $\boldsymbol{y}_{\mathbf{2}}, \boldsymbol{y}_{\mathbf{3}}, \dots, \boldsymbol{y}_{\boldsymbol{M}}$.

4.2.4 ZF/MMSE-OSIC

The ZF/MMSE OSIC algorithm is similar to that of ZF/MMSE-SIC but with an important modification. Instead of extracting the symbols from the ordered set as in SIC, OSIC reorders the detection process through post-detection SNR computation which helps to minimize the error propagation.

Three consecutive phases take place for ZF/MMSE-OSIC are,

1. Linear interference suppression and detection
2. Interference cancellation of the sub streams detected
3. Reordering of the detection process through SNR post-detection

The steps 1 and 2 are same as that of SIC however step 3 changes the order of detection of symbols according to the post detection SNR. Thus the symbols with highest post detection SNR are decoded first than those with low post-detection SNR values. This aids to minimize the error propagation which is inevitable in such scheme but can be minimized. The generalized algorithm for ZF/MMSE-OSIC is presented below:

Initialization:

$$G_1 = W_{ZF} \ or \ W_{MMSE}$$

$$i = 1$$

The recursive phase is composed of the previously described steps as:

$$k_i = arg \ \ min_{J \in S, j \notin \{k_1, k_2, \dots, k_{i-1}\}} \left\| (G_i)_j \right\|^2$$

$$w_{k_i} = (G_i)_{k_i}$$

$$z_{k_i} = w_{k_i}^T . y_i$$

$$\hat{x}_{k_i} = Q(z_{k_i})$$

$$y_{i+1} = y_i - \hat{x}_{k_i}(H)_{k_i}$$

$$G_{i+1} = H_{k_i}^{\pm}$$

$$i = i + 1$$

4.2.5 Sphere decoding

Step 1. (Initialization)

Set $i := 2m, T_i := 0, z_i := 0 \; and \; radius := infinite$(sphere radius)

Step 2. Set $x_i = Q\left[\frac{y_i' - z_i}{r_{i,i}}\right] \; and \; \Delta_i = sign(y_i' - z_i - r_{i,i}x_i)$

Step 3. (Main step)

If $T_i + \left|y_i' - z_i - r_{i,i}x_i\right|^2 > radius$

Go to Step 4 (outside the sphere)

Else if $x_i \notin$ constellation

Go to Step 6 (inside the sphere but outside the signal set boundaries)

Else (inside the sphere and signal set boundaries)

If $i > 1$

$$z_{i-1} = \sum_{j=i}^{2m} r_{i-1,j}x_j$$

$$T_{i-1} = T_i + \left|y_i' - z_i - r_{i,i}x_i\right|^2$$

$$i = i - 1$$

Go to Step 2

Else ($i.e., i = 1$)

Go to Step 5

Step 4. If $i = 2m$

Terminate Algorithm

Else

Set $i := i + 1$. Go to Step 6

Step5. (A valid point is found)

Update $radius = T_i + \left|y_i' - z_i - r_{i,i}x_i\right|^2$

Save $\boldsymbol{x}$. Let $i = i + 1$ and go to Step 6.

Step 6. (Schnorr-Euchner enumeration of level i)

Let $x_i = x_i + \Delta_i$

$\Delta_i = -\Delta_i - sign(\Delta_i)$

Go to Step 3.

4.3 Verification

Verification of different detection schemes are done on the simulation on the MATLAB software. On evaluation the performance, plot has been done between the BER and SNR.

4.4 Finalization

Finally, various detection schemes are compared in terms of BER to find out the optimal detection method among them.

CHAPTER 5: SIMULATION AND RESULTS

To evaluate the performance of various detection algorithms as described in previous sections the simulation is carried out in MATLAB software. MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, Java, and Fortran. The MATLAB programming software is used to plot the BER vs. SNR values computed for the detection schemes.

5.1 Simulation Setup

For the purpose of experimental setup, point to point (single user MIMO) is considered. The evaluation of performance of various detection algorithms is carried out under the variations of following scenarios:

- MIMO System: 2x2 and 4x4
- Modulation: 16QAM
- Channel: Rayleigh Fading Channel
- Noise: AWGN with zero mean and variance $\sigma^{2.}$

The MIMO system has *M* transmit antennas and *N* receive antennas. The modulation scheme used at transmitter is 16-QAM. For 16-QAM, $x_i \epsilon$ {±3±3j ,±3±1j, ±1±j, ±1±3j}

The transmitted symbol vector can be expressed as $\boldsymbol{x} = (x_1, x_2, \dots, x_M)^T$

The transmitted symbols interact with Rayleigh channel before entering the receiver. The Rayleigh channel is characterized as a channel in which fading occurs due to multiple paths between transmitter and receiver, with no distinct dominant path. The NxM channel matrix is given by Eq. 2.4 where each h_{ij} is a complex Gaussian with zero mean and unit variance. In mathematical form a Rayleigh channel is given Eq. 2.5 with each element, $h_{ij} = \alpha + j\beta$, where, both α and β are zero mean Gaussian random variable.

Noise is also added to the transmitted signal. The noise considered is AWGN with zero mean and variance σ^2. The $Nx1$ noise matrix is expressed as

$$\boldsymbol{n} = (n_1, n_2, \ldots, x_N)^T$$

With these assumptions on transmitted signal, channel and noise, the received signal vector of $Nx1$ dimension is expressed as: $\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{n}$

The received signal $\boldsymbol{y}$ is then processed according to the various detection algorithms described in chapter 4 to find the best estimate of the transmitted symbols. The Eb/N0 ranges between -10 to 20 dB in the step of 2 dB. The Block Error Rate (BER) is calculated by performing 10,000 trials for each Eb/N0 value. A new realization of H is chosen in each trail for each Eb/N0 values. Finally the plot of BER vs Eb/N0 values is done.

5.2 Simulation Results:

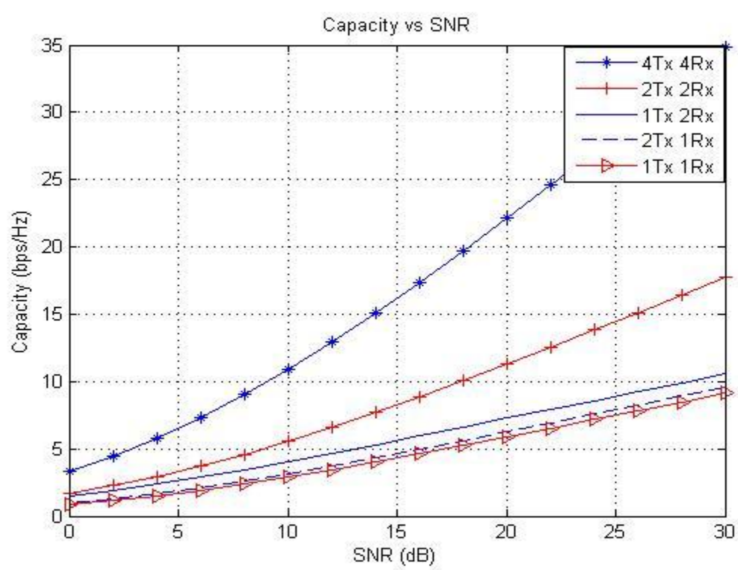

Figure 5.1: Simulation result of comparison of capacity of SISO and MIMO system

Simulation result interpretation:

Figure 5.1 shows the simulation result of comparison of capacity of various antenna configurations system. The channel capacity is least for single antenna, SISO system and increases with increase in the antenna numbers.

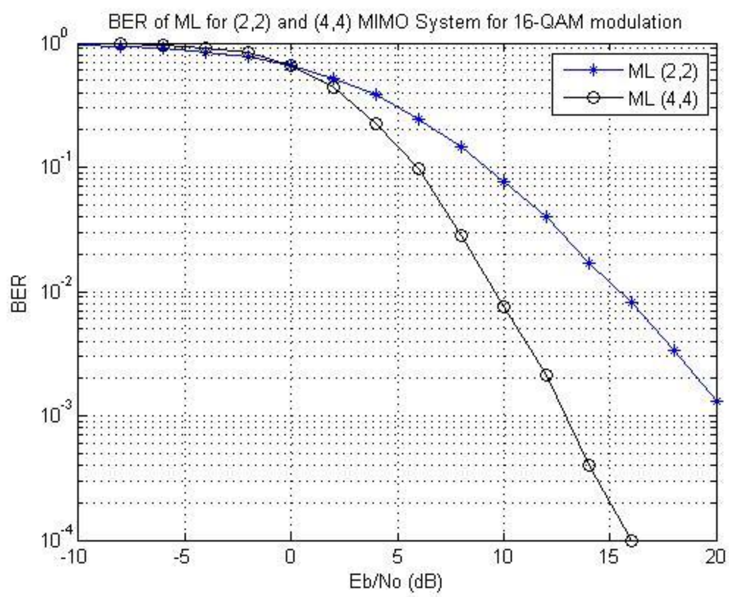

Figure 5.2: BER of ML for (2,2) and (4,4) MIMO System for 16-QAM modulation.

Simulation result interpretation:

Figure 5.2 shows performance curve of ML receivers employed for (2,2) and (4,4) MIMO system with 16-QAM modulation scheme. The ML solution is the optimum detection scheme and act as reference to other detection schemes. It can be seen that the performance is improved when the antenna numbers is increased.

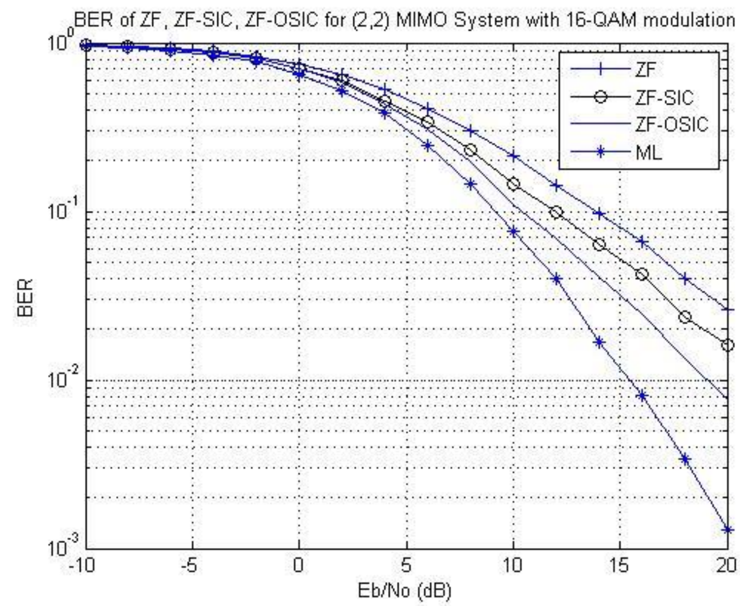

Figure 5.3: BER of ZF, ZF-SIC, ZF-OSIC for (2,2) MIMO System with 16-QAM modulation.

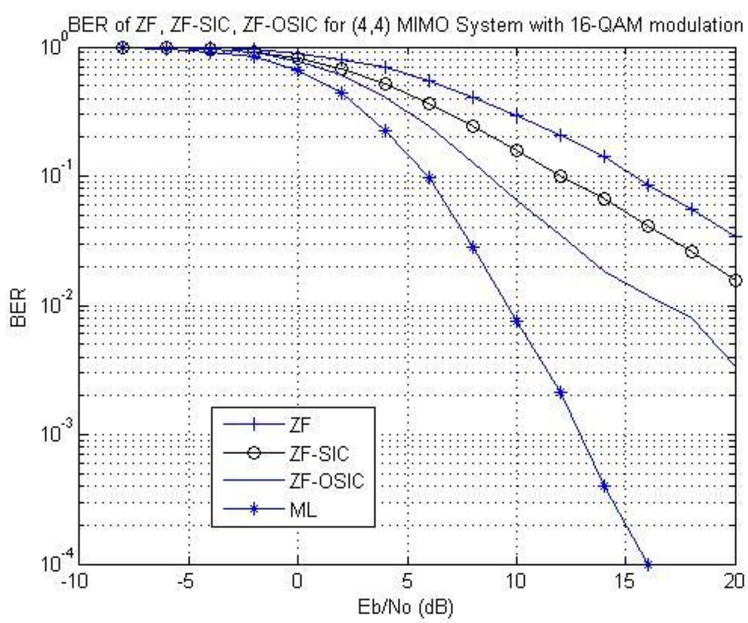

Figure 5.4: BER of ZF, ZF-SIC, ZF-OSIC for (4,4) MIMO System with 16-QAM modulation.

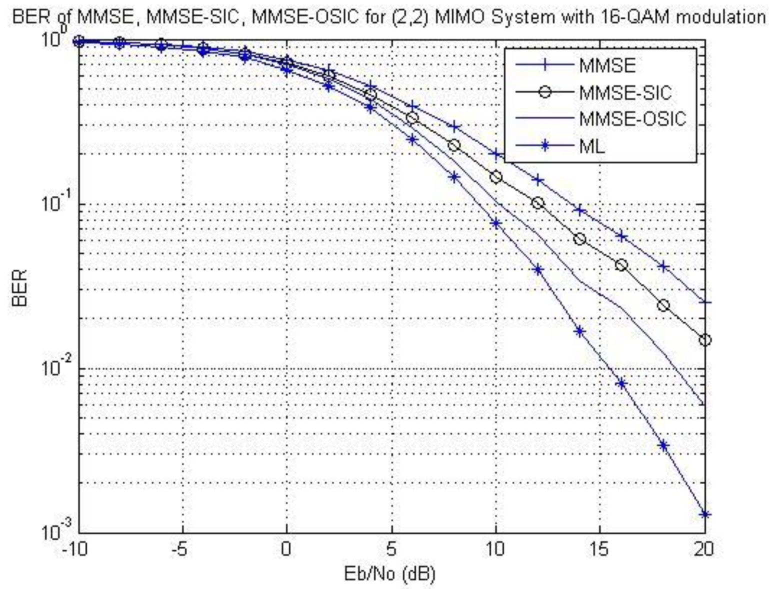

Figure 5.5: BER of MMSE, MMSE-SIC, MMSE-OSIC for (2,2) MIMO System with 16-QAM modulation.

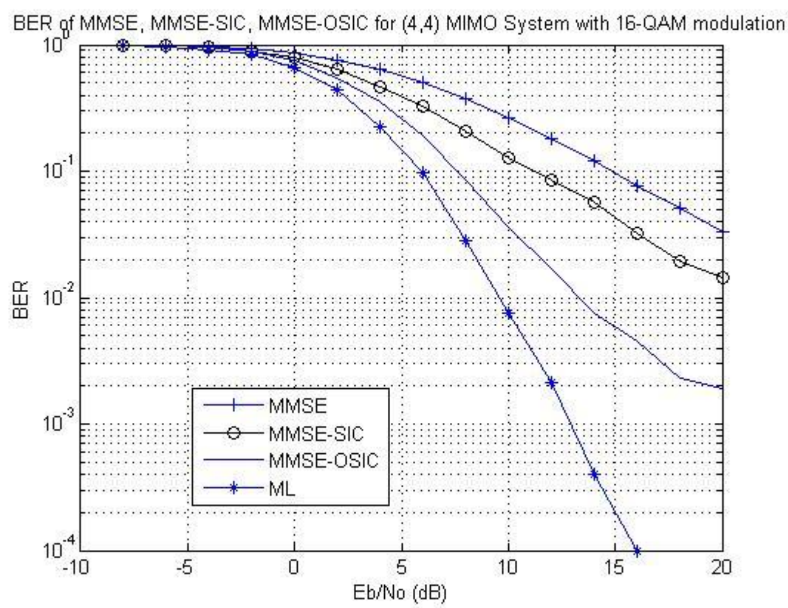

Figure 5.6: BER of MMSE, MMSE-SIC, MMSE-OSIC for (4,4) MIMO System with 16-QAM modulation.

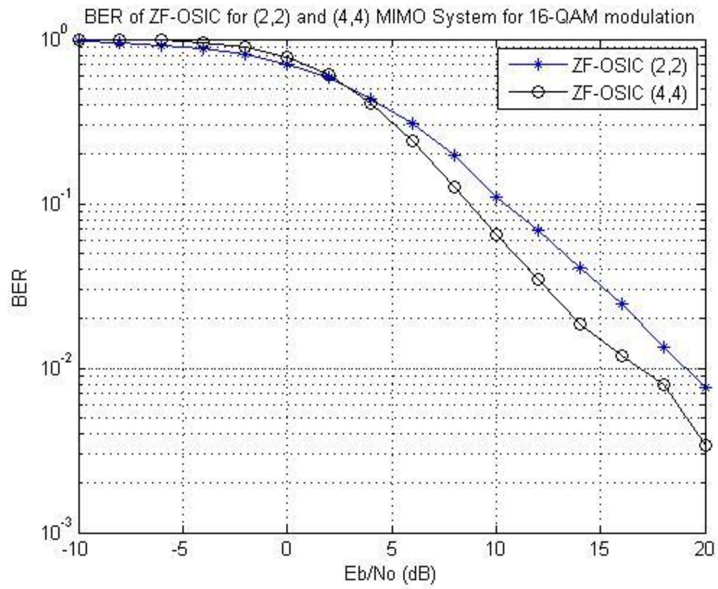

Figure 5.7: BER of ZF-OSIC for (2,2) and (4,4) MIMO System with 16-QAM modulation.

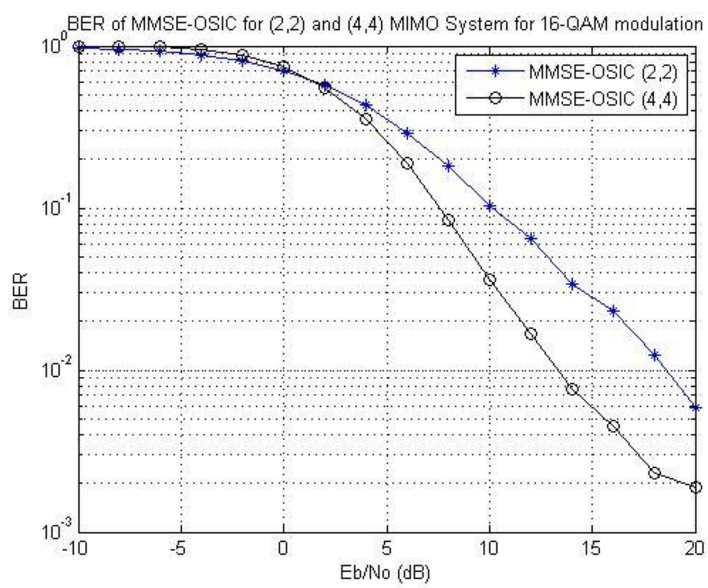

Figure 5.8: BER of MMSE-OSIC for (2,2) and (4,4) MIMO System with 16-QAM modulation.

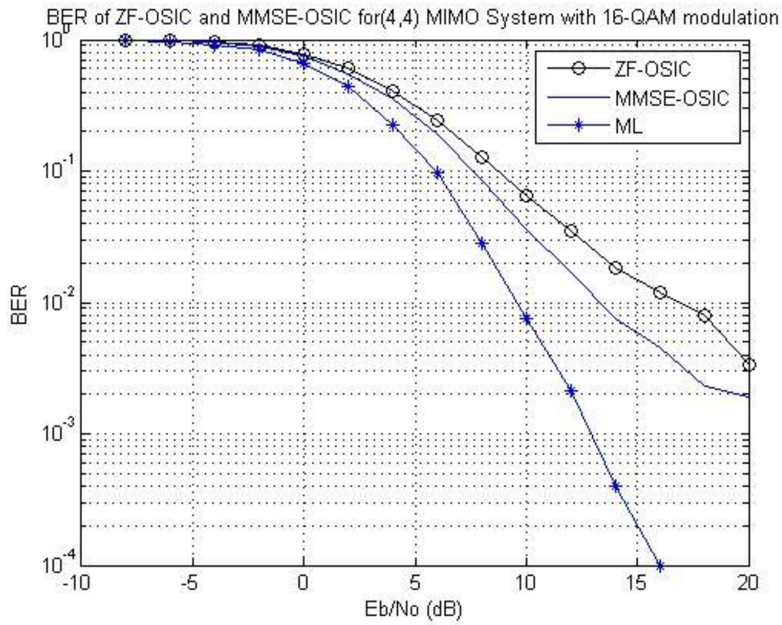

Figure 5.9: BER of ZF-OSIC and MMSE-OSIC for (4,4) MIMO System with 16-QAM modulation.

Simulation result interpretation:

Simulation results from Figs. 5.3 to 5.9 shows performance curve for the various linear and non-linear detection schemes studied in this thesis work. It is observed that the performance of ordered successive interference cancellation detection is superior to others techniques. Also, improvement is observed when antenna size is increased. The performance of MMSE-OSIC was superior to ZF-OSIC. Moreover, the performances of these receivers are still less than that of ML detection scheme.

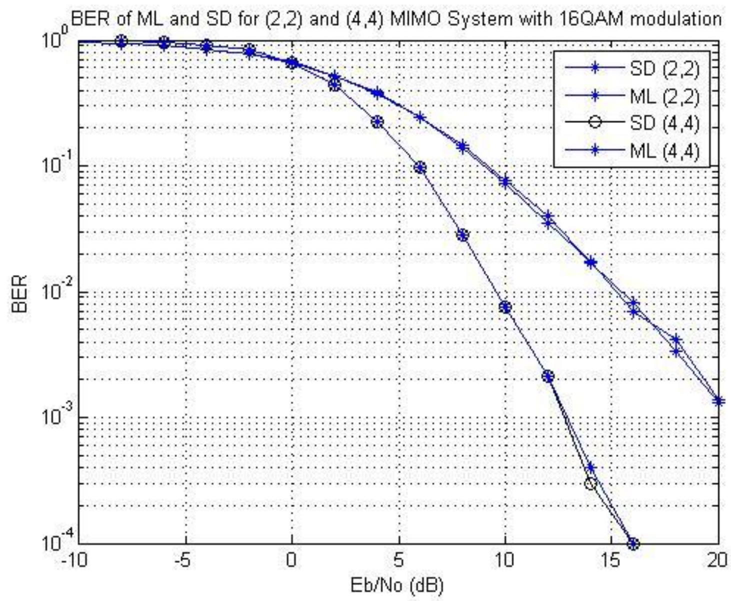

Figure 5.10: BER of ML and SD for (2,2) and (4,4) MIMO System with 16-QAM modulation.

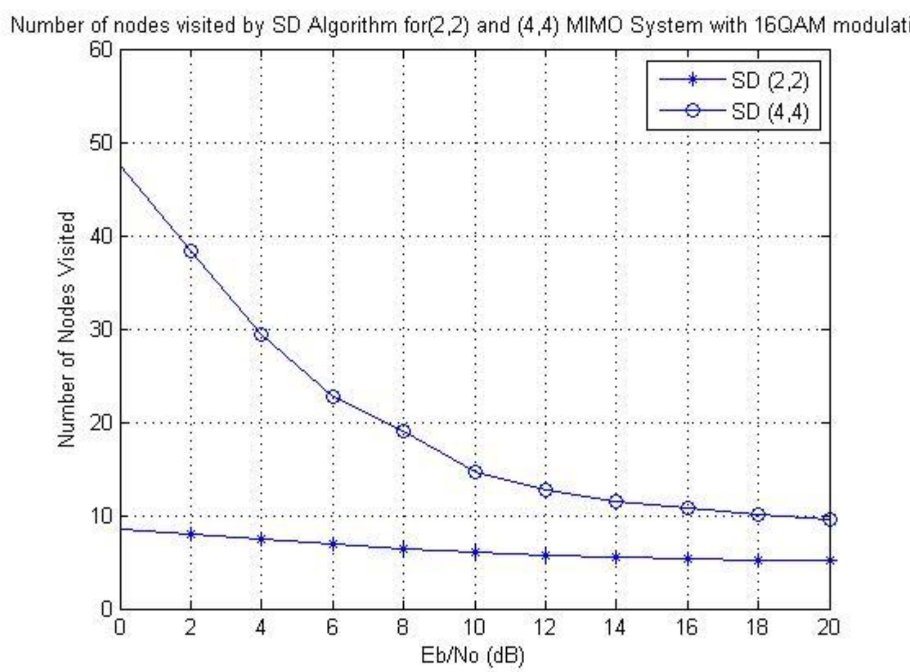

Figure 5.11: Number of nodes visited by SD algorithm for (2,2) and (4,4) MIMO system with 16-QAM modulation.

Simulation result interpretation:

Figure 5.10 shows the performance curve of Sphere Decoder receivers with Schnorr Euchner enumeration in (2,2) and (4,4) MIMO system. It is seen that the output of SD is almost same as that of ML detection scheme. It would have been in exact with ML solution but since these schemes are run in different environment there is a slight deviation from the expected output. Figure 5.11 shows the average number of nodes visited by SD algorithm at various SNR values. It can be observed that the average number of visited nodes decreases as the SNR values increase. This is in fact due to less probability of error in higher SNR regime and the decoder is likely to reach the solution more quickly.

Table 5.1: A comparative table for different detection algorithm at Eb/N0=10 dB for 10000 iterations for (2,2) MIMO with 16 QAM

Detection	**BER**	**Runtime(sec)**	**No. of Nodes visited**
ZF	0.2039	4.6608	NA
MMSE	0.1995	4.8484	NA
ZF-SIC	0.1483	6.0886	NA
MMSE-SIC	0.1465	6.9347	NA
ZF-OSIC	0.1120	6.2029	NA
MMSE-OSIC	0.1080	7.4478	NA
SD	0.0741	8.6564	6.0159
ML	0.0741	20.4768	256

Table 5.2: A comparative table for different detection algorithm at Eb/N0=10 dB for 10000 iterations for (4, 4) MIMO with 16 QAM

Detection Scheme	**BER**	**Runtime(sec)**	**No. of Nodes visited**
ZF	0.2993	7.3172	NA
MMSE	0.2616	7.4534	NA
ZF-SIC	0.1577	10.3566	NA
MMSE-SIC	0.1322	12.8834	NA
ZF-OSIC	0.0623	12.4897	NA
MMSE-OSIC	0.0366	13.4639	NA
SD	0.0063	20.6215	14.94
ML	0.0063	5062.2	65536

The simulations were performed on computer with 2.53 GHz Intel i3 processor and 4GB RAM in windows 7 operating system environment. The BER of SD is same as that of ML detection scheme with much less computational time as well as number of visited nodes.

CHAPTER 6: DISCUSSIONS

This thesis investigated the examples of the different types of receivers utilized within a MIMO spatial multiplexing system. The conventional MIMO detection schemes such as ZF, MMSE, SIC with ZF and MMSE, ordered SIC with both ZF and MMSE and sphere detection have been evaluated against ML detection.

According to the simulation result, it can be observed that the ML detection algorithm has an optimal solution compared to other detection schemes. Unfortunately, the complexity of this grows exponentially with increase in number of antenna or constellation size[10]. Hence it is necessary to find the MIMO signal detection schemes that can achieve performance close to ML detection.

MIMO detection techniques have been studied, analyzed and compared. The performance plots linear receivers: ZF and MMSE are relatively poor with slight improved performance of MMSE over ZF. In an attempt to improve the performance over linear receiver, successive interference cancellation with ZF and MMSE receiver are employed. SIC approach showed an improved performance but is vulnerable to error propagation. In an attempt to decrease the error propagation probability the ordered SIC has been employed which increase the performance. It was found that MMSE-OSIC has the most favorable solution compared to previous detection methods.

Also, sphere decoder based on the Schnorr-Euchner enumeration was implemented to obtain maximum likelihood solution of the transmitted symbol at the receiver. This decoder along with other MIMO receiver was compared to conventional ML receiver by comparing their error performance. The sphere detection scheme was found provide the ML solution with much less computational complexity.

CHAPTER 7: FUTURE ENHANCEMENT

Some of the lists of recommendations to the possible extensions of the work of this thesis research are:

- The Rayleigh fading channel has been considered to characterize the environment in this research. Other channel environment such as Rician fading chanel, Nakagami fading channel etc can be used to characterize the channel.
- In an attempt to reduce the error propagation, other ordering such as Column Norm Ordering, Received Signal Ordering etc can also be employed.
- The sphere detection technique is based on SE enumeration on depth first search algorithm; this can be analyzed on breadth first search algorithm.
- The initial radius was set to infinity to determine the first point in the lattice, a more deterministic way of setting the initial radius can be used to find the solution more efficiently.

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