EVACUATION OPTIMIZATION WITH MINIMUM CLEARANCE TIME

A THESIS SUBMITTED TO THE

CENTRAL DEPARTMENT OF MATHEMATICS

INSTITUTE OF SCIENCE AND TECHNOLOGY TRIBHUVAN

UNIVERSITY NEPAL

FOR THE AWARD OF

DOCTOR OF PHILOSOPHY

IN MATHEMATICS

BY

ISWAR MANI ADHIKARI

SEPTEMBER 2020

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DECLARATION

Thesis entitled “**Evacuation optimization with minimum clearance time**" which is being submitted to the Central Department of Mathematics, Institute of Science and Technology (IOST), Tribhuvan University, Nepal for the award of the degree of Doctor of Philosophy (Ph.D.), is a research work carried out by me under the supervision of Prof. Dr. Tanka Nath Dhamala, Central Department of Mathematics, Tribhuvan University. This research is original and has not been submitted earlier in part or full in this or any other form to any university or institute, here or elsewhere, for the award of any degree.

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Iswar Mani Adhikari

RECOMMENDATION

This is to recommend that **Iswar Mani Adhikari** has carried out the research entitled "**Evacuation optimization with minimum clearance time**" for the award of Doctor of Philosophy (Ph.D.) in **Mathematics** under my supervision. To my knowledge, this work has not been submitted for any other degree.

He has fulfilled all the requirements laid down by the Institute of Science and Technology (IOST), Tribhuvan University, Kirtipur for the submission of the thesis for the award of Ph.D. degree.

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Supervisor
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September, 2020

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LETTER OF APPROVAL

Date: 09/11/2020

On the recommendation of **Prof. Dr. Tanka Nath Dhamala**, this Ph. D. thesis submitted by **Iswar Mani Adhikari**, entitled **"Evacuation Optimization with Minimum Clearance Time"** is forwarded by Central Department Research Committee (CDRC) to the Dean, IOST, T.U.

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Iswar Mani Adhikari

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ABSTRACT

We are under the threats of natural or human-created disasters. Disasters are unavoidable and are mostly uncertain to happen. If occurs, the situation becomes vulnerable, effects badly the society, and its socio-economic status. Its direct impact is on the traffic systems. On the other hand, the increasing number of complex traffic networks brought difficulty in managing the rush hour traffic as well as the large events in urban areas. The optimal use of the vehicles and their assignments to the appropriate shelters from the disastrous zones are highly complicated in emergency situations. The maximum efficiency and effectiveness of the evacuation planning can be achieved by the appropriate assignment of the transit-vehicles during pre- and post-disaster operations.

The evacuation planning problem deals with sending the maximum number of evacuees from the danger zones to the safe zones in minimum time, as efficiently as possible. It can be further classified into microscopic and macroscopic planning. The microscopic planning deals with the individual evacuee's behavior in which some probabilistic laws for individual evacuees movement are presented and mainly based on the simulation approaches. But in macroscopic planning, it is principally based on optimization approaches where the evacuees are treated as the homogeneous group and only the common characteristics are considered. Optimization approaches on such macroscopic evacuation planning can further be classified as a heuristic approach, population optimization, modeling as fluid dynamics, mathematical programmings, traffic management, optimal evacuation destination, and network flow formulation. Among them, the dynamic network flow formulation has been found suitable evacuation optimization approach with the variants of flow maximization and/or time minimization problems. In such formulations, time can be considered as discrete or continuous.

Evacuation planning problems are handled with different prospectives, namely, the transit-based, car-based, and pedestrian movements depending upon the movement of the evacuees on the evacuation scenarios. The transit-based planning problems are to

minimize the duration of evacuation by routing and scheduling a fleet of vehicles, say buses, as the bus-based evacuation planning problem. Such a problem is an important variant of the vehicle routing problem. Traffic route guidance, destination optimization, and optimal route choice are some of the approaches to accelerate the evacuation planning process. Their effectiveness depends upon the evacuee arrival patterns at the pickup locations and their appropriate assignment to the transit-vehicles in the available evacuation network.

An embedded network is composed of two constituent sub-networks, namely, the primary and the secondary sub-networks. Evacuees are to be collected at the pickup locations of the primary sub-network from the danger zone(s) and are to be assigned to transit-vehicles in the secondary sub-network. For time minimization evacuation planning problems, evacuees are to be collected in the earliest arrival flow pattern at zero transit times and is to be followed by dominant vehicle assignments. Transit-vehicles are provided from the bus depot in the secondary sub-network. Pickup locations are taken as the sources for the subsequent process to minimize the overall network clearance time from the danger zone to safety. In our work, we have proposed an integrated optimization approach in such an embedding to achieve the minimum clearance time. The earliest arrival pattern respects the partial lane reversal strategy, whereas the better assignments are based on the dominance relations concerning the evacuation duration.

We use the quickest transshipment partial arc reversal strategy to collect the evacuees in minimum time from the disaster zones to the pickup locations of the primary prioritized sub-network. By treating such pickup locations as sources, the available set of transit-buses is assigned simultaneously in the secondary sub-network to shift the evacuees finally to the sinks with minimum clearance time. The lane reversal strategy significantly reduces the evacuation time and maximizes the flow of evacuees, whereas reversing them only partially has an additional benefit that the unused road capacities can be used for supplying emergency logistics and allocating facilities as well.

LIST OF ACRONYMS AND ABBREVIATIONS

BEPP	Bus-based Evacuation Planning Problem	3
CED	Combined Evacuation Duration	49
EAE	Earliest Arrival Evacuee	3
EAF	Earliest Arival Flow	3
EAT	Earliest Arrival Transshipment	21
ED	Evacuation Duration	45
EPP	Evacuation Planning Problem	3
FPTAS	Fully Polynomial-Tme Approximation Scheme	8
IEPP	Integrated Evacuation Planning Problem	3
Lex-max	Lexicographically-maximum	4
MCF	Minimim Cost Flow	4
MDF	Maximum Dynamic Flow	17
OPT	Optima	8
PTAS	Polynomial-Tme Approximation Scheme	8
QPART	Quickest Partial Arc Reversal Transshipment	82

RBEPP	Robust Bus-based Evacuation Planning Problem	46
TA	Transit-vehicle Assignment	77
TRF	Temporally Repeated Flow	3
VRP	Vehicle Routing Problem	3

LIST OF SYMBOLS

Γ	multi-set of proper chain flows	18
Π	tourlists/tourplans	40
A_i^{in}	set of incoming arcs of i	9
A_i^{out}	set of outgoing arcs of i	9
A	set of arcs	9
B	set of transit-vehicles	33
$D = \{d\}$	a bus depot	36
D	set of bus depots	33
$G = (V, A)$	graph	9
Q	capacity of a bus	33
S	set of sources	10
T	time horizon	10
V	set of vertices	9
Z	set of sinks	10
$[i, j]$	undirected edge	9
Ω_0	effective waiting for EAE at y_0	77
Ω	effective waiting time in $\mathscr{N}^E$	76
$\alpha'(y_k)$	adjusted demand at $y_k \in Y$	88
$\alpha(s)$	total evacuees at s	13
$\alpha(y_k)$	potential demand at $y_k \in Y$	88
α_i	demand at $i \in Y$	34
$\beta(z)$	total evacuees at z	13
β_j	capacity at $j \in Z$	34
$\cup$	union of sets	10
γ	terminal respecting flows	18

$\mathbb{N}$	natural numbers	7
$\mathbb{R}_{\geq 0}$	non-negative real numbers	15
$\mathbb{R}$	real numbers	7
$\mathcal{I}$	an instance of a problem	7
$\mathcal{NP}$	class of non-deterministic polynomial algorithms	7
$\mathcal{O}(f(n))$	running time of an algorithm	7
$\mathcal{O}$	big O notation	7
$\mathcal{P}_1,\ \mathcal{P}_2$	decision problems	8
$\mathcal{P}$	class of polynomially solvable decision problems	7
$\mathcal{T}\text{max}$	duration of evacuation over all buses	34
$\mathscr{A}_\epsilon$	family of algorithms	8
$\mathscr{A}$	algorithm	6
$\mathscr{I}_c$	collection of all valid instances	7
$\mathscr{I}$	set of possible instances	7
$\mathscr{N}^* = (V, A, u, \tau, s^*, z^*, T)$	extended network	10
$\mathscr{N} = (V, A, u, \tau, S, Z, T)$	multi-terminal dynamic network	10
$\mathscr{N}^E = \mathscr{N}_1 \cup \mathscr{N}_2$	embedded network	14
$\mathscr{N}^T$	time-expanded network	10
$\mathscr{N}_1$	primary sub-network	14
$\mathscr{N}_2$	secondary sub-network	14
$\mathscr{P}_c$	computational problem	6
$\mathscr{P}_o$	optimization problem	7
OPT	optima	8
head$(a) = j$	head of $a = (i, j)$	9
tail$(a) = i$	tail of $a = (i, j)$	9
μ	demand/supply function	16
$\nu(Y, \theta)$	flow value reached to Y at θ	70
π	path in network	18
$\subseteq$	subset or equal	9
$\tau(\gamma)$	length of the chain flow	18

$\tau(\pi)$	total length of path	18
τ	transit time	10
$\mathbf{T} = \{0, 1, ..., T-1\}$	discrete time-settings	15
$\theta \in [0, T)$	continuous time-settings	15
$\tilde{A}$	set of arcs in transformed network	14
$\tilde{\mathscr{N}}$	transformed network	14
$\tilde{\tau}$	transit time function in transformed network	14
$\tilde{u}$	capacity function in transformed network	14
ξ_{y_0}	number of evacuees at super-pickup node	46
Ψ	effective waiting at prioritized $\mathscr{N}^E$	88
$a' = (j, i)$	backward arc	14
$a = (i, j)$	an arc	9
$ex_f(i)$	excess flow at i	15
$f(n)$	runtime function	7
f^1, f^2	terminal respecting flows	18
f	flow function	15
s^*	super-source	10
s	single source	10
u	capacity function	10
y_0	supper pickup node	43
z^*	super-sink	10
z	single source	10
$\|A\| = m$	number of arcs	9
$\|V\| = n$	number of vertices	9
$\|\mathcal{I}\|$	size of an instance	7

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Evacuation is an organized, phased, and supervised withdrawal, dispersal of civilians from dangerous, or potentially dangerous areas, and their reception, and care in safe areas in utmost reality. Most of the disasters cannot be predicted and are unavoidable, and the damages caused by them are severe. Past evacuation experiences are to take account of the preparedness and planning strategies which are followed by the response and recovery. Disaster operations can be performed before or after the disasters as pre- and post-disaster operations to bring the people towards the normalcy. Short-notice evacuations, facility location, and stock pre-positioning are carried out as the main pre-disaster operations whereas the relief distribution, logistic support services, and the casualty transportations are the main aspect of the post-disaster operations, [25, 35, 38].

Evacuation planning strategies and their operations may vary due to their applicable geographical scales; behavior, density, and size of the affected population; nature and modes of transportation network; traffic capacity; and the evacuation objectives. Mainly the evacuees may be considered as pedestrian, auto-based, or transit-based, depending upon their movements or the response during such disasters. Pedestrian movements deal about their individual walking characteristics, whereas the auto-based evacuees have their vehicles. For the transit-based system, evacuees need to be sent towards the transit hubs for evacuation that is followed by the scheduling and routing of transit-vehicles.

Evacuation models can be classified into two broad categories, microscopic, and macroscopic. The former emphasizes individual parameters like walking speed, physical ability, reaction time, and the interaction of each evacuee with other evacuees during the movement and are based mainly on simulation, [33, 38]. But in the later, occupants are treated as a homogeneous group and are taken into account of their common characteristics only and can produce mainly the lower bounds, [61]. The macroscopic models are dealt with an optimization approach to produce an acceptable optimal solution. These models can also be combined in an interactive solution process where the output of one is used as input to the other so that both remain stable, as a sandwich approach, [59].

Fluid models and the models based on differential equations capture very well the dynamic behavior of traffic as the continuous quality but comparatively inefficient to handle the large network. Mathematical models seldom represent all the existing characteristics of real-life situations as on their formulation. One has to idealize the real-life problem by making some simplifying hypotheses [85] to handle the problem. So, one has to be careful using the solutions of such models as they tend to be large and exhibit an exponential complexity with the problem size. Its performance and efficiency depend upon the nature of the network, population density, the behavior of the population, and on many other factors. The research work related to flow problems has been carried out in a wide spectrum of mathematical frameworks like optimal control, variational inequality, fluid dynamics, numerical simulations, mathematical programmings, and network flow formulations. The network flow formulation concerning to macroscopic behavior is with a computationally acceptable research domain and has received high attention from the research community, [38].

Problems on dynamic networks were firstly introduced by Ford and Fulkerson [50, 51]. The minimum clearance time evacuation planning problem (EPP) considered here has been modeled in dynamic networks in which each arc has a non-negative flow capacity and integer transit time. This is one of the fundamental problems in the EPP to find the minimum time limit such that all supplies can be sent to the sinks (safe places) from the sources (disastrous zones). There has been a fair amount of work regarding different aspects of the EPPs, including the flow maximization and/or time minimization, as referred by [33, 34, 35, 38].

The $\mathcal{NP}$-hard multi-depot, multi-trip bus-based evacuation planning problem (BEPP) was introduced and analyzed prominently in [17] which is closer to the split delivery multi-depot vehicle routing problem (VRP) with inter-depot routes. However, if there is only a single bus-depot, assuming that the bus pickups the same number of people that equals its capacity, the author in [55] has also proposed the BEPP for the evacuation of a region from a set of collection points to a set of capacitated sinks. Based on such BEPP, Pyakurel *et al.* [117] explored a wide horizon to the research related to the transit-dependent EPP. In their study, the excess exterior points of the disaster region were taken as the pickup locations and the available open spaces as the sinks. Evacuees were supposed to gather themselves from their residents to the nearby pickup locations. Homogeneous buses with the uniform capacity were used for the evacuees' pickup. Their computational analysis for the transit-based evacuation of the densely populated region in Kathmandu valley noticed that the domain of optimal solutions remains on the larger number of buses with higher capacity and speed irrespective of the number of sources and sinks chosen. Most of these prominent BEPPs have considered the evacuees gathered themselves at different pickup locations and were silent about their arrival patterns. However, [117] have considered about their arrivals relative to the population density of the transit-dependent people nearby them.

In our work, we are focused on the new and better-suited form of arrival pattern of evacuees in the earliest arrival flow (EAF) pattern for the integrated evacuation planning problem (IEPP). It will maximize the arrival of evacuees at every possible instance at the pickup locations with zero transit times from a source. We present a polynomial-time earliest arrival evacuee (EAE) algorithm following the principle of temporally repeated flow (TRF) to solve the EAE problem with zero transit times and partial arc reversal capability. Such evacuees collected at different pickup locations of the primary sub-network are considered as the supplies during the subsequent evacuation process of vehicle assignment for the secondary sub-network. The topology of integrated network considered is more general than the network for the EAF and the transit-based network, separately. The assignment of transit-vehicles in such an integrated evacuation network is also carried in a dominating solution approach for the minimum evacuation duration. Different analytical issues are addressed to improve the performance of the evacuation in an integrated framework, though the problem itself, in general, is challenging.

A solution to the maximum flow problem sends the maximal amount of flow from the sources to the sinks for the fixed integer time horizon T. Lexicographically maximum (Lex-max) flow problem with many sources and many sinks was introduced and many efficient algorithms were presented from different aspects in [93, 94, 98] and [134]. In such a problem, the terminals (sources and/or sinks) are ordered with certain priority for a lex-max flow respecting the priority. Such a lex-max flow is not necessarily a maximum flow and vice versa, however, they are equivalent for two-terminal networks, [51]. If a flow is both source-optimal and sink-optimal, then it becomes the lexicographically optimal flow in the prioritized network, [76].

Pyakurel and Dhamala [112] investigated the lex-max dynamic contraflow problem where reversals of arcs are allowed. The partial contraflow with path reversals leads to a significant improvement in increasing the flow values, decreasing the evacuation time, and utilization of the unused capacities of paths for humanitarian logistics, [121]. Authors in [119] have introduced the partial lane reversal strategy and presented efficient dynamic flow algorithms for quickest and maximum flow EPPs. In the quickest transshipment problem, a given number of evacuees has to be shifted in minimum time. Such problems have been studied by the help of the lex-max dynamic flow problem applying the minimum cost flow (MCF) computations as a tool, [69]. An algorithm to find the universally quickest transshipment has been presented in [49].

In this work, evacuees are collected from the disaster zone to the pickup locations of the prioritized primary sub-network in minimum time as the quickest transshipment by using the lex-max flow approach. Considering such pickup locations as the sources, the available set of transit-buses are also assigned in the network to evacuate the evacuees safely to the sinks on the first-come-first-serve basis. This novel approach proposed here is better-suited for the simultaneous flow of evacuees with minimum waiting delay at such pickup locations for the EPP in such prioritized embeddings.

The structure of this dissertation is as follows. Chapter 2 gives a compact overview of the fundamental background about the basic terms, as a basic foundation of the dissertation. It covers the problem complexity, graph, and network including their topologies, different attributes, and the flows in networks. Chapter 3 covers different aspects of evacuation optimization concerning to evacuee arrival patterns, vehicle assignments, and the embedding of the evacuation network. For the better assignments of the vehicle routing, we have proved the dominance concerning evacuation duration on the upper bounds of BEPP including the formulation of a simplified BEPP for a diminished network. With a better-suited arrival pattern in an EPP as the EAE, we have introduced the IEPP in Chapter 4. In such embedding, the EAE at its collection sub-network is assigned dominantly in the assignment sub-network to achieve the minimum clearance time in the embedding.

We have considered the prioritized primary and a bus-based secondary sub-network in Chapter 5. Here the evacuees are collected at the prioritized sub-network as the quickest transshipment and are assigned simultaneously in the embedding on a first-come-first-serve basis for the minimum clearance time. Demands of the pickup locations are also adjusted to have the minimum effective waitings in the embedding. Different algorithms and the solution strategies are also presented for the minimum clearance time respecting the quickest partial arc reversal transshipment. The partial lane reversal strategy significantly reduces the evacuation time, whereas reversing them only partially as requested by the flow has an additional benefit that the unused road capacities can be used for logistics and emergency vehicle movements. Chapter 6 concludes the dissertation with some remarks and further research directions.

1.2 Rationale

Most of the research in the area of evacuation optimization focuses on auto-based emergency evacuation planning and ignores other fundamental modes of an emergency evacuation. In urban areas, the majority of the people are transit-dependent. The dependency of evacuation planning on transit modes becomes crucial, particularly, in the case of a large-scale emergency evacuation. They are to be given special attention due to their ages, language inefficiencies, different health problems, or other physical disabilities. The great loss of people on Hurricane Katrina was due to the lack of proper planning for the transit-based evacuees, [90]. The transit-based evacuation planning problems corresponding to such evacuees are to minimize the duration of evacuation by routing and scheduling a fleet of vehicles, say buses.

The conventional VRP and the traveling salesman problems are based on the truck dispatching problem for the gasoline distribution and the vehicle scheduling problem for delivery of goods, respectively. Such problems were projected in the age of 1960s on two seminal papers as [29, 32]. Since then the research on VRP has grown exponentially. Most of the evacuation optimization strategies are formulated and designed to solve the BEPP were usually based on VRP variants. There is a wide variety of literature evolving into various dimensions that includes the multi-depots, capacitated vehicles, multiple trips, split deliveries, time windows for customers, service choice, pickup and deliveries, backhauls. In most of them, the objectives are to minimize the total bus travel time or the total network clearance time. Some of the other such formulations on somewhat similar fashions for the time minimization on evacuation planning system are also available in the literature, [84, 122, 127, 132]. Some of the BEPP solutions have taken account of the walking time as well as the waiting time of the evacuees during the evacuation process while optimizing the evacuation routes, as in [44, 66, 107, 141].

In most of the BEPP, the fundamental objective is to minimize the duration of evacuation by routing and scheduling a fleet of homogeneous and capacitated buses which were initially located at one or more depots. The number of evacuees at each pickup locations can exceed the capacity of a single bus, which requests the split delivery. The number of available buses might be insufficient to transport all the evacuees without multiple trips and each shelter has a capacity that limits the number of evacuees it can serve. A prominent BEPP has been introduced and analyzed in [17] and its simplified version was presented in [55] considering the number of evacuees be the integral multiple of busloads. Based on such BEPPs, the research related to the transit-dependent EPP concerning to Kathmandu valley network has been presented, [117]. In most of them, the evacuees gathered themselves at different pickup locations and were silent about their arrival patterns.

The effectiveness of the solution of BEPP depends upon the evacuee arrival patterns at the pickup locations and their appropriate assignment to transit-vehicles in the available evacuation network. The earliest arrival pattern is the better-suited for evacuees collection as it maximizes the flow at each time unit from the beginning within the given time horizon. Evacuees are better to collect at the prioritized pickup locations following the quickest transshipment in the lex-max flow approach and can be assigned simultaneously to homogeneous transit-buses for time minimization objectives. The formulation, operation, and the universally acceptable solution methodologies for the EPP in the diversified evacuation networks for large scale disasters is one of the challenging issues in disaster management.

1.3 Objectives

Most of the research work on emergency EPPs focuses on auto-based evacuation plannings and ignores other fundamental modes of emergency evacuation strategies. As in urban areas, the majority of the population are transit-dependent and are to give special attention. The transit-based EPPs corresponding to such evacuees is to minimize the duration of evacuation by routing and scheduling of vehicles in such an evacuation network. In our work, our focus is on the followings:

- To get a better extension on evacuation optimization corresponding to the better-suited arrival pattern of the evacuees in the transit-based evacuation network.
- To get the better and the dominating solution approach for the time minimization aspects of such EPP to achieve the minimum network clearance time.
- To address different analytical issues to improve the performance of EPP in an integrated evacuation framework.

CHAPTER 2

FUNDAMENTALS

This chapter begins with the brief introduction of some important terms and their notations. Besides this, we also recite some important theorems, including some mathematical models and formulations as the basic foundations related to our work. The details about the covered topics are available in the network optimization books like [10, 51].

2.1 Preliminaries

2.1.1 Algorithms and complexity

Here, some of the basics related to the complexity are revisited. For the detail systematic analysis, see for example the books like [53, 133]. An algorithm $\mathscr{A}$ is a list of simple atomic operations like the addition of values or a decision to be performed next to solve a problem. It is a precise and universally understood sequence of instruction for the solution of a computational problem, $\mathscr{P}_c$. It is assumed that all such simple atomic operations in $\mathscr{A}$ demand the one time unit each and the totality of such a number of operations that an algorithm performs is the time complexity of an algorithm and is also named as its running time. The running time of an algorithm is denoted by $\mathcal{O}$-notation. It indicates only the most dominating term in the running time. For example if the running time of an algorithm $\mathscr{A}$ is $100n + n^2 + 0.0001n^3$ then the complexity of such $\mathscr{A}$ is the order of n^3. It does not measure the exact running time but its order.

Let $\mathscr{I} := \{\mathcal{I} : \mathcal{I} \in \mathscr{I}\}$ be the set of all problem instances $\mathcal{I}$ of a computational problem $\mathscr{P}_c$. Then the size of an instance is given by $|\mathcal{I}|$. For an algorithm $\mathscr{A}$, the runtime function $f(n) : \mathbb{N} \to \mathbb{N}$ is bounded by the input $f(n)$, if for all instances $\mathcal{I}$ we have $|\mathcal{I}| = n$. The smallest of such a function having the running time $\mathcal{O}(f(n))$ is the time complexity of $\mathscr{A}$. The instances are not known in advance and the running time is computed as the worst-case running time. Let $\mathscr{I}_c$ be a collection of valid instances for a problem then the worst-case running time is defined by $\max\{\min\{f(\mathcal{I})\} : f$ runtime function for $\mathcal{I} : \mathcal{I} \in \mathscr{I}_c\}$.

An algorithm $\mathscr{A}$ having the runtime function $f(n)$ bounded by a polynomial is said to have the polynomial-time complexity, otherwise, it becomes super-polynomial. An algorithm $\mathscr{A}$ having polynomial-time complexity is said to be efficient. And in addition, it becomes a strongly polynomial, if its time complexity is bounded by a polynomial in the number of integers in the input instance. A polynomial-time complexity which is not that of strongly polynomial-time is with the weakly polynomial-time complexity. In other hand, if the running time of an algorithm $\mathscr{A}$ depends on the size of the number in the input, then that are said to be with pseudopolynomial-time complexity. If the time complexity of an algorithm $\mathscr{A}$ is not bounded by a polynomial then that is said to be with exponential-time complexity.

A problem having only two valid solution, either in yes or no, is a decision problem. So, there are two instances for such a decision problem as the yes-instance or no-instance. An optimization problem might have its decision version. An optimization problem $\mathscr{P}_o$ is a set of all instances $(\mathscr{P}, c)$ of a given problem, where $\mathscr{P}$ represents the set of all feasible solutions and $c : \mathscr{P} \to \mathbb{R}$ is the cost function. The problem becomes discrete/ continuous according as all the variables be considered as discrete/continuous. If the objective of an optimization problem is to maximize (or minimize) a given value, the corresponding decision version answers the question whether a solution of a given value exits. Let $\mathcal{P}$ denotes the class of all polynomially solvable decision problems then $\mathcal{NP}$ denotes the set of problem that can not be solved in polynomial-time but can be verified in polynomial-time. So, for $\mathcal{NP}$-class problem, their solution is in exponential-time form but the verification is in polynomial-time. In another word, a decision problem $\mathcal{P}_1$ is in class $\mathcal{NP}$ if for every yes-instance $\mathcal{I}_1$ of $\mathcal{P}_1$, there is a polynomial-time verification that the instance is a yes-instance. The famous problem like graph colouring problem in which each adjacent nodes are to be colored with different colors is with exponential-time solution but can be verified in polynomial-time, as if the solution is provided, it is easier to check for its correctness in polynomial-time. Hence, $\mathcal{P} \subseteq \mathcal{NP}$. But one of the open problem is whether $\mathcal{P} = \mathcal{NP}$.

Let $\mathcal{P}_1,\ \mathcal{P}_2$ are decision problems. If there exist an algorithm that transforms an instance $\mathcal{I}_1$ for the problem $\mathcal{P}_1$ into an instance $\mathcal{I}_2$ for problem $\mathcal{P}_2$ in polynomial-time then the problem $\mathcal{P}_1$ is said to be polynomially reduced to a problem $\mathcal{P}_2$. A problem $\mathcal{P}_1$ is said to be $\mathcal{NP}$-hard, if every problem in $\mathcal{NP}$ can be polynomially reduced to $\mathcal{P}_1$, e.g. $\mathcal{P}_2 \leq_p \mathcal{P}_1$ for all $\mathcal{P}_2 \in \mathcal{NP}$. In addition, if $\mathcal{P}_1 \in \mathcal{NP}$, then the problem $\mathcal{P}_1$ becomes $\mathcal{NP}$-complete. Hence, the class $\mathcal{NP}$-hard is boarder than the class $\mathcal{NP}$-complete because it includes the class $\mathcal{NP}$ as well the problem that are not in the class $\mathcal{NP}$. Moreover, a polynomial algorithm for any $\mathcal{NP}$-hard problem is found only if $\mathcal{P} = \mathcal{NP}$, which is assumed to be impossible. An $\mathcal{NP}$-hard problem is said to be weakly $\mathcal{NP}$-hard if a pesudopolynomal algorithm for such an $\mathcal{NP}$-hard problem exists. On the other hand, it is said to be strongly $\mathcal{NP}$-hard, if it remains $\mathcal{NP}$-hard if all the numbers in an instance are bounded by $\mathcal{I}$.

It is not possible to find the efficient algorithms to solve $\mathcal{NP}$-hard problems. For this, different approximation algorithms have been developed. Let OPT be an optimal solution of an optimization problem $\mathscr{P}_o$. Let $k > 0$, a k-approximation algorithm for $\mathscr{P}_o$ is a polynomial algorithm that gives a solution whose value lies within a factor of k of the value OPT. For maximization problems, the solution has at least a value of OPT/k. But for the minimization problem, the solution has at most a value of $k \cdot OPT$.

A polynomial time approximation scheme (PTAS) is a family of algorithms $\{\mathscr{A}_\epsilon\}$, where for each $\epsilon > 0$, there is an algorithm $\mathscr{A}_\epsilon$ that is a $(1 + \epsilon)$ approximation algorithm. Such a family of algorithms $\{\mathscr{A}_\epsilon\}$ becomes a fully polynomial time approximation scheme (FPTAS) if for each $\epsilon > 0$ there is an algorithm $\mathscr{A}_\epsilon$ which is a $(1 + \epsilon)$-approximation algorithm having the polynomial running time in the input size and in $1/\epsilon$, [56, 78]. We also refer to the books [131] and [135] for the extensive coverage of approximation algorithms and several solution strategies regarding to it.

2.1.2 Graph and network

A directed graph is a pair $G = (V, A)$ where V is a finite set of nodes or vertices and A is a family of ordered pairs, $A \subseteq V \times V$. Here the elements of A are the directed edges or arcs and can be denoted as an ordered pair (i, j) for $i, j \in V$. Unless otherwise stated, we use an arc (i, j) be the directed arc in such a directed graph G with the node $i \in V$ as the tail node and the node $j \in V$ as the head node. So, for the arc $a = (i, j) \in A$, tail$(a) = i$ and head$(a) = j$. Here, the arc a is emanated from the node i as an outgoing arc of i and is terminated at node j as an incoming arc of j. Whenever, an arc $a \in A$, the node j is an adjacent to node i. For $a \in A$, the sets of outgoing and incoming arcs are denoted by, $A_i^{out} := \{a = (i, j) \in A\}$ and $A_i^{in} := \{a = (j, i) \in A\}$, respectively for the node $i \in V$. The in-degree of a node is the number of incoming arcs of that node and its out degree is the number of its outgoing arcs and their sum represents the degree of a node. If two or more than two arcs are with same tail and head nodes, then there exists the multi-arcs in the network with edge-disjoint paths. If the tail node and head node are same that will form the loop in the network, such cases are rarely considered.

An edge between two nodes $i, j \in V$ in an undirected graph is denoted by $[i, j]$ consists of both pairs (i, j) and (j, i) in the arc set A. Hence, a graph can be categorized as a directed, undirected or mixed depending whether it consists of directed arcs only, or undirected edges only or with directed arcs and undirected edges both in it, respectively. Unless stated otherwise, we consider a graph be a directed graph with $|V| = n$ and $|A| = m$. A network is considered as a graph in which nodes and/or arcs (edges) have associated with certain attributes like capacities, transit times, costs, supplies, demands, gain or loss factors, etc. However, graph and network can also be used synonymously.

A graph $H = (V_1, A_1)$ is a subgraph of $G = (V, A)$ if $V_1 \subseteq V$ and $A_1 \subseteq A$. A walk in G is a subgraph of G consists with a sequence of nodes and arcs $i_1 - a_1 - i_2 - a_2 - \cdots - a_{r-1} - i_r$ such that for all $1 \leq k \leq r - 1$, either $a_j = (i_k, i_{k+1}) \in A$ or $a_j = (i_{k+1}, i_k) \in A$. Such a walk without any repeation of nodes becomes a path. So, a directed path is simply a directed walk without any repetition of nodes. In addition to the path consisting of $a_1 - a_2 - \cdots - a_r$ if the arc (i_r, i_1) or (i_1, i_r) is also included, it yields a cycle as $a_1 - a_2 - \cdots - a_{r-1} - a_r - a_1$.

Depending upon the time parameter, a network can be categorized as a static and a dynamic. The multi-terminal dynamic network is denoted by $\mathscr{N} = (V, A, u, \tau, S, Z, T)$, where u, τ, S, Z, and T represent capacity, transit time, set of sources, set of sinks, and time horizon, respectively. The capacity $u : A \to \mathbb{Z}_{\geq 0}$ restricts the amount of flow on the arc and the transit time $\tau : A \to \mathbb{Z}_{\geq 0}$ represents the amount of time to transverse the respective arc. For a single source single sink (two-terminal) network, we prefer $S = \{s\}$ and $Z = \{z\}$. For multi-terminal networks, an extended network $\mathscr{N}^*$ can also be considered. Here, we are having a glimpse on different network structures related to the evacuation network.

Extended network. A multi-terminal network $\mathscr{N} = (V, A, u, \tau, S, Z, T)$ can be transformed to be a two-terminal extended network $\mathscr{N}^* = (V, A, u, \tau, s^*, z^*, T)$ by adding the super-source s^*, super-sink z^*, arcs (s^*, s_i) to each $s_i \in S$, and arcs (z_i, z^*) to each $z_i \in Z$ with infinite capacity and zero transit time. Hence, the total occupancy and the demands at s_i and z_i can be assumed to be at (s^*, s_i) and (z_i, z^*), respectively, corresponding to such an extended two-terminal network.

Time-expanded network. Let $\mathscr{N} = (V, A, u, \tau, S, Z, T)$ be a network with capacities u and non-negative transit times τ on the arcs. For a given time horizon $T \in \mathbb{Z}_{>0}$, the corresponding time-expanded network $\mathscr{N}^T = (V^T, A^1 \cup A^2)$ can be defined with capacities and costs on the arcs. Each node $i \in V$ has $T + 1$ copies in V^T, denoted $i(0), i(1), \ldots, i(T)$, i.e., the node set on the corresponding network be, $V^T := \{i(\theta) : i \in V, \theta = 0, 1, \ldots, T\}$. Here, the set of holdover arcs A^1 with arc capacity $u(i)$ and the movement arcs A^2 with the arc capacity $u(i, j)$, respectively, are defined by,

$$
\begin{aligned}
A^1 &= \{(i(\theta), i(\theta + 1)) : i \in V, \theta = 0, 1, \ldots, T\}, \\
A^2 &= \{(i(\theta), j(\theta + \tau(i, j))) : (i, j) \in A, \theta = 0, 1, \ldots, T - \tau(i, j)\}.
\end{aligned}
$$

One can transform the static flow in $\mathscr{N}^T$ to a single-source-single-sink problem by introducing a super-source and a super-sink. Thus, finding a flow in a dynamic network can be solved by finding a static flow in time-expanded network. However, the size of the time-expanded network is not polynomial in size of the input, as there are T copies of the network. So, any algorithm based on $\mathscr{N}^T$ can not be polynomial. In fact, the size of the time-expanded network $\mathscr{N}^T$ is linear in T and is pseudopolynomial in the input size. It is still an open problem whether there is a polynomial-time solution for it in general.

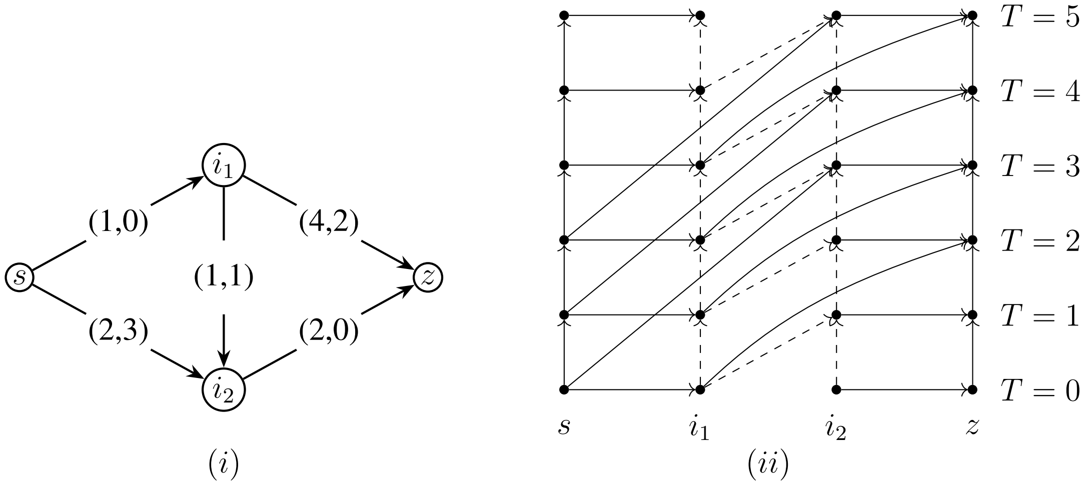


Figure 1: (i) A simple network, (ii) A time-expanded network of (i).

In a simple network $\mathscr{N}$ as in Figure 1 (i), s and z denote the source and sink, respectively. In the arc (s, i_2), capacity is 2 and transit-time is 3, denoted by $(2, 3)$. A time-expanded network $\mathscr{N}^T$ corresponding to the simple network $\mathscr{N}$ that of Figure 1 (i) can be constructed as in Figure 1 (ii) with time horizon T=5, for discrete time-setting.

Condensed time-expanded network. As mentioned earlier, $\mathscr{N}^T$ is not provided with polynomial-time solution. However, if all the arcs are the multiple of $\delta > 0$ such that $\lceil \frac{T}{\delta} \rceil$ is bounded by a polynomial in the input size then the network can be re-scaled corresponding to time to be a condensed time-expanded network $\mathscr{N}^T/\delta$ containing $\lceil \frac{T}{\delta} \rceil$ copies of V. Here, every arc corresponds to a time interval of length δ and the capacities are multiplied by δ.

Series-parallel network. A single arc $a = (s, z)$ is a series-parallel with s be the start-terminal and z be the end terminal, respectively. Consider the network $\mathscr{N}_i$ having s_i and z_i be the start-terminal and end-terminal, respectively, for $i = 1, 2$. Then the network $S(\mathscr{N}_1, \mathscr{N}_2)$ obtained by identifying z_1 as z_2 in the series combination is a series-parallel network with s_1 and s_2 as its terminals. Likewise, the network $P(\mathscr{N}_1, \mathscr{N}_2)$ obtained by identifying s_1 as s_2 and also t_1 as t_2 in the parallel combination is a series-parallel network with $s_1 = s_2$ and $t_1 = t_2$ as its terminals.

Grid network. A network defined on a grid graph is the grid network. For a grid network in $\mathbb{N}^2$, the grid points in the plane be $\{1, 2, \ldots, N\} \times \{1, 2, \ldots, N\}$ for $n = N^2$ be the number of vertices. Here, a vertex $i \in V$ be $i = \{(i_1, j_1) : 1 \leq i_1 \leq N, 1 \leq j_1 \leq N\}$ and the sink z be one of such vertices. Let $i, j \in V$ with $i = (i_1, j_1)$ and $j = (i_1', j_1')$ then their distance is $d(i, j) = |i_1 - i_1'| + |j_1 - j_1'|$. They are connected by an edge iff $d(i, j) = 1$. In such a grid network, capacities of all arcs have the same value u and their transit times also have the same value τ.

Residual network. Let f be a flow in $\mathscr{N} = (V, A, u, \tau, S, Z, T)$. The residual network with respect to the f is denoted by $\mathscr{N}(f) = (V, A(f), r, \tau, S, Z, T)$. The residual capacity for an arc $(i, j) \in A$ is defined by $r(i, j) := u(i, j) - f(i, j) + f(j, i)$, and the arcs set $A(f)$ is defined by $A(f) := \{(i, j) \mid r(i, j) > 0\}$. Here, the edges are allowed to augment their flows while the capacity constraint must be met as their may exist an augmenting path to increase the source-sink flow.

2.2 Evacuation Network Topology

In an evacuation network topology, the evacuation region is represented by a network, nodes represent the locations where the evacuees are gathered or waiting for the transshipment, and the arcs connecting the nodes are the road segments through which the evacuees are transversed, considered as a flow. The term flow in an evacuation network indicates either the evacuees or the vehicles carrying evacuees. One of the approach considering the EPP is the dynamic network flow model, the time can be considered continuous or discrete. In case of static network flow model, the time parameter is absent. In general, the continuous model yields more accurate results over the discrete but are more challenging to compute. Evacuees movement can be considered as pedestrian or car-based or bus-based. During large scale disasters, the transit-based system is more prominent than others. Supplies and demands at the nodes might be known or unknown. If known, then they can be denoted by vectors $\alpha(s)$ and $\beta(z)$, respectively. Then $\alpha(s)$ represents the total unit of evacuees at node s that need to be sent to the node z with its demand $\beta(z)$.

2.2.1 Prioritized network

During the collection of evacuees in an EPP, the nature of pickup locations that means the pickup terminals may differ from each other depending upon their locations, availabilities, etc. On the other hand, the nature and needs of the evacuees might vary. Not only this, the nature of the safe zones are also different, in real practice. Hence, sometimes the terminals in such an evacuation network can also be prioritized. A prioritized network is a multi-terminal network which consists of prioritized terminals. Under the given priority of terminals, two flows can be compared according to departure/arrival flows form/in the sources/sinks.

2.2.2 Integrated evacuation network

Depending upon the nature and needs, different networks can be embedded to have an integrated network. For example, an evacuee collection network$\mathscr{N}_1$ and the transit-vehicle assignment network $\mathscr{N}_2$ as the primary and secondary sub-networks can be embedded as a unit for an integrated evacuation planning approach (cf. Chapter 4 and Chapter 5). In such an embedded network having $\mathscr{N}^E = \mathscr{N}_1 \cup \mathscr{N}_2$, the sub-network $\mathscr{N}_1$ consists of directed two-way arcs respecting the partial arc reversal capability (cf. Section 2.2.3) for the collection of evacuees. By treating such supplies as the sources, the available set of transit-vehicles at depot are assigned to transverse the evacuees to the sinks in $\mathscr{N}_2$. The sub-network $\mathscr{N}_2$ is the mixed network having directed one-way arcs from bus depot to the pickup locations and undirected edges for the transit-vehicle assignment network.

2.2.3 Arc reversal network

The arc reversal is an approach that modifies the orientation of arcs in the network to improve the flow. This significantly increases the flow value to make the traffic smooth and minimize the time during evacuation. A transformed network will be used to extract the improved flow in such networks.

The transformed network of $\mathscr{N} = (V, A, u, \tau, S, Z, T)$ is $\tilde{\mathscr{N}} = (V, \tilde{A}, \tilde{u}, \tilde{\tau}, S, Z, T)$. Let the reversal of an arc $a = (i, j)$ be $a' = (j, i)$, then the transformed network $\tilde{\mathscr{N}}$ of $\mathscr{N}$ consists of the modified arc capacities and symmetric transit times as,

$$\tilde{u}_{\tilde{a}} = u_a + u_{a'}, \quad \text{and} \quad \tilde{\tau}_{\tilde{a}} = \begin{cases} \tau_a & \text{if } a \in A \\ \tau_{a'} & \text{otherwise} \end{cases} \tag{2.1}$$

where an arc $\tilde{a} \in \tilde{A}$ in a transformed network, if $a \vee a' \in A$ in $\mathscr{N}$. The remaining graph structures and data are unaltered.

2.3 Flows in Network

Flows in network are categorized fundamentally as in discrete time-setting flow and the continuous time-setting flow. In discrete dynamic flows, time steps are discretized as $\mathbf{T} = \{0, 1, ..., T-1\}$. But on the other hand, the continuous flow specifies the flow rate for every moment in time for $\theta \in [0, T)$. In discrete time-setting, the dynamic flow is a mapping $f : A \times \{0, 1, ..., T-1\} \to \mathbb{R}_{\geq 0}$, whereas it is a Lebesque-integrable function $f : A \times [0, T) \to \mathbb{R}_{\geq 0}$, for continuous time-setting. Flow entering an arc a with a transit time τ_a at time θ leaves a at time $\theta + \tau_a$.

In such static flow models the time parameters are absent. A static flow $f : A \to \mathbb{Z}_{\geq 0}$ is assigned to be a flow value to each edge such that $f(a) \leq u_a$ for each arc $a \in A$, as the capacity constraint. Moreover, the excess in each node i is given by the difference of the flow reaching the node to leaving the node as,

$$ex_f(i) = \sum_{a \in A_i^{in}} f(a) - \sum_{a \in A_i^{out}} f(a) \tag{2.2}$$

such that $ex_f(i) = 0$ for $i \in V \backslash (S \cup Z)$, $ex_f(i) \geq 0$ for $i \in Z$ and $ex_f(i) \leq 0$ for $i \in S$, as the flow conservation constraints. Then, the maximum flow in such static network is a flow that maximizes the flow value denoted by,

$$|f| = \sum_{z \in Z} ex_f(z). \tag{2.3}$$

In both cases, no flow is left in the edges after the time horizon T that is $f(a, \theta) = 0, \forall\, \theta \geq T - \tau_a$. These are to satisfy the capacity constraints as well as the flow conservation constraints. For the capacity constraints, we need

$$0 \leq f(a, \theta) \leq u_a, \forall\, a \in A. \tag{2.4}$$

The excess of flow in each node i is given respectively as,

$$ex_f(i, \theta) = \sum_{a \in A_i^{in}} \sum_{\xi=0}^{\theta - \tau_a} f(a, \xi) - \sum_{a \in A_i^{out}} \sum_{\xi=0}^{\theta} f(a, \xi). \tag{2.5}$$

Then for the flow conservation constraints on two different time models, it should satisfy,

$$\begin{aligned} ex_f(i, \theta) &= 0 \quad \text{for}\, i \in V \backslash (S \cup Z), && (2.6) \\ ex_f(i, \theta) &\geq 0 \quad \text{for}\, i \in V \backslash Z, && (2.7) \\ ex_f(i, \theta) &\leq 0 \quad \text{for}\, i \in V \backslash S. && (2.8) \end{aligned}$$

Additionally, we have $ex(i) = ex(i, T)$. Moreover, if the dynamic network is provided with demand/supply $\mu : V \to \mathbb{Z}$, then it is natural to have,

$$ex(i) \leq -\mu(i) \text{ for } i \in V \backslash Z \text{ and } ex(i) \geq \mu(i) \text{ for } i \in V \backslash S. \tag{2.9}$$

Either in static or dynamic network flows, a maximum flow is the flow maximizing the flow value given by Equation (2.5). Moreover, the total amount of flow sent to the sinks until the time θ for is given by,

$$|f|_\theta = \sum_{z \in Z} ex_f(z, \theta). \tag{2.10}$$

A flow of evacuees from the source to the sink over time is a non-negative function f on $A \times \mathbb{R}_{\geq 0}$, for given time $\mathbf{T} = \{0, 1, \ldots, T-1\}$ satisfying the flow conservation and capacity constraints (2.11-2.13). The inequality flow conservation constraints allow waiting for flow at intermediate nodes. However, the flow conservation constraints force that flows entering an intermediate node must leave it again immediately.

$$\begin{aligned}
\sum_{\sigma=\tau_a}^{T}\sum_{a\in A_i^{in}} f(a,\sigma-\tau_a) - \sum_{\sigma=0}^{T}\sum_{a\in A_i^{out}} f(a,\sigma) &= 0,\ \forall\, i \notin \{s,z\}, && (2.11)\\
\sum_{\sigma=\tau_a}^{\theta}\sum_{a\in A_i^{in}} f(a,\sigma-\tau_a) - \sum_{\sigma=0}^{\theta}\sum_{a\in A_i^{out}} f(a,\sigma) &\geq 0, \forall i \notin \{s,z\}, \theta \in \mathbf{T}, && (2.12)\\
0 \leq f(a,\theta) &\leq u_a, \qquad \forall\, a \in A,\ \theta \in \mathbf{T}. && (2.13)
\end{aligned}$$

Here, $A_i^{out} = \{a = (i,j) \in A\}$ and $A_i^{in} = \{a = (j,i) \in A\}$ are the sets of outgoing and incoming arcs, respectively for the node $i \in V$. For the source node s, we get the flow value be $\nu_f(s) > 0$, and for the sink z the flow value becomes $\nu_f(z) < 0$, whereas $\sum_{i\in V} \nu_f(i) = 0$. The flow value at θ is defined by,

$$(\nu_f,\theta) = \sum_{\sigma=0}^{\theta}\sum_{a\in A_s^{out}} f(a,\sigma) = \sum_{\sigma=\tau_a}^{\theta}\sum_{a\in A_z^{in}} f(a,\sigma-\tau_a). \tag{2.14}$$

2.3.1 Maximum dynamic flows

Ford and Fulkerson [50, 51] have developed the first dynamic flow problem and solved the maximum dynamic flow (MDF) problem. The problem maximizes the value of Equation (2.15) satisfying Constraints (2.11-2.13),

$$(\nu_f,T) = \sum_{\sigma=0}^{T}\sum_{a\in A_s^{out}} f(a,\sigma) = \sum_{\sigma=\tau_a}^{T}\sum_{a\in A_z^{in}} f(a,\sigma-\tau_a). \tag{2.15}$$

The MDF problem can be transformed into the maximum static flow problem by constructing the time-expanded network for the given dynamic. Then any algorithm of maximum static flow algorithm can be applied to extract the solution. But the size of the time-expanded network depends on T, thus the time complexity becomes pseudopolynomial. The MDF problem can be considered as the MCF problem by assuming the time of the dynamic model as the cost of the MCF problem. This static has been used to construct the TRF. Ford and Fulkerson decompose the static flow into the path flow and repeats the path flow over T. This TRF can be expressed as,

$$(\nu_f, T) = \sum_{\gamma \in \Gamma} (T - \tau_\gamma + 1) f_\gamma = (T+1)f - \sum_{a \in A} \tau_a f_a. \tag{2.16}$$

In this case, the solution can obtained in polynomial-time. This expression depends only on the static flow and is independent with choice of paths.

2.3.2 Lex-max dynamic flows

A flow value is said to be lexicographic if it is compared according to the rank of the terminals. Let $\mathscr{N} = (V, A, u, \tau, S, Z, T)$ be a prioritized network with priority $t_1, t_2, \cdots, t_n;\ t_i \in S \cup Z$. Let

$$|f|_t := \begin{cases} \sum_{a \in A_t^+} f_a, & t \in S \text{ is a source} \\ \sum_{a \in A_t^-} f_a, & t \in Z \text{ is a sink} \end{cases}$$

be the out/in flow value from/in the source and sink, respectively. Suppose f^1, f^2 be the terminal respecting flows, f^1 is said to be lexicographically bigger than f^2 and written as $f^1 \geq_L f^2$ if $\exists\, l \in \{0, 1, \cdots, \delta{-}1\} : \forall\, i \in \{1, 2, \cdots, l\} : |f^1|_{t_i} = |f^2|_{t_i} \wedge |f^1|_{t_{l+1}} > |f^2|_{t_{l+1}}$ or $\forall i \in \{1, 2, \cdots, \delta\} : |f^1|_{t_i} = |f^2|_{t_i}$. The maximum flow respecting the rank of the terminals is said to be lex-max flow.

Author in [98] formulated and solved the problem in static network. The lex-max flow problem with many sources and many sinks was introduced and many efficient algorithms were presented from different aspects in [62, 93, 95, 98]. In such a problem, the terminals (sources and/or sinks) are ordered with certain priority as $s_1, s_2, \ldots, s_\delta$ and $y_1, y_2, \ldots, y_\delta$, respectively, as a prioritized network. Such a lex-max flow respecting the priority is not necessarily a maximum flow and vice versa, however, they are equivalent for two-terminal networks, [51].

Authors in [98, 134] have transformed the dynamic lex-max flow problem into the time-expanded and solved the problem as in static case. This approach takes pseudopolynomial-time complexity. The first polynomial-time solution for the problem is based on chain decomposition of flow, [67, 68]. Starting with zero flow, the lex-max flow algorithm computes the successive layers of minimum cost static flows in the residual network of the previous layers and adds standard chains to the existing one. It takes δ times the complexity of MCF computations, for a given time T.

A chain flow $\gamma = \langle \nu, \pi \rangle$ is a static flow of values $\nu \geq 0$ along the path π in a network $\mathscr{N} = (V, A, u, \tau, S, Z, T)$. The length of chain flow $\tau(\gamma)$ also represents total length of path $\tau(\pi)$. Given the time horizon T no less than $\tau(\gamma)$, any chain flow γ induces a dynamic flow by sending ν units of flow along the path π at every time step until $T - \tau(\gamma)$. A proper chain flow starts and ends at terminals. A multi-set of proper chain flows, $\Gamma = \{\gamma_1, \gamma_1, \ldots, \gamma_k\}$ is a chain decomposition of static flows f if $\sum_{i=1}^{k} \gamma_i = f$. It becomes a standard chain decomposition of f if all chain flows in it use edges in the same direction as of f. A flow decomposition with zero flows on all cycles is the path decomposition. One may assume that there is no flow along any cycle as the opposite flows along all cycles could be canceled.

In the non-standard chain decomposition, the chain flows may use oppositely directed flows on edges. It may use a residual edge with negative transit times. For $a = (i, j)$, a unit of flow sent from i at time θ reaches j at time $\theta + \tau_a$ is nothing other than sending a negative unit of flow from j at time $\theta + \tau_a$ to reach i at time θ. Let γ_1' be the another chain that flows through (j, i), then that cancels the chain flow γ_1 along (i, j). The chain decomposable flows do not violate capacity constraints. Moreover, the non-standard chain decomposition induces the dynamic flows.

2.3.3 Quickest flows

The quickest flow problem can be considered as an inverse problem of MDF problem. For given amount of flow in the dynamic network it seek to find the minimum clearance time, also known as the minimum time network clearing problem. The different network flow problems with time minimization objective have been reviewed in [26]. The quickest flow problem in two-terminal network has been solved in time $\mathcal{O}(m^2 \log^3 n(m + n \log n))$, [23]. They have also established its relation with the MDF problem. Authors in [88, 126] have presented different algorithms for the problem with better complexities.

The problem having demands and supplies at sinks and sources then the problem is known as the quickest transshipment problem. The quickest transshipment problem is to find the minimum clearance time to send a given amount of flow from multiple sources to multiple sinks network. In the quickest transshipment problem, a given number of evacuees has to be shifted in minimum time. Such problem have been studied by the help of the lex-max dynamic flow problem applying the MCF computations as a tool, in such a prioritized network. For a given time T and an ordered set of multi-terminals, the lex-max dynamic flow problem finds a feasible flow that lexicographically maximizes the flow leaving each terminal in the prioritized network.

A network with uniform capacity is practically applicable since the city streets or building corridors are generally standardized. The quickest transshipment in a grid network with uniform arc capacity can be solved in polynomial-time complexity in time $\mathcal{O}(n \log n)$ for n be the number of vertices of the grid network, [77]. It seems that the transshipment in such a grid network in particular is to be more efficient than the first known polynomial-time algorithm of Hoppe and Tardos [69], which is also polynomial but of higher-order.

2.3.4 Earliest arrival flows

Minimizing the network clearance time is one of the common objective in the evacuation literature. When preparing for an evacuation, the time it actually takes is uncertain, and hence it is preferential to plan at each point of time to execute the maximum flow. Equivalent to say the transversal of maximum number of evacuees reaching safety not only at the ultimate clearance time but also in each possible time unit simultaneously, i.e. with the maximum possible flow value for all $\theta \in \mathbf{T}$, which is offered by the EAF. It is better-suited for evacuation planning as it maximizes the flow of evacuees simultaneously at each instance within the given time horizon. Such an evacuee arrival pattern is more appropriate for the integrated evacuation scenario, (cf. Section 3.5.2).

The existence of the EAF has been established in [52] but solution procedure was not provided in the paper. Based on the successive shortest path of [51], authors in [98] and [134] have developed algorithms for EAF but both are pseudopolynomial. Hoppe

[67] presented another algorithm using chain decomposition but it has the higher complexity comparing to previous algorithm. A fully polynomial approximation having complexity $\mathcal{O}(\frac{m}{\epsilon}(m + n \log n) \log U)$ is developed for the problem, where U is the maximum capacity of edges and $\epsilon > 0$ is the deviation, [67, 68]. The TRF having the earliest arrival property is not possible in general. But, this can be possible in series-parallel network. On such a network, there is always a TRF that is maximal and an earliest arrival as established in [125]. Such a flow can be obtained in $\mathcal{O}(nm + m \log m)$, polynomial-time complexity.

If the supply and demand on sources and sinks $\nu_f(i)$ is a fixed value for all $i \in \{s, z\}$, then the EAE problem maximizes value(ν_f, θ) for all $\theta \in \mathbf{T}$, as in Equation (2.14) satisfying the constraints (2.11-2.13). If all the supplies should be shifted with in given time horizon T, then the problem turns into transshipment problem with given supplies and demands. Each of the earliest arrival transshipment (EAT) additionally optimizes the the amount of flow leaving the network at all times and is therefore a quickest transshipment. But not necessarily the converse, i.e., a quickest transshipment is not necessary to be an EAT, as illustrated in Example 2.1. Moreover, the EAT does not necessarily exist in the networks with multiple sinks, but the quickest transshipments do.

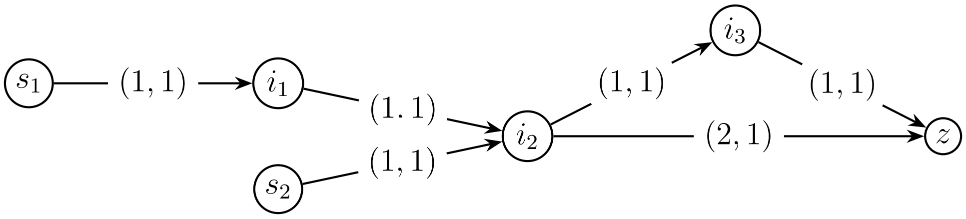


Figure 2: An instance of a dynamic evacuation network.

Example 2.1. Consider a dynamic network $\mathcal{N}$ having (u_a, τ_a) be the capacity and transit time for $a \in A$ as in Figure 2. For s_1 and s_2 be the sources, and z be the sink, let $\mu(s_1) = \mu(s_2) = +3$ and $\mu(z) = -6$.

Consider two different flow patterns with their respective time, path assignments, flow value, and the total flow in different columns, as in Table 1 and Table 2. Here, Table 1 represents a quickest transshipment that is not an EAT. But, Table 2 represents an EAT which is also a quickest transshipment.

Table 1: A quickest transshipment which is not an earliest arrival transshipment.

Time unit	Path assignment	flow value	Total flow reached
3	$s_1 - i_1 - i_2 - p$	1	2
	$s_2 - i_2 - i_3 - p$	1	
4	$s_1 - i_1 - i_2 - p$	1	4
	$s_2 - i_2 - i_3 - p$	1	
5	$s_1 - i_1 - i_2 - p$	1	6
	$s_2 - i_2 - i_3 - p$	1	

Table 2: An earliest arrival transshipment which is also a quickest transshipment.

Time unit	Path assignment	flow value	Total flow reached
2	$s_2 - i_2 - z$	1	1
3	$s_2 - i_2 - z$	1	3
	$s_1 - i_1 - i_2 - z$	1	
4	$s_2 - i_2 - z$	1	5
	$s_1 - i_1 - i_2 - z$	1	
5	$s_1 - i_1 - i_2 - z$	1	6

An integrated approach is presented following the lex-max flow approach of quickest transshipment by the vehicle assignment in an embedded network topology to improve the minimum clearance time of the evacuees in the integrated evacuation network, [7]. The triple optimization problem simultaneously optimizes the three objectives about the quickest transshipment, EAT, and about the minimizing of the average time for the network clearance, [73].

2.4 Contraflow Problems

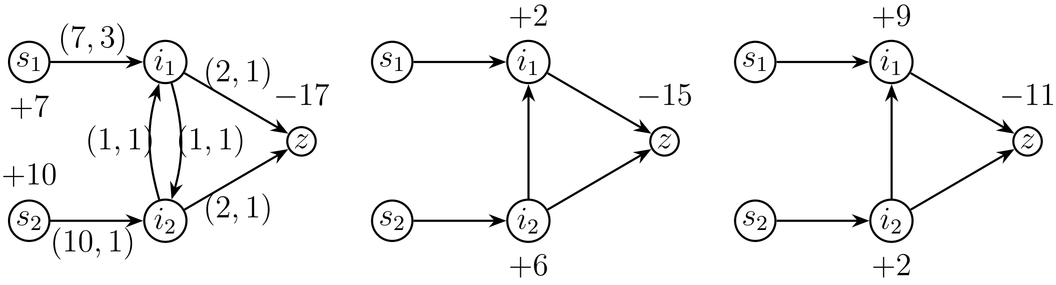
Contraflow has gained a considerable attention in evacuation literature because by finding the ideal direction of lanes of a road network, the flow can be increased and evacuation time can be reduced as compared to the evacuation in the existing road reconfiguration and is applicable to reduce congestion, eliminates the crossing at intersections and traffic jams during the day-to-day rush hours.

Authors in [14, 123] solve the maximum static $s-z$ flow problem on arc reversal in the transformed network by decomposing the obtained flow into paths and cycles, and deleting the latter one assuming that the arcs on either direction will never be used in the optimal flow. An arc $a' = (j,i) \in A$ is reversed iff the flow on (i,j) is greater than $u_{a=(i,j)}$, or if there is a non-negative flow along $(i,j) \notin A$ and the resulting flow is maximum with arc reversal. The maximum static $s-z$ flow problem on arc reversal is solvable in strongly polynomial-time.

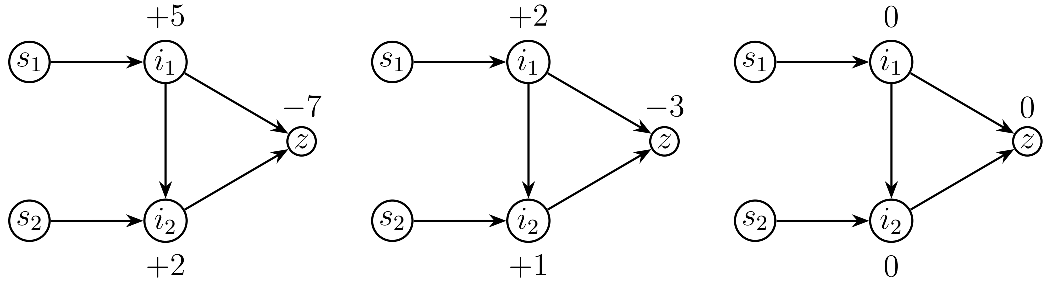
The general maximum static flow problem on arc reversal is reduced to the respective $s^* - z^*$ flow problem, providing the super-source s^* and super-sink z^* as in extended network. Here, s^* is connected to each $s \in S$ having arc capacities equal to their respective surplus and z^* to each $z \in Z$ having arc capacities equal their respective deficits. Hence, the respective static version of the maximum flow problem with arc reversal with multiple sources and multiple sinks, is also polynomially solvable, [14, 123]. The similar situation exits regrading to its complexity in the case of MDF problem on arc reversal in two-terminal networks, though in general, the MDF problem in a multi-terminal network on arc reversal is $\mathcal{NP}$-complete.

Such $\mathcal{NP}$-completeness in a multi-terminal network is so due to the conflict with reverting the intermediate arcs, [112, 123]. Moreover, there is no polynomial-time algorithm for the dynamic transshipment problem with arc reversal having only two suorces and one sink or one source and two sinks, unless $\mathcal{P} = \mathcal{NP}$. Authors in [81, 123] proofs the hardness of the general contraflow problem, is $\mathcal{NP}$-hard. Hence, numerous heuristics and meta-heuristics have been presented and implemented for the solutions of different types of EPPs by arc reversal approach, to increase the flow value and decrease the evacuation time significantly.

In general, the maximum contraflow and maximum dynamic contraflow reverse the arcs on fly and are sightless whether they reverse an arc or not. It does not create any problems for the static or in two terminal dynamic flows, because in standard chain decomposition, one can always derive an optimal solution using only one of the arcs during the complete time horizon. But in the case of multiple sources and multiple sinks network, the potential of using both arcs directs the problem to know whether an arc has been reversed or not. Hence, when one choose arcs, one has to know whether an arc has been reversed or not in every time unit. Such a memory and decision of reversing an arc now or at a later time makes the problem $\mathcal{NP}$-complete. The proofs follow by reductions from the problem 3-SAT and PARTITION, for details about the hardness of contraflow and its proof, we refer to [81], [111] and [123]. Here, we are presenting about the hardness of contraflow problem in a network having two sources and a single sink, based on [123].



(i) 2s1z initial network $(ii)\, T = 2$ $(iii)\, T = 3$



$(iv)\, T = 4$ $(v)\, T = 5$ $(vi)\, T = 6$

Figure 3: Two sources and a single sink dynamic network.

Example 2.2. Let s_1 and s_2 denote two sources with supply $\mu(s_1) = +7$ and $\mu(s_2) = +10$ respectively. Let the sink node z is with demand $\mu(z) = -17$. The capacity and transit time along the arcs on the two sources and one sink network are shown as in Figure 3(i). At time 1, one can switch the arc (i_1, i_2) to increase the capacity across the arc (i_2, i_1) and after time point 3, it can be switched back again to (i_1, i_2) to achieve a feasible flow within a shorter time. Here, after the time point 3, we achieve $\mu(i_1) = +9$,

$\mu(i_2) = +2$, and, $\mu(z) = -11$ is still left as in Figure 3(iii). Then one continue with the change in orientation as in Figure 3(iv) to have the greater feasible flows. Here, the flow is completed at $T = 6$, Figure 3(vi). Such, a possibility of using both arcs time to time leads the problem to be $\mathcal{NP}$-complete.

Authors in [14, 123] have also developed the quickest contraflow problem to minimizes the clearance time for the given amount of flow. The quickest transshipment contraflow problem that satisfies the given demands and supplies of sinks and sources in minimum time, [115]. The quickest contraflow problem with load-dependent travel time has been designed and solved in [120]. They have also presented a computational experiment of their result in the paper. The above mentioned models considers the single commodity flow in their model. An approximation for the multi-commodity quickest contraflow problem has been developed for their novel model in [36].

Shortly after the development of analytical solution, the earliest arrival contraflow problem in series-parallel network has been introduced and developed a polynomial-time algorithm in [37]. The earliest arrival contraflow problem in general network has been solved in [112, 115], and are pseudopolynomial. An approximation for the problem is developed in [42]. The complexity of the algorithm is $\mathcal{O}(m\epsilon^{-1}(m + n\log n)\log U)$, where $\epsilon > 0$ and U is the largest capacity of the arcs. The problem in the network having multi-source, multi-sink, and multi-terminals have been solved in [115]. Moreover, they have also extended and solved the problem for zero transit time. For prioritized terminal, the lex-max contraflow problem and a polynomial-time algorithm have been presented in [112]. The complexity of the algorithm is $\mathcal{O}(\delta(m\log n)(m + n\log n)))$, where $\mathcal{O}((m\log n)(m+n\log n)))$ is the complexity of the MCF problem in the residual network and number of iteration δ.

The lossy network flow model deals with problem with possible flow loss during evacuation process. The maximum dynamic contraflow and earliest arrival contraflow problem in loss network have been introduced and solved in [111, 118]. The contraflow problem with continuous time setting having their respective objective have been considered in [79], [113], and [114]. They have solved their problems by transforming the continuous time into discrete time-setting using the natural transformation, [48]. The complexities of algorithms are the same as in discrete time-setting.

In stead of full arc reversal as mentioned above, arcs are better to reverse upto the necessary capacity only as in the partial arc reversal network as presented lucidly with different models, algorithms, and the solution strategies by Pyakurel *et al.* [121]. In such network, the unused capacities on different arcs can be used for other purposes to facilitate the evacuation. Such flow models on node-arc partial arc reversal network contribute in saving the unnecessary arc reversals and improve the solution corresponding to the complete arc reversal approaches. By introducing the partial lane reversal strategy, the static and the dynamic partial lane reversal problems including, the maximum dynamic, the earliest arrival, the quickest, lex-maximum dynamic, generalized universally maximum, and the partial arc reversal transshipment problems have been solved with efficient algorithms. They have proposed the polynomial-time algorithms to solve these problems with constant transit times on each arc and also introduced different models with variable transit times on each arc. The beauty of partial arc reversal network has been used widely in different research. In a recent work, authors in [9], have integrated it with the EAT and the flows in zero transit times. The EAE problem having zero transit times with partial arc reversal capability follows the principle of TRFs and can be solved in polynomial-time complexity.

Partial arc reversal approach is also extended to abstract network setting with flow on paths and adopt most of the aforementioned arc reversal and partial arc reversal solution techniques to save unused capacities road segments which can be used for supplying emergency facilities and logistics. For the limited resources it is not possible to select all arc reversals as demanded by the optimal arc reversal strategy, as each arc reversal is associated with certain operating cost. To address such issues on the arc reversal problems, the budget constraint version of the problem is investigated in [41] which incorporate the issues of the arc reversal cost in such arc reversal network subject to given budget constraint. They have established different analytical results addressing the issues related to the cost.

The above mentioned models make the use of arc reversal in contraflow. The node-arc model is equivalent to the path-arc model. Based on this, many path-arc models have been developed with different objectives, see in [38]. To deal with the later model, the path reversal approach has been introduced in [116]. Based on this approach, the abstract contraflow models with different objectives like maximum flow, earliest arrival flow, lexicographically maximum flow, and time minimization for the given flow have been discussed in [39], [43], and [121].

The contraflow approach has been integrated with network facility location (FlowLoc) to introduce ContraFlowLoc in [40]. They have introduced maximum ContraFlowLoc problem and developed algorithms for the problem in both static and dynamic network. They have also shown the non-existence of ContraFlowLoc solution with earliest arrival property. The quickest ContraFlowLoc problem to minimize the clearance with facility location have been presented in [104]. The contraflow approach has been applied to locate the sink in evacuation network, [103].

An integrated contraflow strategy has been presented by [70] containing non-contraflow to shorten the strategy set up time, full-lane contraflow to maximize the evacuation network capacity and bus contraflow to realize the transit cycle operation. Here, the routing problem of the transit-based evacuation has been considered as the MCF problem with multiple origin nodes and single super destination node. This is a mixed integer linear programming problem which can be solved in a very efficient way using the branch and bound method. Whereas, the auto-based evacuation method has a bi-level structure and is usually solved by using heuristic algorithms as in [96].

CHAPTER 3

EVACUATION OPTIMIZATION

Summary. Evacuation planning problem deals with sending the maximum number of evacuees from sources to sinks in minimum time as efficiently as possible. The dynamic network flow formulation has been found suitable optimization approach corresponding to different scenarios of the macroscopic evacuation planning. Bus-based evacuation planning is in the core of this chapter. With some extensions, different aspects of such a problem are highlighted concerning to the dominant vehicle assignments and route selections.

Most of the critical decisions to be made in evacuation emergency phases are related to preparedness (planning) phase of the PPRR framework for the disaster management. Here, PPRR stands for prevention (mitigation), preparedness (planning), response, and recovery phases, [31] and are inter-connected to each other. The decision related to the identification of ideal roots in an evacuation network, and the assignment of vehicles in case of evacuation of people to safe places as quickly as possible, distribution of relief supplies, and their optimally, requires the appropriate mathematical tools and swift calculations. Fortunately, operation research models, especially the optimization modeling, has become a powerful tool to carry out such emergency evacuation decisions, since its first adoption in maritime disaster situations in the 1970s, [25].

3.1 Evacuation Planning Problem

Transit has a unique role in evacuating the car-less, elderly, and the needy populations with different disabilities. Even when transit evacuation is planned carefully, logistics and communications issues are taken care of, the behavior, knowledge, attitude, and nature of the evacuees still play a major role in the effectiveness of an EPP. Moreover, the ages, readiness, and/or the willingness of people to evacuate may be different. So, it demands the behavioral analysis. Furthermore, the lack of coordination between the transit agencies and the traffic operators may highly affect the system. The dynamic network flow formulation has been found suitable evacuation optimization approach. There has been a fair amount of work regarding different aspects of network flow formulation related to the EPP, as referred by [5], [6], [8], [33], [34],[35], and [38].

In a survey of about high-risk area of hurricane as in [19] has noticed that about 54 % of households, the traffic congestion was the main reason for not evacuating on such high-risk hurricane strikes and has noticed that more fatalities were caused by the lack of proper evacuation actions than the hurricane. It was noticed that, 71 % of those who died in Hurricane Katrina in New Orleans were age of 60 and 47 % over of 75. This also demands the need of transit vehicles for effective evacuation on EPP, [90].

Simulation models enable transportation planners and practitioners to develop and compare different evacuation plans for different hypothetical situations to predict traffic conditions and the evacuation duration. Such techniques have also been used to investigate how different evacuation scenarios like alternatives exits, number of vehicles changed, and other traffic control plans would affect the evacuation duration. Such models have been presented systematically in [100] to simulate the transit-based evacuation strategies where the average travel time and total evacuation time were used to compare the results of different evacuation time periods. They also proposed comparatively the effective scenario of transit-based evacuation routing plan with reduction in over all travel time as well as the total evacuation time with respect to the peak hour general evacuation scenarios. Simulations are the powerful tools to evaluate traffic scenarios though it misses the optimization potential.

An alternative evacuation route plan strategy is suggested by [86] with mixed integer nonlinear programming formulation for real time evacuation where the traffic network is affected partially or totally for short or long periods of time. Though in a minor incident, one can wait until the incident is cleared to follow the pre-planned route but in a severe incident it is better to have an alternative path to evacuate the outbound flows due to over congestion and minimize the evacuation clearance time. Assuming that a complete evacuation is not possible to evacuate all the transit-dependent citizens during no-notice evacuation, authors in [127] have developed the models to identify the paths for vehicles to have the minimum number of casualties; to minimize the total evacuation time; and to maximize the vehicle utilization on the system.

The transit signal priority method in [87] has given the priority on (i) transit vehicle arrival time estimation, (ii) queuing vehicle dissipation time estimation, (iii) traffic signal status estimation, (iv) transit signal optimization, and (v) arterial traffic signal coordination for transit vehicle in evacuation route. In a survey, with some demographical analysis of the Upstate New York city, the authors in [65] have suggested to the planners, transit providers, emergency management officials and even to the researchers for the development of multi-modal mass evacuation plans with the incorporation of more high-capacity vehicles for the comprehensive and effective emergency management plan for the large scale evacuation.

A multi-modal optimization evacuation framework has been proposed by [3] to optimize simultaneously the minimizing of in-vehicle travel time, minimizing of the at-origin waiting time, and minimizing fleet cost for mass transit evacuation. By the comparative analysis of different evacuation scenarios they claimed that considering only the travel time underestimates the waiting time of the evacuees in no-notice evacuation. In another hand, minimizing of the evacuee waiting time implies evacuating all the population instantly and will ultimately demand their simultaneous evacuation, which may lead to longer travel times in the system and the longer evacuation time with congestion on the evacuation network. Furthermore, minimizing travel time causes the delaying of the evacuees at the origin and will ultimately increases the waiting time. A good compromise and their proper trade-off is always challenging as these two objectives might be conflicting to each other.

In general, the continuous dynamic flow models give more accurate results over the discrete ones but are with high degree of computational complexities and are challenging. The models with flow and/or time dependent attributes convert the problem to be nonlinear and demands the linear relaxation to handle it for computationally possible approximations. Flow models dealing with differential equations give the more accurate results for small size problems in comparison but are complicated and difficult to handle in large instances of real-world problems. Most general problem addressing the diversified and the heterogeneous vehicles including different commodities and networks is rather complicated from the computational point of views. To cope such situations, the BEPP is considered as one of the important tool of transit-based system for the evacuation optimization.

3.2 Bus-based Evacuation Planning Problem

Optimization strategies based on scheduling and routing are one of the fundamental parts of BEPP. Since from the seminal papers of Dantzig *et al.*, [32] and Clarke & Wright, [29] for the truck dispatching problem for gasoline distribution and the vehicle scheduling problem for delivery of goods respectively, the research on VRP has grown exponentially. There is a wide variety of literature evolving into various dimensions and variants that includes the multi-depots, capacitated vehicles, multiple trips, split deliveries, time windows for customers, service choice, pickup and deliveries, backhauls.

Models and algorithms formulated and designed to solve the BEPP were usually based on the conventional VRP or traveling salesman problem, which has been solved and applied in [20, 136, 138, 140]. In most of them, the objectives are to minimize the total bus travel time or the total network clearance time, as usual. Some of the other such formulations on somewhat similar fashions for the time minimization on evacuation planning system are also available in the literature, [84, 122, 127, 132]. Some of the BEPP solutions have taken account of the walking or waiting time of the evacuees during the evacuation process while optimizing the evacuation routes, [44, 66, 107, 141]. They have considered the total evacuee walking time from buildings to the pickup locations and on the total vehicle travel time. Some studies combined the buses with other traveling modes of traffic networks during multi-model evacuation for the urban transport evacuation, [101, 102]. To obtain the high-feasible solutions quickly, different heuristics algorithms were often applied, like simulated annealing algorithm, genetic algorithm, and ant colony algorithm.

A VRP with split deliveries formulation of BEPP incorporating the multiple trips with time windows for the last trips for each pick location has been formulated by [4]. They have considered the total vehicle time, total evacuee waiting time at various pickup locations, and the fleet size. They seek to minimize the total travel time, waiting time of the evacuee, and the fleet size too. However, the proper trade-off between these conflicting objectives is challenging. An integration of automobiles with mass transit-vehicles have been presented in [1, 4]. In their integration, demands for two systems are estimated and are solved in different compartments. Firstly, the automobile compartment is solved and is used as the input parameter for the mass transit component.

The BEPP formulations as a VRP with time windows for uncapacitated shelters have been presented in [27] with an iterative approach for its solution. A round-trip BEPP model with scheduling and routing planning has been presented by [144] where the buses travel back and forth between pickup locations and sinks to pickup and shelter evacuees. During the routing, the same bus can serve different pickup points and shelters. For their model construction, the total evacuee time cost including the in-bus travel time and waiting time is designed as the optimization objective.

In the BEPP formulations in [54] and [84], buses are initially parked at depot and pickup locations, respectively. However, assuming the buses are initially parked at the sinks, authors in [92] have formulated the BEPP to maximize the number of rescued evacuees before an overall evacuation end-time. They also assume that the buses will perform round trips. In the BEPP formulation by [127], the problem seeks to minimize the total travel time of vehicles. Similar to that of [27], they too use a logistic mobilization curve to estimate the number of evacuees at the pickup location. In corporating such a logistic mobilization curve to estimate the number of evacuees at the pickup location, the author in [141] has proposed the shortest path formulation to minimize the total evacuation time for the multi-modal evacuation system considering the pedestrian and transit-based evacuees.

In most of the BEPP, the core objective is to minimize the duration of evacuation by routing and scheduling a fleet of homogeneous and capacitated buses which were initially located at one or more depots. Most often, the number of evacuees at each pickup locations can exceed the capacity of a single bus, which signify the necessity of split delivery. Moreover, the number of available buses is insufficient to transport all the evacuees without multiple trips and each shelter has a capacity that limits the number of evacuees it can serve. Such situations also request the split delivery. In such situations, the author in [17] proposed and analyzed the model for the multi-depot, multi-trip, BEPP, which identifies the optimal route construction and assignment of the vehicle for the optimal route assignment from a set of feasible routes. Unlike to this, a multi-trips, multi-vehicles, multi-periods, soft time windows with an split delivery strategy has been formulated in [89] as a multi-objective integer programming and is solved heuristically by the genetic algorithm followed by decomposition of the original problems.

Here, we recall one of the prominent BEPP, proposed by [17], briefly. It is one of the prominent BEPP formulation for the multi-depot, multi-trip transit-vehicle routing system in the evacuation network.

Consider a network $\mathscr{N} = (N, A)$ where N is the set of nodes and A is the set of arcs. N is composed of three subsets D, Y, and Z where D stands for a depot at which buses are initially located and dispatched from; Y, as a set of demand nodes representing pickup locations demanding the evacuation services; and Z, as a set of shelter (sinks) where the evacuees are to be transported. Let B be the available transit-vehicles, say buses, each having a capacity Q is subdivided into the subsets B_i for $i \in D$, and the bus is initially located at depot. Depot is only the initial location of the buses and does not play significant roles further on the evacuation process and the buses do not return to the depot after the completion of evacuation process. In fact, it could be risky to return to the depot under a threat. Moreover, such depots may not be the best places to store the buses during threats.

Let the demand node i has a demand α_i, $i \in Y$ and sinks j has a capacity β_j, $j \in Z$. Then the arc (i, j) has a non-negative travel cost of τ_{ij} for $(i, j) \in A$. Travel cost is proportional to the travel time or distance; though we mainly use the term time, as our aim is to route and schedule the buses to minimize the evacuation duration. All costs in the network are taken symmetric and are supposed to satisfy the triangle inequality, i.e., $\tau_{ij} = \tau_{ji}$, and $\tau_{ij} \leq \tau_{ik} + \tau_{kj}$ respectively for all arcs $(i,j), (i,k), (k,j) \in A$. Then for $\mathcal{T}_{\text{max}}$ be the duration of evacuation over all buses, then the BEPP formulation becomes,

$$\text{minimize } \mathcal{T}_{\text{max}}, \tag{3.1}$$

$$\text{such that } \mathcal{T}_{\text{max}} \geq \sum_{(i,j)\in A} \sum_{t=1}^{T} \tau_{ij} x_{ij}^{bt}, \ \forall b \in B, \tag{3.2}$$

$$\sum_{i:(i,j)\in A} x_{ij}^{bt} = \sum_{k:(j,k)\in A} x_{jk}^{b(t+1)}, \ \forall j \in Y, b \in B, t = 1, \ldots, T-1, \tag{3.3}$$

$$\sum_{i:(i,j)\in A} x_{ij}^{bt} \geq \sum_{k:(j,k)\in E} x_{jk}^{b(t+1)}, \ \forall j \in Z, b \in B, t = 1, \ldots, T-1, \tag{3.4}$$

$$\sum_{(i,j)\in A} x_{ij}^{bt} \leq 1, \ \forall b \in B, t = 1, \ldots, T, \tag{3.5}$$

$$x_{ij}^{b1} = 1, \ \forall i \in D, j : (i,j) \in A, b \in B_i, \tag{3.6}$$

$$x_{ij}^{bt} = 0, \ \forall i \in D, j : (i,j) \in A, b \in B, t = 2, \ldots, T, \tag{3.7}$$

$$x_{ij}^{bT} = 0, \ \forall j \in Y, i : (i,j) \in A, b \in B, \tag{3.8}$$

$$\eta_j^{bt} \leq \sum_{(i,j)\in A} Q x_{ij}^{bt}, \ \forall j \in N, b \in B, t = 1, \ldots, T, \tag{3.9}$$

$$0 \leq \sum_{j\in Y} \sum_{l=1}^{t} \eta_j^{bl} - \sum_{k\in Z} \sum_{l=1}^{t} \eta_k^{bl} \leq Q, \ \forall b \in B, t = 1, \ldots, T, \tag{3.10}$$

$$\sum_{b\in B} \sum_{t=1}^{T} \eta_j^{bt} \leq \beta_j, \ \forall j \in Z, \tag{3.11}$$

$$\sum_{b\in B} \sum_{t=1}^{T} \eta_j^{bt} = \alpha_i, \ \forall i \in Y, \tag{3.12}$$

$$\sum_{j\in Y} \sum_{t=1}^{T} \eta_j^{bt} = \sum_{k\in Z} \sum_{t=1}^{T} \eta_k^{bt}, \ \forall b \in B, \tag{3.13}$$

$$x_{ij}^{bt} \in \{0,1\}, \ \forall (i,j) \in A, b \in B, t = 1, \ldots, T, \tag{3.14}$$

$$\eta_j^{bt} \geq 0, \ \forall (i,j) \in A, b \in B, t = 1, \ldots, T. \tag{3.15}$$

Constraint (3.2) demands $\mathcal{T}_{\text{max}}$ be greater than or equal to the maximum cost incurred by the bus with the highest travel cost and is to be minimized by Constraint (3.1). Constraint (3.3) ensures that a bus traveling to demand node j on trip t leaves node j on trip $t+1$, the flow-balance constraint for the demand nodes. Constraint (3.4) ensures that the last trip of the bus may end at a shelter, the flow-balance constraint for the shelters. Constraint (3.5) allows a bus to make at most one trip at a time. Constraint (3.6) tells that the first trip of each bus starts from its depot and Constraint (3.7) tells that the buses do not leave the depot for later trips. Constraint (3.8) does not allow the last trip a bus can make to end at a demand node. Constraint (3.9) signifies that a bus can only pick up evacuees from node j, if it is traveling to that node where Constraints (3.10) and (3.11) are the bus capacity and the shelter capacity constraints respectively. Constraints (3.12) and (3.13) signify that all evacuees are picked up and are delivered to a shelter, respectively. Constraints (3.14) and (3.15) are the logical binary and non-negativity restrictions on the x and η variables, respectively.

Similar to the formulations as in [1, 3, 4], the formulations proposed by [17] also allow the split deliveries, intermediate nodes during their tours and visit different shelters multiple times to unload evacuees. In their two formulations, the min-max objective function of the first formulation is modified to a lexicographic equivalent that minimizes the travel time of every vehicle. The author has presented two different heuristic algorithms for the solution of such BEPP. The first one provides a feasible solution whether the movement of each bus is determined before the next bus starts. And, the second one relaxes to identify the preferred routes and are able to provide the near-optimal solution.

Based on the BEPP introduced by [17], authors in [55] has developed a modified version of BEPP as in Problem 3.1. It is for the evacuation of a region from a set of collection points to a set of capacitated sinks with the help of buses in minimum time assuming that the bus pick ups exactly the number of people that equals its capacity when visiting a source and hence, no need of split delivery services.

Problem 3.1. Let $(\tau_{ij})_{i\in Y, j\in Z}$ be a matrix of source-sink travel times, $D = \{d\}$ be a single bus depot, τ_{di} be a vector of depot-source travel times, $(\alpha_i)_{i\in Y}$ be a vector of evacuees number and $(\beta_j)_{j\in Z}$ be a vector of sink capacities. Then the BEPP is to find a tour plan to minimize the maximum travel times overall buses such that all the evacuees are transported to the sinks.

For this, it is assumed that the number of evacuees at every source is known in terms of the integral multiples of the busloads, which do not require any split service. Assume that every bus has a capacity of one unit and are initially at depot d. Moreover, the capacity of the sink is also in terms of busloads. The movement between a source to another source is ignored, and the same situation is considered between the sinks. It is assumed that, the set of tours of the buses cannot be changed anymore after they start to move. Let $\sum_{i\in Y}\alpha_i$ and $\sum_{j\in Z}\beta_j$ be the total number of evacuees and the total sink capacities, respectively. The maximum number of rounds R for the evacuation process is given by $\sum_{i\in Y}\alpha_i$. The non-negative travel cost of τ_{ij} on each edge $e=(i,j)\in A$ is taken symmetric and satisfies the triangle inequality.

The variables τ_{to}^{br} and τ_{back}^{br} give the travel time for the vehicle b within the round r from a source to a sink, and from that sink to another source, respectively. Let $\mathcal{T}_{\text{max}}$ be the duration of evacuation overall vehicles. The problem can be formulated as follows.

$$\text{minimize } \mathcal{T}_{\text{max}}, \tag{3.16}$$

$$\text{such that } \mathcal{T}_{\text{max}} \geq \sum_{i\in Y}\sum_{j\in Z}\tau_{di}x_{ij}^{b1} + \sum_{r\in R}\tau_{to}^{br} + \sum_{r\in R}\tau_{back}^{br},\ \forall b\in B, \tag{3.17}$$

$$\tau_{to}^{br} = \sum_{i\in Y}\sum_{j\in Z}\tau_{ij}x_{ij}^{br},\ \forall b\in B, r\in R, \tag{3.18}$$

$$\tau_{back}^{br} \geq \tau_{ij}\Big[\sum_{k\in Y}x_{kj}^{br} + \sum_{l\in Z}x_{il}^{b,r+1} - 1\Big],\ \forall b\in B, r\in R-1, \tag{3.19}$$

$$\sum_{i\in Y}\sum_{j\in Z}x_{ij}^{br} \geq \sum_{i\in Y}\sum_{j\in Z}x_{ij}^{b,r+1},\ \forall b\in B, r\in R-1, \tag{3.20}$$

$$\sum_{i\in Y}\sum_{j\in Z}x_{ij}^{br} \leq 1,\ \forall b\in B, r\in R-1, \tag{3.21}$$

$$\sum_{j\in Z}\sum_{b\in B}\sum_{r\in R}x_{ij}^{br} \geq \alpha_i,\ \forall i\in Y, \tag{3.22}$$

$$\sum_{i\in Y}\sum_{b\in B}\sum_{r\in R}x_{ij}^{br} \leq \beta_j,\ \forall j\in Z, \tag{3.23}$$

$$x_{ij}^{br} \in \{0,1\},\ \forall\ i\in Y,\ j\in Z,\ b\in B,\ r\in R, \tag{3.24}$$

$$\tau_{to}^{br}, \tau_{back}^{br} \in \mathbb{R}, \forall\ b\in B,\ r\in R, \tag{3.25}$$

$$\mathcal{T}_{\text{max}} \in \mathbb{R}. \tag{3.26}$$

Constraint (3.17) needs $\mathcal{T}_{\text{max}}$ to be greater than or equal to the maximal travel cost incurred by all buses, which is to be minimized on Constraint (3.16). Constraints (3.18) and (3.19) are the measure of travel time for the bus b within the round r from a source to a sink, and from that sink to the next source, respectively. Constraint (3.20) tells that the tours are connected and can stop whenever they like. Constraint (3.21) allows a bus from a source to sink per round. Constraints (3.22) and (3.23) are the bus capacity and shelter capacity constraints, respectively. Constraint (3.26) represents whether the bus b travels from source i to sink j in the round r.

Let τ_{ij}, τ_{di}, α_i and β_j for $i \in Y$ and $j \in Z$ be as in Problem 3.1. For $k \in \mathbb{R}$, is there a tourplan with $\mathcal{T}_{\text{max}} \leq k$, for the complete evacuation? Regarding the complexity of such a decision version of BEPP, the following result is established.

Theorem 3.1. [54] The decision version of BEPP is $\mathcal{NP}$-complete, even if $\tau_{di} = 0$ and $\tau_{ij} = \tau_{i'j}$ for all $i, i' \in Y$ and $j \in Z$.

Proof. The BEPP is reduced to the scheduling problem of scheduling n-jobs on P-parallel machines, which is $\mathcal{NP}$-hard, as mentioned in [54]. Such a scheduling problem with maximum completion time, $C_{\text{max}} \leq k$ for a given k has a yes-instance if and only if the respective bus routing plan of BEPP with $\mathcal{T}_{\text{max}} \leq k$ has a yes-instance. As both the completion time and the feasibility of the given solution can be checked polynomially, the decision version of BEPP is $\mathcal{NP}$-complete. □

Example 3.1. In this instance as in Figure 4, number of evacuees at the demand nodes are assumed as same as the vehicle capacity or its integral multiples with demands $\alpha_i = (3, 3, 1)$, capacities at the sinks be $\beta_j = (3, 4, 3)$ with buses available be 3. The distance of the demands from the depot be $d = (4, 3, 6)$ with the distances between Y to Z as

$$\tau = \begin{pmatrix} 4 & 7 & 9 \\ 10 & 7 & 6 \\ 7 & 6 & 9 \end{pmatrix}.$$

Table 3: Feasible solution of a bus-based evacuation planning problem.

Trip	1	2	3	Route assignment	Evacuation duration
Bus 1	(1,3)	(3,1)	-	$\tau_1, \tau_{13}, \tau_{33}, \tau_{31}$	25
Bus 2	(2,1)	(2,3)	-	$\tau_2, \tau_{21}, \tau_{21}, \tau_{23}$	21
Bus 3	(1,1)	(1,2)	(2,3)	$\tau_1, \tau_{11}, \tau_{11}, \tau_{12}, \tau_{22}, \tau_{23}$	25

Generally, the BEPP is formulated so as to choose the minimum of the evacuation cost of all possibilities where the critical path of the plan is for bus 3 or bus 1 as in Table 3, which shows its feasible solution with the evacuation cost of 25.

3.2.1 Solutions on BEPP

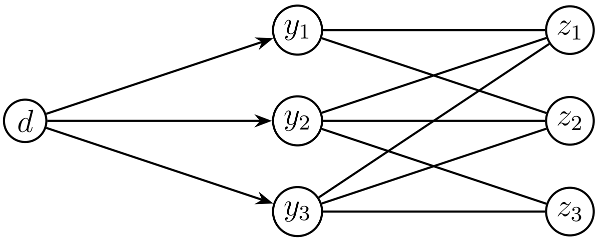


Figure 4: An instance of a bus-based evacuation planning problem.

During the solution of BEPP, authors in [55] have presented the branch and bound frameworks with four different upper bounds and three lower bounds for time, three branching rules to minimize the number of branches, and two tree reduction strategies to avoid the equivalent branches. Upper bounds on such BEPP are constructed in polynomial-time complexity by four different heuristic algorithms. Among them, the first three are based on a greedy distribution of tours on buses with precomputed tourlists, and the last one uses an iterative way without any precomputed tourlists.

Initially, a set of tours (i, j) for $i \in Y$ and $j \in Z$ is to be constructed for each source node i and $\alpha_i \geq 0$ from i to the nearest sink j with $\beta_j > 0$ resulting $\alpha_i \to \alpha_i - 1$ and $\beta_i \to \beta_j - 1$ such that α_i should vanish. One of the randomly chosen tours from the tourlists Π is assigned to a transit-vehicle with the minimum total travel cost. More precisely, as in Algorithm 1.

Algorithm 1: First upper bound on BEPP

```
Input  : An instance of BEPP with partial tourplan $x_b, b = i, \ldots, B$.
Let, $\Pi \leftarrow \phi$
for all $i \in Y$ do
  while $\alpha_i > 0$ do
    $j \leftarrow$ arg min$\{\tau_{ij'} : \beta_{j'} > 0, j' \in Z\}$
    $\Pi \leftarrow \Pi + (i, j)$
    $\alpha_i \leftarrow \alpha_i - 1$
    $\beta_i \leftarrow \beta_j - 1$
  end while
end for
  while $|\Pi| > 0$
    Choose a tour $(i, j)$ on fly from $\Pi$.
    Let $b \in B$ be a bus with minimum total travel time.
    $x_b \leftarrow x_b + (i, j)$
    Remove $(i, j)$ from $\Pi$.
  end while
Output: Feasible solution x.
```

In their second upper bound, a set of all possible tours (i, j) for $i \in Y$ and $j \in Z$ is to be constructed and is sorted on non-decreasing cost provided by $\alpha_i, \beta_j > 0$. Such a set of tours sorted until α or β vanishes. The initially available transit-vehicle is assigned to the tour with the highest cost, and then it continues the next expensive tour, and so on. It is based on the longest processing time first rule [110], as in Algorithm 2.

Algorithm 2: Second upper bound on BEPP

```
Input  : An instance of BEPP with partial tourplan $x_b, b = i, \ldots, B$.
Let, $\Pi \leftarrow \phi$
while $\alpha \neq 0$ do
    $(i, j) \leftarrow \arg\min\{\tau_{i'j'} : \alpha_{i'} > 0, \beta_{j'} > 0, i' \in Y, j' \in Z\}$
    times $\leftarrow \min(\alpha_i, \beta_j)$
    Add times tours $(i, j)$ to $\Pi$
    $\alpha_i \leftarrow \alpha_i -$ times
    $\beta_j \leftarrow \beta_j -$ times
end while
while $|\Pi| > 0$ do
    Choose the last tour $(i, j)$ from $\Pi$.
    Let $b \in B$ be a bus with minimum total travel time.
    $x_b \leftarrow x_b + (i, j)$
    Remove $(i, j)$ from $\Pi$.
end while
Output: Feasible solution x.
```

The third upper bound is obtained by a modification of the upper bound given by Algorithm 2 and needs no guarantee for the improvement of the solution. For the precomputed tourlist (i, j) for $i \in Y$ and $j \in Z$ having $\min\{d_{i'j'}\}$ with $\alpha_{i'} > 0, \beta_{j'} > 0$ for all i' and j' as in Algorithm 2, transit-vehicles are assigned by reversing the set of such tours for each vehicle. For the assignment of such vehicles, the long tour may have a long return tour, which might be beneficial. But, the vehicles having a long tour need not have a long return tour at the end, as designed in Algorithm 3.

Algorithm 3: Third upper bound on BEPP

```
Input  : An instance of BEPP with partial tourplan $x_b, b = i, \ldots, B$.
Run Algorithm 2 to get the feasible solution x.
Reverse the set of tours $\Pi$ in $x_b$ that have been added during the run of Algorithm 2
  for each $b \in B$
Output: Feasible solution x.
```

The fourth upper bound is based on iterative algorithm with out any precomputed tourlists. This begins with the best possibility to bring one evacuee back from the sink to the source and is continued iteratively. For this, let $\mathscr{Y}$ and $\mathscr{Z}$ be the available set of sources and sinks, respectively, provided for $\alpha_i > 0$ and $\beta_j > 0$. Let t_i^b be the distance of the current position of the vehicle to the source and offset_b be the distance of the vehicle $b \in B$, which is already planned. Initially, for all the vehicles in the depot, we get $t_i^b = \tau_i$ and $\mathit{offset}_b = 0$. In such a case, the best possibility to assign the vehicle is with a minimum possible value of the sum of offset_b, τ_i and τ_{ij}, which is given by $\min\{\tau_i + \tau_{ij}\}$, same as the minimum total travel cost. Updating the t_i^b for each $\alpha_i \to \alpha_i - 1$ and $\beta_j \to \beta_j - 1$ in an iterative procedure for the next assignment, and so on, the feasible solution is obtained in minimum possible time, as in Algorithm 4.

Algorithm 4: Fourth upper bound on BEPP

```
Input  : An instance of BEPP with partial tourplan $x_b$, $offset_b$, and
         $t_i^b, i = 1, 2, \ldots, Y, b = 1, 2, \ldots, B$.
Let, $\mathscr{Y} = \{i \in Y : \alpha_i > 0\}$
$\mathscr{Z} = \{j \in Z : \beta_j > 0\}$
while $|\mathscr{Y}| > 0$ do
    $(i', j', b') \leftarrow \arg\min\{offset_b + t_i^b + \tau_{ij} : j \in \mathscr{Z}, b \in B\}$
    $\alpha_{i'} \leftarrow \alpha_{i'} - 1$
    if $l_{i'} = 0$ then
        $\mathscr{Y} \leftarrow \mathscr{Y} \setminus i'$
    end if
    $\beta_{j'} \leftarrow \beta_{j'} - 1$
    if $\beta_{j'} = 0$ then
        $\mathscr{Z} \leftarrow \mathscr{Z} \setminus j'$
    end if
    $offset_{b'} \leftarrow offset_{b'} + t_{i'}^{b'} + \tau_{i'j'}$
    $t_i^{b'} = \tau_{ij'} \quad \forall i \in Y$
    $x_{b'} \leftarrow x_{b'} + (i', j')$
end while
Output: Feasible solution x.
```

All three lower bounds are also computed with polynomial-time complexity. The first lower bound is based on the estimation of the travel times from sources to sinks and from sinks to sources, respectively. The second lower bound is based on the fact that the lower bound for the maximum travel time is the average travel time. To address this, the objective is to minimize the sum of travel times and has been formulated by replacing the relations (3.16) and (3.17) by,

$$\text{minimize} \sum_{i\in Y}\sum_{j\in Z}\tau_{di}x_{ij}^{b1}+\sum_{b\in B}\sum_{r\in R}\tau_{to}^{br}+\sum_{b\in B}\sum_{r\in R}\tau_{back}^{br}. \tag{3.27}$$

The third lower bound is the simplification of model formulation, assuming that sinks are far away from the dangerous zone, and the pickup locations Y are nearby with negligible distances between such pickups. Consider all the pickups be at y_0 as the super pickup node with $\alpha_{y_0} = \sum_{i\in Y}\alpha_i$. Let the sinks $j \in Z$ and the depot d are at a distance of τ_j and τ_d respectively from y_0, where $\tau_j = \min_{i\in Y}\tau_{ij}$, for $(i, j) \in E$ and $\tau_d = \min_{i\in Y}\tau_i$. Here, τ_d is the same for all vehicles available and can be neglected. Let y_j^b be the number of tours for the vehicle b from y_0 to sink Z and z_j^b be the number of tours for the vehicle b from sink Z to y_0, then the model as in Equations (3.16-3.26), can be reformulated as:

$$\begin{aligned}
\text{minimize } \mathcal{T}\text{max}, & & & \qquad (3.28)\\
\text{such that } \mathcal{T}\text{max} &\geq& \sum_{j\in Z}\tau_j(y_j^b+z_j^b), & \qquad (3.29)\\
\sum_{b\in B}\sum_{j\in Z}y_j^b &\geq& l_{y_0}, & \qquad (3.30)\\
\sum_{b\in B}y_j^b &\leq& \beta_j,\ \forall j\in Z, & \qquad (3.31)\\
\sum_{b\in B}z_j^b &=& x_j^b-1,\ \forall j\in Z, & \qquad (3.32)\\
y_j^b, z_j^b &\in& \mathbb{N}, \forall b\in B, \quad j\in Z, & \qquad (3.33)\\
\mathcal{T}\text{max} &\in& \mathbb{R}. & \qquad (3.34)
\end{aligned}$$

By analyzing these four algorithms, (cf. Algorithm 1 to Algorithm 4) used to construct the feasible solutions on their upper bounds, we have proved Theorems 3.2 and 3.3, with respect to their dominating relations corresponding to the evacuation duration. Here, the dominance on these algorithms is followed with respect to the superiority of having minimum evacuation duration for their better performance in the network considered. A solution S_1 is said to dominate another solution S_2 if the solution S_1 is either no worse than S_2 or is strictly better than S_2 for the objective considered. Example 3.2 verifies the dominating relations of these algorithms.

Theorem 3.2. Algorithm 1 dominates Algorithms 2 and 3 in evacuation duration.

Proof. The shortest processing time first dispatching rule is superior over the longest processing time first dispatching rule in minimizing the total completion time criteria, [110]. Nevertheless, the longest processing time first dispatching rule balances the loads on the network and does not guarantee the optimality. Hence, Algorithm 1 dominates Algorithm 2 and Algorithm 3 with respect to the evacuation duration, where Algorithm 3 is a simple modification of Algorithm 2. □

Theorem 3.3. Algorithm 4 dominates Algorithms 2 and 3 in evacuation duration.

Proof. Algorithm 4 is initialized with $t_i^b = \tau_i$ and $\mathit{offset}_b = 0$ for all vehicles in the depot and gives the minimum initialization on the vehicle assignment with respect to the rest with $\min\{\tau_i + \tau_{ij}\}$, i.e. the same as the minimum total travel cost as in Algorithm 1. Routes are considered iteratively with the best possibility to have the link (j, i) for each route (i, j) considered for the better choice of minimum evacuation time for their to-distance and back-distance than that for such distances with respect to Algorithm 2 and Algorithm 3. The updated t_i^b in each iteration with $\alpha_i \to \alpha_i - 1$ and $\beta_j \to \beta_j - 1$ minimizes the route assignment to have a feasible solution in minimum time. Hence, Algorithm 4 dominates Algorithm 2 and Algorithm 3 both.

□

Example 3.2. Let the pickup locations Y be at a distance $\tau_{di} = \begin{bmatrix} 1 & 3 \end{bmatrix}$ from d, and the transit times to the sinks Z from Y be $\tau_{ij} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. Consider a scenario with the demands at Y as $\alpha_i = (2, 1)$ and capacity of Z as $\beta_j = (2, 3)$, respectively.

For the given data in Figure 5, we construct the tourplans Π using Algorithm 1, Algorithm 2, Algorithm 3, Algorithm 4, and calculate the corresponding evacuation duration

(ED) as represented in the second, third, fourth, and fifth columns of Table 4, respectively. It shows that Algorithm 1 dominates Algorithm 2 and Algorithm 3 (as proved in Theorem 3.2). Similar result holds in case of Algorithm 4 as well (cf. Theorem 3.3).

This also helps to estimate the threshold number of the transit-vehicles. Here, the threshold number for $|B|$ is 3 as for $|B| = 4$, the fourth bus is left with no tour.

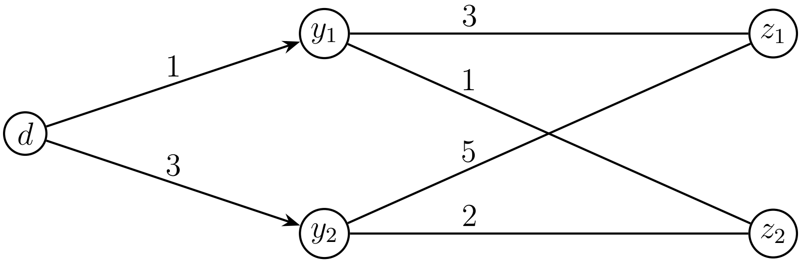


Figure 5: An instance of bus assignment problem.

Table 4: Tourplans with evacuation duration corresponding to different algorithms.

$\lvert B\rvert$	A1: Π & ED	A2: Π & ED	A3: Π & ED	A4: Π & ED
1	$\tau_1 + 3\tau_{12} + 2\tau_{22} = 8$	$\tau_2 + \tau_{21} + 3\tau_{11}$ $+\tau_{12} = 18$	$\tau_1 + 2\tau_{12} + \tau_{11}$ $+2\tau_{21} = 16$	$\tau_1 + 3\tau_{12}$ $+2\tau_{22} = 8$
2	$\tau_1 + \tau_{12} + 2\tau_{22} = 6$	$\tau_2 + \tau_{21}$	$\tau_2 + \tau_{21} = 8$	$\tau_1 + \tau_{12} + 2\tau_{22} = 6$
	$\tau_1 + \tau_{12}$	$\tau_1 + 2\tau_{11} + \tau_{12} = 8$	$\tau_1 + 2\tau_{12} + \tau_{11}$	$\tau_1 + \tau_{12}$
3	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{21} = 8$	$\tau_2 + \tau_{21} = 8$	$\tau_1 + \tau_{12}$
	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{12}$
	$\tau_2 + \tau_{22} = 5$	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{22} = 5$
4	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{21} = 8$	$\tau_2 + \tau_{21} = 8$	$\tau_1 + \tau_{12}$
	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{12}$
	$\tau_2 + \tau_{22} = 5$	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{22} = 5$
	ϕ	ϕ	ϕ	ϕ

Note that, Algorithm 1 construed with precomputed tourlists and assigned to the closest sink approach, and Algorithm 4 constructed iteratively without any precomputed tour lists and assigned as above, are very closed to each other and are dominating the rest. However, for the vehicle assignment in $\mathscr{N}_2$ in an embedded network (cf. Section 4.1), we prefer Algorithm 4 as it does not require precomputed tourlists.

By assuming that the number of evacuees is not known exactly, the BEPP is extended to robust bus-based evacuation planning problem (RBEPP) in [54]. Though the exact number of evacuees are not known in advance, a set of likely scenarios is known and after sometime such uncertainty will be removed. In such instance, it is to decide whether the buses are better to send right now as the here-and-now bus under such uncertainty or to wait as wait-and-see bus until the exact scenario becomes known.

Regarding to the problem complexity of the decision version of such problem, they have state and established the Theorem 3.4 based on Theorem 3.1 as,

Theorem 3.4. [54] The decision version of RBEPP is $\mathcal{NP}$-complete.

3.2.2 BEPP in a diminished evacuation network

Consider a BEPP for a diminished evacuation network close to the real scenario which is equivalent to the third lower bound, [55] as about the simplification of model formulation. Here, sinks are far away from the dangerous zone. The bus depot is in a closed environment to the disaster zone with B be the set of available buses. So, let the super pickup node with the bus depot be y_0 with the evacuees ξ_{y_0}. Assume the capacity of each bus be 1. The movement between Z is ignored. The set of tours of the buses cannot be changed anymore after they start to move and are connected. The maximum number of trips for the evacuation process is given by ξ_{y_0}. The non-negative travel cost of τ_{0j} on each edge $e = (0, j) \in E$ are taken symmetric. Let the sinks $z \in Z$ be at a distance of τ_{0j} from Y_0, where y_j^b denotes a tour that the bus b drives from the source to sink $j \in Z$ and back to the source. Let $\mathcal{T}_{\max}$ be the duration of evacuation overall buses, then the problem becomes,

$$\begin{aligned}
\text{minimize } \mathcal{T}_{\max}&, && (3.35)\\
\text{such that } \mathcal{T}_{\max} &\geq \sum_{j \in Z} \tau_{0j} y_j^b \;\; \forall\, b \in B, && (3.36)\\
\sum_{b \in B} \sum_{j \in Z} y_j^b &\geq \xi_{y_0}, && (3.37)\\
\sum_{b \in B} y_j^b &\leq \mu_j, \forall j \in Z, && (3.38)\\
y_j^b &\in \{0, 1\}, b \in B, j \in Z. && (3.39)
\end{aligned}$$

Constraint (3.36) requires $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all buses and is to be minimized on Constraint (3.35). Constraints (3.37) and (3.38) are the bus and shelter capacity constraints, respectively, which ensure that all evacuees are transported and shelter capacities are respected. Constraint (3.39) represents whether the bus b travels from source to sink j (or travels back from the sink j to the source).

For the solution status and the proof of its decision version, similar to Theorem 3.1 as mentioned earlier, we have,

Theorem 3.5. The decision version of the BEPP in a diminished evacuation network is $\mathcal{NP}$-complete.

As mentioned above in Algorithm 4, the assignment of transit-vehicles begins with the best possibility to bring one evacuee back from the sink to the source and is continued iteratively, respecting the sink capacity constraints. It dominates the rest in evacuation duration and is equivalent to the nearest sink approach as in [9]. So, we prefer it as the dominating vehicle assignment for the network as in Example 3.3.

Observation 3.1. Let ξ_{y_0} and $\beta(z_i)$ be the number of evacuees at the super pickup node Y_0 and the sink capacities, respectively. Consider $\tau_k = \min\{\tau_{01}, \tau_{02}, \ldots, \tau_{0n}\}$ as the nearest sink z_k in the network. Let the extended sink capacity to the nearest sink be $\beta(z_k) \geq \sum_{j=1}^{n} \beta(z_i)$. Then such evacuation network $s - Z$ be reduced to an $s - z$ network with the minimum possible evacuation duration. For this, the estimated evacuation duration becomes $(2\xi_{y_0} - 1)\tau_k$.

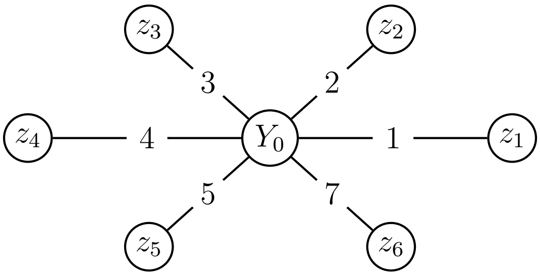


Figure 6: An instance of a bus-based evacuation problem in a diminished network.

Example 3.3. Consider a diminished evacuation network as in Figure 6. Let $\xi_{y_0} = 26$ with $\beta(z_i) = \{7, 6, 5, 4, 3, 2\}$ for $i = \{1, 2, \ldots, 6\}$. Their respective transit times τ_{0j} from Y_0 are shown along with the figure.

Then the tourplan for $|B| = 1$ is given by $14\tau_{01} + 12\tau_{02} + 10\tau_{03} + 8\tau_{04} + 6\tau_{05} + \tau_{06}$ with $\mathcal{T}_{\max} = 137$. But if $|B| \geq 26$, then $\mathcal{T}_{\max} = 7$. For $1 < |B| < 26$, consider $|B| = 13$, as an arbitrary. Then the tourplan becomes $2\tau_{01}+2\tau_{02}$, $2\tau_{01}+2\tau_{04}$, $2\tau_{02}+2\tau_{04}$, $2\tau_{02}+2\tau_{05}$, and $2\tau_{02}+\tau_{07}$ for the buses $\{B_1, B_2, B_3, B_4, B_5\}$, $\{B_6, B_7\}$, $\{B_8, B_9\}$, $\{B_{10}, B_{11}, B_{12}\}$, and $\{B_{13}\}$, respectively. For this, the effective $\mathcal{T}_{\max} = 14$.

If this $s - Z$ network is replaced by an $s - z$ network having sufficient sink capacity at z_1 as requested by the given demand, then the tour plan for $|B| = 1$ be reduced to $51\tau_{01}$ with $\mathcal{T}_{\max} = 51$. But for $|B| = 13$, the respective $\mathcal{T}_{\max} = 3$. However, for $|B| \geq 26$, it becomes 1. This is why a single sink having sufficient capacity is more appropriate for the transit-based EPP.

3.3 Evacuation Duration on Different Scenarios

The structure of the evacuation network adds the complexity to the solutions and also have a close impact to the evacuation duration. There is no an evacuation strategy that can be considered as the best and universally applicable in each and every situation. The performance of the strategies depends not only on the nature of the road structures but also on the appropriate route-shelter selections and on the route-to-vehicle assignment and vice versa, which is more complex in practice. Here, we consider a few evacuation scenarios on different types of simple evacuation structures with their impacts to evacuation duration.

3.3.1 Evacuation duration on edge-disjoint parallel network

Let s and z denote the source and sink corrseponding to an evacuation region, respectively. For u and τ to denote the capacity and transit times of the path from s to z, then the capacity of path is the minimum capacity of the edges in such a path and the travel time of the path becomes the sum of all the travel times of the edges in the path considered. Let $\alpha(s)$ be the number of evacuees at the source s, then the ED is calculated by [97] as,

$$ED = \tau + \lceil \frac{\alpha(s)}{u} \rceil - 1. \tag{3.40}$$

Here, $\lceil . \rceil$ denotes the upper ceiling function. Consider $\pi_1, \pi_2, \ldots, \pi_k$ be the k edge-disjoint paths from the source s to the sink z with u_i and τ_i be the capacity and transit times of paths π_i and $\alpha(s)$ be the number of evacuees at s. Among such k edge-disjoint paths, the path π_i is said to be the quickest path if and only if

$$\tau_i + \lceil \frac{\alpha(s)}{u_i} \rceil - 1 \leq \tau_j + \lceil \frac{\alpha(s)}{u_j} \rceil - 1, \; \forall j \in \{1, 2, \ldots, k\} \backslash i. \tag{3.41}$$

For the k edge-disjoint paths on such $s - z$ parallel network, the combined evacuation duration (CED) is given by

$$CED(\pi_1, \pi_2, \ldots, \pi_k) = \left\lceil \frac{\alpha(s) + \sum_{i=1}^{k} u_i \tau_i}{\sum_{i=1}^{k} u_i} \right\rceil - 1. \tag{3.42}$$

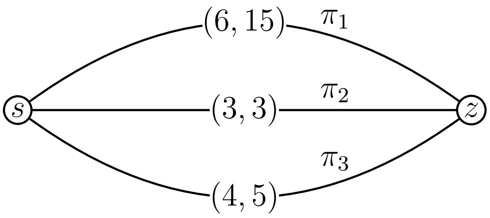


Figure 7: An $s - z$ network with 3 edge-disjoints paths.

Example 3.4. Consider three edge-disjoint paths π_1, π_2, and π_3 in between s and z with (u, τ) as their respective capacity and travel time, respectively as in Figure 7. Let the number of evacuees at s be 52.

Then by using Equation (3.40), the ED through these paths can be calculated as 23, 20, and 17 respectively. Here, π_3 becomes the quickest path considered. If the next path π_1 is added on the evacuation system, then the CED as in Equation (3.42) becomes $CED(\pi_3, \pi_1) = 16$. Here, $CED(\pi_3, \pi_1) < ET(\pi_1)$. But if the path π_2 also added in the evacuation scenario, then the CED becomes $CED(\pi_1, \pi_2, \pi_3) = 13$, further improved. In case if the route with longest travel time is removed from the system, then we get $CED(\pi_3, \pi_2) = 11$. Here, we get, $CED(\pi_2, \pi_3) = 11 < CED(\pi_1, \pi_2, \pi_3)$ and the evacuation time be further reduced.

Note that, the purpose of adding paths into the evacuation route is to reduce the ED by distributing the evacuees in multiple paths and will be terminated when the CED by the current quickest path becomes greater then the previous CED. Running time is determined by the number of iteration which is bounded by the total number of paths in such evacuation network and does not depend much on the number of evacuees, [97].

3.3.2 Split delivery on evacuation system

During evacuation system, the number of evacuees at the demand node might be greater than the capacity of a transit-bus, and it demands the split delivery service within the pickup locations. For example, in the BEPP, presented by Bish in [17] such a scenario exists but in the BEPP formulation by [55] the number of evacuees at the demand node are considered as the integral multiple of the busloads and the evacuation system does not request for the split service. However, the split delivery service does not improve the solution, as illustrated in Figure 8 corresponding to Example 3.5, based on [46].

Example 3.5. Consider an evacuation network in Figure 8, where d, $\{y_1, y_2, y_3\}$, and z are the bus depot, pickup locations, and the sink, respectively. Consider $|B|$ buses are located at d with a homogeneous capacity of 50 evacuees. Let the demands at pickup locations y_1, y_2, and y_3 be 30, 40, and 30, respectively. Consider the pickup locations are at equal distance, τ each, from the depot and are at 5τ from the sink. Here, ϵ is used to denote that the pickup locations are sufficiently close to each other and connected by edges. Consider the sink capacity to be 100.

The tourplan Π and the respective ED without and with split delivery, and their respective bounds are shown in the second, third, and fourth columns of Table 5, where the buses were scheduled simultaneously. Here, the bound denotes the corresponding ratio of the ED obtained in column 2 to that in column 3, for the network considered. The bound greater than 1 indicates that the split delivery service improves the solution. During their route assignments, sometimes the split delivery service is also appropriate though it may not always improve the ED. Here for $|B|$ equals 3 and 4, there is no improvement in the solution by applying the split delivery service.

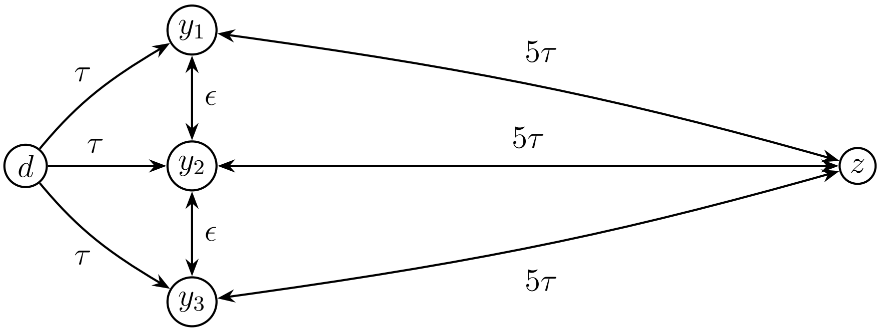


Figure 8: An instance of split delivery network.

Table 5: The split delivery does not always improve the solution.

$\|B\|$	Π and ED without split delivery	Π and ED with split delivery	Bound
1	$d \to y_1 \to z \to y_2 \to z \to y_3 \to z$ $= 26\tau$	$d \to y_1 \to y_2 \to z \to y_2 \to y_3 \to z$ $= 16\tau + 2\epsilon$	1.625
2	$d \to y_1 \to z \to y_2 \to z = 16\tau$	$d \to y_1 \to y_2 \to z = 6\tau + \epsilon$	2.67
	$d \to y_3 \to z$	$d \to y_3 \to y_1 \to z$	
3	$d \to y_1 \to z = 6\tau$	$d \to y_1 \to z = 6\tau$	1
	$d \to y_2 \to z$	$d \to y_2 \to z$	
	$d \to y_3 \to z$	$d \to y_3 \to z$	
4	$d \to y_1 \to z = 6\tau$	$d \to y_1 \to z = 6\tau$	1
	$d \to y_2 \to z$	$d \to y_2 \to z$	
	$d \to y_3 \to z$	$d \to y_3 \to z$	
	ϕ	ϕ	

3.3.3 Route selection on evacuation system

Among all feasible routes on the evacuation network, a route π_1 is said to dominate another route π_2, and is named as the dominant route, if it does not have longer evacuation duration, it does not have the more cost for the demand nodes considered and every unreachable demand nodes on route π_1 is also unreachable for

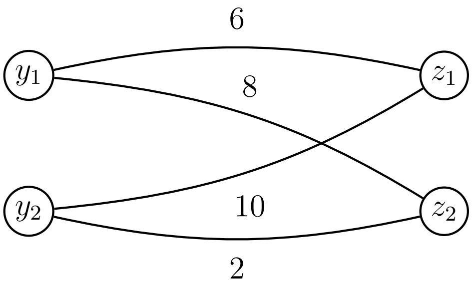


Figure 9: A network with uncapacitated sinks.

π_2. If different routes lead to the same sink and neither of them is better than the others over all criteria, then neither of the route be dominating. There are different factors affecting for the selection of such dominant routes and mainly depends upon nature of the available network. In general, it seems that sending each vehicles to its closest sink to last pickup node is the appropriate route to be assigned for the optimal routing. But, it is not always. Let us consider, an instance,

Example 3.6. Consider an instance as in Figure 9. Let a bus which has picked up a full load of evacuees at y_1 can still pick up a full load at y_2 with uncapacitated sinks z_1 and z_2. Then, if the sink closest to its last pickup node is assigned, then the bus would be on the route $\pi_1 = y_1 \rightarrow z_1 \rightarrow y_2 \rightarrow z_2$ and will have evacuation duration of 18, whereas the optimal solution would be on the route $\pi_2 = y_1 \rightarrow z_2 \rightarrow y_2 \rightarrow z_2$ with evacuation duration of 12.

The solution of a problem is effected not only by its network structure but also by the number of vehicles available in such a scenarios and on many other constraints. The problem is not only on the selection of a route and the selection of a sink for each route but also on the route-to-vehicle assignment and vice versa, which is more complex in practice. Furthermore, the network adds complexity to the solution and impacts on the problem. It needs to modify and the extension on the available model to make it more realistic and applicable to the real life problems.

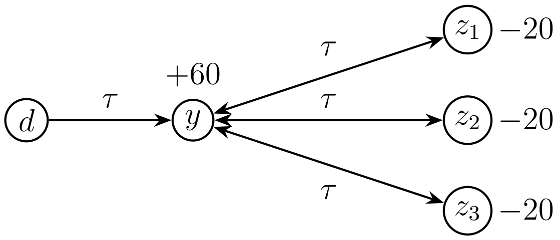


Figure 10: A network with capacitated sinks.

Example 3.7. Consider the fixed demand at the pickup location y be 60, with the vehicles having homogeneous capacity of 40 and the sinks are with capacity 20 each, as the capacitated sinks represented in Figure 10. Then the evacuation duration for different vehicle assignment scenarios including the routeplan Π can be obtained as in Table 6. It indicates that, more vehicles seem to be beneficial in considering the evacuation duration to some extent.

Table 6: Routeplan and ED on the network with capacitated sinks.

|B|	Route plan of bus routing (Π)	ED
1	$d \to y \to z_1 \to y \to z_2 \to y \to z_3$	$6\,\tau$
2	(i) $d \to y \to z_1$, (ii) $d \to y \to z_2 \to y \to z_3$	$4\,\tau$
3	(i) $d \to y \to z_1$, (ii) $d \to y \to z_2$, and (iii) $d \to y \to z_3$	$2\,\tau$
4	(i) $d \to y \to z_1$, (ii) $d \to y \to z_2$, (iii) $d \to y \to z_3$, and (iv) ϕ	$2\,\tau$

Example 3.8. Similar to above example as in Example 3.7, let a bus which has picked up a full load of evacuees at either of y_i can still pick up a full load at y_j with uncapacitated sinks d, as well as the bus depot. Consider the evacuation scenario with one, two, three, and four buses, respectively in the evacuation network as in Figure 11, (i), (ii), (iii), and (iv), respectively . Let the buses start from the depot d assigned to the demand locations y_i, and finally return to the depot. Let the network be symmetric, then the time required for the vehicle routing be denoted by $d(d, y_i) = 2\tau$ for $i = 1, 2, 3, 4$ and $d(y_1, y_2) = d(y_4, y_3) = 2\tau$. Consider, $d(y_4, y_1) = 3\tau$.

The number of buses |B| assigned on the evacuation scenarion and the respective route plan and the corresponding evacuation duration are also shown in different columns of Table 7. Here, the evacuation duration does not always decrease in a convex manner with the number of buses assigned. There is an optimal threshold fleet size corresponding to evacuation. And, increasing the fleet size beyond this threshold does not impact the optimality.

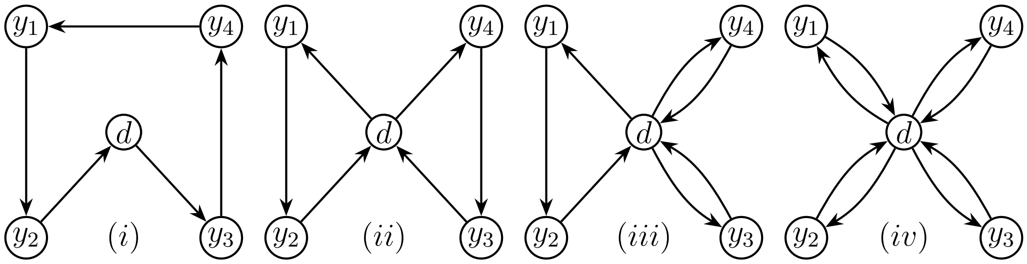


Figure 11: Evacuation scenarios corresponding to different fleet size.

Table 7: Routeplan and ED corresponding to different fleet size.

lBl	Routeplan of the vehicle routing	ED
1	$d \to y_3 \to y_4 \to y_1 \to y_2 \to d$	$11\,\tau$
2	$(i)\, d \to y_1 \to y_2 \to d, (ii)\, d \to y_4 \to y_3 \to d$	$6\,\tau$
3	$(i)\, d \to y_1 \to y_2 \to d, (ii)\, d \to y_4 \to d$, and $(iii)\, d \to y_3 \to d$	6τ
4	$(i)\, d \to y_1 \to d, (ii)\, d \to y_2 \to d, (iii)\, d \to y_3 \to d$, and $(iv)\, d \to y_4 \to d$	4τ
5	$(i)\, d \to y_1 \to d, (ii)\, d \to y_2 \to d, (iii)\, d \to y_3 \to d, (iv)\, d \to y_4 \to d$, and ϕ	4τ

3.4 Factors Affecting on Evacuation Optimization

Evacuation planning strategies, and their operations may vary due to their applicable geographical scales, total affected population size, transportation resources and traffic capacity, evacuation objectives, and the time spans. Evacuation scheduling, location-allocations, traffic route guidance, destination optimization, network structures, and the optimal route choice are some of the prominent factors affecting on evacuation optimization. Here, we revisit two of these factors as the location-allocations and the network structures, briefly.

3.4.1 Location-allocations of resources

The effectiveness of transit-based evacuation system highly depends on the location of pickup facilities, allocation of the resources and evacuation management system. Different mathematical models were developed to address such transit-based evacuation under both predictable and unpredictable disasters and are integrated to the location-allocation design with the routing, assignment and scheduling of the buses on evacuation network. In a real scenario, the evacuee walking time to various pickups, their waiting and loading time at pickups, service delays on the system etc. are also to take account which is rarely been considered in existing literature. Comparatively, a reliable location model has been developed in [11], which integrates the pre-and post-emergency operations in an exponential number of facility disruption scenarios including their interconnection to pickup location, service facility and the vehicle assignment to determine the optimal evacuation. However, the assumptions like constant rescue demand with identical and independent facility disruptions may not always be realistic. The pickup facilities might also be disrupted in real practice. Hence, during designing of the evacuation system, the care should be taken not only on the efficiency but also on the reliability and efficacy.

A location-allocation model for EPPs has been proposed in [83] with a bi-level programming formulation. In their formulations, the locations of shelters are chosen among a given set as an upper level whereas the lower level is to determine the allocation of sources to sinks and the route assignment of vehicles to the corresponding sinks. During disasters some of the facility locations like ambulances or fire stations are for emergency facilities. During large scale disasters some of such facilities may be damaged or might be inoperable. For such situations, a variation of the p-center problem has been proposed, [71].

Dealing with the household behavior under the emergency evacuation scenarios, authors in [99] have provided two formulations to determine the meeting location for household members and their sequence pickup. Vehicles are distributed according to the location of their drivers and are taken heterogeneous depending upon their capacity and is convenient in an emergency. Authors in [142] have proposed a model to determine the pickup locations within several clusters of demand zones for the routing and scheduling of transit-vehicles based on vehicle availability and the time-dependent evacuee demand pattern. By suggesting the equilibrium of the evacuee arrival process and the functioning pickup facility, authors in [11] have presented the optimal resource allocation strategy to balance the trade-off between evacuees' risks and the operation costs, and about its dynamic nature, [64].

In general, minimizing the evacuation time for all pick up locations is desired during evacuation. But if a city is threatened by wildfire then the neighborhoods close to the wildfire are supposed to be evacuated before the ones far away where the minimum network clearance time resource allocation is not the right choice.

3.4.2 Network structures

The network structure adds complexity to different solutions and impacts on the flow maximization and/or time minimization EPPs. Most of the traffic delays and the potential accidents are due to the merging and crossing conflicts at the intersections of the evacuation network. To address this, comparatively a smart traffic routing without crossing and merging conflicts has been proposed by [30] and is further improved by many others like [21, 22, 137], following the assumption that, vehicles have to order in the appropriate lanes that correspond to their subsequent turn before they enter the intersection and the restructuring of traffic routing with regard to a safe evacuation process to minimize the evacuation time. Likewise, the effectiveness of lane based system even in the damage traffic sensors and interrupted communication system has been presented by [13]. Different heuristic approaches have been suggested in [80, 82] to solve the problem of minimization of evacuation egress time with time-dependent node and arc capacity by lane reversal approach. The demand management strategies of staging and routing have been presented in [18] incorporating to some extent for the evacuee behavior aspects. In their model, the objectives are to route the vehicles to their closest evacuation zone exit and to minimize the number of intersection merging-conflicts, which also satisfy the shortest evacuation plan criterion given by [139].

Lane-based reversal design and routing with intersection crossing conflicts elimination for the evacuation has been integrated in a bi-level model in [143] by applying a tabu search algorithm. It helps to find an optimal lane reversal plan in upper-level and the simulated annealing algorithm on the lower level consisting of single arc and multiple arcs. Such a network optimization model with the bi-level scheme has been formulated in [91] with the upper level determining the best sets of signalized and uninterrupted flow at intersections and the lower-level handling routing assignment of evacuation traffic demand. The upper level describes the behavior of the policy makers for minimizing the total evacuation cost whereas, the lower-level problem captures the behavior of evacuees in choosing the evacuation routes under the budget constraints. In fact, such information is critical for emergency managers to allocate the limited resources to the most appropriate location and the mass transit VRP can been solved iteratively between two levels of problems as the transit problem and the passenger problem as in [106].

With the simultaneous consideration of the lane reversal on roadway sections and crossing elimination at the intersections, the authors in [137] have presented comparatively the effective way to use existing network capacity by identifying the emergency vehicle routes. In their approach, they have considered the reconfiguration of the network for evacuees to satisfy the multiple objectives for emergency management. However, an integrated optimal evacuation approach to have a single comprehensive solution to the problem is always challenging and somewhat lacking for real case scenarios as the evacuation network topology adds complexity to the solution and impacts on the problem.

3.5 Integreted Approach on Evacuation

Here, we consider the integraed approach on evacuation planning strategy corresponding to the embedding of different network structures.

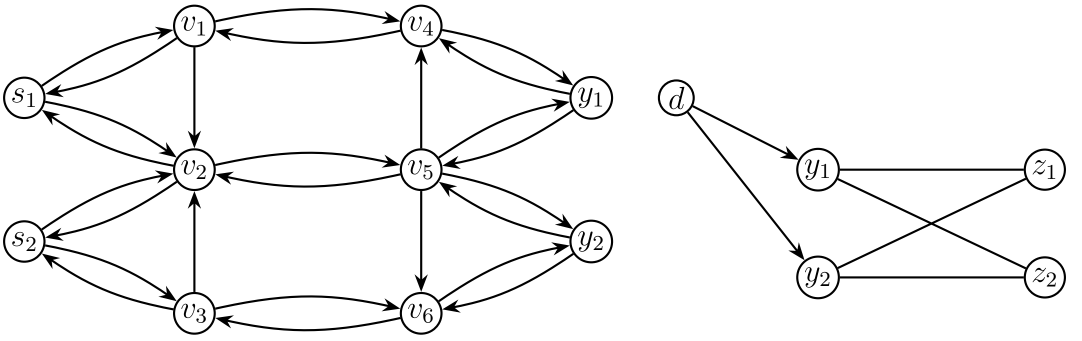
3.5.1 Embedded evacuation network

Consider two separate dynamic networks $\mathscr{N}_1$ and $\mathscr{N}_2$, as a primary and a secondary sub-network. These two are embedded as a unit to form an embedded evacuation network $\mathscr{N}^E$, i.e., $\mathscr{N}^E = \mathscr{N}_1 \cup \mathscr{N}_2$. Among them, $\mathscr{N}_1$ is a directed two-way network and $\mathscr{N}_2$ is the mixed network having directed one-way arcs and undirected edges.

The primary sub-network. Let $\mathscr{N}_1 = (S, V, A, u_a, \tau_a, Y)$ be a primary sub-network, where $S = \{s_1, s_2, \dots, s_n\}$, $V = \{v_1, v_2, \dots, v_n\}$, and $Y = \{y_1, y_2, \dots, y_n\}$ denote the sets of sources, auxiliary nodes, and pickup locations, respectively. The set of arcs joining any two nodes in $\mathscr{N}_1$ are denoted by A where the capacity and transit time are denoted by u_a and τ_a, respectively. The capacity $u_a : A \to \mathbb{Z}_{\geq 0}$ restricts the amount of flow on the arc and the transit time $\tau_a : A \to \mathbb{Z}_{\geq 0}$ represents the time required for the flow to transverse through the respective arc. During evacuee arrival at the pickup locations Y from S, the set of pickup locations Y is the set of sinks.

The secondary sub-network Let $\mathscr{N}_2 = (d, Y, E, \tau_e, Z)$ be a secondary sub-network, where d and $Z = \{z_1, z_2, \dots, z_n\}$ are the bus depot and sink, respectively. A set of transit-buses $B = \{b_1, b_2, \dots, b_n\}$ with uniform capacity are located initially at the bus depot d and are assigned as required during the evacuation procedure. Buses do not return to d even after the completion of the evacuation process as it is risky to return to it under such threats. So, it does not play significant roles further in the system. The set E consists of the one-way arcs $e = (d, y)$ with $y \in Y$ and the undirected edges $e = [y, z]$ with $y \in Y, z \in Z$. Transit times of the respective arcs and edges are denoted by $\tau_e \in \mathbb{Z}_+$ as τ_{di} and τ_{ij} respectively.

An embedding of the integrated network An embedding of the integrated evacuation network is formed as, $\mathscr{N}^E = \mathscr{N}_1 \cup \mathscr{N}_2$. Capacity, transit time, and other related attributes for $\mathscr{N}^E$ are carried over from their respective sub-networks. The node Y works as the sinks concerning $\mathscr{N}_1$ but as the supply for $\mathscr{N}_2$. Evacuees are collected at Y in $\mathscr{N}_1$, the transit-buses are assigned in $\mathscr{N}_2$ and will be continued till the supply is available at Y respecting the capacity of the sinks Z.



(i) A primary sub-network $\mathscr{N}_1$.   (ii) A secondary sub-network $\mathscr{N}_2$.

Figure 12: Sub-networks for evacuee arrival and vehicle assignment.

3.5.2 Arrivals of the evacuees in an embedding

During the evacuation procedure, the collection of evacuees at pickup locations can be categorized to follow different arrival patterns like constant arrival rate or a random variable, probabilistic, deterministic or stochastic, time-dependent or flow-dependent.

A set of appropriate pickup locations among different number of demand points can be determined by constructing the Thessian polygon with different clusters within the region. The number of evacuees on such a pickup location at a specified time can be estimated by $\xi = \sigma_1 \times \sigma_2$ where σ_1 gives the population density of the cluster around the pickup locations and σ_2 denotes the percentage of transit-dependent people in it.

Two of the prominent BEPP formulations as in [17] and [55] do not take account of any specific arrival patterns of the evacuees. Evacuees are assumed to be at the pickup locations. However, they take into account whether the number of evacuees is the integral multiples of the busloads or not. Such a deterministic type of the problem BEPP is extended to an RBEPP by assuming that the number of evacuees is not known exactly but a set of estimates for the number of evacuees at each source is given in [54]. Evacuees have gathered themselves at different pickup locations relative to the population density of the transit-dependent people nearby them in [117] with no specific arrival patterns.

A constant arrival rate of evacuees at the predetermined pickup locations has been considered by the authors in [107]. The constant arrival rate of the evacuees at pickup node Y is denoted by ρ_i. The last scheduled pickup time and the end of arrival process by L_i. The f^{th} pickup is scheduled for node i at time p_i^f. Then the time interval between the schedule pickup times at pickup node i becomes, $q_i^f = p_i^f - p_i^{f-1}$. Let F denotes the number of pickups allowed for the maximum service level. Let each arc $(i, j) \in A$ has an associated traversal time τ_{ij}. Then, assuming the evacuation to begin in zero time the minimum exposure schedule for a given parameter F taken to minimize the total exposure,

$$\text{minimize} \sum_{i \in Y} \frac{\rho_i}{2} \sum_{i=1}^{F} (q_i^f)^2, \tag{3.43}$$

$$\text{subject to } \sum_{i=1}^{F} q_i^f = L_i, \forall i \in Y, \tag{3.44}$$

$$q_i^1 \geq \tau_{0i}, \forall i \in Y, \tag{3.45}$$

$$q_i^f \geq 0, \forall i \in Y, f = 1, \ldots, F. \tag{3.46}$$

In fact, such assumption of constant arrival rate is still unrealistic as the true arrival process is probabilistic and will likely vary over time.

The cumulative percentage of total evacuees loaded in the evacuation network by time θ since the start of evacuation can be estimated via a logistic mobilization curve as in [72] by,

$$\xi(\theta) = \frac{1}{1 + exp[-\alpha(\theta - h)]}. \tag{3.47}$$

The parameter α, loading rate of evacuees, represents the response of the public to the disaster. The parameter h is the half loading time, represented by the mid-position of such curve. Lower loading rate are on the early stage of evacuation process. Let $\xi^i(\theta_k)$ and $\xi^i(\theta_{k-1})$ are the cumulative percentage of the evacuees arrival at the pickup location i at the end of time interval k and $k-1$, respectively. Let δ_i be the number of evacuees arriving at pickup location i during evacuation time frame and θ_k is the end of the evacuation time interval k; then the number of evacuees arriving at pickup location i during time interval k can be expressed as,

$$\delta_i^k = [\xi^i(\theta_k) - \xi^i(\theta_{k-1}] \times \delta_i. \tag{3.48}$$

For d_i^k be the number of demand of vehicles at pickup location i during the time interval k and Q be the maximal passenger capacity of the vehicle; then we get,

$$\delta_i^k = d_i^k \times Q. \tag{3.49}$$

Hence, the number of evacuees arriving at pickup location i during the evacuation time frame be,

$$\delta_i = \frac{d_i^k \times Q}{\xi^i(\theta_k) - \xi^i(\theta_{k-1})}. \tag{3.50}$$

In a large scale transit-based evacuation model, evacuees at each demand point were guided to nearby pickup locations according to their closeness. Transit-vehicles are started from the depot to pickup and dropped off at the shelter, after reloading, they are headed to other pickup locations until all evacuees were picked up. A linear programming mathematical model using binary variables was developed in [75] to select the most suitable location and the number of bus stops in such a network. Each bus stop scenario that contained a greater number of bus stop locations performed superior with lower delay, travel, and stop times.

When preparing for an evacuation, the time it actually takes is uncertain. Hence, it is preferential to plan at each point of time to execute the maximum flow, which is offered by the EAF. It is better-suited for an EPP as it maximizes the flow of evacuees simultaneously at each instance within the given time horizon. Such an evacuee arrival pattern is more appropriate for the integrated evacuation scenario. In our work, we have considered such an arrival pattern in an integrated evacuation network, (cf. Section 4.1).

3.5.3 Vehicle routings in an embedding

Initially, the VRPs were used to design an optimal route of a set of vehicles to service the given set of customers under given constraints. It started with the seminal papers of [32] and [29] for the truck dispatching and the vehicle scheduling problems for the delivery service. Routing of vehicles plays a vital role in supply chain management, scheduling, pickup and delivery service, day to day traffic, and of course in disaster management. Different variants of the VRPs have been used extensively in the optimization literature with respect to different aspects of EPP. In such an EPP on the embedded network, we seek to minimize the clearance time, as the network clearance time, which stands for the total evacuation time from the beginning of the evacuation process to the time required to transverse the last evacuee to the sink. However, the problem is $\mathcal{NP}$-hard. For such $\mathcal{NP}$-hard problems during sufficiently large problem instances, finding an optimal solution is not practicable. Therefore, different heuristics and meta-heuristics have emerged which quickly produce solutions of reasonable quality.

3.6 Applications on Evacuation

Mathematical models seldom represent all the characteristics of real-life situations because when formulating the mathematical models, one has to idealize the real-life problems by making some simplifying hypothesis, [85]. So one has to be careful using the solutions of the mathematical models to real world problem. Various approaches were carried out as the applications of emergency planning optimization. With focus on the logistics aspects of such problems, authors in [12] have presented a systematic survey of contributions on relief distribution networks in response to disaster management concerning to their theoretical and practical implementations. Logistics support in emergency with capacity restrictions has been addressed in [74]. Goerigk and Grün

[54] consider two instances of (i) finding a bomb within city center of Keiserslautern Germany and (ii) an earthquake with a subsequent flood in the area of Nice, France to test the applicability of their modeling of a comprehensive evacuation planning.

Four notable applications of lane-evacuation routing were effectively conducted in Salt Lake City, Utah as in [30] for different situations by creating pedestrian evacuation zone and vehicle evacuation zone. Where the evacuation of Yokosuka City by [139], and the evacuation of Knox County, as a county-wide evacuation scenario for Tennessee in [63] were carried out using an MCF network model.

Among others, authors in [57], have applied the numerous optimization and simulation approaches to model the trip route assignment to maximize the flow of evacuees from the risk area to safety or to minimize the travel time based on Dijkstra algorithm on time-space network. For this, firstly all residents or other evacuees are evacuated safely to temporary safe stop by foot and secondly they are picked up by public transit which also signifies the importance of public transit-dependent emergency evacuation in developing countries, like Nepal. Moreover, a transit-based evacuation model for metropolitan areas was tested significantly on the city of Baltimore's downtown road network in a study by [142] assuming an incident of sudden terrorist attack. The applicability of the proposed model and its advantages compared with the available models in the literature was found significant.

The effective and imperative study on the emergency evacuation planning model for the specific need populations addressing the optimal location of bus stops has been presented by [75] and was applied to a real-life to evacuate the effects of the location, number, and distribution of optimal evacuation bus stops. The proposed methodology was applied to the real-life case of the downtown Washington, D.C. to select the most suitable location and number of bus stops. It suggests not only for the need of different evacuation routes, head-ways, and frequencies in which the buses depart or pick up the evacuees, but also to explore the new, possible, and appropriate evacuation bus stop locations within the network. Reroute and restructuring of evacuation planning approach has been implemented in [86] to find a set of alternative paths and their corresponding flow rates. This approach was tested successfully on the actual evacuation network of the Greater Houston area.

To test the integrated contraflow strategy for the multi-modal evacuation, authors in [70] consider the evacuation network of Ningbo city, located on the east coast of the Pacific Ocean. An arterial sub-network, as the road segment without direct connection to the origin nodes and the local roads which are connected to the origin nodes as the access to arterials are considered on the network aggregation for the two-stage evacuation process. For this, the origin is located as an arterial access point with in the local road network and then the destination point is achieved from that point via the arterial network. They have presented the separate evacuation models for the transit-based and auto-based evacuees where the transit-based evacuation problem is solved with an MCF model in first priority and then only the auto-based evacuation problem is addressed with a bi-level network flow model. The approximate optimal evacuation plan of the evacuation network has been obtained at the top level, where the traffic volumes and travel times in streets were derived from equilibrium traffic assignment in the bottom level. However, it is almost impossible to optimize an evacuation network containing all the arterials and local roads, simultaneously.

To evacuate the area surrounding a nuclear power point, [24] has successfully applied the k-shortest path method. For the mass evacuation of the areas surrounding the sites of nuclear power points, [130] has used the macroscopic traffic simulation model to simulate the traffic patterns to have minimum clearance time. Capacitated network-flow approach for the evacuation-location problems were investigated by the authors in [47] to determine the safest sinks to maximize the number of evacuees. They have used an exact as well as some heuristics to solve their proposed problem to determine the single or multiple sinks.

Evacuation problems were combined with location analysis to reduce the evacuation time by facility location approach in [60] that are supportive to establish and maintain the facility locations without destroying too many source-sink paths. In a modeling by [28] on three different types of road networks like a grid network, a ring road structure, and a real road network the overall benefit of the chosen bus stops located within the evacuation area has been maximized by dividing the area into smaller sub-section. Such sub-sections of different zones can be grouped together and a minimum number of bus stops can be set for the sub-section of evacuation bus stops where the total benefit will increase as the total number of optimum bus stops increased. Assuming that a complete evacuation is not possible, authors in [127] used to maximize the number of evacuees served, which incorporates traffic flow dynamics from the simulation package with a logistic function to estimate the number of evacuees at each pickup location. Likewise, for solving the problem of refuge location through facilitating buildings to provide shelter to the victims with the quality service after a disaster has also been highlighted in [108].

One of the most prominent applications of the transit-vehicles with their optimal routing and scheduling has been applied to evacuate the entire city of Toronto [2], with a population of about 2.37 million. The model generates optimal scheduling and timetable for each train on the subway lines and the big shuttle buses on the evacuation network. Based on two objectives as the minimizing of routing time and the minimizing of total system evacuation each of the subway line was evaluated for average in-vehicle travel time, average total in-vehicle time, average travel distance and average waiting time. The results show that the Toronto Transit Commission fleet is capable of evacuating the transit-dependent population of about 1.34 million within 2 hours on average.

An application of BEPP is presented by [117] in a hypothetical case study of the evacuation planning of transit-dependent people of Kathmandu valley by using branch and bound and tabu search algorithms. Different formulations and solution methodologies have been applied to generate an evacuation plan for different regions on different disasters. Some of such prominent plannings used for the evacuation systems are like that of [141] used for the south-central region of Beijing, China, and that of [54] for Kaiserslautern, Germany. For large instances of the disaster situations, the mass transit component formulation and solution methodology presented by [4] have been applied to generate a transit-based evacuation plan for downtown Toronto, Canada, whereas [1] extends the evacuation area to the entire city of Toronto, Canada. Some of the transit-based evacuation models were also applied for the evacuation plannings for different disasters situations at the United States of America like that of [27], [84], [92], [124], and [127]. These are applied for College Park, Maryland; Sioux Falls, South Dakota; Miami-Dade, Florida; Gulfport, Mississipi; and Fortworth, Texas; respectively.

To address the problem of distributing different products with different priorities for example, oxygen distribution to different hospitals, to industrial customers and medical facilities, generally the priorities are given to the hospitals. For such situations, the authors in [15] have applied the inventory routing problem with priorities corresponding to the heterogeneous fleet of vehicles to satisfy the customers demand considering inventory levels and priority constraints while minimizing the inventory and routing cost.

CHAPTER 4

EARLIEST ARRIVAL TRANSSHIPMENT IN AN EMBEDDED NETWORK

Summary. Evacuees are shifted to the pickup locations from the source for the evacuation. Such pickup locations are taken as the sources for the subsequent evacuation process to minimize the overall network clearance time where the transit-vehicles are provided from the bus depot to the embedding. Here, an integrated optimization approach is proposed in such an embedding to achieve the minimum clearance time. The earliest arrival pattern during the collection of evacuees at primary sub-network respects the partial lane reversal strategy, whereas the better assignments of transit-buses in the embedding are based on the dominance relations concerning the evacuation duration.

For a multi-terminal dynamic network with given supplies and demand, a transshipment over time is said to accomplish the earliest arrival property to maximize the amount of flow that has reached the sinks until time θ for any $\theta \geq 0$ such that the supplies and demands must not be exceeded. Such a transshipment on the multi-terminal dynamic network is called the EAT. Here, we are dealt with such an EAT in an embedding.

4.1 A Topology of an Embedding

An embedding on the evacuation network can be formed as $\mathscr{N}^E := \mathscr{N}_1 \cup \mathscr{N}_2$ such that $\mathscr{N}_1 := (s, V, A, u_a, \tau_a, Y)$ and $\mathscr{N}_2 := (d, Y, E, \tau_e, Z)$. Here, $s := \{s\}$ a single source; d := a bus depot having $B := \{b_1, b_2, \ldots, b_n\}$, a set of homogeneous bus with capacity Q. The set of auxiliary nodes, pickup locations, and set of sinks are denoted by $V := \{v_1, v_2, \ldots, v_n\}$, $Y := \{y_1, y_2, \ldots, y_n\}$, and $Z := \{z_1, z_2, \ldots, z_n\}$, respectively. In an embedding, we are using $v_i \in V$ as the auxiliary nodes rather than the nodes $i \in V$ as before in simple flow networks. Moreover, $A := \{a \mid a = (s, v) \vee (v, y),\ \text{where } v \in V,\ y \in Y\}$ and $E := \{e \mid e = (d, y) \vee [y, z],\ \text{where } y \in Y, z \in Z\}$ are the set of arcs in the the constituent sub-networks $\mathscr{N}_1$ and $\mathscr{N}_2$, respectively. Here, (d, y) and $[y, z]$ are used to denote the one-way arcs and the undirected edges, respectively. Here, the capacity $u_a : A \to \mathbb{Z}_{\geq 0}$ restricts the amount of flow on the arc and the transit time $\tau_a : A \to \mathbb{Z}_{\geq 0}$ represents the amount of time to transverse the respective arc. The bus depot d does not play significant roles further on the solution procedure as the buses do not return to it even after the completion of the evacuation plan because of risks under threat. In such an embedded network, as in Figure 13, the partial arc reversal capability is applicable in $\mathscr{N}_1$ for the collection of evacuees. The set of nodes Y works as the sink for $\mathscr{N}_1$ during the evacuees' collection is considered as the set of sources for $\mathscr{N}_2$ on the vehicle assignments.

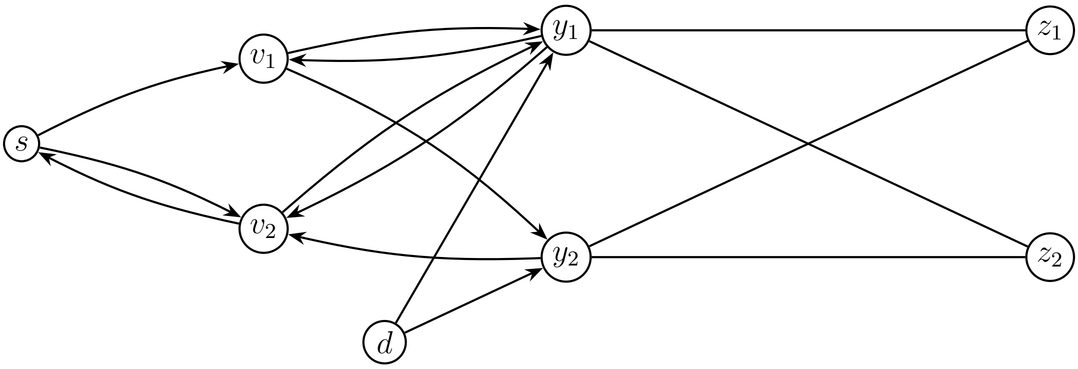


Figure 13: A topology of an embedding.

4.2 The Earliest Arrival Flows in an Embedding

Shortly after the introduction of maximum flows over time on [50], an $s - z$ earliest arrival flow problem has been studied on [52]. For a dynamic network $\mathscr{N}$ with a single source s and a single sink z, the aim is to find a flow over time in the network such that the amount of flow which has reached the sink z up to time θ is maximized simultaneously for all $\theta \geq 0$. Such a flow is said to have the $s - z$ earliest arrival property and is called an $s - z$ earliest arrival flow. In their study, it is shown that such a flow always exists in a discrete-time setting. The existence of the continuous-time setting has been presented, [109]. For computing $s - z$ earliest arrival flow, Minieka [98] and [134] both gave pesudopolynomial-time algorithms and are based on the successive shortest path. The computing of such an $s - z$ earliest arrival flows is weakly $\mathcal{NP}$-hard, [45].

The EAT with the earliest arrival property does not exist in general. It does not exist even in one source and two sinks for arbitrary time setting [16]. However, for multiple sources with given supplies and a single sink z, such a transshipment does always exist. The first polynomial-time algorithm for the EAT problem in networks with zero transit times has been presented in [58].

In most of the EPPs, evacuees are collected at some nearby pickup locations out of the disaster zone and are finally traverse to the safe zone by using transit-vehicles. For the EAF over time reached to Y in $\mathscr{N}_1$, the net amount of flow given by Equation (2.14) should be maximum at every point in time within the given time horizon under the constraints as in Equations (2.11-2.13). Here, for the embedded network N^E, as the pickup locations Y of the primary sub-network works as the sink during EAE collection on $\mathscr{N}_1$, the $s - z$ EAF as mentioned earlier in Section 2.3, is computed as the $s - y$ EAF. Such a flow over time will simultaneously maximize the flow that has already reached the pickup locations Y for all points in time.

Here, we consider a flow over time problem with zero transit time function $f : A \times \mathbb{Z}_{\geq 0} \to \mathbb{R}_{\geq 0}$. For zero transit times $\tau_a = 0$, the total flow amount out of the source s that reached to the pickup locations Y for all time up to $\theta' \in \mathbb{Z}_{\geq 0}$, is given by,

$$|\nu_f|_{\theta'} = \sum_{\theta=1}^{\theta'} |\nu(Y, \theta)|. \tag{4.1}$$

For the given time bound T, the value in Equation 4.1 becomes,

$$|\nu_f| = \sum_{\theta=1}^{T} |\nu(Y, \theta)|. \tag{4.2}$$

Here, $\nu(Y, \theta)$ represents the flow value reached to the pickup location Y at θ for $\tau_a = 0$.

In an integrated evacuation scenario, evacuees collected at the pickup locations Y in $\mathscr{N}_1$ are assigned to transit-buses in the appropriate route across $\mathscr{N}_2$ and are finally sent to the sinks. In our embedded evacuation approach, evacuees are collected in the earliest arrival pattern in $\mathscr{N}_1$ respecting the partial arc reversal capability and are assigned to the vehicles in $\mathscr{N}_2$. Here, we introduce the EAE problem respecting the partial arc reversal capability.

Problem 4.1. Given a flow over time sub-network $\mathscr{N}_1 = (s, V, A, u_a, \tau_a, Y)$ with supplies at s, auxiliary nodes V, arc capacity u_a, and arc transit time τ_a for $a \in A$. The EAE problem is to find the earliest arrival of evacuees at Y with partial arc reversal capability.

In the case of arbitrary transit times, no EAT exists even in a one-source-two-sinks network as mentioned in [16]. However, every in-or out-tree with the depth of at most two always allows for the EAT for every choice of capacities and flow values for zero transit times. For details, we refer to [115, 129].

4.2.1 Existence of the earliest arrival flows with zero transit times

As mentioned above, the existence of EAFs with zero transit times is based on the depth of the network considered. A directed graph with exactly one path from v_1 to v_2 for every node v_1 is the in-tree with root v_2. Likewise, the directed graph with exactly one path from v_2 to v_1 for every node v_1 is the out-tree with root v_2. The depth of an in-or out-tree is the number of edges on the longest path contained in it.

Lemma 4.1. [129] Consider $\mathscr{N}_1$ with zero transit times. Then every in-or out-tree with a depth of at most two in $\mathscr{N}_1$ always allows for the EAT regardless of capacities and balance values.

Theorem 4.1. There exists an EAT with zero transit times in a type $\mathscr{N}_1$ network of single-source and multi-sink, where the depth is at most two.

Proof. The earliest arrival $s - y$ flow exists for a single-source-single-sink network as in [52]. However, the EAT does not exist for arbitrary transit times, even in a single-source and double-sink network. But from Lemma 4.1, the EAT with zero transit times exists for all networks with a depth of at most two, and is satisfied for the single-source and multi-sink network. □

4.2.2 Networks permitting the EAF at zero transit times

EAF will simultaneously maximize the flow that reaches the sinks in each and every time unit but such flows do not exist in arbitrary network neither in constant or/and variable time nor in zero transit times. For the special case of the flow over time with zero transit times, some characteristic networks always permit for the EAF pattern regardless of their capacities, supplies and demands, [129]. This can be summerized as follows:

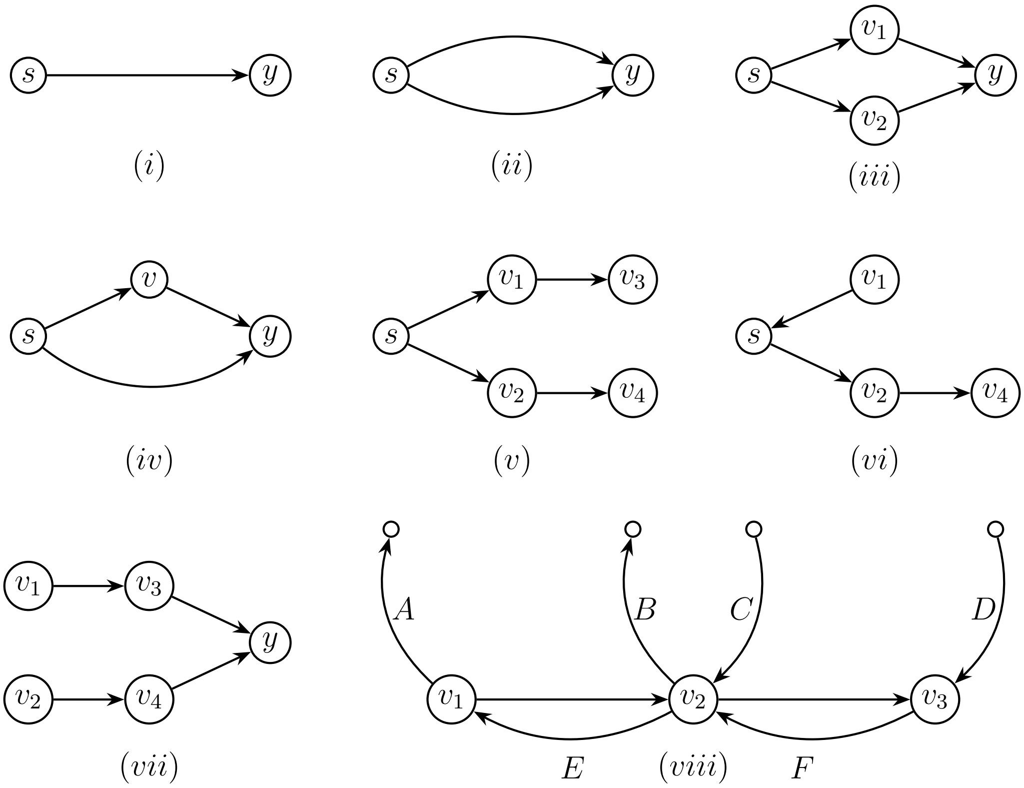

(i) (ii) (iii) (iv) (v) (vi) (vii) $(viii)$

Figure 14: Networks permitting earliest arrival transshipments at zero transit times.

1. A simple $s - y$ network with a single edge, Figure 14 (i).
2. An $s - y$ network with two parallel paths of length two starts in a same node and ends to the next common node, Figure 14 (ii).
3. In addition to the network of the type Figure 14 (ii), that can also have an intermediate node in each paths, Figure 14 (iii).
4. A network containing a parallel edge $s - y$ and a path of length two with an intermediate node, Figure 14 (iv).

5. A network containing two paths starting in the same node s but continuing disjointly which leads to a larger class of graph of out-tree on root s, Figure 14 (v).

6. A network for $v = s$ and the edges be $(v_1, s), (s, v_2), (v_2, v_4) \in A$, Figure 14 (vi).

7. A network containing two paths of length two that starts in different nodes but continue to ends in the same node, Figure 14 (vii).

8. Finally, let us consider a network containing at least one path of length two. Note that, in above mentioned cases $(1) - (7)$, as illustrated in Figure 14$(i) - (vii)$, each of the networks is either with no path of length two or two paths of length two starting at the same node or end to the same node such that the in-or out-tree is with the depth of $d \leq 2$, as demanded by the Lemma 4.1 for the existence of earliest arrival transshipments in zero transit. In case, if there is at least one path of length two then it can be considered that the ends of the edges with path two are (v_1, v_2) and (v_2, v_3) where the end points are v_1, v_2, v_3. Then all the possibilities of the additional edges for the connection of the additional nodes $u, v \in \{v_1, v_2, v_3\}$ as in Figure 14(viii). For $d \leq 2$, it can not share certain edges at the same time and we need to exclude certain edges. For such scenarios, the EAT can be constructed by considering either of the following.

   (a) No edges of type A or D.

   (b) At least one edge of type A and prohibits the edges of type C, D and F.

   (c) At least one edge of type D and prohibits the edges of type A, B and E.

But, this need not hold in multi-source multi-sink networks. Nevertheless, even in zero transit times it is not necessary to exist the EAT in each network, as illustrated in Figure 15 with respect to Example 4.1.

Example 4.1. For the adjoining graph as in Figure 15, consider the capacities and balances be $u(y_1, y_2) = u(y_3, y_4) = 4$, $u(y_2, y_3) = 2$ and $\mu(y_1) = 8$, $\mu(y_2) = -4$, $\mu(y_3) = 4$, $\mu(y_4) = -8$.

Initially, as in left-half of Figure 15, by sending two units each from (y_1, y_2), (y_1, y_4), and (y_3, y_4) will get only 6 units of flow in first time step but can balance all the nodes

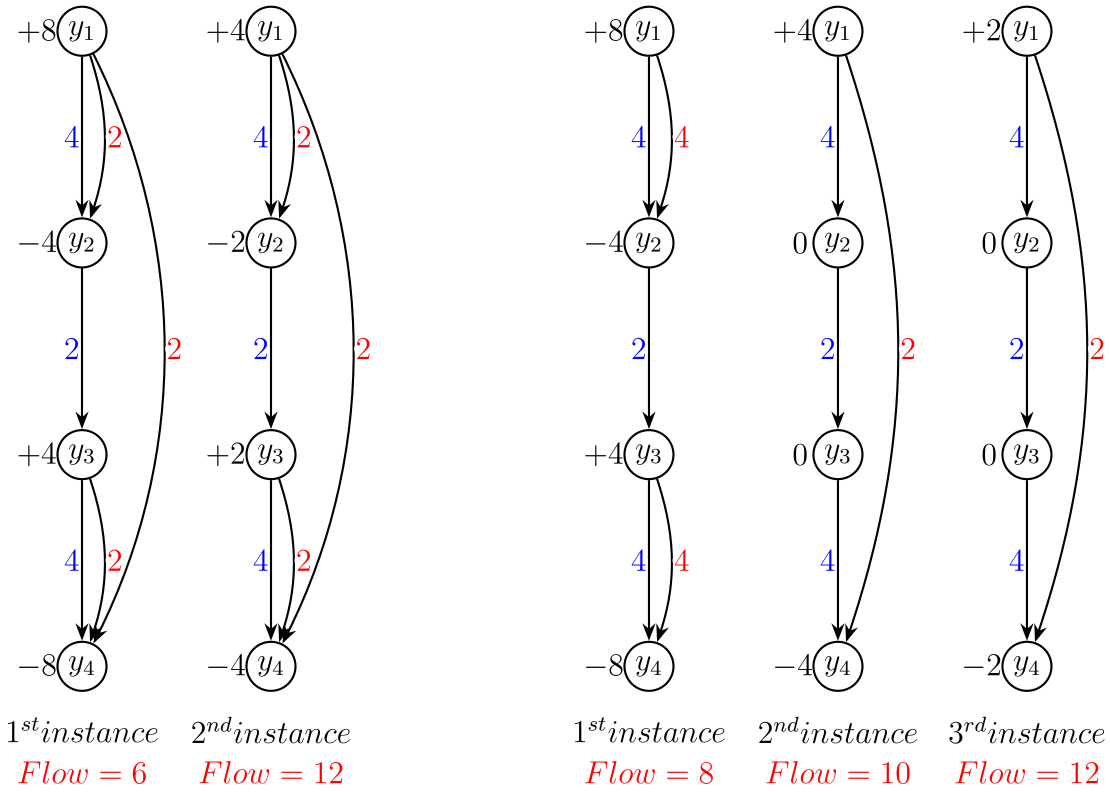


Figure 15: Non-existence of earliest arrival flow even in zero transit times.

in second step with the same repetition of flows with the total flow of 12 units in second instance.

But in another setting, as in right-half of Figure 15, four units of parallel flows on (y_1, y_2) and (y_3, y_4) will balance completely the y_2 and y_3 with the total flow of 8 units in first instance itself, but the next 4 units can only be sent in two different time steps which needs three time steps for the total flow of 12 units. Hence, no EAT does exist in such a network, even in zero transit times.

Now, an EAE algorithm is designed as in Algorithm 5 to solve the EAE problem as introduced in Problem 4.1 with zero transit times with partial arc reversal capability in polynomial-time complexity.

Algorithm 5: Earliest arrival evacuee algorithm

```
Input  : A flow over time sub-network $\mathscr{N}_1 = (s, V, A, u_a, \tau_a, Y)$ with $\tau_a = 0$ for
         each $a \in A$.
1 Construct a transformed network $\tilde{\mathscr{N}}_1$ as in Equation (2.1).
2 Determine the maximum number of evacuees at every possible time instance at
  each $Y$ from $s$ as in [129].
3 For each $\theta \in \mathbf{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ if and only if $c_a > u_a$, $u_a$
  replaced by 0 whenever $a \notin A$, in $\mathscr{N}_1$, where $c_a$ denotes the static $s - y$ flow value
  in each $a \in A$ for such sub-network.
4 For each $\theta \in \mathbf{T}$ and $a \in A$, if $a$ is reversed, $\kappa_a = u_{\tilde{a}} - c_{a'}$ and $\kappa_{a'} = 0$. If neither $a$
  nor $a'$ is reversed, $\kappa_a = u_a - c_a$, where $\kappa_a$ is saved capacity of $a$, [119].
Output: Earliest arrival of evacuees at $Y$ with $\tau_a = 0$ for each $a \in A$.
```

Theorem 4.2. The EAE algorithm as in Algorithm 5 sends the evacuees at the earliest arrival time to Y at each instances and saves the unused arc capacity.

Proof. The construction of a transformed network for a given network in Step 1 is feasible. Steps 2 and 4 are feasible. As there is no cycle flow in Step 2, the flow is either on arc a or a' but never in both directions simultaneously. And such a flow is not greater than the modified capacities of each arc in the transformed network. So, Step 3 is also feasible. Hence, the EAE algorithm as in Algorithm 5 is feasible.

Now, we show that the EAE algorithm given by Algorithm 5 gives an optimal solution. In the transformed network, we compute the maximum number of evacuees reached to each pickup location in Y at every possible time point with zero transit times on each arc using the algorithm of [129]. As in Theorem 4.1, there are some characteristic networks with the depth bounded by two in which the EAF exists. As the maximum amount of flows are assigned from the source s across different auxiliary nodes to Y at each instance, for the maximum flow value, it gives the evacuees at the earliest arrival time to such pickup locations at each instance. Moreover, the obtained solution is equivalent to the solution of the EAE problem in the original sub-network $\mathscr{N}_1$ with the arcs reversed up to the necessary capacity as in Step 3, [119]. The capacities of the arcs not used by the flow after partial arc reversals are recorded in Step 4. This completes the proof. $\square$

Theorem 4.3. The EAE problem having zero transit times with partial arc reversal capability follows the principle of TRFs and can be solved in polynomial-time complexity using EAE algorithm given by Algorithm 5 in the sub-network $\mathscr{N}_1$.

Proof. As Steps 1, 3, and 4 of EAE algorithm given by Algorithm 5 are solved in linear time, its time complexity is dominated by the time complexity of computation of the EAEs at the pickup locations Y with zero transit times on each arc as in [129] in Step 2, which is solved in polynomial-time. Thus, we solve the EAE problem in polynomial-time complexity in the sub-network $\mathscr{N}_1$.

The flow over time problem having zero transit times that reached to each of the pickup locations determines the maximum number of evacuees at every possible time instance from the beginning in the primary sub-network $\mathscr{N}_1$ as in [129]. That means the EAEs at Y from s with zero transit times on the transformed network follows the principle of TRFs which is equivalent to the solution with arc reversals capability on the original network, [115]. Hence the theorem is proved. □

4.3 Assignment of Vehicles in an Embedding

For large scale disasters with a sufficiently large number of evacuees, all the evacuees may not arrive at Y at the same time, and it requests certain waiting time at Y before to start the bus assignment in $\mathscr{N}_2$. Those who are delivered to Y earlier will have comparatively more waiting time. Meanwhile, for the evacuees, waiting at Y is comparatively better than to be at s. On the other hand, buses available at d request a certain time to be assigned to Y and are given by τ_{di}. Hence the effective waiting time in $\mathscr{N}$ can be denoted by $\Omega = \max\{\omega_i, \tau_{di}\}$, for ω_i be the waiting at $y_i \in Y$. To address this in an integrated evacuation network, where the evacuees collected in $\mathscr{N}_1$ are to be assigned in $\mathscr{N}_2$, the objective function given by Equation (3.17) is modified. For this, let $\mathcal{T}_{\max}$ be the duration of evacuation overall vehicles, then the model for IEPP can be reformulated as follows:

$$\text{minimize } \mathcal{T}_{\max}, \tag{4.3}$$

$$\text{such that } \mathcal{T}_{\max} \geq \Omega + \sum_{r \in R} \tau_{to}^{br} + \sum_{r \in R} \tau_{back}^{br} \; \forall b \in B. \tag{4.4}$$

$$\text{with the constraints} \quad (3.18 - 3.26). \tag{4.5}$$

Constraint (4.4) needs $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all vehicles and is to be minimized in (4.3). Other constraints are the same as in Equations (3.18-3.26).

For the solution of our integrated model, we adopt the branch and bound algorithm of [55] in which the computation of upper bounds, lower bounds, branching rules, and reduction strategies are discribed in Section 3.2. However, based on the effective waiting time at each pickup locations, the objective as in Equation (3.27) in the second lower bound becomes,

$$\text{minimize } \Omega + \sum_{i \in Y} \sum_{j \in Z} \tau_{di} x_{ij}^{b1} + \sum_{b \in B} \sum_{r \in R} \tau_{to}^{br} + \sum_{b \in B} \sum_{r \in R} \tau_{back}^{br}. \tag{4.6}$$

Whereas, for the third lower bound, the relations (3.28) and (3.29) can be replaced by,

$$\text{minimize } \mathcal{T}_{\max}, \tag{4.7}$$

$$\text{such that } \mathcal{T}_{\max} \geq \Omega_0 + \sum_{j \in Z} \tau_j (y_j^b + z_j^b), \tag{4.8}$$

where, $\Omega_0 = \max\{\omega_0, \tau_j\}$, for ω_0 be the waiting time for the earliest arrival of evacuees at the super pickup node y_0. As the problem (4.3-4.5) is not easier than the problem (3.16-3.26), we state the following result.

Theorem 4.4. The decision version of the IEEP is $\mathcal{NP}$-complete.

4.4 Solution Approach in an Embedding

As discussed in Section 3.2, the number of evacuees at each pickup location is given, but their approach does not speak about the arrived pattern at Y. The main aim of our work is to present an analytical investigation of an appropriate network flow model and give an efficient solution algorithm that provides information on the number of evacuees arriving at the pickup locations for the BEPP. Evacuees are collected on the EAF pattern from s. Such evacuees at Y are considered as the supplies for $\mathscr{N}_2$ and are to be assigned to the available transit-vehicles in an integrated evacuation network. Here, we introduce the transit-vehicle assignment (TA) problem as in Problem 4.2 and develop the TA algorithm as in Algorithm 6 to assign the transit-vehicles in such an embeddeding with minimum clearance time.

Problem 4.2. Given an evacuation network $\mathscr{N}^E = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$, having supplies and demands at s and Z, respectively. The TA problem is to assign the vehicles for evacuees transshipment with minimum clearance time.

Algorithm 6: Transit-vehicle assignment algorithm for minimum clearance time.

```
Input : An embedded evacuation network $\mathscr{N}^E = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$.
1 In $\mathscr{N}_1 = (s, V, A, u_a, \tau_a, Y)$, consider $Y$ as the sinks and determine the earliest
    arrival of evacuees for $\tau_a = 0$ at different $Y$ from $s$, by using Algorithm 5.
2 Assign the transit-vehicles from $d$ to $\mathscr{N}_2 = (d, Y, E, \tau_e, Z)$ for the supplies
    provided by Step 1 at $Y$, as guided by the dominant vehicle assignment approach
    as in Section 3.2.
3 Stop, if all the supplies available at each of $Y$ are fulfilled, respecting the capacity
    constraints of $Z$.
4 Otherwise, return to Step 2.
Output: Transit-vehicle assignment with the minimum clearance time from
        $s \to Z$.
```

Theorem 4.5. The TA algorithm as in Algorithm 6 gives the dominating solution for the TA problem as in Problem 4.2 with minimum clearance time.

Proof. Step 1 is feasible, since $\mathscr{N}_1$ is constructed in which the EAF of evacuees exists for $\tau_a = 0$. As $\mathscr{N}_2$ is embedded in $\mathscr{N}_1$ for the appropriate vehicle assignment in $\mathscr{N}$, Step 2 is feasible. Feasibility of Step 3 is obvious as the flow respects the supplies as well as the demands in the network $\mathscr{N}^E$. Step 4, is of course, feasible. Hence, the TA algorithm as in Algorithm 6 is feasible.

Now, we show that Algorithm 6 gives a dominating solution. The maximum amount of flows are assigned from s across different auxiliary nodes to Y in $\mathcal{N}_1$, as in the form of the earliest arrivals of evacuees by using the EAE algorithm as in Algorithm 5. So, it gives the maximum possible flow of evacuees at each instance in the earliest arrival time and saves the unused capacity by Theorem 4.2. These resulting flows of such evacuees arrived at Y are taken as an input in $\mathcal{N}_2$ for the transit-vehicle in the subsequent evacuation process with the dominating vehicle assignment approach as in Section 3.2. Such an assignment of vehicles is continued until the last evacuee on such pickup locations reached to the sinks without violating their capacities. Hence, the resulting vehicle assignment in the integrated evacuation network gives the dominating solution with minimum clearance time. Hence, the theorem is proved. □

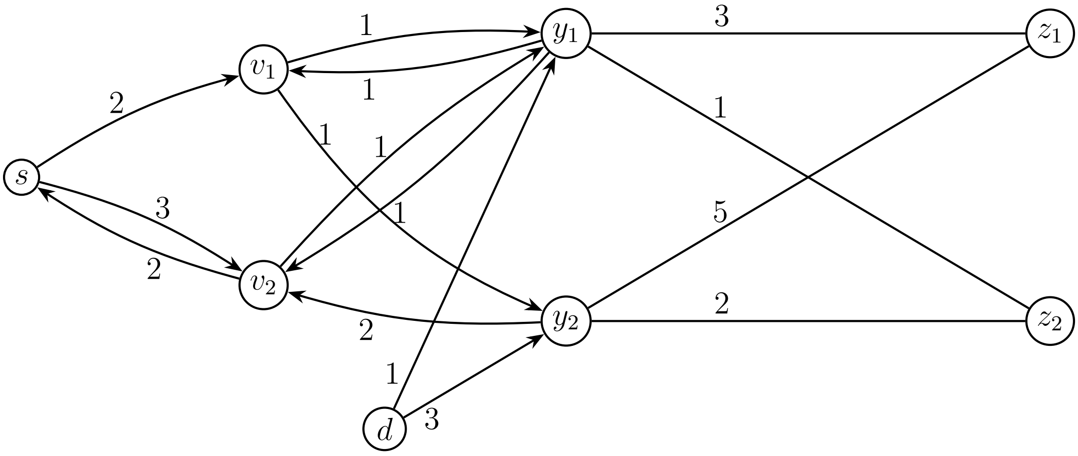


Figure 16: An instance of an integrated evacuation network.

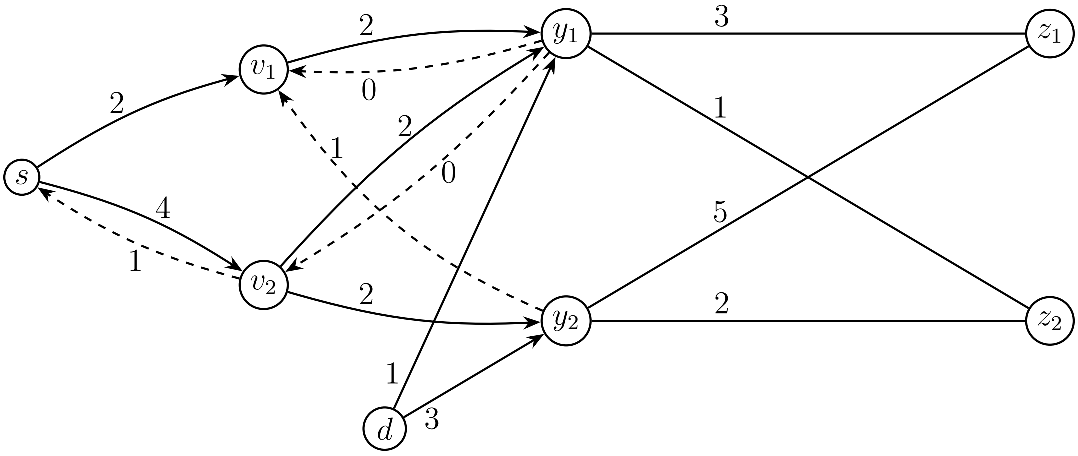


Figure 17: An instance of an integrated evacuation system with partial arc reversal.

Example 4.2. Consider an instance of integrated evacuation system in $\mathcal{N}^E$, as in Figure 16. Consider a scenario with 12 units of evacuees at s, and with the pickup demands 8 and 4 for y_1 and y_2, respectively. Let the capacity of the sinks be 6 and 10 for z_1 and z_2, respectively.

Each of the paths $s \rightarrow v_1 \rightarrow y_1$, $s \rightarrow v_2 \rightarrow y_1$, and $s \rightarrow v_1 \rightarrow y_2$ can be assigned with 1 unit of flow as the EAF of evacuees at zero transit times. Such flow is TRF as in Theorem 4.3 and will collect 8 and 4 units of flows at y_1 and y_2, respectively, after 4-time units. Similarly, for the arc reversal capability, 2 units of flow can be assigned on each of the above-mentioned paths at zero transit times and are also TRF. It can collect 8 and 4 units of flows at y_1 and y_2 respectively after 2-time units by using the EAE algorithm as in Algorithm 5 and also helps to improve the waiting instance.

We seek to find an optimal tour plan of this scenario using the dominating vehicle assignment approach as demanded by Algorithm A4 (cf. Section 3.2), without violating the demands and the capacity constraints. Let the transit-vehicles be located at the depot d. Then their respective $\mathcal{T}_{\max}$ on $\mathcal{N}^E$, without and with arc reversal capability in $\mathcal{N}_1$, can be estimated as 41 and 39, respectively for $|B| = 1$.

Here, we have $\mathcal{T}_{\max}$ is 7 and 5 respectively, concerning to without and with arc reversal capability, for all $|B| \geq 12$. It also gives the threshold number of transit-buses. Moreover, the saved capacities obtained from partial arc reversals are shown along with the dotted arcs in Figure 17.

CHAPTER 5

MINIMUM CLEARANCE TIME ON THE PRIORITIZED NETWORK

Summary. In this work, evacuees are collected from the disaster zone to the pickup locations of the primary sub-network in minimum time as the quickest transshipment by using the lex-max flow approach. Considering such pickup locations as the sources, the available set of transit-buses are also assigned in the network to evacuate the evacuees safely to the sinks on the first-come-first-serve basis. This novel approach proposed here is better-suited for the simultaneous flow of evacuees with minimum waiting delay at such pickup locations to achieve the minimum clearance time in such a prioritized embedded evacuation network.

The EPP can be viewed as different variants of dynamic flow maximization and/or time minimization problems. An optimal solution to the time minimization EPP sends a given amount of flow from disaster zones to safe zones in minimum time. The clearance time in an EPP stands for the total evacuation time from the commencement of evacuation process until to the clearance of the last evacuee to the sink. We seek to minimize it in a prioritized embedding.

5.1 A Prioritized Embedding

Consider an embedding, $\mathscr{N}^E = (S, V, A, u_a, \tau_a, Y, d, u_e, \tau_e, Z)$, *i. e.*, $\mathscr{N}^E = (\mathscr{N}_1 \cup \mathscr{N}_2)$, as defined in Section 4.1. Here, $\mathscr{N}_1$ is a prioritized primary and $\mathscr{N}_2$ is a bus-routed secondary sub-networks. A prioritized network is a multi-terminal network that consists of prioritized terminals. Under the given priority of terminals, flows can be compared according to their departure (or arrival) flows, form (or in), the sources (or sinks).

We use the quickest transshipment partial arc reversal strategy to collect the evacuees in minimum time from the disaster zones to the pickup locations of $\mathscr{N}_1$. The lane reversal strategy significantly reduces the evacuation time, whereas reversing them only partially has an additional benefit that the unused road capacities can be used for supplying emergency logistics and allocating facilities as well.

5.2 The Quickest Arrival of Evacuees

In the quickest transshipment problem, a given number of evacuees has to be shifted in minimum time. Such problem have been studied by the help of the lex-max dynamic flow problem applying the MCF computations as a tool, in such a prioritized network. For a given time T and an ordered set of multi-terminals, the lex-max dynamic flow problem finds a feasible flow that lexicographically maximizes the flow leaving each terminal in the prioritized network, as mentioned in Section 2.2.1. Hoppe and Tardos [69] deals with a chain decomposable flows in such a network corresponding to their standard as well as non-standard chain decompositions.

To have the quickest arrival of evacuees at such pickup locations Y of the primary sub-network with partial arc reversals capability, we introduce the quickest partial arc reversal transshipment (QPART) problem as in Problem 5.1 and design an algorithm, the QPART algorithm as in Algorithm 8.

Problem 5.1. Given $\mathscr{N}_1 = (S, V, A, u_a, \tau_a, Y)$ with supplies at S, demands at Y, auxiliary nodes V, arc capacity u_a, and arc transit time τ_a for $a \in A$. The QPART is to find the quickest arrival of evacuees at Y with partial arc reversals capability.

Consider the set of sources and sinks be prioritized as $s_1, s_2, \ldots, s_\delta$ and $y_1, y_2, \ldots, y_\delta$, respectively. Let s^* be the super source connected to such s_i with arcs having the infinite capacity and zero transit time. Let the sinks in $\mathscr{N}_1$ be prioritized with the highest priority to the nearest by determining their shortest distances concerning s^*. The lex-max flow within the specified time horizon T entering the sinks Y in $\mathscr{N}_1$ in that order can be computed in polynomial-time, as in Algorithm 7, based on [69].

Algorithm 7: Lex-max dynamic flow of evacuees in $\mathscr{N}_1$.

```
Input : A dynamic sub-network $\mathscr{N}_1 = (S, V, A, u_a, \tau_a, Y)$. Let, $V = V \cup \{s^*\}$
        and $A^{\delta+1} = V \cup \{(s^*, s_i)\} : s_i \in S$, where $u_a(s^*, s_i) = \infty$ and
        $\tau_a(s^*, s_i) = 0$ for $\{s^*\}$ be the supersource. Let, $g_0^{\delta+1} = 0$ be the zero flow
        and the set of hain be $\Gamma^{\delta+1} = \phi$.
For $i = \delta, \ldots, 1$, set the arc be $A^i = A^{i+1}$.
    a. If $s_i$ sink, add the arc $s_i s^*$ with $u_a(s^*, s_i) = \infty$ and $\tau_a(s_i, s^*) = -(T+1)$. Then
       get, $f^i$= minimum cost circulation in $\mathscr{N}_{1g^{i+1}}^{i}$ using $\tau_a$ as the arc cost.
    b. If $s_i$ source, delate the arc $s^* s_i$ from $A^i$ and get, $f^i$= minimum cost maximum
       $s^* s_i$-flow in the network $\mathscr{N}_{1g^{i+1}}^{i}$ using $\tau_a$ as the arc cost.
Update the dynamic flow, $g^i = g^{i+1} + f^i$.
Let $\Delta^i$= standard chain decomposition of $f^i$, then the chain decomposition set
  becomes $\Gamma^i = \Gamma^{i+1} + \Gamma^i$.
Finally, return $\Gamma = \Gamma^1$.
Output: The arrival of the lex-max dynamic flow of evacuees at $Y$ in $\mathscr{N}_1$.
```

As a *mutatis mutandis* to the results in [69], we are here with two more results.

Theorem 5.1. Algorithm 7 constructed for the lex-max dynamic flow in $\mathscr{N}_1$ gives the feasible solution for the lex-max number of evacuees at Y.

Theorem 5.2. [105] For any lex-max dynamic flow problem, a lex-max dynamic flow can be computed via δ MCF computations in $\mathcal{O}(\delta \text{MCF}(m, n))$ time, where $\text{MCF}(m, n) = \mathcal{O}(m \log n(m + n \log n))$ on a network with n nodes and m arcs.

Hoppe and Tardos in [69] studied the lex-max flow problem in dynamic networks by applying the MCF computations and have shown the quickest transshipment problem is equivalent to it. Different algorithms to solve such quickest transshipment have been presented based on the chain-decomposable flows and the minimization of submodular functions but are with high-order time complexity, though are polynomial-time solvable. Such chain decomposable flows help to generalize the stationary dynamic flows and the TRFs. The TRFs were refered as the standard chain-decomposable flows. Such chain flows in a nonstandard chain decomposition may use a backwards arc with negative transit time. Based on this landmark paper [69], authors in [128] have also presented the quickest transshipment algorithm to determine the quickest transshipment as a convex combination of simple lex-max dynamic flows.

The arc reversal strategy is performed to collect the quickest lex-max evacuees by constructing a transformed network for a given network where the flow is either on arc a or a' but never in both directions simultaneously. There is no cycle flow and such a flow is not greater than the modified capacities of each arc in such a transformed network. Here, we are presenting the QPART algorithm to get the quickest arrival of evacuees at Y in $\mathscr{N}_1$ corresponding to the arc reversal capability.

Algorithm 8: Quickest partial arc reversal transshipment algorithm

```
Input  : A dynamic sub-network $\mathscr{N}_1 = (S, V, A, u_a, \tau_a, Y)$, with the supply and
         demand.
1 Construct a transformed dynamic sub-network $\tilde{\mathscr{N}_1}$ as in Equation (2.1).
2 Solve the quickest transshipment problem [69] in the transformed network of
    Step 1.
3 For each $\theta \in \mathbf{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ iff $c_a > u_a$, $u_a$ replaced
    by $0$ whenever $a \notin A$, in $\mathscr{N}_1$, where $c_a$ denotes the static $s - y$ flow value in each
    $a \in A$ for such sub-network.
4 For each $\theta \in \mathbf{T}$ and $a \in A$ , if and only if $a$ is reversed, $\kappa_a = u_{\tilde{a}} - c_{a'}$ and $\kappa_{a'} = 0$.
    If neither $a$ nor $a'$ is reversed, $\kappa_a = u_a - c_a$, where $\kappa_a$ is saved capacity of
    $a$, [119].
Output: The quickest arrival of evacuees at Y in $\mathscr{N}_1$ with partial arc reversal
         capability.
```

Theorem 5.3. The QPART algorithm as in Algorithm 8 constructed for the QPART problem gives the optimal solution for the quickest arrival of evacuees at Y and saves the unused capacity.

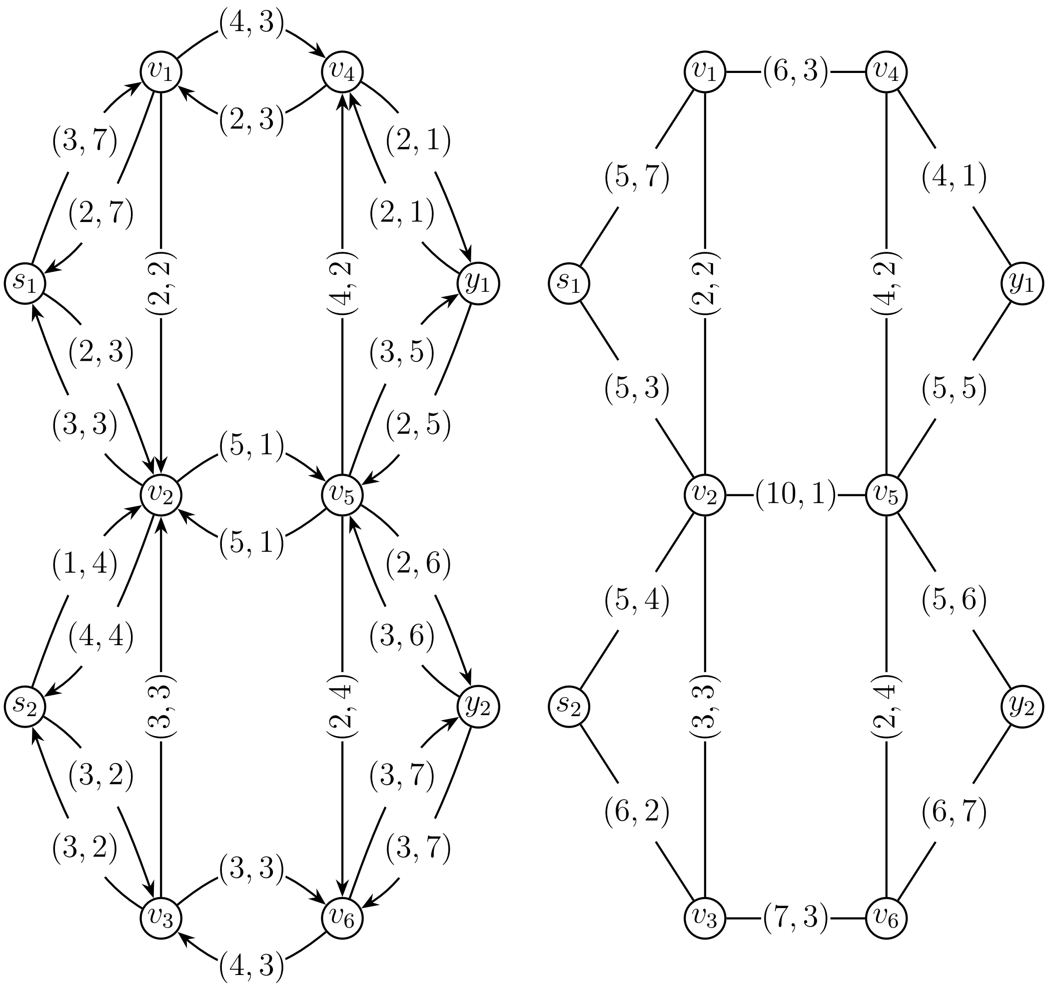
Proof. The construction of a transformed dynamic network $\tilde{\mathscr{N}_1}$ for the given network $\mathscr{N}_1$ in Step 1 is feasible. From the quickest transshipment problem [69], Step 2 is feasible. Moreover, the flow is either on arc a or a' but not in both directions simultaneously. And such flow is not greater than the modified capacities of each arc in the transformed network. So, Step 3 is also feasible. Step 4 is also feasible and helps to compute the saved capacity by arc reversal capability. Hence, the QPART algorithm as in Algorithm 8 is feasible.

Now, we show that the QPART algorithm as in Algorithm 8 gives the optimal solution. In the transformed dynamic network $\tilde{\mathscr{N}_1}$, we compute the lex-max number of evacuees reached to each of the prioritized pickup locations Y as the lex-max dynamic flow as in [69]. Moreover, the obtained solution is equivalent to the solution of the quickest arrival of evacuees at Y in $\mathscr{N}_1$ with the partial arc reversal up to the necessary capacity as in Step 3, [121]. Capacities of the arcs not used by the flow after partial arc reversals are computed in Step 4. This completes the proof. □

Theorem 5.4. For the quickest partial arc reversal transshipment in $\mathscr{N}_1$, the QPART problem can be computed in polynomial-time complexity via δ MCF computations in $\mathcal{O}(\delta \text{MCF}(m,n))$ time, where $MCF(m,n) = \mathcal{O}(m \log n(m + n \log n))$ on a network having n nodes and m arcs.

Proof. Steps 1, 3, and 4 related to the arc reversal capability as in QPART algorithm given by Algorithm 8 are solved in linear time. So their time complexity is dominated by the time complexity of the computation of the quickest evacuee arrival at Y in $\mathscr{N}_1$ and is solved in polynomial-time in $\mathcal{O}(\delta\text{MCF})(m,n)$, where MCF$(m,n) = \mathcal{O}(m \log n(m + n \log n))$ on a network having n nodes and m arcs as in Theorem 5.2. Hence, the theorem is proved. □

Example 5.1. Consider the primary sub-network with sources s_i, intermediate nodes v_i, and sinks y_i for evacuee arrival as in Figure 18 (i). Consider the evacuees at s_1 and s_2 be 16 and 13, respectively. Let the potential demands of Y be $\alpha(y_1) = 20$ and $\alpha(y_2) = 9$. Consider s^* be the super source connected to s_1 and s_2 in $\mathscr{N}_1$ having infinite-capacity and zero-transit time, then the highest priority is to be assigned for y_1 as specified. Then the respective arrivals of the evacuees at y_1 and y_2 can be determined



(i) (capacity, transit time)

(ii) (capacity, transit time)

Figure 18: (i) Primary sub-network, (ii) auxiliary network to (i).

as in Table 8 (using Algorithm 7) and Table 9 (using Algorithm 8) with their respective paths assignment without and with arc reversal capability in $\mathscr{N}_1$, respectively.

Table 8: Arrival of evacuees at Y in $\mathscr{N}_1$ without partial arc reversal capability.

Released time	Reached time	Paths assignment	Flow at y_1	Flow aty_2	Total flow
0	7	$s_1 - v_2 - v_5 - v_4 - y_1$	2		2
1	8	$s_1 - v_2 - v_5 - v_4 - y_1$	2		4
2	9	$s_1 - v_2 - v_5 - v_4 - y_1$	2		6
3	10	$s_1 - v_2 - v_5 - v_4 - y_1$	2		8
0	10	$s_2 - v_2 - v_5 - y_1$	1		9
4	11	$s_1 - v_2 - v_5 - v_4 - y_1$	2		11
1	11	$s_2 - v_2 - v_5 - y_1$	1		12
5	12	$s_1 - v_2 - v_5 - v_4 - y_1$	2		14
2	12	$s_2 - v_2 - v_5 - y_1$	1		15
0	12	$s_2 - v_3 - v_6 - y_2$	-	3	18
6	13	$s_1 - v_2 - v_5 - v_4 - y_1$	2		20
3	13	$s_2 - v_2 - v_5 - y_1$	1		21
0	13	$s_1 - v_1 - v_4 - v_5 - y_1$	2		23
1	13	$s_2 - v_3 - v_6 - y_2$	-	3	26
2	14	$s_2 - v_3 - v_6 - y_2$	-	3	29
Total			20	9	29

Table 9: Arrival of evacuees at Y in $\mathscr{N}_1$ with partial arc reversal capability.

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 - v_2 - v_5 - v_4 - y_1$	4		4
1	8	$s_1 - v_2 - v_5 - v_4 - y_1$	4		8
2	9	$s_1 - v_2 - v_5 - v_4 - y_1$	4		12
3	10	$s_1 - v_2 - v_5 - v_4 - y_1$	4		16
0	10	$s_2 - v_2 - v_5 - y_1$	4		20
0	12	$s_2 - v_3 - v_6 - y_2$		6	26
1	13	$s_2 - v_3 - v_6 - y_2$		3	29
Total			20	9	29

5.3 Assignment of Vehicles in a Prioritized Embedding

Transit-buses having uniform capacity Q are assigned from d which are sufficiently nearer to Y in $\mathscr{N}_2$, on the first-come-first-serve basis. Such assignment begins only after $\alpha_1 \geq Q$, for α_1 be the number of evacuees arrived at the highest pickup demand. For the subsequent assignments or even for the initial assignment, the effective waiting instance Ψ is almost negligible. However, waiting at pickup locations is comparatively better than to wait at disaster zone itself.

Buses are assumed to pickup their full capacities. For this, the potential demands of the pickup locations are adjusted to be the integral multiple of busloads. Let the potential demand of the pickup location $y_k \in Y$ be $\alpha(y_k)$. Then the demands can be adjusted to be the integral multiple of busloads as the adjusted demands $\alpha'(y_k)$ by using the following demand adjustment,

$$\alpha'(y_k) = \Big\lfloor \frac{\alpha(y_k) + \sum_{q=1}^{k-1}[\alpha(y_q) - \alpha'(y_q)]}{Q} \Big\rfloor .Q. \tag{5.1}$$

But if the k^{th} pick up location is the last one with the least priority, then it is given as in Equation (5.2), that is taken as an integral multiple of busloads as no more evacuees are left to be collected there in the sub-network.

$$\alpha'(y_k) = \alpha(y_k) + \sum_{q=1}^{k-1}[\alpha(y_q) - \alpha'(y_q)]. \tag{5.2}$$

Then the model for the IEPP for prioritized embedding can be reformulated as follows:

$$\begin{aligned} &\text{minimize } \mathcal{T}_{\max}, && (5.3)\\ &\text{such that } \mathcal{T}_{\max} \geq \Psi + \sum_{r \in R} \tau_{to}^{br} + \sum_{r \in R} \tau_{back}^{br} \; \forall b \in B, && (5.4)\\ &\text{with the constraints} \quad (3.18 - 3.26). && (5.5) \end{aligned}$$

Constraint (5.4) needs $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all buses and is to be minimized in (5.3). Other constraints are the same as in Equations (3.18-3.26).

As problem (5.3-5.5) is not easier than problem (3.16-3.26), we state the following.

Theorem 5.5. The decision version of the IEPP in a prioritized embedding is $\mathcal{NP}$-complete.

Example 5.2. By using Equation (5.1), the arrivals of evacuees at Y of $\mathscr{N}_1$ as in Figure $18(i)$ is shown as in Table 8 and Table 9 can be adjusted to be the integral multiple of busloads as in Table 10 and Table 11, with their respective paths assignment without and with arc reversal capability in $\mathscr{N}_1$, respectively. Such evacuees are to be assigned on the integrated evacuation network (cf. Example 5.3) where the transit-buses are assumed to pickup their full capacities except for the last trip. For the last trip, it is assumed to be the integral multiple of busloads by the demand adjustment principle as in Equation (5.2) for the remaining evacuees for the complete evacuation by convention.

Table 10: Arrival of evacuees after demand adjustment without partial arc reversals.

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 - v_2 - v_5 - v_4 - y_1$	2		2
1	8	$s_1 - v_2 - v_5 - v_4 - y_1$	2		4
2	9	$s_1 - v_2 - v_5 - v_4 - y_1$	2		6
3	10	$s_1 - v_2 - v_5 - v_4 - y_1$	2		8
0	10	$s_2 - v_2 - v_5 - y_1$	1		9
4	11	$s_1 - v_2 - v_5 - v_4 - y_1$	2		11
1	11	$s_2 - v_2 - v_5 - y_1$	1		12
5	12	$s_1 - v_2 - v_5 - v_4 - y_1$	2		14
2	12	$s_2 - v_2 - v_5 - y_1$	1		15
0	12	$s_2 - v_3 - v_6 - y_2$	-	3	18
6	13	$s_1 - v_2 - v_5 - v_4 - y_1$	2		20
2	13	$s_2 - v_2 - v_5 - y_1$	1		21
1	13	$s_2 - v_3 - v_6 - y_2$	-	3	24
2	14	$s_2 - v_3 - v_6 - y_2$	-	3	27
3	15	$s_2 - v_3 - v_6 - y_2$	-	2	29
Total			18	11	29

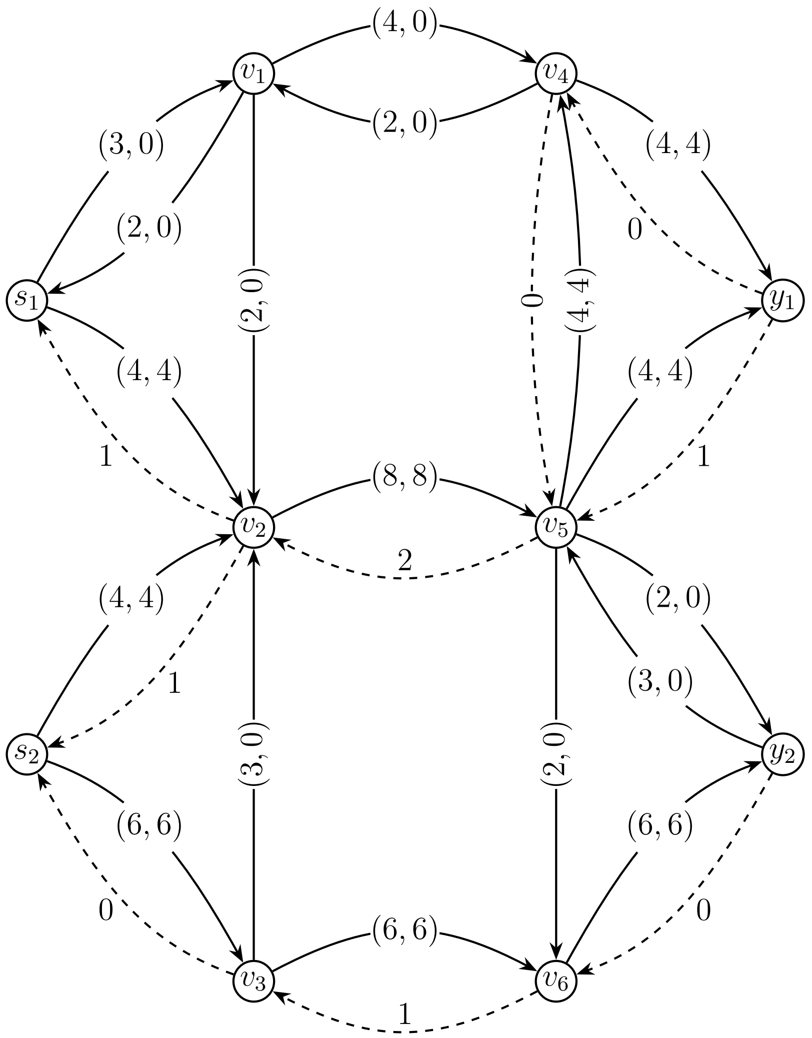


Figure 19: Flow patterns with respect to adjusted demands.

Table 11: Arrival of evacuees after demand adjustment with partial arc reversals.

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 - v_2 - v_5 - v_4 - y_1$	4		4
1	8	$s_1 - v_2 - v_5 - v_4 - y_1$	4		8
2	9	$s_1 - v_2 - v_5 - v_4 - y_1$	4		12
3	10	$s_1 - v_2 - v_5 - v_4 - y_1$	4		16
0	10	$s_2 - v_2 - v_5 - y_1$	2		18
0	12	$s_2 - v_3 - v_6 - y_2$	-	6	24
1	13	$s_2 - v_3 - v_6 - y_2$	-	5	29
Total			18	11	29

The partial arc reversal strategy allows the reversal of only the necessary part of the road segments along with the direction of traffic in the evacuation network, saving the remaining part of the road segments. Due to such phenomena, in some arcs, certain parts are partially reversed as necessary and some remains saving. Flow patterns with respect to adjusted demands respecting the partial arc reversals are shown as in Figure 19. In this network, occupied arc capacities and flows are shown along with the arcs and the dotted arrows are to show the saved capacities. Here, the road segments (y_1, v_5),(v_5, v_2), (v_2, s_1), (v_2, s_2), and (v_6, v_3) are saving the arc capacities of 1, 2, 1, 1, and 1 respectively. But no such arc saving on (v_4, v_5), (y_1, v_4), (y_2, v_6), and (v_3, s_2) due to their full reversal, as demanded. Saved arc capacities are beneficial not only for the humanitarian logistics and facility locations but also for the emergency vehicle movements within the network during such evacuation.

5.4 Solution Approach in a Prioritized Embedding

In an integrated evacuation approach, the quickest transshipment of the evacuees arrived at Y in $\mathscr{N}_1$ as in the form of lex-max dynamic flows of evacuees with respect to the adjusted demands are assigned to the transit-buses in $\mathscr{N}_2$ in the embedded prioritized network. Here, an IEPP in a prioritized embedding is introduced as in Problem 5.2 and an algorithm is developed as an integrated evacuation planning algorithm in an prioritized embedding as in Algorithm 9.

Problem 5.2. Given an evacuation network $\mathscr{N}^E = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$, having supplies and demands at s and Z, respectively. The IEPP in a prioritized embedding is to assign the vehicles for evacuees transshipment with minimum clearance time.

Algorithm 9: Integrated evacuation planning algorithm in a prioritized embedding

```
Input : An embedding $\mathscr{N}^E = (S, V, A, u_a, \tau_a, Y, d, u_e, \tau_e, Z)$, i.e.,
        $\mathscr{N}^E = (\mathscr{N}_1 \cup \mathscr{N}_2)$ provided with given supply and demand.
1 Consider $\mathscr{N}_1 = (G, u_a, S, \tau_a, Y)$, having their pickup locations be $Y$.
2 Construct a priority ordering of $Y$ assigning the highest priority to the nearest from
    $S$.
3 Determine the arrival of evacuees at $Y$ of $\mathscr{N}_1$ from $S$ using Algorithm 8.
4 Assign the transit-buses from $d$ to $Y$ in $\mathscr{N}_2 = (d, Y, u_e, \tau_e, Z)$ for the supplies
    obtained in Step 3, to the nearest sink $Z$, on the first-come-first-serve basis.
5 Begin the assignment with $\alpha_1 \geq Q$, for $\alpha_1$ be the collection of evacuees at $y_1 \in Y$
    and Q be the homogeneous capacity of each transit-buses and is continued for the
    adjusted demands at $Y$ provided by the Equation (5.1).
6 Stop, if all the supplies at each $Y$ be fulfilled, respecting the capacity constraints of
    $Z$.
7 Otherwise, return to Step 4.
Output: Transshipment of evacuees finally to $Z$ in minimum clearance time.
```

Theorem 5.6. Algorithm 9 constructed for integrated evacuation planning algorithm in a prioritized embedding gives the feasible solution to send the evacuees to Z in minimum clearance time.

Proof. In Step 1, $\mathscr{N}_1$ is constructed with the pickup locations be Y and is feasible. In this prioritized sub-network by construction, Step 2 is also feasible. The arrival of evacuees determined at Y provided by Algorithm 8 gives the feasibility as well as the validity of Step 3 of Algorithm 9. Two more Steps 4 and 5 are about the transit-buses assignment in the integrated network and are governed not only by the availability of the buses at d but also by the supply available at Y. It is continued in $\mathscr{N}$ for the available evacuees respecting the capacity constraints of Z and are feasible. Hence, the algorithm is feasible.

Now, we show that Algorithm 9 gives the feasible solution in minimum time. The arrival of the lex-max number of evacuees at Y from S by using Algorithm 8 gives the quickest transshipment of evacuees in the given priority by [69] and saves the unused capacity by Theorem 5.3. These resulting flows at Y are taken as the input in $\mathscr{N}_2$ for the required transit-buses assignment. Such assignment to nearest sink approach respecting the priority is with almost negligible waiting delay for these evacuees at Y in the embedding. Such an assignment is continued till the last evacuees are reached to the sink without violating their capacities. Hence, it gives a feasible solution with minimum clearance time. Hence, the theorem is proved. □

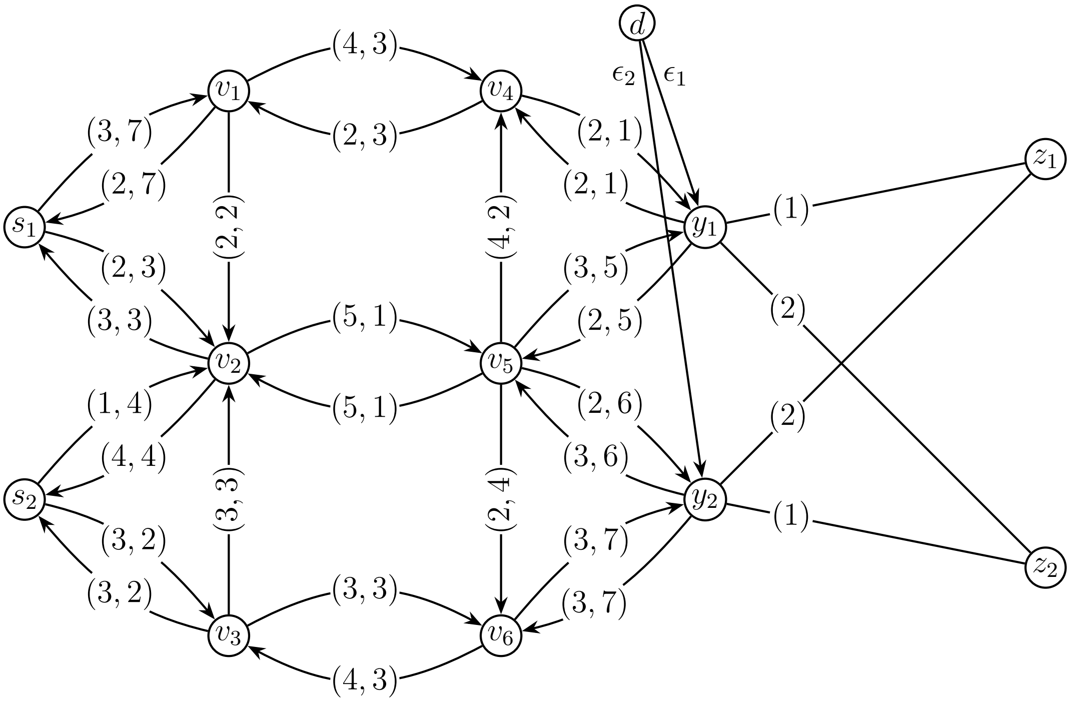


Figure 20: An embedding showing an instance of integrated evacuation scenario.

Example 5.3. Let the adjusted demands as in Table 10 and Table 11 be the available supplies for the bus assignment. Consider the available transit-buses be $|B| = 2$ as B_1 and B_2 and are with uniform capacity of $Q = 6$. The assignment concerning without and with path reversal capability in $\mathscr{N}_1$ are illustrated briefly in Table 12 and Table 13, respectively. Here, the column θ represents different time instances in the integrated network, B_1 and B_2 as the assignment of transit-buses with their respective positions in $\mathscr{N}_2$. Here, M_{12} and M_{21} are used to denote the mid-way of the road segments connecting y_1 to z_2 and y_2 to z_1, respectively. The columns for y_i, y'_i, and z_i for $i = \{1, 2\}$ are denoting the total flow arrived at Y, released from Y and reached to Z, respectively.

The quickest transshipment at Y and their assignment to Z are beneficial in the network having arc reversal capability than to the network without arc reversal capability, i.e., the sooner the better. It is 14, if the partial arc reversal is allowed otherwise 17, as illustrated with respect to θ. Here, the buses B_1 and B_2 have the route plan of $d \to y_1 \to z_1 \to y_1 \to z_1 \to y_2 \to z_2 \to y_2 \to z_2$ and $d \to y_1 \to z_2$, respectively concerning without arc reversal at $\mathscr{N}_1$. But, with respect to arc reversal at $\mathscr{N}_1$, the route plan of the buses are $d \to y_1 \to z_1 \to y_1 \to z_2 \to y_2 \to z_2$ and $d \to y_1 \to z_1 \to y_2 \to z_2$, respectively.

Table 12: Integrated approach for the demand adjusted evacuees.

θ	B_1	B_2	y_1	y_1'	y_2	y_2'	z_1	z_2
9	y_1		6	6				
10	z_1		9				6	
11	y_1		12	12				
12	z_1		15		3		12	
13	M_{21}	y_1	18	18	6			
14	y_2	M_{12}			9	6		
15	z_2	z_2			11			12
16	y_2					11		
17	z_2							17

Table 13: Integrated approach for the demand adjusted evacuees with partial arc reversals.

θ	B_1	B_2	y_1	y_1'	y_2	y_2'	z_1	z_2
8	y_1		8	6				
9	z_1	y_1	12	12			6	
10	y_1	z_1	18	18			12	
11	M_{12}	M_{21}						
12	z_2	y_2			6	6		6
13	y_2	z_2			11	11		12
14	z_2							17

CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 Summary

Evacuation planning problem deals with sending the maximum number of evacuees from sources to sinks in minimum time as efficiently as possible. The dynamic network flow formulation has been found suitable optimization approach corresponding to different scenarios of the macroscopic evacuation planning. A compact overview of the fundamental backgrounds and basic denotation for the results presented in the thesis is addressed in Chapter 2, as a basic foundation. Various types of evacuation planning optimization problems, their mathematical formulations, and the solution strategies on different evacuation networks are presented briefly with appropriate examples and illustrations in Chapter 3. With some extensions, different aspects of BEPP are highlighted concerning evacuees collections, vehicle assignments, network structures respecting their embeddings, and the dominant route selections. Among four different upper bounds on the evacuation duration corresponding to the BEPP as in [55], we have proved their dominance analytically, and the better one is applied for the transit-vehicle assignment approach in the integrated evacuation network.

Evacuees are shifted to the pickup locations from the source in the primary sub-network for the evacuation. Such pickup locations are taken as the sources for the subsequent evacuation process corresponding to the secondary sub-network to minimize the overall network clearance time where the transit-vehicles are provided from the bus depot to the embedding. An embedding consists of primary and secondary sub-networks as the evacuees' collection and vehicle assignment sub-networks, respectively. An integrated optimization approach is proposed in such an embedding to achieve the minimum clearance time. The earliest arrival pattern during the collection of evacuees at the primary sub-network respects the partial lane reversal strategy, whereas the better assignments of transit-buses in the embedding are based on the dominance relations concerning the evacuation duration.

In this work, evacuees are collected from the disaster zone to the pickup locations of the primary sub-network in minimum time as the quickest transshipment by using the lex-max flow approach. Considering such pickup locations as the sources, the available set of transit-buses are also assigned in the network to evacuate the evacuees safely to the sinks on a first-come-first-serve basis. This novel approach proposed here is better-suited for the simultaneous flow of evacuees with minimum waiting delay at such pickup locations to achieve the minimum clearance time in such a prioritized embedded evacuation network.

6.2 Conclusions

The effectiveness of the solution of BEPP depends upon the evacuee arrival patterns at the pickup locations and their appropriate assignment to transit-vehicles in the available evacuation network. BEPP is an important tool for transit-based EPP. Most of the available BEPPs in literature have considered the evacuees gathered themselves at different pickup locations and were silent about their specific arrival patterns. In our work, we have considered the earliest arrival patterns on an embedded evacuation network for the better-suited arrival pattern of the evacuees as it maximizes the collection of evacuees at every time units from the beginning. However, the EAF does not exist in the network with multiple sinks for general transit times. Under some characterizations, some of the specific networks always permit for the EAT regardless of all choices of the capacities and balances in zero transit times. In our work (cf. Chapter 4), such an EAT is considered. We have integrated the EAE collection network with partial arc reversal strategy to a bus-based mixed network and introduced a model for IEPP. We present a polynomial-time EAE algorithm following the principle of TRF to solve the EAE problem with zero transit times and partial arc reversal capability. Such evacuees collected at different pickup locations of the primary sub-network are considered as the supplies during the subsequent evacuation process. The transit-vehicles in the embedding are also assigned dominantly to achieve the minimum clearance time in such embedding.

However, for large-scale disasters with a sufficiently large number of evacuees, all the evacuees may not arrive at the pickup locations at the same time, and it requests a certain waiting time before starting the bus assignment. Those who are delivered earlier will have comparatively more waiting time at the pickups. Meanwhile, for the evacuees, waiting at pickups is comparatively better than to be in a danger zone. Such waiting is reduced to some extent in the prioritized embedded network, (cf. Chapter 5). The assignment of transit-vehicles in such a prioritized embedding is also carried in a dominating solution approach by adjusting the potential demands of the pickup locations to have minimum effecting waiting in the resulting in the minimum evacuation duration.

For such an approach, evacuees are collected at the prioritized pickup locations of the primary sub-network following the quickest transshipment in the lex-max flow and are assigned simultaneously to the homogeneous transit-buses in the secondary sub-network on the first-come-first-serve basis. The waiting delay at the pickup locations is almost negligible, (cf. Chapter 5). The arc reversal capability of the primary sub-network is beneficial to improve the minimum clearance time of the evacuees in the embedding and also in the prioritized embedding where the saved unused arc capacities are useful for logistics. For such a prioritized embedding, we have introduced a model for an IEPP with a different solution approach. Different analytical issues are addressed to improve the performance of the evacuation in an integrated framework, though the problem itself, in general, is challenging.

6.3 Recommendations for further work

In our work, we have considered the earliest arrival patterns of the evacuees on an embedded evacuation network as the better-suited arrival pattern as it maximizes the collection of evacuees at every time units from the beginning. But such a transshipment does not exist for general transit times in the network with multiple sinks, though some of the specific networks permit it regardless of all choices of the capacities and balances at zero transit times. Its extensions for the arbitrary time setting and their different approximation approaches, including the experimentation and different case studies closer to the real scenarios, are the further research extensions.

Here in our problems, the road segments are considered with symmetric transit times and will satisfy the triangle inequality, which might not be in real practice. Usually, most of the network structures will partially or fully be damaged in mega-disasters that demands network clearance first rather than evacuation planning. Like others, we too have considered the homogeneous group of transit-vehicles in our system. It is interesting to extend these techniques for the heterogeneous or mixed model transit-buses respecting their multi-commodity flows, their cost factors, and also for the disparate group of evacuees in different network topology, like lossy or gainy network, grid network, etc. On a slightly different track, the problem will be more interesting if the congestion due to personal vehicle movements, crossing and merging effects in the networks, availabilities of resources during the evacuation including other uncertainties as in real-life situations are also addressed.

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Appendix A

INDEX

Appendix B

B.1 List of Publications

1. Dhamala, T. N., & Adhikari, I. M. (2018). On evacuation planning optimization problems from transit-based perspective. *International Journal of Operation Research*, 15(1), 29-47.
2. Adhikari, I. M., Pyakurel, U., & Dhamala. T. N. (2020). An integrated solution approach for the time minimization evacuation planning problem. *International Journal of Operation Research*, 17(1), 27-39.
3. Adhikari, I. M., & Dhamala, T. N. (2020). Minimum clearance time on the prioritized integrated evacuation network. *American Journal of Applied Mathematics*, 8(4), 207-215.
4. Adhikari, I. M., & Dhamala, T. N. (2020). On the transit-based evacuation strategies in an integrated network topology. *The Nepali Mathematical Sciences Report, 37 (1 & 2), 1–13.*
5. Dhamala, T. N., Adhikari, I. M., Nath, H. N., & Pyakurel, U. (2018). Meaningfulness of OR Models and Solution Strategies for Emergency Planning, In: Living Under the Threat of Earthquakes, 175-194, Springer Natural Hazards, Springer.
6. Adhikari, I. M., & Dhamala, T. N. (2018). An insight on the evacuation planning optimization problems on transit-based system. In: 11th Triannual International Conference on Operation Research, 132-135.
7. Adhikari, I.M., & Dhamala, T. N. (2017). Dominance vehicle routing in transit dependent evacuation scenario. In: National Conference on Mathematics and Its Applications, 54-60.

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On Evacuation Planning Optimization Problems from Transit-based Perspective

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Abstract: Increasing number of complex traffic networks and disasters today has brought difficulty in managing the rush hours traffic as well as the large events in urban areas. The optimal use of the vehicles and their assignments to the appropriate shelters from the disastrous zones are highly complicated in emergency situations. The maximum efficiency and effectiveness of the evacuation planning can be achieved by the appropriate and significant assignment of the transit dependent vehicles during pre and post-disaster operations.

This paper presents a comprehensive overview of the evacuation planning optimization techniques developed over the years, emphasizing the importance of their formulation and the solution strategies on disaster management from the transit-based perspective. Each technique is briefly described and presented lucidly with some of its known applications, significances, and solution strategies expecting that it should be able to guide much more interest into this important and growing area of research.

Keyword — Disaster management; evacuation planning; transportation network; vehicle assignment.

1. INTRODUCTION

Evacuation planning is an important aspect of disaster management. Emergency evacuation is the immediate and urgent movement of people away from threat, from the danger zone to the safety zone. According to (DHS, 2004) an evacuation is "organized, phased, and supervised withdrawal, dispersal, or removal of civilians from dangerous or potentially dangerous areas, and their reception, and care in safe areas." It concentrates mainly to find the optimal use of vehicles and the routes as effectively as possible with utmost reliability. The most fundamental necessity of human beings is saving lives which should be the core objective of the planning. But the life of a human being is always in danger and under the threat because of natural or man-made disasters. Most of the disasters cannot be predicted and are unavoidable, and the damages caused by them are severe. The increasing rate of such disasters demands the comprehensive analysis and planning for the evacuation management. Past evacuation experiences on different situations are to take account of planning and mitigation strategies which are followed by the response and recovery to normalize the situation. Disaster operations can be performed before or after the disasters as pre-and post-disaster operations. Short-notice evacuations, facility location, and stock pre-positioning are carried out as the main pre-disaster operations whereas the relief distribution, logistic support services, and the casualty transportations are the main aspect of post-disaster operations (Caunhye et al., 2012; Dhamala et al., 2018). In evacuation planning, auto-based and transit-based evacuees can be categorized like high and low-mobility populations, respectively. The former are supposed to withdraw the hazardous area by using their own vehicles whereas the latter need to be sent to the transit hubs for further evacuation. In large cities of developing countries, many people fall into low-mobility population and are to be given a special attention due to their ages, language inefficiencies, different health problems, or other physical disabilities. The great loss of people on Hurricane Katrina was due to the lack of proper planning for the transit-based evacuees (Litman, 2006).

The traditional vehicle routine problem (VRP), deals for the distribution of goods from different depots to customers to design the least cost delivery is the main root of transit dependent evacuation planning. It has several variations depending on different contexts, among them relief distribution, logistic support and management, and evacuee transportation are of great importance on emergencies. Among its different extensions, the Split Delivery Multi Depot Vehicle Routing Problem with Inter-depot Routes is relatively similar and applicable form of VRP in evacuation scenarios for transit dependent vehicles. For an overview of VRP variants and their different applications, we refer to (Kumar and Panneerselvam, 2012; Laporte, 2007).

Basically, evacuation models can be classified into two broad categories, microscopic and macroscopic. The

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former emphasizes individual parameters like walking speed, physical ability, reaction time, and the interaction of each evacuee with other evacuees during the movement and are based mainly on simulation (Dhamala, 2015). But in later the occupants are treated as a homogeneous group and are taken account of their common characteristics only and are able to produce mainly the lower bounds (Hamacher and Tjandra, 2002). These two aforementioned models can also be combined in an interactive solution process as in (Hamacher et al., 2011), where the output of one is used as input to other so that the output of both remains stable and is named as sandwich method. The effectiveness of transit-based evacuation highly depends on the location of pick-up facilities, allocation of the resources and evacuation management system. Different mathematical models were developed to address such transit-based evacuation under both predictable and unpredictable disasters and are integrated to the location-allocation design with the routing, assignment and scheduling of the buses on evacuation network. In a real scenario, the evacuee walking time to various pick-ups, their waiting and loading time at pick-ups, service delays on the system etc. are also to take account which is rarely been considered in existing literature. Comparatively, a reliable location model has been developed in (Shi et al., 2013) which integrate the pre-and post-emergency operations in an exponential number of facility disruption scenarios including their interconnection to pick-up location, service facility and the vehicle assignment to determine the optimal evacuation, however, the assumptions like constant rescue demand with identical and independent facility disruptions may not always be realistic. Moreover, not only the evacuation network, pick-up facilities might also be disrupted in real practice. Hence, during designing of the evacuation system, the care should be taken not only on the efficiency but also on the reliability and efficacy.

Evacuation problems were combined with location analysis to reduce the evacuation time by facility location approach in (Hamacher et al., 2013). In a modeling by (Chen and Zhan, 2008) on three different types of road networks like a grid network, a ring road structure, and a real road network the overall benefit of the chosen bus stops located within the evacuation area has been maximized by dividing the area into smaller sub-sections, zones can be grouped together, and a minimum number of bus stops can be set for the sub-section of evacuation bus stops where the total benefit will increase as the total number of optimum bus stops increased. Assuming that a complete evacuation is not possible, authors in (Sayyady and Eksioglu, 2010) used to maximize the number of evacuees served, which incorporates traffic flow dynamics from the simulation package with a logistic function to estimate the number of evacuees at each pickup location, likewise for solving the problem of refuge location through facilitating buildings to provide shelter to the victims with the quality service after a disaster has also been highlighted in (Pèrez-Galarce et al., 2017). Dealing with the household behavior under the emergency evacuation scenarios, authors in (Murray-Tuite and Mahmassani, 2003) have provided two formulations to determine the meeting location for household members and their sequence pick-up. Vehicles are distributed according to the location of the drivers and are taken heterogeneous depending upon their capacity which is more convenient in an emergency.

Evacuation planning strategies, models, methods, and their operations may vary due to their applicable geographical scales, total affected population size and density, behavioral and organizational situations, modes of transportation, traffic capacity, evacuation objectives, and the time spans. An emergency situation caused by different factors like fire, nuclear reactor accident, terrorist attack, hurricanes, earthquakes, floods or landslides are all different in nature. Evacuation scheduling, traffic route guidance, destination optimization, optimal route choice, and other various approaches have significantly contributed to accelerate the evacuation process, even then the integrated optimal plan to have a single comprehensive solution to the problem is lacking for real case scenarios. Different models and strategies have been developed so that the solution methods can be applied effectively to the realistic networks of reasonable size and also with possible extension of further improvements on different parameters to enhance the performance of evacuation process. Significant contributions have been made by many researchers in the scientific field of evacuation planning utilizing the highly prominent transit like in (Abdelgawad and Abdulhai, 2012; Chiu et al., 2007; Hobeika and Kim, 1998; Shayti and Mahmassani, 2006). An overview of the mathematical modeling and algorithms of evacuation problems has been presented in (Bretschneider, 2012; Hamacher and Tjandra, 2002) whereas, different surveys of discrete dynamic network flows are in (Aronson, 1989; Dhamala, 2015).

Contraflow has gained a considerable attention in evacuation literature because by finding the ideal direction of lanes of a road network, the flow can be increased and evacuation time can be reduced as compared to the evacuation in the existing road reconfiguration and is applicable to reduce congestion, eliminates the crossing at intersections and traffic jams during the day-to-day rush hours.Depending upon the objectives, different contraflow variants on the evacuation models like maximum dynamic contraflow, lexicographic contraflow, the earliest arrival contraflow and many more have been studied by the authors in (Dhamala and Pyakurel, 2013; Pyakurel et al., 2015; Pyakurel, 2016; Pyakurel and Dhamala, 2014; Pyakurel and Dhamala, 2015).

A critical review on the evacuation planning of network design problem has been presented in (Abdelgawad and Abdulhai, 2009) which has reviewed and compiled the main evacuation strategies, network design problem formulation, traffic simulation and the optimization tools. Depending upon the nature and circumstances of the disasters a survey in (Xiongfeil et al., 2010) has suggested for the improvements of models with more reasonable and realistic assumptions including travel behaviors. There are many uncertain factors in disasters like evacuee's route choice behavior, departure time, road capacity and so on, which demands the stochasticity and robustness.

In this paper, we present a comprehensive overview of the most important transit dependent evacuation approaches focused on disaster management that were not covered widely in an organized form in the literature. Section 2 presents the transit-based evacuation model from different perspectives. Different solution strategies will be discussed in section 3 whereas the applications in section 4 with the conclusions in section 5.

2. TRANSIT-BASED EVACUATION MODEL

Transit has a unique role in evacuating the car-less, elderly, and the needy populations with different disabilities. Even when transit evacuation is planned carefully, communications and logistics issues are taken care of, the behavior, knowledge, attitude and nature of the evacuees still play a major role in effective emergency evacuations. Moreover, the knowledge, ages, readiness, and/or the willingness of people to evacuate may be different, it also demands behavioral analysis. Furthermore, the lack of coordination between the transit agencies and the traffic operators may highly affect the system. In a survey of about high-risk area of hurricane as in (Blendon et al., 2005) have noticed that about 54 % of households, the traffic congestion was the main reason for not evacuating on such high-risk hurricane strikes and has noticed that more fatalities were caused by evacuation than the hurricane. Whereas, in a survey (Litman, 2006) it was noticed that, 71 % of those who died in Hurricane Katrina in New Orleans were age of 60 and 47 % over of 75. This also demands the need of transit vehicles for effective evacuation.

To cope the situations, a prominent bus-based evacuation problem (BEP) model, as a unique variant of VRP is proposed by (Bish, 2011), with the objective to minimize the time of evacuation in case of a short notice using given number of homogeneous buses. It is formulated as a mixed integer linear program in which the decision variables determine the assignment of routes to buses and assignment of buses to the evacuees so that the evacuation time of the last evacuee to reach the safe destination is minimized for the given number of evacuees at the sources. For the formulation of BEP network, let N be the set of nodes and A be the set of arcs in the networks (N, A), where N is composed of three subsets Y, P and S where Y stands for the set of yards at which buses are initially located and dispatched from; P, as a set of demand nodes representing pickup locations requiring the evacuation services; and S, as a set of shelters (sinks) where the evacuees are to be transported. Let V be the available vehicles (say buses) each having a capacity Q is subdivided into the subsets V_i for $i \in Y$, and the bus $j \in V_i$ is initially located at yard i. Let the demand node j has a demand $D_j, j \in P$ and shelter i has a capacity $C_i, i \in S$. Then the arc(i, j) has a non-negative travel cost of τ_{ij} for $(i, j) \in A$. In fact, the travel cost is proportional to the travel time and distance. All costs in the network are taken symmetric and are supposed to satisfy the triangle inequality for all arcs.

Decision variable

x_{ij}^{mt} : 1, if trip t for bus m transverse arc(i, j), else 0, $\forall (i, j) \in A, m \in V, t = 1, 2, .., \theta$.

b_j^{mt} : No. of evacuees assigned to bus m after trip t (or, released from, if j is a shelter),

$\forall j \in N, m \in V, t = 1, 2, ..., \theta.$

Γ_{evac} : duration of evacuation

BEP formulation

$$minimize \quad \Gamma_{evac} \tag{1}$$

$$such\ that \quad \Gamma_{evac} \geq \sum_{(i,j)\in A} \sum_{t=1}^{\theta} \tau_{ij} x_{ij}^{bt}, \forall m \in V \tag{2}$$

$$\sum_{i:(i,j)\in A} x_{ij}^{mt} = \sum_{K:(i,j)\in A} x_{jk}^{m(t+1)}, \forall j \in P, m \in V, t = 1, 2, \ldots, \theta - 1 \tag{3}$$

$$\sum_{i:(i,j)\in A} x_{ij}^{mt} \geq \sum_{K:(j,k)\in A} x_{jk}^{m(t+1)}, \forall j \in S, m \in V, t = 1, 2, \ldots, \theta - 1 \tag{4}$$

$$\sum_{(i,j)\in A} x_{ij}^{mt} \leq 1, \forall j \in P, m \in V, t = 1, 2, \ldots, \theta - 1 \tag{5}$$

$$x_{ij}^{m1} = 1, \forall j : (i, j) \in A, m \in V_i \tag{6}$$

$$x_{ij}^{mt} = 0, \forall j : (i, j) \in A, m \in V, t = 2, \ldots, \theta \tag{7}$$

$$x_{ij}^{m\theta} = 1, \forall j \in P, (i, j) \in A, m \in V \tag{8}$$

$$b_j^{mt} \le \sum_{(i,j) \in A} Q x_{ij}^{mt}, \forall j \in N, m \in V, t = 1, \ldots, \theta \tag{9}$$

$$0 \le \sum_{j \in P} \sum_{l=1}^{t} b_j^{mt} - \sum_{k \in S} \sum_{l=1}^{t} x_k^{ml} \le Q, m \in V, t = 1, \ldots, \theta \tag{10}$$

$$\sum_{m \in V} \sum_{t=1}^{\theta} b_j^{mt} \le C_j, \forall j \in S \tag{11}$$

$$\sum_{m \in V} \sum_{t=1}^{\theta} b_j^{mt} \le D_j, \forall j \in P \tag{12}$$

$$\sum_{j \in P} \sum_{t=1}^{\theta} b_j^{mt} = \sum_{k \in S} \sum_{t=1}^{\theta} b_k^{mt}, \forall m \in V \tag{13}$$

$$x_{ij}^{mt} \in \{0, 1\}. \forall (i, j) \in A, m \in V, t = 1, 2, \ldots, \theta \tag{14}$$

$$b_j^{mt} \ge 0. \forall (i, j) \in A, m \in V, t = 1, 2, \ldots, \theta \tag{15}$$

Constraint (2) needsthe evacuation duration be greater than or equal to the maximum cost incurred by the bus with the highest travel cost and is to be minimized by (1) as the "min–max" objective. Constraint (3) ensures that a bus traveling to demand node j on trip t leaves node j on trip $t + 1$, the flow-balance constraint for the demand nodes. Constraint (4) ensures that the last trip of the bus may end at a shelter, the flow-balance constraint for the shelters. Constraint (5) allows a bus to make at most one trip at a time; constraint (6) tells that the first trip of each bus starts from its yard; constraint (7) tells that the buses do not leave the yard for later trips; and constraint (8) does not allow the last trip a bus can make to end at a demand node. Constraint (9) signifies that a bus can only pick up evacuees from node j ,if it is traveling to that node where constraint (10) and constraint (11) are the bus capacity and the shelter capacity constraints respectively. Where, constraint (12) and constraint (13) signify that all evacuees are picked up and are delivered to a shelter, respectively. Moreover, constraints (14) and (15) are the logical binary and non-negativity restrictions on the x and b variables, respectively.

Example 1: *In this instance as in Figure 1 with one yard, three demand nodes and three sinks, it is assumed that the number of evacuees at the demand nodes be as same as the vehicle capacity or its integral multiples where the demands be* $l_i = (3, 3, 1)$, *capacities at sinks be* $u_j = (3, 4, 3)$ *with buses available be* 3. *The distance of the demands from the yard be* $d = (4, 3, 6)$ *with the distances between* P *to* S *as*

$$\tau = \begin{bmatrix} 4 & 7 & 9 \\ 10 & 7 & 5 \\ 7 & 6 & 9 \end{bmatrix}$$

In general, BEP is formulated so as to choose the minimum of the evacuation times of all possibilities where the critical path of the plan is for B3 or bus B1 as in Table 1, which shows the feasible solution of BEP with the evacuation duration of 25.

Table 1. Table of feasible solution of BEP

Trip	1	2	3	Tour plan	Time
B1	(1,3)	(2,1)	-	$\tau_1 + \tau_{13} + \tau_{33} + \tau_{31}$	25
B2	(2,1)	(2,3)	-	$\tau_2 + \tau_{21} + \tau_{21} + \tau_{23}$	21
B3	(1,1)	(1,2)	(2,3)	$\tau_1 + \tau_{11} + \tau_{11} + \tau_{12} + \tau_{22} + \tau_{23}$	25

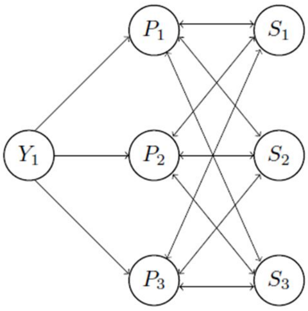

Figure 1: A simple BEP

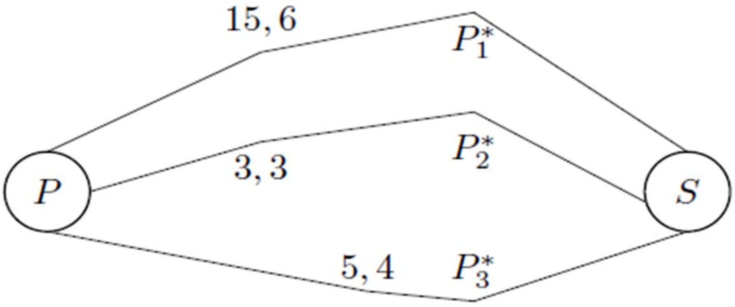

Figure 2: A simple evacuation network

Authors in (Goerigk and Grün, 2014) have presented lucidly the robust bus evacuation problem (RBEP), in which the exact numbers of evacuees are not known in advance though a set of likely scenarios is known and after sometime such uncertainty will be removed. In such instance, it is to decide whether the buses are better to send right now as the *here-and-now* bus under such uncertainty or to wait as *wait-and-see* bus until the exact scenario becomes known.

Considering both the transit time and capacity on each path, the concept of combined evacuation time (CET) and the quickest paths has been introduced by (Min and Neupane, 2011). For T be the travel time of the path (the sum of travel time of edges in the path), C be the capacity of path (the minimum capacity of the edges in the path), and x be the number of evacuees at the source P, the evacuation time (ET) is given by,

$$ET = T + \left\lceil \frac{x}{C} \right\rceil - 1. \tag{16}$$

The path P_1^* is said to be the quickest path if and only if,

$ET = T + \left\lceil \frac{x}{C_i} \right\rceil - 1 \le ET = T + \left\lceil \frac{x}{C_j} \right\rceil - 1, \forall j \in \{1,2,\dots,k\} \setminus \{\mathrm{i}\}$. Let $P_1^*, P_2^*, \dots P_k^*$ be edge-disjoint paths from source P to sink S with C_i and T_i be the capacity and transit times of paths P_i^* and x be the number of evacuees at P then the combined evacuation time is given by

$$CET\left(P_1^*, P_2^*, \dots P_k^*\right) = \left\lceil \frac{x + \sum_{i=0}^{n} C_i T_i}{\sum_{i=0}^{n} C_i} \right\rceil - 1. \tag{17}$$

Example 2: *Consider three possible paths* $P_1^*, P_2^*\ P_3^*$ *in between demand node* P *and the sink* S *with their respective travel time* T *and capacity* C *as in Figure 2. Suppose the evacuees at demand node be* 52 *then the* ET *through these paths can be calculated by using equation (16) be* 23, 20 *and* 17 *respectively among them* $ET\left(P_3^*\right)$ *be chosen as the quickest path. But if the next path* P_1^* *be added on the evacuation route then the* CET *as in equation (17) becomes* $CET\left(P_3^*, P_1^*\right) = 16$ *which is shorter than the current evacuation time. Moreover, by adding the next path* P_2^* *also on the route* CET *becomes* $CET\left(P_1^*, P_2^*, P_3^*\right) = 13$ *further improved, which is smaller even than the route* P_1^**, with longest travel time. Hence, we can remove the route* P_1^* *with longer travel time from the evacuation route, since* $CET\left(P_2^*, P_3^*\right) = 11 < CET$ *and the evacuation time be reduced even more by* 2*. Note that, the purpose of adding paths into the evacuation route is to reduce the* ET *by distributing the evacuees in multiple paths and will be terminated when the* CET *by the current quickest path becomes greater than the previous* CET*. Running time is determined by the number of iteration which is bounded by the total number of paths in and does not depend much on the number of evacuees.*

2.1 Dominant Vehicle Assignment on Transit Routes.

Among all feasible routes on the evacuation network, a route p is said to dominate another route p' and is named as the dominant route, if it does not have longer evacuation duration, it does not have the more cost for the demand nodes considered and every unreachable demand nodes on route p is also unreachable for p'. If different routes lead to the same shelter and neither of them is better than the others over all criteria, then neither of the routes is dominating. There are different factors affecting for the selection of such dominant routes and mainly depends upon nature of the available network. The problem is not only on the selection of a route and the selection of a shelter for each route but also on the route-to-vehicle assignment and vice versa, which is more complex in practice. Furthermore, the network adds complexity to the solution and impacts on the problem. It needs the modification and extension on the model to make it more realistic and applicable.

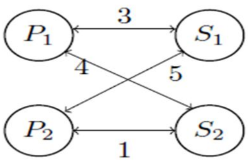

Figure 3: A simple network

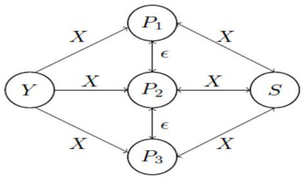

Figure 4: A split delivery network

In general, it seems that sending each vehicle to its closest shelter to last pickup node is the appropriate route to be assigned for the optimal routing. Likewise, one may assume that the split delivery network will improve the evacuation duration. But, it is not always (Dror, 1990).

Example 3: *In this simple network as in Figure 3, let a bus which has picked up a full load of evacuees at* P_1 *can still pick up a full load at* P_2 *with uncapacitated shelters* S_1 *and* S_2*. Then, if the shelter closest to its last pickup node is assigned, then the bus would be on* $P_1 - S_1 - P_2 - S_2$ *and will have the cost of 9, whereas the optimal solution would be on the route* $P_1 - S_2 - P_2 - S_2$ *with cost of 6.*

Example 4: *Consider a simple split delivery network as in Figure 4 where Y and S are yard and sink respectively, with* P_1*,* P_2 *and* P_3 *be three different demand nodes. The evacuation durations with and without split delivery and their respective bounds are shown as in Table 2, where the vehicles were scheduled simultaneously. This signifies that the split delivery network will not always improve the evacuation duration.*

Table 2: Table showing the observations.

No. of Vs.	With SD	Without SD	Bound	Remarks
1	4X+ϵ	6X	1.5	Improved
2	2X+ϵ	4X	2	Improved
3	2X	2X	1	Not improved

Moreover, in Figure 4, the evacuation cost is 6X for 3 vehicles without split delivery where the cost becomes approximately the same, i.e., $6X + \varepsilon$, only for 1 vehicle. Which signifies that 1 vehicle covering all the routes will have approximately the same cost as multiple vehicles covering the same routes in such simple evacuation network.

2.2 Location Allocations of Transit Vehicles

Authors in (Zhang and Chang, 2014) have proposed a model to determine the pick-up locations within several clusters of demand zones for the routing and scheduling of transit vehicles based on vehicle availability and the time-dependent evacuee demand pattern. By suggesting the equilibrium of the evacuee arrival process and the functioning pick-up facility, authors in (An et al., 2013) have presented the optimal resource allocation strategy to balance the trade-off between evacuees' risks and the operation costs, and about its dynamic nature in (He and Peeta, 2014) addressing of when, for how long, and where to assign to improve the evacuation efficiency.

Let $\left|P_i\right|$ be the number of evacuees in pick-up location who need to be transported one of the shelters; $A = \left|f_{ij} : 1 \le i \le n, 1 \le j \le m\right|$ be the allocation of fleet; f_{ij} be the amount of fleet assigned to transferring the evacuees from P_i to S_i with τ_{ij} be the time for round trip between them including the boarding and alighting time. Define $\eta_{ij} = \dfrac{f_{ij}}{\tau_{ij}}$ be the number of evacuees transferred per unit time from P_i to S_i and $\tau_i = \sum_{j=1}^{m} \eta_{ij} = \sum_{j=1}^{m} \dfrac{f_{ij}}{\tau_{ij}}$ the rate at which evacuees are transferred from P_i to any of the shelters. Then maximum evacuation rate resource allocation is to maximize the number of evacuees who reach safety by any given deadline after the evacuation, under some capacity constraints, and is given by,

$$maximum \quad s\left(\Gamma\right) = \Gamma \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{f_{ij}}{\tau_{ij}} \tag{18}$$

For S^* be the optimal solution of (18) as in (Aalami and Kattan, 2017), the minimum network clearance time resource allocation is to evacuate the whole endangered population to shelters in the shortest possible time, i.e.

$$minimum \quad \left\{ S^*\left(\Gamma\right) \ge \sum_{i=1}^{n} \left|P_i\right| \right\} \tag{19}$$

Theorem 1. (Aalami and Kattan, 2017), Let $\kappa_i = \dfrac{\left|P_i\right|}{\tau}$ denote the clearance time of P_i. Then in the minimum clearance time resource allocation, $k_i = k_j \forall i, j \in \left\{1, 2, \ldots, n\right\}$.

Proof: Let $K_x = \left\{ i : k_i = \max_j k_j \right\}$ and $K_y = \left\{ i : k_i = \min_j k_j \right\}$ be the set of indices of the pick-up locations with largest and smallest clearance time respectively. K_x and K_y both are non-empty. Assume the contrary, $K_x \cap K_y = \varnothing$. If not, then, $x_j k_j = \min_i k_i$, proof becomes obvious. So for, $K_x \cap K_y = \varnothing$ the network clearance time can be reduced by taking a small portion of the resonance from the pick-up locations with indices in K_x and allowing them to pick-up locations with indices in K_y which contradicts the assumption. Hence, $K_x \cap K_y = K_x = K_y$. ■

However, in general, minimizing the evacuation time for all pick up locations is desired during evacuation, but if a city is threatened by wildfire then the neighborhoods close to the wildfire are supposed to be evacuated before the ones faraway where the minimum network clearance time resource allocation is not the right choice.

2.3 Lane-based Vehicle Assignment for Transit Vehicles during Congestion

Most of the traffic delays and the potential accidents are due to the merging and crossing conflicts at the intersection. To address this, comparatively a smart traffic routing without crossing and merging conflicts has been proposed by (Cova and Johnson, 2003) and is further improved by many others like (Bretschneider, 2012; Bretschneider and Kimms, 2011; Xie and Turnquist, 2011), following the assumption that, vehicles have to order in the appropriate lanes that correspond to their subsequent turn before they enter the intersection and the restructuring of traffic routing with regard to a safe evacuation process to minimize the evacuation time. Likewise, the effectiveness of lane based system even in the damage traffic sensors and interrupted communication system has been presented by (Ardekani and Hobeika, 1988). Different heuristic approaches have been suggested in (kimms and Massen, 2011; Kim et al., 2007) to solve the problem of minimization of evacuation egress time with time-dependent node and arc capacity by lane reversal approach. The demand management strategies of staging and routing have been presented in (Bish et al., 2013) incorporating to some extent for the evacuee behavior aspects subjected to route the vehicles to their closest evacuation zone exit and to minimize the number of intersection merging-conflicts, which also satisfy the shortest evacuation plan criterion given by (Yamada, 1996).

Lane-based reversal design and routing with intersection crossing conflicts elimination for the evacuation has been integrated in a bi-level model in (Zhao et al., 2016) by applying a tabu search algorithm to find an optimal lane reversal plan in upper-level and the simulated annealing algorithm on the lower level consisting of single arc and multiple arcs approaches on lane based route plans with intersection crossing conflict elimination to minimize the total evacuation time on the network. Such network optimization model with the bi-level scheme has been formulated in (Liu and Luo, 2012) with the upper level determining the best sets of signalized and uninterrupted flow intersections and the lower-level handling routing assignment of evacuation traffic demand. The upper level describes the behavior of the policy makers or planners for minimizing the total evacuation cost whereas, the lower-level problem captures the behavior of evacuees in choosing the evacuation routes under the budget constraints. In fact, such information is critical for emergency managers to allocate the limited resources to the most appropriate location and the mass transit VRP can been solved iteratively between two levels of problems as the transit problem and the passenger problem as in (Pages et al., 2006), where the transit problem has been taken as the initial problem and its initial solution is used to improve by assigning the passengers on the routes. The transit signal priority method in (Lin and Gong, 2016) has given the priority on (i) transit vehicle arrival time estimation, (ii) queuing vehicle dissipation time estimation, (iii) traffic signal status estimation, (iv)transit signal optimization, and (v) arterial traffic signal coordination for transit vehicle in evacuation route. In a survey, with some demographical analysis of the Upstate New York city, the authors in (Hess and Gotham, 2007) have suggested to the planners, transit providers, emergency management officials and even to the researchers for the development of multi-modal mass evacuation plans with the incorporation of more high-capacity vehicles for the comprehensive and effective emergency management plan for the large scale evacuation.

2.4 Cost Objective and Min-max Objective on Transit Vehicle Assignment

(Bish, 2011) illustrates the impact to the optimality of the solution on the min-max objective given by (1) and the cost objective, where the cost objective is taken as,

$$minimize \quad \sum_{(i,j)\in A} \sum_{m\in V} \sum_{i=1}^{\theta} \tau_{ij} x_{ij}^{mt} \tag{20}$$

Both the min-max and the cost objective can have multiple optimal solutions, some of which are better than others and become the dominating solutions. In case of multiple optimal solutions, there might be some bottle-neck vehicles where some of the solutions may include the undesirable or unnecessary routes and may increase the costs and trips on evacuation process. An alternative lexicographic min-max objective with the lexicographic constant L has been introduced as,

$$minimize \quad \Gamma_{evac} + \frac{1}{L} \sum_{(i,j)\in A} \sum_{m\in V} \sum_{i=1}^{\theta} \tau_{ij} x_{ij}^{mt} \tag{21}$$

The first term denotes the evacuation duration and it lexicographically dominates the second term denoting the total evacuation cost on cost objective. (Sherali, 1982) has considered the lexicographic constant, $L = \sum_{m\in V} \sum_{t=1}^{\theta} \max \left\{ \tau_{ij} : \left(i, j\right) \in A \right\}$ as the maximum possible value of the second term such that the first term will lexicographically dominates the second term. As a dual, the cost and duration can be minimized lexicographically as,

$$minimize \quad \sum_{(i,j)\in A}\sum_{m\in V}\sum_{i=1}^{\theta}\tau_{ij}x_{ij}^{mt} + \frac{1}{L}\Gamma_{evac} \tag{22}$$

By various approaches including some empirical evidences (Bish, 2011) has concluded some useful results on BEP regarding the fleet size and different objectives:

●*For a single vehicle as the fleet size, the min-max and cost objectives have equivalent optimal solutions.*

●*For the min-max objective there is an optimal threshold fleet size. Increasing the fleet size beyond this threshold does not impact optimality.*

●*For the min-max objective, the evacuation duration does not always decrease in a convex manner with the number of vehicles.*

In another approach, the authors in (Campbell et al., 2008) concentrate on the min-max objective to minimize the time until the last delivery is made for the relief supplies in a disaster. In general, when the cost objective is taken, it becomes the route selection problem and if min-max objective, then it becomes the route selection as well as the route-to-vehicle assignment, and is obviously more complex.

As already mentioned above, the VRP objective minimizes the total routing cost for the entire vehicle considered where the minimization of the duration of evacuation, i.e., the min-max objective for the routing cost is concerned on BEP. Basically on evacuation, minimizing the cost should not be the primary concern as the most common objective is to minimize the time till the last delivery, including the safety of drivers and evacuees. Besides this common mini-max objective on BEP, authors in (Sayyady and Eksioglu, 2010) have addressed to identify the number of public transit vehicles needed to evacuate all transit-dependent citizens during no-notice evacuation. Not only this, the model is concentrated to identify paths for vehicles to have the minimum the number of casualties; to minimize the total evacuation time; and to maximize the vehicle utilization on the system. Moreover, their model maximizes the number of evacuees served assuming that a complete evacuation is not possible and thus the objective function is quite different from the common min-max objective.

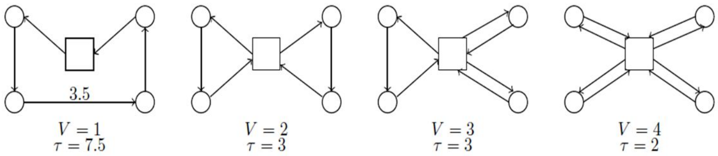

Figure 5: Different scenarios of vehicle assignment

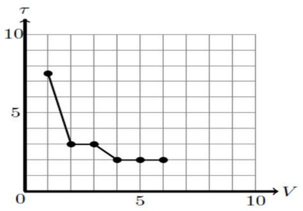

Figure 6: Relation between τ & V

Example 5: *Consider a simple scenario with four different assignments with one, two, three and four vehicles as in Figure 5 where vehicles start from the depot S denoted by square and assigned to the demands P_i denoted by circles and finally return to the depot again. Let all the unmark arcs are with length unity, the relation between the number of vehicles V, and the respective maximum route length τ is shown in the Figure 6.*

2.5 Integrated Contraflow Approach of the Vehicle Assignment on Transit-based Evacuation

Authors in (Kim et al., 2007), study the microscopic model for the reconfiguration of transportation network and provide a pair of heuristic approaches as the *greedy* and *bottleneck relief* for the high quality solution with significant performance and for the large scale evacuation, respectively and they improve the evacuation egress time by about 40 percent or above in different experimental results. In a recent work, different analytical solutions of continuous time contraflow problems has been presented in (Pyakurel et al., 2017), with an extension of dynamic contraflow to more general setting where the given network is replaced by a two terminal abstract contraflow network with each element having symmetric capacity and established a remarkable theorem with its analytical proof, on the flow value.

Theorem 2.(Pyakurel et al., 2017) If the minimum d*ynamic abstract cut capacities are symmetric for a two terminal abstract contraflow network, the flow value can be increased up to double with contraflow reconfiguration.*

An integrated contraflow strategy has been presented by (Hua et al., 2014)containing non-contraflow to shorten the strategy set up time, full-lane contraflow to maximize the evacuation network capacity and bus contraflow to realize the transit cycle operation. Here, the routing problem of the transit-based evacuation has been considered as the minimum cost flow problem with multiple origin nodes and single super destination node. This is a mixed integer linear programming problem which can be solved in a very efficient way using the branch and bound method. The auto-based evacuation method has a bi-level structure and is usually solved by using heuristic algorithms as in (Miandoabchi and Farahani, 2011). The evacuation model has been formulated with the objective to route the transit vehicles to their closest evacuation destination as follows:

$$minimize \quad Z_D = \sum_{(i,j)\in A_R} x_{ij}^{*} l_{ij} \tag{23}$$

$$subject\ to \quad \sum_{j\in\Gamma_i} x_{ij}^{*} - \sum_{j\in\Gamma_i'} x_{ki}^{*} = 0, \forall i \in N_A \cup N_I \tag{24}$$

$$\sum_{j\in\Gamma_i} x_{hi}^{*} = d_h^{*}, \forall h \in N_S \tag{25}$$

$$x_{ij}^{*} \le c_{ij}^{*} z_{ij}, \forall \left(i,j\right) \in A_R \tag{26}$$

$$0 \le x_{ij}^{*}, \forall \left(i,j\right) \in A_R \tag{27}$$

Constraints (24) and (25) describe the equality of inflow and outflow volume i.e., the flow conservation. Constraint (26) ensures the proper amount of flow on each link $\left(i,j\right)$ i.e., if $z_{ij} = 1$ then the bus contraflow configuration is applied on the link, otherwise the link is not served on evacuation for N and A be the set of nodes and links, as usual for $N = N_S \cup N_A \cup N_I \cup N_D$ as different nodes like super origin, access, intersection and destination nodes. The lower bounds on all transit-based flow variables are provided by constraint (27).

A multi-modal optimization evacuation framework has been proposed by (Abdelgawad et al., 2010) to optimize simultaneously the minimizing of in-vehicle travel time, minimizing of the at-origin waiting time and minimizing fleet cost for mass transit evacuation. By the comparative analysis of different evacuation scenarios they claimed that considering only the travel time underestimates the waiting time of the evacuees in no-notice evacuation. In another hand, minimizing of the evacuee waiting time implies evacuating all the population instantly and will ultimately demand their simultaneous evacuation, which may lead to longer travel times in the system and the longer evacuation time with congestion on the transportation network. Furthermore, minimizing travel time causes the delaying of the evacuees at the origin and will ultimately increases the waiting time. A good compromise and their proper trade-off is always challenging as these two objectives might be conflicting to each other.

3. SOLUTION STRATEGIES

Mathematical models seldom represent all the existing characteristics of real-life situations as on their formulation, one has to idealize the real-life problem by making some simplifying hypotheses (Lancaster, 1976). So, one has to be careful using the solutions of such models as they tend to be large and exhibit an exponential complexity with the problem size. Its performance and efficiency depends upon the nature of road network, population density, the behavior of the population and on many other factors.

So far as the BEP is concerned, the objective is to minimize the duration of evacuation by routing and scheduling a fleet of homogeneous and capacitated buses which were initially located at one or more yards. Most often, the number of evacuees at each pickup location can exceed the capacity of a single bus, which signifies the necessity of split delivery service. Moreover, the number of available buses is insufficient to transport all the evacuees without multiple trips and each shelter has a capacity that limits the number of evacuees it can serve. Such situations also demand the split delivery service. In such situations, the author in (Bish, 2011) has proposed and analyzed two alternative models for the multi-depot, multi-trip, bus-based evacuation problem, at which the first simultaneously identifies optimal route construction and assignment of the vehicle where the next identifies the optimal route assignment from a set of feasible routes. Unlike to this, a multi-items, multi-vehicles, multi-periods, soft time windows with a split delivery strategy has been formulated in (Lin and Luo, 2011) as a multi-objective integer programming model and is solved heuristically by the genetic algorithm (GA) followed by decomposition of the original problems. Whereas, (Goerigk et al., 2013) have developed a simplified version of BEP model for the evacuation of a region from a set of collection points to a set of capacitated shelters with the help of buses in minimum time assuming that the bus pick-ups exactly the number of people that equals its capacity when visiting a source and hence, no need of split delivery services. By assuming that the number of evacuees is not known exactly, the BEP is extended to RBEP in (Goerigk and Grün, 2014) as mentioned above.

The development of large scale simulation-optimization approach as a decision support tool optimizing network performance and logistics during emergencies has been presented in (Cavusoglu et al., 2013) for the evacuation of car-less populations via different transits. Considering the preferences of the evacuees for their departure times, routes, and destinations, authors in (Huibregtse et al., 2011) have developed an iterative solution technique where the objective function and the simulation model can be chosen by the analyst, and can be applied to an arbitrary region and hazard. However, the output depends upon the choice of the objective function. Simulation models also enable transportation planners and practitioners to develop and compare different evacuation plans for different hypothetical situations to predict traffic conditions and the evacuation duration. Such techniques has also been used to investigate how different evacuation scenarios like alternatives exits, number of vehicles changed, and other traffic control plans would affect the evacuation duration. Such models have been presented systematically in (Naghawi and Wolshon, 2010) to simulate the transit based evacuation strategies where the average travel time and total evacuation time were used to compare the results of different evacuation time periods. They also proposed comparatively the effective scenario of transit based evacuation routing plan with reduction in overall travel time as well as the total evacuation time with respect to the peak hour general evacuation scenarios. Simulations are the powerful tools to evaluate traffic scenarios though it misses the optimization potential. Fluid models and the models based on differential equations capture very well the dynamic behavior of traffic as the continuous quality but comparatively inefficient to handle the large network. The authors in (Xie and Turnquist, 2011) have presented comparatively the effective way to use existing network capacity by identifying the candidate emergency vehicle routes and then the reconfiguration of the network for evacuees to satisfy the multiple objectives for emergency management.

An intelligent algorithm, by embedding the GAhas been developed in (Deai et al., 2011) to solve the optimization model of a real evacuation network having 19 pickups and 4 shelters. A hybrid type of GA has been formulated in (Song et al., 2009) for the solution of a location routing problem to get its optimal transit routing in the system. Various constraints have been satisfied in its initialization and reproduction process. The proposed hybrid GA has also been tested in a small evacuation network and found to be better than the traditional GA. An alternative evacuation route plan strategy is suggested by (Lim et al., 2016) with mixed integer nonlinear programming formulation for real time evacuation where the traffic network are affected partially or totally for short or long periods of time. Though in a minor incident, one can wait until the incident is cleared to follow the pre-planned route but in a severe incident it is better to have an alternative path to evacuate the outbound flows due to over congestion and minimize the evacuation clearance time. Unlike to this the authors in (Sayyady and Eksioglu, 2010) have considered the case of minimum casualties within minimum time with maximum use of vehicles.

As the exact methods tend to perform poorly on large size instances and demands the heuristics. Two heuristic algorithms have been used to solve BEP in (Bish, 2011), the first is to produce quickly the feasible solution and is also to improve the solution by route swapping and assignment based on a simple search technique whereas the next based on mathematical programming formulation. Authors in (Goerigk et al., 2013) have presented branch and bound algorithms for various computational results to find lower and upper bounds with several node pruning techniques and branching rules. Four greedy algorithms are also presented to construct the feasible solutions and three algorithms to find lower bounds, though the greedy algorithms cannot give always the optimal solution. These bounds have been integrated into the branch and bound framework to obtain the near optimal solution.

Authors in (Pereira and Bish, 2014) have presented a spatial-temporal synchronization of vehicles with customer-oriented objective function to mitigate the evacuation risk for maximum service level with the routing and scheduling having a constant evacuee arrival rate, BEP-CA. They signify the dynamic relationship between the maximum service level and the fleet size for the development of more efficient transit based regional evacuation plan. Assuming the evacuation to begin in zero time the objective is taken as to minimize the total exposure,

$$\sum_{j\in P} \frac{D_j}{2} \sum_{J=1}^{F} (q_j^f)2 \tag{28}$$

Here, D_j is the CA rate of the evacuee at pickup node and $q_j^f = p_j^f - p_j^{f-1}$ be the time intervals between pickups. For a given parameter F the minimum exposure schedule, is the pickup schedule to minimize (28), i.e. where the total exposure is dominated by the largest interval between pick-ups and is given as,

$$minimize \quad \sum_{j\in P} \frac{D_j}{2} \sum_{J=1}^{F} (q_j^f)2 \tag{29}$$

$$\textit{subject to} \quad \sum_{J=1}^{F}\left(q_j^f\right) = L_j, \forall j \in P, \tag{30}$$

$$\sum_{J=1}^{F}\left(q_j^1\right) \geq t_{oj}, \forall j \in P, \tag{31}$$

$$q_j^f \geq 0, \forall j \in P, f \in 1,2,\ldots,F. \tag{32}$$

Unlike to such constant arrival rate, the arrival pattern of the evacuees at a pick-up locations have been represented as a mobilization curve in (Jamei,1984) by

$$\xi_t = \frac{1}{1+\exp\left[-LR\left(t'-h\right)\right]} \tag{33}$$

where ξ_t is the cumulative percentage of evacuees loaded in the network by time t', LR is the loading rate of the evacuees to disaster and is referred as the slope of (33) and h be the half loading time. With reference to different LR, there are different evacuation scenarios, as the low LRs is during the early stage of evacuations for the no notice evacuation scenarios and the high LRs during short notice evacuation scenarios.

A simplified version of one of the earliest algorithms, Capacity Constrained Route Planer (CCRP) algorithm is also presented in (Mishra et al., 2015) and is claimed better than most of the heuristic algorithms.

Algorithm 3.1: Simplified CCRP Algorithm

```
1. P is added to the priority queue.
2. The nodes in priority queue are ordered based on its distance from P
3. While the evacuees are in P, find a path P* having minimum destination arrival from P to S taking the capacity of nodes and edges into consideration.
4. Find the capacity of P* and reserve capacity along the path for a group size equal to minimum capacity.
5. If evacuees left at P, go to step 3.
```

Authors in (Min and Neupane, 2011) have presented the simple version of Quickest Path Evacuation Routine (QPER) algorithm for R be the set of paths in the evacuation routes with $RC\left(e\right), CP\left(e\right)$ and $TT\left(e\right)$ be the reserved capacity, original capacity and the travel time respectively of the edge e then the capacity and travel time of the paths for $p^* \in P$ becomes $CP\left(p^*\right) = \min_{e\in p^*}\left(CP\left(e\right) - RC\left(e\right)\right)$ and $TT\left(p^*\right) = \min_{e\in p^*}\left(TT\left(e\right)\right)$. Then based on above mentioned equation (16), the combined evacuation time by the route R becomes

$$CET\left(R\right) = \left\lceil \frac{x + \sum_{p^*\in R} CP\left(p^*\right) TT\left(p^*\right)}{\sum_{p^*\in R} CP\left(p^*\right)} \right\rceil - 1. \tag{34}$$

Algorithm 3.2:QPER Algorithm

```
●Initialization
    1. Set $R = \varnothing$ and $RC\left(e\right) = 0, \forall e \in E$.
    2. Set $CET = \infty, p^*CET = \infty$.
● Iteration
    3. Repeat the following while $CET \leq \text{p}^*CET$ :
        (i). Find the quickest path $p^*$ with the minimum combined evacuation time $T = CET\left(R \cup \left\{p^*\right\}\right)$.
        (ii). If $T = p^*CET$ do the following:
```

```
        (a). Set $R = \left(R \cup \left\{p^*\right\}\right), p^*CET = C$ and $CET = T$.
        (b). For each edge $e$ on $p^*$ set $RC\left(e\right) = RC\left(e\right) + CP\left(p^*\right)$.
● Path removal
    4. Repeat the following while there is a path $p^* \in R \ s.t. \ TT\left(p^*\right) > CET\left(R\right)$.
        (i). Set $R = R \setminus \left\{p^*\right\}$.
```

The running time of QPER algorithm is determined by the number of iterations and is bounded above mainly by total number of paths rather than the number of evacuees and makes it suitable for large scale networks. A slightly modified and simple version of this algorithm has been presented and applied for the single source single sink evacuation (SSEP) problem in (Mishra et al, 2015) for k edge-disjoint paths $P_1^*, P_2^*, \ldots, P_k^*$ from P to S in ascending order of their transit times with $T_1 \le T_2 \le \ldots \le T_k$. For $S_i = P_1^*, P_2^*, \ldots, P_k^*$ paths to the set of routes R are added as a simple SSRP algorithm as in Algorithm 3.3.

Algorithm 3.3: SSEP Algorithm

```
(i). $R = \left\{P_1^*\right\}$.
(ii). $CET = CET\left(S_1\right)$.
(iii). Start with $i = 1$. Execute step (iv) and (v) till $i \le k$ and $T_{i+1} \le CET$.
(iv). Add path $P_{i+1}$ to $R$.
(v). $CET = CET\left(S_{i+1}\right)$ and $i \leftarrow i + 1$.
(vi). Return $R$.
```

For such edge disjoint paths $\{P_1^*, P_2^*, \ldots, P_i^*\}, i \le 1,$ the next path P_{i+1}^* is discovered in residual graph if and only if $T_{i+1} \le CET\left(S_i\right)$. As the saturated nodes and edges in each iterations are deleted in maximum $m + n$ iterations are carried out but not more than x as each path can evacuate at least an evacuee. So, at most $\min\left(m + n, x\right)$ paths are disconnected. For $m = O\left(n\right)$, its time complexity is at most $O\left(xn \log n\right)$. For such single source single sink problem the SSEP algorithm has been developed to the evacuation route planner algorithm.

Algorithm 3.4: Evacuation Route Planner Algorithm for $P - S$.

```
●Input: A network $G\left(V, E\right)$ with designated $P, S \in V$. Every node $v \in V$ has an occupancy and maximum capacity. Every edge $e \in E$ has a maximum capacity and transit time. Initially, all evacuees are in $P$.
●Output: Evacuation route plan for each evacuee.
begin
    Initialize $R = \varnothing$ and $CET = \infty$.
    Initialize $i \leftarrow 0$.
    While $S$ is reachable from $P$ and number of discovered paths $p^* - 1$ do
        Find the shortest path $p_{i+1}^*$ from $P$ to $S$ for $T_{i+1}, C_{i+1}$ be its transit time and capacity.
        if $T_{i+1} \le CET$ then
            $R = R \cup \left\{p_{i+1}^*\right\}$
            $CET = CET\left(S_{i+1}\right)$
            Reduce the capacity of each node and each edge of $P_{i+1}$ by $C_{i+1}$.
            $V = V \setminus \left\{v : v \ is \ a \ saturated \ node \ of \ P_{i+1}\right\}$.
            $E = E \setminus \left\{e : e \ is \ a \ saturated \ edge \ of \ P_{i+1}\right\}$.
```

```
            end
        else
            break
        end
    end
    Let $R = \{p_1^*, p_2^*, \dots, p_k^*\}$.
    Send $x_i$ evacuees via $p_i^*$ for $1 \le i \le k$, where $T_i + \left\lceil \frac{x_i}{C_i} \right\rceil - 1 = CET$.
end
```

The idea of CET in (Min and Neupane, 2011) is extended to the probabilistic behavior of the evacuees in (Mishra et al, 2015) assuming that the evacuees do not follow the path suggested as in Algorithm 3.1. For this, let α and $1-\alpha$ be the probabilities that for suggested and the next (those, who will try to reach their nearest exit), then the total number of evacuees following p_1 and p_i becomes $x_i + \sum_{i=2}^{k}(1-\alpha)x_i$ and αx_i, $i \neq 1$ respectively with the expected time at which the last person arrive at the destination through such paths be $T_1 + \frac{x_i + \sum_{i=2}^{k}(1-\alpha)x_i}{C_1} - 1$ and $T_i + \frac{\alpha x_i}{C_i} - 1, i \neq 1$ respectively. Thus the expected evacuation time in this scenario becomes,

$$E[T] = \left(T_1 + \frac{(1-\alpha)n}{C_1} - 1, \max_{2\le i\le k}\left(T_i + \frac{\alpha x_i - 1}{C_i} \right) \right) \tag{35}$$

It has the lower bound as $T_1 + \frac{(1-\alpha)n}{C_1} - 1$.

Emergency evacuation strategies based on the spatial and temporal information of the evacuees has been formulated in (Zheng, 2014) where the buses run continuously on the basis of the where-and-when information and according to the needs of the evacuee, rather than the fixed routing in order to minimize the exposed casualty time rather than the operating cost. For the solution, a Lagrangian-relaxation-based algorithm was proposed where the model was formulated as usual as a mixed-integer linear programming formulation.

4. APPLICATIONS

Two instances of (i) finding a bomb within city center of Keiserslautern Germany and (ii) an earthquake with a subsequent flood in the area of Nice, France were taken in (Goerigk and Grün, 2014) to test the applicability of their modeling of a comprehensive evacuation planning using genetic solution algorithm. Four notable applications of lane-evacuation routing were effectively conducted in a similar manner in Salt Lake City, Utah as in (Cova and Johnson, 2003) for different situations by creating pedestrian and vehicle evacuation zone. The evacuation of Yokosuka City by (Yamada, 1996) and the evacuation of Knox County, as a county-wide evacuation scenario for Tennessee in (Han et al., 2006) were carried out using a maximum cost flow network model. An application of BEP is presented by (Pyakurel et al., 2015) in a hypothetical case study of the evacuation planning of transit dependent people of Kathmandu valley to evacuate the population of around 25,672 within the area of 1.45 km2 using branch-and-bound and tabu search algorithms. The best results obtained for an instance are; evacuation time of 29 minutes with 6 or 5 sources and 5 sinks for evacuation of 50 percent population using 140 buses having 90 evacuees per bus capacity and 15 km/hr speed.

By using an optimal spatio-temporal evacuation (OSTE) model, authors in (Abdelgawad et al., 2010) have investigated, analyzed and purposed the multiple time-structure model for the transit vehicle routing and scheduling from a multi-objective perspective with real-life constraints and also suggested for the need of other modes like cycling and walking. For large-scale multi-modal emergency evacuation, authors in (Abdelgawad and Abdulhai, 2010) have also used it to optimize the routing and scheduling of mass-transit vehicles on the city Toronto by using only 1320 transit shuttle buses and 4 rapid transit lines to evacuate efficiently the transit-dependent population of about 1.34 million within 2 hours.

Among others, authors in (Zhang et al., 2010) have applied the numerous optimization and simulation approaches to model the trip route assignment to maximize the flow of evacuees from the risk area to safety or to minimize the travel time based on Dijkstra algorithm on time-space network. For this, firstly all residents or other evacuees are evacuated safely to temporary safe stop by foot and secondly they are picked up by public transit which also signifies the importance of public transit-dependent emergency evacuation in developing countries, like Nepal. Moreover, a transit based evacuation model for metropolitan areas was tested significantly on the city of Baltimore's downtown road network in a study by (Zhang and Chang, 2014) assuming an incident of sudden terrorist attack. There were about 40 pedestrian demand points, 2 transit depots, and 10 safe destinations. The applicability of the proposed model and its advantages compared with the available models in the literature was found significant.

The effective and imperative study on the emergency evacuation planning model for the specific need populations addressing the optimal location of bus stops has been presented by (Kaisar et al., 2012) and was applied to a real-life to evacuate the effects of location, number, and distribution of optimal evacuation bus stops. By selecting 20, 40, and 60 bus stop scenarios; they experienced that 20-bus-stop scenario has a high delay because of the congestion of the buses; 40-bus-stop scenario yields the highest delay, travel, and stop times since the bus stop locations still requires a large number of evacuation trips whereas the 60-bus-stop scenario produced the most efficient evacuation time. Its delay, travel, and stop times were all the lowest compared to the others. The proposed methodology was applied to the real-life case of the downtown Washington, D.C. to select the most suitable location and number of bus stops. It suggests not only for the need of different evacuation routes, headways and frequencies in which the buses depart or pick up, but also to explore the new, possible and appropriate evacuation bus stop locations within the network. Restructuring of evacuation planning approach has been implemented in (Lim et al., 2016) including a network preprocessing algorithm and a network flow optimization approach and was developed to find a set of alternative paths and their corresponding flow rates. This approach was tested successfully further on the actual evacuation network of the Greater Houston area.

To test the integrated contraflow strategy for the multi-modal evacuation, authors in (Hua et al., 2014) have considered the evacuation network of Ningbo city, located on the east coast of the Pacific Ocean where there are on an average of 3.1 typhoons per year. They present a plan to evacuate 350,000 people with 69,000 auto vehicles each vehicle with an average capacity of 2.9 and buses with each 35 seats. An arterial sub-network, as the road segment without direct connection to the origin nodes and the local roads which are connected to the origin nodes as the access to arterials are considered on the network aggregation for the two-stage evacuation process. They have presented the separate evacuation models for the transit-based and auto-based evacuees where the transit-based evacuation problem is solved with a minimum cost flow model in first priority and then only the auto-based evacuation problem is addressed with a bi-level network flow model. The approximate optimal evacuation plan of the evacuation network has been obtained at the top level, where the traffic volumes and travel times in streets were derived from equilibrium traffic assignment in the bottom level. However, it is almost impossible to optimize an evacuation network containing all the arterials and local roads, simultaneously. But, the network aggregation method has maintained the balance between the accuracy and efficiency though the management of arterial-arterial intersections and the transit priority at the intersections are not considered which may further improve the transit-based evacuation. To evacuate the area surrounding a nuclear power point, (Campos et al., 2000) have successfully applied the k-shortest path method. For such disasters situations, like nuclear accidents, hurricanes, and floods etc. different approaches of simulation tools has also been applied. For the mass evacuation of the areas surrounding the sites of nuclear power points (Sheffi et al., 1982) have used the macroscopic traffic simulation model to simulate the traffic patterns to have minimum clearance time.

The most prominent applications of the transit vehicles with their optimal routing and scheduling has been applied in (Abdelgawad and Abdulhai, 2012) to evacuate the entire city of Toronto, Canada with a population of about 2.37 million. The model generates optimal scheduling and timetable for each train on the subway lines and the big shuttle buses on the transportation network. The results show that the Toronto Transit Commission fleet is capable of evacuating the transit-dependent population of about 1.34 million within 2 hours on average. The four subway lines of the city of Toronto carry approximately 0.62 million people and can evacuate these people in less than 3 hours of average whereas, 1320 shuttle buses of the Toronto Transit Commission can evacuate the remainder of the transit dependent population of about 0.72 million in approximately 1.5 hours on average.

5. CONCLUSIONS

This paper has attempted to provide a comprehensive review of the fundamental and prominent transit-based approach of evacuation planning optimization problems. With some highlights on various types of the evacuation models on different basis, we are concentrated mainly on the bus-based evacuation. At the meantime, different mathematical models and algorithms which were developed over the years to address such transit-based evacuation under both predictable and unpredictable disasters with the location-allocation design, pick-up locations, their routing, assignments and scheduling on the evacuation network are also addressed. The determination of optimal fleet size, their assignment for optimal routing, and the appropriate network structure and their impact to the optimality of the solution on the mini-max objective and the cost objective are also addressed with some highlights on the traffic control delays at the intersections, the lane reversal and crossing elimination strategies including their different characteristics. Additionally, some representative real-world applications of each approach have also been presented so far, expecting that it should be able to guide much more interest into this important and growing area of research and some of the extensions might be as follows:

- Therouting modification, management of arterial sub-networks, and transit priority at the intersections can be considered for the further improvements of the lane reversal and crossing elimination strategies.
- In practice, people may be of limiting resources during the evacuation planning. So, different resource-constraint version of the problem can also be expected.
- Most of the problems have been tackled with various heuristics. In the networks of practical size, alternative solution methodologies, such as problem decomposition and use of meta-heuristics and different other relevant algorithms can be performed to improve and adjust the solutions.
- Most of the evacuation models are with several assumptions like symmetric networks, constant evacuation demand, constant evacuee rate, and homogeneous distributions of evacuees, identical and independent facility disruptions and many more; which are not always realistic.
- Various simulation approaches are developed on various situations with different parameters for different objectives and constraints and might be the better choice for further modifications.
- The location planning of the collection points, selection of the optimal pick up locations, appropriate shelters, planning of logistic surroundings and the provisioning and medical shelters can also be considered moderately for the better evacuation planning in practice.
- Public transit shuttle buses, rapid transit vehicles and different automobiles are used on evacuation planning in a single platform demands the multi-modal evacuation and can also be integrated with walking, cycling etc. The coordination between transit modes, route choice and the evacuees' behavior and some specific needs are still lacking. It demands the further investigation.
- Furthermore, more theoretical and analytical studies, relevant lower bounds, algorithms with performance guarantee and their dominance, complexity results, etc. are still lacking and insufficient.
- Many of the parameters are not known and mostly non-linear with full of uncertainties. So, various limitations may exist on the findings and always expecting for the further improvements.

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An integrated solution approach for the time minimization evacuation planning problem

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Abstract: Traffic route guidance, destination optimization, and optimal route choice are some of the approaches to accelerate the evacuation planning process. Their effectiveness depends upon the evacuee arrival patterns at the pickup locations and their appropriate assignment to transit-vehicles in the network. Here, the integrated evacuation network topology is composed of two constituent sub-networks, namely, the primary and the secondary sub-networks. We are focused on the collection of evacuees at the pickup locations of the primary sub-network from the danger zone in the earliest arrival flow pattern, and then their assignment to the transit-vehicles in the secondary sub-network.
Transit-vehicles are provided from the bus depot in the secondary sub-network. Pickup locations are taken as the sources for the subsequent process to minimize the overall network clearance time from the danger zone to safety. In this paper, we have proposed an integrated optimization approach in such an integrated network to achieve the minimum clearance time. The earliest arrival pattern respects the partial lane reversal strategy, whereas the better assignments are based on the dominance relations concerning the evacuation duration.

Keyword — Integrated network, arrival pattern, vehicle assignment, clearance time, partial lane reversals, dominance relations.

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1. INTRODUCTION

The massive loss of human life and the socio-economic damage caused by different disasters draw increasing attention from society and also researchers towards disaster management. Effective evacuation planning helps to save the life of people from such disasters. In most of the large cities, many people depend on transit-vehicles. The great loss of people in disasters is due to a lack of proper planning for transit people and vehicles rather than the disaster itself.

Mostly, evacuation planning solutions are based on network flow models. A network consists of nodes and edges. Each node corresponds to the intersection of streets, and each edge, connecting a pair of nodes, corresponds to a road or street segment in the region. Commodities flow between nodes, transported by edges having a capacity constraint, which restricts the amount it can transverse and compromises the balance and flow of the process. The locations where evacuees are situated initially are the source nodes and the safe locations where the evacuees are to be transported to are sink nodes. The transit time is the amount of time it takes for the flow to travel through the edges. Network flows have many applications on transportation modeling. For example, the evacuation planning problems deal with shifting the maximum number of evacuees from disastrous areas or potential danger zones to safe destinations as quickly as possible with utmost reliability. For dynamic network flow problems, not only the amount of flow transmitted but also the time needed for the flow plays an important role. The flow may be auto-based, transit-based or the pedestrian movements, and the time may be discrete or continuous. Such problems arise in many applications of the evacuation planning problems. There has been a fair amount of work in this area, as referred by Dhamala and Adhikari (2018); Dhamala, Pyakurel, and Dempe (2018).

The pioneering work of Ford and Fulkerson (1962) opened a wide horizon for the flow over time problems, which seek the optimal maximum flows over time in the given time horizon T. The quickest flow problem minimizes the time to transfer a given amount of flow value from an initial position to the destination Chen and Chin (1990). The earliest arrival flow maximizes the amount of flow units reached a sink at each point in time simultaneously. Such a flow may not necessarily exist in every network, though it exists for a single-source single-sink Gale (1959).

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A prominent bus-based evacuation planning problem BEPP is proposed by Bish in Bish (2011) to minimize the time of evacuation in case of a short notice using a given number of homogeneous buses satisfying all evacuee demands, without violating both the sink and vehicle capacity constraints. In this, the number of evacuees at the demand node might be greater than the capacity of a bus, and it demands the split delivery within the pickup locations. But if the number of evacuees at every source is known in terms of the integral multiples of the busloads, then it does not demand the split Goerigk, Grün, and Heßler (2013). The majority of the evacuation planning problems in the literature had used the homogeneous fleet of vehicles though some of them like Baou, Koutras, Zeimpekis, and Minis (2018) have used the heterogeneous fleet of vehicles.

Pyakurel, Goerigk, Dhamala, and Hamacher (2015) explored a broad horizon to the research related to the transit-dependent evacuation planning problem. Kathmandu, one of the densely populated city, has been considered as the disaster region for their case study and have drawn different findings on evacuation planning problem. In such a problem, evacuees were supposed to gather themselves from their residents, depending on the disasters scenario, to the nearby pickup locations. The excess exterior points of the endangered region were taken as the pickup locations and the available open spaces as the sinks. Evacuees were supposed to be brought at such sinks from the pickup locations by using the homogeneous buses having the uniform capacity for the evacuees' pickup.

A deterministic and a stochastic formulation are proposed by Goerigk and Grün (2014), where the exact number of evacuees is not known in advance, although a set of possible scenarios is provided. However, after some reckoning time, such uncertainty is removed with exact figures. The problem is to decide for each bus, whether it is better to move right now on such uncertainties as a here-and-now bus, or to wait till the uncertainties are removed as a wait-and-see bus. Their approach aims to minimize the total network clearance time, that is, the time needed until the last evacuee is brought to safety.

Hua, Ren, Cheng, and Ran (2014) had presented an integrated contraflow strategy for multimodal evacuation. Their strategy contains non-contraflow to shorten the strategic set-up time, full-lane contraflow to maximize the evacuation network capacity and bus contraflow to realize the transit cycle operation. Pyakurel and Dhamala (2015) had investigated the lexicographically maximum dynamic contraflow problem in which the flow is maximized in a given priority ordering. By introducing the continuous network contraflow approach, they have addressed different analytical and theoretical aspects for evacuation planning problems at arbitrary and zero transit times, Pyakurel and Dhamala (2017a). The quickest continuous contraflow problem on single-source-single-sink arbitrary networks and the continuous earliest arrival contraflow problem on single-source-single-sink series-parallel networks with undefined supply and demand have been solved in Pyakurel and Dhamala (2016). Pyakurel, Dhamala, and Dempe (2017) have considered the value approximate earliest arrival transshipment contraflow approach for the arbitrary and zero transit in arcs. Recently, the authors in Pyakurel, Nath, Dempe, and Dhamala (2019) have introduced the partial contraflow model and addressed different issues on it with constant transit times and inflow-dependent transit times.

Here, in the integrated evacuation approach, the network topology is composed of two constituent sub-networks, namely, the primary and the secondary sub-networks. Evacuees are collected at the pickup locations of the primary sub-network in the earliest arrival flow pattern, and then they are assigned to the transit-vehicles, in the secondary sub-network. It is more general than the network for the earliest arrival flow and the transit-based network, separately. It is an integrated approach to solve for the time minimization evacuation planning problem.

The rest of this paper is organized as follows. Section 2 gives few preliminary concepts with the network topology for the integrated evacuation planning problem. Section 3 presents an integrated evacuation scenario in three different subsections. Firstly, we introduce the earliest arrival evacuee problem in a network of a single source and multiple pickup locations. Considering the flow model with zero transit time, we present a polynomial-time algorithm following the principle of temporally repeated flows to transship given flow value from the source to the pickup locations by saving all unused arc capacities of arcs in Subsection 3.1. We study the BEPP, and prove the dominance relation of different heuristics with respect to the evacuation duration in Subsection 3.2. Then combining these two approaches, we present an integrated evacuation model in Subsection 3.3. The solution approach in an integrated evacuation network is presented in Section 4. Finally, Section 5 concludes the paper.

2. PRELIMINARIES

We consider a network $\mathcal{N}$, obtained by combining two of its components $\mathcal{N}_1$ and $\mathcal{N}_2$ representing a primary and a secondary sub-network, respectively. The first part $\mathcal{N}_1$ contains directed two-way road segments and the partial arc reversals is applicable. The second part $\mathcal{N}_2$ contains directed one-way road segments, connecting the bus depot to the pickup locations, and undirected edges connecting such pickup locations to the sinks for the bus routing. The network topology of such an embedded network, $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ is illustrated in Figure 1.

The primary sub-network is denoted as $\mathcal{N}_1 = (s, V, A, u_a, \tau_a, Y)$ with a single source s, set of auxiliary nodes $V = \{v_1, v_2, \dots, v_n\}$, set of pickup locations $Y = \{y_1, y_2, \dots, y_n\}$, set of arcs $A = \{a \mid a = (s, v) \vee (v, y) \text{ where } v \in V,\ y \in Y\}$, capacity u_a and transit time τ_a for each $a \in A$. Here, Y is also considered as a set of multiple sinks. The capacity $u_a : A \to \mathbb{Z}_{\geq}0$ restricts the amount of flow on the arc and the transit time $\tau_a : A \to \mathbb{Z}_{\geq}0$ represents the amount of time to transverse the respective arc. As we consider τ_a to be constant for all $a \in A$, it is assumed to be zero.

Also, secondary sub-network is denoted as $\mathcal{N}_2 = (d, Y, E, \tau_e, Z)$, where d is the bus depot at which a set of transit-buses $B = \{b_1, b_2, \dots, b_n\}$ having the homogeneous bus capacity are located initially and are assigned as required during the evacuation process. This node d does not play significant roles further on the solution procedure as the buses do not return to it even after the completion of the evacuation plan because of risks under threat. The set of nodes Y with respect to $\mathcal{N}_1$ is considered as the set of sources for $\mathcal{N}_2$. The set of sinks is denoted by $Z = \{z_1, z_2, \dots, z_n\}$. In this mixed sub-network, the set E consists of the one-way arcs $e = (d, y)$ with $y \in Y$ and the undirected edges $e = [y, z]$ with $y \in Y, z \in Z$. Transit times of the respective arcs and edges are denoted by $\tau_e \in \mathbb{Z}_+$.

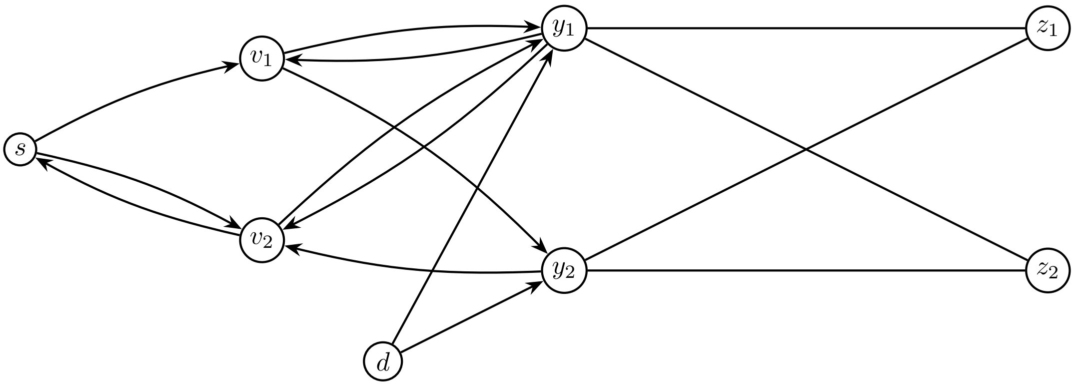

Figure 1: A topology of an integrated evacuation network.

3. AN INTEGRATED EVACUATION SCENARIO

In an integrated evacuation scenario, evacuees collected at the pickup locations Y in $\mathcal{N}_1$ are assigned to transit-buses in the appropriate route across $\mathcal{N}_2$ and are finally sent to the sinks. In such embedding, the set Y works as the sink for $\mathcal{N}_1$ but as the source in $\mathcal{N}_2$.

The arrivals of evacuees at different pickup locations are usually probabilistic, which are categorized by a constant arrival rate or by a random variable that is, deterministic or stochastic, time-dependent or flow-dependent. Evacuees have gathered themselves at different pickup locations relative to the population density of the transit-dependent people nearby them in Pyakurel et al. (2015) with no specific arrival patterns. Pereira and Bish have considered the constant arrival rate of evacuees at the predetermined pickup locations in Pereira and Bish (2014). However, such an assumption is still unrealistic as the actual arrival process is probabilistic and will likely vary over time. Two of the prominent BEPP formulations as in Goerigk et al. (2013) and Bish (2011) have considered the evacuees at the pickup locations, but these approaches do not speak about the specific arrival patterns. However, they take into account whether the number of evacuees is the integral multiples of the busloads or not. Such a problem BEPP is extended to a robust BEPP by assuming that the number of evacuees is not known exactly but a set of estimates for the number of evacuees at each source Goerigk and Grün (2014).

In this section, we are presenting the earliest arrival pattern of evacuees in $\mathcal{N}_1$, their assignment to vehicles in $\mathcal{N}_2$ and the mathematical model in $\mathcal{N}$.

3.1 The earliest arrival pattern of evacuees

When preparing for an evacuation, the time it actually takes is uncertain, and hence it is preferential to plan at each point of time to execute the maximum flow, which is offered by the earliest arrival flow. It is better-suited for evacuation planning as it maximizes the flow of evacuees simultaneously at each instance within the given time horizon. Such an evacuee arrival pattern is more appropriate for the integrated evacuation scenario.

An s-y flow of evacuees over time is a non-negative function f on $A \times \mathbb{R}_+$, for given time $\mathbf{T} = \{0.1. \dots, T\}$ satisfying the flow conservation and capacity constraints (1-3). The inequality flow conservation constraints allow waiting for flow at intermediate nodes. However, the flow conservation constraints force that flows entering an intermediate node must leave it again immediately.

$$\sum_{\sigma=\tau_a}^{T}\sum_{a\in A_i^{in}} f(a,\sigma-\tau_a)-\sum_{\sigma=0}^{T}\sum_{a\in A_i^{out}} f(a,\sigma) \quad = \quad 0,\ \forall\, i\notin\{s,y\}, \tag{1}$$

$$\sum_{\sigma=\tau_a}^{\theta}\sum_{a\in A_i^{in}} f(a,\sigma-\tau_a)-\sum_{\sigma=0}^{\theta}\sum_{a\in A_i^{out}} f(a,\sigma) \quad \geq \quad 0, \forall i\notin\{s,y\},\theta\in\mathbf{T}, \tag{2}$$

$$0 \ \leq \ f(a,\theta) \ \leq \ u_a, \qquad \forall\, a\in A,\ \theta\in\mathbf{T}. \tag{3}$$

Here, $A_i^{out}=\{a=(i,j)\in A\}$ and $A_i^{in}=\{a=(j,i)\in A\}$ are the sets of outgoing and incoming arcs, respectively for the node $i\in V$. For the source node s, we get the flow value be $\nu_f(s)>0$, and for the sink y the flow value becomes $\nu_f(y)<0$, whereas $\sum_{i\in V}\nu_f(i)=0$. If the supply and demand on sources and sinks $\nu_f(i)$ is a fixed value for all $i\in\{s,y\}$, then the earliest arrival evacuee problem maximizes value(ν_f,θ) for all $\theta\in\mathbf{T}$, as in Equation (4) satisfying the constraints (1-3).

$$(\nu_f,\theta)=\sum_{\sigma=0}^{\theta}\sum_{a\in A_s^{out}} f(a,\sigma)=\sum_{\sigma=\tau_a}^{\theta}\sum_{a\in A_y^{in}} f(a,\sigma-\tau_a) \tag{4}$$

We consider a flow over time problem with zero transit time function $f: A\times\mathbb{Z}_+\to\mathbb{R}_+$.

Problem 1. Given a flow over time sub-network $\mathcal{N}_1=(s,V,A,u_a,\tau_a,Y)$ with supplies at s, auxiliary nodes V, arc capacity u_a and arc transit time τ_a for $a\in A$. The earliest arrival evacuee problem is to find the earliest arrival of evacuees at Y with partial arc reversal capability.

Let the reversal of an arc $a=(i,j)$ be $a'=(j,i)$, then the transformed network of $\mathcal{N}_1$ consists of the modified arc capacities and constant transit times as,

$$b_{\overline{a}}=b_a+b_{a'}, \quad \text{and} \quad \tau_{\overline{a}}=\begin{cases}\tau_a, & \text{if } a\in A\\ \tau'_a, & \text{otherwise}\end{cases} \tag{5}$$

where an edge $\overline{a}\in\overline{A}$ in a transformed network, if $a\vee a'\in A$ in $\mathcal{N}_1$. The remaining graph structure and data are unaltered. For the sake of simplicity, we use $\mathcal{N}_1$ for a transformed network, in which $\overline{a}$ and $\overline{A}$ are replaced by a and A, respectively, in the rest of the works. In the transformed network $\mathcal{N}_1$, we have solved the earliest arrival transshipment problem with zero transit times on each arc as in Schmidt and Skutella (2014) and saved all unused arc capacity as in Pyakurel et al. (2019).

The total flow amount out of the source s that reached to the pickup locations Y in $\mathcal{N}_1$ for all time up to $\theta'\in\mathbb{Z}_+$, with zero transit times $\tau_a=0$, is given by

$$|\nu_f|_{\theta'}=\sum_{\theta=1}^{\theta'}|\text{value}(Y,\theta)|. \tag{6}$$

For the given time bound T, the value in 6 is denoted by $|\nu_f|=\sum_{\theta=1}^{T}|\text{value}(Y,\theta)|$.

For the earliest arrival flow over time reached to Y in $\mathcal{N}_1$, the net amount of flow given by Equation (6) should be maximum at every point in time within the given time horizon. Such a flow over time will simultaneously maximize the flow that has already reached the sinks for all points in time. For details, we refer to Pyakurel and Dhamala (2017b); Schmidt and Skutella (2014).

In the case of arbitrary transit times, no earliest arrival transshipment exists even in a one-source-two-sinks, Baumann and Skutella (2009). However, every in-or out-tree with the depth of at most two always allows for the earliest arrival transshipment for every choice of capacities and flow values for zero transit Schmidt and Skutella (2014).

Hence, the existence of such flow with zero transit times is based on the depth of the network considered. A directed graph with exactly one path from i to j for every node i is the in-tree with root j. Likewise, the directed graph with exactly one path from j to i for every node i is the out-tree with root j. The depth of an in-or out-tree is the number of edges on the longest path contained in it.

Lemma 1. Consider $\mathcal{N}_1$ with zero transit times. Then every in-or out-tree with a depth of at most two in $\mathcal{N}_1$ always allows for the earliest arrival transshipment regardless of capacities and balance values, Schmidt and Skutella (2014).

Theorem 1. There exists an earliest arrival transshipment with zero transit times in a type $\mathcal{N}_1$ network of single-source and multi-sink, where the depth is at most two.

Proof. The earliest arrival $s - y$ flow exists for a single-source-single-sink network as in Gale (1959). However, the earliest arrival transshipment does not exist for arbitrary transit times, even in a single-source and double-sink network. But from Lemma 1, the earliest arrival transshipment with zero transit times exists for all networks with a depth of at most two, and is satisfied for the single-source and multi-sink network. ☐

Now, Algorithm 1 is presented to solve the earliest arrival evacuee problem with zero transit times with partial arc reversal capability in polynomial-time complexity.

```
Algorithm 1: The earliest arrival evacuee algorithm
  Input : A flow over time sub-network $\mathcal{N}_1 = (s, V, A, u_a, \tau_a, Y)$ with $\tau_a = 0$ for each $a \in A$.
1 Construct a transformed network $\mathcal{N}_1$ as in Equation 5.
2 Determine the maximum number of evacuees at every possible time instance at each $Y$ from $s$ as in Schmidt
    and Skutella (2014).
3 For each $\theta \in \mathbf{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ iff $c_a > u_a$, $u_a$ replaced by 0 whenever $a \notin A$,
    in $\mathcal{N}_1$, where $c_a$ denotes the static $s - y$ flow value in each $a \in A$ for such sub-network.
4 For each $\theta \in \mathbf{T}$ and $a \in A$, if $a$ is reversed, $\kappa_a = u_{\overline{a}} - c_{a'}$ and $\kappa_{a'} = 0$. If neither $a$ nor $a'$ is reversed,
    $\kappa_a = u_a - c_a$, where $\kappa_a$ is saved capacity of $a$, Pyakurel et al. (2019).
  Output: Earliest arrival of evacuees at $Y$ with $\tau_a = 0$ for each $a \in A$.
```

Theorem 2. Algorithm 1 sends the evacuees at the earliest arrival time to Y at each instances and saves the unused arc capacity.

Proof. The construction of a transformed network for a given network in Step 1 is feasible. Steps 2 and 4 are feasible. As there is no cycle flow in Step 2, the flow is either on arc a or a' but never in both directions simultaneously. And such a flow is not greater than the modified capacities of each arc in the transformed . So, Step 3 is also feasible. Hence, Algorithm 1 is feasible.

Now, we show that Algorithm 1 gives an optimal solution. In the transformed network, we compute the maximum number of evacuees reached to each pickup location in Y at every possible time point with zero transit times on each arc using the algorithm of Schmidt and Skutella (2014). As in Theorem 1, there are some characteristic networks with the depth bounded by two in which the earliest arrival flow exists. As the maximum amount of flows are assigned from the source s across different auxiliary nodes to Y at each instance, by using Equation (6) for the maximum flow value, it gives the evacuees at the earliest arrival time to such pickup locations at each instance. Moreover, the obtained solution is equivalent to the solution of the earliest arrival evacuee problem in the original sub-network $\mathcal{N}_1$ with the arcs reversed up to the necessary capacity as in Step 3, Pyakurel et al. (2019). The capacities of the arcs not used by the flow after partial arc reversals are recorded in Step 4. This completes the proof. ☐

Theorem 3. The earliest arrival evacuee problem with zero transit times can be solved in polynomial-time complexity using Algorithm 1 in the sub-network $\mathcal{N}_1$.

Proof. As Steps 1, 3, and 4 of Algorithm 1 are solved in linear time, its time complexity is dominated by the time complexity of computation of the earliest arrival evacuees at the pickup locations Y with zero transit times on each arc as in Schmidt and Skutella (2014) in Step 2, which is solved in polynomial-time. Thus, we solve the earliest arrival evacuee problem in polynomial-time complexity in the sub-network $\mathcal{N}_1$. ☐

Theorem 4. The earliest arrival evacuee problem having zero transit times with partial arc reversal capability follows the principle of temporally repeated flows and can be solved in polynomial-time complexity.

Proof. The flow over time problem having zero transit times that reached to each of the pickup locations determines the maximum number of evacuees at every possible time instance from the beginning in the primary sub-network $\mathcal{N}_1$ as in Schmidt and Skutella (2014). That means the earliest arrival of evacuees at Y from s with zero transit times on the transformed network follows the principle of temporally repeated flows which is equivalent to the solution with arc reversals capability on the original network, Pyakurel and Dhamala (2017b). It can be obtained in polynomial-time complexity as in Theorem 3. Hence the theorem is proved. ☐

3.2 Assignment of vehicles

Transit-vehicles are assigned in an appropriate route across $\mathcal{N}_2$ to send the evacuees to the sinks. It is similar to the BEPP as presented by Bish (2011) and Goerigk et al. (2013) to minimize the duration of evacuation by using a given number of homogeneous buses satisfying all evacuee demands respecting the sink and vehicle capacity constraints. In such a problem formulated as in Bish (2011), the number of evacuees at the demand node might be greater than the capacity of a bus, and it demands the split delivery (SD) service within the pickup locations. However, the split delivery service does not improve the solution, as illustrated in Example 1, based on Dror and Trudeau (1990).

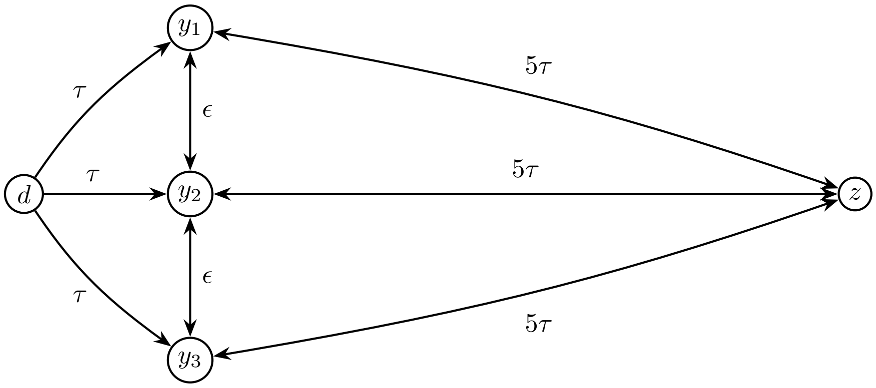

Figure 2: An instance of evacuation network.

$\lvert B\rvert$	Π & ED without SD	Π & ED with SD	Bound
1	$d-y_1-z-y_2-z-y_3-z=26\tau$	$d-y_1-y_2-z-y_2-y_3-z=16\tau+2\epsilon$	1.625
2	$d-y_1-z-y_2-z=16\tau$	$d-y_1-y_2-z=6\tau+\epsilon$	2.67
	$d-y_3-z$	$d-y_3-y_1-z$	
3	$d-y_1-z=6\tau$	$d-y_1-z=6\tau$	1
	$d-y_2-z$	$d-y_2-z$	
	$d-y_3-z$	$d-y_3-z$	
4	$d-y_1-z=6\tau$	$d-y_1-z=6\tau$	1
	$d-y_2-z$	$d-y_2-z$	
	$d-y_3-z$	$d-y_3-z$	
	ϕ	ϕ	

Table 1: The split delivery does not always improve the solution in transit-based evacuation.

Example 1. Consider an evacuation network in Figure 2, where d, $\{y_1, y_2, y_3\}$, and z are the bus depot, pickup locations, and the sink, respectively. Consider $|B|$ buses are located at d with a homogeneous capacity of 50 evacuees. Let the demands at pickup locations y_1, y_2, and y_3 be 30, 40, and 30, respectively. Consider the pickup locations are at equal distance, τ each, from the depot and are at 5τ from the sink. Here, ϵ is used to denote that the pickup locations are sufficiently close to each other and connected by edges. Consider the sink capacity to be 100.

The tour plan Π and the respective evacuation duration (ED) without and with SD, and their respective bounds are shown in the second, third, and fourth columns of Table 1, where the buses were scheduled simultaneously. Here, the bound denotes the corresponding ratio of the ED obtained in column 2 to that in column 3, for the network considered. The bound greater than 1 indicates that the SD service improves the solution. During their route assignments, sometimes the SD service is also appropriate though it may not always improve the ED. Here, no improvement in the solution by applying the SD service for $|B|$ equals 3 and 4.

Here, we consider the modified version of the BEPP as in Problem 2, as formulated in Goerigk et al. (2013).

Problem 2. Let $(\tau_{ij})_{i\in Y, j\in Z}$ be a matrix of source-sink travel times, τ_{di} be a vector of depot-source travel times, $(_i)_{i\in Y}$ be a vector of evacuees number and $(\mu_j)_{j\in Z}$ be a vector of sink capacities. Then the BEPP is to find a tour plan to minimize the maximum travel times overall buses such that all the evacuees are transported to the sinks.

For this, it is assumed that the number of evacuees at every source is known in terms of the integral multiples of the bus loads, which do not require any split delivery service. Assume that every bus has a capacity of one unit.

Moreover, the capacity of the sink is also in terms of bus loads. The movement between a source to another source is ignored, and the same situation is considered between the sinks. It is assumed that, the set of tours of the buses cannot be changed any more after they start to move. Let $\sum_{i\in Y} l_i$ and $\sum_{j\in Z} \mu_j$ be the total number of evacuees and the total sink capacities, respectively. The maximum number of rounds R for the evacuation process is given by $\sum_{i\in Y} l_i$. The nonnegative travel cost of τ_{ij} on each edge $e = (i, j) \in E$ is taken symmetric and satisfies the triangle inequality.

The variables τ_{to}^{br} and τ_{back}^{br} give the travel time for the vehicle b within the round r from a source to a sink, and from that sink to another source, respectively. The binary variable $x_{ij}^{br} \in \{0, 1\}$ denotes whether the vehicle b travels from source i to sink j in the round r. Let $\mathcal{T}_{\max}$ be the duration of evacuation overall vehicles. The problem can be formulated as follows.

$$
\begin{aligned}
\text{minimize} \quad & \mathcal{T}_{\max} && (7)\\
\text{such that} \quad \mathcal{T}_{\max} &\geq \sum_{i\in Y}\sum_{j\in Z} \tau_{di} x_{ij}^{b1} + \sum_{r\in R} \tau_{to}^{br} + \sum_{r\in R} \tau_{back}^{br}, \quad \forall b \in B, && (8)\\
\tau_{to}^{br} &= \sum_{i\in Y}\sum_{j\in Z} \tau_{ij} x_{ij}^{br}, \quad \forall b \in B, r \in R, && (9)\\
\tau_{back}^{br} &\geq \sum \tau_{ij} [\sum_{k\in Y} x_{kj}^{br} + \sum_{l\in Z} x_{il}^{b,r+1} - 1], \quad \forall b \in B, r \in R-1, && (10)\\
\sum_{i\in Y}\sum_{j\in Z} x_{ij}^{br} &\geq \sum_{i\in Y}\sum_{j\in Z} x_{ij}^{b,r+1}, \quad \forall b \in B, r \in R-1, && (11)\\
\sum_{i\in Y}\sum_{j\in Z} x_{ij}^{br} &\leq 1, \quad \forall b \in B, r \in R-1, && (12)\\
\sum_{j\in Z}\sum_{b\in B}\sum_{r\in R} x_{ij}^{br} &\geq {}_{i}, \quad \forall i \in Y, && (13)\\
\sum_{i\in Y}\sum_{b\in B}\sum_{r\in R} x_{ij}^{br} &\leq \mu_j, \quad \forall j \in Z, && (14)\\
x_{ij}^{br} &\in \{0, 1\}, \quad \forall \tau_{to}^{br}, \tau_{back}^{br}, \mathcal{T}_{\max} \in \mathbb{R}. && (15)
\end{aligned}
$$

Constraint (8) needs $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all buses, which is to be minimized on (7). Constraints (9) and (10) are the measure of travel time for the bus b within the round r from a source to a sink, and from that sink to the next source, respectively. Constraint (11) tells that the tours are connected and can stop whenever they like. Constraint (12) allows a bus from a source to sink per round. Constraint (13) and (14) are the bus capacity and shelter capacity constraints, respectively. Constraint (15) represents whether the bus b travels from source i to sink j in the round r.

Let τ_{ij}, τ_{di}, ${}_i$ and μ_j for $i \in Y$ and $j \in Z$ be as in Problem 2. For $k \in \mathbb{R}$, is there a tour plan with $\mathcal{T}_{\max} \leq k$, for the complete evacuation? Regarding the complexity of such a decision version of BEPP, the following result is .

Theorem 5. The decision version of BEPP is $\mathcal{NP}$-complete, even if $\tau_{di} = 0$ and $\tau_{ij} = \tau_{i'j}$ for all $i, i' \in Y$ and $j \in Z$.

During the solution of BEPP, authors in Goerigk et al. (2013) have presented the branch and bound algorithms with four different upper bounds and three lower bounds for time, three branching rules to minimize the number of branches, and two tree reduction strategies to avoid the equivalent branches. Upper bounds are constructed in polynomial-time complexity by four heuristics. Among them, the first three heuristics are based on a greedy distribution of tours on buses with precomputed tour lists, and the last one uses an iterative way without any precomputed tour lists.

1. **Heuristic H1:** Initially, a set of tours (i, j) for $i \in Y$ and $j \in Z$ is to be constructed for each source node i and $l_i \geq 0$ from i to the nearest sink j with $\mu_j > 0$ resulting $l_i \to l_i - 1$ and $\mu_i \to \mu_j - 1$ such that l_i should vanish. One of the randomly chosen tours from the tour list is assigned to a transit-vehicle with the minimum total travel cost. It is continued in a similar fashion to have a complete evacuation.

2. **Heuristic H2:** Initially, a set of all possible tours (i, j) for $i \in Y$ and $j \in Z$ is to be constructed and is sorted on non-decreasing cost provided by $l_i, \mu_j > 0$. Such a set of tours sorted until l or μ vanishes. The initially available transit-vehicle is assigned to the tour with the highest cost, and then it continues the next expensive tour, and so on. It is based on the longest processing time first rule as in Pinedo (2008).

3. **Heuristic H3:** For the precomputed tour list (i, j) for $i \in Y$ and $j \in Z$ having $\min\{d_{i'j'}\}$ with $l_{i'} > 0, \mu_{j'} > 0$ for all i' and j' as in heuristic $H2$, transit-vehicles are assigned by reversing the set of such tours for each vehicle. For the assignment of such vehicles, the long tour may have a long return tour, which might be beneficial. But, the vehicles having a long tour need not have a long return tour at the end. It is a simple modification of $H2$ and needs no guarantee for the improvement of the solution.

4. **Heuristic H4:** This begins with the best possibility to bring one evacuee back from the sink to the source and is continued iteratively. For this, let Y and Z be the available sources and sinks, respectively, provided for $l_i > 0$ and $\mu_j > 0$. Let t_i^b be the distance of the current position of the vehicle to the source and *offset*$_b$ be the distance of the vehicle $b \in B$, which is already planned. Initially, for all the vehicles in the depot, we get $t_i^b = \tau_i$ and *offset*$_b = 0$. In such a case, the best possibility to assign the vehicle is with a minimum possible value of the sum of *offset*$_b$, τ_i and τ_{ij}, which is given by $\min\{\tau_i + \tau_{ij}\}$, same as the minimum total travel cost. Updating the t_i^b for each $l_i \to l_i - 1$ and $\mu_j \to \mu_j - 1$ in an iterative procedure for the next assignment, and so on, the feasible solution is obtained in minimum possible time.

All three lower bounds are also computed with polynomial-time complexity. The first lower bound is based on the estimation of the travel times from sources to sinks and from sinks to sources, respectively. The second lower bound is based on the fact that the lower bound for the maximum travel time is the average travel time. To address this, the objective is to minimize the sum of travel times and has been formulated by replacing the relations (7) and (8) by,

$$\text{minimize} \quad \sum_{i\in Y}\sum_{j\in Z}\tau_{di}x_{ij}^{b1} + \sum_{b\in B}\sum_{r\in R}\tau_{to}^{br} + \sum_{b\in B}\sum_{r\in R}\tau_{back}^{br} \tag{16}$$

The third lower bound is the simplification of model formulation, assuming that sinks are far away from the dangerous zone, and the pickup locations Y are nearby with negligible distances between such pickups. Consider all the pickups be at y_0 as the super pickup node with $l_{y_0} = \sum_{i\in Y} l_i$. Let the sinks $j \in Z$ and the depot d are at a distance of τ_j and τ_d respectively from y_0, where $\tau_j = \min_{i\in Y}\tau_{ij}$, for $(i, j) \in E$ and $\tau_d = \min_{i\in Y}\tau_i$. Here, τ_d is the same for all vehicles available in the network and can be neglected. Let y_j^b be the number of tours for the vehicle b from y_0 to sink Z and z_j^b be the number of tours for the vehicle b from sink Z to y_0, then the model as in Equations (7-15), can be reformulated as:

$$\begin{aligned}
\text{minimize} \quad & \mathcal{T}_{\max} && && (17)\\
\text{such that} \quad & \mathcal{T}_{\max} &\geq\ & \sum_{j\in Z}\tau_j(y_j^b + z_j^b), && (18)\\
& \sum_{b\in B}\sum_{j\in Z} y_j^b &\geq\ & l_{y_0}, && (19)\\
& \sum_{b\in B} y_j^b &\leq\ & \mu_j, \quad \forall j \in Z, && (20)\\
& \sum_{b\in B} z_j^b &=\ & x_j^b - 1, \quad \forall j \in Z, && (21)\\
& y_j^b, z_j^b &\in\ & \mathbb{N}, \forall b \in B, \quad j \in Z, && (22)\\
& \mathcal{T}_{\max} &\in\ & \mathbb{R}. && (23)
\end{aligned}$$

By analyzing these four heuristics used to construct the feasible solutions on their upper bounds, we have proved Theorems 6 and 7, with respect to their dominating relations. Here, the dominance on heuristics is followed with respect to the superiority of having minimum evacuation duration for their better performance in the network considered. A solution S_1 is said to dominate another solution S_2 if the solution S_1 is either no worse than S_2 or is strictly better than S_2 for the objective considered. Example 2 verifies the dominating relations of these heuristics.

Theorem 6. Heuristic H1 dominates heuristics H2 and H3 in evacuation duration.

Proof. The shortest processing time first dispatching rule is superior over the longest processing time first dispatching rule in minimizing the total completion time criteria, Pinedo (2008). Nevertheless, the longest processing time first dispatching rule balances the loads on the network and does not guarantee the optimality. Hence, H1 dominates H2 and H3 with respect to the evacuation duration, where H3 is a simple modification of H2. ☐

Theorem 7. Heuristic H4 dominates heuristics H2 and H3 in evacuation duration.

Proof. Heuristic H4 is initialized with $t_i^b = \tau_i$ and $\textit{offset}_b = 0$ for all vehicles in the depot and gives the minimum initialization on the vehicle assignment with respect to the rest with $\min\{\tau_i + \tau_{ij}\}$, i.e. the same as the minimum total travel cost as in H1. Routes are considered iteratively with the best possibility to have the link (j, i) for each route (i, j) considered for the better choice of minimum evacuation time for their to-distance and back-distance than that for such distances with respect to H2 and H3. The updated t_i^b in each iteration with $l_i \to l_i - 1$ and $\mu_j \to \mu_j - 1$ minimizes the route assignment to have a feasible solution in minimum time. Hence, heuristic H4 dominates H2 and H3 both. $\square$

Example 2. Let the pickup locations Y be at a distance $\tau_{di} = \begin{bmatrix} 1 & 3 \end{bmatrix}$ from d, and the transit times to the sinks Z from Y be $\tau_{ij} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. Consider a scenario with the demands at Y as $l_i = (2, 1)$ and capacity of Z as $\mu_j = (2, 3)$, respectively.

For the given data in Figure 3, we construct the tour plans Π using heuristics H1, H2, H3, and H4 and calculate the corresponding ED as represented in the second, third, fourth, and fifth columns of Table 2, respectively. It shows that heuristic H1 dominates heuristics H2 and H3 (as proved in Theorem 6). Similar result holds in case of heuristic H4 as well (cf. Theorem 7).

This also helps to estimate the threshold number of the transit-vehicles. Here, the threshold number for $|B|$ is 3 as for $|B| = 4$, the fourth bus is left with no tour.

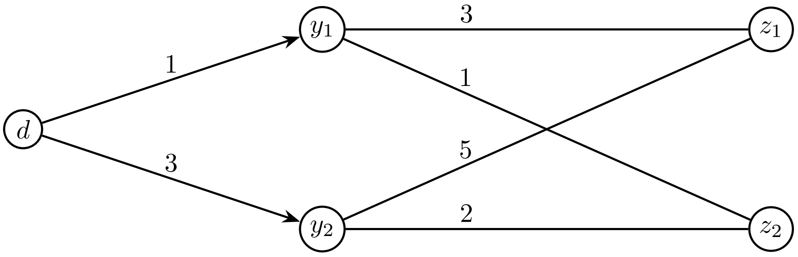

Figure 3: An instance of bus assignment problem.

$\lvert B\rvert$	Π & ED w.r.t. H1	Π & ED w.r.t. H2	Π & ED w.r.t. H3	Π & ED w.r.t. H4
1	$\tau_1 + 3\tau_{12} + 2\tau_{22} = 8$	$\tau_2 + \tau_{21} + 3\tau_{11} + \tau_{12} = 18$	$\tau_1 + 2\tau_{12} + \tau_{11} + 2\tau_{21} = 16$	$\tau_1 + 3\tau_{12} + 2\tau_{22} = 8$
2	$\tau_1 + \tau_{12} + 2\tau_{22} = 6$	$\tau_2 + \tau_{21}$	$\tau_2 + \tau_{21} = 8$	$\tau_1 + \tau_{12} + 2\tau_{22} = 6$
	$\tau_1 + \tau_{12}$	$\tau_1 + 2\tau_{11} + \tau_{12} = 8$	$\tau_1 + 2\tau_{12} + \tau_{11}$	$\tau_1 + \tau_{12}$
3	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{21} = 8$	$\tau_2 + \tau_{21} = 8$	$\tau_1 + \tau_{12}$
	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{12}$
	$\tau_2 + \tau_{22} = 5$	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{22} = 5$
4	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{21} = 8$	$\tau_2 + \tau_{21} = 8$	$\tau_1 + \tau_{12}$
	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{11}$	$\tau_1 + \tau_{12}$
	$\tau_2 + \tau_{22} = 5$	$\tau_1 + \tau_{12}$	$\tau_1 + \tau_{12}$	$\tau_2 + \tau_{22} = 5$
	ϕ	ϕ	ϕ	ϕ

Table 2: Tour plan Π with evacuation duration ED w.r.t. different heuristics.

Note that, heuristic H1 construed with precomputed tour lists and assigned to the closest sink approach, and heuristic H4 constructed iteratively without any precomputed tour lists and assigned as above, are very closed to each other and are dominating the rest. However, for the vehicle assignment in N_2, we prefer H4 as it does not require precomputed tour list.

3.3 A mathematical model in an integrated network

For large scale disasters with a sufficiently large number of evacuees, all the evacuees may not arrive at Y at the same time, and it requests certain waiting time at Y before to start the bus assignment in $\mathcal{N}_2$. It is obvious that those who are delivered to Y earlier will have comparatively more waiting time. Meanwhile, for the evacuees, waiting at Y is comparatively better than to be at s. On the other hand, buses available at d request a certain time to be assigned to Y and are given by τ_{di}. Hence the effective waiting time in $\mathcal{N}$ can be denoted by $\Omega = \max\{\sum \omega_i, \tau_{di}\}$, for ω_i be the waiting at $y_i \in Y$. To address this in an integrated evacuation network, where the evacuees collected in $\mathcal{N}_1$ are to be assigned in $\mathcal{N}_2$, the objective function given by Equation (8) is modified. For this, let $\mathcal{T}_{\max}$ be the duration of evacuation overall vehicles, then the integrated evacuation planning problem can be reformulated as follows:

$$\begin{aligned} &\text{minimize} && \mathcal{T}_{\max} && && (24)\\ &\text{such that} && \mathcal{T}_{\max} && \geq \Omega + \sum_{r\in R} \tau_{to}^{br} + \sum_{r\in R} \tau_{back}^{br} \quad \forall b \in B. && (25)\\ &\text{with the constraints} && && (9-15). && (26) \end{aligned}$$

Constraint (25) needs $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all vehicles and is to be minimized in (24). Other constraints are the same as in Equations (9-15).

For the solution of our integrated model, we adopt the branch and bound algorithm of Goerigk et al. (2013) in which the computation of upper bounds, lower bounds, branching rules, and reduction strategies are described in Subsection 3.2. However, based on the effective waiting time at each pickup locations, the objective as in Equation (16) in the second lower bound becomes

$$\text{minimize} \quad \Omega + \sum_{i\in Y}\sum_{j\in Z} \tau_{di} x_{ij}^{b1} + \sum_{b\in B}\sum_{r\in R} \tau_{to}^{br} + \sum_{b\in B}\sum_{r\in R} \tau_{back}^{br} \tag{27}$$

Whereas, for the third lower bound, the relations (17) and (18) can be replaced by

$$\begin{aligned} &\text{minimize} && \mathcal{T}_{\max} && && (28)\\ &\text{such that} && \mathcal{T}_{\max} && \geq \Omega_0 + \sum_{j\in Z} \tau_j (y_j^b + z_j^b) && (29) \end{aligned}$$

where, $\Omega_0 = \max\{\omega_0, \tau_j\}$, for ω_0 be the waiting time for the earliest arrival of evacuees at the super pickup node y_0.

As the problem (24-26) is not easier than the problem (7-15), we state the following result.

Theorem 8. The decision version of the integrated evacuation planning problem is $\mathcal{NP}$-complete.

4. SOLUTION IN AN INTEGRATED EVACUATION NETWORK

In this section, we deal with an integrated evacuation network $\mathcal{N}$. As discussed in Subsection 3.2, the number of evacuees at each pickup location is given, but their approach does not speak about the arrived pattern at Y. The main aim of our work is to present an analytical investigation of an appropriate network flow model and give an efficient solution algorithm, as considered in Subsection 3.1 that provides information on the number of evacuees arriving at the pickup locations for the BEPP. Evacuees are collected on the earliest arrival flow pattern from s. Such evacuees at Y are considered as the supplies for $\mathcal{N}_2$ and are to be assigned to the available transit-vehicles in an integrated evacuation network.

Problem 3. Given an evacuation network $\mathcal{N} = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$, having supplies and demands at s and Z, respectively. The transit-vehicle assignment problem is to assign the vehicles for evacuees transshipment with minimum clearance time.

Algorithm 2: The transit-vehicle assignment algorithm for minimum clearance time.

```
Input  : An embedded evacuation network $\mathcal{N} = (s, d, V, Y, A, E, u_a, \tau_a, \tau_e, Z)$.
1 In $\mathcal{N}_1 = (s, V, A, u_a, \tau_a, Y)$, consider $Y$ as the sinks and determine the earliest arrival of evacuees for
    $\tau_a = 0$ at different $Y$ from $s$, by using Algorithm 1.
2 Assign the transit-vehicles from $d$ to $\mathcal{N}_2 = (d, Y, E, \tau_e, Z)$ for the supplies provided by Step 1 at $Y$, as
    guided by the dominant vehicle assignment approach as in Subection 3.2.
3 Stop, if all the supplies at each of $Y$ are fulfilled, respecting the capacity constraints of $Z$.
4 Otherwise, return to Step 2.
Output: Transit-vehicle assignment with the minimum clearance time from $s \to Z$.
```

Theorem 9. The transit-vehicle assignment algorithm as in Algorithm 2 gives the dominating solution for the transit-vehicle assignment problem as in Problem 3 with minimum clearance time.

Proof. Step 1 is feasible, since $\mathcal{N}_1$ is constructed in which the earliest arrival flow of evacuees exists for $\tau_a = 0$. As $\mathcal{N}_2$ is embedded in $\mathcal{N}_1$ for the appropriate vehicle assignment in $\mathcal{N}$, Step 2 is feasible. Feasibility of Step 3 is obvious as the flow respects the supplies as well as the demands in the network $\mathcal{N}$. Step 4, is of course, feasible. Hence, the algorithm is feasible.

Now, we show that Algorithm 2 gives a dominating solution. The maximum amount of flows are assigned from s across different auxiliary nodes to Y in $\mathcal{N}_1$, as in the form of the earliest arrivals of evacuees by using Algorithm 1. So, it gives the maximum possible flow of evacuees at each instance in the earliest arrival time and saves the unused capacity by Theorem 2. These resulting flows of such evacuees arrived at Y are taken as an input in $\mathcal{N}_2$ for the transit-vehicle in the subsequent evacuation process with the dominating vehicle assignment approach as in Subsection 3.2. Such an assignment of vehicles is continued until the last evacuee on such pickup locations reached to the sinks without violating their capacities. Hence, the resulting vehicle assignment in the integrated evacuation network gives the dominating solution with minimum clearance time. Hence, the theorem is proved. $\square$

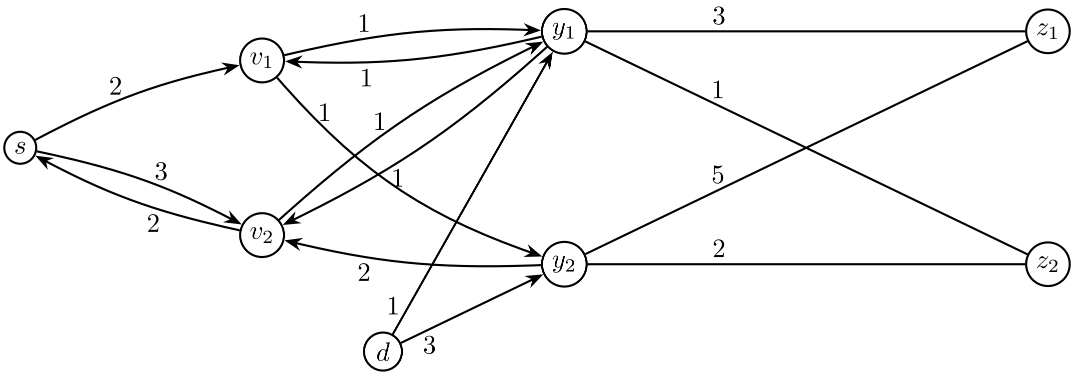

Figure 4: An instance of an integrated evacuation network $\mathcal{N}$

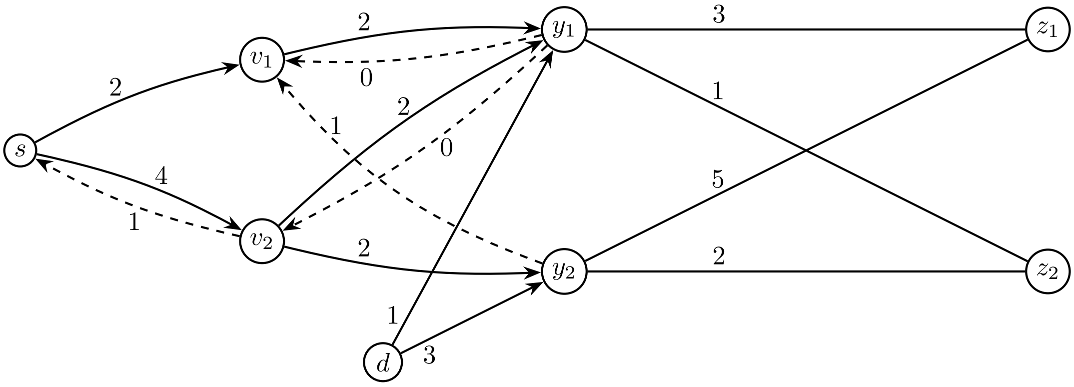

Figure 5: An instance of an integrated evacuation system with partial arc reversal in $\mathcal{N}_1$.

Example 3. Consider an instance of integrated evacuation system in $\mathcal{N}$, as in Figure 4. Consider a scenario with 12 units of evacuees at s, and with the pickup demands 8 and 4 for y_1 and y_2, respectively. Let the capacity of the sinks be 6 and 10 for z_1 and z_2, respectively.

Each of the paths $s - v_1 - y_1$, $s - v_2 - y_1$ and $s - v_1 - y_2$ can be assigned with 1 unit of flow as the earliest arrival flow of evacuees at zero transit time. Such flow is temporally repeated as in Theorem 4 and will collect 8 and 4 units of flows at y_1 and y_2, respectively, after 4-time units. Similarly, for the arc reversal capability, 2 units of flow can be assigned on each of the above-mentioned paths at zero transit time and are also temporally repeated. It can collect 8 and 4 units of flows at y_1 and y_2 respectively after 2-time units by using Algorithm 1 and also helps to improve the waiting instance.

We seek to find an optimal tour plan of this scenario using the dominating vehicle assignment approach as demanded by heuristic H4 (cf. Subsection 3.2), without violating the demands and the capacity constraints. Let the transit-vehicles be located at the depot d. Then their respective $\mathcal{T}_{\max}$ on $\mathcal{N}$, without and with arc reversal capability in $\mathcal{N}_1$, can be estimated as 41 and 39, respectively for $|B| = 1$.

Here, we have $\mathcal{T}_{\max}$ is 7 and 5 respectively, concerning to without and with arc reversal capability, for all $|B| \geq 12$. It also gives the threshold number of transit-buses. Moreover, the saved capacities obtained from partial arc reversals are shown along with the dotted arcs in Figure 5.

5. CONCLUSIONS

A solution of an evacuation planning problem is preferential to plan within the available time horizon to execute the maximum flow in each possible time unit and is offered by the earliest arrival flows. The earliest arrival flow does not exist in the network with multiple sinks for general transit times. Under some characterizations, some of the specific networks always permit for the earliest arrival transshipments regardless of all choices of the capacities and balances in zero transit times.

In our work, we are focused on the new and better-suited form of arrival pattern of evacuees in the earliest arrival flow pattern for the integrated evacuation planning problem. It will maximize the arrival of evacuees at every possible instance at the pickup locations with zero transit times from a source. We present a polynomial-time earliest arrival evacuee algorithm following the principle of temporally repeated flows to solve the earliest arrival evacuee problem with zero transit times and partial arc reversal capability. Such evacuees collected at different pickup locations of the primary sub-network are considered as the supplies during the subsequent evacuation process of vehicle assignment for the secondary sub-network. The partial arc reversal approach for the collection of evacuees also reduces the waiting instances at different pickup locations by collecting them earlier and helps to improve the solution of our integrated evacuation model. The assignment of transit-vehicles in such an integrated evacuation network is also carried in a dominating solution approach for the minimum evacuation duration. Different analytical issues are addressed to improve the performance of the evacuation in an integrated framework, though the problem itself, in general, is challenging. Its extensions for the arbitrary time setting and their different approximation approaches, including the experimentation and different case studies closer to the real scenarios, are the further research interests.

Conflict of interest. There is no conflict of interest regarding the publication of this paper.

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Minimum Clearance Time on the Prioritized Integrated Evacuation Network

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Abstract: The evacuation planning problem can be viewed as different variants of dynamic flow maximization and time minimization problems. An optimal solution to the latter problem sends a given amount of flow from disaster zones to safe zones in minimum time. We solve this problem on an embedded integrated network of a prioritized primary and a bus-routed secondary sub-networks. For a lexicographically maximum (lex-max) dynamic flow problem, we are given a time horizon and a prioritized network, where we need a feasible dynamic flow that lexicographically maximizes the flow amount leaving each terminal respecting the priority. Here, we use the quickest transshipment partial arc reversal strategy to collect the evacuees in minimum time from the disaster zones to the pickup locations of the primary sub-network. By treating such pickup locations as sources, the available set of transit-buses is assigned in the secondary sub-network to shift the evacuees finally to the sinks on the first-come-first-serve basis. This novel approach proposed here is better suited for the simultaneous flow of evacuees with minimum waiting delay at such pickup locations in the integrated evacuation network topology. The lane reversal strategy significantly reduces the evacuation time, whereas reversing them only partially has an additional benefit that the unused road capacities can be used for supplying emergency logistics and allocating facilities as well.

Keywords: Evacuation Planning, Integrated Network, Minimum Clearance Time, Lexicographic Flow, Partial Arc Reversal

1. Introduction

Problems on dynamic networks were firstly introduced by Ford and Fulkerson [1, 2]. The minimum clearance time evacuation planning problem considered here has been modeled in dynamic networks in which each arc has a nonnegative flow capacity and integer transit time. This is one of the fundamental problems in evacuation planning to find the minimum time limit such that all supplies can be sent to the sinks (safe places) from the sources (disastrous zones). There has been a fair amount of work regarding different aspects of the evacuation planning problems, including the quickest one for minimum clearance time, as referred by [3-6]. These problems are usually handled with a different perspective, namely, the transit-based, car-based, and pedestrian movements depending on evacuation scenarios. The transit-based planning problems are to minimize the duration of evacuation by routing and scheduling a fleet of vehicles, say buses. The NP-hard multi-depot, multi-trip bus-based evacuation planning problem (BEPP) was introduced and analyzed prominently in [7] which is closer to the split delivery multi-depot vehicle routing problem with inter-depot routes. However, if there is only one bus-depot, assuming that the bus pick ups the same number of people that equals its capacity, the author in [8] has also proposed the BEPP for the evacuation of a region from a set of collection points to a set of capacitated sinks. Based on such BEPP, Pyakurel et al. [9] explored a wide horizon to the research related to the transit-dependent evacuation planning problem. In their study, the excess exterior points of the disaster region were taken as the pickup locations and the available open spaces as the sinks. Evacuees were supposed to gather themselves from their residents to the nearby pickup locations. Homogeneous buses with the uniform capacity were used for the evacuees' pickup. Their computational analysis noticed that the domain of optimal solutions remains on the larger number of buses with higher capacity and speed irrespective of the number of sources and sinks chosen.

A solution to maximum flow problem sends the maximal amount of flow from the sources to the sinks for the fixed integer time horizon T. Lexicographically maximum (lex-max) flow problem with many sources and many sinks was introduced and many efficient algorithms were presented from different aspects in [10-13]. In such a problem, the terminals (sources and/or sinks) are ordered with certain priority for a lex-max flow respecting the priority. Such a lex-max flow is not necessarily a maximum flow and vice versa, however, they are equivalent for two-terminal networks [2]. If a flow is both source-optimal and sink-optimal, then it becomes the lexicographically optimal flow in the priority network [14].

The lex-max dynamic contraflow problem was investigated in [15] where reversals of arcs are allowed. The partial contraflow with path reversals leads to a significant improvement in increasing the flow values, decreasing the evacuation time, and utilization of the unused capacities of paths for humanitarian logistics and vehicle movements, [16]. Authors in [17] have introduced the partial lane reversal strategy and presented efficient dynamic flow algorithms for quickest and maximum flow evacuation planning problems. In the quickest transshipment problem, a given number of evacuees has to be shifted in minimum time. Such problems have been studied by the help of the lex-max dynamic flow problem applying the minimum cost flow computations as a tool [18]. An algorithm to find the universally quickest transshipment has been presented [19]. They also have used the minimum cost flow computations.

In this work, evacuees are collected from the disaster zone to the pickup locations of the primary sub-network in minimum time as the quickest transshipment by using the lex-max flow approach. Considering such pickup locations as the sources, the available set of transit-buses are also assigned in the network to evacuate the evacuees safely to the sinks on the first-come-first-serve basis. This novel approach proposed here is better suited for the simultaneous flow of evacuees with minimum waiting delay at such pickup locations for the evacuation planning problem in the integrated network.

In Section 2, we explain an integrated network topology. The arrival of evacuees at the primary sub-network in Section 3. The assignment of vehicles in the secondary sub-network including its embedding to the primary sub-network is described in Section 4. An integrated solution approach for the proposed problem is presented in Section 5. Section 6 concludes the paper.

2. An Integrated Network Topology

Consider two separate dynamic networks N_1 and N_2 as a primary and a secondary sub-network, respectively. These two are embedded as a unit to form an integrated evacuation network as $N = N_1 \cup N_2$. Among them, N_1 is a directed two-way network and N_2 is the mixed network having directed one-way arcs and undirected edges.

2.1. The Primary Sub-network

Let $N_1 = (S, V, A, u_a, \tau_a, Y)$ be a primary sub-network, $S = \{s_1, s_2, \dots, s_n\}$, $V = (v_1, v_2, \dots, v_n)$ and $Y = (y_1, y_2, \dots, y_n)$ denote the sets of sources, auxiliary nodes, and pickup locations, respectively. The set of arcs joining any two nodes in N_1 are denoted by A where the capacity and transit time are denoted by u_a and τ_a , respectively. The capacity $u_a: A \to \mathbb{Z}_{\geq 0}$ restricts the amount of flow on the arc and the transit time $\tau_a: A \to \mathbb{Z}_{\geq 0}$ represents the time required for the flow to transverse through the respective arc. During evacuee arrival at the pickup location Y from S, the set $Y = \cup \ \{Y_i\}$ is the set of sinks.

2.2. The Secondary Sub-network

Let $N_2 = (d, Y, E, \tau_e, Z)$ be a secondary sub-network, where d and $Z = \{z_1, z_2, \dots, z_n\}$ are the bus depot and sink, respectively. A set of transit-buses $B = \{b_1, b_2, \dots, b_n\}$ with uniform capacity are located initially at the bus depot d and are assigned as required during the evacuation procedure. Buses do not return to d even after the completion of the evacuation process as it is risky to return to it under such threats. So, it does not play significant roles further in the system. The set E consists of the one-way arcs $e = (d, y)$ with $y \in Y$ and the undirected edges $e = [y, z]$, with $y \in Y$ and $z \in Z$. Transit times of the respective arcs and edges are denoted by $\tau_e \in \mathbb{Z}_+$ as τ_{di} and τ_{ij}, respectively.

2.3. An Embedding of the Integrated Network

In an embedding, $N = N_1 \cup N_2$, capacity, transit times, and other related attributes for N are carried over from their respective sub-networks. The node Y works as the sinks concerning N_1 but as the supply for N_2. Transit-buses are assigned in N_2 from d, which are sufficiently closer to it, on the first-come-first-serve basis, i.e., the evacuees collected earlier will be assigned earlier to the appropriate sink and will be continued till the supply is available at Y respecting the capacity of the sinks Z.

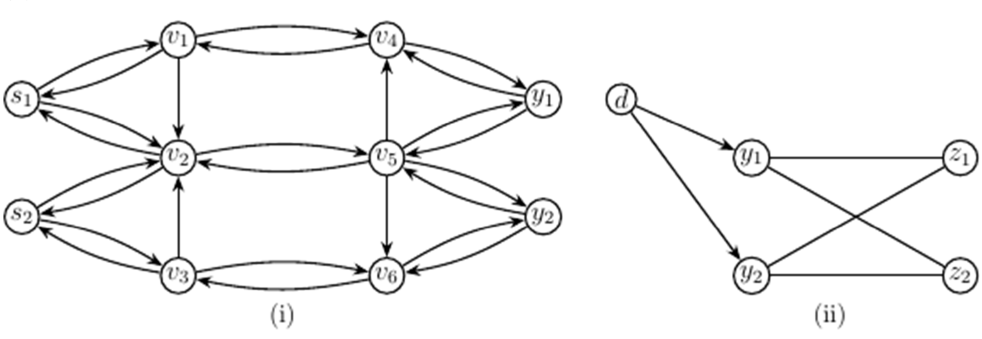

Figure 1. *(i) A primary sub-network, and (ii) a secondary sub-network.*

3. Arrival of Evacuees

The nature of the evacuees and the intensity of the disasters may vary at different sources. Mostly, the evacuees collected at pickup locations have to wait long for being assigned to transit-buses in the network. On the other hand, the nature of the pickup locations may also differ from each other depending upon their locations, availability, etc. So, the network is better to be prioritized. For such formulations, the lexicographic approach is better suited as different priorities can be assigned to various attributes. Such an approach can be applied to achieve the quickest transshipment with several sources and sinks, provided with the given supply and demand [18].

An $s-y$ flow of evacuees over time is a non-negative function f on $A \times T$ for given time $T=\{0,1,\dots,T\}$ satisfying the flow conservation and capacity constraints (1-3). The inequality flow conservation constraints allow it to wait for flow at intermediate nodes, however, the equality flow conservation constraints force that flows entering an intermediate node must leave it.

$$\sum_{a\epsilon\, A_i^{in}} \sum_{\sigma=\tau_a}^{T} f\ (a,\sigma-\tau_a)-\sum_{a\epsilon\, A_i^{out}} \sum_{\sigma=0}^{T} f\ (a,\sigma) = 0, \forall\, i \in V\backslash(S \cup Y) \tag{1}$$

$$\sum_{a\epsilon\, A_i^{in}} \sum_{\sigma=\tau_a}^{\theta} f\ (a,\sigma-\tau_a)-\sum_{a\epsilon\, A_i^{out}} \sum_{\sigma=0}^{\theta} f\ (a,\sigma) \geq 0, \forall\, i \in V\backslash(S \cup Y), \theta \in T \tag{2}$$

$$0 \leq \mathrm{f}\,(\mathrm{a},\theta) \leq u_a\ \forall\, a \in A, \theta \in T \tag{3}$$

The sets of outgoing and incoming arcs for the node $i \,\epsilon\, V$ are denoted by, $A_i^{out}=\{a=(i,j)\,\epsilon\, A\}$ and $A_i^{in}=\{a=(j,i)\,\epsilon\, A\}$, respectively. Not stated otherwise, for all $y \,\epsilon\, Y$ and $s \,\epsilon\, S$, we assume that $A_i^{out}=A_i^{in}=\phi$ in the case without arc reversals. However, for the source node s and sink node y, we get the flow value be $\upsilon_{\mathrm{f}}(\mathrm{s})>0$ and $\upsilon_{\mathrm{f}}(\mathrm{y})<0$, respectively, where $\sum_{i\,\epsilon\, V} \upsilon_{\mathrm{f}}(\mathrm{i})=0$.

Problem 1. Given an evacuation sub-network $N_1=(S,V,A,u_a,\tau_a,Y)$ *with supplies at S, demands at Y auxiliary nodes V, arc capacity* u_a*, and arc transit time* τ_a *for* $a \,\epsilon\, A$*. The quickest partial arc reversal transshipment problem is to find the quickest arrival of evacuees at Y with partial arc reversals capability.*

Let the reversals of an arc $a=(i,j)$ be $a'=(j,i)$. Then the transformed network of N_1 consists of the modified arc capacities and constant transit times as,

$$u_{\bar{a}}\ =\ u_a+\ u_{a'} \text{ and } \tau_{\bar{a}}=\tau_a \text{ if } a \in A \text{ and is } \tau_{a'} \text{ for } a \notin A. \tag{4}$$

Here, an edge $\bar{a} \in \bar{A}$ in transformed network $\overline{N_1}$ if $a \vee a' \in N_1$. Concerning the auxiliary reconfiguration, it is allowed to redirect the arc in any direction with the modified increased capacity but with the same transit time in either direction. The remaining graph structure and data are unaltered. In the transformed network $\overline{N_1}$, we have solved the lex-max dynamic flow problem on each arc as in [18] and saved all unused arc capacity as in [17].

Consider the set of sources and sinks be prioritized as $\{s_1,s_2,\dots,s_n\}$ and $\{\mathrm{y}_1,\mathrm{y}_2,\dots,\mathrm{y}_\mathrm{k}\}$, respectively. Let s^* be the super source connected to such s_i with arcs having the infinite capacity and zero transit time. Let the sinks in N_1 be prioritized with the highest priority to the nearest by determining their shortest distances concerning s^*. The lex-max flow within the specified time horizon T entering the sinks Y in N_1 in that order can be computed in polynomial-time, as in Algorithm 1, based on [18].

Algorithm 1. Lex-max dynamic flow of evacuees in N_1*.*

Input: A dynamic sub-network $N_1=(S,V,A,u_a,\tau_a,Y)$. Let $V=V\cup\{s^*\}$ and $A^{\delta+1}=V\cup\{(s^*,s_i)\}: s_i \in S$, where $u_a(s^*,s_i)=\infty$ and $\tau_a(s^*,s_i)=0$ for $\{s^*\}$ be the super source. Let $N_1^{\delta+1}$ denotes the resulting network with $g_0^{\delta+1}=0$ be the zero flow and the set of chains be $\Gamma^{\delta+1}=\phi$.

1. For $i=\delta,\dots,1$, set the arc be $A^i=A^{i+1}$,

a. If s_i is sink, add the arc $s_i s^*$ with $u_a(s^*,s_i)=\infty$ and $\tau_a(s^*,s_i)=-(T+1)$. Then get $f^i=$ minimum cost circulation in the resulting network using τ_a as the arc cost.

b. If s_i is source, delete the arc $s_i s^*$ from A^i and get $f^i=$ minimum cost maximum $s^* s_i$ – flow in the resulting network using τ_a as the arc cost.

2. Update the dynamic flow $g^i=g^{i+1}+f^i$.

3. Let $\Delta^i=$ standard chain decomposition of f^i, then the chain decomposable set becomes $\Gamma^i=\Gamma^{i+1}+\Gamma^i$.

4. Finally, return $\Gamma=\Gamma^1$.

As a *mutatis mutandis* to the results in [18], we are here with two more results.

Theorem 1. Algorithm 1 constructed for the lex-max dynamic flow in N_1 *gives the feasible solution for the lex-max number of evacuees at* Y*.*

Theorem 2. For any lex-max dynamic flow problem, a lex-max dynamic flow can be computed via k minimum cost flow (MCF) computations in $O(k(MCF)(m,n))$*, where* $O\big(k(MCF)(m,n)\big)=O(m\log n\,(m+n))$ *[20].*

Hoppe and Tardos in [18] studied the lex-max flow problem in dynamic networks by applying the MCF computations and have shown the quickest transshipment problem is equivalent to it. Different algorithms to solve such quickest transshipment have been presented based on the chain-decomposable flows and the minimization of submodular functions but are with high-order time complexity, though are polynomial-time solvable. Such chain decomposable flows help to generalize the stationary dynamic flows and the temporary repeated flows. The temporally repeated flows were referred to as the standard chain-decomposable flows. Such chain flows in a nonstandard chain decomposition may use a backwards arc with negative transit time. Based on this landmark paper [18], authors in [21] have also presented the quickest transshipment algorithm to determine the quickest transshipment as a convex combination of simple lex-max dynamic flows. Now, we are presenting the quickest partial arc reversal transshipment algorithm to get the quickest arrival of evacuees at Y in N_1 corresponding to the arc reversal capability.

Algorithm 2. Quickest partial arc reversal transshipment algorithm.

Input: A dynamic sub-network $N_1 = (S, V, A, u_a, \tau_a, Y)$, with the supply and demand.

1. Construct a transformed dynamic sub-network $\overline{N_1}$ as in Equation (4).
2. Solve the quickest transshipment problem [18] in the transformed network of Step 1.
3. For each $\theta \in \mathrm{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ if and only if $c_a > u_a$, u_a replaced by 0 whenever $a \notin A$, in N_1, where c_a denotes the static $s-$ y flow value in each $a \in A$ for such sub-network.
4. For each $\theta \in \mathrm{T}$ and $a \in A$, if a is reversed, $k_a = u_{\bar{a}} - c_{a'}$ and $k_{a'} = 0$. If neither a nor a' is reversed, $k_a = u_a - c_a$ where k_a is saved capacity of a, [17].
5. Output: The quickest arrival of evacuees at Y in N_1 with partial arc reversal capability.

Theorem 3. Algorithm 2 constructed for the quickest partial arc reversal transshipment gives the optimal solution for the quickest arrival of evacuees at Y and saves the unused capacity.

Proof. The construction of a transformed dynamic network $\overline{N_1}$ for the given network N_1 in Step 1 is feasible. From the quickest transshipment problem [18], Step 2 is feasible. Moreover, the flow is either on arc a or a' but not in both directions simultaneously. And such flow is not greater than the modified capacities of each arc in the transformed network. So, Step 3 is also feasible. Step 4 is also feasible and helps to compute the saved capacity by arc reversal capability. Hence, Algorithm 2 is feasible.

Now, we show that Algorithm 2 gives the optimal solution. In the transformed dynamic network $\overline{N_1}$ we compute the lex-max number of evacuees reached to each of the prioritized pickup locations Y as the lex-max dynamic flow as in [18]. Moreover, the obtained solution is equivalent to the solution of the quickest arrival of evacuees at Y in N_1 with the partial arc reversal up to the necessary capacity as in Step 3, [17]. Capacities of the arcs not used by the flow after partial arc reversals are computed in Step 4. □

Theorem 4. For the quickest partial arc reversal transshipment in N_1, the quickest evacuee arrival problem can be computed in polynomial-time complexity via k minimum cost flow (MCF) computations in $O(k(MCF)(m,n))$ time, where $MCF(m,n) = O(m \log n\ (m + n \log n))$.

Proof. Steps 1, 3, and 4 related to the arc reversal capability as in Algorithm 2 are solved in linear time. So their time complexity is dominated by the time complexity of the computation of the quickest evacuee arrival in N_1 and is solved in polynomial-time in $\mathrm{O(k(MCF)(m,n)}$ where $MCF(m,n) = O\big(m \log n\ (m + n \log n)\big)$ as in Theorem 2. □

Example 1. Consider the primary sub-network for evacuee arrival as in Figure 2(i). Consider the evacuees at s_1 and s_2 be 16 and 13, respectively. Let the potential demands of Y be $\alpha(Y_1) = 20$ and $\alpha(Y_2) = 9$. Consider s^* be the super source connected to s_1 and s_2 in N_1 having infinite-capacity and zero-transit time, then the highest priority is to be assigned for y_1 as specified. Then the respective arrivals of the evacuees at y_1 and y_2 can be determined as in Table 1 (using Algorithm 1) and Table 2 (using Algorithm 2) with their respective paths assignment without and with arc reversal capability in N_1, respectively.

Table 1. *The arrival of evacuees at Y without partial arc reversal capability.*

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		2
1	8	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		4
2	9	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		6
3	10	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		8
0	10	$s_2 \to v_2 \to v_5 \to y_1$	1		9
4	11	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		11
1	11	$s_2 \to v_2 \to v_5 \to y_1$	1		12
5	12	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		14
2	12	$s_2 \to v_2 \to v_5 \to y_1$	1		15
0	12	$s_2 \to v_3 \to v_6 \to y_2$	-	3	18
6	13	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		20
3	13	$s_2 \to v_2 \to v_5 \to y_1$	1		21
0	13	$s_1 \to v_1 \to v_4 \to v_5 \to y_1$	2		23
1	13	$s_2 \to v_3 \to v_6 \to y_2$	-	3	26
2	14	$s_2 \to v_3 \to v_6 \to y_2$	-	3	29
Total			20	9	29

Table 2. *The arrival of evacuees at Y with partial arc reversal capability.*

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		4
1	8	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		8
2	9	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		12
3	10	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		16
0	10	$s_2 \to v_2 \to v_5 \to y_1$	4		20
0	12	$s_2 \to v_3 \to v_6 \to y_2$	-	6	26
1	13	$s_2 \to v_3 \to v_6 \to y_2$	-	3	29
Total			20	9	29

4. Assignment of Vehicles

In this section, we investigate the BEPP to propose an integrated evacuation planning approach for such a problem.

4.1. Bus-based Evacuation Planning Problem

Problem 2. Let $i \in Y, j \in Z$ with τ_{ij} as the source-sink travel times. Let τ_{di}, l_i and μ_j be the depot-source travel times, number of evacuees and sink capacities, respectively. Then the BEPP is to find a tour plan to minimize the maximum travel times overall buses such that all the evacuees be transported to the sink.

Let the number of evacuees at every source be known. Assume Q be the uniform bus capacity, as a unit. The movement between the pickup locations Y is ignored and the same situation in between the sinks Z. The set of tours of the buses cannot be changed anymore after they start to move. Let $\sum_{i\in Y} l_i$ and $\sum_{j\in Z} \mu_j$ be the total number of evacuees and the total sink capacity, respectively. The maximum number of rounds R for the evacuation process is given by $\sum_{i\in Y} l_i$. The nonnegative travel cost of τ_{ij} on each edge $e = (i, j) \in E$ are taken symmetric and satisfies the triangle inequality. The variables τ_{to}^{br} and τ_{back}^{br} give the travel time for the bus b within the round r from source to sink, and from the sink to the next source, respectively. Let T_{max} be the duration of evacuation overall buses. The problem can be formulated to minimize $T_{\max}$ such that,

$$T_{max} \geq \sum_{i\in Y} \sum_{i\in Z} \tau_{di}\ x_{ij}^{b1} + \sum_{r\in R} \tau_{to}^{br} + \sum_{r\in R} \tau_{back}^{br}\ , \forall\ b\ \in\ B, \tag{5}$$

$$\tau_{to}^{br} = \sum_{i\in Y} \sum_{i\in Z} \tau_{ij}\ x_{ij}^{br}, \forall\ b\ \in\ B, r \in R, \tag{6}$$

$$\tau_{back}^{br} \geq \sum \tau_{ij} \left[\ \sum_{k\in Y} x_{kj}^{br} + \sum_{l\in Z} x_{il}^{b,r+1} - 1\right], \forall\ b \in B, r \in R-1, \tag{7}$$

$$\sum_{i\in Y} \sum_{i\in Z}\ x_{ij}^{br} \geq\ \sum_{i\in Y} \sum_{i\in Z}\ x_{ij}^{b,r+1}, \forall\ b\ \in\ B, r \in R, \tag{8}$$

$$\sum_{i\in Y} \sum_{i\in Z}\ x_{ij}^{br} \leq 1, \forall\ b \in B, r \in R-1, \tag{9}$$

$$\sum_{i\in Y} \sum_{i\in Z}\ \sum_{r\in R} x_{ij}^{br} \geq\ l_i, \forall\ i \in Y, \tag{10}$$

$$\sum_{i\in Y} \sum_{b\in B}\ \sum_{r\in R} x_{ij}^{br} \leq\ \mu_j, \forall\ j \in Z, \tag{11}$$

$$x_{ij}^{br}\ \in \{0,1\}, \forall\ \tau_{to}^{br}, \tau_{back}^{br}, \mathrm{T}_{max}\ \in\ \mathbb{R}. \tag{12}$$

Constraint (5) needs T_{max} to be greater than or equal to the maximal travel cost subject to all bus movements and is to be minimized on $T_{\max}$. Constraints (6) and (7) are the measure of travel time for the bus b within the round r from source to sink, and from that sink to the next source, respectively. Constraint (8) tells that the tours are connected and can stop whenever they like. Constraint (9) allows a bus from a source to a sink per round. Also (10) and (11) represent the bus and shelter capacity constraints, respectively. Constraint (12) represents whether the bus b travels from source i to sink j in the round r.

For $i \in Y, j \in Z$. Let τ_{di}, τ_{ij}, l_i and μ_j as in Problem 2. For $k \in \mathbb{R}$, is there a tour plan with $T_{max} < k$, for the complete evacuation? Regarding the complexity for such a decision version, the following result is established. [22]

Theorem 5. The decision version of BEPP is NP-complete, even if $\tau_{di} = 0$ *and* $\tau_{ij} = \tau_{i'j}\ \forall\ i, i' \in Y$ *and* $j \in Z$.

For a solution, the branch and bound algorithms with four upper bounds and three lower bounds for time, three branching rules to minimize the number of branches and two tree reduction strategies to avoid the equivalent branches have been presented in [8]. Upper bounds have been constructed in polynomial-time complexity by four heuristic algorithms. Among the lower bounds, the first one is based on the estimation of the travel times from sources to sinks and from sinks to sources, respectively. The second lower bound is based on the fact that lower bound for the maximum travel time is the average travel time. The third one is about the simplification of model formulation.

4.2. An Integrated Evacuation Planning Approach

Transit-buses having uniform capacity Q are assigned from d which are sufficiently nearer to Y in N_2 *on the first-come-first-serve basis. Such assignment begins only after* $\alpha_1 \geq Q$ *for* α_1 *be the number of evacuees arrived at the highest pickup demand. For the subsequent assignments, the effective waiting instance* ξ *is almost negligible. However, waiting at pickup locations is comparatively better than to wait at the disaster zone itself. Buses are assumed to pick up their full capacities. For this, the potential demands of the pickup locations are adjusted to be the integral multiple of busloads. Let the potential demand of the pickup location* $y_k \in Y$ *be* $\alpha(y_k)$*. Then the demands can be adjusted to be* $\alpha'(y_k)$ *by using the following demand adjustment,*

$$\alpha'(y_{k-1}) = \lfloor\{\ \alpha(y_{k-1}) + \alpha(y_{k-2}) - \ \alpha'(y_{k-2}) + \cdots + \alpha(y_1) - \ \alpha'(y_1)\}\ /Q\rfloor . Q \tag{13}$$

But if the k^{th} *pickup location is the last one with the least priority, then it is taken as,*

$$\alpha'(y_k) = \alpha(y_k) + \alpha(y_{k-1}) - \ \alpha'(y_{k-1}) + \cdots + \alpha(y_1) - \ \alpha'(y_1) \tag{14}$$

With the constraints (5-12), the integrated evacuation planning problem can be reformulated to minimize $T_{\max}$ such that:

$$T_{max} \geq \xi + \sum_{r\in R} \tau_{to}^{br} + \sum_{r\in R} \tau_{back}^{br} \quad \forall\, b \in B, \quad (15)$$

As such an integrated problem is not easier than the BEPP in Section 4.1, we state the following result.

Theorem 6. The decision version of the integrated evacuation planning problem is NP-complete.

Example 2. By using Equation (13), the arrivals of evacuees at Y of N_1 in Figure 2(i) is shown as in Table 1 and Table 2 can be adjusted to be the integral multiple of busloads as in Table 3 and Table 4, with their respective paths assignment without and with arc reversal capability in N_1, respectively. Such evacuees are to be assigned on the integrated evacuation network (cf. Example 3) where the transit-buses are assumed to pick up their full capacities except for the last trip. For the last trip, it is assumed to be the integral multiple of busloads for the remaining evacuees (if any) for the complete evacuation by convention.

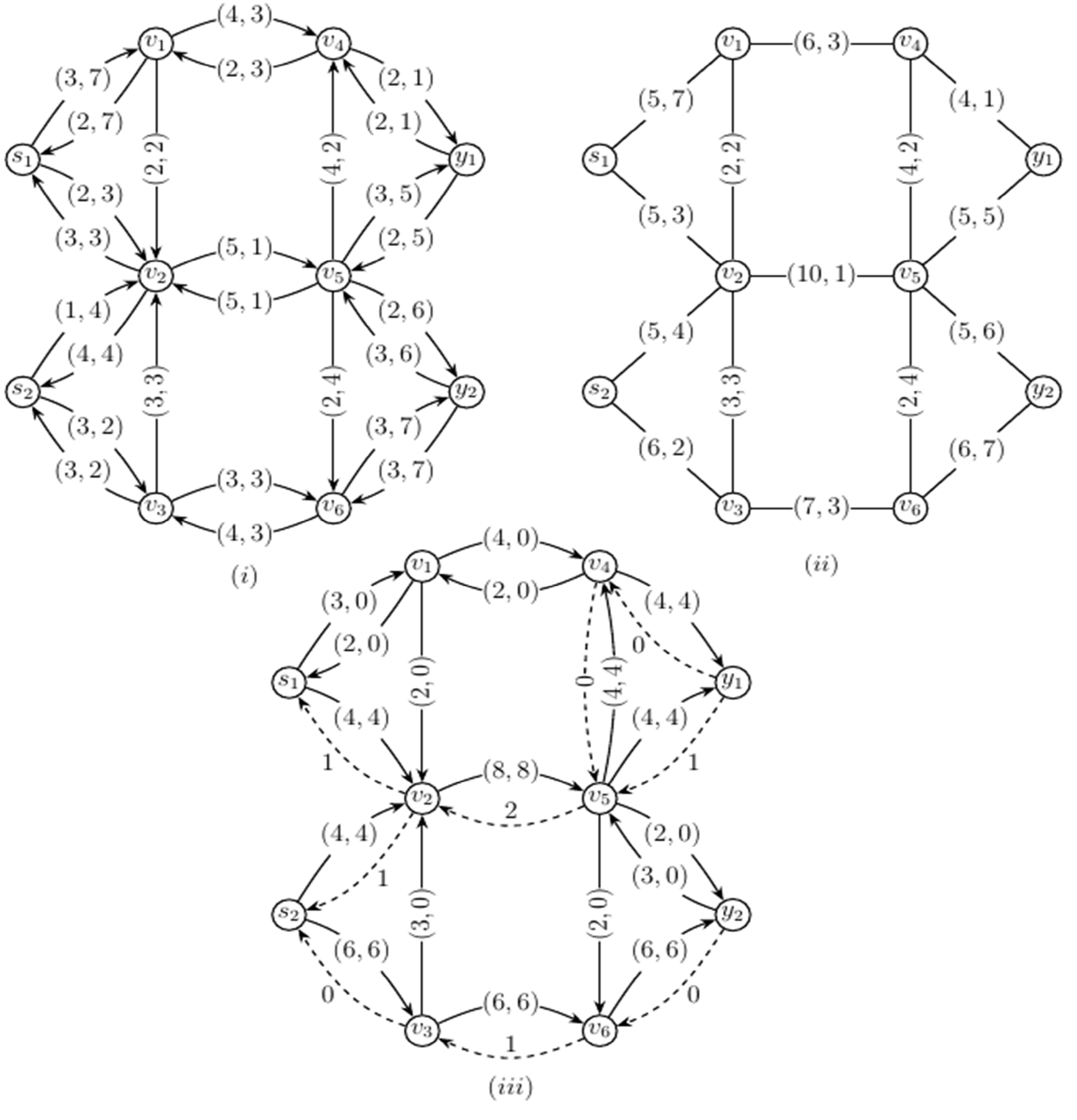

Figure 2. *(i) Primary sub-network with (u_a, τ_a), (ii) auxiliary network to (i), and (iii) network showing the arc capacities, flows and the saved arc capacities due to partial arc reversal concerning to adjusted demands.*

Table 3. *The arrival of evacuees at Y after demand adjustment without partial arc reversal capability.*

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		2
1	8	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		4
2	9	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		6
3	10	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		8
0	10	$s_2 \to v_2 \to v_5 \to y_1$	1		9
4	11	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		11
1	11	$s_2 \to v_2 \to v_5 \to y_1$	1		12
5	12	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		14
2	12	$s_2 \to v_2 \to v_5 \to y_1$	1		15
0	12	$s_2 \to v_3 \to v_6 \to y_2$	-	3	18
6	13	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	2		20
2	13	$s_2 \to v_2 \to v_5 \to y_1$	1		21
1	13	$s_2 \to v_3 \to v_6 \to y_2$	-	3	24
2	14	$s_2 \to v_3 \to v_6 \to y_2$	-	3	27
3	15	$s_2 \to v_3 \to v_6 \to y_2$	-	2	29
Total			18	11	29

Table 4. *The arrival of evacuees at Y after demand adjustment with partial arc reversal capability.*

Released time	Reached time	Paths assignment	Flow at y_1	Flow at y_2	Total flow
0	7	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		4
1	8	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		8
2	9	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		12
3	10	$s_1 \to v_2 \to v_5 \to v_4 \to y_1$	4		16
0	10	$s_2 \to v_2 \to v_5 \to y_1$	2		18
0	12	$s_2 \to v_3 \to v_6 \to y_2$	-	6	24
1	13	$s_2 \to v_3 \to v_6 \to y_2$	-	5	29
Total			18	11	29

The partial arc reversal strategy allows the reversal of only the necessary part of the road segments along with the direction of traffic in the evacuation network, saving the remaining part of the road segments. Due to such phenomena, in some arcs, certain parts are partially reversed as necessary and some remain saving. The dotted arcs represent the arc saving along with their measure as in Figure 2(iii). Here, the road segments $(y_1, v_5), (v_5, v_2), (v_2, s_1), (v_2, s_2)$, and (v_6, v_3) are saving the arc capacities of 1, 2, 1, 1, and 1 respectively. But no such arc saving on $(v_1, v_5), (y_1, v_4), (y_2, v_6)$ and (v_3, s_2) due to their full reversal, as demanded. Saved arc capacities are beneficial for the humanitarian logistics, facility locations, and also for the emergency vehicle movements within the network during such evacuation.

5. An Integrated Solution Approach

In an integrated solution approach for the evacuation planning problem, the quickest transshipment of the evacuees arrived at Y in N_1 as in the form concerning the adjusted demands are assigned to the transit-buses in the embedded network.

Algorithm 3. Evacuation planning algorithm in an integrated network topology.

Input: An embedding $N = (S, V, A, u_a, \tau_a, Y, d, u_e, \tau_e, Z)$, provided with given supply and demand.

1. Consider $N_1 = (S, V, A, u_a, \tau_a, Y)$ having their pickup locations be $\cup\, y_i = Y$.
2. Construct a priority ordering of Y assigning the highest priority to the nearest from S.
3. Determine the arrival of evacuees at Y of N_1 from S using Algorithm 2.
4. Assign the transit-buses from d to Y in $N_2 = (d, Y, u_e, \tau_e, Z)$ for the supplies obtained in Step 3, to the nearest sink Z, on the first-come-first-serve basis.
5. Begin the assignment with $\alpha_1 \geq Q$ for α_1 be the collection of evacuees at $y_1 \in Y$ and Q be the homogeneous capacity of each transit-buses and is continued for the adjusted demands at Y provided by Equation (13).
6. Stop, if all the supplies at each Y be fulfilled, respecting the capacity constraints of Z.
7. Otherwise, return to Step 4.

Output: Transshipment of evacuees finally to Z in minimum clearance time.

Theorem 7. Algorithm 3 constructed for the evacuation planning problem in an integrated network gives the feasible solution to send the evacuees to Z in minimum clearance time.

Proof. In Step 1, N_1 is constructed with the pickup locations be $\cup\, y_i = Y$, and is feasible. In this prioritized sub-network by construction, Step 2 is also feasible. The arrival of evacuees determined at Y provided by Algorithm 2 gives the feasibility as well as the validity of Step 3 of Algorithm 3. Two more Steps 4 and 5 are about the transit-buses assignment in the integrated network and are governed not only by the availability of the buses at d but also by the supply available at Y. It is continued in N for the available evacuees respecting the capacity constraints of Z and are feasible. Hence, the algorithm is feasible.

Now, we show that Algorithm 3 gives the feasible solution in minimum time. The arrival of the lex-max number of evacuees at Y from S by using Algorithm 2 gives the quickest transshipment of evacuees in the given priority by [18] and saves the unused capacity by Theorem 3. These resulting flows at Y are taken as the input in N_2 for the required transit-buses assignment. Such assignment to nearest sink approach respecting the priority is with almost negligible waiting delay for these evacuees at Y in the embedding. It is continued till the last evacuees are reached to the sink without violating their capacities. Hence, it gives a feasible solution with minimum clearance time. □

Example 3. Let the adjusted demands as in Table 3 and Table 4 be the available supplies for the bus assignment in the embedding as shown in Figure 3. Consider the available transit-buses be $|B| = 2$ *as* B_1 *and* B_2 *and are with uniform capacity of* 6. *The assignment concerning without and with path reversal capability in* N_1 *are illustrated briefly in Table 5 and Table 6, respectively. Here, the column* θ *represents different time instances in the integrated network,* B_1 *and* B_2 *as the assignment of transit-buses with their respective positions in* N_2. *Here,* M_{12} *and* M_{21} *are used to denote the mid-way of the road segments connecting* y_1 *to* z_2 *and* y_2 *to* z_1 *respectively. The columns for* y_i, y_i' *and* z_i *are denoting the total flow arrived at* Y, *released from* Y, *and reached to* Z, *respectively.*

The quickest transshipment at Y *and their assignment to* Z *be beneficial in the network having arc reversal capability than to the network without arc reversal capability, i.e., the sooner the better. It is 14, if the partial arc reversal is allowed otherwise 17, as illustrated with respect to* θ. *Here, the buses* B_1 *and* B_2 *have the route plan of* $d \to y_1 \to z_1 \to y_1 \to z_1 \to y_2 \to z_2 \to y_2 \to z_2$, *and* $d \to y_1 \to z_2$, *respectively concerning without arc reversal at* N_1. *But, with respect to arc reversal at* N_1, *the route plan of the buses are* $d \to y_1 \to z_1 \to y_1 \to z_2 \to y_2 \to z_2$ *and* $d \to y_1 \to z_1 \to y_2 \to z_2$, *respectively.*

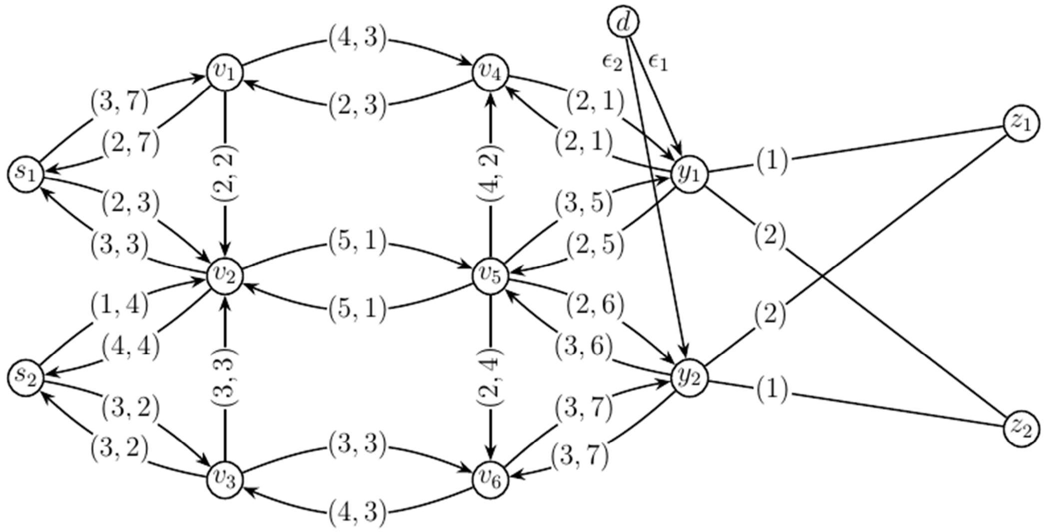

Figure 3. *An embedding* $N = N_1 \cup N_2$ *showing an instance of integrated evacuation scenario.*

Table 5. *Integrated evacuation planning approach in N for the demand adjusted evacuees.*

θ	B_1	B_2	y_1	y_1'	y_2	y_2'	z_1	z_2
9	y_1		6	6				
10	z_1		9				6	
11	y_1		12	12				
12	z_1		15		3		12	
13	M_{21}	y_1	18	18	6			
14	y_2	M_{12}			9	6		
15	z_2	z_2			11			12
16	y_2					11		
17	z_2							17

Table 6. *Integrated evacuation planning approach in N for the demand adjusted evacuees with partial arc reversal.*

θ	B_1	B_2	y_1	y_1'	y_2	y_2'	z_1	z_2
8	y_1		8	6				
9	z_1	y_1	12	12			6	
10	y_1	z_1	18	18			12	
11	M_{12}	M_{21}			6	6		
12	z_2	y_2			11	11		6
13	y_2	z_2						12
14	z_2							17

6. Conclusions

The quickest transshipment problem is to find the minimum clearance time to send a given amount of flow from multiple sources to multiple sinks network. For a lex-max dynamic flow problem, we are given a time horizon and a dynamic network with an ordered set of terminals, where we need a feasible dynamic flow that lexicographically maximizes the flow amount leaving each terminal in the given priority. The quickest transshipment problem in a dynamic network can be reduced to an equivalent lex-max flow problem and is solved in polynomial-time complexity.

Evacuees are collected at the prioritized pickup locations of the primary sub-network following the quickest transshipment in the lex-max flow approach and are assigned simultaneously to the homogeneous transit-buses in the secondary sub-network on the first-come-first-serve basis. The waiting delay at the pickup locations is almost negligible. On the other side, if there is some waiting, it is preferable to wait at such pickup locations rather than to be at the danger regions. The arc reversal capability of the primary sub-network is beneficial to improve the minimum clearance time of the evacuees in the integrated evacuation network where the saved unused arc capacities are useful for emergency facility locations and logistics. It is interesting to extend these techniques for the heterogeneous or mixed model transit-buses and also for the disparate group of evacuees in a different network topology.

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ON THE TRANSIT-BASED EVACUATION STRATEGIES IN AN INTEGRATED NETWORK TOPOLOGY

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Abstract: Evacuation planning problem deals with sending the maximum number of evacuees from the danger zone to the safe zone in minimum time as efficiently as possible. The dynamic network flow models for various evacuation network topology have been found suitable for the solution of such a problem. Bus-based evacuation planning problem (BEPP), as an important variant of the vehicle routing problem (VRP), is one of the emerging evacuation planning problems. In this work, an organized overview of this problem with a focus on their solution status is compactly presented. Arrival patterns of the evacuees including their transshipments at different pickup locations and their assignments are presented. Finally, a BEPP model and a solution for a special network are also proposed.

Key Words: Evacuation planning problem; evacuation network topology; bus-based evacuation
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1. INTRODUCTION

Evacuation planning problem deals with sending the maximum number of evacuees from source (danger zone) to sink (safe zone) in minimum time as efficiently as possible. It can be further classified into microscopic and macroscopic planning. The microscopic planning deals with the individual evacuee's behavior in which some probabilistic laws for individual evacuees movement are presented and mainly based on the simulation approaches. But in macroscopic planning, they are principally based on optimization approaches where the evacuees are treated as the homogeneous group and only the common characteristics are considered. Optimization approaches on such macroscopic evacuation planning can further be classified like a heuristic approach, population optimization, modeling as fluid dynamics, traffic management, optimal evacuation destination, and network flow formulation. Among them, the dynamic network flow formulation has been found suitable evacuation optimization approach. There has been a fair amount of work regarding different aspects of network flow formulation related to the evacuation planning problem, as referred by [2], [3], [6]- [8]. For detail about the time minimization on such problems with minimum clearance time in an integrated evacuation network topology, we refer to [1], [4] and the references therein.

For an evacuation network, a remarkable BEPP model was formulated by Bish in [5] to minimize the time of evacuation using a given number of homogeneous buses satisfying all evacuee demands, without violating both sink and vehicle capacity constraints. Here,

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the number of evacuees at the demand node might be greater than the capacity of a bus and requests the split delivery within the pickup locations. But for a BEPP in [12], the number of evacuees at every source node be known in terms of the integral multiples of the busloads and does not request for the split delivery service. However, the split delivery need not improve always the evacuation duration, as mentioned in [4].

A robust bus-based evacuation planning problem RBEPP has been presented in [11] in which only some possible scenarios of the evacuees are provided rather than the exact number in each of the collection points. Based on such BEPP and RBEPP, Pyakurel *et al.* [23] explored a wide horizon to the research related to the transit-dependent evacuation planning problem. Kathmandu, one of the densely populated city, has been considered as the disaster region. In their solutions, they have used the branch and bound approach presented in [12], and the tabu search heuristic from [11], respectively. In their results, the domain of optimal solutions remains on a large number of buses with higher capacity and speed, irrespective of the population chosen where the choice of the number of sources and sinks does not play a significant role. A risk-based bus schedule technique for pickup location with concerns of disaster dynamics and time-varying supply-demand conditions is proposed in [17].

This paper considers optimization problems in evacuation planning. However, covering the broad horizon of the evacuation strategies respecting to such problems in a single paper is almost impossible. Here, we are focused fundamentally on the time minimization aspect of the transshipment problems in the transit-based evacuation scenarios in the integrated evacuation network topology. A compact and systematic overview of the evacuation strategies based on network flow formulation with regard to their solution status is presented. Besides this, it has a glimpse of the arrival patterns of the evacuees in such the evacuation network and their assignment with reference to a bus-based evacuation planning problem.

The paper is organized as follows. Section 2 presents an organized overview of evacuation network topology, different variants the network flow models and their solution strategies. Collection of evacuees at different pickup locations in an evacuation scenario is presented Section 3. Bus based evacuation planning problem is in Section 4. Finally, Section 5 concludes the paper.

2. Evacuation Network Topology

Consider a dynamic network $\mathcal{N} = (G, u, \tau, S, Z)$ that consists of a directed graph $G = (V, A)$ in which an arc $e = (i, j) \in A$ has a flow capacity function $u : A \to \mathbb{R}_+$ and a transit time function $\tau : A \to \mathbb{R}_+$. Here, S and Z are used to denote the set of sources (danger zones) and sinks (shelters or safe zones), respectively. Their union, i.e. $S \cup Z$ denotes the terminals. It may additionally provided with supply/demand function $\nu : S \cup Z \to \mathbb{R}_+$ with supplies $\nu(s) > 0$ for all sources $s \in S$ and demands $\nu(z) < 0$ for all sinks $z \in Z$ such that $\sum_{s \in S \cup Z} \nu(s) = 0$. One of the fundamental problems in the dynamic network is the evacuation planning problem.

In a dynamic network flow model, the time can be considered continuous or discrete. In the case of a static network flow model, the time parameter is absent. In general, the continuous model yields more accurate results over the discrete but are more challenging to compute. Evacuees movement can be considered as pedestrian or car-based (auto-based) or bus-based (transit-based). In most of the large cities, many people depend on transit-vehicles, say buses, and are not provided with their own vehicles. The great loss of people in disasters is due to a lack of proper planning of transit-vehicles rather than the disaster itself. In such situations, transit-based evacuation system is more effective than others, in general, and can be operated in an integrated approach in a different network topology. However, it may depend upon the nature of the evacuation scenario. The nature of the pickup locations may also differ from each other. Such pickup locations can also be prioritized depending upon their locations and availabilities.

2.1. **Prioritized network.** A prioritized network is a multi-terminal network which consists prioritized terminals. Under the given priority of terminals, two flows can be compared according to departure/arrival flows form/in the sinks or sources. A flow value is said to be lexicographic if it is compared according to the rank of the terminals. Let $\mathcal{N} = (G, u, \tau, S, Z)$ be a prioritized network with priority $t_1, t_2, \ldots, t_n$; $t_i \in S \cup Z$. Let

$$|f|_t := \begin{cases} \sum_{e \in A_t^+} f_e, & t \in S \text{ is a source} \\ \sum_{e \in A_t^-} f_e, & t \in Z \text{ is a sink} \end{cases}$$

be the out/in flow value from/in the source and sink, respectively. Suppose f^1 and f^2 be the terminal respecting flows, f^1 is said to be lexicographically bigger than f^2 and written as $f^1 \geq_L f^2$ if $\exists l \in \{0, 1, \ldots, k-1\} : \forall\, i \in \{1, 2, \ldots, l\} : |f^1|_{t_i} = |f^2|_{t_i} \wedge |f^1|_{t_{l+1}} > |f^2|_{t_{l+1}}$ or $\forall i \in \{1, 2, \ldots, k\} : |f^1|_{t_i} = |f^2|_{t_i}$. The maximum flow respecting the rank of the terminals is said to be lexicographically maximum flow.

A solution to maximum flow problem sends the maximal amount of flow from the sources to the sinks for the fixed integer time horizon T. Lexicographically maximum (lex-max) flow problem with many sources and many sinks was introduced and many efficient algorithms were presented from different aspects in [19–21]. In such a problem, the terminals (sources and/or sinks) are ordered with certain priority for a lex-max flow respecting the priority, considering the set of sources and sinks be prioritized as $s_1, s_2, \ldots, s_k$ and $y_1, y_2, \ldots, y_k$, respectively, as a prioritized network. Such a lex-max flow is not necessarily a maximum flow and vice versa, however, they are equivalent for two-terminal networks [10].

In the quickest transshipment problem, a given number of evacuees has to be shifted in minimum time. Such problem have been studied by the help of the lex-max dynamic flow problem applying the minimum cost flow computations as a tool, in such a prioritized network. For a given time T and an ordered set of multi-terminals, the lex-max dynamic flow problem finds a feasible flow that lexicographically maximizes the flow leaving each terminal in the prioritized network. Hoppe and Tardos [13] deals with a chain decomposable flows in such a network.

A chain flow $\gamma = \langle \nu, \pi \rangle$ is a static flow of values $\nu \geq 0$ along the path π in a network $\mathcal{N} = (G, u, \tau, S, Z)$. The length of chain flow $\tau(\gamma)$ also represents total length of path $\tau(\pi)$.

Given the time horizon T no less than $\tau(\gamma)$, any chain flow γ induces a dynamic flow by sending ν units of flow along the path π at every time step until $T - \tau(\gamma)$. A proper chain flow starts and ends at terminals. A multiset of proper chain flows, $\Gamma = \{\gamma_1, \gamma_1, \dots, \gamma_k\}$ is a chain decomposition of static flows f if $\sum_{i=1}^{k} \gamma_i = f$. It becomes a standard chain decomposition of f if all chain flows in it use edges in the same direction as of f. A flow decomposition with zero flows on all cycles is the path decomposition. One may assume that there is no flow along any cycle as the opposite flows along all cycles could be canceled.

In the non-standard chain decomposition, the chain flows may use oppositely directed flows on edges. It may use a residual edge with negative transit times. For $e = (i, j)$, a unit of flow sent from i at time θ reaches j at time $\theta + \tau_e$ is nothing other than sending a negative unit of flow from j at time $\theta + \tau_e$ to reach i at time θ. Let γ_1' be the another chain that flows through (j, i), then that cancels the chain flow γ_1 along (i, j). The chain decomposable flows do not violate capacity constraints. Moreover, the non-standard chain decomposition induces the dynamic flows.

Starting with zero flow, the lex-max flow algorithm computes the successive layers of minimum cost static flows in the residual network of the previous layers and adds standard chains to the existing one. It takes k times the complexity of minimum cost flow computations, for a given time T.

Theorem 2.1. *[13] A k-terminal lex-max dynamic flow problem can be solved in polynomial-time complexity with $O(k.g(mn))$, where $O(g(mn))$ is required for one minimum cost flow computation on a network with n nodes and m edges.*

2.2. **Arc reversal network.** The arc reversal is an approach that modifies the orientation of arcs in the network to increase outbound flow with reduced time on the evacuation. It is an effective and widely accepted approach for the optimal use of available road network in evacuation management that increases the outward road capacities from the disastrous areas towards the safe destination.

Let the reversal of an arc $e = (i, j)$ be $e' = (j, i)$, then the transformed network of $\mathcal{N}$ consists of the modified arc capacities and constant transit times as,

$$u_{\overline{e}} = u_e + u_{e'} \text{ and } \tau_{\overline{e}} = \begin{cases} \tau_e, & \text{if } e \in A \\ \tau_{e'}, & \text{otherwise} \end{cases} \tag{2.1}$$

where an edge $\overline{e} \in \overline{A}$ in a transformed network, if $e \vee e' \in A$ in $\mathcal{N}$. The remaining graph structure and data are unaltered. For the solution status of a problem, we have

Theorem 2.2. *[28] The maximum static $s - z$ flow problem with arc reversals is polynomially solvable.*

Proof. The maximum static $s - z$ flow problem on arc reversal in the transformed network can be solved by decomposing the obtained flow into paths and cycles, and by deleting the latter one assuming that the arcs on either direction will never be used in the optimal flow. An arc $e' = (j, i) \in A$ is reversed if and only if the flow on (i, j) is greater than $u_e(i, j)$, or if there is a nonnegative flow along $(i, j) \notin A$ and the resulting flow is maximum with arc reversal. Such a flow is feasible and optimal too with polynomial-time complexity. □

And a similar result follows in a multi-terminal network by a simple reduction.

Theorem 2.3. *[28] The maximum static flow problem on the arc reversal network with multiple sources and sinks is polynomially solvable.*

Proof. As the general maximum static flow problem on arc reversal can be reduced to the respective $s^* - z^*$ flow problem, providing the super-source s^* and super-sink z^*. Here, s^* is connected to each $s \in S$ having arc capacities equal to their respective surplus and z^* to each $z \in Z$ having arc capacities equal their respective deficits. Hence, the respective static version of the maximum flow problem with arc reversal with multiple sources and multiple sinks is also polynomially solvable. □

The similar situation exists regarding the maximum dynamic flow problem on arc reversal in two-terminal networks, though in general, we have,

Theorem 2.4. *[28] The maximum dynamic flow problem in a multi-terminal network on arc reversal is $\mathcal{NP}$-complete.*

Such $\mathcal{NP}$-completeness in a multi-terminal network is so due to the conflict with reverting the intermediate arcs, [28]. Hence, numerous heuristics and metaheuristics have been presented and implemented for the solutions of different types of evacuation planning problems by arc reversal approach, [26]. Instead of full arc reversal, arcs are better to reverse up to the necessary capacity only, as the partial arc reversal. Such an approach has presented lucidly with different models, algorithms, and solution strategies by Pyakurel *et al.* [24,25]. In recent work, authors in [4], have integrated it with the earliest arrival transshipment and the flows in zero transit times as,

Theorem 2.5. *[4] The earliest arrival evacuee problem having zero transit times with partial arc reversal capability follows the principle of temporally repeated flows and can be solved in polynomial-time complexity.*

Proof. The flow that reached to each of the pickup locations at zero transit times determines the maximum number of evacuees at every possible time instance from the beginning, as in [29]. So, it follows the principle of temporally repeated flows on the transformed network which is equivalent to the solution with arc reversals capability on the original network, as in [27]. Hence, it can be solved in polynomial-time complexity. □

For the limited resources it is not possible to select all arc reversals as demanded by the optimal arc reversal strategy, as each arc reversal is associated with certain operating costs. To address such issues, its budget constraint version is investigated in [9]. They have solved such a problem in a time-expanded network so it is pseudo-polynomial.

Theorem 2.6. *[9] The maximum dynamic flow problem with budget constraint switching cost can be solved optimally in pseudo-polynomial-time.*

2.3. **Integrated evacuation network.** Depending upon nature and needs, different networks can be embedded to have an integrated network. For example, the authors in [14] had presented an integrated contraflow network for multimodal evacuation. They integrate the non-contraflow, the full-lane-contraflow, and the bus-contraflow networks to shorten the strategy set up time, to maximize the evacuation network capacity, and to realize the transit cycle operation, respectively. Adhikari *et al.* [4] have presented an integrated evacuation strategy in an embedded network topology, $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$, where $\mathcal{N}_1$ consists of directed two-way network respecting the partial arc reversal capability for the collection of evacuees. By treating such supplies as the sources, the available set of transit-vehicles at the depot are assigned to transverse the evacuees in the dominant routing to the sinks in $\mathcal{N}_2$. In such an approach, evacuees are collected at the pickup locations from the sources in the earliest arrival flow pattern at zero transit times and then they are assigned to the transit-vehicles in the embedded network with minimum clearance time.

Theorem 2.7. *[4] The transit-vehicle assignment algorithm for an integrated evacuation network gives the dominating solution for the transit-vehicle assignment problem with minimum clearance time.*

3. Collection of evacuees at pickup locations

The collection of evacuees at pickup locations can be categorized to follow different arrival patterns. But two of the prominent BEPP formulations as in [12] and [5] have assumed the evacuees to be at the pickup locations with no specific arrival patterns. Such a problem is extended to an RBEPP by assuming that the number of evacuees is not known exactly but a set of estimates for the number of evacuees at each source is given [11]. Evacuees have gathered themselves at different pickup locations relative to the population density of the transit-dependent people nearby them in [23] with no specific arrival patterns. A constant arrival rate of evacuees has been considered by the authors in [22]. Such an assumption is still unrealistic as the true arrival process is probabilistic. A linear programming mathematical model using binary variables was developed in [16] to select the most suitable location and the number of bus stops.

The cumulative percentage of total evacuees loaded in the evacuation network by time θ since the start of the evacuation can be estimated as [15],

$$\xi(\theta) = \frac{1}{1 + exp[-\alpha(\theta - h)]}. \tag{3.1}$$

Here, α is the loading rate of evacuees representing the response of the public to the disaster. The parameter h is the half loading time, represented by the mid-position of such curve. Let $\xi^i(\theta_k)$ and $\xi^i(\theta_{k-1})$ be the cumulative percentage of the evacuees arrival at the pickup location i at the end of time interval k and $k-1$, respectively. Let δ_i be the number of evacuees arriving at pickup location i during the evacuation time frame and θ_k is the end of the evacuation time interval at k and Q be the maximal passenger capacity of the vehicle. Let d_i^k be the number of demand of vehicles at each pickup locations i during the time interval k, then the number of evacuees arriving at pickup location i during the evacuation time frame is,

$$\delta_i = \frac{d_i^k \times Q}{\xi^i(\theta_k) - \xi^i(\theta_{k-1})}. \tag{3.2}$$

The flow of evacuees from the disaster zone $s \in S$ to the pickup locations $p \in P$ over time is a non-negative function f on $A \times \mathbb{R}_+$, for given time $\mathbf{T} = \{0, 1, \ldots, T\}$ satisfying the flow conservation and capacity constraints (3.3-3.5). The inequality flow conservation constraints allow waiting for flow at intermediate nodes. However, the flow conservation constraints force that flows entering an intermediate node must leave it again immediately.

$$\sum_{\sigma=\tau_e}^{T} \sum_{e \in A_i^{in}} f(e, \sigma - \tau_e) - \sum_{\sigma=0}^{T} \sum_{e \in A_i^{out}} f(e, \sigma) \;=\; 0, \;\forall\, i \notin \{S, P\}, \tag{3.3}$$

$$\sum_{\sigma=\tau_e}^{\theta} \sum_{e \in A_i^{in}} f(e, \sigma - \tau_e) - \sum_{\sigma=0}^{\theta} \sum_{e \in A_i^{out}} f(e, \sigma) \;\geq\; 0, \forall i \notin \{S, P\}, \theta \in \mathbf{T}, \tag{3.4}$$

$$0 \;\leq\; f(e, \theta) \;\leq\; u_e, \qquad \forall\, e \in A,\ \theta \in \mathbf{T}. \tag{3.5}$$

Here, $A_i^{out} = \{e = (i, j) \in A\}$ and $A_i^{in} = \{e' = (j, i) \in A\}$ are the sets of outgoing and incoming arcs, respectively for the node $i \in V$. For s, the flow value be $\nu_f(s) > 0$, and for p it becomes $\nu_f(p) < 0$, whereas $\sum_{i \in V} \nu_f(i) = 0$. If the supply and demand on such terminals be fixed for all $i \in \{s, p\}$, then the earliest arrival evacuee problem maximizes value(ν_f, θ) for all $\theta \in \mathbf{T}$, as in Equation (3.6) satisfying the constraints (3.3-3.5).

$$(\nu_f, \theta) = \sum_{\sigma=0}^{\theta} \sum_{e \in A_s^{out}} f(e, \sigma) = \sum_{\sigma=\tau_e}^{\theta} \sum_{e \in A_p^{in}} f(e, \sigma - \tau_e). \tag{3.6}$$

Let $\nu(P, \theta)$ stands for the flow amount out of source s that reaches to the pickup location P at time $\theta \in Z_+$ with zero transit times, then the total flow amount out of s that reached to P for all time up to $\theta' \in \mathbb{Z}_+$, with $\tau_e = 0$, is given by

$$|\nu_f|_{\theta'} = \sum_{\theta=1}^{\theta'} |\nu(P, \theta)|. \tag{3.7}$$

For the given time bound T, the value in Equation 3.7 becomes

$$|\nu_f| = \sum_{\theta=1}^{T} |\nu(P, \theta)|. \tag{3.8}$$

3.1. **Transshipment problems.** When preparing for an evacuation, it is uncertain how much time that will take to enact it. So, it is preferential to plan for the transversal of a maximum number of evacuees reaching safety not only at the ultimate clearance time but also in each possible time unit, i.e. with the maximum possible value of the value(ν_f, θ) for all $\theta \in \mathbf{T}$, as in Equation (3.6). It is offered by the earliest arrival flows. On the other hand, the quickest transshipment problem is to find the minimum clearance time to send a given amount of flow from multiple sources to multiple sinks. Each of the earliest arrival transshipment additionally optimizes the amount of flow leaving the network at all times and is therefore the quickest transshipment. But not necessarily the converse, as illustrated in Example 3.1. Moreover, the earliest arrival transshipment does not necessarily exist in the networks with multiple sinks, but the quickest transshipments do.

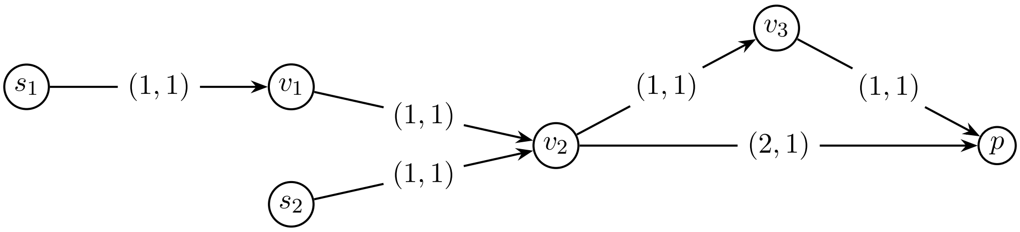

FIGURE 1. An instance of a dynamic evacuation network.

Example 3.1. Consider a dynamic network $\mathcal{N}$ having (u_e, τ_e) be the capacity and transit time for $e \in A$ as in Figure 1. For s_1 and s_2 be the sources, and p be the pickup location, let $\nu(s_1) = \nu(s_2) = 3$ and $\nu(p) = -6$.

Consider two different flow patterns with their respective time, path assignments, flow value, and the total flow in different columns, as in Table 1 and Table 2. Here, Table 1 represents a quickest transshipment that is not an earliest arrival transshipment. But, Table 2 represents an earliest arrival transshipment which is also a quickest transshipment.

TABLE 1. A quickest transshipment which is not an earliest arrival transshipment.

Time unit	Path assignment	flow value	Total flow reached
3	$s_1 - v_1 - v_2 - p$	1	2
	$s_2 - v_2 - v_3 - p$	1	
4	$s_1 - v_1 - v_2 - p$	1	4
	$s_2 - v_2 - v_3 - p$	1	
5	$s_1 - v_1 - v_2 - p$	1	6
	$s_2 - v_2 - v_3 - p$	1	

4. BUS BASED EVACUATION PLANNING PROBLEM

In large cities of the developing countries, many people depend on transit-vehicles, say buses. They are to be given a special attention due to their ages, language ineffciencies, different health problems, or other physical disabilities. The great loss of people in disasters is due to a lack of proper planning of transit-vehicles rather than the disaster itself. It was

Table 2. An earliest arrival transshipment which is also a quickest transshipment.

Time unit	Path assignment	flow value	Total flow reached
2	$s_2 - v_2 - p$	1	1
3	$s_2 - v_2 - p$	1	3
	$s_1 - v_1 - v_2 - p$	1	
4	$s_2 - v_2 - p$	1	5
	$s_1 - v_1 - v_2 - p$	1	
5	$s_1 - v_1 - v_2 - p$	1	6

noticed that, the great loss of people on Hurricane Katrina was due to the lack of proper planning for the transit-based evacuees an mentioned in [18]. In an integrated evacuation network, the formulation of a prominent BEPP and its dominant assignment plays a vital role. Based on the BEPP formulation by [5], one of the prominent version of BEPP has been considered by [12] as follows:

Let $(\tau_{ij})_{i\in P,\, j\in Z}$ *be a matrix of source-sink travel times. Let the vectors* τ_{di}*,* $(l_i)_{i\in P}$*, and* $(\mu_j)_{j\in Z}$ *be the depot-source travel times, number of evacuees, and sink capacities, respectively. Then the BEPP is to find a tour plan to minimize the maximum travel times overall buses such that all the evacuees be transported to the sinks.*

For this, it is assumed that the number of evacuees at every source node be known in terms of the integral multiples of the busloads and is so for the sink. For the solution of such BEPP, the branch and bound algorithms with four upper bounds and three lower bounds for time, three branching rules to minimize the number of branches and two tree reduction strategies to avoid the equivalent branches have been presented by Goerigk *et al.* in [12]. Upper bounds have been constructed in polynomial-time complexity by four heuristic algorithms. Among the lower bounds, the first one is based on the estimation of the travel times from sources to sinks and from sinks to sources, respectively. The second lower bound is based on the fact that lower bound for the maximum travel time is the average travel time. The third one is about the simplification of model formulation.

For $i \in P$ and $j \in Z$, consider τ_{ij}, τ_{di}, l_i and μ_j be as in such BEPP as formulated in [12]. For $k \in \mathbb{R}$, is there a tour plan with $\mathcal{T}_{\max} \leq k$? Regarding its complexity, we have,

Theorem 4.1. *[11] The decision version of BEPP is* $\mathcal{NP}$*-complete, even if* $\tau_{di} = 0$ *and* $\tau_{ij} = \tau_{i'j}$ *for all* $i, i' \in P$ *and* $j \in Z$*.*

Proof. The BEPP is reduced to the scheduling problem of scheduling n-jobs on P-parallel machines, which is $\mathcal{NP}$-hard, as mentioned in [11]. Such a scheduling problem with maximum completion time, $C_{\max} \leq k$ for a given k has a yes-instance if and only if the respective bus routing plan of BEPP with $\mathcal{T}_{\max} \leq k$ has a yes-instance. As both the completion time and the feasibility of the given solution can be checked polynomially, the decision version of BEPP is $\mathcal{NP}$-complete. ☐

4.1. **BEPP in a diminished evacuation network.** Consider a BEPP for a diminished evacuation network close to the real scenario which is equivalent to the third lower bound, [12] as about the simplification of model formulation. Here, sinks are far away from the dangerous zone. The bus depot is in a closed environment to the disaster zone with B be the set of available buses. So, let the super pickup node with the bus depot be Y_0 with the evacuees ξ_{y_0}. Assume the capacity of each bus be 1. The movement between Z is ignored. The set of tours of the buses cannot be changed anymore after they start to move and are connected. The maximum number of trips for the evacuation process is given by ξ_{y_0}. The nonnegative travel cost of τ_{0j} on each edge $e = (0, j) \in E$ are taken symmetric. Let the sinks $z \in Z$ be at a distance of τ_{0j} from Y_0, where y_j^b denotes a tour that the bus b drives from the source to sink $j \in Z$ and back from the sink $j \in Z$ to the source. Let $\mathcal{T}_{\max}$ be the duration of evacuation overall buses, then the problem becomes,

$$\begin{aligned}
&(4.1) && \text{minimize} \quad \mathcal{T}_{\max} && \\
&(4.2) && \text{such that} \quad \mathcal{T}_{\max} &\geq& \sum_{j\in Z} \tau_{0j} y_j^b \ \ \forall\, b \in B, \\
&(4.3) && \sum_{b\in B}\sum_{j\in Z} y_j^b &\geq& \xi_{y_0}, \\
&(4.4) && \sum_{b\in B} y_j^b &\leq& \mu_j, \forall j \in Z, \\
&(4.5) && y_j^b &\in& \{0,1\}, b \in B, j \in Z.
\end{aligned}$$

Constraint (4.2) requires $\mathcal{T}_{\max}$ to be greater than or equal to the maximal travel cost incurred by all buses and is to be minimized on Constraint (4.1). Constraints (4.3) and (4.4) are the bus and shelter capacity constraints, respectively, which ensure that all evacuees are transported and shelter capacities are respected. Constraint (4.5) represents whether the bus b travels from source to sink j (or travels back from the sink j to the source).

For the solution status and the proof of its decision version, similar to Theorem 4.1 as mentioned earlier, we have,

Theorem 4.2. *The decision version of the BEPP in a diminished evacuation network is $\mathcal{NP}$-complete.*

For the upper bound of the evacuation duration on the BEPP as in [12], four different heuristics algorithms were presented, three with the precomputed tourlists and the fourth without any precomputed tourlists. In the fourth algorithm, the assignment of transit-vehicles begins with the best possibility to bring one evacuee back from the sink to the source and is continued iteratively, respecting the sink capacity constraints. It dominates the rest in evacuation duration and is equivalent to the nearest sink approach as in [4]. So, we prefer it as the dominating vehicle assignment for the network as in Example 4.4.

Observation 4.3. *Let ξ_{y_0} and $\mu(z_i)$ be the number of evacuees at the super pickup node Y_0 and the sink capacities, respectively. Consider $\tau_k = min\{\tau_{01}, \tau_{02}, \ldots, \tau_{0n}\}$ as the nearest sink z_k in the network. Let the extended sink capacity to the nearest sink be $\mu(z_k) \geq \sum_{j=1}^{n} \mu(z_i)$. Then such evacuation network $s-Z$ be reduced to an $s-z$ network with the minimum possible evacuation duration. For this, the estimated evacuation duration becomes $(2\xi_{y_0} - 1)\tau_k$.*

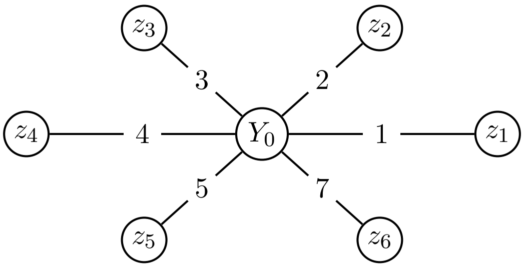

FIGURE 2. An instance of a bus-based evacuation problem in a diminished network.

Example 4.4. Consider a diminimshed evacuation network as in Figure 2. Let $\xi_{y_0} = 26$ with $\mu(z_i) = \{7, 6, 5, 4, 3, 2\}$ for $i = \{1, 2, \dots, 6\}$. Their respective transit times τ_{0j} from Y_0 are shown along with the figure.

Then the tour plan for $|B| = 1$ is given by $14\tau_{01} + 12\tau_{02} + 10\tau_{03} + 8\tau_{04} + 6\tau_{05} + \tau_{06}$ with $\mathcal{T}_{\max} = 137$. But if $|B| \geq 26$, then $\mathcal{T}_{\max} = 7$. For $1 < |B| < 26$, consider $|B| = 13$, as an arbitrary. Then the tour plan becomes $2\tau_{01} + 2\tau_{02}$, $2\tau_{01} + 2\tau_{04}$, $2\tau_{02} + 2\tau_{04}$, $2\tau_{02} + 2\tau_{05}$ and $2\tau_{02} + \tau_{07}$ for the buses $\{B_1, B_2, B_3, B_4, B_5\}$, $\{B_6, B_7\}$, $\{B_8, B_9\}$, $\{B_{10}, B_{11}, B_{12}\}$ and $\{B_{13}\}$, respectively. For this, the effective $\mathcal{T}_{\max} = 14$.

If this $s - Z$ network is replaced by an $s - z$ network having sufficient sink capacity at z_1 as requested by the given demand, then the tour plan for $|B| = 1$ be reduced to $51\tau_{01}$ with $\mathcal{T}_{\max} = 51$. But for $|B| = 13$, the respective $\mathcal{T}_{\max} = 3$. However, for $|B| \geq 26$, it becomes 1. This is why a single sink having sufficient capacity is more appropriate for the transit-based evacuation planning problem.

5. CONCLUSIONS

Proper planning of transit-vehicles within an integrated evacuation network might be helpful to reduce the massive loss of the people and the socio-economic damage during different disasters. It is beneficial for their normalcy. Planning a bus-based evacuation is an extremely rich problem. In general, a challenging task.

We have presented a compact overview concerning different solution strategies and the optimization approaches for the transit-based evacuation planning problem in an integrated network topology. Flow maximization and/or time minimization on the transshipments are highly affected by the arrival and assignment pattern of the evacuees. However, depending upon the optimization objectives and the nature of the network topology, it needs several extensions and should be investigated further.

Regardless of many directions of evacuation strategies, we have restricted mainly to the computationally acceptable research domain on the network flow optimization for the transit-based evacuation system in an integrated network topology. Most of the problems in this category are $\mathcal{NP}$-hard in nature and demand the various computational techniques with acceptable approximations. Such $\mathcal{NP}$-hard transit-dependent models and the solution strategies demanded by them have rarely been considered in the literature however most of the evacuation regions rely on transit-vehicles. In a real evacuation scenario, an optimizer has to address the characteristics of the diversified and heterogeneous vehicles including different commodities that are rather complicated from their computational aspects.

Conflict of Interest: The authors declare no conflict of interest.

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B.2 List of Presentations

1. *Transit Based Optimization for Evacuation Planning*, In: International Conference on Applications of Mathematics to Nonlinear Sciences (AMNS-16), Nepal, May 26-29, 2016.
2. *Meaningfulness of OR Models and Solution Strategies for Emergency Planning*, In: Mathematics and Science Research Colloquium, MSU IIT, Philippines, August 17, 2016.
3. *Facility Location as A Basic Component of Evacuation Planning*,
   In: Südwest-workshop, TU kaiserlautern/Fraunhofer, ITWW, Germany, Oct. 28, 2016.
4. *Dominance Vehicle Routine in Transit Dependent Evacuation Scenario*, In: National Conference on Mathematics and Its Applications (NCMA-2017), Nepal, Jan 11-13, 2017.
5. *Transit Dependent Evacuation Planning*, In: Workshop on Bilevel Optimization, AvH Research Linkage Program, Germany & TU, Nepal, Feb 28-March 7, 2017.
6. *Transit Dependent Vehicles on Evacuation Planning*, In: National Conference on History and Recent Trends of Mathematics (NCHRTM-17), Nepal, June 2-4, 2017.
7. *Some Aspects on Transit Dependent Evacuation Planning*, In: Emergency Management Research Workshop, Lancaster University, U.K. & CDM, TU, Nepal, 1-2 September, 2017.
8. *An Insight on the Evacuation Planning Optimization Problems on Transit-based System* In: 11th Triennial Conference of Association of Asia Pacific Operational Research Society, Kathmandu, Nepal, 2018.
9. *Evacuation Planning Problems on Transit-based Networks*, In: Modern Algebraic Geometry Silkroad Mathematics Center, China, July 23-26, 2018.

10. *Minimum Clearance Time with Earliest Arrival Pattern for Transit-based Evacuation*, In: Second International Conference on Applications of Mathematics to Nonlinear Sciences, Pokhara, Nepal, June 27-30, 2019.

11. *Evacuation Optimization with Minimum Clearance Time*, In: Applied Mathematics in Science & Engineering, Udisa, India, Oct 24-26, 2019.

12. *Evacuation Optimization in an Integrated Network Topology*, In: International Conference on Computational Sciences-Modeling and Soft Computing, Calicut, Kerala, India, Sept 10-12, 2020.

13. *Time Minimization Aspect on the Transit-based Evacuation System*, In: International Conference on Emerging Trends in Mathematical Sciences and Computing (IEMSC-21), Kolkata, India, February 5-7, 2021.

B.3 Certificates of Participations

TRIBHUVAN UNIVERSITY

CENTRAL DEPARTMENT OF MATHEMATICS
KIRTIPUR, KATHMANDU
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Institute of Science and Technology

Certificate of Attendance

Ishwar Mani Adhikari
Central Department of Mathematics, Tribhuvan University, Nepal
has attended the workshop
on
Bilevel Optimization
from 28th February until 7th March of 2017

Trainers
Prof. Dr. Stephan Dempe and Dr. Maria Pilecka
Technical University Bergakademie, Freiberg, Germany

Organizer
Central Department of Mathematics, Tribhuvan University, Nepal

Support
Research Group Linkage Program
Alexander von Humboldt Foundation, Germany

Tanka Dhamala
Prof. Dr. Tanka Nath Dhamala
Cooperation Partner
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Tribhuvan University, Kathmandu, Nepal

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Prof. Dr. Stephan Dempe
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TU Bergakademie, Freiberg, Germany

KNuprety
Dr. Kedar Nath Uprety
Professor & Head

National Conference on

Mathematics and Its Applications

(NCMA-2017)

January 11-13, 2017, Chitwan, Nepal

CERTIFICATE

This certificate is awarded to

Iswar Mani Adhikari

Tribhuvan University

for participating and presenting
a paper entitled
Dominance Vehicle Routing in Transit Dependent Evacuation Scenario
in the National Conference on
Mathematics and Its Applications
organized by
Nepal Mathematical Society.

Prof. Dr. Tanka Nath Dhamala
President
Nepal Mathematical Society

Prof. Dr. Ishwari Prasad Dhakal
Vice-Chancellor
Agriculture and Forestry University, Nepal

Date: January 13, 2017

The **Department of Mathematics**
Technische Universität Kaiserslautern, Germany

Central Department of Mathematics
Institute of Science and Technology, Tribhuvan University, Nepal

and

Department of Mathematics and Statistics
Mindanao State University - Iligan Institute of Technology, Philippines

in cooperation with

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present this

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to

Iswar Mani Adhikari

Central Department of Mathematics, Institute of Science and Technology, Tribhuvan University, Nepal

for actively participating the Seminar-Workshop on Graph Theory and Optimization with Applications in Industry and Society held at Mindanao State University – Iligan Institute of Technology, Iligan City, Philippines on June 13-24, 2016.

Given this 24th day of June 2016 at Mindanao State University, Iligan Institute of Technology, Iligan City, Philippines.

PROF. DR. FERDINAND P. JAMIL
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"Meaningfulness of OR Models and Solution Strategies for Emergency Planning"
during its

MATHEMATICS AND STATISTICS RESEARCH COLLOQUIUM

held on 17 August 2016 at Lecture Hall B,
College of Science and Mathematics, MSU-Iligan Institute of Technology,
Iligan City, Philippines.

Given this 17th day of August, 2016.

JULIUS V. BENITEZ, Ph.D.
CHAIRPERSON
Dept. of Mathematics and Statistics

MARK NOLAN P. CONFESOR, Ph.D.
DEAN
College of Science and Mathematics

International Conference on
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May 26-29, 2016, Kathmandu, Nepal
http://www.anmaweb.org/ANMS-2016

Certificate of Poster Presentation

This certificate is awarded to

Mr.ISWAR MANI ADHIKARI

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Transit Based optimization for Evacuation Planning

in the International Conference on
Applications of Mathematics to Nonlinear
Sciences
(AMNS-2016)

on May 26-29, 2016, Kathmandu, Nepal.

Tanka Dhamala
Prof. Dr. Tanka Nath Dhamala
(on behalf of the organizers)

President
Nepal Mathematical Society

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Department of Mathematics
University of Kaiserslautern, Germany

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Prithvi Narayan Campus, Pokhara, Nepal

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held at Central Department of Mathematics, Institute of Science and Technology, Tribhuvan University
March26-April 6, 2017

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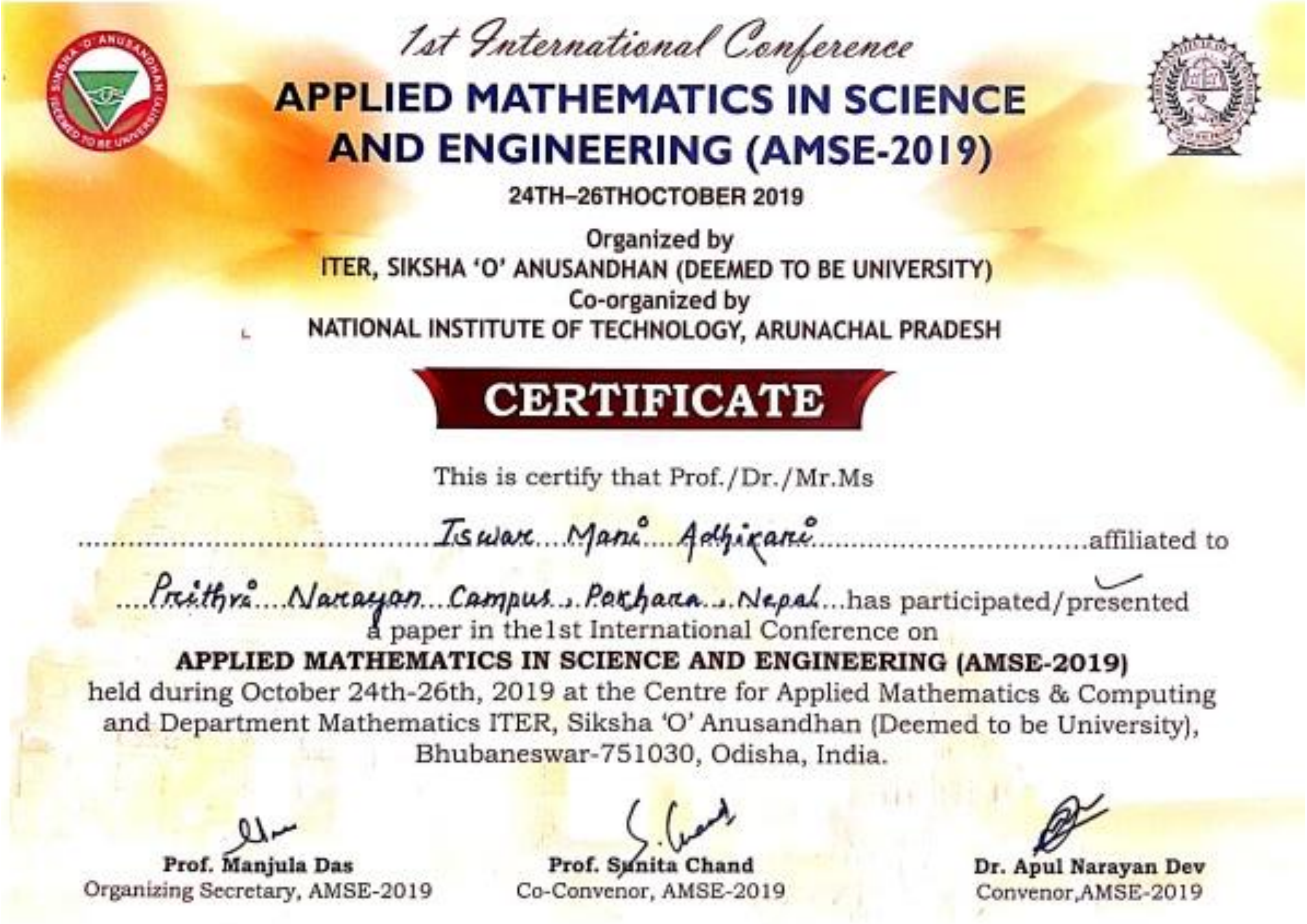

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He /She has also presented the paper entitled **Evacuation Optimization in an Integrated Network Topology.**.

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Time Minimization Aspect on the Transit-based Evacuation System

Track-6 (Applied Mathematics -II)

IN INTERNATIONAL CONFERENCE ON EMERGING TRENDS IN MATHEMATICAL SCIENCES AND COMPUTING (IEMSC-21) AT KOLKATA, INDIA DURING 5TH-7TH FEBRUARY, 2021.

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