DIFFICULTIES IN LEARNING VECTOR GEOMETRY AT SECONDARY LEVEL

A

Thesis

BY

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Letter of Certification

This is to certify to **Mr. Purushottam Adhikari** a student of academic year 2071/72 with campus Roll No. 223, exam Roll No. 28710315, thesis number 1393 and T.U. Regd. No. 9-2-375-180-2009 has completed his thesis under supervision of Mr. Dipak Mainali during the period prescribed by the rule and regulation of Tribhuvan University, Nepal. The thesis entitled **"Difficulties in Learning Vector Geometry at Secondary Level"** has been prepared based on results of his investigation. I, here by recommended and forward that his thesis be submitted for evaluation as the partial requirements to the degree of Master of Mathematics Education.

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Assoc. Prof. Laxmi Narayan Yadav

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Letter of Approval

A thesis by Purushottam Adhikari entitled **"Difficulties in Learning Vector Geometry at Secondary Level"** has been approved for the partial fulfillment of the requirements for Degree of Master of Mathematics Education.

Committee for Viva-Voce	**Signature**	**Signature**
Assoc. Prof. Laxmi Narayan Yadav (Chairman)		…….
Prof. Dr. Hari Prasad Upadhyay (External)		…….
Mr. Dipak Mainali (Member)		…….

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Recommendation for Acceptance

This is to certify that **Mr. Purushottam Adhikari,** has completed his thesis entitled **"Difficulties in Learning Vector Geometry at Secondary Level"** under my supervision during the period prescribed by the rules and regulation of Tribhuvan University, Kirtipur, Kathmandu, Nepal. I recommend and forward his thesis to the Department of Mathematics Education to organize final viva-voce.

…….

Dipak Mainali

Supervisor

Date: …….

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DEDICATIONS

Honestly dedicated

To

My wife

Ranjana Aryal

Declarations

This thesis contains no material which has been submitted for the award of other degree in any institution. To the best of my knowledge and belief this thesis contains no material previously published by any authors except due acknowledgement has been made.

Date:……. …….

Purushottam Adhikari

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Abstract

This is a case study related to difficulties of students in learning vector geometry at secondary level. The main objectives of the study were to explore the learning difficulties in vector geometry at secondary level and to analyze the major causes related to the problem in learning vector geometry in secondary level. This was based on qualitative nature of data. I selected own my working school Pragati Shikshya Sadan Secondary School in Lalitpur district. Test was conducted in 16 students at grade X. Among them five respondents was selected for the interview. The data were collected from paper pencil test and face to face interview with the students. There were only seven questions used to conduct the test. The collected information was analyzed with four different (difficulties in types and notations, vector additions and graphical representation, application of scalar multiplication, theorem proved by vector method) themes.

In this study, student had many difficulties on mathematical terms concepts, facts applications, and learning transformations. Types of vector and its classifications students recognize in example but could not define easily. In vector addition difficulties on application following the rules and representing graph and it showing reverse arrowhead of entire vectors. Similarly, in application of scalar product unable to follow the correct formula in correct applications. Finally they were unable to justify correct reason to prove the theorem by vector method. So, it gives meaning of mathematical term properly and had unable to choose appropriate operation in vector geometry. Lack of pre-requisite background of knowledge, basic operation, terms and definitions makes not fruitful on learning vector geometry. Student can learn mathematics effectively when giving clear instruction, connection with daily life and teaching through problem based. So, focus to student interest, ability and challenges.

Table of Content

Chapter I

Introduction

Background of the Study

Many students feel that mathematical knowledge skill and fact connect to solve the problem is uneasy. Good Therapy, (2016) states that, learning difficulties, which are neurological challenges, affect the way the brain receives, processes, stores, and analyzes information. Because a learning difficulty often affects an individual's ability to develop reading, writing, and math skills, a learning difficulty is typically recognize and diagnose while an individual is in school. However, some of those affects by a learning difficulty may not have it discovered or diagnosed until they are in college or after they have joined the workforce. Others never have their condition diagnosed and may continue to experience difficult processing information as they progress through life. Kid Sense, (2018) states that, "Learning difficulties are a group of disorders involving significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning or mathematical abilities. These disorders are specific to the individual and are presumed to be due to central nervous system dysfunction". Rittey (2018) states that's," Learning difficulties have a very significant effect on individuals and on society. The affected individual will have difficulties in thinking, acquisition, and processing of new information and knowledge".

Many students have learning difficulties in symbols of vector, formulation of vector, verbal problems examples and theorem proved by vector method (Sah, 2016). Students have difficulties to understanding and using of vector representation on the different levels and different conditions as well as teachers also feel that different problem in teaching vector geometry (Karki, 2016).

Vector is “A quantity having direction as well as magnitude, especially as determine the position of one point in space relative to another” as well as scalar is “A quantity having only magnitude not direction". It’s refers that to both the direction of the medium’s as well as the measurement of the scalar quantity, increase and decrease in temperature and velocity are the two examples of vectors. Where the measurement of the medium’s temperature is a scalar quantity; the measurement of the increase or decrease in the medium’s temperature is a vector quantity. The velocity refers measurement of the rate at which an object changes position is a vector quantity. Moreover, the entire directional element that may be applied to the vector quantity is also, different between horizontal and vertical movement.

The parallelogram law for the addition of vector is also intuitive but its origin is unknown. Now a day it is believed that it may lose work of Aristotle (384 -322 B. C.) and mechanics of Herons at Alexandria. Its first corollary was found in Newton’s Principia Mathematica (1687). In Principia Newton dealt extensively with what are now considered vector entities but not asked vector even using vector geometrical concept. The systematic study and use of vectors were 19^{th} century and early 20^{th} century phenomenon.

A Euclidean vector is frequently represented by a line segment with a definite direction, graphically as an arrow, connecting an initial point A to terminal point B, and denoted by $\overrightarrow{AB}$. Its concern is what is needed to carry the point A to the point B. Vectors plays important role in physics the velocity and acceleration of a moving object. Many physical quantities are useful through of as vectors. Although most of them doesn’t represent distances, their magnitude and direction can be represented by the length and direction of arrow. The mathematical representation of a physical vector depends on the co – ordinate system used to describe it. The term vector also

has generalized to higher dimensions and to more formal approaches with much wider applications.

Statement of the Problem

Many students and teacher have feeling that vector is difficult chapter in secondary level. Most of the optional mathematics students did not attempt the vector geometric question even and had pre-mentality to solve all question. They asked many times without solving vector geometric question also obtained A+ or A grade in SEE exam. The less motivation of many students as well as some students remained the solutions of vector geometry problem by route learning is also problem of learning vector geometry. Rarely students only attempt the vector geometry question in the exam.

Vector geometry stands the connection of geometry, co- ordinate with its direction. Scientifically, it was a quantity with magnitude and direction. Most of the student feels difficulty in geometry only because of its logic and instruction. Vector geometry is new chapter for secondary level students so, they does not motivate to study basic concepts of vector like as types of vectors, magnitude of vectors, direction of vectors, negative vector, parallel vectors, scalar products of vectors, angle between two vectors, geometrical theorems proved by vector method and so on.

When I reviewed the many literatures related to difficulties in vector geometry I found that many students had misconception to represent magnitude of vector, students wrote both sides arrow heads to represent the vector, difficulties in unit vector and scalar multiplication, wrong indication of angle in vector, misunderstanding of graphical vector addition. Even though they had problem in basic concept like as points, straight line, plan, notation of vector approaches. For these causes it was very difficult to solve the vector geometry problem in secondary level.

Therefore I liked to study on "difficulties in learning vector geometry at secondary level.

Research questions

This study was mainly concern with the problem faced by secondary school optional mathematics students in vector geometry. This study sought to answer the following research questions.

- What were the difficulties in learning vector geometry at secondary level?
- Why students felt difficulties in learning vector geometry?

Significance of the Study

In the context of Nepal low achievement in mathematics due to many causes. The low achievement of mathematics is highly remarkable so, in this context many public school students as well as private schools students also do not select the optional mathematics subjects. Those students who have selected the optional mathematics also do not try to solve the vector geometric problem. Most of the students do not able to understating the connection of the topic vector and geometry due to lack of their pre knowledge of geometry.

Mathematics is related to human life and their behavioral activities so, this problems creates students feeling as mathematical others topic also difficult. The researcher believes that all stakeholder; student, their parents, teachers, educators, curriculum developer and government sector seriously concern with this field.

The main significances of this study are as follows:

- It finds out the main problem of the learning difficulties in vector geometry which helps to make the new strategy to improve the teaching learning activities.
- It aware the entire stakeholders to improve their related work own their place.

- Its helps to conduct learning by doing method in teaching, problem based teaching and research based teaching.

Objective of the Study

The main focuses of the study were to identify the difficulties of students in learning vector geometry for secondary level students. The following objectives were formulated for this study

- To explore the difficulties in learning vector geometry at secondary level.
- To analyze the major causes related to the problem in learning vector geometry in secondary level.

Delimitation of the Study

For any research the researcher had some boundaries is called delimitations of the study. This study was limited to the following facts.

- The study was limit to only at Lalitpur district.
- The study focused only Grade X students.
- The study covered only vector part of optional mathematics of grade IX and X.
- The study was based in purposive sampling method.
- The study was based on students difficulties and causes of difficulties in learning vector geometry.
- The design of the study was case study.
- The study was based on test paper and interview.
- The necessary data only presented on the report.
- The result may not be applicable in another context.

Definition of Key Terms

The following terms are defined for this study.

Difficulties. In this study difficulties mean problem of students or disabilities of students to understand terms, meaning and condition of vector geometry at secondary level.

Difficulties in learning. In this study difficulties in learning means “those difficulties which creates the problems to fulfill the curricular objectives, which are related to concepts, language, transformation, organization, computation and lack of pre-knowledge in vector geometry.

Conceptual difficulties. In this study conceptual difficulties mean "the difficulties which creates the problems to understand basic structures, basic terms, and concept related to vector geometry".

Language difficulties. In this study language difficulties mean "the difficulties which creates the problem to identify its own terms and terminologies, notations, axioms and postulates as well as reading system.

Comprehensive difficulties. In this study comprehensive difficulties mean " the difficulties which creates the problem related to understand the meaning and relation of the problem.

Organizational difficulties. In this study organizational difficulties mean "the difficulties which creates the problem on problem situation, solving order and finding solution.

Difficulties in vector geometry. In this study difficulties in vector geometry mean " the difficulties which creates the problem in learning vector geometry on the basis of its basic concepts, term terminologies, notation, mathematical connection to discipline, transformations, mathematical relation, transformation as well.

Difficulties in notation and types of vector geometry. In this study difficulties in notation and of vector geometry mean "the difficulties which creates the problem in symbolically represent.

In this study difficulties in types of vector geometry mean "the difficulties which creates the problem on to recognized the various types of vector in its classification".

Difficulties vector addition and graphical representation. In this study difficulties in vector addition mean "the problem creates on addition of two or more vectors or problem on applying its addition rules.

In this study difficulties in graphical representation mean " the problem on vector represent on graph and showing its direction.

Difficulties in application of scalar multiplication. In this study difficulties in application of scalar multiplication mean " the problem on vectors scalar multiplication and its uses on vectors problem.

Difficulties in theorem proved by vector method. In this study difficulties in theorem proved by vector method mean "problem creates on to prove the theorem by using vectors rules, definition and proposition and its connections. All kinds of theorem can be proved by vector method by applying its properties mainly in secondary mathematics curriculum were based on plan geometry which was also proved by vector method.

Chapter II

Review of Related Literature

Review of Literature deals to know how already had been done to determine and what is to be done in the concern Area to be studied. It Helps to clear the concept of study, and directed to analyze as well as interpreted the data. Its criteria were Number of books Research report papers, curriculum, teaching material, methods and so on.

The effective Research Must be based upon past knowledge this steps helps to eliminate the duplication of what have been done in past.

Empirical Review

Foldesiva (2003) conducted a research on the topic "sequence analytical and vector geometry at teaching of solid geometry at secondary level". For this research he made four hypotheses and tests it. The aim of the experiment is also attest and conform the hypothesis. The preparation of experiment was tenets theory of deductive situation frame. For that, they made analyze of math test books for secondary school, various mathematical materials, they give with instructing of possible strategy of students solutions and analyze a priori of problem destination to experiment. They are synthetic approach, analytic solution, vector calculus- exploitation collinearity of vectors and vector calculus exploitation barycenter. After that they conduct conclude experiment to the students at grammar-school in Nitra (17-18 years old). The students had a review lesson on solid geometry in their final grade after that they take time 25 minutes time to solve the problem. Among them 14 students only solved the problem and all solutions had a synthetic approach. They found that student's problem is basic concepts like as points, straight line, plane, notation of vector and vector approaches. Which make the solid geometry problem is difficult.

Wutchana and Emarat (2011) research on a topic "Students understanding of graphical vector addition of one and two dimensions". The objective of study was "to explore student's qualitative understanding of graphical vector addition in one dimension and two dimensions. They research in 63 grades ten high school students were categories for conducting a research in Thailand. Among them they collated data from 2 classes of grade ten students and 31 students from another school. Also they conduct research in medium size public schools containing approximately 900 students and 48 teachers. They re-search also in those students who were ready to study physics. They investigated student's understanding of vector addition, magnitude direction for presenting problem in graphical form. For this research they administrated a seven item quiz including free response problem, they observed teachers classes, asked question to student and teacher and they teach meaningfully to the some classes. They found most of the students were not able to add or subtract vector graphically after traditional instructional method. In this method students learn passively listening taking notes and rarely asking questions. Occasionally the teacher would ask some students to show how to solve problems in front of the class but some student ready to solve but not confidently. They conclude that there is verity of students misunderstanding of graphical vector addition, but the main problem is they can add graphically without any physical content. The students did not gasp the important concept about the vector direction and how a vector may be moved while its magnitude and direction are preserved. Many students are still confused about predicates that might cause future problems to their physics problem.

Aniol and Zavala (2012) conducted research on the topic "student's difficulties with unit vectors and scalar multiplication of vectors". The study focuses the investigation on students understanding. The objectives of the study were students

understanding on unit vector and students understanding on scalar multiplication of vectors. For this study they administrate two different tests to a total 850 students after taking their first introductory physics course on mechanics at large Mexican university. The first test about unit vectors was taken by 270 students and second test about scalar multiplication of a unit vectors by 580 students. On the basis of this research they find students difficulties to represent the unit vector in correct direction only 22% students shown correctly misconception to represent magnitude of vector, students write both sides arrow heads to represent the vector, some of the students find sum of vectors when question asked find the unit vector of given two vectors. Similarly when they investigate positive and negative scalar multiplication of vectors they sketch same type of figure in graph only students write correct numerical value. They conclude that many students have difficulties in unit vector and scalar multiplication, even after a calculus based course of mechanics students will have difficulties with unit vectors and negative scalar multiplication of vector. They suggest that instructors should be aware of these specific difficulties to plan the instruction.

Mikula and Hecker (2013) conduct a research on the topic "student's difficulties with trigonometric vector components persist in multiple students' population" in the Ohio State University Columbus. The study focuses on the investigation of students difficulties with the use of trigonometric functions on simple vector decomposition problem. The data were collected after relevant instruction over three separate terms from four inductor physics student population. They conduct the research on algebra-based mechanics (N=636), algebra-based E & Q (N =636), calculus based mechanics (N = 229) and calculus based E &M (N = 188). Data was presented for all population. They conducted different simple problem of

trigonometric represent action of vectors and they found sin/cos error (students switches sine and cosine effectively answering for the wrong component), sign error (students differ from correct response from sign), triangle error (students answers based on a triangle in which the weight vector is not the hypotenuse) and showing wrong indication of angle in vector. They conclude that difficulties on inclined plane problems, students frequently omit necessary signs, confuse sine and cosine, drawn triangle and placed angle.

Sah (2016) conducted a thesis “Entitled Learning difficulties in vector Geometry a case study of grade ten students”. The objectives were "To identify the difficulties on learning vector geometry at secondary level and to analyze the cause of difficulties in learning vector geometry at secondary level. He used qualitative research design using purposing method. In his study, he conduct research about head teacher mathematics and six students in grade X of keshobati Anrud secondary Rajbiraj saptari district. In his study, he used classroom observation form, in-depth interview and mathematical achievement test. He used primary data for interpreting and analyzing by using descriptive method, cross match approach and triangulation method. He finds students were weak irregular, lack of teaching material, lack of familiar with student teacher, teacher were untrained, lack of physical facilities, classroom arrangement and students teachers regularity. He conclude that most of many students have learning difficulties in symbols of vector, formulation of vector, verbal problems examples and theorem proved by vector method. The causes of becoming difficulties in learning vector due to lack of time and irregular.

Karki (2016) conducted Research on "Problem faced by teachers in teaching vector in secondary level". The objectives of study were "To identify major problems in teaching vector geometry and to analyze the major caused related to the problem in

teaching vector geometry. The research design was quantitative authenticated by qualitative. Therefore, he used descriptive survey method and quantitative approach as well as exploratory were used. The sample of the study was fifty optional mathematics teachers were selecting randomly from Ramechhap district. The tools of study are oppinnaire, observation, and interview schedule. He used primary and secondary both source of data. For quantitative data used table and simple statistical methods, descriptive technique to analyze qualitative data using likert scale. To maintain reliability and validity of the study triangulation could adopted. His find were low motivation to learn mathematics geometrically and meaningfully, computer and teaching materials were not available, no mathematics lab in school, students irregularity, poor geometrical background of students, adequate teachers teaching skills. He conclude that teacher needed to be aware the students, school provide sufficient material and mathematical lab, the students.

Gautam (2016) conducted a thesis "Difficulties in learning vector geometry at secondary level". A case study of problem based research. The objectives of study were "to explore the different kinds of difficulties in learning vector geometry and secondary level and to find out the way for minimizing the difficulties in learning vector geometry at secondary level. The design if study was qualitative so, he used qualitative descriptive research design and he used purposive sampling method. He research in a Mulpani higher secondary school, Mulpani Baglung. In his study he research about students, mathematics teacher and head teacher. He used in-depth interview, observation, and test for tools of the study and also used triangulation theory for data re-presentation. He carefully noted his purposive questions answer and interpreted by using descriptive method. Primary and secondary source both were used in his research and used cross matched approach for validation of the data. He

finds that students are confused by the language or terminology to prove vector geometric theorems, unable to made figure according to the question due to poor pre-knowledge, meaningful connections, teachers were also feel difficulty to teach vector geometry and there were not inclusive environment for remedial teaching. He concludes that both teacher and students was passive agent in learning. There were attention difficulties, computational difficulties organizational difficulties organizational difficulties, language difficulties, connectional difficulties output difficulties and difficulties due to lack of pre knowledge.

Kwon (2016) conducted a research on the topic "Conceptualizing vectors in collage geometry: A new framework for analysis of student approaches and difficulties". The goal of framework is to give a new way to discuss the complexity of vectors both conceptual and pedagogical that students may grapple with in order to understand vector in geometry effectively. He conducts four surveys and interviews were carried out. For data collection he used ninety –eight students who are pre-service secondary and elementary/middle level teachers from students located in Midwest public university. The researcher focus on physical to mathematical combinations, analytic to synthetic combinations, geometric to symbolic abstraction, process- object duality in geometric representations and the prevalence of analytical approaches to the synthetic approaches while developing mathematical abstraction. He found problem in students work there is difference between physical and mathematical vector problem in definitions and transformation and transformation process, object duality on geometric representation of vectors transformation of school known to other concepts.

Theoretical Review

Many learning theories could be used in teaching learning process of vector Geometry. They are behavior-learning theories, cognitive learning theories, constructivism learning theories and social critical learning theories. Among them, I was used Paget's cognitive constructivism theory and R. Gagne's learning theory.

Cognitive constructivism and difficulties in learning. According to the Piaget, cognitive development is a comprehensive theory about nature and development of human intelligence. Children as little philosophers they have own individual theory of knowledge. This theory believes that cognitive development is a progressive re- organization where, mental process as result of biological maturation and environmental experience. Children were contrast the knowledge between what they already know and what they discover in their environment. Child-centered classrooms and are direct applications of Piaget's views. Piaget, divide four developmental stages and three mental structures.

Learning difficulties refers that problem in learning. Learning is the constructing new idea from previous knowledge, skill and result of the active involvement of the time. Some of the student have problem of learning ability so, we cannot discuss here that types learning disabilities. In case of normal child his/her learning difficulties depends on their opportunity. Children's family background, education status, social-cultural status, economic status and living standards also directly indirectly affect his/her learning difficulties. But student's difficulties determine curricular dimension, teaching learning activities, connection of subject matters, students pre-knowledge and teacher presentation plays important role in learning. In one words learning difficulties in unable to construct new knowledge.

The main cognitive development explains how a child constructs the knowledge. He was not agreed with intelligence was fixed traits and regarded cognitive development as process which occurs due to biological maturation and interaction with environment (Leod 2018). Children give wrong answer of reasoning from his logical thinking. Paget's believes that these in correct answers revealed important difference between the thinking of adults and children. It is also related to constructing their schema. At the time of children born, he has very basic mental structure on which all subsequence learning and knowledge are based. He believes that fundamental concepts like the very idea of number, time, quantity causality, justice and so on emerge. Difficulties can affects all age people. In particularly the most of problematic students in learning activities depends on key literacy skill required for students to excel in all area of curriculum. Adult learners struggle which spelling or reading skills may be people who faced learning difficulties as children but did not receive proper support. How can we measure the student's difficulties in learning? This question is raised mainly; we cannot have exact answer but the performing issues such as slow reading or below average writing abilities. It is also common for conditions to go un-diagnosed which can lead to students not receiving the help or attention they need and result in low self-steam and negative attitude towards learning.

The cognitive process in learning according to Jean Piaget' depends on their mental construction like as from his mental schema, assimilation/accommodation and equilibration process. The level of secondary schools students in the period of adolescences so, that period also affects the learning activates. The mental cognition of pre knowledge creates the new knowledge and many problems are solving by active involvement, concentration and thinking actively.

Gagne's learning theory and difficulties in learning. Gagne's nine levels of learning provide a step by step approach and many learning opportunities to the students. The subject of mathematics learning requires the learners' conceptual knowledge and skill based activities as well as prior knowledge. Every learner needed prerequisites to complete the higher level of learning. According to Gagne all learner have to passes through these levels in order and no learner can skip a level. So, students also have difficulties in learning vector geometry if they have lack of gaining attention, stating the objectives, stimulating the prior knowledge, presenting the information, eliciting the performance, proving guidance, providing feedback, Assessing performance and enhancing retention and transfer to other contents.

The vector is a subject of logically complicated and abstract structure. Many professors, teachers and resource persons already knew students difficulties from their experience of teaching. Students should be encouraged to freely explore their view and experience. Learners are constructing new knowledge according to his previous knowledge skill and attitude. It declare that teaching and learning process depends on the methods simple to complex, known to unknown and concrete to abstract also. So, teachers were taught as well as encouraged to students with the cognitive learning chain or not when student learned vector geometry. This study also studying cognitive role of signal learning, stimulus response learning, chain learning, verbal association learning, multiple discrimination learning, concept learning, rule learning with related to difficulties in learning vector geometry.

Conceptual framework

In this study the researcher use conceptual matter related to vector geometry in secondary level. It have constructed the conceptual framework on the basis of the research studies carried by: Sah (2014), Karki (2016), Gautam (2016), Wutchana and

Emarat (2011), Krown (2016), Aniol and Zavala (2014), Mikula and Hecker (2013) and Foldesiva (2003). These review guided me to search similar types of conceptual framework according to the similar study of research problem. On the basis of Bloom's taxonomy test questions were prepared and research was conducted.

Conceptual review is the most important part in literature review. It helps to make well conclusion from finding data. The strong conceptual frameworks, capture something real, remember and application. They are different method of problem solving as well as teaching learning also. I conducted following conceptual framework for this study.

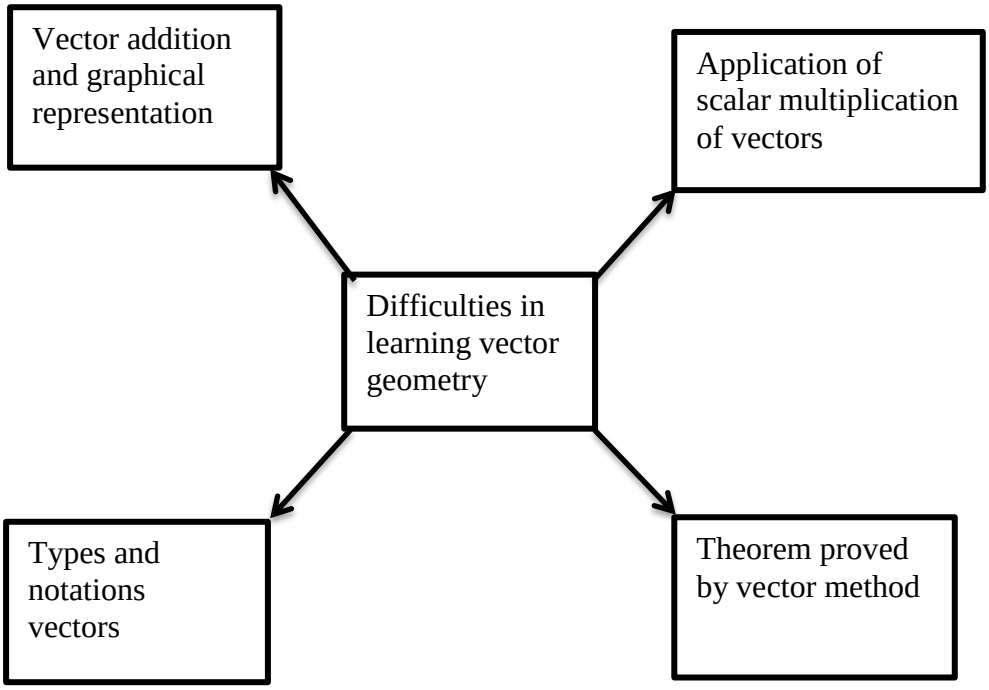


Figure 1: Dimensions of difficulties in learning vector geometry

Addition and subtraction of vector is dependents on its graphical representations. In which the determination of its co-ordinate points represents the position vectors. Types and notions of vectors classify its basic nature and its representations; like as column vectors, magnitude of vectors, unit vectors, like

vectors, unlike vectors, equal vectors parallel vectors etc. Application of scalar multiplication is depends on its fundamental principles. By the uses of this fundamental principle scalar product is derived and by the use of this multiplication the direction of the vector can be derived. Finally the topic of vector geometry of secondary level is concern the geometrical theorem proved by vector method. In this proved the method of triangle law and scalar product is mostly used.

The above conceptual framework was designed from empirical review and secondary level optional mathematics syllabus of Nepal. Among the four conceptual reviews first one was difficulties on types and notation of vector, which was derived from the research of Foldesiva (2010) "sequence analytical and geometry at teaching of solid geometry at secondary level'. Whose conclusion on difficulties on concepts like as points, straight line, plan, notation of vector, vector approaches etc. The second one was difficulties on vector addition and graphical representation of vectors, which was derived from research of Wutchana and Emarat (2011) on the topic of "students understanding of graphical vector addition of one and two dimensions". Whose conclusion on difficulties on magnitude, direction preserving, vector addition and graphical representation. The third one was difficulties on application of scalar multiplication, which was derived from research of Aniol and Zavala (2012) on the topic of "student's difficulties with unit vectors and scalar multiplication of vectors". Which conclusion were student's difficulties on unit vectors, positive negative scalar multiplication of vectors. Last one was difficulties on theorem proved by vector method, which was derived from research of Kwon (2016) on the topic of "conceptualizing collage geometry". Which conclusion were student's difficulties in mathematical combinations, geometric to symbolic abstractions, geometric representations of concept of vector geometry.

Chapter III

Methods and Procedures

This chapter presents the methods and procedure of the study which concern to identify the difficulties in learning vector geometry at secondary level and to analyze the major causes related to the problem in learning vector geometry at secondary level. Research method describes the design, selection of respondents,, sampling technique, data collection tools, method of the collection of data, analysis of and interpretations of data etc.

The Design of the Study

The design of the study was a case study based on descriptive qualitative in nature on which meaning derive from the total picture of the description. Karlsson (2016) states that "the case study is an investigation of a phenomenon in a real-life situation the study should intensively investigate a small sample of either one or a few cases and the cases can be of various sizes". Mitchell (1983) defined a case study as a “detailed examination of an event which the analyst believes exhibits the operation of some identified general theoretical principles”. This was a case study depends on the students existence, feelings and action found by the researcher as a teacher also. This study was concerned with difficulties in learning vector geometry at secondary level. So, this complex situation was described from descriptive method.

In this research I had collected the data from multiple sources as direct or participant observations, interview, archival records or documents and audiovisual materials. I spend time for interaction with participants. This approach had a potential to deal with simple to complex situations. Mostly used "how" and "why" type of question in the conduction of research in entire groups. The report was prepared including the lesson learnt from study connecting with the theory.

Selection of Respondents

For this case the researcher were take government school of Lalitpur district. The main respondent of this study were grade ten student of Pragati Shikshya Sadan Secondary school Kopundole Lalitpur. I had selected this school on the method of purposive sampling of own my students. So, I believed that in- depth information was taken from familiar institution.

The selection of respondent of this research was student's difficulties in vector geometry at secondary level. There were sixteen students in grade ten for the purpose of this study I took exam/test of these students in that school. Four short question and three long questions were took place in the examination. The researcher selected the highly difficulties occurred answer sheet after checking the all answer sheet. Among them five students were chosen for interview.

Tools/Instrument of the Study

The outcomes and validity of the study depends on the technique of data collections. There were many tools for the qualitative research to get information from depends about their knowledge, skill, attitude, experience and beliefs. This study was qualitative in nature so the researcher collected the information only by the primary source and analyze from appropriate literature review. The major tools of this research were test, interview and reliability and validity tools.

Tests paper. In the qualitative research the test is important tool for fulfill the objectives of research. This was also related to the conclusion driven. Generally a kind of assessment given to the students for measuring their learning knowledge skill and attitude is known as tests. The purpose of that study was finding students difficulties in vector geometry. The researcher was conducted four short questions and three long questions. The set of question was constructed from different categories of

Bloom's taxonomy and selected from related topic to fulfill the objective of this research. The taxonomy is divided into six levels knowledge, comprehension, application, analysis, synthesis and evaluation. So, written test were designed 2 comprehension level, 2 application levels, 1 analytic level, 1 synthesis level and 1 evaluation level question. After making the questions it was administrated in grade X students. After conducting the test for sixteen students I analyzed the students difficulties in theirs response. The test paper is given in appendix-A

Interview. Interview is the important tool for measuring the student's attitude concepts and overview of subject matters. It helps to face to face contact with students, teachers and researcher. Best and Kahn (2013) state that "interviews are used to gather information regarding an individual's experiences and knowledge; his or her opinions, beliefs, and feeling". This was being for qualitative information. After administrating the test the researcher was selected five students for interview. The interview was be design on the basis of reading and learning difficulties according to the vector addition and graphical representations, theorem proved by vectors, vector notations and application of vector multiplying. In interview the researcher used more flexibility for asking questions for students and teacher also. The interview guidelines for students are given in Appendix-B.

Reliability and Validity of the Tools

The reliability refers to the consistency of the result and the validity refers to actual measurement or equivalent to the international standards. Best and Kahn (2014) state that "reliability is the degree of consistency that the instrument or procedure demonstrates. Whatever it is measuring, it does consistently. Validity is that quality of a data-gathering instrument or procedure that enables it to measure what it is supposed to measure.

The aim of this study was finding difficulties so, I did not establish validity by administrating the pilot test but for validity tries to gather the accurate instrument and collecting different types of data from different perspectives on the basis of Paget's cognitive constructivism theory. After taking suggestion of supervisor, subject expert or specialists were accepting the test question.

Data Collection Procedure

The process of data collection is depends on getting information from variable of interest and systematic theoretical procedure. I had experience of frequently visiting that school, where was collect the data. That was my own working school so; it was easy to collect data. The essential data and information could get immediately. In this research, I had used written test and interview guideline. At first I had conducted test by using four short question and three long questions and conducted to sixteen students. After conducting the test I was selected five students for interview. An in-depth interview conducted to five students and essential data was collected. The interview guideline notes the important things, student's curiosity were recoded.

Data Analysis and Interpretation

Qualitative data analysis is wide variety in approaches to qualitative research matched by the many approaches to data analysis. The data analysis was in the form of words, which are relatively imprecise, diffuse and context based. It is the data analysis is in the form of words, which are relatively imprecise, diffuse and abstract concepts (Bitstream, 2017). Thematic method was applied for analyzing and interpreting the data. Braun and Clark (2006) state that "thematic analysis is a method for identifying, analyzing, and reporting patterns (themes) within data" (page no. 6). Thought focusing on meaning across a dataset thematic analysis allows the researcher to see and make sense of collective or shared meaning and experience identifying

unique and idiosyncratic meaning and experience found only within a single data item". According to this primary source of data such as test interview and discussion, the research report was prepared analyzing by thematic method.

In this study I conducted a test for optional mathematics students and interviews were conducted for those students who had maximum difficulties found by test. On the basis of answer sheet of test and their interview data were analyzed. After collecting the data I were coding decoding and taking conclusion tougher with analyzing and describing the all responses. The collected data were classified into: vector addition and graphical representation, types of notation, application of scalar Multiplication, and theorem proved by vector addition types of difficulties. At last the main themes were analyze with the above literature review as well as analyzed according to Piaget's learning theory and Gagne's learning theory. According to Gagne mathematical learning were divided into two objects direct and indirect. So, data were analyzed on the basis of direct objects fact, skill, concepts (Notations) and principal (themes). Similarly it was focused on his indirect object transfer of learning, problem solving, self-discipline etc. from these theoretical reviews were the key guidance of data analyzing.

Chapter IV

Analysis and Interpretation of the Data

The main body of the research is analysis and interpretation of the data. So, this chapter helps to achieve the objectives of the study on the basis of methods and procedures. Ngulube (2015) states that qualitative research "Provides naturally occurring information and assists in answering why and how questions, while documenting the interventions of the researcher during the whole research process". This was a case study related to difficulties in learning vector geometry at secondary level of Lalitpur district in mathematics. Only one school Pragati Shikshya Sadan Kopundole Lalitpur was purposively selected.

There were altogether 17 students in class ten, optional mathematics. First I had planned to administrate the exam/test for all students due to absent of one student it was administrated to the 16 students of the class. Then I checked the all answer sheets of the students and selected the answer sheet which has highly difficulties occurred. Among them I had selected five students for the interview on the basis of their mistake in their answer sheets.

I checked the all answer sheet and categories the difficulties of all seven questions into types and notation of vectors, vector addition and graphical representation, application of scalar multiplication and theorem proved by vector method. Among those seven questions designed from Secondary Education Exam model. The test was designed on the basis of cognitive domain through according to blooms taxonomy. Cullinane (2009) states that "there are many applications of Bloom's taxonomy, It was initially designed to be applied when setting examinations paper at third level but it now has been used as a basic reference for educators globally. The data was interpreted by thematic method. After checking these answer-

sheets of students response in-depth interviews were conducted. Then I was analyzed all collected data by coding decoding and derived conclusion from whole information with the above literature review. On the basis of analysis of answer sheet of the students, I had collected their difficulties and tabulated below.

S.N.	Types of difficulties	No. of students had Difficulties	Main difficulties
1.	Types and notations	12	Many of them does not care about basic rules notation and definition of vector terminologies because knowledge type question does not ask in SEE exam
2.	Vector addition and graphical representation	10	Lack of contract and variation about graphical representation of geometrical shape by co-ordinate geometry and vector geometry.
3.	Application of scalar multiplication	13	Students have confusion about calculating scalar multiplication and magnitude. Those reverse processes were applied.
4.	Theorem proved by vector method	9	Students were properly used vectors rules to prove the theorem but many of them had rote learning, those student only proved the theorem if they had properly remained it.

Above table shows that there were 12 students had difficulties in type and notations, 10 students had difficulties in vector addition and graphical representation, 13 students had difficulties in application of scalar multiplication and 9 students had difficulties in theorem proved by vectors method. Barnilo, P. & Zavala, G. (2012) "The most common error was to choose a vector with incorrect magnitude. In our study we also found that no student translated the vector with the positive scalar multiplication problem. However, we did find this error and other students' difficulties in the negative scalar multiplication of a vector".

Difficulties in Learning Vector Geometry at Secondary Level

Vector is the new concept of secondary level students. Its concepts become with connection with geometrical concepts and plan co-ordinates concepts begin at class nine. Lack of pre-knowledge and mathematical skill many of the students were discouraged to this subject. In this level curriculum included in optional mathematics subjects also. People believe that those students only selected optional mathematics he/she solved all mathematical problems as well as without solving vector geometric problem students obtained minimum acceptable grade.

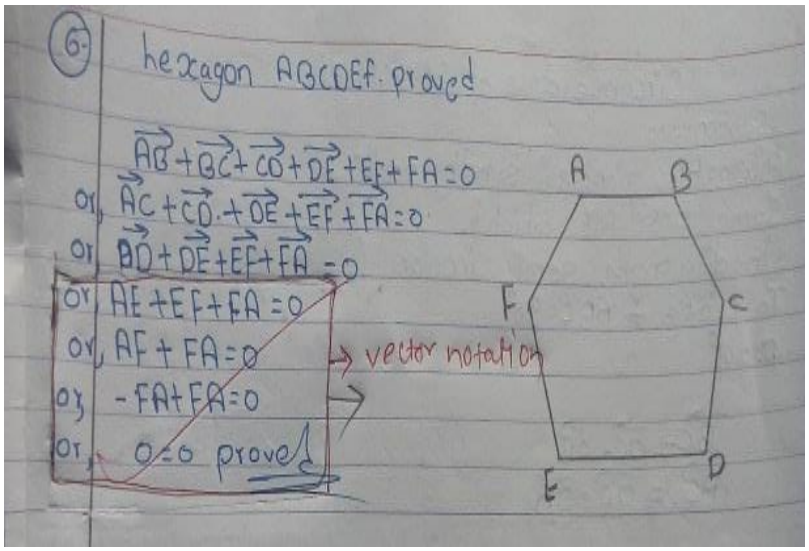
If we ignore the some carelessness of students in their exam, many of them had not well understood about the conceptual meaning of the problem. In vector geometry, I found so many problems areas like as difficulties in types and notion, vector addition and graphical representation, application of scalar multiplication and theorem proved by vector method. In this area the researcher found some sub-areas on difficulties in vector geometry. Difficulties were like as conceptual difficulties, language difficulties, comprehensive difficulties and organizational difficulties (Khanal, 2018).

Difficulties in Notation and Types of Vector

Mathematizing any concept, discipline or any quantity connection with its own symbol is known as mathematical notation. Generally, it is a specific symbol in mathematics as well as vector geometry also. The notation gives guideline to study the concept of its written testimonies. Kwon (2016) states that difficulties in notation mean "the problem in five important transitions, physics to mathematics, arithmetic to algebraic, analytic to synthetic, geometric to symbolic, concrete to abstract, and corresponding student difficulties along epistemological and ontological axes". Similarly, the types of vector are classification of difference in comparison between two or more vectors. For this concepts student have specific differentiate ability to check the problem situation.

In this section, mainly focus on whether students were able to recognize the world problem and symbol or not, if found then I categorized it as difficulties in notation and types of vector geometry. For finding difficulties in notation and types of vector, I took seven word problems and it was assigned to every student which had related notation and type related problem. After taking test/exam I checked the answer sheet and categories the problem related to types and notation of vector.

Types and notation of vector is not directly related to exam points of view for Secondary level students. Without knowledge of basic concept, student cannot derive meaningfully concept of vector geometry. Students had following type of difficulties in vector notations.



Above solution was question no. 6 proof, which was logically proved. But when the starting of this proof student use correct notation but from fourth step student cannot used correct notation so, in this situation had comprehension problem on notation of vector they had simply understand about line AE and vector $\overrightarrow{AE}$.The correct notation of magnitude notation of vector is | | but student did not use this symbol and they wrote magnitude of vectors in words which found by me in other answer sheet.

For justification of student's difficulties in vector types and notation of vector, I had had taken interview to related student. On the basis of interview guideline then I had asked question at the different point of view and listened the students answer. The common difficulties of those focused students answer were notated the dialoged form (answers were not noted the unique students' conversation forms).

The following dialog took place. (I: interviewer; S; student).

I: What is vector? do you ask?

S: Vector is mathematical topic.

I: What is called position of vector? What does represent the O in vector geometry?

S: I don't know sir position of vectors and O-represent origin in vector geometry.

I: How can you calculate magnitude of vector?

S: Magnitude of vector is $\sqrt{x^2 + y^2}$

I: What is called zero vectors?

S: Zero vectors no vector.

I: Can you give me an example of zero vectors.

S: 0 vector = $\vec{a} - \vec{a}$.

I: What is unit vector?

S: Unit vector is 1 unit.

I: What are equal vectors? Give one examples of equal vectors.

S: If two vectors having same direction and magnitude are called equal vectors and example is if $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ *and* $\overrightarrow{CD} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

Above conversation student had problem on recognizing vector definition where student asked vector is a mathematical topic did not define vector is a quantity which have both magnitude and direction. So, they did have differentiated ability of scalar to the vectors. Position of vectors did not know but known about notion of origin. They were familiar about magnitude of vector but not define zero vectors. But they had concept of zero vectors. Student had concept of unit vectors but not had clear definition of unit vectors. So, did not ask vector having magnitude is 1 unit in this situation student had difficulties in conceptual understanding of unit vectors.

Moreover, in vector notation and types of geometry some other questions also asked. Most of the students asked very clearly about the concepts of using arrow, how denote unit vectors, what is called like vector?, what is parallel vectors?, what is mean by negative vectors etc. but some of them not asked anything due to lack of study so,

for these questions students had no difficulties. Among the 16 students 12 had some difficulties on types and notations of vector geometry but 4 students had no difficulties measured by my tools. The main problem was students did not care about basic rules notation and definition of vector terminologies because knowledge type question does not ask in SEE exam.

Students have conceptual, language and organizational were the main difficulties on types and notation of vector geometry. Wanda (2001)" Learning is much more meaningful if the child is allowed to experiment on his own rather than listening to the teacher/lecture. The teacher should present students with materials and situations and occasions that allow them to discover new learning. In the book "*To Understand Is to Invent"* Piaget said the basic principle of active methods can be expressed as follows: "to understand, to discover, or to reconstruct by rediscoveries and such conditions must be complied with if in the future individuals are to be formed who are capable of production and creativity and not simply repetition. In active learning, the teacher must have confidence in the child's ability to learn on his own".

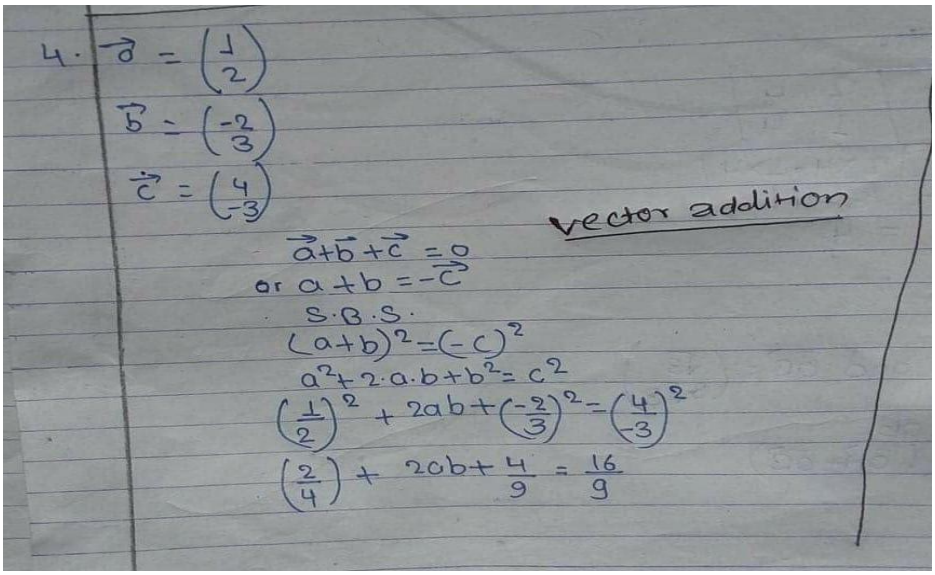
Difficulties in Vector Addition and Graphical Representation

In vector geometry, generally the point of co-ordinate axis refers the position vectors. If two position vectors were given, I had combined new vectors with the help of two position vectors. Also we had to fill it in graph and shown its direction with the help of its instructions.

Lack of reading ability, comprehensive ability, transformation ability students would not have clear the concept of basic instruction to find out the problem and its correct solutions. Nguyen and Meltzer (2003), state that "students have difficulties

Performing basic vector operation they were not able to add or Subtract vectors graphically after traditional instruction, and could not answer qualitative questions about vector addition and subtraction.

I used different questions to identifying the student difficulties on vector addition and graphical representation. The problem in this level was If $\vec{a} = \begin{pmatrix}1\\2\end{pmatrix}$ $\vec{b} = \begin{pmatrix}-2\\3\end{pmatrix}$ *and* $\vec{c} = \begin{pmatrix}4\\-3\end{pmatrix}$ then, find the value of $(\vec{a} + \vec{b} + \vec{c})$ and$(\overrightarrow{a} - \vec{b} + \vec{c})$. In this problem four students committed such types of difficulties. The answer sheet of a student is given below for example:



Above solution shows the addition difficulties of the students. The students tried to solve the problem without understanding the meaning of the problem. In this problem $\vec{a},\ \vec{b},\ and\ \vec{c}$ already given but students cannot solve the problem $(\vec{a} + \vec{b} + \vec{c})$ and$(\overrightarrow{a} - \vec{b} + \vec{c})$ meaning fully. It was a conceptual problem in vector addition. Therefore, there was conceptual difficulty in this answer sheet. When question asked $(\vec{a} + \vec{b} + \vec{c})$ and$(\vec{a} - \vec{b} + \vec{c})$ but student writes $\vec{a} + \vec{b} + \vec{c} = 0$ it is not correct. Student used squaring the both sides, which is not required condition. Lack of preliminary concepts students cannot used correct process.

Moreover, for justification of student's difficulties in this problem, I took face to face interview with the students and researcher.

The following dialog took place (I: interviewer; S; student).

I: What is called vector addition?

S: Addition of two or more vector.

I: In question $\vec{a} - \vec{b} + \vec{c}$*, what does it mean?*

S: $\vec{b}$ *is subtracted from* $\vec{a}$ *then adding* $\vec{c}$.

I: Why you did not do it in test?

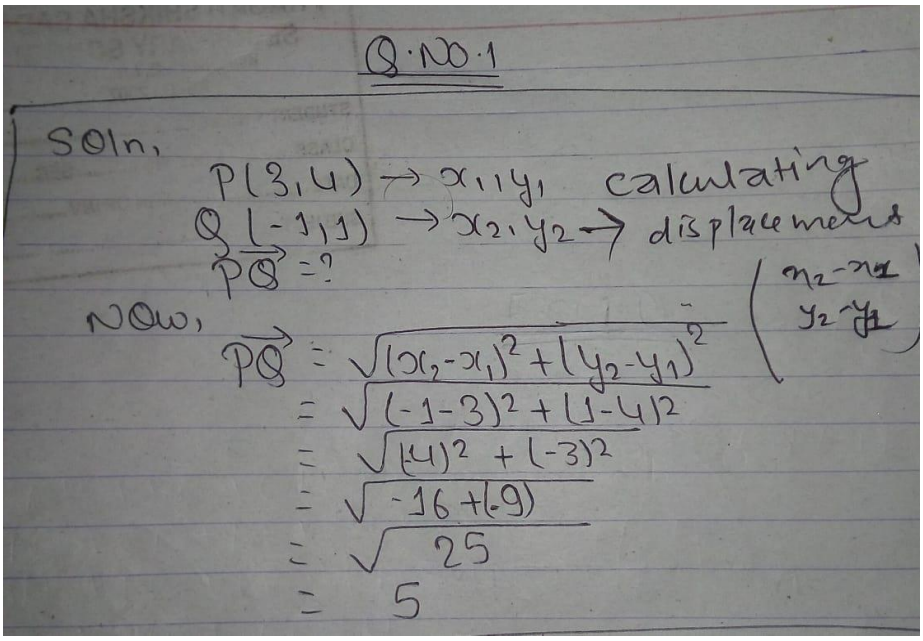
S: I could not remind the formula to add the two or more vectors.

In this problem, student was not reminding the vector addition formula. Form this fact of interview I known that student knowledge and comprehension level also determines the difficulties in vector addition. After that, I was given a question if $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ *and* $\overrightarrow{RS} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, then find $\overrightarrow{PQ} + \overrightarrow{RS}$, with the help of book student using formula and easily calculate this question answer. Observing many classes of student's behaviors I felt that lack of concentration of student's activities to learning activities creates the students difficulty in learning.

Sah (2016) states that "the skill and knowledge of mathematics is regular ongoing process, the previous knowledge and skills are the source improving current mathematics. In this sense, mathematics is to be taught by applying the skill and knowledge. The concept (knowledge and understanding) that are helpful to learn the new topic is pre-knowledge. Students should have the well concept of basic properties of geometry, co-ordinate, etc. as pre-knowledge for vector geometry.

The difficulties on graphical representation was measuring from the question, $\overrightarrow{PQ}$ Displaces P (3, 4) to Q (-1, 1) then find the direction of $\overrightarrow{PQ}$. Also represent in graph. On this question students have two types of difficulties. Students calculated

displacement $\overrightarrow{PQ}$ with incorrect formula, in vector geometry if two position vector given P = $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $Q = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ then $\overrightarrow{PQ}$ =$\begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$ and $|\overrightarrow{PQ}|$ =$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. But students used wrong formula to calculate it. They had reverse formula were used. So, calculation is not used properly. So they have confusion about it. An answer of a student is given below.



Displacement directly related to vector subtraction of its component. Which relation representing on graph is directly related to graphical representation. Lack of this misconception of calculating displacement $\overrightarrow{PQ}$ made representing graph was uneasy so five of them did not representing graph also. Those students who had calculated displacement of $\overrightarrow{PQ}$ also had confusion about showing direction also. Among them 3 students represent direction of $\overrightarrow{PQ}$ in reverse order. So, it was created the difficulties in learning for students in vector geometry.

Moreover, for justification of student's difficulties in vector addition and graphical representation, I had taken interview to the related student. After the student

had heard the question and asked the answer to the interviewer.

The following dialog took place (I: interviewer; S; student).

I: What are the difference between vectors addition and vector addition by triangle law?

S: If $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ then $\vec{a} + \vec{b} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$ *and vector triangle law means in any triangle* $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

I: When we use this different addition rules in vector geometry?

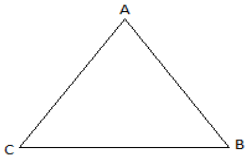
S: I did not know sir.

I: What is the difference between representing $\overrightarrow{PQ}$ *vector in graph and PQ line in graph?*

S: Actually I don't know sir.

I: In vector addition sometime given, if $\overrightarrow{OP} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ *and* $\overrightarrow{OQ} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ *then find* $\overrightarrow{OP} + \overrightarrow{OQ}$ *and sometimes given* $\overrightarrow{OP} = 2\vec{\imath} - 4\vec{\jmath}$ *and* $\overrightarrow{OQ} = 4\vec{\imath} + 3\vec{\jmath}$ *= then find*$\overrightarrow{OP} + \overrightarrow{OQ}$*. What is the difference between them?*

In these above questions students had difficulties which were discussed question wise. In first question only one type of difficulties, some of students asked its addition rule of position vectors with example. i.e. if $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ then $\vec{a} + \vec{b} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$. And some of the students were justified with vector addition on triangle law. i.e. in a given triangle $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



But they had not clear about difference between addition of position vector and vector addition on triangle law. After the complement question they did not ask addition of

position vector and vector addition on triangle law were different situation of addition on vector geometry. In third question, no one student asked difference between $\overrightarrow{PQ}$ and line PQ representing on graph, where they have difficulty in special meaning of graphical representing$\overrightarrow{PQ}$. In fourth question no Student asked fact answer, they had difficulty on recognizing position vector and vectors with combination of unity. They did not ask it is only different expression on vector addition and its expression.

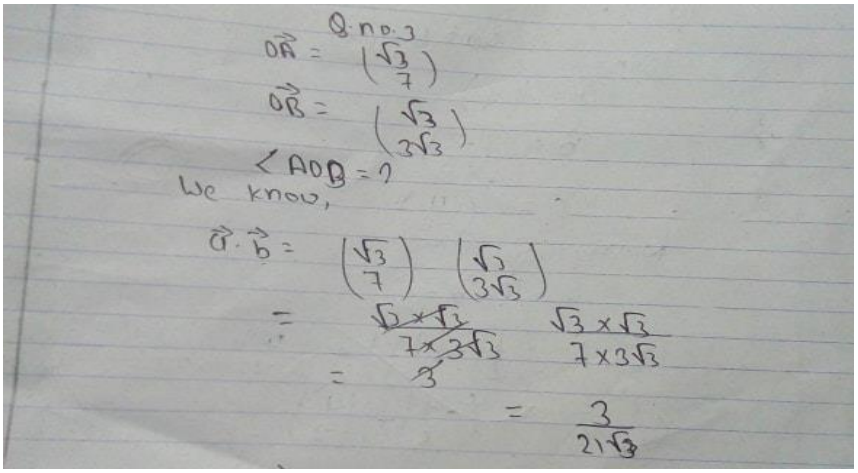
Students had incorrect addition schema on their minds so; it reflects their answer sheet also. So, conceptual clearness must need for them also. New method should apply to change their misconception. Lafa (2014) states that "In piaget view a schema includes both a category of knowledge and process of obtaining the knowledge. As experience happen, this new information is used to modify and add to change previous existing information or schemas. Wutchana, U. & Emarat, N.(2011). The students did not grasp the important concept about the vector direction and how a vector may be moved while its magnitude and direction are preserved. Many students are still confused about the “tip-to-tail” strategy.

Difficulties in Application of Scalar Multiplication

Difficulties in application of scalar multiplication define as the problem on vectors scalar multiplication and its uses on vectors problem. Barniol, P. & Zavala, G.(2014) states that "students still have difficulties with unit vectors and negative scalar multiplication of a vector. Instructors should be aware of these specific difficulties to plan the instruction".

Many of the students had problem on process in scalar multiplication. The researcher had looked and observed the solution of students test and interview for identifying the problem of solving scalar multiplication and its application. Scalar product is used, to find the angle between two points as well as if scalar product of

two vectors is 0 then those given vectors were orthogonal also. Theorems proved by vector method must needed application of scalar product of vectors. In this purpose one of the test item included in question no. 3. Which is $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\7\end{pmatrix}$ And $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\3\sqrt{3}\end{pmatrix}$, Calculate the angle ∠AOB. In this question student must had calculated the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ as well as direction of $\overrightarrow{OA}$ and$\overrightarrow{OB}$. In this question student had following types of difficulty in Application of scalar multiplication. The example was given below.



Above the solution shows the level of student's difficulty to combine the terms where the correct solution was $\vec{a}.\vec{b} = \sqrt{3}.\sqrt{3} + 7.3\sqrt{3} = 3 + 21\sqrt{3}$ but student get answer in fraction. Students were aware about sign but they did mistake in magnitude. Barniol, P. & Zavala, G. (2012) states that "students answered correctly and no student chose the options with vectors being translated. The most common error was to choose a vector with incorrect magnitude. In our study we also found that no student translated the vector with the positive scalar multiplication problem. However, we did find this error and other students' difficulties in the negative scalar multiplication of a vector".

Moreover, for clear justification of student's difficulty in learning application of scalar multiplication the researcher took face to face contact question to the related student. The following dialog took place (I: interviewer; S; student).

I: What is the formula to find the angle between two vectors?

S: If $\vec{a}$ *and* $\vec{b}$ *are given then angle between two vector is* $cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

I: What is mean by $\vec{a}.\vec{b}$*?*

S: It means scalar product of $\vec{a}$ *and* $\vec{b}$.

I: What is the formula to calculate the$\vec{a}.\vec{b}$*?*

S: If $\vec{a} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ *and* $\vec{b} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$*then* $\vec{a}.\vec{b} = x_1y_1 + x_2y_2$.

I: How to calculate$|\vec{a}|$ *and*$|\vec{b}|$*?*

S: $\sqrt{(x-component)^2 + (y-component)^2}$ *this gives the value of magnitude of either vector.*

I: Why you write $\begin{pmatrix} \sqrt{3} \\ 7 \end{pmatrix}\begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix} = \begin{pmatrix} 3 \\ 21\sqrt{3} \end{pmatrix}$.

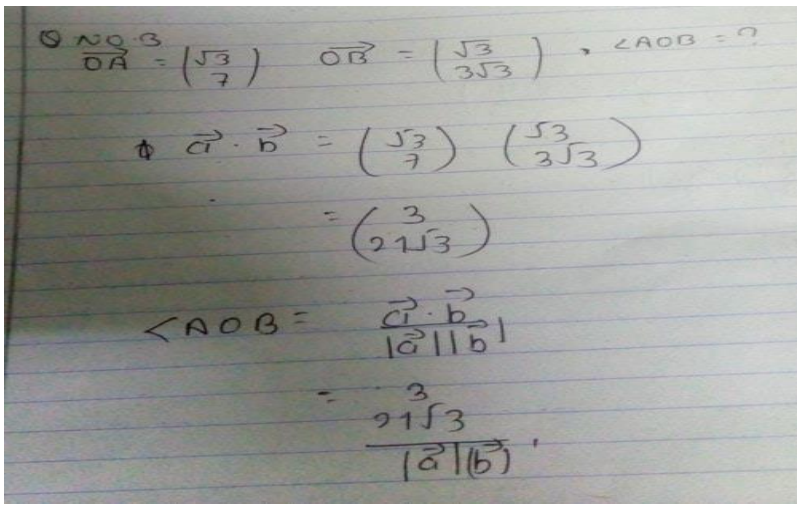
S: When $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ *and* $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ *then* $\vec{a} + \vec{b} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$.

So, sometimes it makes confusion.

Above conversation represents student's difficulties where students gave all question answer theoretically but they have confusion when applying the rules. The correct scalar product of $\begin{pmatrix} \sqrt{3} \\ 7 \end{pmatrix}\begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix}$ =3 +21 $\sqrt{3}$ was the correct scalar product but student write $\begin{pmatrix} \sqrt{3} \\ 7 \end{pmatrix}\begin{pmatrix} \sqrt{3} \\ 3\sqrt{3} \end{pmatrix} = \begin{pmatrix} 3 \\ 21\sqrt{3} \end{pmatrix}$. They know about rules of scalar product each and every premilinary were familiar but lack of connecting theory to application, the role of carelessness makes that transformational difficulties in application of scalar product. Here lack of conceptual understanding student remaind formulae leaning

rotly. Khanal(2018) state that "the main causes of conceptual difficulties are lack of student's practice, connecting world probem and symbol as well as directly solve problem without understanding the problem correctly".

I found another types of difficulties in application of scalar multiplication, student would write correct formula for calculating the angle between two vectors but scalar multiplication could not differentiate from addition of position vectors. Student multiplies the corresponding terms but was not added and wrote the answer in scalar number. The example was given below.



Above solution shows the difficulties on application of scalar multiplication. The student tried to solve without understanding the meaning of the problem. In this solution student doesnot remained calculation idea of $|\vec{a}|$ and $|\vec{b}|$. But using the concept of multiplication not used the concept of scalar multiplication. So, the student wrote the scalar product of two vector $\vec{a}$ and $\vec{b}$ $= \vec{a}.\vec{b} = \begin{pmatrix}\sqrt{3}\\7\end{pmatrix}\begin{pmatrix}\sqrt{3}\\3\sqrt{3}\end{pmatrix} = \begin{pmatrix}3\\21\sqrt{3}\end{pmatrix}$ which was incorrect. Student unable to understand the meaning of scalar product of two vectors. In this fact student was written the scalar product of $\vec{a}$ and $\vec{b}$ is

$\vec{a}.\vec{b} = \sqrt{3}.\sqrt{3} + \sqrt{3}.\ 7\sqrt{3}$ $= 3 + 21\sqrt{3} = 39.373$

$|\vec{a}| = \sqrt{\ (\sqrt{3})^2 + 7^2\ } = \sqrt{3 + 49} = \sqrt{51}$ and

$$|\vec{b}| = \sqrt{(\sqrt{3})^2 + (3\sqrt{3})^2} = \sqrt{3+27} = \sqrt{30}$$

This calculation was much needed to calculate angle between two vectors. Thus the student had comprehensive type of difficulty found in application scalar multiplication of vector geometry. Which means that student had unable to understand the actual meaning of the problem.

Moreover, for clear justification of student's difficulty in learning application of scalar multiplication the researcher took face to face contact question to the related student. The following dialog took place (I: interviewer; S; student).

I: How to calculate angle between two vectors?

S: If $\vec{a}$ *and* $\vec{b}$ *are given then angle between two vector is* $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$.

I: What is the formula to calculate the $\vec{a}.\vec{b}$*?*

S: If $\vec{a} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ *and* $\vec{b} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ *then* $\vec{a}.\vec{b} = x_1y_1 + x_2y_2$.

I: How to calculate $|\vec{a}|$ *and* $|\vec{b}|$*?*

S: I don't know sir.

I: Magnitude of $\vec{a}$ *is denoted by* $|\vec{a}|$ *and defined by*

$|\vec{a}| = \sqrt{(x-component)^2 + (y-component)^2}$ *isn't it?*

S: Yes sir but some times $|\vec{a}|$ *means absolute value of* $\vec{a}$*. So, it made me confusion.*

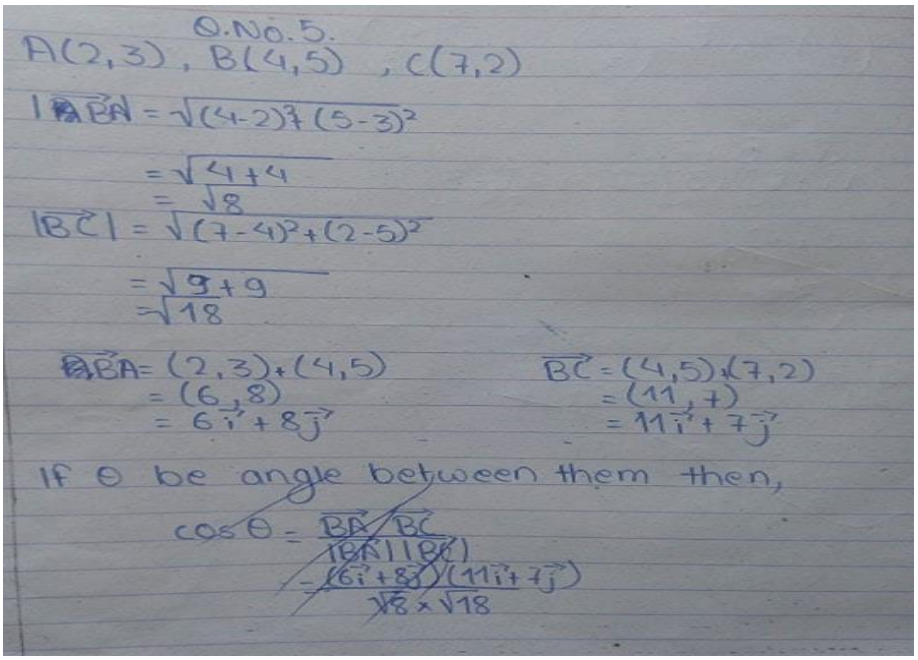
I: You know exactly what is mean by x- component and y component in position vector?

S: x component mean $x_2 - x_1$ *and y- component mean* $y_2 - y_1$.

From the above interview student had difficulties on finding magnitude of vector where notation of magnitude use | |, absolute value of mathematical

component and determinat of vector also uses same notation so, it made student confusion. When student seen the formula he/she was esaly calculate the magnitude of vectors. Student understand part part question but did not connection about the full problem situation.

In this purpose, anther of the test item also included, which is in question 5. The question was if A (2, 3), B (4, 5) and C (7, 2) Find the measurement of angle ABC. In this second problem, seven students did not attempt the question 5 student gave in correct answer and only 4 gave correct answer. Students' written solution was analyzed through their answer sheets. In this question position of three points were given. In this problem student had following type of difficulty.



Above solution shows student difficulty about application of scalar difficulties. Where question asked find the measurement of ∠ABC for calculating ∠ABC we must need $\overrightarrow{AB}, \overrightarrow{BC}$, $\left|\overrightarrow{AB}\right|, \left|\overrightarrow{BC}\right|$ and $\overrightarrow{AB}.\overrightarrow{BC}$. For calculating $\overrightarrow{AB}$ must need

$$\overrightarrow{AB} = \begin{pmatrix} x\ component\ of\ point\ \ B - x\ component\ of\ point\ A \\ y\ component\ of\ point\ \ B - y\ \ component\ of\ point\ A \end{pmatrix}$$

Similar process was used for finding $\overrightarrow{BC}$ also. Here calculating magnitude of vector $\overrightarrow{AB}$ and $\overrightarrow{BC}$ were correctly. Angle between two vectors according to this question was written correctly. When the first step for calculating $\overrightarrow{AB}$ and $\overrightarrow{BC}$ is wrong so student could not gave the correct answer.

Moreover, for clear justification of student's difficulty in learning application of scalar multiplication I took face to face contact question to the related student. The following dialog took place (I: interviewer; S; student).

I: How to calculate angle between two vectors if $\vec{a}$ *and* $\vec{b}$*were given.*

S: Angle between two vectors is $cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$.

I: How to calculate angle between two vectors (∠ABC) if position vector of points A, B and C were given.

S: The measurement of ∠ABC is $cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{BC}}{|\overrightarrow{AB}|\,|\overrightarrow{BC}|}$

I: How to calculate $\overrightarrow{AB}.\overrightarrow{BC}$ *?*

S: If $\overrightarrow{AB} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ *and* $\overrightarrow{BC} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$*then* $\overrightarrow{AB}.\overrightarrow{BC} = x_1y_1 + x_2y_2$

I: How to calculate $\left|\overrightarrow{AB}\right|$*?*

S: $\sqrt{(x-component)^2 + (y-component)^2}$

I: How to calculate $\overrightarrow{AB}$ *if point A and point B were given.*

S: I do not know sir?

From the above information I known about that student had difficulties in application of scalar product in vector geometry and I reached lack of remanding the rule of scalar product creates the student difficulties. The view of cognitive constructivism is connecting pre-knowledge to new knowledge with the systematic

chain creates the new knowledge. So, lack of connecting scalar product and magnitude of vector students unable to find angle between vectors.

Difficulties in Theorem proved by Vector Method

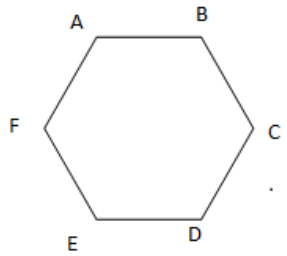
Difficulties in theorem proved by vector method mean "problem creates on to prove the theorem by using vectors rules, definition, proposition and its connections. Kuperberg (2018) state that "in a math course, it's easy to cling to the idea that the heart of every problem is to calculate something, and proving it means simply showing your work. In a proof-based course, it's not true. The heart of a proof problem is reasoning, not calculation. Any proof of anything should have at least some explanation in English".

In mathematics there were many ideas to prove the theorem. Basically theorem were proved by inductive and deductive logic, it was also categories in proof by logic or calculation. The vector geometries theorems were proof by formal deduction from its basic concept of vector like as triangle law, notions, definitions rules of scalar product etc.

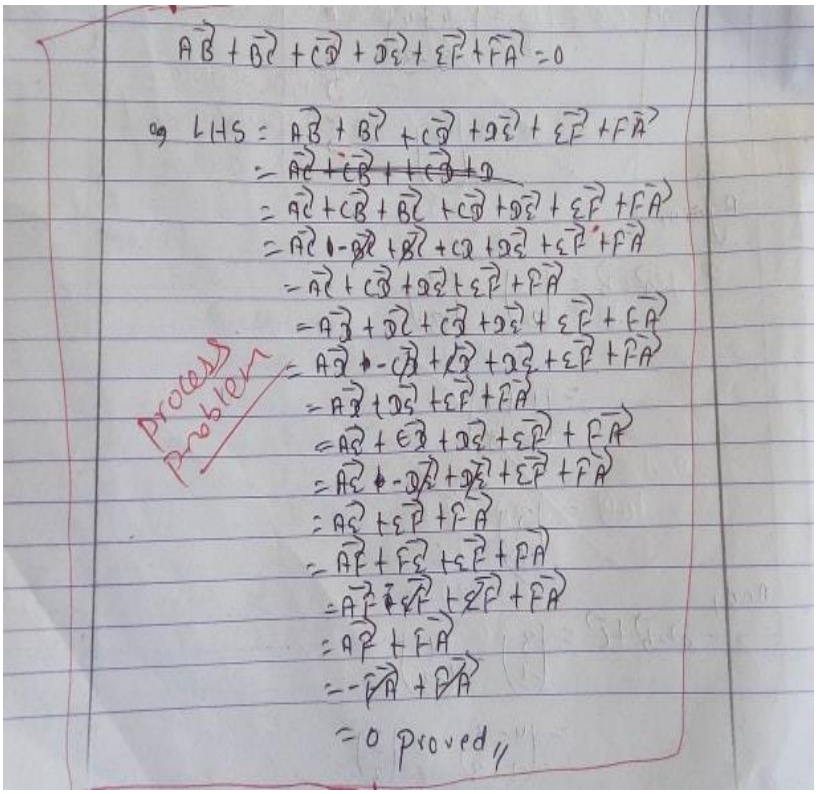
Theorem proving had certain rule and process. In this process students used wrong way and carried unnecessary things which made difficulties in process. The process was classified solving process skill difficulties when the student were able to identify the difficulties operation but did not know the procedures to carry out these operations correctly. By observing the solution of students solving problem I identified the student's difficulties in theorem proved by vector method. Inside this framework two questions were included which were respectively analyzed below.

The first problem in this level was in given hexagon ABCDEF, prove that:

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA} = 0$$



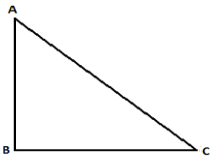
In this problem 3 students committed such type of the difficulties and the highest level solving difficulties in this problem. The answer sheet of a student is given below for example:



The above solution shows that the solving process skill level difficulties in theorem proved by vector method. In this problem the student did not derived correct logic of construction in figure. First and last step write correct but between the first and last step did not had correct in process. This type of difficulties can be considered as solving process difficulties. This shows that students usually face difficulties in the process skill.

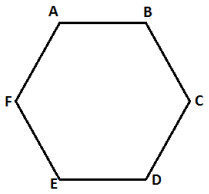
Moreover, for clear justification of student's difficulties in theorem proved by vector method I took face to face contact question to the related student. The following dialog took place (I: interviewer; S; student).

I: What is triangle law in vector geometry?



S: In any triangle $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

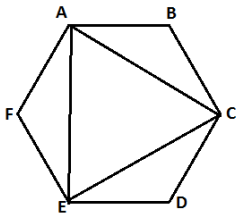
I: In test/exam there was given a hexagon how can you use triangle law in this figure? Which diagonals are joined for derived construction first?



S: I don't know sir.

In this question student did not replied the construction idea. So, I was given hint for constructing proof idea. So I was asked next question. That was as follows.

I: If I joined diagonal AC, CE and EA how can you use triangle law for to proof the theorem $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA} = 0$



S: No, sir,

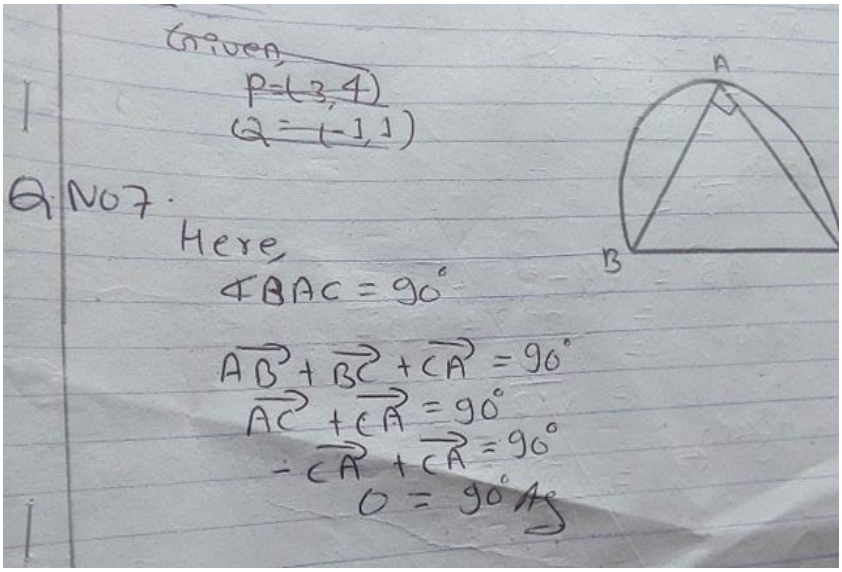
I: How can you conform two vectors are orthogonal.

S: No, sir.

Above dialog and solution of theorem proof represented student difficulties in vector geometry. This theorem can be easily proved by vector method but student cannot have basic ideas for proof the theorem from this dialog I had found there types of difficulties which were face in derive construction, systematically follow the chain of vector addition and basic concepts

Derived construction was basic requirement for proof the theorem. But some student cannot construct the line or without drawing the figure started to prove the theorem. Similarly, student did not systematically follow the chain of vector addition rule then theorems were not proved. Student did not carry out correct reason and process so, it makes process difficulties. In another hand basic concepts were play vital role to proof the theorem in vector geometry lack of basic knowledge student could not proof the theorem correctly. Above dialog was represented the students difficulties in vector geometry. Where students did not know how to proof two vectors are orthogonal.

In another, question was also used to find the students difficulties in theorem proved by vector method which was evaluation level question according to the blooms taxonomy. I believed that it find out all level difficulties on the basis of evaluation level question. In the wider sense theorem proved by vector geometry were uses all concepts of vector geometry. The problem was in question no. 7 and it was the last question of the test also. Which was "prove that the angle at the circumference of semicircle is a right angle" by using vector method". 9 students had difficulties in theorem proved by vector method among exam attempted 16 students. Where 3 students could not solved both 2 questions. The answer sheet of a student is given below for exam.



In the above theorem proved by vector method represent student difficulties. Student were familiar with angle in semicircle is right angle but they didn't connection with vector definition and addition by triangle law. By the solution after calculating scalar product of $\overrightarrow{AB}$ *and* $\overrightarrow{AC}$ student can reach the right angle. Student were aware in vector triangle law in addition so, he/she write $\overrightarrow{AB}$ + $\overrightarrow{BC}$ +$\overrightarrow{CA}$ =0. Also he/she aware about $\vec{a}.\vec{b}$ = 0 then angle between $\vec{a}$ and $\vec{b}$ is right angle but he/she did not have connection with it. Student did not construct center of semicircle which make difficulties to prove the theorem. So, I said that students had compression difficulties in theorem proved by vector method. If student could have written the following the answer was correct.

$$\overrightarrow{AC}.\overrightarrow{BA} = (\overrightarrow{CO} + \overrightarrow{AO}).(\overrightarrow{BO} + \overrightarrow{AO})$$

$$= (\overrightarrow{AO} - \overrightarrow{OC}).(\overrightarrow{AO} + \overrightarrow{OC})$$

$$= \overrightarrow{(AO)}^2 - \overrightarrow{(OC)}^2$$

$$= AO^2 - OC^2 \text{ [square of vector represent scalar value]}$$

$$= AO^2 - AO^2 \text{ [AO and OC is radius of semicircle]}$$

Lack of connecting the basic concepts of the theorem was not proved. Moreover, for clear justification of student's difficulty in theorem proved by vector method I took face to face contact question to the related student. The following dialog took place (I: interviewer; S; student).

I: What was the question did ask to you?

S: It was asked " angle at the semicircle is right angle by vector method.

I: What are the difference methods to prove the theorem by vector and other geometrical method?

S: I don't know sir?

I: Prove that "the angle at the circumference of semicircle is a right angle" by using vector method.

S: Sir I don't know

I: Why you don't know this question was already asked in test also?

S: Sir I don't remember this solution.

From the above dialog student should not prove the theorem by vector method even though same question asked many times in test/ exam also. Lack of derived construction; systematically follow the vector addition, lack of reminding basic concepts and rote learning makes students difficulties on theorem proved by vector method. Gautam (2016) states that "many students, despite a good understanding of mathematical concepts but inconsistently computing. They did not follow the sequential order of the solution in vector geometric theorems. Confusing mathematical terminologies despite of lack of pre-knowledge make student difficulty to prove the theorems. The theory of cognitive constructivism also deal with it and Awwad, (2013) states that "Piaget confirmed in his interpretation of the learning process on the previous information, each growth stage depends on learned

information from the preceding stage, there is no doubt that the earlier information constitutes a indispensable thinking process consists of four elements, namely the reality and senses, this reality transferred to the brain, and then the previous information that explain the fact".

Chapter V

Summary, Findings, Conclusions and Recommendations

This chapter is related with summary, finding, and conclusion and recommendations. After the analysis and interpretation of collected data, an attempt has been made to summarize and list the finding, conclusion and some recommendation for further study which are deal separately sections.

Summary of the Study

This study had focused in difficulties in learning vector geometry at secondary level. The objectives of this study were –to explore the learning difficulties in vector geometry at secondary level and to analyze the major causes related to the problem in learning vector geometry at secondary level. Design of the study was case study. All the class ten government school's students of Lalitpur district were the population of the study where only academic year of 2018/2019 A.D. were taken. Pragati Shikshya Sadan Secondary school was purposively selected for this study. In class ten, there were altogether 30 students among them 17 students were in optional mathematics group so, conclusion were derived from this sample. In the administrated test, 16 students were presented and five students were purposively selected among them for interview that showed highly difficulties in test. The researcher analyzed the test result and interviewed on the basis of above reviewed literature. Many student were gave incorrect answer and few students only gave correct answer. Lack of understanding the mathematical terms, connection the concepts, ignoring the process, lack of linking the relation between variables were main problem.

Finding & Discussion

Difficulties in learning vector geometry at secondary level were the main concern of this study. In this study the researcher research the difficulties on vector

geometry which were type and notation, vector addition and graphical representation, application of scalar multiplication and theorem proved by vector method. The result of data analysis showed that felt difficulties are in the following areas.

Difficulties in Notation and Types of Vector. Notation is the symbol represented to special term in vector geometry which is called mathematical language also. The space of language is reducing and vague, sentence is minimizing by mathematical notation. If student understanding the mathematical term, he/she feels easy to understand the mathematical term otherwise it is so difficult and does not derived solution even though reading the problem also. In this study I found that many students were able to recognize the notations in vector geometry but some students did not asked answer due to lack of concentration in learning. Types of vector geometry mean special classification of vector with their different features. It was a heart of vector geometry study lack of types of vector makes difficulties to study vector relation and connection between its terms and symbol as well as difficult to prove theorem also.

Among sixteen students 12 students have difficulties on notation and types of vectors. Students did not write vector notation where $\overrightarrow{AE}$ is simply wrote AE, for representing magnitude of vector did not write it. The notation of magnitude of vector notation is | | also not written. The definition of vector was not correctly defined they only asked mathematical topic, position of vector also did not clearly defined. Student had miss conception of zero vector I found that zero vector is no vectors also.

Difficulties in Vector Addition and Graphical Representation. Vector addition is categorized in to two types of addition in vector geometry. Among of them one is addition of position of two or more than two vectors, it means subtraction of position of vector also. Another is triangle law in vector addition.

Addition/subtraction is establishing between positions vectors adding/subtracting its abscissa and ordinate separately. In this addition 12 students had given correct answer and 4 had given incorrect answer in vector addition, they did not calculate displacement also. Question clearly asked to calculate displacement but student calculates magnitude. Student did not clear about different context of using position vector addition and vector addition by triangle law.

Graphical representations mean the position vector was plotted in graph, where joining the line between positions of vector and constructed the geometrical shapes as well as representing the arrow head by its instructions. In this study 8 students only given correct answer, 5 students did not represent in graph, 3 students shows reverse of its direction. 2 students had difficulties on both vector addition and graphical representations.

Difficulties in Application of Scalar Multiplication. Application of scalar product means the necessity condition of scalar product between two vectors where scalar product is a product of two vectors which value comes in single measurement. In polar coordinates, vectors are expressed in terms of magnitude and direction. When expressed in this format, the dot product of two vectors is equal to the product of their lengths, multiplied by the cosine of the angle between them. So, application of scalar product is much need to calculate angle between two vectors as well as proof the angle between two vectors is orthogonal also.

Among these 16 students 13 students had difficulties on application of scalar multiplication. The main findings were using incorrect formula to calculate scalar product, answer wrote in incorrect structure, confusing on magnitude formula and displacement formula, displacement is calculating distance. These difficulties were

occurred due to incomplete explanation, not clarifying problem and lack of choose appropriate operation.

Difficulties in Theorem proved by Vector Method. Theorem proved by vector method mean theorems were proved by vectors properties, definitions and rules of its basic concepts. In vector geometry statements of theorem were same as geometrical statement but proof process were different. Two theorem were given in test first theorem were proved by 13 student correctly and 3 student did not proved correctly. In that student 3 students were using same vector triangle law and problem in connection of process and derived proof were not valid. In second statement 10 students were proved correctly and 6 students did not proved correctly. So, altogether nine students had difficulties occurred.

In this theme, theorem proved by vector method I found that students had using triangle law without derived construction, applying triangle law incorrectly, unable to connect scalar multiplication for proving the theorem, and unable to prove by idea without reminding.

Conclusion

The major finding of this study shows there were many difficulties. Those difficulties were categorized in the above finding. Difficulties in notation and types of vector are in basic concepts, term, terminologies and notations. Difficulties in addition and graphical representation were using incorrect formula, showing arrow in reverse order and wrongly plotted in graph. Difficulties in application of scalar product were written in incorrect structure, confusing on magnitude formula and displacement formula, displacement is calculating distance. Difficulties in theorem proved by vector method were using triangle law without derived construction, applying triangle

law incorrectly, unable to connect scalar multiplication for proving the theorem, and unable to prove by idea without reminding.

I conclude that student can learn mathematics effectively when giving clear instruction, connection with daily life and teaching through problem based. If instructor should provide students interest, thinking ability and focusing their challenge then they can gain more knowledge. So, teacher provide realistic mathematical problem using symbolic mathematical notation, perform the necessary operations, interpreting the result and provide a meaningful solution to the original problem. Theorem is formal deduction so, teaching instruction clear from basic term, connecting necessary pre-knowledge, derive construction, systematically follow the chain of rule solved the problem.

Educational Implication

Before staring the chapter, teachers should have given fundamental concepts of vector geometry which makes study ready to selecting way to solve the problem. The conceptual clearness increases student's participation in learning and actively interact the classes. In the class activities, the teacher should try to find out the reasons about committing the difficulties. In class, there are different kinds of difficulties level to the students. So, teacher should treat on the basis of student's difficulties level.

- Teacher should encourage his/her student for sequential study of vector geometry.
- Teacher should give pre-knowledge and other chapters relation before starting a new chapter.
- Teacher should conduct diagnostic assessment to identifying the student's difficulties and after that give feedback to his/her students.

- The teacher and administration follow the weak students and conduct them special moral class to pick up.

The teacher suggest to administration for finding other factor which distract students learning and minimize it also.

Recommendation for Further Study

In the researcher point of view, suggested for further study which provides the better mathematical learning activities and comfortable techniques for learning mathematics. The following suggestion could be beneficial for further study.

- The teacher conducts action research in class time to time.
- The policy maker should be make a plan for this types of research in other state or parts country also.
- The government focuses educational research and provides opportunity to universities students with proper time, sufficient material and economic facilities for conducting the research.
- Further study can be done on "factor affecting vector geometric achievement of secondary level".

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Appendix- A

Pragati Shiksha Sadan Secondary School

Kopundole Lalitpur

Opt. Mathematics

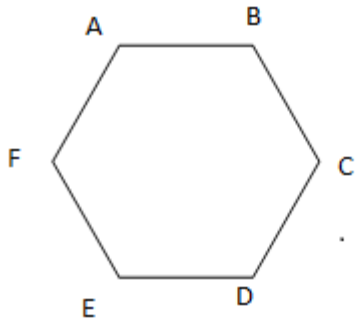
Class: X **Time: 40 minutes**

1. $\overrightarrow{PQ}$ Displaces P(3,4) to Q(-1,1) then find the direction of $\overrightarrow{PQ}$. Also represent in graph.
2. A (2, 1) and B (4, 4) are the endpoints of line segments. Find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$
3. $\overrightarrow{OA} = \begin{pmatrix}\sqrt{3}\\ 7\end{pmatrix}$ And $\overrightarrow{OB} = \begin{pmatrix}\sqrt{3}\\ 3\sqrt{3}\end{pmatrix}$, Calculate the angle ∠AOB.
4. If $\vec{a} = \begin{pmatrix}1\\ 2\end{pmatrix}$ $\vec{b} = \begin{pmatrix}-2\\ 3\end{pmatrix}$ and $\vec{c}\begin{pmatrix}4\\ -3\end{pmatrix}$ then, find the value of $(\vec{a}+\vec{b}+\vec{c})$ and $\vec{a}-\vec{b}+\vec{c}$).

Group B [3 x 4 = 12]

5. If A (2, 3), B (4,5) and C(7,2), Find the measurement of angle ABC.
6. In given hexagon ABCDEF, prove that :

   $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA} = 0$



7. Prove that "the angle at the circumference of semicircle is a right angle" using vector method.

Appendix B

Interview Guidelines for the students

The interview to the mathematics students will be taken on the basis of following main topics

1. Types and notations difficulties:
    - Vocabulary and unfamiliar words.
    - Reading ideas
    - Understanding the problem
    - Mathematical terminologies
2. Addition and graphical representations difficulties
    - Graphical representation
    - Vectors direction shown in graph
3. Difficulties in application of scalar multiplication
    - Calculating magnitude and scalar multiplication
    - Connection between words and symbol
    - Mathematical procedure
    - Correct mathematical operation or procedure
4. Difficulties theorem proved by vector method
    - Understanding the meaning of problem and sentence
    - Connection of definition and theorems from previous knowledge
    - Strategic to approach to solve the problem.