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Free Vibration Analysis of a Cantilever Beam with Stiffeners

by

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ABSTRACT

The present paper deals with the free vibration analysis of a cantilever beam containing various types of stiffeners, by employing the analytical, numerical, and experimental approaches. The three different models i.e. Normal Cantilever beam, Cantilever beam with midline stiffener, and Cantilever beam with diagonal stiffener are analyzed. The frequencies of the free vibration of beam are computed by the analytical method based on the Euler–Bernoulli beam theory. The natural frequencies are calculated by performing the numerical modal analysis using the finite element approach and modeling in ANSYS Mechanical. In addition, these frequencies are also calculated by independent modeling in MATLAB. The experimental modal test is performed using fabricated specimens and a vibration test is conducted by impact hammer as the external excitation. The analyzed signals through FFT (Fast Fourier Transform) are used to obtain the natural frequencies. Results obtained after the comparison shows that the diagonal stiffener provides the greatest increment in the natural frequency among all the configurations. The experimental, numerical and analytical method closely predicts the natural frequency of the beam, conforming the reliability of developed models.

Keywords: ANSYS, MATLAB, Natural Frequency, Stiffener, Finite Element Method.

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LIST OF ABBREVIATIONS

E	Young's modulus of Elasticity
ρ	Density in Kg/m^3
t	Thickness of the beam
L_e	Effective Length
L_{sd}	Length of stiffener along the diagonal at the bottom
W_s	Width of Stiffeners
T_s	Thickness of Stiffeners
V	Volume in m^3
m	Mass in Kg.
K_e	Element Stiffness Matrix
M_e	Element Mass Matrix
K	Global Stiffness Matrix
M	Global Mass Matrix
ω	Angular frequency in radians per second (rad/s)
f	Natural Frequency
ϕ	Phase Angle
$w(x,t)$	Transverse Displacement
I	Second moment of Area
A	Cross-sectional Area
$\boldsymbol{c}$	Constant

CHAPTER 1 INTRODUCTION

1.1 Background

Vibration of structures is an important consideration during the design of engineering structures. The mechanical and civil structures experience the vibration during the operation and to prevent the deflection, noises and fatigue failure, the properties of the vibration have to be studied. The beam is one of the most commonly used structural components and can be found in bridges, aircraft components, machine parts and other building components. The cantilever beam is the most commonly used beam and the easiest to support, fixed at one end. The added stiffeners in the beams are used for the purpose of increasing the stiffness of the plate like structures so as to reduce the degree of deformation of such structures.

Free vibration of a system is an important aspect of understanding the inherent dynamic characteristics and its response to external excitation. Knowledge of natural frequencies and modes shapes helps in identifying the probable excitation frequency that may cause resonance thereby resulting in large amplitude of vibration which can cause fatigue failure, add to noise and cause damage to a structure. The study of the free vibration natural frequency and the mode shapes help to prevent the resonance during the operational time. The resonance occurs when the natural frequency matches the resonance frequency, the mechanical and civil structure vibrates excessively causing the fatigue failure and structural damage. So, accurate prediction of the response of the cantilever beam is very necessary for safe and sound design.

1.2 Statement of Problem

Cantilever beams are common structural components and are widely used in many areas of engineering. However, many beams used in practice have relatively low vibration frequency and are subjected to very large deflection and there is the risk of buckling. Many researchers tried to enhance the dynamic behavior of cantilever beam by adding stiffener. Examples of stiffeners such as rib, T-stiffeners and diagonal stiffeners have been studies by some researchers. By increasing the moment of inertia of beam, added stiffeners can increase the dynamic stiffness, thus enhance the frequency of natural vibration and decrease the vibration amplitude. So, it can be used to prevent the resonance and increases the beams stability. This paper presents the free vibration analysis of cantilever beams with different types of stiffeners. The study has been carried using numerical, analytical and experimental methods. FEA by using ANSYS and numerical solution by MATLAB was conducted which is validated using experimental test results.

1.3 Objectives of Research

Main objective

1. To study the free vibration characteristics of a cantilever beam, with and without stiffeners, using a numerically, and an experimentally based procedure.

Specific objectives

1. Fabrication of Cantilever Beam with stiffeners and without stiffeners
2. Developing a finite element model of a cantilever beam with and without stiffeners using ANSYS Mechanical and analysis of natural frequencies and mode shapes by free vibration analysis using MATLAB.
3. To compare and validate numerical results with experimental data.

1.4 Rationale of the study

This paper focuses on the vibration behavior of cantilever beams subjected to dynamic loads and examines the optimum design of a stiffener to reduce vibrations. Cantilever beams are typical structural components which are susceptible to resonance, fatigue and damage from dynamic loads. Previous research had studied the behaviors of the beam using numerical and analytical solutions. The study has been conducted only limited on numerical simulations and modeling. The experimental validation has been rarely conducted for such study. Though some studies have been conducted but the experimental validation of the cantilever beam with the stiffener and with the non-uniform stiffness has not been fully explored. The effects of the different stiffener configuration such as midline and diagonal stiffeners have not been studied properly. So, to validate a numerical, analytical approach, the experimental verification was necessary.

CHAPTER 2 LITERATURE REVIEW

Cantilever beam is common structural component that is fixed from one end and free at another end. Such beams are found useful in various branches of industry and engineering practice. In many applications, such as robotic arms, machine frames, cranes and air craft wings, beams are fixed from one end and exposed at the other end, making them susceptible to possible vibrations. As such structures have large number of possible vibration modes, their frequencies are highly sensitive to physical properties of the beam material, geometrical dimensions and end supports. To overcome general problem of excessive deflection, stiffness is added to simple beams by adding stiffeners. Generally, addition of stiffeners also adds mass but in practice, stiffness addition is much more pronounced than mass addition and therefore, beams with stiffeners require careful analysis to study their dynamic behavior. Mass and stiffness added to beam by stiffeners change both in a complex manner and therefore, require modal analysis experiment and computational Finite Element Method (FEM) for their free vibration studies.

Stiffeners are the elements added to the beams and plates with an aim to increase the relative strength and reduce the bending of the structures. Use of the stiffeners can be seen in almost every branch of engineering sectors. They can be seen used widely in the bridges, ship partitions, machinery frames, airplanes wings designs, sky walks and similar projects. Stiffeners in a cantilever increase the moment of inertia of the beam, thereby increasing the natural frequency of vibration, which decreases the amplitude. As with many other design parameters, stiffeners can be designed, to some extent, to be efficient. However, adding stiffeners to a beam does change the configuration of the structure and, therefore, its dynamic characteristics, which can make the vibration analysis of such a structure a difficult task.

Free vibration of a structure is defined as the vibration of a structure after being excited to some pre-determined level of response. The free vibration from the name too it indicates the free response of the beam after certain initial excitation. The natural frequency of an object is mainly determined by a structure’s stiffness and mass. The natural frequency increases if the stiffness values of a structure increase. This further is characterized by a mode shape which is the shape assumed by the structure under vibration. The knowledge regarding the free vibration characteristics is an important aspect for the design, if undertaken for the design the structure can vibrate violently reaching the resonance level depending on the excitation energy. This can lead to very large vibrations causing structural failures. Hence it is important to know the natural frequency and the corresponding mode shapes of the structure be it a simple cantilever beam or a complex industrial plant. This knowledge helps an engineer to design the plant such that the operating frequency of the plant does not match the natural frequency of vibration thus avoiding resonance and associated dangers.

Vibrations of structures constitute a fundamental issue in design and analysis of modern engineering structures subjected to dynamic loading. Many experiments and numerical analyses are performed to study the dynamic behavior of structures under dynamic loadings. One of such numerical analysis methods is FEM approach. FEM analysis divides a structure into sub-assembling, or sub-elements, in order to study the stiffness, the mass distribution and the overall dynamic behavior of the structure using an engineer's point of view.

Modal analysis is a common simulation task for the dynamical characterization of structures and components in Mechanical Engineering. Commercial GUI based software such as ANSYS acts as a powerful tool for the numerical solutions which can shows the natural frequencies and mode shapes of the complex geometries also. This information is used to calculate resonance locations and to exclude the operating speed of mechanical drives from the resonance locations of a structure or part. Even though the calculation options are very powerful, the reliability of the results of numerical calculations can only be confirmed through experimental verification of model assumptions. The structural dynamic response is recorded using accelerometers, vibration exciters, impact test transducers and a Data Acquisition System commonly known as DAQ. The obtained signals from the DAQ systems can be analyzed to obtain displacement, velocity and acceleration with a number of techniques such as Fast Fourier Transform method (FFT). The peaks in the FFT Curves indicate natural frequency. Studying the combined numerical modeling and experimental results Combining numerical modeling and experiments results helps to study the structural dynamics properly.

Natural frequencies and modes of vibration of cantilever beams during free vibration have long been studied. Theoretical models have been derived, and a number of tests performed to verify these models. Results of numerical models are compared with experimental test results obtained by placing beams under certain loadings and using vibration sensors such as accelerometers. Time histories of the acceleration, velocity, and displacement of the beam obtained from the vibration tests were analyzed using an FFT (Fast Fourier Transform) into a software package such as LabView from National Instruments. Theoretical, numerical, and experimental models have been shown to accurately determine frequencies and modes of vibration of simple beams made of aluminum subjected to certain forces. The natural frequencies so obtained from the experimental frequency values obtained from experimental tests is validated with theoretical and numerical models for a cantilever beam. This approach is the suitable approach for the cantilever beams(Sura et al., n.d.).

The dynamic characteristics for the mechanical system for extracting the natural frequency and mode shapes can be studied by a technique called modal analysis. To study the behavior of the cantilever beams subjected to the initial excitation, simulation-based study from the computer commercial software's such as ANSYS and ABAQUS can be used to study the response of natural frequency. The results from such analysis are then compared with experimental modal tests.

Frequency Response Function (FRF) is obtained through the experiments which can be used to examine dynamic behavior of the structures with changing the response to variable excitation frequency. The results obtained through the experiments were close to the results from the FEA. Thus, modal analysis is an effective approach to understand the dynamic characteristics of a structure and prevents the structures from resonating, hence enabling engineers to design better(Dr. Babasaheb Ambedkar Technological University, Lonere, Maharashtra, India et al., 2025).

In the process of engineering structure design, vibration analysis becomes a significant factor because natural frequency and mode shape becomes significant elements. The natural frequency and mode shape can be obtained through modal analysis. Some useful information about the mechanical system behavior under dynamic loads can be obtained. Studies on free vibration of rectangular cantilever beams have been carried out. The results for free vibration of two mild steel rectangular cantilever beams have been determined. One side of the beam is free and the other side is attached. The results obtained can be used for structural design optimization, can be a basis for safety and enhancing efficiency during dynamic loading conditions. Hence, free vibration analysis of a cantilever beam is very significant(Naval Materials Research Laboratory (DRDO), Ambernath, Maharashtra, India et al., 2025).

A three-dimensional Finite Element Model (FEM) has been developed to predict the natural frequencies and mode shapes of rotating cantilever beams. The FEM formulates the stiffness matrix of the beam by the principle of virtual work; a stiffness matrix for the non-rotating beam and increment stiffness matrix for the increase in stiffness of the beam due to rotation were incorporated into the analytical model. The model is generally applicable to beams of any configuration and orientation to the axis of rotation. Results have shown that the FEM with an accurate mass matrix predicts the dynamic characteristics of a rotating beam well. Numerical results obtained for rotating cantilever beams using the ANSYS software show a remarkable increase in the effect of increment stiffness on the modal characteristics with increase in velocity; this is critical in predicting the dynamic behavior of rotating structures.(Ekene et al., 2024).

The knowledge of the dynamic properties of structures during design phase is a basic requirement for the correct dimensioning and for avoiding failures, especially due to resonant loads. The modal properties (natural frequencies and mode shapes) of cantilevers have traditionally been determined by experimental, FEA, or numerical methods. There have been many researches about this topic and it has been observed that the experimental approach using software like DEWE soft for the analysis of the structure's FRF (Frequency Response Function) is very effective and provides a real view of the dynamic behavior of the structure by means of graphical animation. The experimental values obtained from the study have been analyzed to obtain the amplitude and phase values of corresponding modes, which can be used to generate an animation of the modes.(Patil & Vibhute, n.d.)

The Euler-Bernoulli beam model is common for theoretical and numerical determination of modal characteristics of a cantilever beam. In the article, the Finite

Element Analysis method in software ANSYS Workbench is used for calculation of natural frequencies. The results are compared with results obtained using simple analytical relations, which were computed in MATLAB environment. Analytical relations can be used for verification of modes shapes. Such a level of analysis is very important for design of many mechanical structures which have to resist to dynamic loads caused by vibrations.(Chaphalkar et al., 2015).

The natural frequency and modes of stiffened plates have been investigated using modal analysis in numerous studies by Finite Element Analysis. An extensive study was conducted to investigate the effects of various parameters on dynamic behavior of plates with stiffeners. Various parametric study has been studied such as stiffener type, its configuration, angle and boundary conditions, aspect ratios, and stiffener thickness to beam thickness. For simply supported plates with clamped boundary conditions, different forms of eccentric stiffened plates have been reported in literature and a number of nondimensional design charts have been presented for accurate calculation of natural frequencies of stiffened plates. These design charts are convenient from engineering practice point of view because they enable efficient analysis of vibrations of stiffened plates with minimum computational effort.(Nayak et al., 2018).

The influence of mass of accelerometers on natural frequencies of cantilever beams was studied experimentally, theoretically and numerically. During study, slight changes were observed in first natural frequency based on experimental results for various beams made of different materials including magnesium alloy AZ61. Results from experimental studies were measured by vibration analyzers, where impulse excitation technique was used and hammer of small mass was used. These results were further validated by finite element modeling, which was carried out by making models in ANSYS for the similar studies. Good correlation was observed during the studies for the effect of mass of accelerometers on natural frequencies of cantilever beams.(Yadav & Singh, 2019)

Fixed cantilever beams have been studied extensively in their ability to respond to dynamic loads. Natural frequency of a beam is primarily a function of its material and geometrical properties and boundary conditions. Additional mass including accelerometers, motor and 3D printed springs attached to the end of the beam have also been investigated and confirmed that the triangular patter dampens the maximum vibration.(Mahdi et al., 2025)

Experiments performed on cantilever beams include free vibration analysis and rotationally unbalanced vibration. The experiment was conducted for the natural frequency, damping ratio and the possible resonance values were identified for various loadings conditions. The experimental values nearly coincide with a theoretical value. Importantly, hands-on experiments like these are valuable because they enable a practical understanding of the effect of mass loading, damping, excitation, etc., as applied to cantilever beams.(Ma et al., 2020)

The dynamic behavior of a structure can be established through both a modal analysis approach and a harmonic approach. The impulse modal test can be a simplest way to do a modal testing. This relationship for a structure can be used to predict the dynamic behavior using FRF for each degree of freedom. FRFs can be done using test hardware such as impact hammer and magnetic shakers. The impact test establishes the rows of the FRF and the shaker test establishes the columns of the FRF(Rani, 2018).

Spectral dynamic stiffness approach is employed for the vibration analysis of stiffened plate structures. Various boundary conditions and beam stiffeners (open and closed sections) including eccentric joints are considered. Plate stiffness matrix equation is established and combined with the corresponding stiffeners stiffness matrix equation. The Wittrick-William's algorithm is utilized for the modal analysis, resulting in an efficient and accurate solution methodology. The results are validated using ANSYS, and the technique has good accuracy, and reliability, and computational economy. It can also be used for static and dynamic structural analysis and design(Liu et al., 2021).

The modal analysis of the cantilever beam reveals the vibration characteristics and aids in the mechanical systems design by identifying the natural frequencies and modes in the low, mid and high frequency ranges. The frequency was obtained using both ANSYS by a method of FEA and numerical methods such as MATLAB and the results from both methods closely agrees with each other. This supports the accuracy of FEA and the applicability of modal analysis as well (Chaphalkar et al., 2015).

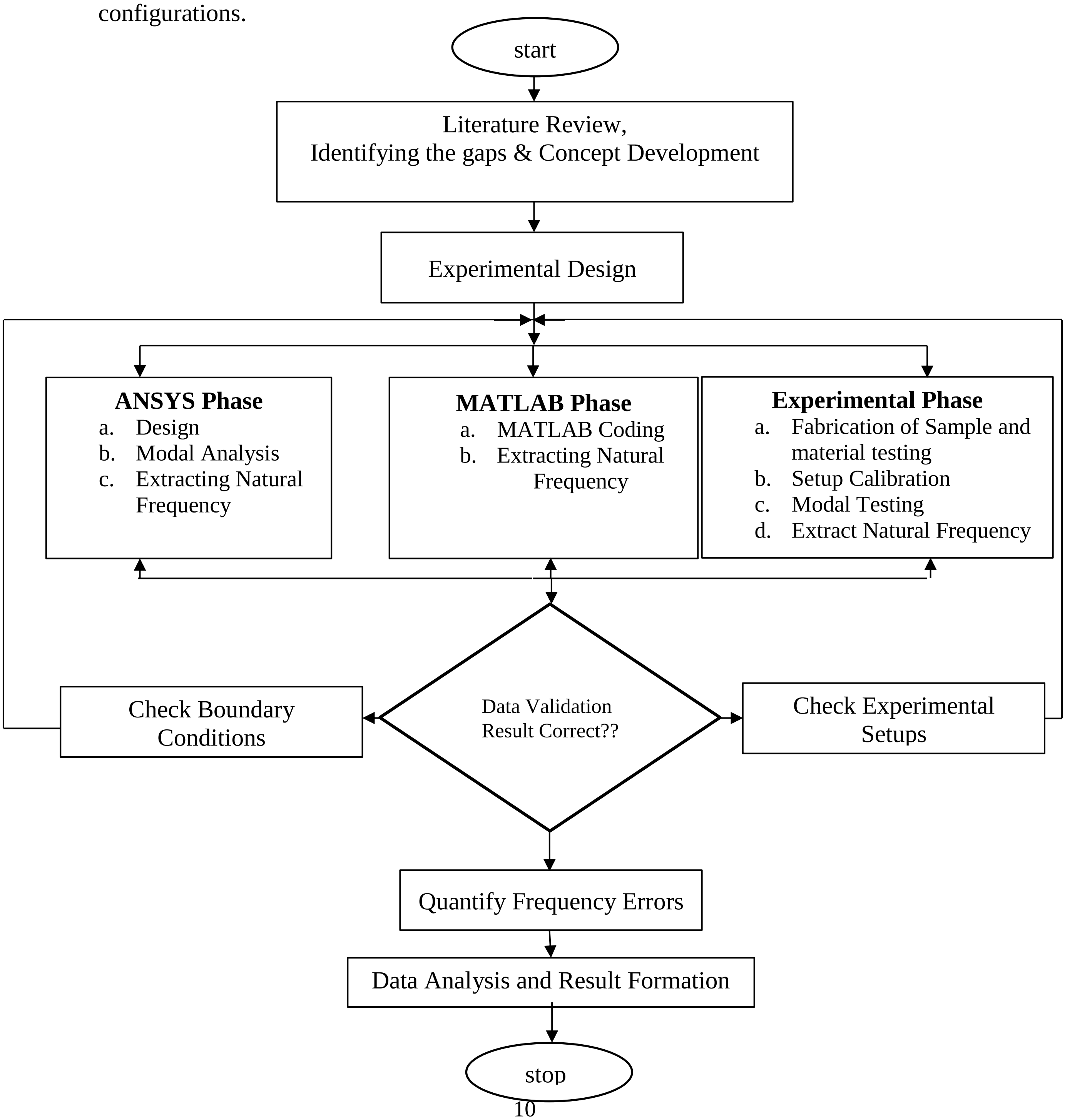
2.1 Research Gap

Vibration characteristics of stiffened cantilever beams by free vibration have been investigated by both the mathematical and the finite element methods, with certain assumptions on boundary conditions. However, experiments have rarely been performed, especially those due to stiffness non-uniformity and mass stiffness due to stiffeners. In practical studies using finite element method, large discrepancies between numerical results and experimental results are usually ignored due to the weight of the sensors, poor fixture constraint, and variation in production processes. A detailed study is conducted by combining the numerical and experimental methods in order to investigate cantilever beams with stiffeners.

CHAPTER 3 METHODOLOGY

3.1 Conceptual Framework

In this study a combined analytical, numerical and experimental approach is adopted to study the free vibration of normal and stiffened cantilever beams. The numerical models were developed and analyzed using finite element software package ANSYS and a software package MATLAB. Free vibration experiments of Normal and stiffened cantilever beams were performed with help of FFT. The obtained results will be then compared to validate the proposed models and to study the effect of various stiffener configurations.



3.2 Literature Review

In the introduction to the current research, a literature survey is conducted and results obtained in previous studies on vibration of cantilever beams are discussed. Analytical results for both free vibration and natural frequency of uniform cantilever beams using both modal approach and the finite element method are discussed in detail. Very less study related to the effect of stiffeners has been conducted to investigate the effects of various stiffener patterns in the literature. The literature survey made in the introduction helps define the problem definition and guides the researcher to identify the research gaps.

3.3 Identifying the gaps & Concept Development

This report begins with a literature review followed by an identification of the research gaps and the generation of the concept for the new research works. Simple cantilever beams have been studied previously with respect to their vibration characteristics however there is a very low study related to the study of effect of stiffeners, as well as their configuration and pattern. This leads to the main research question of the work, how do different stiffener patterns affect the vibrational characteristics of a simple cantilever beam. So, an experimental and numerical study has to be developed and implemented for the investigation about the stiffeners effect on the free vibration characteristics of the cantilever beam. This is carried out using the ANSYS and MATLAB software packages, along with experimental testing.

3.4 Experimental Design

After defining the research gap, an experimental design is needed for defining the parameters for the investigation. Choosing the right materials, lengths of the specimens, their orientations, thicknesses, and the pattern of stiffeners which affects the natural frequency are some of the key design parameters. All the specimens should have same dimension to allow variation in design of stiffeners only. We also need to know how to design our test setup for the measurement and how to simulate that in our finite element model using MATLAB and ANSYS. The section is divided into three phases which run in parallel. The ANSYS phase, MATLAB Phase, and the Experimental phase cross check the values for validating the results obtained from the study.

3.4.1 ANSYS Phase

In the ANSYS phase finite element models for the cantilever beams were created with stiffeners included as well as without. The models were designed in the ANSYS Space claim and mesh generation was developed for each case after which the experimental boundary conditions were applied during the modal analysis. The behavior of the beam was simulated and modal analysis was performed to determine the natural frequencies and modes for comparison with experimental results. The material properties in the Material sections were based on data obtained during the material testing such that the models accurately captured the real specimen behavior.

Detailed attention was given to the development of the stiffener patterns as slight variations in the stiffener location and position can alter the natural frequencies of the beam as the dimensions, angle and position of the stiffeners have a large effect on the stiffness of a beam and its behaviors in natural frequency.

3.4.2 MATLAB Phase

During this phase finite element models of the cantilever beams studied in the experimental parts and analyzed using ANSYS are developed. The beam models are defined with exact dimensions, boundary conditions, and material properties of the experimental specimen, with both stiffened and unstiffened configurations included. The moment of inertia for each case was calculated for each case and also the centroid of each structure is calculated using neutral axis theorem which helps to predict the centroid of the overall structure using parallel axis theorem and after running the sets of codes, the mode shapes of the beam will be obtained using eigen vector. The results so obtained using the developed MATLAB program are then compared with results from ANSYS and experimental results.

3.4.3 Experimental phase

The cantilever beam specimens were fabricated using the CNC process as per their precise dimensions and configurations .The three specimens of plain cantilever beam was fabricated and one midline stiffener of width 3mm was fabricated using CNC process for the accuracy , and two additional stiffener for the diagonal stiffener was also fabricated using the CNC machine , so that the consistency remains same throughout the study , after which the stiffeners were welded for the midline and diagonal case .Similarly the tensile specimens were fabricated of the same material by forming the dog bone shape to determine the Young's modulus of the test specimens which will be used in ANSYS simulation and MATLAB models .

After the material testing, calibration of experimental setup has to be done for proper reading with calibrator before conducting the experimental test. The accelerometer was mounted at the calibrator and the circuit diagram for the calibration was designed using LabVIEW, which shows the acceleration response. Before conducting the modal test, the experimental setup will be calibrated using the calibrator to ensure accurate data acquisition. Once secured, the beams will be fixed at one end using C clamp.

The testing of the experimental beam specimen will be conducted through using an equipment known as impact hammer to create an excitation to the beam while the accelerometer measures the responses. The dynamic excitation is supplied to the beam through an impact hammer, while the accelerometer captures the signals or voltages that have been created from the excitation at regular intervals as per the sampling rate. The test points will be selected equally spaced at four points on the beam and data capturing is to be continued till all parts of the cantilever beams have been tested; the same will be repeated several times and averages data will be considered. The data captured in the time domain was transformed to frequency domain using the technique called Fast Fourier Transform (FFT). The frequency response plot will be examined in order to get the dominant peaks, which represent the natural frequencies. The natural frequencies from different modes were calculated and compared with the ANSYS and MATLAB simulation results. In this respect, after gathering all the information from both numerical and experimental analysis, validation will be conducted.

3.5 Quantify Frequency Errors

After obtaining all comparisons among the sets of data of MATLAB, ANSYS, and experimental results, we have to calculate the error analysis in the form of percentage error, which represents the deviation of numerical readings from experimental readings. For each type of vibration, it must be calculated in terms of frequency error of the first three transverse natural frequency. Frequency error calculation will be used to calculate the deviation of the natural frequency in relation to the numerical solution. Besides, through this analysis, we may understand the sensitivity of the cantilever beam due to the existence of stiffeners and their configuration. Likewise, this error analysis will help us understand why the higher mode's natural frequency of the cantilever beam decreases due to neglect of accelerometer sensor mass, damping, and other assumptions like ideal boundary conditions and stiffener configurations.

Once the error percentage calculations have been done and the error percentage lies within the permissible range, then the numerical model can be considered to be valid and can be used to study the dynamic responses of the cantilever beams which can then further be used in the designing of the new systems taking into account the error percentage and deviation from the practical results. This can help in further optimization of design as well as in the study of various types of stiffener arrangements.

3.6 Data Analysis and Result Formation

The numerical models and the experimental results when show the closeness, the data analysis phase will be started. This process involves comparing the data's obtained from the ANSYS, MATLAB and experimental findings for the adopted configurations such as normal cantilever beam without any stiffener, the one with a midline stiffener and the last one with the diagonal stiffener. The change in the frequency pattern across various modes and patterns are examined, in this stage and how much is the change in natural frequency of the beam when different configurations are considered which means that how much the frequency changes from each other by the study called frequency ratio analysis and also includes the study for the better configurations among the selected specimens.

After analyzing the frequency ratio, we may be able to compare these results for the selected designs through charts and graphs, which will give us an understanding regarding the differences between the natural transverse frequencies, and thus help us choose the design that has the greatest improvement in vibrations.

CHAPTER 4 DESIGN AND FABRICATION

4.1 FEM Approach

The cantilever beam can be studied using Finite Element Method (FEM) using Euler Bernoulli beam theory which is an engineering method to calculate the load carrying capacity and study of the deflection of the beams which assumes the cross-section plane and perpendicular to neutral axis and ignore the shear deformation, which is an idealized case for the cantilever beam. This method discretizes the beam into smaller elements and forms a element and mass matrix of each element and when assembled together forms a global stiffness and consistent mass matrices and when solved together generates a transverse natural frequency(Augarde, 1998).The governing equation so called strong form of the Euler Bernoulli equation is given as:

4.1.1 Governing Equation/Strong Form

The transverse free vibration of a uniform Euler--Bernoulli beam is governed by

$$EI\frac{d^4w(x,t)}{dx^4} + \rho A\frac{d^2w(x,t)}{dt^2} = p(x,t) \tag{1}$$

where,

E : Young's modulus of Elasticity

I : Second moment of area

ρ : Density

A : Cross-sectional area

$w(x,t)$: Transverse displacement

$p(x,t)$: Transverse Load

For static stiffness, the inertia term is ignored and the equation transforms to the following equations

$$EI\frac{d^4w(x)}{dx^4} = p(x) \tag{2}$$

4.1.2 Weak Formulation

The weak formulation of the above equations can be obtained by multiplying by a weight function $v(x)$ and integrating over the entire element domain $[0, L]$

$$\int_0^L v\,(EIw'''' + \rho A\ddot{w} - p)dx = 0 \tag{3}$$

Integrating the bending term by parts twice:

$$\int_0^L v\ EI\, w''''dx = [EIvw''']_0^L - \int_0^L EI\, v'w'''dx \tag{4}$$

$$= [EIvw''']_0^L - [EIv'w'']_0^L + \int_0^L EI\, v''w''dx \tag{5}$$

Hence, the weak form becomes:

$$\int_0^L EI\ v''w''dx + \int_0^L \rho A\, v\ddot{w}dx = \int_0^L v\, pdx + [v'M - vV]_0^L \tag{6}$$

where $\mathrm{M} = \mathrm{EI\, w''}$ is the bending moment and $\mathrm{V} = \mathrm{EI\, w'''}$ is the shear force.

4.1.3 Finite Element Approximation

For a 2-node beam element it has 4 degrees of freedom, here w represent the transverse displacement and θ is the rotation of the element

$$q_e = [w_1 \quad \theta_1 \quad w_2 \quad \theta_2\ ]^T \tag{7}$$

The transverse displacement is approximated using cubic Hermite shape functions as

$$w(x,t) = \sum_{i=1}^{4} N_i(x)q_i = [N]q_e \tag{8}$$

The Hermite shape functions are calculated as

$$N_1 = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3$$

$$N_2 = x - 2\frac{x^2}{L} + \frac{x^3}{L^2}$$

$$N_3 = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3$$

$$N_4 = -\frac{x^2}{L} + \frac{x^3}{L^2} \tag{9}$$

4.1.4 Second Derivatives

The curvature $w''(x)$ is

$$\mathrm{w''(x) = [B]q_e, \quad [B] = [N_1{}'' \quad N_2{}'' \quad N_3{}'' \quad N_4{}''\ ]} \tag{10}$$

$$\mathrm{N_1'' = -\frac{6}{L^2} + \frac{12x}{L^3}}$$

$$N_2'' = -\frac{4}{L} + \frac{6x}{L^2}$$

$$N_3'' = \frac{6}{L^2} - \frac{12x}{L^3}$$

$$N_4'' = -\frac{2}{L} + \frac{6x}{L^2} \tag{11}$$

4.1.5 Element Stiffness Matrix

The element stiffness matrix is given as

$$[K_e] = \int_0^L EI\,[B]^T[B]dx \tag{12}$$

4.1.5.1 Integration of Stiffness Terms

$$k_{11} = \int_0^L EI\,(N_1'')^2 dx = EI \int_0^L \left(-\frac{6}{L^2} + \frac{12x}{L^3}\right)^2 dx$$

$$= EI \int_0^L \left(\frac{36}{L^4} - \frac{144x}{L^5} + \frac{144x^2}{L^6}\right) dx$$

$$= EI \left[\frac{36}{L^4} L - \frac{144}{L^5}\frac{L^2}{2} + \frac{144}{L^5}\frac{L^3}{3}\right] = \frac{12EI}{L^3}$$

$$k_{12} = \int_0^L EI\,(N_1'' N_2'')dx = EI\frac{6EI}{L^2}$$

$$k_{13} = \int_0^L EI\,(N_1'' N_3'')dx = -\frac{12EI}{L^3}$$

$$k_{14} = \int_0^L EI\,(N_1'' N_4'')dx = \frac{6EI}{L^2}$$

$$k_{22} = \int_0^L EI\,(N_2'')^2 dx = \frac{4EI}{L}$$

$$k_{23} = \int_0^L EI\,(N_2'' N_3'')dx = -\frac{6EI}{L^2}$$

$$k_{24} = \int_0^L EI\,(N_2'' N_4'')dx = \frac{2EI}{L}$$

By symmetry

$k_{ij} = k_{ji}$,$k_{33} = k_{11}$, $k_{44} = k_{22}$,$k_{34} = k_{43} = -k_{12}$……..

Thus, the final stiffness matrix:

$$K_e = \int_0^{L_e} EI\, B^T B dx = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \tag{13}$$

4.1.6 Consistent Mass Matrix

The element consistent mass matrix is given as

$$[M_e] = \int_0^{L} \rho A\, [N]^T [N] dx \tag{14}$$

Using normalized coordinate $\xi = x/L$, $dx = L\, d\xi$, the shape functions become:

$$\begin{gathered} N_1 = 1 - 3\xi^2 + 2\xi^3 \\ N_2 = L(\xi - 2\xi^2 + \xi^3) \\ N_3 = 3\xi^2 - 2\xi^3 \\ N_4 = L(-\xi^2 + \xi^3) \end{gathered} \tag{15}$$

4.1.7 Integration of Mass Terms

$$m_{11} = \rho AL \int_0^1 N_1^2 d\xi = \rho AL \int_0^1 (1 - 3\xi^2 + 2\xi^3)^2 d\xi$$

$$= \rho AL \int_0^1 (1 - 6\xi^2 + 4\xi^3 + 9\xi^4 - 12\xi^5 + 4\xi^6) d$$

$$= \rho AL \left[1 - 2 + 1 + \frac{9}{5} - 2 + \frac{4}{7}\right] = \frac{13}{35}\rho AL$$

$$m_{12} = \rho AL \int_0^1 N_1 N_2 d\xi = \rho AL^2 \int_0^1 (1 - 3\xi^2 + 2\xi^3)(\xi - 2\xi^2 + \xi^3) d\xi = \rho AL^2 \frac{11}{210}$$

$$m_{13} = \rho AL \int_0^1 N_1 N_3 d\xi = \rho AL \frac{9}{70}$$

$$m_{14} = \rho AL^2 \int_0^1 N_1 N_4 d\xi = -\rho AL^2 \frac{13}{420}$$

Continuing similarly for all entries, the consistent mass matrix is

$$M_e = \int_0^{L_e} \rho A\, N^T N dx = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{16}$$

4.1.8 Assembly and Boundary Conditions

Assemble global stiffness and mass matrices for n elements

$$K = \sum_{e=1}^{n} K_e \,, \quad M = \sum_{e=1}^{n} M_e \tag{13}$$

Apply cantilever Boundary Condition by removing DOFs at the fixed end we get Eigen values as,

4.1.9 Eigen value /Free Vibration

$$(K - \omega^2 M)\phi = 0 \tag{14}$$

4.1.10 Natural Frequencies

$$f = \frac{\omega}{2\pi} \tag{15}$$

Mode shapes are given by the eigen vectors ϕ.

4.2 Design Considerations

4.2.1 Material Testing

The testing of the mild steel specimen was conducted at the Central Material Testing Laboratory, Pulchowk, Lalitpur. The thickness of the test specimen was measured at three different locations as 2.779 mm, 2.795 mm, and 2.784 mm, while the width of the reduced section in the dog bone was recorded as 34.99 mm, 34.87 mm, and 35.02 mm at three different locations. As per these measurements, the average thickness and width were found to be 2.78 mm and 34.96 mm. The tensile test specimen prepared had an initial gauge length of 55 mm. The yield load and ultimate load were found to be 3300 kg and 3900 kg after material testing. After fracture of the specimen after crossing ultimate load the final gauge length was measured to be 76 mm. The three rectangular specimens of dimensions 450mm×60mm×2.7888mm were considered for density evaluation. The volume of each specimen was calculated as:

$$V = 450mmX60mmX2.788mm = 75{,}297.6mm^3 = 7.52976x10^{-5}m^3$$

The average mass of the specimen was measured to be 0.53212 kg. Therefore, the density of the material was calculated using:

$$\rho = \frac{m}{v} = \frac{0.53212}{7.52976x10^{-5}} = 7{,}067.00Kg/m^3$$

The thickness of the beam at three sections: 2.779mm, 2.795mm and 2.784mm

The width of the beam at three sections of dog bone: 34.99mm, 34.87mm and 35.02mm

Gauge Length: L_0 =5.5cm =55mm

Yield Load: P_y=3300 Kg

Ultimate Load: P_U=3900Kg

Final Gauge Length:$L_f = 7.6$ cm $= 76$mm

Young's Modulus $E = 196.00$ GPa

The average thickness of the tensile specimen was calculated as

$$t_{avg} = \frac{2.779 + 2.795 + 2.784}{3} = 2.786 \text{ mm}$$

And the average width of the tensile specimen was found to be

$$b_{avg} = \frac{34.99 + 34.87 + 35.02}{3} = 34.96 \text{ mm}$$

Similarly, the total cross-sectional area of the specimen was calculated as

$$A_0 = 2.7917x34.8633 = 97.33mm^2$$

And the yield Load during the testing was obtained as:

$$P_y = 3300 \times 9.80665 = 32{,}361.95 \text{ N}$$

And the Ultimate Load during the testing was obtained as:

$$P_u = 3900 \times 9.80665 = 38{,}245.94 \text{ N}$$

And the Yield Strength was calculated as,

$$\sigma_y = \frac{P_y}{A_0} = \frac{32361.95}{97.33} = 332.26\ MPa$$

And the Ultimate Tensile Strength was calculated as,

$$\sigma_u = \frac{P_u}{A_0} = \frac{38245.94}{97.33} = 392.67\ MPa$$

The Strain during the fracture was calculated as,

$$\varepsilon_f = \frac{L_f - L_0}{L_0}$$

$$\varepsilon_f = \frac{76 - 55}{55} = 0.3818$$

The Percentage Elongation of the specimen was calculated as

$$\% \text{ Elongation} = \varepsilon_f\ x\ 100$$

$$\% \text{ Elongation} = 0.3818\ x\ 100$$

$$= 38.18\%$$

4.2.2 Normal Beam without Stiffeners

The normal cantilever beam without stiffeners, with a Young's modulus of 196 GPa and a material density of 7,067 kg/m³. The beam has a total length of 450 mm, a width of 60 mm, and a thickness of 2.7888 mm. It is fixed at one end at x = 0 mm and constrained up to x = 50 mm, making an effective free length of 400 mm. Based on these dimensions, the below are the properties of the Normal Cantilever Beams without Stiffeners.

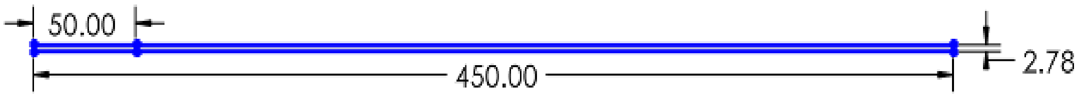


Figure 4.1: Dimensions of normal cantilever beam from front view

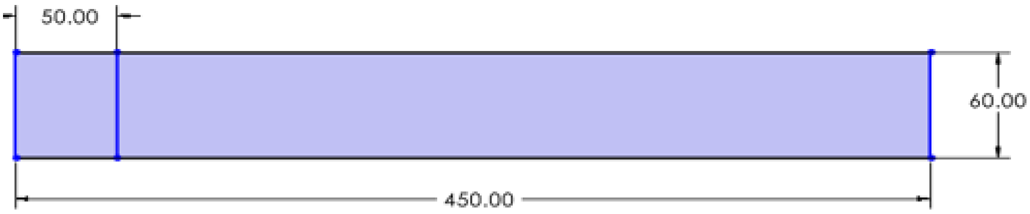


Figure 4.2: Dimensions of normal cantilever beam from top view

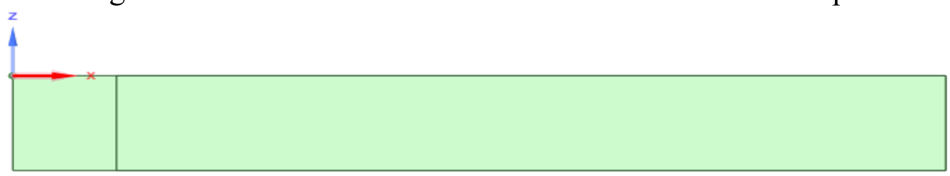


Figure 4.3: Normal cantilever beam from top view in spaceclaim

Material Properties of the beam

Youngs Modulus of Elasticity (E) =196GPa

Density of Material (ρ) = 7,067.00 Kg/m^3

Total Length (L_t) = 450mm

Width of the beam (W) =60mm

Thickness of the beam (t) =2.7888mm

Boundary Conditions

At $x = 0\ mm\ Fixed\ End$

At $x = 50\ mm\ Fixed$

Effective Length (L_e) = 400mm

Effective Volume (V) =8.1x 10^{-5} m^3

4.2.3 Cantilever Beam with Stiffener along midline at bottom

The cantilever beam with midline stiffeners, with a Young's modulus of 196 GPa and a material density of 7,067 kg/m³. The beam has a total length of 450 mm, a width of 60 mm, and a thickness of 2.7888 mm and a midline stiffener of 3mm width at the midline starting from the length of 50mm from the fixed end. The beam is fixed at one end x = 0 mm and constrained up to x = 50 mm, resulting in an effective free length of 400 mm. Based on these dimensions, the below are the properties of the Cantilever Beams with Midline Stiffeners.

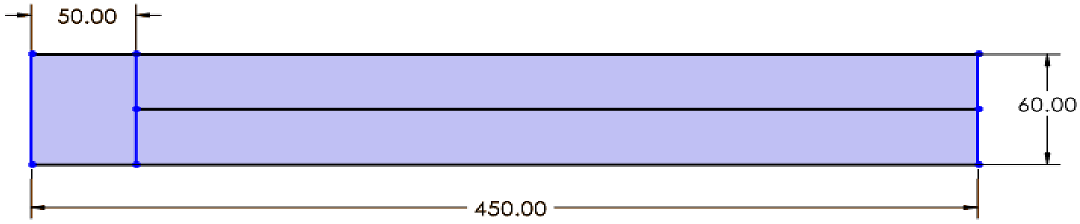


Figure 4.4: Beam with stiffener along midline at Bottom

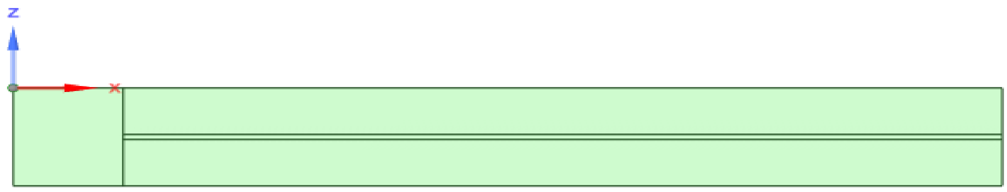


Figure 4.5: Beam with stiffener along midline modelled in ANSYS

Youngs Modulus of Elasticity (E) =196GPa

Density of Material (ρ) = 7,067.00 Kg/m^3

Dimensions of Beam

Total Length (L_t)= 450mm

Width of the beam (W) =60mm

Thickness of the beam (t) =2.7888mm

Boundary Conditions

At $x = 0\ mm\ Fixed\ End$

At $x = 50\ mm\ Fixed$

Effective Length (L_e) = 400mm

Length of Stiffeners along midline at the bottom (L_{sb}) =400mm

Width of Stiffeners (W_s) =3mm

Thickness of Stiffeners (T_s) =2.7888mm

Effective Volume (V) =8.46x 10^{-5} m^3

4.2.4 Cantilever Beam with Stiffener along diagonal at bottom

The cantilever beam with midline stiffeners, with a Young's modulus of 196 GPa and a material density of 7,067 kg/m³. The beam has a total length of 450 mm, a width of 60 mm, and a thickness of 2.7888 mm and a midline stiffener of 3mm width at the midline starting from the length of 50mm from the fixed end. The beam is fixed at one end x = 0 mm and constrained up to x = 50 mm, with an effective free length of 400 mm. Based on these dimensions, the below are the properties of the Cantilever Beams with Diagonal Stiffeners.

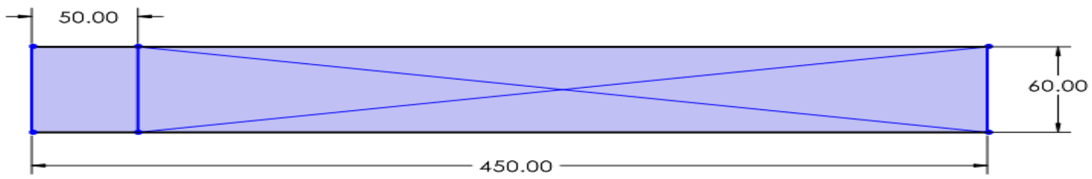


Figure 4.6: Beam with diagonal stiffener at the bottom of the beam

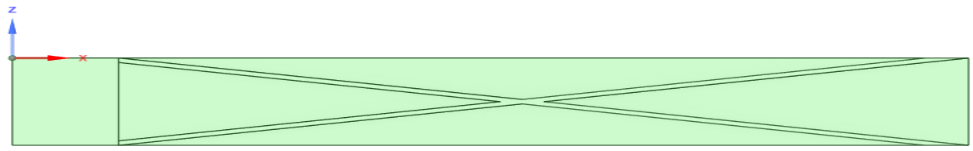


Figure 4.7: Beam with diagonal stiffener modelled in ANSYS

Youngs Modulus of Elasticity (E) =196GPa

Density of Material (ρ) = 7,067.00 Kg/m^3

Dimensions of Beam

Total Length (L_t)= 450mm

Width of the beam (W) =60mm

Thickness of the beam (t) =2.7888mm

Boundary Conditions

At $x = 0\ mm\ Fixed\ End$

At $x = 50\ mm\ Fixed$

Effective Length (L_e) = 400mm

Length of Stiffeners along the diagonal at the bottom (L_{sd}) =404.47mm

Width of Stiffeners (W_s) =3mm

Thickness of Stiffeners (T_s) =2.7888mm

Volume (V) =8.80x 10^{-5} m^3

4.3 Experimental Setups

Fixturing setup for the experiment includes specimen fixed at one side. Accelerometer model 4371 DeltaShear Unigain by Brüel & Kjær were used in this experiment. The accelerometer works on the principle of piezoelectric transducer. The accelerometer has a weight of 11 gm and frequency range is 0.1 Hz to 12,600.00 Hz. It forms a closed circuit with charge amplifier, which was produced by the Brüel & Kjær of model number 2635. Charge amplifier is a four-stage amplifier and needs an external power supply. The charge amplifier requires 4.5V for functioning and draws a current of 0.5 Amp

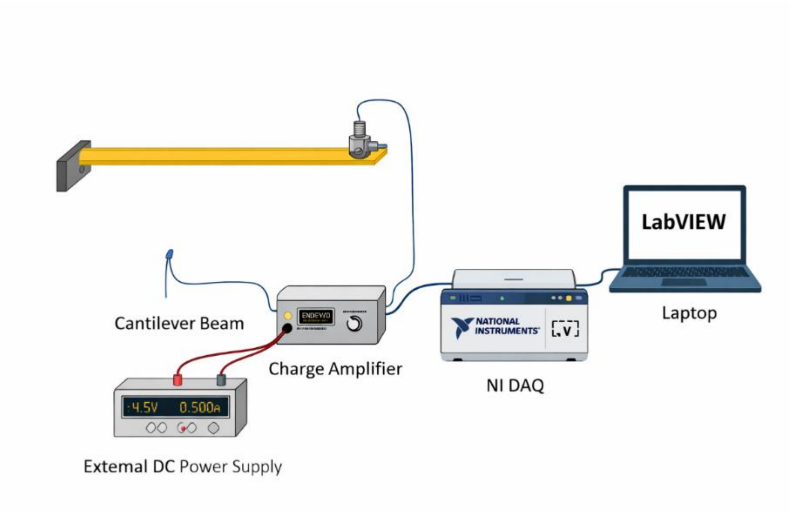


Figure 4.8: Schematic drawing of experimental setup

The use of Brüel & Kjær Type 2635 charge amplifier is aimed at converting the electrical charges generated by an accelerometer into raw voltage. The accelerometer generates millivolts, but with a high charge output impedance that cannot be read by the DAQ. The charge amplifier converts the charge into a voltage, which will be stable and with low impedance. The Type 2635 charge amplifier is used for signal conditioning through amplification, filtration, and noise reduction among others, and allows adjusting the sensitivity and gains. The voltage output is then passed on to the NImyDAQ device through LabVIEW software.

The NI myDAQ, is a portable hand-held device that is used to convert the analog signals to the digital. The output voltage from the charge amplifier is analog signal and is digitized to digital signal using LabVIEW. The digital signal so obtained from the LabVIEW, is transformed into frequency domain using Fast Fourier Transform and the results obtained with the peaks resembles the natural frequency and resonance frequency which can be used to study the dynamic characteristic of the structure.

Experimental Procedures

1. The Cantilever beam specimens were made using CNC Machine for normal beam, midline stiffener, and diagonal stiffener.
2. The beam was fixed rigidly up to 50mm by using C-Clamp to satisfy cantilever condition.
3. The accelerometer sensitivity was calibrated from 1.004 $pC/m/s^2$ to 0.974 $pC/m/s^2$ by using calibrator and calibrator circuit diagram.
4. After calibration, an accelerometer was installed at a point to obtain the maximum vibration response from it.
5. Charge amplifier was supplied by 4.5 V DC power supply.
6. Output of the accelerometer was sent to the input of the Brüel & Kjær Type 2635 charge amplifier.
7. Amplifier output was interfaced to NI myDAQ system.
8. Data acquisition was done using LabVIEW with a sampling rate of 2000 Hz and 25,000 samples using Nyquist Theory of Sampling.
9. The beam was excited by using an impact hammer and its vibration response was recorded, and location of the accelerometer was changed for four equal spacing locations.
10. Acceleration versus time data acquired from my DAQ was saved in excel through Write to Measurement File in LabVIEW.
11. First three transverse natural frequencies were obtained from FFT analysis of acquired data using MATLAB.

4.4 Circuit Diagram for the Data Acquisition and Calibration

The Figure 4.10 shows the calibration circuit of the accelerometer. The output from the DAQ reads the raw voltage signal and is converted into acceleration using accelerometer sensitivity factor. When the calibrator acceleration and the output acceleration from DAQ corresponds with each other then it is ready for the reading of the experimental setup and the Figure 4.9 shows the circuit diagram for the data acquisition, which is used to receive the data from the DAQ ,when calibrated using the sensitivity factor, it passes through the Dynamic data attributes circuit and the output from the dynamic data attributes is then passes to the Write to measurement file ,where it is saved with a well-defined filename and allows saving and can be used for the post-processing for the FFT to generate the natural frequency peaks.

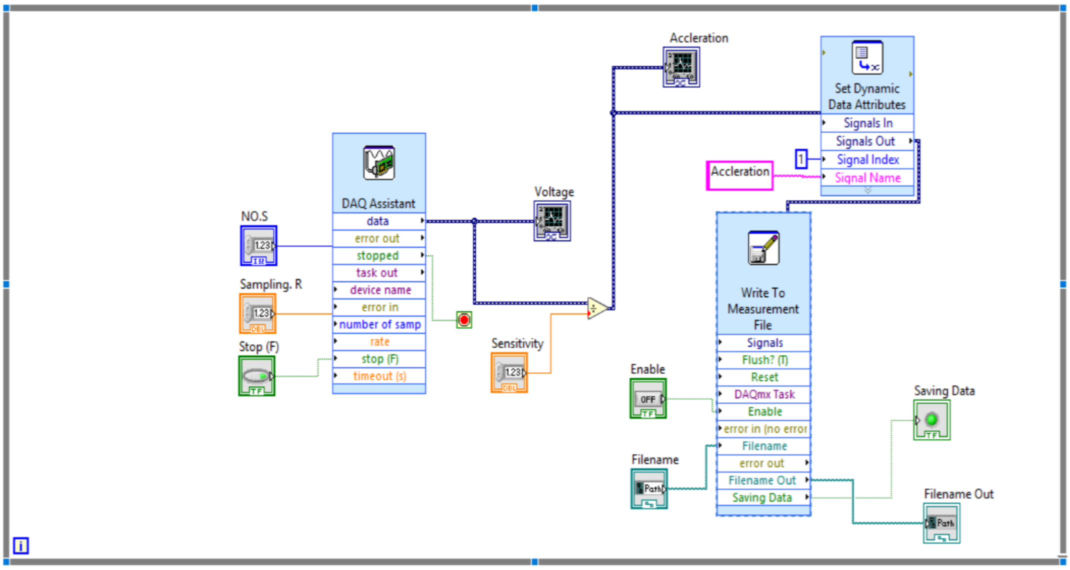


Figure 4.9: Data acquisition block diagram of LabVIEW

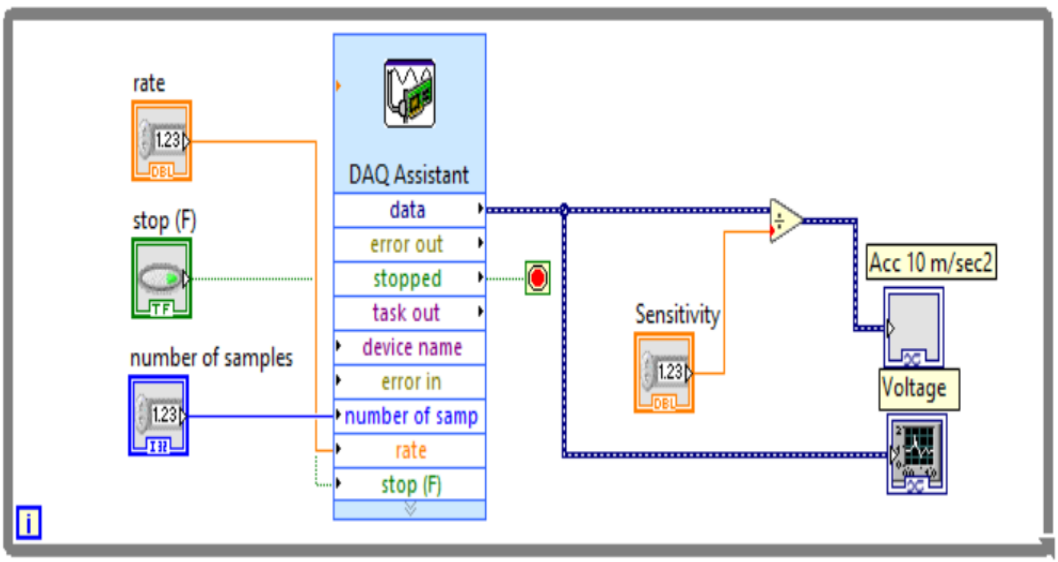


Figure 4.10: Calibration diagram of LabVIEW for accelerometer calibration

CHAPTER 5 RESULTS AND DISCUSSION

5.1 Cantilever beam without stiffener

5.1.1 ANSYS Validation of cantilever beam without stiffener

The modal analysis of the normal cantilever beam without stiffener was performed using ANSYS to determine its natural frequencies and corresponding mode shapes. The Figure 5.1 shows the first mode shape of the beam with a natural frequency of 13.75 Hz. The first mode exhibits a characteristic bending pattern with single smooth bending along the beam length. From the figure we can see that there is a maximum displacement at the free end and zero displacement at the fixed end.

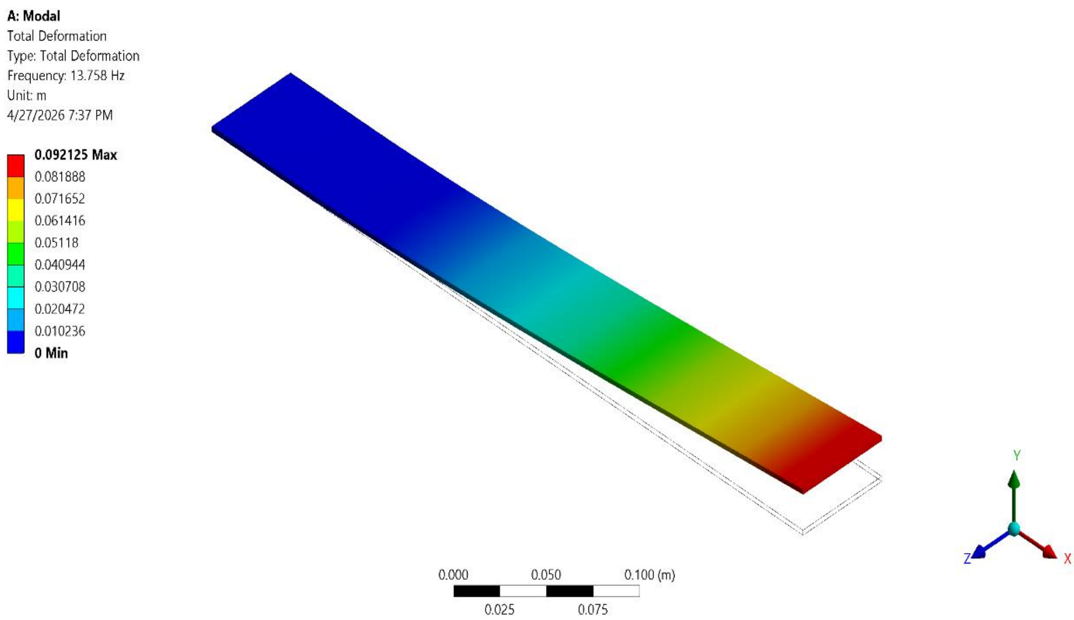


Figure 5.1: First natural frequency of the normal cantilever beam

Similarly, the modal analysis of the normal cantilever beam was performed using ANSYS to determine its transverse second natural frequencies and corresponding mode shapes. The Figure 5.2 shows the second mode shape of the beam with a natural frequency of 86.14 Hz. The second mode exhibits a characteristic bending pattern with one nodal point along the beam length. The mode shapes of the cantilever beam reflect the pattern at which the beam vibrates at the natural frequency. There is a slight increase in the curvature with the presence of nodal points, forming a complex deformation pattern.

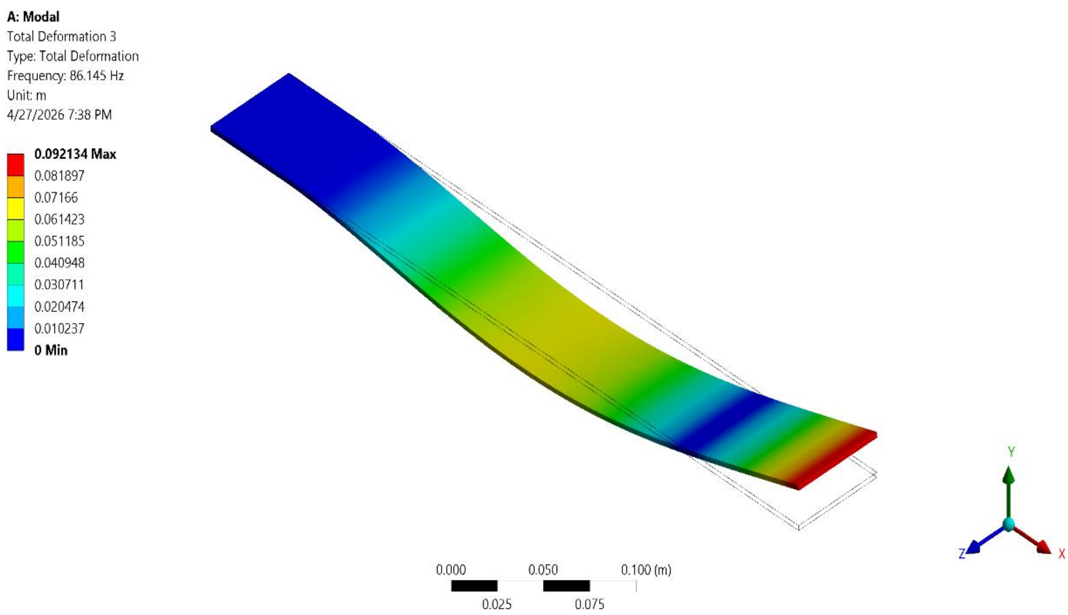


Figure 5.2: Second natural frequency of the normal cantilever beam

The modal analysis of the cantilever beam was performed using ANSYS to determine its natural frequencies and corresponding mode shapes. The Figure 5.3 shows the third mode shape of the beam with a natural frequency of 241.44 Hz. The third mode exhibits a characteristic bending pattern with two nodal point along the beam length and forms more complex deformation pattern.

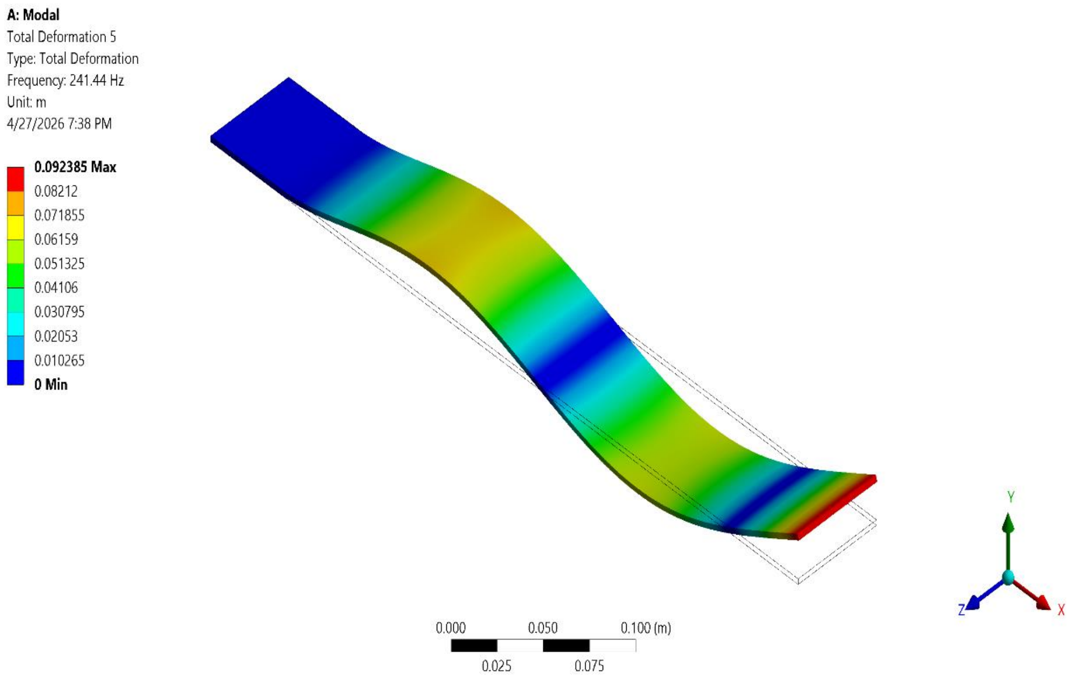


Figure 5.3: Third natural frequency of the normal cantilever beam

5.1.2 MATLAB validation of normal cantilever beam

The ANSYS results from the modal analysis has to be validated using the numerical methods, so a commercial software called MATLAB, was taken into the account and the natural frequencies of the cantilever beam without any stiffener was obtained using finite element formulation based on Euler Bernoulli beam theory. The results obtained after FEM method indicates that the first three transverse natural frequency of the normal cantilever beam were 13.64Hz for the first mode ,85.51Hz for the second mode and 239.45Hz for the third mode. Figure 5.4 shows the MATLAB outputs using the finite element formulation showing the three mode shapes. The natural frequency obtained from the finite element formulation shows a good agreement with that of the simulation results. From the readings it was confirmed that the Finite Element Method predicts the bending behavior of the beam also as represented by the figure 5.4. Minor differences between the results may arise due to variations in mesh discretization, numerical approximations, and modeling assumptions used in the two methods.

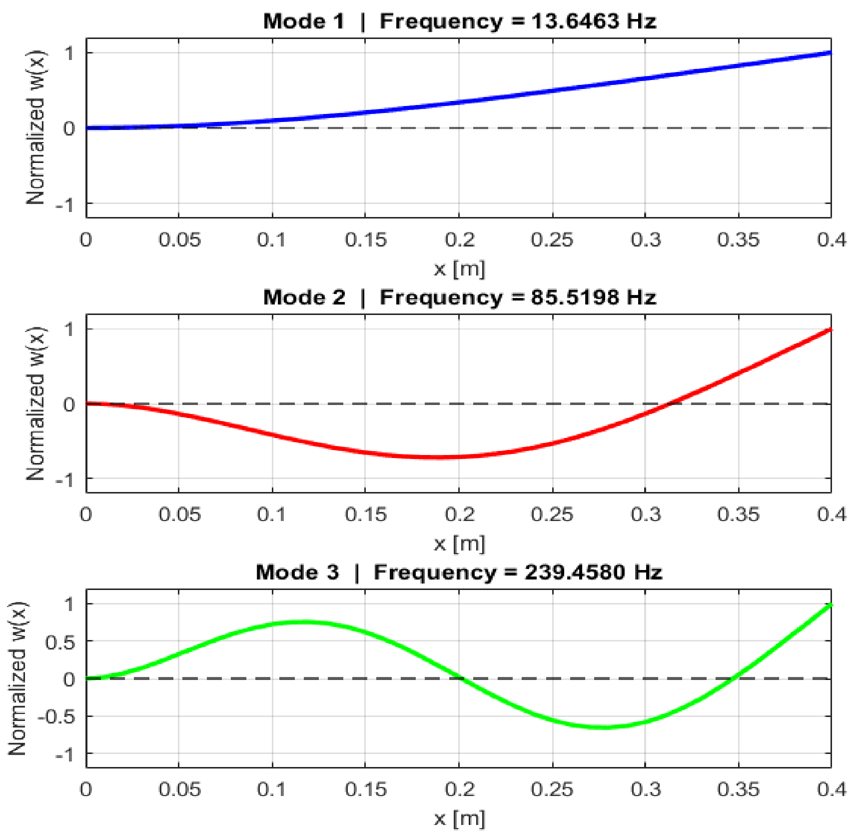


Figure 5.4: Mode shapes of cantilever beam without stiffeners

From the Figure 5.4 , it was found that the natural frequency of the ANSYS for the first mode was 13.75 Hz against a MATLAB Result of 13.64 Hz, which closely matches the ANSYS Result, followed by second natural frequency of 86.14Hz against 85.51 Hz and third natural frequency of 241.44Hz against 239.45 Hz.

5.1.3 Experimental Validation of Cantilever Beam without Stiffeners

The experiment for the normal cantilever beam was performed and the readings were taken at the multiple equally sectioned of the normal cantilever beam. Four readings were taken at each section of the normal cantilever beam to ensure the repeatability and consistency of the readings and again the accelerometer position is changed to another location for better pattern readings of the experimental data. The average data from four different locations were averaged to effectively replicate the peaks for the natural frequency. Only minor variations were observed across the different location as the experimental values slightly depends upon the accelerometer position and placement locations. The collected data from the experiments were then taken into account for the Fast Fourier Transform and the average natural frequency of the four locations were noted as 13.64 Hz for the first mode, similarly for the second mode it was calculated as 79.54 Hz and that for the third mode it was obtained as 187.56 Hz which is represented by the Table 5.1 and the FFT of the experimental data is shown by the Figure 5.5: Natural frequency of normal cantilever beam after FFT and the dominant peaks represents the corresponding natural frequency.

Table 5.1: Results from experiment for cantilever beam without stiffener at four different locations

Location	Mode 1	Average Value	Mode 2	Average Value	Mode 3	Average Value
1	13.29		79.37		187.49	
2	13.86	13.64	80.27	79.54	188.76	187.56
3	13.70		79.24		187.06	
4	13.72		79.27		186.92	

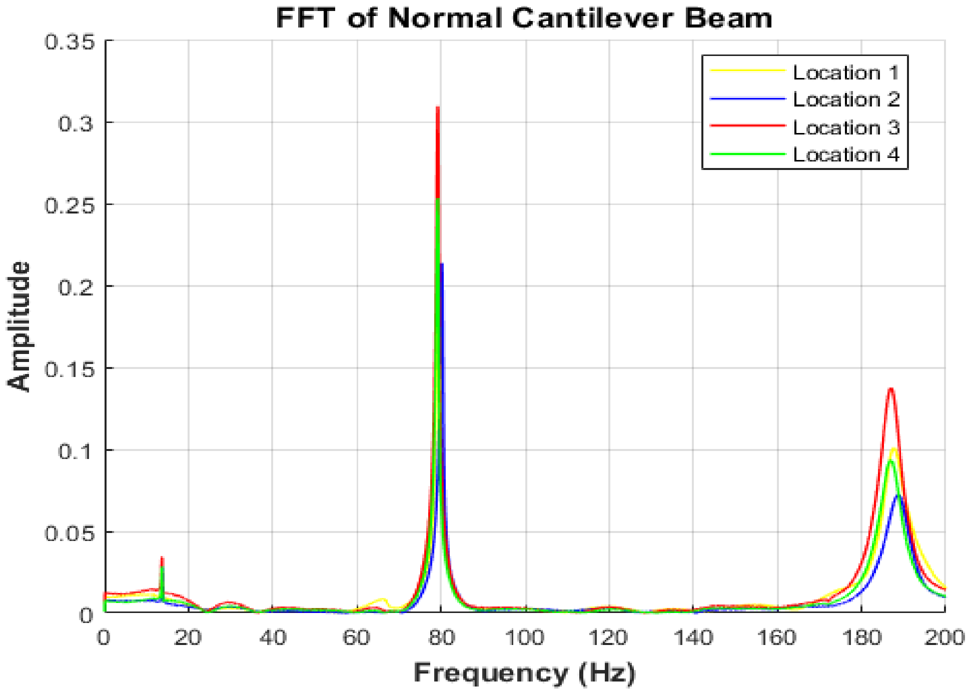


Figure 5.5: Natural frequency of normal cantilever beam after FFT

5.2 Cantilever beam with midline stiffener at the bottom

5.2.1 ANSYS validation of cantilever beam with midline stiffener

The modal analysis of the cantilever beam with midline stiffener was performed using ANSYS to determine its natural frequencies and corresponding mode shapes. The specimen was designed in the space claim, the length of the beam was 450mm, width was 60mm, and a midline stiffener at the bottom of the beam with thickness 2.7888mm and width of 3mm.The material properties as obtained from the Material Testing were used as the material properties in ANSYS. After applying the mesh generation and applying the necessary boundary conditions the first natural frequency of the midline stiffened cantilever beam so obtained was 16.84 Hz. The first mode exhibits a characteristic bending pattern with single smooth bending along the beam length. The Figure 5.6: First natural frequency of the cantilever beam with midline stiffener shows the first mode shape of the beam with a single smooth bending along the beam length. From the figure we can see that there is a maximum displacement at the free end and zero displacement at the fixed end. The modal analysis of the cantilever beam with stiffener along midline at the bottom was performed using ANSYS Mechanical to determine its natural frequencies.

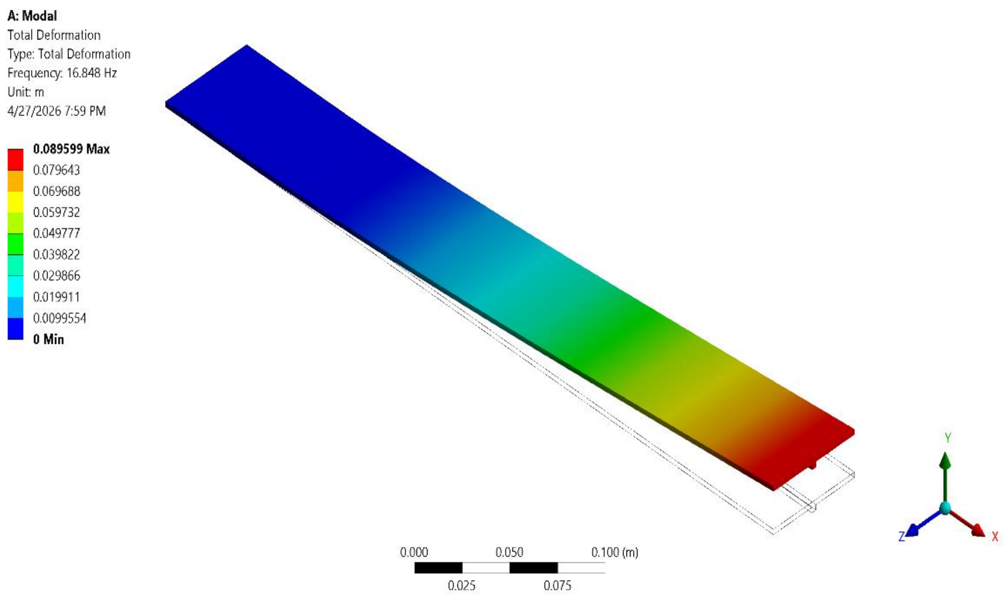


Figure 5.6: First natural frequency of the cantilever beam with midline stiffener

Similarly, the modal analysis of the normal cantilever beam was performed using ANSYS to determine its transverse second natural frequencies and corresponding mode shapes. The Figure 5.7: Second natural frequency of the cantilever beam with midline stiffener shows the second mode shape of the beam with a natural frequency of 105.37 Hz. The second mode exhibits a characteristic bending pattern with one nodal point along the beam length. The mode shapes of the cantilever beam reflect the pattern at which the beam vibrates at the natural frequency. There is a slight increase in the curvature with the presence of nodal points, forming a complex deformation pattern.

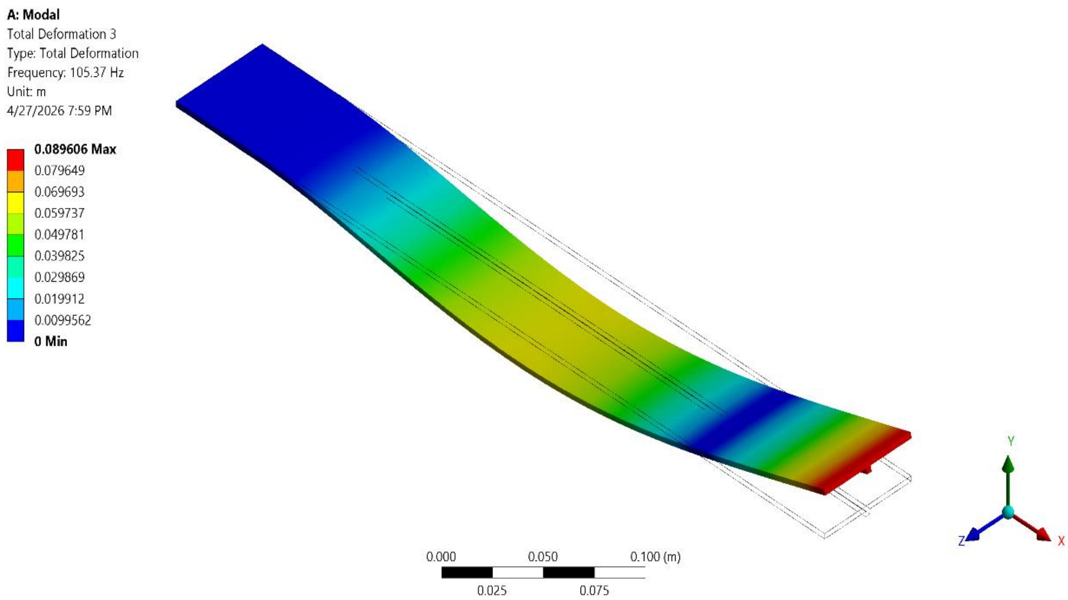


Figure 5.7: Second natural frequency of the cantilever beam with midline stiffener

The modal analysis of the cantilever beam with Stiffener along midline was performed using ANSYS to determine its natural frequencies and corresponding mode shapes. The Figure 5.8 Third natural frequency of the cantilever beam with midline stiffener shows the third mode shape of the Cantilever beam with midline Stiffener with a natural frequency of 294.08 Hz. The third mode exhibits a characteristic bending pattern with two nodal point along the beam length.

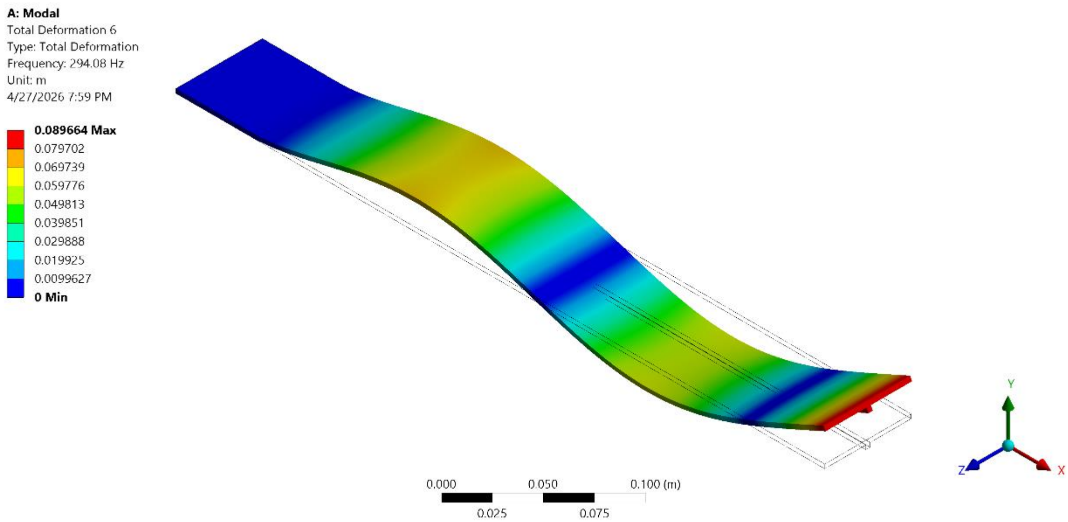


Figure 5.8 Third natural frequency of the cantilever beam with midline stiffener

5.2.2 MATLAB validation of cantilever beam with midline stiffener

The ANSYS results from the modal analysis has to be validated using the numerical methods, so a commercial software called MATLAB, was taken into the account and the natural frequencies of the cantilever beam with midline stiffener was obtained using finite element formulation based on Euler Bernoulli beam theory. The results obtained after FEM method indicates that the first three transverse natural frequency of the normal cantilever beam were 16.93 Hz for the first mode ,106.14 Hz for the second mode and 297.22 Hz for the third mode. Figure 5.10 shows the MATLAB outputs using the finite element formulation showing the three mode shapes. The natural frequency obtained from the finite element formulation shows a good agreement with that of the simulation results. From the readings it was confirmed that the Finite Element Method predicts the bending behavior of the beam also as represented by the Figure 5.9: Mode shape of cantilever beam with midline stiffener. Minor differences between the results may arise due to variations in mesh discretization, numerical approximations, and modeling assumptions used in the two methods.

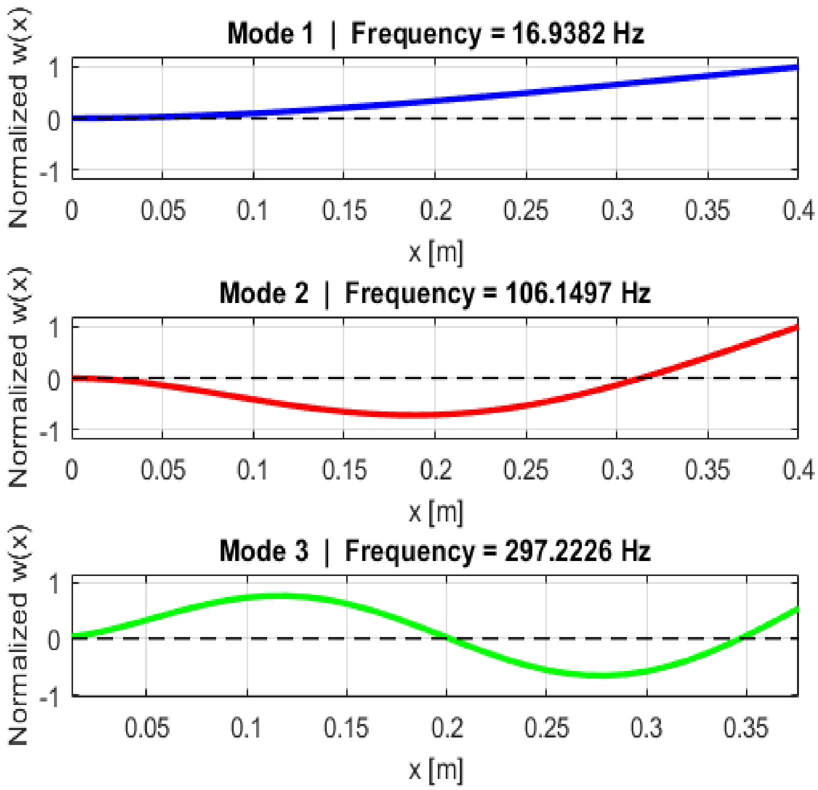


Figure 5.9: Mode shape of cantilever beam with midline stiffener

5.2.3 Experimental validation of cantilever beam with midline stiffener

The experiment for the cantilever beam with midline stiffener was performed and the readings were taken at the multiple equally sectioned of the normal cantilever beam. Four readings were taken at each section of the cantilever beam with midline stiffener to ensure the repeatability and consistency of the readings and again the accelerometer position is changed to another location for better pattern readings of the experimental data. The average data from four different locations were averaged to effectively replicate the peaks for the natural frequency. Only minor variations were observed across the different location as the experimental values slightly depends upon the accelerometer position and placement locations. The collected data from the experiments were then taken into account for the Fast Fourier Transform and the average natural frequency of the four locations were noted as 16.42 Hz for the first mode, similarly for the second mode it was calculated as 82.95 Hz and that for the third mode it was obtained as 236.31 Hz which is represented by the Table 5.2 and the FFT of the experimental data is shown by the Figure 5.10: Natural frequency of cantilever beam with midline stiffener after FFT and the dominant peaks represents the corresponding natural frequency.

Table 5.2: Results from experiment for cantilever beam with midline stiffener at four different locations

Location	Mode 1	Average Value	Mode 2	Average Value	Mode 3	Average Value
1	16.14		83.22		231.93	
2	16.49	16.42	83.54	82.95	237.68	236.31
3	16.51		83.37		237.96	
4	16.55		81.69		237.69	

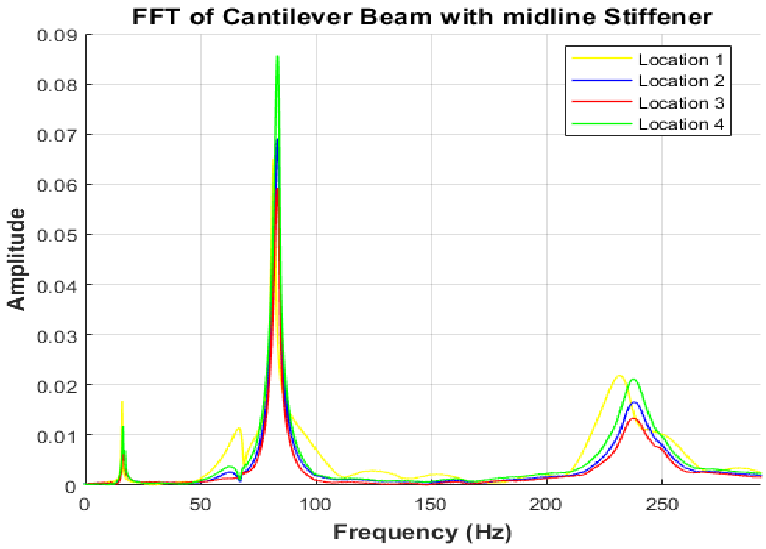


Figure 5.10: Natural frequency of cantilever beam with midline stiffener after FFT

5.3 Cantilever beam with diagonal stiffener at the bottom

5.3.1 ANSYS validation of cantilever beam with diagonal stiffener

The modal analysis of the cantilever beam with diagonal stiffener was performed using ANSYS to determine its natural frequencies and corresponding mode shapes. The specimen was designed in the space claim, the length of the beam was 450mm, width was 60mm, and two stiffeners arranged diagonally at the bottom of the beam with thickness 2.7888mm and width of 3mm. The first natural frequency of the midline stiffened cantilever beam so obtained was 18.68 Hz. The first mode exhibits a characteristic bending pattern with single smooth bending along the beam length. The Figure 5.11: First natural frequency of the cantilever beam with diagonal Stiffener shows the first mode shape of the beam with a single smooth bending along the beam length. From the figure we can see that there is a maximum displacement at the free end and zero displacement at the fixed end. The modal analysis of the cantilever beam with diagonal stiffener at the bottom was performed using ANSYS Mechanical to determine its natural frequencies.

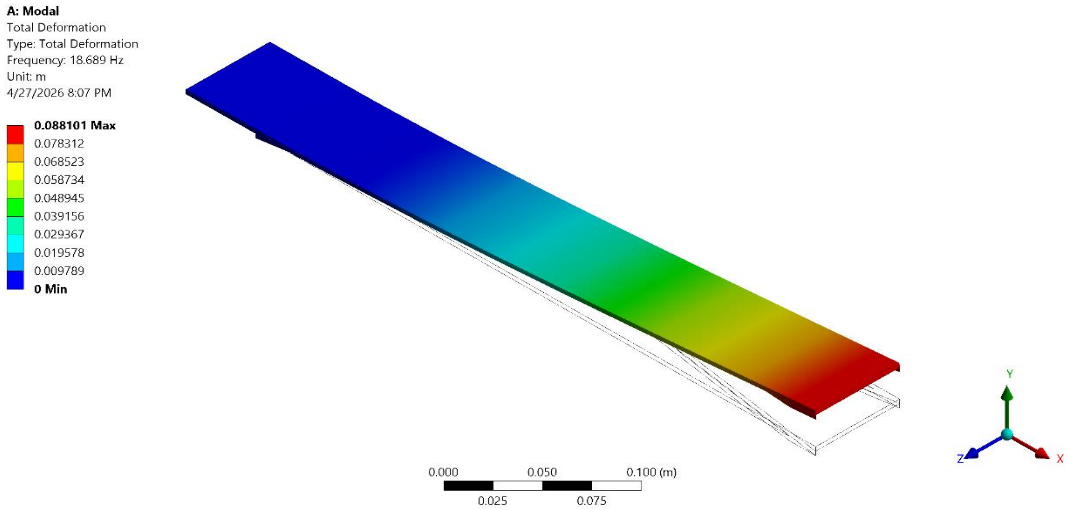


Figure 5.11: First natural frequency of the cantilever beam with diagonal Stiffener

Similarly, the modal analysis of the normal cantilever beam was performed using ANSYS to determine its transverse second natural frequencies and corresponding mode shapes. The Figure 5.12 shows the second mode shape of the beam with a natural frequency of 116.21 Hz. The second mode exhibits a characteristic bending pattern with one nodal point along the beam length. The mode shapes of the cantilever beam reflect the pattern at which the beam vibrates at the natural frequency. There is a slight increase in the curvature with the presence of nodal points, forming a complex deformation pattern.

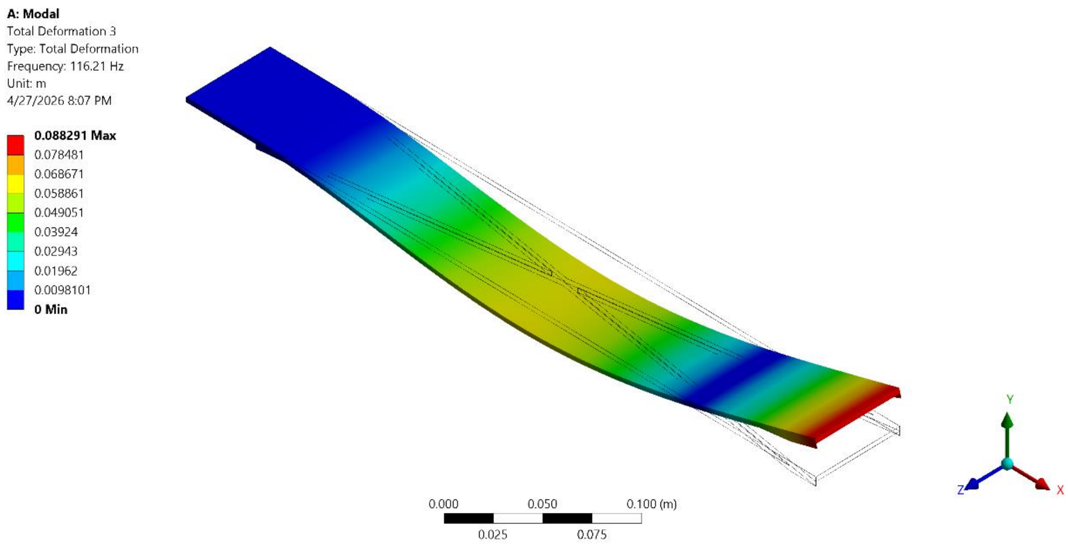


Figure 5.12: Second natural frequency of the cantilever beam with diagonal Stiffener

The modal analysis of the cantilever beam with a diagonal stiffener was performed using ANSYS to investigate its dynamic behavior in terms of natural frequencies and corresponding mode shapes. The Figure 5.13 illustrates the third mode shape of the stiffened beam, which occurs at a natural frequency of 325.63 Hz. In this mode, the beam exhibits a more complex bending pattern characterized by two nodal points along its length, where the transverse displacement becomes zero. Compared to lower modes, the deformation in the third mode is more localized and sensitive to stiffness variations. The addition of the diagonal stiffener significantly influences the stiffness distribution, which causes the increase in natural frequency and a modification of the bending characteristics. This mode provides important information related to the dynamic response and structural stability of the stiffened cantilever beam.

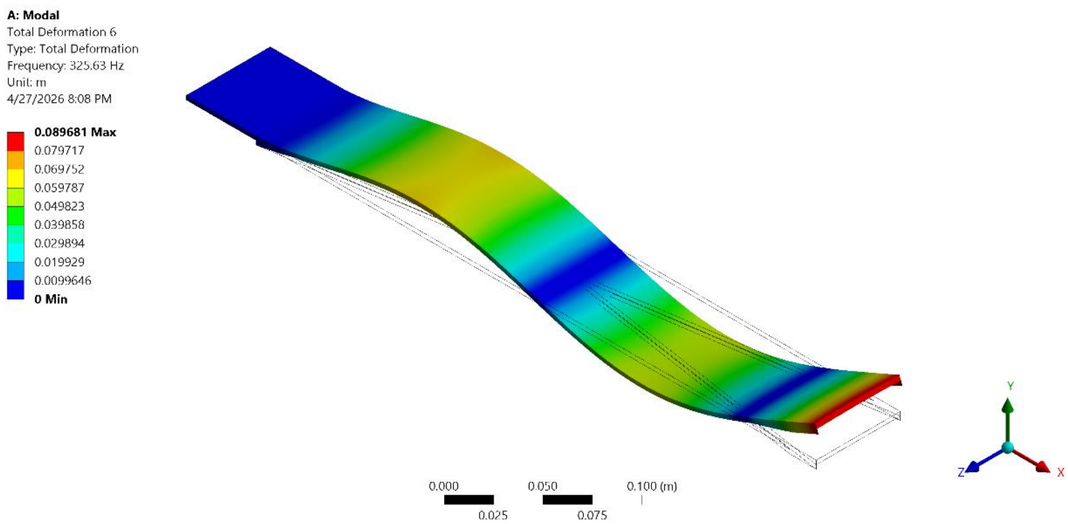


Figure 5.13: Third natural frequency of the beam with diagonal Stiffener

5.3.2 MATLAB Validation of Cantilever Beam with Diagonal Stiffener

The ANSYS results from the modal analysis has to be validated using the numerical methods, so a commercial software called MATLAB, was taken into the account and the natural frequencies of the cantilever beam with midline stiffener was obtained using finite element formulation based on Euler Bernoulli beam theory. The results obtained after FEM method indicates that the first three transverse natural frequency of the cantilever beam with diagonal stiffeners were 19.19 Hz for the first mode ,120.31 Hz for the second mode and 336.87 Hz for the third mode. Figure 5.14shows the MATLAB outputs using the finite element formulation showing the three mode shapes. The natural frequency obtained from the finite element formulation shows a good agreement with that of the simulation results. From the readings it was confirmed that the Finite Element Method predicts the bending behavior of the beam also as represented by the Figure 5.14. Minor differences between the results may arise due to variations in mesh discretization, numerical approximations, and modeling assumptions used in the two methods.

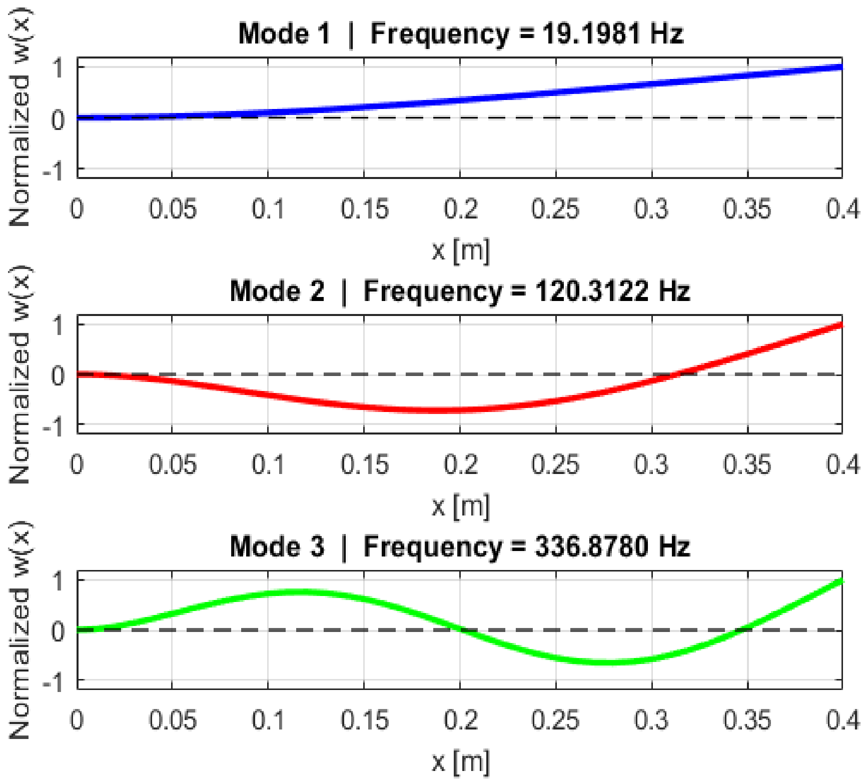


Figure 5.14: Mode shapes of cantilever beam with diagonal stiffener

5.3.3 Experimental validation of cantilever beam with diagonal stiffener

The experiment for the cantilever beam with diagonal stiffener was performed and the readings were taken at the multiple equally sectioned of the normal cantilever beam. Four readings were taken at each section of the cantilever beam with diagonal stiffener to ensure the repeatability and consistency of the readings and again the accelerometer position is changed to another location for better pattern readings of the experimental data. The average data from four different locations were averaged to effectively replicate the peaks for the natural frequency. Only minor variations were observed across the different location as the experimental values slightly depends upon the accelerometer position and placement locations. The collected data from the experiments were then taken into account for the Fast Fourier Transform and the average natural frequency of the four locations were noted as 17.01 Hz for the first mode, similarly for the second mode it was calculated as 97.66 Hz and that for the third mode it was obtained as 274.05 Hz which is represented by the Table 5.3 and the FFT of the experimental data is shown by the Figure 5.15 Natural frequency of cantilever beam with diagonal stiffener after FFT and the dominant peaks represents the corresponding natural frequency.

Table 5.3: Results from experiment for cantilever beam with diagonal stiffener at four different locations

Location	Mode 1	Average Value	Mode 2	Average Value	Mode 3	Average Value
1	17.05		97.93		274.73	
2	16.97	17.01	97.68	97.66	274.30	274.05
3	17.03		97.67		273.88	
4	16.98		97.35		273.29	

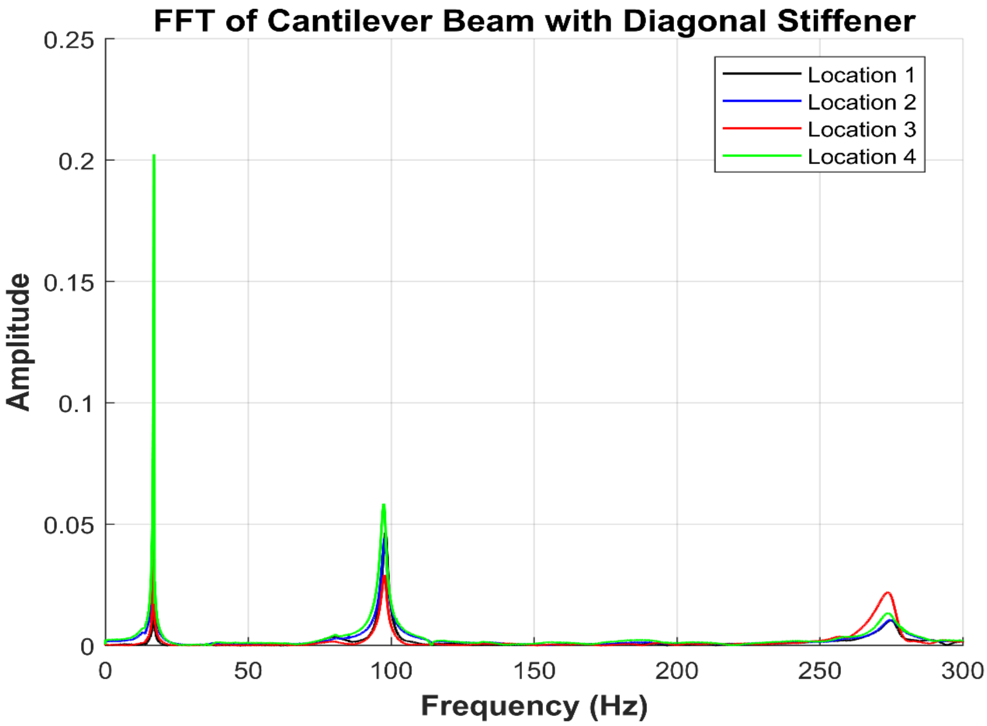


Figure 5.15 Natural frequency of cantilever beam with diagonal stiffener after FFT

The natural frequency between the three conditions of Normal Cantilever beam without stiffeners, a beam with stiffener along the midline and a diagonal stiffener is tabulated below and is for the comparison of free vibration behavior of cantilever beam in three cases.

5.4 Comparison between ANSYS, MATLAB & experimental findings

The comparison between the natural frequency obtained from the numerical, analytical and experimental testing for the normal cantilever beam, beam with midline stiffener and the one with diagonal stiffener configuration can be used for identifying the pattern and trends. The natural frequency of the cantilever increases as the addition of the stiffener patterns reflecting the improvement in the structural stiffness and natural frequency with a minimal weight addition. Among the three cases the diagonal stiffened cantilever beam proved to be the one which produces highest natural transverse frequency in all modes, indicating the effectiveness of the diagonal configuration.

The comparison between ANSYS and MATLAB results shows a very close result among each other, which validated the numerical model, since ANSYS is a graphics user interface whereas MATLAB runs on one dimensional FEM modeling. The results from the numerical modeling and simulations shows a close agreement with the experimental values mainly for the lower modes, however there is a slight deviation in case of higher modes due to real-world effects such as material damping, boundary condition imperfections, and measurement errors. The Table 5.4: Comparison of natural frequency between ANSYS, MATLAB & experimental results shows a clear comparison between the ANSYS, MATLAB and experimental results.

Table 5.4: Comparison of natural frequency between ANSYS, MATLAB & experimental results

Particulars	**Mode No**	**ANSYS**	**MATLAB**	**Experimental**
Normal Cantilever Beam	1	13.75	13.64	13.64
	2	86.14	85.51	79.54
	3	241.44	239.45	187.56
	Mode No	**ANSYS**	**MATLAB**	**Experimental**
Cantilever Beam with Midline Stiffener	1	16.85	16.93	16.42
	2	105.37	106.14	82.95
	3	294.08	297.22	236.31
	Mode No	**ANSYS**	**MATLAB**	**Experimental**
Cantilever Beam with Diagonal Stiffener	1	18.68	19.19	17.01
	2	116.21	120.31	97.66
	3	325.63	336.87	274.05

The graphical figure can be used to compare the results obtained from ANSYS, MATLAB & experimental results for the normal cantilever beam. The Figure 5.16: Comparison between ANSYS, MATLAB & experimental results for normal cantilever beam gives a clear understanding about the trend and the deviation of the values among ANSYS, MATLAB and experimental results. The graph shows that the natural frequencies of normal cantilever beam without any stiffeners for first three modes. After plotting the natural frequency vs mode number, it was found that the natural frequency in case of the first mode closely matches with each other with only a slight deviation indicating the experimental procedure to be correct. Similar trend can be seen in the second mode but a slight deviation in case of experimental results as well as third mode. The possible reasons could be the accelerometer mass effect, its damping and the imperfect boundary conditions and slight errors in the fabrication period.

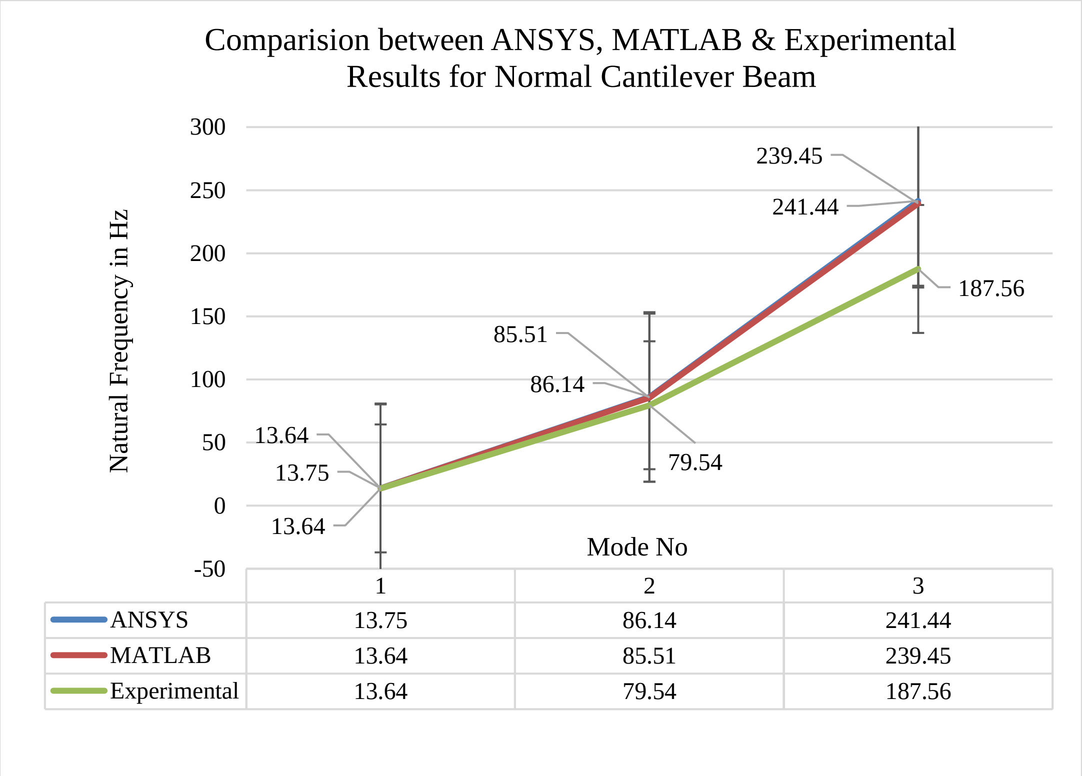
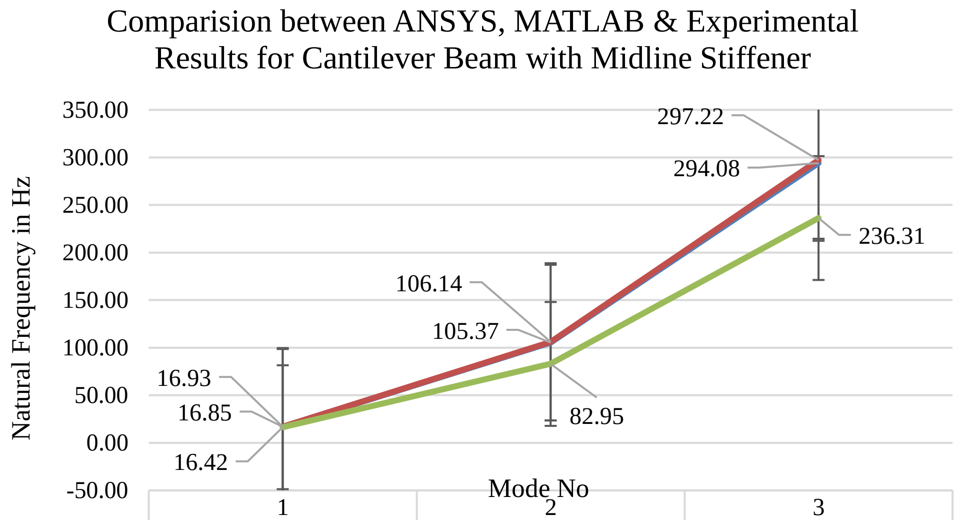


Figure 5.16: Comparison between ANSYS, MATLAB & experimental results for normal cantilever beam

The Figure 5.17 shows the comparison of the values from ANSYS, MATLAB and Experimental Results of Cantilever beam with midline stiffeners. The figure shows the natural frequency of cantilever beam with midline stiffener values in three different modes. The natural frequencies in case of the first mode closely matches with each other with a slight deviation also similar trend can be seen in case of second mode too but a little more deviation from ANSYS and MATLAB. However, in case of third mode it can be seen that there is a noticeable deviation in case of natural frequency between ANSYS, MATLAB & Experimental results. The ANSYS and MATLAB however closely matches in all three corresponding modes however the experimental values seem to be slightly deviated from the ANSYS and MATLAB. The possible reasons could be the accelerometer mass effect, its damping and the imperfect boundary as the perfect boundary conditions is ideal in case of numerical and simulations and also the errors in the fabrication period such as welding of the part, which is perfectly assumed in case of ANSYS & MATLAB.



	1	2	3
ANSYS	16.85	105.37	294.08
MATLAB	16.93	106.14	297.22
Experimental	16.42	82.95	236.31

Figure 5.17 Comparison between ANSYS, MATLAB & experimental results with midline stiffener

The Figure 5.18 shows the comparison of the values from ANSYS, MATLAB and Experimental Results of Cantilever beam with diagonal stiffeners. The figure shows the natural frequency of cantilever beam with diagonal stiffener values in three different modes. The natural frequencies in case of the first mode closely matches with each other while a slight deviation can be seen in case of second mode too but a little more deviation from ANSYS and MATLAB. However, in case of third mode it can be seen that there is a noticeable deviation in case of natural frequency between ANSYS, MATLAB & Experimental results. The ANSYS and MATLAB however closely matches in all three corresponding modes however the experimental values seem to be slightly deviated from the ANSYS and MATLAB. The possible reasons could be the accelerometer mass effect, its damping and the imperfect boundary as the perfect boundary conditions is ideal in case of numerical and simulations and also the errors in the fabrication period such as welding of the part, which is perfectly assumed in case of ANSYS & MATLAB.

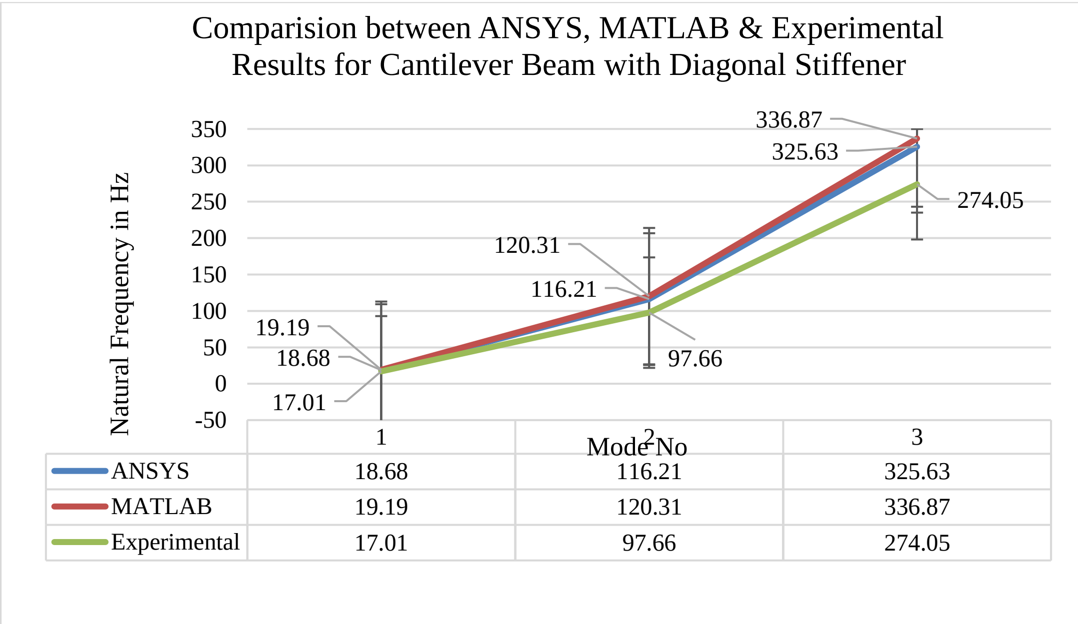


Figure 5.18: Comparison between ANSYS, MATLAB & experimental results of cantilever beam with diagonal Stiffener

From the figures above, it can be seen that the addition of stiffeners creates an increase in the natural frequencies of the cantilever beam.

5.5 Frequency Ratio Analysis for Experimental Data

The natural frequency of an object depends on the stiffness of an object. Same object with same mass, however stiffness values different may result in the increase in natural frequency of beam. The addition of a stiffener to a cantilever beam makes the beam stiffer. To understand how much is the natural frequency raised after altering the stiffness pattern, it is important to study the frequency ratio analysis and is given as the ratio of natural frequency of the stiffened to the non-stiffened beam which can be used to find the effectiveness of the stiffener. The frequency ratio is given as,

$$Frequency\ Ratio = \frac{f_{stiffened}}{f_{normal}}$$

Table 5.5: Frequency ratio of cantilever beam with midline stiffener

Mode	Normal (Hz)	Midline (Hz)	Frequency Ratio
1	13.64	16.42	1.204
2	79.54	82.95	1.043
3	187.56	236.31	1.260

The frequency ratio of the cantilever beam with midline stiffener is shown in the Table 5.5, the natural frequency in the first three modes of the normal cantilever beam and cantilever beam with midline cantilever beam are compared with each other. For the first mode it was observed that the frequency ratio of 1.204 was observed which means that the natural frequency increased by 20.4% and for the second mode it was observed that the natural frequency increased by 4.3% and for the third mode it was found to increase by 26.00%, which suggests that a midline stiffener can effectively raise the stiffness of the beam compared to the normal cantilever beam. The percentage increase in case of the second mode is due to the presence of a nodal points as it has one internal node and the strain energy is splitted into two lobes and the point from where the second mode shifts, the stiffener may not have been aligned well with peak curvature zones.

Table 5.6: Frequency ratio of cantilever beam with midline stiffener

Mode	Normal (Hz)	Diagonal (Hz)	Frequency Ratio
1	13.64	17.01	1.247
2	79.54	97.66	1.228
3	187.56	274.05	1.461

The frequency ratio of the cantilever beam with diagonal stiffener is shown in the Table 5.6, the natural frequency in the first three modes of the normal cantilever beam and cantilever beam with diagonal stiffened cantilever beam are compared with each other. For the first mode it was observed that the frequency ratio of 1.247 was observed which means that the natural frequency increased by 24.7% and for the second mode it was observed that the natural frequency increased by 22.8% and for the third mode it was found to increase by 46.10 %, which suggests that a diagonal stiffener can effectively raise the stiffness of the beam compared to the normal cantilever beam. The percentage increase in case of the second mode is comparatively low due to the presence of a nodal points as it has one internal node and the strain energy is splitted unevenly and the point from where the second mode shifts, the stiffener may not have been aligned well with peak curvature zones.

Table 5.7 Comparison of midline ratio and diagonal ratio of cantilever beam

Mode	Midline Ratio	Diagonal Ratio	Better Configuration
1	1.204	1.247	Diagonal
2	1.043	1.228	Diagonal
3	1.260	1.461	Diagonal

The Table 5.7 compares the frequency ratio analysis among the midline ratio and diagonal ratio analysis and the superiority of the configuration types. From the table it can be concluded that for all modes it was observed that the diagonal stiffened cantilever beam holds the better stiffness causing a gradual increment in the natural frequency. For the first mode, it can be noted that the diagonal stiffener priories the global bending resistance of the structure causing a higher natural frequency. However upon clear inspection in case of second mode it can be noted that there is a very less increment in the natural frequency of the midline stiffened cantilever beam due to the presence of internal nodal point which gradually decreases the effectiveness of the midline stiffener but in case of diagonal configuration it was found that the frequency ratio is more than that of the midline stiffened frequency ratio analysis because the diagonal stiffener distributes stiffness away from nodal region evenly .

The third mode, where curvature and strain energy are more localized, the diagonal configuration makes multiple high-curvature regions, producing the largest increase in frequency ratio.

The natural frequencies of three designed beams normal cantilever beam, cantilever beam with a midline stiffener, and cantilever beam with a diagonal stiffener were determined by numerical analysis using ANSYS and MATLAB software as well as experimentally. The result obtained from these analyses were compared with each other to check the accuracy of the numerical methods. Upon comparisons, the effect of stiffeners on the vibration of beams were also discussed. In case of the normal cantilever beam, the first natural frequency obtained by ANSYS was 13.75 Hz, by MATLAB was 13.64 Hz and experimentally obtained was 13.64 Hz. Therefore, the results from both numerical methods are accurate since their percentage errors are 0.800% and 0% respectively as represented in the Table 5.8

Table 5.8: Error percentage calculation between ANSYS, MATLAB & experimental data

Particulars	**Mode No**	**ANSYS**	**Experimental**	**% Error**
Normal Cantilever Beam	1	13.75	13.64	0.800%
	2	86.14	79.54	7.662%
	3	241.44	187.56	22.316%
	Mode No	**MATLAB**	**Experimental**	**% Error**
	1	13.64	13.64	0.000%
	2	85.51	79.54	6.982%
	3	239.45	187.56	21.670%
	Mode No	**ANSYS**	**Experimental**	**% Error**
Cantilever Beam with Midline Stiffener	1	16.84	16.42	2.494%
	2	105.37	82.95	21.277%
	3	294.08	236.31	19.644%
	Mode No	**MATLAB**	**Experimental**	**% Error**
	1	16.93	16.42	3.012%
	2	106.14	82.95	21.849%
	3	297.22	236.31	20.493%
	Mode No	**ANSYS**	**Experimental**	**% Error**
Cantilever Beam with Diagonal Stiffener	1	18.68	17.01	8.984%
	2	116.21	97.66	15.962%
	3	325.63	274.05	15.840%
	Mode No	**MATLAB**	**Experimental**	**% Error**
	1	19.20	17.01	11.397%
	2	120.31	97.66	18.828%
	3	336.88	274.05	18.650%

For the second mode in the simulation, the transverse natural frequency obtained from ANSYS compared to the experimental value is 86.14 Hz and 79.54 Hz. This indicates moderate percentage error, which increases as the mode number increases. In the third mode, there is noticeable deviation in the simulation result from the experiment. The frequencies obtained from ANSYS and MATLAB are 241.44 Hz, 239.45 Hz respectively which are high than the experimental value 187.56 Hz. The error in estimating the natural frequencies in this mode is more than 22%.

Similar conclusions can be drawn in case of the cantilever beam with a midline stiffener. It can be noted that the natural frequencies are increased compared to the normal cantilever beam after adding the midline stiffener. The natural frequency for the first mode as per the simulation study shows that the first transverse natural frequency is 16.84Hz, from the MATLAB coding the value was 16.93Hz compared to experimental value of 16.42Hz, and the percentage error varies from 2-3%, which indicates that the simulation-based study and experimental values correlate with each other very precisely. Similarly, the percentage errors in the second mode are the highest up to 21.849% and for the third mode the error percentage varies from 19.644% to 20.493%.

The diagonal stiffener is the one with the highest increase in the stiffness meaning that it has the highest increase in the value of transverse natural frequency .From the simulations study the first transverse natural frequency was noted as 18.69Hz , the finite element modeling gives a value of 19.20Hz and the experimental results was obtained as 17.01Hz , showing us a percentage error varying from 8.984% to up to 11.39% in case of the first mode and the second mode shows a percentage error varying from 15.962% in case of ANSYS and experimental results , however the percentage error increases up to 18.828% in case of comparison between MATLAB and Experimental results. And similar trend can be observed in case of third mode; the percentage error varies from 15.840% to 18.650% .

CHAPTER 6 CONCLUSIONS AND RECOMMENDATION

6.1 Conclusions

Free vibration characteristics of cantilever beams with stiffeners was studied using a combined numerical and experimental approach. The results obtained from the numerical ANSYS and MATLAB simulation as well as experimental study have been compared and validated.

The cantilever beams with and without stiffeners were fabricated and tested for the free vibration study, providing a basis for the validating numerical results. The finite elements developed in ANSYS and the analytical formulations adopted in the MATLAB were used to determine the natural frequency and mode shapes of cantilever beams with and without stiffeners. The numerical and analytical formulation showed a good agreement among each other.

The results from the numerical and experimental findings indicates the addition of stiffeners increases the overall stiffness of the cantilever beam after validation. The mode shapes resemble the bending behavior of the cantilever beam, which remains unchanged, only the presence of the stiffeners modifies the deformation pattern by changing the deflection and curvature across beam length.

6.2 Recommendation

The results from the cantilever beam with and without stiffeners can be extended to a complex stiffeners pattern. Comparison can be made between commonly used stiffeners configurations, to find the optimal increase in natural frequency without significantly increasing the mass. Future researches can be done for all possible stiffener configuration and study their effects on dynamic behavior of the beam. Similarly, the parametric study such as types of stiffeners, its thickness, widths and positions alternation can be made and studied for the optimal results. Similarly, the study can be extended to high performance lightweight composite materials. The damping effect can be studied and forced vibration response can be further studied. The use of the more accurate measuring methods such as laser vibrometers can be used to measure the natural frequency correctly and precisely and can be extended for the structural health monitoring of the system and the best configuration among normal cantilever beam, midline and diagonal configuration is diagonal configuration which is recommended to use in similar structure.

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Appendix I: Material Testing and Fabrication

Figure 1: Test specimen for Tensile Testing

Figure 2: Test Specimen after Tensile Testing

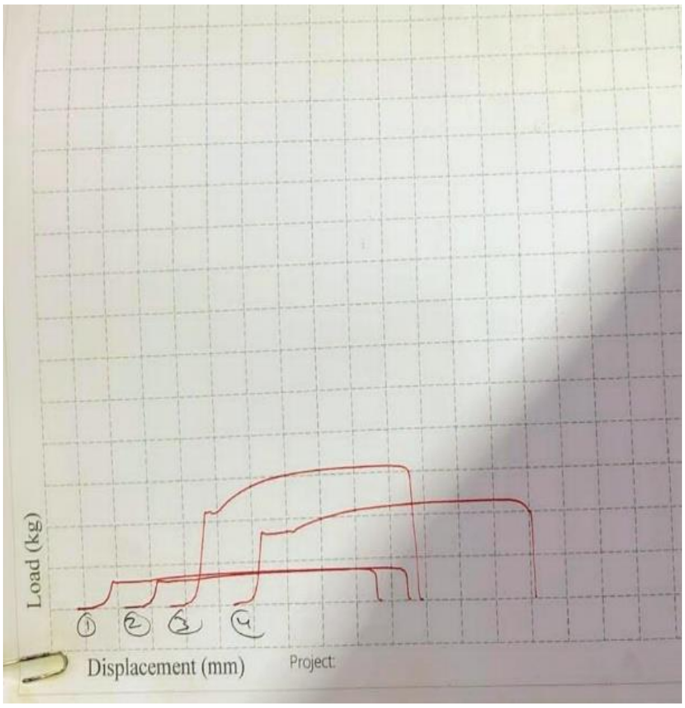

Figure 3: Stress vs Strain Graph after Material Testing (Figure 1 and 2)

Figure 4: Test Specimen for Experiment

Appendix II Experimental Devices Photographs

Figure 5: Accelerometer Calibration using Calibrator

Figure 6: Accelerometer with its sensitivity

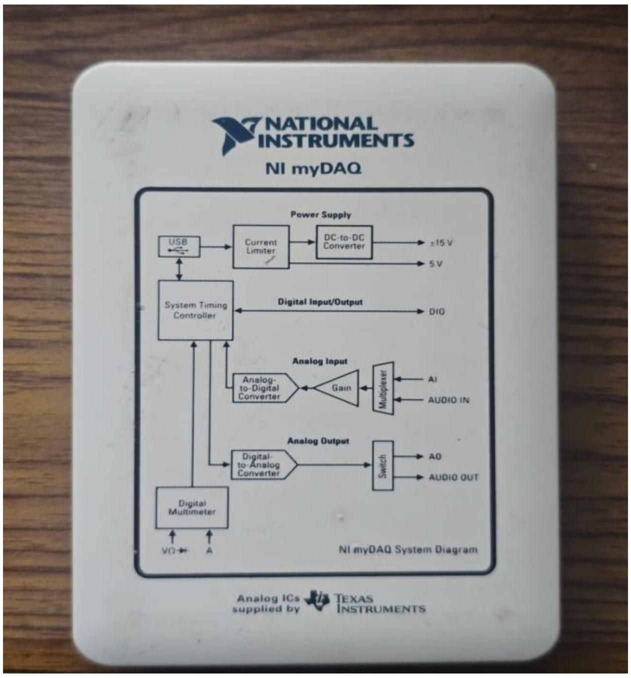

Figure7:NI myDAQ from National Instruments for Data Acquisition

Figure 8: Charge Amplifier after adjusting the Sensitivity

Appendix III Experimental Photographs

Figure 9: Experimental Setup

Figure 10: Experimental Setup for Diagonal Stiffener

Figure 11: Applying Boundary Conditions to the specimen

Figure 12: Photograph taken during the experiment

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6th May, 2026

Letter of Acceptance

Pawan Kumar Budhathoki, Prof. Dr. Mahesh Chandra Luitel, Asst. Prof. Er. Kamal Darlami

The Editorial Board of Journal of Advanced College of Engineering and Management (jacem) (ISSN No: 2392-4853), is pleased to inform you that your manuscript **"FREE VIBRATION ANALYSIS OF A CANTILEVER BEAM WITH STIFFENERS"** has been reviewed by the referee and accepted for the publication. Your article will be published in the coming issue of Journal of Advanced College of Engineering and Management, Vol. 13.

We are delighted and thankful for considering this Journal as a venue of your valuable research work.

With Regards

Er. Ajaya Shrestha
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