

Chapter I

INTRODUCTION

Background of the Study

Recognizing the procedure-based curricula, National Council of Teachers of Mathematics (NCTM, 1989) recommended that the teaching of mathematics emphasizes the conceptual understanding, multiple represents and connection, mathematical modelling and problem solving (as cited in, 28th ICTM). Teaching by multiple representations of the contents, relational teaching may help to make mathematics as a popular subject (Hana shatila, 28th ICTCM). In the context of Nepal, if teaching mathematics emphasizes the conceptual understanding, connect with real life problem multiple representation of content may help the mathematics teaching learning effective.

There are so many national and international organizations they have focused in the deep and conceptual understanding of mathematics and the popularization of mathematics is one of the key issues of today. The effective teaching and learning are the main slogan of today. Calculus is seen as difficult subject because of the weak presentation and understanding (Park, 2012). The concept of limit in calculus is difficult (Williams, 1991). The concept of limit is an important and basic notion among others in a calculus course. Students learning limits of functions perceive and treat limits differently (Jalter, 2006) as cited in (sebsibe, 2019).

Limit is included in the curriculum of mathematics of class XI. According to curriculum given by curriculum development center for Mathematics class XI specific objectives are, students will be able to define limit of a function, establish properties of limits and define continuity of a function using the concept of limit. It can be seen that

limit is widely introduced in the field of engineering, economics, science etc. In my experience, students of grade XI take calculus as a difficult area of mathematics. Almost calculus concepts require a good knowledge of the limit of the function (Adhikari, 2020). Some research has shown that teachers with many years of experience teaching calculus have difficulty with the concept of limit (Simonsen, 1995) as cited in (Blaisdell, 2012). Various researches show that many students feel that the limit is a difficult concept and they always remember the formulas and the process of finding limit. Even though the students can correctly solve the differentiation problems, they may not be actually making the sense of what the concept of limit truly mean? Thus, in these contexts my study tried to explore the difficulties in learning limit in high school students.

In mathematics, a limit is the value that a function (or sequence) "approaches" as the input (or index) "approaches" some value. Limits are essential to calculus (and mathematical analysis in general) and are used to define continuity, derivatives, and integrals.

The concept of a limit of a sequence is further generalized to the concept of a limit of a topological net, and is closely related to limit and direct limit in category theory.

In formula, a limit of a function is usually written as $\lim_{x \rightarrow c} f(x) = L$ and is read as "the limit of f of x as x approaches c equals L ". The fact that a function f approaches the limit L as x approaches c is sometimes denoted by a right arrow (\rightarrow), as $\inf(x) \rightarrow L$, as $x \rightarrow c$. Suppose f is a real-valued function and c is a real number. Intuitively speaking, the expression $\lim_{x \rightarrow c} f(x) = L$ means that $f(x)$ can be made to be as close

to L as desired by making x sufficiently close to c . In that case, the above equation can be read as "the limit of f of x , as x approaches c , is L " (Steward, 2012).

According to Cajori (1923) Augustin-Louis Cauchy in 1821, followed by Karl Weierstrass, formalized the definition of the limit of a function which became known as the (ϵ, δ) -definition of limit. The definition uses ϵ (the lowercase Greek letter *epsilon*) to represent any small positive number, so that " $f(x)$ becomes arbitrarily close to L " means that $f(x)$ eventually lies in the interval $(L - \epsilon, L + \epsilon)$, which can also be written using the absolute value sign as $|f(x) - L| < \epsilon$. The phrase "as x approaches c " then indicates that we refer to values of x whose distance from c is less than some positive number δ (the lower case Greek letter *delta*)—that is, values of x within either $(c - \delta, c)$ or $(c, c + \delta)$, which can be expressed with $0 < |x - c| < \delta$. The first inequality means that the distance between x and c is greater than 0 and that $x \neq c$, while the second indicates that x is within distance δ of c .

The above definition of a limit is true even if $f(c) \neq L$. Indeed, the function f need not even be defined at c .

Generally, difficulties refer to something that is hard to do, understand or, surmount (Dictionary.com, 2017). The concept of limit in calculus is difficult (Williams, 1991). Some researches have shown that teachers with many years of experience teaching calculus have difficulty with the limit concept (Simonsen, 1995) as cited in (Blaisdell, 2012). There are limited researches in the topic of difficulties in learning limit in class XI in the context of Nepal although the maximum numbers of students feel it difficult. It is field of my interest and I wanted to explore what are the difficulties in learning it?

Statement of Problem

Formulating a research problem as the first and most important step in the research process. Research problem identifies our destination, it should tell us, our research supervisor and our readers what we intend to research (Kumar, 2011). Curriculum Development Center included limit of 10 teaching hour in mathematics curriculum of class XI. It can be seen that Limit teaching is essential part of higher secondary and different fields of higher education. In this way without knowledge of limit, we can't learn more in every field. Teachers and students are facing many problems during teaching and learning limit. Previous studies have given some evidences for the existence of the problems such as lack of pre knowledge about function and relation etc., lack of appropriate teaching methods and materials, lack of student's motivation as observed by the teacher, lack of proper knowledge in teaching strategies, lack of discipline, etc. The problems of this study are to find the causes of problems during teaching learning limit. My experience as student teacher of teaching practice of M. Ed; I noticed that the students did not understand what the Limit of a function meant. Especially they felt difficult while asking the problem related indeterminate form. While I was a higher secondary student, I also felt difficult to understand limit. I was unable to comprehend the formal definition and its other symbolize form. So, teaching and learning Limit become one of the rigors & difficult tasks in our context for both teacher and students. Because of lack of conceptual understanding of Limit the students feel it so much difficult at secondary level. So, I wanted to find the difficulty of learning limit in class XI. The following questions were the main research problems-

- Why students feel difficult to understand limit in Class XI?
- What are the causes of difficulties in learning limit in Class XI?

Objectives of the Study

The main objectives of this study are to explore the difficulties in learning limit and to analyze the causes of these difficulties. Following were the purpose of this study:

- To analyze the causes of difficulties of students in learning limit in Class XI.
- To explore the difficulties of students in learning limit in Class XI.

Significance of the Study

Mathematics is an essential part of school curriculum. So, it is taught as a compulsory subject at school level. Mathematics is also included as an optional subject at secondary level for the students who want to learn more about mathematics. Although mathematics has been given an important place in the curricula of all levels of school education, many students were weak in mathematics. However, it is a fact that most of the students dislike mathematics. Learning limits requires skills from many mathematical areas. Students need to be able to understand formal expositions, perform algebraic manipulations, understand the meanings of quantifiers and absolute values, which students found problematic, and link theory to their everyday problem solving. This means that their concept images need to be well developed both in depth and width, i.e., they need abstraction skills and strong links between numerous topic areas. They also need to find inspiration and reasons to go through the hard work to make the knowledge meaningful in their concept images. Now a day's limit has become a difficult to learn and in such circumstances, my research is fruitful and supportive document to teach maths in a productive way. My study helps to all mathematics teachers who teach in the high school calculus courses. From my study, they would become aware about the student's difficulties of learning limit and they

could improvise their teaching and learning. Also, this study helps to those who are interested to research in the field of limit. In short, the following are the significance of my study to stakeholders:

- This research helps to do self- reflection for students so that they could improve their achievements.
- This research helps to teachers for making their class effective & productive.
- This research helps various stakeholders to guide in curriculum planning, textbook writing, making teaching strategy etc.
- This research helps as a literature for those who want to study on the topic.

Delimitation of the Study

Delimitation is the boundaries created by the researcher. The delimitation are those characteristics that limit the scope and define the boundaries of the study. They are created before any investigations are carried out in order to reduce the amount of time or effort spent in certain unnecessary, perhaps even unrelated, areas to the overall study. This study delimited under the following aspects:

- Only one school was selected.
- It was only bounded on limit inclass XI.
- This study was based on qualitative data.
- The study was conducted inthe Kapilvastu district.

Operational definition of terminology

The operational definition and terminology clarifies the meaning of the terms used in the study. It gives the operational meaning of variables of the study. The following are the operational definitions of some terms:

Secondary Level. In this study, secondary level is school education from class IX-XII

Difficulties. In this study, difficulties are hindrance which affect the learning the clear picture of some concepts. The causes and factors that hinders student to understand the concept of limit and process of finding limit is the difficulty.

Misconception. In this study, the meaning of misconception is the misapplication of a rule, an over- or under- generalization, misbelieve about limit.

Conceptual difficulty. In this study, conceptual difficulty means difficulty to constructing concepts, on developing reasoning skills on different terms and meaning of notations related to limit.

Procedural difficulty. In this study, procedural difficulty means the difficulty of process to finding limit to use different tricks and techniques.

Language. In this study, language means way to present views, problems and solutions.

Difficulty in understanding. In this study, difficulty in understanding are the factors and hindrances that affect in the learning the proper use of algorithms, formulas, rules of solving problem etc. of limit.

Chapter II

Review of the Related Literatures

This chapter is devoted to the discussion of the previous researches relevant to the academic writing. The purpose of this literature review is to examine the existing research about how student perceive about limit. This research also investigates the difficulties in the conceptual understanding about limit. There have been limited studies on these concepts.

Empirical Literature

The review of empirical literature pertains the systematic summary of scientific researches and true investigations including their topics, the reasons why this study was conducted, methods of the study, data collection tools and methods of ensuring their validity and reliability and key findings in the related field.

Areaya and Sidelil (2012) studied on “Student’s difficulties and misconceptions in learning concept of limit, continuity and derivative.” The main purpose of this study was to examine student’s difficulties and misconceptions in learning calculus. The researcher used descriptive survey method. 135 students were involved in this study. Achievement test was used to collect data by the researcher. The results revealed that gaps between aspiration of the math’s syllabi prepared by Ethiopian Federal Ministry of Education and students’ actual achievement. It also indicated the presence of misconception on the part of students regarding the basic concept of calculus. Difficulties embodied in the language, infinitesimal concept, infinite numbers, procedural knowledge and so on.

The study of Orhun (2012) studied on “Graphical Understanding in Mathematics Education: Derivative functions and students difficulties.” The purpose

of this study was to investigate that the students were not successful in analyzing limit function. The study was conducted during the full semester in two calculus classes. The data of research has been collected from five diagnostic the graph of derived function. To collect data researcher developed test and interview. The result indicated that students have lack in conceptual understanding .This case could be the result of the traditional teaching methods .In the study the students haven't interpreted the graph of the limit function. The students find it difficult to make connections between the graph of derived function and the original function. Usually, they were to interpret the graph of derived function as the graph of function. The students did not use the mathematical language to describe the graph of derived function. Effective teaching in calculus course should be informal, intuitive and conceptually based on graphs and functions in order to improve the quality of learning and develop the understanding of calculus concepts.

Pangali (2012) conducted research on “Difficulties in Learning Arithmetic in grade IX”. The study used qualitative research design. The main purpose of this study was to identify the difficulties and its causes in learning arithmetic in grade IX students. The researcher used purposive sampling method to collect data from field. He used written test, face to face interview and observation as research tools. The collected data were analyzed by cross match approaches. He found that there were many difficulties in learning arithmetic content grade IX students such as: students were not clear handwriting and slow to write, difficulties in languages of arithmetic, they were out of attention in mathematics class. Also many organizational difficulties in arithmetic. The students had some causes of difficulties which were student participation in learning arithmetic, teacher-students interaction interpersonal relation and so on. There was no proper interaction between teacher and students. Most of the

students were not sufficiently practice a home, because they were not clear concept of arithmetic, also their parents were illiterate and they were not commanding to their child for learning mathematics.

Denbel (2014) studied on “Students’ Misconceptions of the Limit Concept in a First Calculus Course”. The researcher used survey design. Questionnaire and interview were the data collection tools. This research was conducted on 130 pre engineering students of Dilla University, Ethiopia. The findings of the study show that students think that when a function has a limit, then it has to be continuous at that point. Other misconceptions were: The limit is equal to the function value at a point, i.e. a limit can be found by a method of substitution, when one divides zero by zero, the answer is zero. Many students’ knowledge and understanding rest largely on isolated facts, routine calculation, memorizing algorithm, procedures and that their conceptual understanding of limits, continuity and infinity is deficient. Students see a limit as unreachable due to the language used in many books to describe limits for example ‘tends to’ and ‘approaches’. Another view of a limit that the students have is that a limit is a boundary point.

Joshi (2014) conducted thesis on “Students’ Enrolment Trend in Major Mathematics at Higher Secondary Level.” The study attempted to find out the enrolment rate of students in major mathematics and why the students are less enrolled major mathematics at higher secondary level and it was concluded to attend the objectives. The purposes of this study were to find out the causes low enrolment of students in major mathematics at higher secondary level. The researcher used qualitative research. Sample was selected from only one school Tikapur Multiple Campus, Tikapur, Kailali and tools were interview, observation and school documents. Face to face interview with students, campus chief, mathematics teachers

and parents and observed the classroom reality. And then, the researcher used the school documents to find out the causes of low enrolment in major mathematics for the collection of primary data. The collected information was analyzed by triangulation method. The finding of the study showed the causes of low enrolment in mathematics which were lack of prior knowledge, insufficient support of family, lack of decisional process, weak educational background and poor economic condition of parents, and teacher's behavior towards students. The campus policies were not sufficient to address low enrolment of students.

Chiluwal (2014) worked on "Language Problem Faced by Students in Learning Mathematics at Higher Secondary Level." This study was qualitative study. It tried to explore the understanding of mathematics and English language influence in teaching and learning mathematics at higher secondary level. More specifically, this study tried to find out the hidden feelings, learning difficulties, understanding of mathematics teaching and classroom practices faced by mathematics teacher and how students perceived it in the real classroom situation. The researcher used ethnographic method of enquiry to explore experiences and images of mathematics which the mathematics teachers and students carried. In this study, the researcher presented the teacher's and students' views about the mathematics teaching and learning in English at higher secondary level. The findings revealed that medium of instruction were the major reason of mathematics teaching and learning. English language instruction promoted the mathematics in understanding, avoid mathematical anxiety and bring pleasure and make real classroom situation in teaching and learning process.

Beynon and Zollman (2015) worked on "Lacking formal concepts of limit". This is mixed-methods study examines the conceptual understanding of limit among 22 undergraduate of Indiana University, Southeast, USA. The main purpose of this

study was to examine the conceptual understanding of limit. Written test and in-depth interview were the data collection tools. The findings indicate that the students do not consider employing a formal definition of limit to solve limit based mathematical problem and students personal concept definitions of limit are both in operable for solving limit problems and inconsistency with the formal definition of limit.

Pokhrel(2017) did a thesis on “Problems Faced by Students and Teachers on Limit.” The main target of this research was to find out the causes of problems faced by mathematics teachers and students while teaching learning limits at Grade XI of higher secondary school. The research was conducted in six +2 colleges of Arghakhanchi district. The tools used in data collection procedure were class observation form and face to face interview. The descriptive survey research design was adopted to conduct the study. The researcher finds above problems are due to student’s weak pre- knowledge about functions, geometry and co-ordinate etc., unique cognitive structure and lack students project works. Teachers do not access modern teaching techniques, materials and evaluation at teaching, due to lack of idea to generate locally available materials.

Bhandari(2017) did a thesis on “Difficulties in Learning Group Theory.” The main objective of this research was to explore the learning difficulties in group theory. The design of the study was qualitative case study 5 students were selected from the Kathmandu valley. Interview and observation was used for data collection. The researcher found that students have difficulties in proving process of abstract ideas. They were unable to sequential logic to reach in up to conclusions. They were also unable to use proper symbols as well as quantifiers.

Limbu (2018) conducted research on “Students’ Difficulties in Learning Algebra at grade VIII”. This was a case study on lower secondary school students’

difficulties in learning algebra with view to explore the students' difficulties in learning algebra at grade VIII and to find the causes of difficulties in learning algebra at grade VIII. The study used mixed method research design. The study selected two schools from Kathmandu district by using convenience sampling. Written test, face to face interview and class observation were used as the tools to collect the data. The result indicated number of difficulties categories under each area; under variable, students were unable to differentiate like and unlike term and their simplification. The abstract structure of algebraic expressions posed many problems to students such as distributive errors in bracket expansion, difficulties in operating with negative integers. Inadequate understanding of the uses of equal sign and its properties were major difficulties in solving equation, students were confused not only lack of technical words and vocabulary but also translating algebraic form with proper sign. Under transition from arithmetic to algebra students were unable to substitute the value in expression and equation. The causes of difficulties in algebra were due to the lack of pre requisite knowledge, solving ways and rules, traditional teaching method and did not use teaching materials.

Khanal (2018) conducted research on "Difficulties in Solving Word Problem in Algebra". This was a case study. The study used qualitative research design. The aim of this study was to explore and analyze the difficulties in solving word problem of algebra at grade IX. He selected one school from Kathmandu district by purposive sampling. He took written test and face to face interview with students and teacher. The collected information was analyzed in five different levels (Reading and language, conceptual, comprehension, transformation and solving process difficulties). The data were analyzed with the help of theories and related literature. He concluded that many students were unable to understand the mathematical

concepts, facts and skills. The major difficulties of students were deficiency of comprehension difficulties, transformation difficulties and solving process difficulties. The finding of this study showed that there were many difficulties in solving algebra word problem in grade IX students such as: unable to give the meaning of mathematical term properly and unable to choose appropriate operation to solve the problem, inability to use alternative method for solving the problem, students always emphasis on product only rather than process, lack of pre-requisite knowledge, lack of the basic knowledge about algebra, unable to translate into correct mathematical form, traditional trend of problem solving style, students did not have the sufficient practices of mathematics.

Kafle, (2019) conducted a study on "Conceptual and Procedural difficulties in Learning Derivative". The study was case study. The study was conducted on Makwanpur Multiple Campus, Makwanpur and 40 students of Grade XI were selected as sample by using purposive sampling. The data were collected through test and interview guidelines. The finding of this study indicates that students had weak concept to understand the derivative as a rate of change, unable to understand the clear geometrical meaning of derivative, unable to make exact sense of limit necessary to study the derivative, unable to use power rule to find the derivative. Hence, it can be concluded that the teacher are focusing only procedural understanding of derivative and students becomes exam oriented which makes the derivative as one of the difficult topic. Thus, both teacher and students should focus the conceptual teaching and learning of derivative and pre knowledge of derivative such that they could easily understand the concept of derivative. Thus, teacher should change their teaching style so they can make their classroom very fruitful and learning derivative become a meaningful which avoids the rote learning.

Joshi (2019) conducted thesis on investigating students' mathematical difficulties with quadratic equation at grade IX. The main purpose of this study was to explore the students' difficulties while learning quadratic equations and to find out the causes of difficulties while learning quadratic equations at grade IX. The study was a case study. The researcher selected the one school from Kanchanpur district. The study was conducted in 24 students at grade IX. The data were collected from students' written test and in-depth interview with students at grade IX. Also in-depth interview was taken with mathematics teacher. The findings of this study was, the students have conceptual and procedural difficulties in solving quadratic equations at grade IX students such as unable to factorize, unable to apply factorization principle appropriately, inability to use completing square for solving quadratic equation, students always emphasis on product only rather than process, students remembered the quadratic formula incorrectly, had difficulty in determining the factors through cross multiplication and completing square principles. The causes of difficulties were lack of pre- requisite ix knowledge, lack of the basic knowledge about quadratic equation, irregular in school, not sufficient interaction on teacher and student's carelessness in study and students did not have sufficient practices of mathematics

Sebsibe (2019) conducted thesis on "Overcoming Difficulties in Learning Calculus Concepts: The Case of Grade 12 Students". The main purpose of the study is to overcome student's difficulties in learning calculus concepts by developing a literature informed intervention model. A design-based research approach of three phases was conducted. Grade 12 natural science stream students in one administrative zone in Ethiopia were used as the study population. The study concluded with Implications for practice that includes the use of student's errors and misconceptions as an opportunity for progression. Besides, students should be assisted to make sense

of concepts through real-life problems, including training teachers in problem-solving approaches and mathematical thinking practice.

Adhikari (2020) conducted a study on "Difficulties and Misconceptions of Students in Learning Limit". The purpose of this study is to explore the difficulties and misconceptions of secondary level students in learning limit of the function. The study used mix-method design. Test was used on 200 grade XII students of Kathmandu Metropolitan City of Nepal, and interviews were conducted on 12 purposively selected students from 200 students. The data obtained from the test was categorized as the category of the responses and interpreted using a percentage score. In addition to that, the data obtained from the interview was analyzed and interpreted by using a general qualitative approach. The finding of the study revealed that student's difficulties in solving a limit problem have aroused because of the lack of conceptual clarity on the limit of the function. Similarly, the student has misconception on indeterminate and infinite, limiting and functional value of the function, and symbolic representation of infinity.

Reflection on Literatures

These reaches show students have difficulties and misconception in limit. Most of the researches regarding difficulty with limit were done outside the Nepal. There are the few studies about the difficulties in limit in the context of Nepal. The above literatures are mainly focused on problems of teachers and students. Many researches focused on how to overcome from the misconception. My experience of teaching practice the students felt the limit one of the difficult concepts and they always rote the process of finding limit. So, in the above study the difficulties of the students and the causes of failure in finding limit are not mentioned. The specific difficulties that are faced by the students while learning limit are not mentioned in the

above literature. So, by using this gap I want to find out what are the difficulties in learning limit in higher secondary courses of Nepal.

Theoretical Framework

There are so many theories which can be used to learning difficulties in mathematics. Besides the above theory researcher used Action-Process-Object-Schema (APOS) theoretical framework. APOS is a framework that describes different levels of students go through when learning mathematical concepts is progression through action- process- object- schema hierarchy (Dubinsky,1991). APOS theory arose out of an attempt to understand the mechanism of reflective abstraction, introduced by Piaget to describe the development of logical thinking in children, and extended this idea more advanced mathematical concept (Dubinsky,1991). This theory begins with hypothesis that mathematical knowledge consists in individuals' tendency to deal with perceived mathematical problem situation by constructing mental actions, process, objects and organizing them in schemas to make a sense of the situation and solve the problems. Many theories such as, cognitivism approach theory and constructivism approach theory have different view about the difficulty of the students. In this study conceptual understanding of class management will be analyzed according to, APOS theory and Vygotsky's the social Constructivist.

APOS Theory can be used directly in the analysis of data by a researcher. In very fine grained analyses, the researcher can compare the success or failure of students on a mathematical task with the specific mental constructions they may or may not have made. If there appear two students who agree in their performance up to a very specific mathematical point and then one student can take a further step while the other cannot, the researcher tries to explain the difference by pointing to

mental constructions of actions, processes, objects and/or schemas that the former student appears to have made but the other has not.

An action is a transformation of objects perceived by the individual as essentially external and as requiring, either explicitly or from memory, step-by-step instructions on how to perform the operation.

When an action is repeated and the individual reflects upon it, he or she can make an internal mental construction called a process which the individual can think of as performing the same kind of action, but no longer with the need of external stimuli. An individual can think of performing a process without actually doing it, and therefore can think about reversing it and composing it with other processes.

An object is constructed from a process when the individual becomes aware of the process as a totality and realizes that transformations can act on it.

Finally, a schema for a certain mathematical concept is an individual's collection of actions, processes, objects, and other schemas which are linked by some general principles to form a framework in the individual's mind that may be brought to bear upon a problem situation involving that concept. As with encapsulated processes, an object is created when a schema is thematized to become another kind of object which can also be de-thematized to obtain the original contents of the schema (as cited in Dubinsky, 1991). Hence the APOS theory used as a theoretical framework of my study. The tools were constructed on the basis of APOS which explores the difficulties in learning limit.

According to the social constructivism, knowledge is the best constructed when learners collaborate together. Vygotsky states "Every function's in the child's cultural development appears twice, first on the social level and later on the individual

level: first between people (inter psychological), and then inside the child (intra psychological). This applies equally to voluntary, attention, to logical memory and to the formulation of concepts. All the higher functions originate as actual relations between human individuals.” In social constructivism, there are three main things which are presented in below:

Making meaning: Learners built up any things meaning from the society. So, there is great role of society in learning.

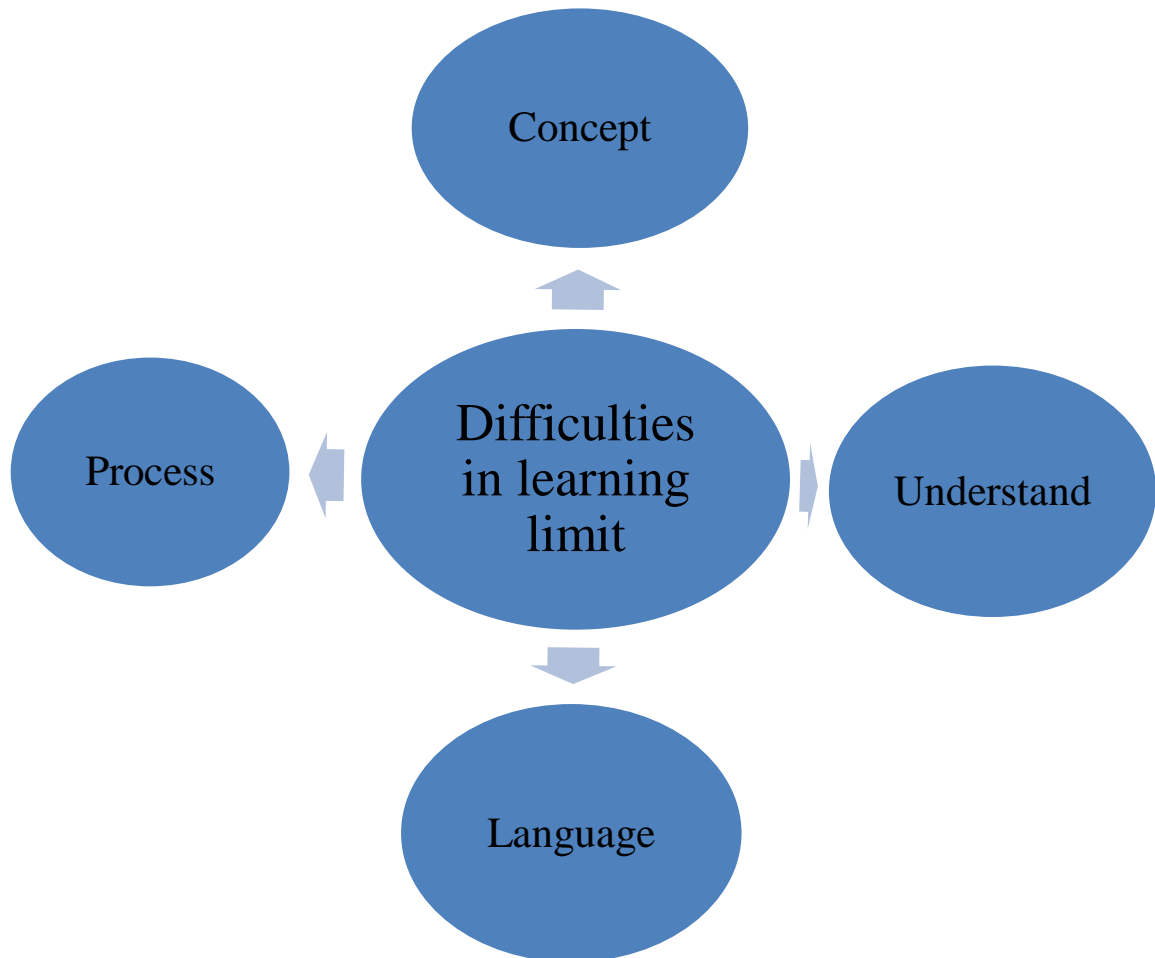
Tools of cognitive development: Culture, language and ancestors are the tools of knowledge from them a learner can get a lot of knowledge in learning their mobilization and their nature towards education affects the learning.

The zone of proximal development (ZPD): ZPD means the difference between capacities of solving problem by learners themselves and solving problem from the supports of others. The full development of the ZPD depends upon social interaction, in that the range of skill can be developed with social guidance or collaboration, thus it often exceeds what can be attained alone. To ensure development in the ZPD, the assistance given must have certain features and they are inter- subjectivity, scaffolding and guided participant (Acharya, 2015).

Conceptual Framework

Conceptual framework helps to see clear picture of the thesis. It is also called the operational road map. The conceptual framework of the study shows the clear picture of the study. The theoretical framework consists of the theories or issues in which study is embedded; the conceptual framework describes the aspect selected from theoretical framework to become the basis of enquiry. The problem of my study is the difficulties in learning limit. In the literature mentioned above, there is no exact

aspect of learning difficulties in limit. On the basis of the study of Beynon and Zollman (2015), Areaya and Sidelil (2012) and Orhun (2012) related to difficulties in learning limit the following was the conceptual framework for my study.



I found the result on the basis of above conceptual framework. How student feel difficulty in learning limit and why they feel the difficulty. That was found by using two tools i.e. test and interview guidelines. The researcher found the result with help of student's performance in the written test. To find the difficulties in learning limit researcher categorized it into four types of difficulty they are difficulties in learning concept, difficulties in understanding the concept difficulties in language and last one is difficulties in process in finding limit. On the basis of APOS theory the researcher constructed the test and on the basis of test result researcher selected the

respondent and conducted the interview and examined the above four types of difficulties on concept, understand language and process.

Under the difficulties of concept of limit the study was found difficulty in definition of limit, indeterminate form and two sided limits. Similarly, understanding is the second dimension of my conceptual framework. This study also tried to find the difficulties on understanding meaning of that $|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$, value of function and limit of function. Under the difficulty in language the study was found difficulty in meaning of $\lim_{x \rightarrow 2^-}$, meaning of phrases approaches and infinity, undefined terms, exist or does not exist. Finally, under the difficulties in process the study was found difficulty in finding limit of square root function, trigonometric function and exponential function.

Chapter III

RESEARCH METHODS AND PROCEDURES

This section explains the design of the study in detail. Research design is the specification of the methods and procedure. A research methodology gives research legitimacy and provides scientifically sound findings. It also provides a detailed plan that helps to keep researchers on track, making the process smooth, effective and manageable. It includes the detail description of the manner in which decision has been made about the type of data needed for the study, the tools, devices and methods used in collecting data.

Design of the Study

Research designs are plans and the procedures for research that span the decisions from broad assumptions to detailed methods of data collection and analysis (Creswell, 2009). Research design is the conceptual structure, strategy of logical, systematic plan and direction of research. It is the way through which the researcher reaches to the goal of research. It is a qualitative research model. According to Bhandari(2020), "Qualitative research design deals with words and meaning while quantitative deals with numbers and statistics. Qualitative research involves collecting and analyzing non-numerical data (e.g., text, video, or audio) to understand concepts, opinions, or experiences". The ontology of the interpretive is there is multiple reality and epistemology is the subjective i.e. Knowledge is constructed through the underlying meanings of the individuals, events or activities and the methodology is the qualitative (Patel, 2015) as cited in (Joshi, 2017).

I adopted the case study design among the qualitative researches for my study. According to McCombes (2019) case studies are often a good choice in a thesis or

dissertation. They keep our project focused and manageable when we don't have the time or resources to do large-scale research. According to (Creswell, 2009), Case study is a strategy or inquiry in which the researcher explores in depth a program, event, activity, process or one or more individuals. McCombes (2019) said "a case study is a detailed study of a specific subject, such as a person, group, place, event, organization, or phenomenon. Case studies are commonly used in social, educational, clinical, and business research.". The single instance is of a bounded system, for example a child, a class, a school, a community. It provides a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply presenting them with abstract theories or principles (Cohen, 2007). Cohen (2007) also mentioned that Case studies can establish cause and effect, indeed one of their strengths is that they observe effects in real contexts, recognizing that context is a powerful determinant of both causes and effects.

Case studies are set in temporal, geographical, organizational, institutional and other contexts that enable boundaries to be drawn around the case; they can be defined with reference to characteristics defined by individuals and groups involved; and they can be defined by participants' roles and functions in the case (Hitchcock and Hughes 1995,p319, as cited in Cohen, 2007). Various researches show that the learning limit is difficult for students. Such problem is also in the students of the Kapilvastu district. Thus the students of class XI in the public school of the Kapilvastu district were taken for the case of the study. Three students having poor achievement in the test (based on APOS Theory) were taken as case of the study. This difficulty in learning the limits was studied in depth interview in this study.

Study Area and Selection of Respondents

Kapilvastu district was the area of study and students of grade XI. 28 students were selected as sample for the study by using purposive sampling. The case of this study was the students of the grade XI. In this study the students of the one of the public school of grade XI of Kapilvastu District was chosen. Test was conducted to the each students of the class and for the purpose of the study researcher selected three students on the basis of result test for the interview. Also the interview was conducted to collect the data according to purpose.

Data Collection Tool

Research tool is the most important part for the data collection in the study. On the basis of data collection technique we can study and analyze every aspect of the study. For the data collection, I made use of the following tools:

Tests

Test is a kind of assessment which would be taken as a paper pencil test with students. It is widely used to evaluate the students' achievement. The purpose of the study was to find the difficulties in learning limit. To find out the difficulties in learning limit the researcher made one test having four questions. The instrument that was administered was organized by the APOS (Action, Process, Object, Schema) framework (See Appendix-A). The first problems were action problems, then the process problems, then object and lastly a schema problem. Problems on the assessment were arranged from basic level of knowledge to the highest level of conceptual problems. Students were instructed to solve the problems to the best of their ability. Questions were made on the basis of the APOS model for observation of the student's knowledge of limit.

There were a total of four problems. Problem no 1 is the action level problem. These problems are straight forward and are basic for students to do. Action questions required students to execute the steps to solve problems explicitly. On the test, the problem no 1 contains seven problems related to definition of limit, indeterminate forms, two sides of limit, limit of a function and existence of limit.

Problem no 2 demonstrates the process component of APOS theory. The process part of this theory is for students to reflect on the action. The procedure of the process level is similar to the action; however, the process level creates more of thought process for a student. This means that, students would have to think about the questions before applying the action technique. On the test, the problem no 2 contains five problems related to difficulties in limit.

Problem no 3 represents an object questions for APOS theory. Once a student is comfortable with an action and is proficient with the process level of this theory, then a student can move up to the object level. The object level is where a student can think more analytically about a problem. There were three questions mentioned in a Test.

Problem number 4 was a schema questions. According to APOS theory this is a highest level of understanding for students. Schema links the action, process, and object steps together. Schema questions show that students have deeper understanding of the topic. There were only two questions mentioned on test.

It helps to examine the knowledge of students acquired from certain content. Some questions were made on the basis of the APOS model for observation of the students understanding of limit. There were three questions related to action and five questions related to Process and one –one questions related to the object and schema in test. The

test was administered in the whole class. The test consisted one question related to concept, two questions were related to understanding, one question was language related and two were based on finding the limit and the related questions on the basis of APOS theory.

In-depth Interview

After conducting the test the interview was conducted to the participants. Semi-structure interview was adopted to find the difficulty. The subject related questions were asked. Apart from these questions, other situational questions were asked. These interviews involve unstructured and generally open-ended questions that are few in number and intended to elicit views and opinions from the participants (Cresswell, 2009). The interview is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard. The order of the interview maybe controlled while still giving space for spontaneity, and the interviewer can press not only for complete answers but also for responses about complex and deep issues. In short, the interview is a powerful implement for researchers. On the other hand, the researcher using interviews has to be aware that they are expensive in time, they are open to interviewer bias, they may be inconvenient for respondents, issues of interviewee fatigue may hamper the interview, and anonymity may be difficult (Cohen, 2007).

After conducting the test, interview was conducted with the help of interview guidelines (See Appendix-B) to the participants. Semi-structure interview was used for finding the difficulty in learning limit. The necessary questions were asked to the participants although these are the main questions. Except these questions, other situational questions were asked. I tried to explore the difficulty by conducting the

depth interview. The interview was kept by recording and interview result was compared to the current literature.

Reliability and Validity of Tools

Reliability and validity is the one of the important characteristics which the data collection tool should consist. Qualitative validity means that the researcher checks for the accuracy of the findings by employing certain procedures, while qualitative reliability indicates that the researcher's approach is consistent across different researchers and different projects (Gibbs, 2007: as cited in Creswell, 2009). For the reliability and validity of test, the pilot testing was conducted in one of the school of Kapilvastu district and afterwards, its reliability and validity was evaluated.

Qualitative researchers use many methods for gathering information and interviewing is one of those methods with a research base .When conducting interviews, relationships and rapport must be established, and couple with trust: The purpose of interviewing is to find out what is in and on someone else's mind. Active listening and nonjudgmental behavior are two of the common practices that should be prioritized when interviewing for case study research. Thus reliability and validity of interview schedule of my study was calculated from the consistency in response, multiple response, prolonged engagement, Triangulation etc.

Goodness and Trustworthiness

Because qualitative research entails the researcher taking an active role in the collection and interpretation of others' meaning making, to be credible, qualitative researchers must be good and trustworthy. To ensure the goodness and trustworthiness, I used the triangulation of data, member check, confidentiality, and peer review, rich thick description etc.

Credibility

This concept replaces the idea of internal validity; by which researcher seek to establish confidence in the truth of their findings. Guba and Lincoln (1998) as cited in Joshi(2017) recommend several techniques inquirers may use to enhance the credibility of their research: persistent observation, triangulation, peer debriefing, negative case analysis, progressive subjectivity checks and member checking. To maintain credibility of my research I tried to spend as much as time for case respondents and selected school. The data were collected from multiple sources and got several information on related with research problem. After getting information I again met participants to make result realistic. I also asked question to other people and tried to find real difficulties from those information.

Transferability

Transferability replaces the concept of external validity. This criterion refers to the applicability of findings in one context (where the research is done). To maintain transferability I had included photos of taking examination, interview and related person of my research. I had tried to capture most of scenario by using thick description of interviews and my meaning making.

Dependability

This concept replaces the ideas of reliability. This is the third standard for judging qualitative standards and refers to the stability or consistency of the inquiry processes used over time. To maintain it I had taken written test and logic used for selecting participant, interview and include in the study. I would try to maintain credibility and transferability to ensure dependability standard.

Confirmability

A fourth standard is confirmability, which refers to the quality of the results produced by an inquiry in terms of how well they are supported by informants (members) who are involved in the study and by events that are independent of the inquirer. This is sometimes referred to as audit trail (a record of how decisions were made throughout the study). I am also a part of mathematics students. So, to maintain confirmability before concluding information, I reviewed those information myself several times sometimes and I conform those information to my other students/friends before concluding information as well.

Data Collection Procedure

The study was conducted in a sample selected from related population. The required data was collected from the related school and through the help of the head teacher of the school, I got in touch of my respondents for the test. The data was collected through the interview and was tested after the valued consent from the head teacher and then from the participants and eventually from the guardians, if necessary.

The test questions made on the basis of APOS theory were given to the students and the result was measured using Rubric. The interview was conducted with the participants and the questions which were related to the difficulties in solving the problem were asked. The interview was recorded transcribed and then coded.

Data Analysis and Interpretation Procedures

Data analysis is described as a systematic search for meaning such that the qualitative and /or quantitative data observed may be communicated to others in understandable ways. Analyzing qualitative data requires understanding make sense of text and images so that can form answers to your research questions (Creswell,

2009). This research qualitative research is mostly associated with words, language and experience rather than statistics. The collected information from the primary source such as test and interview were analyzed and interpreted. The researchers collect the data only through the primary sources.

In this study, researcher used the test and interview with student to find out the different difficulties, while the collected the data student was provided with test to find out the difficulties in their answer sheet. The interview was carried out with the three students having maximum difficulty in the test. The general inductive approach by Thomas (2006) was used for analysis of the data obtained from interview. According to Thomas (2006) the inductive approach is a systematic procedure for analyzing qualitative data in which the analysis is likely to be guided by specific evaluation objectives (p.8). The inductive approach clarifies the data reduction process by describing a set of procedures for creating meaning in complex data through the development of summary themes or categories from the raw data. -

In this study, I analyzed the interview result according as general inductive approach by Thomas (2006). At first interview was recorded and then transcripts of interview was made. After continuous reading of transcript so many times, the transcripts were coded. I categorized so many code into different category. Finally, the themes were generated such as students had weak concept on definition of limit, difficulty on finding of two sided limit, unable to differentiate between value of function and limit of function and misconceptions on terms used on limit.

Chapter IV

ANALYSIS AND INTERPRETATION

This chapter deals with the analysis and interpretation of the collected information derived from the case study during the research period. Analysis of data is a process of inspecting, cleaning, transforming, and modeling with the goal of highlighting useful information, suggestions, conclusions and supporting decision making (Best and Khan, 2017). According to Cohen (2007), Qualitative data analysis involves organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities.

The process of data analysis involves making sense out of text and image data. It involves preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data (some qualitative researchers like to think of this as peeling back the layers of an onion), representing the data, and making an interpretation of the larger meaning of the data. Data analysis involves collecting open-ended data, based on asking general questions and developing an analysis from the information supplied by participants (Creswell, 2009).

After the collection of data with the help of the tools and techniques, the next step is to analyze and interpret them with the view to arrive at empirical solution to the problem. For the data collection, I used first the Test. From the result of the test I selected those students who have difficulty in learning limit and the interview was conducted with them. The data obtained from interview was analyzed analytically, descriptively and qualitatively. This is the case study about learning difficulty in limit at XI. To meet the objective of the research I collected data from the only one school

of Kapilvastu district, named as Shree Gautam Buddha Secondary School. There were 28 students in the class XI. First I conducted the exam by using test paper. From the test result the researcher had selected three students whose results was very weak. The researcher asked the guideline questions (Appendix- B) for each student.

The interview is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard. The order of the interview maybe controlled while still giving space for spontaneity, and the interviewer can press not only for complete answers but also for responses about complex and deep issues. In short, the interview is a powerful implement for researchers. On the other hand, the researcher using interviews has to be aware that they are expensive in time, they are open to interviewer bias, they may be inconvenient for respondents, issues of interviewee fatigue may hamper the interview, and anonymity may be difficult (Cohen, 2007).

After the result of test I selected the three students on the basis of weak performance in the test. My cases were the three students read in the civil engineering faculty of class XI. They were permanent resident of out of Kapilvastu district. Researcher asked the questions on the basis of guideline questions (Appendix-B) for selected participant.

Finally, I categorized the collected data for analysis and interpretation in the following main four headings as difficulty in learning concept, difficulty in understanding, difficulty in language and difficulty in solving process in learning limit which can be present as below on the basis of conceptual framework.

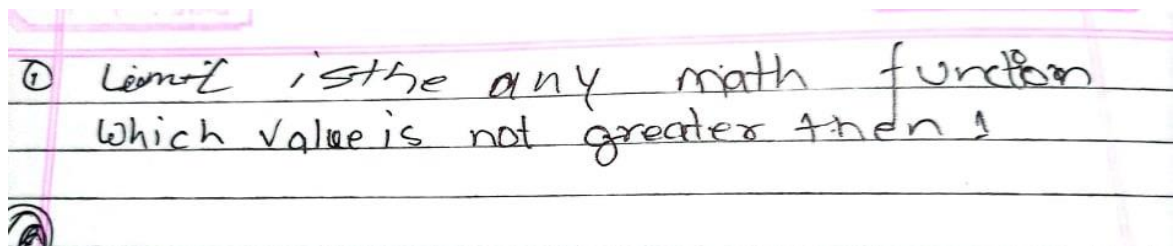
Difficulty in Learning Concept

Limits occur in many different mathematical contexts. The limit concept is fundamental to mathematical thinking however it proves to be difficult for many students to use as a basis of their thinking. The limit concept should be approached in such a way that previous knowledge that students have is taken into consideration. Limits occur in many different mathematical contexts including the limit of a series and in connection with functions. The difficulty in learning concept of limit refers the difficulties in the definition of limit, difficulties in the meaning indeterminate forms and difficulty in the meaning of left-hand limit & right-hand limit and difficulties in connection between them. It also includes the misconceptions. According to Heibert&Carpentar (1992) Conceptual knowledge is a network of knowledge where the connections are meaningful. Conceptual knowledge is a network of associations of mathematical procedures, integrated with the mathematical principles (as cited in Abbey, 2008). Difficulties in making concept are the factors that hinders in learning the basic and fundamental elements in the larger structure of the content. I used three problems by which are in the action level questions based on APOS framework. On the basis of test result the answer and mistakes done by respondents were analyzed by taking an interview. To explore the difficulties in concept I categorized the three main concepts that definition of limit, indeterminate form and left hand limit & right hand limit.

Definition of limit. Definition is a statement of the exact meaning or significance of a word, phrase, idiom, etc. This difficulty refers the difficulties related to defining the limit. This difficulty refers to the difficulty related to what exactly the limit. It also includes the misconception about the term. Also at my higher secondary level, due to the lack clear concept about limit I felt difficult to define limit by using

symbol like delta, epsilon etc. There are various ways to define limit, but it should give right meaning of the limit. Students may have the difficulty to define limit or misconception to give exact meaning of the limit. Before defining limit students must have clear idea about the limit too well define the limit. Researcher analyzed the difficulties related defining limit on the basis of function concept, meaning limit, limit value. If the respondents are able to explain the above concept then there is no any difficulty but if they cannot explain about the function concept, meaning limit, limit value. Then such type of difficulty are categorized as a difficulty in the defining limit. According to Williams (1991) students have misconception on Confusion over whether a function can reach its limit, whether limits are inherently tied to motion concepts, Confusion over whether a limit is actually a bound and Confusion whether limits are dynamic processes or static objects.

For finding difficulty in defining limit, by the APOS theory one action level question was asked. The question was "Define limit on your own words". All of the students attempt the question. That means they have some idea on definition of limit, but they didn't show idea that a function approaches to the limiting value as variable moves towards tending value. Most of the Students did not use symbols of delta, epsilon etc to define limit. To explore these difficulties first, a test taken. On the basis of students answer researcher conducted the interview. Answer of one of the interviewer who is trying to define limit is given below:



① Limit is the any math function which value is not greater than 1

Analyzing the above definition of limit, it shows the students had more difficulty to define a limit. Student A defined it as any math function whose value is not greater than 1. From the definition given by student A we can easily understand that the students have no clear concept about the limit. The study of Pokhrel (2017) also found that problems are due to student's weak pre-knowledge about functions. They were unable to define what the limit exactly. They have a concept that limit is a math function, but they were struggling to understand the meaning of limit. In order to confirm the difficulties related to the concept of limit, the researcher has conducted the interview with all respondents. The interview taken by the researcher with student A can be mentioned as below:

I: Do you know what the limit is?

Student A: Don't know clearly.

I: Is a limit of a function always less than one?

Student A: I don't know.

I: But you wrote limit of a function is not greater than 1 in your answer sheet?

Student A: I wrote it carelessly.

I: Can you find the limit of a function $f(x) = x+3$ as x tends to 0.

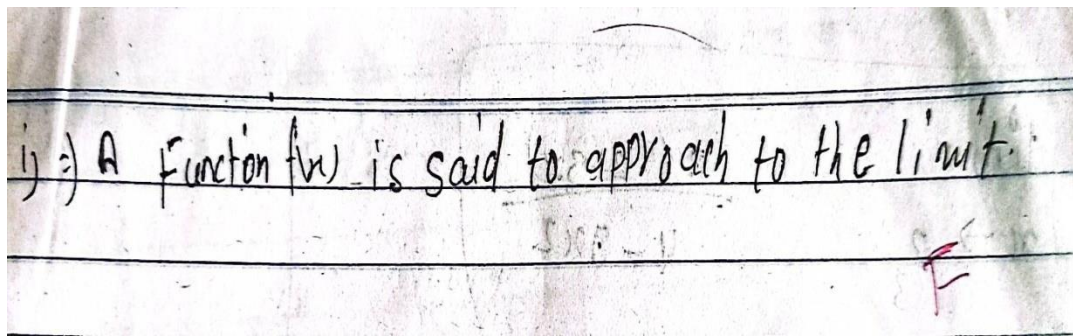
Student A: Sure. The limit of given function is 3.

I: But how?

Student A: Put the value of $x=0$ in given function.

From the above discussion, it can be seen that the student has very much difficulty in the definition of limit. Students were unable to define the limit of a function. It seems that the students had very much difficulty about the definition of limit. The

answer given by student “I wrote carelessly” shows students have no clear idea on limit. They were not able to understand what the limit is exactly. Also, In a study conducted by Tarmizi (2010), found that students did not fully understand function notation, memorized a procedure, and could not verbalize what they were doing in the problem. Hence from the above interpretation, it can be easily seen that students had very much difficulty in understanding limit. Answer of one of the interviewer who is trying to define limit is given below:



Above answer of the student showed that student have difficulties in defining limit. Student B tried to define limit but he had not complete approach about limit. He tried to say function approach to the limit but unable to define properly. Similar type of result found by Denbel (2014) that student had misconception the limit is equal to the function value at a point. This shows student have difficulty on definition of limit. To explore the difficulties on definition of limit further discussion can be presented below:

I: Do you know what the limit is?

Student B: Don't know exactly.

I: You wrote function is said to approach to limit. Can you explain?

B: I don't know sir.

I: Is a function limit?

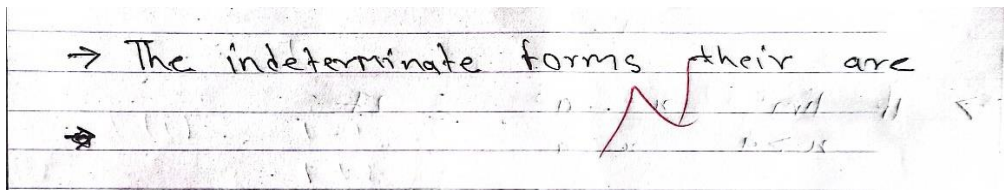
B: I think so, sir.

From the above discussion, it can be seen that the student had very much difficulty in definition of limit. Students were unable to define limit of a function. This showed students had very much difficulty about the define limit. They had a misconception about limit of a function is a particular value or the limit of a function is again a function. They were unable to clarify limit and function. Similar type of result was found by study of Beynon and Zollman (2015) that students personal concept definitions of limit are both in operable for solving limit problems and inconsistency with the formal definition of limit. But it was found that student have lacking of personal concept no definition of limit. Student didn't have clear conceptual clarity on limit.

Furthermore discussion, I asked student *anywhere you found the answer that you answered for the questions?* Student A answered *"for a function given in the text I think I saw that answer."* Again I questioned him that can you tell me *"where you saw that answer on your text book"*. He replied *"I don't know exactly but I think that is before the first exercise on the text book"*. Again I asked him to show from book and he showed me a sequence is increasing 0.9,0.99,0.999... and for the example there was limit of the these increasing term is less than 1.

From above discussion it can be seen that student were not reading the learning material carefully. They were careless on the definition of limit. Students did not give sufficient work on reading and understanding the definition of limit. They use an example as definition of limit. Similarly Khanal (2018) concluded that *"students did not have the sufficient practice of mathematics"*. Due to lacking of practice, they had misconception and they felt difficulties on defining the limit. Student did not read carefully the mathematical terms and ideas.

Indeterminate forms. An indeterminate form is an expression involving two functions whose limit cannot be determined solely from the limits of the individual functions. These forms are common in calculus; indeed, the limit definition of the derivative is the limit of an indeterminate form. There are seven indeterminate forms which are typically considered in the literature. For finding difficulty in indeterminate forms of limit, by the APOS theory one action level question was asked. The question was "What are indeterminate forms? State their forms". Only 23 students attempted the question. On the basis of students answer, researcher conducted the interview. Answer of one of the interviewer is given below:



The above answer of student showed that student have no clear idea on indeterminate forms of limit. Also study of Adhikari (2020) found that student had misconception on indeterminate forms of limit. Student tried to write indeterminate forms but could not write the complete answer. This seems student had difficulties on indeterminate forms. To explore the difficulties indeterminate forms following discussion conducted as presented below:

I: Could you tell me what you found on finding limit a function,

$$f(x) = \frac{x^2 - 4}{x - 2} \text{ as } x \rightarrow 2 ?$$

Student A: It is 0/0 form so we factorize the $x^2 - 4$.

I: What we call these types of forms?

Student A: Indeterminate forms.

Student B: Indeterminate forms

I: Is only form $0/0$ indeterminate form?

Student A: No sir, the form $0/0$, ∞/∞ , $\infty-\infty$, 0^0 , ∞^0 , $0.\infty$ and ∞ power of 1.

Student B: Not only $0/0$ something upon zero not determined.

Student C: Don't know exactly.

I: Then what is indeterminate forms mean?

Student A: Don't know clearly.

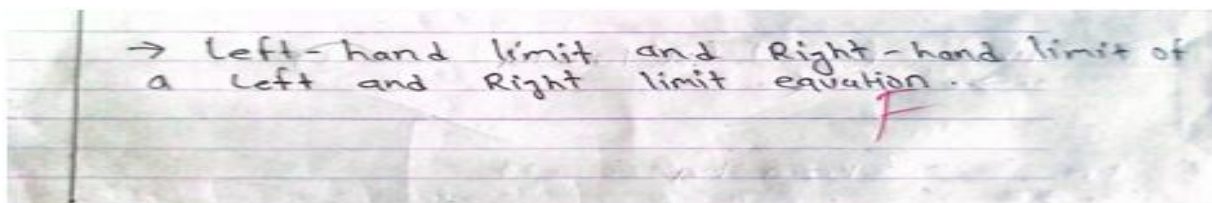
From the above discussion, it can be seen that student had some concept about indeterminate forms. Student A gives right answer but Student B had misconception that only something upon zero is indeterminate form. Student C did not answer the questions. From this, it seemed that students were unknown about other indeterminate forms like form $0/0$, ∞/∞ , $\infty-\infty$, 0^0 , ∞^0 , $0.\infty$ and ∞ power of 1. This shows that student feel difficult about indeterminate forms. Mainly students were confused about $0/0$ is only indeterminate form. These all facts justified that the students had difficulties in indeterminate forms. Students were unaware on why these forms are called indeterminate forms. They were unable to answer what is indeterminate forms. They were unaware on limiting value of these forms cannot be determined so these forms are indeterminate forms. Adhikari (2020) found that student had misconception on indeterminate and infinite, limiting and functional value of the function, and symbolic representation of infinity. As similar, Study of Denbel (2014) concluded that students had misconceptions that the limit is equal to the function value at a point, i.e. a limit can be found by a method of substitution, when one divides zero by zero, the answer is zero. Many students' knowledge and understanding rest largely

on isolated facts, concept of limits, continuity and infinity is deficient. Student had misconception on indeterminate forms.

Furthermore discussion, student C was asked “why you did not try to get answer? He replied “I find difficulty on this topic.” I again asked him why you feel this difficult. He answered that “*mathematics is compulsory on our course but I don’t like to study mathematics. I just study mathematics to get only pass marks. Due to which I could not give enough time on mathematics. So, I felt difficult.*”

From this it can be seen that student *felt* difficult because they did not give appropriate time on mathematics. They mostly focused on exam oriented questions. Similar types of result were found by Kafle(2019) that students become exam oriented due to which they felt difficult. Also, Pangali (2012) found that most of the students did not sufficiently practice at home. One of the reasons behind this was found that students study mathematics to get pass only and they didn’t give enough time on mathematics.

Two sided Limit. In Calculus, sometimes functions behave differently depending on what side of the function that they are on. By definition, a one-sided limit is the behavior on only one side of the value where the function is undefined. If the two one-sided limits are not equal, the two-sided limit does not exist. For finding difficulty in two sided limit, by the APOS theory, one object level question was asked on the test (see question no 3 on Appendix-B), Only 6 students attempted the question but none of the interviewer attempted the question. One action level question and one object level question were asked. The action level question was "What are left hand limit and right hand limit of a function?". Only 17 students attempted the question. On the basis of students answer, researcher conducted the interview. Answer of one of the interviewer is given below:



When the answer of the student was analyzed, It seem that he had some idea of the limit form right side and left side is two sided limit. Also it can be seen that student had idea that left side and right side give different equation but student were not totally aware on two sided limit. They did not understand why this happened. Students were unaware that closer from above and closer form lower are the two sided limit of the function. To explore the difficulties on two sided limit, following interview was conducted which is represented below:

I: What do you mean by “x approach to a” from left?

Student C: I don't know exactly.

I: But you define as “left hand limit and right hand limit of a left and right limit

equation”, what does it meant?

Student C: I don't know sir..

I: Does it sufficient for a function $f(x)$ to have limit at $x=a$ is Left Hand Limit and Right Hand limit be equal?

Student C: No, sir, There most function value also equal to Left Hand Limit and Right Hand limit.

From the above discussion it was found that students felt difficult to connect definitions with examples. Students were memorizing the definition of Left Hand Limit and Right Hand limit but they didn't know exact meaning of it. They tried to

answer that they had memorized but were unable to write the total answer of the question. They mostly focused on memorizing the definitions. They were unable to connect memorized definitions with examples, similar types of result was also found by Denbel (2014) was that most student's knowledge and understanding rested largely on memorizing algorithm. Similar type of result was also found by the study of Areaya and Sidelil (2012), that students had misconception of learning limit, continuity and derivatives. The condition for continuity is sufficient to limit but only Left Hand Limit and Right Hand limit are equal gives limit. But the students had misconceptions that functional value is also required to be equal with Left Hand Limit and Right Hand limit for the limit. To find the difficulty furthermore discussion was done as presented below:

I: Can you say what the limiting value of the function is f at x tends to 3 from right side on the given graph?

Student A: I don't know sir

Student B: I saw first time this question

Student C: Ummm.... I don't know sir.

I: Can you tell me what is the right hand limit and left hand limit of the function at x tends to 3?

Student A: I never did such types of questions.

Student B: I saw first time

Student C: Ummm....

I: Did you never saw graphical questions on limit on the classroom?

Student A: no sir

Student B: maybe I was absent at that day

From above discussion it seems that student had difficulties on graphical questions. They were surprised on graphical questions. None of them attempted these graphical problems. Student were unable to answer the left hand limit and right hand limit. All interviewer said that they saw this type of question first time. They never solved such types of questions. Students had difficulty on graphical questions. This showed students had difficulties in graphical representation on two sided limit. The study of Orhun (2012) also found that students had not interpreted the graph of the limit function. The students felt it difficult to make connection between the graph of derived function and the original functions. Student had difficulty on left hand limit and right hand limit. They were unable to get enough idea on such left hand limit and right hand limit. To find the causes of difficulties furthermore discussion was done as presented below.

Students were asked “what is the reason that you didn’t solve such types of problem before?” Student A replied “*Such types of question are not asked in our examination so we didn’t practice.*” Furthermore, I asked “did you see such graphical question on your test book or practice materials?” student answered that “*mostly we use exercise of text book and old questions solution for practice.*”

From above discussion, it seems that students mostly focused on exam oriented problems due to which they felt difficult on limit. On a study Kafle(2019) students become exam oriented. Students were mostly focused on exam oriented problem solving and didn’t get enough knowledge on the concept.

As a result of the study, it can be seen that students had difficulty to define the limit. They were not able to understand what the limit is exactly. They had a

misconception about limit of a function is a particular value or the limit of a function is again a function. Similar types of result were found by Adhikari (2020) was that student having difficulties on limit because of conceptual clarity on the limit of function. Students had difficulties in indeterminate forms. Students were unaware on why these forms are called indeterminate forms. They were unable to answer what are indeterminate forms. They had misconception that something upon zero is indeterminate form. Students were memorizing the definition of Left Hand Limit and Right Hand limit but they didn't know exact meaning of it. Such types of difficulty was also found by the study of Adhikari (2020), he found that the student had misconception on indeterminate and infinite, limiting and functional value of the function, and symbolic representation of infinity. They mostly focussed on memorizing the definitions. Students had difficulties in graphical representation on two sided limit. They were unable to comprehend these memorize definition with graphical representation. The reason behind this was they mostly focused on the exam oriented problem solving. They focused on marks and getting pass on the exam.

Difficulty in Understanding

Understanding is a psychological process related to an abstract or physical object, such as a person, situation, or message whereby one is able to think about it and use concepts to deal adequately with that object. Understanding difficulties means those difficulties in which students are unable to comprehend the limit concepts. Understanding involves a range of different skills. A student may have difficulty on one or more areas of subject matters. According to Bhandari(2017) these areas include following:

Listening and Attention. Paying attention and listening to teachers can be difficult for some students. They may easily get distract.

Understanding Words and Concepts. Every student need to understand the word of a teacher is speaking. Some student finds it difficult to learn and remember new words such as tends to, left hand limit & right hand limit, indeterminate forms, limit at infinity, limiting value, functional value etc. Moreover, it is necessary to understanding the meaning of word or idea behind it. Many students find difficult to understand the abstract concepts.

1. Understanding The Abstract Statement. Mathematics faculty and students generally consider calculus to be the most troublesome course. Students feel that calculus is one of the difficult subjects (Tall, 1992). Thus the statements given in the limit are considered to be difficult and challenging to understand.

Memory. Memorization is the process of committing something to memory. Memorization is a key cognitive process of the brain because almost all human intelligence is functioning based on it. Memorization without understanding does not last longer. Thus memorizing limit concepts without comprehension is another problem of understanding.

Symbolic representations of limit also create the difficulties in understanding. Students must know the meaning of symbol and their proper use. For example “approaches” (\rightarrow), “less than” ($<$), “does not equals to” (\neq), ϵ “epsilon”, δ “delta” etc. These symbols have their own particular meaning. Many students’ rote the statement and note provided by the teachers. They even don’t try to understand the meanings of these symbols have. According to the literature, there are two different kinds of notions of the limit, a dynamic limit notation and a formal limit notion. The dynamic limit notion is considered as $x \rightarrow a \Rightarrow f(x) \rightarrow L$ and read, “as x approaches a,

$f(x)$ approaches L ". The formal limit definition is given as follows: If the condition, for any $\varepsilon > 0$, there exists $\delta > 0$ such that $|x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$ is satisfied, the limit of the function of f at the point of a is L .

These symbols used on limit have their own meaning as above definition for an epsilon as difference between x and a became smaller the difference between function and limit goes smaller that indicates as x goes to a function goes near to L . But on my school level I also just tried to remember the formula. I didn't understand the meaning of the definition. In this study, difficulty on understanding is difficulty regarding the meaning of $|x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$ of formal definition, value of function and limit of function. If students were able to give right meaning of $|x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$ of formal definition and right answer of value of function and limit of function. It would be assumed that there are no difficulty in understanding the limit.

Meaning of formal definition of limit. For any $\varepsilon > 0$, there exists $\delta > 0$ such that $|x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$ is satisfied, the limit of the function of f at the point of a is L is called formal definition of limit. To explore the difficulties of students on meaning of $|x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$ of formal definition one action level question according to APOS model were asked on the test. The question was "write the formal definition of limit (ε, δ)". 16 students were attempt the question. The answer of one of the interviewer was given below:

ff formal definition: for (ε, δ) / some positive number $|x - a| < \varepsilon$ and $|f(x) - L|$

According to above answer of the student, it can be seen that students do not have enough knowledge on formal definition of limit. He wrote that "for (ε, δ) there

exist some positive number $|x - a|$ and $|f(x) - L|$. Student was trying to write the formal definition of limit using (ϵ, δ) but he/she was not able to give right solution. He/she was unable to write full definition and he or she forgot some symbol. So it could be seen that student had difficulties on meaning of $|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$ of formal definition. To explore such difficulties following interview was taken which is represented below:.

I: Can you define limit of a function without using symbol like epsilon, delta etc?

Student C: no sir, I tried to write the memorized from book..

I: What is the meaning of “for any $\epsilon > 0$, there exists $\delta > 0$ such that $|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$ ”?

Student A: I don't know exactly.

Student C: Ummm....., it is definition of limit for x tends to a , but I don't know the meaning of δ .

I: The above condition must satisfy to define the limit, why?

Student C: I have no idea.

I: what do you understand by these symbol ϵ, δ ?

Student A: they are some positive number.

Student C: don't know exactly.

I: why they are introduced on the formal definition of limit?

Student A: I don't know sir.

From the above discussion, it can be seen that the student had very much difficulty in the meaning of symbol. Students were trying to remember definition from book but they were unable to explain the meaning of definition. Student A said, "I don't know sir" and answer of Student C did not have clear idea about the symbols that were used to define limit. But both of them answered that ϵ, δ are positive numbers. Student C also had no idea why the condition is required? They were not fully aware of the meaning of these symbol and use of these symbols. Students were unaware why this situation must satisfied. In a study of Adhikari(2020) found that "concept test reveal that none of the students have a clear concept on the meaning of limit, ϵ , δ definition, and relation between. ϵ and δ ". They were unaware that (ϵ, δ) are two positive number such that the modulus of difference between x and a is less than δ whenever modulus of difference between function and limit is less than ϵ . The value of δ depends upon value of ϵ . So, It seemed that the students had very much difficulty on using these symbols. They were just focussing on remembering the definition. Also, in a study Beynon and Zollman (2015), found that students do not consider employing a formal definition of limit to solve limit based mathematical problem and student's personal concept definitions of limit are both in operable for solving limit problems and inconsistence with the formal definition of limit. Also, in a study of Tarmizi(2010), found that students did not fully understand function notation, memorized a procedure, and could not verbalize what they were doing in the problem. Hence from the above interpretation it can be easily concluded that students had very much difficulty in understanding symbols like ϵ , δ etc. use in limit. They were unable to interpret the meaning of notations and symbols.

To find the causes of difficulties furthermore discussion was conducted as presented. Students are asked that what you feel difficult on the formal definition of limit? Student B replied that:

Sir, I felt difficult on symbols. These symbol makes me confused.

They are not easier as words to remember and their meaning make me confused. Sentence gives some idea to memorize but it is so difficult to understand the meaning of combination of such symbols.

I feel these symbol as unknown language.

Again I asked what could be the reason that these symbols makes you difficult. And the student replied that “*without knowing the meaning and why they are used it makes difficult to understand?*” The same question student C answered that “*I don't like to remember definition on mathematics solution. I mostly like to solve problem with using formula. So, I didn't focus more on definitions.*”

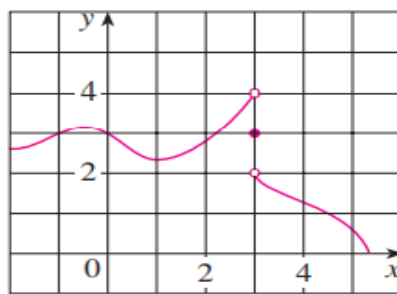
From above discussion, it seemed that students felt difficulties due to the symbols and their combination. These symbols and notions makes difficulties for the students. According to Tall and Vinner (1981) the definition of the limit is very difficult for students to understand because of the ε , δ approach as cited in (Blaisdell, 2012). So, it seems that student feel difficulties in understanding the limit due to unable to make relation between ε , δ . student mostly focused on exam oriented problems and they focus on problems solve with formulas than the definition and understanding the notions and symbols used in the definitions.

Value of function at a point and limit of function at same point. If f is function from X and Y and $x=a$ is an element in the domain of f then the image $f(a)$ corresponding to $x=a$ is said to be value of function at $x=a$. And again a function $f(x)$

is said to tend to a limit 'l' when $x \rightarrow a$ if the numerical difference between $f(x)$ and l can be made as small as we please by making x sufficiently close to a and we write $\lim_{x \rightarrow a} f(x) = l$. For finding difficulty value of function at a point and limit of function at same point, by the APOS theory, one action level and one schema level questions were asked on the test. The schema level question was "for the function f from the given graph if it exist. If it does not exist. If it does not exist, explain why?"

a) $\lim_{x \rightarrow 3} f(x)$

b) $f(3)$



Only 6 students attempted the question. None of the interviewer attempted the question. The action level question was "distinguish between limit of a function and value of a function at a point". 19 students attempted the questions. Rest of 9 students didn't attempt the questions. None of the interviewer attempted the question showed difficulty on the value of function at a point and limit of function at same point. To explore the difficulties following interview was conducted:

I: what you do to find the value of a function?

Student A: we put the value of given variable and calculate.

I: what you do to find the limit of a function at a pint?

Student A: same as finding the value of a function.

I: Is the value of function and limit of function same at a point?

Student A : umm.... I think, yes sir.

Student C: I think no sir.

I: How do they differ? Can you give an example of it?

Student C: But how, I don't know sir.

From the above discussion, it can be seen that the student was unable to define limit of a function and value of function. Student C had not a clear idea that the difference between value of function at point and limit of the function at that point. Students were unaware about these terms. From above discussion, it is clear that students had a lack of pre-knowledge for the understanding. According to the Google search: "The value of a function is the actual calculation done at a certain point. The limit is - roughly speaking - the value at points that are "arbitrarily close" to the same point". But Students are not able to clarify how the value of that the found causes of low enrolment in mathematics which were lack of prior knowledge, insufficient support of family, lack of decisional process, weak educational background and poor economic condition of parents, and teacher's behavior towards students. The result was found by Pokhrel(2017) in his thesis on "Problems Faced a function is differing from limit of the function at a point. In a study of Joshi (2014) it was by Students and Teachers on Limit." It was found that problems are due to student's weak pre-knowledge about functions, geometry and co-ordinate etc., unique cognitive structure and lack students project works. So, it concluded that the students had difficulties in limit of the function and value of the function. They saw both of them as same way of solving problems.

I: Why did not you differentiate between the value of function and limit of the function on your answershit?

Student C: Umm..., I don't know about this exactly.

I: Can you define value of a function at a point?

Student C: I don't know sir.

I: In the given graph what is the value of $f(3)$?

Student A: I don't know sir

Student B: I saw first time these question.

Student C: don't know

I: In the given graph what is the value of $\lim_{x \rightarrow 3} f(x)$?

Student A: ummm.....

Student B: I saw first time these question.

Student C: I don't know sir, I feel difficult on graph problems.

From above discussion, it seems that students had difficulty on graphical questions. They were surprised on graphical questions. None of them attempted these graphical problems. This showed students have difficulties in graphical representation on value of function at a point and limit of function at same point. Students were not able to give the value of given function at $x = 3$. They didn't spoke and not given my answer. They also did not give answer of the what is the value you found of $f(3)$ in the given graph. From this it seems that student are unable to find the limit and of the graphical function. The study of Orhun (2012) also found that students have not interpreted the graph of the limit function. Thus, it seems that students find it difficult on finding limit of the function at a point and value of the function at that point.

To find the causes of such difficulties further discussion was conducted. Which is presented as: *“I asked the students, what makes you difficult for solving the graphical problem on finding value of the function at a point and limit of a function at same point?”* Student A answered that: *“In our classroom we did not solve such types of questions. Our teachers did not focused to solve such types of questions.”* Answer of other students on this question was as same as the student A. Again I asked *“which types of problem you feel easy to solve either graphical or numerical?”* On this question student answered that *“I found numerical problems easier”*.

Form above discussion, it seems that student felt more difficult graphical questions than the numerical problems. In a study of Orhum(2012), similar type of result found that students did not use the mathematical language to describe the graph of derived function. The reason behind this was found that students did not practice the graphical questions on the classroom.

All of the above discussion justified that students are unable to comprehend the definition of limit. They have the difficulties on symbols using to define limit. They are also unaware about the difference between value of function and limit of the function. Student felt difficulty on the value of function at a point and limit of the functional at same point on graphical representation. The result found by study of Blaisdell (2012) *“Students in general had less difficulty with graphical representations than mathematical notation or definition questions”*. The reason behind this was found that students focused more on the exam oriented problems than the other and in the classroom teacher are not focused to teach the graphical problems.

Difficulty in Language

Language is of vital importance in communicating knowledge because it carries the context which allows for meaning and re-contextualization. Language is composed of words and notions, which allows for the articulation of the meaning. Language not only just enhances the knowledge but it also affects the way one thinks. It affects how a certain individual sees things because he knows the main ways of knowing. Furthermore, he knows because of the knowledge perceived through the use of sense has to be expressed through weapon of language. Many of us start with our native tongue and the learning process is largely depended on the language. This is however an important process as language is not only a person's main medium of communication but spoken language is a large part of any classroom and workplace. As a child grows, native language teaching becomes more explicit and reading and writing instruction begins. It is at this point that some children discover difficulties with what the majority of able minded children can generally learn quite easily.

Mathematics has its own language with symbols and notation. Limit also consists lots of symbols and notations like “approaches” (\rightarrow), “less than” ($<$), “does not equals to” (\neq), ϵ “epsilon”, δ “delta” etc. One statement can be represented by various ways. Due to which student become confused. Huillet (2005) States that the limit concept can be studied in many different settings: geometrical (area and volumes), numerical (sequences, decimals and real numbers, series), cinematic (instantaneous velocity and acceleration), functional (maximum and minimum problems), graphical (tangent line, asymptotes, sketching the graph of a function), formal (definition), topological (topological definition, concept of neighborhood), linguistic (link between natural and

symbolic languages of limits), algebraic (limits calculations). Each of these settings underscores a specific feature of the limit concepts cited in (Jaffar, Dindyal, 2011).

Meaning of $\lim_{x \rightarrow 2}$ and Meaning of phrases approaches. Those various ways are represented by various symbols and notions. These notations and symbols give some meaning. The composition of these symbols and notations makes a sense but understanding that sense becomes difficult for the students. On the other hand, mostly higher school level Mathematics works on English language. But in context of our nation, not all the students are from English background school. The students from Nepali background school find difficulties due to the mathematics in English. According to Mamona-Downs (1990) the concept limit has a special place in mathematics as being almost symbolic of the first cross-over from naive mathematics to rigor and as a result has attracted a fair amount of attention in educational research as cited in (Sebsibe, 2019). Similarly Denbel (2014) found that Students see a limit as unreachable due to the language used in many books to describe limits for example ‘tends to’ and ‘approaches’. Another view of a limit that the students have is that a limit is a boundary point. In higher school days I also felt difficulties due to the English language. I also felt difficulties due to symbols and their combination. So, researcher was trying to find the difficulties due to English language and difficulties due to limits own symbols and notations and their compositions. To found such difficulties one action level question was asked on the test. The question was 1(a) on the Appendix-A. 23 students were attempt the question. Answer given by one of the student was given below:

According to above answer it can be seen that students were unable to solve the problem correctly. Student represent as a symbol but did not explain that the x is moving closer to 2. Tends to is a phrase and student could not find the meaning of this

phrase. This shows students have difficulty and to explore these difficulties of students on phrases, notions and symbols of limit on the basis of conducted test paper following interview were observed.

I : What do you understand by $\lim_{x \rightarrow 5}$?

Student A: x meet to 5.

Student B: x tends to 5.

Student C : x approaches to 5

I : Do x meet 5?

Student B : Ummmmm.....?

Student C : I don't sir?

I: How would you clarify the meaning of $\lim_{x \rightarrow 2}$?

Student C: I don't know exactly.

I: But you define as "x approach to a" from left is left hand limit, how?

Student C: I think I saw that on the book.

I : Consider two sequences 0.9, 0.99, 0.999, 0.9999, and 1.1, 1.01, 1.001, 1.0001, what you found from these sequence?

Student A: ummmmm.....

Student B: I don't know.

Student C : don't know

I : Above sequence converges to 1. Can we say limit of the function giving these different values at different points is 1?

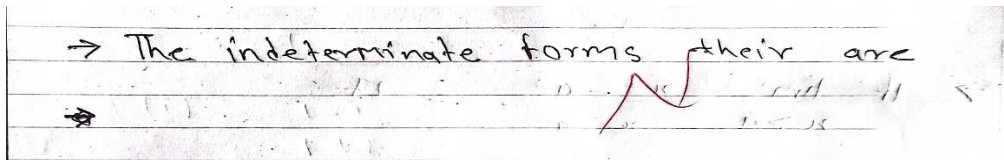
Student B: I don't know.

Student C : may be.

From above discussion it can be concluded that students have misconception on symbols and they are unaware to generalize the meaning of symbols. Students are unknown on either it x touches 5 or not? Student define left hand limit on their answershit but in interview they are unable to elaborate. Students C was unknown whit the symbol $\lim_{x \rightarrow 2^-}$ but it define Left hand limit as the limit of function from left side. Student remembering definition but he is unaware about the meaning of definition. Student are unable to generalize the meaning of symbol $\lim_{x \rightarrow 2^-}$. With the help of two sequences researcher is trying to get their knowledge on limit but students are unknown with the limit of sequence. They are unaware about the limit of sequences is 1. Students are also unaware about the words tends to or approaches to can be replaced by converges to. In a Study of Monaghan, (1991) it was concluded that "To the mathematician, the phrases tends to, approaches, converges and limit are interchangeable, that student are more familiar with the phrases tends to and approaches than converges". To a large extent, this is seen by the students but there are many disturbances in the pattern. Approaches appears to present the least difficulty to students because it is a vague term. Tends to is often seen as similar in meaning to approaches in mathematical contexts although its everyday use does not suggest limit situations. Both phrases are given a dynamic interpretation. Monaghan, (1991) "Converges is confusing in that its everyday meaning is strongly associated with lines converging". Students are unaware on many symbols and their meaning they are also unaware that mathematician the phrases tends to, approaches, converges and limit are interchangeable. From above I had concluded that they have difficulties on

generalizing the meaning of symbols used in limit. In the study it was seen that students have difficulties on phrases tends to, approaches, converges and limit are interchangeable.

Infinity, undefined term and indeterminate forms. Infinity, undefined terms and indeterminate forms. Various researches shows that student found similar meaning of these terms. These terms are used in limit and students found difficulties either these exist or not? For finding difficulty in indeterminate forms of limit, by the APOS theory one action level question was asked. The question was "What do you mean by indeterminate forms? State their forms". Only 23 students attempt the question. On the basis of students answer researcher conducted the interview. Answer of one of the interviewer is given below:



The above answer of student showed that student have no clear idea on indeterminate forms of limit. The student was not completed the answer of the question. Student tried to attempt the question but he didn't complete it. From this it is clear that student have difficulty on the indeterminate form. Also, according to a study of Adhikari(2020) student has misconception on indeterminate and infinite, limiting and functional value of the function, and symbolic representation of infinity. So, it can be concluded that student have difficulty in indeterminate forms. To find the such difficulties following interview was conducted:

I: What do you understand by the symbol ∞ ?

Student A: Infinity

Student B: It is a very big number, which we are not able to count.

Student C: It is undefined term.

I: Does ∞ exists?

Student A: I don't know

Student C: umm....

I: What do you understand by indeterminate forms?

Student A: I don't know.

Student C: Some forms like $0/0$ are indeterminate forms.

I: Why these forms are called indeterminate forms?

Students C: I don't know

I: If a non-zero number is divided by zero. Is that indeterminate form?

Student C: Yes, sir, it is infinity so it is indeterminate form.

From above discussion it is clear that students have misconceptions on infinity and indeterminate forms. Students unable to say whether infinity exist or not? It seems that students have difficulty that infinity does exist or not? Jutler (2006) beliefs that with respect to infinity, a point of debate among earlier mathematicians and still a challenge for students as cited in (Sebsibe, 2019). They are also unaware that infinity does not exist. Student C says that infinity and indeterminate form is same. It seems that student c has no complete information on infinity and indeterminate forms. He has misconception that infinity is indeterminate form. According to Jones, (2015) "One difficulty with the concept "infinity" is considering it as an object or plugged in infinity as a number to calculate the limit at infinity while process view is required" as cited in (Sebsibe, 2019). These words that infinity, undefined terms,

indeterminate forms have misconception on student's mind. In a Study of Jaffar&Dinyal (2011) found that the students possess varying interpretations of the words infinite, does not exist, indeterminate, undefined, and non-applicable. They concluded that the students in the present study have made generalizations of their own when they surmised that classifying as undefined or as does not exist means the same thing. According to (Dictionary.com, 2017) an indeterminate form is an expression involving two functions whose limit cannot be determined solely from the limits of the individual functions. Infinity, the concept of something that is unlimited, endless, without bound. In mathematics, the term undefined is often used to refer to an expression which is not assigned an interpretation or a value". Students are unable to define what the exact mean of these words is and what connections between these words is. The seven forms are called indeterminate forms due to their value cannot be determined or value is not fixed so indeterminate forms are undefined. Infinity is boundless, unlimited and endless. It is imaginary number so it does not exist.

To find such cause of such difficulties following interview was conducted as I asked the "student *why did you find confused on these terms on infinity, undefended term and indeterminate form*"? Student A replied: "*May be I was not in the classroom at that time of teaching*". On the same question student C told that "*Teachers teach the problem with explanation and ask the general and regular question like, understood? Most of the question is answered in group, yes sir even I dint have any idea on that*". Again I asked to student C "*why did you asked again with the teacher*" student replied "*I feel afraid to ask again*"

From above discussion it seen that student feel difficulties due to not regularity on the class room and some student didn't asked to teacher that they have

confusing even teacher asked with total classroom either they understand or not. Study of Joshi(2019) found that Students are afraid to be mocked at by their peers during the lessons, carelessness. So, it seems that student feel difficulties due to students are not regular on the class, some student could not ask with teacher about their doubts.

All of the above discussion justified the fact that students have misconception on notions and symbols. Students are unable to generalize the mathematical meaning of that combinations of the symbols. They are unaware to generalize the meaning of $\lim_{x \rightarrow 2}$. The combination of symbols gives a mathematical meaning. Students are unable to say whether infinity exist or not? They are also unaware that infinity does not exist. They are also unaware that mathematician the phrases tends to, approaches, converges and limit are interchangeable. Students have misconceptions on infinity and indeterminate forms. Study of Jaffer add Denyal (2011) found that “the respective students could have acquired knowledge of the terms such as indeterminate, by acquiring the knowledge on their own. The misconception that indeterminate has the same meaning as undefined, is a misconception that might have been constructed by the students themselves”. Parameswaran (2007) “Students tend to have a dynamic process view but not an object static view of limit, with the consequence that they cannot interplay between two” as cited in (Tarmizi, 2010). Adhikari (2020) found that student has misconception on indeterminate and infinite, limiting and functional value of the function, and symbolic representation of infinity. Similar types of result was seen in my study. So, It can be concluded that they had difficulties on objectify the meaning of phrases like infinity, indeterminate forms, undefined terms, exist or does not exist. Students have also difficulties on connecting relations of these phrases. Reason behind this was found students did not get attention on the classroom.

Students are not regular on the class, some student could not ask with teacher about their doubts.

Difficulty in Process

Process is the way of solving problems. If students are unaware about the way of solving the problem then they could not find the right solution of the problems. In this section researcher were trying to find difficulties regarding to finding limit of square root function, trigonometric function and exponential function. Solving routine problems is performed with minimal understanding of the mathematical principles involved (Heibert& Carpenter, 1992) as cited in (Abbey, 2008). If the students are able, to find limit of square root function, trigonometric function and exponential function then it is considered that they have no difficulty but if they could not find the limit of such problems then it can be considered that student have difficulty in finding limit. To find out the in process of finding limit I used the one question related to square root function, two questions related to trigonometric function and one question related exponential function on the test.

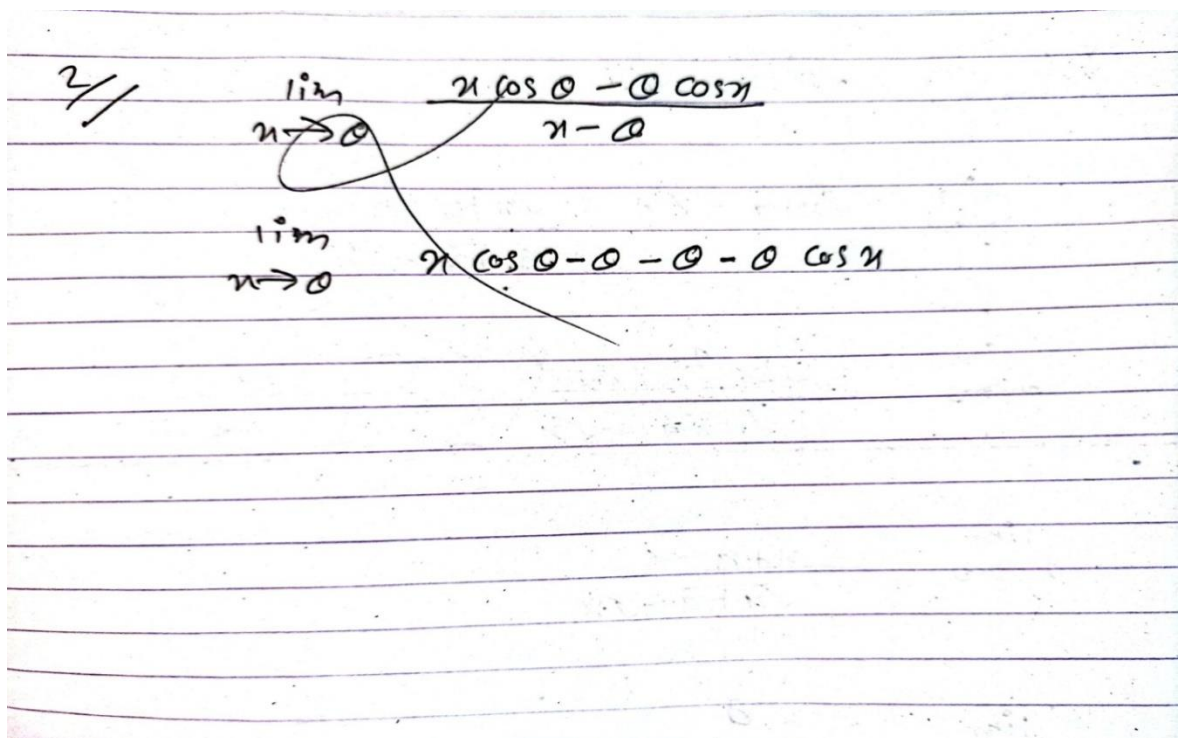
Difficulties in square root functions. This difficulty refers the difficulty in finding limit of square root function. Only the process of finding limit was seen to find difficulty in this category. If the student cannot able to find the correct answer of the given problem then such difficulty is categorized as difficulty in in square root function. To find out the difficulty one process level question was to the student.

Which is " Find the limit of $\lim_{x \rightarrow \infty} (\sqrt{x+a} - \sqrt{x})$ ".

In this case, it was found that the respondents had correctly found the limit of the given problem. From this I came to conclude that there were no difficulties in the finding limit of square root function. Because I had mentioned the difficulties as

"Whether the students can or cannot find out correct answer of the problem. In this way all the respondents find out the correct answer of the given square root function. All of the fact mentioned above justified that there was no difficulty in finding limit of square root function.

Difficulty in Trigonometric function. This difficulty refers to difficulty in finding limit of the trigonometric functions. Only the process of finding limit was seen to find difficulty in this category. If the student cannot able to find the correct answer of the given problem then such difficulty is categorized as difficulty in in trigonometric function. To find out the difficulty two process level question were to the student on the test. Which were " Evaluate: $\lim_{x \rightarrow \theta} \frac{x \cos \theta - \theta \cos x}{x - \theta}$ and Find the limiting values of $\lim_{x \rightarrow 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta}$." The student A and student C had no difficulty in finding limit of trigonometric function. But student B had some difficulty. He easily done the second question but he could not find the limit of the first trigonometric function. The answer given by the student B is given below:



When the above solution of the student was analyzed, The student was trying to solve the problem but he was unable to get idea to solve the problem. It can be concluded that student had difficulties in trigonometric functions. To ensure the difficulty, I conducted interview which can be presented as below:

I : What is limiting value of $\sin x/x$ if x tends to 0?

Student B: 1

I : What is the limiting value of $\cos \theta$ if θ tends to 0?

Student B : 1

I: What is the range of $\sin x$?

Student B: I don't know sir.

I: Is that possible that limiting value of $\sin x/x$ is greater than 1 or less than 1?

Student B : sorry ...

I: why do you incomplete your answer for the question.

Student B: Well I don't know any idea about this. I was absent when my teacher taught this chapter.

The above discussion shows that even some student did not have any difficulty but some student had difficulty in finding limit of trigonometric function. It can be seen that student B had idea on limiting value of trigonometric function but he had difficulty to generalize the idea. The study of Adhikari (2020) found that student's difficulties in solving a limit problem have aroused because of the lack of conceptual clarity on the limit of the function. Due to lacking of conceptual knowledge required to solve the trigonometric function student felt difficult. Student B was unaware on that range of $\sin x$ is between -1 and 1. So, some students had difficulty on finding limit of

trigonometric function. The reason that the student felt difficult was found that students were irregular on the classroom.

Difficulty in Exponential function. This difficulty refers the difficulty in finding limit of exponential function. Only the process of finding limit was seen to find difficulty in this category. If the student cannot find the correct answer of the given problem, then such difficulty is categorized as difficulty in exponential function. To find out the difficulty one process level question was asked to the student on the test, which was " Evaluate: $\lim_{x \rightarrow 0} \frac{e^{px}-1}{e^{qx}-1}$."

In this case, it was found that the respondents had correctly found the limit of the given problem. From this, I came to conclude that there were also no difficulties in the finding limit of exponential function. Because I had mentioned the difficulties as "Whether the students can or cannot find out correct answer of the problem". In this way, all the respondents gave the correct answer of the given exponential function. All of the fact mentioned above justified that there was no difficulty in finding limit of exponential function.

Finally, from all of above discussion, students had only some difficulties regarding the process. They were able to solve the problem related to finding limit of exponential function and square root function. Blaisdell (2012) "Students in general had less difficulty with graphical representations than mathematical notation or definition questions". This shows that student had more difficulties on mathematical definition and notation than the problem solving. Problem solving is the process of finding answer, they away some way according to questions to find the answer. I have concluded that student no difficulties on finding limit of exponential function and square root function. But some student had difficulty on trigonometric function. In the

case of trigonometric function, student knows the formula but they could not generalize the idea to solve the problem.

Chapter V

FINDINGS, CONCLUSIONS AND IMPLICATIONS

This chapter deals with the summary, findings, conclusion and recommendation concerning the student's difficulties in learning limit. After rigorous analysis and interpretation of data, the findings of the study have been derived and conclusions have been made based on findings, the implications have been forwarded in different levels. This chapter is divided into three sections findings, conclusion and recommendation for educational implication.

This study was entitled "Exploring Learning Difficulties of Grade XI Students on Limit." The purpose of this study was to explore the difficulties of students on learning limit. The design of the research was case study. The study was conducted by purposely chosen case students of grade XI of Gautam Buddha Secondary School of Kapilvastu District. To achieve the objective of the study, data and information were collected through test and interview was conducted with three students selected on the basis of test result. The researcher analyzed the interview data according to General inductive approach by Thomas (2006). Cross matching and triangulation ways were used to analyze the collected data. Hence, from the deep analysis and interpretation of the collected data, the derived findings can be presented below:

Findings and Discussion

The findings of the study were drawn from the deep and systematic analysis and interpretation of the collected data. This study was case study about the learning in limit. The main purpose of this study was to explore the difficulties in limit. For this purpose, I included four types of difficulties. First one is difficulty in concept of

concept of limit the study was found difficulty in definition of limit, indeterminate form and two-sided limits. Similarly, understanding is the second dimension of my conceptual framework. This study also tried to find the difficulties on understanding meaning of that $|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$, value of function and limit of function. Third one is difficulties in language to found the difficulties on language used to express limit and its problem presented by different ways by using symbol or words. The difficulty in language the study was found difficulty in meaning of $\lim_{x \rightarrow 2}$, meaning of phrases approaches and infinity, undefined terms, exist or does not exist. Last one is the difficulties in process of solving problems that tried to found difficulties on finding limit of square root function, trigonometric function and exponential functions. From this case study the results of the data analysis showed that students felt difficulties in following areas:

Weak concept on definition of limit.

Students could not define limit on their own words. Students had no clear concept about the limit. They were unable to define what the limit exactly. They had a concept that limit is a math function, but they were struggling to understand the meaning of limit. They were not able to understand what the limit is exactly. They had a misconception about limit of a function is a particular value or the limit of a function is again a function. In the case of formal definition, they are unaware about the situation $|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$ gives limit of $f(x)$ is L for x tends to a . Students were unable to make relation between ϵ , δ . They were unable to found that the value of δ is depend upon value of ϵ .

Difficulty on finding of two-sided limit.

From the deep analysis of the data, it was found that students felt difficult to connect the definitions with examples. Students were memorizing the definition of Left-Hand Limit and Right-Hand limit but they didn't know exact meaning of it. They were trying to answer that they had memorized but were unable to write total answer of the question. They mostly focused on memorizing the definitions. They were unable to connect memorized definitions with examples. Student had difficulties on graphical questions. They were surprised on graphical questions. None of them attempted these graphical problems. Student were unable to answer the left-hand limit and right-hand limit. All interviewer said that they saw this types of question, first time. They never solved such types of questions. Students had difficulty to find of two-sided limit of graphical questions.

Unable to differentiate between value of function and limit of function.

From deep analysis, it was found that student had not a clear idea that the difference between value of function at point and limit of the function at that point. Students were unable to define limit of a function and value of function. They felt these term as same. Students had difficulty on graphical questions. They were surprised on graphical questions. None of them attempted these graphical problems. This showed students had difficulties in graphical representation on value of function at a point and limit of function at same point. Students were not able to give the value of given function at $x = 3$. They didn't speak and give my answer. They also did not give answer of the what is the value they found of $f(3)$ in the given graph. From this, it seems that students were unable to find the limit and of the graphical function.

Misconceptions on terms used on limit.

From the deep analysis of the data it was found that students have misconception on symbols and they are unaware to generalize the meaning of symbols. Students are unknown on either it x touches 5 or not? Student define left hand limit on their answers but in interview they are unable to elaborate. Students were unknown with the symbol $\lim_{x \rightarrow 2^-}$ but it define Left hand limit as the limit of function from left side. Student remembering definition but he is unaware about the meaning of definition. Student are unable to generalize the meaning of symbol $\lim_{x \rightarrow 2^-}$. With the help of two sequences researcher is trying to get their knowledge on limit but students are unknown with the limit of sequence. They are unaware about the limit of sequences is 1. Students are also unaware about the words tends to or approaches to can be replaced by converges to. To a large extent, this is seen by students but there are many disturbances in the pattern. Approaches appears to present the least difficulty to students because it is a vague term. Tends to is often seen as similar in meaning to approaches in mathematical contexts although its everyday use does not suggest limit situations. Both phrases are given a dynamic interpretation.

Students have misconceptions on infinity and indeterminate forms. Students unable to say whether infinity exist or not? It seems that students have difficulty that infinity does exist or not? They are also unaware that infinity does not exist. Student found infinity and indeterminate form is same. Student has no complete information on infinity and indeterminate forms. He has misconception that infinity is indeterminate form. These words that infinity, undefined terms, indeterminate forms have misconception on student's mind. Students are unable to define what the exact mean of these words is and what connections between these words is.

Conclusion

From the deep analysis, interpretation and findings, I concluded that the students have more difficulty in learning limit. Based on above findings, I concluded that the students had no any difficulties on process of finding limit although one respondent had weak understanding on solving the problem of trigonometric function. Also, I concluded the main causes of difficulties are that the students were focusing more on solving the problem using formula, lack of graphical understanding of function, irregularities of students, lack of pre knowledge, weak performance of teachers etc. Hence, it can be concluded that the teacher are focusing only numerical problem besides providing the concept of the terms of limit and students become exam oriented which makes the limit as one of the difficult topic. Thus, both teachers and students should focus the concept of the limit and related terms and pre knowledge of function such that they could easily understand the concept of limit and teacher must change their teaching style by which they can make their classroom very fruitful and learning limit become a meaningful which avoids the rote learning.

Educational Implications

- Teacher should teach limit by applying the basic concepts.
- Before starting the chapter, the teacher has to give the fundamental knowledge about the topic, conceptual teachings are must needed.
- Teacher should encourage his/ her students to focus on process rather than product.
- The teacher should try to find out the reason about the difficulties.

- Teacher should do the diagnostic test to identify the area of difficulty and must use remedial teaching to avoid the difficulties.
- It is important for students to make clear connections between symbolic representations relating to limit problem. .
- Students should be regular in their classes.
- Students should also be given opportunities to explore a range of situations in which they are required to construct, interpret.

Recommendation for Further Study

The researcher had tried to make some suggestion for further study in this field.

The following suggestions could be beneficial for further research study:

- This type of research can be conducted in the higher level.
- Further research can be done in the area of function and derivatives.

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Appendix-A

Written Test

The objective of my study is to find the difficulties in learning limit so attempts all the questions as far as you can.

Answer the following questions.

1.

- What is the meaning of x tends to 2. Define limit on your own words.
- What are the indeterminate forms? State their different forms.
- What do you mean by Left-hand limit and Right-hand limit of a function? Write formal definition of limit (ϵ , δ).
- Show that $\lim_{x \rightarrow \frac{2}{3}} \left(\frac{2}{2-3x} \right)^1$ does not exist.
- Distinguish between limit of a function and value of a function
- If $\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = 27$, find all the possible value of a .
- Do the $f(x) = \frac{x-1}{x+2}$ define for the value at $x=1$.

2.

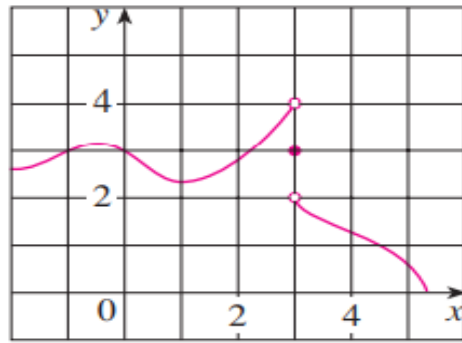
- Find the limit of $(2x^2 + 3x - 14)$ when x approaches to 2.
- Evaluate: $\lim_{x \rightarrow \theta} \frac{x \cos \theta - \theta \cos x}{x - \theta}$
- Find the limit of $\lim_{x \rightarrow \infty} (\sqrt{x+a} - \sqrt{x})$
- Find the limiting values of $\lim_{x \rightarrow 0} \frac{1 - \cos 4\theta}{1 - \cos 6\theta}$
- Evaluate: $\lim_{x \rightarrow 0} \frac{e^{px} - 1}{e^{qx} - 1}$

3. For the function f whose graph is given, state the value of each quantity.

a) $\lim_{x \rightarrow 3^-} f(x)$

b) $\lim_{x \rightarrow 3^+} f(x)$

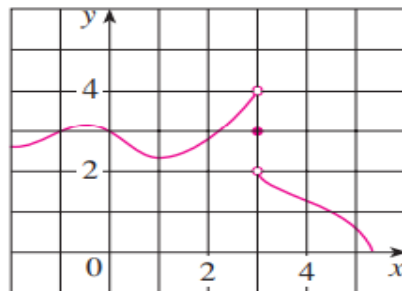
c) $\lim_{x \rightarrow 0} f(x)$



4. For the function f from the given graph if it exists. If it does not exist. If it does not exist, explain why.

a) $\lim_{x \rightarrow 3} f(x)$

b) $f(3)$



Appendix-B

Interview Guidelines

The following are the guidelines for interview after students completed the test. On the basis of following area the interview will be conducted.

1. The definition of limit.
2. Right hand limit and Left hand limit.
3. Phrases used in limit as: infinity, indeterminate forms undefined term, approaches, tends to etc.
4. The difficulties while finding limit of the function.

Some sample questions are as given below:

1. Which problem was the easiest and why?
2. Where there any problem where you did not know about what to do?
3. For the problem where you did not know how to solve, how did you come about your answer? Did you guess or was their reasoning behind your response?
4. What do think about limit as x tends to a ?