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Vibration Analysis of a Simply Supported Beam with a Central Mass with and without Dynamic Vibration Absorbers

by

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The undersigned certify that they have read, and recommended to the Institute of Engineering for acceptance, a thesis entitled **"Vibration Analysis of a Simply Supported Beam with a Central Mass with and without Dynamic Vibration Absorbers"** submitted by **Harikant Yadav (079MSMDE010)** in partial fulfillment of the requirement for the degree of Master of Science in Mechanical Systems Design and Engineering.

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ABSTRACT

This paper presents an analytical and simulation-based approach to understanding the behavior of a system of a simply supported beam, considering both cases with and without a dynamic vibration absorber. The model was formulated based on Euler-Bernoulli beam theory and solved using the Galerkin method.

The analytical solution for a beam without a dynamic vibration absorber resulted in a mid-span deflection of 1.62×10^{-4} m, which was found to be in close agreement with the simulation model, giving a result of 0.42×10^{-3} m. This confirms the correctness of the formulation. Based on tuning conditions, a dynamic vibration absorber was included in the system, consisting of a mass of 0.2 kg and a spring stiffness of 50000 N/m.

The inclusion of a dynamic vibration absorber resulted in a reduced mid-span deflection to 0, as obtained using the analytical model, and 4.6×10^{-5} m, as obtained using the simulation model, thus validating the model.

Keywords: Dynamic Vibration Absorber, Harmonic Analysis, Galerkin Method, Simply Supported Beam

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LIST OF SYMBOLS

L	Length of the beam	m
ω	Angular frequency	rad/s
A	Cross Section Area	m^2
I	Mass Moment of Inertia	m^4
ρ	Density of Beam	kg/m^3
K	Stiffness	N/m
E	Modulus of Elasticity	N/m^2

LIST OF ABBREVIATIONS

FEM	Finite Element Method
DOF	Degree of Freedom
DVA	Dynamic Vibration Absorber
P.E	Potential Energy
K.E	Kinetic Energy
Hz	Hertz
ANSYS	Analysis System (Software)
FEA	Finite Element Analysis
BC	Boundary Condition
ODE	Ordinary Differential Equation
SDOF	Single Degree of Freedom
N	Newton
N/m	Newton per meter

CHAPTER ONE : INTRODUCTION

1.1 Background

Most human activities daily include different forms of vibrations. For example, human beings can sense the vibrations of light waves in order to see, and the vibrations of eardrums in order to hear. Vibrations are of great importance when designing machines, buildings, engines, turbines, and other structures in the field of engineering.

Vibration and oscillation are terms that characterize any repetitive movement at regular intervals of time. The vibration system includes elements where potential energy is stored, elements storing kinetic energy, and elements through which energy dissipates. Therefore, vibrations represent a cyclic transformation of potential energy into kinetic energy and vice versa. Thus, the vibration process occurs until the system is damped; in such cases, energy dissipation takes place in each cycle of vibration and must be supplied externally to sustain vibrations (Rao, 2011).

The first method to reduce the vibrations is to reduce them at the source, which can be done by designing the equipment so that control is maintained over the tolerances while manufacturing the equipment. Others method which can reduce the vibrations which are generated due to the machinery is to make the system in such a manner that the natural frequencies are not close to the operating speed, so that the responses are not high due to the addition of damping, the addition of isolating devices in the system, and the addition of an auxiliary mass in the equipment (Ramamurti, 2008).

Tunable vibration neutralisers are presently being utilised to control harmonic vibration in structures at the point where they are connected and the manner in which the characteristics affect the neutralisers' performance in this capacity is well understood. However, the application of tunable vibration neutralisers in the control of global vibration in a structure has not been sufficiently explored, and in this article, the parameters of the neutralisers that affect the kinetic energy of the vibrational motion are highlighted. However, as is shown, the tuned vibration neutralisers must be positioned appropriately on the structure (Brennan & Dayou, 2001).

1.2 Problem Statement

In mechanical systems, unwanted vibrations arise, which are caused by dynamic loading, external forces, or internal interactions. These unwanted vibrations cause inefficiency, increased noise, discomfort, fatigue, and a short lifespan of mechanical components. The conventional approach of increasing stiffness or damping may not be feasible, especially in flexible or light structures. This has given rise to the need for a more efficient solution to overcome unwanted vibrations without making major changes to the original system. The concept of a vibration absorber, typically in the form of an auxiliary mass-spring-damper system, has been shown to be an effective solution in eliminating resonant vibrations. The aim of this work is to investigate and design an efficient vibration absorber to reduce unwanted vibrations in the original system subjected to harmonic excitations.

1.3 Thesis Objectives:

1.3.1 Main Objective:

The objective of this thesis is to study dynamic behavior of a simply supported beam with a central mass, with or without a vibration absorber.

1.3.2 Specific Objectives:

- To develop a mathematical model for the simply supported beam with a central mass with and without vibration absorber.
- To determine analytical solution for harmonic response of system with and without vibration absorber.
- To compare and analyze the analytical result with simulation result for the given system,

1.4 Limitations

The limitations of research works are:

- The analytical approach is based upon the assumptions that the material behavior is linear, small deflections are involved, and the boundary conditions are perfect, which might fail to represent the reality of structural elements during large deformations or varied loads.

- The vibration absorber is considered as a mass spring damper system. In practical approaches, the components of the absorber might have nonlinear stiffness, variable damping or defects due to manufacturing process.
- Other effects, including temperature variations, humidity, wear, or external disturbances other than harmonic, are ignored, which may affect the performance of the system.
- The study only considers the effect of harmonic loading, while in practice, the structures may experience random, transient, or other types of loading, which may necessitate the use of more sophisticated types of vibration absorbers.
- Nonlinear effects, including geometric stiffening, friction, or contact, are ignored, which may affect the performance of the system, especially when the amplitude of the vibrations is high.

1.5 Structure of the Thesis

The thesis has been structured in seven chapters. Below is a brief outline of the contents of each chapter.

- Chapter 1 outline basic introduction, problem statement, objectives, and limitations of the thesis work.
- Chapter 2 presents extensive literature survey of related past works presents.
- Chapter 3 is for Methodology adopted for thesis.
- Chapter 4 considers developing the equation of motion for the system, assuming the beam to be a Euler Bernoulli beam.
- Chapter 5 includes analyzing the analytical solution obtained through the mathematical model and validating the model through ANSYS workbench.
- Chapter 6 draws conclusions from the results of the study and highlights recommendations for future research.
- Appendix is for providing supporting information of the thesis work.

CHAPTER TWO : LITERATURE REVIEW

2.1 Overview of a Simply Supported Beam

A simply supported beam is a structural member supported at both ends, usually by a pin and a roller, allowing rotation but preventing vertical displacement. This implies that there will be no deflection and no bending moment along the support locations. The vibration behavior of simply supported beams is widely studied due to their practical applications in structures like bridges and machine components. Classical beam theories such as Euler–Bernoulli provide the basis for analyzing their dynamic response. These models usually assume uniform cross-section and small deflection conditions. Because of their simple boundary conditions, simply supported beams are commonly used in vibration analysis to study natural frequencies, mode shapes, and response under harmonic excitation.

2.1.1 Mathematical Modeling

When creating a mathematical model, it might be essential to make certain assumptions to eliminate insignificant effects from the analysis and to simplify the problem while maintaining suitable accuracy (Luintel, 2024).

2.1.2 Mathematical Solution

The governing equation of the motion is subsequently solved utilizing a suitable mathematical approach. An analytical solution in closed form is favored when the governing equation is in a straightforward and standard format (Luintel, 2024).

2.1.3 Assumed Mode Method

The assumed mode method can be applied alongside the Lagrange equation to transform the governing equations of any continuous system represented by partial differential equations into a set of ordinary differential equations that only have time as the independent variable.(Luintel, 2024).

2.1.4 Previous Researches

(Idris, 2015), studied the principle of dynamic vibration absorber implemented to a simply supported beam system with multiple absorbers in order to reduce vibration or amplitude. In this study, two main approaches are adopted namely analytical equations and finite element analysis. MATLAB software was applied to graphically demonstrate the analytical equations and also to validate the results of ANSYS finite element simulation. The results obtained showed that the analysis of a simply supported beam using analytical equations and finite element simulation of ANSYS produced an almost similar result. The frequency range considered was from 5Hz to 1000Hz and four modes of resultant shapes were generated. In addition to this, other studies were performed by changing the absorber location configurations. Moreover, a single absorber and multiple absorbers were also placed in order to determine their effect on the reduction of vibration's amplitude percentage. A total average reduction of vibration percentage achieved through the multiple absorber systems was 88.6% while only 8.21% was attained through single absorber system.

(Jaini, 2014), studied the fixed end beam is the structure element held from two ends that undergoes flexure while carrying load since it is subjected to vibrational force due to its capacity to carry vertical force and gravity. It was established that the dynamic vibration absorber can lower the vibrations in the beam while the third condition emerges as the most suitable configuration considering the continuous decrease of 95% for the first DVA and 99% for the second DVA. The acquired information from this study can be applied to lessen vibration amplitude in structures and machinery, prolonging their life span at once.

(Jusoh, 2015), based the dynamic vibration absorbers have been attached to the fixed-fixed end beam under four different conditions in accordance with their placements. Comparison between vibration amplitudes of the beam structure without DVA and with the DVAs attached to it was made. It could be seen from the simulation results that there is a reduction in vibration amplitude of the beam structure at its natural frequency near zero while the amplitude of DVAs is increasing. Vibration amplitudes of both, the fixed-fixed end bean and DVAs can be seen from the graphs plotted. It shows that the DVAs can absorb vibrations in the beam structure.

(Zainulabidin et al., 2012), founded experimental analysis on the transverse vibration of a fixed-fixed end beam with dynamic vibration absorbers (DVA) has been conducted. The DVA consists of a flexible beam with two masses located symmetrically on both sides of the beam. The fixed end beam was held in a static frame structure and the DVAs were then attached to it. One end of the beam was harmonically excited by an electric shaker. An accelerometer was attached to the beam center for measurement purposes. The vibration response amplitudes and the natural frequencies of the beam were noted. The dynamic vibration absorbers were evaluated in four different situations depending on their location. Comparison between the amplitudes of the beam vibrations before and after the attachment of the absorbers was done. The results showed that the DVA was effective in reducing the vibration of the beam.

(Yang et al., 2011), discussed the application of dynamic vibration absorbers in controlling vibrations of a structural system in the frequency domain both in narrowband and wideband. It was revealed that the coupling effect between the plate and the absorbers through the reaction forces of the absorbers controls the process in the narrowband whereas in the wideband the dissipation effect through the absorbers' damping is more important. In the case of increasing the bandwidth of control, the placement of the absorbers will not only depend on the target mode but also on other plate modes. These placements should be considered only when a compromise is achieved between the target mode and the other interacting modes. The numerical results were analyzed using a simply supported plate and good correlation between the predicted and experimental results was achieved.

(Webster et al., 1992) describes the effective use of the tuned mass damping system in addressing the steady state vibrations of the long span, cantilevered, composite floor structure of the Terrace on the Park Building in New York. The successful application of the tuned mass damper system in addressing the problem of the building provides a basis for applying TMD in other composite structures. The applicability of the TMD in other composite floors is examined, as well as areas for future research.

(Ghazali, 2015), studied uncontrolled vibrations in machines and structures can lead to increased stress, energy losses, wear, bearing loads, fatigue, and discomfort, ultimately damaging equipment and reducing its lifespan. This research focuses on developing a tuned vibration absorber (TVA) to mitigate these issues. Using finite element analysis and experimental validation, two TVA designs were suggested and evaluated. Design 1, selected for manufacturing based on superior FEA performance, was tested using DEWEsoft-201 equipment. The experimental results closely matched the FEA data, confirming the effectiveness of the TVA. The new absorber, weighing 620.6 kg, effectively reduces vibrations and is suitable for mobile applications.

2.2 Research Gap

Although a lot of research work has been supported out on vibration absorbers for suppressing vibrations in structural systems, there are still a number of important research gaps that need to be addressed. The major cavity is that most researchers have concentrated their analysis on simple models of vibrations, while real structural systems may exhibit nonlinear behavior. The nonlinear behavior may be initiated by large deformation, material nonlinearity, or a complex loading condition and this may affect the performance of the vibration absorbers. Another gap is that most researchers have concentrated on simple mass representation, while the interaction of the vibration absorbers with structural systems such as a beam, plate, or a rotating shaft is a continuous system, and this representation is not simple.

In addition, little research work has been done on the optimal placement of a dynamic vibration absorber and additional masses to a beam for maximum vibration reduction.

CHAPTER THREE : RESEARCH METHODOLOGY

3.1 Conceptual Framework

The research will be carried out with the following methodology:

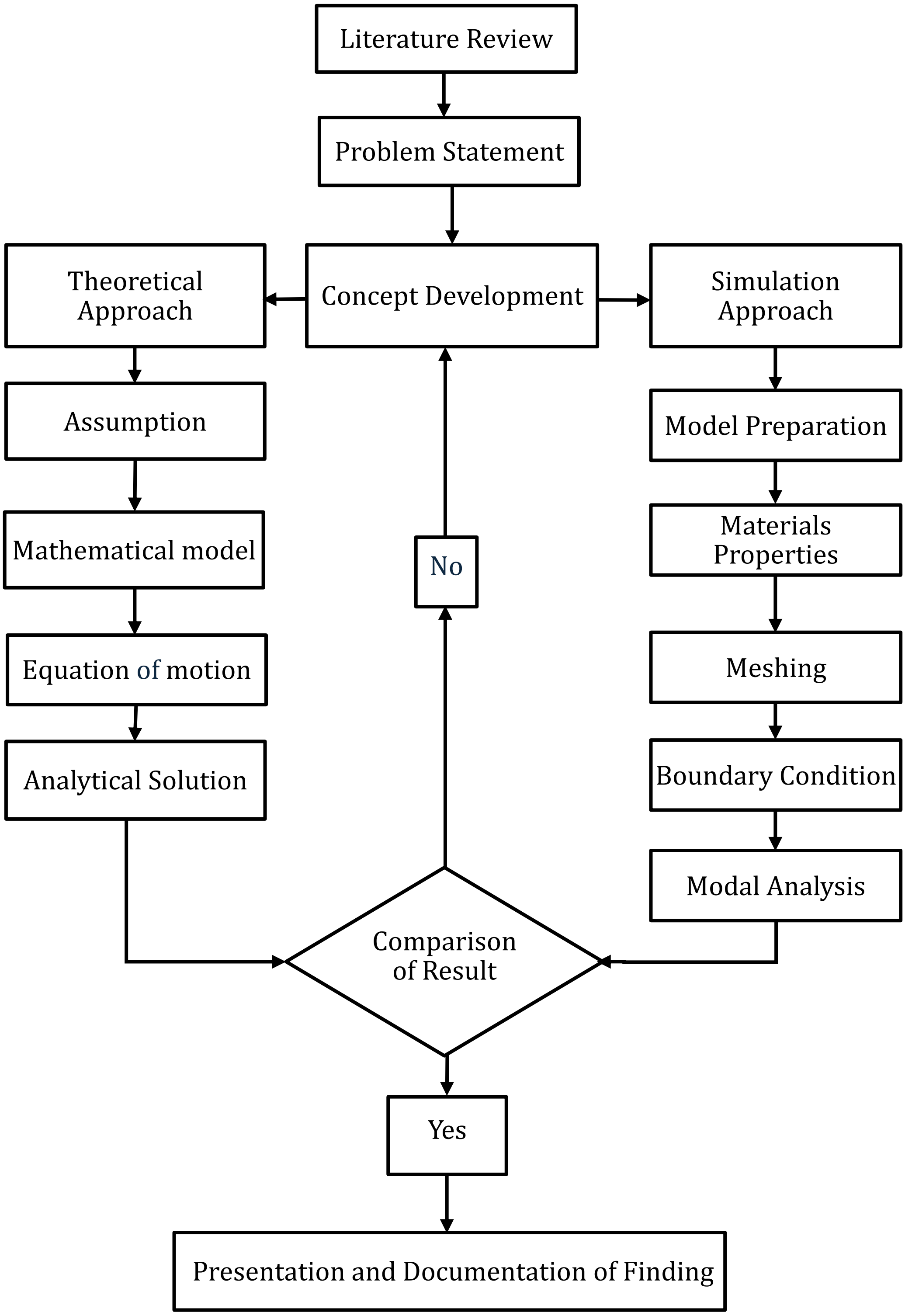

Figure 3.1 Flowchart of research methodology

This chapter describe the methodology vibration analysis of a simply supported beam. The research follows a structured approach as shown in figure 3.1 Flowchart of research methodology. The methodology includess of both theoretical and simulation-based analysis to predict the dynamic response of the beam under harmonic excitation.

3.2 Problem Statement

The research focuses on the vibration analysis of a simply supported beam with a central mass and a dynamic vibration absorber under harmonic excitation. While many studies consider beams without control systems, limited work addresses vibration reduction using absorbers. Therefore, this study investigates the effect of absorber parameters on reducing beam displacement and improving dynamic performance.

3.3 Literature Review

For the purpose of developing an understanding of the vibration behavior of simply supported beam and the use of dynamic vibration absorbers, a thorough literature review was conducted. Various sources of literature, including books, academic reports, previous theses, and international journal publications, were referred to and included in the review process. They made use of the IOE's previous thesis and their access to numerous international journals. The relevant prior literature was reviewed and grouped based on the type of issue they addressed, the approach they took, and an extensive evaluation of their main conclusions.

3.4 Concept Development

Based on the literature review and identified research gap the conceptual framework for this study was developed to systematically analyzed the vibration behavior of simply supported beams.

3.5 Theoretical Approach

Mathematical models have Developed based on “Euler-Bernoulli beam theory” for simply supported beam. The differential equations describing the dynamics were formulated and solved under suitable boundary conditions.

3.5.1 Assumption

- Linearity: The beam experience small deflection, so geometric nonlinearity is negligible.
- The beam consists of Mild Steel with uniform material properties.
- Uniform cross-sectional area along the axial direction.
- Damping Effects are not considered.

3.5.2 Mathematical Model

The Euler-Bernoulli beam theory is used to represent the simply supported beam. The cross-sectional area A and Moment of Inertia I are expressed as functions of axial position.

3.5.3 Equation of motion

Equations of motion for Euler Bernoulli beam model were developed by using assumed mode method and Lagrange's equations into the expressions of kinetic and potential energies of the system

3.5.4 Analytical Solution

The Maple software were utilized to calculate natural frequency for first mode for simply supported beam.

3.6 Simulation Approach

The Ansys designed modeler were used to developed 3D model of simply supported beam. Properties such as density, Young's modulus, and Poisson's ratio of mild steel material of simply supported beam were defined in the model. Simply supported boundary condition was applied to the 3D model having mesh refinement of 0.00127m.

3.6.1 Modal Analysis

Model analysis was performed to determine the mode shape and natural frequency for first mode.

3.7 Comparison of Results

The comparative analysis was carried out based on the analytical and numerical values obtained to check the accuracy and reliability of the developed mathematical model. The values obtained from the analytical solution for transverse displacement were checked at the center point and compared with the values obtained from the ANSYS simulation results. The results obtained from the problem statement were satisfactory, and a few differences were noted during certain points. This difference may be due to the ideal conditions assumed during the analytical modeling, such as boundary conditions and shear deformation, and mass distribution conditions. Although different conditions were assumed during modeling, the percentage errors were within acceptable limits.

3.8 Report Preparation

The findings of the research work will present in the conferences. Similarly, all findings are compiled as a thesis report as per the requirement of the department of Mechanical and Aerospace engineering and will be submitted to the department.

CHAPTER FOUR : ANALYTICAL AND SIMULATION MODELS

This Chapter describes the mathematical and Simulation framework that can be used to investigate the vibration response of a beam system, which has been fitted with a dynamic mass vibration absorber. The development of the governing partial differential equation begins by establishing the influence of geometric parameters, material properties and the boundary conditions on the dynamic response of the beam as it undergoes harmonic excitations. This establishment will provide an appropriate starting point for developing more complex and advanced solution techniques and enable comparisons to be made between numerical solutions and analytical solutions developed within this chapter.

4.1 Mathematical Modeling of Beam without Vibration Absorber

Development of a mathematical model of the beam without a vibration absorber begins by representing the vibrational behavior of the beam using classical beam theory. In this representation, the beam is modeled as a continuous, linearly elastic system and the vibrational behavior of the beam is described using the Euler-Bernoulli beam equation where only bending vibrations are considered. The resulting differential equations represent both the deformation and vibration responses of the system depicted in Figure 4.1 over time as influenced by the geometry of the beam, the properties of the materials from which it is fabricated, the specified boundary conditions and the applied external excitations

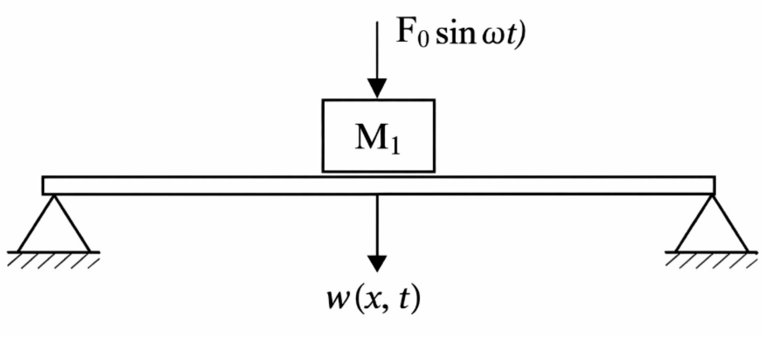

Figure 4.1 Beam without Vibration Absorber

The schematic represents a simply supported beam with supports at both ends, allowing rotation while stopping vertical displacement. A concentrated mass M_1 is located at the center of the beam. A harmonic downward force $F_0 \sin(\omega\, t)$ acts on the central mass, exciting transverse (bending) vibrations of the beam. The vertical displacement of the beam is represented by w(x, t), which denotes the represents the displacement of the beam as a function of the position along the beam and time. The use of this configuration is to provide a simple model that can be used to analyse the effects of dynamic loading on simply supported beams that are subjected to harmonic loading with a central mass attached

4.1.1 Kinetic Energy

The kinetic energy of a simply supported beam with a point mass located on its length is given by the sum of kinetic energy of the beam and point mass.

The kinetic energy of the beam, denoted as T_b,, is expressed as:

$$T_b = \frac{1}{2}\rho A \int_0^L (\dot{w})^2 dx \quad \text{Eq.(4.1)}$$

Similarly, the kinetic energy of a point mass M_1 located at a distance $x = \frac{L}{2}$ from the left support is given by are:

$$T_{M1} = \frac{1}{2} M_1(\dot{w})^2\Big|_{x=\frac{L}{2}} \quad \text{Eq.(4.2)}$$

Hence, the total kinetic energy of the system is:

$$T_1 = \frac{1}{2}\rho A \int_0^l (\dot{w})^2 dx + \frac{1}{2}M_1(\dot{w})^2\Big|_{x=\frac{L}{2}} \quad \text{Eq.(4.3)}$$

4.1.2 Strain Energy

The strain energy of the simply supported beam due to bending is expressed as:

$$Vs = \frac{1}{2}EI \int_0^l (w'')^2 dx \quad \text{Eq.(4.4)}$$

4.1.3 Non Conservative Energy

The kinetic energy of a simply supported beam with a point mass located on its length is given by the sum of kinetic

$$\mathrm{W_{nc}} = \mathrm{F_0 \sin(\omega t) w}\Big|_{x=\frac{L}{2}} \quad \text{Eq.(4.5)}$$

$$\delta\mathrm{W_{nc}} = \mathrm{F_0 \sin(\omega t) \delta w}\Big|_{x=\frac{L}{2}} \quad \text{Eq.(4.6)}$$

4.1.4 System Modelling

This area describes the system model used to depict the dynamics of a simply-supported beam with an applied point mass at a center $\frac{L}{2}$ and subjected to harmonic loading, as depicted Figure 4.2.

The model includes:

- The geometric and material properties of the beam,

- The inertia properties of the point mass,
- The simply supported beam boundary conditions, and
- The sinusoidal forces acting on the beam

This model is crucial in developing equations that describe the dynamic response characteristics of the system.

4.1.5 Equations of Motion

The equation of motion for the simply supported beam is developed with the Euler-Bernoulli beam theory and taking into consideration:

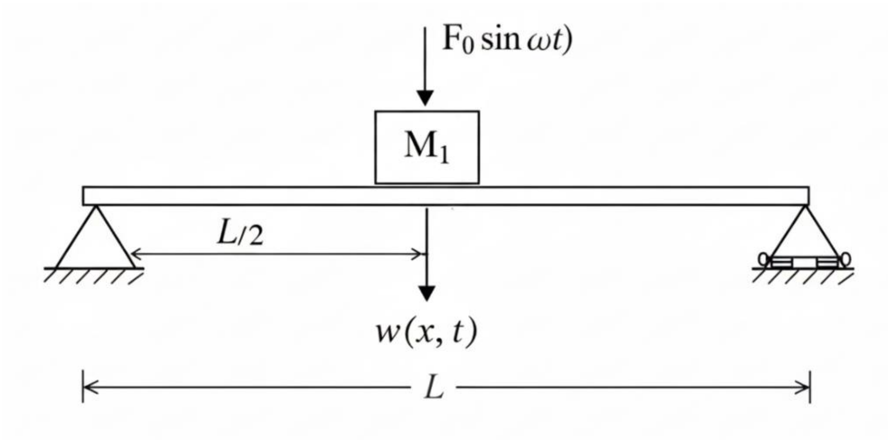

Figure 4.2 Simply Supported Beam without Vibration Absorber

Using Hamilton's principle, the governing equation of motion is expressed as:

$$\delta \int_{t_1}^{t_2} (T - V + W_{nc}) dt = 0 \qquad \text{Eq.(4.7)}$$

$$\frac{1}{2}\delta \int_{t1}^{t2} \int_0^l \rho\, A(\dot{w})^2\, dxdt + \frac{1}{2}\delta \int_{t1}^{t2} M_1(\dot{w})^2\Big|_{x=\frac{L}{2}} dt - \frac{1}{2}\int_{t1}^{t2} \int_0^l EI(w'')^2\, dxdt + \delta \int_{t1}^{t2} F_0 \sin(\omega t) w\Big|_{x=\frac{L}{2}} dt = 0 \qquad \text{Eq.(4.8)}$$

$$\rho\, A \int_{t1}^{t2} \int_0^l \dot{w}\, \delta\dot{w}\, dxdt + \int_{t1}^{t2} M_1(\dot{w})^2\Big|_{x=\frac{L}{2}} dt - EI \int_{t1}^{t2} \int_0^L w''\delta w''\, dxdt - F_0 \sin\omega t \int_{t1}^{t2} \delta w\Big|_{x=\frac{L}{2}} dt = 0 \qquad \text{Eq.(4.9)}$$

$$\rho\ A\int_0^L \dot{w}\ \delta|_{t1}^{t2}\ \mathrm{dx} - \rho\ A\int_{t1}^{t2}\int_0^l \ddot{w}\ \delta \mathrm{w\ dxdt} + M_1 w^{\cdot}\delta\ |_{t1}^{t2}|_{x=\frac{L}{2}}$$

$$- M_1\int_{t1}^{t2}\ddot{w}\delta \mathrm{w}|_{x=\frac{L}{2}} - EI\int_{t1}^{t2} w''\delta w'\ |_0^L\ \mathrm{dt} \qquad \text{Eq.(4.10)}$$

$$+ EI\int_{t1}^{t2}\int_0^L w'''\delta w'\ \mathrm{dxdt} + \mathrm{F_0 sin\omega t}\int_{t1}^{t2}\delta \mathrm{w}|_{x=\frac{L}{2}}\ \mathrm{dt} = 0$$

$$\rho\ A\int_0^L \dot{w}\ \delta|_{t1}^{t2}\ \mathrm{dx} - \rho\ A\int_{t1}^{t2}\int_0^l \ddot{w}\ \delta\dot{w}\ \mathrm{dxdt} + M_1 w^{\cdot}\delta\ |_{t1}^{t2}|_{x=\frac{L}{2}}$$

$$- M_1\int_{t1}^{t2}\ddot{w}\delta \mathrm{w}|_{x=\frac{L}{2}} - EI\int_{t1}^{t2} w''\delta w'\ |_0^L\ \mathrm{dt}$$

$$+ EI\int_{t1}^{t2} w'''\delta w\ |_0^L\ \mathrm{dt} - EI\int_{t1}^{t2}\int_0^L w^{iv}\delta w\ \mathrm{dxdt} \qquad \text{Eq.(4.11)}$$

$$+ \mathrm{F_0 sin\omega t}\int_{t1}^{t2}\delta \mathrm{w}|_{x=\frac{L}{2}}\ \mathrm{dt} = 0$$

$Now, \delta w\ |_{t1}^{t2} = 0$

$$\rho\ A\int_{t1}^{t2}\int_0^l \ddot{w}\ \delta \mathrm{w\ dxdt} + M_1\int_{t1}^{t2}\ddot{w}\delta \mathrm{w}|_{x=\frac{L}{2}} + EI\int_{t1}^{t2} w''\delta w'\ |_0^L\ \mathrm{dt}$$

$$+ EI\int_{t1}^{t2} w'''\delta w\ |_0^L\ \mathrm{dt} + EI\int_{t1}^{t2}\int_0^L w^{iv}\delta w\ \mathrm{dxdt} \qquad \text{Eq.(4.12)}$$

$$- \mathrm{F_0 sin\omega t}\int_{t1}^{t2}\delta \mathrm{w}|_{x=\frac{L}{2}}\ \mathrm{dt} = 0$$

$$\int_{t1}^{t2}\int_0^L \left[\rho A\ddot{w} + M_1\ddot{w}\delta(\mathrm{x} - \frac{\dot{L}}{2}) + EIw^{\iota v} - \mathrm{F_0 sin\omega t}\delta(\mathrm{x} - \frac{L}{2})\right]\delta \mathrm{wdxdt}$$

$$+ EI\int_{t1}^{t2} w''\delta w'\ |_0^L\ \mathrm{dt} = 0 \qquad \text{Eq.(4.13)}$$

Hence, the equation of motion is given by:

$$\rho A w^{\cdot\cdot} + M_1\ddot{w}\delta\left(\mathrm{x} - \frac{\mathrm{L}}{2}\right) + EIw^{iv} - \mathrm{F_0 sin\omega t}\delta\left(\mathrm{x} - \frac{\mathrm{L}}{2}\right) = 0 \qquad \text{Eq.(4.14)}$$

With the help of boundary conditions of simply supported beam.

A) Either $EIw'' = 0$ or $\delta w' = 0$ at x = L
B) Either $\mathrm{EI}w'' = 0$ or $\delta w' = 0$ at x = 0
C) Either $EIw''' = 0$ or $\delta w = 0$ at x = L
D) Either $EIw''' = 0$ or $\delta w = 0$ at x = 0

4.2 Shape Function

A shape function defines how a structure moves along its spatial coordinates. It is a vibration mode shape and is important for finding the system's natural frequencies. The shape functions are developed according to the boundary conditions of the system, ensuring continuity of displacement and slope, so that the beam's deformation can be accurately modeled.

In the case of a simply supported beam with simple boundary conditions at both ends, polynomial shape function is formulated based on the boundary conditions at both ends for displacement and moment. The following are the first three mode shapes for the simply supported beam (Luintel, 2021):

$$\phi_1 = x^4 - 2Lx^3 + L^3x \qquad \text{Eq.(4.16)}$$

$$\phi_2 = x^5 - \frac{5}{2}Lx^4 + \frac{5}{3}L^2x^3 - \frac{1}{6}L^4x \qquad \text{Eq.(4.17)}$$

$$\phi_3 = x^6 - 3Lx^5 - \frac{2073}{682}L^2x^4 + \frac{368}{341}L^3x^3 - \frac{27}{682}L^5x \qquad \text{Eq.(4.18)}$$

In this study, only the first mode shape is considered as the mode function as the bending vibration responses are primarily governed by the fundamental mode.

4.3 Mathematical Modelling of Beam with Vibration Absorber

The mathematical modeling of the beam absorber system can be performed by considering the beam as an idealized Euler-Bernoulli beam, to which a discrete vibration absorber is attached at a particular point. By considering the deformation of the beam, as well as the dynamic interaction of the absorber, the equations of motion of the coupled system can be determined by applying appropriate boundary conditions, considering the effects of external harmonic excitations, as well as the mass-spring-damper characteristics of the absorber. This type of model can be used to investigate the vibration response of the system, as well as the effectiveness of the absorber for a harmonic excitation, as presented Section 4.3.

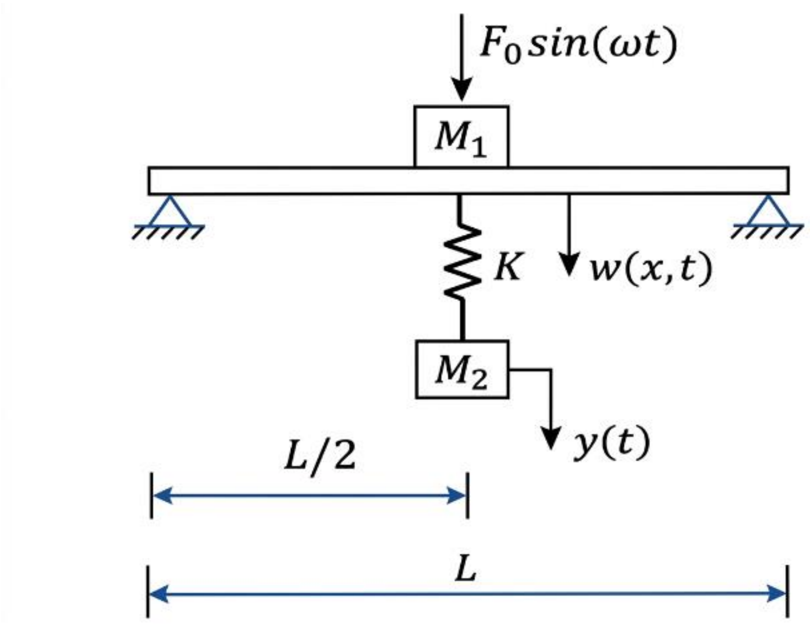

Figure 4.3 Simply Supported Beam with vibration absorber

4.3.1 Kinetic Energy

The total kinetic energy of the primary system, where there is a simply supported beam with a point mass and a vibration absorber located at the midspan, will be the sum of the kinetic energy of the beam and the point mass located at the center. The kinetic energy of the secondary system is the kinetic energy of the vibration absorber.

The kinetic energy of the simply supported beam T_b , is expressed as:

$$T_b = \frac{1}{2}\rho\, A \int_0^L (\dot{w})^2 dx \qquad \text{Eq.(4.29)}$$

The kinetic energy of the point mass at the center of the beam T_{M1}, is:

$$T_{M1} = \frac{1}{2} M_1 (\dot{w})^2 |_{x=\frac{L}{2}} \qquad \text{Eq.(4.20)}$$

Similarly, the kinetic energy of the vibration absorber (secondary mass)T_{M2} , positioned at the center, is

$$T_{M2} = \frac{1}{2} M_2 (\dot{y})^2 |_{x=\frac{L}{2}} \qquad \text{Eq.(4.21)}$$

4.3.2 Expression of Kinetic Energy for Simply Supported Beam with Central Mass and Vibration Absorber

The total kinetic energy of the system with the beam, the point mass at the center, and the vibration absorber is given by the sum of the kinetic energy of the beam, the point mass at the center, and the vibration absorber kinetic

$$T = \frac{1}{2}\rho A \int_0^L (\dot{w})^2 dx + \frac{1}{2} M_1 (\dot{w})^2|_{x=\frac{L}{2}} + \frac{1}{2} M_2 (\dot{y})^2|_{x=\frac{L}{2}} \qquad \text{Eq.(4.21)}$$

4.3.3 Strain Energy

The strain energy of the beam will be the sum of the strain energy of the beam because of the point mass at the center and the vibration absorber. The strain energy of the beam arises because of the deformations that occur in the beam. The deformations in the beam arise because of the second derivative of the beam displacements in space. This strain energy arising because of the deflections in the beam is called the strain energy of bending. The strain energy of bending is the resistance that the beam offers to the bending deformations.

There will also be strain energy associated with the vibration absorber because of its displacement. This arises because of the displacement of the vibration absorber from its equilibrium position. We obtain the total potential energy for applying Hamilton's principle.

The strain energy of the beam due to bending is given by

$$V_b = \frac{1}{2} EI \int_0^l (w'')^2 \, dx \qquad \text{Eq.(4.22)}$$

The strain energy of the spring element of the vibration absorber attached at the center of the simply supported beam is:

$$V_s = \frac{1}{2} K (y - w)^2|_{x=\frac{L}{2}} \qquad \text{Eq.(4.22)}$$

4.3.4 Expression of Strain Energy for simply Supported Beam with Center Mass and Vibration Absorber

The total strain energy expression is given by the sum of the strain energy in the beam and the strain energy in the vibration absorber.

$$V = \frac{1}{2}EI\int_{0}^{l}(w'')^2\,dx + \frac{1}{2}K(y-w)^2|_{x=\frac{L}{2}} \qquad \text{Eq.(4.23)}$$

4.3.5 Equation of Motion

The equation of motion for a simply supported beam with point mass at center and vibration absorber can be obtained by applying Hamilton's principle to the combined system of the continuous and discrete systems. The total action integral of the system can be represented as

- The beam bending strain energy
- The elastic energy of the vibration absorber and
- The kinetic energy of both the beam and the vibration absorber, as well as the work done by any external forces

The vibration absorber system is a mass spring system located at the center of the beam and can be represented as a lumped system. This system affects the vibration characteristics of the beam.

By applying the principle of minimum action and satisfying the boundary conditions of a simply supported beam and the continuity conditions at the center of the beam where the vibration absorber system is located, the differential equations of motion of the system can be obtained. This forms the fundamental basis of analysis of the vibration behavior of the system. Using Hamilton principle

$$\delta\int_{t_1}^{t_2}(T - V + W_{nc})dt = 0 \qquad \text{Eq.(4.24)}$$

$$\begin{aligned}
&\frac{1}{2}\delta\int_{t1}^{t2}\int_{0}^{l}\rho A(\dot{w})^2\,dxdt + \frac{1}{2}\delta\int_{t1}^{t2}M_1(\dot{w})^2|_{x=\frac{L}{2}}\,dt \\
&\qquad + \frac{1}{2}\delta\int_{t1}^{t2}M_2(\dot{y})^2|_{x=\frac{L}{2}}\,dt + \frac{1}{2}\int_{t1}^{t2}\int_{0}^{l}EI(w'')^2\,dxdt \\
&\qquad - \frac{1}{2}\int_{t1}^{t2}K(y-w)^2|_{x=\frac{L}{2}}\,dt + \delta\int_{t1}^{t2}F_0 sin(\omega t)w|_{x=\frac{L}{2}}\,dt \\
&\qquad = 0
\end{aligned} \qquad \text{Eq.(4.25)}$$

$$\rho\, A\int_{t1}^{t2}\int_{0}^{l}\dot{w}\,\delta\dot{w}\;dxdt\;+M_1\int_{t1}^{t2}\dot{w}\delta\dot{w}|_{x=\frac{L}{2}}dt+M_2\int_{t1}^{t2}\dot{y}\delta\dot{y}|_{x=\frac{L}{2}}dt$$

$$-EI\int_{t1}^{t2}\int_{0}^{L}w''\delta w''\;dxdt-K\int_{t1}^{t2}(\delta y-\delta w)|_{x=\frac{L}{2}}dt \qquad \text{Eq.(4.25)}$$

$$+F_0 sin\omega t\int_{t1}^{t2}\delta w|_{x=\frac{L}{2}}dt=0$$

$$\rho\;\;A\int_{t1}^{t2}\int_{0}^{l}\ddot{w}\,\delta wdxdt\;+M_1\int_{t1}^{t2}\ddot{w}\delta w|_{x=\frac{L}{2}}dt+M_2\int_{t1}^{t2}\ddot{y}\delta y|_{x=\frac{L}{2}}dt$$

$$-EI\int_{t1}^{t2}\int_{0}^{L}w'''\delta w'\;dxdt+EI\int_{t1}^{t2}w''\delta w'\;|_0^L\;dt$$

$$+K\int_{t1}^{t2}y\delta y|_{x=\frac{L}{2}}dt-K\int_{t1}^{t2}w\delta w|_{x=\frac{L}{2}}dt \qquad \text{Eq.(4.26)}$$

$$-F_0 sin\omega t\int_{t1}^{t2}\delta w|_{x=\frac{L}{2}}dt=\;0$$

$$\rho\;\;A\int_{t1}^{t2}\int_{0}^{l}\ddot{w}\,\delta wdxdt\;+M_1\int_{t1}^{t2}\ddot{w}\delta w|_{x=\frac{L}{2}}dt+M_2\int_{t1}^{t2}\ddot{y}\delta y|_{x=\frac{L}{2}}dt$$

$$+EI\int_{t1}^{t2}w''\delta w'\;|_0^L\;dt-EI\int_{t1}^{t2}\int_{0}^{L}w'''\delta w'\;dxdt$$

$$+EI\int_{t1}^{t2}\int_{0}^{L}w^{iv}\delta w\;dxdt+K\int_{t1}^{t2}y\delta y|_{x=\frac{L}{2}}dt \qquad \text{Eq.(4.27)}$$

$$-K\int_{t1}^{t2}w\delta w|_{x=\frac{L}{2}}dt-F_0 sin\omega t\int_{t1}^{t2}\delta w|_{x=\frac{L}{2}}dt=\;0$$

Hence, the equation of motion after applying boundary condition of simply supported beam will be

$$\rho A\ddot{w}+M_1\ddot{w}\delta\left(x-\frac{L}{2}\right)+\;EIw^{iv}-Ky\delta\left(x-\frac{L}{2}\right)+Kw\delta\left(x-\frac{L}{2}\right)$$

$$-F_0 sin\omega t\delta\left(x-\frac{L}{2}\right)=0 \qquad \text{Eq.(4.28)}$$

Hence, the equation of motion for beam is given by:

$$\rho A\ddot{w}+M_1\ddot{w}\delta\left(\mathrm{x}-\frac{\mathrm{L}}{2}\right)+\;EIw^{iv}-\mathrm{F_0sin\omega t\delta}\left(\mathrm{x}-\frac{\mathrm{L}}{2}\right)=0 \qquad \text{Eq.(4.29)}$$

Where the boundary conditions are.

At the left end (x=0)

A) Either $EIw'' = 0$ or $\delta w' = 0$

B) Either $EIw''' = 0$ or $\delta w = 0$

At the right end (x=L)

A) Either $EIw'' = 0$ or $\delta w' = 0$

B) Either $EIw''' = 0$ or $\delta w = 0$

For the absorber system M_2 , equation of motion is given by

$$M_2\ddot{y} + ky - kw\delta_d\left(x - \frac{L}{2}\right) = 0 \qquad \text{Eq.(4.30)}$$

4.4 Simulation Model Development

The figure 4.4 shows the finite element model of the beam that was used for modal analysis using ANSYS has been provided. In the model, there is support at both ends, and hence the boundary conditions are those of a simply supported beam. The rest of the span is free to vibrate, which is how a classical simply supported beam works. This model doesn't come with a dynamic vibration absorber or any other attachments, so you can see how the bare beam behaves on its own. This setup is the starting point for figuring out the beam's natural frequencies and the shapes of the modes that go with them.

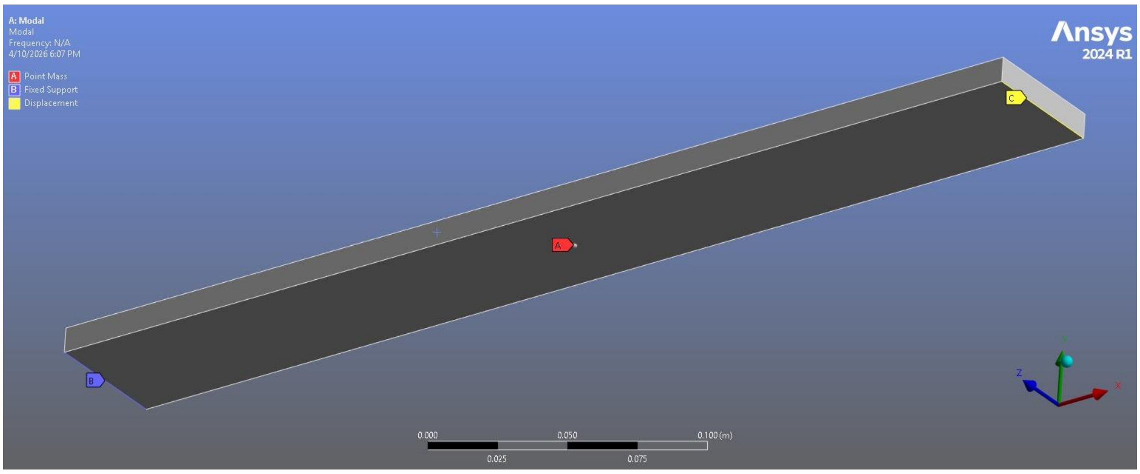

Figure 4.4 Model of System without Absorber

4.4.1 Mesh Generation

The first step for the computational field simulation process is the discretization of the field of interest, which is known as mesh generation,(Maharjan & Luintel, 2025). The Finite Element tetrahedral mesh discretization model employed in this study is that of a simply supported beam for the modal analysis. It consists of 1225777 nodes and 868521 tetrahedral elements with an element size of 1.27×10^{-3} m, providing highly accurate vibrational analysis. Mesh refinement has been done for both edge ends. The mesh refinement was performed at both end edges. Details of mesh generation are illustrated in Figure 4.5.

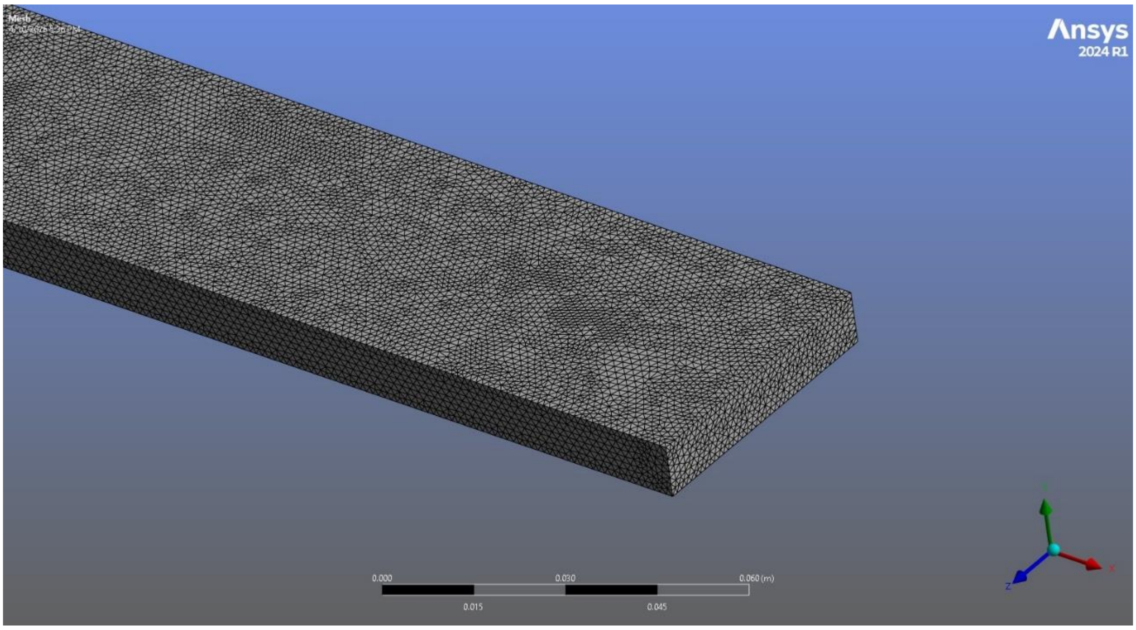

Figure 4.5 Mesh Generation of System

4.4.2 Boundary Conditions

The validation of the analysis was done via numerical simulation through Ansys, considering the boundary conditions given in Figure 4.6. In particular, the simulation concentrates on the vibration behavior of the beam both with and without vibration absorber at the center.

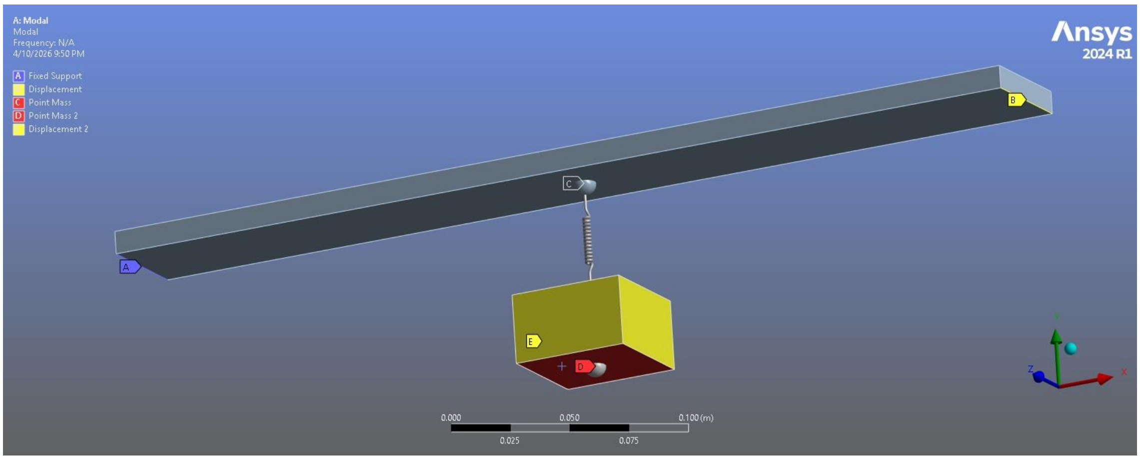

Figure 4.6 Model of Beam with Absorber

The simulation considers the beam with simply supported boundary conditions wherein both ends of the beam are pinned to avoid vertical displacement. This is similar to the boundary condition employed in the Euler-Bernoulli beam theory. A precise FE model for the system was created, with a spring connection between the beam and the dynamic vibration absorber. In the case where the beam is loaded, a concentrated mass element that represents the mass added to the middle position of the span of the beam. Given that there are no variations in the cross-sectional area along the length of the beam, the FEA model can predict the modal shapes and frequencies of the system.

CHAPTER FIVE : RESULTS AND DISCUSSION

The results obtained from the mathematical modeling and numerical analysis offer a detailed understanding of the behavior of the simply supported beam system with the added dynamic vibration absorber and the central mass. From the response profile of the system, the transverse deflection of the beam due to harmonic excitation is shown, and the influence of the added dynamic vibration absorber on the system's behavior is highlighted at various excitation frequencies. From the system's behavior with and without the added dynamic vibration absorber, effectiveness of absorber's tuning and influence of the absorber's placement at the center of the beam are highlighted.

From the transverse deflections obtained in the simply supported beam with and without the absorber, the parameters in the Table 5.1 are considered

Table 5.1 Parameters of the system.

S. N	Parameter	Symbol	Values
1	Material	-	Mild Steel
2	Beam Length	L	0.415 m
3	Beam Breadth	B	0.05 m
4	Beam Height	H	0.0098 m
5	Density of Beam	ρ	7850 kg/m³
6	Young's Modulus	E	200 GPa
7	Mass per unit length	m	3.84 kg/m
8	Point Mass of Primary Beam	M_1	1.6 kg
9	Frequency of Harmonic Force	ω	500 rad/sec
10	Spring Stiffness	K	50000 N/m
11	Harmonic Force Applied	F	10 N
12	Cross-sectional Area Beam	A	$4.9 \times 10^{-4} m^2$
13	Moment of inertia of Beam	I	$3.92 \times 10^{-9} m^2$

Table 5.1 summarized the key geometric and material parameters used to for numerical calculation and Ansys simulation.

5.1 Analytical Results

The research aims to analyze the suppression of vibrations for a simply supported beam with a central mass and subjected to a harmonically varying force using a dynamic vibration absorber. The analytical formulation yields expressions for the transverse displacement at the center point of the beam, both in the presence and absence of the absorber. The results were verified numerically for a beam made of aluminum alloy material under given conditions.

5.2 Galerkin Method

The Galerkin method is an approach for transforming continuous problems described by differential equations into discrete problems for easy analysis. In this approach, an approximation of the solution is sought by expressing it as an expansion of admissible functions satisfying boundary conditions for a simple-supported beam. The residual between the exact and approximate solutions is minimized when weighting functions are taken as the assumed basis functions. This process generates algebraic equations, which can be solved to find the unknowns.

5.3 Calculation of Transverse Deflection of Beam without Vibration Absorber

The response of the simply supported beam with a mass at the center, without the vibration absorber, is taken as a reference to assess the improvement in response for the control strategies implemented subsequently. From the results in the time domain, it is observed that there are considerable oscillations at the center point of the beam, signifying low damping and high sensitivity to harmonic.

5.3.1 Analytical Approach

Using the Galerkin method and a polynomial single mode approximation, the transverse displacement of the simply supported beam is postulated to be a function of space and time.

The displacement is given by,

$$w(x,t) = B\phi_1(x)\sin(\omega t) \qquad\qquad \phi_1 = x^4 - 2Lx^3 + L^3x$$

Governing Residual Equations

$$\rho A\ddot{w} + M_1\ddot{w}\delta\left(x-\frac{L}{2}\right) + EIw^{iv} - F_0\sin(\omega t)\delta\left(x-\frac{L}{2}\right) = 0 \qquad \text{Eq.(5.1)}$$

By applying the Galerkin Method, the governing equation is multiplied by the weight function and integrated from 0 to L,

$$\int_0^L \phi_1 R dx = 0 \qquad \text{Eq.(5.2)}$$

Substituting the assumed solution $w(x,t)$ into the residual equation and evaluating the time-dependent term $\ddot{w} = -Bw^2\phi_1(x)\sin(\omega t)$, the following governing integral equation is obtained,

$$\begin{aligned} -\rho AB\omega^2 \int_0^L \phi_1{}^2(x)dx \\ - M_1 B\omega^2 \int_0^L \phi_1{}^2(x)\delta_d\left(x - \frac{L}{2}\right)dx \\ + EIB \int_0^L \phi_1(x)\phi_1{}^{iv}(x)dx - F_0 \int_0^L \phi_1(x)\delta_d\left(x - \frac{L}{2}\right)dx \\ = 0 \end{aligned} \qquad \text{Eq.(5.3)}$$

Distributed Mass Integral

The integral for kinetic energy of the beam material is

$$\int_0^L \phi_1{}^2(x)dx = \frac{31}{630}L^9$$

Point Mass and Force Terms (at midpoint x = L/2)

Displacement and its square at the mid-point of the beam, (where the mass M_1 and force $F_0$are applied) is given by,

$$\phi_1(L/2) = \frac{5}{16}L^4$$

$$\phi_1{}^2\left(\frac{L}{2}\right) = \frac{25}{256}L^8$$

Bending Stiffness Integral

The internal strain energy term, using the fourth derivative ${\phi_1}^{iv}(x) = 24$ is,

$$\int_0^L \phi_1(x)\phi_1{}^{iv}(x)dx = \frac{24}{5}L^5$$

Algebraic expression for Amplitude (B) becomes,

$$B\left(-\frac{31}{630}\rho A\omega^2 L^9 - \frac{25}{256}M_1\omega^2 L^8 + \frac{24}{5}EIL^5\right) = \frac{5}{16}F_0 L^4 \qquad \text{Eq.(5.8)}$$

Rearranging to solve for B

$$B = \frac{\frac{5}{16}F_0 L^4}{-\frac{31}{630}\rho A\omega^2 L^9 - \frac{25}{256}M_1\omega^2 L^8 + \frac{24}{5}EIL^5} \qquad \text{Eq.(5.9)}$$

The expression for the transverse beam displacement as a function of both position and time is

$$w(x,t) = \frac{\frac{5}{16}F_0L^4(x^4 - 2Lx^3 + L^3x)}{-\frac{31}{630}\rho A\omega^2L^9 - \frac{25}{256}M_1\omega^2L^8 + \frac{24}{5}EIL^5}\sin(\omega t) \qquad \text{Eq.(5.10)}$$

Using the specific beam parameters and forcing conditions defined in Table 5.1, the transverse displacement at the center point (x = L/2) is calculated as,

$$w(L/2,t) = \pm 1.62 \times 10^{-4} \sin(500t)$$

This shows the significant amount of vibration occurring at the specific frequency, which is the standard response of the primary system. This is the standard response of the primary system, and we will use the vibration absorber to reduce the amount of deflection occurring at this point.

5.3.2 Simulation Approach

The simulation approach was employed in the numerical validation of the analytical models developed for the simply supported beam system in the presence and absence of the dynamic vibration absorber. The finite element model was created to represent the beam system in terms of its geometric properties and material properties. The boundary conditions were specified to represent the simply supported beam system.

The simulation was performed to determine the dynamic response of the system under harmonic excitation to evaluate the deflection of the beam at the center point and along the entire length. For the system with the dynamic vibration absorber, the mass and spring constant obtained from the analytical model were specified at the center point of the beam to evaluate the effectiveness of the absorber in suppressing the vibrations. The results from the simulation were compared with the results from the analytical model to evaluate the accuracy of the model.

5.3.3 Comparison of Natural Frequency

The natural frequency of the simply supported beam is determined analytically and compared with the results obtained from simulation to validate the developed model.

For a simply supported beam, the angular natural frequency is given by:

$$\omega_n{}^2 = \frac{\frac{24}{5}EIL^5}{\frac{31}{630}\rho AL^9 + \frac{25}{256}M_1L^8} \quad \text{Eq.(5.11)}$$

The natural frequency obtained in Hertz is

$$f_n = \frac{\omega_n{}^2}{2\pi} \quad \text{Eq.(5.12)}$$

By substituting the beam parameters from Table 5.1 the fundamental natural frequency (at first mode, n= 1) is calculated as,

$$f_n = 75.3 \text{ Hz}$$

For the first mode, the simulation result is 74.86Hz, while the analytical solution is 75.3 Hz, resulting 1% deviation as shown in below table 5.2. This confirms the validity of the analytical model and ensures that the system behavior is accurately captured, particularly near the fundamental mode which dominates the dynamic response.

Table 5.2 Natural frequency from Analytical and Simulation Comparison

Mode	Analytical (Hz)	Simulation (Hz)	Values
1	75.3	74.86	1%

5.3.4 Modal Analysis

Modal analysis was carried out in order to obtain the natural frequencies of the system and the corresponding mode shapes of the simply supported beam system. The modal analysis is very important in providing the required information about the dynamic behavior of the system. The modal analysis lays the basis upon which the dynamic behavior of the system can be understood in the presence of external excitations. The dynamic vibration absorber system can be well understood with the help of modal analysis. The modal analysis helps in the accurate formulation of the dynamic model of the system.

The modal analysis results obtained from ANSYS for the simply supported beam are shown in Figure 5.1. In the figure, it can be seen that the first mode of bending deformation occurs in the Y-axis, and the maximum displacement is at the center of the beam, with the least displacement at the supports. This is the fundamental mode for a simply supported beam. In the figure, it can also be seen that the color contour indicates the amount of displacement, from blue for the least amount of displacement to red for the highest amount of displacement. In addition, the natural frequency for the first mode is found to be 74.86 Hz, while the natural frequencies for the first tenth modes are shown in the bar chart and table, where it can be seen that there is a gradual increase in natural frequency from the first mode to the ten mode.

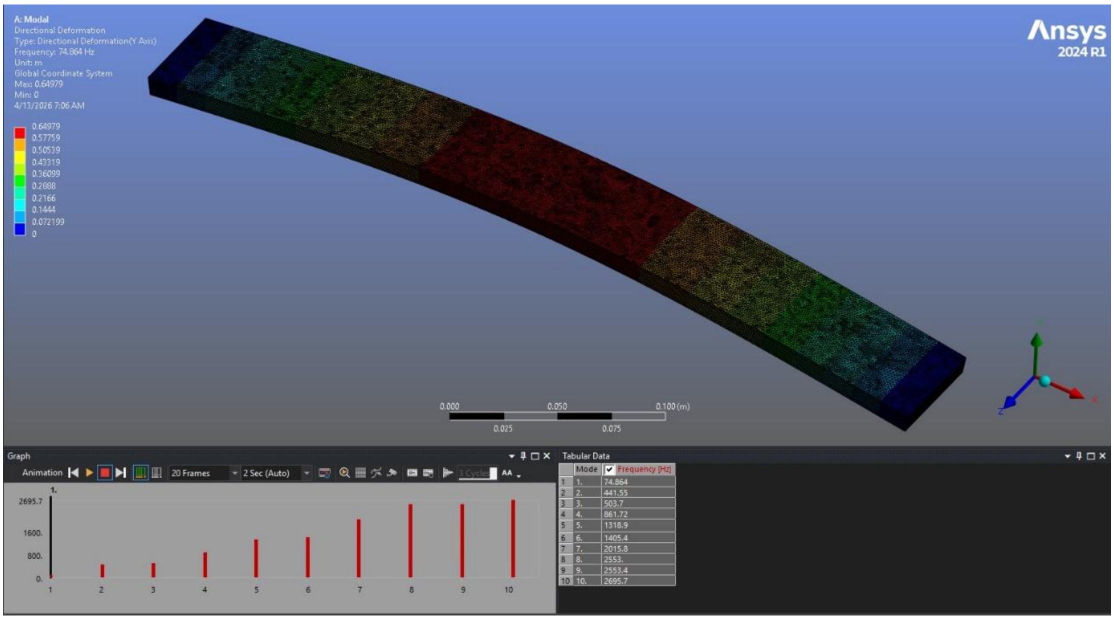

Figure 5.1 Modal Analysis

The amount of transverse displacement at the center of the beam for the given parameters and harmonic loading has been found to be 6.4 mm. This shows a significant amount of vibration at the applied frequency, and thus an appropriate method for controlling the same is needed.

5.3.5 Applied Harmonic Force

Figure 5.2 shows the harmonic force used in the numerical simulation is depicted. A 10 N force is applied in the negative Y-direction at the center of the simply supported beam, and the arrow represents the direction and magnitude of the force. Although the graphical user interface displays the load as being 0 Hz, it is worth noting that the actual load will be considered over the entire range in the study.

The above figure ensures that the loading conditions have been correctly defined, thereby allowing the accurate determination of the harmonic responses.

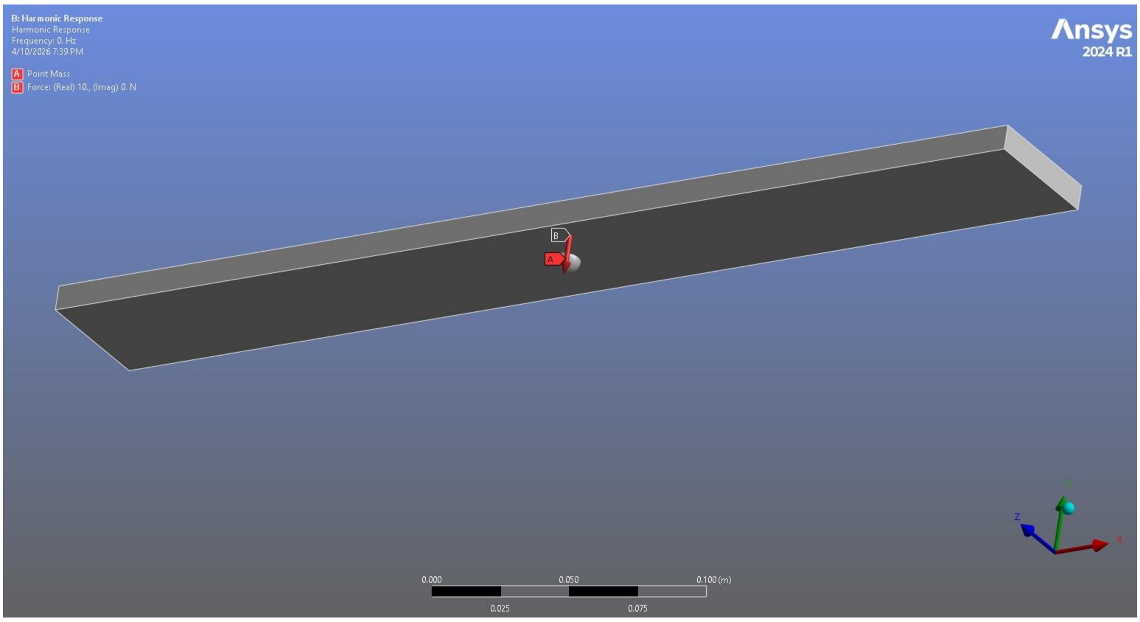

Figure 5.2 Harmonic Load Applied

5.3.6 Directional Deformation without Absorber

The directional deformation of the simply supported beam obtained using the harmonic response analysis in ANSYS as per Figure 5.3. The deformation has been evaluated along the Y-axis, which represents the main direction of bending for the beam. As depicted in the figure using the color contour, the maximum value of 4.2 mm has been obtained at the center of the beam, while the values at the supports remain zero. This ensures the correct application of the boundary conditions. This represents the dynamic response of the beam for the excitation frequency of 79.57 Hz. The same values for the maximum and minimum have been obtained because the plot represents the response amplitude and the phase angle of 0°. This figure clearly represents the vibratory behavior of the simply supported beam.

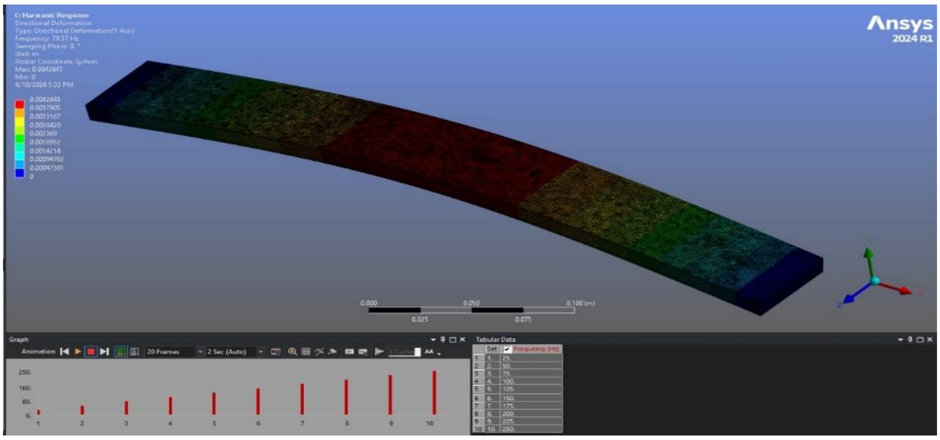

Figure 5.3 Directional Deformation of beam without absorber

5.3.7 Frequency response analysis without Absorber using FEA

The frequency response function of the simply supported beam, which shows the variation in the amplitude of the vibration with the excitation frequency, is presented in Figure 5.4. The response curve shows the resonance peak in the range of 70 to 80 Hz, which shows the first natural frequency of the beam. The amplitude of the displacement at resonance is approximately 3.28 mm. The response decreases sharply after this range. The tabulated data presented in the figure shows the amplitude values for the corresponding frequency steps. This FRF plot is an important tool in identifying the natural frequencies of the beam.

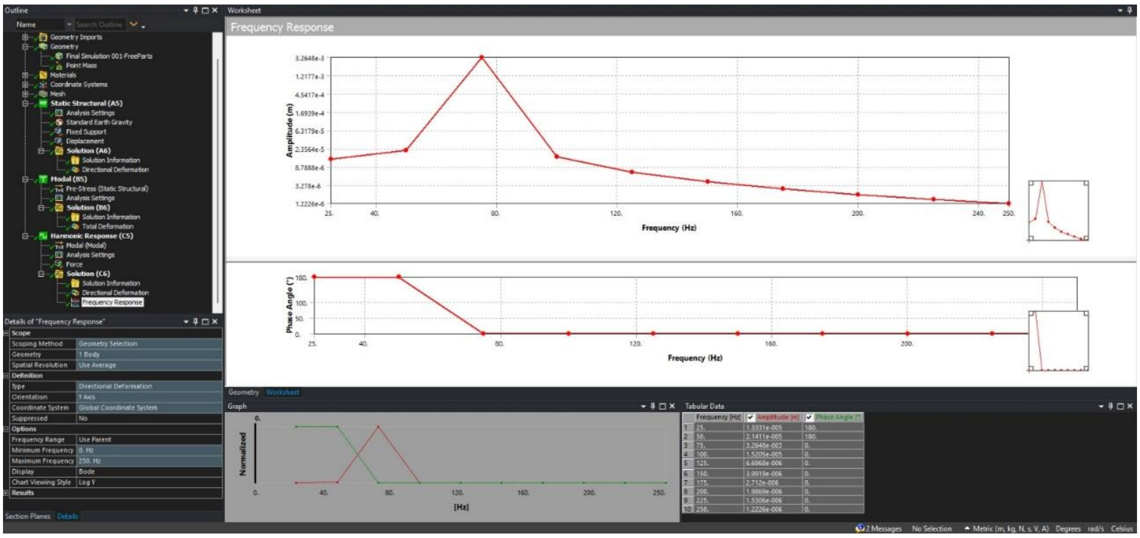

Figure 5.4 Frequency response analysis without absorber using FEA

5.4 Calculation of Transverse Deflection of Beam with Vibration Absorber

The addition of a DVA to the simply supported beam results in a significant change in the overall dynamic response of the beam. The addition of the DVA results in a new degree of freedom for the beam, allowing for the dissipation of vibrational energy into the DVA mass. This results in a highly effective means for reducing oscillations within the beam, especially when forces are applied near the natural frequency. From observations based on the time-domain data, it can be seen that there is a drastic reduction in the peak displacement at the center of the beam compared to when there is no DVA. This shows the effectiveness of the DVA in improving stability within the beam.

5.4.1 Analytical Approach

The analytical approach employed in this study is based on the classical theory of vibration for flexible beams with lumped parameter attachments. The simply supported beam is modeled based on Euler-Bernoulli beam theory, with transverse displacement described by the Galerkin method.

For the primary beam, the assumed mode shape is $\phi_1 = x^4 - 2Lx^3 + L^3x$. The transverse displacement of the beam is $w(x,t) = B\phi_1(x)\sin(\omega t)$, and the displacement of the absorber mass is $y(t) = Y\sin(\omega t)$.

Governing Residual Equations

$$\rho A\ddot{w} + M_1\ddot{w}\delta\left(x - \frac{L}{2}\right) + EIw^{iv} - Ky\delta\left(x - \frac{L}{2}\right) + Kw\delta\left(x - \frac{L}{2}\right) - F_0 \sin\omega t\,\delta\left(x - \frac{L}{2}\right) = 0 \qquad \text{Eq.(5.13)}$$

The governing equation for the primary system is expressed by integrating the residual R_1,

$$\int_0^L \phi_1 R_1 dx = 0 \qquad \text{Eq.(5.14)}$$

The governing equations of primary mass becomes

$$-\rho AB\omega^2 \int_0^L \phi_1{}^2(x)dx - M_1B\omega^2 \int_0^L \phi_1{}^2(x)\delta_d\left(x - \frac{L}{2}\right)dx + EIB\int_0^L \phi_1(x)\phi_1{}^{iv}(x)dx + KB\phi_1{}^2\left(x - \frac{L}{2}\right) - KY\phi_1\left(x - \frac{L}{2}\right) - F_0\int_0^L \phi_1(x)\delta_d\left(x - \frac{L}{2}\right)dx = 0 \qquad \text{Eq.(5.14)}$$

The equation has been simplified by considering the values of the Dirac delta function $\delta(x - L/2)$, the equation has been written in terms of the beam amplitude B and the absorber amplitude Y,

$$B\left[-\rho A\omega^2 \int_0^L \phi_1{}^2dx - M_1\omega^2\phi_1{}^2\left(\frac{L}{2}\right) + EI\int_0^L \phi_1\phi_1{}^{iv}dx + K\phi_1{}^2\left(\frac{L}{2}\right)\right] - KY\phi_1\left(\frac{L}{2}\right) - F_0\phi_1\left(\frac{L}{2}\right) = 0 \qquad \text{Eq.(5.15)}$$

Simplifying we get

$$B\left[-\frac{31}{630}\rho A\omega^2L^9-\frac{25}{256}M_1\omega^2L^9+\frac{24}{5}EIL^5+\frac{25}{256}KL^8\right]-\frac{5}{16}KYL^4 -\frac{5}{16}F_0L^4=0 \quad \text{Eq.(5.16)}$$

We can write it as

$$BX-\frac{5}{16}KYL^4-\frac{5}{16}F_0L^4=0 \quad \text{Eq.(5.17)}$$

Where,

$$X=\left[-\frac{31}{630}\rho A\omega^2L^9-\frac{25}{256}M_1\omega^2L^9+\frac{24}{5}EIL^5+\frac{25}{256}KL^8\right] \quad \text{Eq.(5.18)}$$

For the secondary mass (absorber), the residual R_2 is calculated as,

$$\int_0^L \phi_1 R_2 dx=0 \quad \text{Eq.(5.19)}$$

The governing equation of absorber is

$$M_2\ddot{y}+ky-kw\delta_d\left(x-\frac{L}{2}\right)=0 \quad \text{Eq.(5.20)}$$

By substituting harmonic assumptions above equations becomes,

$$\int_0^L \phi_1[-M_2\omega^2Y+KY-KB\phi_1(x)]\delta(x-L/2)dx=0 \quad \text{Eq.(5.21)}$$

By applying property of the Dirac delta function, integral evaluates the expression specifically at the center point of attachment $x=L/2$.

$$\phi_1(L/2)[-M_2\omega^2Y+KY-KB\phi_1(L/2)=0 \quad \text{Eq.(5.22)}$$

Since displacement at center $\phi_1(L/2)$ is non-zero, equation simplifies for secondary mass as,

$$Y(K-M_2\omega^2)-KB\phi_1(L/2)=0 \quad \text{Eq.(5.23)}$$

Substituting value from Table 5.1,

$$Y(K-M_2\omega^2)-\frac{5}{16}KBL^4=0 \quad \text{Eq.(5.24)}$$

Solving the coupled equations gives,

$$B=\frac{5L^4F_0(K-M_2\omega^2)}{M+N} \quad \text{Eq.(5.25)}$$

$$Y=\frac{25KL^8F_0}{16(M+N)} \quad \text{Eq.(5.26)}$$

Where,

$$M = 16X(K - M_2\omega^2)$$

$$N = -\frac{25}{16}K^2L^8$$

The integration of the dynamic vibration absorber into the system significantly changes the dynamic response of the beam. The steady-state response of the dynamic vibration absorber mass is found by solving the coupled differential equations of motion for the simply supported span of length 0.415m. The steady-state response of the dynamic vibration absorber mass is found as,

$$y(t) = -2 \times 10^{-4} \sin(500t)$$

While, the resultant transverse deflection at the center of the beam is effectively suppressed to

$$w(0.2075, t) \approx 0$$

This reduction in the amplitude further verifies the absorber's capability to counteract the external harmonic force. These results clearly indicate the effectiveness of the dynamic vibration absorber in maintaining stability in the structure.

5.4.2 Directional Deformation at the center of Beam with Absorber

The directional deformation of the simply supported beam with the integrated dynamic vibration absorber has shown significant changes in the dynamic behavior of the system. In this case, the deformation, measured in terms of the Y-axis ($\approx 4.6 \times 10^{-5}$m) , has shown the effect of the dynamic vibration absorber in reducing the amplitude of the beam at the center compared to the uncontrolled system. With the additional degree of freedom provided by the dynamic vibration absorber, the amplitude of the beam has been reduced, especially at the resonant frequency of the beam.

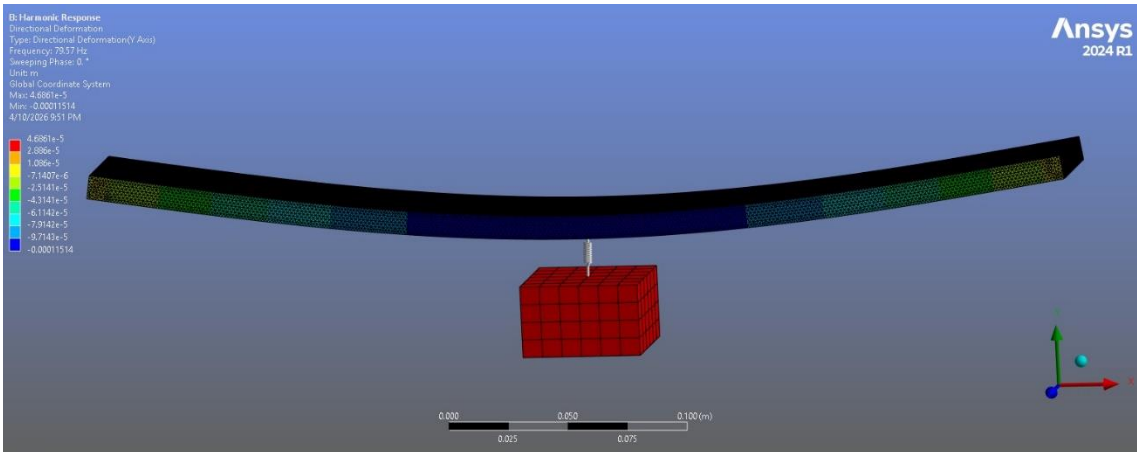

Figure 5.5 Directional Deformation at the center of Beam with Absorber

The deformation contour also represents a reduced maximum deflection, which confirms that indeed the maximum deflection is reduced as a result of the effect of the absorber, which absorbs and counteracts some of the harmonic excitations. This visual representation confirms the improved performance of the combined continuous-discrete system in terms of vibration and the effectiveness of the absorber in reducing beam deformation.

5.4.3 Frequency Response of Beam with Absorber

The frequency response for the simply supported beam system after the dynamic vibration absorber has been tuned to target the dominant natural frequency for the primary structure is shown in Figure 5.6. There is a clear peak observed at approximately 75 Hz on the frequency response curve due to the anti-resonance effect caused by the addition of the dynamic vibration absorber. After this point, there is a significant decrease in the vibration response amplitude. This shows that the addition of the dynamic vibration absorber has resulted in a significant reduction in the amount of energy transferred to the primary beam. The curve then plateaus for higher frequencies, showing improved dynamic stability.

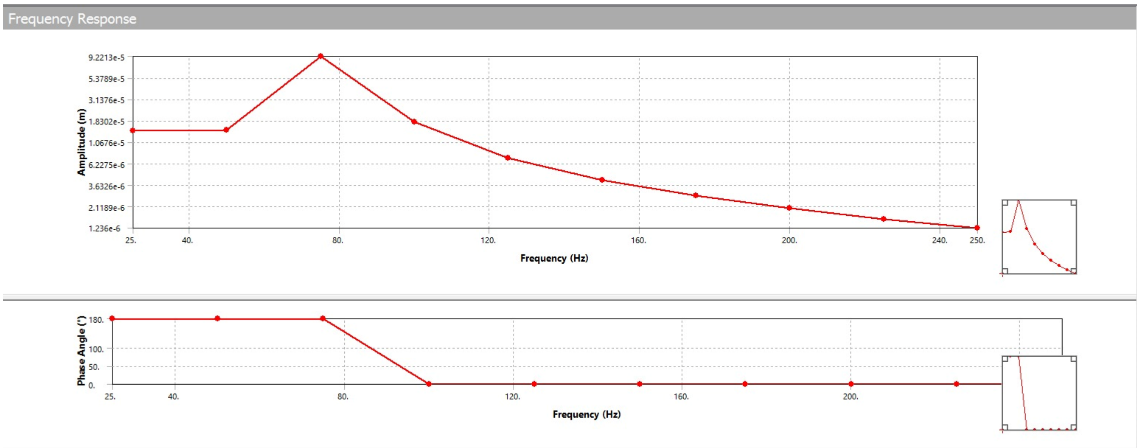

Figure 5.6 Frequency response analysis with Absorber using FEA

The simulation results show that the displacement at the center of the beam is reduced to $w(L/2) = 0$.

Effect of Absorber Mass on Beam Deflection

The transverse displacement at the center point of the simply supported beam is reduced to nearly zero, while the maximum transverse motion occurs at the dynamic vibration absorber. This shows that the dynamic vibration absorber is effective in reducing the vibrational motion, thereby reducing the amplitude of the primary beam.

In order to achieve optimal vibration suppression, the mass of the dynamic vibration absorber M_2 was tuned based on the spring constant K=50000 N/m. The analytical expression for M_2 and K is used to determine the optimal mass for M_2 as 0.2 kg. This condition results in maximum energy transfer between the primary structure and the dynamic vibration absorber, thereby minimizing the vibrational motion.

5.5 Validation with Analytical Model

To validate the accuracy of the finite element model, the numerical results were compared with the analytical solution for the simply supported beam absorber system. Important parameters such as natural frequencies, mode shapes, and steady-state responses in the harmonic domain were evaluated.

Minor variations can arise from variations in mesh density, boundary condition definitions, and/or how the absorber is modeled as a lumped mass. In such instances, better results were obtained by refining the mesh, modifying the boundary condition definitions and modifying the parameters of the absorber. Iterative refinements were made until there was close correspondence between the results from the FEM and the analytical model, validating the model for further design and vibration mitigation work.

5.6 Comparative Analysis

- Without Absorber: For the transverse displacement at the center of the beam without a dynamic vibration absorber, the value was found to be approximately equal to 1.62×10^{-4}m by the analytical method, and by the finite element method, the value was found to be 0.42×10^{-3}m.
- With Absorber: By using the analytical method, the value of the optimum value of the absorber mass to be placed on the beam in order to minimize the transverse deflection at the center was found to be $M_2 = 0.2$ kg. By modelling the beam with the absorber having the same value of mass and calculating the value of the transverse deflection, the value was found to be $w = 4.6 \times 10^{-5}$, which is nearly equal to zero.

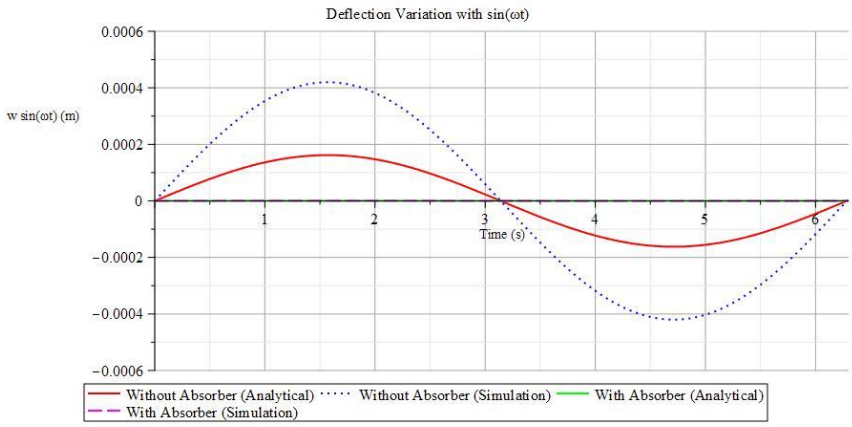

Figure 5.7 Deflection comparison of beam with and without Absorber

5.7 Discussion

The results show the effectiveness of the proposed method in the reduction of the beam's responses to the harmonic excitation. This effect is achieved when the absorber's natural frequency coincides with the excitation. This enables the efficient transmission of energy from the beam to the absorber, thus effectively reducing the deflection.

The similarity in the results obtained using the proposed method and the FEM verifies the accuracy of the developed mathematical and numerical models. Hence, the proposed method offers a reliable basis for the development of a passive control system for a wide range of structural and mechanical engineering applications

CHAPTER SIX : CONCLUSION AND RECOMMENDATION

6.1 Conclusion

The analytical model of the simply supported beam system, in both forms, with and without the dynamic vibration absorber is derived to capture the dynamics behavior of the structure efficiently. In the case of the uncontrolled beam, the governing equations developed by using Hamilton's principle and Euler-Bernoulli beam theory adequately describe the behavior of the beam. Therefore, this model can be considered appropriate to be used as a reference to compare with other similar models.

The addition of the dynamic vibration absorber to the system introduces an additional degree of freedom, which is accurately captured by the coupled continuous-discrete system. This enables an efficient understanding of the interaction mechanisms between the incident beam and the added dynamic vibration absorber. Therefore, the models developed can be considered appropriate to be used to describe the behavior of the system and understand the mechanisms related to the suppression of vibrations.

The results obtained by the analytical method and the finite element method for the beam without the absorber show a high degree of accuracy and similarity. This confirms the accuracy and appropriateness of the mathematical model. For instance, the value of the center displacement, $\mathrm{w} = 1.62 \times 10^{-4}\mathrm{m}$, obtained by the analytical method shows a high degree of similarity to the value obtained by the simulation method, $\mathrm{w} = 0.42 \times 10^{-3}\mathrm{m}$. However, the two values show a negligible difference. When the absorber is introduced, the value of the center displacement approaches zero in both the analytical and simulation methods. This confirms the accuracy and appropriateness of the absorber. For instance, the value of the absorber mass, $\mathrm{M}_2 = 0.2$ kg, and the value of the absorber stiffness, K = 50000 N/m, used in the simulation and analytical methods, respectively, were derived using the analytical method.

As can be seen from the obtained results, the dynamic vibration absorber is found to be an effective tool in the reduction of the response of the beam structure, particularly when the natural frequency is the same as the excitation frequency. The reduction in the center point deflection suggests that the variables such as mass and stiffness of the dynamic vibration absorber need to be utilized in the system. With regard to the design aspect, the proposed approach can be improved using parametric investigations such as damping ratios or the placement of the dynamic vibration absorber. It is also found that the analytical results obtained in the present study show good agreement with the FEM simulations carried out in the ANSYS environment. The proposed model can be used as an effective tool in the design of passive control systems.

6.2 Recommendation

Based on the results obtained in this study, the following recommendations are proposed for improving the efficiency of dynamic vibration absorbers in structural systems.

- Parametric Analysis: An in-depth study of the influence of parameters like mass, spring stiffness, and damping in the dynamic vibration absorber on the efficiency of the system in controlling the vibrations should be conducted. This will help in identifying the best possible arrangement for maximum efficiency.
- Incorporation of Damping: Damping should be incorporated in the dynamic vibration absorber system for better efficiency in real-world problems, as damping is always present in real systems.
- Experimental Verification: Experimental verification of the beam-absorber system should be carried out for validation of the numerical and analytical results obtained in this study, for better efficiency in real-world problems.
- Practical Applications: This study should be extended for real-world problems in various fields like bridges, aerospace structures, and machine parts, where controlling the vibrations is of utmost importance.

By following these recommendations, it is possible for future studies to improve the efficiency of dynamic vibration absorbers and thus enhance the efficiency of passive control systems in various fields of engineering.

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APPENDIX

Code for Analytical Solution without Absorber

```
#Define parameters
L := 0.415;
rhoA := 3.84;
M1 := 1.6;
omega := 500;
F0 := 10;
EI := 784;
phi1 := L^3*x - 2*L*x^3 + x^4;
top1 := 5/16*F0*L^4;
bot1 := -31/630*rhoA*omega^2*L^9 - 25/256*M1*omega^2*L^8 + 24/5*EI*L^5;
B_val := evalf(top1/bot1);
phi_mid := evalf(subs(x = L/2, phi1));
w_mid := evalf(B_val*phi_mid);
printf("top1  = %.6e\n", evalf(top1));
printf("bot1  = %.6e\n", evalf(bot1));
printf("B_val = %.6e\n", B_val);
printf("phi_mid = %.6e\n", phi_mid);
printf("w(L/2,t) = %.4e sin(500t) m\n", abs(w_mid));
```

Code for Analytical Solution with Absorber

```
#Define parameters
L     := 0.415:
rhoA  := 3.84:
M1    := 1.6:
omega := 500:
F0    := 10:
EI    := 784:
K     := 50000:
M2    := K/omega^2:
# MODE SHAPE TERMS
```

```
phi_mid  := (5/16)*L^4:
phi_mid2 := (25/256)*L^8:
# X TERM
X := -(31/630)*rhoA*omega^2*L^9
    -(25/256)*M1*omega^2*L^8
    +(24/5)*EI*L^5
    +(25/256)*K*L^8:
# M AND N TERMS
Mterm := 16*X*(K - M2*omega^2):
Nterm := -(25/16)*K^2*L^8:
# AMPLITUDE B
B_val := evalf((5*L^4*F0*(K - M2*omega^2))/(Mterm + Nterm)):
# AMPLITUDE Y
Y_val := evalf((25*K*L^8*F0)/(16*(Mterm + Nterm))):
# DISPLACEMENTS
w_mid := evalf(B_val*phi_mid):
y_amp := evalf(Y_val):
printf("M2        = %.6f kg\n",       evalf(M2)):
printf("X         = %.6e\n",          evalf(X)):
printf("B_val     = %.6e m\n",        B_val):
printf("Y_val     = %.6e m\n",        Y_val):
printf("w(L/2,t)   = %.4e sin(500t) m\n", abs(w_mid)):
printf("y(t)       = %.4e sin(500t) m\n", abs(y_amp)):
```

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