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Reliability Analysis for Tunnel Supports System by Using Finite Element Method– A Case Study: Sunkoshi Marin Diversion HRT

by

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ABSTRACT

This study presents a reliability-based support design analysis for the Sunkoshi Marin Diversion Multipurpose Project (SMDMP) headrace tunnel in Nepal. Rock mass properties were characterized from face mapping and laboratory testing, providing mean and variance of Geological Strength Index (GSI), uniaxial compressive strength (UCS), and Hoek–Brown constant mi. Numerical modeling was performed in Rocscience Phase2 using the core replacement technique. Deterministic analysis, with average parameter values, indicated adequate tunnel stability, as all points satisfied the factor of safety FOS >1.4. In contrast, reliability analysis using Rosenblueth's two-point Point Estimate Method (PEM) revealed markedly different behavior. By varying GSI, UCS, and mi across eight parameter sets, the probabilistic model showed plastic zones nearly twice as large and wall deformations up to six times greater than in the deterministic case. The concrete liner exhibited a probability of failure of 5–5.5 % at both the crown and sidewall. These findings demonstrate that deterministic design alone underestimates risks in variable rock masses. The study highlights the importance of adopting reliability-based methods in Nepalese tunneling practice, particularly for TBM-driven hydropower tunnels, to achieve safer and more resilient support systems.

Keywords: Reliability Analysis, Point Estimate Method, Core Replacement Technique

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DECLARATION

I hereby declare that this study titled "**Reliability Analysis for Tunnel Supports System by Using Finite Element Method– A Case Study: Sunkoshi Marin Diversion HRT**" is based on my original research work. Related works on the topic by other researchers have been duly acknowledged. I owe all the liabilities relating to the accuracy and authenticity of the data and any other information included hereunder.

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LIST OF ACRONYMS

2D	:	Two Dimensional
CRT	:	Core Replacement Technique
FOS	:	Factor of Safety
FORM	:	First Order Reliability Method
GSI	:	Geological Strength Index
LDP	:	Longitudinal Displacement Profile
PEM	:	Point Estimate Method
Pf	:	Probability of failure
RMR	:	Rock Mass Rating
RQD	:	Rock Quality Designation
SMDMP	:	Sunkoshi Marin Diversion Multipurpose Project
SORM	:	Second Order Reliability Method
UCS	:	Uniaxial Compressive Strength
X*	:	Normalized Support Installation Distance
R*	:	Normalized Tunnel Radius
mi	:	Intact Rock Constant
u_0*	:	Normalized Tunnel Closure at the Face
β	:	Reliability Index
γ	:	Unit Weight

1. INTRODUCTION

1.1 Background

In case of Nepal the construction of tunnels has a key role for the development of hydropower, irrigation, transportation. Considering its multipurpose application, particularly in hydropower and irrigation projects, diversion tunnels play a key role in transferring water from surplus basins to water-deficient basins. There are various component of underground structures, among them support system plays key role for providing the stability and assuring safe and reliable operation. The interaction of rock mass and the tunnel structure is complex because of uncertainties resulting from variations in geological formation, and mechanical properties of rock mass. The inherent variability on properties of rock mass needs to be taken care as it directly influence the response of excavation and support system performance. When deterministic analysis employs average values of ground parameters, it produces a single factor of safety (FOS) as a result, which conceals the associated risks and can result in either insufficient or excessive design.

To address these uncertainties, probabilistic methods have been introduced into tunnel analysis and design practices to quantify the likelihood of failure and to provide a measure of reliability for design decisions. The reliability-based approach integrates the variability of rock mass parameters into numerical analyses, allowing engineers to estimate not only the factor of safety but also the associated probability of failure. In this context, the Point Estimate Method (PEM), proposed by Rosenblueth (Rosenblueth, 1975) and later implemented in Rocscience's Phase2 (RS2) software, provides an efficient and computationally feasible technique for evaluating reliability without the need for extensive Monte Carlo simulations. The method approximates statistical moments of output parameters such as deformation or factor of safety by evaluating a limited number of combinations of input variables, making it particularly suitable for geotechnical problems where computational resources are limited.

Tunnel excavation involves a three-dimensional stress redistribution around the opening. Accurately representing this three-dimensional effect in two-dimensional numerical models is challenging. To simulate this behavior, the core replacement technique has been developed as an alternative to the convergence-confinement method (Periku et al., 2000). In this technique, the modulus of the material inside the tunnel core is gradually reduced from the rock mass modulus to zero to represent the progressive advancement of the excavation face. By combining this approach with the empirical Vlachopoulos & Diederichs, 2009 relationships for tunnel wall deformation, the pre-support deformation stage can be determined with good accuracy. The method has been used in reliability-based analyses of tunnels (Bukaçi et al., 2017; Pandit & Sivakumar Babu, 2021; Periku et al., 2000).

In Nepal, several large hydropower projects are under construction or in operation where tunneling is carried out through weak and heterogeneous rock masses. The Sunkoshi Marin Diversion Multipurpose Project (SMDMP) is one of the major water diversion schemes, comprising a headrace tunnel excavated mainly through moderately weathered sandstone. During excavation, significant variability in the rock mass conditions was observed, reflected in the wide range of RMR and UCS values recorded along the tunnel alignment. For such conditions, deterministic analyses may not adequately represent the actual behavior of the tunnel support system. A reliability-based evaluation is therefore necessary to quantify the possible range of responses and to assess whether the existing support system provides an acceptable level of safety.

This study focuses on the reliability analysis of the existing tunnel support system of the SMDMP headrace tunnel using FEM based RS2 (Phase2) program with the core replacement technique for staged excavation simulation and the PEM for probabilistic evaluation. The analysis aims to evaluate the adequacy of the support by computing the reliability index (β) and corresponding probability of failure (Pf) for key points such as the tunnel crown and sidewalls. By comparing deterministic and probabilistic results, the study seeks to highlight the effects of rock mass uncertainty on predicted deformation, plastic zone development, and factor of safety. The findings contribute to improved understanding of tunnel support performance under uncertainty and promote the adoption of reliability-based assessment in Nepal's hydropower tunneling projects.

1.2 Statement of the Problem

Tunnel construction in weak and variable rock masses poses significant challenges due to the uncertainties associated with mechanical properties of rock mass, in-situ stresses, and construction conditions. In the traditional deterministic design practice, mean values of parameters are taken to evaluate the fos of tunnel supports. However, these mean values may not represent the actual range of geological conditions encountered in the field. As a result, deterministic analyses often produce misleadingly optimistic predictions of tunnel stability and may overlook scenarios that could lead to localized failure.

In the Sunkoshi Marin Diversion Multipurpose Project, the headrace tunnel passes through sandstone formations exhibiting high variability in UCS (20–60 MPa) and GSI (25–45). Although the existing support system, consisting of an M50 concrete liner of 0.3 m thickness, has been designed based on deterministic analysis, it is uncertain whether it achieves the desired level of reliability under the actual range of rock mass conditions. Previous studies (Bukaçi et al., 2017, Valley & Duff, 2010) have shown that probabilistic analyses can yield significantly higher estimates of deformation and plastic zone radius compared to deterministic models. Therefore, there is a need for the reliability analysis under the parameter variability.

The core replacement technique offers a realistic means of simulating the three-dimensional face advance effect in two-dimensional models, while the PEM approach allows for efficient probabilistic assessment. Yet, their combined application for reliability analysis of real hydropower tunnels in Nepal has not been extensively documented. This gap motivates the present research, which aims to integrate these two methods in RS2 modeling to assess the reliability of the existing SMDMP headrace tunnel support system.

1.3 Research Objectives

The aim of this study is to analyze the reliability of the existing tunnel support system of the SMDMP headrace tunnel using the core replacement technique and Point Estimate Method (PEM) within the RS2 numerical modeling framework.

The specific objectives are:

a. To collect and statistically characterize rock mass parameters (UCS, GSI,) from field and laboratory data, determining their mean values and standard deviations.
b. To perform deterministic analysis of tunnel excavation and support installation using the core replacement technique to evaluate deformation, plastic zone radius, and factor of safety.
c. To conduct reliability analysis using Rosenblueth's Point Estimate Method (PEM) in RS2 by varying the input parameters based on their statistical distributions and compute reliability index (β), probability of failure (Pf) for critical points and also assess the adequacy of the support from deterministic and probabilistic approaches.

1.4 Scope of the Study

This study focuses on the numerical and probabilistic analysis of tunnel stability for the Sunkoshi Marin Diversion Headrace Tunnel. It aims to evaluate tunnel deformation, plastic zone development, and support performance under varying rock mass conditions using both deterministic and reliability-based approaches.

The scope includes the following aspects:

I. Geological Site Conditions: The study considers the headrace tunnel section based on the geological data obtained from exposed tunnel faces. Field and lab parameters like RMR, UCS, mi are evaluated. These data are statistically analyzed to determine the mean and standard deviations, which serve as input parameters for subsequent analyses.
II. Numerical Modeling: Numerical analysis is carried out using the Rocscience Phase2D (version 8) finite element method (FEM)-based software. The tunnel excavation process is simulated through the core replacement technique, which effectively replicates the three-dimensional advance of the tunnel face in a 2D model. This approach enables the estimation of ground response prior to and after support installation.
III. Deterministic Analysis: The tunnel excavation and support installation is first analyzed using a deterministic approach, where the mean values of the

selected parameters are applied as a single set of inputs. The pre-support tunnel wall deformation is computed using the Modified Longitudinal Displacement Profile (LDP) proposed by *Vlachopoulos and Diederichs (2009)* to estimate the deformation at the time of support installation.

IV. Reliability analysis: To incorporate the variations in rock mass properties, a reliability analysis is performed using Rosenblueth's PEM. This method enables the calculation of the reliability index (β) and the probability of failure (Pf) for the tunnel support system, thereby quantifying the uncertainty.

1.5 Limitations of the Study

The project report is prepared under the following limitations:

a. The analysis is based on two-dimensional plane-strain modeling, which assumes uniform conditions along the tunnel axis and does not fully capture three-dimensional stress redistribution or face effects beyond those simulated by the core replacement technique.
b. The input parameters (UCS, GSI, and m_i) were treated random variables; correlations among them were not explicitly considered.
c. The reliability analysis was limited to the Point Estimate Method (PEM) due to its computational efficiency. More advanced probabilistic methods such as FORM, SORM, or Monte Carlo simulation were not applied.
d. Single section is considered for the analysis.

2. LITERATURE REVIEW

2.1 Design Approach:

The sound understanding of the mechanical behavior of rock masses and their interaction with underground openings is essential for design of tunnels. Upon tunnel excavation in-situ stresses gets disturbed and redistributed with the establishment of plastic zones around the opening and displacements of tunnel wall. The magnitude of this response depends primarily on the engineering behavior of rock mass, loading conditions and the sequence of excavation and support installation.

Traditional design approach is based on assumptions and simplified geometry however the results obtained will not be reliable because of the complexity of the ground conditions. For handling the complex ground conditions and its uncertainty numerical modeling tool with probabilistic approach is used. This chapter presents an overview of key theoretical and empirical developments relevant to this study, including rock-mass characterization, estimation of deformation modulus, methods for pre-support deformation evaluation, numerical modeling approaches, and reliability-based analyses in tunnel engineering.

2.2 Rock-Mass Behavior and Classification:

Rock mass behave differently from the intact rock because of presence of discontinuities. Rock mass characterization is done using classification systems such as RMR, Q-system, and GSI, which were developed by various researchers (Bieniawski, 1973; Barton et al., 1974; Hoek & Brown, 1997).

2.2.1 Rock Mass Rating:

The RMR system widely used for classification is based on the six parameters: the UCS of the intact rock, RQD, the spacing of discontinuities, the condition of joints, groundwater conditions, and the orientation of discontinuities. The first five parameters are assessed and added to find RMR value ranging from 0 to 100. The sixth parameter is the adjustment parameter for orientation of discontinuities. The RMR based classifications is given below as given by: (Bieniawski, 1988).

Table 2.1 Rock Mass Classes based on RMR (Adapted from Bieniawski, 1988)

RMR	81-100	61-80	41-60	21-40	0-20
Class Number	I	II	III	IV	V
Description	Very good rock	Good rock	Fair rock	Poor rock	Very poor rock

At excavated and exposed tunnel face, the geotechnical team performs a detailed face-mapping survey, during which all the parameters required for the RMR system are assessed and combined to determine the RMR value corresponding to that specific chainage.

2.2.2 Geological Strength Index:

GSI, another classification system for rock mass is based on structure of rock and surface condition. From these qualitative parameters a GSI value is assigned using the Figure 2.1 (Hoek, 1994).

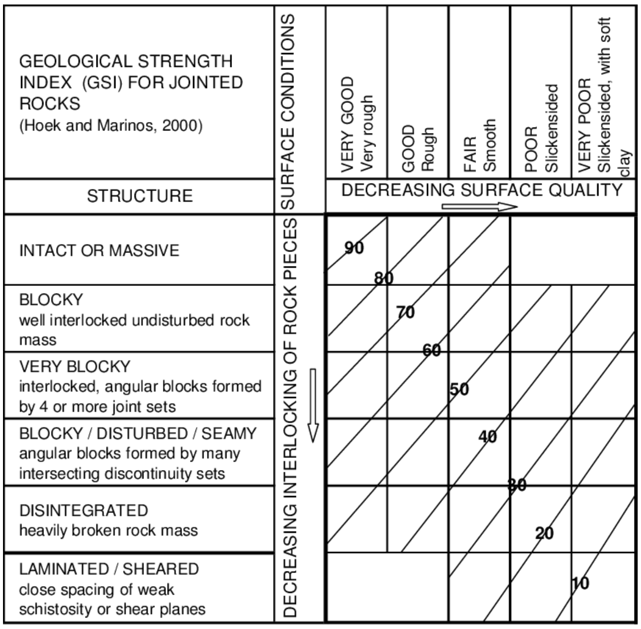

Figure 2.1 GSI classification chart as given by Hoek,1994

The GSI is used as an input in the numerical modeling software because it characterize the rock mass behaviour from intact rock mass properties using HB criterion. The GSI can be correlated with observed RMR using accepted empirical formula, while the intact-rock constant (m_i) and UCS are obtained from laboratory tests. Together, these parameters are used to estimate the rock mass parameters (m_b, s, and a) and the E_{rm}, which serve as key inputs to numerical models. Accurate estimation of these parameters is essential for realistic simulation of tunnel deformation and support performance. GSI can be estimated from RMR as follows:

$$GSI \approx RMR_{89}-5 \quad (1)$$

where RMR_{89} is the value computed according to Bieniawski (1989).

2.3 Hoek and Brown Failure Criterion

The Hoek–Brown Failure Criterion first developed by Hoek and Brown (1980) and further improved (Hoek & Brown, 1997),(Carranza-Torres & Fairhurst, 2000), is an empirical criterion widely used to represent the nonlinear failure behavior of rock masses. This criterion is equally applicable to the rock masses along with intact rock.

The generalized form of the criterion is expressed as

$$\sigma_1 = \sigma_3 + \sigma_{ci}(m_b \frac{\sigma_3}{\sigma_{ci}} + s)^a \quad (2)$$

where σ_1 and σ_3 are the major and minor effective principal stresses at failure, σ_{ci} is the uniaxial compressive strength of the intact rock, and m_b, s, and a are empirical constants that describe the rock-mass properties. These constants are derived from the intact-rock parameter m_i and the Geological Strength Index (GSI) as follows :

$$m_b = m_i e^{(GSI-100)/(28-14D)}, s = e^{(GSI-100)/(9-3D)} \quad (3)$$

$$a = 0.5 + \frac{1}{6}(e^{-GSI/15} - e^{-20/3})$$

where D is the disturbance factor representing the degree of blast damage or stress relaxation (0 = undisturbed, 1 = heavily disturbed) (Hoek & Brown, 1997) .

Not only on intact rock HB criterion is also applicable to rock mass. Thus HB criterion along with GSI is inbuilt into modern FEM based tools like Rocscience to model the non linear behavior of them.

2.4 Estimation of Rock-Mass Deformation Modulus:

The deformation modulus of a rock mass represents its elastic modulus values and and used to find the stress and deformation. Direct in-situ determination of E_{rm} through plate-loading or flat-jack tests is often limited by high cost, long testing time, and difficulties in data interpretation. Therefore, empirical relationships linking E_{rm} to rock-mass classification indices have been developed.

(Hoek & Diederichs, 2006) reviewed a large database of deformation-modulus measurements and proposed empirical equation for rock mass from intact rock . This is useful when field test is not conducted to find the deformation modulus.

$$Erm = Ei(0.02 + \frac{1-\frac{D}{2}}{1+e(60+15D-GSI)/11}) \quad (4)$$

where E_{rm} is in MPa and D is the disturbance factor (0 for undisturbed, 1 for heavily blasted rock).

For cases where the intact modulus (E_i) is known, a more detailed relationship provides improved accuracy.

Intact rock modulus Ei can be obtained from the intact UCS using the modulus ratio as given by Deere et. al (1968) as:

$$(E_i) = MR*\sigma_{ci} \quad (5)$$

2.5 Estimation of Pre-Support Deformation Using the Improved LDP Method

When a tunnel face advances, ground deformation develops progressively ahead of and behind the face. The deformation that occurs before support installation—known as pre-support or unsupported deformation—directly influences the load acting on the lining.

To quantify this deformation, (Vlachopoulos & Diederichs, 2009) proposed the Improved Longitudinal Displacement Profile (LDP), derived from three-dimensional finite-element simulations. The LDP provides an exponential relationship between the distance of a tunnel section from the face and the fraction of total radial displacement that has already occurred:

$$u_0^* = \frac{u_0}{u_{\max}} = \frac{1}{3}e^{-0.15R^*} \quad (6)$$

where u_0^* is the normalized deformation at the tunnel face.

u_o - radial displacement at the face location

u- maximum deformation for far face condition$_{max}$

R^*- Normalized plastic radius given by $\mathrm{R}^* = \frac{R_p}{R_t}$ (7)

where R_p is plastic radius and R_t is tunnel radius

For the support installation distance x, normalized displacement at the support installation distance $u^*\left(\frac{u}{u_{max}}\right.$ is given as$\left.\right)$

$$u^* = 1 - (1 - u_0^*)e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0 \tag{8}$$

$$where\ X^* = \frac{X}{R_t} \text{ is the normalized support installation distance}$$

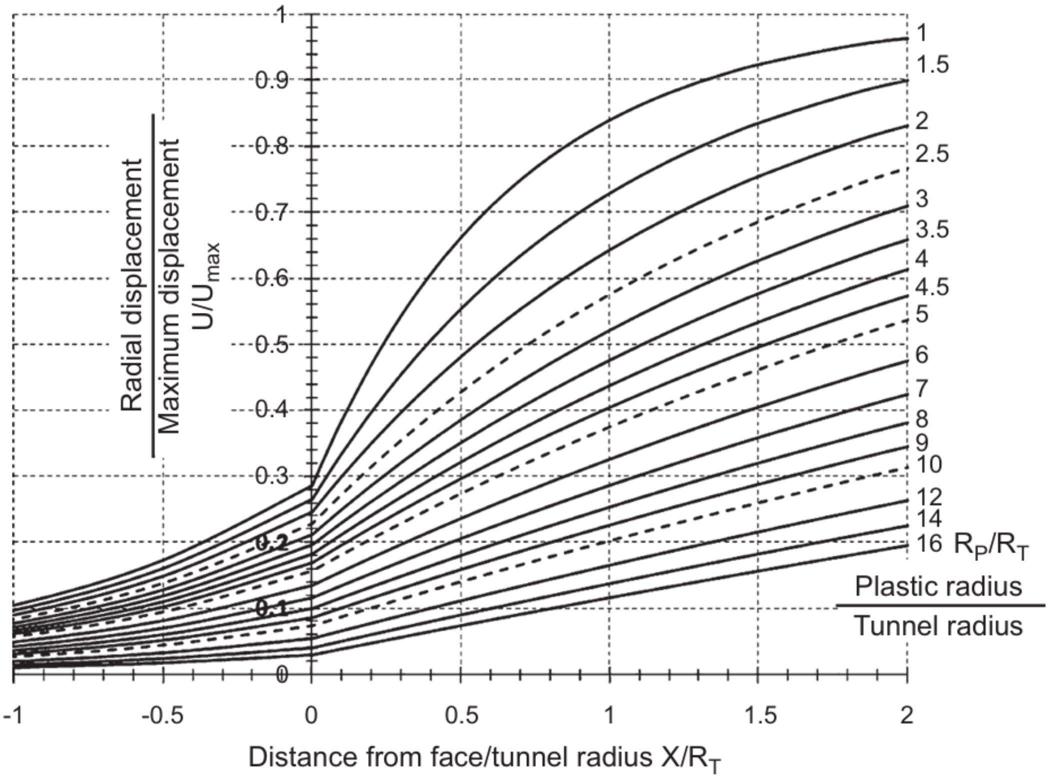

Figure 2.2 LDP template given by Vlachopoulos and Diederichs (2009)

The above template is used as an alternative to the above equations for the calculation of deformation prior to support installation.

The improved LDP enables engineers to estimate the amount of deformation at any stage of excavation and, therefore, to identify the proper stage for support installation in numerical models. It effectively links the three-dimensional face-advance effect with two-dimensional analyses, ensuring that the modeled support activation reflects actual field conditions.

Hoek and Diederichs modulus relationship in conjunction with LDP is used to find the support installation state. This information is essential for the Core Replacement Technique used in this study.

2.6 Numerical Modeling

Numerical modeling is the useful computational technique to find the responses under loading for complex geometry and ground condition. In tunnel engineering, it is used to simulate the stress redistribution, deformation, and potential failure zones that develop around an opening. Analytical solution is based on simplifications and useful only for small ground domain while numerical modeling technique provides solution for complex and large ground domain.

By employing numerical methods such as FEM or FDM, engineers can represent complex geological conditions, non-linear rock mass behavior, anisotropy, and staged excavation sequences that closely resemble actual construction processes. This capability enables optimization of the support system design, assessment of stability and deformation control. Hence, numerical modeling has become an indispensable part of modern tunnel design and analysis

Rocscience Phase2 (v8) is an advanced two-dimensional finite element modeling software widely applied in geotechnical and underground engineering. It allows simulation of the stress redistribution, deformation, and failure behavior of rock and soil masses during excavation and subsequent support installation. One of its key advantages is its ability to perform staged analyses, enabling the sequential modeling of excavation, stress relief, and support application to realistically replicate tunneling conditions in the field.

Phase2 was selected for this study due to its robust capability to analyze nonlinear rock mass behavior through HB failure criterion. Also it allows to use various supports for tunnel like concrete liners, shortcrete, bolts etc. The software also facilitates interaction analysis between ground and support, helping evaluate the structural adequacy of supports under changing stress conditions. Furthermore, its integration with probabilistic modules and user-defined material properties makes it a reliable tool for both deterministic and reliability-based analyses of tunnel stability and support performance.

2.6.1 Core Replacement Technique

The excavation of a tunnel represents a four-dimensional problem when time effects are taken into account, and a three-dimensional problem in their absence, since the progression of the tunnel face leads to ongoing redistribution of stresses within the surrounding rock mass. Ideally, a full three-dimensional numerical model would capture this behavior accurately. However, such modeling is associated with high computation and time-consuming. To overcome these limitations, two-dimensional (2D) plane strain modeling in conjunction with appropriate technique can be used for tunnel analysis and design. This approach simplifies the problem while maintaining sufficient accuracy, provided that the influence of the advancing face is properly simulated.

In 2D analyses, the stress redistribution because of advancing face can be modeled using two main approaches:

Convergence Confinement Method(CCM)

Core Replacement Technique(CRT)

Both methods are used to represent the gradual stress release and deformation at the point as the tunnel face advances. Near the tunnel face, the rock mass carries a portion of the original in-situ stresses. As the tunnel advances, these stresses are redistributed, and the boundary deformation occurs . The values of deformation with the face advance is essential for computing the support forces and support response.

In the Convergence–Confinement Method, the load factor on the tunnel boundary is decreased from value 1 to 0 gradually to represent the excavation sequence. In the Core Replacement Technique, inside core modulus is gradually reduced and finally removed to simulate the face advance.

In the CRT, the numerical model begins with the rock mass in its original, undisturbed condition, represented as a full 2D plane strain domain. The central region corresponding to the tunnel (the "core") initially possesses the same elastic properties as the surrounding rock mass. Then series of stage analysis is done. At each stage, the modulus of the core material is gradually reduced from its surrounding rock mass deformation value to lower values, and finally to zero, which corresponds to a completely excavated tunnel. This sequential reduction in deformation modulus effectively models the advance of the tunnel face and the accompanying redistribution of stresses along with deformations.

The final stage represents the condition where the tunnel face is sufficiently far from the section of interest, and the stresses have fully adjusted to the new equilibrium-this is often referred to as the far-face condition. From this stage, the model provides key parameters such as the maximum wall deformation, the location of the maximum displacement, and the radius of the plastic zone. These results form the basis for subsequent support design and performance assessment.

Before installing the tunnel support, it is necessary to estimate the amount of wall deformation that occurs prior to support installation, as the tunnel face typically advances a certain distance beyond the installation point. The empirical relationships proposed by Vlachopoulos and Diederichs (2009) are commonly used for this purpose. They provide a link between the distance of the advancing face and the corresponding deformation. Using this relationship, the deformation value at the time of support installation can be determined.

A stage versus deformation plot is generated from the modeling results. The stage that produces deformation equal to the empirically calculated pre-support deformation is identified as the point of support installation. After this stage, the appropriate support system (e.g., a shotcrete or concrete liner) is added to the model. The model is then re-run to evaluate the interaction between the ground and the support, and the Factor of Safety (FOS) of the support system is obtained based on the resulting stress and deformation distributions.

Conceptually, the Core Replacement Technique is similar to the Convergence–Confinement Method in that both represent the progressive nature of tunnel excavation and the associated ground relaxation. However, the CRT differs fundamentally in its modeling mechanism: while the CCM simulates excavation through boundary pressure reduction, the CRT achieves it by modifying the stiffness of the tunnel core. This makes CRT particularly advantageous in finite element analyses, such as those performed in Rocscience Phase2, where staged reduction of material modulus is straightforward to implement and effectively captures the nonlinear behavior of rock mass relaxation.

A schematic representation of the Core Replacement Technique is typically used to illustrate this process, showing the sequential reduction of the core modulus, the resulting deformation of the tunnel boundary, and the stage of support installation.

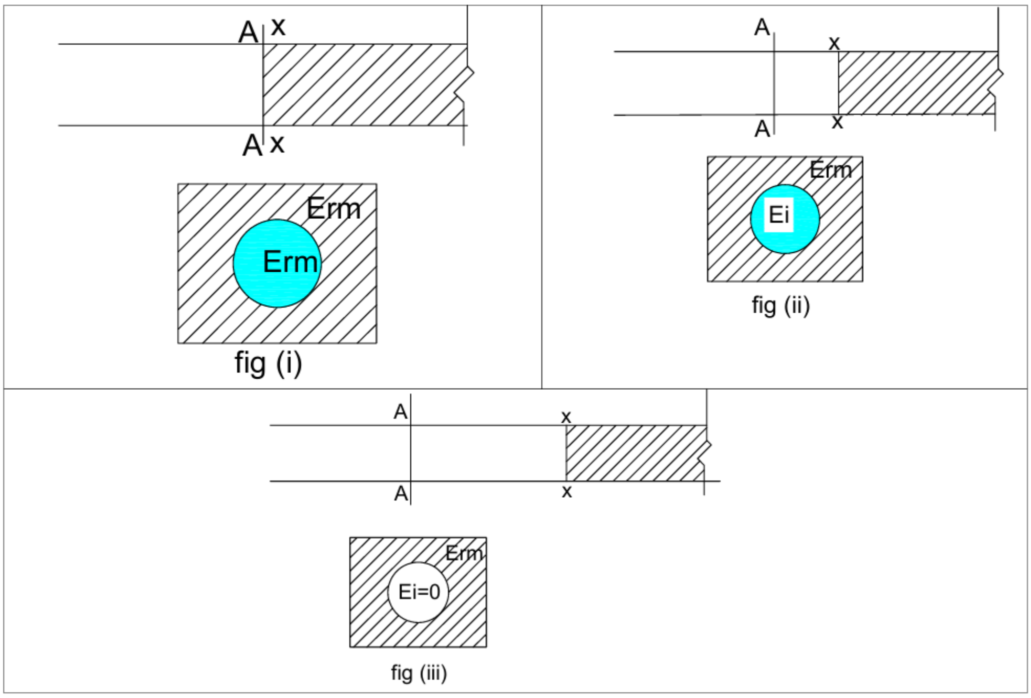

Figure 2.3 Schematic representation of CRT technique

Schematic representation of the Core Replacement Technique showing A–A′ as the point of consideration and X–X′ as the tunnel face. Fig. (i) shows when the face and analysis point coincide, so both have the same modulus (E = Erm); fig. (ii) shows the face advanced by some distance, reducing the core modulus (Ei < Erm); and fig. (iii) shows the face sufficiently far away, where the core modulus becomes zero (Ei = 0).

This technique provides a practical, computationally efficient, and conceptually sound approach for simulating tunnel excavation in 2D, enabling engineers to evaluate both ground response and support performance with sufficient accuracy for design applications.

2.7 Probabilistic and Reliability-Based Analyses in Tunneling

Uncertainty is inherent in rock engineering due to natural variability in material properties and limited site investigation data.

Uncertainty in rock mechanics arises naturally from the variations on the geological formations and artificially because of limited knowledge, accessibility for field and laboratory investigations. According to (Myers, 2005), uncertainties that influence risk and reliability analysis in geotechnical engineering can be broadly classified into three major categories: natural variability, knowledge uncertainty, and decision-model uncertainty (Figure 2.4).

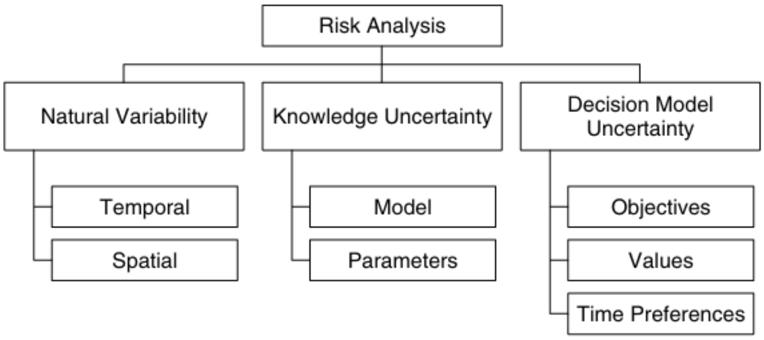

Figure 2.4 Categories of uncertainty entering risk analysis," adapted from Myers, 2005

Natural variability refers to the inherent randomness of geological formations, which causes spatial and temporal variations in parameters such as strength, stiffness, and in-situ stress. This variability is a result of natural geological processes and cannot be eliminated, only described statistically through sufficient sampling and probabilistic modeling.

Knowledge uncertainty comes from incomplete understanding of the subsurface conditions and the mechanical behavior of rock masses. It includes three main subcategories:

- Site characterization uncertainty, due to limited or inaccurate data from field and laboratory investigations, measurement errors, or insufficient spatial coverage;
- Model uncertainty, reflecting the inability of analytical or numerical models to perfectly represent the actual physical behavior of rock masses; and
- Parameter uncertainty, which results from imprecision in estimating model input parameters (e.g., UCS, GSI, modulus) because of limited observations or data scatter.

Finally, decision-model uncertainty is for the factors, such as assumptions made during design, and construction practices that will be different from the modeling.

In rock-engineering applications, these uncertainties collectively affect predictions of tunnel stability, deformation, and support performance. Recognizing and quantifying them through probabilistic and reliability-based methods—such as the Point Estimate Method (Rosenblueth, 1975) or Monte Carlo simulation-provides a rational basis for incorporating risk into tunnel design.

Traditional deterministic analyses produce a single factor of safety, which does not reflect the probability of failure. To overcome this limitation, reliability-based methods quantify the likelihood that a structure or support system will perform satisfactorily under uncertain conditions (Park et al., 2012).

Different Probabilistic analysis methods are such as Monte Carlo Simulation (MCS), and analytical approximation methods, such as the First-Order and Second-Order Reliability Methods (FORM/SORM) and the Point Estimate Method (PEM). There are several practices for reliability analysis of tunnel support design like Monte Carlo Simulations, analytical approximation methods like FORM, SORM. In probabilistic analysis of tunnel support systems, FORM, SORM and MCS are often limited by their computational demands. They rely on iterative procedures and, in the case of second-order approaches, additional curvature evaluations, which make them impractical when coupled with complex numerical models. Bukaçi et al., 2017 analyzed the diversion tunnel at the Rrëshen Hydropower Plant in Albania using Monte Carlo simulations. Their approach involved generating random input values and performing a large number of iterations, which, although computationally intensive, provided a probabilistic assessment of tunnel performance. To overcome the computational challenges associated with FORM and SORM in finite element analyses, a simplified yet efficient probabilistic technique was introduced by Rosenblueth (1975) the Point Estimate Method (PEM). The fundamental idea of this method is to approximate the statistical moments of output variables using a limited number of strategically selected evaluation points, rather than performing extensive numerical integrations. As highlighted by McGuffey et al., 1981, the PEM approach offers a significant advantage in geotechnical applications because it does not require complex mathematical formulations or large computational resources, while still capturing the essential effects of input variability on system performance. Its simplicity and compatibility with finite element modeling make it particularly suitable for reliability analysis of tunnel support systems in programs such as RS2.

2.7.1 Point Estimate Method:

The Point Estimate Method (PEM) is a probabilistic approach developed by Rosenblueth (1975) and later applied to geotechnical problems by Mc Guffey et al. (1978) for estimating probability moments of engineering responses. The method provides an efficient way to propagate uncertainty in input parameters through analytical or numerical models without requiring extensive Monte Carlo simulations or complex derivative calculations such as those used in FORM or SORM.

In this approach, each random variable is represented not by a continuous probability density function (pdf) but by a discrete two-point probability mass function. Equal weighting is given to the two values equally spaced from their mean one at one standard deviation above the mean and other at one below the mean. Equations below adapted from McGuffey et al. 1978:

$$X_+ = \bar{X} + \sigma_x, X_- = \bar{X} - \sigma_x \quad (9)$$

$$P_+ = P_- = \frac{1}{2} \quad (10)$$

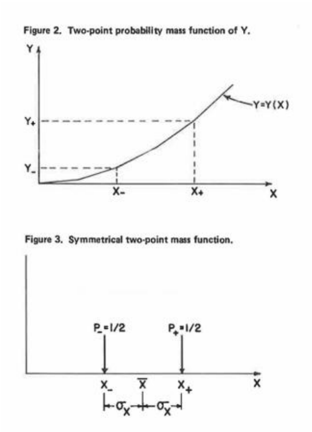

Figure 2.5Adapted from the McGuffey et al. 1978

If the performance function $Y = f(X_1, X_2, \dots, X_n)$ depends on *n* uncorrelated random variables, the function is then approximated by 2^n combinations of these upper and lower point values. Each combination is evaluated in the numerical model, and the resulting outcomes are combined to compute the mean and variance of *Y* as:

$$E(Y) = \sum P_i Y_i, Var(Y) = \sum P_i Y_i^2 - [E(Y)]2 \quad (11)$$

where Y_i is the model output (e.g., Factor of Safety) obtained from each combination of input variables and P_i is its associated weight. Because only 2^n model runs are needed, PEM is highly efficient and readily compatible with finite-element software such as RS2, which already performs staged analyses. In this study, the PEM framework was used to quantify the effect of uncertainty in three key geotechnical parameters UCS, GSI, and intact constant (m_i) on the computed Factor of Safety (FOS) of the tunnel support system. Each parameter was varied, and the deterministic RS2 model was evaluated for all eight possible combinations of (X+,X-) values. From the resulting mean and standard deviation of FOS, the β and P_f were derived representing variations of properties to the likelihood of support failure.

The key advantage of the PEM is its simplicity: it captures the first two statistical moments of model outputs with minimal computational cost while preserving the nonlinear relationships among parameters. This makes it especially practical for reliability analysis of complex geotechnical systems, where full probabilistic simulations are computationally prohibitive.

Several recent studies have demonstrated the growing significance of incorporating probabilistic approaches into tunnel design and analysis to capture the uncertainties inherent in geological and geotechnical parameters. Periku et al., 2000 performed a *reliability analysis of tunnel support systems* using FEM combined with PEM for the *Energy Tunnel 2* in the *Rrëshen Hydropower Project, Albania*. Their study employed the CRT to simulate 3D excavation in 2D space and evaluated the β and P_f by considering the variability in key rock mass parameters such as UCS, GSI, and intact rock parameter (m_i). The results highlighted that deterministic safety evaluations often underestimate the risk of instability, whereas probabilistic methods provide a more realistic measure of tunnel performance under uncertainty.

Similarly, Paudel et al., 2022 applied a *probabilistic finite element approach* to assess the stability of the tunnel lining at the *Phukot-Karnali Hydroelectric Project, Nepal*. Their study compared deterministic and probabilistic analyses using the Rocscience Phase2 software. The probabilistic analysis, performed using Rosenblueth's PEM, considered the variability of UCS, GSI, Poisson's ratio, and Young's modulus. The findings revealed that while deterministic analysis showed a stable support system with a factor of safety greater than one, probabilistic analysis identified failure probabilities up to 5% for shotcrete and 4% for steel ribs, depending on tunnel location. The authors concluded that deterministic methods alone may lead to an overly optimistic interpretation of tunnel safety and emphasized the need for reliability-based assessment in the Nepalese tunneling context.

More recently, Sebbeh-Newton et al., 2025 conducted a comprehensive *probabilistic analysis of a Malaysian water tunnel*, quantifying rock mass parameter uncertainties through Monte Carlo simulation and employing the built-in PEM module in RS2 to predict displacement and plastic zone evolution. Their results demonstrated that the inclusion of parameter uncertainty leads to more accurate estimates of deformation and support loading.

Collectively, these studies illustrate that reliability-based tunnel analysis, integrating probabilistic methods such as the Point Estimate Method with finite element modeling, provides a more rational framework for understanding tunnel behavior under uncertainty. They further establish that while deterministic design remains useful for preliminary evaluation, probabilistic approaches yield deeper insights into the variability of ground conditions and support system performance-an approach directly adopted and extended in the present study to evaluate the reliability of the SMDMP headrace tunnel support system under Himalayan rock mass conditions.

2.8 Summary and Research Gap

While significant advancements have been made internationally in the development of empirical, numerical, and probabilistic approaches for tunnel design, their application in the Nepalese tunneling context remains limited. Methods such as the Hoek–Diederichs (2006) correlation for estimating rock-mass modulus, the Vlachopoulos–Diederichs (2009) approach for pre-support deformation, and reliability-based analyses using the Point Estimate Method (PEM) have been successfully integrated into several hydropower and transportation tunnel studies worldwide. However, within Nepal, the majority of tunnel stability assessments still rely predominantly on deterministic analyses or empirical classifications such as RMR and Q-system, with little emphasis on quantifying the effects of geological uncertainty on support performance.

Despite the country's rapidly growing portfolio of underground hydropower projects, there exists very limited research focusing on reliability-based design or performance evaluation of tunnel support systems under local Himalayan rock-mass conditions. Previous works, such as Paudel et al. (2022), have demonstrated the feasibility of probabilistic analysis using FEM and PEM, but the approach has yet to be extended to assess the reliability of existing support systems installed in operational or ongoing hydropower tunnels.

Therefore, this study addresses that critical gap by applying a combined Finite Element – Core Replacement Technique (CRT) and Point Estimate Method (PEM) framework to the Sunkoshi Marin Diversion Multipurpose Project (SMDMP) headrace tunnel. The research aims to quantify the reliability level of its M50 concrete lining, accounting for the uncertainty in key rock-mass parameters (UCS, GSI, and m_i) and to demonstrate how probabilistic evaluation can supplement conventional deterministic analyses in the context of Nepal's emerging underground construction sector.

3. METHODOLOGY

3.1 Overview

The methodological framework adopted in this study integrates collection and statistical analysis of field data, numerical modeling, and reliability analysis to evaluate the tunnel excavation responses and support characteristics under ground variability. The overall workflow followed throughout the study is illustrated in the flowchart presented in Figure 3.1.

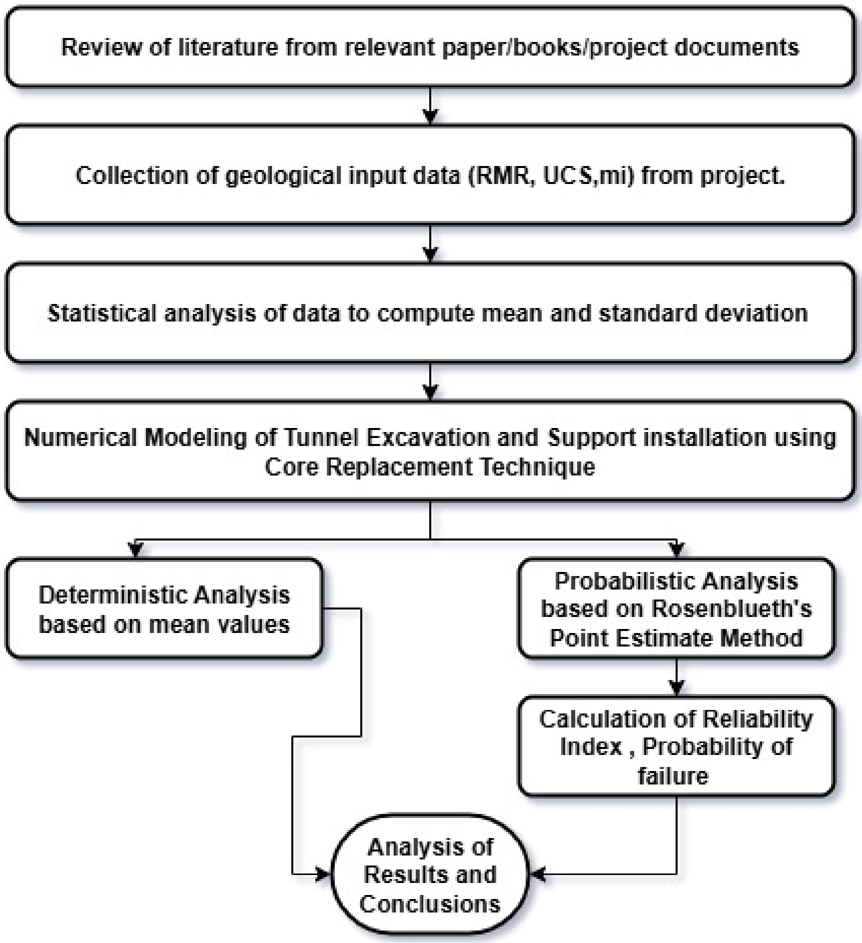

Figure 3.1 Flowchart of the methodology adopted in this study

The approach begins with the collection and statistical characterization of input data, followed by the development of deterministic and reliability-based numerical models using the core replacement technique in Rocscience Phase2 (v8). The deterministic analysis establishes baseline deformation and safety conditions, while the reliability-based approach quantifies the effects of uncertainty in key rock mass parameters on tunnel stability.

3.2 Input Data and Site Description

Numerical modeling of tunnel excavation in rock mass requires both the geometric configuration of the tunnel and the mechanical properties of the surrounding rock mass. The present study focuses on SMDMP Headrace Tunnel, which serves as the case study.

As per the Geotechnical Baseline Report the SMDMP site is located in Siwaliks, Lesser Himalayas, and Higher Himalayas. Major thrusts of Nepal Himalaya such as the Main Boundary Thrust (MBT) and Mahabharat Thrust (MT) lie within this project area. The study area is a hilly region with rugged topography and altitude more than 1950 masl.

The tunnel, having a finished diameter of 6.4 m, is being excavated using a Double Shield Tunnel Boring Machine (DS-TBM) supplied by Robbins. Geological characterization was based primarily on face mapping data collected during excavation through manholes at accessible chainages as shown in the fig Figure 3.2 below. However, in several locations, the tunnel face could not be observed directly due to rockfalls and unfavorable conditions.

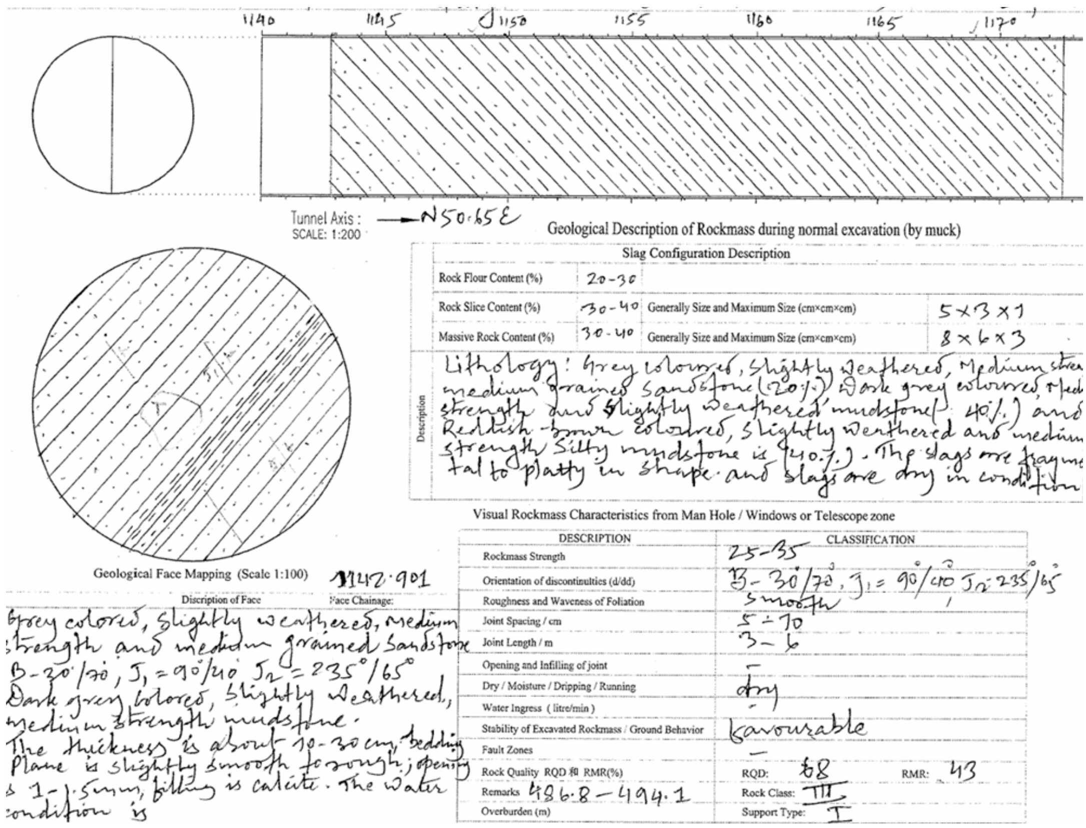

Figure 3.2 Face Mapping at 1+140 m chainage (SMDMP Report)

A total of 120 sets of Rock Mass Rating (RMR) data were compiled from chainages 0+040 m to 4+759 m. The corresponding Geological Strength Index (GSI) values were derived using standard empirical correlations, and their statistical parameters (mean and standard deviation) were computed. Similarly, 23 UCS test results were obtained from laboratory testing of intact rock samples collected along the tunnel alignment. The datasets obtained from face mapping along the tunnel alignment are compiled and presented in Appendix A, which provides detailed records of RMR observations and their corresponding GSI derivations. Likewise, the results of UCS laboratory testing on intact rock samples are summarized in Appendix A for reference and verification. The mean and standard deviation values of the GSI and UCS datasets, which serve as the key input parameters for the numerical analysis, are presented below.

Table 3.1Mean values and standard deviation of key input parameters

	Mean Value	Standard Deviation
UCS (MPa)	39.32	17.38
GSI	34.13	7.79

Face mapping and sample analysis revealed that the dominant lithology in the investigated stretch is moderately weathered sandstone. For this lithological class, the Hoek–Brown material constant (mi) was taken as 17 with a standard deviation of 4, based on established literature values for similar rock types.

3.3 Numerical Modeling Using Rocscience Phase2

Numerical analysis was carried out using Rocscience Phase2 (Version 8), which is a two-dimensional finite element software commonly applied for evaluating stress, deformation, and plastic behavior around underground excavations. The program allows simulation of staged excavation and sequential installation of support systems, making it suitable for analyzing ground response during tunneling operations.

This study investigates the Headrace Tunnel (HRT) of the SMDMP through both deterministic and reliability-based approaches. While tunnel excavation is fundamentally a three-dimensional phenomenon, a two-dimensional plane strain approximation was adopted using the Core Replacement Technique (CRT) to simulate the influence of the advancing tunnel face.

In the CRT approach, excavation is modeled by gradually decreasing the stiffness (Young’s modulus) of the core material from its initial rock mass value (Erm) to zero, thereby representing the transformation from intact rock to an excavated cavity. This stepwise reduction enables the simulation of stress redistribution and deformation as excavation progresses. Once the deformation prior to support installation is obtained, the support system is incorporated into the model, and its interaction with the surrounding rock mass is evaluated to determine the Factor of Safety (FOS).

In the deterministic analysis, the mean values of input parameters namely UCS, GSI, and Hoek–Brown material constant (mi) were considered. Other modeling parameters, such as tunnel geometry, in-situ stress conditions, and support characteristics, were adopted from the project data. The numerical model was developed in Rocscience Phase2 (Version 8) using the Core Replacement Technique with multiple excavation stages, starting from the intact condition (E = Erm) to the fully excavated stage (E = 0). The model was reanalyzed after support installation to obtain the Factor of Safety (FOS) along the liner and the surrounding rock mass, representing the deterministic stability condition.

The reliability analysis extends the deterministic model to include the natural variability and uncertainty in rock mass parameters. The same model geometry, boundary conditions, and support configuration were used, but the input variables (UCS, GSI, and mi) were treated as random variables. Each parameter was varied by mean ± standard deviation, and based on Rosenblueth's PEM, a total of $2^3 = 8$ models were developed and analyzed using the CRT. Using the FOS as the performance function, the reliability index (β) was calculated from the set of deterministic outputs. The P_f was then determined by assuming a normal distribution of the FOS. This approach allowed quantification of the uncertainty effects on tunnel stability and support performance, providing a more realistic measure of design reliability compared to deterministic evaluation.

The CRT effectively modeled both the excavation process and the support interaction through staged modulus reduction in a 2D finite element environment. The deterministic model provided the baseline tunnel response, while the reliability analysis showed the impact of parameter variability on tunnel, enabling probabilistic assessment of the designed support system.

3.4 Deterministic Analysis

The deterministic analysis established the baseline performance of the tunnel under mean input parameters. The following sequential steps were adopted:

i. Computation of pre-support deformation using the Longitudinal Displacement Profile (LDP) approach proposed by Vlachopoulos and Diederichs (2009).

ii. Identification of equivalent core modulus using the core replacement method.

iii. Introduction of M50 grade reinforced concrete liner and evaluation of Factor of Safety (FOS).

The overall modeling procedure adopted in this study for deterministic analysis is described as follows:

Step 1: Model Geometry and Boundary Conditions

- The geometry of the tunnel was modeled based on the actual dimensions of the Sunkoshi Marin Diversion Multipurpose Project (SMDMP) headrace tunnel. Diameter of excavation is taken as 6.4 m. Since the CRT is used for modeling the model is prepared with 12 number of stages.
- A circular tunnel section was modeled under plane strain condition, assuming a homogeneous, isotropic rock mass domain large enough to minimize boundary effects. External boundary was created as Box Type with expansion factor of 5.
- Characteristics of setup mesh is of mesh type graded, element type 6 Noded Triangles.
- The lateral, bottom and top boundaries were fixed in both horizontal and vertical directions.
- In-situ stress conditions were defined using the principal stress values obtained from field data (horizontal = 21 MPa, vertical = 15 MPa, and out-of-plane = 21 MPa).

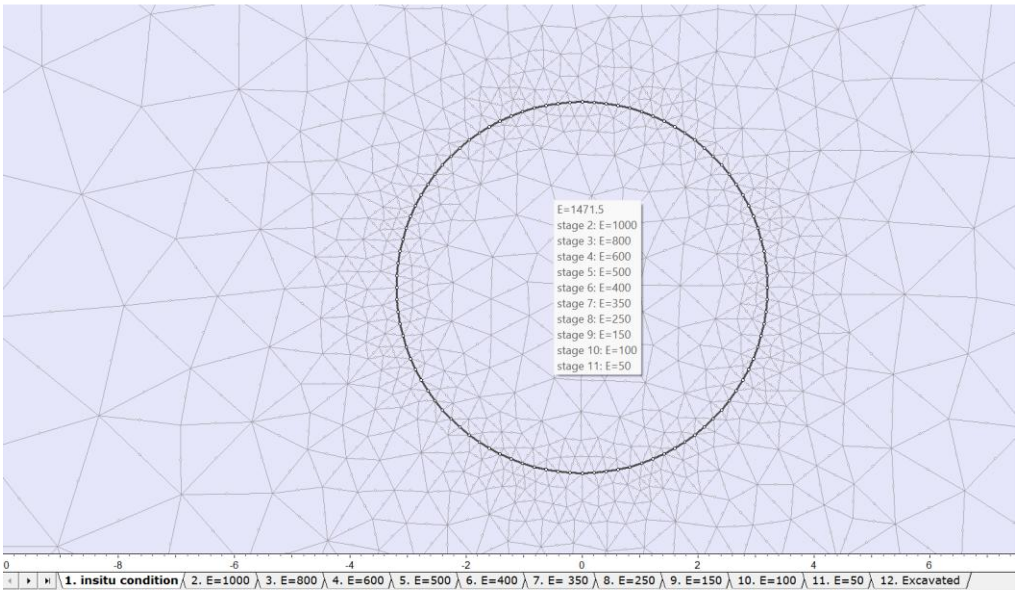

Figure 3.3 Staged modeling for deterministic analysis

Step 2: Material Properties and Rock Mass Parameters

- The rock mass behavior was represented using the Hoek–Brown failure criterion
- The input parameters for the model were obtained from laboratory testing and field observations, consisting of the UCS, GSI, and the Hoek–Brown material constant (mi). For the deterministic analysis, the mean values of these parameters were used as input to evaluate the tunnel's response under the representative rock mass conditions. Input parameters for the rock mass are:

Table 3.2Mean values of input parameters

Intact UCS	39.32
Geological Strength Index	34
m_i	17

- The rock mass modulus (Erm) was estimated using the Hoek–Diederichs (2006) empirical correlation, and modulus ratio was assigned based on lithological type. Using the modulus ratio as proposed by Deere[1968] ,

  Intact Modulus , $E_i = MR \times \sigma_{ci}$

  MR value for sandstone is taken as 350.

  $E_i = 350 \times 39.32$
  $E_i = 13762 Mpa$

  Rock mass modulus (Erm) as suggestd by Hoek and Diederichs (2006) is given by:

$$E_{rm} = E_i(0.02 + \frac{1 - \frac{D}{2}}{1 + e^{(60+15D-GSI)/11}}) \quad (12)$$

$$E_{rm} = 13762\left(0.02 + \frac{1}{1 + e^{(60-34)/11}}\right)$$

$$E_{rm} = 1458.6 Mpa$$

These parameters served as the initial values for the surrounding rock and the core region in the unexcavated model.

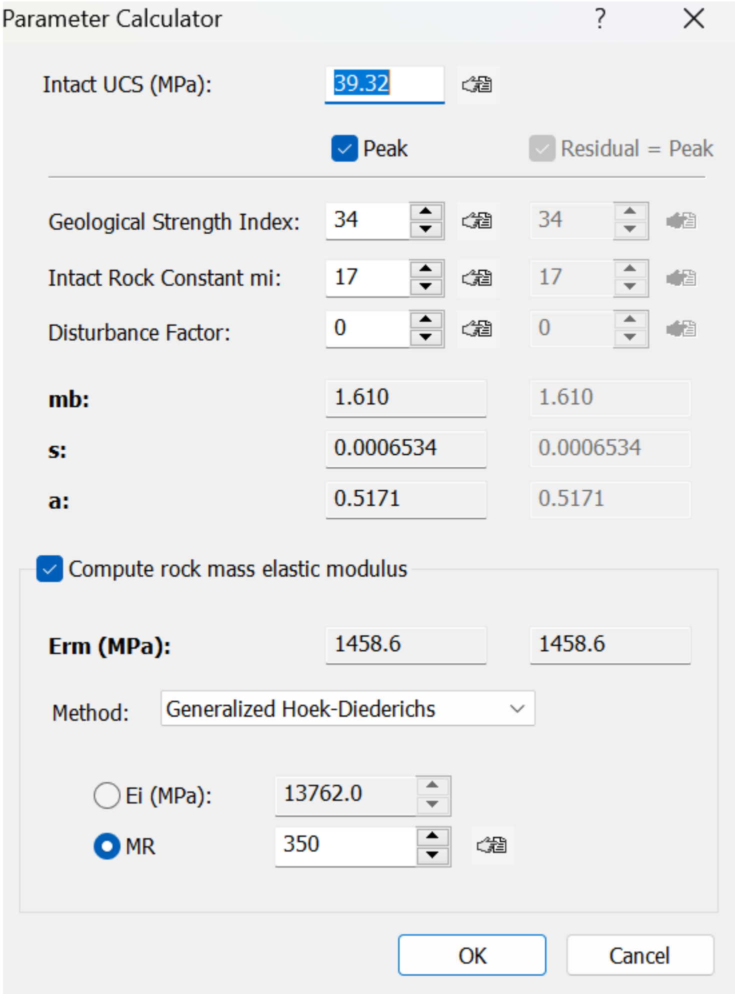

Figure 3.4 Material Properties input for deterministic analysis

Step 3: Initial In-situ Model Setup

- A full-section model representing the pre-excavation condition was created where both the tunnel core and the surrounding rock were assigned the same modulus (E = Erm).
- This represents the stage when the tunnel face coincides with the section of analysis (Figure 2.3), and no stress redistribution has yet occurred.

Step 4: Staged Excavation Simulation (Core Replacement)

- To simulate the progressive advancement of the tunnel face, the Young's modulus of the tunnel core was gradually reduced in 12 stages from the rock mass value (Erm) as above to zero.

- This staged reduction replicates the stress relief and deformation that occur as the face moves away from the section under consideration.
- Each intermediate stage corresponds to a face distance (x) from the point of consideration (A–A′).
- Typically, 12 stages were defined, ensuring smooth modulus transition to capture the face effect adequately.
- The final stage (E = 0) represents the fully excavated condition when the face is sufficiently far away.

Each excavation stage was labeled according to its corresponding core modulus value, and a total of ten material models were defined with Young's modulus values matching those stage-specific moduli. The material inside the excavation boundary was then assigned the appropriate material property for each stage, ensuring that the core region reflected the corresponding modulus (E) at every step of the simulation.

This gradual replacement and reduction of modulus enable the tunnel boundary to deform progressively with each stage of excavation. Across the ten defined stages, the material within the excavation core is successively substituted with one having no initial in-situ stress (i.e., *Initial Element Loading = None*) and a lower Young's modulus than that of the previous stage. In the final stage, the core material is completely removed, representing a fully excavated tunnel section. This staged process effectively simulates the advancement of the tunnel face, where each stage and its associated modulus correspond to a specific face distance relative to the point of analysis. The final stage signifies the condition when the tunnel face is sufficiently distant, such that its influence on stress redistribution and wall deformation becomes negligible.

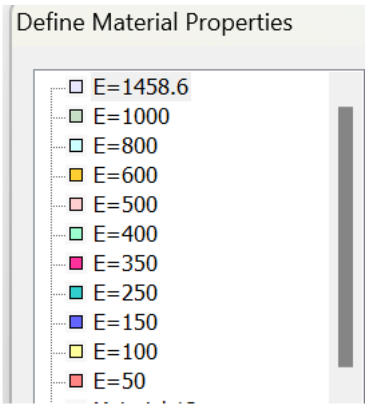

Figure 3.5 Defined material properties

Step 5: Determination of Wall Deformation and Plastic Zone

- The analysis was run for all stages to obtain the displacement contours, plastic zone radius, and maximum tunnel wall deformation (u_{max}).
- The far-face condition (final stage) results were used to determine the total deformation (u_{max}) and radius of plastic zone Rpl.
- This deformation was normalized to estimate the pre-support deformation corresponding to the distance of support installation, using the Vlachopoulos and Diederichs (2009) relationship for the improved Longitudinal Displacement Profile (LDP) as given in equations (5), (6) and (7):

Support installation distance (X)= 1.4 m is taken for the analysis.

Step 6: Determination of Support Installation Stage

- From the deformation versus stage plot, the stage corresponding to the calculated pre-support deformation was identified.
- This stage represents the condition at which the support installation occurs during actual excavation, when the tunnel face has advanced by the installation distance (X).
- Beyond this stage, support elements were introduced in the model to analyze their interaction with the surrounding rock.

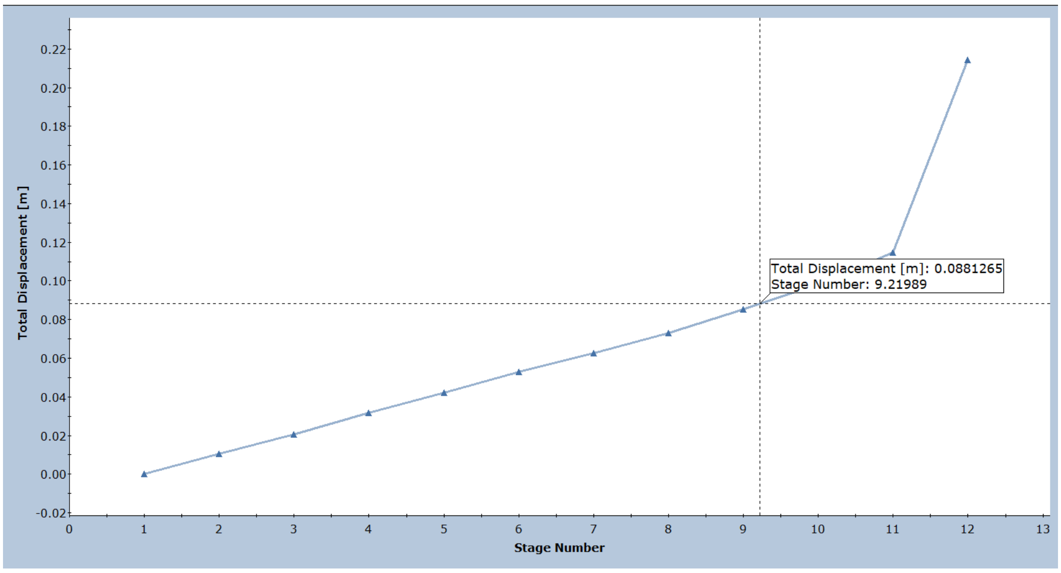

Figure 3.6 Deformation vs stage obtained from the model

Step 7: Support Modeling and Analysis

- The support system used in the analysis consisted of an M50 concrete liner, modeled as a linear elastic material with:
    - Thickness = 0.3 m
    - Elastic modulus = 35,000 MPa
    - Compressive strength = 50 MPa
    - Tensile strength = 4.1 MPa
- The support was activated at the identified after 9th stage as shown in the figure below:

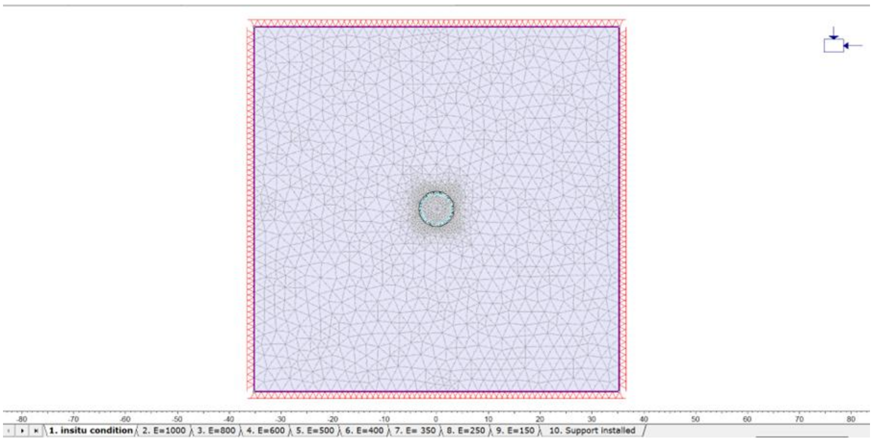

Figure 3.7 Support installed after 9th stage

- The analysis was then rerun to evaluate the deformation control, stress redistribution, and Factor of Safety (FOS) along the liner and surrounding rock.

After incorporating the concrete liner into the model as the tunnel support system, the analysis was extended to evaluate the overall performance and stability of the ground–support interaction. The inclusion of the liner allowed simulation of how the structural support responds to redistributed stresses following excavation and deformation of the surrounding rock mass. Once the support elements were activated in the model, the system was reanalyzed under the same boundary and loading conditions to assess the resulting stress distribution, deformation pattern, and structural response of both the liner and the adjacent rock.

The outcomes of this analysis were interpreted primarily through support capacity plots, which graphically depict the relationship between the applied load and the support's resisting capacity. From these plots, the Factors of Safety (FOS) at various critical points around the tunnel periphery were calculated, enabling identification of the most vulnerable zones and evaluating whether the installed support provides sufficient resistance against potential failure. These FOS values reflect the ratio of the liner's available strength to the induced stresses acting on it, offering a direct measure of its adequacy.

By examining the variation of FOS and stress contours along the tunnel boundary, the model provided a clear understanding of the liner's performance and its effectiveness in maintaining tunnel stability. The support capacity plots thus served as a visual and analytical tool to confirm whether the selected support configuration—under the given rock mass and loading conditions—was structurally safe and functionally appropriate for the excavation stage being simulated.

3.5 Reliability Analysis

Upon reliability analysis UCS, GSI, and the material constant (m_i) were taken as variables. Variations in these inputs propagate through the Hoek–Brown rock-mass parameters (m_b, s, a), the equivalent rock-mass modulus (E_{rm}), and ultimately the Factor of Safety (FOS) at critical tunnel locations. Rosenblueth's Point Estimate Method is used for the reliability analysis.

3.5.1 Rosenblueth's PEM:

In this study, the reliability of the tunnel support system was evaluated using a probabilistic approach based on Rosenblueth's Point Estimate Method (PEM). The objective of this analysis was to quantify the influence of geological variability on the tunnel's stability and the performance of its support system. While the overall modeling sequence followed the same procedure as in the deterministic analysis, the key distinction here lies in the treatment of the input parameters as random variables rather than fixed mean values.

The selected random variables for this reliability analysis were: UCS, GSI, m_i.

These parameters were chosen because they represent the most influential factors governing the mechanical behavior of the rock mass and hence the performance of the tunnel support system. To capture their natural variability the parameters were varied by $\mu \pm \sigma$, resulting in two representative values for each variable (upper and lower bounds).

Since three variables were considered, a total of $2^3 = 8$ combinations were generated, forming eight independent reliability models (REL_1 to REL_8). Each combination shall be considered a case of scenario as shown in Table 3.3.

Table 3.3 Different combinations used in reliability analysis

Name	Analysis type	Reliability analysis parameter		
		UCS	GSI	mi
REL_1	+,+,+	56.70	41.92	21.00
REL_2	-,+,+	21.94	41.92	21.00
REL_3	+,-,+	56.70	26.35	21.00
REL_4	-,-,+	21.94	26.35	21.00
REL_5	+,+,-	56.70	41.92	13.00
REL_6	-,+,-	21.94	41.92	13.00
REL_7	+,-,-	56.70	26.35	13.00
REL_8	-,-,-	21.94	26.35	13.00

Each reliability model was analyzed following the same modeling steps as the deterministic case, using the Core Replacement Technique (CRT) in Rocscience Phase2. The modeling process can be summarized as follows:

i. Model Setup – The tunnel geometry, boundary conditions, and staged excavation configuration were same to those used in the deterministic analysis.

ii. Assignment of Variable Values – For each of the eight reliability models, the corresponding sets of UCS, GSI, and m_i values from Table 3.3 were assigned to the rock mass to reflect the variability of geological conditions.

iii. Staged Excavation Analysis (Core Replacement Technique) – Each model was analyzed through multiple stages, with the Young's modulus of the core region progressively reduced from its rock mass value to zero, representing the excavation process and the advancing tunnel face.

iv. Deformation Evaluation – From each model, the deformation versus stage curve was generated to determine the wall deformation prior to support installation using the empirical Longitudinal Displacement Profile (LDP) approach proposed by Vlachopoulos and Diederichs (2009).

v. Support Simulation – The M50 concrete liner (thickness 0.3 m, modulus 35 GPa, compressive strength 50 MPa, tensile strength 4.1 MPa) was introduced at the equivalent deformation stage corresponding to the distance of support installation. The model was then reanalyzed with the support activated.

vi. Safety Evaluation – The FOS for the tunnel support was obtained at two critical monitoring locations: the tunnel crown (Point 1) and the tunnel sidewall (Point 2), as shown in Figure 3.8. These two points represent the most stress-sensitive regions in the tunnel cross-section.

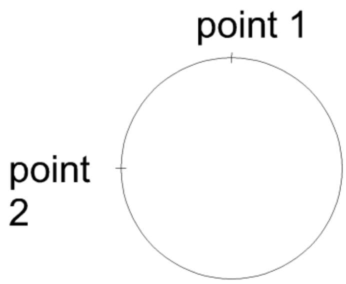

Figure 3.8 Points of analysis

From the eight reliability models, a total of eight FOS values were obtained for each analysis points i.e. point 1 and point 2. Obtained values reflects the effects of uncertainites on parameters on the response of the support.

3.6 Reliability Index and Probability of Failure

The performance function for the reliability analysis for this study is as:

g=FOS-1

where $g = 0$ is the boundary between safety and failure, $g > 0$ indicates safe condition, and $g < 0$ denotes failure.

The computed FOS values are assumed to follow a normal distribution upon reliability analysis using PEM, the reliability index (β) can be expressed as:

$$\beta = \frac{\mu_{FOS} - 1}{\sigma_{FOS}} \quad (13)$$

Where, μ_{FOS} = mean value of FOS obtained from the eight PEM models, and σ_{FOS} = standard deviation of FOS obtained from the same dataset. The reliability index (β) represents the number of standard deviations by which the mean FOS lies above the failure threshold (FOS = 1). A higher β value implies a lower probability of failure and a more reliable support system.

Once β is determined, P_f is calculated using the standard normal cumulative distribution function (Φ) as:

$$P_f = \Phi(-\beta) \quad (14)$$

This expression gives the probability that the FOS will fall below 1, i.e., the likelihood that the tunnel support system will fail under the combined effect of parameter variability. The computed **β** and **P_f** values provide a quantitative measure of the reliability of the tunnel support system. A higher reliability index corresponds to a smaller probability of failure. Conversely, a lower β implies a greater likelihood of failure, suggesting that the support system is sensitive to parameter uncertainty.

By comparing the deterministic FOS with the probabilistic β and P_f, the analysis highlights how geological variability influences tunnel safety and demonstrates the advantage of using reliability-based design over purely deterministic approaches for underground structures.

4. RESULTS AND DISCUSSIONS

This chapter presents the results of both the deterministic and reliability-based analyses performed for the Sunkoshi Marin Diversion Multipurpose Project (SMDMP) headrace tunnel. Numerical simulations were carried out using Rocscience Phase2 (v8), incorporating the Core Replacement Technique (CRT) to represent the excavation sequence, deformation behavior, and subsequent support performance. The analyses aimed to evaluate the adequacy of the designed tunnel support system and to assess the influence of rock mass variability on tunnel stability through probabilistic methods.

4.1 Deterministic Analysis

In the deterministic analysis, the mean values of the input parameters — UCS (σ_{ci}), GSI, and Hoek–Brown constant m_i — were used to represent the average ground condition along the studied section of the tunnel. The model incorporated the actual geometry, in-situ stress conditions, and support characteristics obtained from the project data. Mean values of the parameters taken are:

Table 4.1 Input Parameters

UCS (mean)	GSI (mean)	m_i (mean)
39.32	34.13	17

4.1.1 Tunnel Deformation and Plastic Zone

The staged excavation analysis showed progressive deformation of the tunnel boundary with each reduction of core modulus. At the final excavated (far-face) stage, the maximum radial deformation (u_{max}) at the crown was found to be 0.214 m, while the radius of the plastic zone (R_p) extended to 7.84 m around the tunnel periphery. These values represent the complete stress redistribution after the tunnel face moved sufficiently away from the section under consideration.

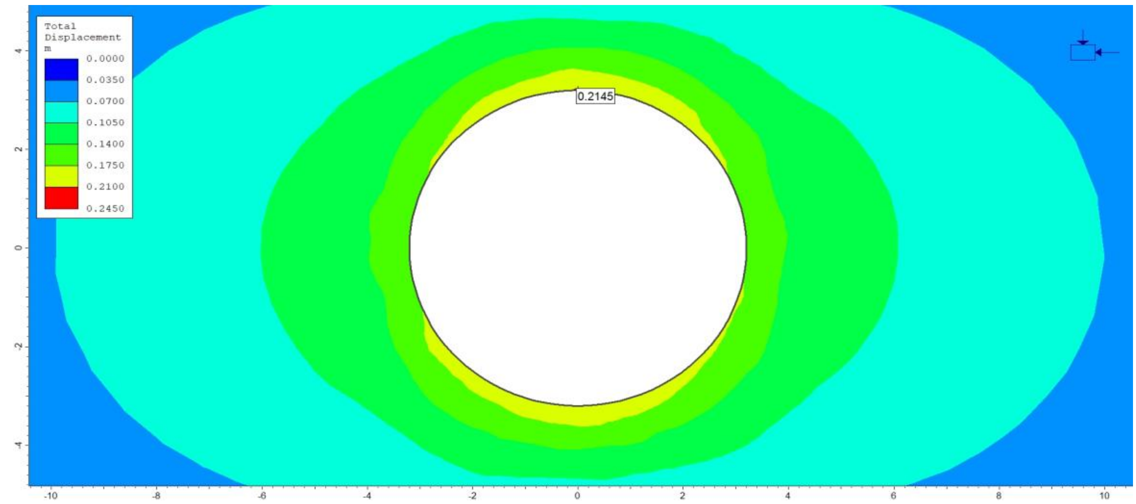

Figure 4.1 Maximum deformation at the crown for final excavated stage for deterministic analysis

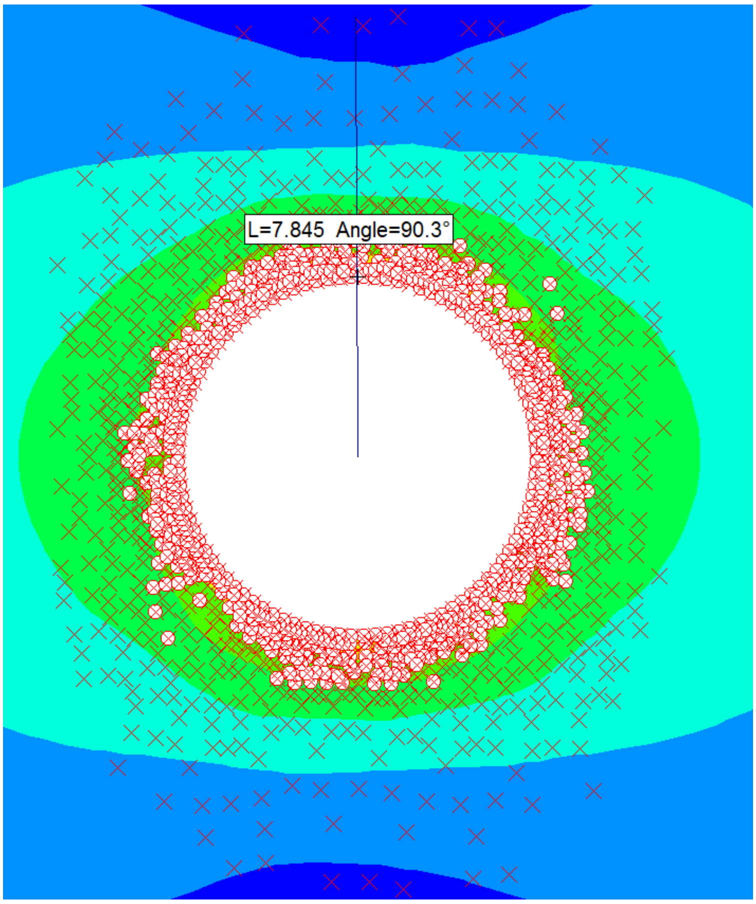

Figure 4.2 Plastic zone radius observed for deterministic analysis model

4.1.2 Support Installation and Performance

Using the Vlachopoulos and Diederichs (2009) empirical relationship for the improved Longitudinal Displacement Profile (LDP), the *normalized support installation distance (X)** was determined as 0.4375, which corresponds to a pre-support deformation of 0.088 m.

Pre support deformation was obtained by using the equations proposed by Vlachopoulos and Diederichs (2009) as given above in equations (6) , (7) and (8) . The involved calculation result are:

Table 4.2 Calculation result for deterministic analysis

Tunnel Radius	3.2m		
Radius of plastic zone	7.84m		
Normalized Radius	2.45		
u max	0.214m	locations	crown
support installation distance	1.4m		
Normalized distance	0.4375		
From Vlachopoulos and Diederichs equations			
uo*(normalized closure at tunnel face)	0.231m		
u*(u/umax)	0.412		
u	0.088m		

where u is the deformation before the support installation. This value computed using the equation is helpful for the determination of the stage at which the support is added in the RS2 model.

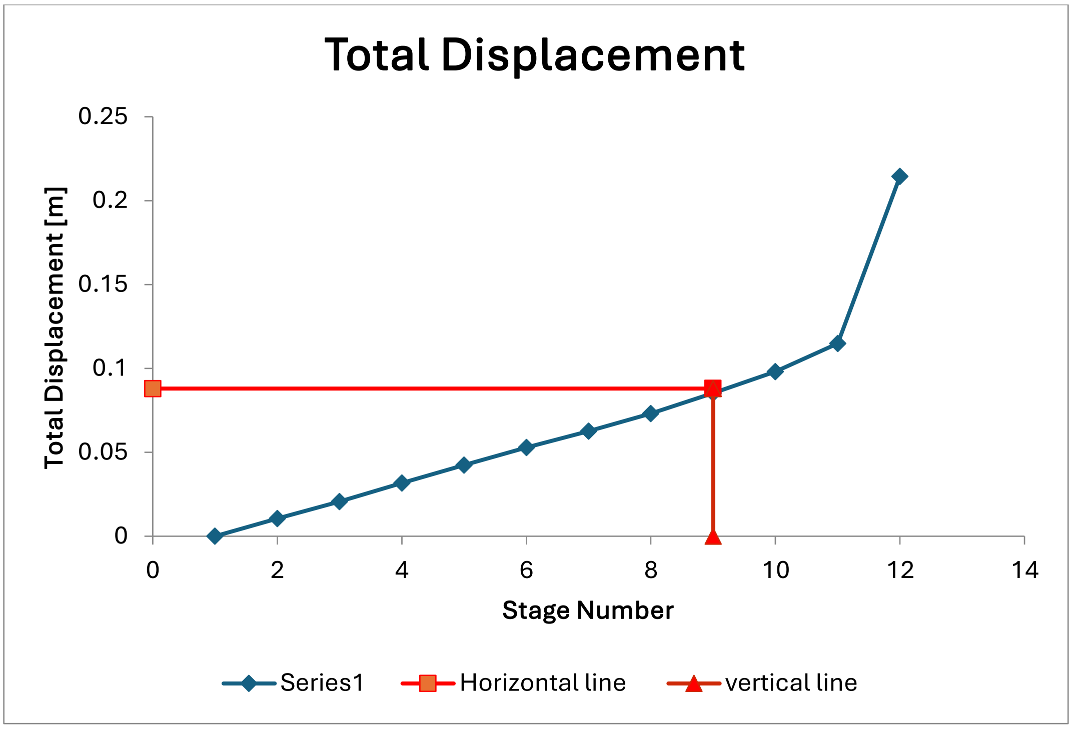

Figure 4.3 Stage vs displacement plot from model

From the stage–displacement relationship obtained in the RS2 analysis, this deformation corresponds to the ninth stage of core modulus reduction. Accordingly, the support was installed after the ninth stage of the simulation as shown Figure 4.4 below:

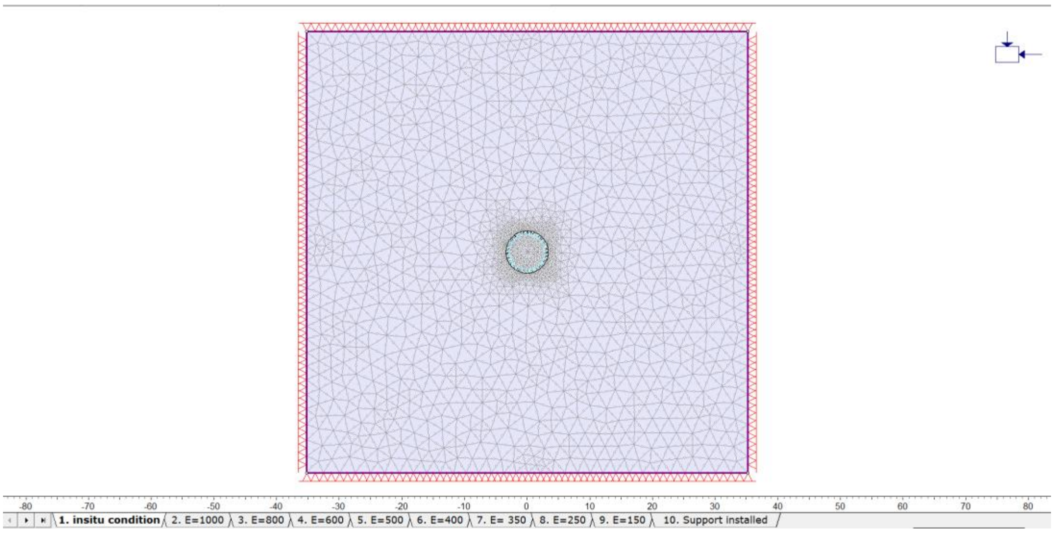

Figure 4.4 Support added after ninth stage

The adopted support system consisted of an M50 reinforced concrete liner, with a thickness of 0.3 m, elastic modulus of 35 GPa, compressive strength of 50 MPa, and tensile strength of 4.1 MPa. After adding the liner at the designated stage, the model was reanalyzed to study the ground–support interaction and compute the Factor of Safety (FOS) at critical locations.

4.1.3 Deterministic Results Interpretation

The support capacity plot generated from the analysis as shown in fig [4-5] showed that all evaluated points fell within the FOS = 1.4 envelope, confirming that the designed liner provides sufficient confinement to the surrounding rock mass under mean conditions. The results demonstrated that the deterministic model predicts stable tunnel behavior, with no indication of overstressing or failure within the support or rock mass. Thus, the deterministic evaluation verified the adequacy of the M50 liner system for the modeled section of the SMDMP headrace tunnel.

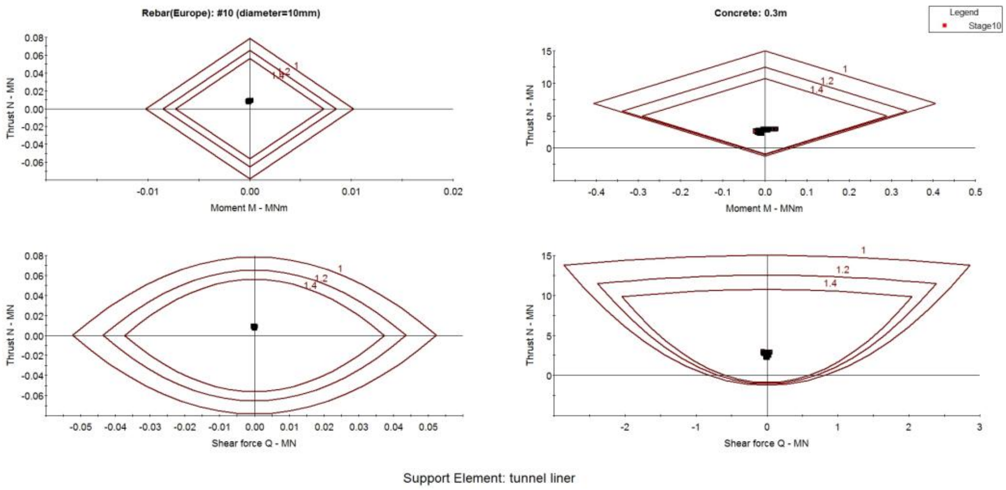

Figure 4.5 Support capacity plot for the tunnel support system for deterministic analysis

4.2 Reliability Analysis

The reliability-based analysis was conducted to assess the performance of the tunnel support system under the inherent uncertainty and variability of geological parameters. The Rosenblueth Point Estimate Method (PEM) was employed to propagate uncertainty in the key input variables UCS, GSI, and Hoek–Brown constant m_i and to evaluate their combined effect on tunnel response and support safety.

Each of the three parameters varied by ±1 standard deviation from its mean value, producing a total of $2^3 = 8$ models. Table 3.3 summarizes the different parameter combinations used for the reliability analysis (REL_1 to REL_8). Each of the eight models was analyzed following the same staged CRT procedure:

i. Excavation simulation using core modulus reduction until the far-face condition.
ii. Evaluation of wall deformation and identification of support installation stage using the LDP relation.
iii. Addition of support at the corresponding stage and reanalysis.
iv. Extraction of FOS at two critical points — the tunnel crown (Point 1) and sidewall (Point 2).

4.2.1 Observed Parameters and Model Response

From each reliability model, the following parameters were extracted:

- Radius of the plastic zone (R_{pl}),
- Deformation before support installation (u), and
- Core modulus (E) at the support installation stage.

A summary of the observed and computed parameters is presented in Table 4.3. The results revealed considerable variability in deformation and plastic radius among the eight realizations. The plastic zone radius ranged from approximately 5.4 m to 15.7 m, indicating that weaker parameter combinations (lower UCS, lower GSI, lower m_i) lead to significantly larger yielding zones and deformations. High variations are observed in deformations before support installations having its value of 0.028 m for strong combination analysis to 0.407 m for worst combination analysis. Gradual reduction of support installation stage and corresponding core modulus at which it is done is observed from the analysis. The observed values gives the ideas about the sensitivity of the parameters involved indicating the influence of particular variable on the responses observed.

Table 4.3 Summary of parameters calculated and observed from the model

Parameters / Name	Analysis type	Radius of plastic zone(Rpl)	Deformation before support installation (u)(m)	Support installation stage value of core modulus (E) (Mpa)
REL_1	+,+,+	5.435	0.0283	375
REL_2	-,+,+	7.809	0.1093	100
REL_3	+,-,+	6.891	0.0855	150
REL_4	-,-,+	11.694	0.3413	50
REL_5	+,+,-	6.05	0.0296	300
REL_6	-,+,-	9.901	0.1251	150
REL_7	+,-,-	8.651	0.1076	150
REL_8	-,-,-	15.712	0.4077	50

Details of the calculations for the above summary Table 4.3 is presented in appendix B.

In addition to the reliability analyses performed using the Point Estimate Method, the in-built probabilistic analysis module available in Rocscience RS2 was also utilized to further examine the variability of the tunnel response. This integrated probabilistic feature automates the stochastic simulation of multiple realizations by varying the input parameters (UCS, GSI, and m_i) according to their defined probability distributions. The analysis produces a combined visualization of the plastic zone occurrence across all simulated models, thereby providing a spatial representation of the likelihood of material yielding within the tunnel periphery.

The results of this built-in probabilistic modeling are illustrated in Figure 4.6, which presents the plastic zone distribution for the eight model realizations. In the figure, the darker shaded zones correspond to regions where yielding occurred across many models, indicating a higher probability of failure or overstressing of the rock mass. Conversely, the lighter grey areas represent zones that yielded in only a few analysis, suggesting that those regions are less sensitive to parameter variability and remain relatively stable under different conditions.

This is the visual result of different scenarios analysis which provides the valuable result than just represented by single deterministic result. By superimposing the results from multiple probabilistic simulations, the figure highlights critical regions around the tunnel that are more susceptible to failure due to uncertainty in rock mass properties. The use of this probabilistic representation thus enhances the understanding of how variability in the geological parameters influences the extent and intensity of the plastic zone, supporting a more comprehensive reliability-based evaluation of tunnel stability.

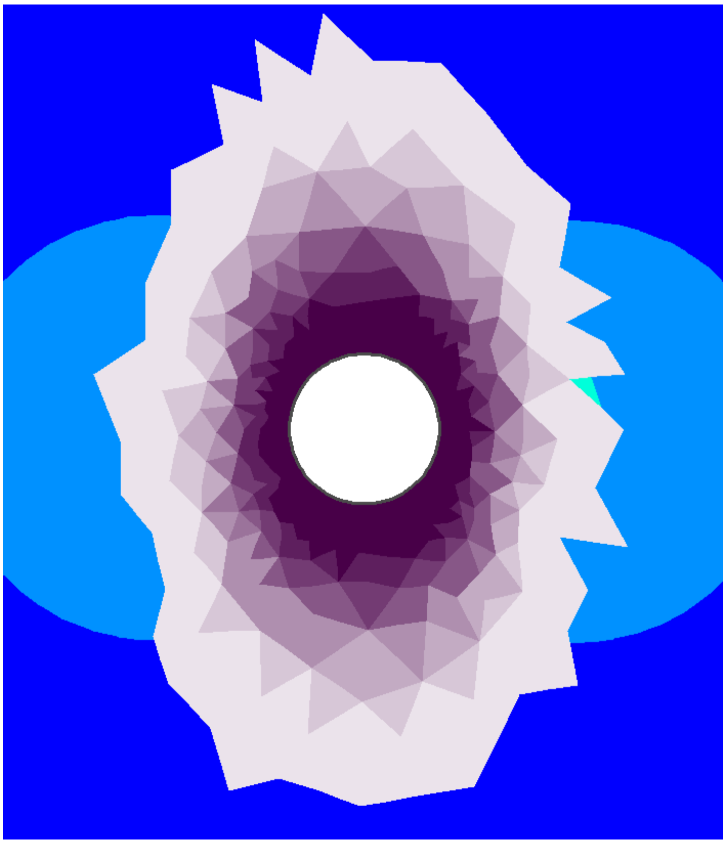

Figure 4.6 Yield Zone from Probabilistic Analysis

4.2.2 Factor of Safety and Reliability Evaluation

After support installation, the FOS was computed at the two monitoring points for each of the eight reliability models. The minimum FOS values at the crown and sidewall were then used to determine the mean (μ_{FOS}) and standard deviation (σ_{FOS}) of the safety response. Using these parameters, the reliability index (β) was calculated as :

$$\beta = \frac{\mu_{FOS} - 1}{\sigma_{FOS}} \quad (15)$$

Assuming the performance function (g = FOS − 1) follows a normal distribution, the probability of failure (P_f) was obtained from the standard normal cumulative distribution function:

$$\mathbf{P_f} = \Phi(-\beta) \quad (16)$$

Table 4.4 presents the FOS for the points 1 and 2 as shown in the Figure 3.8 and the computed value of reliability index and P_f values for both observation points.

Table 4.4 FOS for the points 1 and 2 from eight PEM models

x	y	Minimum FOS	Point	Analysis
0.1	3.19	7.31	1	+,+,+
-0.1	3.19	7.62	1	
0.1	3.19	4.88	1	-,+,+
-0.1	3.19	4.99	1	
0.1	3.19	4.43	1	+,-,+
-0.1	3.19	4.31	1	
0.1	3.19	2.12	1	-,-,+
-0.1	3.19	2.12	1	
0.1	3.19	5.04	1	+,+,-
-0.1	3.19	5.04	1	
0.1	3.19	2.97	1	-,+,-
-0.1	3.19	2.97	1	
0.1	3.19	3.02	1	+,-,-
-0.1	3.19	3.03	1	
0.1	3.19	1.53	1	-,-,-
-0.1	3.19	1.53	1	
-3.196	0.104	11.19	2	+,+,+
-3.196	-0.104	11.25	2	
-3.196	0.104	7.1	2	-,+,+
-3.196	-0.104	7.09	2	
-3.196	0.104	7.29	2	+,-,+
-3.196	-0.104	7.27	2	
-3.196	0.104	2.75	2	-,-,+
-3.196	-0.104	2.75	2	
-3.196	0.104	8.54	2	+,+,-
-3.196	-0.104	8.56	2	
-3.196	0.104	3.85	2	-,+,-
-3.196	-0.104	3.85	2	
-3.196	0.104	4.42	2	+,-,-
-3.196	-0.104	4.43	2	
-3.196	0.104	1.55	2	-,-,-
-3.196	-0.104	1.54	2	

Calculation of standard deviation of FOS using the equation as suggested by Phoon and Ching ;

$$\sigma_f = \sqrt{\sum P_i F_i^2 - F_{mean}{}^2} \quad (17)$$

Table 4.5 Mean and standard deviation of FOS

Point 1			
FOS values	Pi*Fi*Fi	Mean FOS	Standard Deviation
7.465	6.966	3.93	1.79
4.935	3.044		
4.37	2.387		
2.12	0.562		
5.04	3.175		
2.97	1.103		
3.025	1.144		
1.53	0.293		
sum	18.673		
Point 2			
FOS values	Pi*Fi*Fi	Mean FOS	Standard Deviation
11.22	15.736	5.83	3.03
7.095	6.292		
7.28	6.625		
2.75	0.945		
8.55	9.138		
3.85	1.853		
4.425	2.448		
1.545	0.298		
sum	43.335		

Table 4.6 Reliability index and probability of failure

Reliability Index & Probability of failure Calculation				
point	Mean	standard dev	Reliability index	Probability of failure
1	3.93	1.79	1.637	0.051
2	5.83	3.03	1.594	0.055

The results show that the reliability index at the tunnel crown and sidewall were 1.637 and 1.594, respectively, corresponding to probabilities of failure of 5.1% and 5.5%. These values indicate that the support system performs consistently across the cross-section, with a slightly higher risk of failure at the sidewall due to stress concentration.

4.2.3 Comparison with Deterministic Analysis

The reliability-based analysis provided deeper insight into how parameter uncertainty affects tunnel performance. While the deterministic analysis predicted stable behavior with FOS ≥ 1.4, the probabilistic assessment revealed:

- A plastic zone nearly twice as large as that from the deterministic case, and
- A maximum far-face deformation approximately six times greater than in the deterministic analysis.

These results demonstrate that variability in rock mass properties can significantly influence the extent of yielding and deformation, emphasizing the need to incorporate probabilistic methods in tunnel design rather than relying solely on mean-value deterministic evaluations.

The results from both deterministic and reliability analyses confirm that the M50 concrete liner provides adequate support for the SMDMP headrace tunnel under the expected range of rock mass conditions. However, the probabilistic analysis highlights that uncertainty in input parameters can lead to notable differences in deformation and plastic zone development.

The moderate reliability indices (β ≈ 1.6) correspond to acceptable failure probabilities (<5.5%) for typical hydropower headrace tunnels, aligning with the reliability targets suggested in tunneling literature for permanent linings. The analysis also underscores the importance of geological characterization particularly in defining accurate GSI and UCS distributions as these variables strongly influence tunnel performance.

Overall, the study demonstrates that integrating reliability-based design with numerical modeling provides a more realistic and defensible evaluation of tunnel stability. For future projects, adopting probabilistic approaches at the design stage can enable optimized support selection, risk-informed decision making, and improved safety assurance in complex geological conditions.

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This study focused on the numerical and probabilistic assessment of the tunnel support system for the SMDMP headrace tunnel. Numerical analyses were conducted using Rocscience Phase2 (v8), adopting the CRT to simulate the progressive excavation process and evaluate support performance. Both deterministic and reliability-based (probabilistic) analyses were carried out to compare the behavior of the tunnel under mean conditions and under the influence of parameter variability.

The following key conclusions can be drawn:

i. Adequacy of the Designed Support System:

The deterministic analysis indicated that the installed M50 concrete liner provides adequate stability to the tunnel under average rock mass conditions. All points in the support capacity plot were found within the FOS = 1.4 safety envelope, confirming that the adopted support configuration effectively controls deformation and prevents overstressing of the lining and surrounding rock.

ii. Progressive Deformation and Plastic Zone Development:

The deformation observed with each stage is gradually differentiated with a maximum crown deformation of 0.214 m and a plastic zone radius of 7.84 m at the fully excavated stage. The observed deformation trend is in line with expected tunnel behavior, validating the accuracy of the modeled result.

iii. Influence of Parameter Variability on Tunnel Performance:

The reliability analysis, performed using Rosenblueth's Point Estimate Method (PEM), revealed that variations in UCS, GSI, and Hoek–Brown constant m_i significantly affect the deformation response and extent of the plastic zone. The probabilistic models showd plastic zones nearly two times and deformations nearly six times greater than the deterministic case, demonstrating the considerable effect of geological uncertainty on tunnel stability.

iv. Reliability Index and Probability of Failure:

The calculated reliability indices (β) ranged between 1.59 and 1.63. It corresponds to failure probabilities of approximately 5–5.5% at the tunnel crown and sidewall.

These values represent acceptable safety levels for permanent hydropower headrace tunnels and align well with the values in the literature.

v. Effectiveness of the PEM Approach:

The Point Estimate Method proved to be an efficient and practical reliability tool, capable of providing realistic results without the computational complexity of Monte Carlo or FORM/SORM simulations. It enabled the assessment of uncertainty effects with only eight deterministic model runs, making it an attractive method for preliminary risk assessment and design verification in engineering practice.

vi. Importance of Geological Characterization:

The study highlights that uncertainties in GSI and UCS have more influence on tunnel response. Therefore, accurate and adequate field mapping, laboratory testing, and statistical analysis of geotechnical data are required to reduce uncertainty in design inputs and improve the reliability of numerical analysis.

vii. Integration of Probabilistic Methods in Tunnel Design:

This study shows the significant impact of predicted parameter variations on responses of tunnel and support. Thus probabilistic approach of design should be used somehow to know the variations on response.

5.2 Recommendations

Based on the findings of this research, the following recommendations are made for future tunneling projects and studies:

i. Adoption of Reliability-Based Design:

Design practice should gradually shift from deterministic base to probabilitstic base. As use of probabilistic approach helps to prevent under-design or over-design.

ii. Utilization of Point Estimate Method for Practical Design:

The PEM approach is highly recommended for engineering applications where computational efficiency is important. It provides a reliable estimate of mean and variance of performance metrics with minimal model runs, allowing quick evaluation of design robustness under parameter uncertainty.

iii. Improved Geotechnical Data Collection:

Adequate geotechnical data is necessary to reduce uncertainty thus the spatial frequency of geotechnical data collection should be increased.

iv. Integration with Advanced Reliability Methods:

While PEM offers simplicity, future work can employ First-Order Reliability Method (FORM) or Monte Carlo Simulation (MCS) to cross-validate results and assess tail-risk behavior for extreme scenarios..

v. Continuous Model Calibration and Monitoring:

Adequate instrumentation and calibration while excavation and after construction is necessary to understand the response and useful for the future aspects as it serve as the design record and helps in design decisions.

vi. Policy-Level Recommendations:

For governmental and client agencies, it is recommended to establish design guidelines that include reliability-based safety assessment as a standard requirement for underground projects. This will promote consistency, safety, and economy in the long-term development of hydropower and transport tunnels.

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ANNEXES

Appendix A Field RMR, UCS Data

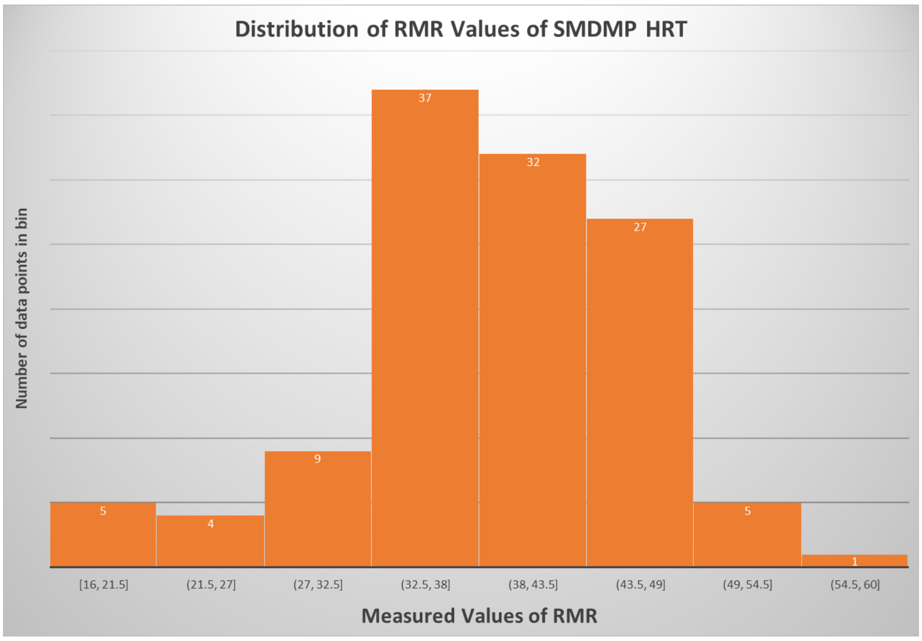


Figure A. 2 Distribution of RMR values of SMDMP HRT

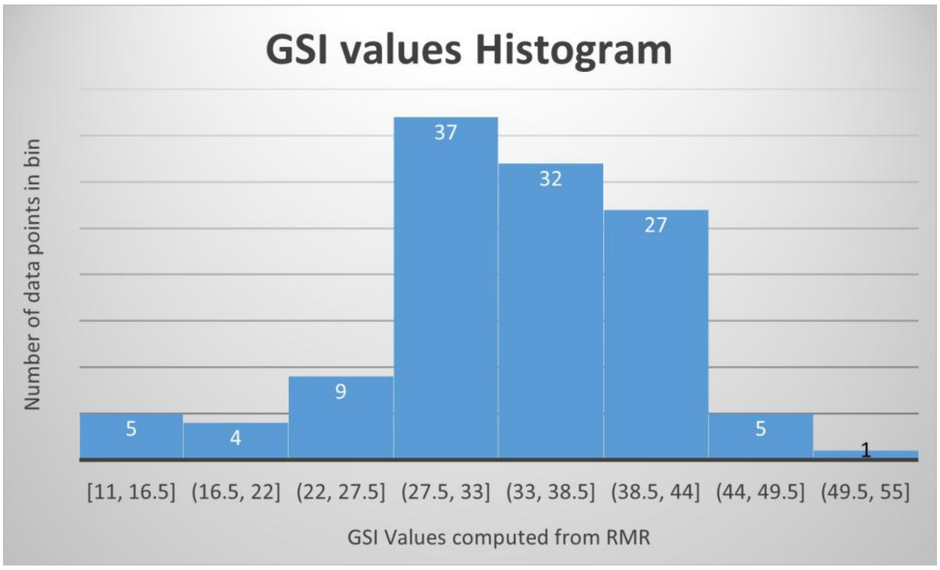


Figure A. 1 Computed GSI Values Histogram

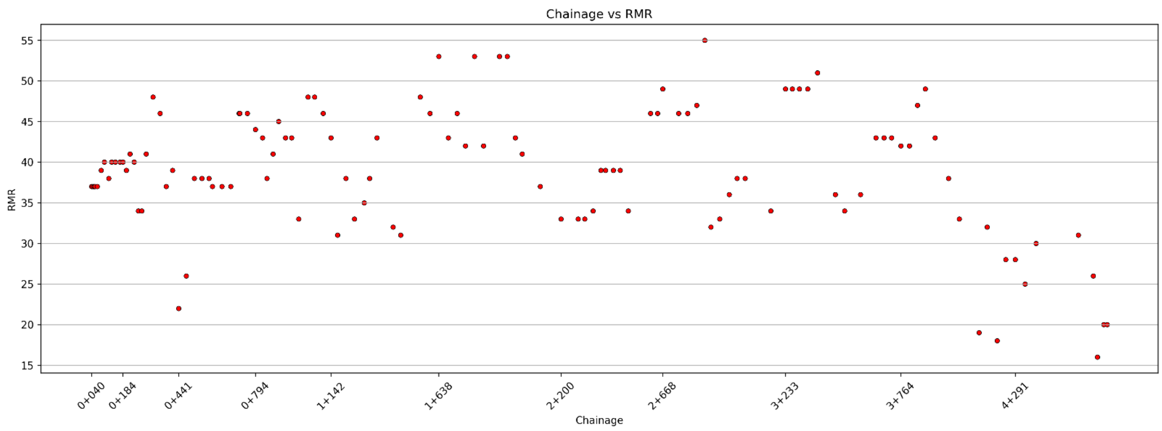


Figure A. 3 Observed RMR values for different chainages of SMDMP HRT

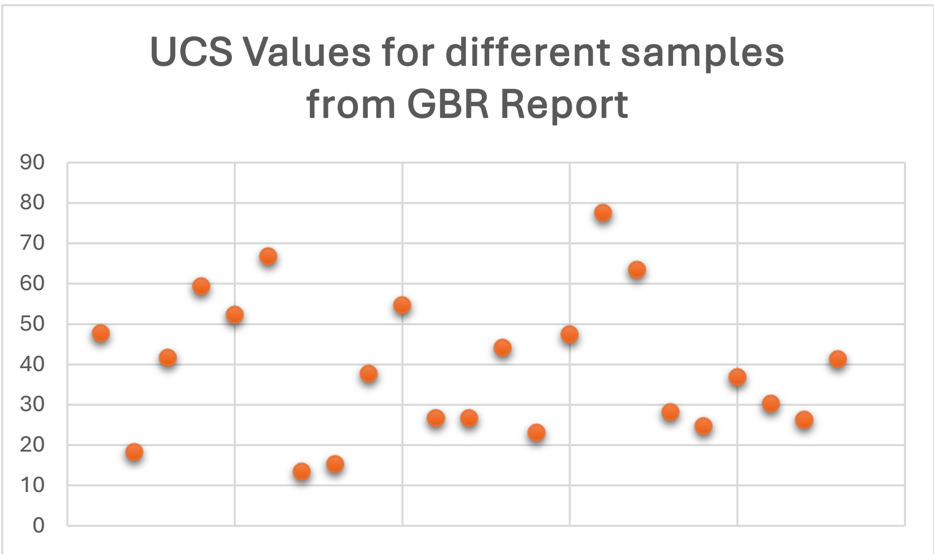


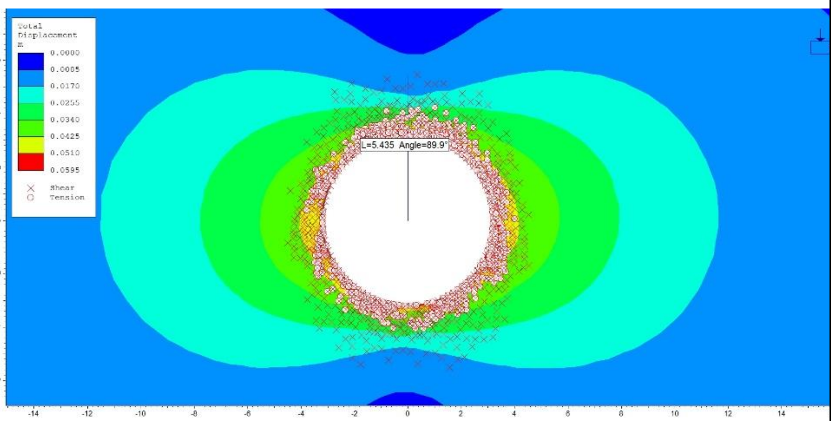
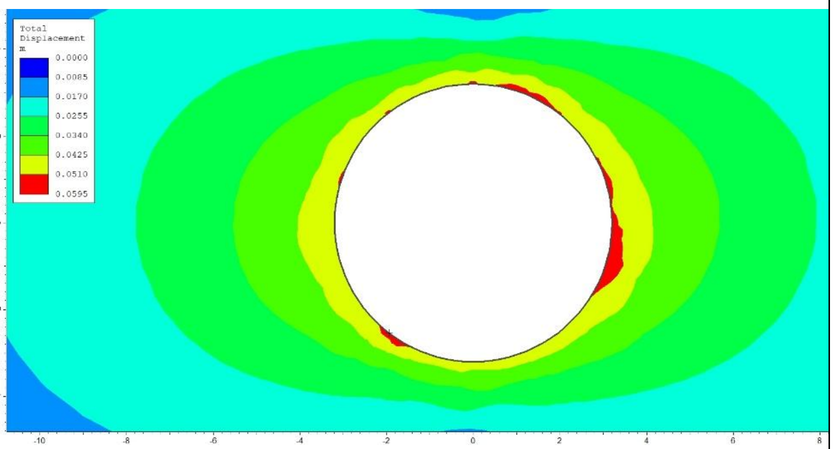
Figure A. 4 UCS values from GBR report

Appendix B: Computed Parameters for Different Reliability Models

Mean and Standard Deviation of Input Parameters:

	Mean	Standard Deviation
GSI	34.13	7.79
UCS	39.32	17.38
mi	17	4

Name of analysis: REL_1

<table>
<tr><td>Analysis Type</td><td>+,+,+</td></tr>
<tr><td>Input Parameters</td><td>UCS= 56.7, GSI= 41.92 and m_i= 21</td></tr>
<tr><td>Tunnel Radius</td><td>3.2 m</td></tr>
<tr><td>Radius of plastic zone observed from the model at final excavated stage (Rpl)</td><td>5.435m

</td></tr>
<tr><td>Maximum displacement(u_{max}) observed</td><td>0.057m

</td></tr>
</table>

Location of maximum deformation	-1.948,-2.548
Support installation distance(X)	1.4m
Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

From Vlachopoulos and Diederichs equation:

Normalized closure at tunnel face (u_0^*)

$$= \frac{u_0}{u_{max}} = \frac{1}{3} e^{-0.15R^*}$$

$$= 0.258$$

normalized displacement at the support installation distance $u^* (\frac{u}{u_{max}})$

$$= 1 - \left(1 - u_0^*\right) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$$

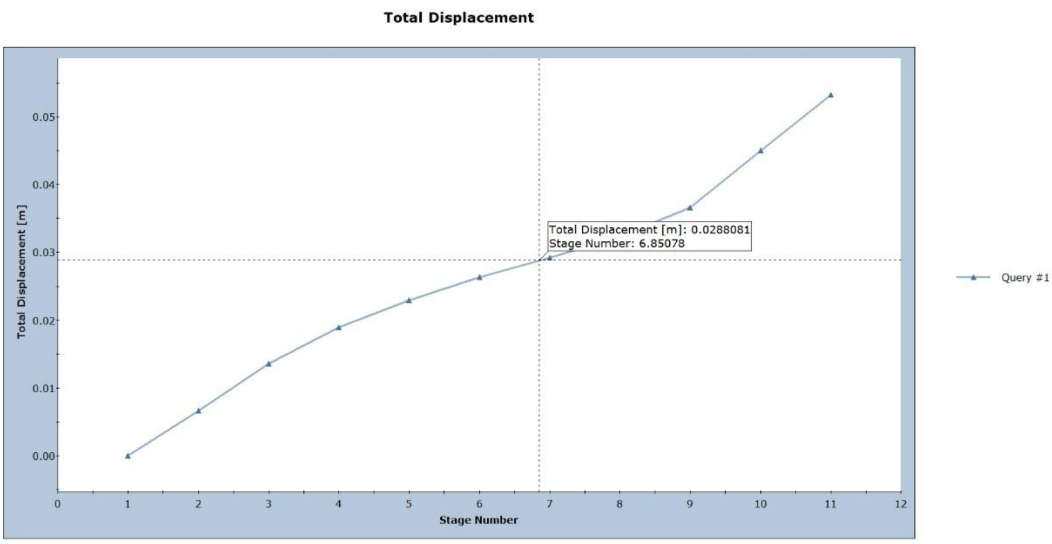
$$= 0.496$$

Displacement at the support installation distance

$$u = 0.028m$$

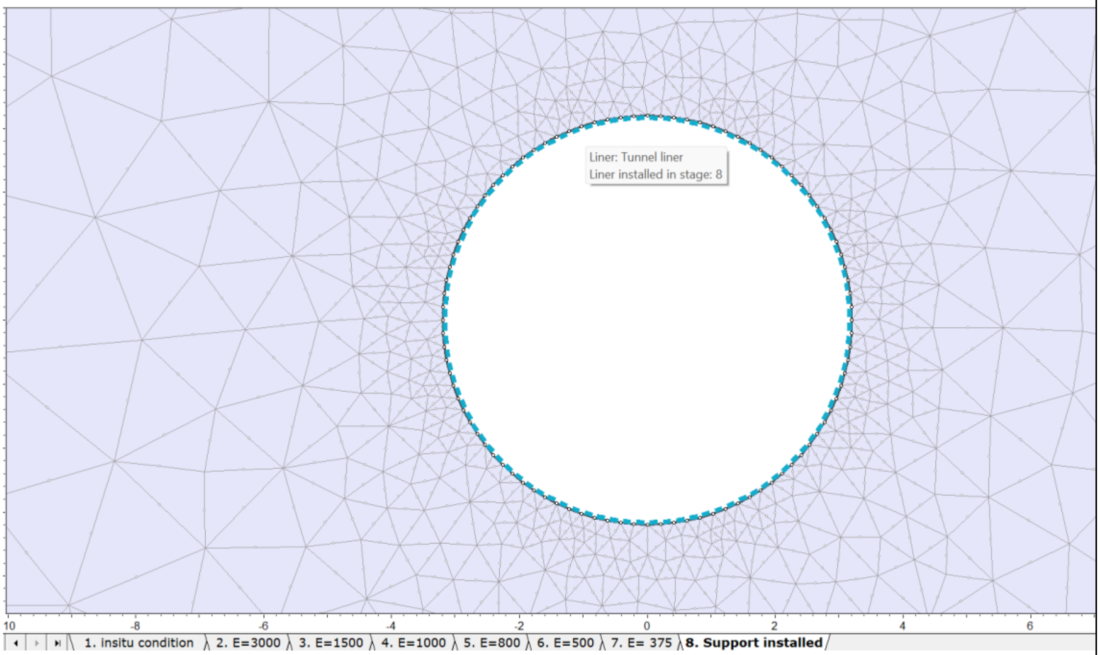
Determination of support installation stage at the model:

Deformation vs stage plot obtained from the model is:

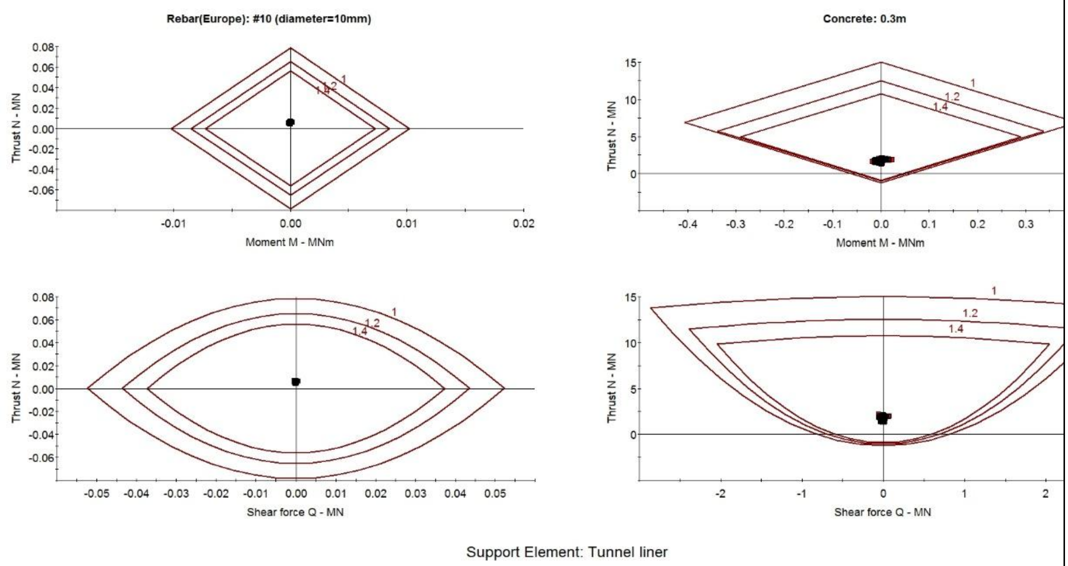


Total displacement 0.028m at stage no. 6.8 equivalent to 7 with E= 375 Mpa

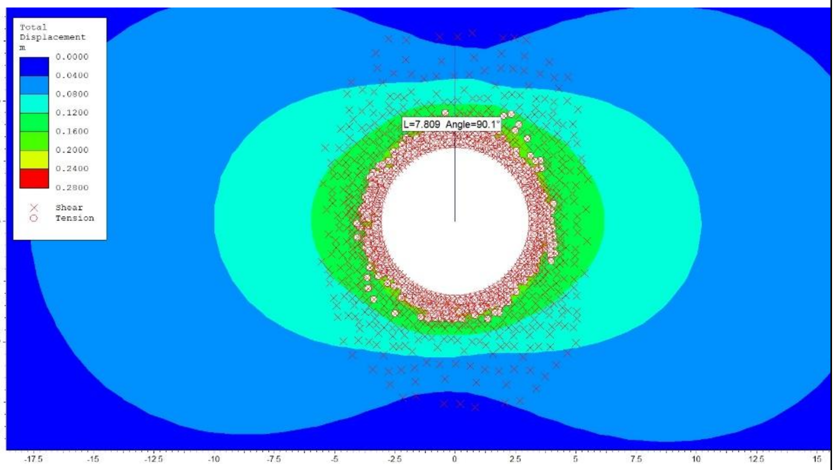
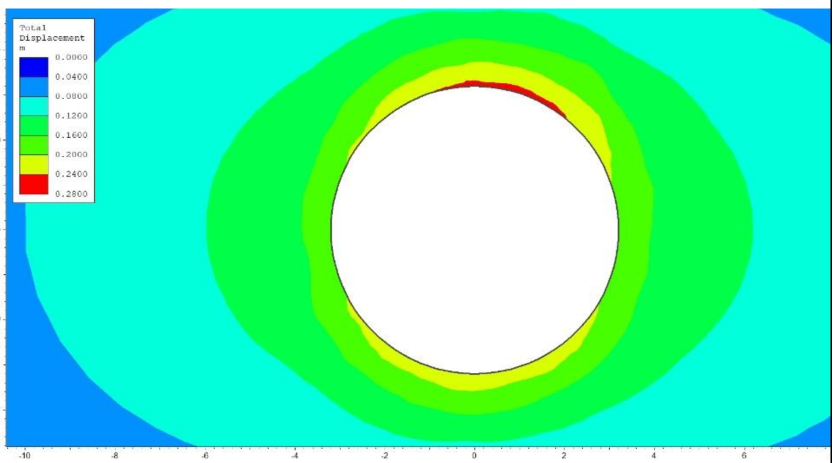
Support added at the stage 7:

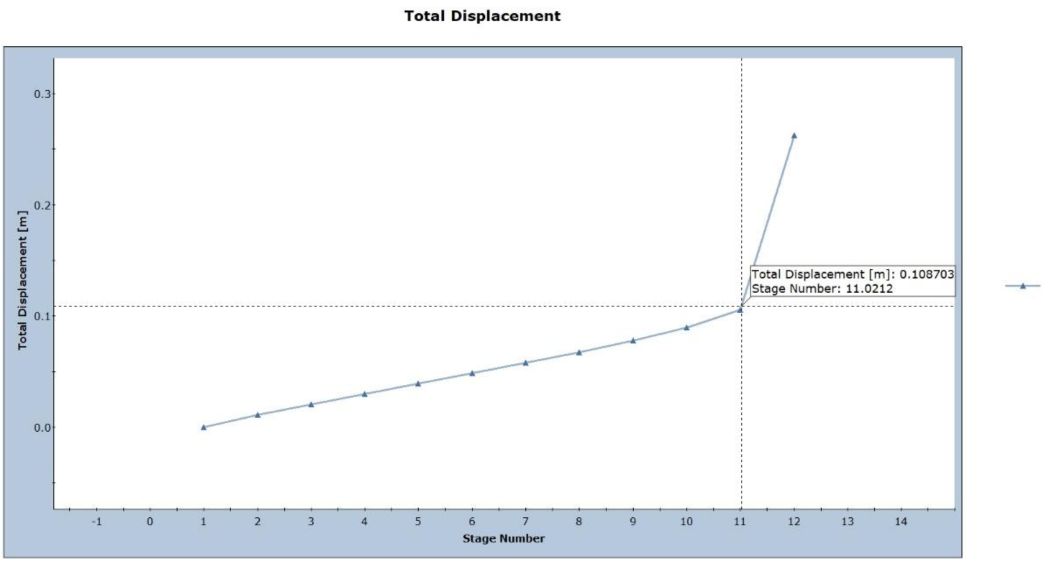


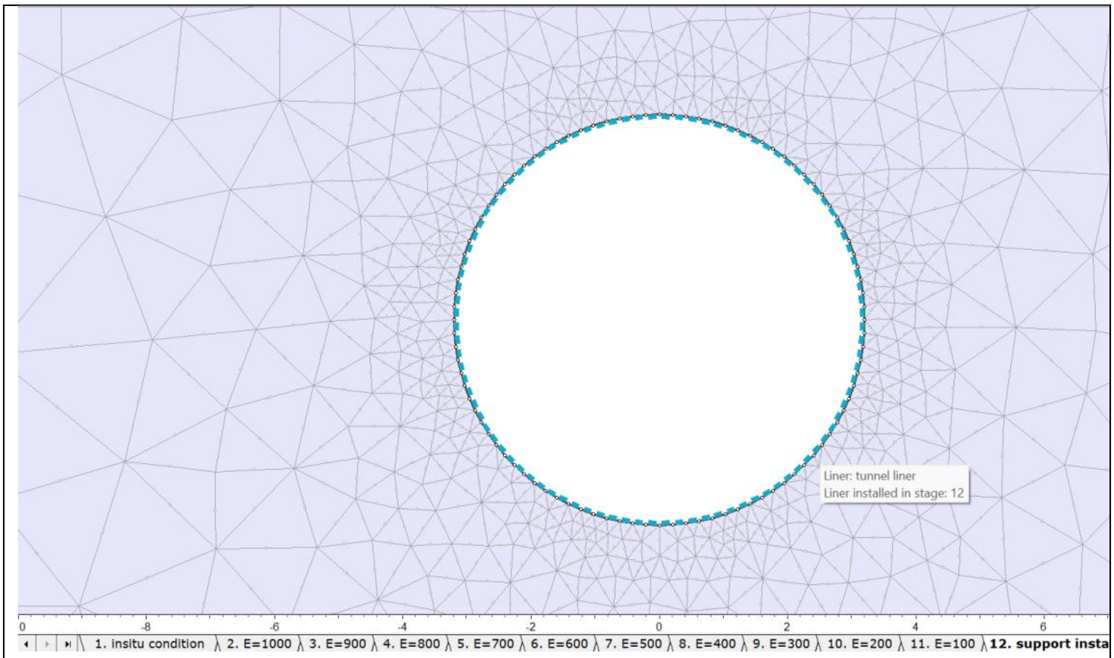
Support Capacity Diagram:



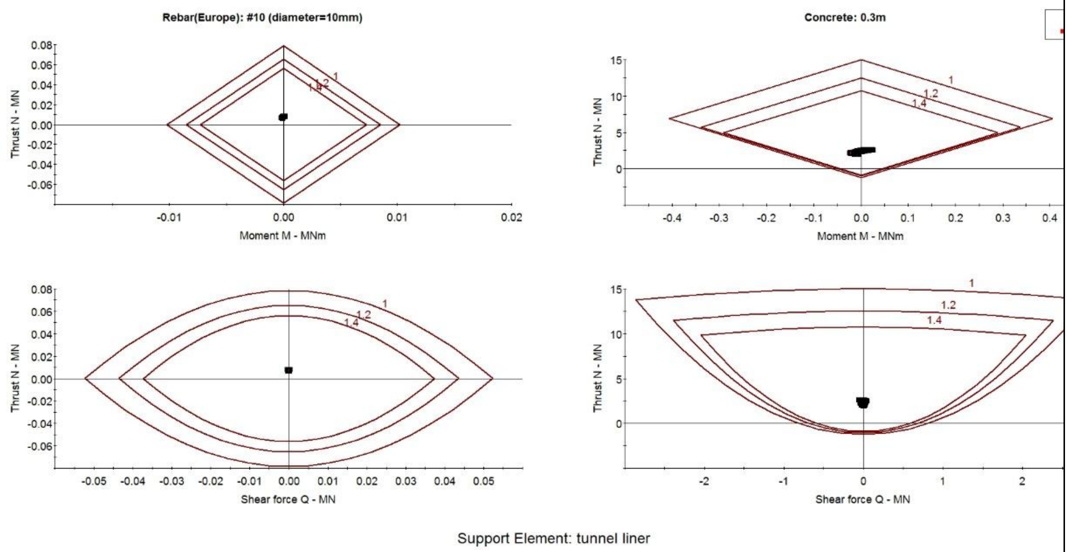
Name of analysis: REL_2

Analysis Type	-,+,+
Input Parameters	UCS= 21.94, GSI= 41.92 and m_i= 21
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	7.809 m 
Maximum displacement(u_{max}) observed	0.262m 
Location of maximum deformation	crown
Support installation distance(X)	1.4m
Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

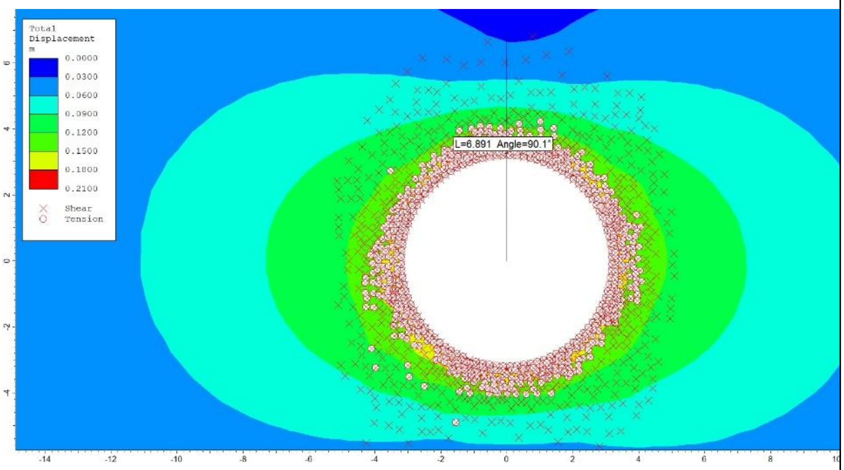
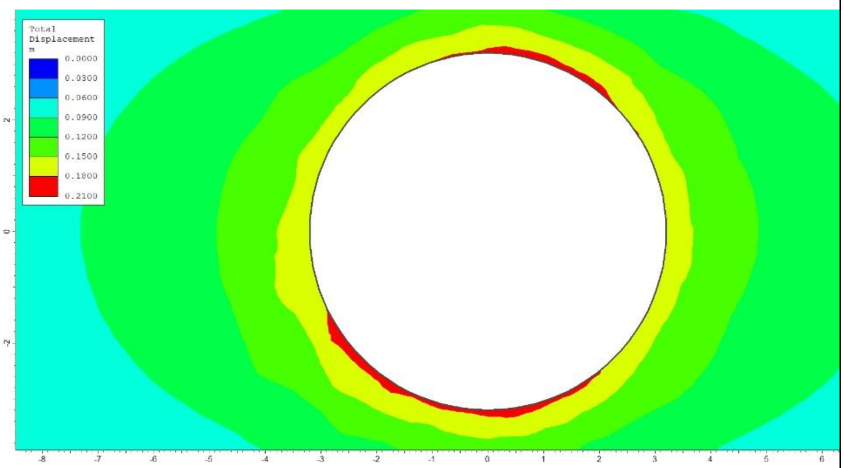
From Vlachopoulos and Diederichs equation:
Normalized closure at tunnel face (u_0^*) $= \frac{u_0}{u_{max}} = \frac{1}{3} e^{-0.15R^*}$ $= 0.231$ normalized displacement at the support installation distance $u^* (\frac{u}{u_{max}})$ $= 1 - (1 - u_0^*) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$ $= 0.412$ Displacement at the support installation distance $u = 0.108m$
Determination of support installation stage at the model: Deformation vs stage plot obtained from the model is:  Total displacement 0.108m at stage no. 11 with E= 100 Mpa Support added after the stage 11:

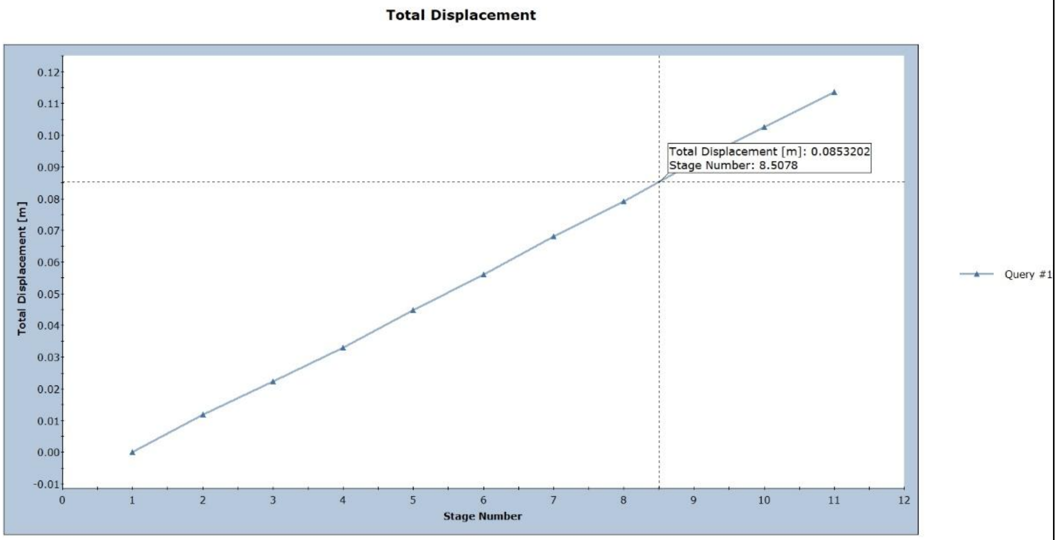


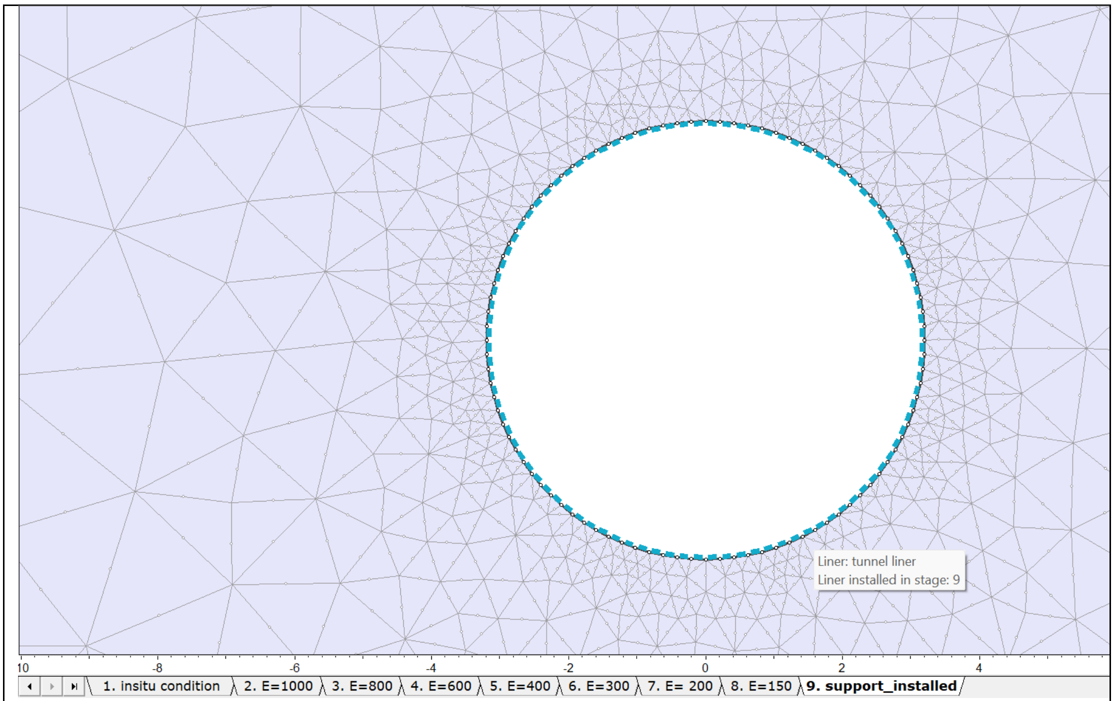
Support Capacity Diagram:



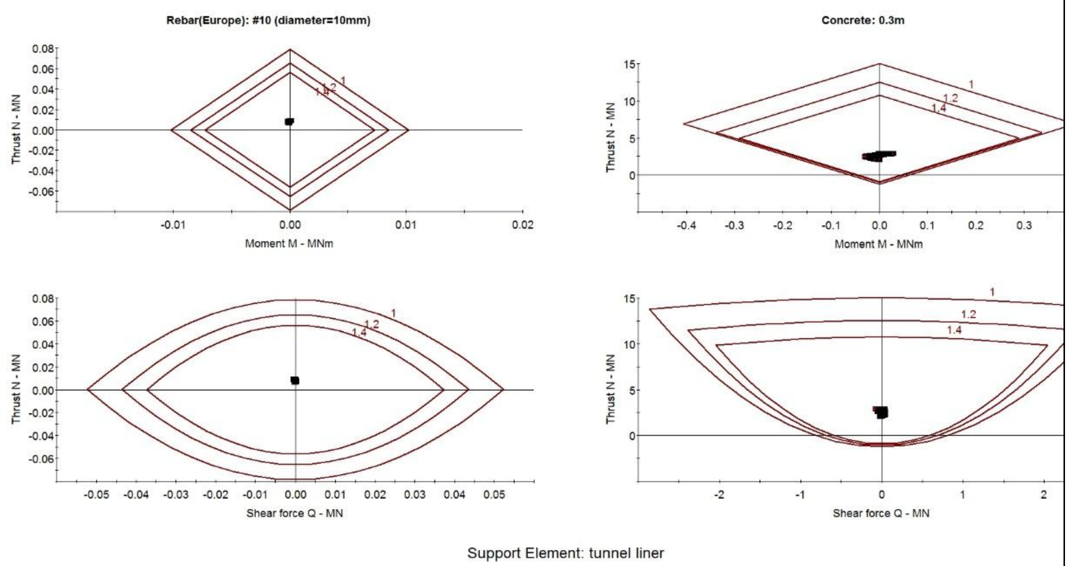
Name of analysis: REL_3

Analysis Type	+,-,+
Input Parameters	UCS= 56.7, GSI= 26.35 and m_i= 21
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	6.891 m 
Maximum displacement(u_{max}) observed	0.194m 
Location of maximum deformation	crown
Support installation distance(X)	1.4m
Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

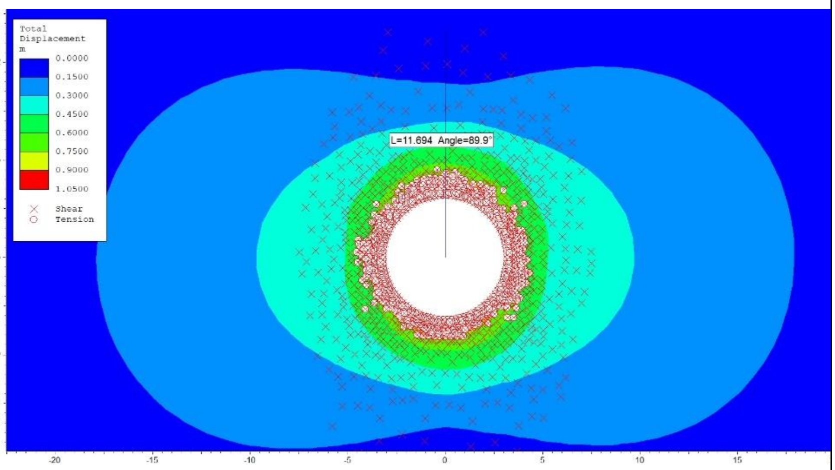
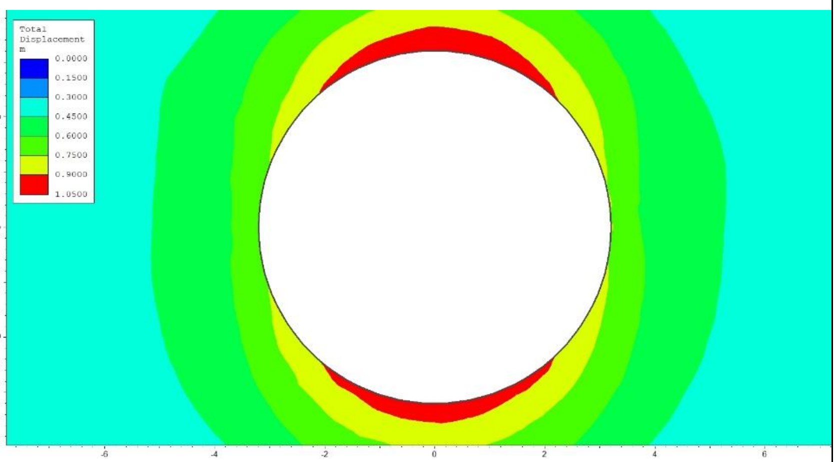
From Vlachopoulos and Diederichs equation:
Normalized closure at tunnel face (u_0^*) $= \frac{u_0}{u_{max}} = \frac{1}{3} e^{-0.15R^*}$ $= 0.241$ normalized displacement at the support installation distance $u^* (\frac{u}{u_{max}})$ $= 1 - (1 - u_0^*) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$ $= 0.44$ Displacement at the support installation distance $u = 0.085m$
Determination of support installation stage at the model: Deformation vs stage plot obtained from the model is:    Total displacement 0.085 at stage no. 8.5 with E= 150   Support added after the stage 10:

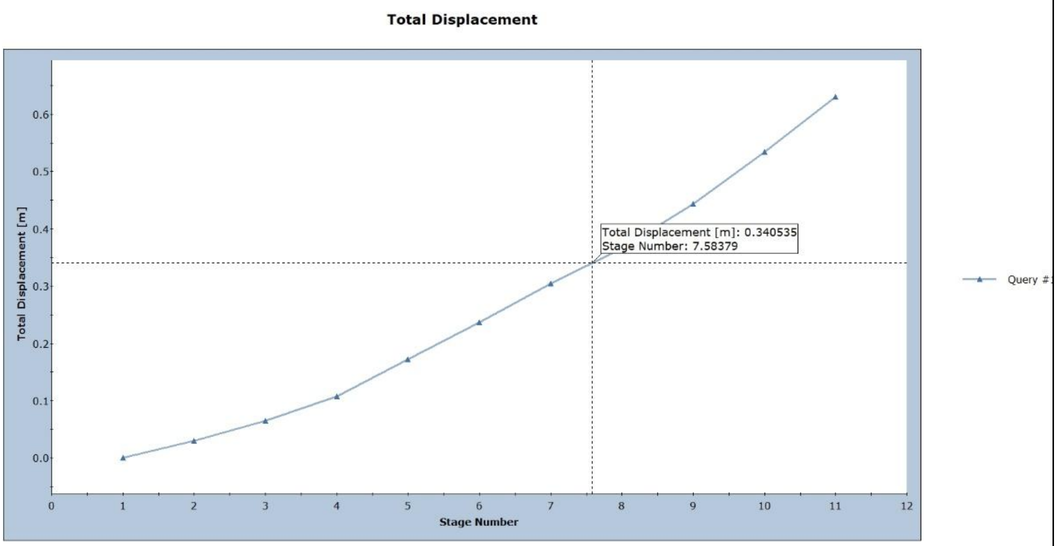


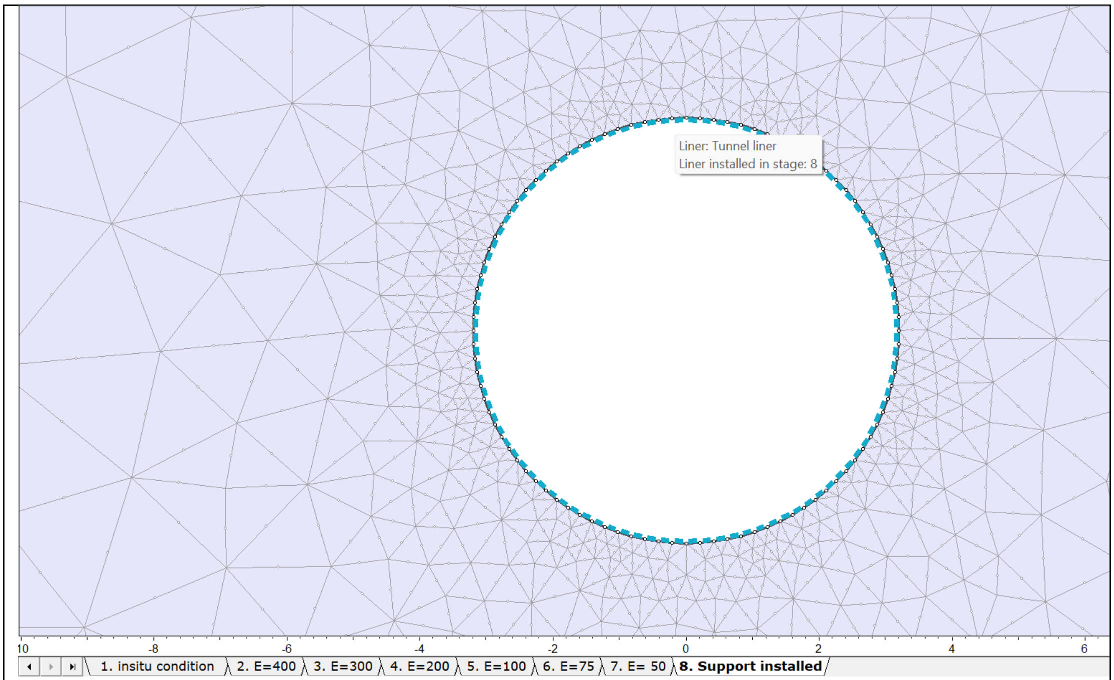
Support Capacity Diagram:



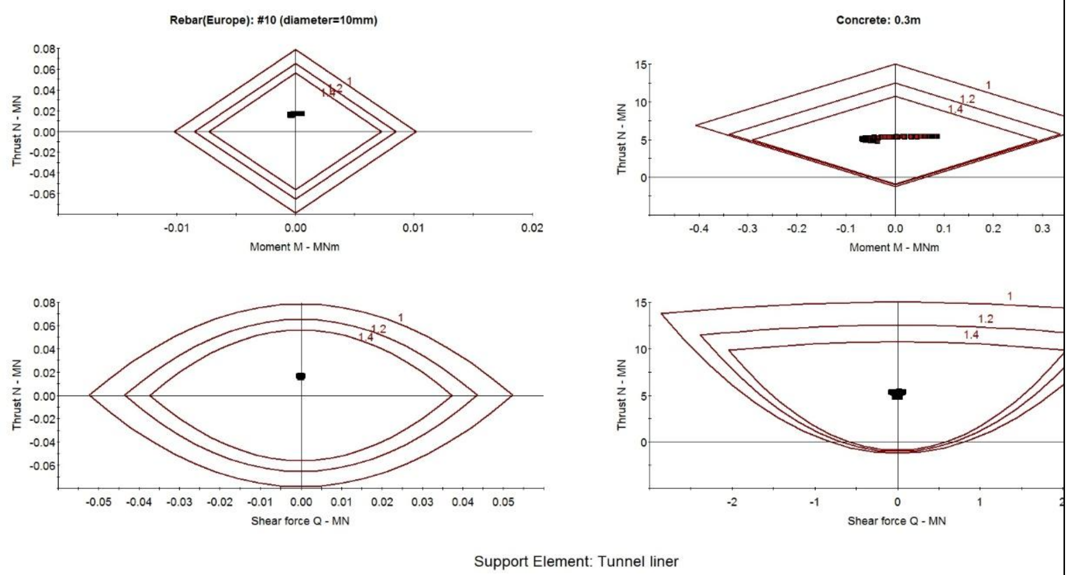
Name of analysis: REL_4

Analysis Type	-,-,+
Input Parameters	UCS= 21.94, GSI= 26.35 and m_i= 21
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	11.694 m 
Maximum displacement(u_{max}) observed	1.049m 
Location of maximum deformation	(0.624,-3.138)
Support installation distance(X)	1.4m
Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

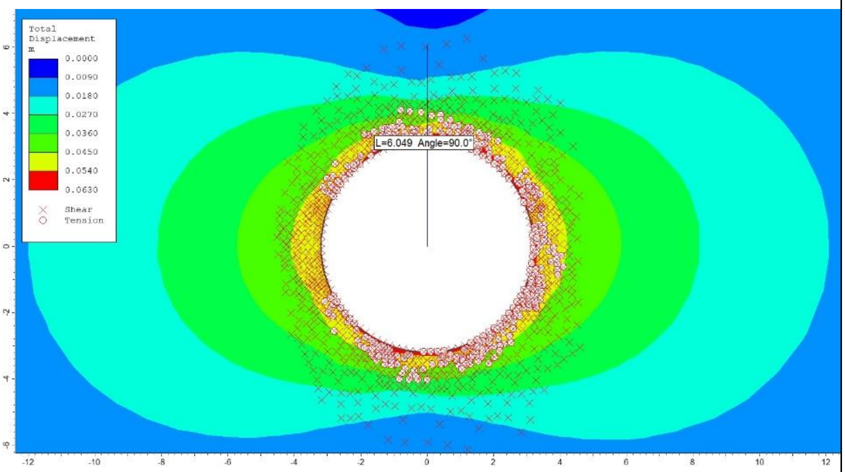
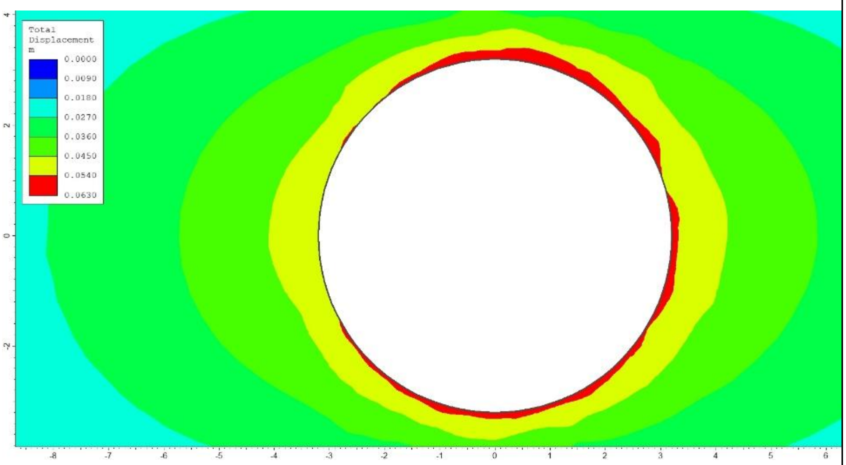
From Vlachopoulos and Diederichs equation:
Normalized closure at tunnel face (u_0^*) $= \frac{u_0}{u_{max}} = \frac{1}{3} e^{-0.15R^*}$ $= 0.192$ normalized displacement at the support installation distance $u^* (\frac{u}{u_{max}})$ $= 1 - \left(1 - u_0^*\right) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$ $= 0.325$ Displacement at the support installation distance $u = 0.341m$
Determination of support installation stage at the model: Deformation vs stage plot obtained from the model is:  Total displacement 0.34 at stage no. 7.5 with E= 50 Support added after the stage:



Support Capacity Diagram:



Name of analysis: REL_5

Analysis Type	+,+,-
Input Parameters	UCS= 56.7, GSI= 41.92 and m_i= 13
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	6.05 m 
Maximum displacement(u_{max}) observed from model at final excavated stage	0.06m 
Location of maximum deformation	(1.028,3.03)
Support installation distance(X)	1.4m
Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

From Vlachopoulos and Diederichs equation:

Normalized closure at tunnel face (u_0^*)

$$= \frac{u_0}{u_{\max}} = \frac{1}{3} e^{-0.15R^*}$$

$$= 0.25$$

normalized displacement at the support installation distance $u^* (\frac{u}{u_{\max}})$

$$= 1 - \left(1 - u_0^*\right) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$$

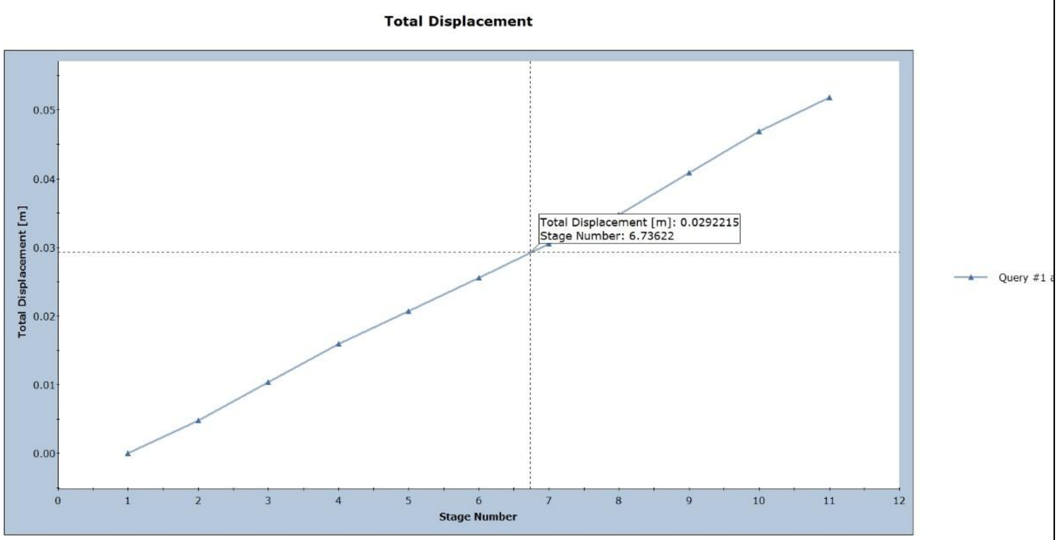
$$= 0.47$$

Displacement at the support installation distance

$$u = 0.029m$$

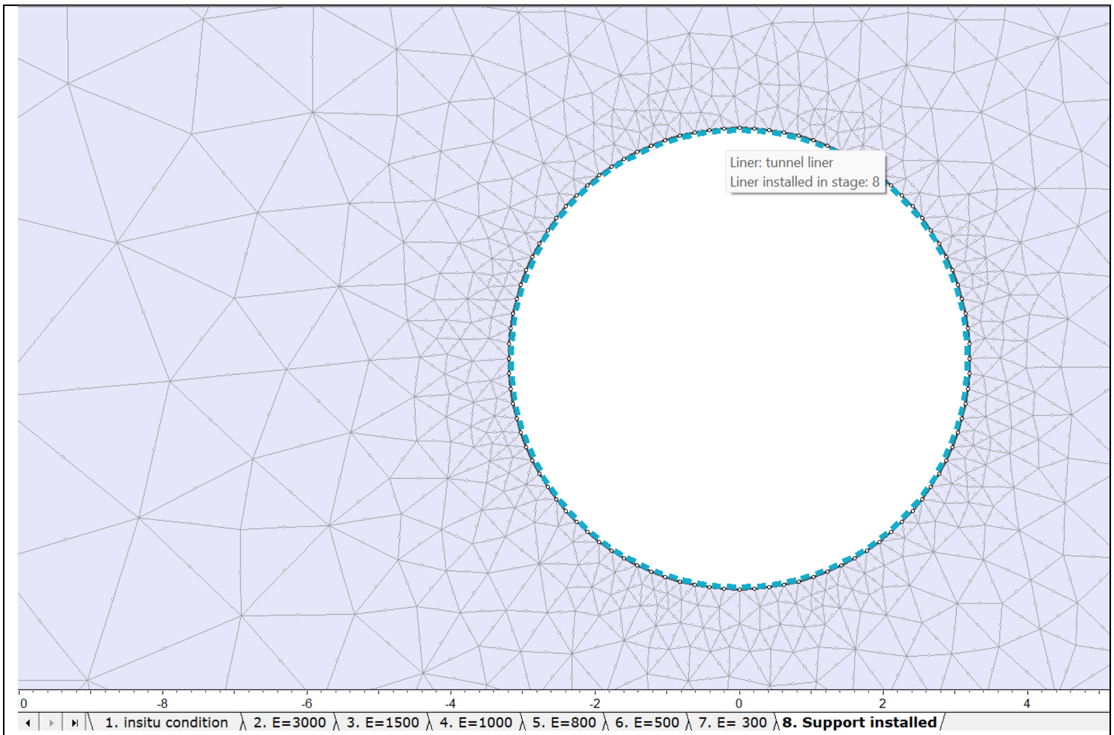
Determination of support installation stage at the model:

Deformation vs stage plot obtained from the model is:

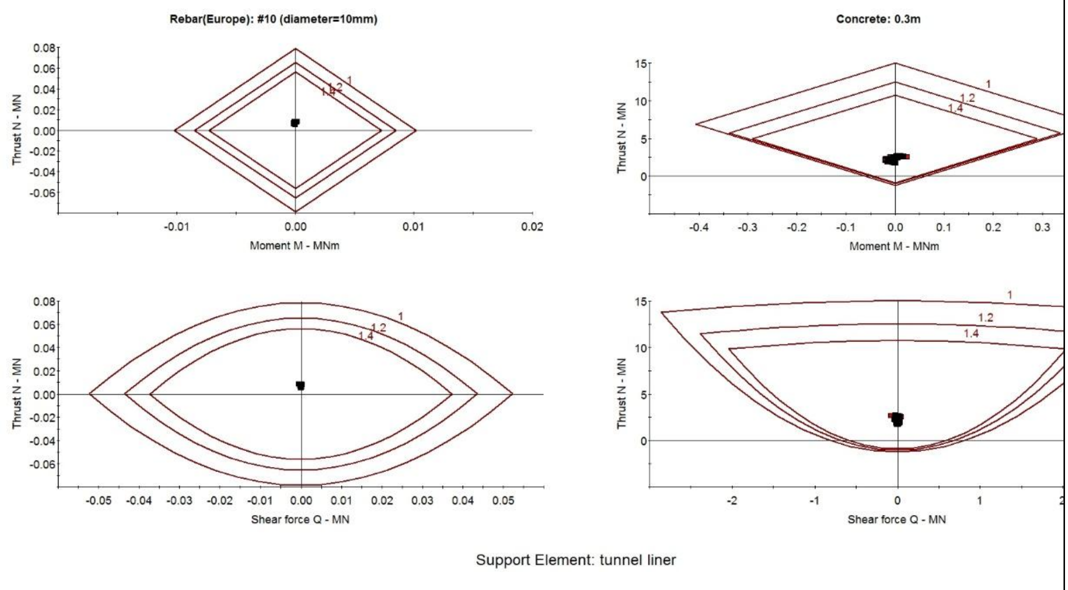


Total displacement 0.029 at stage no. 6.7 with E=300

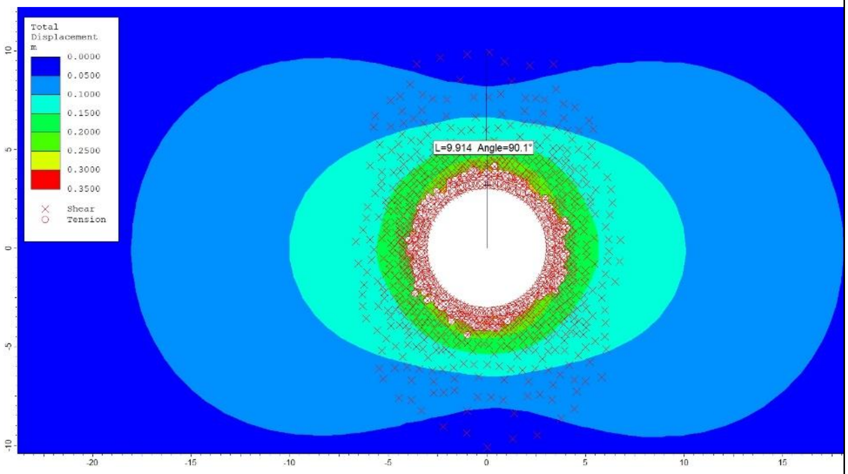
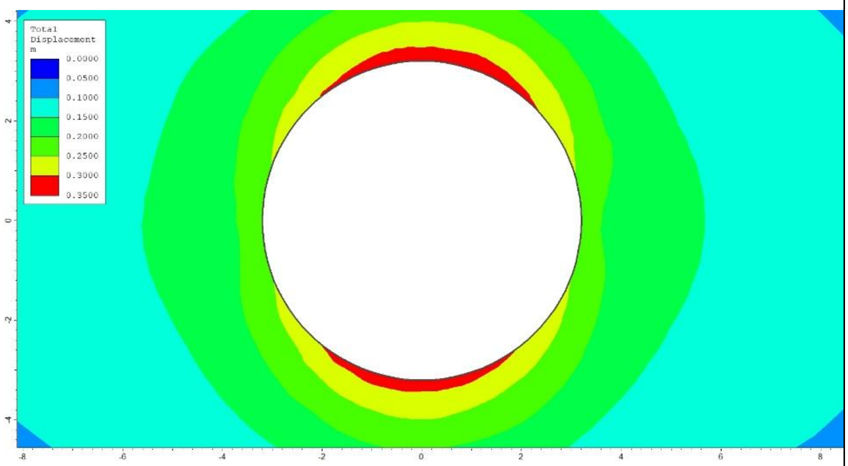
Support added after the stage:



Support Capacity Diagram:



Name of analysis: REL_6

Analysis Type	-,+,-
Input Parameters	UCS= 21.94, GSI= 41.92 and m_i= 13
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	9.9 m 
Maximum displacement(u_{max}) observed from model at final excavated stage	0.347m 
Location of maximum deformation	crown
Support installation distance(X)	1.4m
Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

From Vlachopoulos and Diederichs equation:

Normalized closure at tunnel face (u_0^*)

$$= \frac{u_0}{u_{\max}} = \frac{1}{3} e^{-0.15 R^*}$$

$$= 0.209$$

normalized displacement at the support installation distance $u^* \left(\frac{u}{u_{\max}}\right)$

$$= 1 - \left(1 - u_0^*\right) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$$

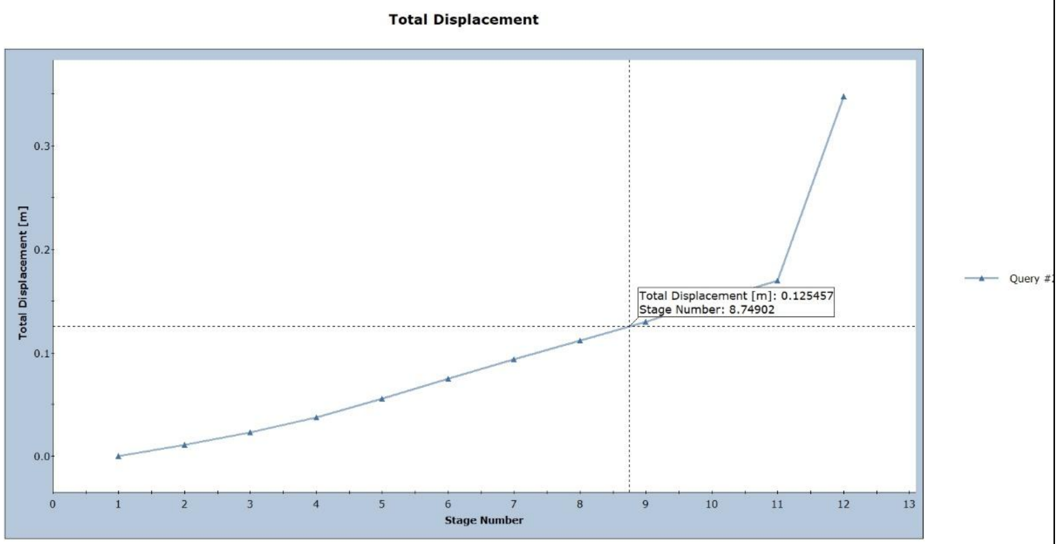
$$= 0.36$$

Displacement at the support installation distance

$$u = 0.125m$$

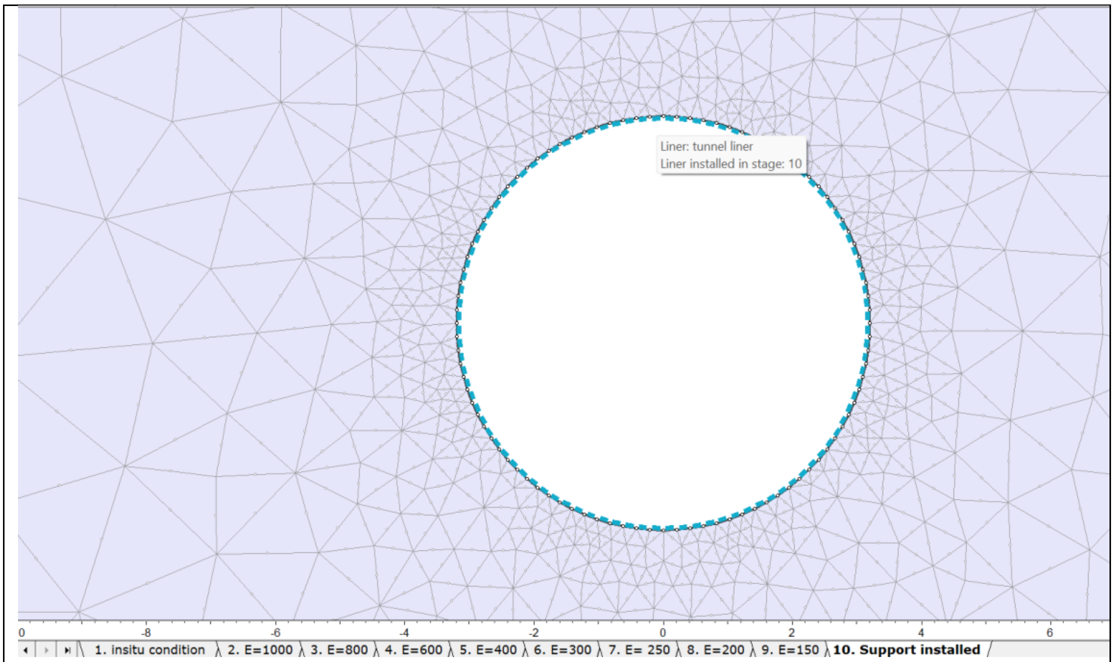
Determination of support installation stage at the model:

Deformation vs stage plot obtained from the model is:

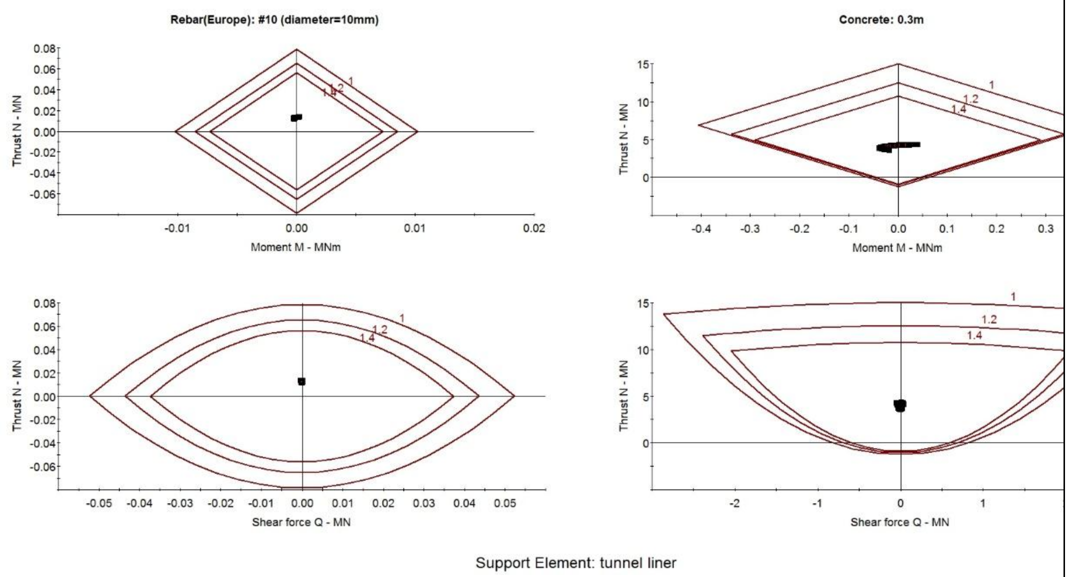


Total displacement 0.125 at stage no. 9 with E= 150

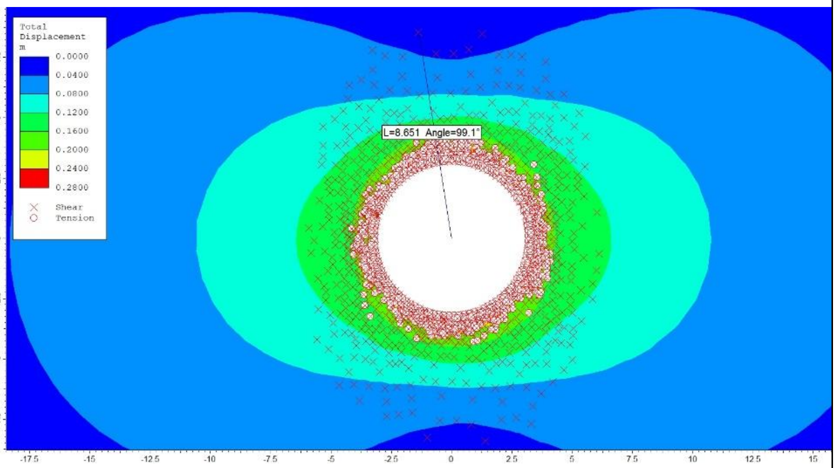
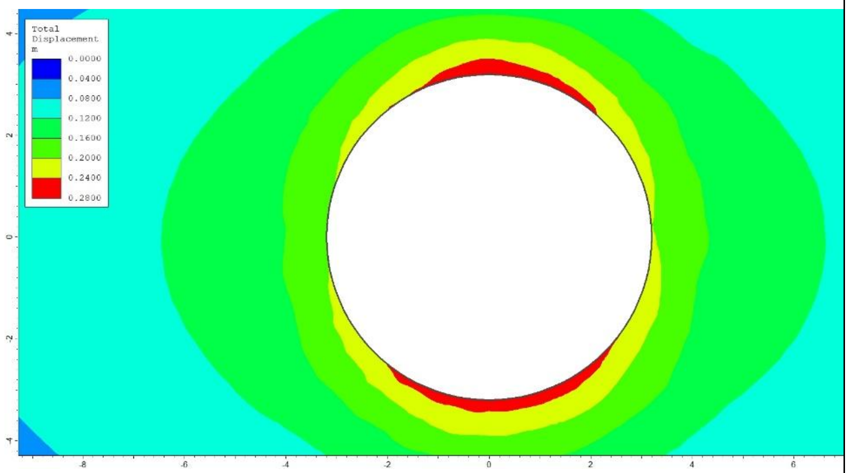
Support added after the stage:



Support Capacity Diagram:



Name of analysis: REL_7

Analysis Type	+,-,-
Input Parameters	UCS= 56.7, GSI= 26.35 and m_i= 13
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	8.65 m 
Maximum displacement(u_{max}) observed from model at final excavated stage	0.276m 
Location of maximum deformation	crown
Support installation distance(X)	1.4m

Normalized distance (X*)	$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$

From Vlachopoulos and Diederichs equation:

Normalized closure at tunnel face (u_0^*)

$$= \frac{u_0}{u_{\max}} = \frac{1}{3}e^{-0.15R^*}$$

$$= 0.222$$

normalized displacement at the support installation distance u^* ($\frac{u}{u_{\max}}$)

$$= 1 - \left(1 - u_0^*\right)e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$$

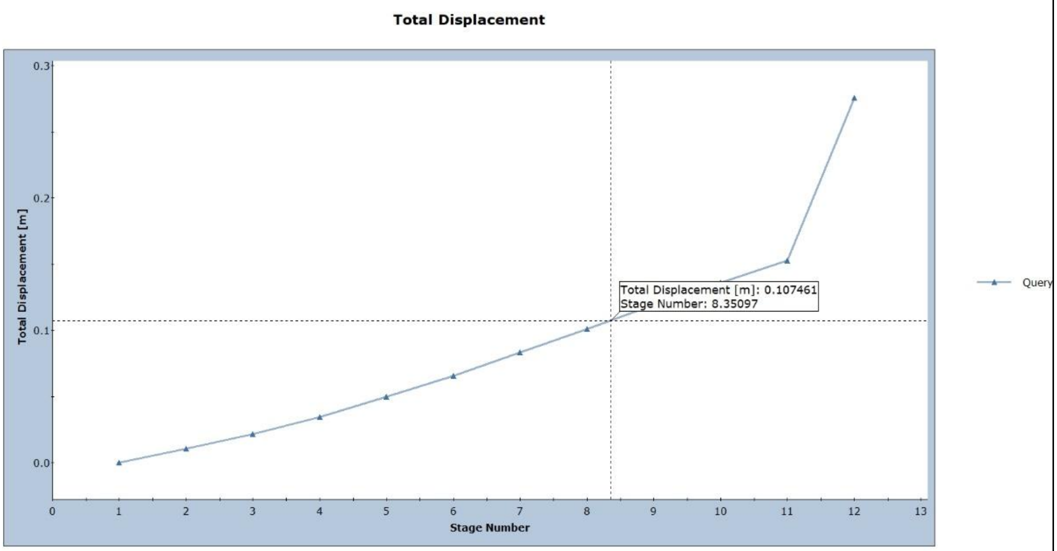
$$= 0.39$$

Displacement at the support installation distance

$$u = 0.107m$$

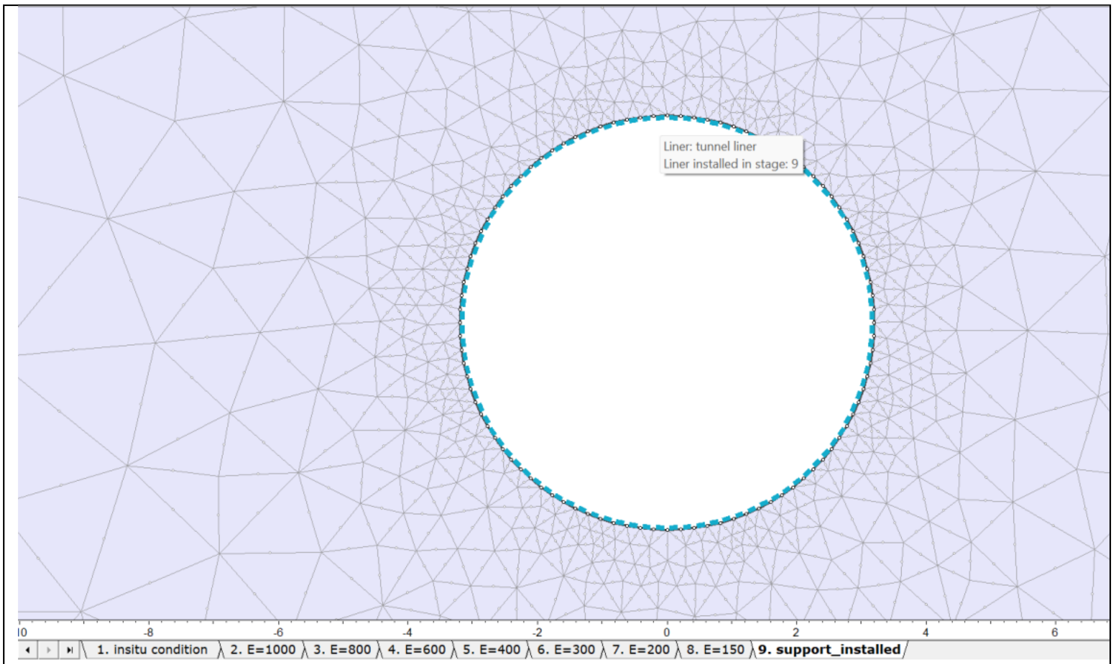
Determination of support installation stage at the model:

Deformation vs stage plot obtained from the model is:

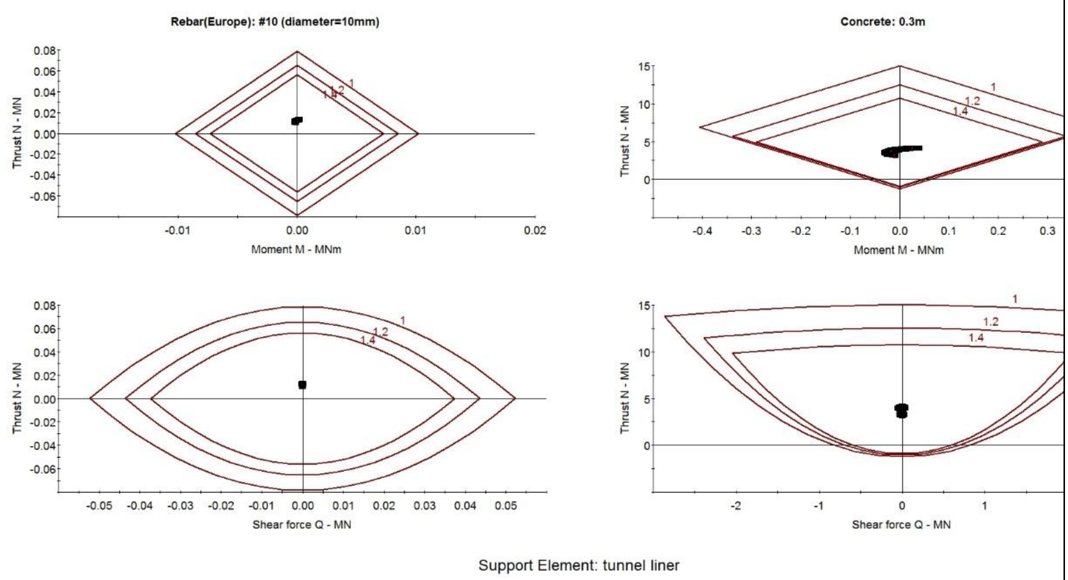


Total displacement 0.107 at stage no. 8.35 with E= 150

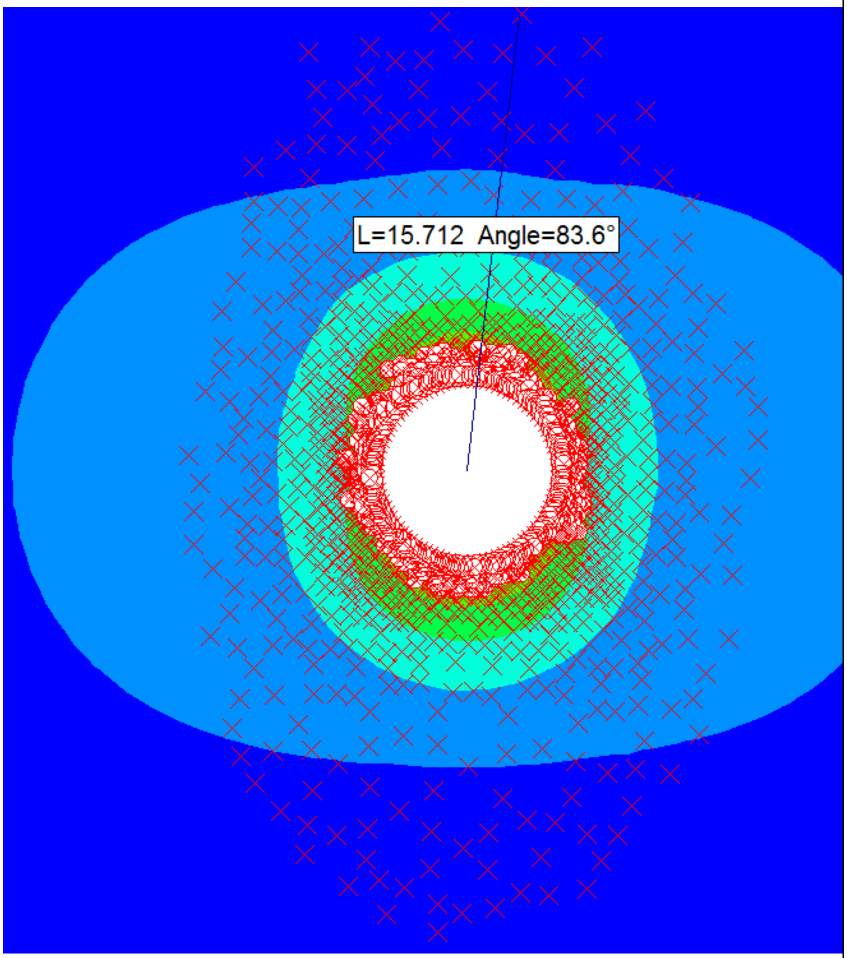
Support added after the stage:

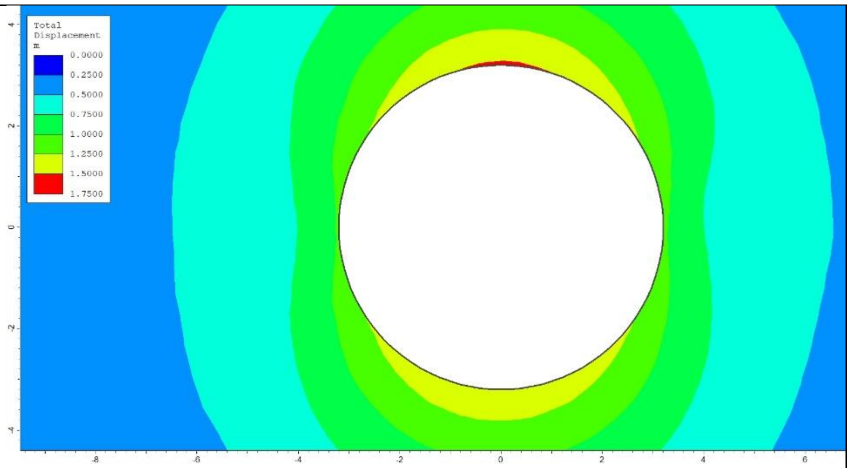


Support Capacity Diagram:



Name of analysis: REL_8

Analysis Type	-,-,-
Input Parameters	UCS= 21.94, GSI= 26.35 and m_i= 13
Tunnel Radius	3.2 m
Radius of plastic zone observed from the model at final excavated stage (Rpl)	15.71 m 
Maximum displacement(u_{max}) observed from model at final excavated stage	1.54 m

<table>
<tr><td></td><td>
</td></tr>
<tr><td>Location of maximum deformation</td><td>crown</td></tr>
<tr><td>Support installation distance(X)</td><td>1.4m</td></tr>
<tr><td>Normalized distance (X*)</td><td>$X^* = \frac{X}{R_t} = \frac{1.4}{3.2} = 0.4375$</td></tr>
<tr><td colspan="2">From Vlachopoulos and Diederichs equation:</td></tr>
<tr><td colspan="2">
Normalized closure at tunnel face (u_0^*)

$= \frac{u_0}{u_{max}} = \frac{1}{3} e^{-0.15R^*}$

$= 0.16$

normalized displacement at the support installation distance $u^* (\frac{u}{u_{max}})$

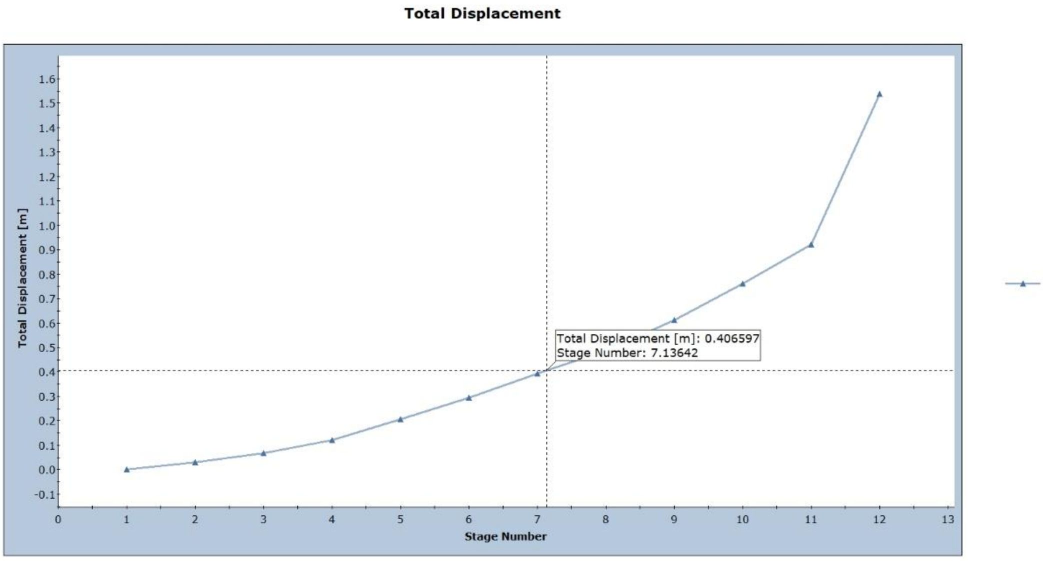
$= 1 - (1 - u_0^*) e^{-\frac{3X^*}{2R^*}}, \quad X^* \geq 0$

$= 0.264$

Displacement at the support installation distance

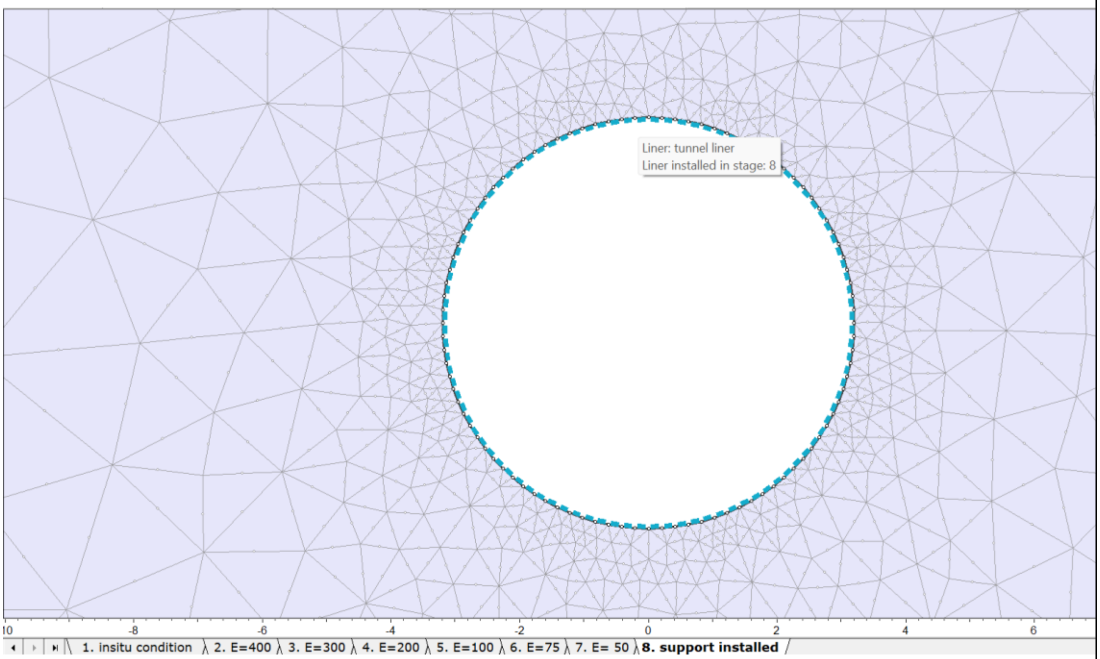
$u = 0.407m$
</td></tr>
<tr><td colspan="2">
Determination of support installation stage at the model:

Deformation vs stage plot obtained from the model is:
</td></tr>
</table>

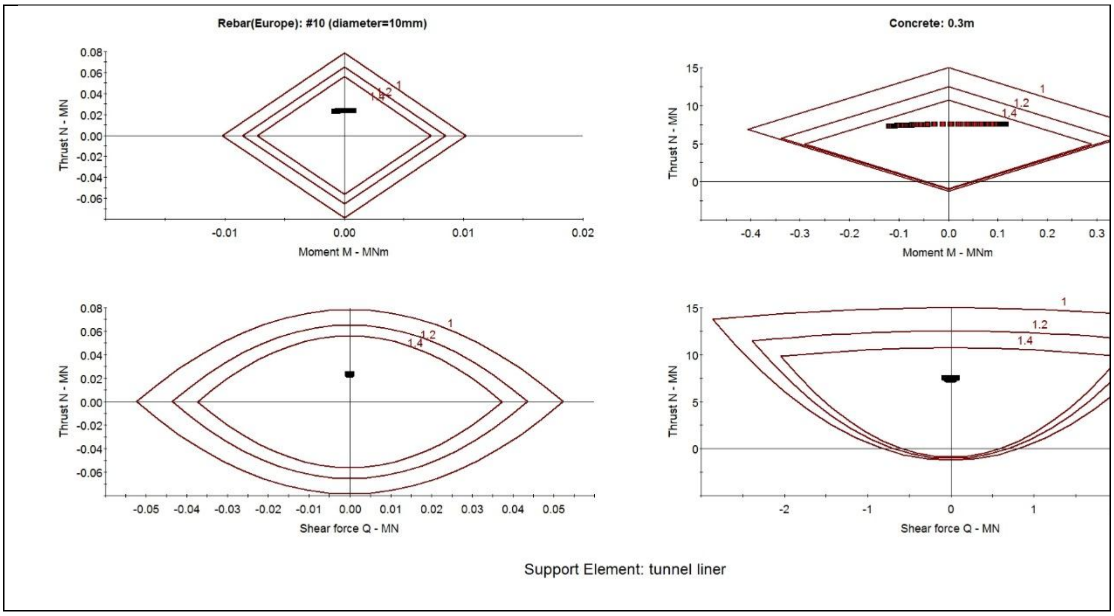


Total displacement 0.407 at stage no. 7.13 with E=50

Support added after the stage:



Support Capacity Diagram:



Rebar(Europe): #10 (diameter=10mm)
Concrete: 0.3m
Thrust N - MN
Moment M - MNm
Shear force Q - MN
1
1.2
1.4
Support Element: tunnel liner

Appendix-C: Analytical Validation

Validation for my work:

The loading condition (far field stress) I have assumed for my model is as below:

Figure C. 1 Stress used in model for deterministic analysis

For the deterministic analysis the observed values of maximum deformation and radius of plastic zone are as follows:

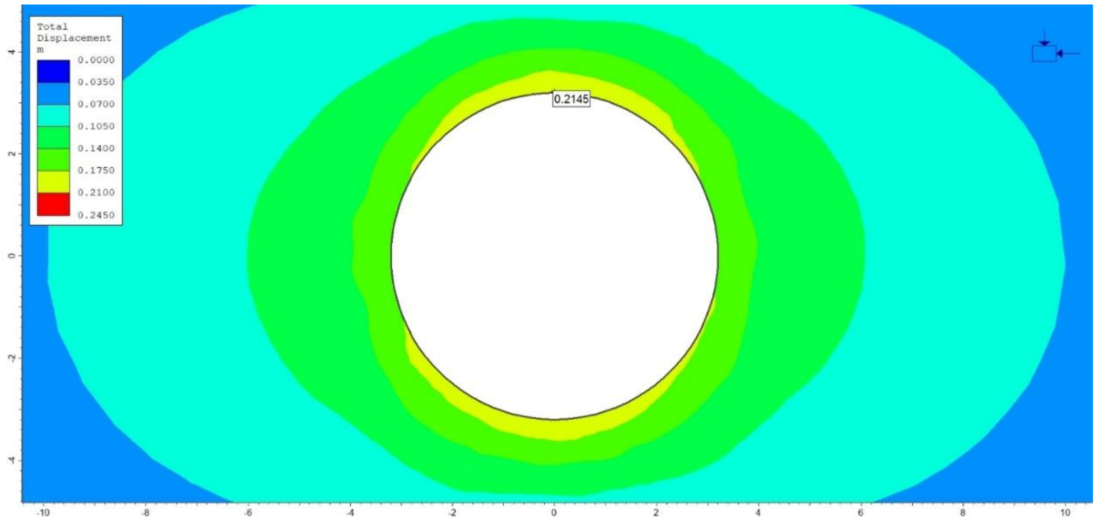


Figure C. 2 Maximum deformation for deterministic case

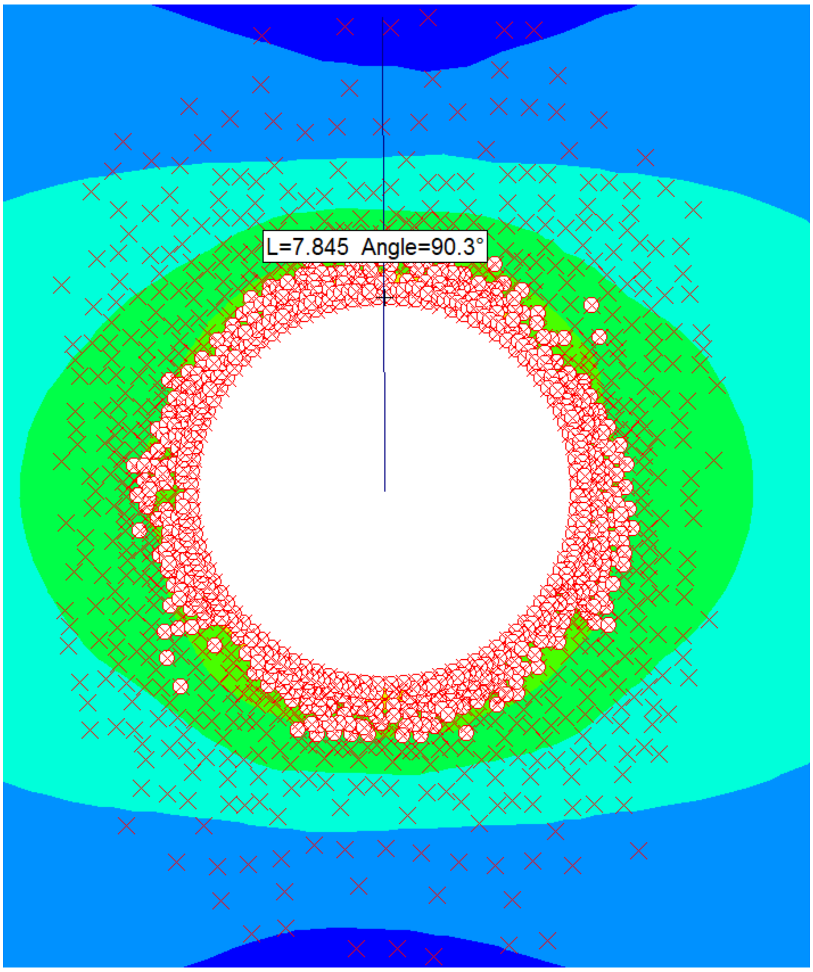


Figure C. 3 Radius of plastic zone observed for deterministic analysis

The analytical framework established by Carranza-Torres and Fairhurst (2000) provides a integration of closed-form solutions previously developed by the authors in 1999, which enable the precise calculation of the radius of the plastic zone and the associated radial deformation under hydrostatic stress conditions (Carranza-Torres & Fairhurst, 1999, 2000).

Analytical Solution:

This is based on the general form of Hoek and Brown Criterion proposed by Londe (1988). Rock mass is assumed to satisfy the Hoek-Brown failure criterion.

Scaled internal pressure (Pi) and far field stress (So) as:

$$P_i = \frac{p_i}{m_b \sigma_{ci}} + \frac{s}{{m_b}^2}, \text{ for final excavated stage } p_i \text{ is taken as zero.} \quad (18)$$

m_b = rock mass constant $= m_i \times e^{\frac{GSI-100}{28}}$

$= 17 \times e^{\frac{34.13-100}{28}}$

$= 1.617$

For GSI $\geq$ 25,

$s = e^{\frac{GSI-100}{9}} = 0.000662$
$a = 0.5$
$P_i = \frac{0.000662}{1.617^2} = 0.000253$
Average stress for different stress system$(\sigma_o) = \frac{15+21}{2} = 18Mpa$
Another scaled parameter S_o

$$S_o = \frac{\sigma_o}{m_b\sigma_{ci}} + \frac{s}{{m_b}^2} \quad (19)$$

$$= \frac{18}{1.617 * 39.32} + \frac{0.000662}{1.617^2} = 0.283$$

Scaled critical internal pressure(elastic limit)

$$P_i^{cr} = \frac{1}{16}\left[1 - \sqrt{1+16S_o}\right]^2 \quad (20)$$

$$= 0.114$$

Radius of plastic zone for far face case(Rpl)

$$R_{pl} = R \times e^{\left[2\left(\sqrt{P_i^{cr}} - \sqrt{P_i}\right)\right]} \quad (21)$$

$$= 3.2 \times e^{\left[2\left(\sqrt{0.114} - \sqrt{0.000253}\right)\right]}$$

$$= 6.00m$$

Deformation Calculation:

Elastic deformation:

$$u_r^{el} = \frac{\sigma_o - p_i}{2G_{rm}} \times R \quad (22)$$

$$= \frac{18 - 7.25}{2 \times 561} \times 3.2$$

$$= 0.03m$$

Plastic deformation:

Non scaled critical internal pressure (p_i^{cr}):

$$p_i^{cr} = \left[P_i^{cr} - \frac{s}{{m_b}^2}\right] * m_b\sigma_{ci} \quad (23)$$

$$= \left[0.114 - \frac{0.000662}{1.617^2}\right] * 1.617 * 39.32$$

$$= 7.25Mpa$$

Shear Modulus of Rock Mass:

$$G_{rm} = \frac{E_{rm}}{2(1+\nu)} \quad (24)$$

E_{rm} is found out using Hoek and Diederichs (2006) as given by following equation:

$$Erm = Ei(0.02 + \frac{1 - \frac{D}{2}}{1 + e(60 + 15D - GSI)/11}) \quad (25)$$

$E_i = MR * \sigma_{ci}$
$E_{rm} = 1458.6Mpa$
Poisson's ratio is taken as 0.3.

$$G_{rm} = \frac{E_{rm}}{2(1+\nu)} = 561Mpa$$

Equation for the plastic deformation calculation :

$$\frac{u_{pl}}{R} \cdot \frac{2G_{rm}}{\sigma_o - p_i^{cr}} = \left[\frac{1-2\nu}{2} \cdot \frac{\sqrt{P_i^{cr}}}{S_o - p_i^{cr}} + 1\right] \left(\frac{R_{pl}}{R}\right)^2 + \frac{1-2\nu}{4(S_o - P_i^{cr})} \left[ln\left(\frac{R_{pl}}{R}\right)\right]^2 - \frac{1-2\nu}{2} \cdot \frac{\sqrt{P_i^{cr}}}{S_o - P_i^{cr}} \left[2\,ln\left(\frac{R_{pl}}{R}\right) + 1\right] \quad (26)$$

$$\frac{u_{pl}}{3.2} \cdot \frac{2*561}{18-7.5} = \left[\frac{1-2*0.3}{2} \cdot \frac{\sqrt{0.114}}{0.283-0.114} + 1\right] \left(\frac{6}{3.2}\right)^2 + \frac{1-2*0.3}{4(0.2833-0.114)} \left[ln\left(\frac{6}{3.2}\right)\right]^2 - \frac{1-2*0.3}{2} \cdot \frac{\sqrt{0.114}}{0.2833-0.114} \left[2\,ln\left(\frac{6}{3.2}\right) + 1\right]$$

$u_{pl} = 0.15m$

u_{max}=0.18 m.

The values obtained for plastic radius and maximum deformation for the set of parameters used in deterministic analysis are thus in close agreement with the values obtained from the model. The obtained result after the analysis thus validates the model result.

The slight difference in model result and above analysis is because of the fact that in my model I have used that anisotropic stress condition as shown in the above Figure C. 1 and above analysis is based on the average istropic far field stress condition.

Again I modeled in phase 2d for the same isotropic stress as shown in the fig below other parameters remaining the same.

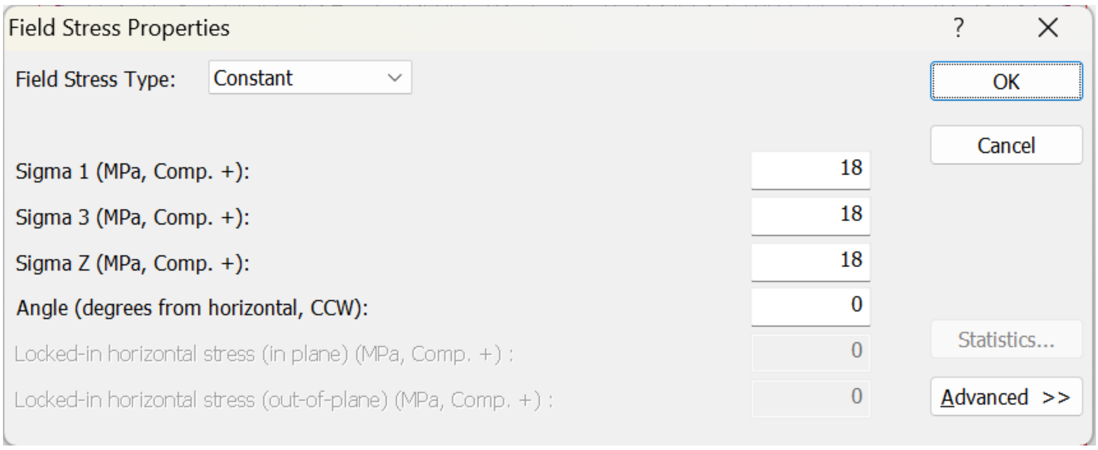


Figure C. 4 Loading condition for the validation model (for isotropic stress)

The model output result is as below:

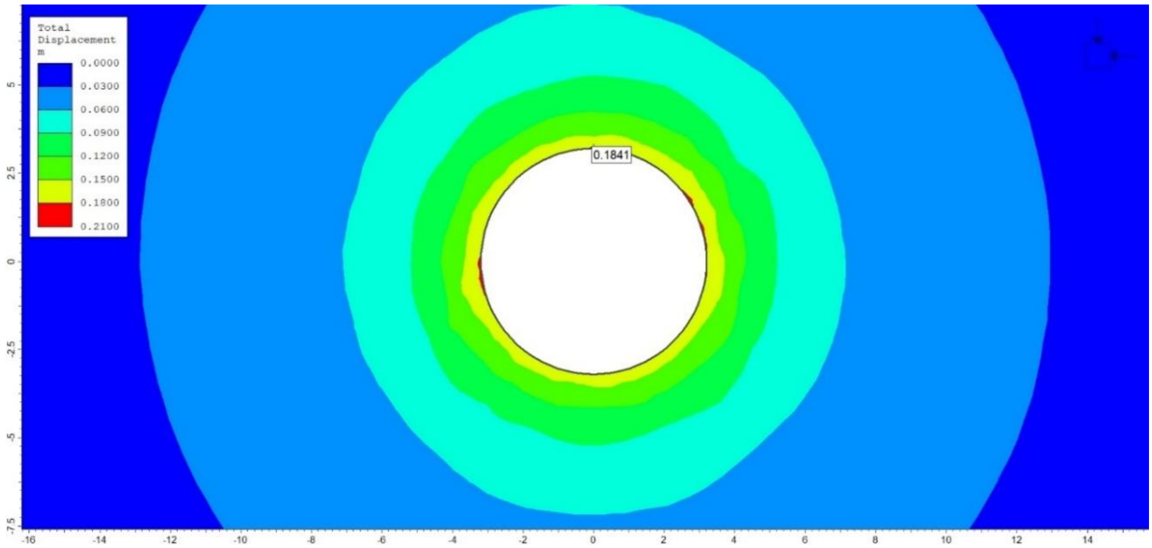


Figure C. 5 Maximum deformation for validation model

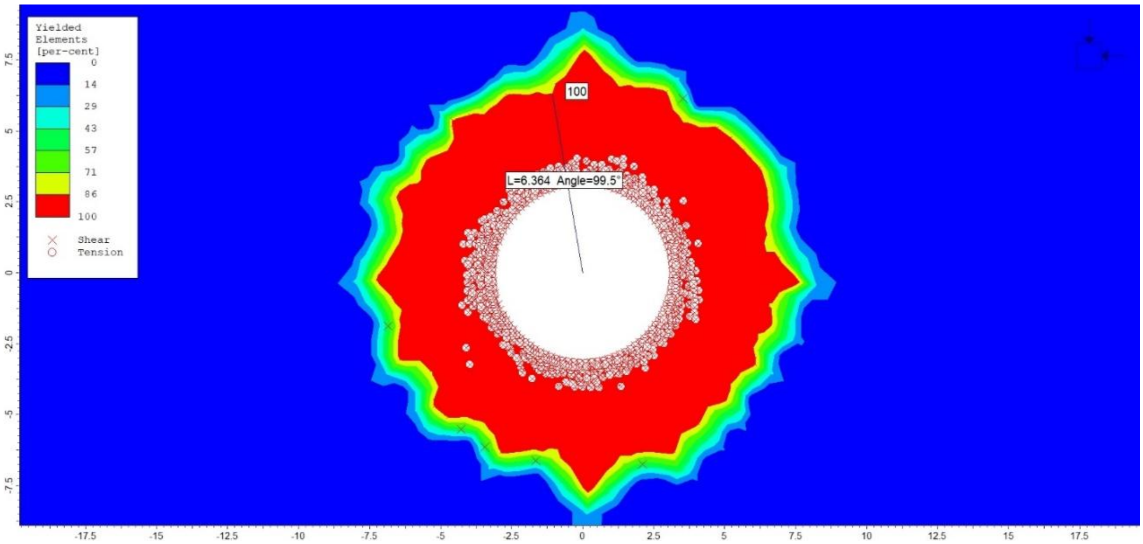


Figure C. 6 Radius of plastic zone

The obtained result from the model and the analysis result is tabulated below:

Table C. 1: Analytical vs Model Result

	Analysis Result	Model Result	Difference (%)
Deformation(far face)	0.17m	0.184m	8%
Radius of plastic zone(Rpl)	6.00m	6.364m	6%