CHAPTER - I

INTRODUCTION

Background of the Study

Mathematics is a mirror of civilization. It is the subject that has significant impact on people. Every people need mathematics to solve the problems in the daily activities. Mathematics is the study of numbers, shapes and space using reason and usually a special system of symboles and rules for organizing them (Cambridge Advanced Learner's Dictionary).

Mathematics has been tought as a compulsory subject at all levels of school system in nepal. Compulsory mathematics and optional mathematics are offered to willing and worthly students in secondary level. In both subject has geometry part is tought separately as an important part of the whole school mathematics curriculum. In compulsory mathematics geometry has included covering 24% of weights in secondary level (CDC,2064).

Geometry is one of the mother structure of mathematics. The word 'Geometry' is derived from the Geek word 'Geo' and 'Metry' which means measurement of the earth. It is one of the most applicable and daily useful branch of mathematics. It explains about quantity, change, structure and spatial relationship. The origin of geometry is Babylonian, Egyptians and Greek. Greek geometrician Plato advocated that "Let no man ignorant of geometry enter" for the development of the geometry development. The concept of geometry in mathematics was developed by Kelly and Ladd. At that time formally discovered the geometry by 'Euclid'.

Geometry is the growing body of knowledge with ever widening application and inherent beauty in its systematic structure and organization. It is not known the full extent to which geometric and numerical relationships were understood and use in very ancient times.

But evidence indicates that until about 600 BC, all mathematical knowledge consisted of various “rule of thumb” formulas or procedures that were obtained from experience and that gave useful approximation, at least in special cases. Later it was replaced by the rule of reasoning. However, it is not certain who first had the idea of trying to prove a mathematical rule by reasoning rather than by testing it in different cases (Pandit, 2010, p.1).

Goemetry is one of the understanding part of mathematics. The shape,size and other properties of figures and the nature of space are the measurement of geometry. It is the branch of mathematics that deals with the measurements and relationship of line, angle surface, and solid. The measurement of line, angle surface area etc. is particular object or shape. Euclidean geometry is concerned with the axomatic system of study of polygons, conic section, spheres, polyhedral and related geometric objects in two and three diamensions (Wallace and West, 1989 “Second edition”)

In the late 1950s in the Natherland, two mathematics teachers, Pierre Van Hiele and Dina Van Hiele – Geldof , husband and wife, put forth theory of development in geometry based on theory own teaching and research. They observed that in learning geometry based on theory own teaching and research. They observed that in learning geometry, student progress through a sequence of five reasoning levels, from wholistic thinking to analytic thinking to rigorous abstract mathematical deduction (Pandit, 2010, p.453).

The problem regarding the teaching and learning of geometry was identified by two dutch mathematics educator Pierre Van Hiele and Dina Van Hiele – Geldof, who due to their frustrations investigated possible reason that could have created this problem in their classroom. The findings of their invistigations resulted in the development of a theory. The theory distinguishes five different thought levels (Level 0 : visualization, Level 1: analysis, Level 2: informal deduction, Level 3: formal deduction Level 4: rigor) that a student should go through when learning geometry. Van Hiele developed five levels of thoughts on the issues why students feel difficulties in geometry. Each level tests achievement of students in geometry. This method is necessary for determining; how is the geometric achievement of students in lower level and how their weeknesses and increment is in level wise geometric knowledge in arriving from lower level geometric achievement to higher level geometric achievement.

Statement of the Problem

In the context of Nepal most of the teacher and students feel geometry as a topic dificult to teach and learn. When teachers enter in the classroom they start traditional method and limit their method in board and marker. It consequently hinders the students to obtain good reslt in the topic. To overcome such problem Van Hiele model is believed to be highly effective.

In this regard, the present study intends to answer the following questions:

1. How Van Hiele model promote students' achievement in Geometry?
2. In what extent Van Hiele method appropriate to use in the context of Nepal?

Rationale of the Study

The current study has following rationale in the field of teaching geometry:

- It makes teacher to be able to assign students various Van Hiele levels and prepare suitable directions more appropretely. Teacher can amend the way they teach the cope the challenges emerged from a diverse group of learners.
- It would be helpful for the textbook writers and curriculum designers to align their materials according to Van Hiele model.
- The researchers in the field of geometry would get benefit to further study the impact of Van Hiele model.

Objectives of the Study

The study will have the following objectivies:

1. To dig out the level of mathematics students of secondary level compared with Van Hiele model.
2. To explore the attitude of the students about Van Hiele model of teaching geometry at grade nine.

Delimitation of the Study

The study will have limited in the following aspects:

- The study will be delimited to Bhageshwor rural municipality in Dadeldhura disrrict.
- The study will be concerned only with the students of grade nine.
- The study will only be conducted for the area of geometry especially: Triangle, Parallelogram, Construction, Similarity and circle.
- The study will be based on Van Hiele Model.

Definition of the Related Terms

Van Hiele Model (VHM). VHM is the method of teaching geometry, which is developed by Pierre Van Hiele and his wife Dina Van Hiele-Geldof.

Van Hiele Geometry Test (VHGT). VHGT was used by Van Hiele. It consist of different multiple choice questions in the order with 0-4, in order to find the level of students.

CHAPTER – II

REVIEW OF RELATED LITERATURE

Review of related is an important part of research for the researcher. Literature review paves a way to the research since it clearly visions the status of the past research. The theoretical background of the ongoing research and the importance of the proposed aspect should meet the interest. The need of the proposed research is also clearly advocated in this section. Specially the review of literature has been described into three parts: empirical literature, theoretical literature and conceptual framework. In this study the researcher reviewed the considerable related and relevant literature carried out by various researchers in the field of Geometry and Van Hiele's method of geometric instruction.

Eimpirical Literature

Joshi (2017), condected a research on " Effectiveness of Van Hiele approach in teaching geometry" with the objectives to compare the Van Hiele approach and treditional approach of teaching geometry at grade seven students. He focused his study in Janata Higher Secondary School, Vidyapur-4 Surkhet. His sample population of study was 60 students studying in grade seven. He found that the achievement of the grade seven students who were tought geometry with using Van Hiele approach of teaching achieved better than the students who were tought using traditional method. He conclude that Van Hiele approach in teching geometry was effective approach in teaching experimental verification of geometry at lower secondary level.

Thapa(2017), studied on "Students' Van Hiele level of geometric though and it's relationship to their achievement in mathematics" with the objectives to determine the Van Hiele levels of geometric thinking of grade ten students. She focused his study in Tanahu district. Her sample population of the study was 203 students of grade ten. The of her study indicated that the absence of students at level 4 and only few students were at level 3, implies that most of them didn't possess the experience necessary of the formal study of secondary school geometry. Also, the result indicated that the majority of the students who took VHGT were at level 0 on the Van Hiele geometric scale, which implies their knowledge of school was poor. She concluded that VHGT is one of the most importaant tool to find out that achievement of students in geometry and supports Van Hiele theory is one of the best framework in exploring students' geometric reasoning.

Awasthi (2020), conducted a research on "Level of thinking of grade X students in geometry: Van Hiele perspective" with objectives to find whether the selected grade X students are or are not functioning at a level of geometric thinking fitting with their mathematics curriculum. He focused his study in Bedkot Municipality of Kanchanpur district. His population of study was 1625 students in 19 secondary school. He found that the result of the test indicated that of 139 students who participated 58 (41.77%) at level 1, 38 (27.24%) at level 2, 23 (16.55%) at level 3 and 14 (10.07%) at level 4. He conclude that the students who participated in the study are functioning at a level of geometyic thinking no fitting with their mathematics curriculum.

Lamsal (2005), conducted a study on "A study on the effecfiveness of Van Hiele approach in teaching geometry at lower secondary level" aimed at exploring the effectiveness of the Van Hieles' approach in teaching geometry. The population of his study was eight grade students enrolled in the public school in Syanja district. He experimented on the forty nine pupils of the sample with the help of teaching module as a research tool. The achievement students are analysed statistically by using Mean, Standard deviation, t-test at 0.05 level of significance. In his study he found that mean achievement score of the student taught by Van Hiele approach was higher than that of taught by conventional approach.

Meteya (2008), conducted a study on “Using Van Hiele theory to analyze geometrical conceptualization in grade 12 students: A Namibian Perspectives”. The result of his study indicated that many of the students who participated in the research had weak conceptual understanding of geometric concepts. The findings of his study also highlighted the issues of how the namibian grade 12 geometry syllabuses should be aligned with the Van Hiele level of geometric thinking as well as the use of the appropraite and correct language in geometrical thinking and problem solving.

Usiskin (1982), studied on “Van Hiele level and achievement in secondary school geometry”. He developed a multiple choice test to measure a student’s Van Hiele level of reasoning. He wanted to find out if these test could at all predict student achievement in geometry. The population for this studied consists of 2699 students in the United States enrolled in a one–year geometric course in 13 schools. This course is taken by about 56% of male and 55% of females who are or become seniors in high school. Schools were selected on the basis of meeting certain socio-economic criteria and offering a high probability of success in obtaining reliable data consistent with the testing requirements. The findings was that the poor performance of many students either in a geometry contant test or in proof writing was strongly associated will being at the lower Van Hiele levels.

A study carried by Oli (2013) entitled “ Van Hiele of geometry though and mathematics achievement of student” to analyses and exploring the Van Hiele level of geometric thought of secondary level students and the relationship between mathematics achievement and Van Hiele level of geometric thought in this study. The researcher using by survey design in this study conducted Rukum district. The sample population of study was 52 schools of class 10 students. Van Hiele geometry achievement test and school mathematics test was used as a main tools for the data collection. The finding result of the student who participated in the research have a work conceptual understanding of geometry concepts 28.67% at level 0, 26.67% at level 1, 20.67% at level 2 and 8.67% at level 3 of total students. No students were found to be thinking at level 4. Further more and interview schedule for teacher was development to get their reflection about VHGT. The scores obtained by students on VHGT and SMT where correlated and correlation coefficient was found to be 0.68. The result indicated that there was a positive correlation between Van Hiele model of geometric thought and mathematics achievement. Also supported that study the positive relation Van Hiele level of thinking in geometry and its achievement of student.

Theoretical Literature

In mathematics education, the Van Hiele model is a theory that describes how students learn geometry. The theory originated in 1957 in the doctoral dissertations of Dina Van Hiele - Geldof and Pierre Van Hiele (wife and husband) at Utrecht University, in the Netherlands. The Soviets did research on the theory in the 1960s and integrated their findings into their curricula. American researchers did several large studies on the Van Hiele theory in the late 1970s and early 1980s, concluding that students' low Van Hiele levels made it difficult to succeed in proof-oriented geometry courses and advising better preparation at earlier grade levels. Pierre Van Hiele published "Structure and Insight" in 1986, further describing his theory. The model has greatly influenced geometry curricula throughout the world through emphasis on analyzing properties and classification of shapes at early grade levels. In the United States, the theory has influenced the geometry strand of the standards published by the national council of teachers of mathematics and the new common core standards.

Van Hiele describes how children think about geometric concept in different level. The basis of the theory is the idea that a student's growth in grometry takes place in terms of distinguishable levels of thinking. Geometry insteuctions should be designed with these levels in mind (Choi-koh 1199). The Van Hiele believes that written proofs requires thinking at comparatively high level, and that may students need more experiences in thinking at lower levels before learning formal geometric concept. This description of how children view shapes dominate in many later publication by Clements and his colleagues (Clements & Sharma, 2000). The Van Hiele model for the theory of geometry through consists of five levels (Burger and Shaughnessy 1986).

The Van Hiele theory of geometric through describes the different level of understanding through which student's progress when learning geometry.

Table: 1

Van Hiele Theory of geometric thought

Level	**Description**	**Ability of Students**
0	Visualization	Describes shapes on the basis of their appearance.
1	Analysis	Describes shapes on the basis of their properties.
2	Informal Deduction	Recognizes the importance of properties and the relationships among them which assist students in logically ordering the properties of the shape.
3	Deduction	Attains logical reasoning ability and proves theorems deductively.
4	Rigor	Establishes and analyzes theorems in different postulation systems.

Van Hiele Model of Geometric Thinking

The student learns by rote to operate with [mathematical] relations that he/she does not understand, and of which he/she has not seen the origin. Therefore, the system of relations is an independent construction having no rapport with other experiences of the child. This means that the student knows only what has been taught to him/her and what has been deduced from it. He/She has not learned to establish connections between the system and the sensory world. He/She will not know how to apply what he has learned in a new situation (Pierre Van Hiele, 1959).

The best known part of the Van Hiele model are the five levels which the Van Hieles postulated to describe how children learn to reason in geometry. Students cannot be expected to prove geometric theorems until they have built up an extensive understanding of the systems of relationships between geometric ideas. These systems cannot be learned by rote, but must be developed through familiarity by experiencing numerous examples and counterexamples, the various properties of geometric figures, the relationships between the properties, and how these properties are ordered. The five levels postulated by the Van Hieles describe how students advance through this understanding.

The five Van Hiele levels are sometimes misunderstood to be descriptions of how students understand shape classification, but the levels actually describe the way that students reason about shapes and other geometric ideas. Pierre Van Hiele noticed that his students tended to "plateau" at certain points in their understanding of geometry and he identified these plateau points as levels. In general, these levels are a product of experience and instruction rather than age. This is in contrast to Piaget's theory of cognitive development, which is age-dependent. A child must have enough experiences (classroom or otherwise) with these geometric ideas to move to a higher level of sophistication. Through rich experiences, children can reach Level 1 in elementary school. Without such experiences, many adults (including teachers) remain in Level 0 all their lives, even if they take a formal geometry course in secondary school. The levels are as follows:

Level 0 : Visualization / Reorganization

At this level, the focus of a child's thinking is on individual shapes, which the child is learning to classify by judging their holistic appearance. Children simply say, "That is a circle," usually without further description. Children identify prototypes of basic geometrical figures (triangle, circle, square). These visual prototypes are then used to identify other shapes. A shape is a circle because it looks like a sun; a shape is a rectangle because it looks like a door or a box; and so on. A square seems to be a different sort of shape than a rectangle, and a rhombus does not look like other parallelograms, so these shapes are classified completely separately in the child’s mind. Children view figures holistically without analyzing their properties. If a shape does not sufficiently resemble its prototype, the child may reject the classification. Thus, children at this stage might balk at calling a thin, wedge-shaped triangle (with sides 1, 20, 20 or sides 20, 20, 39) a "triangle", because it's so different in shape from an equilateral triangle, which is the usual prototype for "triangle". If the horizontal base of the triangle is on top and the opposing vertex below, the child may recognize it as a triangle, but claim it is "upside down". Shapes with rounded or incomplete sides may be accepted as "triangles" if they bear a holistic resemblance to an equilateral triangle. Squares are called "diamonds" and not recognized as squares if their sides are oriented at 45° to the horizontal. Children at this level often believe something is true based on a single example.

Thus level students use visual perception and non-verbal thinking. They recognize geometric figures by their shapes as “a whole” and compare the figures with their properties.

This means that the identification of shapes is based on a certain prototype. They use simple language. They do not identify the properties of geometric figures.

Level 1 : Analysis / Description

At this level, the shapes become bearers of their properties. The objects of thought are classes of shapes, which the child has learned to analyze as having properties. A person at this level might say, "A square has 4 equal sides and 4 equal angles. Its diagonals are congruent and perpendicular, and they bisect each other." The properties are more important than the appearance of the shape. If a figure is sketched on the blackboard and the teacher claims it is intended to have congruent sides and angles, the students accept that it is a square, even if it is poorly drawn. Properties are not yet ordered at this level. Children can discuss the properties of the basic figures and recognize them by these properties, but generally do not allow categories to overlap because they understand each property in isolation from the others. For example, they will still insist that "a square is not a rectangle." (They may introduce extraneous properties to support such beliefs, such as defining a rectangle as a shape with one pair of sides longer than the other pair of sides.) Children begin to notice many properties of shapes, but do not see the relationships between the properties; therefore, they cannot reduce the list of properties to a concise definition with necessary and sufficient conditions. They usually reason inductively from several examples, but cannot yet reason deductively because they do not understand how the properties of shapes are related.

Thus, students see figures ascollections of properties. They can recognize and name properties of geomertic figures, but they do not see relationship between these properties. When describing an object, a student operating at this level might list all properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.

Level 2 : Informal Deduction / Ordering / Abstraction

At this level, properties are ordered. The objects of thought are geometric properties, which the student has learned to connect deductively. The student understands that properties are related and one set of properties may imply another property. Students can reason with simple arguments about geometric figures. A student at this level might say, "Isosceles triangles are symmetric, so their base angles must be equal." Learners recognize the relationships between types of shapes. They recognize that all squares are rectangles, but not all rectangles are squares, and they understand why squares are a type of rectangle based on an understanding of the properties of each. They can tell whether it is possible or not to have a rectangle that is, for example, also a rhombus. They understand necessary and sufficient conditions and can write concise definitions. However, they do not yet understand the intrinsic meaning of deduction. They cannot follow a complex argument, understand the place of definitions, or grasp the need for axioms, so they cannot yet understand the role of formal geometric proofs.

Thus, students perceive relationship between properties and figures. At this level, students can create meaningfull definition and give informal arguments to justify there reasoning. Logical implication and class inclusions, such as square being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.

Level 3 : Deduction

Students at this level understand the meaning of deduction. The object of thought is deductive reasoning (simple proofs), which the student learns to combine to form a system of formal proofs (Euclidean geometry). Learners can construct geometric proofs at a secondary school level and understand their meaning. They understand the role of undefined terms, definitions, axioms and theorems in Euclidean geometry. However, students at this level believe that axioms and definitions are fixed, rather than arbitrary, so they cannot yet conceive of non-Euclidean geometry. Geometric ideas are still understood as objects in the Euclidean plane.

Thus, students can construct proofs, understand the role of axoms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

Level 4 :Rigor

At this level, geometry is understood at the level of a mathematician. Students understand that definitions are arbitrary and need not actually refer to any concrete realization. The object of thought is deductive geometric systems, for which the learner compares axiomatic systems. Learners can study non-Euclidean geometries with understanding. People can understand the discipline of geometry and how it differs philosophically from non-mathematical studies.

Thus, This is the highest level of thought in the Van Hiele hierarchy. Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. Students at this level can work in different axiomatic systems and would most likely be enrolled in a college or university level cource in geometry.

American researchers renumbered the levels as 1 to 5 so that they could add a "Level 0" which described young children who could not identify shapes at all. Both numbering systems are still in use. Some researchers also give different names to the levels.

Properties of Van Hiele Level

The Van Hiele levels have five properties:

Fixed sequence. The levels are hierarchical. Students cannot "skip" a level. The van Hiele claim that much of the difficulty experienced by geometry students is due to being taught at the deduction level when they have not yet achieved the abstraction level.

Adjacency. Properties which are intrinsic at one level become extrinsic at the next. (The properties are there at the visualization level, but the student is not yet consciously aware of them until the analysis level. Properties are in fact related at the analysis level, but students are not yet explicitly aware of the relationships.)

Distinction. Each level has its own linguistic symbols and network of relationships. The meaning of a linguistic symbol is more than its explicit definition; it includes the experiences the speaker associates with the given symbol. What may be "correct" at one level is not necessarily correct at another level. At Level 0 a square is something that looks like a box. At Level 2 a square is a special type of rectangle. Neither of these is a correct description of the meaning of "square" for someone reasoning at Level 1. If the student is simply handed the definition and its associated properties, without being allowed to develop meaningful experiences with the concept, the student will not be able to apply this knowledge beyond the situations used in the lesson.

Separation. a teacher who is reasoning at one level speaks a different "language" from a student at a lower level, preventing understanding. When a teacher speaks of a "square" she or he means a special type of rectangle. A student at level 0 or 1 will not have the same understanding of this term. The student does not understand the teacher, and the teacher does not understand how the student is reasoning, frequently concluding that the student's answers are simply "wrong". The Van Hiele believed this property was one of the main reasons for failure in geometry. Teachers believe they are expressing themselves clearly and logically, but their level 3 or 4 reasoning is not understandable to students at lower levels, nor do the teachers understand their students' thought processes. Ideally, the teacher and students need shared experiences behind their language.

Attainment. The Van Hiele recommended five phases for guiding students from one level to another on a given topic:

- ***Information or inquiry***. students get acquainted with the material and begin to discover its structure. Teachers present a new idea and allow the students to work with the new concept. By having students experience the structure of the new concept in a similar way, they can have meaningful conversations about it. (A teacher might say, "This is a rhombus. Construct some more rhombi on your paper.")
- ***Guided or directed orientation.*** students do tasks that enable them to explore implicit relationships. Teachers propose activities of a fairly guided nature that allow students to become familiar with the properties of the new concept which the teacher desires them to learn. (A teacher might ask, "What happens when you cut out and fold the rhombus along a diagonal? the other diagonal?" and so on, followed by discussion.)
- ***Explicitation.*** students express what they have discovered and vocabulary is introduced. The students' experiences are linked to shared linguistic symbols. The Van Hiele's believe it is more profitable to learn vocabulary after students have had an opportunity to become familiar with the concept. The discoveries are made as explicit as possible. (A teacher might say, "Here are the properties we have noticed and some associated vocabulary for the things you discovered. Let's discuss what these mean.")

- ***Free orientation:*** students do more complex tasks enabling them to master the network of relationships in the material. They know the properties being studied, but need to develop fluency in navigating the network of relationships in various situations. This type of activity is much more open-ended than the guided orientation. These tasks will not have set procedures for solving them. Problems may be more complex and require more free exploration to find solutions. (A teacher might say, "How could you construct a rhombus given only two of its sides?" and other problems for which students have not learned a fixed procedure.)
- ***Integration:*** students summarize what they have learned and commit it to memory. The teacher may give the students an overview of everything they have learned. It is important that the teacher not present any new material during this phase, but only a summary of what has already been learned. The teacher might also give an assignment to remember the principles and vocabulary learned for future work, possibly through further exercises. (A teacher might say, "Here is a summary of what we have learned. Write this in your notebook and do these exercises for homework.") Supporters of the Van Hiele model point out that traditional instruction often involves only this last phase, which explains why students do not master the material.

For Dina van Hiele-Geldof's doctoral dissertation, she conducted a teaching experiment with 12-year-olds in a Montessori secondary school in the Netherlands. She reported that by using this method she was able to raise students' levels from Level 0 to 1 in 20 lessons and from Level 1 to 2 in 50 lessons.

Conceptual Framework

A conceptual framework is mental map which is makes by researcher to reach in goal. This study will had used theory of Van Hiele method of teaching geometry at grade nine students.

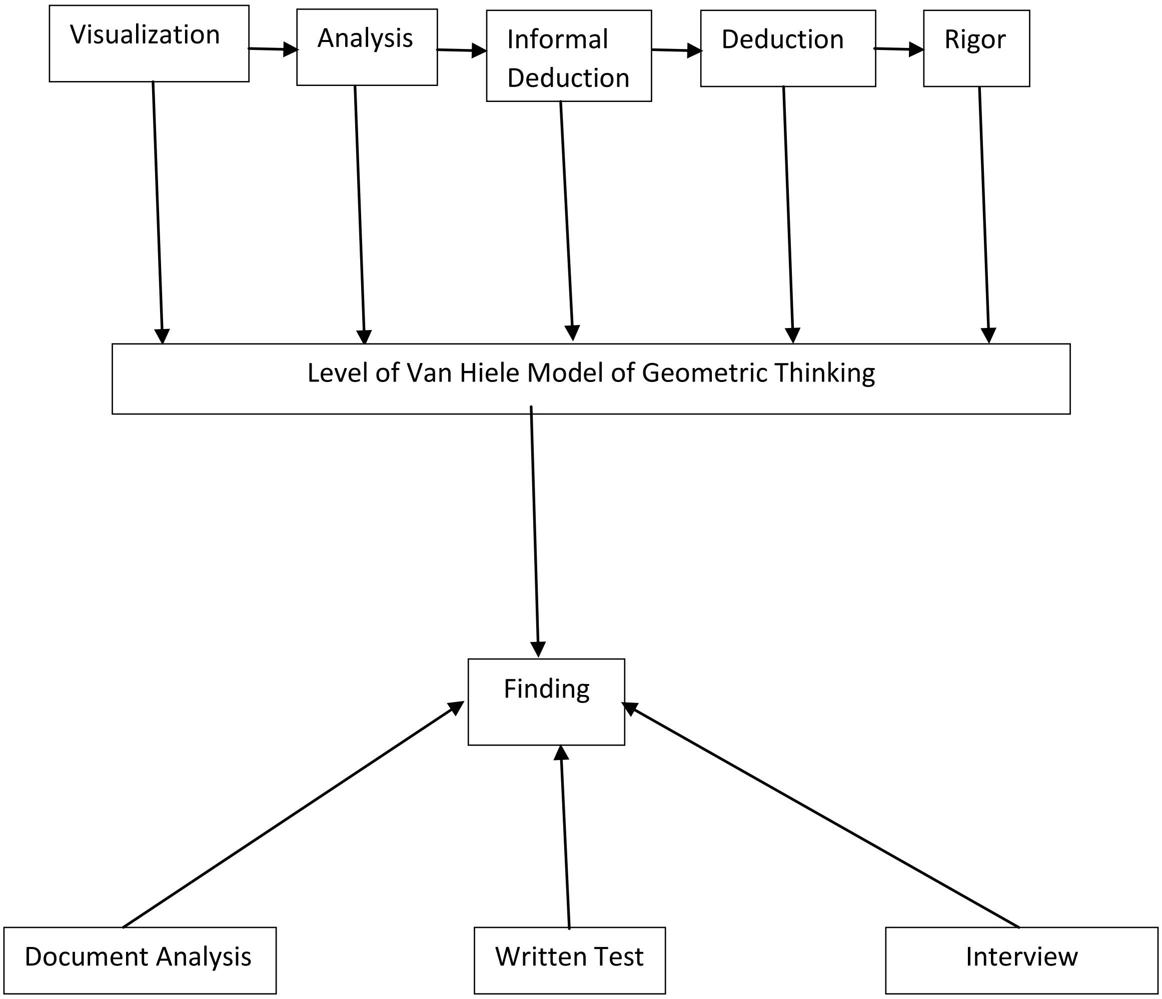


Figure 2.1: Conceptual Framework

CHAPTER – III

METHODS AND PROCEDURES

Design of the Study

This research is quantitatively followed by qualitative (mixed) research in nature. The results of the test used to determine the number of participants at each Van Hiele level. The use of interview in this study is aim at allowing the participants to freely express their views with regard to geometric concepts. The result of the interview assisted in revealing the participants conceptual understanding of the geometric concepts. The test was analyzed quantitatively and the interview and documents be analyzed qualitatively.

Population of the Study

The population of the study consisted all students at grade nine in academic year 2077 in Bhageshwor Rural Municipality of Dadeldhura district. There were about 305 grade nine students in 7 secondary school in Bhageshwor Rural Municipality of Dadeldhura district constitutes population of the study.

Sample of the Study

The research design was mixed method approach. For this study two government secondary school out of 7 secondary schools in Bhageshwor Rural Municipality were selected for the Van Hiele Geometry test by purposive sampling according as convenience.

Tools of the Study

The researcher was developed the written test or subjective test on geometrical problem. It was help to meet the objectives. The tools were used for data collection as described as follows:

Document Analysis. Analysis of syllabus issued by CDC of grade nine and analysis of model questions set.

Van Hiele Geometry Test. The Van Hiele geometry test consisted of four subsets, each with five multiple choice items based on each of the Van Hiele levels except the level 3, level 3 consisted two multiple choice items and 3 theorem proving problem. That is, there were 20 items in all, with number 1-5 testing the attainment of Van Hiele level 0 (visualization), 6-10 level 1 (analysis), 11-15 level 2 (informal deduction) and 16-20 level 3 (formal deduction).

Interview. Interview was taken of 8 selected students from each school based on the Van Hiele geometry test paper. And manipulative sorting activities were done for the interview. Also identifying and naming shape task, defining shape task and scoring by class inclusion task were done for interview.

Reliability and Validity of Tools

The reliability of test was determined by test method and reliability coefficient by piloting. Content validity and construct validity were insured by selecting questions of each level according to the syllabus of grade nine.

Data and Collection Procedure

The research followed by following process to collect the data:

Phase 1: Document Analysis

For this research study, mathematics syllabus of geometry for grade nine, model question sets were collected and analyzed.

Phase 2: Van Hiele Geometry Test

First of all, the researcher informed to the principal of selected school. Then researcher visited all selected two schools one by one. The researcher selected all the students of grade nine of all selected school. The research informed students way of answering method. The time allocated for test was 45 minutes. After the duration of examination, the answer sheets were collected and checked.

Phase 3: Interview

One week after the test, 8 students from each selected school including 2 from level 0, 2 from level 1, 2 from level 2, 2 from level 3 and 2 students who wrote the correct answer of upper level without answering the previous level were informed through their mathematics teacher one by one school. The interview was taken based on Van Hiele geometric test paper and allowed them to express their opinions freely.

Data Analysis Procedure

Data collected was analyzed both qualitatively and quantitatively. Promotion of students' achievement in geometry using Van Hiele's model in secondary level was analyzed and interpreted on the basis of conceptual framework that researcher already developed in the literature review. For this purpose, the geometry syllabus content of grade 9 was analyzed. It was analyzed to establish how geometry concepts are developed and to promote of students' achievement required by the grade 9 mathematics curriculum in Nepal. Researcher used the general objectives in relation to the features of the Van Hiele levels.

From the test, the numerical data was obtained. The analysis of the test result was done based on the success criteria suggested by Usiskin. Under this criterion, if a students answered correctly at least 3 out of 5 items in the given subset, he or she was considered to have mastered the level.

The result of interview was not used to assign the levels to the selected participants. Instead it was used to establish whether or not the participants could portray the geometric reasoning they had shown in the Van Hiele geometry test. The result of interview was analyzed qualitatively.

CHAPTER- IV

ANALYSIS AND INTERPRETATION OF DATA

This chapter deals with the analysis and interpretation of the collected data. Data presentation refers to the manner in which the collected information is displayed to the reader of the report.

In this study, I have used tables, figures and graph. The data presented is that of documents perused, the test, as well as clinical interview result. Data analysis involves working with data organizing it breaking it down, synthesizing it searching for patterns, discovering what is important and what is to be learned and deciding what you will tell others. The analysis started off with an analysis of geometry syllabus content for grade 9 and yearly examination 2076 including geometry questions only.

The second instrument used was the Van Hiele Geometry Test. The result of this test were analyzed in order to assign the research participants to the Van Hiele levels of geometric thinking. This was followed by an analysis of the individual item of the Van Hiele Geometry Test to find out how the research participants tried to answer each them. The latter was followed by analysis of the result of clinical interview. The discussion some of the findings follow towards the end of this chapter.

Documents Analysis

Document refers to records of past events in the form of letters, diaries anecdotal notes, and document usually presented in collection. The analysis started with an analysis of geometry syllabus content for grade 9 and end of year geometry examination questions.

Syllabus (Issued by CDC) of grade 9

1. **Verification of properties of triangle**
   - The sum of three angles of triangle is two right angles (Theoretical proof)
   - The exterior angle formed by producing a side of a triangle is equal to the sum of two opposite interior angles (Theoretical proof)
   - The base angles of an isosceles triangle are equal (Theoretical proof)
   - The sides opposite to two equal angles of a triangle are also equal (Theoretical proof)
   - The angular bisector of vertical angle of an isosceles triangle is perpendicular to the base and bisects the base and its converse theorem (Theoretical proof)
   - The sum of any two sides of a triangle is greater than the third side (Experimental verification)
   - In any triangle, the side opposite to the greater angle is longer than the side opposite to the smaller angle and its convers theorem (Experimental verification)
   - Of all straight line segments drawn to a given line from an external point, the perpendicular is the shortest one (Experimental verification)
   - Pythagoras Theorem (Experimental verification)
   - Discussion on the interrelationship of the properties of sides and angles of triangle
   - A straight line segment joining the mid-points of any two sides of a triangle is parallel to the third side and its conversion theorem (Theoretical proof)
2. **Verification of properties of parallelograms**
   - The straight line segments that join the ends of two equal and parallel line segments towards the same sides are also equal and parallel (Theoretical proof)
   - The opposite sides of a parallelogram are equal (Theoretical proof)
   - The opposite angles of a parallelogram are equal (Theoretical proof)
   - The diagonals of a parallelogram bisect each other (Theoretical proof)
   - The theoretical proofs of the converse of theorems mentioned above
   - Discussion of the interrelationship of quadrilaterals (parallelogram, rectangle, square, rhombus)
3. **Construction**
   - Construction of quadrilaterals (square, rectangle, rhombus, parallelogram and trapezium)

4. **Similarity**
   - Review of similarity
   - Simple problems on similar polygons
5. **Circle**
   - The perpendicular drawn from the centre of a circle to a chord, bisects the chord (Theoretical proof)
   - The line joining the mid-point of a chord and the centre of a circle is perpendicular to the chord (Theoretical proof)
   - Equal chords of a circle are equidistant from the centre and its converse theorem (Theoretical proof)

The content of syllabus above mentioned basic geometric concepts as simple shapes, primarily by means of visual consideration are involved in level 0. For example, student recognized triangle, parallelogram, quadrilateral, arc, centre, chord, circumference etc.

The properties of triangle, quadrilaterals, parallelograms, circle etc are content of level 1. The relationship among these properties are the content of level 2. Like as square has all properties of rectangle.

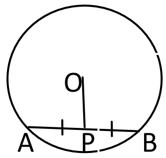
Theorem proving is the content of level 3. As the proof of The interior angle of triangle is two right angle; The exterior angle of a triangle is equal to the sum of the two opposite interior angles; The base angle of isosceles triangle are equal; The opposite angles and sides of a parallelogram are equal etc.

That is why the geometry syllabus content of grade 9 consists Van Hiele level 0,1,2 and 3 means the highest possible Van Hiele can be taught is level 3.

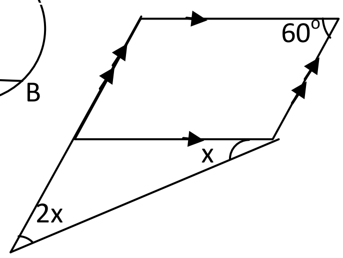
Past Geometry Examinations Questions

Past geometry examination questions were extracted from end of year class 9 yearly examination 2076 papers. These questions are as follow:

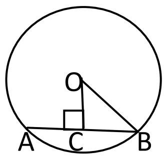
Q1. In the adjoining figure, O is the centre of circle and AB is the chord. If AP = PB, write the relation between OP and AB.



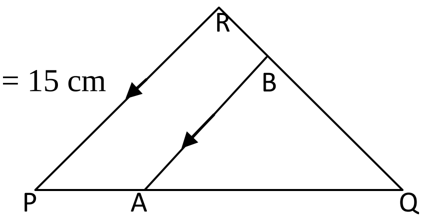
Q2. In the adjoining figure, calculate the size of unknown angle.



Q3. In the adjoining figure, O is the centre of circle with radius 5cm. If OC = 3cm, find the length of the chord



Q4. In the given figure, $\Delta PQR \sim \Delta AQB$. If AP = 6 cm, PQ = 15 cm and QB = 3cm, find the length of BR.

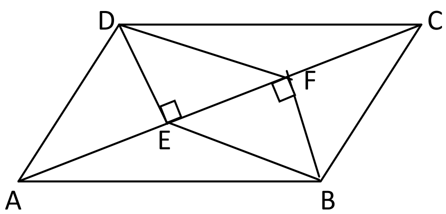


Q5. If any two sides of a triangle are equal prove that the angles opposite to them are equal.

Q6. Prove that the chords which are equidistant from the centre of a circle are equal.

Q7. Construct a parallelogram ABCD in which diagonals AC = 5cm, BD = 5.8cm and they bisect each other making an angle of 30^{o}.

Q8. ABCD is a parallelogram. DE ⊥ AC and BF ⊥ AC. Prove that BEDF is a parallelogram.



Analysis of the Van Hiele Geometry Test Result

To collect the data researcher was visited all selected two schools. The Van Hiele Geometry Test was taken in two purposively selected two government schools. Researcher used the notation A and B for these selected two school, where A for Shree Kailpal Secondary School Paniut, Bhageshwor 1 and Shree Ashigram Secondary School Rupal, Bhageshwor 2 Dadeldhura. The total number of students taken under research study was 83 of which 43 students from school A and 40 students from school B.

The Van Hiele Geometry Test consisted of four sub-test each with five multiple choice items based on each of the Van Hiele levels except the level 3, level 3 consisted two multiple choice items and 2 theorem proving problem. That is there were 20 items in all, with numbers 1- 5 testing the attainment of Van Hiele level 0(visualization), 6 - 10 level 1(analysis), 11 - 15 level 2 (informal deduction) and 16 - 20 level 3 formal deduction.

Table 4.1 ***Number and Percentage of Students at each Level Van Hiele Level***

Levels	School A		School B	
	N	%	N	%
0	21	48.83	20	50
1	11	25.58	12	30
2	5	11.63	5	12.5
3	1	2.33	2	5
Total Fitting	38	88.37	39	97.5
No Fit	5	11.63	1	2.5
Total	43	100	40	100

Table 4.1 shows that the number and percentage of students at each of Van Hiele levels from selected two school A and B. Table shows that the maximum number of students, more than 48% were assigned at Van Hiele level 0. Maximum number of students are assigned at level 0, because the students at each level reasons about basic geometric concept, such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicitly regard to properties of its components. For example, students recognize triangles, square, parallelograms, rectangle and quadrilateral but they do not explicitly identify the properties of these figures. Minimum of students from both schools were functioning at level 3 because in level 3 formal deductions are required.

The results were presented in bar graph as below:

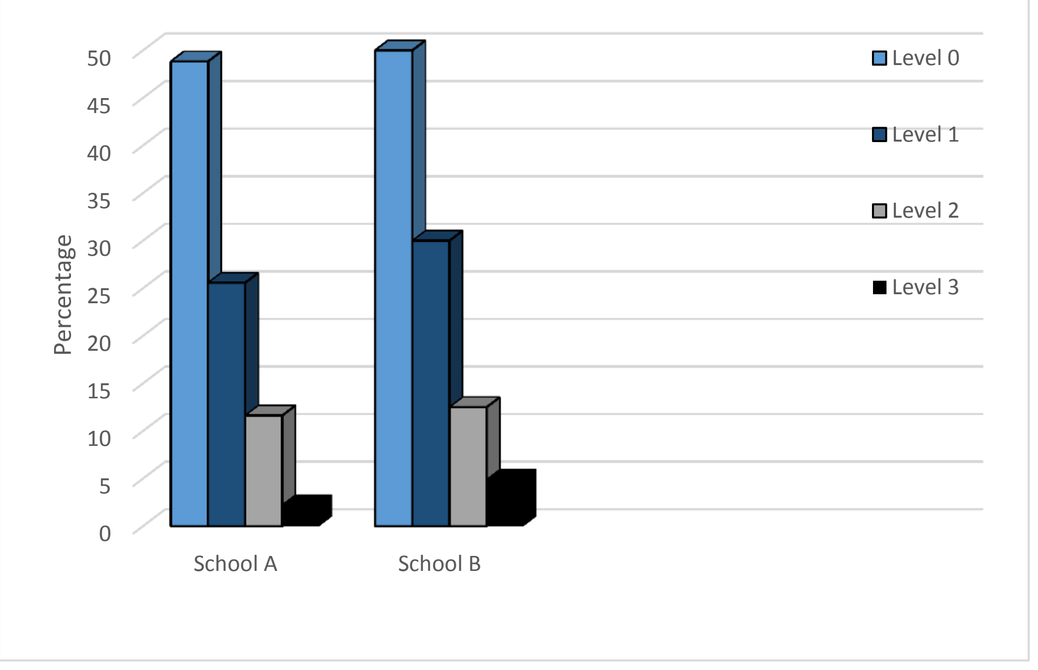


Figure 4.1 **Bar graph of the performance of the participants at each Van Hiele Level in Percentage.**

The bar graph in figure 4.1 presents the performance of the research participants in each subset of the Van Hiele Geometry Test. The presentation indicates the Van Hiele levels and the percentage of the score obtained by the students per level.

The bar graph shows that school A and B performed better in subset of van Hiele level 0. There is a small difference between the performance of participants from both schools.

The graph shows that participants from school B performed better than school A and participants of school A had weak performance in level 1 comparatively. In level 2 had slightly difference between school A & school B participants. At level 3 school B performed better than school A.

In level 3 school A had week performance comparatively to school B.

Table 4.2 ***Number and Percentage of Students at each Van Hiele Level***

Level	Number of Students	Percentage
0	41	49.39
1	23	27.71
2	10	12.04
3	3	3.61

The results were presented in bar graph as follow:

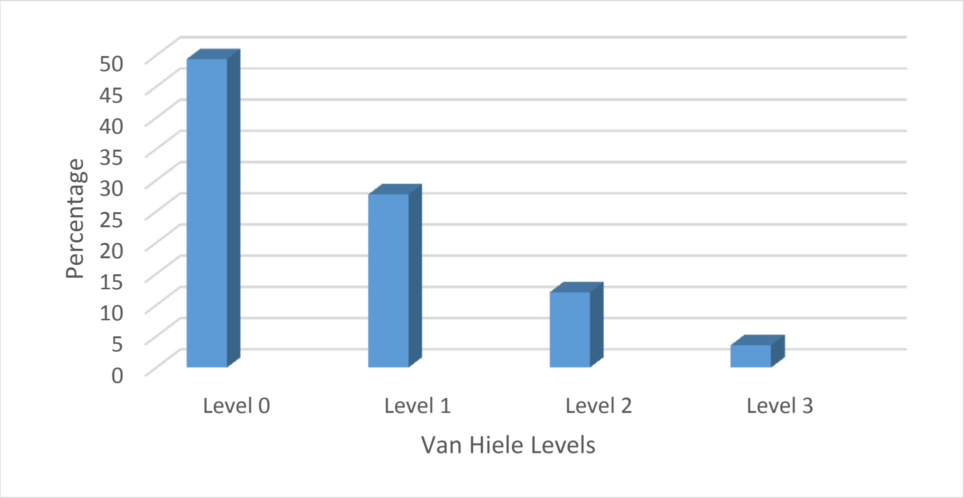


***Figure 4.2*: Bar graph of Percentage of Participants at each Van Hiele Level (In Total)**

From the above graph and table 41(49.39%) participants performed in Van Hiele level 0 and similarly 23(27.71%), 10(12.04%) & 3(3.61%) of participants performed in Van Hiele level 1, 2 and 3 respectively. In total it can be seen that the participants from school A and B performed better in the subsets of Van Hiele Level 0 and 1 than in Van Hiele Level 2 and 3. This is because the items for Van Hiele level 0 and 1 are straight forward compared to that of level 2 and 3 where informal and formal deductions are required.

Teaching and learning in geometry is mainly focused on Van Hiele level 0 and 1, with a small amount of geometry work being done at level 3. Result from this study shows that most participants only operate on the pre-recognition, 0 and 1 level.

Researcher now provide a detailed analysis of each items of the Van Hiele Geometry Test. The figure in bold represent the total number of students who answered that item correctly. The letter A & B represent school A and school B respectively. (Appendix D)

Table 4.4 ***Number and percentage of School A & B at each item of Van Hiele Level***

Level	Item	School A		School B	
		N	%	N	%
0	1	42	97.67%	39	97.5%
	2	24	55.81%	38	95%
	3	21	48.83%	20	50%
	4	6	13.95%	12	30%
	5	8	18.6%	15	37.5%
1	6	17	39.53%	17	42.5%
	7	5	11.62%	3	7.5%
	8	12	27.9%	34	85%
	9	8	18.6%	12	30%
	10	14	32.55%	16	40%
2	11	16	37.2%	12	30%
	12	14	32.55%	7	17.5%
	13	13	30.23%	12	30%
	14	8	18.6%	24	60%
	15	13	30.23%	7	17.5%
3	16	11	25.58%	8	20%
	17	4	9.3%	2	5%
	18	1	2.32%	2	5%
	19	0	0%	0	0%
	20	0	0%	0	0%

Table 4.4 indicated that at level 0 the school A had 42(97.67%) students choose the correct answer of item 1, 24(55.81%) students choose the correct answer of item 2, 21(48.83%) students choose the correct answer of item 3, 6(13.95%) students choose the correct answer of item 4, 8(18.6%) students choose the correct answer of item 5. Similarly, in school B there were 39(97.5%) students choose the correct answer of item 1, 38(95%) students choose the correct answer of item 2, 20 (50%) students choose the correct answer of item 3, 12(30%) students choose the correct answer of item 4, 15(37.5%) students choose the correct answer of item 5. As a whole School B performed better than school A in level 0.

In level 1 the school A had 17(39.53%) students choose the correct answer of item 6, 5(11.62%) students choose the correct answer of item 7, 12(27.9%) students choose the correct answer of item 8, 8(18.6%) students choose the correct answer of item 9, 14(32.55%) students choose the correct answer of item 10. Similarly, in school B there were 17(42.5%) students choose the correct answer of item 6, 3(7.5%) students choose the correct answer of item 7, 34(85%) students choose the correct answer of item 8, 12(30%) students choose the correct answer of item 9, 16(40%) students choose the correct answer of item 10. As a whole School B performed better than school A in level 1.

In level 2 the school A had 16(37.2%) students choose the correct answer of item 11, 14(32.55%) students choose the correct answer of item 12, 13(30.23%) students choose the correct answer of item 13, 8(18.6%) students choose the correct answer of item 14, 13(30.23%) students choose the correct answer of item 15 Similarly in school B there were 12(30%) students choose the correct answer of item 11, 7(17.5%) students choose the correct answer of item 12, 12(30%) students choose the correct answer of item 13, 24(60%) students choose the correct answer of item 14, 7(17.5%) students choose the correct answer of item 15. As a whole School B performed better than school A in level 2.

In level 3 the school A had 11(25.58%) students choose the correct answer of item 16, 4(9.3%) students choose the correct answer of item 17, 1(2.32%) student wrote the correct answer of item 18, none of them wrote the answer of item 19& 20. Similarly, in school B had 8(20%) students choose the correct answer of item 16, 2(5%) students choose the correct answer of item 17, 2(5%) student wrote the correct answer of item 18, none of them wrote the answer of item 19& 20. As a whole School B performed better than school A in level 3.

Analysis of Clinical Interview

For this research study, researcher have used the clinical interview to help researcher establish whether the 16 selected participants (8 from each school including each level) would confirm similar patterns of geometric reasoning as they did in the Van Hiele Geometry Test. The 16 selected participants were purposefully drawn from the large population of research participants who were already assigned to Van Hiele levels using their results for the Van Hiele Geometry Test performance.

Exploratory Analysis

To analyze the result of clinical interview, I adopted an exploratory analytic method. The exploratory analytic method enabled me to establish patterns in the way participants geometrically reasoned about the geometric concepts they have dealt with. In order to explore the patterns of geometric reasoning that could possibly be displayed by the participants, students carried out certain activities. These activities included identifying and naming of geometric shapes, defining shapes and sorting geometric shapes by class inclusion. This meant that the participants were first given geometric shapes to identify and name. Therefore, the results of the activities were used to establish the type of reasoning each of the participants had shown. The result of each activity was recorded in a table.

Table 4.5 ***Number of Students Who Named Geometric Shapes Correctly and Who Stated the Correct Reason for Naming Each Shape***

Shape No.	Name of Shapes	No. of Students correctly naming shapes		No. of Students Stating correct reason	
		School A N = 8	School B N = 8	School A N = 8	School B N = 8
1	Square	8	7	7	6
2	Scalene triangle	4	3	4	3
3	Rhombus	7	6	6	6
4	Rectangle	6	8	6	7
5	Circle	7	8	7	8
6	Right angled triangle	3	3	2	3
7	Parallelogram	6	6	5	5
8	Isosceles triangle	7	6	7	6
9	Equilateral triangle	5	6	5	5
10	Trepezium	7	6	6	6
11	Rhombus	6	6	6	6
12	Right angled triangle	2	3	2	3
13	Rectangle	6	7	6	7
14	Trepezium	7	7	6	6
15	Square	8	7	7	7

Table 4.5 shows some important misconceptions about geometric concepts among the research participants. Most of participants identified shapes by the property of sides. The

table shows that students can easily identify a shape when it is an orientation that they are familiar with. For example, shape no. 1& 15 are square and shape no. 5 is circle. It was easy for the students to recognize shape no. 1 as a square. This was different with shape no. 15.

The most commonly given reason for the identification of triangle is/are “It is having three sides, three sides are equal, two sides are equal, no sides are equal and it is having one right angle, three angles are acute & one angle is obtuse.” The first reason refers to a triangle according to side and angle in general. The second reason refers to equilateral triangle, isosceles triangle, scalene triangle and third reason refers to right angled triangle, acute angled triangle & obtuse angled triangle. The following are some of the common reasons to used when quadrilaterals were identified “It’s having four sides, the opposite sides are equal & parallel, a pair of sides are parallel, opposite sides are parallel & having one right angle, all sides are equal & parallel, all sides are equal & having four right angle”. These reason refers to quadrilateral, parallelogram, trapezium, rectangle, rhombus, square. The circle identify that have equal radius from the centre O.

Defining Shapes Task

The activities expected the participants to define the given shapes by means of their properties.

Table 4.6 ***Defining Shape Task***

S.N.	Shapes	No. of Stating Correct Definition	
		School A (n = 8)	School B (n = 8)
1	Equilateral triangle	8	8
2	Isosceles triangle	7	7
3	Scalene triangle	7	6
4	Right angled triangle	6	5
5	Rectangle	6	6
6	Parallelogram	6	6
7	Rhombus	6	5
8	Square	8	8
9	Trapezium	5	4
10	Circle	8	8

Table 4.6 shows how the students performed in the task of defining shapes. It reflects that all the 16 students managed to define an equilateral triangle and circle. This shows that most of the students are more familiar with equilateral triangle and circle than the other shapes. The table further shows that students of school A had a better understanding of defining the given shapes than students of school B. There is a weak understanding among students with regard to the concept of trapezium. All the definitions above reveal that even though that the research participants are grade 9 students, they still have a problem in using the appropriate description to describe most of the geometric shapes. The definitions further show that most of the research participants had lack of fundamental geometrical conceptualization.

Sorting by Class Inclusion Task

This task required the participants to identify common properties among them given shapes and sort them by class inclusion.

Table 4.7 ***An Example of Possible Class Inclusion: Triangle***

SN	Triangle	No. of Stating the Correct Possible Class Inclusion	
		School A (n = 8)	School B (n = 8)
1	Equilateral triangle	**7**	**7**
2	Isosceles triangle	6	5
3	Scalene triangle	5	5
4	Right angled triangle	2	3

Table 4.8 ***An Example of Possible Class Inclusion: Quadrilateral and Circle***

SN	Quadrilateral and circle	No. of Stating the Correct Possible Class Inclusion	
		School A (n = 8)	School B (n = 8)
1	Trapezium	5	4
2	Parallelogram	5	5
3	Rectangle	4	3
4	Square	4	4
5	Rhombus	6	5
6	Circle	6	6

From above table 4.8, it is clear that class inclusion is one of the major problems as far as student's geometrical conceptualization is concerned.

The outcomes of the manipulatives showed that none of the selected students correctly identified and named all the 15 shapes. Most of them showed the line of reasoning at either van Hiele 0 or 1, because when they were asked to describe the given shapes, they only described them by the property of sides. Most of the 16 participants did not know the properties of the given shapes they dealt with. They have lack of the knowledge required by Van Hiele level 2. The participants were unable to from class inclusion which is one of the features or properties of Van Hiele level 3. The results of the manipulatives are consistent with that generated with the Van Hiele level geometry test. The majority of the participants were mainly operating at Van Hiele levels 0 & 1, and a smaller but significant number at Van Hiele levels 2 & 3.

According to Van Hiele, the theory is hierarchical in that a student can't operate with understanding on one level without having been thought the previous level. For example, a student who has attained level n may not understand thinking of level n+1.

But in this research study researcher found the condition that some students who didn't have answer correctly questions of lower level, had answered the questions of high level. Researcher tried to found the answer of that situation. According to participant responses, it is found that by rote learning, guessing, learning rules or definitions by they didn't understanding.

CHAPTER –V

FINDINGS, CONCLUSION AND IMPLICATION

This chapter provides the conclusion of the whole research study. It includes a summary, findings, conclusion and recommendation for further study.

Findings

The purpose of the study was to dig out the level of mathematics students of secondary level compared with Van Hiele model and to explore the attitude of the students about Van Hiele model of teaching geometry at grade nine.

The pilot test paper was administered to the Shree Kailpal Secondary School, Paniut. For this study, population was the students who studied at grade 9 in Bhageshwor Rural Municipality. Researcher selected two school of Bhageshwor by using purposive sampling. Among them both school are government school. The tools used in this data collection procedure were document analysis, Van Hiele Geometry Test and clinical interview.

The researcher visited both selected school. The researcher informed the purpose to both the school headmaster and mathematics teacher before administrating test. Also researcher explained the answering method for the students. The time, allocated for completing the test 45 minutes. After the time duration of test, the answer sheet were collected and checked by researcher. Also researcher conduct clinical interview of class 9 students from both school consisting each level of Van Hiele Geometric thinking.

Finding of this research study are discussed in detail in chapter IV. These findings were generated by using three instruments: documents analysis, test and clinical interview.

- By comparing the contents of grade 9 geometry syllabus with the features of the Van Hiele levels of geometric reasoning, it emerged that there is a possible relationship between too.
- The result of the test indicated that of 83 students who participated 36(43.37%) at level 0, 27(32.53%) at level 1, 11(13.25%) at level 2 & 3(3.61%) at level 3.
- The result shows that the students who participated in the study are functioning at a level no fitting with their curriculum.
- The teacher did not used student centered method for teaching geometry in the classroom. So the teacher was active and students were passive in teaching learning process.
- Some students wrote correct answer of upper level without answering the previous level by guessing, rote learning, learning rule of definitions by rote or by applying routine algorithms that they didn't understand.
- The main challenge found by this study was that the participants lacked conceptual understanding of geometric concepts. Researcher found that the participants knew the names of most or all geometric figures or shapes, could not state all or some of the properties. The participants could define the given shapes by properties of sides only.
- Most of the students are at Van Hiele level 0 and 1 and few numbers of students were operating at Van Hiele level 2 and 3.

Conclusion

The study shows that the students are grade 9 were not adequately prepared to understanding the concept of geometry. The research indicates that the Van Hiele model can be a way of characterize the teaching phases in geometrical proof. Due to lack of knowledge about Van Hiele theory, many issues remain unclear, including how the phases of teaching relate to subject matter.

Most of the students of this study were the Van Hiele level 0 and 1, which indicated that the grade 9 in Bhageshwor Rural Municipality students' geometric knowledge is poor. They have poor conceptual understanding in geometry possibly due to their emphasis in mechanical and procedural learning. The poor performance of lower level of Van Hiele was strongly associated with being on mathematics, which reveals that the students' levels of geometric thinking plays vital role in the learning of mathematics.

This study revealed that Van Hiele level of thinking is model, which helps to promote the students achieve the geometric knowledge consequently and saves in each levels knowledge also knowledge of previous level. The main challenge found by this research study was that the participants lacked conceptual understanding of geometric concepts.

Implication

The results of this research study could not be generalized due to the limitations in the study. On the basis of this study the following implication and suggestions are made:

- The teaching and learning of geometry should involve more hands on activities that will actively engage the students. This will enhance students conceptual understanding of geometric concepts.

- When teaching about geometric concepts, teachers should ensure that students understand the know the properties of all geometric shapes. Students can only recognize, describe and distinguish geometric shapes from each other by knowing their properties.
- When teaching about geometric shapes concepts, teachers should ensure that the proper geometric terminologies are used by both the teachers and students. This involves correct spelling of the concepts, proper pronunciations and using the correct names of the geometric shapes.
- Teacher should focus to student to study the structure of geometry, student should be self-motive to learn geometry, generate rules and regulations side of school, student and teacher to promote the achievement in geometry.
- This study was conducted only in Bhageshwor Rural Municipality of Dadeldhura district. To get more valid and reliable result it would extent to province wise.
- A similar study will be appropriate for all class of lower and secondary of higher level.

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APPENDIX A

Table 3.1 ***Sample of selected school and students***

S.N.	Name of School	Boys	Girls	Total
1	Shree Kailpal Secondary School Paniut, Bhageshwor 1	15	28	43
2	Shree Ashigram Secondary School Rupal, Bhageshwor 2	13	27	40
Total		28	55	83

APPENDIX B

Ministry of Education

Curriculum Development Centre, Sanothimi, Bhaktapur

Syllabus of Compulsory Mathematics Grade - 9

Area	Subject Matters	No. of Periods
1. Set	• Review of Set • Union, intersection, difference of two sets, complement of set • Problem solving of simple verbal problems in two sets by using Venn-diagram	8
2. Arithmetic	• Problems related to profit and loss • Problems related to commission, discount, tax, dividend • Household arithmetic (Electricity bill, water bill, telephone bill, taximeter)	20
3. Mensuration	• Simple problems related to area (carpeting the floor, paving stones, construction paths, coloring and plastering, ete.) and expense on it. • Surface area, area of cross-section and volume of prism, cube, cuboid • Project work an problem collections related to mensuration in real life situations	24
4. Algebra	• Factorization of the expression of the form $a^4 + a^2b^2 + b^4$ • Problems related to indices (simplification of negative and fractional indices) • Exponential equations (excluding the quadratic form) • Simple problems related to ratio and proportion • Solution of Simultaneous linear equations (substitution method, elimination method and graphical method) • Solution of quadratic equations (factorization method, completing square method and by using formula)	30
5. Geometry	**Verification of properties of triangle (use of Experimental and Theoretical verifications)** • The sum of three angles of triangle is two right angles (Theoretical proof) • The exterior angle formed by producing a side of a triangle is equal to the sum of two opposite interior angles (Theoretical proof) • The base angles of an isosceles triangle are equal (Theoretical proof) • The sides opposite to two equal angles of a triangle are also equal (Theoretical proof) • The angular bisector of vertical angle of an isosceles triangle is perpendicular to the base and bisects the base and its converse theorem (Theoretical proof)	55

	• The sum of any two sides of a triangle is greater than the third side (Experimental verification) • In any triangle, the side opposite to the greater angle is longer than the side opposite to the smaller angle and its convers theorem (Experimental verification) • Of all straight line segments drawn to a given line from an external point, the perpendicular is the shortest one (Experimental verification) • Pythagoras Theorem (Experimental verification) • Discussion on the interrelationship of the properties of sides and angles of triangle • A straight line segment joining the mid-points of any two sides of a triangle is parallel to the third side and its conversion theorem (Theoretical proof) **Verification of properties of parallelograms** • The straight line segments that join the ends of two equal and parallel line segments towards the same sides are also equal and parallel (Theoretical proof) • The opposite sides of a parallelogram are equal (Theoretical proof) • The opposite angles of a parallelogram are equal (Theoretical proof) • The diagonals of a parallelogram bisect each other (Theoretical proof) • The theoretical proofs of the converse of theorems mentioned above • Discussion of the interrelationship of quadrilaterals (parallelogram, rectangle, square, rhombus) **Construction** • Construction of quadrilaterals (square, rectangle, rhombus, parallelogram and trapezium) **Similarity** • Review of similarity • Simple problems on similar polygons **Circle** • The perpendicular drawn from the centre of a circle to a chord, bisects the chord (Theoretical proof) • The line joining the mid-point of a chord and the centre of a circle is perpendicular to the chord (Theoretical proof) • Equal chords of a circle are equidistant from the centre and its converse theorem (Theoretical proof)	
6. Trigonometry	• Introduction of trigonometric ratios on the basis of right-angled triangle • Measurement and problem solving related to the ratios sin, cosine and tangent • Problems related to sine, cosine and tangent of the angle 0^o, 30^o, 45^o, 60^o and 90^o in right-angled triangles.	12

7. Statistics	• Introduction and construction of line graph, pie-chart, histogram and ogive by using the collected data • The other information of the data from histogram and ogive • Calculation of mean, median, mode and quartiles of ungrouped data	13
8. Probability	• Introduction to probability • Definition of Probability and basic concepts • Use of simple probability and introduction of probability scale (0-1) • Introduction and use of formula to estimate probability a) Empirical probability $P = \frac{\text{Number of observed outcomes}}{\text{Total number of experiments}}$ b) Probability of an event $P = \frac{\text{Favourable nuumber of cases}}{\text{Exhaustive number of cases}}$	8

APPENDIX C

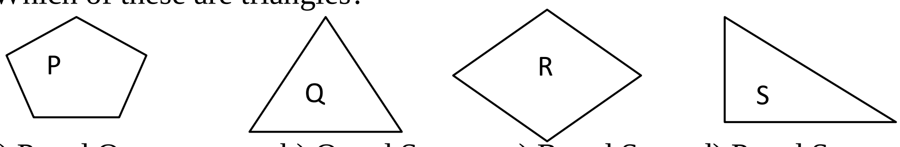
The Van Hiele Geometry Test

Time: 45 min. Class: 9 Date:

Name: .... Roll No.: ....

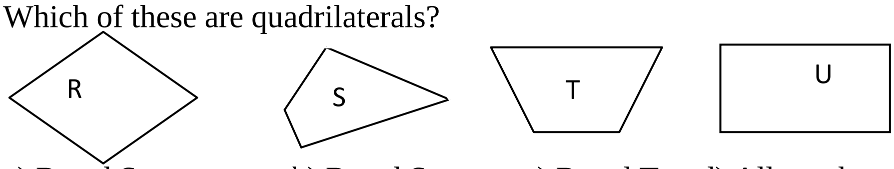
School's Name: ....

1. Which of these are triangles?



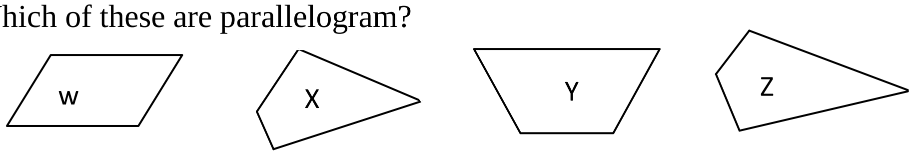
a) P and Q b) Q and S c) R and S d) P and S

2. Which of these are quadrilaterals?



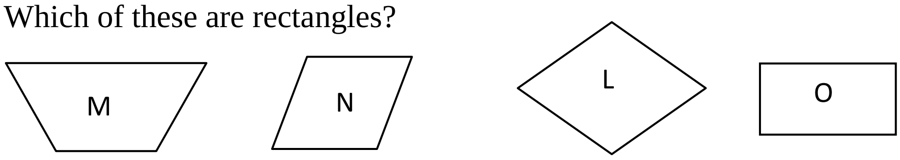
a) R and S b) R and S c) R and T d) All are the quadrilaterals

3. Which of these are parallelogram?



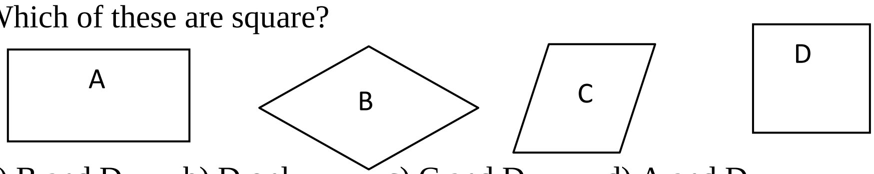
a) W only b) X and Z c) W and Y d) None of them

4. Which of these are rectangles?



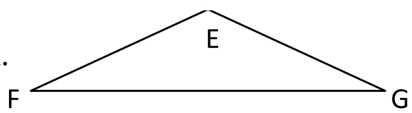
a) O only b) L and O c) M andN d) N only

5. Which of these are square?

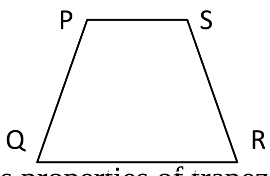


a) B and D b) D only c) C and D d) A and D

6. EFG is an isosceles triangle. Which relation is true?
   a) EF and EG are equal in length.
   b) FG, EF and EG are equal in length.
   c) EF and EG are not equal.
   d) ∠G , ∠F and ∠E are equal.



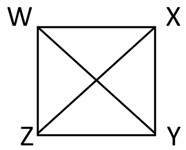
7. PQRS is a trapezium. Which of the following is properties of trapezium?
   a) Length of PS and QR are equal.
   b) PQ and SR are parallel.
   c) PQ and RS are perpendicular.
   d) PS and QR are parallel.



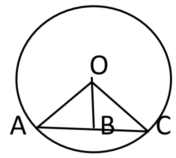
8. ABCD is a rhombus. Which of the following is properties of trapezium?

a) Only length of AB and CD are equal.
b) Length of AB, BC, CD and AD are equal.
c) AB and BC are perpendicular.
d) ∠B and ∠C are equal.

9. WXYZ is a square. Which of the following property of square?
a) Length of WY and ZX are equal.
b) WY and ZX are perpendicular.
c) Length of WX, XY, YZ and WZ are equal.
d) All of the above.



10. O is the centre of given circle. If OB ⊥ AC then which relation is true?
a) Length of AB and AC are equql.
b) Length of AB and BC are equql.
c) Length of OA and OB are equql.
d) Length of OC and OB are equql.



11. Which sentence is false.
a) All properties of rectangle are properties of quadrilateral.
b) All properties of square are properties of rectangle.
c) All properties of rhombus are properties of parallelogram.
d) All of the above.

12. A property of rectangle which does not lies in parallelogram?
a) Opposite sides are equal.
b) Opposite angles are equal.
c) Diagonals are equal.
d) Opposite sides are parallel.

13. A property of square which does not lies in rectangle.
a) All angles are equal and right angle.
b) Diagonals are bisects to each other at perpendicularly.
c) Diagonals ere equal.
d) Opposite sides are equal.

14. There are two Statements:
Statement 1 : In ΔABC three sides are equal.
Statement 2 : ΔABC have ∠B and ∠C are equal.
Which of the following is true:
a) If the statement 1 is true then statement 2 is false.
b) If the statement 1 is true then statement 2 is true.
c) If the statement 2 is true then statement 1 is false.
d) Statements 1 & 2 can not be true.

15. There are two Statements:
Statement 1 : ∠B and ∠C are complementary angles of ΔABC.
Statement 2 : ΔABC is a right angled triangle.
Which of the following is true:
a) If the statement 1 is true then statement 2 is false.
b) If the statement 1 is true then statement 2 is true.
c) If the statement 2 is true then statement 1 is false.
d) Statements 1 & 2 can not be true.

16. There are the properties of a figure:

Property D: Its diagonals are equal.
Property R: It is a rectangle.
Property S: It is a square.
Which statements is true?
a) D implies S and S implies R.
b) D implies R and R implies S.
c) S implies R and R implies D.
d) R implies S and S implies D.

17. There are two Statements:
Statement I: If a triangle is an isosceles then it’s base angles are equal.
Statement II: If two angles of a triangle are equal then it is isosceles triangle.
Which of the following is true?
a) To prove I, it is enough.
b) To prove II, it is enough to prove that I.
c) To prove I and II, it is enough to find the bisector of vertical angle.
d) None of the above.

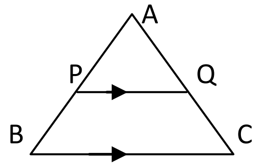
18. Prove that the sum of the interior angle of triangle is 180^o.
Given:
To prove:
Construction:
Proof:

S. N.	Statements		Reasons

19. Prove that if opposite sides of parallelogram are then it is rectangle.
    Given:
    To prove:
    Construction:
    Proof:

S. N.	Statements		Reasons

20. In the given figure AB=AC and PQ∥BC then prove that AP=AQ
    Given :
    To prove:
    Construction:
    Proof:



S. N.	Statements		Reasons

APPENDIX D

Analysis of Items

In this section student's performance per items is looked at.

Table 4.3 ***Van Hiele Geometry Test: Item analysis for each level for school A & school B.***

Level	Item / Choice										
O	Item Choice	1A	1B	2A	2B	3A	3B	4A	4B	5A	5B
	A	0	0	14	2	**21**	**20**	**6**	**12**	10	7
	B	**42**	**39**	0	0	1	0	6	13	**8**	**15**
	C	0	1	5	0	9	0	8	2	9	6
	D	1	0	**24**	**38**	12	20	25	13	16	12
1	Item Choice	6A	6B	7A	7B	8A	8B	9A	9B	10A	10B
	A	**17**	**17**	9	3	9	4	15	**6**	7	5
	B	16	4	16	16	**12**	**34**	8	6	**14**	**16**
	C	6	11	13	18	8	0	12	16	9	9
	D	9	8	**5**	**3**	14	2	**8**	**12**	13	10
2	Item Choice	11A	11B	12A	12B	13A	13B	14A	14B	15A	15B
	A	5	6	14	4	11	10	12	13	11	0
	B	17	18	4	9	**13**	**12**	**8**	**24**	**13**	**7**
	C	5	4	**14**	**7**	8	15	14	0	7	24
	D	**16**	**12**	11	20	11	3	9	3	12	9
3	Item Choice	16A	16B	17A	17B						
	A	8	21	16	1						
	B	**11**	**8**	17	19						
	C	10	9	**4**	**2**						
	D	14	2	6	18						

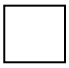
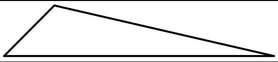
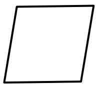
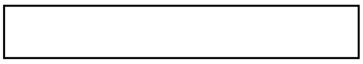
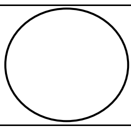
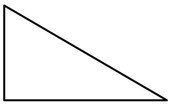
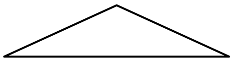
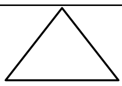
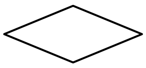
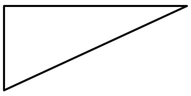
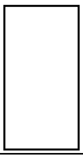
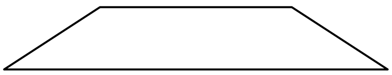
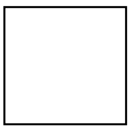
APPENDIX E

The Manipulatives: Sorting Activities

Name: Class: Roll No.:

School:

Q. 1 Answer the questions in the spaces provided for each questions:

Shape No.	Name of shape	Reason
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Q.2 Short the shapes into two groups. List the shape number only:

Group A shapes	Group B shapes

a) Which is the common name for all the shape in group A?

Ans:

b) What is the common name for all the shapes in group B?

Ans: