

Chapter I

INTRODUCTION

Background of the Study

Algebra includes being able to represent a single quantity or a relationship between two quantities. The ability to symbolize and to represent mathematics in several ways may help students learn from early elementary grades through middle school and beyond. For example, Brenner et al. (1997) showed that students who practiced representing problems in multiple ways performed better than a control group on function word problems and representation tasks, such as translating word problems into tables and graphs. Participants in this study were grade students from six classes in three junior high schools. Gender and socioeconomic.

The ability to interpret mathematical meaning through symbols is a skill that mathematics teachers often try to develop in their students. The term symbol refers specifically to mathematical symbols. These include letters, numbers, equal signs, plus and minus signs, parentheses, square root signs, etc. Symbols are a type of representation, which is a term that refers to “a configuration of some kind that, as a whole or part by part, corresponds to, is referentially associated with, stands for, symbolizes, interacts in a special manner with, or otherwise symbolizes something else”. The use of the word representation in this study can be interpreted as what Goldin & Kaput (1996) call external representations, that is, “physically embodied, observable configurations such as words, graphs, pictures, equations” (p. 400).

Symbolic or algebraic representations involve symbols and manipulations of symbols. Other possible representations will include graphical representations and numeric representations in the form of graphs and tables respectively. Although graphs and tables are often referred to as forms of mathematical signs (Sfard, 2000),

they will not be included as symbols here, because the researcher wishes to distinguish students' use of graphs and tables from their use of mathematical symbols in problem solving. Symbols are what allow learners to communicate mathematically, and need to be understood so that one can read, interpret and share information in mathematics classes. However, students often struggle with using and understanding mathematical symbols and notation. Algebraic symbols, in particular, are complicated components of the language of mathematics. Symbol sense, as described by Arcavi (1994), is an understanding and appreciation of symbols necessary at all stages of problem solving.

Symbols are the components of mathematical language that make it possible for a person to communicate, manipulate, and reflect upon abstract mathematical concepts. The usefulness of mathematical symbols is interpreted in many different ways in the research literature. For example, symbols name, label, signify, communicate, simplify, represent, reveal structure, and display relationships. With so many interpretations and purposes, it is no wonder that the symbolic language of mathematics is often a cause of great confusion for students (Rubenstein & Thompson, 2001). The expert mathematician or mathematics teacher is able to work with and to interpret mathematics through its symbolic representations, whereas students often struggle in this endeavor.

The ability to interpret mathematical meaning through symbols is a skill that mathematics teachers often try to develop in their students. Researchers suggest that students often need to be told what to see and how to reason with mathematical symbols. From a constructivist viewpoint, however, it is not possible for students to know about symbols by simply telling them what to know; symbols and syntactic

rules of mathematics do not have meaning for students until they are interpreted by the individual (Goldin & Kaput, 1996)

According to Thompson (2002), “it can be appropriate for teachers to describe explicitly to their students the understandings they hope their students will have. But after so describing those understandings, and after teaching for them, teachers should not presume that what students understand is what was intended” (p. 199). Too often, teachers assume that students interpret mathematical notation in the same way that they do and that students approach mathematical problems with the same goals that the teacher has for the problem. If students are not able to successfully solve the problem, the teacher may conclude that the students need more practice or more time to understand. However, confusion with interpreting and understanding mathematical symbols may only be exacerbated by more practice. Students bring their own interpretations for symbols to the classroom based on their previous interactions, successes, and failures with symbols in past math classes (Stacey & MacGregor, 1999). They also rely on their understanding of the rules and syntax of natural language to try to help them make sense of mathematical language. Simply telling students that the rules are different is not guaranteed to change students’ understand of symbolic notation. A lack of understanding of mathematical language can have a negative effect on students’ problem solving skills and reasoning abilities (Kieran, 2007; Neria & Amit, 2004), causing students to search for ways of dealing with mathematical problems without attending to the meaning communicated by the formal symbols. In this case the topic of my thesis influence of symbol on basic level students in solving algebraic problems. There are many mathematical problem of students, one of them is symbol sense. Therefore, misconception or misunderstanding to symbol, that's why one cannot apply properly and used symbol in an effective

manner while learning mathematics. In mathematical learning different types of symbol gives different meaning of mathematics view. Thus, especially this research tries to find out the problem on symbol and its error generally students committed as an error.

Statement of the Problem

The problem to be address in this study relates to how students' interpretations and uses of the language, symbols, and syntax of mathematics affect their problem solving and reasoning skills in college mathematics courses. Mathematical communication is strongly related to students' problem solving and reasoning abilities (Kieran, 2007; Neria & Amit, 2004). Thus, teachers and researchers need to be aware of how students interpret mathematical notation, and how symbols help or hinder students' understanding of the mathematical language. A large percentage of students come to college unprepared to take required courses like calculus. These students often begin their math requirements with a pre-calculus course for which they might not receive credit. In many cases, these pre-calculus classes are not working to prepare students for higher mathematics courses (Schattschneider, 2002); students are spending time and money retaking classes to a point where they may become blocked from reaching their educational goals (McGowen, 2000). The typical approach for helping students who are failing is to provide them with more examples and more practice problems. However, if instructors are not aware of the networks of understanding (or misunderstanding) that students have constructed, then more practice may reinforce misinterpretations and frustrations (DeMarois 1996; Dubinsky, 1991; Tall & Razali, 1993). Misunderstandings related to the symbols and syntax of mathematics could lead some students to actually develop their own (incorrect) techniques and goals in problem solving due to their personal interpretations of the

symbols involved (Gray & Tall, 1994). For example, the researcher has observed students canceling *log* or *ln* in logarithmic equations to reach their goals of solving for a variable based on their interpretation that the *log* notation can stand alone and be treated as representing a quantity. The problem address in this research was:

- How students are communicate the nature of symbols and to the importance of communicating their work?
- How are students' uses and understandings of symbols related to students' algebraic assessment?

Significance of the Study

Here, the researcher was try to study the symbol understand by students in algebra learning. This study is about the students' understanding in algebra on problem solving. Besides several variables influencing students learning and achievement in mathematics, errors committed in by the students in solving algebraic problems is one. Followings are the significant of this study:

- The teacher educators, mathematics teacher, policy maker, mathematics educationist, curriculum planner and students.
- To know the errors committed by the students and to choose the best way of minimizing it.
- The results of this study intended to determine the influence of symbols on algebraic problems. So this study is help to determine the symbol sense of basic level students.
- The results of the study showed the different types of symbols in algebra, so it help to strengths the symbolic understanding in algebra.
- It helped to improve the teaching learning situation in the content of algebra in basic level.

Research Question

The research questions for this study were as the following:

- How do common algebraic symbols perceive and manipulate by students in solving algebraic problems ?
- How do students communicate with symbol?
- Where do students make error in solving algebraic problems ?

Objective of the Study

The main objective of the study is to find out the influence of symbols on basic level students in solving algebraic problem. The specific objectives of this study are stated as follows:

- To explore the symbol sense of basic level students' in algebra.
- To identify the students' error in an operating algebraic problems.

Delimitations of the Study

Every study has delimitations due to the constants of time, budget and other related factors. In like manner this study was also has some limitations.

- This study was limited at Gorkha district.
- This study was conduct in grade seven only.
- The sample of the study was taken from only one community school.
- Only seven students' was selected for cases.

Operational Definition of the Key Terms**Symbols**

Symbol refers specifically to mathematical symbols. These include letters, numbers, equal signs, plus, minus, multiple, division signs, parentheses, square root signs and inequality.

Algebraic representations

Algebraic representations involve symbols and manipulations of symbols. Other possible representations will include graphical representations and numeric representations in the assessment question.

Real world problem

The problem related to the language and verbal word as perceives by grade VII curriculum of mathematics.

Real world solution

The solution of real world problems carried by students'.

Mathematical problem

The problem related to the application of mathematics and mathematical symbols as perceives by grade VII curriculum of mathematics.

Mathematical solution

The solution of mathematical problem carried by students'.

Chapter II

REVIEW OF RELATED LITERATURES

The intention of this study is to examine the way in which students deal with and interpret symbols and symbolic representations in mathematics problems. Symbols are what allow learners to communicate mathematically, and need to be understood so that one can read, interpret and share information in mathematics classes. However, students often struggle with using and understanding mathematical symbols and notation. The following review addresses the roles and uses of symbols and provides insights into reasons for students' difficulties with symbols when solving problems

Role of symbols in mathematics

As mentioned in the beginning, symbols are identified in the literature as useful and necessary to mathematics for a variety of reasons.

Empirical Literature

According to Pimm (1995), "one central reason for symbolizing is that symbols allow us to manipulate by proxy things that are not easily handled, or which are even impossible to handle, by our physical senses" (p. 109). It is, for example, possible for students to work easily and efficiently with symbols without paying much attention to their referents (Arcavi, 1994; Pimm 1995). Symbols used as substitutes for some abstract idea can be viewed as one of the strengths of symbol use.

Rubenstein and Thompson (2001) have constructed a list of some of the different roles for mathematical symbols, although they find it impossible to provide an exhaustive definition of the roles that symbols play. Their list includes symbols that (a) name concepts, (b) state relationships, (c) indicate operations, (d) abbreviate words, and (e) indicate grouping. They provide examples of symbols in each role in

several content areas. One thing they fail to highlight is the multiple roles that symbols play within a single mathematical statement.

Stacey and MacGregor (1999), Usiskin (1988), and Driscoll (1999) remind us that letter symbols in algebra can be used as generalized numbers, parameters, unknown numbers, and variables. For example, in representing the equation of a line as $y=mx+b$, the students must differentiate the letters y and x as variables and the letters m and b as parameters that define a specific linear function. It is important to be able to understand the different roles played by letters, operators, and other notational devices in order to be able to communicate fluently in mathematics. Teachers and researchers have observed that many difficulties in mathematics can be attributed to students' problems with manipulating and understanding algebraic symbols (Driscoll, 1999; Gray & Tall, 1994; Kinzel, 1997; Pimm, 1995; Stacey & Macgregor, 1999, Usiskin, 1988). Arcavi (1994) and Fey (1990) have described the underlying understanding of algebraic symbols and their uses as symbol sense.

Caldwell (1994) addressed the importance of other mathematical symbols, not only variables, when using graphing calculators. In his study, he made some interesting observations about what students need in order to be able to work with the calculator in the first place. He reminds readers that correctly entering expressions involves using, for example, nested parentheses, exponents, and roots; in other words, students must be able to use algebraic symbols proficiently when working with the technology. This interaction between algebraic manipulation and skillful use of the calculator can take time to develop with students, which, Caldwell suggests, may be a reason that many teachers continue to use more traditional, less time-consuming, teaching strategies in their classrooms.

Fey (1990) describes symbol sense as an “informal skill required to deal effectively with symbolic expressions and algebraic operations” (p. 80), while Arcavi (1994) explains it as “a quick or accurate appreciation, understanding, or instinct regarding symbols” (p. 31) that is involved at all stages of mathematical problem solving. Working fluently with symbols in mathematics requires developing strong symbol sense. Neither Arcavi nor Fey attempt to formally define symbol sense, claiming that to do so is difficult because it interacts with other senses such as number sense or function sense. Instead, they provide examples of what it would mean for a student to have symbol sense.

Arcavi’s (1994) examples of symbol sense include: understanding when to use symbols for problem solving and when to abandon them for better tools; understanding the need to continuously check symbols meaning and compare with one’s own expectations and intuitions; and having an appreciation of the communicability and power of symbols to display and prove relationships (these and several other examples from Arcavi are described and used in the conceptual framework section of this chapter). Arcavi suggests that many students fail to see algebra and its symbols as a tool for understanding, communicating, and making connections, even after several years of study. He sees development of symbol sense.

Kenney, R. W. (2008) study on the Influence of Symbols on Pre-calculus Students’ Problem Solving Goals and Activities. The purpose of this study is to investigate students’ uses and interpretations of mathematical symbols and the influences that symbols have on students’ goals and activities when solving tasks with

and without a graphing calculator. The researcher conducted a multi-case study of pre-calculus college students with a focus on the goals and activities they selected and the anticipations and reflections they made as they worked on math problems in different settings. Data were collected and analyzed under the conceptual lens of an activity effect relationship framework and a symbol sense framework. Six different student cases were investigated, and both within-case and cross-case data analysis was conducted and reported. The researcher found that some symbols and symbolic structures had strong influences on students' choices in problem solving. Graphing calculators were used as a way to abandon symbolic manipulation, although very few connections were made between symbolic and graphic or numeric forms. Students demonstrated a mixture of instances of symbol sense as they worked on symbolic mathematical problems.

Kirshner (1989) believes that students' understanding of order of operations corresponds with a visual hierarchy of operations: diagonal juxtaposition (exponents) has a higher precedence than horizontal or vertical juxtaposition (multiplication and division), which has a higher precedence than wide spacing (addition and subtraction). Thus, a student can know how to evaluate in an expression like $3x^2 + 2$ by following this hierarchy. Additional support for the effect of space on syntactic reasoning can be found in several other studies. Landy and Goldstone (2007) tested the proposition that people use space to help process syntax. They found that in both written and typed tasks, their participants used spaces to separate addition and subtraction, and constructed other spatial properties to match their own perceptions of the expressions.

Rubenstein and Thompson (2001) suggest that challenges come from trying to read, verbalize, and write mathematical symbols. Their paper shares specific

examples of these challenges with high school teachers. For instance, verbalizing mathematics is difficult because symbols are not always read from left to right, which violates the usual sequence of reading; reading symbols is problematic when the same symbol has different meanings depending on the context; and writing with symbols is especially challenging when translating syntax for use with technology. They stress that the interpretation of symbols in mathematical expressions can be a difficult task at all levels of learning.

Sharma (2010) study on "An error analysis on solving verbal problem of algebra by grade VII student". The purpose of this study as to study the patterns of errors committed by the grade VII students while solving verbal problems in algebra; The objective of this study was to identify and analyze the error on the basis of a recognized theoretical base. The sample in this study consist twenty students, ten students from each sampled school, Mahakavi Devdota higher secondary school and Sakura Memorial English Boarding school Sunwal. The schools were selected purposively for the convenience of the researcher. Neman technique was adopted as the theoretical base of the study. A test consisting of ten problems from grade VII mathematics was administrated to the sampled students to collect the required data. The collected answer sheets of the students were checked and errors on those were identified on the basis of Newman technique of analyzing error. The identified errors were classified into five categories as described by Newman. The frequency of each type of error was tabulated and converted into percentage to make comparison easy. The identified errors according to Newman were: Reading errors, Comprehension error, transformation errors, process sill errors and encoding errors.

The study revealed that students had committed number of errors on solving verbal problem in algebra. The concentration of errors was seen on the phase of

transformation, process skill and comprehension of the problems. In comparison Government school students commit more errors than private school students. The data revealed that students had felt difficulty on verbal problems.

Sharma (2009) conducted on study entitled “an error analysis in solving algebraic problems of grade five students”. The purpose of this study was to study the patterns of errors committed by the grade- Five students while solving verbal problem of simplification and equation of algebra. The objectives of this study were to identify and analyze the errors and to compare the error made by students I knowledge, skill and application and problem solving level of algebra. The samples in this study consisted of 30 students from janata primary school, surkhet. Newman technique was adopted as the theoretical base of the study. A test consisting of 8 problems from grade-five students was administered to the sampled students to collect the required data. The collected answer sheets of the students were checked and errors on these were identified on the basis of Newman technique of analyzing error. The identified errors were classified according to Newaman were Reading error, Comprehension error, Transformation error, Process skills error and Encoding error.

The study revealed that students' have committed numbers of errors on solving verbal problems of simplification and Equation of algebra. This study also revealed that students commit more error in knowledge level while solving verbal problems in algebra. This study showed that the frequency of error were dense in comprehension error and transformation error as in Newman theory of analyzing error.

Steinberg, Sleeman, and Ktorza (1990) studied equivalence of equations. Their sample consisted of 96 eighth- and ninth-grade students who had completed a unit on solving one-variable equations. Students were asked to decide if 21 pairs of equations were equivalent. The results revealed that only about one-third of the problems were

solved through use of transforming equations (e.g. students recognized the second equation was correctly derived from the first, compared elements term by term, etc.) while most problems were solved through computing solutions or through methods used incorrectly. Common misconceptions identified in their analysis included students not understanding like terms, only looking at one side of the equation (not both), and using surface features to base decisions (Steinberg et al., 1990).

Adhikari (2007) Studied on "An error analysis in mensuration of grade-IX students in Kathmandu district" aim to Compare the error made by students in knowledge, skill and application and problem solving of menstruation, Compare the error with respect to gender, Analyze the errors made by the students n problem solving.

Fluorescent higher secondary school was selected as a sample school and 80 out of 120 students were selected by lottery method. T-test was applied under 0.05 of significance and questionnaire made from S.L.C. questions banks and used both quantitative and qualitative research design. The main findings of this study were: the number of non-observable error is greater than number of observable error. Children's have committed more errors in area of problem solving, students have committed more errors in the area of knowledge than in the area of skill and application, students have committed more errors in the area of knowledge than in the area of skill and application, no significant difference can be found between the errors committed by students in the area of problem solving and knowledge. Similarly, the researcher has found no significance difference between the errors committed by the students in the area of problem solving, skill and application, girl committed more errors than boys in the area of knowledge, the role of gender is less important to commit errors in the area of skill and application and problem solving, students have commuted carelessness

error, which don't fall under the category of errors described by Newman. The carelessness error committed by students has resembled with dienes symbol manipulation errors, although the ages of all students were above 11 or 12 and they were on the formal operational stage according to the Piaget stages of intellectual development, they were not functioning completely at the formal operation level.

Theoretical Literature

It is important to consider that priority of education. The method of inquiry for this research was case study. "Case studies are appropriate when the objective is an in-depth data collection involve multiple sources of information rich in context"

(Creswell, 1998, p. 61). In this study, the case was defined as an individual basic level of students. The focus, however; was not on the intrinsic quality of the student, but on the ways in which the student and reflected on a problem based on their interpretation of mathematical symbols.

Constructivist Learning Theory

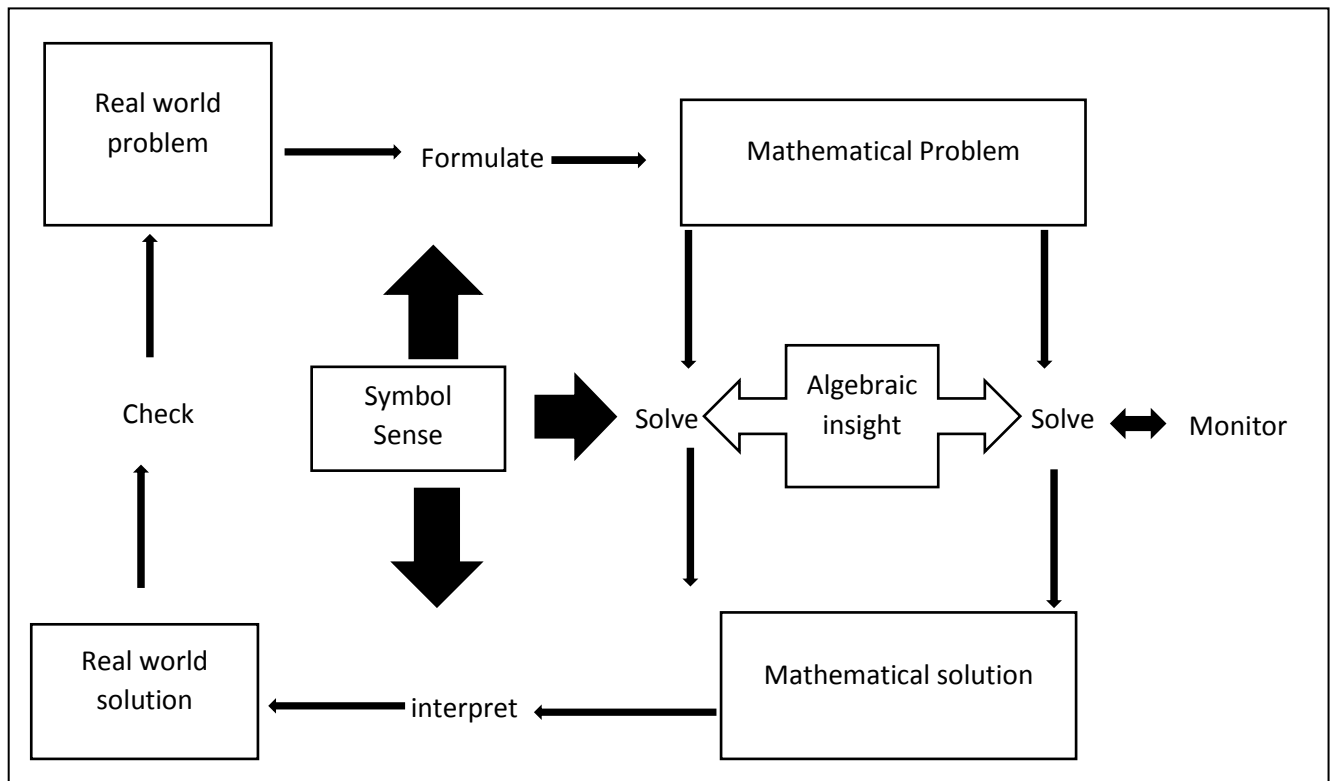
The theoretical perspective used in this study is that of radical constructivism. In this paradigm, reality exists only as far as it fits into the world of the learner's experiences (Von Glasersfeld, 1996). It is a conception-based perspective in which mathematical learning is viewed as "a process by which humans create and adapt conceptions to organize their experiential worlds" (Simon et al., 2000). According to Simon et al., radical constructivism operates under the following assumptions: (a) humans only have access to mathematics that is created through their own activity and is dependent on their way of knowing, (b) individuals can only see, understand, and learn new concepts based on what they already know, and (c) learning is a process of transformation or modification of existing ideas. Research in the constructivist paradigm "attempts to build plausible models of how, by means of reflection and

abstraction, viable concepts could be derived from subjective experience” (Von Glasersfeld, 1996). In this study, the researcher works under the assumption that students’ experiences with symbols and graphing calculators are unique for each learner and influence the development of conceptual understandings of math concepts.

This perspective is suited to a study involving symbol sense because language and signs are only symbols when one assigns them conceptual meaning (Von Glasersfeld, 1996). In order for symbols to be meaningful to students for linking numeric and graphic representations to symbolic representations when calculator use is involved, the representations must already be a part of students’ experiences. The resulting list of the element gives component of what the term of algebraic insight which was detail in the literature reviews.

Conceptual Framework

This study was followed by the following conceptual map firstly the real world problem can be formulated in mathematically problem through algebraic insight by the help of problem solve method students symbol sense were check out and interpreted. The purpose of this study is to explore the symbol sense on basic level student in solving algebraic problem. The assumption that students experience with symbol is unique for each Lerner and influence the development of conceptual understanding of mathematics concept.



Source: Kenney, (2008)

Algebraic insight also involves knowing how to make links between multiple representations. Ampierce and Stacey (2001) divide algebraic insight into two parts: a) algebraic expectation is the aspect needed for working within a symbolic expression, and b) linking representations is the insight needed to make connections between symbolic and graphic forms or symbolic and numeric forms. Algebraic expectation is defined as the thinking that takes place as one considers possible results or outcomes of an algebraic activity. Three key elements of algebraic expectation include: recognition of basic properties; identification of structure, and identification of key features of an expression that determine features expected in the solution.

Chapter III

METHODS AND PROCEDURES

This chapter presents the design of the study, which was carried out to achieve the objectives of the problem. This chapter includes design of the study, study site, selection of participants, data collection tools, data collection procedure and analysis procedure of the study.

Design of the Study

An objective for this study was to examine students' understanding and interpretations of algebraic symbols when solving algebraic problems. A simple record of whether or not a student answers algebraic question correctly does not provide detailed information about what understanding or meaning has been obtained. It is possible that students could use strategies on a calculator to get a correct answer without having any understanding as to why the answer is correct or what mathematical concepts are at work behind the calculations. It is also possible for students to solve a problem using incorrect techniques or working without symbol sense and still come up with an answer that matches the correct solution.

It was necessary for the researcher to play the role of "active learner" in order to look at the relationship between students' uses of symbols and problem solving goals and activities. This involved getting next to the students, either physically or with the video camera, as they worked on tasks in different settings, and engaging them in discussions about their thinking as they did mathematics. According to

Creswell (1998), with a qualitative study, the researcher builds a "complex, holistic picture that takes the reader into the multiple dimensions of the problem or issue and displays it in all of its complexity" (p. 15). There was a need to present a very detailed view of students' ideas and interpretations as they made sense of

symbols and solved mathematical problems. Such details could not be captured by using quantitative surveys or by analyzing test scores. For this reason, the researcher chose to design a qualitative case study to collect and analyze data pertaining to the research questions for the study.

Case Study

The method of inquiry for this research was case study. Case studies are appropriate when the objective is an “in-depth data collection involving multiple sources of information rich in context” (Creswell, 1998, p. 61). In this study, the case was defined as an individual student at basic level of Gorkha district. The focus, however, was not on the intrinsic quality of the student, but on the ways in which the student anticipated, acted, and reflected on a problem based on their interpretation of mathematical symbols in algebraic problems. The cases were bounded within a single class of basic level students.

Study Site

Study site refers to the area of research study, the researcher was selected the study site conveniently in qualitative research. This study aim to explore the symbol sense so researcher select a secondary school of Gorkha district. The reason behind the chosen of this school is due to researcher connivances. There is a large number of students' in grade seven and this school is reach in students' diversity.

Selection of Participants

Firstly the researcher was take algebraic assessment in a class seven and after analysis solution of assessment seven students' were taken among then five are low and two are high performing students.

Data Collection Tools

The data was collected through interview and assessment test. Case studies involve using multiple sources of information in the data collection (Creswell, 1998). The data sources in this study include: interviews with students, and assessment tests.

Algebraic Assessment

Another tools for collection of collected data was students' work on a course test that related to the algebraic test. Tests was be prepare by researcher himself on the basis of grade seven level mathematics curriculum. Researcher was interest in comparing how they had paid attention to symbol use on the assessment and how they used and interpreted symbols on the test. The test covered material on algebraic topics covered in the assessment. Questions was design to accommodate the rule on symbol identification, and students was expected to use mostly algebraic solution methods. This assessment questions is attached in appendix-A.

Reliability and Validity

The researcher used the split-half method in this study to obtain the reliability measure. In this method, the test scores were divided into two halves: scores for odd-numbered items and scores for even-numbered items. Then the correlation between the two halves was determined. The following Spearman-Brown prophecy formula was used to calculate the reliability coefficient of the whole test.

$$r_{total-test} = \frac{2 r_{split-half}}{1 + r_{split-half}} .$$

Validity of the test

The validity of a test instrument is equally important as its reliability. If a test does not serve its intended function well, then it is not valid. According to Best and Kahn (2009) validity is that quality of a data gtering instrument or procedure that

enables into measure what it is supposed to measure. This type of validity is not directly relevant to the current study. Construct validity is about what psychological qualities a test measures. This type of validity is primarily used when the other three types are insufficient. In order to preserve content validity, the content of the test was prepared by consulting the mathematics curriculum: Grade VII (CDC, 2012) as a basis. The content of the test was discussed with a subject expert, two mathematics teachers and thesis supervisor and their suggestions were included prior to the piloting test. Also, similar test construction procedures in the literature were consulted when preparing the test items.

Interview

The researcher conducted personal interviews that contained carefully constructed tasks and questions to identify students' goals, activities, reflections, and symbol sense. Interviews allowed the researcher to explore the phenomenon from a participant's field of perception and to look for the meaning of each participant's experiences. An interview guide for each interview was prepared ahead of time with questions and tasks to present to the participants. However, the interviewer allowed participants to guide the interview to a certain extent, as long as conversation remained within the realms of the study. For this study five students from low performing and two students from high performing are selected for interview. The interview schedule is kept in appendix-B.

Data Collection Procedure

The method of the data collection whole task that takes during the study. In this procedure the data collect, assessment and interview made. The data collection is the method includes how it be was done, which helps the research in analyzing and interpretation of data. The research was be adopted the case study such that the

research have been done with the help of Interview schedule and assessment. Firstly the researcher was take algebraic assessment in a class seven and data was collect after analysis solution of assessment five students from low performing and two students from high performing are selected for case. The interview was be conducted among seven students on the study after assessment test. The researcher was visited the officials of the school under the study of sample school. First of all the researcher collect the data from assessment test of seven class, and interview was be taken.

Procedure of Analysis and Interpretation of the Data

The obtain data were analyzed by transcribing, coding, and theme induction by linking horizontally and vertically. The quality, credibility or validity of data was be ensure by triangulation, conformability, negative case analysis, and thick description. The derivation of codes and the coding process tend to differ in quantitative and qualitative research. In quantitative research, codes are commonly created prior to data collection. Concepts and hypotheses are most often developed in advance, and categories and their codes are derived deductively from theory or borrowed from the extant literature. These predetermined categories are used to structure the data that are collected. Many questionnaires, for example, are in fact preceded (the categories and their dimensions are explicitly listed and the respondent is asked to choose among the options provided). Thematic analysis is one of many methods used to assess whether or not saturation has occurred in the data collecting process. Typically, reoccurrence is a prime means of analyzing data for themes. That is, researchers assess the interview transcripts for repeated statements, phrases, and words.

Coding

The coding process refers to the steps the researcher takes to identify, arrange, and systematize the ideas, concepts, and categories uncovered in the data. Coding consists of identifying potentially interesting events, features, phrases, behaviors, or stages of a process and distinguishing them with labels. These are then further differentiated or integrated so that they may be reworked into a smaller number of categories, relationships, and patterns so as to tell a story or communicate conclusions drawn from the data. A coding frame, a scheme that lays out key concepts, their definitions, and criteria for recognition, is evolved over time during the coding and analysis of the data. It is subject to change and refinement as the researcher proceeds with successive passes through the data. Many researchers keep notes on insights, ideas, patterns, and connections that occur to them as they stressed.

Grounded theory proponents have broken the coding process into stages in an attempt to illuminate the logic that underlies analysis, although they caution that no sharp boundaries exist in actual practice. The labeling of concepts and categories during the early stages of coding is referred to as open coding. During successive stages of coding, the researcher begins to home in on and refine more specific categories and their properties, examining in depth one category at a time. This is spoken of as axial coding. A still further focus on particular links and relationships among a few chosen categories (the integration of categories) is referred to as selective coding. The grounded theory approach tends to emphasize more impersonal, relatively objective processing and reprocessing of data. Although most writers on coding practices acknowledge their debt to the grounded theory approach, many urge relaxing one or another of the recommendations in hopes of stimulating creativity and insight.

Categorization

This process, categories are often created by chunking together groups of previously coded data. This integration or aggregation is based on the similarities of meaning between the individually coded bits as observed by the researcher.

Categories in turn may be abstracted or conceptualized further to discern semantic, logical, or theoretical links and connections between and across the categories. The results of this process may lead to the creation of themes, constructs, or domains from the categories.

Categories can also be seen as an intermediary step in an ongoing process of separating and connecting units of meaning based on the qualitative data being collected. Coding is often the first step in the analytic process as researchers attempt to make meaning of the various bits of information collected in the field or generated during interviews. As a second step in the ongoing process, researchers look for connections between or among these separate codes. This coding of the content can produce categories as researchers discern linking patterns between or among the individual codes. The analytic process continues as researchers' next look for patterns that run through and across the system of categories. The results of this categorization of the categories can lead to the creation of themes, constructs, or domains.

The categorization process encourages researchers to describe overtly what they have observed and to segment the observed phenomena into units. The characteristics or internal properties of the categories are further developed or discovered as researchers continually and transparently note or memo how all coded units of meaning within a particular category are similar and how the coded units within the category contrast with other coded units perceived as being outside the category in question.

Researchers can use a variety of techniques to accomplish this goal, including posing

a priori questions from existing theoretical systems (i.e., a deductive approach) and testing the integrity of the categories by constantly judging the credibility of the categories with further observations based on the data (i.e., an inductive approach). Researchers can also use a combination of both inductive and deductive logic in creating and refining categories.

The process of categorization continues in a research project until saturation (i.e., no further categories are discovered or constructed based on examination of new generated data) or exhaustion (i.e., the existing system of categories accounts for all meaningful or significant aspects of the phenomenon in question). In constructing a system of categories, it is important for researchers to evaluate how each category has internal integrity (i.e., is there a high degree of homogeneity across the individual coded units within the category?) and external integrity (i.e., is there a high degree of heterogeneity or differentiation between or among the array of homogeneous categories?).

Researchers not only must judge the internal and external coherence across the system of categories but also must be cognizant of the coherence between the categories and the phenomenon in question. Researchers should endeavor to create an exhaustive system of categories so that no meaningful feature of the phenomenon under study falls outside the array of categories. In such a fashion, the process of categorization operates along dual planes of focus: horizontality (i.e., category to-category relationships) and verticality.

Chapter IV

ANALYSIS AND INTERPRETATION OF DATA

In this Chapter the collected information from the case students are interpreted. The researcher had visited the sampled school then taken assessment test among twenty six students of grade seven. The assessment task of twenty six students were analyzed first then, five students having poor and two students with high performances were selected for the cases and interview was conducted among them. After coding, the themes of the data have been obtained and this obtained data have been presented and analyzed according to the objectives which has been presented in the form of case as follows:

Case I (P1)

The first case student of this research study is inhabitant of Nareshwor, Gorkha. He was 13 years old now. He had to walk half an hour to reach the school. He had studied in grade Seven in Rastiya secondary school. There were five members in his family. He had to involve in household activities before and after his school. He was struggling with mathematics in the earlier stage of school life.

When the researcher collected the information about the economic condition of his parents, he found that he was from poor economic status. He also whenever gets free, from study he had to engage himself in those actives which could support his parents economically.

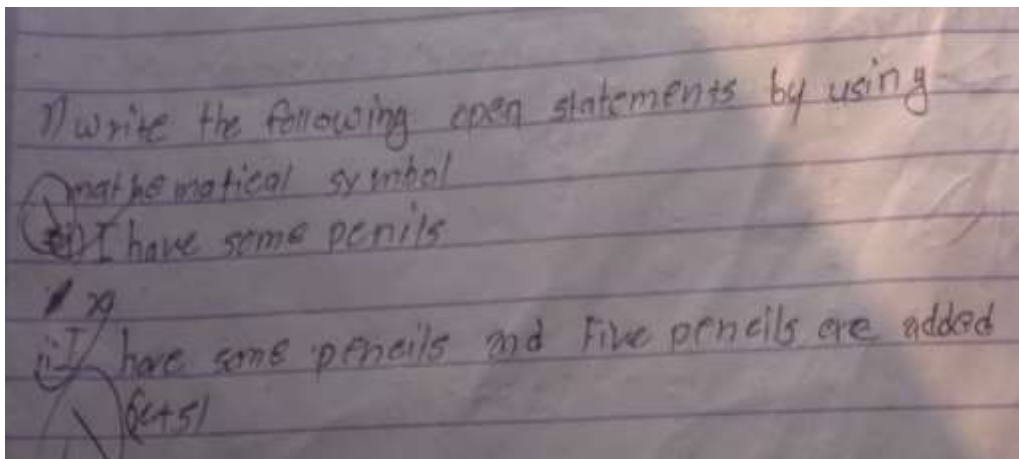
The researcher wanted to be familiar about his educational background. In this regard, it was found that both his father and mother were illiterate. They have not taken any formal education. Being away from the touch of literacy, they were not aware about the importance of education. Though, they have sent their son at school, it seems to be formality not for genuine intentions. They rarely concerned about his

academic activities. When he felt problems related to mathematics, he did not find anyone around his for support and he left it in due.

He described himself as a slow learner in mathematics however he liked mathematics. He did his homework with the help of his friends. He had tried to solve algebraic problems without understanding the formula and its applications. He solve eight problems out of 17 problems related to symbol sense in the assessment conducted by the researcher.

Understand the Unknown Variables

P1 able to know how and when to use symbols and symbolic representations, and knows how symbols are used in different ways in a representation. His ability to understand the ability to unknown shows from the following solution of task



He did unknown by letting x and five more is by adding the 5 in x . He links the symbolic representations back to his own expectations based on his prior experiences, and is capable of linking symbol meaning back to the given problem.

The researcher had conducted interview with P1. The conversation between researcher and P1 was as follows:

Researcher: Why do you use x for given statement?

P1: x is unknown variables as possible for this question to necessary.

Researcher: What is the sum of some pencil and six pencils?

P1: According to the question, let the number of some pencil be x . Then $(x+6)$ pencils.

It is clear that some pencils denoted by x and added other six pencils.

P1: If any variable of the product rule base are same then power is exactly sum. So that result is x^{m+n}

Researcher: What is the result of x^0 ?

P1: When any variables or constant has power zero then the value is given by 1.

Researcher: How do you derivation of the formula according to law of indices of $x^m \times x^n$?

Researcher: How do you construct the value of x^{-m} ?

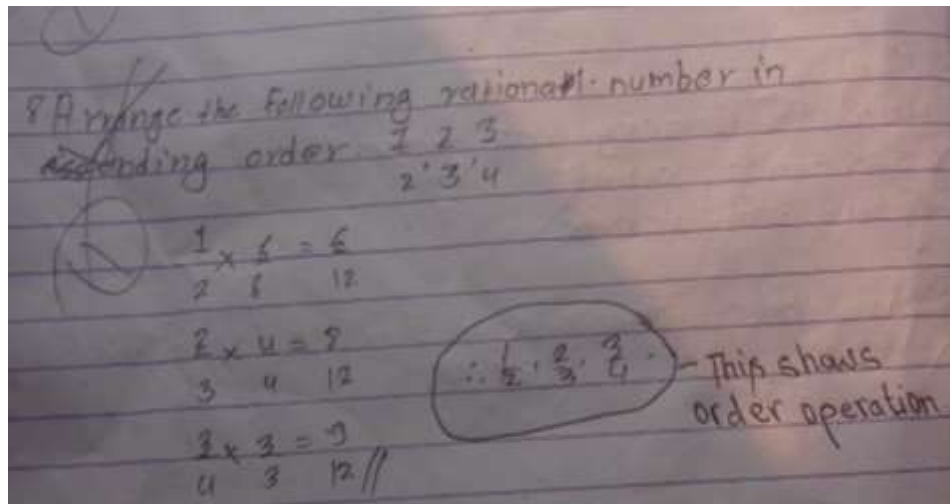
P1: According to the inverse rule when power is given by inverse changed into quotient rule for $\frac{1}{x^m}$, it means that inverse value changed into fraction. It gives the result positive.

From the above tasks P1 symbol sense related to Understand the Unknown Variables is clear. He able to identify the unknown, knowing the position of unknown and find the suitable position for it. P1 began work on the word problem with prescribe rule for the problem at this point P1 was to find some relationships among the relation about the information. He gave the answer of the question about I have some pencils by letting 'x' and adding five in it was by $x+5$. Similarly in the interviewed question P1 gives the true answer of each questions. The unknown about the index law, he know the indexing the unknown number.

Over all result derived from assessment and interviewed result of P1 seems that, he was able used the proper method for computation of given problem, tried to see if required relation, apply the necessary steps and process, find out the perfect solution. This show that the ability of P1 to understand the unknown.

Order Operation, Linking the Symbol to the Problems

P1 had the sense of order of operations, but he also shows that he has a strong grasp of this symbol sense in many cases. In the task of eight his answer shows that he had showed that good concepts of order between the three numbers.



Looking back through the tasks, it can be seen that this element was lacking most often when he chose activities based on a procedure inherent in the symbolic structure instead of considering the overall syntactic structure of the problem or when he was following someone else's suggestions. He does follow a correct order of operations when he takes the time to reflect on the problem to progress toward his goal.

The interview result from this category shows that

Researcher: Which is greater 5 or 3?

P1: *The number of the sequence five greater than three because 5 is more than 2 step by three.*

Researcher: How do you construct symbol of greater than?

P1: *The symbol of greater than our just right hand side folded on right side it means greater than or symbolically $>$.*

Researcher: How can you understanding the concept of ascending order?

P1: *Ascending order means like as bottom to top of ladder.*

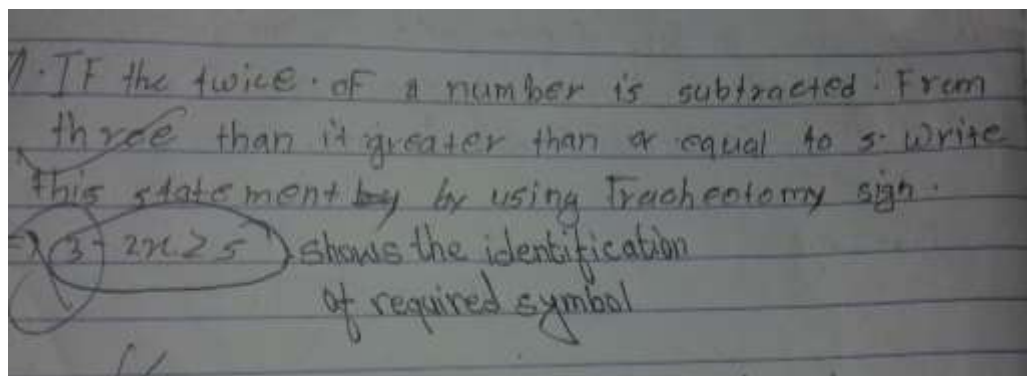
Researcher: $(5-6) < 1$, it is mathematical concept true or false?

P1: *The difference of 5-6 always -1 so that -1 smaller than 1 or symbolically $-1 < 1$ so the sentence is correct.*

A look through the symbol sense on the 17 tasks shows that P1 exhibited some elements of symbol sense repeatedly throughout the problems, but also lacked some necessary or useful symbol sense in several situations. The list of the instances of symbol sense that are seen most often with his work are: as well as the instances that would have been useful but are missing from his work are P1 activities are also guided at times by her habit of extending algebraic rules for some structures to structures in which these rules do not hold true. For example, he treats the absolute value symbol in the same manner as he should treat only answer without logic and distributes an exponent over multiplication of even number of negatives.

Identify the Required Symbol

P1 initial tasks are influenced by something other than his own understanding of the problem. For example, on the task of 7 he showed that the good sense of symbol identification, he refers to steps that need to be followed.



On the initial assessment questions, he is influenced by some specific manipulation inherent in the symbols and not by attention to the overall meaning of the symbolic problem. An exception to this may be found in Task 3.

His interview data shows also good identification of tracheotomy symbol.

Researcher: Why do use ' \geq ' signs in the given problem?

P1: *Specially compromise two or more variables known to smallest number or greatest number with variables.*

Researcher: Why do you write $3-2x$ in the above statement?

P1: *In this question, at first when the twice of number subtracted by 3. It is written by mathematical symbol $3-2x$.*

Researcher: What is symbol for greater than?

P1: *> There is this symbol indicates greater means folded by right sides. It is said to be greater.*

Researcher: What are symbol for less or equal?

P1: *< There is this symbol indicates greater means folded by left sides. It is said to be less.*

Researcher: What are Tracheotomy symbol?

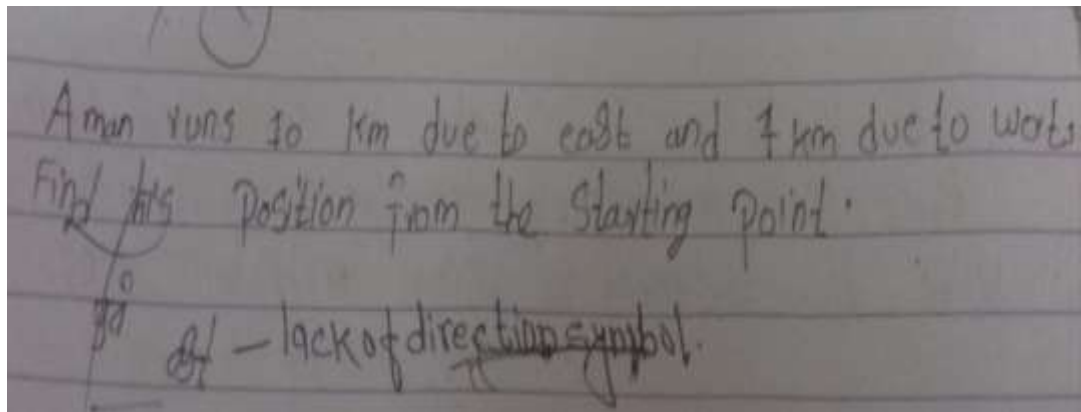
P1: *The symbol of greater than, less than or equal to are the tracheotomy symbol.*

Above examples from assessment and interview takes showed that's P1 able to exhibit the identification of required symbol. It seems from the takes of students solving the researcher questions. P1 had the sense of good identification of required symbol as required in the assessment question.

Directional Symbol

P1 has to first construct a symbolic representation and must reflect on the meaning of the problem. There is evidence in her work and discussion that many of P1 activities were guided by his familiarity and experiences with the symbols instead of by symbol sense or attention to order of operations. P1 activity choices were guided by what he was seeing in the symbols, and these activities often led his to

succeed in reaching his personal goals despite a lack of some useful instances of symbol sense that is directional symbol.



Interview episode of this question shows

Researcher: What are the relation between east, west, south, north and mathematical symbols?

P1: *They all are direction.*

Researcher: What mathematical symbol is possess by east?

P1: *I don't know.*

Researcher: What mathematical symbol is possess by left hand side?

P1: *I don't know*

Researcher: Compare the four direction with Cartesian axis?

P1: *Left, below negative and above, right positive.*

However, P1 also extends more conceptual ideas to symbolic structures and tries to make his prior experiences work on new forms without reflecting on the meaning of the structure or the problem. This is seen in task six and seven when he insists that he should be able to factor and cancel to solve a rational equation. P1 is a successful math student who understands many of the procedures and concepts behind the tasks he completes, but who does not always pay attention to the rules of algebraic manipulation when his goal is to get a correct answer for a problem. The next case

presented is that of P1, who has a very different level of understanding of the procedures, concepts, and language of mathematics.

Symbol Sense of P1

During the assessment task of P1 reaction or answer to the assessment questions are analyzed here. His answer to the assessment question which was related to the definition of the symbol given as was a letter or picture but not given an illustrative example. His work as task two that was the examples of tracheotomy symbols gave two examples but he had completely unknown about the directional symbols as in the task three in the assignment questions which was distance between east and west direction. He gave the correct answer of task four as involving the identification of unknown variables and task five which involving the identification but he was completely unknown about similar pattern construction which was in task six. In the task seven involving both tracheotomy and unknown identification. His answer to the task eight related to the identification of order symbol as in the task nine and ten as solving for X or equality sign he done both the assessment question correctly. As in the task eleven his did not care the symbol as in the product of even number of negative sign is positive and has a good identification of square root sign and indexing as in the task twelve and thirteen.

Case II (P2)

The second case student of this research study was inhabitant of Nareshwor, Gorkha. She was 13 years old now. She had to walk 15 min to reach the school. She studied in grade seven in Rastiya secondary school. There were eight members in her family. She had to involve in household activities before and after her school.

The researcher wanted to be familiar about their educational background. In this regard, it was found that both her father and mother were illiterate. They had not

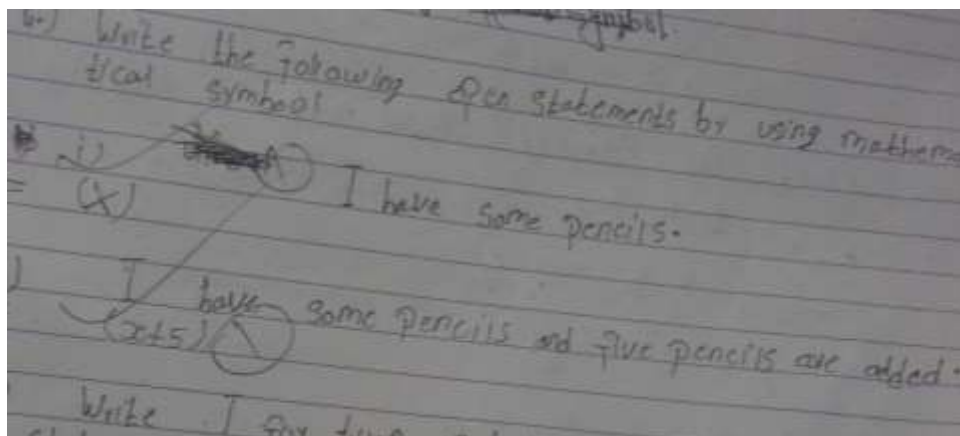
taken any formal education. They rarely concerned about her academic activities.

When she had problems related to mathematics, she did not find anyone around her and she leaves it in due.

The teacher told me that she got more difficulty in division algebraic expression multiplication by minus (-) word problem, taking L.C.M simplification and construction of angles etc. She had more theoretical knowledge rather than practical knowledge. This was her main difficulty. She felt difficulty in algebraic expression. Since they don't use it in their daily life. Following are the symbol facts found in P2.

Understand the Unknown Variables

P2 is able to manipulate the unknown she had written because she was concerned about mixing two different data types. Her interpretation of the symbol both guided and restricted her activities for the problem. Her goals and activities are influenced by what the symbols in a problem "tell" her to do. She is not a very strong mathematics student, and she when working with others she allows their ideas and instructions to dominate the thinking on the task. But she showed the unknown identification sense as follow



In the absence of other people, she depends on the problem itself to lead her toward some manipulation to complete. She is able to reflect on a problem to compare

it to similar work that she has done, but only if the similar experience occurred recently.

The interview data is

Researcher: What is the result of three times of x ?

P2: According to question it means that thrice of x than the result means $3x$.

Researcher: What is the sum of $4x$ and 5 ?

P2: The case is binomial term that is unlike term it gives the result sum means add two or more term. It is symbolically written as $4x + 5$.

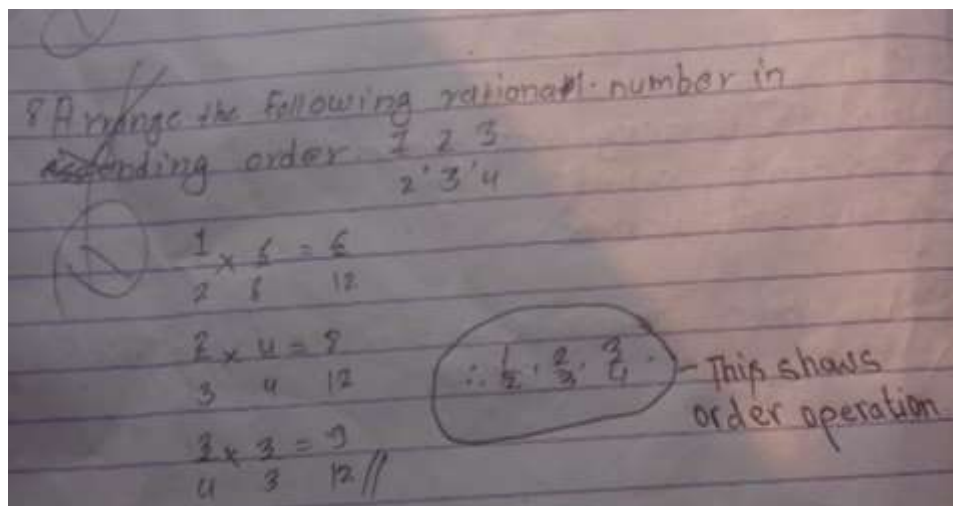
Researcher: The sum of two numbers is 20, how to write it mathematically?

P2: Here sum of two variables is x and y then $x+y = 20$ which is sufficient.

These assessment and interview result shows that P2 had good sense of understanding the unknown. She able to identify the required condition as identify the assessment question and able to link the symbol with number.

Order Operations, link symbol to the problem

P2 also seems the sense of operation sense with interpreting and translating someone else's discussion of mathematics to a symbolic representation on paper. This is evident in task three, six and seven.



Some specific symbols directly affect P2 understanding of the material. Two of these are the equal and inequality signs. Inequality signs, for example, intimidate P2. She is not comfortable with the symbol, and she blames her trouble with inequality problems specifically on the presence of the inequality sign. The equal sign affects P2 work in a different way.

The interview result of P2 related to order operation are present below.

Researcher: Which one is greater among $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$?

P2: *The number of fraction denominator is less after it gives the result greater than denominator of the number.*

Researcher: In above which one is greater?

P2: $\frac{1}{2}$

Researcher : Why $\frac{1}{2}$?

P2: *Because $\frac{1}{2}$ has least denominator*

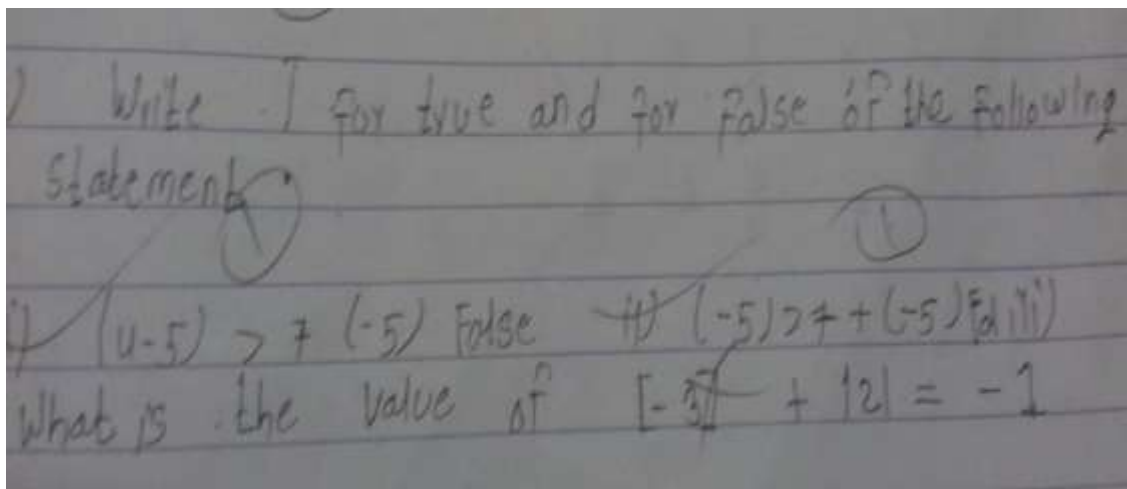
Researcher: In which condition least denominator fraction is greater?

P2: I don't know.

The exhibited symbol sense of P2 in assessment task are:, are the instances of symbol sense that were seen most often in P2 work as well as the instances that were missing in P2 works are. Given a symbolic representation, she was able to manipulate it into a form that made sense to her and that helped her think about the problem, even if it was not always a form that would help her solve the problem. She had no trouble with order of operations when shown a symbolic structure, but she symbolic representations and choosing the correct part of a symbolic structure to manipulate.

Identify the Required Symbol

A great deal of P2 difficulty with symbol sense comes from her ability to make connections between different representations. When representation or word problem, she had deciding how to use symbols and identifying what types of symbols were useful for the problem. For example, when solving the word problem in her mistakenly choose the relationship. The task of six and seven are given below



The interview task of P2 question is

Researcher: What does $<$, $>$, \leq , \geq , \neq refers to?

P2: These symbol indicates that tracheotomy sign.

Researcher: Is $5 < -6$, is true?

P2: In above 5 more than always -6 because it is wrong.

Researcher: Insert the proper symbol 5 5 , 7 -8 ,

P2: Compromise of these number 5 and 5 it means that both are equal it is symbolically written as $5 = 5$ and similarly 7 and 8 into the next case. Hence, is two condition positive and negative it shows that positive number always greater than negative number symbolically we write $7 > -8$.

P2 worked for different numeric representations before considering using a variable in the problem. P2 also had a great deal of trouble connecting symbolic data.

She has difficulty visualizing mathematics in her head and struggles to understand something not written down on paper. For example, in Task 1 the researcher asked to meaning of mathematical symbol, P2 think of a word or a value, but it is not until she wrote the examples.

Symbol Sense of P2

During the assessment task of P2 reaction or answer to the assessment questions are analyzed here. Her answer to the assessment question which is related to the definition of the symbol given as is a group of words but not given an illustrative examples. Her work as task two that is the examples of tracheotomy symbols give two examples but he has completely unknown about the directional symbols as in the task three in the assignment questions which is distance between east and west direction. She give the correct answer of task four as involving the identification of unknown variables and task five which involving the identification but she is completely unknown about similar pattern construction which is in task six. In the task seven involving both tracheotomy and unknown identification. Her answer to the task eight related to the identification of order symbol as in the task nine and ten as solving for X or equality sign she done both the assessment question correctly. As in the task eleven his do not care the symbol as in the product of even number of negative sign is positive and has a good identification of square root sign and indexing as in the task twelve and thirteen.

Case III (P3)

The third case student of this research study was inhabitant of Nareshwor, Gorkha. He was 13 years old now. He had to walk half an hour to reach the school. He studied in grade Seven in Rastiya secondary school. There were five members in

his family. He had to involve in household activities before and after his school. He was the struggling with mathematics in the earlier stage of school life.

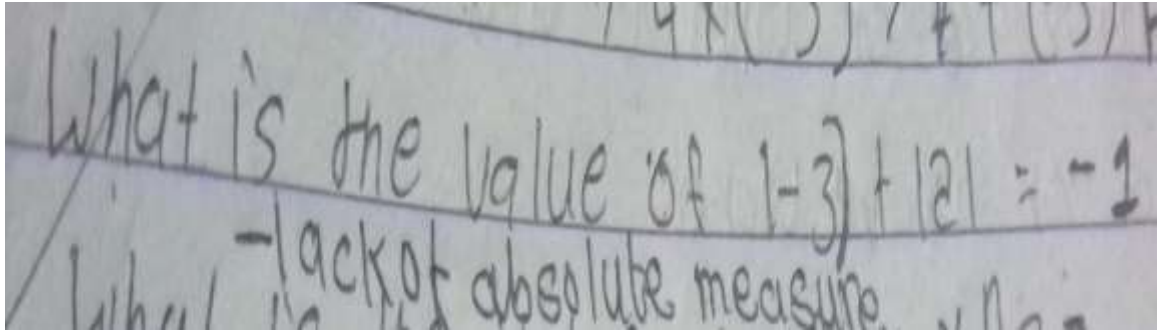
When the researcher collected the information about the economic condition of his parents, he found that he is from average economic status. The case-I also whenever gets free, engages himself in those actives which could support his parents economically.

The researcher wanted to be familiar about their educational background. In this regard, it was found that both his father and mother are illiterate. They have not taken any formal education. Being away from the touch of literacy, they are not aware about the importance of education. Though, they have sent their son at school, it seems to be formality not for genuine intentions. They rarely concern about his academic activities. When he has problems related to mathematics, he does not find anyone around his and he leaves it in due.

He describe himself as a slow learner in mathematics but he liked mathematics. He did his homework with the help of his friends. He try to solve algebraic problems without understanding the formula and its applications. He solve eight problems out of 17 problems related to symbol sense.

Directional Symbol

The instances present indicate that his recognizing the absolute sign a need for symbols and for creating a particular symbolic representation. However, having the symbol sense to know that he needed symbols and representations did not always result in P3 choosing a correct representation or symbol. This was primarily due to the overwhelming number of times that he tried to link the meaning of the symbols back to her prior experiences with similar types of problems, as well as the overwhelming lack of linking meaning to the question being asked.



In the interview data on the on the absolute measure on the participant shows the following

Researcher: Do you know about mathematical symbol?

P3: In my view mathematical symbol are +, -, \times , \div .

Researcher: Only +, -, \times , \div are symbols or there are other also?

P3: I already or pre class about of these symbol learn to teacher. But I do not know about other mathematical symbol.

Researcher: What is the value of $|-3|$

P3: I don't know.

Researcher: How do you apply for the symbol is used in three time of x ?

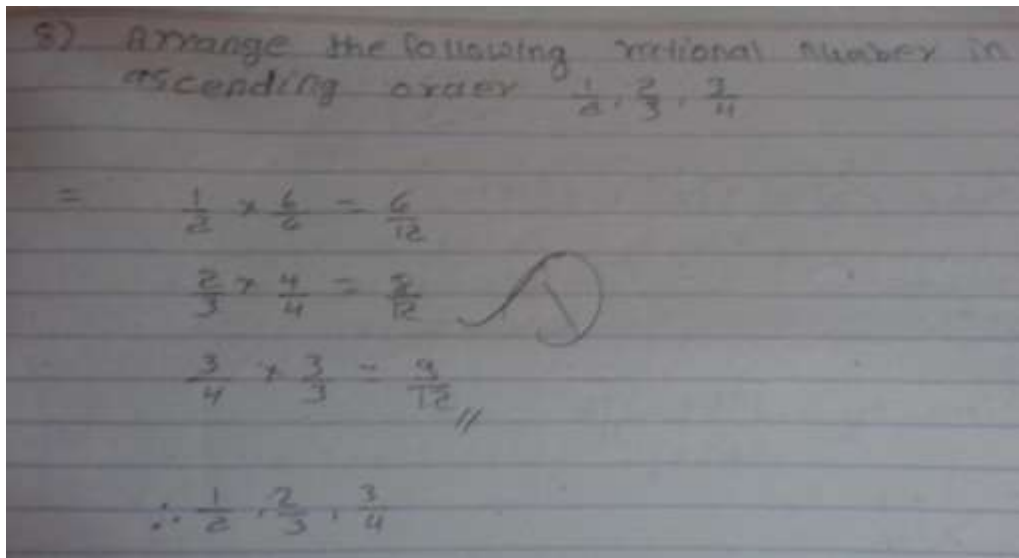
P3: It means product or multiple of fx . Symbolically write x .

He did not make a connection to how the homework problem differed in structure from his previous experience. From the above result it was found that the lack of directional symbol in P3. It was also found that directional symbol sense is difficult for P3 than other symbol concept.

Order Operations, link the Symbol to the problem

P3 goals throughout the eight analyzed tasks were influenced by his expectations for the result to have a specific form. Expectation and thinking ahead to the end result is a desired characteristic for learners. The problem for P3 was that his

expectations were often dependent on ideas from previous problems that he had abstracted as being needed in all situations.



Researcher: Which one is greater among $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$?

P3: $\frac{1}{4}$

Researcher: Why $\frac{1}{4}$?

P3: Because $\frac{1}{4}$ has least denominator.

Researcher: In which condition least denominator fraction is greater?

P3: Don't know.

Researcher: How do you do about the mathematical rule of simplification?

P3: In the rule of mathematics firstly working the brackets and secondly rule of Division, Multiple, addition and subtraction

Researcher: What mathematical rule of simplification?

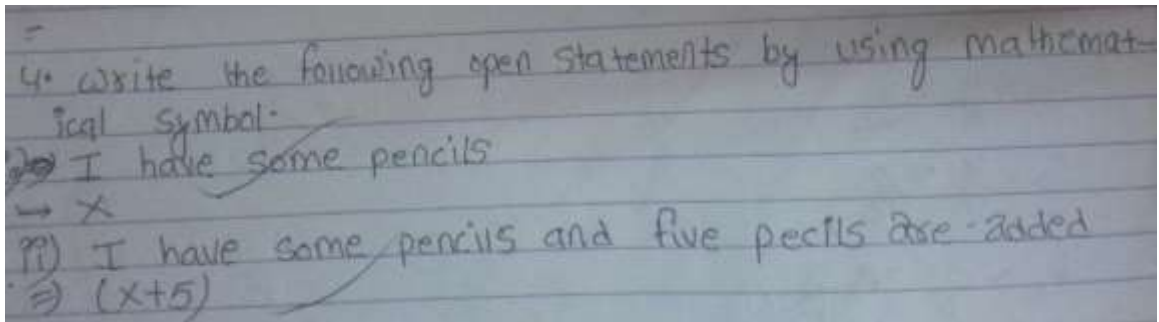
P3 : BODMAS.

The analyzed data shows that at first comparison the ascending fraction. Then after understanding the problem on ascending order. Thus, the researcher concluded that the result gives positive response of participants. This practice may have been

affected by the fact that he had taken the same course just a few months earlier and had many similar examples fresh in his memory.

Understand the Unknown Variables

He showed some growth in his understanding of how problems differed in structure and setup and in recognizing a need for different approaches and activities in different situations.



Researcher: What are the mathematical symbol did you learn?

P3: *The mathematical symbol are +, -, \times , \div*

Researcher: Write the open statements by using mathematical symbol. I have some banana and 8 banana are added..

P3: *Here is some banana represent variables and 8 banana represent fixed number of banana. It gives the result $x + 8$ banana*

Researcher: What teaching method did teacher used for symbol teaching?

P3: *He did not using any fixed teaching method for mathematics teaching, but his aim is how children received the knowledge and understanding the mathematical symbol.*

Researcher: What is the result of 2 times of x ?

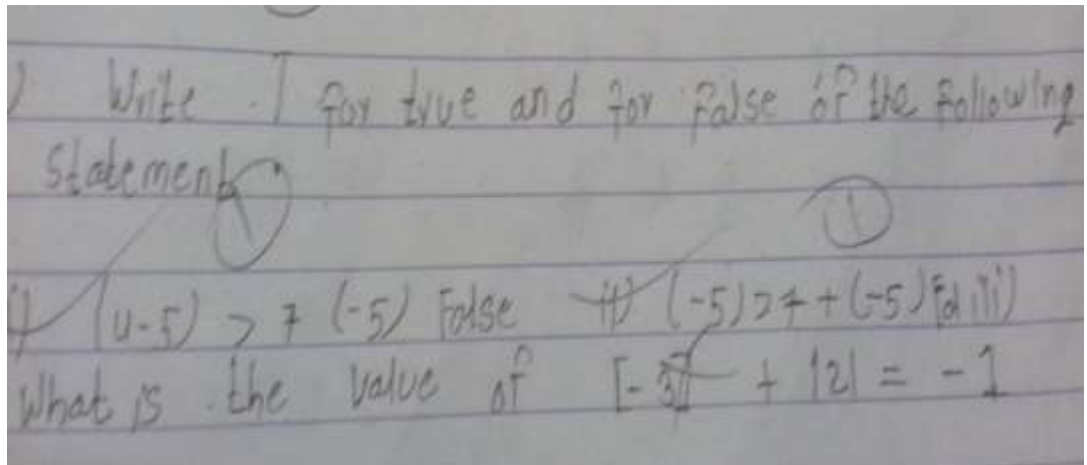
P3: *According to the question, it means that twice of x then the result means $2x$.*

However, he continued to abstract relationships between the symbolic results of a problem and his expectations for the procedures needed to produce a similar

looking result in different situations without paying attention to important differences between two problems.

Identify the Required Symbol

The instances of symbol sense that were seen most often in P3 work as well as the instances that were missing in P3 works are, all these sense depicts the symbol sense most often observed in P3 work.



This idea is also evident in the initial interview when P3 made comments such as “I feel like have to keep everything on one side,” and “I feel comfortable with the work but I’m not sure if my answer is right”. In the latter statement, it is possible that he felt comfortable with his work because it looked familiar to her recollection of previous work, and that he was not confident in the answer because he was not willing or able to link back to the given problem. The researcher had conducted interview with P3.

The conversation between researcher and P3 was as follows:

Researcher: How can you write above statement false?

P3: *It is simple case already solving this problem. thus this question also related to pre-knowledge learn in mathematics. In the above question, when $4-5 = -1$ and $7-5 = 2$, then 2, then $2 > -1$ because positive number always greater than negative number. So the given student which one is false.*

Researcher: How do you change the sign of modules of subtraction?

P3: *For this question generally there is no change sign addition and subtraction but the rules of modules sign when change of this case I don't understand about modules.*

Researcher: What is the result of $|-4| + |3|$?

P3: *In this question generally $-4 + 3 = -1$, but I don't know about rule of modules.*

Researcher: The sum of three times a number and 4 is less than and equal to changed 19 inequality ?

P3: *For this question, the number be x then according to question 3 times the number $= 3x$. after that in my view $3x + 4 \leq 19$.*

Researcher: How do you solve in the following inequality is $2x \geq 14$.

P3: *For this question respondent said in his pre-knowledge about inequality for this condition $2x \geq 14$ at first in this case dividing both side 2 it gives the result $x \geq 7$.*

P3 also had the sense of symbolic data. He had difficulty visualizing mathematics. He head able to identify the symbol was used in which condition of mathematics. For example, in Task 1 the researcher asked to meaning of mathematical symbol, P3 think of a word or a value, but it is not until he wrote the examples.

Symbol sense of P3

During the assessment task of P3 reaction or answer to the assessment questions are analyzed here. His answer to the assessment question which is related to the definition of the symbol given as is a letter or picture but not given an illustrative examples. His work as task two that is the examples of tracheotomy symbols give two examples but he has completely unknown about the directional symbols as in the task three in the assignment questions which is distance between east and west direction. He give the correct answer of task four as involving the identification of un known variables and task five which involving the identification but he is completely unknown about similar pattern construction which is in task six. In the task seven

involving both tracheotomy and unknown identification. His answer to the task eight related to the identification of order symbol as in the task nine and ten as solving for X or equality sign he done both the assessment question correctly. As in the task eleven he do not care the symbol as in the product of even number of negative sign is positive and has a good identification of square root sign and indexing as in the task twelve and thirteen.

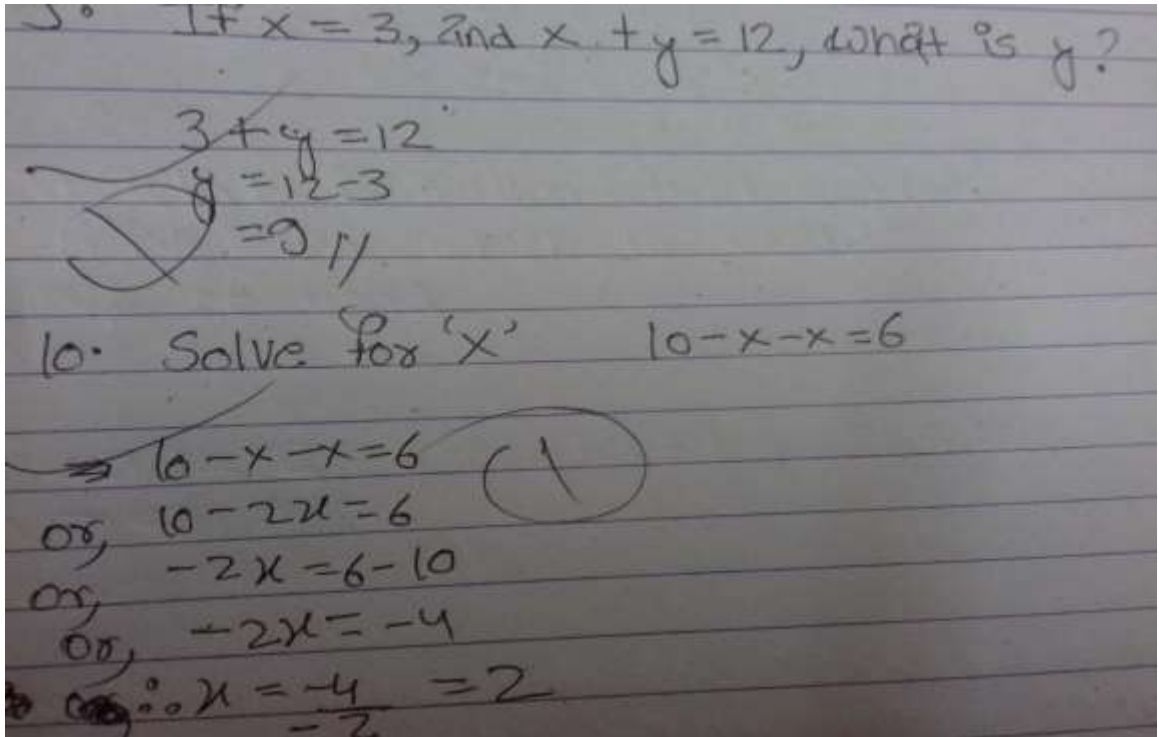
Case IV (P4)

The fourth case student of this research study is inhabitant of Nareshwor, Gorkha. She was 12 years old now. She had to walk half an hour to reach the school. She studied in grade seven in Rastiya secondary school. There were eight members in her family. She had to involve in household activities before and after her school.

The researcher wanted to be familiar about their educational background. In this regard, it was found that both her father and mother were illiterate. They have not taken any formal education. They rarely concern about her academic activities. When she had problems related to mathematics, she did not find anyone around her and she leaves it in due.

Understand the Unknown Variables

The symbol sense instance that was most often lacking from P4 work was the ability to identify strategic groups of components. Looking through the analyzed tasks, it is evident that P4 struggled with this instance when rational structures were included in the problem. P4 told the researcher that she often struggled when there was a unknown term. The task of her work related to this sense is given below



When the researcher take interview on P4 shows:

Researcher: In the above statement, why the statement results $y = 9$?

P4: *In this give condition if $x = 3$, then by the question $x + y = 12$. It is clearly shows the value of x is given then subtracting the value of x on both sides $y = 12 - 3$, which gives the result exactly $y = 9$.*

Researcher: Suppose the sum of two numbers are 10 and one number is 7. Can you find another number?

P4: *Let the number be x then $x + 7 = 10$. After that according to the question subtracting both side 7 also $x + 7 - 7 = 10 - 7$, it gives the result or value of $x = 3$.*

Researcher : Find the sum of double of number and 5 is equal to 15.

P4: *Suppose the number be x then double the number = $2x$. According to the question $2x + 5 = 15$.*

Focusing on meaning of the equation. Similarly her confusion with the rational structure with a non-factored quadratic denominator kept her from recalling details about finding the domain and asymptotes of the function.

Order Operation, Linking the Symbol to the Problems

P4 used many different activities on most of the tasks that she completed. She seemed to jump to a new activity as soon as she ran into a problem with the one she was using, often without determining why her current activity was not working. The task of her work related to this sense is given below

Handwritten mathematical work on lined paper:

$$6^\circ$$

$$\rightarrow -5x - 1 = +5$$

$$-5x - 2 = +10$$

$$-5x - 3 = +15$$

$$-5x - 4 = +20$$

$$9.0 - x - = + 11$$

Researcher: How do you construct the rules of simplification ?

P4: *In this question, I am reading about simplification at first working the brackets and second properly use of four types of symbol division, multiplication, addition and subtraction. or I can say short cut language to memories BODMAS.*

Researcher: What is the product of -5×-3 ?

P4: *In this condition the rules of symbol I can say that $-5 \times -3 = 15$ because the product of two negative number always gives positives.*

Researcher: How do you solve the multiple of $+5$ and -3 ?

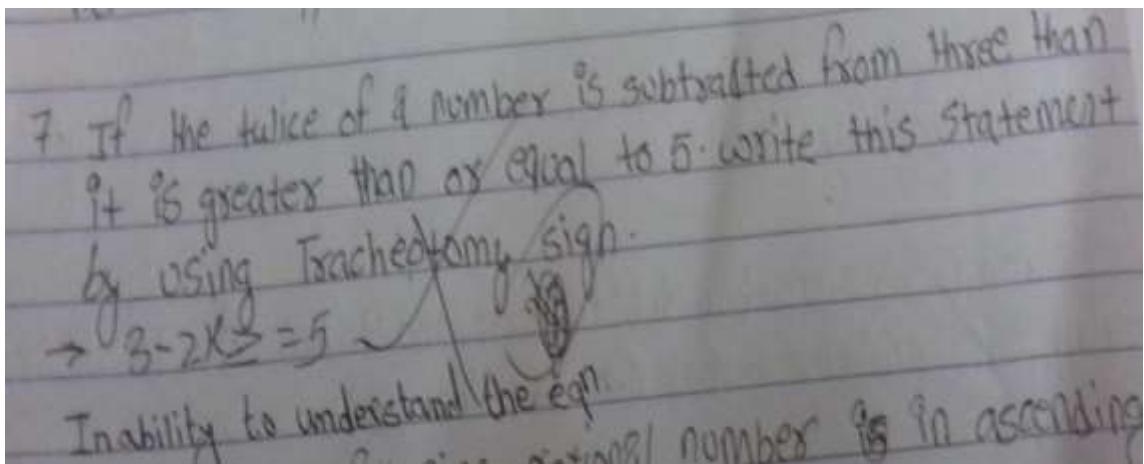
P4: *The product of one positive integers and one negative integers gives the result negative. Similarly in this question $5 \times -3 = -15$.*

In some cases, her difficulty involved knowing how to continue working with a particular symbol or symbolic structure. For example, in her work on the word problem in Task 3, P4 repeatedly wrote a symbolic relationship to try to work with the

given information, but quickly gave up on each equation when she could not think of a way to manipulate to solve the problem.

Identify the required Symbol

In some other instances, the difficulties that caused her to jump to a new activity came from her attempts to use the graphing calculator. She often expected that it would be able to work with symbols to create equations, solve the problem, or tell her an appropriate symbolic form.



When the researcher take interview on P4 shows

Researcher: What are symbol for greater than or equal to ?

P4: It means that greater or equal to denoted by \geq .

Researcher: Thrice of a number is less than 9, what is its expression?

P4: In this word problem exact I don't know about one mathematical language because in this word problem I don't understand of this question.

Researcher: If the twice of a number is subtracted from five then it is greater than or equal to 8. Write these statements by using tracheotomy sign

P4: In my view let the twice of a number be $2x$, then $5 - 2x > = 8$

Researcher: What are Tracheotomy symbol?

P4: The symbol greater ($>$), smaller ($<$) and equal ($=$) are sue to compare the number are called tracheotomy. In symbolically $a > b$, $a < b$ and $a = b$.

The analyzed data show that majority of students were found that to be with knowledge but understanding the word problem in particular content of algebra, generalized of rules while solving the problem, comprehending, expressing the word problem into algebraic equation and inequality and application level. The analysis indicated that P4 was misinterpretation even not correctly transforming algebraic word problem into equation. Thus researcher concluded properties of tracheotomy if a and b are two integers $a > b$, $a < b$, $a = b$ only one of these statements are true at a time are known as property of integers.

Symbol Sense of P4

During the assessment task of P4 reaction or answer to the assessment questions are analyzed here. Her answer to the assessment question which is related to the definition of the symbol given as is a group of words but not given an illustrative examples. Her work as task two that is the examples of tracheotomy symbols give two examples but he has completely unknown about the directional symbols as in the task three in the assignment questions which is distance between east and west direction. She give the correct answer of task four as involving the identification of unknown variables and task five which involving the identification but she is completely unknown about similar pattern construction which is in task six. In the task seven involving both tracheotomy and unknown identification. Her answer to the task eight related to the identification of order symbol as in the task nine and ten as solving for X or equality sign she done both the assessment question correctly. As in the task eleven his do not care the symbol as in the product of even number of negative sign is positive and has a good identification of square root sign and indexing as in the task twelve and thirteen.

P4 was often able to use methods such as guess and check or numeric manipulations to find an answer to her problem. She struggled with successfully creating and using symbolic equations and formulas. In several cases, the symbols in a problem caused P4 to reflect on her expectations for the problem. She considered that her activity was incorrect because she was supposed to be solving for x . Similarly in the linear form of her area equation caused concern because she anticipated finding an unknown.

Case V (P5)

The fifth case student of this research study was an inhabitant of Nareshwor, Gorkha. He was 13 years old now. He had to walk half an hour to reach the school. He studied in grade Seven in Rastiya secondary school. There were five members in his family. He had to involve in household activities before and after his school. He was struggling with mathematics in the earlier stage of school life.

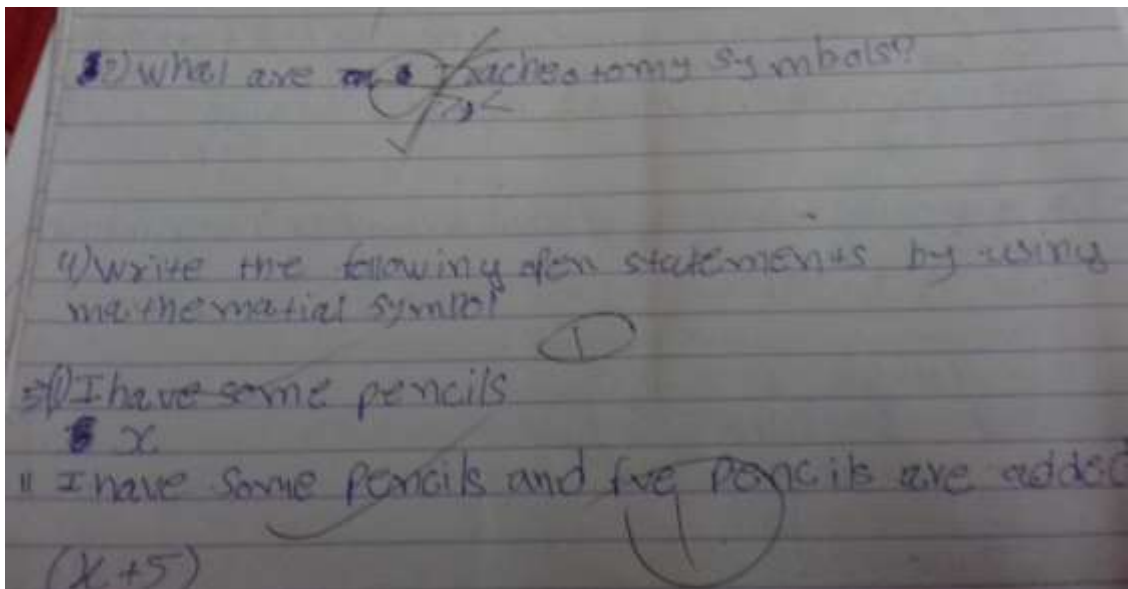
When the researcher collected the information about the economic condition of his parents, he found that he was from average economic status. He also whenever gets free, engages himself in those activities which could support his parents economically.

The researcher wanted to be familiar about their educational background. In this regard, it was found that both his father and mother are illiterate. They had not taken any formal education. Being away from the touch of literacy, they are not aware about the importance of education. Though, they had sent their son at school, it seems to be formality not for genuine intentions. They rarely concerned about his academic activities. When he had problems related to mathematics, he did not find anyone around him and he leaves it in due.

He describe himself as a slow learner in mathematics but he liked mathematics. He did his homework with the help of his friends. He try to solve algebraic problems without understanding the formula and its applications. He solve eight problems out of 17 problems related to symbol sense.

Understand the Unknown Variables

P5 work as well as the instances that were missing in P5 works are, and absent in his work. It is important to point out that a close inspection of the coding shows that the majority of times when he was identified to be lacking the indicated instances of symbol sense occurred when he was working on the four initial interview tasks. In fact, he was only seen to be lacking symbol sense for knowing order of operations and properties of operations one time in the homework and test tasks.



To be fair, at the time of the initial interview, he had not been engaged in algebraic problem solving for at least a year, and so it is not surprising that he did not remember all of the rules of algebraic manipulation.

Researcher: What are the mathematical symbol did you learn?

P5: *In my view at first I know about addition (+), it is clearly that the sum of two or more number and difference of two number means subtracted from first and second if*

it symbolically (-) and multiple rules mean the product of two or more quantity and it is symbolically (x) and divided represent divided into to work more parts of the quantity it is denoted by (\div)

Researcher: *Only +, -, \times , \div are symbols or there are other also?*

P5: Exactly, I don't know about these other symbol but there are other symbol. I don't know about other symbol.

Researcher: What is variable in mathematics?

P5: In this question generally I know that letters are variables like as x, y x,

Researcher: If five is added to sum marbles then what is the result?

P5: Let the number of sum marbles be x. By the question $x + 5$ means that the sum of give marbles and some marbles.

Researcher: If three is subtracted to double of sum marbles then what is the result?

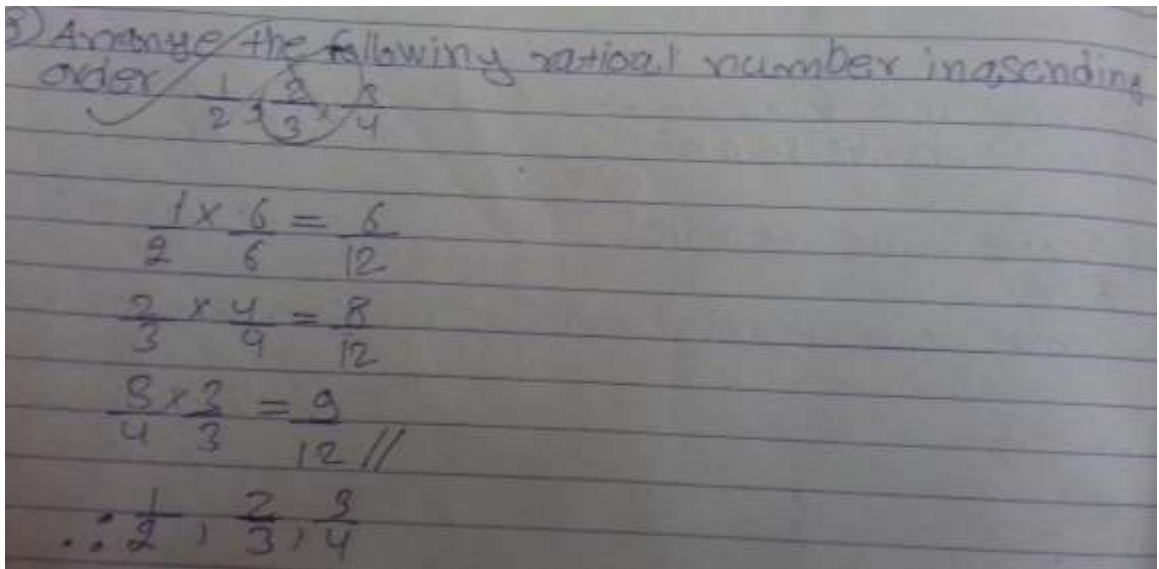
P5: Suppose number of marbles be x then $2x$ it is clear that double of x by the question I can say $2x - 3$.

In the other interviews, he was working on problems very similar to ones he had recently reviewed in class. However, at no time in any of the interviews did he demonstrate an understanding of why his procedures were working, or a desire to reflect on the meaning of the symbols he was using.

Order Operations, linking the symbol to the problem

The presence of instances of symbol sense was usually coupled with a referral to his notes for help or with a recollection of a memorized procedure from class. He only recalled using the notation because he thought it was what the teacher had done, and did not know how it related to objects he was trying to find. Similarly, he was able to link the symbolic structures to his prior experiences, and identified strategic groups to find roots and build a number line, but he did not know how to relate what

he had done to the question.



Researcher: Which one is smaller among $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}$?

P5: If 2, 3 and 4 are integers such that $2 < 3$, $3 < 4$ then there are integers such that

$\frac{2}{7}, \frac{3}{7}$ and $\frac{4}{7}$. In symbolically $\frac{2}{7} < \frac{3}{7} < \frac{4}{7}$.

Researcher: Which is the sign for less than?

P5: $<$.

Researcher: What do you know about ascending order?

P5: Generally ascending order means bottom to top or smallest to biggest.

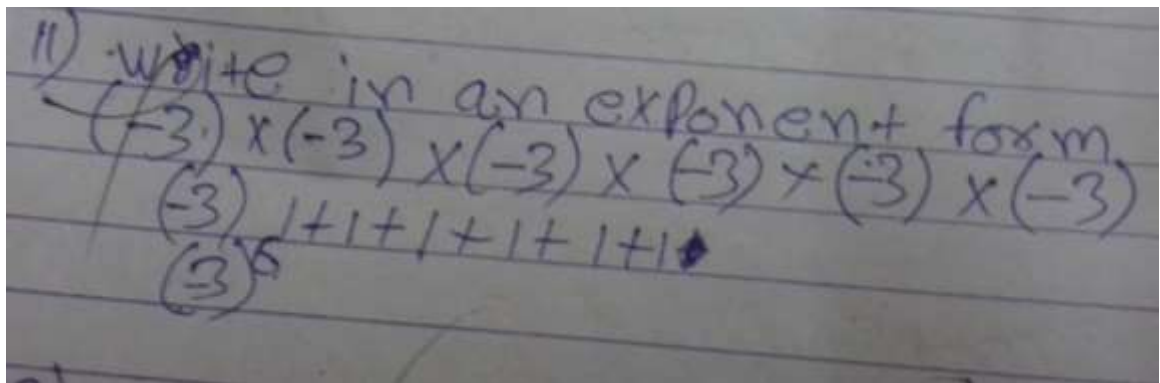
Researcher: What are Tracheotomy symbol?

P5: Tracheotomy symbols means greater, or less.

He chose his answer based on what looked familiar, and had no understanding of how to check and see if his answer was correct because he could not link the result to the problem. It seems evident that in most cases He did not understand the mathematics behind the symbolic manipulations that he completed, but was merely trying to reconstruct procedures to match his recollection of what others had done in similar symbolic settings.

Identify the required symbol

His goals and activities were often connected to a desire to avoid doing certain symbolic manipulations or to eliminate certain symbolic structures from the problem. He told the researcher that he could possibly factor the denominator, but he was not willing to try that. Instead, he manipulated without regard for any algebraic rules and managed to get the equation into a form that was more comfortable for her, but his actions resulted in a product of three factors that he had neither the desire nor knowledge for expanding.



He was relieved when he realized that he could work on this problem with a number line and avoid “solving” the problem with symbolic manipulations. He did not often make connections to symbolic manipulations that he completed in problems that were not set up in the same way.

In the test tasks, P5 in many cases, he successfully completed the problems because he was either following an example or recalling learned steps for a problem. However, he demonstrated an ability to think about and explain why certain steps produced the intended results based on his understanding of the symbols. Explain why the he could explain why finding the vertex would produce the desired result by correctly interpreting the effect of the symbols in the function on the function.

Researcher: How do you construct the following question in exponential form

$$5^4 \times 5^3 \times 5^{-1}.$$

P5: According to the question the law of indices base are same by the product rule power is sum then it is clearly shows that $5^{4+3-1} = 5^6$.

Researcher: Write in an exponent form $(-5) \times (-5) \times (-5) \times (-5)$.

P5: In this case, as the above question according to the law of indices here is power of each base no one means 1 it is clear that $5^{1+1+1+1} = 5^4$.

Researcher: What is the exponential form of $x^5 \div x^3$?

P5: For this question according to the quotient rule if the base are same of these quantity change into power rule $\frac{x^5}{x^3}$. it is clearly that $x^{5-3} = x^2$ which is correct

P5 case represents a student whose anxiety toward certain symbols and certain symbolic manipulations inhibits his ability to understand the reasons associated with mathematical formulas and procedures. He has learned to succeed in mathematics courses by watching and performing procedures repeatedly, but he does not seem to trust himself to think mathematically about his work. This is evident in his dependence on the, and in the lack of reflection evident in his work. His anxiety or fear of mathematics has not been debilitating enough to keep his from earning A and B grades in his mathematics classes, but he dislike of the subject and he fear of certain symbols does restrict he desire to engage in reflective thinking. Based on he work in the initial interview, it seems reasonable to assume that P5 is a student who is engaged in mathematics at only a participatory stage, and who would be unable to solve a problem that differs too much from one he was seen worked out several times.

Symbol Sense of P5

During the assessment task of P5 reaction or answer to the assessment questions are analyzed here. His answer to the assessment question which is related to

the definition of the symbol given as is a letter or picture but not given an illustrative examples. His work as task two that is the examples of tracheotomy symbols give two examples but he has completely unknown about the directional symbols as in the task three in the assignment questions which is distance between east and west direction. He give the correct answer of task four as involving the identification of un known variables and task five which involving the identification but he is completely unknown about similar pattern construction which is in task six. In the task seven involving both tracheotomy and unknown identification. His answer to the task eight related to the identification of order symbol as in the task nine and ten as solving for X or equality sign he done both the assessment question correctly. As in the task eleven his do not care the symbol as in the product of even number of negative sign is positive and has a good identification of square root sign and indexing as in the task twelve and thirteen.

Similarly the researcher take the two cases of showing high symbolic sense in the assessment that is Case-VI and Case- VII.

Case VI (P6)

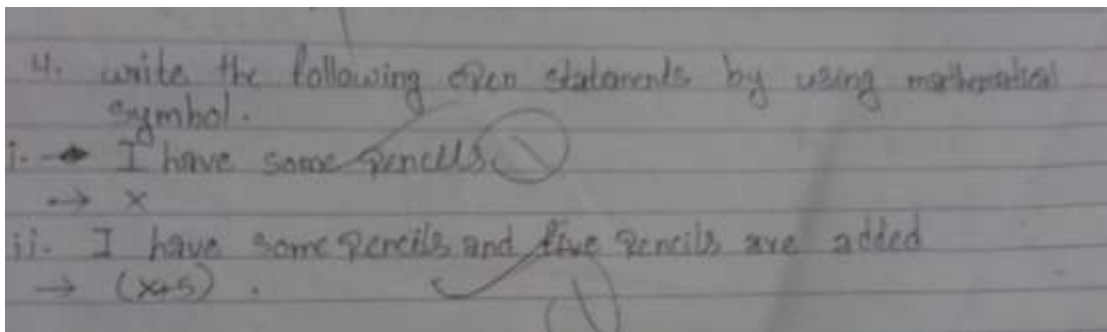
The sixth case student of this research study was inhabitant of Nareshwor, Gorkha. She was 13 years old now. She studies in grade Seven in Rastiya secondary school. There were five members in his family. She had good symbol sense thought the assessment.

When the researcher collected the information about the economic condition of her parents, the researcher found that she is from average economic status. The researcher wanted to be familiar about their educational background. In this regard, it was found that both her father and mother were literate. She described herself as a medium learner in mathematics and she liked mathematics. She tried to solve

mathematics (algebraic problems) in home also. She solve 16 problems out of 17 problems related to symbol sense. The detail about P6 symbol sense are.

Understand the Unknown Variables

P6 seems to know how and when to use symbols and symbolic representations, and knows how symbols are used in different ways in a representation meaningfully and truly. She links the symbolic representations back to his own expectations based on her prior experiences, and is capable of linking symbol meaning back to the given problem if necessary.



Researcher: What are the mathematical symbol did you learn?

P6: *Mathematical symbol like as +, -, ×, ÷*

Researcher: If there is only +, -, ×, ÷ are mathematical symbol or there are other also?

P6: *In my view I know about +, -, ×, ÷ are mathematical symbol. Therefore I don't know about more mathematical symbol.*

Researcher: What letters is used for unknown variables?

P6: *The letter like as x, y, z..... variables. It is said to be also letters.*

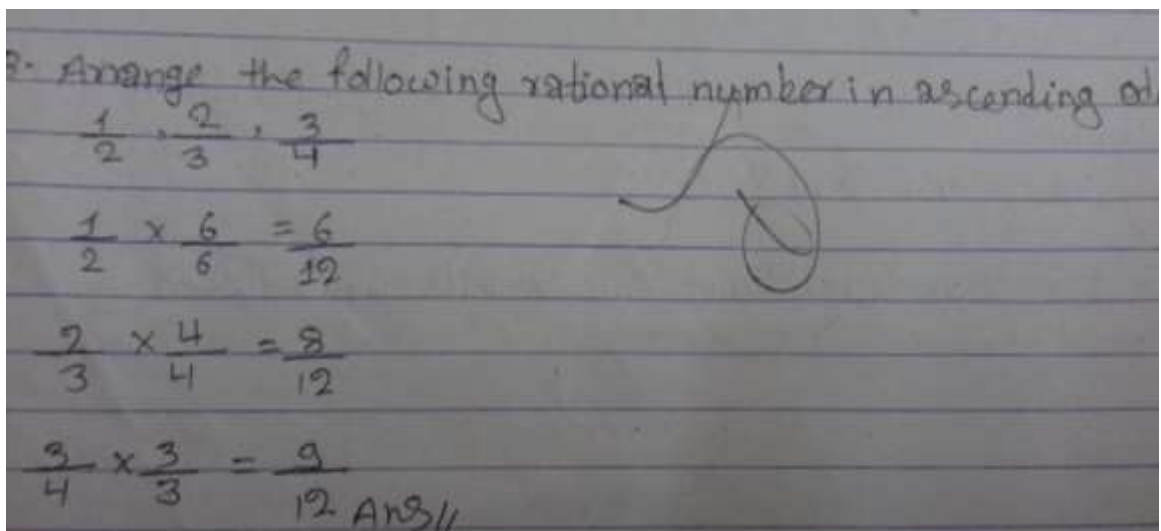
Researcher: If two is added to some marbles then what is the result?

P6: *For this question at first let the number of some marbles be x. Then by the question the number some marble + two marbles = x + 2 marbles.*

The above result showed that P6 was able to cope with the mathematical problem related to the unknown and variable identification. The assessment tasks and interview tasks related to the unknown identification was solved by P6. P6 was also able to identify the required symbol as required in assessment question. The episode of P6 answer to the assessment question 4 is shown in the above, similarly the interview result was also presented. All these episode showed that P6 had good understand of unknown symbol.

Order Operation, linking the symbol to the problems

P6 succeeds in mathematics by learning to recognize the effect of slight differences in symbolic structures. Who learns mathematics by seeing one problem worked repeatedly, P6 learns by doing as many different practice problems as she can until she feels she had mastered different variations of the same type of problem. Her work on the homework and test tasks above provides evidence that she is able to think reflectively about algebraic properties and rules and about connections between symbolic, numeric, and graphical representations. Her work on the initial interview tasks, however, supports his claim that she made mistakes when working on specific type of problem for the first time.



3. Arrange the following rational number in ascending order

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$$
$$\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$$
$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$$
$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \text{ Ans//}$$

Researcher: Which one is greater among $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$?

P6: *In this question actually the greater fraction is $\frac{1}{2}$ because more number divided by numerator. It gives the result less. Therefore $\frac{1}{2}$ is greater fraction. Because $\frac{1}{2}$ has least denominator.*

Researcher: When $a > b$ and $ac < bc$ then what type of number is c ?

P6: *It is not a simple case because there is apply tracheotomy law. So I don't know about this question.*

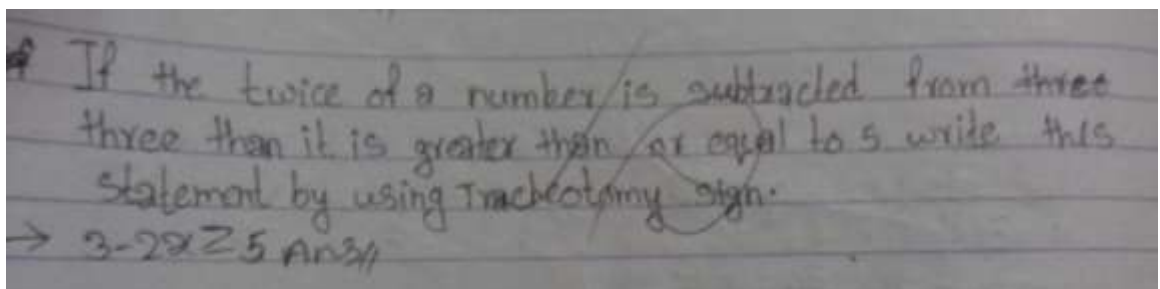
Researcher: What do you mean by descending order?

P6: *Descending order represented biggest to smallest or top to bottom.*

From the above tasks analysis it was found that P6 able to identify the simple order operation. The operation sense as demanded by the curriculum was to operate the symbol between the two whole numbers. P6 was able to identify the required symbol used between two numbers as given in tasks.

Identify the required symbol

Out of all of the participants, Shawn had the greatest potential to be identified as working at an anticipatory stage of problem solving. He admitted that given a problem type that he had never seen before, he was likely to make some mistakes, but he also shared that she went beyond the graded homework and did many practice problems so that she could experience many different variations of the same problem type.



Researcher: How can you apply or use of tracheotomy law?

P6: If a and b are two integers then $a < b$, $a > b$ $a = b$ only one of these statement are true at a time.

Researcher: What teaching method did teacher used for symbols teaching?

P6: Teacher only apply traditional method like as he was done example and solve exercise.

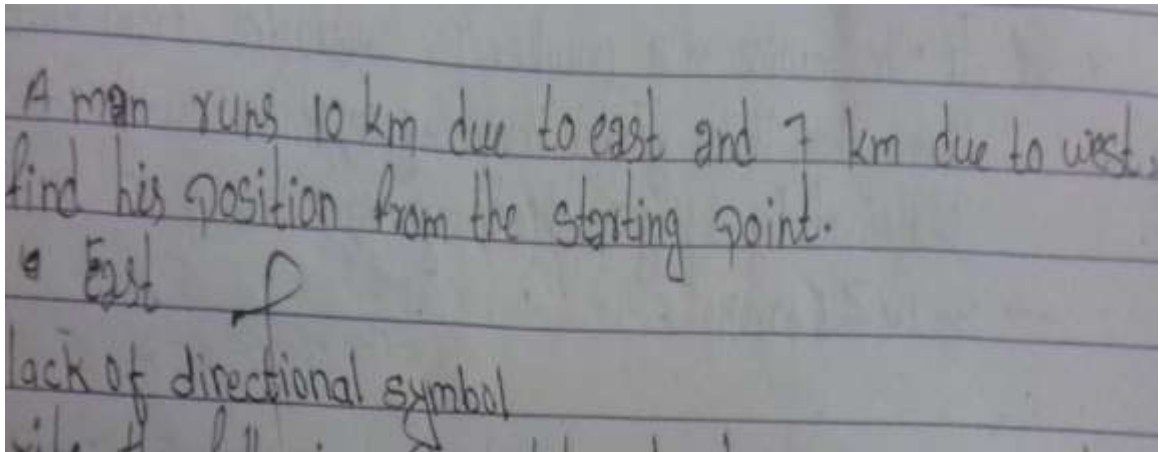
Researcher: When 5 is subtracted from the double of a number the remainder is greater than or equal to three. Write this statement by using tracheotomy sign.

P6: In the above question suppose the number be x . then double time of the number = $2x$. According to the question $2x - 5 \geq 3$.

P6 exhibited a good understanding of the mathematical concepts in the problems he completed, and she was more successful at setting and reaching goals that were useful and in line with the teacher's or researcher's goals for a problem. It is not surprising that a strong showing of symbol sense would accompany a successful student's work.

Directional symbol

A look through the symbol sense on the 17 tasks, P6 exhibited good sense of symbol throughout the problems, but also lacked directional symbol. The list of the instances of symbol sense that are seen most often with her work are as well as the instances that would have been useful but are missing from her work are



Researcher: What are the relation between east, west, south, north and mathematical symbols?

P6: They all are direction

Researcher: How can you use for mathematical term on implication of these direction?

P6: For this question there is a wider area used for direction so there is not fixed used of this application of this mathematical symbol. Thus I don't about clearly.

Researcher: How to write 2 meter above the sea level?

P6: In this question actually here is pointed sea level. It is clearly that sea level from upside 2m represent to 2m give the value really +2m.

Researcher: Lakhan is 5 meter right from me, write it with proper symbol.

P6: For the above question according to four direction scale Lakhan goes to 5 meter right side it is clearly he goes 5 meter just our right hand side. Thus, it gives the result 5 meters. .

She had the same goal in both problems, but made no connection between the different activities she used. On possible influence on the different activities is that the first problem began with an x on the right-hand side, whereas the second problem had only a constant on the right. She had obviously struggled with mathematics in the past, and she had learned to compensate for her difficulties by memorizing

procedures. When a symbolic structure looked very similar to a problem she had memorized, she was able to work through the steps to get an answer, even if she had little understanding of how the procedure worked or of how the answer related to the original problem.

When the symbolic structure was not familiar, she turned to tutors to tell her how to start a problem, and then added a new symbolic structure to her list of memorized examples. Although she received a successful grade in the course, the researcher concludes that she had little understanding of the meaning behind most mathematical symbols or structures or the procedures for working with these objects.

P6 case represents a student whose successful in mathematics toward certain symbols and certain symbolic manipulations inhibits her ability to understand the reasons associated with mathematical formulas and procedures. She has learned to succeed in mathematics courses by watching and performing procedures repeatedly, but she does not seem to trust herself to think mathematically about her work. This is evident in her and in the lack of directional symbol. She had no anxiety or fear of mathematics has and not been debilitating enough to keep his from earning good marks in her mathematics classes, but her dislike of the subject and her fear of certain symbols does restrict her desire to engage in reflective thinking. Based on her work in the initial interview, it seems reasonable to assume that P6 is a student who is engaged in mathematics at only a participatory stage, and who would be unable to solve a problem that differs too much from one she was seen worked out several times.

Symbol Sense of P6

During the assessment task of P6 reaction or answer to the assessment questions are analyzed here. Her answer to the assessment question which is related to the definition of the symbol given the examples as $+$, $-$, \times , \div . Her work as task two

that is the examples of tracheotomy symbols give two examples but she has completely unknown about the directional symbols as in the task three in the assignment questions which is distance between east and west direction. She give the correct answer of other task correctly.

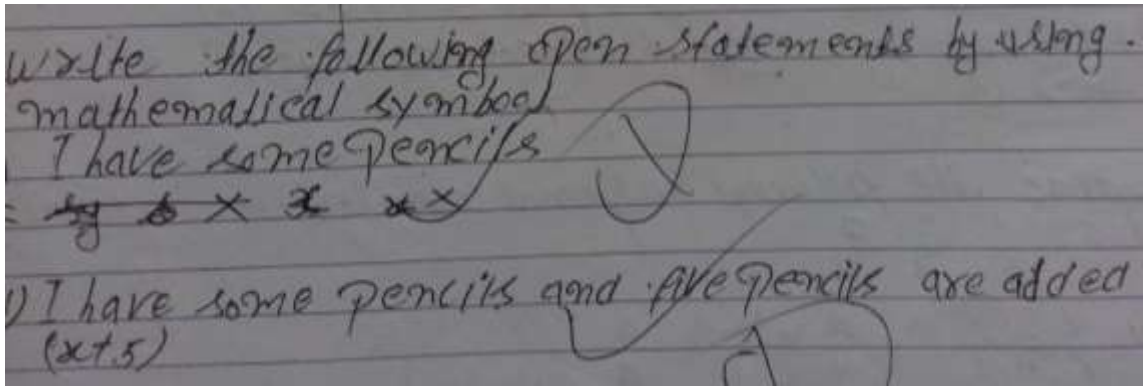
Case VII (P7)

The seventh case student of this research study was inhabitant of Nareshwor, Gorkha. She was 13 years old now. She studied in grade Seven in Rastiya secondary school. There are six members in her family. She had good symbol sense thought the assessment.

When the researcher collected the information about the economic condition of her parents, and found that she is from average economic status. The researcher wanted to be familiar about their educational background. In this regard, it was found that both her father and mother were illiterate. She described herself as a medium learner in mathematics and she liked mathematics. She tried to solve mathematics (algebraic problems) in home also. She solve 15 problems out of 17 problems related to symbol sense. The detail dialogue about student and researcher are.

Understand the Unknown Variables

The task analysis of P7 shows that that her ability to show the ability to understand the unknown. The unknown identification play important role for understand the symbol for mathematics. The unknown here means the dealing the mathematical situation with proper sign.



Researcher: What are the mathematical symbol did you learn?

P7: There is many cause un-use of mathematical symbol but I know about +, -, \times , \div

Researcher: Do you know about only +, -, \times , \div are symbols or there are other also?

P7: I learn already or learning from small class or pre-determine knowledge it is clearly four symbol about use in mathematical problem but I don't know about other symbol.

Researcher: What letters is used for unknown variables?

P7: There are specially like as x, y, z, are unknown variables which are letters.

Researcher: If two is added to sum marbles then what is the result?

P7: In this question, my view let number of some marble box. Then by the question it gives the result is $(x + 2)$ marbles.

In the interviewed, she was working on problems very similar to ones she had recently reviewed in class. However, at no time in any of the interviews did she demonstrate an understanding of why her procedures were working, or a desire to reflect on the meaning of the symbols he was using.

Order Operation, linking the Symbol to the Problems

Order symbol is important concept in mathematics it play to relative understand the object clearly and logically. Her ability to understand the order sense is shown by the following examples.

P7 able to create rules that would allow the sign to change in the case of equation and equality she followed the BODMAS rule while solving the equation solving procedure. However, she had the sense that the symbol implied finding while solving.

Researcher: What is the value of $x + 4 = 8$?

P7: For this question, in generally to find the value of x . Solve this equation subtraction both side 4 then it gives $x = 4$.

Researcher: Which one is greater among $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$?

P7: Actually rule of fraction greater number is $\frac{1}{4}$ because which fraction has denominator is least. So, $\frac{1}{4}$ is greater.

Researcher: Where can you use rules of BODMAS?

P7: Generally used of BODMAS to solving the simplification. In other content solving the word problem and algebraic term.

In some cases, her difficulty involved knowing how to continue working with a particular symbol or symbolic structure. For example, in her work on the word problem in Task 3, P7 repeatedly wrote a symbolic relationship to try to work with the given information, but quickly gave up on each equation when she could not think of a way to manipulate to solve the problem.

Identify the required symbol

Pattern is heart of heart of algebra. The similar pattern construction of given pattern P7 with appropriate symbol and its understanding shows by the given task analysis of the following.

6) $-5x - 1 = +15$
 $-5x - 2 = +10$
 $-5x - 3 = +15$
 $-5x - 4 = +20$
 $11c - x - = +11$

Researcher: What is the product of (-4) and (-5)?

P7: *If the product of any two integer it gives the result always positive, the product of $(-4) \times (-5) = +20$.*

Researcher: What is the sum of (-4) and (+2) ?

P7: *Here according to operations of integer.. The sum of $(-4) + (+2)$ it gives the result greater number is negative and smaller number is positive. So that result gives negative. Thus $-4 + (+2) = -2$.*

Researcher : What are Tracheotomy symbol?

P7: *Tracheostomy symbol represent greater than*

Researcher: What are Tracheotomy symbol?

P7: *Tracheotomy symbols means greater, or less than or equal to.*

Researcher: What do you understand by greater than symbol?

P7: *In this question, greater symbol represent combine for two number. For example*

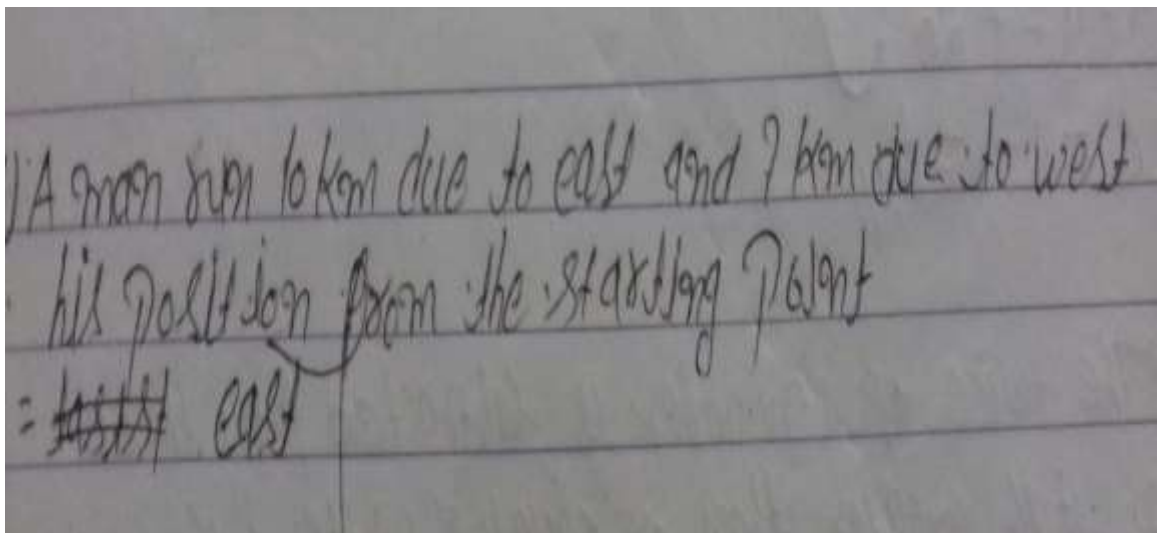
$4 > 2$.

Except all other task P7 show the perfect example of power of negative sign.

Her understanding of this symbol is showing by given examples. She was able to understand the even power of negative sign positive and odd power of negative sign is negative.

Directional symbol

The directional symbol was also influential to students' work on this problem, although they usually did not pay it much attention until they neared the end of their work. P7 was able to compute all tasks other than directional symbol as all other students.



In interview episode she showed

Researcher: What are the relation between east, west, south, north and mathematical symbols?

P7: *They all are direction*

Researcher: Why direction is important in mathematics?

P7: Direction is a kind in mathematical symbol. It is important to know directional aspect.

Researcher: The height of an ice bag on the sea is 30 ft. above the sea level and submarine is 20 ft. below the sea level. What is the difference between their heights.

P7: Here sea level is considered at 0 ft. height. Height of an iceberg above the sea level = +30ft. Depth of submarine of the seal level = 20 ft. Thus their difference

$$= +30 - (-20)$$

$$= 30 + (+20)$$

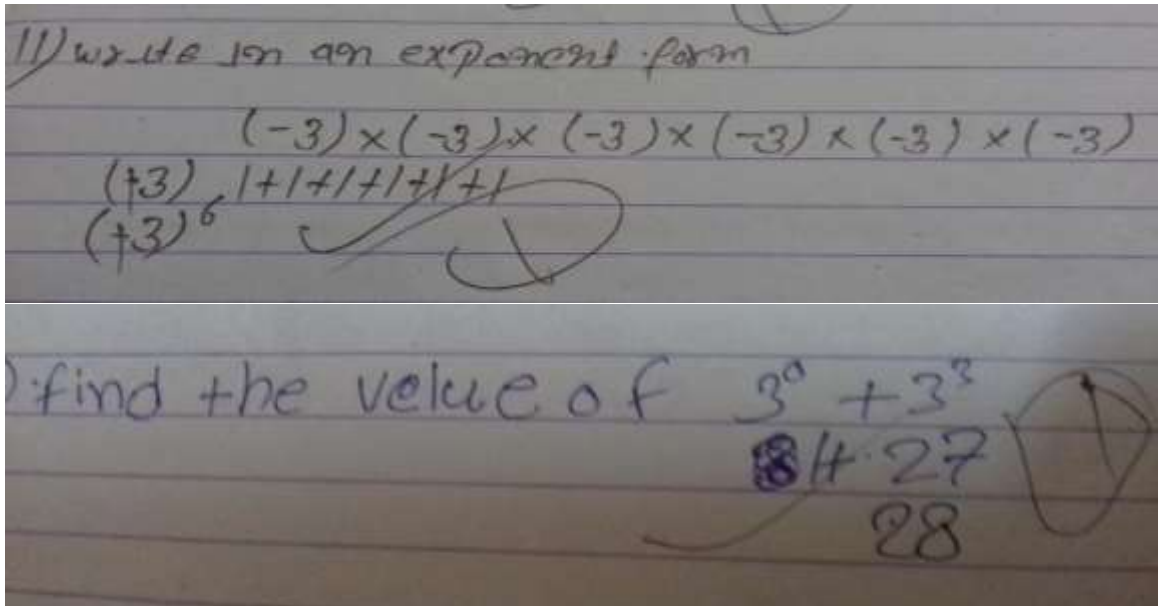
$$= 50 \text{ ft.}$$

The difference between their heights is 50ft.

Directional sense was difficult then the other symbol skills as perceived by students. While questioning the directional related problems the student were confused and ignored the problems. The students are perceived that directional problems are not related to mathematics although it is important mathematical aspect.

Index of law problem.

As well as the instances that would have been useful but are missing from her work are: P7 seems to know how and when to use symbols and symbolic representations, and knows how symbols are used in different ways in a representation meaningfully and truly.



Interview data related to this symbol shows

Researcher: What is the value of x^0 ?

P7: According to the law of indices at first $x \div x$. Here $x^1 \div x^1$ it is clearly that $x^{1-1} = x^0$. Thus any condition power of 0. It gives the result 1.

Researcher : What is the value of $3^4 \times 3^5$.

P7: For this question base are same. According to product rules of indices $a^m \times a^n = a^{m+n}$. Similarly $3^4 \times 3^5 = 3^9$

Researcher: Write in an expanded form of $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$?

P7: For this question this case also same as above case actually law of indices according to product value base are same then power is sum. It is clearly that $3^{1+1+1+1+1+1+1+1+1+1} = 3^{10}$.

She links the symbolic representations back to his own expectations based on her prior experiences, and is capable of linking symbol meaning back to the given problem if necessary.

P7 case represents a student whose successful in mathematics toward certain symbols and certain symbolic manipulations inhibits her ability to understand the

reasons associated with mathematical formulas and procedures. She has learned to succeed in mathematics courses by watching and performing procedures repeatedly, but she does not seem to trust herself to think mathematically about her work. This is evident in her and in the lack of directional symbol. She had no anxiety or fear of mathematics has and not been debilitating enough to keep his from earning good marks in her mathematics classes, but her dislike of the subject and her fear of certain symbols does restrict her desire to engage in reflective thinking. Based on her work in the initial interview, it seems reasonable to assume that P7 is a student who is engaged in mathematics at only a participatory stage, and who would be unable to solve a problem that differs too much from one she was seen worked out several times.

Symbol Sense of P7

During the assessment task of P7 reaction or answer to the assessment questions are analyzed here. Her answer to the assessment question which is related to the definition of the symbol given the examples as $+$, $-$, \times , \div . Her work as task two that is the examples of tracheotomy symbols give two examples but she has completely unknown about the directional symbols as in the task three in the assignment questions which is distance between east and west direction and index law.

A look through the symbol sense on the 17 tasks, P7 exhibited good sense of symbol throughout the problems, but also lacked directional symbol and index law problem.

Chapter V

SUMMARY, FINDINGS, CONCLUSION AND RECOMMENDATIONS

This chapter is basically concentrated in deriving some findings from the discussion of chapter IV. Besides findings and conclusions, it also comprises some educational implications.

Summary

The purpose of this study was to explore students' interpretations of the symbols and symbolic structure of different mathematical problems and to describe how these interpretations when solving problems. Using a combination of two conceptual frameworks and analysis of students' work and discussions on carefully chosen tasks, the researcher focused on clearly articulating, to the extent possible, how or why students solve a problem the way they do. The previous chapters in this study have presented an analysis for each case addressing the research questions for this study, research and answer the research questions for this study, implications for teachers, and suggestions for future research.

Findings and Discussion

The researcher identified many influences for goals and activities that related directly to students understanding, familiarity, and experiences with particular symbols and symbolic structures such as fractions or rational expressions, equal and inequality signs, and parentheses. Linchevski and Linveh (1999) and Norton and Irvin (2007) have previously identified students' understanding of these symbols as causes for algebraic difficulties. Norton and Irvin (2007) claim that errors made with these types of symbols can be traced back to misconceptions in arithmetic. In this study, the researcher was able to present a closer analysis of some ways in which these symbols guide students' decisions in problem solving.

Students often have trouble recalling rules for working with these items and create their own rules to deal with their presence in a problem. For example, P3 and P1 both needed to understand the tracheotomy sign in different tasks because they were uncomfortable with it, and created their own techniques for working with the problem in a form that was more familiar. When the absolute value sign was present, most of the students knew that it indicated a positive result, but none of them understood how to work with it in an inequality problem. All of the students showed evidence of not understanding the meaning of the inequality sign. Students commented on their lack of experience with some of these symbols or with a teacher putting several complicated symbols into one problem.

The equal sign has different meanings for students and is not often interpreted to mean that equivalence has to be maintained. P3, for example, often saw the equal sign as an indication that she was supposed to do something, as evident in his tasks where he tried to just get an x by itself. Three of the students used the equal sign in a different way in the linear word problem when they recorded the statement. The symbol might be seen here as a way of keeping track of given information. P2 explained her use of the symbol in this way as separating different types of information and did not feel she could combine different units on one side of the equal sign.

When results did not match anticipations, students sometimes created new ways of accomplishing their goals that did not follow mathematical rules. This was evident, for example, when students expected to have an x term in the word problem. P4 just transfer by x but not by symbol. Another example can be seen in P5 work on a inequality where he expected to have parentheses in her work, redid his problem until he had them, and then ignored rules of algebra to identify symbol. Similarly the

higher achiever P6 and P7 are able to solve all the problems except the directional problems. They had good sense about the symbol as given by the curriculum.

Students often worked within one representation type without making connections to any other type. Students often struggled to understand how a symbol could be used to help make sense of a symbolic representation on problems. This may have been related to difficulties with understanding how and when to use algebraic problems.

Conclusion

Symbol sense includes understand that there is a constant need to check symbol meaning and to compare meanings with one's own expectations and intuitions. Teachers can demonstrate such reflective habits in the classroom and help students learn the importance of constantly reflecting on the effect of an activity as simple as writing an equal sign. It might be helpful for teachers to ask students to read mathematical statements aloud from a textbook and to discuss the meanings of the symbols involved. Building symbol sense can help build students fluency with the complicated language of mathematics.

It was clear from this study that students lacked many of the desired instances of symbol sense that making in mathematics in general. However, it is not too late even at the basic level to start to help students build symbol sense by engaging students in conversations about the meaning of symbols and how meanings link to students' prior experiences and to specific problems. It is not helpful to just tell a student what a symbol means or how symbolic structures can be manipulated without understanding that students have their own interpretations based on their own experiences with the symbols e.g. students learn to think of the equal sign when they see the word "is" and multiplication when they see the word "of" in word problems.

The meaning “is” for the equal sign easily translates to “gives” or “produces,” which is how many students interpret this symbol, instead of interpreting it meaning balance or equivalence of expressions.

Recommendations

The research in this study was necessary for deeper insight into students’ thinking when using and working with mathematical symbols. Using the collected data, researchers could design new studies for looking specifically at what processes are involved when changes in students’ ways of knowing take place, and design teaching experiments to test the use of carefully designed tasks to promote conceptual change. Since linking representations was an area of symbol sense that was particularly problematic for students, this would be an interesting question to incorporate into future research.

One of the researcher’s original intentions with this study was to address the question: how algebraic symbol be perceive and manipulated by students'? This question could not easily be addressed with the data collected because students made such little use of graphing calculators for problem solving. This area of research can provide useful information for gauging the effectiveness of assessments in symbol understand.

One area of research directly related to the current study that needs further exploration is students’ and teachers’ understandings of the symbolic transition in arithmetic and algebra.

Other suggested extensions of this study include research of the connections between students’ understandings of their spoken language and the understandings of the symbolic language of mathematics. Are difficulties with communicating in mathematics related to difficulties with writing or reading natural language? What

kind of connections can teachers and researchers help students make between the two languages to make reading mathematical textbooks or online tutorials more useful for students? What attention do teachers' pay to students' written and verbal language when mathematical symbols are involved?

Many studies acknowledge that the language of mathematics is difficult because it differs from natural language, but it might be useful to explore students' perceptions of these differences and develop strategies for teaching students to better understand this important language.

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Appendix-A

ALGEBRAIC ASSESSMENT

Dear all,

This is a non-evaluative assessment for partial work of my master's thesis. I would be grateful by your appreciation of this questionnaire. Please solve the given problems in your original manner.

Name:

Roll No:

.....

Knowledge Level

- 1). What is mathematical symbol?
- 2). What are Tracheotomy symbols?

Understanding level

- 3). A man runs 10 km due to east and 7 km due to west, find his position from the starting point.
- 4) Write the following open statements by using mathematical symbol
 - i) I have some pencils
 - ii) I have some pencils and five pencils are added
- 5) a) Write T for true and F for false of the following statement
 - i) $(4 - 5) > 7 - (-5)$
 - ii) $4 + (-5) > 7 + (-5)$
- 5) b) What is the value of $| -3 | + 2$
- 5) c) What is the value of $x^m \times x^n$?

Application level

- 6) Read the following pattern

$$(+5) \times (+3) = 15$$

$$(+5) \times (+2) = 10$$

$$(+5) \times (+1) = 5$$

$$(+5) \times (+0) = 0$$

$$(+5) \times (-1) = -5$$

$$(+5) \times (-2) = -10$$

$$(+5) \times (-3) = -15$$

Therefore $(+ \times -) = -$

Similarly, construct the patterns for and verify $(- \times - = +)$ i.e. $(-5) \times (-2) = +10$.

Problem solving

7). If the twice of a number is subtracted from three than it is greater than or equal to

5. Write this statement by using Tracheotomy sign.

8). Arrange the following rational number in ascending order $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

9. If $x = 3$, and $x + y = 12$, what is y ?

10. Solve for 'x' $10 - x - x = 6$

11). Write in an exponent form

$$(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$

Analysis level

12. Simplify: $\sqrt{9} + \sqrt{16}$

13). Find the value of $3^0 + 3^3$

Appendix-B

INTERVIEW GUIDELINES

Students Demographic Information

- What is your Name?
- How old are you?
- Cast/Ethnicity
- Family members

What do you feels about mathematics?

- Positive/Negative/ Usefulness

What are the difficulties' are of mathematics?

Student's symbol sense

- What are the mathematical symbol did you learn?
- Only $+$, $-$, \times , \div are symbols or there are other also?
- If yes, what are they?
- What are Tracheotomy symbol.
- What did you understand by mathematical symbol?
- What are the relation between east, west, south, north and mathematical symbols?
- What is the relation between east, west, south, north and Cartesians' coordinates?
- What is the result of $(+ \times +)$, $(+ \times -)$, $(- \times +)$, $(- \times -)$.
- Give an example of $(- \times -)$

Mathematical Problems

- What's' about the assessment question.
- Did you face such question before?

-
- When Ram jump from 1m above the ground to 2m dig hole then find the total distance covered by him.
 - What is the value of x^0 ?
 - Give the illustration of $x^0 = 1$
 - What does $<$, $>$, \leq , \geq , \neq refers to?
 - What is the absolute value of -3 ?

Real world Problems

- What is your learning habit in mathematical symbol?
- What teaching method did teacher used for symbols teaching.
- Learning habit/style (listening, interacting, and seeing) of students?
- How teacher behave towards his/her or class?
- What is the Home environment for learning?
- What method of teaching is useful for teaching mathematical symbol?
- How to improve your symbol understanding?
- Your opinion about mathematical symbol and its learning