

# An Empirical Evaluation of Algorithms 

 for solving Subset Sum Problem in terms of Total Bit Length
## Dissertation

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In partial Fulfillment of the requirements for the Degree of Master of Science in Computer Science \& Information Technology

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August, 2015

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## Student's Declaration

I hereby declare that I am the only author of this work and that no sources other than the listed here have been used in this work.

Rajendra Shrestha
Date: 12 August, 2015


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## Supervisor's Recommendation

I hereby recommend that the dissertation prepared under my supervision by Mr. Rajendra Shrestha entitled "An Empirical Evaluation of Algorithms for solving Subset Sum Problem in terms of Total Bit Length" be accepted as in fulfilling partial requirement for the completion of Masters Degree of Science in Computer Science \& Information Technology.

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## LETTER OF APPROVAL

We certify that we have read this dissertation work and in our opinion it is appreciable for the scope and quality as a dissertation in the partial fulfillment of the requirements of Masters Degree of Science in Computer Science \& Information Technology.

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I have done my best to complete this research work. Suggestions from the readers are always welcomed, which will improve this work.


#### Abstract

Subset Sum Problem is an important decision problem in complexity theory and cryptography. Subset sum problem can simply be described as: given a set of positive integers $S$ and a target sum $t$, is there a subset of $S$ whose sum is $t$ ? The complexity of subset sum can be viewed as depending on two parameters: $n$, the number of values, and $m$, the precision of the problem (number of bits required to state the problem). Backtracking algorithm for Subset Sum Problem can be modeled as a binary tree where each node represents a single activation of the recursive code. The worst-case time complexity is $\mathrm{O}\left(2^{\mathrm{n}}\right)$ when n is used as the complexity parameter [7]. Dynamic Programming breaks a problem down into smaller problems and solves them recursively as divide-and-conquer technique. It solves the problem in $\mathrm{O}\left(\mathrm{m} . \mathrm{n}^{2}\right)$ time. Dynamic Dynamic Programming is the extension of the Dynamic Programming with a dynamically allocated list of target sums. It has the time complexity of $2^{\mathrm{O}(\mathrm{x})}$ when the total bit length x of the input set is used as the complexity parameter.


The empirical analysis shows that time complexity of DP and DDP increase sub-exponentially when bit length, m , is increased by 1 . At the same time BT is not sensitive to m and its time complexity increases exponentially when number of inputs, $n$, is increased by 1 .

Keywords: Subset Sum Problem, Backtracking, Dynamic Programming, Dynamic Dynamic Programming, Total Bit Length

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## List of Abbreviations

| DDP | - | Dynamic Dynamic Programming |
| :--- | :--- | :--- |
| DP | - | Dynamic Programming |
| SSP | - | Subset Sum Problem |
| BT | - | Backtracking |
| .. | - | ranges (eg 1..5 ranges from 1 to 5) |

