METACOGNITIVE STRATEGIES IN MATHEMATICS EDUCATION:

AN ACTION RESEARCH

A

THESIS

BY

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IN THE PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

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Letter of Certificate

This is to certify that **Mr. Nirmal Sitikhu** a student of the academic year **2073/74** with campus Roll No. **106** exam Roll No. **7328411** Thesis Number **1694** and T.U. Registration No. **9-2-191-14-2013** has completed this thesis under the supervision and guidance of Ms. Sarala Luitel during the period prescribed by the rules and regulations of Tribhuvan University Kirtipur, Kathmandu, Nepal. The thesis entitled **"Metacognitive Strategies in Mathematics Education: An Action Research"** has been prepared based on the result of his investigation conducted during the prescribed period under the department of Mathematics Education, Central Department of Education, University Campus, Kirtipur, Kathmandu, Nepal. I recommend that his thesis be submitted for evaluation as the partial requirement to award the degree of Master of education.

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Letter of Approval

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Thesis

By

Nirmal Sitikhu

Entitled

Metacognitive Strategies in Mathematics Education: An Action Research

Has been approved in partial fulfilment of the requirements for the

Degree of Master of Education.

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Recommendation for Acceptance

This is to certify that **Mr. Nirmal Sitikhu** has completed his thesis entitled **"Metacognitive Strategies in Mathematics Education: An Action Research"** under my supervision during the period prescribed by the rules and regulations of Tribhuvan University Kirtipur, Kathmandu, Nepal. I recommend and forward his thesis to the department of Mathematics Education to evaluate in the final viva voice.

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Ms. Sarala Luitel

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Date: 26 December, 2022

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Dedication

Honestly dedicated

To

My Parents

Declaration

This research paper contains no materials, which has accepted for the award of other degree in any institution. To the best of acknowledgement and belief this research paper contains no materials previously published by any others except due acknowledgement has been make.

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Nirmal Sitikhu

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January, 2023 (Nirmal Sitikhu)

Abstract

This research is based on the study of **"Metacognitive Strategies in Mathematics Education: An Action Research".** The objectives of this research were to identify the effectiveness of metacognitive strategies to improve achievement in mathematics and to trace behavioural changes of learner's during the teaching and learning using metacognitive strategies. This study was an action research. This study was carried out by using the primary source of data. The researcher used the math achievement test, interview schedule and memo writing through observation as research tools. In the research, only grade X was included in which there were ten students. The researcher used the metacognitive strategies in the teaching mathematics for one month. Mainly the researcher had taken one test before using metacognitive strategies covering theorems and problems related to area of triangle and quadrilateral under the geometry section. After using metacognitive strategies another test was taken with the similar test items.

In this study, the researcher had collected the data for the first objective from the score of the test items and researcher calculated the mean, variance and tested the hypothesis using t-test in 0.01 level of significance. The researcher used the quantitative data analysis method for the first objective and qualitative data analysis method for the second objective. The researcher concluded that there was increment in achievement of students in mathematics and there was positive behaviour, attitude and participation of the students after using the metacognitive strategies. Therefore, using the metacognitive strategies in mathematics classroom is one of the best way to improve the achievement of mathematics and to change the learner's behaviour, attitude and participation positively.

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Chapter I

Introduction

This study was an attempt to determine the effectiveness of the metacognitive strategies in mathematics. Learning without student's actual understanding is meaningless. This study had been selected with believes that metacognitive strategies have positive effect in learner's cognition. In this section, the background of the study, statement of the problem, objectives of the study, rationale of the study, delimitations of the study and operational definition of terms were discussed.

Background of the study

One thing, we all should noticed is that, we think without thinking. Thinking allows humans to make sense of, interpret, represent or model the world that they experience. Moreover, thinking also help to make predictions about that world. If we try to think about thinking, other dimensions of life will opens for us. According to Costa (1984),"Students can learn to understand and articulate their mental processes if teachers specifically encourage thinking about thinking". Thinking about thinking can generally termed as 'metacognition'. Metacognition is knowledge of own cognition and skill to regulate own cognition. It is deep level of understanding the inner self-cognition. According to flavell (1979) the metacognition are highly stimulated at the case in which highly carefulness and conscious thinking are required. Especially, at examination hall students are experiencing metacognition. Self-inquiry into self-memory is direct and easy way to metacognition. Metacognition does not appear abruptly from nowhere, it emerges early in life and it becomes more explicit, more powerful and effective and later it comes to operate increasingly under the one's conscious control (Kuhn, 2000).

However, too much priority given to thinking becomes disease. When enough attention paid into our inner-self then we realize that we do not think, in actual thinking happens to us (Tolle, 2005). Thinking is activity of mind and mind is our superb instrument if used rightly otherwise it is very destructive (Tolle,1997). Being in and aware of ‘now’ (meditation) excess us to our awareness on mind and emotion. If someone hear him/herself talking dialogue with him/herself inside own brain and he/she evaluate his/her own decision, then he/she is experiencing metacognition (Costa, 1984). In metacognition, we deals with how we understands our own cognition process. Metacognition is cognition about cognition in general, is not about some particular subject. In other words, metacognition is about understanding how one understands anything. Therefore, talking metacognition in only mathematics is indeed incomplete. However, this research conduct an experiment on mathematics only due to constraints on resources.

Indeed, nature of mathematics is really differ from other discipline and its action in real world is essential to understand. Modern mathematics is too much abstract and its foundational axiomatic basis are consistent, independent and completeness, which makes mathematical knowledge more powerful and logical. Obviously now, it is difficult to teach and learn mathematics completely. Rote learning does not make sense and meaningful learning is difficult, in understanding modern mathematics. Now students’ learning should be in higher dimension with metacognitive strategy. In metacognitive strategy, teacher focus on learners’ understanding about own thinking process. Such strategy enables students to control their own thinking processes. Understanding the mental strategies used by students when they solve problems and they proceed with the “learn to learn” movement are strands of metacognition (Yoong, 2002).

In our daily life, we humans use language to communicate with each other. Similarly, mathematics is a medium of communicating physical and non-physical things for our own purpose; such purpose may differ from person to person. For some it may be advancing scientific knowledge about earth, stars and galaxies, computer systems, mathematics, etc.; for some developing algorithms for maximizing profit and sales, minimizing cost and risk; for some it may be to developed logical reasoning ability within human; for some it may be to predict future events etc. It is not compulsory for every human to like flowers. Somebody may refuse to like flower but he/she has to miss the smell and beauty of flower. Similarly, somebody may refuse to learn mathematics but he/she has to miss the beauty of mathematics. The complex nerve system of human being is gift given by creator. Many researchers noticed that learning mathematics positively effects on well-functioning of the nervous system.

One special beauty in mathematics is abstraction. Mathematics enjoys high level of abstraction. We fail to understand mathematics properly because we try to find where the physical things that mathematics is telling. If proper awareness arises, we understand imagination is bridge between the physical world and mathematical knowledge in which some are perfectly fit and some are not. Imagination is the special ability with in human beings. According to (Morin, 1999); “Entry and exit routes connection the organism to the outside world make up only 2% of the entire neuro cerebral system; the remaining 98% is devoted to inner functions. The brain constructs a quite independent psychic world where fantasies, needs, images, ideas, desires, and dreams ferment, and this world infiltrates our vision or conception of the outside world.”(p.6). Indeed, vagueness character (assumption that we human actually do not know the creator of all existence in world and world itself) of world is striking human’s consciousness. Due to this vagueness, human beings suffer from countless errors and illusions by making guesses (logical or illogical). In German ideology, Marx and Engels observed that men always misunderstood about themselves, about what they are doing and what they ought to do, and about the world in which they live (Morin, 1999). However, one clear aim of human seem to be live well life in universe through expanding consciousness. Mathematical knowledge is expect to serve human being for our good life.

When we deeply focus on anything, we can realize that actually, we are surround by so many mental assumptions and imaginations. Mathematics is similar to that. So that there are, axioms (assumed initially true) in mathematics. Every word has actually no meaning in itself but it has some meaning with in human mind because he/she can imagine and assumes that this word has that meaning, this word has that meaning, etc. We mentally link words with anything. Similarly, in mathematics numbers like 1, 2, 3… These are just symbols and it has no meaning in itself. We human have to make imagination and have to acquire sense of number in order to use mathematics properly. So, one able to write 1, 2, 3…in paper is not actually learning numbers. Learner must know sense of number as it can connect with any physical object or non- physical things after numbers are learned. That means he/ she must able to count, men like 1 man, 2 men, 3men…,pens like 1 pen, 2 pens…, days like 1 day, 2days…frequency of teachers' smile in a day like 3 times in a day, 5 times in a day…,etc. Moreover, numerologist treat numbers in different way. Every object and living being within the universe can be explained, revealed and/or reflected through numbers (Sharma & Jaipuria, 2011). By viewing numbers like these, one understands that several other mathematical objects can view in broad sense by extending own believe.

Mind is abstract. Considering human's mind as a complex super computer, anything can programmed as core language and anything can grasp according to these core programming language. Therefore, the concept (philosophy) of life that human believe determines and influences somehow the action and reaction of humans. Therefore, we should not focus only on subject matter. We need to think about system of mind that is primarily based on own believes and assumption that can be adjustable.

Educational researchers and developmental psychologists spent lot of time in searching 'how learning occurs'. Rate of learning influenced by what learner consider self as. Learning should not considered as occur able in predesigned manner. Uncertainty should remain for creative learning. At least learner should aware that everything he/she learnt, based on human believes and assumption that words linked with something and hence what one consider true can be false for other in other circumstances. Todays' expansion of knowledge challenging us to be smart learner. Which type of knowledge will be important in future, is now being hard to identify and hence in such situation self-awareness in education is very important.

Statement of the problem

Looking back to history of educational psychology, many researcher efforts to recognize patterns of learning activity especially in humans. Diverge results were found and were categorized in different schools of thought mainly three broad categories are behaviourism, cognitivism and constructivism. However, central focus of all is to identify details about learning like 'How learning occurs?', 'Does learning follow some predictable pattern?', 'How attitude of learning can increase?' etc.

Metacognition mainly stress on self-thinking about own learning. Here learning mean general learning about anything in real world. Metacognition primarily focus on learning or knowing or understanding or aware about and way of self-learning. Metacognitive strategies refers to any process or method of learning that enhance learners' metacognition.

Metacognition can considered as a high-level thinking skill that actually related with making learner as successful problem solver.

In this twenty first century, science and technology are changing rapidly but unfortunately teaching methods is not advancing as required. Focusing on knowing some particular things or idea is now ineffective strategy. Building learners' own learning capabilities by realizing self-feeling of knowing is important and effective strategy for learning. Until now, we do not know fully about such strategy. However, researches on it is growing day by day. In context of Nepal, researches relating with metacognition is very low. Therefore, this research is result of positive thinking that we can explore metacognition deeply.

We are spending lots of time to learn systematic structure of teaching and learning. In other side, science and technologies are developing rapidly and hence we do not know which specific type of knowledge will be necessary in future. Students spent approximately one and half decade in schools and colleges for education. If schools and colleges are unable to give suitable education to students their whole life will not quality life. Researcher is irritate with such teachings which does not focus on students' ability to think about own knowledge and feel of knowing. Researcher believed that learning comes only if student himself or herself can feel and can think about own learning level. However, researcher did not found such learning in his real academic life. As per his expectations and feeling about learning, he want to explore deeply about teaching and learning. This study is initial stage for it. Viewing broadly, this study will sketch some rough on metacognition and strategies for learning by enhancing metacognition of learner.

This study will show focusing on learner's understanding about own thinking approach of teaching is effective teaching. This study come into existence because of the unsatisfied teaching strategies throughout researcher's academic life. At the time of teaching at school, researcher realized there is something wrong with our strategies followed in teaching. Researcher realized then metacognitive strategies seems to be effective for teaching. This study then set following research questions:

- Does metacognitive strategies better for teaching learning activities?
- How to apply metacognitive strategies in classroom by teachers?
- What effect does metacognitive strategies make on mathematical achievement?
- What changes in behavior of learner's can occur after teaching with metacognitive strategies?

Objectives of the study

This study has main purpose to introduce a higher-level thinking and identifying the ways of teaching with understanding. Further, the specific objectives of the study are as follows.

- To identify effectiveness of metacognitive strategies in mathematical achievement
- To trace behavioral changes of learner's during the teaching and learning with metacognitive strategies

Significance of the Study

The following rationale of the study are point out by researcher:

- This study trace different dimension of learning which deals with learner's self-cognition. This gives clear indication to effective learning.
- This study gives a glance to students, about feeling the higher level of thinking that students can follow for higher cognitive development.
- This study can be reason of rising interest in cognitive theories of learning.
- Metacognitive learning strategies can foster an individuals' lifelong learner attitude.
- Metacognition can enhance strength of an individual to cope with everyday new social and psychological problems. This study create some guidance to enhance metacognition.
- Metacognitive strategies can make an individual into successful problem solver (not any particular problem) that is an ability to solve problems in new unknown context. This study can help to begin step in journey of metacognition.

Delimitation of the study

The study will have the following delimitations:

- This study focus metacognition in mathematics achievement only. However, it can done in other subjects also.
- This research was conducted in a public school of Ramechhap district only with limited sample size.
- This research was conducted using limited tool for data collection.
- This study included only limited students of the school's tenth level.
- Less number of students and limited time devoted study may not generalized to all cases.
- The research was conducted in lower aged students only.

Hypothesis of the study

Research Hypothesis

Teaching with metacognitive strategies will provide better result in terms of students' achievement in mathematics achievement test.

Statistical Hypothesis

H_0: There is no significant difference between the achievements of mathematics using metacognitive strategies in teaching learning.

i.e. H_0: $\mu_1 = \mu_2$(Null Hypothesis)

H_1: There is significant difference between the achievements of mathematics using metacognitive strategies in teaching learning

$i.e. H_1$: $\mu_1 \neq \mu_2$ (Alternative Hypothesis)

where μ_1 and μ_2 are the corresponding parametric means of achievement of the students before using and after using metacognitive strategies respectively.

Operational Definition of Terms

Cognition: Mental action or process of acquiring knowledge through thinking, experiences and senses.

Metacognition: knowing about own cognition and understand to monitor own cognitive actions, thinking about thinking, cognition about cognition

Metacognitive Strategies: The methods used to help students for understanding the way they learn or methods that primarily focus on enhancing metacognition. Planning, Self-questioning, Good communication, Encouragement, Paraphrasing / Reflection and self-evaluation are metacognitive strategies.

Thinking*:* An act of being consider on something or reasoning about something.

Chapter II

Review of Related Literature

Human life surrounded by many problems and not all problems have solution. One positive view can be developed that is we must try to search knowledge about solution of problems and recorded our founds, so that at least in future our children, grandchildren and great grandchildren will use it effectively. In normal view, this process is research. If we spent time on search for, what our ancestors had already found, is meaningless. To determine what our ancestors had already found, literature review is required. Further, we expect to get ideas for our research and assures research is useful. In this section, various concepts about metacognition, typology of metacognition, Flavell's model of cognitive monitoring, two components of metacognition (Knowledge of cognition and regulation of cognition), metacognitive theories, metacognitive strategies, Evaluation of growth in metacognitive activities, review of some empirical studies and conceptual framework were discussed.

Metacognition

Metacognition is 'cognition about cognition', 'thinking about thinking', 'knowing about knowing', becoming aware about one's awareness and higher order thinking skill. American developmental psychologist John H. Flavell in 1979, first used the term Metacognition to refer high level cognition. Metacognition is our ability to know that we know and what we don't know (Costa, 1984). Knowledge about cognition and skills that enabling the self-regulation of cognitive process, is metacognition (Toit & Kotze, 2009). Metacognitive according to (Schraw & Moshman, 1995) is an individual's ability to manage, monitor and take action on organized thinking and in the correct order. The three aspects: knowledge of one's knowledge; the conscious monitoring and regulating of one's knowledge; and cognitive and affective states should include in the definition of metacognition (Toit & Kotze, 2009). “Metacognition refers to people’s knowledge of their own information processing skills, as well as knowledge about the nature of cognitive tasks and of strategies for coping with such tasks. Moreover, it also includes executive skills related to monitoring and self- regulation of one’s own cognitive activities” (Schneider & Artelt, 2010,p.149). Metacognition refers to reflexive thinking about learning, thinking about learning to raises questions related with truth, trust, openness, and intrinsic worth and even about how one should to spend one’s time (Ellis, Denton, & Bond, 2014).

From a historical perspective, the concept of Meta memory explored first. Flavell’s (1971) conception of Meta Memory was global, encompassing knowledge of all possible aspects of information storage and retrieval. According to Schraw & Moshman (1995), Metacognition consists of two major components knowledge and regulation.

Knowledge of Cognition

Individual’s knowledge about their own cognition or about cognition in general is known as knowledge of cognition. Schraw & Moshman (1995) classify knowledge cognition in three types of metacognitive awareness namely: declarative knowledge, procedural knowledge and conditional knowledge. Knowledge about things is known as declarative knowledge. It includes knowing self as learner and knowing about factors that influence one’s performance. Knowledge about how to do things is known as procedural knowledge. It is knowing about how to perform the procedural skills. A person having good procedural knowledge use skillful actions automatically and they use qualitatively different strategies to solve problems. Knowledge about when and why to apply various cognitive actions, is known as conditional knowledge. It is knowing 'why' and 'when' aspects of cognition. It may be thought of as declarative knowledge about the relative utility of cognitive procedures. Conditional knowledge continues developing at least through middle childhood as suggest by recent studies. Many theorists believe that metacognitive knowledge appears early and continues to develop at least throughout adolescence.

Regulation of Cognition

Regulation of cognition refers to metacognitive activities that help control one's thinking or learning. Regulation of cognition includes three skills: planning, monitoring and evaluation. Planning involves the selection of appropriate strategies and the allocation of resources that affect performance. Experienced learners have more knowledge about cognition and use it to regulate their learning.

Monitoring refers to one's on- line awareness of comprehension and task performance. For example, the ability to engage in periodic self-testing while learning. Monitoring ability develops slowly and is quite poor in children and even adults. However many studies have found link between metacognitive knowledge and monitoring accuracy. Training and practice improves the monitoring ability.

Evaluation refers to appraising the products and regulatory processes of one's learning. Its' typical example include re-evaluating one's goals and conclusions. Metacognitive knowledge and regulatory skills such as planning are relate to evaluation.

Flavell's model of cognitive monitoring

Flavell (1979) presents a model of cognitive monitoring. Through the actions of and interactions among the following four classes of phenomena Metacognitive Knowledge, Metacognitive experiences, Goals (or Tasks) and Actions (or strategies) monitoring of cognitive monitoring occurred. The knowledge about what factors or variables act and interact in what ways to affect the course and outcome of cognitive enterprises is metacognitive knowledge. Further, the three major categories of these factors or variables are person, task and strategy. Fundamentally, metacognitive knowledge are stored on long-term knowledge same as the other knowledge. Metacognition can activated by conscious search in memory. It activated unintentionally and automatically by retrieval cues in the task situation.

Metacognitive experience is experience of momentary sense of puzzlement that subsequently ignore or may wonder of sometime whether really understand something. These experiences can be brief or lengthy in duration, simple or complex in content. These experiences can occur at any time before, after, or during a cognitive enterprise. For example: An individual may feel that he/she is liable to fail in some upcoming enterprise or that he/she did very well indeed in some previous one. Metacognitive experiences are especially likely to occur in situations that stimulate a lot of careful, highly conscious thinking. Metacognitive experiences occur in the situations where every step requires planning beforehand and evaluation afterwards, where decisions and actions are at once weighty and risky. In Such situations many opportunities for thoughts and feelings about own thinking arise. Metacognitive experiences can have very important effects on cognitive goals or tasks, metacognitive knowledge, and cognitive actions or strategies. Metacognitive experiences can affect metacognitive knowledge base by adding to it, deleting from it or revising it. Metacognitive experiences can activate strategies aimed at either of two types of goals: cognitive and metacognitive. Cognitive strategies are invoke to make cognitive progress and metacognitive strategies to monitor it.

Hence, according to this model, the monitoring of cognitive enterprises proceeds through the actions of and interactions among metacognitive knowledge metacognitive experiences, goals/tasks, and actions/strategies.

Metacognitive theories of Schraw & Moshman

According to Schraw & Moshman (1995), metacognitive theories refer to how individuals combine variety of metacognitive knowledge and regulatory skills into systematic cognitive frameworks. Metacognitive theories are theories that integrate one's knowledge about cognition and regulation of cognition. Metacognitive theories are a subset of theories of mind. Metacognitive theories are those theories of mind that focus on cognitive aspects of mind beside these, theories of mind address other mental phenomena such as emotion, personality and so on.

It is important to distinguish phenomena the theory is about from the structured knowledge that comprises a theory. During theorizing about cognition, individuals create and synthesize metacognitive knowledge. All theories are cognitive in that they are structures of knowledge but not all theories are about cognition. Metacognitive theories are theories about cognition and it comprise metacognitive knowledge. Theories about metacognition would constitute meta-metacognitive knowledge but such theories represent only a subset of metacognitive theories. The main characteristic of Metacognitive theory is that it allows an individual to integrate diverse aspects of metacognition within a single framework.

Types of Metacognitive Theories

1) Tacit
2) Informal
3) Formal

Tacit: Simply, tacit theories are any theory that an individual understood without being stated. An individual mentally construct several theories about any things or activities. Theories that constructed without any explicit awareness are consider as tacit theories. Tacit theories are theories about anything. Tacit theories about own cognition or metacognition is tacit metacognitive theories. These theories are implicit organizational frameworks that systematize one's metacognitive knowledge. These theories are gradually constructed. An individual makes beliefs about cognition that form the core of one's metacognitive theory. These beliefs can acquired from peers, teachers, family or cultures. Moreover, an individual's metacognitive theory may constructed tacitly based on personal experience or adaptions from others. However, these theories are persistent even these are false and maladaptive until they remain tacit. Tacit theories may be difficult to change even when individuals are encouraged explicitly to do so.

Informal: Informal theories are those in which individuals are aware of some of their beliefs and assumptions regarding a phenomenon, but have not yet constructed an explicit theoretical structure that integrates and justifies these beliefs. Informal theorists may have only a rudimentary awareness of their own metacognitive knowledge. Increasing the depth and breadth of metacognitive theories over time may allow informal theorists to better understand and direct constructive processes. Emerging recognition and control of constructive processes as an essential feature of informal metacognitive theories that is not founds in tacit theories. Awareness of the constructive nature of knowledge and theories is important. It helps individuals being able to modify strategically their theories and consequently should be able to regulate their cognition and learning.

Formal: Formal theories consist of highly systematized accounts of a phenomenon involving explicit theoretical structures such as those encountered in university classes in physics, music or statistics. Formal theories about one's performance or anything else are rare. Presently, it is unclear what constitutes a formal metacognitive theory of one's cognition. It is likely that formal theorists possess some explicit awareness of the constructive nature of theorizing and engage in purposeful efforts to construct and modify metacognitive theories. One potential advantage of a formal metacognitive theory is that it allows the individual to make informed choices about self-regulatory behaviours.

In summary, Metacognitive theories form a naturally occurring hierarchy of knowledge about cognitive and metacognitive processes. Tacit theories provide limited guidance and explanatory power. These theories are characterize by loosely systematized knowledge and postulates that are not known consciously by the theorist. Informal theories are partially accessible to the theorist and presumably play a greater role in self-regulation. Formal theories provide an explicit frameworks for understanding and regulating one's cognition.

Metacognitive strategies

Metacognitive strategies refer to the conscious monitoring of individual's own cognitive strategies to achieve specific goals (Ellis, Denton, & Bond, 2014). "Metacognitive learning strategy is a strategy that employs students naturally, actively and constantly observing metacognitive skills and behaviours, knows how to learn effectively, be sensitive to strengths and weaknesses, and is efficient in planning, monitoring and evaluating current learning." (Bakar & Ismail, 2020). Metacognitive strategy is the method used to help students for understanding the way they learn or method that primarily focus on enhancing metacognition. Encouraging students to reflect on and evaluate their activities can support the metacognitive development of students (Kuhn & Dean, 2004).

Following strategies can enhance metacognition according to Costa (1984,pp.59-62).

1) Planning Strategy
2) Generating Questions
3) Choosing Consciously
4) Evaluating with multiple Criteria
5) Taking Credit
6) Outlawing “I Can’t”
7) Paraphrasing or Reflecting Back Students’ Ideas
8) Labeling Students’ Behaviors
9) Clarifying students’ Terminology
10) Role playing and Simulations
11) Journal Keeping
12) Modelling

Planning Strategy: Students should allow planning for solving problem or some cognitive task. For this, first teacher should point out the strategies and steps for attacking problems, rules to remember and all basic requirements. Further, teacher should make clear about reactions to follows, time constraints, purposes and ground rules under which students must operate. Moreover, teacher can invite students to share their progress thought processes and perceptions of their own behaviour. Further teacher ask students to define alternative problem solving pathways. It provides teachers with a diagnostic cognitive map of students thinking Then, after the learning activity, teachers can invite students to evaluate how well the rules are obeyed, how productive the strategies were etc.

Generating Questions: learners should ask themselves what they know and what they do not know before learning activity. Students have to generate questions about textual material related with their own prior knowledge. They should inquire their comprehension level in themselves. If they know the main concept and idea of subject matter. They can give other examples and can predict what may come next. They must then decide what strategic action to take to remove any obstacles to their comprehension.

Choosing Consciously: Teacher should help students to explore the consequences of their choices and decisions prior to and during the act of deciding students will then be able to perceive casual relationships among their choice, their actions and the results they achieved. Teacher should provide non-judgemental feedback to students about the effects of their behaviours and decisions on others. By non-judgemental feedback, students are not taking command/or order of teachers but taking a glance of their actions and results that shall happen.

Evaluating with multiple Criteria: Students themselves should try to create criteria to evaluate their own thinking and actions. Initially, teacher and students jointly create criteria of evaluation. Metacognition can enhance by reflecting upon and categorizing their actions according to two or more sets of evaluative criteria. Learner should ask self to identify helpful aspects and hindering aspects in learning something. Teacher should guide to form self-evaluation about students' actions, plans and even their self-constructed criteria.

Taking Credit: Teacher can help students to identify what they have done is well and can invite them to seek feedback from their peers. Teacher might ask, 'what have you done that you're feeling proud?' and 'How would you like to be recognized for doing that?' (Name on the board, applause from group etc.). Students will become more conscious of their own behaviour and apply a set of internal criteria for those behaviours that they consider good.

Outlawing "I can't": Many students use the terms like, "I Can't…", "I do not know...", "I never able to do …", "I am too slow to…" etc. Teacher should inform students that these behaviours are strictly unacceptable in classroom. Instead of it, teacher should suggest students to identify what information is required for it, what skills are lacking in their ability to perform the desired behaviour. Students should focus on 'How can I know or do it?'.

Paraphrasing or Reflecting Back Students' Ideas: Inviting the students to restate, translate, compare and paraphrase each other's ideas causes them to become not only better listener of other's thinking, but better listeners to their own thinking as well.

Labelling Students' Behaviours: When teachers place labels on students' cognitive processes, students become conscious of their own actions. Like, 'What I see you doing is … making out a plan of action for…'. 'What you are doing is called an experience…"

Clarifying Students' Terminology: Students often use hollow, vague and nonspecific terminology. Teacher needs to clarify the students' such terminologies. It is also helpful to clarify students' problem processes causing students to describe their thinking while they are thinking seems to beget more thinking. Teachers can invite students to talk aloud as they are solving problem, discuss what is going in their heads. After a problem solved, teachers can invite clarification of the processes used.

Role Playing and Simulations: Role-playing can promote metacognition because when students assume the roles of other persons, they consciously maintain the attributes and characteristics of that person. Dramatization of how that person would react in a certain situation. Taking on another role contributes to the reduction of ego-centred perceptions.

Journal Keeping: Writing and illustrating a personal log or a diary throughout an experience causes students to synthesize thoughts and actions and to translate them to symbolic form. The record also provides an opportunity to revisit initial perceptions to compare changes in those perceptions with addition of more data, to chart the processes of strategic thinking and decision making, to identify the blind alleys and pathways taken, and to recall the successes and the tragedies of experimentation.

Modelling: Of all the instructional techniques suggested, the one with the probability of greatest influence on students is that of teacher modelling. Since students learn best by imitating the adults around them, the teacher who publicly demonstrates metacognition will probably produce students who metacognitate. Some indicator of teachers' public metacognitive behaviour might be sharing their planning- describing their goals and objectives and giving reasons for their actions, making human errors and then illustrating recovery from those errors by getting back on track, admitting they do not know an answer but designing ways to produce an answer, seeking feedback and evaluation of their actions from others, having a clearly stated value system and making decisions consistent with that system, being able to self-disclose-using adjectives that describe their own strengths and weaknesses, demonstrating understanding and empathy by listening to and accurately describing the ideas and feelings of others.

Evaluation of growth in Metacognitive Abilities

Costa (1984) suggest the following aspects in evaluating the growth in students' metacognitive abilities:

We can determine if students are becoming more aware of their own thinking as they are able to describe what goes on in their heads when they are thinking. When asked, they can list the steps and tell where they are in the sequence of a problem solving strategy. They can trace the pathways and dead ends they took on the road to a problem solution. They can describe which data is lacking and their plans for producing those data. We should see students becoming more perseverant when the solution to a problem is not immediately apparent. This means that they have systematic methods of analysing a problem, knowing ways to begin, knowing what steps must performed and when they are accurate or are in error. We should see students taking more pride in their efforts, becoming self-correcting and accuracy in their products and becoming more autonomous in their problem solving abilities.

Review of Some Empirical Study

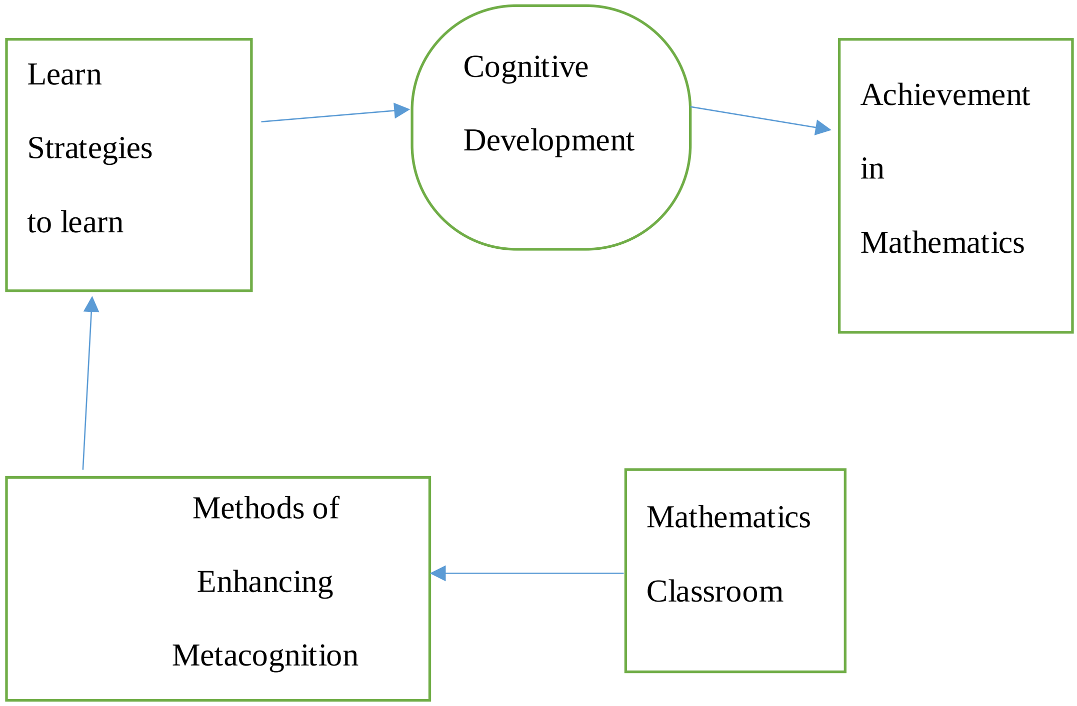
Study of Gautam (2015) entitled "Attitude of Secondary level mathematics teachers towards Action Research" with objectives: to find the attitude of secondary level mathematics teachers towards action research and to find the teacher's experience on action research. To meet objectives of the study, researcher gather data by the method of questionnaire survey and likert type five points attitudes scale as tools. Researcher uses t- test, mean attitude score and percentage to determine the attitude of mathematics teacher towards action research. The major findings of the study was the secondary level mathematics teachers had positive attitude towards action research, teachers feel the action research as a tool to solve classroom problems and teachers have expectation of extra facilities and payment for performing action research.

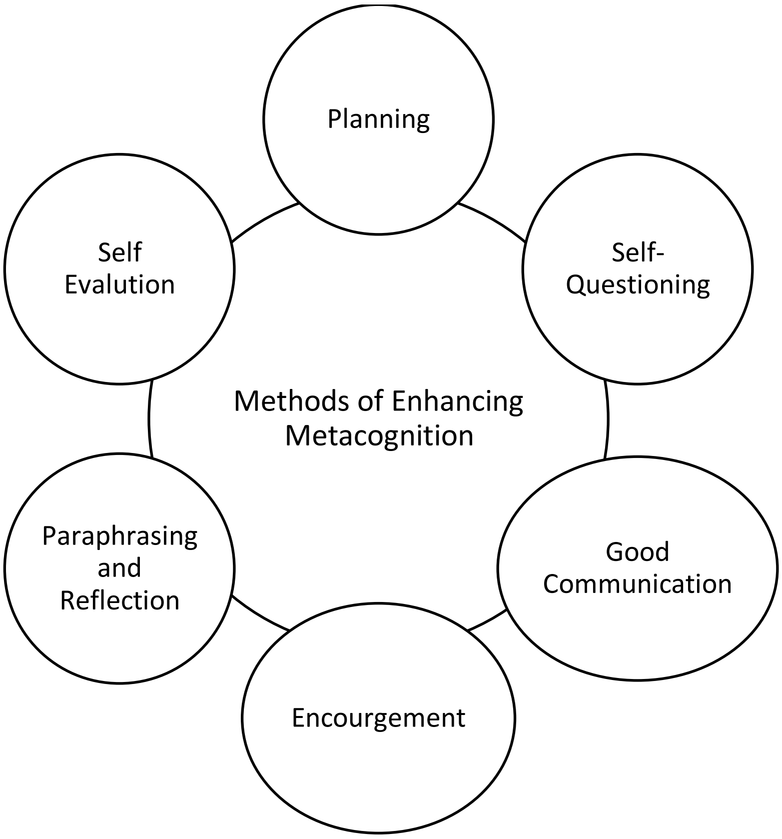
Rayamajhi (2019) conduct a research entitled “Tracing the change in learning Mathematics using the concept of ZPD: An action research”. The main objectives of the study was to trace the change in achievement using the concept of ZPD in classroom teaching and to explore the change in learner’s behaviour, attitude and participation in classroom. The study based on the Vygotsky’s social constructivism theory. This research was qualitative and qualitative (Mixed design). This study uses pre-test and post-test design and use t- test to compare mean scores before and after using ZPD. Moreover, interview with students and memo- writing helps to lead the conclusion. Study shows the mean score after using the concept of ZPD is significantly greater than before using the ZPD and positive responses showed by students. Further, Rayamajhi (2019) conclude that teacher need to conduct action research to identify new and effective learning ways.

Research of Kunasaraphan (2018) entitled ‘Classroom Action Research for teaching Mathematics in Secondary Schools’ was conducted for purpose of investigating compatibility among class levels, mathematics contents, and teaching and learning strategies of classroom action research conducting in secondary schools. The participants were 22 graduate students and the researcher collect data from 22 theses conducted by classroom action research method and then analysed the data by explaining compatibility among class levels, mathematics contents, and teaching and learning strategies. The results showed that students applied different teaching strategies to conduct classroom action research in secondary schools in order to increase students’ mathematics achievement.

Conceptual Framework

The conceptual framework of this study represents the relationships with metacognitive strategies and achievement in mathematics. In mathematics classroom, the researcher will apply methods of enhancing metacognition and hence students learn strategies of how they learn. Once student learn how to learn, they are able to develop their cognition. At last, it results to better achievement in mathematics. By methods of enhancing metacognition planning, self-questioning, good communication, encouragement, paraphrasing / reflection and self-evaluation will be consider throughout this research. The strategies of enhancing metacognition are mainly base on ideas of (Costa, 1984). The better achievement in mathematics will be verify by constructing pre-test and post-test design with mean comparison by using t-test. The researcher make this concept for this research.





a) Planning: First teacher should instruct about the ground rules to follow, steps to perform and all basic requirements. Now, students should allow making plan of the proving theorem or making solutions of the problems. Initially, teacher can show the process of making plan through discussion and asked students to make alternative version of the plan. It employs student in searching ideas on own mind about possibilities.

b) Self-Questioning: Self-question is very important aspect of entering into own metacognition. Teacher should well-manage classroom environment so that students may motivate to self-questioning about subject matter, own feelings and thoughts. Further students should asked to formulate answers and solutions of self-generated questions. Such self-generated questions and answers can discussed in classroom and teacher should provide or ask students to guess results of what their question and answer shows. Learners should ask themselves what they know and what they do not know before learning activity. Students have to generate questions about textual material related with their own prior knowledge. They should inquire their comprehension level in themselves. If they know the main concept and idea of subject matter. They can give other examples and can predict what may come next. They must then decide what strategic action to take to remove any obstacles to their comprehension.

c) Good Communication: Students and teacher should have good communication so that students will share their thought process and feeling to teacher. Here, Good communication means the two-way communication between teacher and student in which sharing of views and beliefs for sake of metacognitive development of students. Students often use hollow, vague and nonspecific terminology. Teacher have to clarify the students' such terminologies. Teachers can invite students to talk aloud as they are solving problem, discuss what is going in their heads. After a problem solved, teachers can invite clarification of the processes used.

d) Encouragement: Teachers' encouragement can employ students to focus on inner voice of students. Teacher should encourage students to remove egocentric behaviours, to identify own construction strategy about solving problem, to cooperate with peers etc. Teacher can place labels on students' cognitive processes so that students become conscious of their own actions. Many student uses the terms like, "I Can't…", "I do not know...", "I never able to do …", "I am too slow to…" etc. Teacher have to encourage students to remove such behaviours. Further, teacher should suggest students to identify what information is required for it, what skills are lacking in their ability to perform the desired behaviour. Students should focus on 'How can I know or do it?'. Teacher can help students to identify what they have done is well and can invite them to seek feedback from their peers. Teacher might ask, ‘what have you done that you’re feeling proud?’ and ‘How would you like to be recognized for doing that?’ (Name on the board, applause from group etc.). Students will become more conscious of their own behaviour and apply a set of internal criteria for those behaviours that they consider good.

e) Paraphrasing and Reflection: Students’ ideas can paraphrase in order to enhance metacognition. Inviting the students to restate, translate, compare and paraphrase each other’s ideas causes them to become not only better listener of other’s thinking, but better listeners to their own thinking as well. Teacher should motivate students to do reflection about own thinking process. Further, role-playing and dramatization can be helpful in metacognition enhancement because when students assume the roles of other persons, they consciously maintain the attributes and characteristics of that person.

f) Self-evaluation: Students themselves should try to create criteria to evaluate their own thinking and actions. Initially, teacher should help students to create criteria of evaluation. Metacognition can enhance by reflecting upon and categorizing their actions according to two or more sets of evaluative criteria. Learner should ask self to identify helpful aspects and hindering aspects in learning something. Teacher should guide to form self-evaluation about students’ actions, plans and even their self-constructed criteria. Moreover, Writing and illustrating a personal log or a diary throughout an experience causes students to synthesize thoughts and actions and to translate them to symbolic form. The record also provides an opportunity to revisit initial perceptions to compare changes in those perceptions with addition of more data, to chart the processes of strategic thinking and decision making, to identify the blind alleys and pathways taken, and to recall the successes and the tragedies of experimentation.

Chapter III

Research Methodology

This section describes the design of the study, action research, research site and sample selection, data collection tools and instruments for the action research, test items, Memo-writing, Interview schedule, reliability and validity of tools, data collection procedure and data analysis process.

Design of study

This research was based on an action research design. The main purpose of the study is to identify effectiveness of metacognitive strategies in mathematical achievement and tracing behavioural changes of learner's during the teaching and learning with metacognitive strategies. This study was based on qualitative and quantitative both nature of data. Quantitative data was obtained from test and qualitative data was obtained from interview and memo writing.

Action research

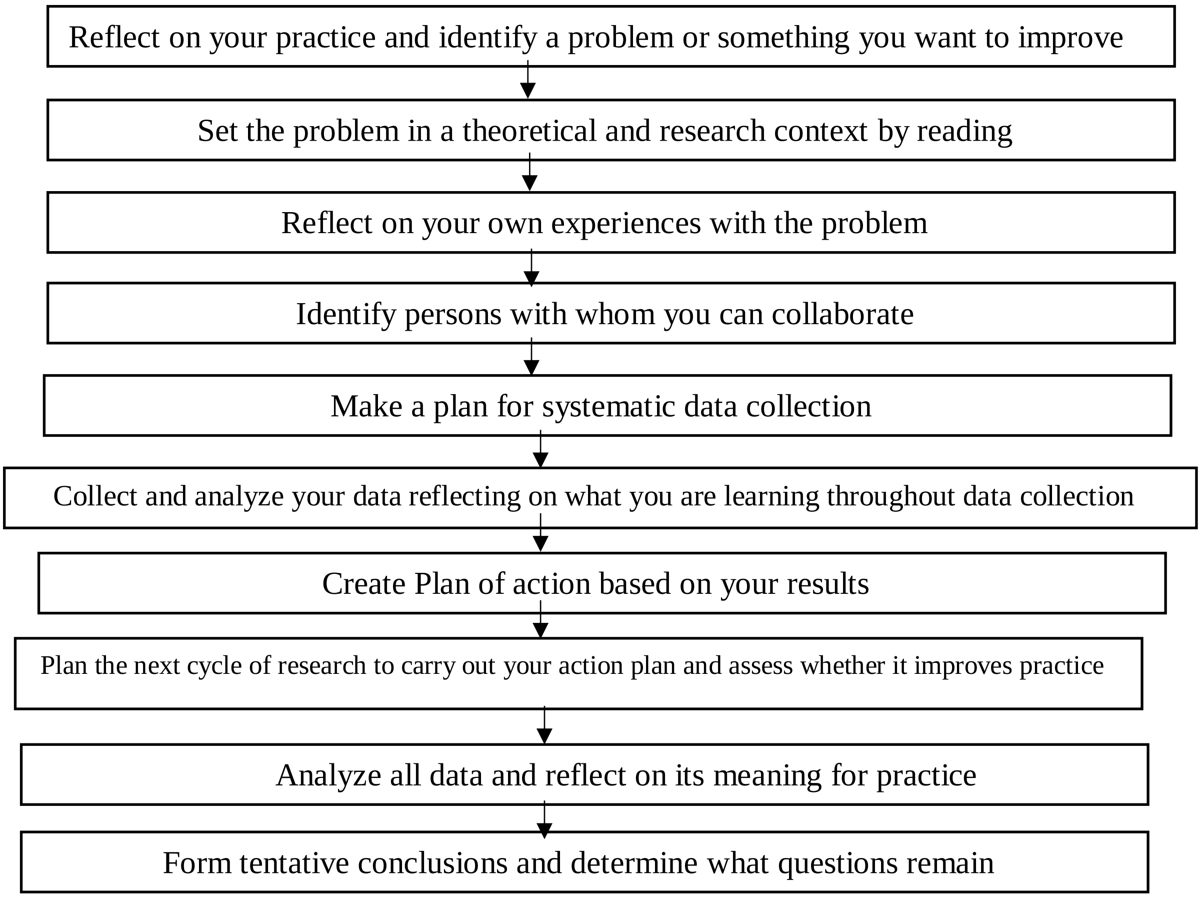
According to (Mills, 2003), action research can defined as gathering information by any systematic inquiry that conducted by teacher researchers to improve their teaching.

Action research is a systematic inquiry into a self-identified teaching learning problem to understand better its complex dynamics and to develop strategies geared towards the problem's environment. The action research process can result in professional development, education change, enhanced personal awareness, improved practice and new learning (Khanal, 2074). The action research is always trying to improve the phenomena of the surroundings (Upadhyay, Pradhan, & Dhakal, 2067). An ongoing process of reflection and action to produce the possible most effective learning environment is an action research (Kunasaraphan, 2018). Action research is a form of disciplined inquiry made by teacher with intent that the research will inform and change his/ her practices in the future. It is a process of monitoring own teaching and taking steps to improve it.

According to (Lodico, Spaulding, & Voegtle, 2006) characteristics of action research are as follows:

> Action research should conducted in the practitioner-researcher's own educational setting and the practitioner takes an active part in the research. It involves collaboration with other educators and persons involved in the educational process.
>
> It focuses on taking action to change and improve educational practices. It is ongoing and includes several waves of data collection, reflection and action.

Further, (Lodico, Spaulding, & Voegtle, 2006) presents following steps involved in action research:



Research site and sample selection

For the research, researcher had selected a public school from Ramechhap. The researcher himself had taught in the school and the research had conduct for class ten mathematics. Duration of research has been one month.

To identify effectiveness of metacognition strategies in mathematical achievement, researcher had conduct class for tenth grader students applying metacognitive strategies for one month. Comparison between mean of achievement on Pre-test and post-test was done using t-test for data analysis. To trace behavioural changes of learners during the teaching and learning with metacognition, researcher did careful observation on activities of learners during the teaching and with metacognitive strategies.

Data Collection Tools

Researcher used mainly following tools for data collection:

a) Memo Writing

b) Interview Schedule

c) Test

Test items

Test is a tool for collecting data from students. A test item is the basic unit of interaction on a test. In this study, five subjective questions relating the theorem proving and five questions relating the application of the theorems. The test was constructed from the course Mathematics of class ten. Test was taken from ten students. Actual number of students in the class was 12 but because of irregularity of two students, only regular ten students were included in this study. The content of test covered from area of triangle and quadrilaterals chapter from geometry section.

Memo-writing

Observation is active acquisition of information from primary source. In present context, source refer to student from whom researcher collect information. Observation is the way of observing closely the activities of the students in the context of this study. Mainly the observation was done to see the change of attitude, behaviour and participation of the students in the classroom. Researcher observed the students then kept the record in form of memo- writing. Memo is a written message that needed to memorise later. Researcher wrote memo about activities and behaviours of students. Moreover, teacher's reflection were written in memo.

Interview schedule

Interview is two-way conversations between two or more persons, which conducted based on pre-planned questions in order to get information from interviewee. Interview schedule is the basic list of set of structured questions that constructed according to pre-plan.

The questions of interview was constructed in such that they experienced metacognitive experience. The interviews of the students was conducted in order to find out the real problems of the students and feelings of the students about the teaching learning activities such as interest, opinion, behaviour and so on. The area of problems was relate to the learning mathematics with metacognitive experience.

Reliability and validity of tools

For reliability of the tools, the discussion with the supervisor of this thesis and experts were done. After the suggestion of supervisor and experts, the tools were selected for the research. Researcher consult with the supervisor and experts to prepare the test items, to write meme, to test the hypothesis and interview schedule. Content validity was preserved through teacher made test.

Data collection

First prior knowledge of students was checked through the pre-test. Then researcher made lesson plan to teach through metacognitive strategies. After that teaching was done according to lesson plan. After teaching, teacher wrote memo about behaviours and activities of students during teaching with metacognitive strategies. After teaching and memo writing, a mathematics achievement test was conduct to identify effectiveness of teaching with metacognition. Then pre-test and post-test were compared through statistical methods. Whenever achievement in post-test is not better then again planning was done for re-teaching with metacognitive strategies.

Data analysis

Two types of data: nominal and ordinal were obtained from the research. Nominal data was obtained from interview with students and memo writing. Such nominal data was analysed through descriptive way. Ordinal data was obtained from mathematics achievement test. Such ordinal data was analysed through statistical methods. After collection of data, data were tabulated and analysed according to objectives of the study. The score of tests were used to find the mean marks, test the hypothesis and used the quantitative data analysis to full fill the first objective. To full fill the second objective, researcher used triangulation analysis method. Triangulation analysis is the method used by researcher to establish and enhance validity and credibility of findings. In this triangulation method, researcher used multiple methods, data sets or theories to address research questions. This research is methodologically triangulated. In this study, triangulation method included observation, test and interview.

Chapter IV

Results and Discussion

This chapter describes about the analysis and interpretation of qualitative and quantitative collected data obtain from the study. The analysis and interpretation of the collected data were describes based on two objectives. In this study, quantitative data which is related to first objective (i.e. to identify effectiveness of metacognitive strategies in mathematical achievement) were analyse and interpreted first. After that qualitative data which is related to second objective (i.e. to trace behavioral changes of learner's during the teaching and learning with metacognitive strategies) were analyse and interpreted.

Analysis and interpretation of the information obtained from the test

This section mainly deals with the change in achievement using metacognitive strategies in teaching. Two tests one, before using metacognitive strategies and second one, after using metacognitive strategies in classroom teaching are considered here. The achievement of the each students before using metacognitive strategies and after using metacognitive strategies and analysis are shown below.

Analysis and interpretation of the information obtained from the test of each students

In table 2, which is shown in Appendix B showed the result of test shown in Appendix-N which were taken before using the metacognitive strategies and after using the metacognitive strategies. Scoring were based on marking guidelines shown in Appendix A. Scores obtained on pre-test refers to scores obtained before using metacognitive strategies and scores obtained on post-test refers to scores obtained after using metacognitive strategies.

The table shows that each students get higher marks in post-test. That means metacognitive strategies increases the achievement in mathematics.

Comparison of achievement in mathematics before and after using metacognitive strategies in teaching mathematics (Pre-test – Post –test achievement comparison)

The following is marks obtained by ten students on test items shown in appendix B, before and after teaching through metacognitive strategies. Name of the students are imaginary.

Name of Students	***X (Pre-test)***	***Y (post-test)***
A	61	100
B	60	100
C	47	86
D	32	66
E	31	67
F	32	66
G	24	59
H	20	50
I	12	48
J	9	52

Source: From classroom

Above table is used to test of hypothesis to determine the effectiveness of metacognitive strategies in teaching mathematics. The tabulated value at 0.01 level of significance with 9 degree of freedom is $t_{0.005,9} = 3.250$. The calculated value of t-test before using metacognitive strategies and after using metacognitive strategies is 28.98. Here, the calculated value is greater than the tabulated value for two tailed test.

That is $28.98 > 3.250$. So, the value does not fall in the acceptance region. Thus the null hypothesis is rejected. So that there was significance difference between the test taken before using metacognitive strategies and after using metacognitive strategies in teaching mathematics. So researcher concluded that after using the metacognitive strategies in teaching mathematics, the achievement of the students in the mathematics was significantly increased.

Hence, using the metacognitive strategies in teaching mathematics is effective. Test of hypothesis is kept in Appendix C.

Data from memo writing

For the evaluation of the learner's behaviour, attitude and activeness in the classroom while using the metacognitive strategies in teaching mathematics. The observation was done by researcher to see the attitude, behaviour and participation of the students then the researcher kept the record in the memo. The memo of each class including the activities of all students. In memo writing, researcher wrote the daily activities, expression of the students, reactions in the classroom participation in learning, changes of the students. Researcher used to write the memo everyday while using metacognitive strategies and the evaluation of the behaviour, attitude and participation of the students. The main noticeable behaviours and activities were as follows:

"In initial classes, students were not showing interest towards the subject matter of mathematics. After teacher taught basis requirements and informing about overall plan, students initiate to pay attention toward teacher. Moreover, after instructing all ground rules and steps to follow students showed extremely different behaviour such as listening to teacher carefully, No one were making noise in the classroom, behaving like being hopeful etc. Students were actively participate in

planning stage. Even in later classes, students themselves start to make plan. Some of them started to do act of teacher in later classes. After that in self-questioning stage, some students seems to deal with self-conflict. Students were trying to compare with their prior knowledge. Students showed strange behaviour like they connected with their prior knowledge and hence their confidence increased in later classes. The students seems little bit confused what to do and how to do. Some students especially chess players are highly active in this stage. Sometimes students made teacher irritate by asking so many question. They many times did not able to answer self-queries. Some students did not try to answer self-queries. In communication stage, students seems to be highly active in discussion and showed positive response. Teacher tries to ask students to share their real feeling and thinking. In starting classes many students did not show any noticeable response. However, in later classes, students try to share their feels and thoughts. Students feel good and showing happiness in their faces during the encouragement stage. They were inspired to learn more mathematics. In paraphrasing stage, students hesitate to say what they understands. Some students were saying to friends in classroom that "The theorem I already understand but I cannot say it in my own way". While students tries to paraphrase, they do it with missing concepts in initial classes. However, in later classes some students paraphrase perfectly. Students were not able to pay enough focus on their own knowledge in initial class. They feel disconnected with their own knowledge in initial classes. However in some extend they evaluate their own knowing."

The planning stage was taken positive by students. Recalling basic requirements for the lesson and ground rules to follow throughout the lesson creates good environment in classroom. Students' curiosity were increased and hence students seems ready for the learning. Students were able to plan themselves after practicing it for more days. Students focusing on their own ideas about lesson was low in earlier days but after practicing it more students were able to questioning to self. During self-questioning stage students usually try to compare and connect the lesson with prior knowledge. In planning stage, all students shows positive response to the teacher. This stage helps teacher to make positive impression of students toward teacher and the subject matter. This stage helps to make mental roadmap for the lesson and hence becomes curious about subject matters. In self-questioning stage, students shows strange behaviours. This stage enables students to frequently ask question. Self-questioning very important to enhance one's metacognition. Class ten students seems to lower participation in self-questioning. In this stage, students search for their memory. Teaching through discussion and question asking process helps to grasp the concept with minimizing distractions of the students. Encouraging students really impressed the students and they were highly motivated to learning mathematics. During encouragement, all students showed positive attitude and behaviours. Students feel difficult to translate something in their own word because it required higher mental action. However, some students easily translate the lesson in their own simple way. In self-evaluation stage, students feel difficult how to do. Exactly knowing 'what I know and what I do not know' was hard for many students. However, writing what's going on in students mind became helpful to students for memorizing the lesson.

Data from interview

Teaching for enhancing learner's conscious about own cognition is most important and difficult as well. It mainly depends upon the students' own reflection and metacognition. Metacognition increases as student honestly pay attention on what going on in the mind.

For the questions in interview schedule shown in appendix O following responses were obtained:

Student A:*"I think teacher starting with instruction of basic rules and all necessary pre-requirements is good. It makes class interesting. Telling about overall plan gives us direction to think about the lesson.*

I usually never self-question and answered by myself in classroom. At beginning, questioning to self and answering by my self seems very difficult to me but by practicing it become easier now. It becomes easier also because of my chess playing habit. It actually enables me to questioning every time on every things. Some time, I make my teachers, friends and parents angry by too much questions. However, I feel my brain is really working at the time of self-questioning. I think self-questioning actually develops our intuition. Materials and visual illustration is really helpful in self-questioning and answering it because it gives us some mental image about theorem. So that we can visualize theorem.

Teacher and students should have good communication, so that students may never hesitate to ask about their doubts in lesson. Good communication of teacher and students actually positively impact on learning. It makes friendly with teacher and subject matter. In such situation, I feel every lesson is very easy and I understands every lessons that taught through metacognitive strategies.

Encouragement by teacher is very good that really give us positive energy to learn mathematics more.

In the beginning, I feel very difficult to transfer the whole lesson in my own words. I understands lesson but I cannot translate it in my own words. At the time of paraphrasing and reflection, so many doubts are arises in my mind. Reflecting with own thought is really very difficult. However after doing it many times it initially *makes me an honest person. Then, many ideas with doubts comes in my mind simultaneously. Sometime, the theorems that I already understands but my inner feeling not accepting that I understands it.*

Self-evaluation required careful and honest attention toward what's going on in our mind. It is difficult task but very important for good understanding the lesson. Writing whatever I understands about the lesson is very helpful to memorize it later".

Student B:

"I love such classes where teacher focus on basic things because we already forgot what we need to remember. Telling about overall plan of proving theorem makes us concentrated and creates interest. Even it helps us to arise our new ideas. It makes clearly understandable to us.

Self-questioning is very important in learning process. By practicing self-questioning, we become more conscious about everything. However, always questioning about everything can creates depressed feeling because there is never end of questions. Overthinking habit can be the possible negative aspect of questioning self. Furthermore, it helps to develops good guessing skill. Materials and visual illustration actually makes easy to students for self-questioning because it create base for imagination about lesson.

Discussion with students at the time of teaching plays vital role in understanding. The concept learnt through discussion plays vital role in long-term memory.

Encouragement by teacher is very important in learning because it enhance positive energy of students. Lack of self-confidence can be one great reason of failure for students. So, encouragement is vital for students to be successful.

Allowing us to paraphrase and reflection enables our actual understanding. Translating in own way requires higher mental action. It is not easy to do but very important in learning.

Self-evaluation is actually necessary action to do for correct intellectual judgement. Actually the concept are known or unknown to learner can know honestly from self-evaluation".

Student J:

"At first, what are required rules should taught. Creating our base in mathematics is necessary. Telling overall plan of proving theorem is also good to us. It makes our correct mind set about lesson. Yes, I think at beginning of lesson, teacher must instruct all ground rule and steps to follow. It makes us active in class so that it positively enhance our rate of learning.

I think in classroom it is not possible to self-question and answer it. We have so many questions but we are unable to answer it. If we are too much force to answering for self-questions, it's just time wasting thing. Instead of that teacher should focus on teaching us. Material or visual illustrations of theorems helps us to understand theorem. I don't know how it effect on self-questioning. If talented students do self-questioning, it may enhance their learning but for me self-questioning is non-sense and I cannot do it in math class.

Yes, teacher should be very friendly with students. They should not shout on us even when we do some minor mistakes. If teacher is friendly with us we becomes fearless and stress free so that we can concentrate on learning. While teaching through discussion with students about subject matter, we understands what teacher is teaching to us. So, good communication with teacher positively impact our learning process.

I feel very motivated at the time of encouragement by teacher for learning. I think every teacher should learn how to encourage students to learn.

I think restating and translating of lesson in our own words is touching our heart. It may be right or wrong but it is truth which is located inside us. At that time, I only afraid that it may be wrong and teacher will shout at me. Otherwise I feel no difficulties.

Self-evaluation is a state in which we understands what we know and do not know about lesson. By writing what we understand, how honestly we can see our thoughts are".

The responses from interview of students were their feelings and thought about teaching using metacognitive strategies. From the interview it is clear that students actually believe that planning stage of metacognitive strategies is important to create interest on students toward the subject, building mental base for the lesson, create positive attitude towards teacher, participate students actively in learning, helps to arrive new ideas in mind about lesson etc. The Second stage self-questioning, seems little bit difficult to students. However, students believe that it helps them to develop intuition, become conscious about everything, develop guessing skills, activate brain's function etc. Some students believe that playing chess help to self-question. Materials and visual illustrations are very useful to create mental image about theorem and it is basis for imagination and hence answering self-queries becomes easier. The students practicing answering self-queries seems to face metacognition highly. Neglecting importance of self-questioning by students face fear and anxiety during learning and there is actually no understanding of lesson by them. The third stage of metacognitive strategies, communication is positively taken by students. Students accept that classroom environment should well maintained so that students may not hesitate to ask to teacher about their confusions. Students believed that teacher should be friendly with students and it helps to create fearless and stress free environment in classroom so that it is helpful to familiar with teacher as well as subject matter. Students thought that good communication can positively impact on learning with understanding. In fourth stage encouragement, students' behaviours were positive toward teacher and the subject matter. They feel positively energies and becomes self-confident. The fifth stage paraphrasing and reflection became little bit difficult for students. However, students felt performing higher mental action during paraphrasing. It became arising doubt and arising honesty phase for students. The sixth stage self-evaluation requires student's honest and careful attention towards their thought and feeling. Students feels that self-evaluation is important for learning with understanding. Students believed that writing their thought kept in long term memory.

Analysis of students' reflection on metacognitive strategies

The planning stage creates interest in students to learn. It provides mental roadmap of lesson to students. It is building mental base of lesson to students. In the journey of experiencing metacognition this planning stage plays important role because after teaching basic requirements students tries to search in memory about knowledge which they acquired in previous classes. Actually planning stage create positive mind-set of students towards subject matter as well as teacher. Self-question is one of the best way to experiencing metacognition. In this stage, students goes to search in prior knowledge's memory to connect or compare with new knowledge. Because of ability to search in old memory students notice the similarities in old and new knowledge and hence they feel they are familiar with the concepts. Materials and visual illustrations support and geared to student's ability for experiencing metacognition. In comparison of chess player students and non-chess players, chess players have more ability in self-question because a good chess players ask themselves for the further steps to perform in chess. That means the metacognition level of chess players is higher than other non-chess players. However, in lack of correct understanding about lesson creates fear and anxiety in students. In the experiencing metacognition students must be honest himself. Communication stage helps students to share their thoughts and feeling with teacher. The concepts formed by students during the class were recalled in paraphrasing stage. In this stage students deal with their skill to regulate their thinking.

Analysis of behavioural presentation of students

Daily memo was writing in order to kept record of behaviour and attitude of students shown in different level of metacognitive strategies. Primarily, researcher noticed that students seem confused and feels difficulties to express inner feelings. Before using metacognitive strategies in teaching mathematics, students are not interested in mathematics especially in geometry. Applying metacognitive strategies in classroom is difficult and important task to teacher that researcher feel during teaching mathematics with metacognitive strategies.

During teaching mathematics through metacognitive strategies, researcher faced many difficulties. Actually, thinking about thinking is not an easy job. Educating from metacognitive perspective requires enough attention towards 'what's going on the mind'. Actually, maturity of students also influence the ability to thinking about thinking. Using metacognitive strategies in teaching can be difficult in initial stage but after practicing it more better results in enhancing students' metacognition hence increase in level of understanding.

Initially, students have created tacit theories that they realise they understand but they cannot state it. In Self-questioning, reflection and self-evaluation stages students focusing on their own cognition so students having good metacognition level feel ease in comparison to students having low metacognition level. The self-questioning and self-evaluation closely related with Meta memory. The paraphrasing process deals with the regulation aspect of metacognition.

Chapter V

Conclusions and Implications

This chapter deals with the summary, findings, conclusion and recommendation of the study. The researcher was tried to elaborate the finding conclusion and recommendation for further study. The researcher tried to identify effectiveness of metacognitive strategies in mathematics classroom and to trace out the behaviours of students during teaching with metacognitive strategies. After interpretation and analysis of the obtain data through test, memo writing and interview schedule. The researcher concluded and summarized this chapter on the following topic: summary, findings, conclusion and recommendations for further study. Which were given below:

Summary

This study in the title “Metacognitive Strategies in Mathematics Education: An Action Research” has main two objectives to identify effectiveness of metacognitive strategies in mathematics classroom and to trace out the behaviours of students during teaching with metacognitive strategies. For the purpose to achieve the objectives of this study, the researcher has selected ten sample student from a government school of Ramechhap.

This study was based on the Costa’s metacognitive strategies. Researcher believed that in mathematics classroom teaching should done through method of enhancing metacognition of students. Then, students learns the strategies to learn. It develops the students’ cognition. And hence, their achievement in mathematics become better. This study was based on believe that planning, self-questioning, communication, encouragement, paraphrasing and self-evaluation are methods to enhance students' metacognition and hence these are considered as metacognitive strategies.

In this study, the researcher used the convenience sampling method. Test items, memo writing, interview schedule, testing the hypothesis, marking guidelines etc. were used as the tools to find the data. After the analysis and the interpretation of the data the researcher found the findings.

Findings

On the basis of data analysis and interpretation of the results, the researcher was pointed the findings of the study as follows:

- The mean scores before using the metacognitive strategies was 32.8 and after using metacognitive strategies was 69.4. So that after using the metacognitive strategies in teaching mathematics brings the positive change in the learning mathematics. There was significance difference between the test which was taken before and after using the metacognitive strategies in teaching mathematics. Because after using t-test with 0.01 level of significance with 9 degree of freedom, it was found that calculated value is 28.98 and tabulated value is 3.250. So, that the researcher found the better result after using the metacognitive strategies in teaching mathematics.
- The learner's behavior, attitude and participation of the learners in classroom was good after researcher used the metacognitive strategies in teaching mathematics.
- In Initial classes students have poor metacognition experience but after practicing more in later classes students have good metacognition experience.
- Chess players have good ability to understand their cognition.

- Material or visual illustration of topic enhance students' ability to understand self-cognition.

Conclusion

Based on the study, the researcher found that the use of metacognitive strategies is effective in teaching mathematics. The metacognitive strategies are here supposed to enhance students' metacognition. The researcher had selected only six ways to enhance metacognition. Beside that there are so many other ways to enhance metacognition. In order to update our educating system we need to search many other ways of metacognitive strategies. There are various levels of understanding. Just knowing the concept is lower level of understanding and understanding how one can understand the concept, is higher level of understanding. It is product of thinking about own thinking. However practicing it more, the level of understanding being increased. Moreover, maturity of students also affect the experience of metacognition.

Metacognitive strategies are important for developing higher dimension of self. Entering into self-world, transfer focus from external things to internal feelings and thought is primary stage in developing metacognition. Focusing on self-questioning develops the imagination power of students and thinking about own cognition develops intuition.

So, Studying through metacognitive strategies makes students creative and open minded. Metacognitive strategies helps to develop a doubting and truth searching habit.

Educational implications

On the basis of findings of obtained from the analysis and interpretation of the data, the recommendations with pedagogical implications are given below.

- To enable students to be actively engaged in their leaning with the future prospect of becoming self-directed and lifelong learner.
- To enable students focusing on self- understanding rather than knowledge without understanding
- To improve long-term memory power of students
- To make open minded and creative learner.
- To encourage critical thinking
- To focus on meta memory by students

Recommendations for further study

- The study can be done on achievement in mathematics by adding meditation on metacognitive strategies.
- Metacognition seem higher in senior students than junior students. So study can done on high school level or on more senior students.
- The study can be done effect of metacognitive strategies in learning mathematics: An experimental research.
- The chess players seems good in experiencing metacognition. So, the comparative study can be done between chess players and non-chess players in experiencing metacognition.

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Appendices

Appendix A

Table 1 Marking Guidelines in Test

S.N	Activities	Marks	Analysis and Interpretation
1.	Drawing correct figure	1	Recall theorem (search in memory)
2.	Initiation with writing given or known conditions and criteria	1	Knowledge of basic rule and basic step (search of condition that attached with terms used in statement)
3.	Writing what to do or which condition should be at last + required construction	2	Understands goals (search what to reach)
4.	Applying Valid procedure to prove statement with valid reasons + Students' additional valid self-adjustment in proving theorem(adding figures and description with own ideas)	5	Know logical process (Search How to do) + Conscious about rule + Realizing I can add something here (Listening inner constructive voice)
5.	Declaring End with conclusion	1	Know the finishing

Appendix B

Table 2: Result of tests before and after using metacognitive strategies in teaching mathematics

Students		**Question No. (Marks)**										**Total**
		1	2	3	4	5	6	7	8	9	10	
A	pre-test	8	8	6	10	10	3	3	1	6	6	61
	post-test	10	10	10	10	10	10	10	10	10	10	100
B	pre-test	8	8	7	10	10	3	3	1	5	5	60
	post-test	10	10	10	10	10	10	10	10	10	10	100
C	pre-test	6	6	6	8	8	3	3	1	3	3	47
	post-test	9	9	7	10	10	7	8	6	10	10	86
D	pre-test	5	5	5	4	4	2	0	1	3	3	32
	post-test	9	9	6	7	6	5	5	5	7	7	66
E	pre-test	5	5	5	4	4	1	0	1	3	3	31
	post-test	8	8	7	8	9	5	5	5	6	6	67
F	pre-test	5	5	5	4	4	1	1	1	3	3	32
	post-test	8	7	8	7	7	5	5	5	7	7	66
G	pre-test	5	5	5	2	2	2	0	1	1	1	24
	post-test	5	5	7	7	7	5	5	5	7	6	59
H	pre-test	3	3	2	5	5	0	0	0	1	1	20
	post-test	6	8	5	7	6	3	4	3	4	4	50
I	pre-test	3	3	2	2	2	0	0	0	0	0	12
	post-test	6	7	5	6	6	3	4	3	4	4	48
J	pre-test	3	3	1	1	1	0	0	0	0	0	9
	post-test	8	8	5	6	6	5	3	3	4	4	52

$$n_1 = n_2 = 10$$

$$\bar{x} = \frac{\sum x}{n_1} = 32.8$$

$$\bar{y} = \frac{\sum y}{n_2} = 69.4$$

$$\sum(x-\bar{x})^2 = 2981.6$$

$$\sum(y-\bar{y})^2 = 3422.4$$

$$\sum(x-\bar{x})(y-\bar{y}) = 3139.8$$

$${s_x}^2 = \frac{\sum(x-\bar{x})^2}{n_1-1} = 331.2889$$

$${s_y}^2 = \frac{\sum(y-\bar{y})^2}{n_2-1} = 380.2667$$

$$s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n_1-1}} = 18.2$$

$$s_y = \sqrt{\frac{\sum(y-\bar{y})^2}{n_2-1}} = 19.5$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{(n-1)s_x s_y} = 0.98$$

$$t = \frac{\bar{y}-\bar{x}}{\sqrt{\frac{{s_x}^2}{n_1}+\frac{{s_y}^2}{n_2}-2r\left(\frac{s_x}{\sqrt{n_1}}\right)\left(\frac{s_y}{\sqrt{n_2}}\right)}} = 28.98$$

Appendix C

Test of Hypothesis

The null and alternative hypothesis are:

H_0: There is no significant difference between the achievements of mathematics using metacognitive strategies in teaching learning.

i.e. H_0: $\mu_1 = \mu_2$(Null Hypothesis)

H_1: There is significant difference between the achievements of mathematics using metacognitive strategies in teaching learning

$i.e. H_1$: $\mu_1 \neq \mu_2$ (Alternative Hypothesis)

Where μ_1 and μ_2 are the corresponding parametric means of achievement of the students before using and after using metacognitive strategies respectively.

Level of Significance: $\alpha = 0.01$

Critical Region: It is two tailed test. Reject H_0 if $t_{0.005,9} \geq 3.250$ and $t_{0.005,9} \leq -3.250$ at 0.01 level of significance.

Computation:

$F = \frac{s_y{}^2}{s_x{}^2}$=1.1478$> 1$, so variances are homogeneous.

Now, $t = \frac{\bar{y}-\bar{x}}{\sqrt{\frac{s_x{}^2}{n_1}+\frac{s_y{}^2}{n_2}-2r\left(\frac{s_x}{\sqrt{n_1}}\right)\left(\frac{s_y}{\sqrt{n_2}}\right)}}$

$\therefore t = 28.98$

Decision: Since computed value of $t = 28.98$ which is greater than table value of $t = 3.250$. Reject H_0 and conclude that at 0.01 level there is a significance difference between the post-test and pre-test scores. That is, there is a significant gain in achievement of mathematics after using metacognitive strategies in teaching mathematics.

Appendix D

Lesson Plan 1

Subject: Mathematics

Topic : Geometry (Area of Triangle and Quadrilateral)

Subject Matter : Theoretical Proof of ‘ Opposite sides and angles of parallelograms are equal’

Behavioural Objectives : After completion of lesson, students will be able to

- define parallelogram
- identify pair of opposite sides and angles
- write theoretical proof of ‘ Opposite sides and angles of parallelograms are equal’

Teaching Learning Activities : (Use of Metacognitive strategies)

Step 1 : Planning

- First recall the concept of parallel lines, quadrilaterals then defining parallelograms.
- Teach about opposite sides and angles of parallelogram as non-adjacent sides and angles respectively, through examples.
- Recall the process of theoretical proof that they follow in previous class.
- Recall the condition with meaning of being two triangles congruent.
- Tell the overall plan of proof as first any two opposite vertices should be join and the formed triangle are need to show congruent.

Step 2 : Self-Questioning

- Allow students to think about shapes of parallelograms and does their opposite side and angles are same or not?

- Ask students to cut a parallelogram shaped paper with the help of Set Square to make parallel opposite side.
- Verify theorem through paper folding process.
- Let students to think how opposite sides and opposite corners of parallelograms are of same measure.

Step 3 : Good communication

- Ask students, what they think about how opposite sides and opposites corners of parallelograms are of same measure.
- Now, teacher write theoretical proof of 'Opposite sides and angles of parallelograms are equal' through the discussion with students.
- In every statements of proof, students will asked to tell about reasons for the statement.
- Further, what can be our next step will be asked to the students.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to say in their own way, whatever he/she think and feel about the how opposite sides and opposite's corners of parallelograms are of same measure.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.
- After students, teacher also tell about overall lesson in common layman language.

Step 6 : Self- evaluation

- Let students to think about does they understand the lesson.
- Allow students to write in their exercise book about whatever they understand in the class.
- Now, ask students that, can they write theoretical proof from the variation of figures and name.

Appendix E

Lesson Plan 2

Subject: Mathematics

Topic : Geometry (Area of Triangle and Quadrilateral)

Subject Matter : Theoretical Proof of 'Diagonal of parallelogram divides the parallelogram into two equal parts'

Behavioural Objectives :

After completion of lesson, students will be able to

- define diagonal
- identify pair of diagonals of parallelogram
- write theoretical proof of 'Diagonal of parallelogram divides the parallelogram into two equal parts'

Teaching Learning Activities : (Use of Metacognitive strategies)

Step 1 : Planning

- First recall the concept of diagonals of any polygon as line segment joining any non-adjacent vertices. Specially, discuss on diagonals of parallelogram with examples.
- Recall about relation between the alternate angles form after two parallel lines cut by a transversal line.
- Recall the process of theoretical proof that they follow in previous class.
- Recall the condition with meaning of being two triangles congruent so that area of congruent figures are equal.
- Telling the overall plan of proof.

Step 2 : Self-Questioning

- Allow students to think about triangles formed by dividing a parallelogram by its diagonal. Can students imagine that these triangles are of same shape and size?
- Ask students to cut a parallelogram shaped paper with the help of Set Square to make parallel opposite side.
- Verify theorem through paper folding process.
- Let students to think how diagonal of parallelogram divides the parallelogram into two triangles of same shape and size.

Step 3 : Good communication

- Ask students, what they think about how diagonal of parallelogram divides the parallelogram into two triangles of same shape and size.
- Now, teacher write theoretical proof of 'Diagonal of parallelogram divides the parallelogram into two equal parts' in whiteboard through the discussion with students.
- In every statements of proof, students will asked to tell about reasons for the statement.
- Further, what can be our next step will be asked to the students.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to say in their own way, whatever he/she think and feel about the diagonal of parallelogram divides the parallelogram into two triangles of same shape and size.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.
- After students, teacher also tell about overall lesson in common layman language.

Step 6 : Self- evaluation

- Let students to think about does they understand the lesson.
- Allow students to write in their exercise book about whatever they understand in the class.
- Now, ask students that, can they write theoretical proof from the variation of figures and name.

Appendix F

Lesson Plan 3

Subject: Mathematics

Topic : Geometry (Area of Triangle and Quadrilateral)

Subject Matter : Theoretical Proof of 'Parallelograms standing on same base and between the same parallels are equal in area.'

Behavioural Objectives :

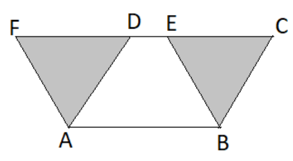
After completion of lesson, students will be able to

- write theoretical proof of 'Parallelograms standing on same base and between the same parallels are equal in area.'

Teaching Learning Activities : (Use of Metacognitive strategies)

Step 1 : Planning

- First recall statements of previous theorems 'Opposite sides and angles of parallelograms are equal' and 'Diagonal of parallelogram divides the parallelogram into two equal parts'.
- Discuss about the meaning of standing on same base and between the same parallel lines with examples.
- Recall the process of theoretical proof that they follow in previous class.
- Recall the condition with meaning of being two triangles congruent so that area of congruent figures are equal.
- Telling the overall plan of proof.



Step 2 : Self-Questioning

- Allow students to think about shaded triangles in given figure. Can students imagine that these triangles are of same shape and size?
- Let students to think if two shaded triangles are congruent then how parallelogram (ABCD and ABEF) are of equal area.

Step 3 : Good communication

- Ask students, what they think about how shaded triangles are congruent and how this fact can use to show parallelogram (ABCD and ABEF) are of equal area.
- Now, teacher write theoretical proof of 'Parallelograms standing on same base and between the same parallels are equal in area.' in whiteboard through the discussion with students.
- In every statements of proof, students will asked to tell about reasons for the statement.
- Further, what can be our next step will be asked to the students.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the theorem in their own way.

- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they know and do not know from the lesson.
- Allow students to write in their exercise book about whatever they understand in the class.
- Now, ask students that, can they write theoretical proof from the variation of figures and name.

Appendix G

Lesson Plan 4

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Subject Matter: Theoretical Proof of 'The area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.'

Behavioural Objectives:

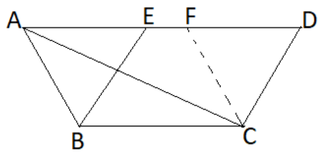
After completion of lesson, students will be able to

- Write theoretical proof of 'The area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.'

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First recall statements of previous theorems 'Diagonal of parallelogram divides the parallelogram into two equal parts' and 'Parallelograms standing on same base and between the same parallels are equal in area.'
- Construction of a line FC (parallel to AB) will be discussed.
- Recall the meaning of standing on same base and between the same parallel lines with examples.
- Tell the overall plan of proof.



Step 2 : Self-Questioning

- Allow students to think about how construction of a line FC (parallel to AB) can use to show area of triangle ABC is half the area of Parallelogram BCDE.

Step 3 : Good communication

- Ask students, what they think about how construction of a line FC (parallel to AB) can use to show area of triangle ABC is half the area of Parallelogram BCDE.
- Now, teacher write theoretical proof of 'The area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.' in whiteboard through the discussion with students.
- In every statements of proof, students will asked to tell about reasons for the statement.
- Further, what can be our next step will be asked to the students.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the theorem in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they know and do not know from the lesson.

- Allow students to write in their exercise book about whatever they understand in the class.
- Now, ask students that, can they write theoretical proof from the variation of figures and name.

Appendix H

Lesson Plan 5

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Subject Matter: Theoretical Proof of 'Triangles on the same base and between the same parallel lines are equal in area.'

Behavioural Objectives:

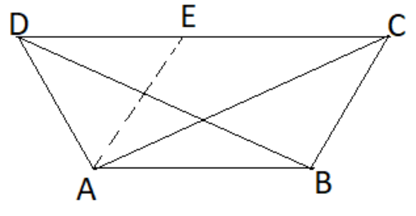
After completion of lesson, students will be able to

- Write theoretical proof of 'Triangles on the same base and between the same parallel lines are equal in area.'

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First recall statements of previous theorems 'Diagonal of parallelogram divides the parallelogram into two equal parts' and 'The area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.'
- Construction of line AE (parallel to BC) will be discussed.
- Recall the meaning of standing on same base and between the same parallel lines with examples
- Tell the overall plan of proof.



Step 2 : Self-Questioning

- Allow students to think about how construction of a line AE (parallel to BC) can use to show area of triangle ABC is equal to area of triangle ABD.

Step 3 : Good communication

- Ask students, what they think about how construction of a line AE (parallel to BC) can use to show area of triangle ABC is equal to area of triangle ABD.
- Now, teacher write theoretical proof of ‘Triangles on the same base and between the same parallel lines are equal in area.’ in whiteboard through the discussion with students.
- In every statements of proof, students will asked to tell about reasons for the statement.
- Further, what can be our next step will be asked to the students.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the theorem in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they know and do not know from the lesson.
- Allow students to write in their exercise book about whatever they understand in the class.
- Now, ask students that, can they write theoretical proof from the variation of figures and name.

Appendix I

Lesson Plan 6

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Area: Application of Theorem

Subject Matter: Proving the area of parallelograms ABCD and PQRD are equal.

If ABCD and PQRD are two parallelograms such that P and C lie on AB and QR respectively.

Behavioural Objectives:

After completion of lesson, students will be able to prove following problem:

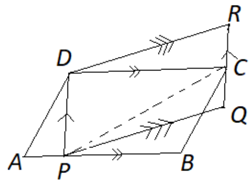
- ABCD and PQRD are two parallelograms such that P and C lie on AB and QR respectively. Show that area of parallelograms ABCD and PQRD are equal.

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First ask students to make figure as given information from question then teacher will draw figure through discussion with students.
- Recall the theorem that the area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.

- Recall the theorem that the parallelograms standing on same base and between the same parallels are equal in area.
- Required construction join P with C will discussed.
- Tell overall plan of proof.
- Moreover, tell students to draw alternative variation of figure.



Step 2 : Self-Questioning

- Allow students to think about how construction of joining P with C can use to show area of parallelogram ABCD is equal to area of parallelogram PQRD.
- Tell students to think what may be relation between area of triangle DPC with area of parallelograms ABCD and PQRD.

Step 3 : Good communication

- Ask students about what they think about how construction of joining P with C can use to show area of parallelogram ABCD is equal to area of parallelogram PQRD.

- Discussion about the area of triangle DPC is half of both parallelograms ABCD and PQRD.
- Now teacher write whole proof of the problem in white board through discussion with teacher.
- Allow students to tell if they know some steps of solution.
- In every step of solution ask students to tell reasons behind the step.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the solution in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they know and do not know from the lesson.
- Let students solve the following problem :

UVWX and MNOX are two parallelograms such that M and W lie on UV and NO respectively. Show that area of parallelograms UVWX and MNOX are equal.

Appendix J

Lesson Plan 7

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Area: Application of Theorem

Subject Matter: Proving the area of triangles PSN and QRM are equal.

If in a trapezium PQRS, M is some point between PS and N is some point between QR such that PQ∥MN∥ SR.

Behavioural Objectives:

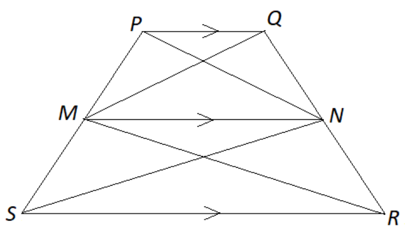
After completion of lesson, students will be able to prove following problem:

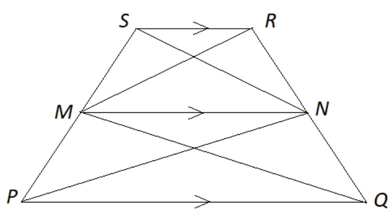
- In a trapezium PQRS, M is some point between PS and N is some point between QR such that PQ∥MN∥ SR. Prove that area of triangles PSN and QRM are equal.

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First ask students to make figure as given information from question then teacher will draw figure through discussion with students.
- Recall the theorem that the area of triangles on the same base and between the same parallel lines are equal.
- Recall the axiom that if equal quantities are added to equal quantities then resulting quantities must be equal. i.e. If a=b and c=d then a+b=c+d.
- Tell overall plan of proof.
- Moreover, tell students to draw alternative variation of figure.





Step 2 : Self-Questioning

- Allow students to think about what may be the relation between area of triangles PMN and QMN
- Similarly what may be the relation between area of triangles SMN and RMN.

Step 3 : Good communication

- Ask students about what they think about relation between areas of triangles PMN and QMN. similarly what about the relation between area of triangles SMN and RMN.

- Now teacher write whole proof of the problem in white board through discussion with teacher.
- Allow students to tell if they know some steps of solution.
- In every step of solution ask students to tell reasons behind the step.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the solution in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they know and do not know from the lesson.
- Let students solve the following problem :

- In a trapezium ABCD, X is some point between AD and Y is some point between BC such that AB$\parallel XY \parallel DC$. Prove that area of triangles ADY and BCX are equal.

Appendix K

Lesson Plan 8

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Area: Application of Theorem

Subject Matter: Proving the area of triangles PQT and PRS are equal.

If QRST is a trapezium in which TS is longer than and parallel to QR. All the vertices of Trapezium QRST are Joined with some exterior point P such that PR||QT and PQ||RS. If PR and PQ meets TS at U and V respectively.

Behavioural Objectives:

After completion of lesson, students will be able to prove following problem:

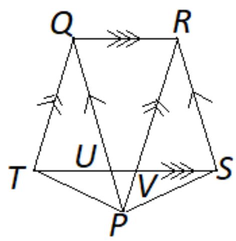
- QRST is a trapezium in which TS is longer than and parallel to QR. All the vertices of trapezium QRST are Joined with some exterior point P such that PR||QT and PQ||RS. If PR and PQ meets TS at U and V respectively, show that area of triangles PQT and PRS are equal.

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First ask students to make figure as given information from question then teacher will draw figure through discussion with students.
- Recall the theorem that the parallelograms on same base and between the same parallels are equal in area.

- Recall the theorem that the area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.
- Tell overall plan of proof.
- Moreover, tell students to draw alternative variation of figure.



Step 2 : Self-Questioning

- Allow students to think about what may be the relation between area of parallelograms QRVT and QRSU
- Allow students to think about what may be the relation between area of triangle PQT with parallelogram QRSU and area of triangle PRS with parallelogram QRVT.
- Similarly what may be the relation between area of triangles PQT and PRS.

Step 3 : Good communication

- Ask students about what they think about relation between area of parallelograms QRVT and QRSU
- Ask students about what they think about relation between area of triangle PQT with parallelogram QRSU and area of triangle PRS with parallelogram QRVT.
- Ask students about what they think about relation between area of triangles PQT and PRS.

- Now teacher write whole proof of the problem in white board through discussion with teacher.
- Allow students to tell if they know some steps of solution.
- In every step of solution ask students to tell reasons behind the step.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the solution in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they understand and do not understand from the lesson.
- Let students solve the following problem :

  ABCD is a trapezium in which DC is longer than and parallel to AB. All the vertices of trapezium ABCD are joined with some exterior point N such that NB$\parallel AD$ and NA$\parallel BC$. If NA and NB meets DC at P and Q respectively, show that area of triangles AND and BNC are equal.

Appendix L

Lesson Plan 9

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Area: Application of Theorem

Subject Matter: Proving the area of triangles APB and BQC are equal. If ABCD is a parallelogram and P, Q are any point on CD and AD respectively.

Behavioural Objectives:

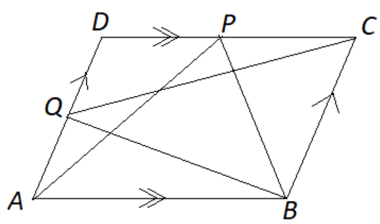
After completion of lesson, students will be able to prove following problem:

- ABCD is a parallelogram and P, Q are any point on CD and AD respectively. Show that triangles APB and BQC are equal in area.

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First ask students to make figure as given information from question then teacher will draw figure through discussion with students.
- Recall the theorem that the area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.
- Tell overall plan of proof.
- Moreover, tell students to draw alternative variation of figure.



Step 2 : Self-Questioning

- Allow students to think about what may be the relation between area of triangle APB with parallelogram ABCD and area of triangle BQC with parallelogram ABCD.
- Similarly what may be the relation between area of triangles APB and BQC.

Step 3 : Good communication

- Ask students about what they think about relation between area of triangle APB with parallelogram ABCD and area of triangle BQC with parallelogram ABCD.
- Ask students about what they think about relation between area of triangles APB and BQC .
- Now teacher write whole proof of the problem in white board through discussion with teacher.
- Allow students to tell if they know some steps of solution.
- In every step of solution ask students to tell reasons behind the step.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the solution in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they understand and do not understand from the lesson.
- Let students solve the following problem :

  PQRS is a parallelogram and A, B are any point on RS and PS respectively. Show that triangles PAQ and QBR are equal in area.

Appendix M

Lesson Plan 10

Subject: Mathematics

Topic: Geometry (Area of Triangle and Quadrilateral)

Area: Application of Theorem

Subject Matter: Proving the sum of area of triangles XAB and XCD is half of area of parallelogram ABCD. If ABCD is a parallelogram and X is any point within it.

Behavioural Objectives:

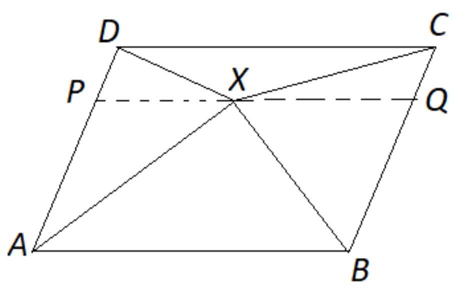
After completion of lesson, students will be able to prove following problem:

- ABCD is a parallelogram and X is any point within it. Prove that the sum of area of triangles XAB and XCD is equal to half of area of parallelogram ABCD.

Teaching Learning Activities: (Use of Metacognitive strategies)

Step 1: Planning

- First ask students to make figure as given information from question then teacher will draw figure through discussion with students.
- Recall the theorem that the area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.
- Tell about construction of PQ through X (parallel to AB and DC)
- Tell overall plan of proof.
- Moreover, tell students to draw alternative variation of figure.



Step 2 : Self-Questioning

- Allow students to think about how construction of PQ can be used to solve the problem.
- Allow students to think about what may be the relation between area of triangle XAB with parallelogram ABQP and area of triangle XCD with parallelogram PQCD.

Step 3 : Good communication

- Ask students about what they think about how construction of PQ can be used to solve the problem.
- Ask students about what they think about what may be the relation between area of triangle XAB with parallelogram ABQP and area of triangle XCD with parallelogram PQCD.
- Now teacher write whole proof of the problem in white board through discussion with teacher.
- Allow students to tell if they know some steps of solution.
- In every step of solution ask students to tell reasons behind the step.

Step 4 : Encouragement

- After listening students thought and feelings, teacher give applause for their thinking that motivate them to think about thinking.
- Teacher will praise students for their answering behaviour.

Step 5 : Paraphrasing and Reflection

- Ask students to restate the solution in their own way.
- Ask students to tell about what they know from overall lesson, in informal and their normal language.

Step 6 : Self- evaluation

- Let students to think about they understand and do not understand from the lesson.
- Let students solve the following problem :

  WXYZ is a parallelogram and A is any point within it. Prove that the sum of area of triangles WAX and ZAY is equal to half of area of parallelogram WXYZ.

Appendix N

Sample of Test Items

A. Theorems

- Opposite sides and angles of parallelograms are equal.
- Diagonal of parallelogram bisect it.
- Parallelograms standing on same base and between the same parallels are equal in area.
- The area of triangle is equal to half of the area of parallelogram on the same base and between the same parallel lines.
- Triangles on the same base and between the same parallel lines are equal in area.

B. Application of Theorem

- ABCD and PQRD are two parallelograms such that P and C lie on AB and QR respectively. Show that area of parallelograms ABCD and PQRD are equal.
- In a trapezium PQRS, M is some point on PS and N is some point on QR such that PQ∥MN∥ SR. Prove that area of triangles PSN and QRM are equal.
- QRST is a trapezium in which TS is longer than and parallel to QR. All the vertices of Trapezium QRST are Joined with some exterior point P such that

  PR∥QT and PQ∥RS. If PR and PQ meets TS at U and V respectively, show that area of triangles PQT and PRS are equal.
- ABCD is a parallelogram and P, Q are any point on CD and AD respectively. Show that triangles APB and BQC are equal in area.

- ABCD is a parallelogram and X is any point within it. Prove that the sum of area of triangles XAB and XCD is equal to half of area of parallelogram ABCD.

Appendix O

Sample for open ended

Interview Schedule

Name of Student:

Class:

Address:

Respondents are requested to response with honesty and all responses are expected to be true. However your response about questions (positive or negative) it should come from your inner self.

Stage 1 planning

1. What are you thinking at the time of teaching basic requirements of lesson?
2. What are you thinking while teacher is telling about overall plan of proving theorem?
3. Do you think at beginning of lesson teacher should instruct all ground rule and steps to follow? How such things impact your learning process?

Stage 2 Self-Questioning

1. Share your experience of answering your own questions?
2. How some material or visual illustration of theorem are necessary in self-questioning?
3. How self-questioning impact your learning process?

Stage 3 Communication

1. Do you think teacher should teach through discussion with students about subject matter? Why?
2. How good communication with teacher impact your learning process?

Stage 4 Encouragement

1. Share your experience at the time of encourage by teacher for learning?

Stage 5 Paraphrasing and Reflection

1. What are your feelings while you are restating and translating lesson in your own words?
2. What difficulties did you faced in paraphrasing and reflection?

Stage 6 Self- Evaluation

1. Share your experience at the time of self-evaluation?