Chapter I

INTRODUCTION

Background of the Study

Number is basic and fundamental concept of mathematics on which arithmetic and algebra was developed standing on number through the logical analysis. The various human daily-life activities on number create the sense of mathematics. So, it can say number is the synonyms of the mathematics.

The number is a sense of ability to recognize quantity of single or collection of objects. Numbers, operation of numbers and their function have occupied important role in mathematics There developed so many number system in the world, among them Hindu-Arabic, Devanagari, Roman etc. are the famous and useable number system in the world. The ancient Egypt, Babylonian, Greek, China, India etc. has the main role to develop the number system. The ancient Babylonian used the sexagesimal (base 60) number system (Boyer, 1968).

The first mathematics originated around five thousand years ago along with Egyptian civilization, Babylonian civilization, Greek civilization, Chinese civilization and Hindu civilization (Boyer, 1968). Thousand years ago there was no number to represent two or three. Instead fingers, rocks, sticks, or eyes were used to represent umber. There were neither clocks nor calendar to help keep track of time. Evidence of the fact that mathematics was developed from culture and civilization spread into another civilization can be found everywhere (Dantzing, 1930).

Primitive people had well practices and struggle to develop the modern number system. They used different strategies to count the objects. The concept of

number and counting developed so long before the time of recorded history and it is imagined that primitive. Human had same number sense of recognizing objects that have belonged to then, with the gradual development of society, simple counting become in portative (Eves, 1985). Ancient people used simple rally method of counting by employing the, on singer per sheep, goat etc. Letter canting was maintained by making collection of pebbles, sticks or making scratches in the wall or on the stone (Eves, 1985).

According to the book, *The Language of Science* (Dantzing, 1930), many birds have a good number sense. If a nest contains four eggs, one can safely be taken but when two are removed the bird generally deserts. The birds can distinguish two from three. An experiment done with a goldfinch showed the ability to distinguish-piles of seed: There four from three and six from three. The goldfinch almost always confused five and four, seven and five, eight and six, and ten and six (Dantzing, 1930).

In the process of the development of society, it was realized to make mane extensive counts and then gradually the counting had to be systematized. This was done by arranging their numbers onto convenient words, symbols, narrations and groups. Perhaps in the early stages, the vocal or spoken counting was widely used. In addition of vocal numbers, finger numbers and finger signs were used together as well (Boyer, 1976).

There various number systems were developed. A millions or more people, they built vast cities, developed expensive road system in developing numeral. Many ancient era and country has deep contribution to developing different numeral system. Mainly, Egypt, Greek, Babylonian, and American civilization has the great contribution on developing the number and numerical system.

The Mayan number system dates back to the fourth century and way approximately 1,000 year more advanced than the Europeans of that time (McLeish, 1991). This system is unique from our current decimal number system which has the base 10, in that the Mayan's used a vigesimal system, which has a base 20. This system is believed to have been used because, since the Mayan's lived in such a warm climate and there was rarely a neat to wear shoes, 20 was the total number of singers and so on. Thus, making this system workable.

It was the Greek who first recognized the natural (or counting) numbers as farming infinite collections on which basic mathematic operation of addition and multiplication could be performed (Datta & Singh, 1935). So, Greek appears to have been the first to develop a mathematical theory of mathematics. The Irion School and Pythagorean School developed extensive theory of both geometry and mathematic.

Babylonians began a numbering system, in about 5,000 years ago (Boyer, 1976). It is one of the oldest numbering systems in ancient country of Babylon, during the third millennium B.C. Tables were the Babylonians must outstanding accomplishment which help them in calculating the problems. One of the Babylonian tablets, Plimpton 322, which is dated between 1900 BC and 1600 BC, contains tables of Pythagorean triples (Jordan, 2009). It is currently in a British museum.

The Babylonian had a very advance number system even for today's standards; it was a base 60 system *(sexagesimal)* rather than a base 10 *(decimal).* Babylonian divided the day into twenty-four hour, each hour into sixty minutes and each minutes into sixty seconds (Jordan, 2009). This form of counting has survived for four thousand years ago.

Nabu-rimannni and Kidinu are two of the only known mathematicians from Babylonian. However, not much is known about them. Historians believe Nabu-rimanni lived around 490 BC and Kidinu lived around 480 BC (http://www.math.edu).

Hindu-Arabic Number System

Hindu-Arabic number is a decimal place-value number system that consists the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0. These numbers was first used by Hindu civilization and after Arabian mostly follow. Hindu-Arabic number system first used in about 250-300 B.C by King *Asoka* (Cajori, 1894). This 'number system' was derived from Hindu civilization and transgressing in western society. During this time 'zero' and place-value was not used in Hindu-Arabic number system. The zero was first used in about 825 A.D by Brahamagupta (Menninger, 2013). After, *Al-Khwarizmi* started positional value. The today's numeral zero was taken from Latin word *Zephirum* which means empty (Boyer, 1976). It make easier to take decimal place-value form in Hindu-Arabic number system.

Devanagari Number System

A Sanskrit number system which is represented by symbols; १, २, ३, ४, ५ , ६, ७, ८, ९,and ०. Devanagari is also decimal number system like Hindu-Arabic. This number system was developed from the ancient Nagari script also a branch of Brahmi numeral (Datta & Sign, 1962). It was a gradually developed of the figure from Brahmi numeral in about 6^{th} to 7^{th} century. After, it extended in Arab and Alexigendria (Sherestha, 2013).

The Hindu had used a decimal system for a long time and even had the zero before this time, but it was not until these two astronomers came up with their system for forming numbers that the system was more or less complete. As all Hindu literatures were written on verse form, they also come up with a system to form numbers that was well in verse.

Number is one of the fundamental concepts of mathematics which was developed in ancient time and it was undergone expansion over centuries. Number is so common in mathematics that for many people's number is a synonym of mathematics. In Sanskrit and Nepali languages the term *Ganita* is used to represent *mathematics* whose etymological meaning is the science of counting. According to History of Hindu Mathematics (Datta & Singh, 1935), *Ganita* is the science of number and counting. Since the number has gone expansion and generalization over centuries, the numbers have extended from finite counting numbers to countable infinite and to uncountable infinite along the line of cardinality due to George Cantor's contribution. On the other hand, the number has extended from counting numbers to rational, irrational and to complex numbers.

The Origin and Discovery of Number

Are animal able to count? The research had been done on the domestic animal. It does happen that a domestic animal a dog, ape, or elephant perceives the disappearance of an object in a restricted ensemble with which it is familiar in many species, the mother shoes by unmistakable sign that she knows that one of her little ones has been taken from her levy Brusl. This another experiment on *goldfish* seen that, chooses food from two small piles of seeds, generally successes in distinguishing three from one, three form two, four from two, four form three and six from three, but

nearly allure confuses five and four, seven and five, eight and six, ten and six (Cook, 1997).

Even another remarkable experiment on the crow and the magpie, which are apparently able to distinguish concrete quantities ranking from one to four. At last, the researcher had got conclusion that animal are not able to count. It seems safe to assume that counting is an exclusively human ability closely related to the development of intelligence and involving a mental process more complex than the number sense (Dantzing, 1930).

The study was concern on the main reason that prompted people developed the notation on number. There are still good reasons for believing that, there was a time when people did not know how to count. There does not imply that they had no notation of number, but only that this notion was limited to a kind of number sense, that is, to what direct perception enable them to recognize at a glance. They find number concept was a concrete reality in separable from objects and that is manifested only in direct perception of physical plurality. They were probably unable to conceive of numbers in themselves, as abstractions. If so, they must not have been aware that such greatest as a brace of hares, the wings of a birds, or eyes, ears, arms, or legs of a person have the common characteristics of "being two".

In the process of development of more complicated society, numbers become eternal components of number symbols should be recorded by this way. Various written number symbols, notations, and other words gradually evolved from these ancient efforts to makes permanent number records. Thus, ancient different civilization and culture are the foundation of number.

Number is most essential things for human life. Number is existence and used everywhere; number has the advance role in the ethno-mathematics. Uneducated people also used the numbers by their ethno mathematics. The concept of number and symbol has taught in preliminary phase of learning. Number and symbols are the fundamental basis of mathematics and mathematics learning. The number is used formally and informally in daily human-life activities. Few peoples follows mathematics professionally but Number has the immense link in each individuals life and culture.

Number is the sense and symbols are script of mathematics. So, mathematics is the language and body of the numbers. The number and numerals' system is foundation of the mathematics. Number is the essential part of the mathematics as well as human life. So, I would like to study on the basis of *"Development of number system."*

Statement of the Problem

Many of the problems confronted in the classroom, the school or research documents on community. Also, technological changes and curricular development are constantly bringing new problems for research .The research problems are come from related research areas. Best and Kahn (10^{th} Ed.) suggested that , research area would be multiculturalism in classroom, distance instruction, charter school, evaluation of program evaluation of learning, homework policies and practices , extra curriculum program , class size, sociometry and soon. In this way, the researcher determine area ‘’Development of Number System’’.

Number is very essential component for human life. It can believe that, primitive people used the numbers but they used their own classical methods. Here,

researcher curious about their classical number system and their counting strategy. At all, researcher meets some debate about number perception in primitive people.

The child of fourteen month old has ability to perceive number differences in the people or objects around him or her are very limited when the number goes beyond three or four (Ifrash, 1985). An experiment has also shown that, the average people have number sense that is around four (Danting, 1930). Another remarkable experiment shows that, the crow and magic which are apparently able to distinguish concrete quantities ranging from one to four (Dantzing , 1930). So, the George Ifrash and Dantzing focused that men and animal have the direct number sense. Men have the direct number sense up to 4 and animal have the different direct number sense This we, conclude that , men can recognized the collection of objects up to 4 without systematic counting .

Ancient people used materials and objects of all sorts (pebbles, shells, bones, sticks, clay objects), notches in bone or wood, knotted strings, intuitive or conventional gestures' (use of fingers, toes etc.) for counting. The concrete term directly imply the notation of number: "sun", "moon" for 1,"birds wings" for 2, "clover of leaf for 3, "animal legs" for 4, "fingers of one hands" for 5 etc (Ortenzi, 1964).

According to (Ortenzi, 1964), ancient people used the concrete numeral to count the number. They used notches on bones, pebble, shells, sticks and fingers on hands for counting and perceive the number one by the uniqueness of objects like "sun", "moon" etc. Also, two by couple objects, three by unique collection of three objects like tripod, four by legs of animal etc. It shows controversy to perceive the number in primitive people.

This shows there is not uniformity about perception of the number on primitive people that they count by using their direct number sense or using on concrete objects. Researcher wants to clear about ancient number perception in primitive people and their counting techniques. So, it is a problem in this research.

The Mayan number system was used around the 4^{th} century and approximately 1000 years of the time to still use. They used vigesimal system, which had base 20 (Ortenzi, 1964). This number was derived by using fingers and toes. There number was based on multiples of 20. They use $1(20^0)$, $20(20^1)$, $400(20^2)$, $8000(20^3)$ and $160000(20^4)$. In Arabic form we use place values of 1,10,100,1000 and 10000. They used calendar where each month contain 20 day with 18 month to year (Ortazi, 1964).

The Babylonians system of mathematics was sexagesimal (base 60) number system (Boyer, 1968). This system is advance applicable in determine the time on a day they divided the day into twenty- four hour , each hour into sixty minutes , and each minute into sixty seconds . This form of counting had survived for four thousand years (Boyer, 1968).

In ancient time both of number system had parallel importance on that civilization. Maya well discovered the systematic vigesimal number system. Example, each month contain 20 day with 18 month on a year (Ortazi, 1964). Again, Babylonian formation of the time is still important on now days. The 360^0 on a circle is a still used which was developed standing on sexagesimal number system. Both of number system has well positional value. But regarding on this perspective, new place-value decimal number system take famous place in current day. This number system has advanced application in mathematics. It creates a curious for me. Regarding in this perspective, how to develop the modern decimal place-value

number system from ancient primitive counting opposite to famous vigesimal and sexagesimal number system. It was a major problem in this research. So, I want to search and add more evidence to verify this problem. So, I try to find out the answer of the following questions:

- How did primitive people perceive the number?
- How were the development phases of the Hindu-Arabic, and Devanagari number system?

Objective of the Study

Hindu-Arabic is the most applicable number system in the world. It is a place-value decimal number system. Researcher wants to carry out the evidences regarding on developments of Hindu-Arabic and Devanagari number system. So, in this study the researcher is dedicated to find the result of the following objectives:

- To explore about the number perception on ancient people.
- To analysis the historical development of Hindu-Arabic and Devanagari number system.

Significance of the Study

This study is based on document analysis research design. This study focuses how to perceive the number in ancient human mind and how to drive the today's place-value decimal number system from ancient one-to-one counting system. This study is tries to carry out the developing phase of Hindu-Arabic and Devanagari number system.

- This study depicts the ancient counting system and development of Hindu-Arabic and Devanagari number system.
- This study explores the different evidence and fact about Hindu-Arabic and Devanagari number system.
- This study might be supportive document for those who are interested about number system and who still working in this field.
- This study provides an idea about document analysis approach.

Delimitation of the Study

This study is related with the topic "Development of number system". The area of the number is very wider (or not limited). But researcher is only curious on the problem regarding on Hindu-Arabic and Devanagari number system. Thus, area of this study delimitated on developing phase of Hindu-Arabic and Devanagari number system.

The primary and secondary information are main source of this study. The researcher cannot manipulate any event and neither controls the environment. So, the design of this study is bounded on qualitative research and historical descriptive. Researcher collects historical document (journals, article, thesis etc), visit website and administrate in-depth interview and counseling with research resource person.

Document analysis was the main methods of this study. Researcher conducts the inductive and triangulation methods for analysis and interpretation the data. So, the tools of this research are delimited on in-depth interview with resource person and document analysis. Interview is delimited with five resource person. This study

focuses only on ancient counting and development of Hindu-Arabic and Devanagari number system.

Operational Definition of the Key Terms

Key terms refer those specific terms which are frequently used in the thesis. The operational definitions of key terms visualize their specific meaning consisting on the thesis. Some of the terms which are used in this thesis are defined as follows:

Number System

It is a counting sense on the objects. Number system refers the counting strategies concern on different numerals system. It pointed counting system of Hindu-Arabic and Devanagari numerals.

Number Perception

It is a pre-concept of counting on human mind toward number system. The individual understanding of counting of different numeral. The counting ideologies of Hindu-Arabic and Devanagari number system.

Hindu-Arabic Number

Hindu-Arabic number is well place-value decimal numeral system that consist the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0. This numbers first introduced by Hindu civilization and after Arabian mostly follow. Brahmi and Devanagari numeral is the foundation of this number system.

Devanagari Number

Devanagari number is Sanskrit numeral system which is representing by symbols; १, २, ३, ४, ५ , ६, ७, ८, ९,and ०. Devanagari number is also decimal place-value number system like Hindu-Arabic. This number system was developed from the ancient Nagari script and Brahmi numeral.

Vigesimal Number System

Vigesimal number system is a systematic counting by developing on the 10 finger and 10 toes. It is a well-developed from contemporary French and Latin word "Vingt" for 20. They use "deux" for 2 and "dix" or "decim" for 20. Ancient Mayan discovery and well use this number system in Central America.

The Sexagesimal System

It is a famous number system (base 60) which was discovered by Babylonian. Over that time, Sexagesimal system was still used for measuring time (minutes, second), arcs, and angles (degree, minute, seconds).

Manuscript

It is the handwritten text recorded on the paper, palm leaf, bitch leaf, stone, clothes etc. Here, manuscripts represent the recorded documents from Lichhavi, Sharada, Brahmi manuscript etc.

Numerals

It is the symbolic and representative form of number system. In this study numerals represent the Hindu-Arabic and Devanagari number system.

Concrete Numerations

Objects of all sorts (pebbles, shells, bones, sticks, clay objects and so on), notches in bone or wood, knotted string, intuitive or conventional gestures for counting.

Primitive Counting

An ancient counting before the systematic counting (or place-value counting). The counting by using pebbles, notches on bones, sticks, sun, moon etc. After people count by associated one object for one as one-to-one correspondence methods.

Civilization

In this study civilizations refer those primitive communities, culture, time and place which provide the evidence about number system. The different phenomena of Hindu-Arabic and Devanagari number system.

Chapter II

REVIEW OF RELATED LITERATURE AND CONCEPTUAL FRAMEWORK

Clearly, the related literature review provides the strong knowledge about the related topic. This related literature would be previous research report, articles, thesis, teachings materials, theories, paper and others booklets can be found that concern with curriculum and so on. We consider the development of number system as a root to help the learner understand the path of development to mathematical process or concept. From the review of literature, we must identify the study what has been established and what has not been try to be found yet. It also provides knowledge to find out the different facts in research for further study of task. It helps to conduct the research programs and give the better ideas for the researcher to reach the goal.

This literature review basically concern on the topics "Development of Number System." The researcher interested to find the pre-existing of number and development of Hindu-Arabic and Devanagari number system from historical phenomena.

Historical Research

History is the meaningful recorded of human activities. It is not merely a list of chronological events but a truthful integrated account of relationship between person, events, times and places. History is used to understand the past and to try to understand the present in light of past events and developments. Historical analysis may be directed toward an individual, an idea, a moment, or an institution. However, none of these can be considered in isolation (Best & Kahn, 2006, pp: 84).

Theoretical Literature Review

Any theory or principle provides the methodology, idea and guide line to the research and make study strong and valid. Number is an important part of social phenomena. Number is so much applicable things in human daily-life. In history of the human civilization, number was developed consisting on the base two, five, ten, twenty etc. Ancient people applied rule based on finger of one hand, two hand, hand and legs etc. for counting and creation the number. Today we follow this system as the principle of number creation. Thus, I would like to review the following principle of base low.

The Principle of the Base Law

When people feel to need to symbolize the nation of number, they can choose between two different procedures. One, which can be called "Cardinal" consists in adopting a standard symbol for 1. The other which can be called "Ordinal" Consists in assigning to the consecutive whole numbers, beginning with 1, distinct symbols unrelated to each other. The number consisting only on two digits, five digits, ten digits etc. are systematic and define method of developing the number. Which is analyzed the follows;

Binary Number System

The modern binary number system was first derived by Gottfried Leibniz in 1979 (Ifrash, 1985). The system related to binary number have appeared earlier in multiple culture including ancient Egypt, China, and India. Arithmetic values represented by part of the Eye part of Horus. Horus-Eye fraction are a binary numbering system which express as a sum of binary fraction 1/2, 1/4, 1/8, 1/16, 1/32,

and 1/64. This system can be found in the document from the fifth Dynasty of Egypt, approximately 2400 BC (Ifrash, 1985).

Binary describe a numbering scheme in which there are only two possible values for each digit: 0 and 1. A bit is a smallest unit of data on a computer each bit has a single value of either 1 or 0 (on and off).

Generally, logic "1" represents a higher voltage, such as 5 volts, which is commonly referred to as a HIGH value, while logic "0" represents a low voltage, such as 0 volts or ground, and is commonly referred to as a LOW value. These two discrete voltage levels representing the digital values of "1's" (one's) and "0's" (zero's) are commonly called: **BI**nary digi**TS** and in digital and computational circuits and applications they are normally referred to as binary **BITS**(http/www.math.wichita.number.edu/history).

Quinary Number System

The base 5 number system drive from the Maharashtra in India (Subedi, 2073), through the some merchants still use on interesting finger technique. In ancient language of Sumer (South Mesopotamia), the words for certain number is kept traces of an earlier use of base 5 (Ifrah, 1985). Eves (1985) said that some South American tribes count by hands: one, two, three, and four, hand and so on. The Aztec oral numeration up to 9was quinary scale which is describe as below:

1 *ce,* 2 *ome,* 3 *yey,* 4 *nanu,* 5 *chia or macuilli,* 6 *chica-ce,* 7 *vhi-ome,* 8*chu-ey,* 9 *chi-nanu.* Sumerian number words shows trace of an earlier quinary system: 6=*chica*+*ce*=5+1, 7= *imin*= *i*+ *min* =5+2, 9=*chica*+*nanu*=5+4 (Subedi, 2017). It means that quinary scale used in earlier history of numeral system. Also the numerical

notation for 4, 5, 6, 7, 8, of Roman system have quinary base such as: IV, V, VI, VII, and VIII.

By the all historical evidence, the grouping system is individual idea of each civilization which depend upon the own culture. We can assume that decimal and quinary system was developed by using the hands. So, it was a well number system in ancient time.

Decimal Number System

The system of oral numeration now widely used can be described as follows; the old sign value decimal number system consist the number from 1 to 10 and modern place-value decimal number system consist the digit 0 to 9 (Connor & Robertson). And each power of 10 has an independent individual name. This number system is formulated around the middle of 3rd century BC in the language of Brahmi numeral (Connor & Robertson, 2000), where the place-value developed however later. The mathematicians Connor & Robertson says, the Brahmi numerals have been found in inscriptions in cave and on coins in region near Pune, Mumbai, and Utter Pradesh. During the Gupta period (early 4th to late 6th century), Gupta numeral developed from Brahmi numeral and after 7th century Gupta numeral developed into Nagari numeral (Connor & Robertson, 2000).

After numbers were express using a "named place-value notation" using a name for power of 10: *kat* (1), *dasa* (10), *sata*(100), *sahara* (1000*), auta* (10,000), *niyuta* (100,000), *Prayuta* (1,000,000,000), *madhya* (10,000,000,000), *anta* (1,000,000,000,000), and *parardha* (1,000,000,000,000) (Datta and Singh, 1962).For example, the number 26432 was expressed as " 2 *ayuta* 6 *sahasra* 4 *shstha* 3 *dasa* 1 *eka.* The process has been carried out in group of ten, with a single word having been

spoken, and it is therefore man's ten fingers that have imposed the base ten. In their use of numeral seem to present to the conditions and needs of their respective societies. The word drive from each power of ten as 10^0, 10^1, 10^2, 10^3, … .This number system was developed by individual's society of ancient Egyptians, Greek, Babylonian and Hindu civilization.

The Vigesimal Number System

Ancient Europe had explored this system first approximately 1000 BC (McLeish, 1991). Vigesimal system originated in the habit of counting on the 10 finger and 10 toes (Ifrah, 1985: 41). The Maya calendar had "month" of twenty days each and could be use for calculating cycles of twenty years (*khatun*), 400 years (*boktun*), and 8000 years *(pictun*). The Aztes also connected by twenties and power of 20 as 20, 20^2, 20^3. The Mayan number systems are as below:

1: hun	11: buluc
2: ca	12: lahca (lahun+ ca=10+2)
3: ox	13: ox-lahun(3+10)
4: ho	14: can-lahun (4+10)
5: can	15: ho-lahun(5+10)
:	:
10: lahun	20: hunkal

21: hun-tu-ku(91+20)

22: *ca tu-kal*(2+20) … 30: *luhan ca kal,* 40: ca *kal*…

80: *can kal*… 100: *ho kal*, 400: *hunbak*, 800: *hun pic*, 160,000: *huncalab.*

(Subedi, 2017)

The number words for 20 to 29 drive from *hun* and *kal*. *Hun* and *kal* means one twentieth = 21, the word for 30 is *lahunca kal,* It means ten-two-twenty. The number-words: up to10 derived independent, from 13 to 19, they have compound name containing the word for 10 and units, with except 11, which *buluc*, not *hun-lahun*(one-ten). The words 21 to 39 are composed with two exceptions, by inserting the prefix*tu*between the word for "score" and the word for the corresponding units. The exception case are 30 "ten-two-twenty" instead of "ten twentieth," and 35, which is "fifteen-two-twenty" instead of "fifteen twentieth."

We can see irregularities in information of number word but we have seen that Mayas had oral numeration system that assigned an individual name to each power of 2o. Also, Maya formulate the concept of zero as a sense of nothing. They use well notation about nothing (zero). This we conclude that, Maya was a well systematic positional number system that period of the time.

The Sexagesimal Number System

Over that time, sexagesimal system was still use for measuring time (minutes, second), arcs, and angles (degree, minute, seconds). For example at 9: 08 Am imply at sexagesimal system, 9 hour, 8 minutes and 43 second be seen, $(9\times60^2) + (36\times60) + 7$ = 92,167" north of the equater.

The origin of sexagesimal number system is on Greek and after it become famous in Babylon (Ifrash, 1985). By astronomers used only for expression fractions. And earlier in Babylon, It was used for expressing whole numbers as well as

fractions. It was a complete numeration system employed by Babylonian mathematicians and astronomers. This system, which proceeds by successive power of 60, has a major drawback resulting from the high value of it base. Theoretically its only units are 1, 60^1, 60^2, 60^3 etc. and using this base requires knowledge of sixty different words for number 1 to 60. It was a complete numeral system, employed by Babylonian mathematicians and astronomers. The Sumerians introduce such units, taking as an intermediate legal between the sexagesimal units of their numeral system (Ifrash, 1985).

Actually, Sumerian system had a decimal form in second level(See Appendix-C) and some of these words are compound. The word of 30 is formed by combining the words for 3 and 10: 30=*ushu*=esh.u=3 x 10; 40= *nishmin*=20 x 2; 50= *ninu = nin + u* = 40 + 10

Another level reached in the Sumerian oral numeration: number 60 to 600 are expressed by treating 60 as a new unit: 60: gesh; 120: gesh-min (60 x 2), 180: gesh-esh(60 x 3), … . The next level reached 600 to 3600; 3600 to 36000; 36000 to 10,800,000, 216000 to 216,000 x 9. The Sumerian oral numeration used 60 as a base and had consecutive level built alternatively on the auxiliary base 6 and 10 such as: 1, 10, 60, 60 x10, 60^2, 60^2 x 10 etc. (See Appendix- C) (Subedi, 2017).

The well description after above sexagesimal number system, the fact that people first learn to count on their ten finger account for the predominance of the decimal system. The less common vigesimal system originates in the practices of counting on ten fingers and ten toes. This number system has its own mathematics. But it is hard to imagine how the sexagesimal system came to adopted by the Sumerian.

The above all number systems are much famous, applicable and authentic in the history of number system. Regarding on this perspective, how to developed and make famous the modern place-value decimal number (Hindu-Arabic and Devanagari) system from ancient to today is major study of this research.

Empirical Literature Review

Cajori (1894) published a book “History of mathematics” focused the Greek geometry and different schools like as iconic, school of Pythagoras, Sophist school, platonic school etc. Also, discussed about the Romans, Hindu’s, Arabs, and Europe’s ancient mathematical contributions. First Babylonian use the vertical wedge ٧for 1, while٢ for 10. By great erudition and phenomenal activity Gerbert infused new life into the study not only of mathematics, but also of philosophy. In that time philosophy of Aristotle are famous which writing of Boethius. Greek text was wanting. But the Latin heard that the Arabs, too, were great admirers of *Peripatetic* and that they passed translation of Aristotle works and commentaries thereon. A little letter John of Seville flourished Gerard of Cremona in Lombardy being desirous to gain possessing of the almagest, he want to loll and there in 1175, translated this great this work of Ptolemy. Inspired by their richness of Mohammedan literature, he gave himself up to study. He translated into Latin over 70 Arabic works. This led them finally to search for and translated Arabic manuscripts.

Kelly (1947) had a thesis on "A logical development of the real number system" In this thesis researcher less analysis about the number sense. He said hat, "The process of putting one collection into one-to-one correspondence with another started man on his road to counting." He focused that man began to set up model collections always denoted certain number. "After generation of use, the name and

sound of the collection head degenerated and the new word that involved become the symbol for Hindu number, Hindus losing the significance of its number is based on this principle of correspondence. We now no longer look for model collections, we set one collection into one-to-one correspondence with the natural numbers on other words, and we count it." This thesis gives an idea of counting number from the developing perspective.

Lumpkin (1987) has spill own mathematical feeling and experience through on essay "African and African-American contributions to mathematics". The purpose of this essay was to provide teachers of grade, kindergarten through 13 with some examples of African and African American contribution on modern mathematics. According to recorded history of Africa men named Tallies cut the bone and make notches for count and thousand years after ancient Egyptian was taken with the first *ciphrization* of numerals. According to this essay "Egyptian numerals were based on 10, a fore runner of our decimal system. The Egyptian system was additive and used a different symbol for each power of 10, from 10^1, 10^2, 10^3, $10^4 10^5$, to 10^6. The Mesopotamian numbers used a place-value system based on 60 and style-made tally marks for units and used today combines Principle from these two ancient systems with the important addition of a zero place holder. The zero symbols for a place-value system was first used consistently on the Mayan based 20 systems. Later use of a zero in the Indian base-10 value numerals, and the use of ciphers instead of tallies for the digits let to our modern numerals. They are called Indo-Arabic numerals become Island culture adopted the numerals and repeatedly introduced them to Europe, C. 850 to 1400. Islamic, mathematicians extended this system to include decimal fractions".

Hill (1995) had contributed a thesis on the topic "A mathematical Biography of *Sophic Germain*" in *Orgen* University. This thesis is completely based on biography of a female mathematicians, focused her mathematical accomplishments a number theory and theory of elasticity. This biography tries to sketch the female mathematician's *Strfles's* clashes life to successes in such a male-dominated subject. Even, her parents and this current situation discourage her from her study, but she still more forward.

She has little success on elasticity but her work in number theory was and still very important. Her most famous theorem "*Sophie Germian's Theorem*" has include in many number theory text books published today. She used a mathematical tone as a/b read as "a divides" a/b read as "a does not divides b", a≡b (mod m) read as " a is congruence to b mod m", gcd (a, b) ≡ m read as "The greatest common denominator of a and b is m" and n1 read "n factorial'. The numeral notation congruence is basically use in number theory.

Adhikari, (2002) has done a research on the topic "The development of numeral system of the Newar Civilization". His Study was descriptive nature basically a historical Survey of numerals of Newar Civilization. Researcher had done his research concern on the primary and secondary data which consisted of the study of profiles, study of the journals, periodicals, books, counseling with resource person. Balaram conduct his study standing the objective: (I) study the trend of development of numeral of Newar civilization and (II) Identify the numeral system of Newar civilization.

From this study, he has got the followings: Development of primate counting is not found, all the numerals found in Newar Civilization are developed from the

Brahma numerals, The Newar numerals system is based on decimal scale, Positioned number system has been adopted since 11^{th} century and zero is adopted since 13^{th} century AD, There are word numerals with place-value in inscriptions and manuscripts, The ciphered numeral system of letter numerals is in use since the ancient period.

Clark and Shinn (2004) investigated the reliability, validity, and sensitivity of the early mathematics measures with first graders using the Aims Web TEN. This study became a springboard for a multitude of others that replicated and expanded their work using other kindergarten and first grade mathematics tests and frequent data collection to measure student growth patterns for specific concepts and skills. EM-CBM studies use *number sense theory* to assess early mathematical skills. The assessments measure each student's ability to count, identify numbers, make judgments in quantity comparisons, and his/her use of a concrete and abstract number line (Baglici, 2008).

Shrestha (2004) has an article "Existence of numeral: from a philosophical perspectives concern on development number system from philosophical aspect. According to history of Hindu mathematics (Data and sing, 1935), mathematics is the science of number and counting. The researcher attuned this article standing on the questions to be addressed along such line of think are being "what are numbers and where are numbers?" The question have been examined in the light of platonic thinking, absolutist philosophy of number, social constructivist philosophy of number including humanist and ultimately with respect to *Nielzsche - Foucautposition*. In this article; according to *Vedanga Tyotislya "Ganita*" is a science of number and derived from counting. The number are neither the poem created by poets nor they are

indubitable class of classes representing abstract entities having universal character, rather they are created reality having objective existence as the image of physical existence just as the image of mountain on the lake.

Sreshtha said, the image generally represent an object in same sense and under certain conditions, the image may represent the object more clearly. He compares number with reflected image and shadow. So he reached conclusion that, mathematical object as the image of shadow carry more or the less the characteristics of its parents. It may be that the infinite and transfinite number as proposed by George contour can also be considered to have shadow existence. This study provides a good guideline to analysis of number system from developing perspective.

Namaste (2006) had an article on the topic "The Lichhavi numeral and Changu Narayan manuscript". This article discus the different historical numeral system like as Babylonian, Chinese, Greek (Acrophobic and Alphabetical numerals), Indian subcontinent; Brahmi, Gupta and Lichhavi numeral, ancient finger numerals, different Nepali nujmerals, 7^{th} , 10^{th} , 11^{th} Lichhavi numerals etc. this article more focused about different Lichhavian numerals and their developing period. They say, number is Lichhavi script were found in a broken pillar from Pashupati Mandir also in the famous Manadevas's Changu Narayan inscription. The number in the Changu Narayan was read as the number 386. It is believed that Brahmi numeral, Gupta numeral, Nepali Bhasa (Bhujimol) numeral were also used in the Shaka year 107(A.D. 185) in the period of kind Jaya Varman. The comparison of the first nine Lichhavi numerals with the first nine numerals of Brahmi and Gupta numeration system shows that the first six numeral of three system are almost alike.

Pant (2008) wrote an article based on the topic "*Prachin Kalma Nepalma Chaleko Sankhya Lekhan Padhdati"*in Nepali script. The writer analysis about two inscription which is very oldest inscription in the Nepal. Those inscriptions was written by Asoka and use the Brahmi script in about 836 A.D., where also use the Lichhavi numeral 386. In the inscription of Lichhvi there not use zero and use individual symbol for 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80 and 90. Also they use different sign for 100, 200, 300, 400, 500 and 600. They use the symbol of 300, 80 and 3 together to denote the number 386. There was not drive the sign greater than 600 in Lichhavi script. Mainly this article focused about the Lichhavi numeral system.

Jordan (2009) studied the predictive relationship between early number competence in kindergarten and later mathematics achievement in third grade student performance on criterion assessments. This study used the Number Sense Brief (NSB) to measure 204 kindergarteners' number sense and measured the same children again in the beginning of first grade. NSB is a research-based untimed multiple-skills assessment that takes approximately15 minutes to administer and consists of 33 number sense items.

Based on the results of the NSB, strong predictability was shown to correlate to performance on the Woodcock Johnson Calculation and Applied Problems (WJ-AP) assessment at the end of both first grade and third grade. This also was the outcome of the correlation when comparing a fourth year of data on the same students generated from the Delaware Student Testing Program (DSTP), the state criterion high stakes test. Repeated measures analysis of variance (the test was conducted 7 times throughout the year) showed a statistically significant main effect for the group

which revealed that children who met the DSTP mathematics standard at the end of third grade consistently obtained higher NSB scores across time than those who did not meet the mathematics standard.

Pandey (2009) purpose a research about the "Sharada" script. It is a comparative and analytical study. The researcher analysis and explore the Sharada script and manuscript in different civilization and era. Sharada is a major historical Brahmi-based script of Kasmiri from 8^{th} century to 20^{th} century CE. This script was used to write Sanskrit, Kasmiri and other language of north – south Asia. From the 20^{th} century, Sharada use to write Manuscript of Vedic and classical Sanskrit text in about 19^{th} century. It is used to growth the Devanagari (Pandey, 2009). In about 16^{th} century Sharada manuscript is appear in Kalidasa's Sakuntala the Mahabharal and other classical Sanskrit text . It occurrence in Devanagari and other script. It may be unify these with existing Devanagari characteristics and Vedic tone. The structure of Sharada is identical that of Devanagari

Sharada digit represents the values of positional decimal system. The notation system of Sharada is unique among the Bhrami-based system in that a dot represents zero (Sharada digit zero) and circle use in other notation system to represent 0, is used to represent the digit one (Pandey, 2009, PP: 22). This shows that, the decimal counting system was existing in ancient Sharada script as well as zero with symbol dot.

Acharya (2011) has conducted a research on "An analytical study of Nepalese history of mathematics". The objectives of this study were to identify the historical development of mathematics in Nepal. In this research, Acharya explained development of mathematics in Nepal in different times, community and regions by

Nepalese people using various books, articles, manuscript, biographies, inscriptions etc.

Burland (2011) has studied on "Statistical relationship among number system, conceptual fluency and Montana comprehensive assessment system" standing on the certain research questions. The purpose of this study was; to investigate the long-terms predictive validity of TEN (test of early numeration) measures that include oral counting, number identification, missing number and quantity discrimination. The population was taken approximately 264 from grade 8 and approximately from high school. He conducted qualitative study based on correlation tools. He got the finding that; A two-part examination of TEN by the high predictability broken down by each TEN skill (OC, NI, QD, and MT) variable and secondly by grade level. Virtually all four of the Ten had some degree of predictability for educational achievement as defined by the grade 3 Mon CAS. Kindergarten Number Identification (NI) and Quantitative Discrimination (QD) had the most explained variance following by kindergarten Oral Counting (OC).

Subedi (2013) has expressed a view toward the developments of number system. He submitted this information through the survey article basically concerned with the development of the elementary number system. His article deals with the how number systems are developed in the sense of addressing real life problems and solving the polynomial equations concerning on this study, fact that two fold impetuses; one is the need of invention of new number system that deal with real life problems and another is finding the way of solving equations in which the number system become the playgrand.

On his articles, "Various positions of singers and hands were used for the expressions of numbers symbols or number names. In the middle age, finger numbers were extended to include the large numbers occurring in the business and other transactions. In the ultimate development, the numbers 1, 2,3,..., 9 and 10, 20,..., 90 were exposed in the left hand and the numbers 100, 200, ..., 900 and 1000, 2000,... ..., 9000 on the right hands."

Bhusal (2013) had an article concerning on the objective of "to explaining the different types of numbers in mathematics." He introduces the different numbers system and their applications. He said that;

> *"The uses of some of them can be explained as follows: the obsolete number is used in all branches of mathematics. A bounded number, amicable numbers, counting, positive and negative numbers, prime and composite numbers, rational number, square, whole number etc. are mainly used in Arithmetic, Algebra, and Analysis etc. Abstract number is use in space, Cayley number, imaginary number etc. are used in complex analysis cardinal number, Linville number etc. are used in Analysis, Topology, and Algebra etc…" (pp: 43-46).*

Acharya (2015) had an article on the tile "Evidence of manuscript of mathematics" regarding on analysis of *Bakshali* manuscript, *Samati Sidhanta, Kulchakra, Sankhyakramadi, Akonvishopadesyu, Dhanabajraachorya, Shishubodhtarangini*, *Vaktachandrika* and soon. Acharya explore historical evidence of different numeral system with their concrete and visual event. He visited the different library, places, Archeological Department, sections, Sides. This gives a good idea to explore and study about number system.

Adhakari (2015) done a thesis on "Mathematical Contribution of first Nepali Women Mathematicians *Chandrakala Devi Dhananjaya*" with objective; to explore mathematical contribution of *Chandrakala Devi Dhananjaya* and analysis her book about *Shishubudha Tarangini*.He used the qualitative research based on historical descriptive. The area of this study was biographic and exploration of mathematical proof and explanations. At last Adhilari conclude that ; Chandrakala first use the measurement trams *Mana, Pathi, Aana, Ropani, Pisa, Angula, Bitta* etc. which provide the ethno-mathematical evidence. Also, inductive method found in her book *Shishubudha Taranini* use first mathematical calculation.

Acharya (2015) has done research on the topic "*Naya Raj Pantka Ganitya Kritiharuko Adhyayan.*" In this research, Acharya explored the details biography and contributions of Nepali mathematicians late Prof. Naya Raj Pant. The research shows that late Prof. Naya Raj Pant was the great mathematician has well contribution on number system. Acharya found that; late Pant experimentally test about the cube root number, study about different number script and says Brahmi is the base of all Nepali number, work on different number system and more other. This literature provides evidence about different number system in Nepal and gives well idea in this research.

Rai (2015) has done a research on "Development of numerical script n Rai Ethnic" based on the objective; identify numerical use in representing number by *Kiranti(Rai).* It was a historical research with qualitative type. He used open types of questionnaire about development of numerals script of *Kiranti (Rai),* published and unpublished books. He reached in conclusion that; *Kirati (Rai)* is influencing by *Bhrami* script and numeral system are based on decimal scales with position number system.

Rai (2015) has studied on "Development of Number System in the *Bantwa Rai*" with objective to identify the numerical system of *Bantuwa Rai* in present time. His study was based on primary data and secondary data consisting of the face to face interview with the resource person, the n study of journal, books, and related published and unpublished literature. He had got the finding as Rai script also have also been identified which have Hope=0, Ee=2, Ha=2, Sum=3, … , Nun=9 in the cardinal number. He reached the conclusion that; *Bantwa Rai*are using the numeral system from *Madival* age. The numerical system of *Banawa Rai* and Hindu- Arabic number both are based on 10 but written numeral and pronunciation are different.

Subedi (2017) has done research on the topic “The development of numeral system in ancient Nepal” with objective; to expiation and origin of numeral system, to identify the structural of numeral system of Nepal, to explore the stage of development of numeral system and identify the invention or diffusion of Nepalese numerals. She followed the material collection and analysis methods in this research. She reviewed the different civilization of numeral history of world. At last, she has got the major finding as; before 10th century, all number system has base-10, no sign for zero, all system are ciphered, Nepalese numeral system does not replaced by another system over the period, Hieratic, Greek, Brahmi and Nepalese system are similar structure, numeral-sign for 9 is identical in Hieratic, Brahmi and Nepalese, Egypt, China and Nepalese are identical base. This provides the much evidence for research concern on number system.

An article from the side (http://www.wikipidia.org/History, 2014s) about "Concept of content Analysis in Research" has been analysis here. Content analysis is a research tool used to use to determine the presence of certain words or concept with

in text or sets of texts. Researchers quantify and analyze the presence, meaning and relationship of such words and concepts, then make in faience about the message with in texts, the writers, the audience and even the culture and time of which these are a par. Palmquist (1990) Studied of two composition classes, in which he analyzed student and teacher interviews, writing journals, Classroom discussions and lecture, and out - of- class in traction sheets. To conduct a cannot analysis on any such text, the text is closed or broken down into manageable categories on a verity of level- word, word sense, phrase, sentence or them and then examine using one of content analysis, basic methods and conceptual analysis or related analysis.

"Content analysis refers to a family of techniques oriented to the study of "mute evidence", that is texts and artifacts. On the other side content analysis also trace and artifacts document. Which conform communication process "Later Sense" Commonly referred to as "Signification" in *Seonitoties*. So, mathematics is a popularly used social subject, where comes analysis way is a suitable way for classical document analysis.

Many researchers have been done different rear number system but regarding on famous Hindu-Arabic and Devanagari number and its foundation, there has not done any research yet. So, I feel there is necessary to add much evidence and provide validation concern on Hindu-Arabic and Devanagari number system. Therefore, I ‘am interested to do the research on topic “Development of Number System”.

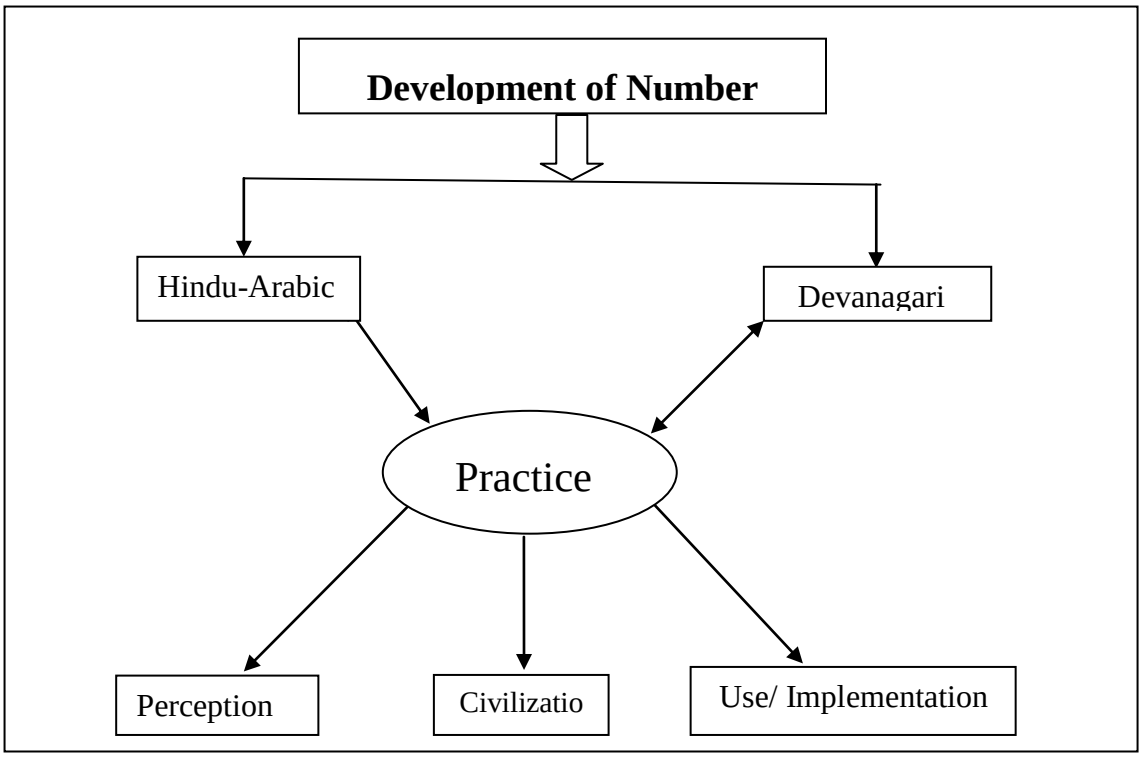
Conceptual Framework

This study is type of qualitative research consisting content analysis approach. The researcher tries to explore the perception of primitive counting and find

development phase of Hindu-Arabic and Devanagari number system. These above literature review provide following conceptual framework to study systematic order.

The researcher focused this study to find concept about number in primitive people and their counting and different turning phase of Hindu-Arabic and Devanagari number system from this ancient counting. From the above literature, numbers are birth from daily human activities and different social phenomena. The human activities and their impact in perception of number in human mind is the main study of this research. The researcher tries to explore the foundational practice of Hindu-Arabic and Devanagari number system. And what was the Hindu-Arabic and Devanagari number's practice in different civilization? Here, the researcher analysis the contribution of different civilization on Hindu-Arabic and Devanagari number system.

Fig No. 1: Conceptual Framework



Source: Bhusal, (2013)

This conceptual framework based on empirical literature review, tries to explain an idea for the stu

dy of this research. Specially, researcher studied the different practices concern on ancient number and counting. The analysis of practices on number perception, practices on different civilization and practices on its implementation is the main strategy of this research. So, the this study is conducted the perspective of different practices of Hindu-Arabic and Devanagari number system from ancient up to today. This study is based on content analysis approach. The researcher analysis different authorized document, visit website and library. At last, researcher takes interview with authentic resource person to make authentic this study.

Chapter III

DOCUMENT ANLYSIS METHODS AND PROCEDURE

Before entering in a research, the researcher should be sure about appropriate methods and procedures. This chapter deals with the methodologies, area of study, information collection tools and procedure, information analysis and interpretation procedure, which has need to fulfill the objectives of the study. At each operational step in the research process, procedure and models of research methodologies, which would help them to best, achieve their objectives. Methods means the way of collecting information from the chosen study and interview, observation, group discussion, document analysis are the popular methods practices in qualitative research (Sharma, 2011). The research questions are using the qualitative data. This chapter has included research design, population of the study, sampling procedures, data collections tools and techniques as well as data analysis procedure.

Design of the Research

This study is based on development of Hindu-Arabic and Devanagari number system. Researcher explored the number perception on ancient time as well as development of Hindu-Arabic and Devanagari number. The researcher analysis and interpreted authentic documents of C. B. Boyer, Datta & Sign, F. Cajori, R. Cook, T. Dantzing, H. Eves, G. Ifrash, Jordan, Clarke. He also used the Ph. D research thesis of Eka Ratna Acharya, Nilam Subedi etc. So, the design of this study is historical research based on descriptive and analytical approach. The researcher does not manipulate any events and neither control environments. The document analysis, library study, and interview with resource person were used on this research.

Source of Data

The primary and secondary information are main source of this study. The researcher collected the primary data from visiting manuscripts, inscriptions, biographies and consult with resource persons. Also, researcher collected the secondary data from library, website and other different recorded etc. The required secondary data were; published books and article, journals, records reports, periodicals, bulletins or catalogues, Syllabi, curt decisions, Pictures, micro films, net sides and cartoons etc.

Area of the Study

Number system is the main area of this study. This study is based on development of Hindu-Arabic and Devanagari number system. The number perception of primitive peoples, foundation of ancient numbers, Brahmi numerals, Nagari numerals etc. were logically analyze from the authentic historical books. Ancient Sanskrit and Brahmi numerals were interpreted and analysis here. In this study, the fundamental stage of Devanagari and Hindu-Arabic numerals and their development has been explain and interpretated briefly. Researcher follows the authentic research documents as research thesis, Ph. D thesis, *History of Hindu Mathematics* by Datta and sign, A History of Mathematics by Boyer and so on. Thus, the origin of the Hindu-Arabic and Devanagari number system and their development is the area of this study.

Tools of the Study

This is a historical research based on qualitative data. In this study, the various types of primary and secondary documents were the main source of data. So, the main

tool of this study was the document analysis. In this study, researcher used the major sources and minor sources of data. Where authentic books, research thesis, journals, articles etc. were the major sources of data. Researcher also conduct the unstructured interview and counseling with resource persons which used the miner sources of data.

This study was complete on the basis of the authentic historical books, research thesis, journals with five respond's views. Mainly researcher follows the 'History of Hindu Mathematics' by Datta & Singh, 'A brief History of Mathematics' by C.B. Boyer, 'History of Mathematics' by F. Cajori, A History of Mathematics by R. Cook, 'Fundamental Steps in Development of Numeration' by C.B. Boyer, 'Number: Language of Science' by T. Dantzing, 'An Introduction to the History of Mathematics by H. Eves, 'The Number in Ancient Time' by E.C. Ortenzi, 'Recent trends in Mathematics Education' by R.P. Pandit, Ph. D research thesis by Eka Ratna Acharya and Nilam Subedi and master research thesis for getting well data for the study.

First, researcher focused the document analysis and information basis on the problem number perception on ancient people. Researcher explores the evidence and facts from the related historical documents and triangulates with the respond's views. According to the documents, number perception on ancient people had developed from contemporary daily life activities. In that time people percept the number by associating, "sun" and "moon" for 1, "eyes" and "breast" for 2. "leaf" for 3 and so on. After, this data was triangulate with views of the respond person. The document analysis was regarding on the next problem Historical development of Hindu-Arabic and Devanagari number system. The historical book; 'History of Hindu Mathematics' by Datta & Singh, 'An Introduction to the History of Mathematics' by H. Eves, says

Brahmi and Nagari script is the foundation of Devanagari Number. After 10^{th} to 12^{th} century, the ancient mathematicians Al-Khwarizmi transgression the Devanagari number in the Arab and Europe then it modify into modern Hindu-Arabic number system. Researcher cross match the views of resource person with it then get the conclusion. The most of the respond has the common views that the foundation of Devanagari number is Brahmi numeral and Brahmi numeral is developed in south India in around thousand years ago. They also response that, Hindu-Arabic number is develop from Devanagari number. Researcher follows the counseling of resources persons to analysis and interpretation the data.

Documentation Procedures

First of all, the researcher visited the different libraries, website, consult resource persons and teachers to collect the required information for research. Specially, researcher used the triangulation (multiple methods of data collection and analysis) method to collect the data. The researcher studied the classical documents, books, journals, articles, thesis etc.

Document Analysis and Interpretation

The nature of the study is historical descriptive. The views of historical documents, recorded history, and journals were the data of this study. Also, researcher collected the views with five resources persons regarding on the problems of this study. The researcher interpreted and analysis the views of document and triangulate with views of resource persons.

The related data was interpretation in logical, analytical and descriptive manner. The statistical data of classical development are very rare and not match the historical analysis. So, researcher applied the inductive as well as triangulation methods for analysis and interpretation the data. Triangulation is mixing approaches to get more than two views. Researcher search common views from different civilization then reach in conclusion by inductive approach. So, the study was based on historical documentation and description.

Chapter IV

DOCUMENT ANALYSIS AND INTERPRETATION

The chapter is consisting with brief analysis of those number system which develop standing on Hindu Arabic and Devanagari number system. Researcher collected authorize historical documents, visit the many library, visit the different web sides, consult with resource person and conduct an in-depth interview for getting the reliable and fact data.

Number Perception on Human Mind

There are still good reasons for believing that, there was a time when people did not know how to count. Not knowing how to count, whenever they had no notation of number. It may imagine that, the notion and counting was limited on their natural sense. That is to what direct perception enables them to recognize at a glance. Regarding on this topic, different evidence analyzing at below:

Different civilization has individual practices regarding on number system. The book *History of Mathematics* by R. Cook says that;

> *A-fourteen year old baby can usually reassemble them into a single group. If something is amassing from a small group that is familiar to him, he will immediately notice it. But his abilities are so limited that he fails to perceive numerical differences on equality in the people or objects around him as soon as their number goes beyond three or four. He cannot conceive any absolute quantity because he is not yet capable of counting, in the strict meaning of the terms… (Cook, 1997, p: 54).*

Here the baby is able to knowing the limited familiar persons. He finds missing of person whenever, there are less than three or four person around him. But all people would be familiar to him. This we conclude that, number sense also associated with recognizing ability of people. This concludes that, number and counting is direct associated with human natural sense.

Counting the objects in a collection means assigning to each of them a symbol (a word, a gesture, or a graphic sign) Corresponding to a number in the natural sequence and counting till the collections is exhausted. Each symbol represents the ordinal number of the object to which it is assigned, and the ordinal number of the last object is the number of the elements in the collection. This shows that "number does not come from things, but from the laws of thought working on things (Balmes, 1978). A research concern on the topic *Number: Language of Science* by (Dantzing, 1995) has explained as;

> *Ingenious experiments conducted by specialists in animal behavior have shown that some species apparently have a kind of rudimentary direct perception of concrete quantities, which is not same as the ability to count. This faculty has been called the number sense. An animal ability probably more precise than in domestic animals has been observed in birds. Experiments have shown that a goldfish, trained to choose it is food from two small piles of seeds, generally succeeds in distinguishing three from one, three from two, four from two, four from three, six from three but nearly always confuses five and four, seven and five, eight and six, ten and six…. (*Dantzing, 1995, p: 56*).*

Above experimental text explain that, animal has direct perception of concrete quantities, they have not the ability to count. In experiment, the goldfish confuse five and four, seven and five, eight and six. So, this concludes that all animal has number sense but not ability to count.

The book *Number in Ancient Time* by (Ortenzi, 1997) describe that ancient people has the number sense up to two. According to Ortenzi;

> *Member of the Aranda tribe in Australia had only two basic number words: nitta, (one) and tara (two), Fore three and four they said tara-ma-nita (two one) and tara-ma-tara (two-and-two), Beyond tara-ma-tara they use word meaning "many"…. (Ortenzi, 1997, p: 84).*

This concludes that, human natural number sense is so limited. Without systematic counting technique, there is no possible to know the number.

Early in the century there were still people in Africa, Oceania and America who could not clearly perceive or precisely express numbers greater than 4. To them, numbers beyond that point were vague, general notations related to physical plurality. It is probably significant that, as Leby-Brunhl reports, some oceanic tribes declining and conjugated in the singular, the dual, the trial, the quartile, and finally the plural (www.wikipidia.org/History- math).

The researcher has taken interview with resources persons and the views given by them are required as below:

"The man and all animals have the number sense. They use their natural number sense by the recognizing form. But counting is another perspective which has used the certain technique. Ancient people had number sense up to two beyond two

they use term "many". A man can recognize up to four objects at a glance…"(5th august, 20017, 8:30 am) (See Appendix-B)

Analyzing from above evidence, I conclude that, all animal and men has the number sense. The human natural number sense is used for only recognizing the objects. The human direct number sense is able to recognize the collection less than four objects at a glance.

Counting Practices on Ancient People

In the history of the time, the different civilization has individual counting practices. The primitive counting practices of different civilization are analysis as follows;

The book *Number in Ancient Time* by (Ortenzi, 1964) describe that ancient people has the number sense up to two. He says that;

> *We may assume that to them, the number concept was a concrete reality inspirable from objects and that was manifested only in direct perception of physical plurality. They were…, The wings of a birds or eye, ears, arms, or legs of a person of common characteristics of being "two"....(*Ortenzi, *1964, p: 34).*

This shows that, the number concept is inspired from the concrete reality. That means, the number concept is developing from the practice of concrete objects. The number concept is developing from the common characteristics of objects.

The book *Number in Ancient Time* by (Ortenzi, 1997) also describes as:

Island in Torres Strait, between New Guinea and Australia had only these member words netat (one), he is (two), neis-neat (three, literally "two one") and neis-neis (four, literally, "two-two"), beyond that, they used a word meaning like "a multitude (pp: 56).

It is similar to Aranda tribe but beyond three they use term "a multitude". Similar, among other example of some kind, Botocutdos in Brazil said their word for 'many' they pointed to their hair, as if to say, "Beyond four things are as countless as the hairs on my head".

Since these people were able to use combinations of their two basic number word for the next two numbers- "two one" for 3, "two-two" for 4, but one might wonder why they did not use "two-two-one" for 5, "two-two-two" for 6 and so on. That would be overlooking the fact that to express 3 and 4, numbers that they can recognized by direct perception, people at this stage. "Simply pair 1 and 2, then 2 and 2 for them, These stage." Simply pair 1 and 2, then 2 and 2 for them, these are still pairs, Thought far us they are hole numbers when we designate them as sand 4 (Loren, pp: 34).

According to Ortenzi, ancient people use the concrete numeral to count the objects. They use notches on bones, pebble, shells, sticks and fingers on hands for counting .They perceive the number one by the uniqueness of objects like “sun”, “moon” etc. Also, two by couple objects, three by unique collection of three objects like tripod, four by legs of animal etc. The human natural number sense is used for only recognizing the objects. The human direct number sense is able to recognize the collection less than four objects at a glance.

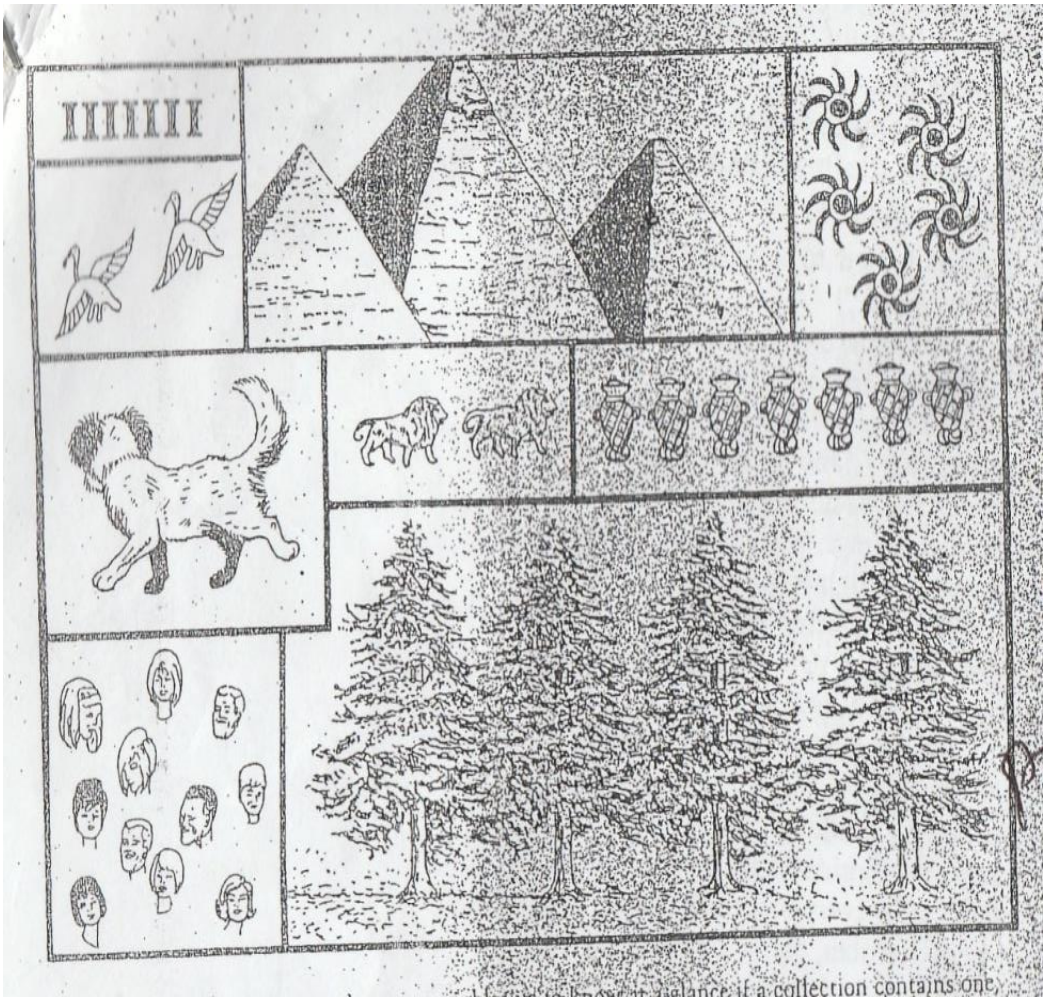


Figure No. 2: Number Sense make from Collection of object (Ortenz, 1997)

This seems that, some ancient people make the number perception by grouping of objects. But before the recorded history, people may recognize the quantity of objects by using their direct number sense. They can recognize the collection of objects up to 4, but beyond 4 hey use "many" and plural. They also able to use combination of two basic number "two-one" for 3, "two -two" for 4, but they did not able to use their perception about using "two-two-one" for 5, "two-two-two" for 6 and so on. After, they perceive the systematic idea on the geographical notations to represent the numeral (Fig: 2).

Ancient people used materials and objects of all sorts (pebbles, shells, bones, sticks, clay objects), notches in bone or wood, knotted strings, intuitive or

conventional gestures' (use of fingers, toes etc.) for counting. The concrete term directly imply the notation of number: "sun", "moon" for 1,"birds wings" for 2, "clover of leaf for 3, "animal legs" for 4, "fingers of one hands" for 5 etc (Ortenzi, 1964).

According to Ortenzi, ancient people use the concrete numeral to count the objects. They use notches on bones, pebble, shells, sticks and fingers on hands for counting .They perceive the number one by the uniqueness of objects like "sun", "moon" etc. Also, two by couple objects, three by unique collection of three objects like tripod, four by legs of animal etc.

One-to-One Correspondence Counting

According to the book "History of the Mathematics" by Boyer "At one time number was thought to be directly concerned with the world of our sense experience".

> *At first primitive notation of number, magnitude and form may have been related to contrasts rather than likeness." The difference them self seem to point to likeness, for the contrast between one wolf and many, between one sheep and a herd, between one three and forest, suggests that one wolf, one sheep and one three have something common their uniqueness. In the some way it would be put into one-to-one correspondence (Boyer, 1968, p: 23).*

The above content from the book "History of Mathematics" by Boyer clears that, which was limited in our experience of sense organ where object can be count by making one-to-one correspondence to another object. Before the record history, people apply a technique of counting which was one-to-one correspondence counting. Ancient people carry the some number of pebbles to count their domestic animal

(http://math.wichita.edu.history/topicsnum-sys.htm). The following figure shows the methods of ancient one-to-one corresponding counting.

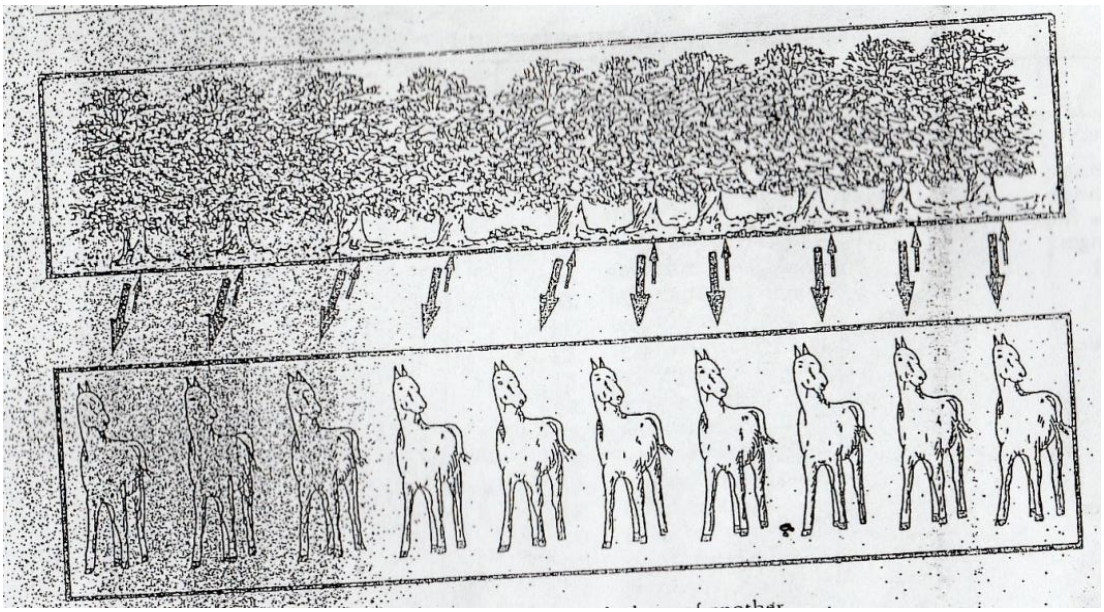


Figure No. 3: Counting through one-to-one correspondence (www.wikipidia.org)

The above counting system has been developed by using the method based on a concept that is highly important in modern mathematics one-to-one correspondence, also known as bisection or matching. Ancient people count the object (seeps) by using one-to-one correspondence. If a men has 10 seeps, then count each his seeps which corresponding his two hands fingers.

According to research thesis *Naya Raj Pantka Ganitiya Kritiharuko Adhyayan* by Acharya:

> *Manishle hajarau barsa pahile ganitko dharana tatha ganti prakiyako suruwat gareko vayata pani yako atihashik dastabj vane 8,8000 bassa purano Esango hadma kudiyaka anka haru nai hun, jun jayarma phela pariyako thiyo….tyaha Hindu syankyako pani ullekh gareko dekhinchha…. (Acharya*, 2015, p: 55). (See Appendix-D)

The responder-A says that, primitive people were count the objects by using pebbles and notches on bones. Pant said that, primitive people make concept of number directly imply the concrete object. The "sun", "moon" for notation 1, "eyes", "breasts" for 2, and so on. Another resource responder-B Express the same views. So, numbers were not drive from the mathematicians, it were the human practices on different culture, civilization. She claimed that, one-to one correspondence counting technique is the foundation of modern counting.

From all above evidence, I conclude that; people use concrete objects for notation the number before the recorded history. Primitive people make number concept from the define unique collection of objects. For Example: "sun" and "moon" for 1, "eyes" and "breast" for 2 and so on. Number was drive from civilization not from mathematicians. The counting of the number was developed from ancient people's one-to-one corresponding counting.

Ancient Practice on Symbolizing the Number

What was the symbol of the number in foundational phase? How much primitive people make the perception behind it? What are the chronological practices? And other many questions arise regarding on the number notation.

> *Counting the objects in a collection means assigning to each of them a symbol (a word, a gesture, or a graphic sign) Corresponding to a number in the natural sequence and counting till the collections is exhausted. Each symbol represents the ordinal number of the object to which it is assigned, and the ordinal number of the last object is the number of the elements in the*

collection. This shows that "number does not come from things, but from the laws of thought working on things (Cajori, 1894, p: 47).

This text is explained that, people use graphical notation to represent the number where each symbol represents by ordinal number. Also the collection of the object is denoted by a single ordinals number. Blame claims, numbers come from law of thought.

In ancient several methods of symbolizing numbers have been used in the course of human development, mainly here discussion as below:

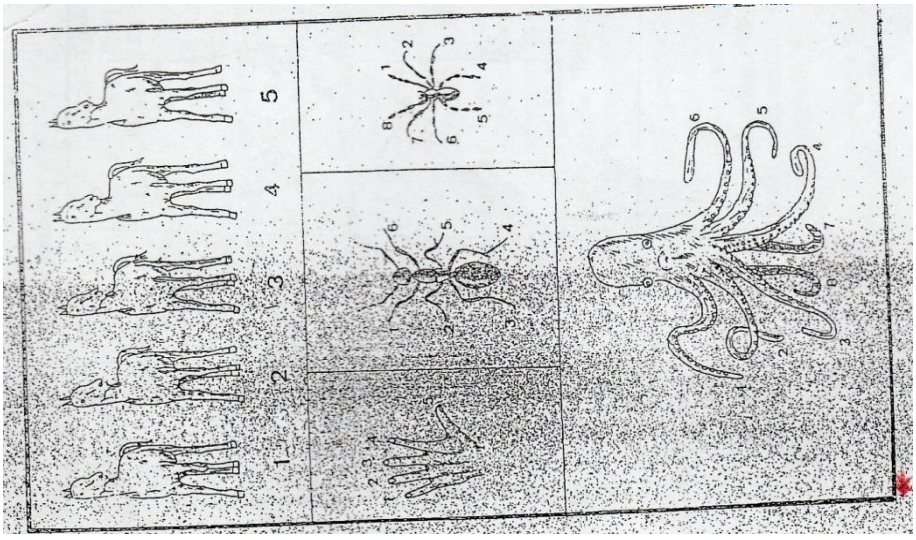


Figure No. 4: Graphical Representation of Number (Dantzing, 1930, pp: 24)

This figure shows the symbolizing the number from counting. There are five goats in first column. Similarly, in second column and third column there are different collections of the objects. Primitive people try to use different techniques for notation the number according to counting. For this, they used one hand for representing “five”, an ant for representing “six” and so on.

The figure: 3 also analyze the human practices on symbolizing the number. This seems that primitive people spend the more time to develop the systematic

symbol of the number. Sometime they used pebble, hand finger, notches on bones etc. to assigning the numbers. From the figure: 4, we conclude that, people use the graphical notation to represent the number. Before the developing systematic counting, people use ordinal notation to represent the collection of objects. Example: symbol octopus is use for ordinal 8.

> *Graphic signs of various kind (drown, painted, or engraved marks, notices, representational signs, letters of the alphabet, signs with no direct visual reference) each called a figure or numeral, taking these terms in the broad sense of any "graphic numerical sign (Cajori, 1978, pp:25-26).*

Analyzing after above contents/documents we can decide that, symbol of number is conform laws of thought working on things. People gesture the "sun", "moon" for count, "eyes" for 2 and etc. Ancient people use "fingers" for count 1, 2, 3, 4, 5, "legs of animal" for 4, "octopus" for 8 as a graphic sign. Notches on bones, different objects were used in graphic sign. Today's numerals are modified and modern form of ancient these graphical sign. So people change the concrete things in mental objects.

Ancient people use graphical notation to represent the number where each symbol represents by ordinal number. Also the collection of the object is denoted by a single ordinals number. Blame claims, numbers come from law of thought.

Hindu-Arabic is more useable place-value decimal number system in the world that uses zero glyph. It glyphs are described from Indian Brahmi numerals. The full system emerged by 8^{th} to 9^{th} centuries, and is first described in Al-Khwarizmi's on the calculations with Hindu-numerals

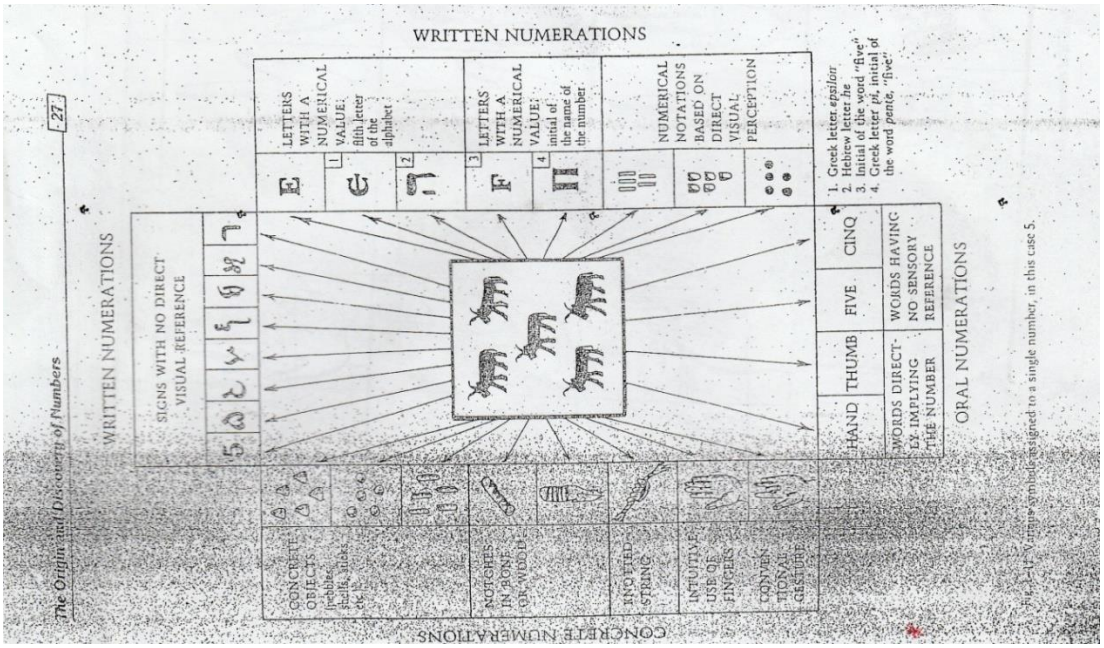


Figure No. 5: Various Symbols Assigned To a Single Number (Dantzing, 1930, pp: 43)

The fig: 5 show the different methods and principal of symbolizing the number. It is a dynamic form technique of symbolizing the number. For counting and notating the five animal primitive people use different symbols, intuitive use of finger, notches in the bone or wood, pebbles, sign with direct visual reference, letter with numerical values etc. this we conclude that, different civilization had different numeration system in ancient time.

Ancient people used the graphical sign and physical objects to represent the number. They draw this sign on bones, stones, clays, skins, tablets, woods etc. This symbol is continuously used the long time and built digit. Before driving the symbol zero, people only left the blank place between the two digits, after they use the symbol dots (.) only for representing the empty later "Sunya" way developed in India as numeral representing the empty. The Maya number system was also explain the about the symbol zero. They also used the symbol zero as the sense of empty. Ancient Babylonian had the concept of zero (Dantzing, 1930).

Making Number by Counting

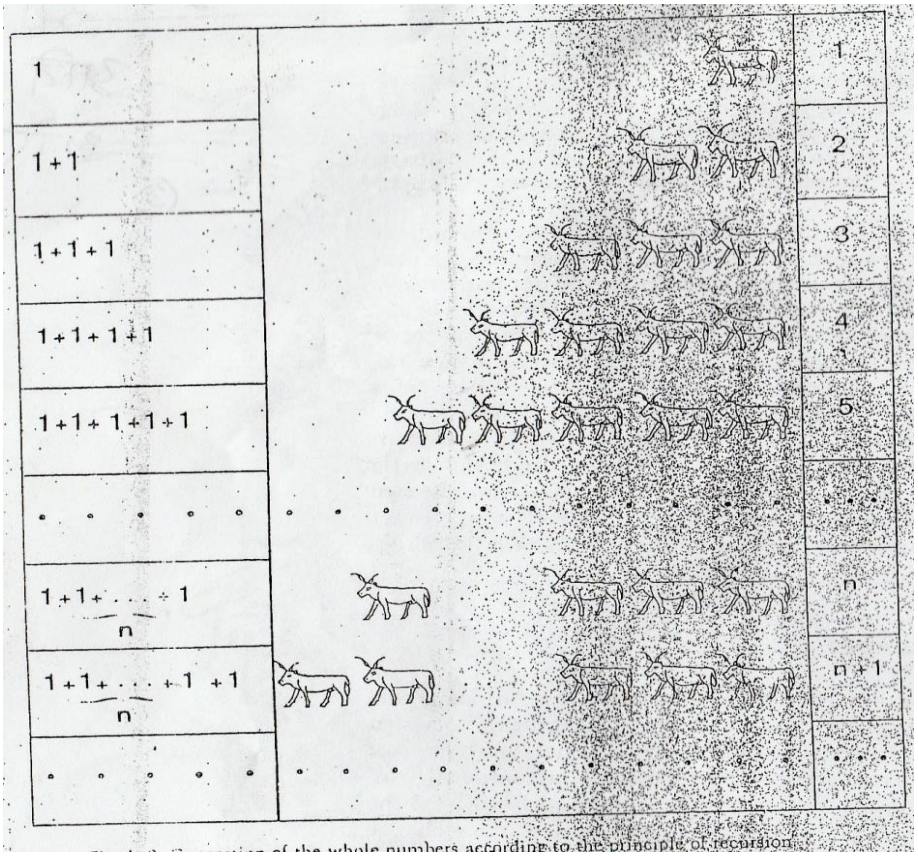


Figure No.6: Generation of Whole Number According To Principal of Recursion. (Ifrash,, 1978)

Decisive progress in the art of abstract counting requires that the whole numbers be arranged in a sequence in which each number, after 1, is obtained by adding 1 to the number before it or counting is developed standing on principle of recursion. In a case of young child, until he has developed to the stage of being able to grasp the principle of recursion, he can recognize the persistence of a given quantity only by means of one-to-one correspondence. But when he has reached that stage-which occurs between the ages of three or four, according to Jean Piaget, he will soon be capable of learning to count and calculate. The first learn to count up to 10, using his

fingers, then to extend his number sequence as he Tome to conceive the abstraction of number (Ifrash, p: 46)).

This fined that, the counting is systematic and technical process. The counting process is developing according to the principle of recursion. In figure No. 6, 2 is come after 1 by adding 1.That is, 1=1, 2=1+1, 3=1+1, 1=2+1, 4=1+1+1+1=3+1 and so on.

The researcher has taken the interview with resource persons and the resource person 'B' has the required views as given below:

"First of all, primitive people use different concrete objects for symbolizing the number. People use the physical objects and graphical sign for representing the number. Ancient people use one hand to represent the five, and other similar manner. Babylonian developed the symbol of number in around 3300-3500 BC. Structure of the number is built from different notches on bones, structure of woods, leaf, sun, moon etc. they scratch symbol on stone, wood and bones for representing some quantity. After, they repeatedly use this symbol and make a number. The symbol of number is also the result of human civilization but not the mathematician's discovery". (5th august, 20017, 8:30 am)

Analyzing and comparing after above evidence, researcher reached in conclusion as; number symbol is contribution of the human civilization but not the mathematician's individual achievement. Ancient people used the graphical sign and physical objects to represent the number. They draw this sign on bones, stones, clays, skins, tablets, woods etc. This symbol is continuously used the long time and built digit.

Origin of Zero and Zero Concept

The formal use of numeral zero was first formulated from south Asian civilization by Brahamagupta in around 6th to 7th century A. D. (Boyer, 1996). The numeral zero first provides the place-value counting system. Regarding on the concept of zero, some document explain as follows;

> *The Mayan's were also the first to symbolize the concept of nothing (or zero). The most common symbol was that of a sell ( ) but there were several other symbols. It is interesting to learn that with all of the great mathematicians and sciences that were around in ancient Greece and Rome, it was the Mayan Indians who independently come up with this symbol which usually meant completions as opposed to zero or nothing… (Ortenzi, 1964, p: 14).*

The Maya number system was famous around 4th century and was 1,000 years more advance than Europeans of that time (MeLish,1991). The Maya were also first to symbolize the concept of nothing (or zero). The Mayan who independently come up with this symbol which meant completion as opposed to zero or nothing.

Again, the zero was first use in about 825 A. D. by Al-Khwarizmi then started to positional value (Boyer, 1976). The Bakshali manuscript (4th to 7th century CE), used a place value system without to denote zero. The dot is called the *sunya-sthana*, "empty place" (www.wikipidia.org).

> *A number such as sixty-five thousand three hundred and two was indicated by the appropriate sign for 6,5,3, & 2 which occupied their due places, while the place which represented the tens rank was left vacant. This*

> *blank place was called "Sunya" in India and "Sifr" in the Arab world, both word meaning is empty. A tenth sign, which way usually in the form of zero symbol (Occasionally it was a dot).” Kani Meanninger, Number words and Number symbols: A Cultural History of Numbers, translated by Paul Broneer from the revised 1958 German edition (Datta and Singh, 1962, p: 15-16).*

Before driving the symbol zero, people only left the blank place between the two digits, after they use the symbol dots (.) only for representing the empty later "Sunya" way developed in India as numeral representing the empty.

Ancient Chinese use the roads made from animals bones or bamboo for base four operation of number. "Roads were used for reckoning science the Warring States period (475-221BC) and possibly earlier, and though their use a numeral system was developed (Yong, 1996; 40-41). This numeral system processed the same intrinsic property as the Hindu-Arabic numeral system. This Chinese place-value and zero representing by blank place use in Hindu-Arabic numeral system.

Also Boyer include that "The Liber-Abachi opens with an idea that should almost modern, but which was characteristics of both Islamic and Christian medieval through- that arithmetic and geometry are connected and support each other. Thesis view is of course reminiscent of al-Khwarizmi’s Algebra, but it was equally accepted in the Latian, Boethian tradition. The Liber-Abchi, never the less is much more connected with number then with geometry. It first describes "The nine Indian figures, together with zero, which is called Zephiram in Arabic". Incidentally, it is from zephiram and it's variants that our words "ciphes" and "zero" are derived. Fibonacci account of Hindu-Arabic numeration was important in the process of transmission. But it was not, as one have seen (Boyer, 1968:278-280).

This explains that, nine figure of numeral together with zero is drive in the India and after it is use in different place in that time of era. It is only variant the word "Cipher" and "Zero" in Arabic word Process of transmission in Fibonacci account.

> *The development stage of "Zero" is an insetting part of number system. Generally, Indian first introduce the numeral zero. "intuitive zero" of Brahamagupta that means "nothing" and which is the difference of two equal numbers. The number zero use the represent of number. The fact that the "intuitive zero" and the "mathematical zero" are represented currently by the "numeral zero" symbol "0" are only consequences of the represent of number. The fact that the "intuitive zero" and the "mathematical zero" symbol "0" are only consequences of the representation of the numeral by the "Ghubars" numerals…. The gradual change from old system without place-value to the new system with zero and the place-value is to be met with in India alone. This fact alone is strong proof of the Hindu origin of the zero and the place value system (Datta and Singh, 1962, p: 17-22).*

So, the book *History of Hindu Mathematics* by Datta and Singh claim that, India first formulate the zero and start place-value system.

The *Brahamagupta* also describe about the numeral "zero" as a meaning of "nothing" or difference of two equal hombres (Dantzing, 1930). This seems that the numeral zero was describe as different perspective on ancient era. In *Gubari* numeral it is representative by symbol "0".

Around 7th century, astronomer-mathematicians *Brahamagupta* wrote his seminal text Brahma *Sphuta Siddhanta* which contained the first mathematical treatment of zero. He defined zero as the result of subtracting a number from itself,

postulated negative number and discussed their properties under arithmetical operations. His word for zero was *Sunya*, the same term previously used for the empty spot in 9-digit place-value system. This provided a new perspective the *shunya-bindu* as a numeral and paved the way for the eventual evolution of a zero digit. The dot continued to be used for at least two years of forwards, and transmitted to Southeast *Asia and Arabia*. *Kashmir's Sharada* script has retained the dot for zero until this day. By the end of 7th century, decimal numbers begin to appear on inscriptions in Southeast Asia as well as in India (www.wikipidia.org/History of Hindu-Arabic numeral system.).

From the above document analysis, before the 7th century there is no place-value number system. The symbol dot (.) was used in empty place to representing he *sunya*(0). Before replacing the symbol zero (0) in dot position, the text Brahma *Sphuta Siddhanta* deign zero as the result of subtracting a number from itself. He also used word "Void" for *Sunya* at least 100 years of this time the notation dot (*Sunya-Bindu*) was used as a zero digit.

From in-depth interview with resource person, they expressed different views regarding on the development of zero. The respond-E has the views that; the concept of number was existing in *"TretaYug"*so about the zero. And respond-A said that the concept of zero was developed in "*Rig-Veda*" era. Also, resource person-B had views that; the concept zero was develop in Babylonian number system, Maya number system, and Greek number system but Brahamagupta first formulate systematically In around 5^{th} to 7^{th} century.

The comparative analytic from above evidence, I conclude that; number symbol is contribution of the human civilization but not the mathematician's

individual achievement. Ancient people used the graphical sign and physical objects to represent the number. They draw this sign on bones, stones, clays, skins, tablets, woods etc. This symbol is continuously used the long time and built digit.

Development of Hindu-Arabic and Devanagari Number

Both Hindu-Arabic and Devanagari both number system are the famous place-value decimal number system in the world. This number system has advance role to develop the mathematical theories. Hindu-Arabic number system first provides foundation to develop modern arithmetic and algebra.

Devanagari Number System

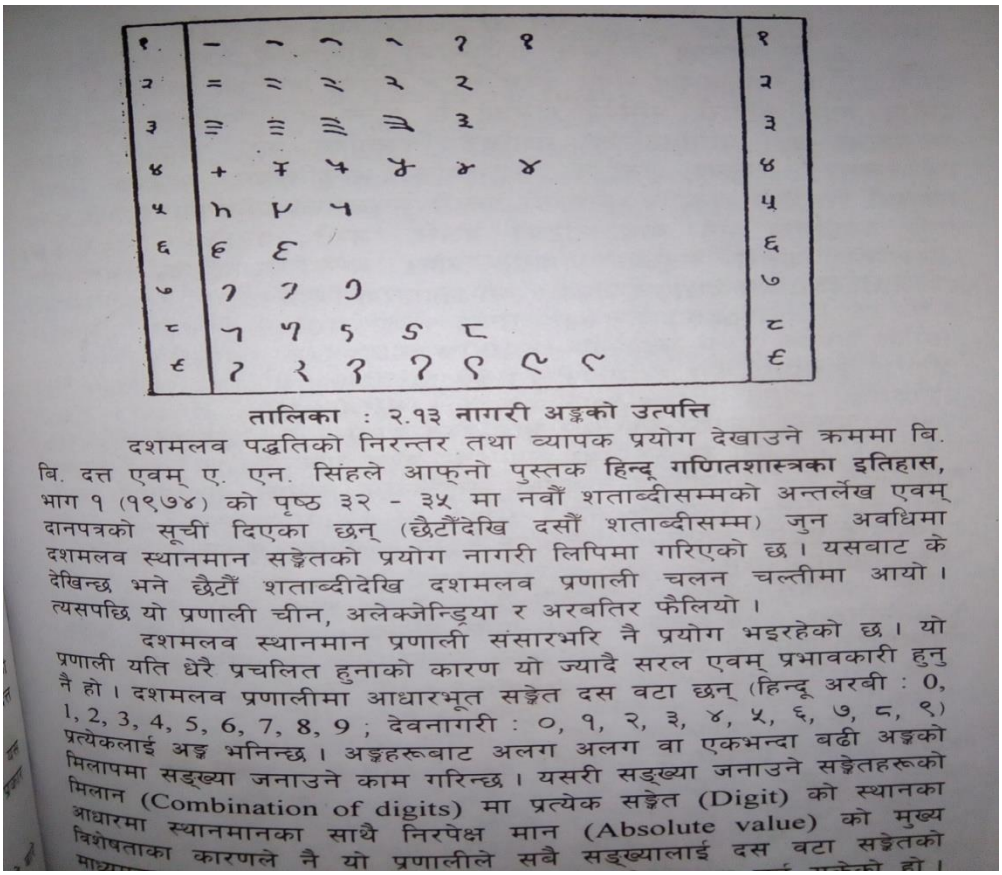
Devanagari is an alphabetical script with some syllabic feature drive from Brahmi, used for the writing of Hindi and many other languages including Sanskrit (British Dictionary). A Sanskrit number system which is develop standing on the Veda representing by symbols; १, २, ३, ४, ५ , ६, ७, ८, ९,and ० (Datta and Singh, 1962). Devanagari number system is developing corresponding Hindu-Arabic number system. It was the ancient *Bhramaboodhani* that give us the ingenious method of expressing all number s by symbol (decimal system). The highest prefix and for using 10 to the power in today's mathematics is "D" for 10 (Boyer, 1968).

The early development of decimal place-value system of numeration is carrying out from ancient Indian civilization or south Asian civilization (Sherestha, 2014). According to book *Ganita Darsan* (Sherestha);

> *Excavation at both Harappa and Mohen Jo-Dara have supported this theory. At the time however a 'complete' place-value have not yet been*

developed and along with symbols for number through nine, there were also symbols for 10, 20 100, and so on (Sherestha, p: 164).

He said that, 1 to 9 digit were develop around 3000 BC in *Harappa* and *Mohen Jo-Daro* civilization. But in this time, place-value system is not developed completely. There were individual sign for 10, 20 and 100.



तालिका : २.१३ नागरी अङ्कको उत्पत्ति

दशमलव पद्धतिको निरन्तर तथा व्यापक प्रयोग देखाउने क्रममा बि. वि. दत्त एवम् ए. एन. सिंहले आफ्नो पुस्तक **हिन्दू गणितशास्त्रका इतिहास,** भाग १ (१९७४) को पृष्ठ ३२ - ३५ मा नवौं शताब्दीसम्मको अन्तर्लेख एवम् दानपत्रको सूची दिएका छन् (छैटौंदेखि दसौं शताब्दीसम्म) जुन अवधिमा दशमलव स्थानमान सङ्केतको प्रयोग नागरी लिपिमा गरिएको छ। यसबाट के देखिन्छ भने छैटौं शताब्दीदेखि दशमलव प्रणाली चलन चल्तीमा आयो। त्यसपछि यो प्रणाली चीन, अलेक्जेन्ड्रिया र अरबतिर फैलियो।

दशमलव स्थानमान प्रणाली संसारभरि नै प्रयोग भइरहेको छ। यो प्रणाली यति धेरै प्रचलित हुनाको कारण यो ज्यादै सरल एवम् प्रभावकारी हुनु नै हो। दशमलव प्रणालीमा आधारभूत सङ्केत दस वटा छन् (हिन्दू अरबी : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 ; देवनागरी : ०, १, २, ३, ४, ५, ६, ७, ८, ९) प्रत्येकलाई अङ्क भनिन्छ। अङ्कहरूबाट अलग अलग वा एकभन्दा बढी अङ्कको मिलापमा सङ्ख्या जनाउने काम गरिन्छ। यसरी सङ्ख्या जनाउने सङ्केतहरूको मिलान (Combination of digits) मा प्रत्येक सङ्केत (Digit) को स्थानका आधारमा स्थानमानका साथै निरपेक्ष मान (Absolute value) को मुख्य विशेषताका कारणले नै यो प्रणालीले सबै सङ्ख्यालाई दस वटा सङ्केतको

Figure No. 7: Invention of Nagari Script (Sherestha, 2014, p: 169) (See Appendix-E)

Sherestha explore that, the place-value system is develop around the 9thcentury (6th to 9th century). After Invention place-value, it was expanding in China, Alexgendriya and Arab. This explain that, *Danpatra-Suchi* of 9th century A.D. explore about the decimal place-value number system in Nagari script. Sherestha explain that, Nagari and Devanagari are the same number which was transgressing around 7th century in China, Alexgendriya and Arab then revised on modern Hindu-Arabic

number. So, it could belief that, Alexgendriya and Arab revised on its sign as; 1 from १, 2 from २, … and so on. And followed same Decimal place-value number system.

In this way, we can conclude that Hindu-Arabic number is the revised and modify of ancient Nagari Script. But Nagari script was developed from Brahmi script around 4^{th} to 5^{th} century. (See Appendix-E)

Place of Devanagari in Sharada Script

Sharada script is developed standing on "*Kutila*" script in about 8^{th} to 9^{th} century (Pandey, 2009: 2). Around 12^{th} century Sharada followed the Sanskrit script which was found in Kashmir (Pandey). Sharada number system also uses the same place-value number system of Devanagari but they have different symbol.

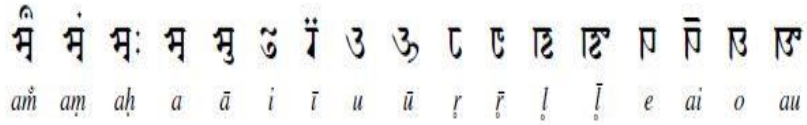
> *Sharada digits represent values of the decimal system. The notation system of Sharada is unique among Brahmi-based systems in that a dot represents zero (Sharada digit zero) and a circle, used in other notation systems to represent 0, is used to represent the digit one (0 Sharada digit one) (Pandey, pp: 22).*

Although, Sharada script is different from Brahmi script where Sharada obvious used the digit zero representing by symbol dot (.). And they represent the one by symbol circle (0). Sharada is a manuscript development in ancient Kashmiri region. Sharada is a local script in about 9^{th} to 12^{th} century in Kasmir. The number system of the Sharada is same as the Devanagari (Fig: 10). From the above authorize evidence; we conclude that Devanagari number is developed before the 9^{th} century.

5.2 Collation

The collating order for Sharada is based on Sanskrit and follows the pattern for Devanagari. Independent vowel letters are sorted before consonant letters. The signs *candrabindu*, *anusvāra*, and *visarga* appear at the head of the vowel order and are written in combination with SHARADA LETTER A.

The collating order for *candrabindu*, *anusvāra*, *visarga*, and independent vowels in Sharada is:



aṁ aṃ aḥ a ā i ī u ū ṛ ṝ ḷ ḹ e ai o au

Dependent vowel signs are sorted in the same position as their independent shape. Consonants with dependent vowels are sorted first by consonant letter and then by the vowel sign (including *candrabindu*, *anusvāra*, and *visarga*) attached to the letter. A consonant with *virāma* is sorted last.

(Pandey, 2009, pp: 22)

These above text provide the information that, Sharada script is developed standing on the Sanskrit language and followed the same pattern as Devanagari. The phonetic system also followed in Sharada. This we conclude that, Devanagari number system is oldest then Sharada script which is developed around 9th century.

NUMERALS

Roman.	Nāgarī.	Śāradā.	Kāshmīrī (Nāgarī).	Kāshmīrī (Śāradā).	Trans-literation.
1	१	०	अख्		*akh*
2	२	१	ज़ह्		*zah*
3	३	३	त्र		*trě*
4	४	४	त्सोर्		*tsōr*
5	५	५	पोंछ्		*pōtsh*
6	६	६	शह्		*shēh*
7	७	७	सथ्		*sath*
8	८	८	ऐठ्		*aiṭh*
9	९	९	नव्		*nav*
10	१०	००	दह्		*dah*
11	११	००	काह्		*kāh*
20	२०	१०	वुह्		*wuh*
30	३०	३०	त्रह्		*trah*
100	१००	०००	हथ्		*hath*
1000	१०००	००००	सास्		*sās*

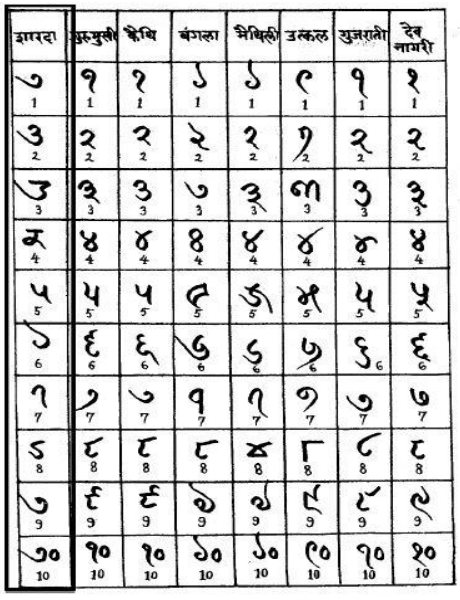


Figure No. 8: Comparison of Ancient Roman, Nagari, Maithili, and Kasmiri With Sarada Script.

(Pandey,2009, pp: 53)

Although, Sharada follows the same counting pattern similar to Devanagari (fig; 8). In ancient time there use the different type of number system which is based on decimal place-value system resembled with Devanagari. But symbols are very differing from each other. There seen that, there are two type of Kashmiri script *Sharada* and *Nagari*. Which were develop together around 7th to 9th century (Pandey, 2009, pp: 53). Where, Devanagari develop later around 9th century (Subedi, 2073).

Ayabhatta's published a book "*Aryabhaty*a" in 455 A. D. (Ifrash, 1985). He first invented that, the earth and planets was taken to be spinning on its axis with respect to the Sun. The book "*Aryabhatiya*" Covered the astronomical and mathematical theories.*Aryabhatiya* translated into Latin in system in 13th century.

In his seminal text of 499 CE, *Aryabhuttya* devise a novel positional number system, using *Sanskrit* consonants for small numbers and vowels for powers of 10. Using the system, number up to billion counts was expressed using short phrase, e.g. *"Khyu-ghr*" representing the number 4,320,000. The system did not catch on because it produced quite unpronounceable phrases, but it driven home the principle of positional number system called *Dasa-Gunottara* (exponents of 10) to later mathematicians (www.wikipedia.org/History of Hindu numeral). So, Aryabhatta use the Sanskrit (Devanagari) script around 5th century.

According to research thesis *Naya Raj Pantka Ganitiya Kritiharuko Adhyayan* by Acharya:

Nepali sanskarma prachalit Brahmi ankako aabiskar(1000ca.-600ca) bhayako painchha. Pandit bhagawanlal Indrajeeko sidhantako pusti as tathya bat

*hunchha ki 19 aakarko aksherma 11 wota ta nichchit rup mai brahmi lipiko aksher sang mildachhan…..***.**(Acharya, 2015, p: 69).

Above document explained that, Brahmi numeral was developed around 1000ca. to 600ca. It also explore that, according to the theory of Pandit Bhagawan Lal, there 11 digits were same as Brahmi numerals among 19 digits.

History of Hindu Mathematics by Datta and Sign

It is generally held that numerical symbols were invented after writing had been in use for some time, and that in the early stages the number were written out in full in words. This seems to be true for the bigger units, but the signs for the smaller units are as add as writing itself. Most of inscriptions of Asoka and the following period are written in a script which has been called Brahmi, while same are in a different script know as Kharosthi (Datta & Singh, 1962).

> *In most of the mathematical works ,the denominations are called "name of places" and eighteen of these are generally enumerated Sridhara (750) gives the following name: eka, dasa, sata, sahasra, ayuta, laksa, prayata, koti, arbuda ,abja, kharva, nikharva, mahasaraja, sarita, sarita-pati, antya, Madhya, parardha for10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000, 100000000, 10000000000, 1000000000000, 10000000000000, 100000000000, 10000000000000, 1000000000000000000 respectively and added that the decuple name proceed even beyond this Mahavira (850) give similar twenty four notational places .Also, the Sanskrit name for the number one to nine are: eka, davi, tri, cuatar, panca, sat, sapta,*

asta, nava. These with the numeral denominations already mentioned suffice to express any required number (Datta & Sign, 1962).

According to the book *History of Hindu Mathematics* by Datta and Singh, Brahmi script is very oldest script in India which found earlier about 1000 B.C. the king Asoka also uses this script in ca. 300 B.C. the number 1, 2, and 3 of Brahmi notation were denoted by horizontal line placed one below and other . This form clearly distinguish the Brahmi notation from Kharosthi and the Semitic system. According to Datta and Singh, thee are separated sign for each number 1, 4, to 9 and 10, 20, 30, …90, 100, 200, 30o,…1000, 2000 etc.

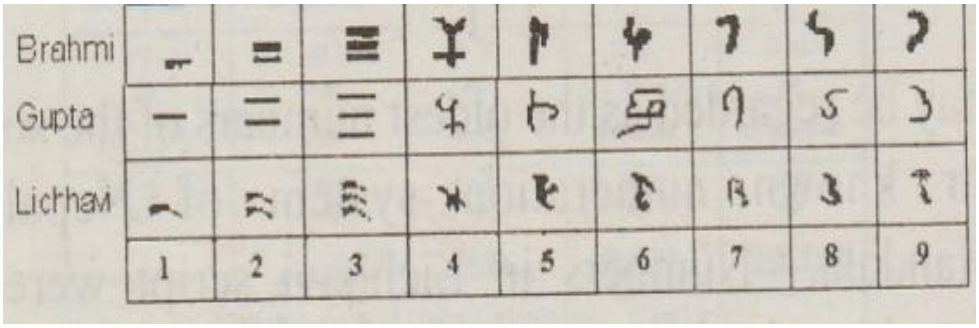


Figure No, 9: Comparing between Brahmi, Gupta and Lichhavi numeral

(Namaste, 2006, Issue: 18) (See Appendix-F & G)

Researcher has taken the interview with resource person. The response of resources person-A is given below;

"*The number was perceive in "Shurti" era / civilization (Before time of manuscript) in Iran and Mishra , where Arya communing was existing . They have also concept of natural number in that time and they use Dave (ancient Sankrit script) language. After, they came to India, when Indian uses the Nagari script. Then Arya expand the Deva language in India and use Nagari language in their own Deva script. This we imagine that they use Nagari digit in Deva script is known as*

Devanagari script. In about 7th century Brahamagupta drive the zero and place-value system which make number so easy and systematic counting. (11th march, 2017, 2:30 p.m.) (See Appendix-B)

Pant explains that, the terminology Devanagari is developing from two separate terms "Deva" and "Nagari". Deva is ancient Sanskrit language from Veda and Nagari is ancient Indian language.

The resource person-B expresses the views toward the Devanagari number system as bellows:

"Hindu civilization has a great contribution on number system which is develop from Veda. The Devanagari number which is come from Magarat, Jaya Brahma period. They used ciphered system" ((5th august, 20017, 8:30 am).

Regarding on Devanagari number system, responder-C has the views as given below:

"Rig-Veda" intensly give place about mathematics and number system.The number terminlogy is also used in "Rig-Veda" as; 1= Eka, Ekm, 2= dwaya, ddua, dwe, 3= trya, tri, etc. "Rig-Veda" has clearly used the terminology "Hajar"(1000). It indicated that, number are so ancient. The famous Brahmi Script of Hindu and Nepali culture was developed in about ca.1000 to ca.600. … After, the decimal place value system of number transgressing in Arab from Bharatvarsa. Because, Arabian put the name "Hindusa" for the number 1 to 9. Same this time, Western called the Hindu-Arabic number for the numreal 0 to 9. (See Appendix-B)

After analyzing all above evidence regarding on Devanagari number, researcher meet conclusion as; the foundation of Devanagari script is ancient Hindu

Nagari script which was a branch of Brahmi script. So, actual foundation of Devanagari script is ancient Brahmi script. After, ancient Arya people combine Dva (ancient Sanskrit script) with Nagari script than each digit revised into Devanagari script. After formulating the zero by Brahamagupta, Devanagari lead the place-value decimal system.

Practices on Hindu-Arabic Number System

Boyer displayed information about Hindu-Arabic number in this book "History of mathematics". He maintained the use of Hindu-Arabic number in three particular books: Alexandre de Villediece (ca. 1225), John of Halifax (ca. 1200-1256), and Leonardo of Pisa (ca. 1180-1250). According to the *History of Hindu Mathematics* of Boyer "*The ceriman de algorism"* of Alexandre is a poem in which the fundamental operation on integers is fully described, using Hindu-Arabic numeral and treating zero as a number."

The Hindu-Arabic numeral system on which arithmetic was built processes the intrinsic property of a place-value process. The intrinsic property of a place-value notation which has ten as bases. The digit of a numeral are arranged from left to right in deciding order of the ranks. Initially, the system employed only nine signs which represented the numbers from one to nine.

Hindu-Arabic is more useable place-value decimal number system in the world that uses zero glyph. It glyphs are described from Indian Brahmi numerals. The full system emerged by 8^{th} to 9^{th} centuries, and is first described in Al-Khwarizmi's on the calculations with Hindu-numerals in about ca. 830. (htt://en.wiwkipidea.org/wiki/history_of_math).

The decimal place-value numeration, it is not known, just how Aryabhatta carried out his calculation, but his phrase "from place to place each in ten times the preceding" is an indication that the application of the principle of position was in his mind. ”Local value" had been an essential part of Babylonian numeration, and perhaps the Hindus were becoming aware of its applicability to the decimal notion for integers in use in India (Ifrash, 193, p: 66).

There found three types of Hindu-Arabic number system explain as following;

1. Western Arabic numerals which was used in western Arabic regions.
2. Eastern Arabic numerals were used in middle and eastern Arabic regions (Egypt and Syria)
3. Eastern Arabic numerals were used in Persian and Urdu speaking regions (Eg. Iran, Pakistan, India)

European (descended from Western Arabic)	0	1	2	3	4	5	6	7	8	9
Arabic-Indic (Eastern Arabic)	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Perso-Arabic variant	٠	١	٢	٣	۴	۵	۶	٧	٨	٩
Urdu variant	٠	١	٢	٣	۴	۵	۶	۷	٨	٩

Figure No. 10: Comparison of Different Number (Dantzing, 1930, p: 32)

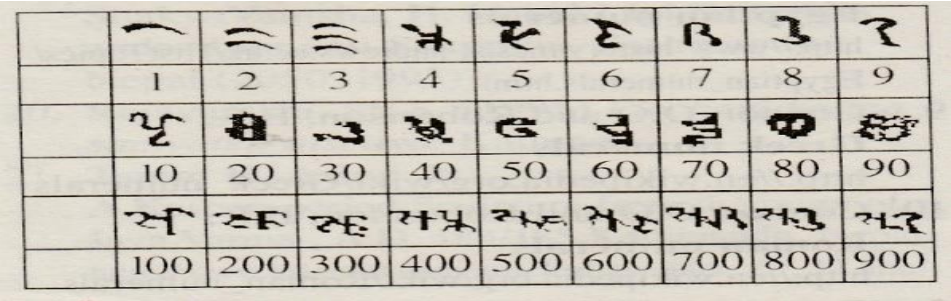
There found different numeration found in ancient Vedic literatures of India. The following table gives one such system used in the Valmiki Ramayana. The decimal Hindu-Arabic number system was developed in India by AD.1000(www.wikipidia.org/History). The development was, gradual, spanning several centuries but the decisive script was provided by *Brahmaputra's* formulation of zero as a number in around 7th century (www.wikipidia.org/History). This system is relevant only by including zero in positional notation "The universally accepted inscription counting the use of the 0 glyph in India is first recorded in 9th century, is an inscription at Gwalior in central India dated to 870 A.D.

European	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
Tamil		க	உ	ங	ச	ஞ	ஙு	எ	அ	கூ

Figure No. 11: Comparison of European, Arabic-Indic, Eastern Arabic and Devanagari. (See Appendix-H)

(www.wikipidia.org/History)

The figure No. 7 shows the ancient practices on Hindu-Arabic number system. There are resemble practices on the Arabic-Indic and eastern Arabic-Indic number. Ancient Arabians also used the numeral zero as notated by dot (٠). Arabic-Indic uses the notation circle (٥) for representing 5. The digit 7 and 8 are the complete different with Nagari digit but the digit 1, 2, and 9 are resemble seen. So, they may have been practice more to revise the Hindu-Arabic number from Devanagari.



	1	2	3	4	5	6	7	8	9
	10	20	30	40	50	60	70	80	90
	100	200	300	400	500	600	700	800	900

Figure No. 12: Comparing Lichhavi number with Hindu-Arabic number

(Mathematics Education Forum, 2006) (See Appendix-G)

Lichhavi is oldest number system in Nepal and that can found almost all ancient inscriptions still available in many parts on Nepal. Basis of the number system that came into uses around the first century A.D. (Namaste, 2006, Issue: 18). This shows that, Lichhavi numeral is oldest number system like as Arabic but symbols are much differ.

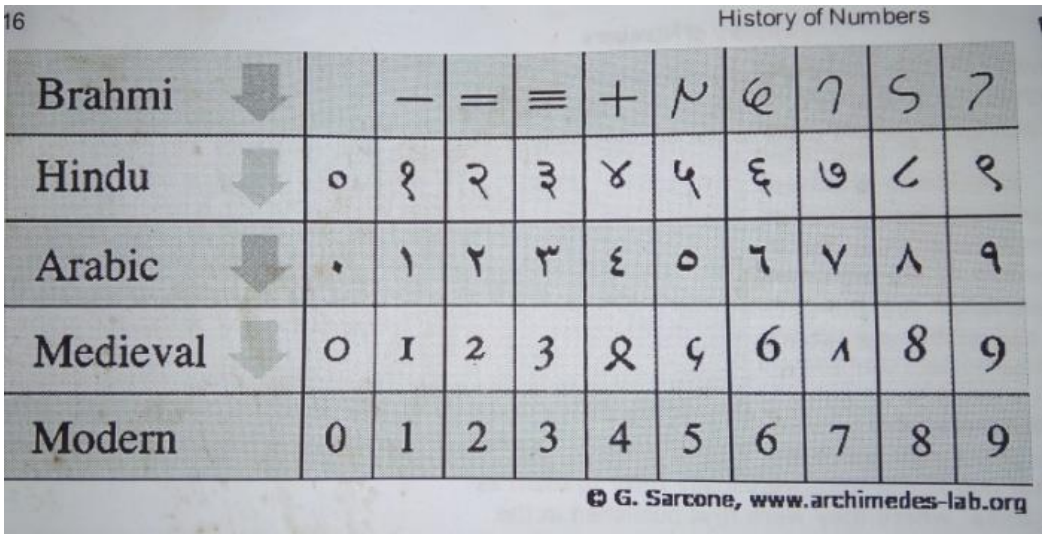


Figure No. 13: Developing phase of Hindu-Arabic Number System

(http://vedicscience.net/articles/history-of-number.html)

(See Appendix-H)

The above figure no.13 shows the trends of modern Hindu-Arabic number system. The Brahmi number system was developed around 2nd cesntury A.D. (Ifrash, 1985). This we conclude that, Brahmi number system is the fundamental basis of

Hindu-Arabic and Devanagari number system. Similar hieroglyphs was explained by Roger Cooke in the book 'A History of Mathematics' (Cooke, 2005).

The article *Origin of the Mathematics,* the responder -C has the view as given below:

> *Mathematics arose from the need of organized society of peoples. Thus even in a primitive society, certain intuitive concept which later developed into mathematics is necessary… this history of mathematics is interwoven with the history of human civilization… (Pandit, 2014, p: 16-17).* (See Appendix-B)

From this text I conclude that, mathematics is the result of the organized society of the people. Mathematics is develop from human intuitive from society.

From the Ph.D. research thesis based on *Naya Raj Pantka Ganitiya Aakirtiharuko Adhyaya*n:

> *Sankhyalai dashako sthamko rupma wa prachalanma lyaune kam bharat barsha bat bhayako hunun pardachha. Kinaki yak(1) dekhi nau (9) sammaka ankalai Hindusa bhaner arabari haeule bhannane gardachhan. Uta pashmi deshharule sunyadekhi nau sammanka ahindu-Arabi bhaninchha. … (Acharya, 2015, p: 70).*

Regarding on this number system, the responder-C says;

"*Arabic and Nahari number develop together. The Hindu Nagari number is a branch of Brahmi script. When Nagari script formulated place-value decimal system then it being so easy to use and Europe and Arab followed this counting technique. In*

this way, the Hindu number transgressing in Europe and Arab and revised in modern Hindu-Arabic." (8th may, 20017, 8:30 am)(See Appendix-B)

Another responder Dr. Shankar Raj Pant expressed views as; the foundation of Hindu-Arabic number is ancient Hindu Nagari script. After around 12th century, it transgression in Arab and transmission into Arabian script. Also, Mr. Eka Raj Pandit sir has the views as; Al-Khwarizmi people carry to well decimal place-value Devanagari number in Europe and transgression in about 10th to 12th century. After 13th to 16th century they revised/modify the Devanagari script in own symbolizing form, which today we called Hindu-Arabic number.

Concern on the Hindu-Arabic number system, the responder-C express the views as given below:

After, the decimal place value system of number transgressing in Arab from Bharatvarsa. Because, Arabian put the name "Hindusa" for the number 1 to 9. Same this time, Western called the Hindu-Arabic number for the numreal 0 to 9. Hindu-Arabic number is developing from combing to number Hindu (Indian Nagari) and Arabic.(8th may 2017, 9:15 am) (See Appendix-B)

After analyzing above evidence, researcher has got the conclusion as; the ancient Brahmi numeral is the foundation of Hindu-Arabic number. Hindu-Arabic number is revised form of ancient Hindu (Nagari) number. Around 10th to 12th century, Indian Devanagari number transgression in Europe and around 13th to 16th century it revised/modify into modern Hindu-Arabic number.

Interview with Resources Persons

The researcher put his problems and curiosities toward "Development of number system" with some related resources persons. Researcher prepares a list of unstructured interview schedule (See Appendix-A) to collect information to make authentic the study. Here, five resource persons have been selected given the name as responder-A, responder-B, responder-C, responder-D and responder-E. Their name put secret for ethical consideration. Unstructured interview was conducted with resources persons to provide authentic and valid the study. Researcher cross matched the response of resources persons with the views of documents. At last, researcher reached in valid conclusion. (See Appendix-B)

Conclusion

Hindu-Arabic and Devanagari number is the place-value decimal number system. Devanagari number was developed around 5^{th} to 7^{th} century together with the formulation of zero by Brahmagupta. The foundation of Devanagari script is ancient Hindu Nagari script which was a branch of Brahmi script. So, actual foundation of Devanagari script is ancient Brahmi script. After, ancient Arya people combine Dva (ancient Sanskrit script) with Nagari script than each digit revised into Devanagari script. After formulating the zero by Brahamagupta, Devanagari lead the decimal place-value system.

Hindu-Arabic number was developed around the 10^{th} to 16^{th} century in Arab and Europe. When Al-Khwarizmi spread the place-value Devanagari number in Arab from India than it become famous. The ancient Brahmi numeral is the foundation of Hindu-Arabic number. Hindu-Arabic number is revised form of ancient Hindu

(Nagari) number. Around 10^{th} to 12^{th} century, Indian Devanagari number transgression in Europe and around 13^{th} to 16^{th} century it revised/modify into modern Hindu-Arabic number. The characteristics of Hindu-Arabic and Devanagari number are as below:

- It is place-value decimal number system.
- The digit zero is exiting in this number system
- Each place-value is determining power of 10 (10^0, 10^1, 10^2, 10^3…).

The number perception is so essential things for mathematics. The human natural number sense is used for only recognizing the objects. The human direct number sense is able to recognize the collection less than four objects at a glance. Symbol of number is conform laws of thought working on things. People gesture the "sun", "moon" for count, "eyes" for 2 and etc. Ancient people use "fingers" for count 1, 2, 3, 4, 5, "legs of animal" for 4, "octopus" for 8 as a graphic sign. Notches on bones, different objects were used in graphic sign. Today's numerals are modified and modern form of ancient these graphical sign. So people change the concrete things in mental objects.

Number symbol is contribution of the human civilization but not the mathematician’s individual achievement. Ancient people used the graphical sign and physical objects to represent the number. They draw this sign on bones, stones, clays, skins, tablets, woods etc. This symbol is continuously used the long time and built digit. Number is birth from daily life necessity of peoples. The concept of number and number was comes from ancient human culture and human civilization. So, the number is invention from different human civilization but was not from mathematicians. This is resemble regarding on numeral system also. All animal and

men have the number sense. The human natural number sense was used for only recognizing the objects. The human direct number sense is able to recognize the collection of less than four objects at a glance. The modern counting was developed from ancient people's one-to-one corresponding counting techniques.

Chapter V

SUMMARY, FINDINGS, CONCLUSION AND RECOMMENDATIONS

This chapter deals with the summary, findings, conclusion, implication drown from the study and recommendation for the further study.

Summary

Number is the ground of mathematics and Hindu-Arabic as well as Devanagari number is well applied number system in mathematics. Mathematics is invention standing on number system and their different operation. Hindu-Arabic and Devanagari are well decimal place-value system. Hindu-Arabic numeral is revised from Devanagari by ancient Arabians and Europeans (Boyer). Devanagari is well famous number system in south Asia develops from Sanskrit script.

According to the History of Hindu mathematics (Datta and Sign, 1935), mathematics the science of number and counting. Number is the origin of branch of mathematics (Boyer, 1968: 1). Mathematics is develop from three human activities; Counting, categorizing shapes, and measuring (Cooke, 1999 : 5), and Counting is depending on number system what is interesting to note is that arithmetic and geometry were not coequal at the origin of mathematics, number was taken to be supreme (Cook, 1996: 6).

Number is the basic and fundamental concept of mathematics which has undergone expansion and generalization over many centuries. Number is the so common in mathematics that for many people it is synonym of mathematics. Number is one of the basic concept of mathematics other the concept of the shape and measurement.

Thousands of years ago there were no number to represent one, two, or three. Instead, people used fingers, rocks, Sticks or eyes to represent the numbers. Most civilizations did not have words for numbers larger than two, so they used terminology familiar to them such as flock of sheep, heaps of grain, piles of sticks or stones or groups of people, hair on the head etc. People had little need for a numeric system until they formed clans, villages and settlement and began a system of bartering and trade that in tern created a demand for currency. Babylonians first drive the numbering system about 5000 years ago (Datta and Sign, 1935). About 500 BCE the Romans developed 9 systems of numerals that used letters from their alphabet rather than special symbols (Example IV for four) (Sherestha, Pp: 5)

The development was gradual, spanning several centuries, but the decisive step was probably provided by Brahma gradual, formulation of zero as a number in AD 628. The system was revolutionary by including zero in positional notation, thereby limiting the number of individual digits to ten. It is consider an important mile stone in the development of mathematics. Hindu-Arabic numerical system is the most common system for the symbolic representation of number in modern world based on the ten digits: 0,1,2,3,4,5,6,7,8,9.

The addition of zero as a tenth positional digit is documented from the 7th century by *Brahmagupta (*Datta and Sign). Though the earlier *Bakhshali Manuscript*, written sometime before the 5^{th} century, also included zero. But it is in *Khmer* numerals of modern Cambodia where the first extant material evidence of zero as a numerical figure designing it's use back to the seventh century, is found (cook, 1996 : 6).

Eventually, to provide validation and make verify this research, the interview with resource person was administrated. The researcher prepared an in-depth interview schedule to make valid and authentic the thesis. The authentic resources persons have been chosen as: Ram Man Sherestha, Shankar Raj Pant, Eka Raj Pandit, Nilam Subedi, and Eka Ratna Acharya who helped to provide the authentic and valid to this study.

Finding of the Study

Basically**,** two objectives were formulated for this study. The first objective of this study was; to explore about number perception on ancient people and the second was historical development of Hindu-Arabic and Devanagari number system. On the basis of analysis of the study, different sources and interviews and consult with resources persons, the researcher has point out the following findings;

- The human natural number sense is used for only recognizing the objects. The number perception is arising from the logical thinking on the objects. Animal has the number sense. For example: crow has number sense up to 4, dog has number sense up to 10, but human have advance number sense.
- Numbers are come from human culture and civilization not the contributions of mathematicians. But numerals are developed from people's insight on objects of graphical sign. A person makes number sense from define collection of objects. They make sense of one from the unique objects likes "sun", "moon" etc., sense of two from the collection of two likes "eyes" and "breast" and so on.

- Ancient people associate one pebble for one sheep, another pebble for another sheep then make counting sense. Thus, the modern counting was developed from ancient people's one-to-one corresponding counting techniques.
- The zero was first formulated by Hindu mathematicians Brahmagupta in around 6^{th} to 7^{th} century. But Arabians obvious used the notation dot (٠) before Al-Khwarizmi spread the Hindu (Nagari) numerals in Arab. There also developed Arabic-Indic number system where the notation circle (٥) used for representing 5. But, first concept of zero, there are controversies up to now.
- Before formulation of zero, Brahmi script also used sign-value number system. There were different symbols for 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100. Lichhvi has the also sign-value number system.
- Modern Hindu-Arabic number was developed from crossing the turning phases; Brahmi, Hindu (Nagari), Arabic and Medival then become Modern (Hindu-Arabic). Arabians first put the name 'Hindusa' for the well Hindu numerals 1 to 9, after Al-Khwarizmi spread the well place-value decimal number system in Arab and Europe than the word 'Hindusa' revised into Hindu-Arabic.
- The foundation of Devanagari script is ancient Hindu (Nagari) script which is the branch of Brahmi script. So, actual foundation of Devanagari script is ancient Brahmi script. Devanagari script is developed from deva (ancient Sanskrit script) and Nagari script. After formulating the zero by Brahmagupta, Devanagari lead the decimal place-value system. Hindu-Arabic number is revised form of ancient Hindu (Nagari) number. Around 10^{th} to 12^{th} century, Indian Devanagari number transgression in Europe and around 13^{th} to 16^{th} century than it revised/modify into modern Hindu-Arabic number.

Conclusion of the Study

The number is the result of social phenomena. On the basis of finding the researcher conclude that, number perception is developed by logical thought on the objects. The concept of number and numerals is comes from ancient human culture and human civilization. So, the number is invention from different human civilization but is not from mathematicians. This is resemble regarding on numeral system also. The human natural number sense is used for only recognizing the objects. The human direct number sense is able to recognize the collection of less than four objects at a glance. The modern counting is develops from ancient people's one-to-one corresponding counting techniques. Numeral is develops by different graphical sign from ancient people.

The ancient Brahmi script is the fundamental script of Devanagari number and Hindu-Arabic number. The decimal place-value Devanagari number system is developing around 5^{th} to 7^{th} century AD. The word Devanagari is formulated by two separated word *Dev*a and *Nagari*. Around 10^{th} to 12^{th} century, Indian Devanagari number transgression in Arab and Europe and around 13^{th} to 16^{th} censtury it revise/modify into modern Hindu-Arabic number. Sharada script also developed from Sanskrit and Nagari script and found around 7^{th} century. So, Devanagari is the fundamental basis of Sharada Script.

Implications of Study

This is a qualitative research and historical descriptive based on content analysis methods. Primary and secondary data were used. The source of data for this study were taken from historical documents, archives, websites, research article, related journals, authorized books, consulting with research persons, libraries,

department of archaeology etc. The researcher used open-ended interview with recourse persons. According to finding and conclusion of this study, researcher expects the implications as:

This study visualized the well information about ancient number perception and counting system. And give good idea about origin of the number.

a) This study explores the different evidence and fact about number system.

b) This study carries out the problems and issues regarding on number system.

c) This study might be supportive document technically and academically basis of the number system.

d) The finding and conclusion of this study, other researchers could be curious to research and documenting the mathematical creations of other number system.

Recommendation for Further Study

Any of the research doesn't give only new finding but also take out new problem and issues. Research also gives the verification of research technique and methodology. Eventually, on the basis of this study the following recommendation have been made.

- It should study comparatively between historical development of Brahmi and Lichhavi numeral system.

- It should study comparatively between historical development of Vigesimal and Sexagesimal number system.

- It should be study about mathematical contribution of Brahamagupta and Aryabhatta in Hindu mathematics.

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Appendix-A

Interview Questions:

- Is number develops from human culture and civilization? Or from mathematicians? Should provide views behind it.
- Numerals are from civilization? Or from mathematicians?
- Ancient people used one-to-one correspondence matching for counting in ancient time, is it true?
- How to symbolize the number in ancient time? Using graphical sign from concrete objects? Or numeral are pre-existing in religious book (Veda)? Should provide evidence.
- Nagari and Brahmi script is foundation for Devanagari number system, regarding on this perspective, express your views.
- Who formulate the zero first, Brahamagupta? Or not?
- The discovery of zero has contribution to develop place-value number system? Should provide reason behind it.
- Is Brahmi script for foundation of Hindu-Arabic number system?

Appendix-B

Tribhuvan University

University Campus

Department of Mathematics Education

T.U. Kirtipur , Kathmandu

Dear Sir/madam

I'm student of Master' Degree in educaion. I have chosen atopic "Development of Number System" for the partial fulfillment of Master Degree. The interview questionnaire are degin as open-end. Your valuable perception will contribute a lots for this study. I, heartiest request for reimburse the permission to use up your response in this research, if required. Your opinion are be used only for the research purpose and will not be made public.

Thanking for your kind cooperating for giving your valuable opinions.

Yours Sincerely

(Yubraj Shahi)

Responder : A

"The number was perceive in "Shurti" era / civilization (Before time of manuscript) in Iran and Mishra , where Arya communing was existing . They have also concept of natural number in that time and they use Dave (ancient Sankrit script) language. After, they came to India, when contemporary Indian used Nagari script. Then Arya transgression the Deva language in India and use Nagari language in their own Deva script. This we imagine that they use Nagari digit in Deva script is known as Devansgari script. In about 7th century Aryabhattya drive the zero and develop place-value system which make number so easy and systematic counting. Arya people also use this place-value decimal system in their Devanagari script about 10th to 12th century. Ancient Arya speak Deva language which is came from Sanskrit. After Devanagari number expand in Arab and Europe which make Hindu-Arabic number. The concept of zero was also developed in the "Rig-Veda" era". (11th march, 2017, 2:30 p.m.)

Responder: B

"Mainly, there are two types of number system; 1) sign-value or positional number system and 2) place-value number system. The ancient number may be developing around 3300 BC to 3500 BC in Sumeria. The concept of number develops from human culture and human civilization by their daily life activity. So, the origin of number is Human culture and civilization not from the mathematicians. In homo-space period, people count only one, two, and many. Beyond two, they use the terminology "many", "hair on the head" etc. Hindu civilization has the a great contribution regarding on number system, which is develop from Veda. Veda also used ka, kha, … Sanskrit (Devanagari) script. In around 14th century, Nepali follow the Devanagari system which was come from Magarat. They used ciphered number

system in Jaya Brahma period. Specially, non-lexical number system was evolution from three stages of origin:- s first stage; one-to-one correspondence, second stage; Ordering and Grouping and third stage; ciphered. Similarly, lexical numeral system has three stage: concrete-(sticks, pebbles, bones), half-abstract and half-concrete (sun, moon for one, eye for two) and abstract stage-(eka, dvi etc.).(8^{th} may, 2017, 8:30 am)

Responder: C

"*First of all, it is belief that number was already exist in Veda. But ancient different culture and civilization has individual practise on number system. According to the Veda, the time interval 1500 BC to 800 BC is called "Vaidik-Kal" and number was develop before this time. Mathematics has the well place in "Veda". Veda is divided into four part, where "Rig-Veda" intensly give place about mathematics and number system.The number terminlogy is also used in "Rig-Veda" as; 1= Eka, Ekm, 2= dwaya, ddua, dwe, 3= trya, tri, etc. "Rig-Veda" has clearly used the terminology "Hajar"(1000). It indicated that, number are so ancient. The famous Brahmi Script of Hindu and Nepali culture was developed in about 1000 BC to.600 BC. "Vaidik-kal" was divided into two period; "Pre-Vaidik" preiod and "Late-Vaidik" period. In "Pre-Vaidik" period, where the english terminology "number" was developed which represent the decimal number system. It may be the concept of zero was in this era. The decimal place-value number system is also used in "Naradhbishnupuran" written by "Vedbyas". This"Naradhbishnupuran" also explained about four fundamental operation of mathematics. After, the decimal place-value system of number transgressing in Arab from Bharatvarsa. Because, Arabian put the name "Hindusa" for the number 1 to 9. Same this time, Western called the Hindu-Arabic number for the numreal 0 to 9. Late-Vaidik period is also called Sulvasutra and "Vedang-*

Jyotisha" period, which is from 1000 BC to 500 BC. This is prove that, number system drive before 1500 BC." (*8th may, 2017, 8:30 am)*

Responder: D

Number is the result of human daily-life activity on a society. Number is the sense on mind which was drive from activity of a multitude of people n a civilization. Not a single man can drive the number. Numerals are representative form of number which arealso result of social phenomena. Devanagari and Hindu-Arabic number were develop from ancient South East Asia. Indo Asian have the great contribution to dvelop the Devanagari number. Devanagari numeral also called the Hindu numerals whose foundation was Brahmi numerals. We may belive, number were existince in Veda which was found in around 6000 B.C. the concept of zero was come out before the Brahmagupta. They used the terminology "Vaccum" that means no value or empty. So, the zero is the reflection of "Vaccum" in which Brahmagupta first formulate zero systematially in around 6000 A.D. Al-Khwaezmi one of the arabian mathematicians who has the contribution to develop the Hindu-Arabic number. Around 10th to 12th century, he protruding Devanagari number in Arab and combine with Arabian scrip. Consequencearound 13th to 16th century Hindu-Arabic was develop. (11th, may, 2017, 3:20 pm)

Appendix-C

Ancient Sumerian Sexagesimal Numeral System

Table No. 2.4: Sumerian Lexical Numerals

Level First (1to 9)	Level Second (10 to 50)	Level third (60 to 600)
1: *gesh, ash, dish* 6: *àsh* 2: *min* 7: *imin* 3: *esh* 8: *ussu* 4: *limmu* 9: *ilimmu* 5: *iá*	*10:u* *20:nish* *30:ushu* *40: nishmin or nimin, nin* *50:ninnû*	*60:gesh* 60×2: *gesh- min* 60×3: *gesh-esh* 60×4:*gesh-limmu* … 60×10:*gesh-u*

Actually, Sumerian system had a decimal form in second level and some of these words are compound. The word for 30 is formed by combining the words for 3 and 10:

30 = ushu = esh.u = 3 × 10; 40 = nishmin = nish.min = 20 × 2; 50 = ninû = nin + u = 40 + 10.

They count the number dividing three level

Table No. 2.5: Mathematical Structure of Sumerian Lexical Numerals

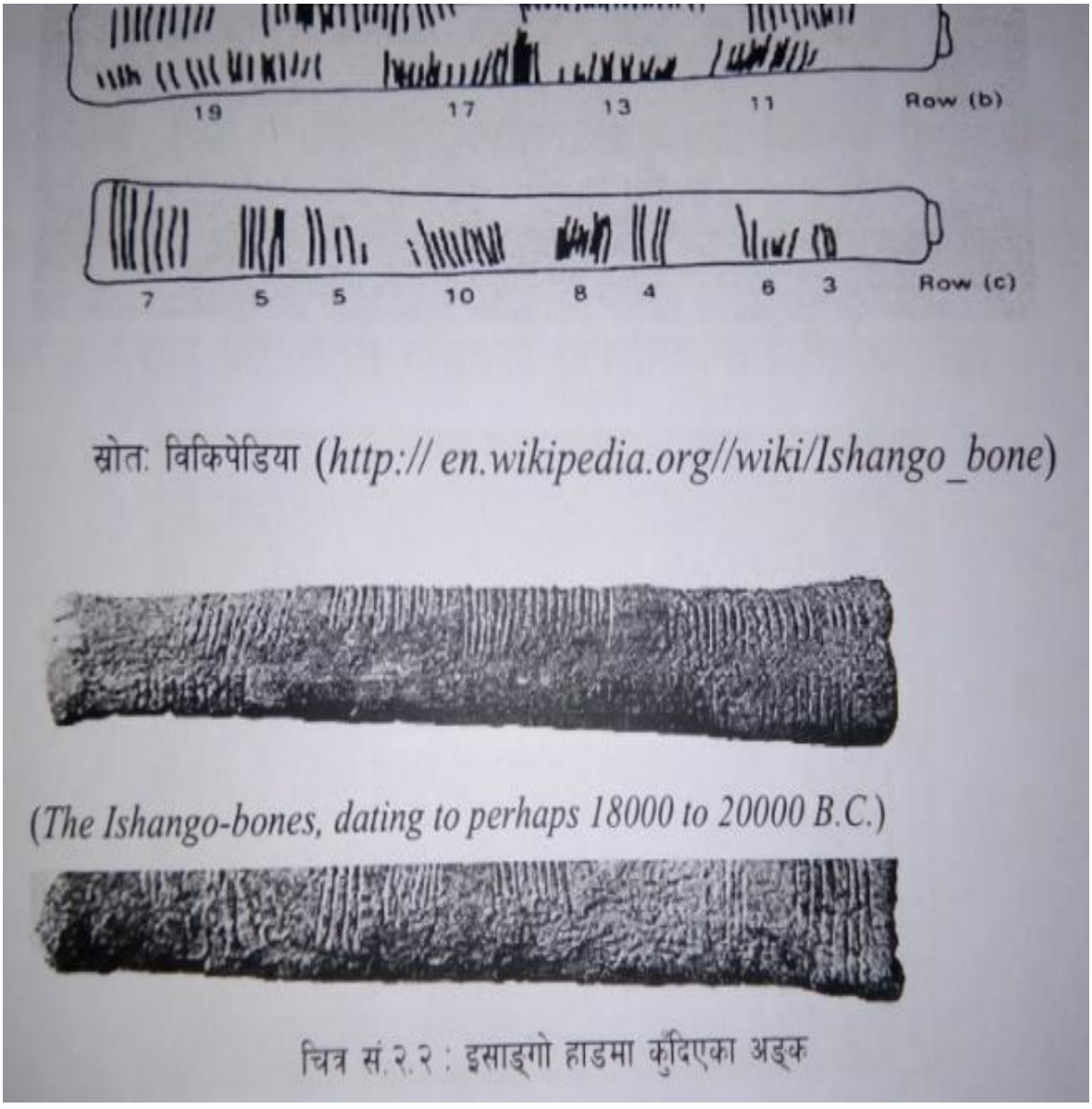
Value	Name	Mathematical Structure	
1	*gesh*	1	1
10	*u*	10	10
60	*gesh*	60	10.6
600	*gesh-u*	60×10	10.6.10
3600	*Shár*	60^2	10.6.10.6
36,000	*shár-u*	60^2×10	10.6.10.6.10
216,000	*shár-gal*	60^3	10.6.10.6.10.6
2,160,000	*shár-gal-u*	60^3×10	10.6.10.6.10.6.10
12,960,000	*shár-gal shu-nu-tag*	60^4	10.6.10.6.10.6.10.6

Smerians follow the auxiuliary base 10 with base 6

Sourse: Subedi, 2017, Ph.D. thesis, p: 90

Appendix-D

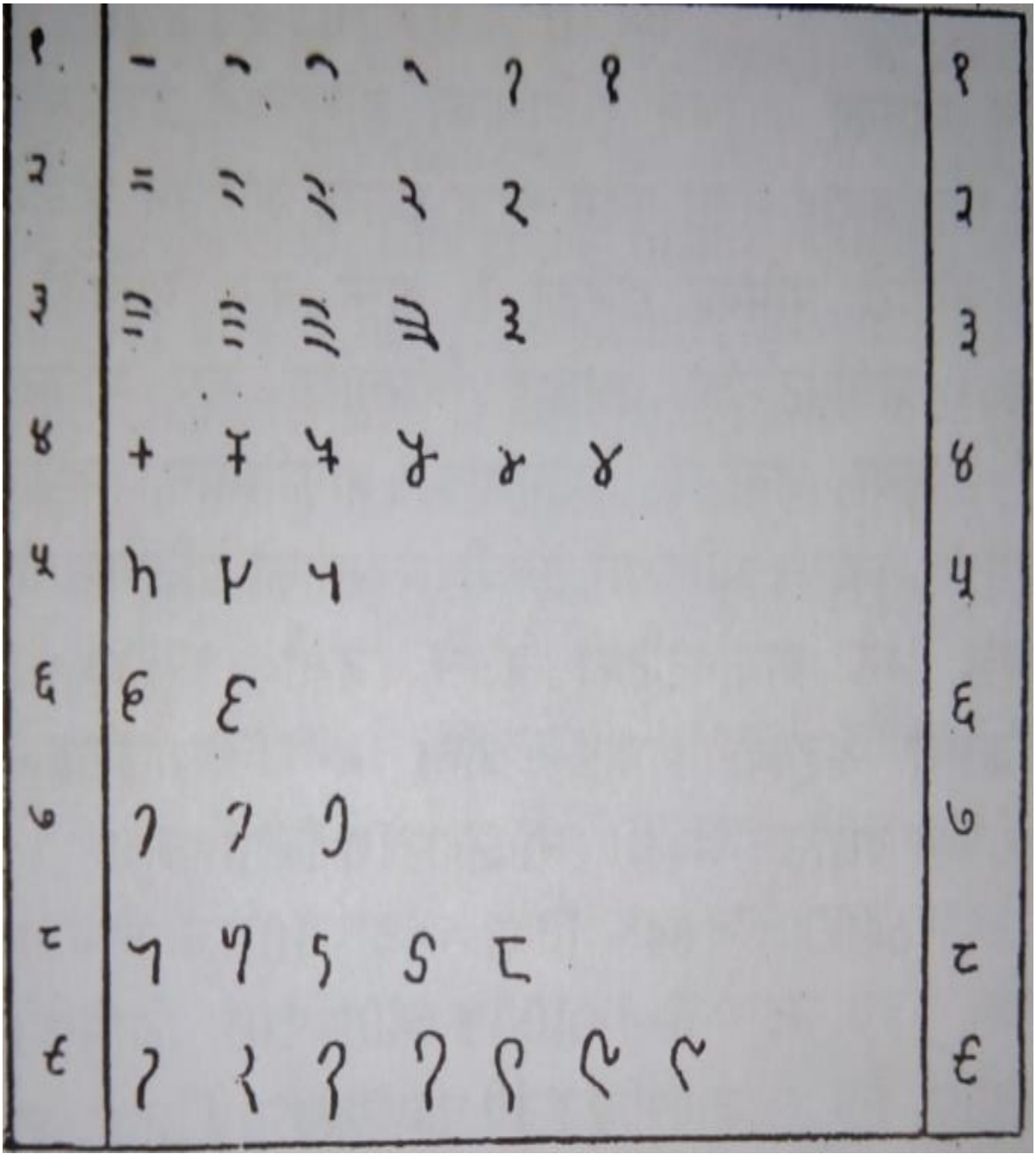
The Ishango-Bone Shows the Ancient Counting, Which was 8000 years Old and Found in Jayar



Source: Ph.D. Thesis, Acharya, 2015

Appendix-E

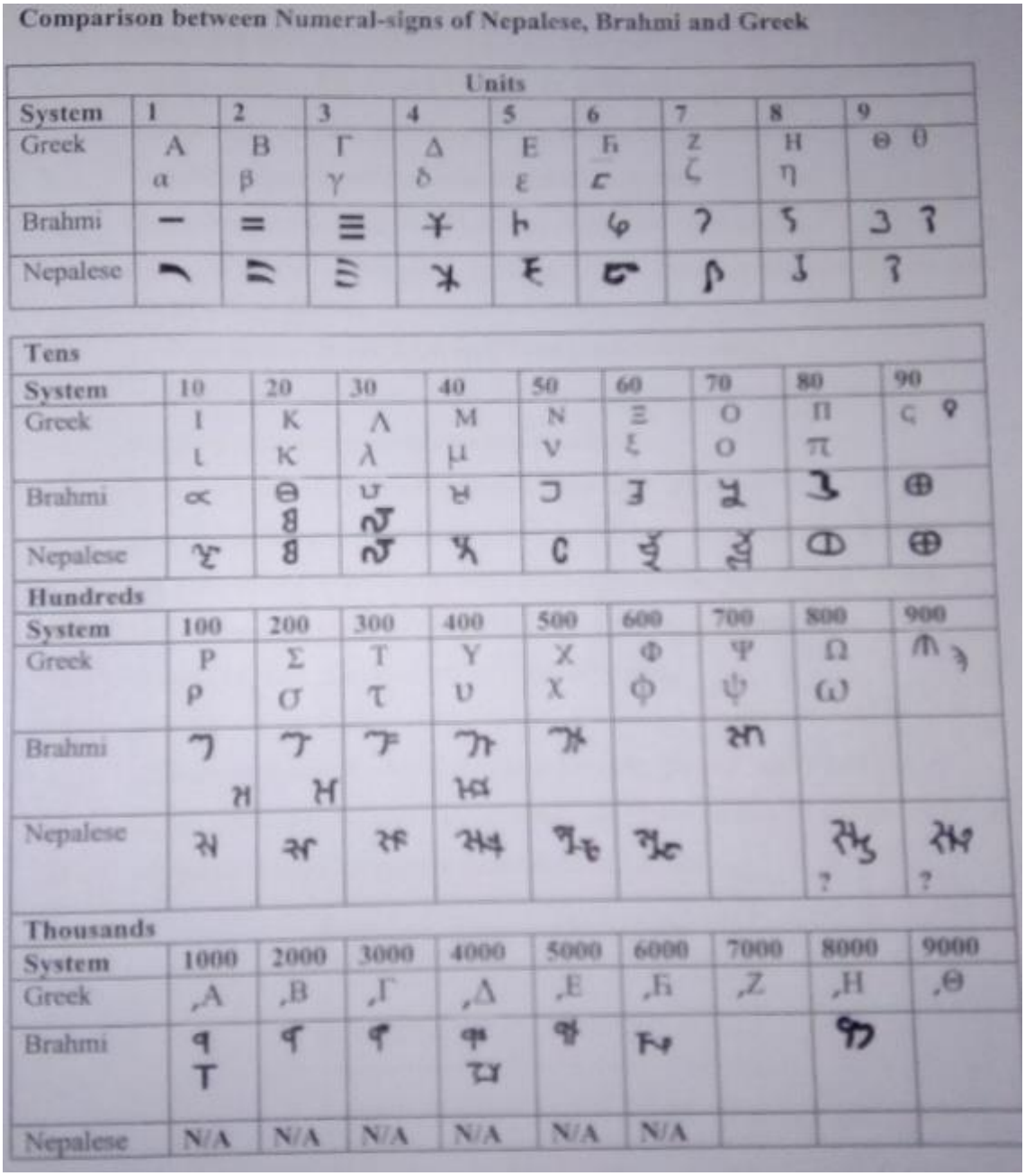
Invention of Devabnagari Script From Brahmi Script



Source: Ganit Darshan, Sherestha, 2070

Appendix-F

Comparison between numeral-sign of Nepalese, Brahmi and Greek in Unit, Ten, Hundred and Thousand prespectives



Comparison between Numeral-signs of Nepalese, Brahmi and Greek

Units									
System	1	2	3	4	5	6	7	8	9
Greek	A α	B β	Γ γ	Δ δ	E ε	F ϛ	Z ζ	H η	Θ θ
Brahmi									
Nepalese									

Tens									
System	10	20	30	40	50	60	70	80	90
Greek	I ι	K κ	Λ λ	M μ	N ν	Ξ ξ	O ο	Π π	ϙ ϟ
Brahmi									
Nepalese									

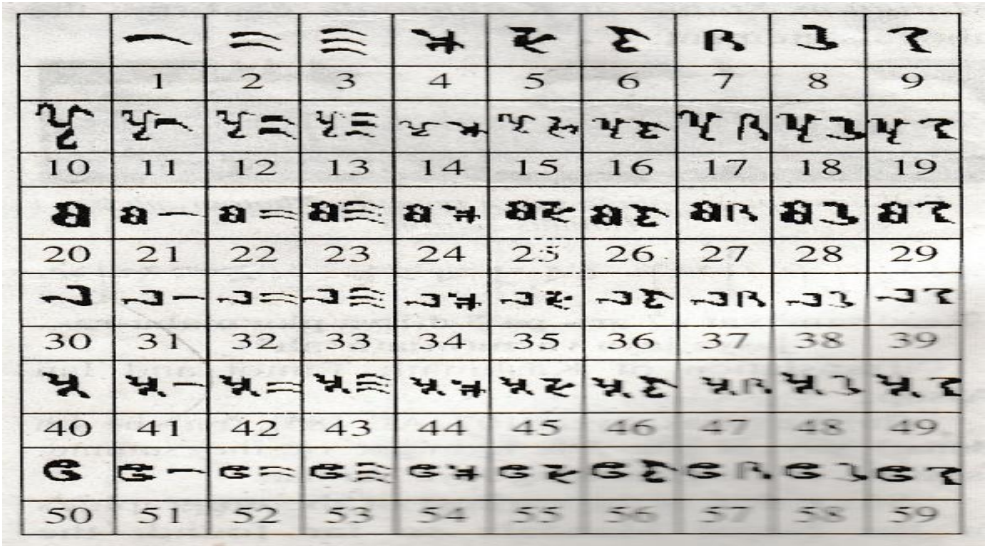
Hundreds									
System	100	200	300	400	500	600	700	800	900
Greek	P ρ	Σ σ	T τ	Y υ	X χ	Φ φ	Ψ ψ	Ω ω	ϡ
Brahmi									
Nepalese								?	?

Thousands									
System	1000	2000	3000	4000	5000	6000	7000	8000	9000
Greek	,A	,B	,Γ	,Δ	,E	,F	,Z	,H	,Θ
Brahmi	T								
Nepalese	N/A	N/A	N/A	N/A	N/A	N/A			

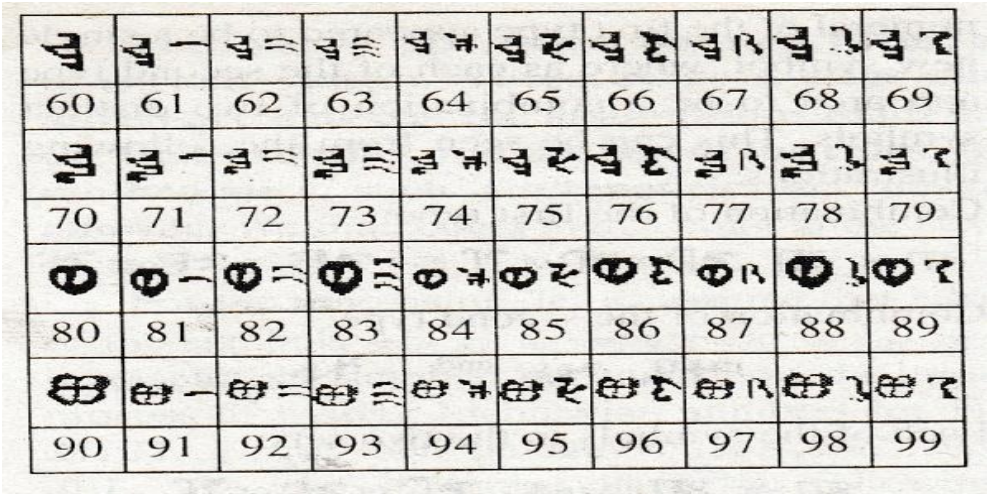
Source: Research Thesis, Subedi, 2017 (Ph.D.)

Appendix-G

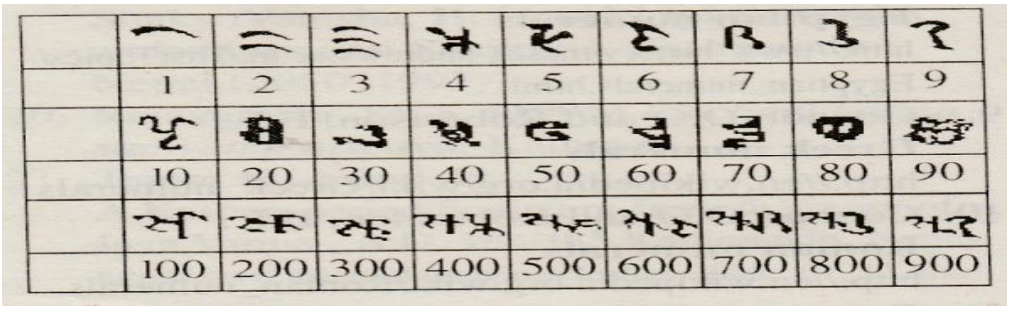
Lichhvi Numeral System



	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59



60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

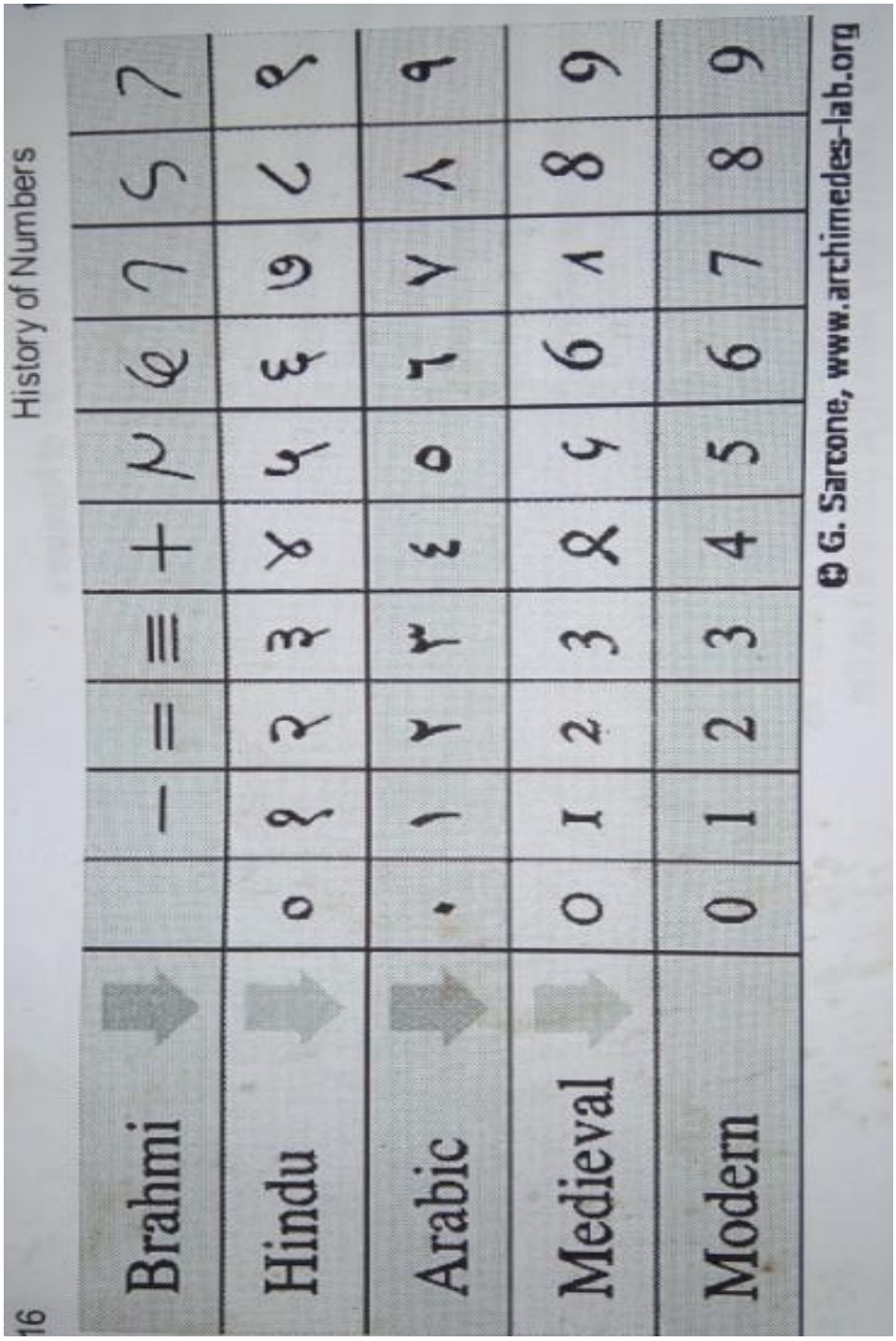


	1	2	3	4	5	6	7	8	9
	10	20	30	40	50	60	70	80	90
	100	200	300	400	500	600	700	800	900

Sources: Mathematics Education Forum, 2006, Issue: 18

Appendix-H

Develping Stage of Brahmi to Hindu-Arabic Number



(http://vedicscience.net/articles/history-of-number.html)