STUDENTS' ERRORS IN LEARNING ALGEBRA AT BASIC LEVEL

A

THESIS

BY

NARESH CHAUDHARY

FOR THE PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTERS OF EDUCATION

SUBMITTED

TO

DEPARTMENT OF MATHEMATICS EDUCATION

CENTRAL DEPARTMENT OF EDUCATION

UNIVERSITY CAMPUS, KIRTIPUR

TRIBHUVAN UNIVERSITY

KATHMANDU

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LETTER OF CERTIFICATE

This is to certify that Mr. Naresh Chaudhary student of the academic year 2073/075 with Exam Roll No.7328408 and T.U. Registration No.9-2-55-53- 2013 has completed his thesis under the supervision of Mr. Krishna Prashad Bhatt during the period prescribed by the rules and regulations of Tribhuvan University, Nepal. The thesis entitled **Students' Errors in Learning Algebra at Basic Level** has en prepared based on the results of his investigation conducted during the period February 2021 to June 2022 under the Department of Mathematics Education, University Campus, Tribhuvan University, Kirtipur, Kathmandu. His thesis number is 1739 I recommend and forward his thesis for evaluation as the partial requirements to award the Degree of Master of Education.

……..

Prof. Dr. Bed Raj Acharya

Head

Date:- 4th May, 2022

RECOMMENDATION FOR ACCEPTANCE

This is to certify that Mr. Naresh Chaudhary has completed his M.Ed. thesis entitled **Students' Errors in Learning Algebra at Basic Level** under my supervision during the period prescribed the rules and regulations of Tribhuvan University, Kirtipur, Kathmandu, Nepal. The study embodies the result of an investigation conducted during the period 2021-2022 under the Department of Mathematics Education, University Campus, Tribhuvan University, Kirtipur, and Kathmandu. I recommend and forward his thesis to the Department of Mathematics Education for the final viva-voice.

…….

Mr. Krishna Prashad Bhatt

Supervisor

Date:4th May, 2022

LETTER OF APPROVAL

This thesis entitled **Students' Errors in Learning Algebra at Basic Level** submitted by Mr. Naresh Chaudhary in partial fulfillment of the requirement for the Master's Degree in Mathematics Education has been approved.

Vice Voce Committee	**Signatures**
Prof. Dr. Bed Raj Acharya (Chairman)	……...
Assoc. Prof. Laxmi Narayan Yadav (External)	……
Mr. Krishna Prashad Bhatt (Member)	…….

Date: 8th June, 2022

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DEDICATION

This work is heartily dedicated to my respected parents. Also, this thesis is dedicated to my friends and relatives.

DECLARATION

This thesis does not contain any other work which is offensive and yond the copy-written norms. To the best of my knowledge and beliefs, this research is truly based on my effort and it does not match with any research that was published earlier in any institutions. I take all the ethical and legal responsibility for submitting this thesis.

Date:…… ……..

Naresh Chaudhary

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…….

Naresh Chaudhary

ABSTRACT

This is a study on "Student's Error in Learning Algebra at Basic Level". The main objective of the study was to identify the students in learning algebra at basic level and to explore the strategies that reduces student's errors in learning algebra at basic level. The research questions were how do students perceive learning algebra? What are the reasons behind students' weak in learning algebra? What are the factors behind students' error in learning algebra at basic level? This research was qualitative research which was based on phenomenology research design. Research site was Shree Durga Secondary School, Shailung-4, Dolakha. As respondent of this study was from only class 6,7, and 8.The major tools used for the study were achievement test , interview and focus group discussion which were clearly mentioned in appendix. To collect the data with respect to meet objective and research question the researcher used interview guidelines for head teacher and mathematics teacher, achievement test and focus group discussion were conducted with students. The researcher was selected as sampling procedure with purposively sampling procedures. Newman's theory was used for Error Analysis in whole study. When the researcher was took the achievement test then the researcher was found variety of errors of participate students in that test which were categorized six types of errors: Reading error, Comprehension error, Transformation error, Process skill error, Careless. Then the researcher conducted Focus Group Discussion with ten students. By the FGD, the researcher found there were many reasons behind the errors. The researcher categorized every reasons contained three factor which were School related factor, Student related factor, and Family related factors. The researcher found the students did error with dealing word problem, error with dealing with comprehending variables, error with dealing with solving equation, error with dealing algebraic expression, error with dealing transition from arithmetic to algebra to arithmetic. Finally, the researcher concluded that instead of all the blame to the stakeholder we can handled it by the deep analyzed those factors and reduces that students' error in learning algebra at basic level.

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Chapter I
Introduction

Background of the Study

Algebra is an influential Mathematics topic in a school curriculum. It is applied in all current branches of Mathematics as well as Science. In support of this observation, MacIntyre (2005) states the success in Mathematics largely depends on algebraic concepts. Menshah (2006) views algebra as forming a large proportion of the final matric examination in Mathematics. In the same line, Christmas and Fay (1990)observed that all Mathematics branches use the fundamental ideas of algebra to reason about and model various phenomena. All these views portray the pivotal role played by algebra in the teaching and learning of Mathematics.

Mamba (2012) analyzed South Africa's Grade 12 November 2008 Mathematics Paper solutions for one of the classes and discovered that algebraic expressions posed many problems to learners. The algebraic skills of learners are very poor as reported by Barry (2014) in the diagnostic reports. The report also stressed the fact that learners struggle with basic Mathematics in Grades 8-10. This results in learners facing challenges in Grade 11 and 12. The Department of Basic Education (DBE, 2014) also indicates that poor performance in higher grades is linked to poor performance in algebra. However, algebra, having such a pivotal role in the learning and development of basic Mathematics aspects, gives learners a challenge at school. Algebra is viewed by Booth (1988) as a source of confusion for learners. Bell (1995) regards algebra as a common problem area for learners. Many learners experience difficulties in understanding algebraic concepts. Learners fail to manipulate algebraic concepts according to accepted rules, procedures, or algorithms. This in turn affects their performance in Mathematics as success in this subject is largely affected by an understanding of concepts in algebra(Mamba, 2012). Learners find it extremely difficult to the extent that some of them drop out of school or if they do not drop out, they struggle to continue with their education (Wellmann, 2008). According to Kinney and Purdy (1952: 59), "Algebra has acquired a reputation among teachers, pupils and parents alike, as one of the most difficult and troublesome courses in the secondary curriculum." Kilpatrick and Izsac (2008), in the same line, also regarded algebra as an evil force wreaking havoc across the land and also as a source of

difficulty and failure. Reeve (1936) in America's National Council of Teachers of Mathematics (NCTM, 1936) eleventh yearbook went to an extent of saying,if there is heaven for subjects, then algebra will not go there. It is one subject in the school that has kept children from finishing school, from developing their special interests, and from enjoying much of their home studying work. It has caused more family rows, more tears, more heartaches, and more sleepless nights than any other subject (p. 2).

This is a serious concern since the South African Mathematics curriculum attaches great importance to algebra as in other countries (Moodley, 2014). Even though algebra plays such a pivotal role in the development of most Mathematics concepts, learners face a plethora of problems in dealing with it. There is clear evidence that enormous efforts have failed to address the issue of improving students' performance in algebra (National Mathematics Advisory Panel (NMAP), 2008). Therefore, there is a need to identify the causes of these problems faced by learners in algebra. The results would potentially provide information on some of the interminable errors committed by learners in algebra. Poor performance in Mathematics in South Africa seems not to be declining. According to Moodley (2014), there is no secrecy in the fact that South Africa trails behind the rest of the world in terms of mathematical achievement. This brings more worries to the Mathematics teachers, subject advisors, district senior managers, and the whole nation at large. Focusing on the Grade 8 learners, it can be seen that they are not performing well in both the national and the international tests. Reddy (2012) sees the curriculum of Grade 8 in South Africa as being on par with the international standards. In the Annual National Assessment (ANA) tests given to Grade 8, the results show 13%, 14%, and 10% pass rates for 2012, 2013, and 2014 respectively. The Trends in Internal Mathematics and Science Studies (TIMSS) administers its tests to Grade 8 learners. But South Africa in 2007 and 2011 fielded Grades 9 learners for these tests. The reason, according to Spaull (2013), was that the tests were too difficult for its Grade 8 learners. However, this did not improve the situation as seen in the TIMSS 2011 results. South Africa after having fielded the Grade 9 learners was still at the bottom together with Honduras and Botswana (Reddy, 2012). Having cited all these problems faced in the Department of Mathematics and also the whole nation, the researcher decided to identify, classify and analyze errors made by learners in simplifying algebraic problems. The aspect of simplifying algebraic problems was

chosen after having discovered the pivotal role played by algebra in the development of most Mathematics aspects. The researcher hoped that learners and the education system, in general, could have a chance to improve from the research findings by using the identified errors and their causes to design better ways of addressing the problems in algebra.

Mathematics arises from the attempt to organize and explain the phenomena of our environment and experience (Bell, 1978). It has been expressed thus: Mathematics is an activity of organizing field of experience. Mathematics is one of the major subjects which help to solve day-to-day needs because it is a science of numbers and space. It is processed by calculating and using numbers. Mathematics helps in creating a social order in this phenomenon. It regulates the functioning of society in many ways. Social conditions like justice, fair play, healthy competition, symmetry, harmony, etc. have often been described in mathematical terms for purpose of clarity. Mathematics is at the heart of many successful careers and successful lives (NTCM,1994).

Algebra is a buzzword in the landmark of mathematics education. Mathematics is recognized as the "science of patterns" and according to Sawyer "Mathematics is the classification and the study of all possible patterns". And the pattern is the heart of algebra. It shows the unparalleled role of algebra in mathematics. Algebra exists as a fundamental field of modern mathematics. Almost all the development in the field of mathematics is due to the algebraic treatment of the existing structure. It is known from history that algebra ultimately from generalized arithmetic (Acharya, 2017). Algebra has been traditionally been introduced when it was considered that students have acquired the necessary arithmetic skills. Besides, algebra has usually been developed separately from arithmetic without taking advantage of their strong link. Usually, in arithmetic we apply operations to numbers and obtain results after each operation; but in algebra, we usually do not start solving a problem using the given numbers, doing calculations with them, and obtaining a numeric result. In algebra, students identify the unknowns, variables, and relations among them, and express them symbolically to solve the problem.

Algebra has been characterized as the most important "gatekeeper" in mathematics. It is widely accepted that to achieve the goal of "algebra for all" students

elementary should have experiences that prepare them for the formal study of algebra in later grades (NTCM, 2000). However, curriculum developers, educational researchers, and policymakers are just beginning to explore the kinds of mathematical experiences. Elementary students need to prepare for the formal study of algebra in the later grades.

In general, why do the students have difficulties in learning algebra? Students struggle to reach procedural and conceptual proficiency in algebra due to a poor understanding of whole numbers, fractions, decimals, and percentages. Additionally, students lack understanding of fundamental algebra concepts such as negativity, variables, equality, and the equal sign.

Learning difficulties and errors in mathematics are directly related to the achievement of students in mathematics which is a great challenge to mathematics teachers. Errors in learning algebra for students may directly be related to the teacher's mathematical knowledge, use of materials, classroom practices and school management, pre-knowledge of students, family education, learning environment, and intellectual capacity. This study focuses on students' difficulties in learning algebra at the lower secondary level. Egodawatte(2011) carried out the main difficulties in learning algebra as difficulties in comprehending variables, difficulties in dealing with algebraic expression, difficulties in solving equations, and difficulties in solving problems. Booth(1988) also identified some of the root causes of students 'difficulty in learning algebra as the algebraic activity to perform, the nature of answers, the use of algebraic notations, and the meaning of letters and variables.

From the above discussion, it seems that more students have been facing several difficulties and errors in learning algebra at the lower secondary level. So, in this study, the researcher tried to find out the Students' Errors in Learning Algebra at Basic Level.

Statement of the Problem

Learners' errors give rise to poor performance in any subject. In Mathematics, most of these errors are attributed to poor algebraic skills. To make matters worse, almost all topics in Mathematics are developed using algebraic concepts. Therefore, there is a need to recognize common errors made by learners in algebra as well as the

causes of those errors. The researcher thinks that analyzing errors encountered by learners in simplifying algebra is one way of achieving this. The identification of these errors will help teachers to come up with better ways of minimizing prominent errors in algebra as well as in many branches of Mathematics. Algebra is a strand of mathematics in which variables are used to express rules about numbers and relationships and part of mathematics that deals with the generalization of other parts of mathematics. Hence, this study intended to identify and analyze the common errors among basic level learners from one secondary school and find out the root causes of these errors. The objective, after identifying the errors, was to come up with a better. The study of the problem should be mentioned in any kind of research so I have tried to state the statement of the problem related to this study in my own experience. Since two years I have teaching in Shree Durga Secondary School, Shailung-4, Magapauwa, Dolakha. According to my experience of teaching at Durga Secondary School, I had faced some difficulties in teaching and learning algebra at basic level students. Most students have the same problem in the same atmosphere in learning algebra in mathematics class. They feel algebra is more difficult and they do more error algebraic topics than other topics therefore, I selected this research topic "A study on students' error in learning algebra at a basic level."Hence the above-mentioned problem encourages me to conduct this study.

Objectives of the Research

The objective of the research is a word to present a clearer destination for any study. So, in this study, I have taken objectives that are helpful for new findings of the students' errors in learning algebra at basic level school. The objectives are

1. To find out the students' errors in learning algebra at a basic level.
2. To explore the strategies that reduce errors in learning algebra at a basic level.

Research Questions

1. How do students perceive learning algebra at a basic level?
2. What are the reasons behind students' weak in learning algebra at the basic level?
3. What are the factors behind Students' Errors in Learning Algebra at Basic Level?

Justification of the Study

Justification of the study means the rationale of the study. This study was concerned with the students' errors in learning algebra at a basic level. Most of the students were weak in algebra because they have no conceptual or structural knowledge of variables, expressions, factorization, and problem-solving way. Thus most of the students did not have a clear concept of algebra. The significance of the study was the following:

- The results of this study were helpful for students, teachers, parents, curriculum makers, and education administration.
- It is more helpful for students to identify the contributing factors and effects of difficulties in learning algebra.
- Its finding would help to reduce the errors in learning algebra and improve mathematics achievement.
- It helps the students and teachers with algebraic teaching and learning.
- This study would open the door for further study about the problem of errors and difficulties in learning algebra at a basic level.

Delimitation of the Study

Delimitation of the study determines the boundary of the study area. In this study, the major delimitations were the following:

- This study is based on basic level students only government school of Dolakha district.
- This study carried only a problem of errors in learning algebra at a basic level.
- This study was completed based on an achievement test (observation) and FGD.
- This result may not be generalized to other school students.

Definition of Key Terms

Algebra. Algebra is a branch of mathematics related to the variable manipulating skill of simplifying, solving an equation, word problems, expression, shapes, etc., is one of the broad parts of mathematics, together with arithmetic, geometrics, and analysis.

Basic Level Students. In this study, student means those who are reading in grades 6,7, and 8.

Observation. In this study, an observation means the action or process of observing or gaining information in grades 6,7 and 8 students to observe their knowledge, understanding of algebraic statements with a symbol, solution process, and present performance

FGD. It means Focus Group Discussion which is a rapid assessment, semi-structured data gathering method in which a purposively selected set of 6-10 participants gather to discuss issues and concerns based on a list of key themes drawn up by the researcher/ facilitator(Khanal,2073).

Errors. It means a mistake, fault, and defect in mathematical problems faced by students in learning algebra.

Learning Error. This is a mistake on mathematical problems or with students in learning algebra. It means being unable to recognize mathematical concepts with appropriate processes and methods.

Reading Error. This is the ability of students to read mathematical problems given and to identify sentences and mathematical symbols used.

Comprehension Error. That is the ability of students to understand mathematical problems.

Transformation Errors. It is the ability of the students to determine the method of mathematical solution.

Process Skills Errors.That ability of the students in doing process skill errors of mathematics correctly or not.

Encoding errors. That is the student's ability to write encoding errors according to the question.

Chapter II
Review of Related Pieces of Literature

Many studies have been carried out on the teaching and learning of Mathematics and in particular, algebra. Numerous studies bring many conceptions about algebra. They bring but the fact that many learners have naive theories, preconceptions, or misconceptions that interfere with their learning of Mathematics which result in them making errors when solving problems. This chapter presents a review of related literature in an attempt to provide the foundation for an organized study of errors encountered by learners in algebra. It helps to conduct the new research systematically by providing the outline for the research and avoiding unnecessary duplication. Some studies related to this study have been reviewed as follows:

Empirical Review

The review of the empirical literature of the study was very important and necessary. If the researcher didn't review this literature then the researcher can't proceed ahead and can't be clear about the research way, concept, design, and analysis process. It provides psychological, sociological, philosophical, contextual, historical knowledge, concept, and traditional way of study. From it, the researcher gains pre-knowledge of study, pre- techniques, pre- methods, and pre-analysis procedures. So, the researcher reviewed the following literature:

Adhikari (2007) conducted research entitled "An error analysis in solving algebraic problems of grade 5 students". The main objectives of this study were to identify the types and causes of errors made by students, mixed design was used to collect data. All students studying in grade v of government schools running classes from grade V situated in the rural location of Gorkha district were considered as the population of this study. He concluded that 75% of errors occurred in the Comprehension, 12%, 5%, and 8% of errors occurs in process skills, coding, and carelessness of studies respectively. And 40%, 34%%, 20% of errors occur on knowledge level, skill and application level, and problem-solving level respectively. The causes behind it are lack of pre-knowledge, technical terms of math, bad relation

between students and teachers, lack of classroom management, lack of teachers' explanation, and lack of classroom discussion.

From the above review of the related literature, it seems that mathematics education studies could not give a certain solution to overcome all the errors in learning and teaching algebra. Many of the works have been considered to address errors in teaching while others focus on improving student achievement. However, there are a few numbers of research to find types and causes of errors. There is still a gap that what are the ways to support to minimize the errors that help to increase the student performance in algebra.

Sharma's (2009) did study "An error analysis in solving algebraic problems of six five students". The objectives of this study were to find the errors made by students in solving the problem of simplification and equations of algebra and to compare the error made by students in knowledge, skill, and application and problem solving of simplification. This research was qualitative and was based on descriptive design. The sample of this study was Janata Primary School of Surkhet and all 30 students of grade VI were selected. Interview schedules were the main tool of this study. Major findings were as: 75% of errors occurred at the comprehension and transmission stage, 12% of errors occurred at the process skill stage, 5% errors occurred encoding stage, 8% errors occurred due to carelessness of students, and 40%,34%, and 26% errors on knowledge level, skill and application level and problem-solving level. The study concludes that poor performance of students in the topics of simplification and equations in different areas such as knowledge, skill, application, and problem-solving.

Shah (2019) studied "Students errors in solving math word problems analysis from the schematic model". The main objective of the study was to identify the errors made by the student and analyze the causes of an error made by them at a basic level based on the Schematic model. The design of the study was the sequential explanatory mixed method. Paper-pencil test was used for quantitative and an interview for qualitative data. 398 students were selected as a sample of the study in the Kathmandu district including both community schools and private schools. This study revealed that students had committed 3398 errors where 1727errors made by boys and 1671 by girls. The main causes of dosing errors were round that schema, carelessness, lack of

conceptual learning, lack of motivation, exam-oriented teaching, less practice for word problems of algebra, lack of basic concepts, and lack of appropriate teaching methods.

According to the Oxford dictionary (2003), algebra is part of Mathematics that uses letters and other symbols to represent quantities and situations. Learners generally lack a sense of algebra. As a result, they fail to appreciate the power of algebra. They do not know when to use algebra or manipulate it in a range of situations. Booth (1988) regards algebra as a source of confusion and is regarded as a common problem area for learners (Bell, 1995).

The challenge in algebra is that most learners fail to understand the main concepts of algebra. Once learners fail to understand the key aspects of algebra, they have difficulties in Mathematics. One of these key aspects is simplifying algebraic expressions. Algebra is a generalized form of arithmetic where letters and both operation and direction signs are used. The use of letters and signs, according to Foster (2007), makes it abstract and difficult. This is because algebraic ideas are based on general ideas instead of real facts or events. Learners possess a serious arithmetic-algebra gap which, as observed by Seng (2010), remains a fundamental cause of learning difficulties. If learners possess a good arithmetic background, they are not likely to face challenges in algebra. This is because algebra knowledge is built upon the foundation of already acquired arithmetical knowledge Learners have many misunderstandings in algebra (Seng, 2010; Mbewe, 2013; MacGregor and Stacey, 1997). These misunderstandings have their impacts on learners.

Li (2006) observed that learners' errors are the symptoms of misunderstanding. According to the free dictionary (2014), an error is, "an act, assertion or belief that unintentionally deviates from what is correct, right or true." Muzangwa and Chifamba (2012) and Donald (2007) view an error as a mistake, blunder, and miscalculation or misjudge. Errors perhaps result from forgetting, confusion, or lack of understanding of key concepts. The idea of learners making errors and/or mistakes is a natural part of learning (Lopez-Valero Fernandez & Clarkson 2008). These errors produced by learners as suggested by Mbewe (2013) play an important role in indicating to teachers the stages at which their learners are at as well as showing where there is a need for further teaching or study. They assist teachers to advise learners for

improvement. According to Radatz (1979), Melis(2004), and Riccomini (2005), there are two main types of errors, namely; systematic and unsystematic errors. Systematic errors are the common errors made by learners over a long period. “They are recurring erroneous responses methodically constructed and produced beyond space and time", (Mamba, 2012: 19). Following Nesher's (1987) and Riccomini (2005)’s views, these errors are symptomatic of a defective cause of thinking. There is a misapplication of rules caused by learners’ failure to grasp concepts or rules. Drews (2005) observed that these systematic errors are not only produced by children needing assistance but also able students to make incorrect generalizations.

Unsystematic errors, as suggested by Riccomini (2005), are non-persistent incorrect responses that learners can easily correct themselves without much intervention from the teacher. They are just random and have no evidence of recurring. Kousathana and Tsaparlis (2002) think that these errors could be a result of overloading the working memory, hastiness, or recklessness. In their view, learners should be able to correct these errors if given another chance. Given the above definitions, this study concludes that most of the errors in algebra are systematic and therefore can be addressed because of their consistency. It is extremely important to identify learners’ errors and their causes. Discovering the errors made by learners and the reasons for making such errors and identifying the most suitable methods of dealing with them is what Luneta (2008) refers to as error analysis or error focusing on the weaknesses of learners and this is meant to help teachers classify mistakes. Looking at the nature of systematic errors, it is possible to do error analysis to identify the diagnosis.

Similarly, Ketterlin-Geller and Yovan off (2009) describe error analysis as reasons for such errors and find ways of helping learners to do away with them. However, students do not come to class with blank minds (Resnick, 1983). Instead, they come with ideas and facts constructed from their everyday experiences. These ideas and facts having been actively constructed provide everyday functionality to make sense of the world (Mestre, 1987). These conjectures to some extent are the causes of misconceptions that lead learners to make errors in solving Mathematics tasks. Learners try to link what they already know to new information and at times they link unrelated things resulting in them making errors. The way these

misconceptions affect learners in learning situations is also evident in algebra. According to Greens and Rubenstain (2008), most students in Grade 8 and 9 struggle to grasp concepts and skills in algebra. This is the reason why most learners discontinue Mathematics at higher levels. If learners do not discontinue and the misconceptions are not remediated, they go up even to colleges making the same mistakes (Gunawardena, 2011). This researcher investigated errors and misconceptions in algebra with the hope to identify their origins. In the investigation, errors and misconceptions were examined in the four main areas of algebra: variable, algebraic expressions, equations, and word problems. From the findings, it was discovered that learners had common misconceptions mostly occurring in algebraic expressions.

Kuchmann (1981) also carried out a study on the 13 to 14-year-olds on their errors and misconceptions in algebra. Kuchmann's (1981) study deduced that learners had difficulties in coping with algebraic letters as unknowns or generalized numbers. The study also identified the conjoining of terms as one of the most prevalent errors in algebra According to Kuchmann (1981), learners seem to have difficulties in accepting the lack of closure. When learners are given an expression like 3y+4 and they think the expression is incomplete; so, they tend to write 7y as their final answer.

Macgregor and Stacey (1997) also conducted a series of studies to investigate the origins of students' misinterpretations of letter usage in algebra. They tried to get explanations for making the errors and also identified the causes of those errors. They deduced that learners can ignore letters while some of them associate them with numerical values. This observation sees learners simplifying m + 4m to 4m as to the appearance of m with no number means there is nothing. The learners who associate the position of a letter in the alphabet with counting numbers think a stands for 1, b for 2, cfor 3, and so on. Errors are caused by misconceptions and the latter is attributed to a lack of conceptualization and understanding.

According to Mbewe (2013), misconceptions are habitual and cannot be solved easily. This was evident from the interviews that Mbewe(2013) conducted with Grade 11 learners after they had written an algebra test. Mbewe (2013) also discovered that learners' errors occur frequently and repeatedly. In concluding his study, Mbewe (2013) then recommended that teachers and learners need to talk about

misconceptions during the teaching and learning process so that ways of doing away with them could be identified. Another study on middle school students was done by Bush (2011). She analyzed Grade 6 and 8 learners' common algebra-related errors and misconceptions. In her research, it was discovered that errors and misconceptions in algebra were just the same as those reviewed in the other kinds of literature. However, she confirmed the need for strong and knowledgeable teachers of Mathematics in elementary and middle grades. There was also a study on college students that was done on student teachers' knowledge and understanding of algebra. It was carried out by Mensah (2006) among final year college of education students in the Eastern Cape. The discovery was that even teachers in training also had misconceptions that they carried from their learning experiences and as they went up there was little change happening. The researcher worried that they would go out of college without well-developed algebraic concepts and therefore would not be good enough to assist learners. Therefore, there would be a cascading effect on teaching and learning in schools resulting in a cycle of errors from their misconceptions in algebra. The issue of the effects of misconceptions from early stages was also discovered by McIntyre (2005), who also investigated college students' misunderstanding of variables. In that research, a pre-test and a post-test were administered to 731 University of Maine students. In the findings, it was deduced that misconceptions are formed as early as pre-algebra when variables are first introduced to learners. They are then carried on if there is no remediation done. These misconceptions are the causes of errors that are always made by learners. Wellman (2008) also carried out a study on 270 freshmen of the school of business at a university by administering a 42-item test to them. The findings were that most of the students performed badly because of their arithmetic and algebraic skills brought from earlier studies. One of the serious learning difficulties in Mathematics is that misconceptions learners may have from previous or inadequate teaching, informal thinking or poor remembrance (Donald, 2007). These are the causes of learners' errors in solving problems. There is a need to reduce if not do away with the misconceptions at early stages before they accumulate and become part of the learners' incorrect conceptions. If these misconceptions are not eradicated, then learners will continuously make errors when solving problems. It is the role of the teacher to let these misconceptions disappear with the framework changes. If misconceptions disappear then errors will also be minimized. It could be seen that

much had been done but still the problem of errors in algebra persisted and it was now the duty of the researcher to give a contribution to what had been done and what had not been done

Theoretical framework

Moodley (2014:11), states “Assimilation occurs when a new idea is interpreted in terms of an existing schema.”An existing mental structure that is available is used to assimilate a new situation. The process of assimilation can be demonstrated using a situation whereby a learner knows that $a^2 – b^2 = (a – b) (a + b)$. When she is asked to evaluate $101^2 – 99^2$ without using a calculator, the learner will be able to express it in the same way used for that$a^2 – b^2$. The learner will have $101^2 – 99^2 = (101 -99) (101 + 99)$. The learner can then simplify what is inside brackets to $2 \times 200 = 400$. The demonstration shows that the learner has interpreted the new situation in terms of the already known aspect of factorization difference of two squares.

The picture of the accommodation process can be explained by looking at a situation where learners have to find products of algebraic terms. If a learner knows that $a \times b = ab$ or ba when a problem requires the same learner to simplify $a \times 5$ then s/he might have a5 as the answer as the learner will be thinking that a5 is the same as 5a.

This means, there is a need for restructuring so that the learner sees when one part of the algebraic term is a number then the number has to be written first. This shows that it is not always possible to connect new ideas to schemas. The failure to link existing schemas to new situations may result in the creation of a new box in the mind of the individual. The learner may find it difficult to link the knowledge in the box to existing schemas which may force him or her to memorize the ideas or rules to learn. In the process of recalling, some of the rules are partially remembered resulting in the learner being confused and making errors. These errors, as suggested by Olivier (1989), are the natural results of learners’ effort to construct knowledge. Labinowicz (1985) also regarded learners’ errors as actually natural steps to understanding. According to Brodie (2014), understanding learner errors is a way of understanding

learner thinking. Therefore, errors must be expected and appreciated in teaching/ learning situations.

Ncube (2016) studied "Analysis of errors made by learners in simplifying algebraic expressions at grade IX". The objectives of this study were to determine learners' errors and to find out the causes of errors in simplifying algebraic expression in grade 1A. She used a sequential explanatory design which is a mixed-method approach that is both quantitative and qualitative method. This study is based on the theory of constructivism. She selected 82 students from 300 students randomly grade 9 learners from secondary school in Ga-Sekgopo village, Mopani district in Limpopo Province in South Africa as the sample for this study. In this study, she found six common types of errors in simplifying algebraic expressions. The errors are: Conjoin error (combining unlike terms)- 16%, misapplication of rules-370, misinterpretation of symbolic notation-60, misuse of distributive property-22%, substituting letters by numbers-10%, and sign error-9%. She concluded that learner's mostly misapplied rules when they were simplifying algebraic expressions. They are failing to deal with direction, operation, sign, and algebraic rules. The main reasons behind the errors are poor arithmetic background, Jack of arithmetic skills, lack of awareness and understanding of the meaning of expressions, and lack of conceptual knowledge.

Laudari (2014) conducted a study entitled "An error analysis of grade V students in solving mathematical word problems". To find out the errors or grade V students on solving a mathematical word problem. This study was quantitative followed by qualitative methods and procedures. All students studying in grade V of government schools running classes from grade V situated in rural location Tanahu district were considered as the population of this study. 10 students from each school including an equal number of boys and girls were selected as the sample of this study. The research design of this study was a small survey. The sample for this study consisted of 40 students by random sampling and 4 teachers for interview. From this study researcher extended the following conclusion that students committed maximum errors in transformation after that comprehension, process, skill, and encoding respectively but didn't commit any error in reading level, and the role of gender is less important to committing the errors it means there is no significant difference between boys and girls to solve word problem in mathematics.

Egodawatte (2011) conducted a study entitled "secondary school students misconception in algebra". This study investigated secondary school students' errors and misconceptions in algebra intending to expose the nature and origin of those errors and to make suggestions for classroom teaching. The study used a mixed-method research design. An algebra test that was pilot-tested for its validity and reliability was given to a sample of grade ll students in an urban secondary school in Ontario. The results indicated several error categories under each area. Some errors emanated from misconceptions. Under variables, the main reason form is conceptions was the lack of understanding of the basic concept of the variable in different contexts. The abstract structure of algebraic expressions posed many problems to students such as understanding or manipulating them according to accepted rules, procedures, or algorithms. The main difficulty in word problems was translating them from natural language to algebraic language. Students used guessing or trial and error methods extensively in solving word problems.

Matt (2010) studied "the analysis of students' error in learning of quadratic equations. The purpose of the study was to determine the students" error in learning quadratic equations. The samples were 30 from three students from a secondary school in Jambi, Indonesia. A diagnostic test was used as the instrument of this study that included three components: factorization, and completing the square and quadratic formula. A diagnostic interview was also used to identify at which level students' errors occur in solving problems. The type of error is based on Newman's error Hierarchy Model which includes reading type error, comprehension, transformation, process skill, and an encoding error. Data were analyzed using descriptive statistics: percentage and frequency. The findings showed that most students make an error in transformation and process skills m solving quadratic equations. There was n error found in the reading. The number of students who made encoding errors and carelessness was small. The student's error in solving quadratic equations was due to their weaknesses in mastering topics such as algebra, fractions, negative numbers, and algebraic expansions.

Sharma (2010) conducted a study entitled "An error analysis on solving verbal problems of algebra by grade VII students" to find the patterns of errors committed by the grade VII students while solving verbal problems in algebra. The objective of this

study was to identify and analyze the errors based on a recognized theoretical base. To analyze the errors committed by the students the researcher developed an achievement test. The sample of this study consisted of twenty students and the school was selected purposively for the convenience of the researcher. From this study, the researcher extended the conclusion those students commitmore errors while solving the verbal problem in algebra, the concentration of errors was seen in the phase of transformation, process skill, and comprehension of the problem.

Chaudhary (2013) Studied "Teachers' belief on teaching algebra and their classroom practices" with the main objective to investigate teacher beliefs on teaching algebra, comparing the belief of rural and urban teachers, comparing the beliefs of experienced and inexperienced teachers about teaching algebra. This study was survey design as well as quantitative in nature and the sample of this study was public lower secondary, secondary, and higher secondary schools of Bara district from 13 were rural and 12 were urban areas by quota sampling. Opinion ire collection was the data collection procedure and the Likert scale was the analysis procedure. The major finding was that 55% of the statements were agreed by the teacher, 35% disagreed and 10%% were undecided and the beliefs of the teacher in rural versus urban shows a statistical difference. The study concludes that it was important for mathematics teachers to attain a proper understanding of algebra.

Poudel (2008) studied "Errors in learning mathematics". The main objectives were to identify the errors in learning mathematics of stone quarries students at school and to find out the cause of errors in learning mathematics of stone carries students at the school level. This study was qualitative and five stone quarries students at the lower secondary level were selected from four public schools in Kathmandu district near Chovar V.D.C. The instruments of this study were interviews and observation. The major finding was there is not sufficient time for mathematics learning at home for stone quarries students and there is a discontinuity between practices of mathematical concepts in school and at home. It concludes that the learning environment at home and school creates errors in mathematics learning.

Reyes (2012) studied "Equal or Not? An exploration of grade-8 students' experience of algebra". The research questions were how to do eight-grade sues to perceive algebra and the grade-8 mathematics curriculum and what difficulties if any,

Students encounter in learning algebra. The research was qualitative and ten students, teachers of Leader middle school of Georgia were selected. The interview was the main instrument of this study and data was analyzed using the constant comparative method. The finding of this study provided several opportunities for the areas curriculum and pedagogy for improving students' experience and it concludes that participants offered great insight into the difficulties with algebra that are experienced by many eighth-grade students.

Tahir (2008) studied "Teaching and learning algebra in the junior secondary years". The main aim of the study was done the multi-faced variable approach leads to a deeper conception of a variable by the student than the traditional approach to teaching algebra in years 7-8. The design of the study was quasi-experimental nature and seven metropolitan higher schools in Sydney were selected based on 54 students in the comparison group and 49 students in the experimental group and teachers. Interviews and questionnaires were the main instruments. The finding was students of the experimental classes demonstrated a deeper understanding of the variable concept compared to the comparison classes and it concludes that the study has provided evidence that it is possible to minimize students' misconceptions by using the MVA in algebra course in years 7-8.

Theoretical Literature

The Australian educator Anne Newman (1997) suggested that when a student attempts a written mathematics word problem then he/she had to be able to pass over several successive hurdles such as reading the word problem, comprehending the word problem, transforming the word problem into mathematical numbers, processing the algorithm & and encoding the answer. Besides this, it this also possible to make a careless error which is due to a lack of motivation or reluctance to attempt at the level of ability. How to do it? Arid, what "to do? Let's see what these successive hurdles areand how these could create errors in solving geometrical problems?

Reading error. Reading error an error is defined as a reading error if the student hadn't been able to read all the words in the questions such that he can't grasp all the information given in the questions. The literature highlighted that children

usually faced difficulty with the vocabulary, language, and the symbolization of mathematics which Newman named as reading error.

Comprehension error. Comprehension error would be classified as comprehension error if the students had been able to read all the words in the questions but hadn't traced the overall meaning of the words and therefore weren't proceeding alone with appropriate problems solving paths. In this category, students may be able to read word problems but are unable to draw mathematical meaning from a scenario (White, 2010).

Transformation error. In this category, a student can read the word problem but is unable to identify the required or suitable algorithms to solve it (White, 2010). An error was classified as a transformation error if the students had understood what the question was asking about but were unable to identify the operation sequences of operation needed to solve problems or unable to transform in a mathematical expression.

Processing skill Error. Here a student can read the question, understand the see, and identified the required algorithms but is unable to process the algorithms to reach the correct result (White, 2010). Process skill error is classified as process skill if the students made the mathematical expression but couldn't identify the correct operation and didn't know the procedure carry out the solution.

Encoding. Now, a student can go a step ahead. That is can read understand and solve but is Unable to transform the mathematical result into a general understandable statement as required by the question (White, 2010). Encoding error was classified as encoding if the students correctly worked out the solution to the problems but express the solution in an acceptable written form**.**

Anne Newman, in the mid-1970s, identifies the sequence of steps (prompts) that a teacher can employ while assisting their learners, It involves interviewing learners, using a series of questions that probe for the exact error that your learner is making.

Newman's error analysis strategy. Newman defined five specific reading skills as crucial performance on mathematical word problems. They are reading(Decoding), comprehension, transformation, process skills, and encoding.

Newman asked students the following questions as they attempted problems.

- She requests their students to read the questions loudly.
- She asked to find the answer appropriately.
- She requests to show what to do to get the answer. "Talk aloud "as you do it so that I can understand how you are thinking.
- Now write down your answer to the question.

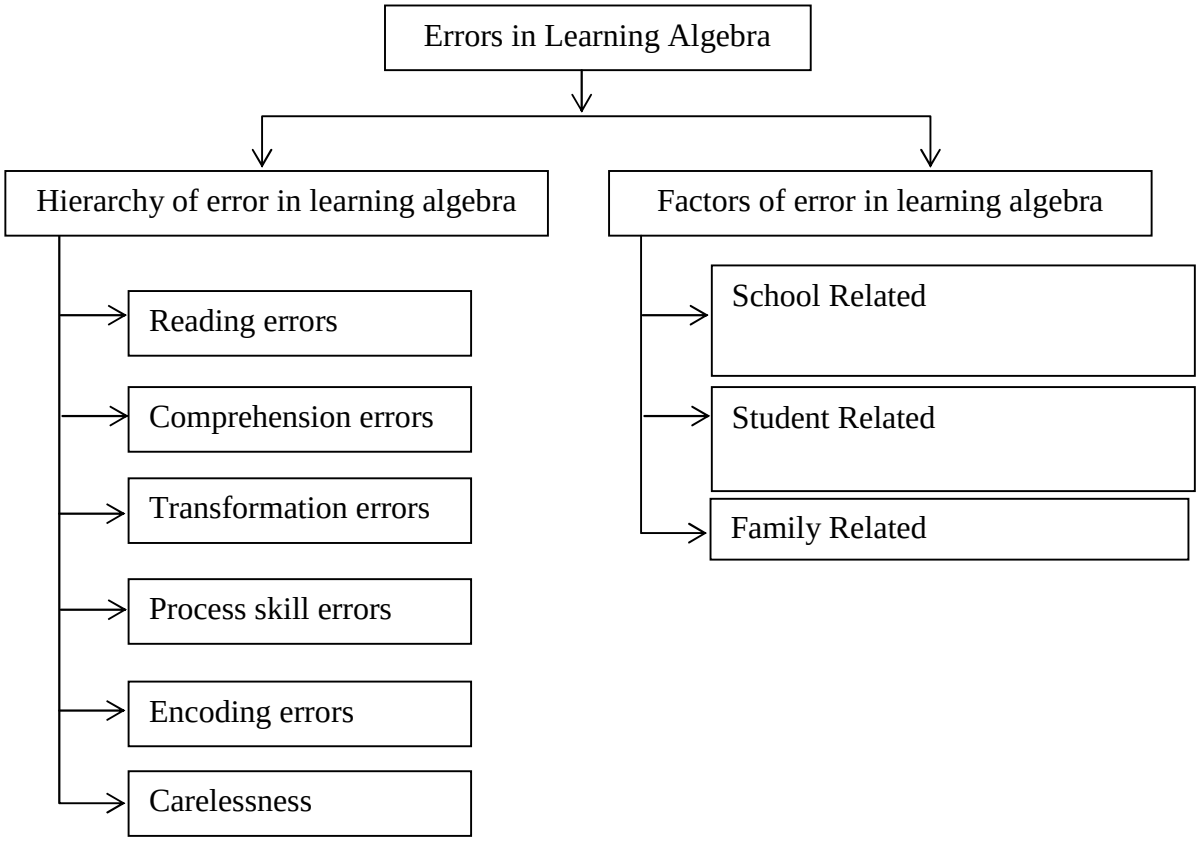
While working through a word problem it was always possible for students to make a careless error and some students deliberately gave incorrect answers due to a lack of motivation to answer to their level of ability (White, 2010). It's noted that this strategy 18 is not a learning approach but a method to identify what the learner knows already and the critical stage where they are making errors (Na,2015).

Newman's research generated a large amount of evidence highlighting that more children experienced difficulty with the semantic structures, the vocabulary, and the symbolism of mathematics than with the standard algorithm. In many Newman studies carried out in schools, the proportion of errors occurring at the comprehension and 'Transformation' stages has been large. Thus, studies regularly reported that approximately 70 percent of errors made by Year students on typical mathematics questions were at the Comprehension or Transformation levels. These researchers also found that Reading (Decoding) errors accounted for less than 5 percent of initial errors and the same was true for process skills errors, mostly associated with standard numerical operations. Also, Newman's research consistently pointed to the inappropriateness of many remedial mathematics programs in schools in which the revision of standard algorithms was overemphasized, while hardly any attention was given to difficulties associated with comprehension and transformation.

Conceptual Framework

A Conceptual Framework of the study deals with the concept of possible areas of this study. In this study, the researcher tried to find out the student's errors in learning algebra at the lower secondary level based on the above empirical and theoretical literature. The conceptual framework has been designed to fulfill the objective of this study based on student errors in learning algebra. The students' errors in learning algebra at a basic level were the following:

The figure of the conceptual framework of this study is drawn below:



(Newman,1983 & Yadav,2017)

This conceptual framework describes the basic level of students' errors in learning algebra. Based on the above framework comprehending variables, algebraic expression, solving an equation, word problems, and transitioning from arithmetic to algebra were the main five errors area in lower secondary level mathematics. These errors were faced by many students at the lower secondary level whose impact is directly related to the student's achievement in mathematics. A conceptual framework

is a representation, either graphically or in narrative form, of the main concept of variables and their presumed relationship with each other. A Conceptual framework covers the main features of research and their presumed relationship. To research an error analysis on solving the verbal problem of algebra by grade 8 students, the researcher has read several other theses, research articles, and journals. After reading and analyzing those studies researcher decided to adopt Newman's procedure for analyzing errors on written mathematical tasks as this conceptual framework. According to Newnan (1977). a person wishing to obtain a correct solution to word problems like this must ultimately proceed according to the following hierarchy:

- Read the problem,
- Comprehend what is read,
- Carry out a mental transformation from the words of the questions to the selections of an appropriate mathematical strategy,
- Apply the process skills demanded by the selected strategy and
- Encode the answer in an acceptable written form

In general, we can see that the element of the step between the three frames relates to each other. Especially, steps of the understanding problem and devising strategies, simultaneously, has likely similar ideas to steps of reading, comprehension, and transformation errors in Newman's analysis, while this idea also appears in mathematical literacy, i.e. formulate. In the early stages of solving mathematical tasks, they end up determining precise mathematical models or strategies before performing further steps of solving the problem. Likewise, each idea of carrying out a step in Polya's process skill errors, process skill errors in Newman, and employ in PISA's mathematical literacy deals with undertaking mathematical to find mathematical results, such as performing arithmetic computations, solving equations, making logical deductions from mathematical assumptions, performing symbolic manipulations, or extracting mathematical information from tables and graph. Furthermore, the last step of Polya's, i.e. looking back, corresponds to the final stage of Newman analysis, i.e. encoding and PISA's mathematical literacy, i.e. interpretation. The idea of this stage is the interpretation of the mathematical result of the initial problem such as checking the reasonableness of the answer or considering other strategies and solutions to the problem. The difference, obviously, only appears

in the type of the tasks examined where PISA's mathematical literacy specifies contextual task (OECD,2013), while Polya and Newman respectively deal with the general mathematical problem and written mathematical task (Clements,1980). Comparing those three frameworks, it is known that Polya's problem-solving steps, which has introduced before the other two frameworks, have an agreement with both Newman's analysis and PISA's mathematical literacy. Thus, the category of Newman errors that researchers will use to analyze the level of student performance in solving problem-based mathematics context problem-solving.

Chapter III
Methods and Procedures

The learners' thinking processes and procedures used in simplifying algebraic had been obtained. Their underlying concepts were revealed through the execution of these procedures. In the quantitative phase, a test instrument was used to identify and classify errors. The findings of the quantitative study were used to determine the type of data that was gathered in the qualitative phase. The study used qualitative data to explain and explore quantitative data and it provided the researcher with information on how learners came up with their answers. Interviews came in the qualitative phase when learners were asked to answer some questions to justify the procedures they had used in obtaining their solutions. Interviews helped to expose learners' thinking processes that were not clear in their work.

Research Design

The research design is the detailed plan of investigation and the blueprint of the research work. The researcher selects the research design to answer the research question objectively, rapidly, and economically as possible(Singh, 2008,p.450)This study was based on descriptive and qualitative because it aims to describe the events or situations addressed in the present activity of students in learning algebra. This research-based on phenomenology research design.

The phenomenological study describes the meaning for several individuals of their lived experiences concepts or phenomena. It focuses on describing what all participants have in common as they experience a phenomenon. The main purpose of phenomenology is to reduce individuals' experience with a phenomenon to a description of the universal essence (Van Manen, 1990). Phenomenology rejects scientific practicality and the view that the empirical sciences have a privileged position in identifying and explaining any phenomenon in the world. This philosophy refocuses inquiry, concentrating not on descriptions of worldly objects but descriptions of experience. The phenomenological perspective encourages careful description of ordinary conscious experience of everyday life which includes perception, believing, remembering, deciding, Feeling, Judging, and evaluating

As per van Manen (1990), to do research from a phenomenological point of view 1s to question the way one experiences the world in which one lives as a human being. The complex, multifaceted philosophy of phenomenology defies simple characterization because it is not a single unified philosophical standpoint. It includes the transcendental phenomenology of Edmund Husserl, the existential forms of Maurice Merleau- Ponty and Jean-Paul Sartre, and the hermeneutic phenomenology of Martin Heidegger. The two major variants of phenomenology that are manifest in contemporary qualitative methodologies are hermeneutic and existential. The former, best known through the work of Hans-Georg Gadamer and Paul Ricoeur tends to focus on the collective or inter-subjective features of sociopolitical life as evident in the primary concern with 15sues of language and the nature and structure of communication. The latter variant, best known through the work of Husserl's follower the phenomenological sociologist Alfred Schutz, I more oriented toward describing the experiences of everyday life asis internalized in the subjective consciousness of individuals(Schwandt, 1997).

According to Patton (1990), phenomenological inquiry focuses on the question: What is the structure and essence of the experience of this phenomenon for these people? (p. 69), while the phenomenon being experienced could be an emotion such as loneliness, jealousy, or fear. Phenomenology asks for the very nature of a phenomenon, for that which a "thing 1s what it 1s and without which it could not be what it is (Merleau-Ponty, 1962). By phenomenolo8y, Husserl meant the study of how people describe things and experience them through their senses. This most basic philosophical assumption was that we can only know what we experience. Thus, phenomenologists focus on how individuals put together the phenomena they experience in such a way as to make sense of the world and, in so doing, develop a world view. There is no separate or objective reality for people. There is only what they know their experience is and means. The subjective experience incorporates the objective thing and a person s reality (Moustakas, 1994).

Another important dimension of a phenomenological approach is the assumption that there is an essence to shared experience. According to Patton (1990), these essences are the core meanings mutually understood through a phenomenon commonly experienced. The experiences of different people are bracketed, analyzed a

compared to identify the essence of the phenomenon. A phenomenologist thus assumes a commonality in human experiences and uses the method of bracketing to search for these commonalities.

Unlike descriptions that depict things and happenings as they exist independently of a person's experience of them, phenomenological research requires descriptions of an experience as appears in a person's consciousness. This further requires the researcher to redirect a person's awareness toward their own experiences. According to Polkinghorne (1989), the problem with such requirements is that consciousness being activity is in a state of continuous flux and therefore differs from natural objects. Consciousness is moreover an integration of perception, memory, and imagination. Access to consciousness is also problematic because the data a researcher collects are several times removed from the actual event. Furthermore, we have direct awareness of only one consciousness, our Own. As phenomenological research is based on descriptions of experience, another difficulty in its acceptance seems to be the fact that the whole enterprise seems too straightforward and too superficial. A common assumption Seemed to be that flimsy things such as descriptions were not worthy of being accepted as scientific knowledge. However, as per Giorgi (1985), despite the still unsolved problems surrounding the use of descriptions in phenomenological search, a quantitative analysis of such descriptions can yield insight value at least equal to what qualitative approaches yield although different in character and style. He goes on to remind us that descriptions pervade science and form the basis of all scientific reports which contain such precise descriptions that the experiment described can be replicated in any branches of science as psychology is dependent upon descriptions

According to Polkinghorne (1989), the general format for phenomenological study entails (a) gathering several descriptions from people who are having or have had the experience under investigation, (b) analyzing these descriptions to get a grasp of the constituents that make the experience what is and (c) producing a report that gives an accurate and articulate description of the experience. The sum of such a procedure was to ensure that a reader of the report could come away and win the feeling that he or she had understood better what it was like for someone to undergo that particular experience. Thus the major obstacle seemed to be not the first-hand

descriptions supplied by the research participants but the post-descriptive analysis that the researcher developed using a qualitative perspective. According to Giorgi (1969), although there are no universally accepted models for such analyses, the bottom line is that the analysis should seem credible. Phenomenology is a philosophical movement attributed to Edmund Husserl (1859-1938). The phenomenological tradition, like any other philosophical tradition, spans many different positions and perspectives. However, there exists a universal understanding that phenomenological studies describe the meanings of lived experiences about the human phenomenon. The underlying assumption of phenomenological research is that every human has a personal and unique perception and perspective of the world, life, and experience. Phenomenology has been characterized as a theory of the unique (van Manen,1990). However, the goal of phenomenological research 1S is the understanding of what is common in the unique manifestations of the phenomenon. As a methodology, phenomenology might be criticized as being Eurocentric. Indeed historically it has been so. Yet the fact that the focus is to uncover the essential nature of human experience means that its methods and techniques can be fruitfully applied to the broadest range of human experience, across cultures and ethnicities. This study employs phenomenology as the methodology design because provides mechanisms. However, the methodological perspectives of phenomenology are not the sole reason for its use. An equally compelling reason is that this methodology aligns closely with own epistemological and pedagogical beliefs(Patton, 1990). In other words, phenomenological research comes known through the significant world and raised the consciousness of the human being. Pedagogy refers to the art of teaching. In a phenomenological study, participants are selected based on their experience of a common phenomenon. The selection criteria were based on accessibility and the student's willingness to participate. It is this overarching interest that prompted this study. The purpose of the phenomenological inquiry is to uncover the essence of human phenomena and is premised on the idea that within the uniqueness of each individual there lays the common essence of the phenomenon under study. Phenomenological research employs in-depth interviews as the primary method of data collection. However, it is not the only method by which to collect data (Creswell, 1998). Qualitative research is rooted in phenomenology (Ary,2002). The phenomenological study is designed to describe and interpret an experience by the people who have participated in it (Ibid, p. 447). The purpose of conducting a

phenomenological study is to describe and interpret the experience of participants to understand the "essence" of the experience as perceived by the participants (McMillan, 2000, p.269). Phenomenology makes no judgment with respect t to the reality status of experience. It merely wants to understand how, through experience, all the events and objects of the world appear to the consciousness (Giorgi, 1988, p. 447).

In this study, the researcher used Newman's techniques as the theoretical base of the study and based on them, the results were analyzed. In this study, as tools the in-depth interview (face-to-face) and focus group discussion areas are the methodologies that may be considered during qualitative research design."Qualitative data collection process is qualitative data to provide a comprehensive analysis of the problem"(Creswell,2009, p.228).

Research Site

Chiromo (2006: 26) defines a population as, "…..all individuals, units, objects or events that will be considered in a research project." In this study, the population of this study was all basic level of Shailung rural municipality in Dolakha district. So the sample of this study was Shree Durga Secondary School of Shailung-4, Dolakha with purposive sampling, and only 10 students of grades 6,7 and 8 had been selected for Focus Group Discussion in this study.

Respondents of the study

In this study, the respondents are only 10 students in grades 6,7 and 8 who had been selected for FGD. This study is based on qualitative inquiry. So, the population and sample of this study were not fixed. There is no hard and fast rule for selecting a sample in qualitative inquiry (Aderson,2011 Cited in Paneru, 2015). The credibility of the study and availability of time and resources. The population of this study will be all the mathematics teachers, school head teachers, and students of Government schools in the Nepalese mountain region. I had selected the Shree Durga Secondary School as my study site and the head teacher, mathematics teacher, and students of such school were selected by purposive sampling method for the respondents of the study.

Sampling Procedure

This is a qualitative research design that's why the population and sample of this research study should be purposive but nonrandom but more representative. Guba and Lincoln (1981) stated that in qualitative research, 'Sampling is seldom representative or random but purposive, intended to exploit competing views and fresh perspectives as fully as possible' (Khanal 2019, p. 130). The researcher identified the population on which data collection methods were to be applied to gather information. As this was a very large population to handle, the researcher decided to work with a sample of the population. McMillan and Schumacher (2010) define a sample as a group of individuals from whom data is collected. Therefore, a sample is part of the entire population which usually represents the whole group under study. Sampling is necessary because it is not always possible or practical to study the whole population. According to Brink (1991), sampling refers to a process of selecting the sample from a population to obtain information regarding phenomena. Once the general problem has been identified, the task becomes to select the unit of analysis (Merriam, 1992). As described by McMillan and Schumacher (2010), the unit of analysis is the object which is to be studied in terms of research variables that constitute the constructs of interest. In this study, learners' errors and the reasons why they made these errors were the units of analysis. The participants in this study were randomly selected.

A sample of the population as random numbers were used to select the participants. The researcher assigns random selection gives every member of the population equal chances of being selected (McMillan and Schumacher, 2010). The justification for selecting students from only one secondary school was as follows: convenient to the researcher because of easy access and the researcher could relate well with the sample resulting in the quality and credibility of research data. A test was administered to the sampled 82 participants. For the explanation of the reasons why learners made errors, FGD was purposively selected from the sampled participants. Tashakkori and Teddie (2003) define purposive sampling as involving selecting certain units or cases based on a specific purpose rather than randomly. The selection of these individuals was based on the specific purpose associated with answering the research study questions. Purposive sampling provided greater depth of

the information from a smaller number of units. One learner was selected to represent each type of identified error. From the results of the test, the researcher. For the accomplishment of the objective and obtaining the data or information for this research study, I selected the population and sample for this study by using non-probability purposive sampling as the method of sampling.

Data Collection Tools

Since the design of the study was qualitative form so that the researcher collected the data using primary sources based on qualitative nature and uses the secondary data if necessary. In this study, the tool as achievement tests, interview guidelines, Focus Group Discussion (FGD), and observation for students to fulfill themselves were used, and collected the necessary primary data or information.

FGD is a rapid assessment, semi-structured data gathering method in which a purposively selected set of 6-10 participants gather to discuss issues and concerns based on a list of key themes drawn up by the researcher/ facilitator(Khanal,2073). Focus groups are especially effective for capturing information about social norms and the variety of opinions or views within a population. Focus groups are more than a collection of interviews. Data are generated by the interaction between group participants. Participant presents their views and experience but they hear also other people. They listen, reflect on what is said, and in the light of this consider their standpoint further. The focus group presents a more natural environment than that of the individual interview because participants are influenced and influenced by others just as they are in real life.

Unlike the group interview, the group discussion stimulates a discussion and uses the dynamics of developing conversation in the discussion as the central source of knowledge (Flick, U, 2006). Participants tend to provide checks and balances on each other which weeds out false or extreme views. The extent to which there is a relatively consistent, shared view can be quickly assessed (Pattom, M.Q. 1990). Focus Group discussions generate diversity and differences in data (Lunt, P. 1996).

Data collection is an important part of the study and tools are important factors for data collection. To attain the objectives and get the responses to research questions, the researcher gathered the data by using such tools. There are many tools

for qualitative research to get the information from the respondents and the study site. In this study, the researcher intended to fulfill objectives by the find the answers to research questions: How do students perceive learning algebra at a basic level? What are the reasons behind students' weak in learning algebra at basic level?, What are the factors behind students' errors in learning algebra at basic school. For this phenomenology research design had used FGD.

Quality Standards of Research Tools

Quality Standards of data collection tools are essential characteristics for the effectiveness of data collection procedures. Reliability is the degree of consistency that the instruments or the procedure and validity are the quality of data collection tools that enables what is supposed to be measured. Reliability is necessary but not a sufficient condition for validity. In interviews, inferences about the validity are made too often based on face validity (Cannell and Kahn, 1968). The most practical way of achieving greater validity is to minimize the amount of bias possible. The purpose of the interview is to find out what is in or on someone's mind and the purpose of open-ended interviewing is not to put things in some owns mind but to assess the perspectives of the person being interviewed. Validity is greater when the interview is based on a carefully designed structure, thus ensuring that significant information is elicited. The critical of the experts in the field of inquiry helps select essential questions (Best and Kahn, 2009).

Similarly, the validity observation guide is ensuring the internal validity of observation and minimization of business. The internal validity is the degree to which observed differences in the dependent variable (Gray et al, 2012).

For quality standards, member checking and triangulation had been applied. Furthermore, to maintain the quality standards Guba and Lincoln (1998) suggest four criteria for 'Naturalistic' research. The categories are following and that's why the researcher had followed these criteria to maintain the quality standards in my research.

Credibility: -Credibility is the concept related to the internal validity by which the researcher seeks to establish confidence in the truth' of their findings. Guba and Lincoln (1998) recommended several techniques inquiries may use to enhance the

Credibility of their researcher: prolonged engagement, persistent observation triangulation, peer debriefing, negative case analysis, progressive subjectivity checks, and member checking. To maintain the Credibility of the research, the researcher tried to spend a long time for observation and engaging with different people with their works.

Transferability: - Transferability replaces the concept of external validity. This criterion refers to the applicability of finding in own context (where the research is done) to other contexts or settings (where the interpretations might be transferred). Providing a rich description of participants' responses (and the researcher's interpretations) makes transferability easier to evaluate. Naturalistic generalization occurs when the findings are in harmony with the experiences of the individual evaluating the research, and thus appear transferable in the eyes of the reader.

Dependability:- This criterion of quality standards refers to the stability or consistency of the inquiry process over time. Triangulation across researchers can be used to investigate dependability. Auditing can also be carried out to allow another researcher to follow the audit trail (ideally) generated by the original researcher.

Conformability: - Are the findings a product of participants' responses and not the researcher's biases, motivations, interests, or perspectives (Lincoln & Guba, 1985:290)? Auditing can be used to evaluate the conformability of findings. A more transparent report or the findings (with signposted reflexivity) makes conformability easier to evaluate.

Authenticity: - Does the research represent a fair range of differing viewpoints on the topic? Do the findings have transformative potential? Is there community consensus that the findings are useful and [have] meaning (especially meaning for action and further steps)'(Lincoln et al., 2011:116)? Member checking can be used to inquire about apparent authenticity with participants or other members of the community in question, sometimes known as 'end-users. These individuals might include practitioners who would potentially change their practice based on the findings.

Ethical Considerations

Qualitative research is frequently concerned with individual cases and unique instances which may involve personal sensitive matters, it raises the question of the identity, confidence, and privacy of the individuals. Ethics refers to well-based standards of right and wrong that prescribe what humans ought to do, usually in terms of rights, obligations, benefits to society, fairness, or specific virtues (Velasquez, Andre, Shanks, & Meyer, 2008). In the research work, numerous ethical norms and values should be considered to make standardization in the data collection procedures and analyze the data in a qualitative research design. The ethical considerations of this research study are:

- **Institutional Approval:** The researcher has been granted permission from institutions before planning or conducting the research work.
- **Informed Consent**: Informed consent means that the participant in the evaluation is fully informed about the evaluation being conducted. Participants need to be aware of the purpose of the study, what group is funding it, and how the finding will be used. Before participating in the interview the objectives and the procedure of the study were verbally explained to each participant individually.
- **Pseudonym**: A pseudonym is a fictional name assigned to give anonymity to a person, group, or place. Pseudonyms are very useful for research on sensitive topics, particularly concerning deviant or criminal behavior. When pseudonyms are used, this must be identified in any dissemination of findings.
- **Voluntary participation:** The principle of voluntary participation requires that people not be coerced into participating in research. Essentially, this means that prospective research participants must be fully informed about the procedures and risks involved in research and must give their consent to participate.

Data Collection Procedures

This study was related to the basic level of Shree Durga Secondary School, Shailung-4, Dolakha. The researcher went to this school with the above tools as achievement tests and Focus Group discussions to collect reliable data. The researcher

conducted the achievement test to observe errors at a basic level and 82 students were selected, and to find out the area of the error related to the teaching and learning activities in algebra. During this period every notable activity was collected. The researcher selected ten students by using observation guidelines and responses were noted carefully as well as conducted the observation properly.

One of the most essential steps of the research study is data collection. To collect the data the researcher had gone to the case school by administrating the interview guideline for mathematics teachers, headteacher, students, and observation guide to observe the errors in learning algebra in the mathematics classroom. Firstly, the researcher had established rapport with the school authority and the mathematics teachers by introducing the researcher himself and stating the purpose and process of the research. Based on the purpose of the study, the researcher requested from the school authority. After that ask the permission and after getting permission the researcher moves toward the mathematics teacher, and to obtain the required information.

To attain the objective of the study, the researcher conducted an in-depth interview with headteachers, mathematics teachers, and students with the help of interview guidelines. The conversation of those participants was recorded by mobile phone to get the originality and nature of the data. Similarly, the mathematics classroom was observed by the researcher to observe applications of such mathematical materials and techniques in mathematics classroom activities even in school also. Moreover, the researcher had used the data from secondary resources from the previous researcher, Books, Articles, Newspapers, and so on.

Data Analysis and Interpretation Procedures

Data analysis is considered an important step of the research study. After collecting the data using different relevant tools and techniques, the next logical step is to analyze and interpret data to arrive at an empirical solution to the problem (Singh, 2009). This study was a qualitative case study that's why, the researcher analyzed them by coding, decoding, and developing themes analytically. In this manner, Khanal(2019), stated that" In a qualitative research study, after collecting the data, the collected data can be analyzed based on recognizing the data, noting data and

recoding the data, building the theme, reporting and finding procedure" After collection the data from a selected sample using interview guidelines and observation checklist. I analyzed and interpreted the data descriptively and analytically. This study was limited to qualitative research therefore the major part of the data analysis was based on descriptive analysis. The data collected from achievement tests and observations and Focus Group discussions had analyzed descriptively on the basic conceptual framework. To analyze the collected data, to follow the procedure organize the data, editing the data, noting, recoding, building theme, reporting, and finding procedure. Firstly, I organized and edit the data. After that, I generated themes from different opinions according to the response of participants and also that recoding the data according to similarities. Alter coding and recoding the data I developed the themes and reported the finding.

Finally, the data was analyzed and interpreted on basis of my conceptual framework and theoretical basis. The researcher analyzed the data by using Newman's error analysis procedures. In this study, the data analysis procedure the data are collected by using observation and finding out the area of the error within the primary source in this study. In the qualitative data analysis procedure, the data were collected by using the tool as observation including 82 students and interviews with 10 students within the area of the different errors and analyzed the data based on Newman's error analysis procedures. In this study class, wise responses were analyzed based on observation as well as main themes were analyzed with the help of five-step of Newman's error analyses. At last, data were analyzed based on integrated mixed inferences.

Chapter IV
Analysis and Interpretation of Data

This chapter provides the findings of the research study in terms of data collected from Grade 6,7 and 8 learners. The data for the study were collected using two methods: achievement test and FGD. In this chapter, data are analyzed and displayed in tables and illustrated verbally.

The test responses were analyzed and quantified to identify the common errors made by participants in learning algebra. The identified errors were categorized into groups. In addition, frequencies for each type of error per item were recorded. From the results of the test, the researcher identified the common errors and also recorded the number of learners who committed those errors. These learners were sampled from the group that committed the most prevalent errors. The observations were held to get a clear understanding of how the learners had arrived at particular solutions. The participants were audio-taped to capture as much accurate information as possible.

In this study, the Analysis of data is a process of inspecting, cleansing, transforming, and modeling to highlight useful information, suggestions, conclusions, and supporting decision making (Best and Khan 2009). The most important part of the study is to analyze the collected data because the essence of the study cannot be found without that. Data analysis involves reducing and organizing the data, synthesizing, searching for significant patterns, and discovering what the important (Khanal, 2019). The data of the present research work was analyzed analytically, and descriptive. This was about the burning issue of information communication Technology in teaching-learning mathematics and its real application in the mathematics classroom.

To meet the objective of the study, the researcher collected the data from one Government school in Dolakha. Shree Durga Secondary School Shailung is one of the suggested and proposed model schools of the Government of Nepal located in the Himalayan Region. Data were collected through Classroom observation Interview Guidelines and Focus Group Discussion. The main respondents of this study were purposefully selected the Head teacher, mathematics teachers, and students of this school. The researcher collected the data in pictorial form by the record on a mobile

phone and the FGD was conducted by two people as moderator and note-taker. A moderator rapport building with the students and started the discussion by showing the observation of seven question-answer papers where their errors in learning algebra. During the discussion, the moderator controlled them, gave opportunities to speak and discuss with friends to get information, and the note-taker noted in a field note the primary data were firstly transcribed respondents' language then translated into English. After that noted all the information then categorize and theme as made. The result of the collected data was analyzed in the following main themes/sections/topics on basis of my conceptual framework.

A Brief introduction to School

This school is located 20km from the headquarters of Dolakha and 135 km from the capital city Kathmandu of Nepal. Firstly, this school was established on 2016/5/5 in B.S., by the named Shree Rashtriya Primary School, Deurali eastward no. 2, Dolakha. After that, the name of the school was changed into Shree Bhagawati Rashtriya Primary School, Deurali in 2025 B.S. With the changing the perspectives of people on education, school, their consciousness, increasing quantity of students, and good leadership of head sir Mr. Jay Bahadur Shrestha had changed the name of the school into Shree Magadeurali Lower Secondary School, Magapauwa Panchayat Dolakhain 2032/03/22 in BS. Finally, again they changed the name of the school to Shree Durga Secondary School in 2040/11/15 BS. and 41 students started SLC from this school from 2043BS. But in this school Higher Secondary School (+2) was established with the department of education and management in 2064/03/32 BS. Nowadays this school is selected on a proposed model school by the ministry of education, science, and technology.

Demography of Respondent Observation

In the age of Globalization mathematical tools becomes an indispensable aspect of the aching learning process. Teacher skill and competency affect the coherent use in mathematics teaching. From the researcher's observation, the senior and experienced teachers were not interested that much as a comparatively younger teachers. They try to teach by a lecture the mathematical content but it also became challenging for them to clarify the concept because they have a lack appropriate

knowledge of tools about how to operate and use them as a medium of instruction. In oppose, younger teachers are familiar with tools and different techniques and mediums of instruction. They want to make a clear concept of content that they can use for a more extended time of their teaching career. They want to collaborate with their colleagues to share their ideas about the modern day teaching-learning process and can improve their skills by sharing and pairing.

Time Constraint

Several recent studies indicate that many teachers have the skill, competence, and confidence the use it in the classroom, but they still make little use of technology because they do not have enough to develop clear content and present them in the mathematics classroom. Dhungel (2020) found in his research study that the most common challenges reported by all the teachers were lack of time to plan the strategies and different plans, explore the different strategies about the relevant pedagogy knowledge.

Mathematics Teacher said

“I had 38 years of experience in the teaching field. But I had not taken any teacher training due to some reasons. When I was very young that time I was very busy with my family problem. During Covid-19, Teacher Professional Development training was conducted online system but I had no online tools like a laptop, android mobile,computer, etc., and I am very poor in technology also. But if I have got an opportunity for teacher training on coming days then I will take the teacher training because I am very interested in teacher training, if I have got the opportunity."

Student;
One student argued that:
"Though we want to study mathematics operating different technological and multimedia type of tools which assists us to clarify the concept. But it's often impossible for us to use tools in the classroom even in school also every day, we have no access to technological devices more than calculators. For the increased use of mathematical tools in learning algebra and further

mathematics, the school has no clear plan, dedication, and commitment to involving all students in teaching-learning activities."

Another student shared her experience:

"We request our head sir to buy the mathematical tools for our classroom. He replied to us that I am trying to buy but the school management committee didn't agree urgently with him to spend more money on buying mathematical tools."

Condition of Mathematics output/ Result

Head Teacher said

“From the initiation of this school, the base of mathematics, condition of mathematics, output/ result of mathematics were critical and still very poor condition in the comparison to another subject"

Mathematics Teacher said that

“According to my experience, 66% of students had no basic knowledge of algebra at a basic level in our school. These students cannot differentiate between natural numbers, whole numbers, integers, and real numbers. They were very poor knowledge about coefficient of variables, base, power, homogeneous, heterogeneous, variable, constant, mathematical operation, derived formula, apply the formula, factor, LCM/HCF, solution process."

Availability of Mathematics Tools for Teaching

Head Teacher.

The role of the headteacher in a school as an administrator, who has the responsibility to maintain the overall management of the school. Headteacher argued that, as the government of Nepal initiate the integration of teaching-learning activities, most governmental schools were facing different types of problems with the effective implementation of it. As it is one of the proposed model schools selected by the Ministry of Education, Science, and Technology, it has well infrastructures constructed with the support of different governmental and non-Governmental organizations (NGOs and INGOs). The

school has a computer lab but we don't have a projector, so we were unable to connect to any of the classrooms.

"Firstly, we have no mathematical materials are available in this school because we had no skillful manpower to use mathematical materials for teaching, such as GeoGebra. This school is a proposed model school and has a large number of students, so difficult to manage a few days because nowadays infrastructures are not sufficient for all students. But we try to provide mathematical materials as soon as possible. This school is located in a geographically arduous area even."

Mathematics Teacher said that,

"In our school, had only tools box for teaching geometry. But I used local materials such as cones, triangles, circles, prisms, cylindrical shapes, and different solid figures which are made of wood. But I had no further mathematical tools for teaching algebra."

Students.

Most of students agreed that the senior and experienced teachers usually useless mathematics materials. They mostly like to teach algebra traditional way in the classroom, using long lectures and writing on a whiteboard if they need. Some students argued that it became easy and interesting to learn mathematics for us if the teacher is confidentially able to use materials in teaching algebra according to the content otherwise it became tough to understand. One student argued that

"Our school had no mathematics tools/materials. Therefore, we were prohibited by mathematics materials in learning algebra and day to day we were felt mathematics much more tough subject in the comparison to another subject."

Appropriateness of Materials in Learning Algebra

Mathematics Teacher. The mathematics teacher agreed that they do not have sufficient physical resources and infrastructure facilities in school to teach mathematical content. Though the Government curriculum included materials for every content schools don't have the necessary equipment.

Mathematics teachers complained about the anxiety problem of students in their school as:

> *"Every teaching materials are undoubtedly largely meaningful and useful. But according to my experience, I thought, due to their small age most students were thinking very tough, quarrelsome, botheration, complex, enigmatic and complicated instead of suitable and appropriate to learning algebra and any mathematical concept. The use of materials in teaching and learning is essential to both the teacher and learners. As a teacher, I use different local materials to teach relevant lessons. The main challenge l faced at my school is that there is only age mostly students were thinking very tough, quarrelsome, botheration, complexity, enigmatic and complicated instead of suitable and appropriateness to learning algebra and any mathematical concept.*"

Student. Almost students expressed their interest in studying mathematics using tools regularly. They argued that mathematics learning became easier and more interesting while teachers teach with materials. But after some timethey had problems forgetting their concept. All of them face the problem of resources in their school. They do not have sufficiently equipped in a classroom, they watch as passive learner whatever their teacher teach in their classroom. Some students also claimed that they couldn't get the proper opportunity to catch whatever teachers were teaching in the classroom.

> One student GSI added that
> *"We have no idea how to remember and how to relate the application and mathematical concept so we were felt boring. Since different strategies assist to make mathematics learning interesting and easy are unable to use them*

regularly because our school doesn't have sufficient modern infrastructure for teaching-learning.

Role of Teacher in Motivated Classroom

Mathematics Teacher said

"In my teaching carrier, I had done many things to make a motivated classroom. The role of the teacher in making a motivated classroom is as followed:

- *Formulae notice patch in classroom*
- *Create formula relation*
- *Class discussion about algebraic terms*
- *Conducted mathematics quiz*

But I did not get an expected improvement of students."

Use of Teaching Methods

Mathematics Teacher said

"*According to experience, I had used lots of teaching methods to teach learning algebra at the basic level. Such as the discussion method, demonstration method, and collaboration method. Among them, the discussion method was mostly used in teaching algebra."*

Challenge of Teaching-Learning Algebra

Mathematics Teacher said

"No available of materials, the economic condition of the school, Lack of time, Comparing mathematics with another subject, different question pattern than another subject"

Administrative Supports to Improve Outcomes of Mathematics

Mathematics Teacher said

"Before I was already a head teacher in this school. So, I knew the detailing the condition of this school. This school had been in pitiable condition from the initiation. Therefore, the administration was unable to support us to improve the result, and outcomes of mathematics. But this school was selected on the proposed model school so administrative could be following support for coming days.

- *Increase time for mathematics class*
- *Extra mathematics curriculum*
- *Manage incentives for teacher*
- *Manage mathematics-related activities*
- *Manage price and reward for students*
- *Manage digital materials*
- *Mediator between students, teachers, and parents"*

The Factor of doing Errors in Learning Algebra

Mathematics teacher. The factor of error in learning algebra is a very important part or aspect or essential characteristic of algebra which is related to Error in solving an equation, Error with dealing with word problems, Error with dealing with comprehension variables, Error with dealing with algebraic expression, Error in transition from arithmetic to algebra to arithmetic.

Mathematics teacher said

"The factors of the student doing errors in learning algebra may be Lack of basic knowledge, Route learning, Lack of parents consciousness, Mathematics anxiety of students, Lack of confidence, Economic condition, Addict to social media, Fall in love, domestic problems."

Students. Most of the Students agreed that the senior and experienced teachers usually use fewer mathematics materials. They mostly like to teach algebra

traditional way in the classroom, using long lectures and writing on the whiteboard if they need. Some students argued that it became easy and interesting to learn mathematics for us if the teacher is confidentially able to use materials in teaching algebra according to the content otherwise it became tough to understand.

Administration Supports

Teacher Motivation. Motivation is a kind of internal encouragement that pushes someone to do things to achieve something (Harmer, 2007). It is the characteristic that helps to shove an individual towards acting, performing the actions, and achieving the result. Teacher motivation is an important factor for the effective implementation of mathematics materials in teaching-learning algebra and further content of mathematics. Fitzallen (2014), Stated that mathematics teachers were motivated to use technology also in their teaching-learning. If the school administration guarantee access to sufficient infrastructures in school, ensure the financial security and teacher professional development opportunities. In this regard, to motivate all teachers in the mathematics classroom, teachers must have access to the materials resources, ensure proper opportunities for professional development and provide extra time and other facilities to teachers.

Mathematics teacher claimed that:

"Teachers do have not enough motivation to use materials or introduce the innovative leaching learning practices in their classrooms. Some teachers try to integrate technology also in their content through their self-motivation, and they understood the benefits of using tools in their classroom. But the school administration does not provide any additional facilities tomotivate them. Most of the teachers claimed that there is no impact of classroom performance order on their promotion and increment of salary. Also, most teachers want their financial security first, they don't want to spend more time on making digital content rather than their time on private tuition. There is almost nothing for teachers to motivate them in the effective use of tools in their teaching-learning activities at their school."

What strategy to reduce the errors problem in learning algebra

Description of mathematics teacher

"The whole education system in school is TEAM (T= together, E=everyone, A=achieve, M=more) work. If our team is strong and integrated then nothing is impossible. We have a strong team in the school for the betterment of improving the condition of mathematics. Having said that we need staff support, administrative support for recommending and implementing our mathematical plan, School Management Support to coordinate among teachers, parents, and students and manage all the mathematics related tools, books, etc."

Test analysis

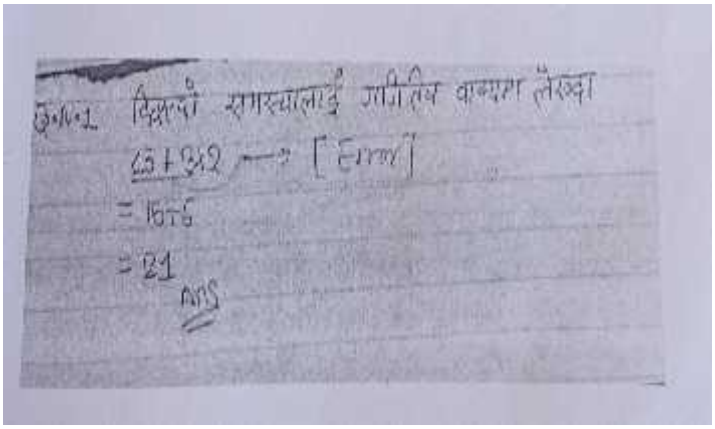
The first phase of the study involved the administering of a 7-item test to eighty-two (82) participants of the study. In the analysis of learners' responses to the test items, the researcher identified six common types of errors displayed by the participants. The main errors observed were: Reading error, Comprehension error, Transformation error, Process skill error, an Encoding error, Careless.

Analysis of the first problem

First problem:

Ram has a total of Rs.15. Ram lent the money Rs.2/2 to 3 people then how many remain with him?

The first problem in which learners were asked to simplify the word problems. The most common solution provided by 83% of the learners was a plus sign instead of the minus sign. However, they



proceeded to inappropriately simplify 15 +3x2 and obtained 21 as the answer. Learners lacked understanding of the concept of like and unlike operations.

By the FGD, we conclude that:

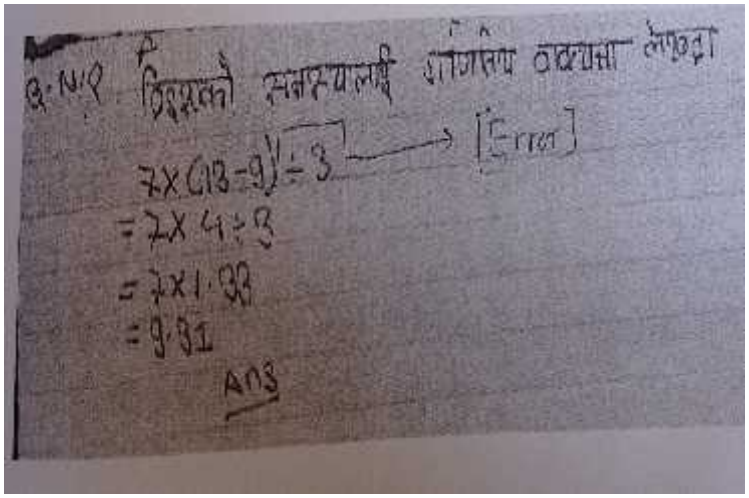
- Do not understand the question properly,
- Translate problems of students,
- Language problems of the understanding question,
- Process mistakes of students,
- Overconfidence of student during solve. When learners have changed the word problem into mathematical terms and simplified. They did a mistake on the first algebraic question.

According to the above picture, we can say that students lack mathematical operation problems.

Analysis of the second problem :

Second problem:

Simplifying: {7x(13-9)÷3}



This error prevailed in most of the questions in the test. Learners misapplied rules in many items. In the second item of the test, learners were asked to simplify, and surprisingly most of them could not get the correct answer. The most common answer is the distributive error in bracket expansion was also a very common error, especially in this error was committed by 23% of the learners in the study. Some of the learners expanded both parts of the bracket. This was evidenced in the expansion of {7x(13-9)}÷3, where the learners gave 1.33as their solution. The other learners did not know the limits of the pre multiplier. Therefore,

when simplifying the given problem, they multiplied the contents of both sides of the brackets. They came up with {7x(13-9)}÷3 =7x13-7x9÷7x3=91-63÷21=28÷21=1.33.The learners over generalized the distributive law. The answer was 9.31 because they were ignoring the bracket instead of applying the BODMAS rule. This error was committed by 37% of the learners.

By the FGD, we conclude that:

- Time management(hurry up) during simplifying,
- Negligence brackets,
- Lack of simplifying rules,
- Unknown to the BODMAS rule,
- Overthinking of students
- Calculation mistake
- Foolish man foolish thinking.

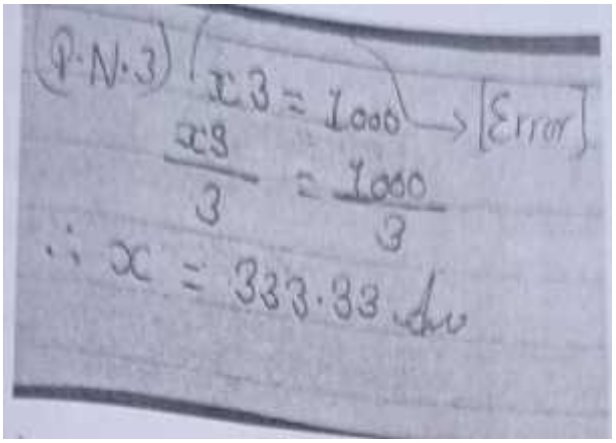
They have no proper idea about the BODMAS rule.

Analysis of the third problem:

Third problem:

Solve: $x^3=1000$

In this solving problem, 35% of students do not deal with cube and cube root. Among them, most students have no idea about base and power.



By the FGD, we conclude that:

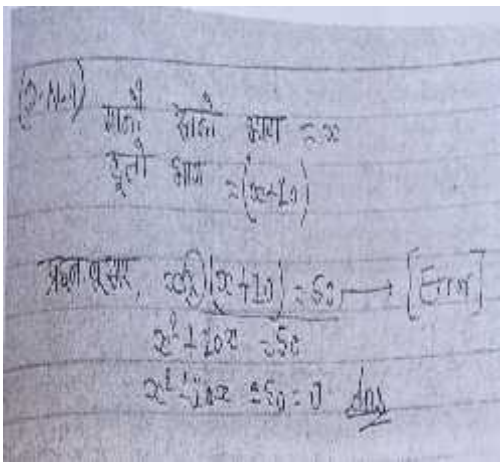
- Unknown to solving process,
- Unknown to the indices role,
- Confusion on power and base,
- A mistake on rewrite question,
- Just copy from a friend,

- Consequences of an irregular student,
- Unknown to cue and cube root,
- Very careless.

Analysis of the fourth problem:

Fourth problem:

If the big number is tenfold that small number and their product is 1000 then find both numbers?



This error was committed by 6% of the learners. Learners made misinterpretations of terms with invisible coefficients. Some students assumed that 0 was the coefficient of terms with invisible coefficients as there were no numbers before the letters. As a result, the coefficients of terms like m were taken as 0. That is why some of the learners when asked to simplify that word problem and when simplifying some student assumed big number as (x+10) instead of 10x then finally trying to solve and their answer was $x^2+10x=50=0$

By the FGD, we conclude that:

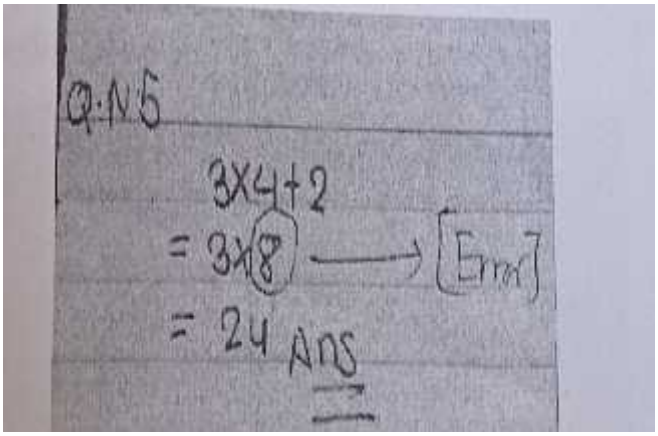
- Understanding the problem properly,
- The question is lengthy,
- Confusion about the question,
- Do not focus during the time of teaching.

Analysis of the fifth problem:

Fifth problem:

Simplify: 3x4+2

In this problem, most of the students do not have an idea, of which mathematical operation is first calculated among multiplication and addition. Learners just multiplied without a mathematical operational rule. These learners relied on unrefined schema which means the learners possessed disorganized information as to their basic structures. Some students presumably had the correct methods in their long-term memory but could not retrieve the information well. The learners had ideas of rules that were supposed to be used but incorrectly adopted the rule.



By the FGD, we conclude that:

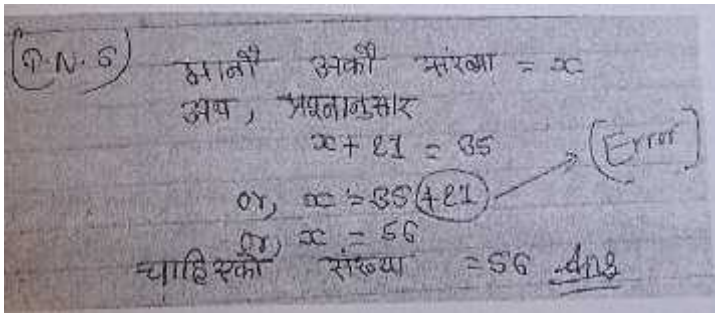
- Unknown to mathematical operation,
- Could not differentiate plus(+) sign and multiplication(x) sign,
- Simplify problem,
- Calculation problem/ mistake,
- Only answer-oriented.

Analysis of the sixth problem:

Sixth problem:

A number in which by adding 21 is 35 then find that number?

In this problem, only 50% of students were able to assume the number as x. Then most of the students had no idea about the mathematical operation and how to change the operation. Most of the students had problems working with integers and operation signs. The errors are due to the inappropriate use of the sign. Sign errors were mainly due to failure to combine operation and direction signs. . So,



they thought x+21=35 and x=35+21 are the same because they ignored the minus sign. Most of the students think that mathematical operations no changed any term of LHS to RHS or RHS to LHS.

By the FGD, we conclude that:

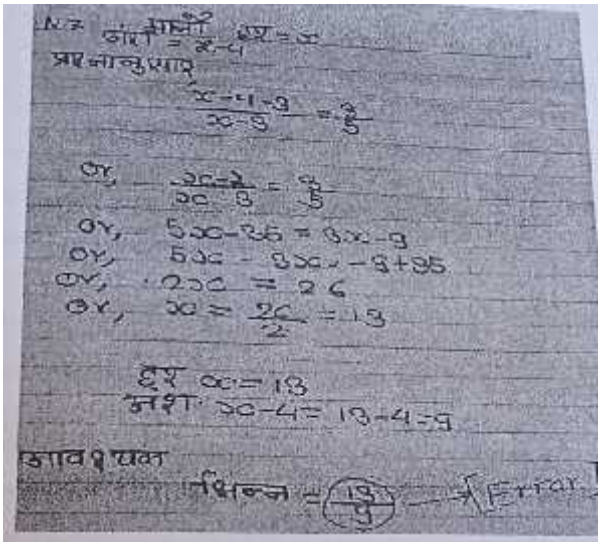
- Unknown to equality/inequality and its property,
- Sign change problem,
- Focus on adding only,
- Language problem,
- Overconfidence

Analysis of the seventh problem:

Seventh problem:

If the numerator is less by 4 than the denominator. If subtracting 3 on the numerator and denominator both then their ratio will be 3:7 then find the fraction?

By the FGD, we conclude that:



- Reading problem,
- Lengthy question,
- Unknown to ratio,
- Unknown numerator, and denominator,
- Convert problem the word problem into mathematical form,
- Unknown to fraction,
- Same thinking about ratio, divide, and fraction.

CHAPTER V
FINDINGS, CONCLUSION AND RECOMMENDATIONS

This chapter presents the conclusions of the study based on the analysis and research findings. The chapter starts by stating the aim and objectives of the research, and research questions. The researcher answers the research questions and provides recommendations for teaching practice and further research.

Discussion of the Research Findings

The main goal of this research study was to identify the student's errors in learning algebra at a basic level. These errors are made by learners in simplifying the algebraic problem and also exploring the strategies that reduce students' errors in learning algebra. For this, find out the possible causes of those errors. Learners committed several errors in learning algebra. This chapter presents a discussion of the findings of the study and relates the findings to the theoretical background and literature review connected to the study.

A sample of 10 learners participated in the study. The participants were given a 7 item test. In the test, learners were expected to write answers and to show their work. Learners' solutions were analyzed to address the first research question of the study which intended to identify the students' errors in learning algebra at a basic level. The researcher analyzed and interpreted the solutions provided by learners. An item analysis was done and errors were coded as shown.

The researcher identified six errors in learners' test responses. The six main errors that were identified according to frequency from highest to lowest were: use of inappropriate rules to simplify algebraic expressions; inappropriate use of the distributive property; conjoin of terms; substituting letters by numbers; sign errors and misinterpretation of algebraic notation. After analyzing the test scripts, 12 learners were selected for FGD about the identified common errors. Find out the causes of errors that had been displayed by learners in learning algebra.

Misapplication of Rules. The findings of the study suggest the misapplication of rules to be the main cause of errors in learning algebra at a basic level. The analysis done in this study shows that learner had misapplied rules frequently in simplifying

the algebraic problem. In this study, 37% of the errors were due to the misapplication of rules. Learners seemed to have got confused and misapplied rules. This was found that learners get confused and misapply or misremember rules for transforming problems. Demby and Kieran also suggest that the terminology and rules of algebra offer little meaning to many learners resulting in learners memorizing algebraic rules with little or no conceptual understanding. Most of the learners only learn the manipulation of rules without reference to the meaning of the expression being manipulated. Therefore, learners find it difficult to keep the rules or apply them appropriately. In other situations, learners create their own rules which work for themselves only. Learners misuse previously learned procedures and rules in situations where they are not appropriate. When asked to simplify 3x4+2, instead of giving 14 as the answer most learners gave 24 as their solution. Learners multiplied the terms instead of adding. In this case, it shows learners failed to differentiate between addition (+) and multiplication (×). This confusion comes about as learners try to construct knowledge. Some researchers believe that errors emanate from misconceptions from prior knowledge as learners try to construct mathematical knowledge meanings. In the process of constructing mathematical meanings, learners get confused. The confusion arises from too much interference coming from learners themselves, other learners, teachers, and also the surrounding environment errors also result from naive concept images that do not measure up to concept definitions. The idea of errors emanating from misconceptions from prior knowledge goes in line with the constructivists.

According to Brodie (2014) positioning errors as the performance of misconceptions, conceptual structures constructed by learners that make sense to learners about their current knowledge. Learners come to a new grade, not as empty vessels but they come with pre-knowledge acquired in previous grades (Hatano, 1996). Learners, as suggested by Olivier (1989) then use that knowledge to assimilate and adapt new mathematical concepts. The problem is that at times prior knowledge conflicts with new knowledge making it difficult for learners to judge what is correct or not. Learners then commit errors because they fail to link new knowledge to prior knowledge. Taking into consideration the explanations provided by learners in the interview, the researcher deduced that this error was mainly due to interference with previously learned concepts. Learners could have applied ideas from the addition of

learners who committed errors because of poor arithmetic background. This is supported by Norton and Irvin (2007), and MacGregor and Stacey (1997) who also indicated that poor arithmetic skills contribute to algebraic errors.

In this study, it has been discovered that learners at the basic level misapply algebraic rules due to interference from other concepts and also because there are many rules in Mathematics. Learners do not make sense of some of the rules; therefore it becomes difficult to keep them in their minds.

Misuse of the Distributive Property. Misapplication of the distributive property was the second most common error contributing 22% of the errors committed by the learners. Learners made errors in trying to remove brackets. Some of the errors were due to a lack of prerequisite facts and concepts as observed by Kieran (1992). Learners displayed instrumental knowledge of the distributive property. The learners then got confused and could not even identify the limits of the brackets. Seng (2010), in his study, discovered that the distributive property was misapplied in many different ways. This was the same case with this study. Errors emanated from invalid or incomplete distribution for possible causes of errors linked to the expansion of brackets. Moodley (2014) sees learners as not knowing the meaning of brackets. According to Moodley(2014), brackets signify multiplication as soon as learners encounter them. In this study, some learners only multiplied the first number in brackets by the pre or post multiplier. For example, when asked to simplify {7x(13-9)}÷3, learners, who saw brackets went on to multiply by a term that was to be added after expansion. This was because they had not seen a visible pre or post multiplier. This means that these learners took brackets to have a different meaning altogether. This is evidenced by items where learners were required to remove brackets.

Moodley's (2014) study also reported similar findings. Moodley (2014) found that learners multiply brackets even in the presence of a plus or a minus sign. Learners displayed a partial understanding of the bracket expansion procedure. The learners were relying on the unrefined schema. In situations where there were two sets of brackets and only the first set had a visible pre-multiplier, learners used the visible pre multiplier for both sides of brackets. This was evidenced in the item which had where some learners gave {7x(13-9)}÷3 =7x13-7x9÷7x3=91-63÷21=28÷21=1.33 as their

answer. Learners simply multiplied without appreciating the limit of the pre multiplier. The researcher discovered that learners had difficulties in bracket expansion. Learners inappropriately used the distributive property in a variety of ways as shown in the analysis of learners' solutions. Interview responses also showed that learners had many misconceptions about bracket expansion. These misconceptions led them to produce errors.

Conjoin error. This error contributed to the third-highest number of errors in the test. The frequency of this error was 16%. Learners made this type of error due to a lack of understanding of the concept of algebraic problems. Learners ignored letters and concentrated on numeric values. They then just added letters to their answers. They also added coefficients and constants and put a letter at the end. For learners, addition is considered 'an action symbol' (Booth, 1999; Davis,1995). The plus sign might have been considered as a signal to conjoin terms. This could be the reason why learners conjoined terms. They thought '+' meant putting terms together whether like or unlike. According to Brodie (2014), structures make sense of learners' current knowledge but are not aligned to convectional mathematical knowledge. In arithmetic, answers are single termed digits but in algebra, this does not apply. Also, support the fact that learners link the idea of single termed answers in arithmetic to algebra. Some learners thought the word simplify meant, reduce to a single term. Most learners displayed simplified 15+3x2 instead of 15-3x2. Therefore, learners decided to complete it in what s/he had an easy way. The main cause of this error was a failure to recognize like terms. Most of the learners just added, unlike terms. They were misled by the plus sign which they took as an instruction to conjoin terms. For learners, the word simplify meant reduce to a single term. This suggests that learners have a problem of failing to accept the lack of closure and therefore complete or finish expressions. This completion of expressions is what was evidenced in the learners' responses.

Substituting letters with numbers. This error contributed to 10% of the errors which were identified in the test. It was mostly identified in item, small part (number) = x and big part (number) = x+10 instead of big pat(number)=10x. This meant the learners thought tenfold means x+10, not 10x. This implies that learners did not take appropriate variables to represent unknown numbers. Substituting letters with

numbers is an error that is produced when learners' responses suggest that the letter has been given a numerical value. According to Christou, Vosniadou & Vamvakoussi (2007), this is because learners tend to use their prior experience with numbers in the context of arithmetic. Learners assign numerical values to variables (Kuchemann, 1978). This was also discovered by MacGregor and Stacey (1997) in the study they carried on learners who were around the age of 15. The cause of this replacement of a letter by a number as suggested by McIntyre (2005) is that learners have a weak understanding of the variable.

Learners displayed a lack of meaning in a variable. Learners possess little knowledge of a variable because the meaning of a variable is often neglected in the teaching and learning of algebra (Usman, 2012). This results in the learners only knowing algebraic manipulation.

Sign errors. This error had a weighting of 9% in the test. Learners failed to subtract integers causing them to make errors in simplifying algebraic expressions. In his study on error analysis of signs was that according to the question, x+21=35 then the second step is x=35+21 and the answer is x=56. In this sense, students had no idea about the function of signs and how to deal with significant change signs. The students used to plus sign instead of the minus sign. Errors resulting from the subtraction of integers prevail because learners have difficulties in operating with a negative sign. In this study, the researcher discovered that learners had a poor background in operating with directed numbers.

Misinterpretation of symbolic notation. The error due to misinterpretation of symbolic notation contributed to 6% of the errors made in the test. Learners misinterpret the symbolic notation cube symbol when they solve the x^3 =1000, x3/3=1000/3 then the answer is 333.33. From this particular test, we can say that if the students misinterpret symbols and notations then they do such types of errors in learning algebra.

Answers to the research questions:

Research question 1

How do students perceive learning algebra at a basic level?

The researcher analyzed and interpreted the solutions provided by the learners. In short, from the test and FGD, most of the students replied that they student's perceived mathematics as a boring subject, threatened subject, anxiety subject, route learning subject, frustrating subject, demoralized subject, etc.

Research Question 2

What are the reasons behind students' weak in learning algebra at a basic level?

By the FGD, may the following reasons behind students' weak in learning algebra: Students threat/ afraid of mathematics, reading (language) problems, calculation mistakes, understanding the formulae problem, route learning, verbal to mathematical convert problem, lack of practice, addiction on mobile, time manage the problem on examination, homework problem, no competition atmosphere, boring classroom, feeling anxiety on mathematics, strict mathematics teacher, daring habit, addict on bad habits, addict on friends, think insignificant mathematics subject, the teacher not well trained, teaching methods problem, lack of school management system, focus on only talent student, demoralized content, no-repeat by the teacher, careless on homework, punishment system, naughtiness, insulting attitude, weak implementation of rules, personal problem, imbalance responsibility and dignity, no attractive mathematics book, students attractive on fashion, careless teacher, teacher-oriented, no discussion with a friend, economic problem, etc.

Research Question 3

What are the factors behind students' errors in learning algebra at basic school?

The factors behind the students' errors in learning algebra at a basic level can be categorized into the following:

School-related factors

- Municipality level Exam (class 8)
- Teacher
- Training
- Technical problem on test item
- Classroom Manage
- Classroom Size
- Discriminate among Student

- Teaching Method
- Number of students
- Curriculum
- Motivation
- Coordination
- Punishment & Reward
- Infrastructure of school
- Administrative Support

Family Related factors

- Parents Consciousness
- Poverty
- domestic problems
- Ignore Education
- Irregular
- Culture
- Home
- Occupation
- Size

Students related factors:

Students related factors may are the following:

- Anxiety / frustrated
- Threat / afraid
- Irregular
- Practice
- Competition
- Demoralized
- Quality
- H.W. &C.W.
- Language/ Reading Problem
- Relation

- Motivation
- Careless
- Misinterpretation of symbolic notation
- Invalid distribution of brackets
- Description Correct answer
- Conjoin error
- Misapplication of rules
- Misuse of the distributive property
- Substitute letter with the value
- Wrong use of signs
- Errors with dealing with comprehending variables
- Errors in with dealing algebraic expression
- Errors in solving the equation
- Errors with dealing with word problem
- Errors in Transition from Arithmetic to Algebra to arithmetic
- Route learning,
- Addict to social media,
- Fall in love,
- No answer

Conclusion

The chapter presented a discussion of the six common errors identified in the study. Related literature has also been used to support the findings of the study. The explanations of the origins of the errors have been related to the existing literature in a way linking them to broader theoretical views. Having discussed the findings of the study, the next chapter provides conclusions and recommendations based on the study's findings:-

Errors with dealing with comprehending variables. Letters represent different meanings in different contexts. When letters are present in algebra entitles, this is a seeming difficulty for students. A letter that represents more than one number or value is called a variable. Understanding letters in a different context is more difficult.

Errors with dealing with an algebraic expression. Letters are used to build up an algebraic expression. Either one letter or a combination of letters could be used in an expression. Therefore, there is a close relationship between understanding the meaning of letters in the context of algebraic expressions. There are many difficulties in expressions like addition, subtraction, closure, distributive property, changing verbal problems into mathematical algebraic statements, and so on.

Errors in solving the equation. When two algebraic expressions combine with an equal sign, it is called an equation. To solve an equation, one most known difficulty was the application of rules of simplifying equations based on given questions. Also use of equal to signs and understanding of the equation solving method were difficult.

Errors with dealing with word problems. Students solving algebraic word problems, and translating the mathematical statement into appropriate algebraic expressions were more difficult for students within assigning a variable, noting constants, and representing relationships among variables. Word problems were related to language, vocabulary, mathematical terminology, and mathematical rules.

Errors in Transition from Arithmetic to Algebra to arithmetic. It means the transition from about a known quantity to thinking about an unknown quantity as they transition from arithmetic thinking to algebraic thinking. So moving from arithmetic to algebraic problem solving is difficult for students. They cannot be clear about an arithmetic strategy to solve algebraic problems.

Recommendations

The results have indicated that learners' errors when simplifying algebraic problems have their root causes. The researcher has learned more from identifying learners' errors and their causes. Any study is fruitless if the findings of the study are not useful for the future. The researcher thinks that the findings of this study are going to benefit the Mathematics Department at a basic level by making teachers aware of the common errors made by learners when learning algebra.

Recommendations for Teaching/Learning

- The findings from this study mainly showed that learners lacked the basics of algebra, and therefore teachers should assist learners to grasp the basics of algebra like collecting like and unlike terms; bracket expansion, addition, and subtraction of directed algebraic terms.
- Knowing the basics of algebra will go a long way in understanding the procedural and conceptual aspects of algebra.
- Teachers should consider the constructivist perspective and be in a position to create a strong arithmetic background for learners so that the arithmetic background could be applied to algebra.
- Teachers are encouraged to use teaching methods that enable learners to gain both procedural and conceptual knowledge.
- The teaching methods should allow learners to give explanations for their answers.
- Teachers should listen carefully to learners' explanations and be able to identify learners' misconceptions and find ways of helping learners to understand algebraic concepts.
- There is a need for teachers to create a classroom environment that allows learners to come up with their conceptions from the procedural and conceptual knowledge taught by the teacher.
- Learners should also be encouraged to share their successes and problems in algebra in a way trying to clear misconceptions.
- At times, learners should receive individual attention to address the issue of individual differences.
- Learners should be given a variety of algebraic problems to simplify.
- Giving learners a variety of algebraic problems, makes learners experience the different ways in which algebraic problems are supposed to be simplified.
- Learners will get used to algebraic manipulation and algebraic representation.

Recommendations for Further Research

- The findings of this study showed that the methods and approaches used by the teacher to teach algebraic concepts affect the way learners grasp the concepts; therefore, the researcher recommends that there is a need to identify the role of the teacher in the errors produced by learners in simplifying algebraic expressions.
- It is good to understand the way the teacher delivers the concepts to the learners.
- This will enable the identification of the teacher's contribution to the commitment of the errors by learners.
- The study also suggests broadening the research by not only concentrating on simplifying algebraic problems but on algebra as a whole.
- This may improve the relevance of the research on the teaching/learning situation.
- The researcher also suggests the use of a bigger sample including participants from several schools.

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Appendices

Appendix-A

Interview Guideline for Head Teacher

Dear Head Teacher,

I am a student of mathematics education at the Central Department of Education, Kirtipur Kathmandu, Nepal. In the partial fulfillment of the requirement for the Degree of Masters in Mathematics Education, I am going to do my research work entitled "Student's errors in learning algebra at a basic level" and the research of this study is to access the

- How do students perceive learning algebra at a basic level?
- What are the reasons behind students' weak in learning algebra at a basic level?
- What are the factors behind students' errors in learning algebra at a basic level?

To fulfill my research study, I need some required data from the school's records related to mathematics. I hope you don't feel any difficulty helping me.

Name:	Interview Date:
Age:	Sex:
Qualification:	Head Teacher From:
School Name:	Type of School:
No. of teachers:	No. of Mathematics Teachers:

The interview with the Head Teacher of the case school was taken under the following points/topics.

- A brief introduction to School
- Mathematics teacher competence and skill
- Mathematics result/output
- Availability of mathematical materials
- Application of mathematics material in teaching
- Responsibility for effective use of materials
- Further planning

Appendix -B

Interview Guideline for Mathematics Teacher

Name: Interview Date:

Age: Sex:

Qualification: Teaching Experience:

School Name:

The interview with the Mathematics Teacher of the case school was taken under the following points/topics.

- Basic knowledge level of student
- Teacher competence and training
- Students' errors in learning algebra
- Condition of mathematical tools and materials
- Use of mathematics materials
- Appropriateness of mathematical material in algebra
- Role of teacher for motivated classroom
- Methods of teaching algebra
- Factors of doing errors in algebra
- Problems/ Challenges in teaching algebra
- Administrative supports for a better outcome
- Comment on mathematics education policy and mathematics practice

Appendix-C

FGD Guideline for Students

Name: Interview Date:

Age: Sex:

School Name: Class:

The interview with the student of the case school was taken under the following points/topics.

- Introduction
- Perception of the mathematics subject
- Perception of the learning algebra
- Mathematics period/class
- Availability of mathematical material
- Use of materials in mathematics class
- Administrative supports for effective mathematics class
- Opportunities and expectations
- Comments and suggestion

Appendix D

To conduct achievement tests to observe the student's errors in learning algebra are on the basis following question.

1. Ram has a total of Rs.15. Ram lent the money Rs.2/2 to 3 people then how much money remains with him?
2. Simplifying: {7x(13-9)÷3}
3. Solve: $x^3=1000$
4. If the big number is tenfold that small number and their product is 1000 then find both numbers?
5. Simplifying: 3x4+2
6. A number in which by adding 21 is 35 then find that number?
7. If the numerator is less by 4 than the denominator. If subtracting 3 on the numerator and denominator both then their ratio will be 3:7 then find the fraction?

Appendix E

Photos