Optimization Models with Exclusive Bus Lanes

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GAURAB CHAND

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RECOMMENDATION

This is to recommend that Mr. **Gaurab Chand** has prepared this thesis entitled "**Optimization Models with Exclusive Bus Lanes**" for the partial fulfillment of the Master in Science (M.Sc.) in Mathematics under our supervision. To our knowledge, this work has not been submitted for any other degree. He has fulfilled all the requirements laid down by the Central Department of Mathematics, Institute of Science and Technology (IOST), Tribhuvan University (TU), Kirtipur for the submission of the thesis for the partial fulfillment of M.Sc. Degree in Mathematics.

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LETTER OF APPROVAL

We certify that the Research Evaluation Committee of the Central Department of Mathematics, Tribhuvan University, Kirtipur approved this research work entitled "**Optimization Models with Exclusive Bus Lanes** " carried out by Mr. **Gaurab Chand** in the scope and generality as a thesis in the partial fulfillment for the requirement of the M.Sc.degree in Mathematics.

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DEDICATION

To

My Beloved wife

Abhisara Singh Thakuri (Pragya)

Along with

My Best Sister

Preeti Chand Thakuri

STUDENT'S DECLARATION

This thesis entitled "**Optimization Models with Exclusive Bus Lanes** ", which has been submitted to the Central Department of Mathematics, Institute of Science and Technology (IOST), Tribhuvan University, Nepal for the partial fulfillment of the Master in Science (M.Sc.) Degree in Mathematics, is a genuine work that I carried out under my supervisor Prof. Dr. Urmila Pyakurel and co-supervisor Asst. Prof. Durga Prasad Khanal and that no sources other than those listed in the references have been used in this work. Moreover, this work has not been published or submitted elsewhere for the requirement of any degree program.

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ABSTRACT

Exclusively reserved lane for public buses in arterial road of the city is called exclusive bus lane (EBL). In this research study, we survey network optimization EBL models, then we review min-max dynamic optimization EBL model with three modes of vehicles. Major upgraded terms on reviewed model have been taken prior origin count of the bus travel time, bureau of public road (BPR) constraint to the car mode and maximum number of motorcycle rider constraint. Among them, BPR constraint has impacted significantly over objective function as well as planning of EBL on the transportation network. Traffic data related to the motorcycle mode had been estimated using statistical tool by increasing the capacity of arcs and without loss of generality with original data of buses and cars. We prefer parallel genetic algorithm (PGA) for the solution of the revised problem and proved that complexity is NP-hard. A numerical example is revealed as a reviewed optimization network model to achieve the feasibility and therefore yield optimal solution.

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LIST OF ACRONYMS AND ABBREVIATIONS

EBL: Exclusive Bus Lane

TSP: Transit Signal Priority

OD: Origin Destination

AFC: Automated Fare Data

GPS: Global Positioning System

BPR: Bureau of Public Road

PCU: Passenger Car Unit

BRT: Bus Rapid Transit

DFS: Depth First Search

PPH: Passenger Per Hour

VPH: Vehicle Per Hour

SD: Standard Deviation

POC: Prior Origin Count

PGA: Parallel Genetic Algorithm

av: Arithmetic Average

et al.: And Others

viz.: Namely

Chapter 1

INTRODUCTION

1.1 Background

In all over the world, urbanization is increasing day by day. That is why, either new cities are formed or existed cities become dense. In many cities, traffic congestion becomes non-terminating problem because of the limited capacity of roads. Many methodologies have been investigated and implemented for the mitigation of urban traffic congestion. Among them, we have chosen exclusive bus lane (EBL) methodology to study and investigate throughout this thesis with following reasons:

1. If the ownership of small vehicles (e.g. motorcycle, car etc.) are increased, then the traffic congestion increases because the road capacity is fixed.

2. Budget to initiate the exclusive bus lane (EBL) is significantly lesser than other mass transportation systems (e.g. mono rail, metro rail, cable car etc.), (Youssef, Alshuwaikhat, & Reza, 2021).

3. Bus transportation is user friendly in terms of the stations and operations.

4. Fares in the bus transportation is comparatively low than other transportation system for the same route.

Definition 1. *Exclusive bus lane (EBL): Exclusive bus lane is transportation planning system for urban cities to ease traffic congestion, whereas lane in the road is reserved exclusively for public buses either at the medium or curb side of the road,(Abdulwahid, 2015; Ho, 2013; Mesbah, Sarvi, & Currie, 2008).*

EBL system can be applied either for the whole day or rush hour only. Further, exclusive bus lane can be separated by either physical barrier or painted line with its

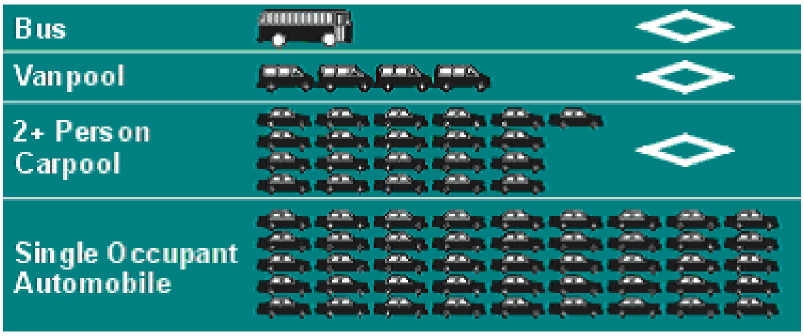

Figure 1.1: Occupying Road Space By Different Modes for 45 Passengers.

adjacent lane. Occupancy of road space capacity can be minimized by using the mass transportation like buses, which is depicted in the Figure 1.1. This figure motivate us to study on EBL and to increase bus transportation system.

Remark 1. *Other mass transportation systems (viz. Priority bus lane, Intermittent/ Dynamic bus lane, High occupancy vehicle lane (HOVL), Bus rapid transit (BRT) and Dedicated bus lane belong to similar class of problem with exclusive bus lane (EBL) but together with distinct objective functions.*

Definition 2. *Origin-destination (OD) Matrix: Let us suppose that the passengers travel from any depart station (origin) to arrival station (destination) within a city. In this circumstances, matrix formed by demand of passengers with origin destination (zone) is called OD matrix of the given city. Classically, for multiple source sink network, origins are kept in the row of matrix and the destinations in the column, (Hussain, Bhaskar, & Chung, 2021), (Mesbah et al., 2008).*

OD matrix is necessity to estimate the passenger demand for the traffic simulation.

Definition 3. *Bus share ratio (Δ): Let, Q_{bus} be the total volume of bus, P_b be an average area occupied by a bus, Q_p be the total passenger demand and F_b be the passenger car equivalent to bus, (Yang, Wang, & Mao, 2020). Bus share ratio in terms of passenger car is defined as following:* $\Delta = \frac{Q_{bus}P_b}{Q_pF_b}$

Definition 4. *Bureau of public road (BPR) function for transportation: BPR function is mostly used for delay time estimating rigoriously of vehicles. Mathematically, this function shows the relationship between travel time and traffic volume, (Treiber, 2016), (Yang et al., 2020), (Mesbah et al., 2008) and (Yang et al., 2020). Let $t(1)$ be travel time for vehicle in particular arc, ν be the flow of vehicles on same arc and $t(0)$ be the free-flow travel time. Then BPR function can be defined as follow:*

$$t(1) = t(0)\left(1 + \alpha\left(\frac{\nu}{u}\right)^{\beta}\right) \tag{1.1}$$

Equation 1.1 called BPR function of transportation.

Here, u is the capacity of discussed arc in terms of vehicle per hour (VPH) i.e. maximum number of vehicle which can pass through. Notice that, α, β and u are not independent variables and there values obtain from estimation by real sample data. Literally, α deals with how road segment travel time increase with small demand flow and β deal how fast converges into capacity.

BPR function for car mode

$$t_k^c(1) = t_k^c(0)\left(1 + \alpha\left(\frac{\nu_c}{u_c}\right)^{\beta}\right) \tag{1.2}$$

BPR function for motorcycle mode

$$t_k^m(1) = t_k^m(0)\left(1 + \alpha\left(\frac{\nu_m}{u_m}\right)^{\beta}\right) \tag{1.3}$$

Equation 1.2 represent relation between travel time and volume of car mode, equation 1.3 reflect relation between travel time and volume of motorcycle mode. Notice that detail meaning of used variables have expressed in Table 3.4 of Chapter-3.

Algorithms and Problems

Applied mathematics is rigorously supported by scientific computation. Fundamental aspects of scientific computation are: 1) algorithms, 2) computational devices and 3) implementation into similar problem. For example, our simple calculator is computational device, whereas instructions are kept fixed to handle the simple arithmetic operations. Overall computing process for rigid solution of any real life problem is depicted in the Figure 1.2:

Definition 5. *Algorithm: An algorithm is sequence of complete instructions to obtain the solution of given problem, (Ahuja, Magnanti, & Orlin, 1993) and (Magnanti & Wong, 1984).*

The lower in efficiency of an algorithm refer to the higher with complexity. In the computational perspective, complexity occurs in terms of space and time taken during solving a problem. Worst-case complexity is used broadly because it is easy to calculate and helpful to comparing other problem. The worst-case complexity of an algorithm refers to maximum number of times for the solution of a given problem.

On the basis of worst-case analysis algorithms are classified as following:

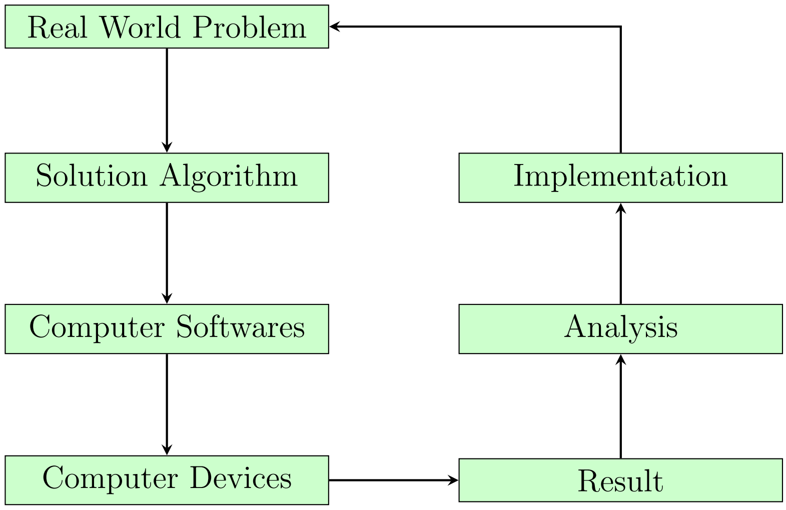

Figure 1.2: Computing Process of Problem Solving

1. Polynomial time algorithm,

2. Pseudo polynomial time algorithm'

3. Exponential time algorithm.

Greedy algorithms, heuristic algorithms and meta-heuristic algorithms belong to the class of exponential time. Hence, expontial time algorithm yield optimizationsolution of given problem, (Ahuja et al., 1993).

Based on complexities, problems are classified as following:

1. **P-class problems** If there exists a polynomial time algorithm (which guaranteed the solution) for well defined problem P_1, then equivalence class of such problem P_1 is known as P-class problems. In addition, P_1 is an element of P-class problems. For example, shortest path problem and maximum flow problem belong to P-class problems, (Ahuja et al., 1993).

2. **NP-class problems** If recognition problem P_1 with only its yes instance should be verified in polynomial time algorithm, then equivalence class of P_1 is known as NP-class problems. Likewise, solution of problem P_1 is guaranteed by non-deterministic polynomial time algorithm rather than polynomial time algorithm. Hamilton cycle problem is an example belonging to this class, (Ahuja et al., 1993).

3. **Class NP-complete problems** If recognition problem P_1 is an element of NP-class and all other problems in NP-class polynomial transfer to P_1, then equivalence class of such problem P_1 is known as class of NP-complete problems. In addition,

this is refined subset of NP-class problems. Hamilton cycle problem is an example of this class, (Ahuja et al., 1993).

4. **Class NP-hard problems** For a given recognition problem P_0, if the rest of problems in NP class are polynomially reduce to P_0, then we say that problem P_0 is NP-hard. An equivalent class of such problem is called NP-hard class problem. In addition, this class include the problems of the class NP and its complement. For example, 0-1 Knapsack Problem is well known NP-hard problem, (Ahuja et al., 1993) and (Wu, Chu, & Che, 2015) and the bus lane reservation problem (BLRP) is also NP-hard problem, (Wu, Chu, Che, & Shi, 2014).

If an algorithm guarantees the solution of a problem in polynomial time, then it is assumed to be good algorithm from user perspective. However, three basic mathematical strategies for measuring the performance of algorithms are discussed as follows, (Ahuja et al., 1993):

1. **Empirical Analysis** This analysis studies the deviation of the solution of an algorithm with real world solution. In addition, small deviation imply good performance by the algorithm.

2. **Average-case Analysis** This analysis determines the average steps taken by an algorithm in order to guarantee solution of a problem.

3. **Worst-case Analysis** This analysis determines the maximum possible number of steps that might be take by an algorithm in order to guarantee the solution of the given problem. In addition, it is expressed in terms of big "O" notation.

Remark 2. *The polynomial time algorithms have higher efficiency than the algorithms of other classes, which imply that they have lower in complexities. That is why, other type of algorithms are reduced to the polynomial algorithms for quicker solution, (Ahuja et al., 1993).*

1.2 Problem Statement of Thesis

Still today, many highly populated cities either have without bus lane or having no any urban mass transportation system. In such cities, passenger may suffer by regular congestion as a daily basis. Especially under-developed and developing countries may not have sufficient budget to initiate infrastructure for mass transportation system (e.g. metro rail, mono rail, bus rapid transit (BRT) etc.). In this scenario, exclusive bus lane (EBL) may be an optimal solution to handle the urban traffic congestion.To review the network optimization design of EBL model including bus share ratio is statement problem of thesis in general.

1.3 Research Objectives

1. To study and investigate various existing EBL models and algorithms.
2. To purpose new type of optimization EBL model and check its efficiency.
3. To familiarize simulation technique in the field of EBL.

1.4 Significance of the Purposed Research

1. It should be beneficial for urban transportation planner by network optimization idea.
2. It aware reader about traffic jam.
3. It might be helpful to investigate optimal solution for transportation network problem.

1.5 Organization of Thesis

In *Chapter-1* ***(Introduction)***, the discussion on the preliminary definitions of the terms related to this research topic like EBL, OD matrix, BPR function of transportation, bus share ratio, algorithm and complexity are included. Moreover, it also includes the justification of a question - why EBL is more necessity than other means of public transportation?. Together with the problem statement of thesis, objectives, and significance of the thesis. *Chapter-2* ***(Literature Review)*** includes the review of ideas presented in research articles related to the topic. After reviewing some articles research gap is drawn.

Chapter-3 ***(Methodology)*** is divided into three sections- problem formulation, optimization of EBL dynamic bi-level model and bus share ratio. This chapter incorporates the central idea of this thesis and demonstrates how we estimate the bus share ratio analytically. We introduce a variable $t_a^{b,POC}$ to capture the real world scenario. *Chapter-4* ***(Solution Technique)*** discusses on a suitable heuristic algorithm, which is taken from (Mesbah, Sarvi, Ouveysi, & Currie, 2011). We include the complexity of problem as NP-hard in this section. In addition, numerical example is reviewed according to revised model by the help of some statistical tools. Finally, interpretation of numerical example is done. Lastly, *Chapter-5* ***Conclusion and Future Work*** presents the conclusion of this thesis including limitation and possible future work.

Chapter 2

LITERATURE REVIEW

2.1 Review of EBL Related Articles

For the sake of simplicity, perspective of EBL model categorized by (Mesbah et al., 2011) is shown in Figure 2.1 herein.

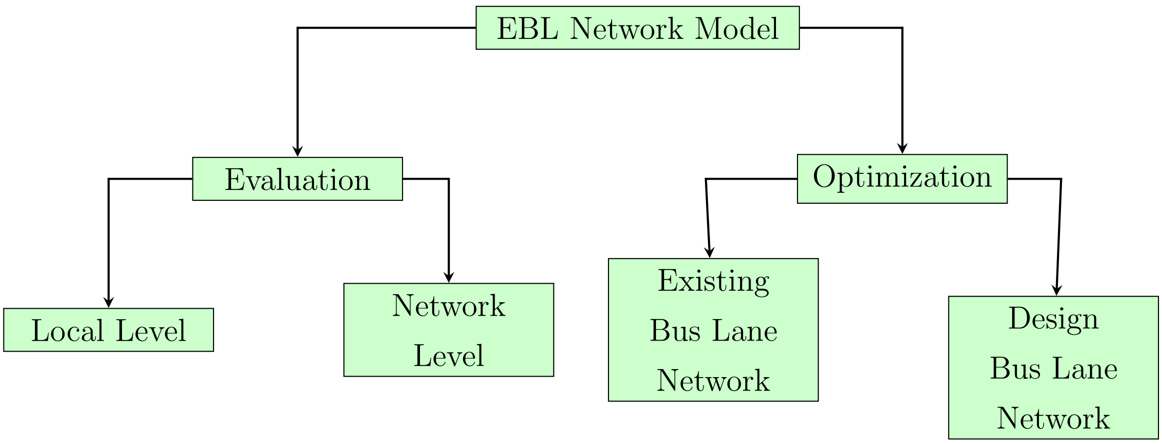

Figure 2.1: Classification of EBL Ideas

Algorithm to solve the non-convex and non-linear network optimization problem is presented in (Nguyen, 1974). EBL planning is wide range problem because of the variation on demand by passengers like safety, punctuality travel time and security of passengers. Comprehensive review of transportation planning with models and algorithms are presented in (Magnanti & Wong, 1984). To reduce the movement of private vehicles in urban cities, qualitative enhancement of the public transportation is very essential. One of the strategies to enhance the bus passengers is to implement the exclusive bus lane (EBL) system. Exclusive bus lane (EBL) system is transportation system, where separate lane is reserved for public buses or mass passenger vehicles. EBL system is implemented more than hundred cities around the world for the mitigation propose of urban traffic congestion (Ho, 2013). At the beginning, EBL project was run experimentally by engineers through arterial road of city. Later on, it was collaborated with other disciplines such as OD matrix estimation, traffic simulation, accuracy of travel time, comfortable of bus service and security of passenger. Is EBL system need for current situation of urban network? If yes, how should it give best optimal solution? These are some of the necessity queries for transportation planner. The simulation method is one the techniques to estimate the future possibility of city with EBL project without investing huge amount of budget and is fundamental aspect to research in transportation network optimization. The network optimization bi-level model with EBL is presented in (Mesbah et al., 2008). The generalized model by incorporating budgetary constraint, waiting time of bus passenger and a numerical example for medium sized network can be found in (Mesbah et al., 2011). Parallel genetic algorithm (PGA) is presented in (Mesbah et al., 2011) to find optimal EBL combination in the transportation network. Numerical example presented in (Mesbah et al., 2008) is used in this thesis with estimated motorcycle parameters. Construction of new EBL roads in the transportation network is dealt by the model in (Mesbah et al., 2011) together with the budgetary constraints. In both of the papers, an objective is to minimize total passenger travel time.

The separately reserved bus lane will reduce the capacity of adjoining non-bus lane simultaneously in same network. That is why, it is difficult task to find optimal result for the bus and non-bus simultaneously, (Ho, 2013) and (Wu et al., 2014). In reality, objective function of the buses and non-buses are contrary in nature. An optimal solution for bi-objective bi-level network optimization model was presented in the thesis of (Ho, 2013), where upper level is analytic framework and lower level is simulation technique. In addition, this optimization problem is proved as NP-hard problem and Pareto solution is discussed in his thesis. Moreover, presented model is not only optimal EBL combination but also the scheduling of vehicles as well. An objective function of (Ho, 2013) is more sensitive due to the bi-objective model in comparison to the models of (Mesbah et al., 2008) and (Mesbah et al., 2011), and is expressed alternatively on the models of (Wu et al., 2014), (Wu et al., 2015) and (Wu, 2018). Models with single objective function are presented in (Wu et al., 2014) and (Mesbah et al., 2008). The objective functions in (Wu et al., 2014), (Wu et al., 2015) and (Wu, 2018) are to minimize negative traffic impact (i.e. time delay on non-bus vehicles after EBL run in the network system). Furthermore, time window constraint, minimum volume of bus to run EBL, budgetary constraint are including in their models. In addition, the minimization of negative uncertain impact (non-bus vehicle type, accident, weather) is an objective function of (Wu, 2018). Minimization of non-bus vehicle travel time is not discussed in (Mesbah et al., 2008) and (Mesbah et al., 2011). As similar to (Magnanti & Wong, 1984), models present in (Wu et al., 2014), (Wu et al., 2015) and (Wu, 2018) are non-linear and mixed-integer. (Wu et al., 2014) proposed a cut-and-solve algorithm and claimed that the proposed algorithm is more efficient then optimization software CPLEX 12.4. During solving process, mixed-integer and non-linear transportation network converted into integer and linear form as shown in (Wu et al., 2015). Models in (Wu et al., 2014), (Wu et al., 2015) and (Wu, 2018) are feasible and yield effective optimal solution by CPLEX software over randomly generated instances .

If network optimization problem gives result in finite polynomial steps, then such network is called polynomial class (P-class) problem. Problem is said to non-deterministic polynomial class (NP-class) problem, if it gives the solution in non-deterministic steps, (Aujha Ravindra K., 1993). The complexity of algorithms in (Mesbah et al., 2008) and (Wu et al., 2014) are belongs to class of NP-hard. Technique to show NP-hardness of given model is shown in (Wu et al., 2014) by reducing its proper sub-problem into 0-1 Knapsack problem. Drawback of cut-and-solve technique is that it is invalid for large scale problem, (Wu et al., 2015).

The EBL project run in the arterial roads does not imply that the traffic congestion is solved forever. Thus the evaluation of EBL is essential frequently in certain period of time. Multi-objective network optimization model is presented in (Sun & Wu, 2017), which is an analogy with models in (Ho, 2013; Mesbah et al., 2011). Upper level model in (Sun & Wu, 2017) is to minimize the total travel time, pollution emission and operating cost, where as the lower level model is to minimize the individual total travel costs. Comprehensive evaluation of EBL policies are discussed in (Ji, 2020), where three ways of evaluations: flow of traffic, vehicle emission and air pollution are discussed. Transportation model in (Sun & Wu, 2017) incorported with OD matrix and BPR function, which is further solved by NSGA II in MATLAB environment together with commercial software GAMS. Meanwhile, microscopic simulators (VERSIT+, SUMO and GRAL) are used in (Ji, 2020). As a result, if we comapare evaluations techniques, then they are mainly differ in terms of simulation software.

Most of the papers argue that the volume of car traffic is reduced wherever EBL run in arterial road of the city. The ratio of shifting of the volume of car traffic (due to EBL run) to volume of total vehicle traffic is called mode shift effect of car. New variety of statistical and simulation way to convert mixed traffic lane into bus lane is presented in (Yang et al., 2020) with simulation technique used for optimal bus share ratio. Nonetheless, this model is not network optimization model like (Mesbah et al., 2008) and (Wu et al., 2015). Mode share is assumed variable in (Mesbah et al., 2008), (Mesbah et al., 2011) and (Yang et al., 2020). Local level transportation technique to evaluate EBL is presented in (Ma, Yuan, Van Oort, & Hoogendoorn, 2020) and (Youssef et al., 2021), whereas sample data is taken from Delf of Netherland in (Ma et al., 2020) and from Riyad Saudi Arbia in (Youssef et al., 2021). Travel demand (passenger per hour) as well as bus share ratio are two key parameter to convert general purposed lane into bus lane. In addition, bus share ratio value satisfy minimum margin between travel time of bus and car for given route. That is why, identification bus share ratio is major concern, (Yang et al., 2020). EBL system is always integrated with other transportation factors (signal priority, passenger demand, road condition, pollution and vehicle type etc.). Without including traffic signal priority in bus lanes, optimal solution of EBL is hard, (Ji, 2020). By using GPS data and smart card, OD matrix is estimated for Suzhou city of China in which they claimed the accuracy of 90% in result with OD matrix simulation, (Huang et al., 2020). Traffic signal priority (TSP) model is presented in (Bagherian, Mesbah, Ferreira, Charles, & Khalilikhah, 2015), which is helpful for decision makers to find optimal signal priority of TSP. Their objective function is similar to (Mesbah et al., 2008). In addition, (Tu, Sano, & Nishiuchi, 2014) discussed the bus signal priority for arterial roads. Proper signal priority for bus lane integrating with EBL system can reduce the average traffic delay times is the conclusion of (Ji, 2020), (Bagherian et al., 2015) and (Tu et al., 2014).

(Hussain et al., 2021) obtained the OD matrix by using automated fare data (AFD), data cleansing and simulation of zone level. The effect in transit time of non-EBL lanes can be obtained by BPR function after the EBL implementation. BPR function always gives the estimated value by some road parameters, (Spiess, 1990) and (Tan, Yang, & Zhang, 2017). In most of the articles cited herein, feasibility and optimality of the solution procedures are presented by help of simulation tools and argue that without proper guidelines of EBL, it is impossible to meet goal of transportation authorities. Moreover, field of traffic signal priority, OD matrix, BPR function of transportation, and air pollution are beyond the limitation of our target in this thesis.

2.2 Research Gap

1. What will happen if we implement three mode of vehicles (e.g. bus, car and motorcycle) in models of (Mesbah et al., 2008) and (Yang et al., 2020)?

2. How are BPR travel time constraint for car and maximum number of motorcycle passengers constraint fit, and what is an impact made by prior origin count (POC) to the bus travel time in the model of (Mesbah et al., 2008)?

Chapter 3

METHODOLOGY

In this chapter, we present the mathematical notations with input variables and explain the model with EBL that is included in this thesis.

3.1 Problem Formulation

Suppose that $G = (N, A)$ be transportation network, where N is the collection of intersection of roads (zones), A is the collection of road segments (arcs) which connects any two adjacent zones. Let O and D be the set of origin and destination zones, respectively. Let K be the set of routes which connect any origin zone to destination zone $od \in OD : o \in O$ & $d \in D$. Detail variables are presented in Tables 3.1, 3.2, 3.3, and 3.4.

Assumptions for transportation model is shown follows:

1. OD matrix estimation is given and keep fixed during analysis period.
2. Only three mode of transportation (i.e. bus, car and motorcycle) allow but exclusive lane is provided for bus only.
3. Cost function, road network layout, arc characteristic, bus routes, frequency and start-stop junctions for buses are already known.
4. There is enough space to construct EBL (building lane) for the given arc in the network, which is prescribed for the EBL.

Table 3.1: Variable Used as Input Parameters

Parameters	Meaning
$G = (N, A)$	: directed transportation network.
N	: Collection of zones (i.e. specific point area in the bus route), $n \in N$.
A	: Arcs/ Collection of road segment which connect two adjacent zones, $a \in A$.
OD	: Set of origin-destination(OD) zone pair, $od \in OD$.
K	: set of all routes which connect all given OD pair, $k \in K$.
R_k	: Collection of all arcs of k routes, such that $\exists a \in A\ \forall a \in R_k$ and $R_k \subset A$.
$x_a^{b/c/m}$	: Passenger flow on arc a for bus/car/motorcycle respectively, $\forall a \in A$.
$t_a^{c/b/m}$	: Travel time on arc a for car/bus/motorcycle respectively, $\forall a \in A$.
$t_a^{b,POC}$	: Travel time of buses with prior origin count (POC) along particular route k in order to cover passenger waiting time, $\forall a \in R_k$. Otherwise this equal to usual travel time of bus.
z_a	: Construction cost of new EBL on arc $a \in R_k$.
ϵ_{bdg}	: Maximum available budget to run EBL project in the network.
$\Gamma_a^{c,bpr}$	: Increment of BPR time over base case time on same arc a, $a \in R_k$ (in terms of percent).
Γ_{fix}	: Maximum upper limit delay to car $\forall a \in R_k$ (in terms of percent).
$f_a^{c/m}$	: vehicle flow on arc a of car/motorcycle respectively for given OD pair.

Table 3.2: Decision Variable Used in Model

ϕ_a	: 1 if arc a of road has exclusive bus lane (EBL), 0 otherwise $\forall a \in R_k$.
λ_a	: 1 if arc a is processed, 0 otherwise $\forall a \in R_k$.

Table 3.3: Variable Used in this Mode Share

Q	: Total traffic volume for given route k in terms of passenger car unit (PCU).
$Q_{bus/car/motorcycle}$	: Traffic volume of bus/car/motorcycle for arc a in terms of passenger car unit (PCU).
Δ	: Percentage of passenger choosing bus.
F_b	: Passenger car equivalent to bus in terms of area cover on the road.
F_m	: Passenger car equivalent to motorcycle in terms of area covered on the road.
Q_p	: Total passenger demand on given route k (in terms of PPH).
$P_{b/c/m}$	: Average area occupancy by bus/car/motorcycle respectively.
Δ_1	: Percentage of passenger choosing car.
Δ_2	: Percentage of passenger choosing motorcycle.

Table 3.4: Variable Used in BPR Functions

$t_a^{c/m}(1)$	: Average travel time for car/motorcycle in arc $a \in R_k$.
$t_a^{c/m}(0)$	: Free flow travel time for car/motorcycle in arc $a \in R_k$.
$\nu_{c/m}$	: Traffic volume of passenger in arc $a \in R_k$ for car/motorcycle.
$u_{c/m}$	: Total capacity of road in arc $a \in R_k$ for car/motorcycle.
α, β	: Constant coefficients of BPR functions.

Interpretation of variable ($t_a^{b,POC}$)

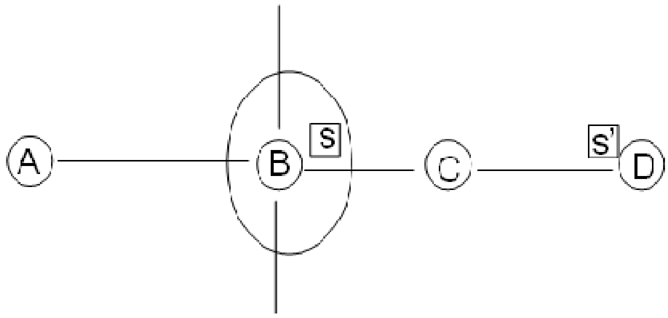

Figure 3.1: Visualization of Video Sensor Zone

Suppose that a passenger has to be departed from station s to station s' through route B-C-D, whose network section is presented in Figure 3.1. Let, the public bus has a route from zone-A for the station s'. An ellipse around zone-B refers the coverage area by video sensor. We add extra time for those passengers who were departed from station s. This extra time is equal to time spent by bus within video sensor zone-B. We call it as prior origin count of bus travel time and denoted by $t_a^{b,POC}$ for given arc a. Main objective to introduce this variable is to count the average passenger waiting time together with bus travel time.

If only few buses are available for many passengers, then flaw occur to value of $t_a^{b,POC}$. This type of flaw can be removed by solving scheduling problem. Furthermore, the collection of waiting time is tedious job as well as might not be free from flaw. The value of $t_a^{b,POC}$ is equal to usual bus travel time, if there is no delay by bus on video sensor zone. It is meaningless to count waiting passenger time for car and motorcycle modes.

3.2 Revised Bi-Level EBL Optimization Planning Model

This formulation is based on Down-Thomson paradox of (Yang et al., 2020), which states that if we promote private vehicles (car, motorcycle etc.) than public buses could lead higher transport cost.

Upper Level Model

This model is revised over the transportation model of (Mesbah et al., 2008).

$$MinZ = \sum_{a \in R_k} \left[x_a^b t_a^{b,POC} + x_a^c t_a^c + x_a^m t_a^m \right] \lambda_a \ \forall k \in K \tag{3.1}$$

Subject to:

$$\sum_{a \in R_k} z_a \phi_a \leq \epsilon_{bdg} \tag{3.2}$$

$$\phi_a = \begin{cases} 0 \text{ for arc a is without EBL.} \forall a \in R_k, \forall k \in K \\ 1 \text{ for arc a is with EBL.} \forall a \in R_k, \forall k \in K \end{cases} \tag{3.3}$$

$$\phi_a \leq \lambda_a, \forall a \in A \tag{3.4}$$

$$x_a^c \geq 0, \forall a \in R_k, \forall k \in K \tag{3.5}$$

$$x_a^m \geq 0, \forall a \in R_k, \forall k \in K \tag{3.6}$$

$$x_a^b \geq 0, \forall a \in R_k, \forall k \in K \tag{3.7}$$

Conditions to satisfy the time variables are as follow:

$$t_a^{b,POC} = t_a^c = t_a^m \ for \ \phi_a = 0, \forall a \in R_k \tag{3.8}$$

$$t_a^{b,POC}, t_a^c, t_a^m \text{ each unique value for } \phi_a = 1, \forall a \in R_k \tag{3.9}$$

$$t_a^c = \begin{cases} t_a^c \text{ If } \forall \phi_a = 0, \forall a \in R_k, \forall k \in K \\ t_a^c(1) \text{ If } \forall \phi_a = 1, \forall a \in R_k, \forall k \in K. \end{cases} \tag{3.10}$$

$$t_a^m = \begin{cases} t_a^m \text{ If } \forall \phi_a = 0, \forall a \in R_k, \forall k \in K \\ t_a^m(1) \text{ If } \forall \phi_a = 1, \forall a \in R_k, \forall k \in K. \end{cases} \tag{3.11}$$

Equation 3.1 entails the minimization of total passenger travel time including bus, car and motorcycle modes. An Equation 3.3 reflects that either arc a is EBL or not and constraint Equation 3.2 is budget constraint that is available to apply new EBL project. Constraint 3.4 means we must choose λ_a before ϕ_a for each a. In addition, Constraints from 3.5 to 3.7 represent the non-negativity of passengers flow, 3.8 and 3.9 are conditions for time count and Equations 3.10 and 3.11 are conditions for time taken to mixed or exclusive bus lane.

Lower Level Models

Bus share ratio:

Optimal bus share ratio is an alternative way to convert general purposed lane into bus lane, (Yang et al., 2020). In addition, the detailed finding process of optimal bus share ratio is the simulation technique. During the process of simulation, transportation demand data, volume of vehicle etc. are real data taken from transportation network. If only three modes of vehicles are present in the network, then total traffic volume in terms of passenger car unit (PCU) is depicted in Equation 3.12:

$$Q = Q_{bus} + Q_{car} + Q_{motorcycle} \tag{3.12}$$

whereas,

$$Q_{bus} = \frac{Q_p \Delta F_b}{P_b} \tag{3.13}$$

$$Q_{car} = \frac{Q_p \Delta_1}{P_c} \tag{3.14}$$

$$Q_{motorcycle} = \frac{Q_p \Delta_2 F_m}{P_m} \tag{3.15}$$

and

$$\Delta + \Delta_1 + \Delta_2 = 1 \tag{3.16}$$

Equation 3.12 is total traffic volume for three modes (bus, car and motorcycle). Equations 3.13, 3.14 and 3.15 entail the relationship between volume and mode share for bus, car and motorcycle, respectively. Equation 3.16 is the property of whole passenger choice ratio.

Instead of mode share model of (Mesbah et al., 2008), we use bus share ratio because it calculates the mode share value as well as optimal bus share ratio. After calculating optimal bus share ratio, which yield mode shift effect to convert general traffic lane into bus lane (Yang et al., 2020). After that, next step of lower level model is depicted as follows:

Car and Motorcycle Assignment Model

$$MaxZ_a^{(c+m)} = \sum_{a \in R_k} (f_a^c + f_a^m)\phi_a, \forall k \in K \tag{3.17}$$

Subject to:

$$\sum_{a \in R_k} \Gamma_a^{c,bpr} \leq \Gamma_{fix}^c, \forall k \in K \tag{3.18}$$

$$\sum_{a \in R_k} x_a^c \leq 3 \sum_{a \in R_k} f_a^c \tag{3.19}$$

$$\sum_{a \in R_k} x_a^m \leq 2 \sum_{a \in R_k} f_a^m, \forall k \in K. \tag{3.20}$$

$$f_a^c \geq 0, \forall a \in R_k, \forall k \in K \tag{3.21}$$

$$f_a^m \geq 0, \forall a \in R_k \text{ and } \forall k \in K. \tag{3.22}$$

where,

$$\Gamma_a^{c,bpr} = \frac{t_a^c(1) - t_a^c}{t_a^c} 100\% \tag{3.23}$$

Equation 3.17 represents the objective function that is to maximize the flow of car and motorcycle vehicle in the EBL implemented arc of network. Constraint in 3.18 guarantees the upper bound of BPR estimation for car travel time due to EBL implementation. The transportation authority has the right to select the value of Γ_{fix}^c. Constraint in 3.19 presents a worst case scenario for all car passengers, whereas variables from Equation 3.21 are the non-negativity of the flow of car passenger on an arc a. Constraint 3.20 indicates the worst case scenario in which at most 2 passengers (including rider) in each motorcycle are allowed for the traffic rule for safety, which is apparently new too.

In the form of chart, upper level model is depicted in Figure 3.2 and the whole methodology is depicted in Figure 3.3.

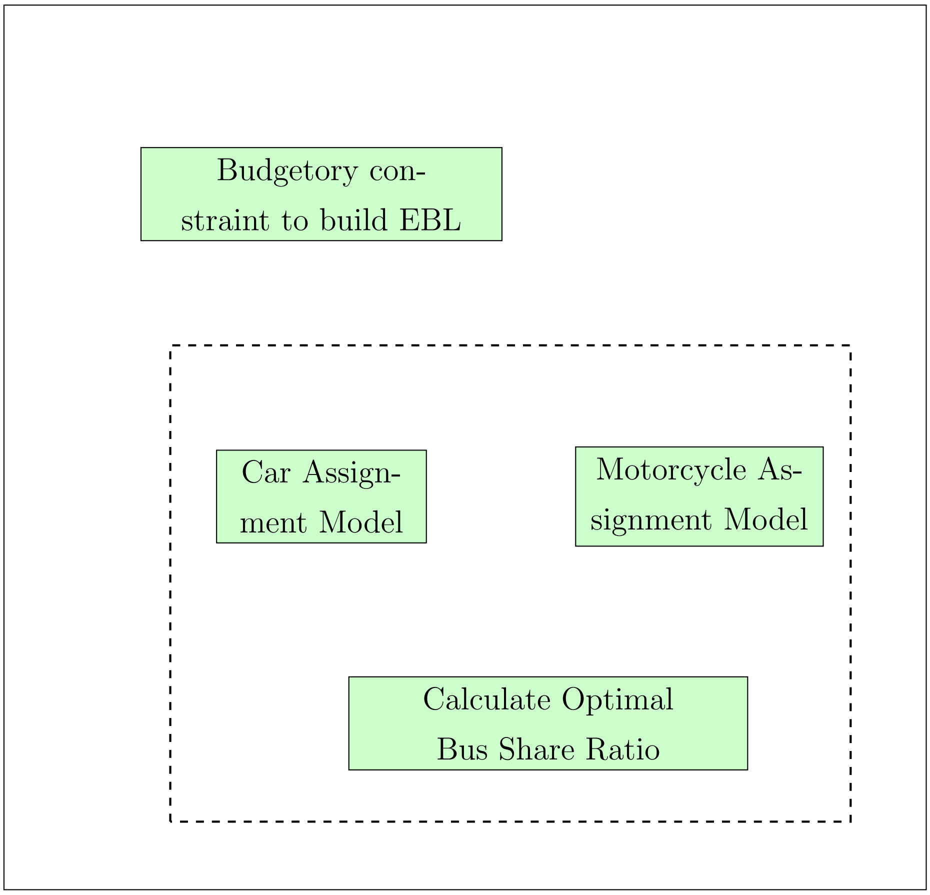

Figure 3.2: Outline of Bi-level Model

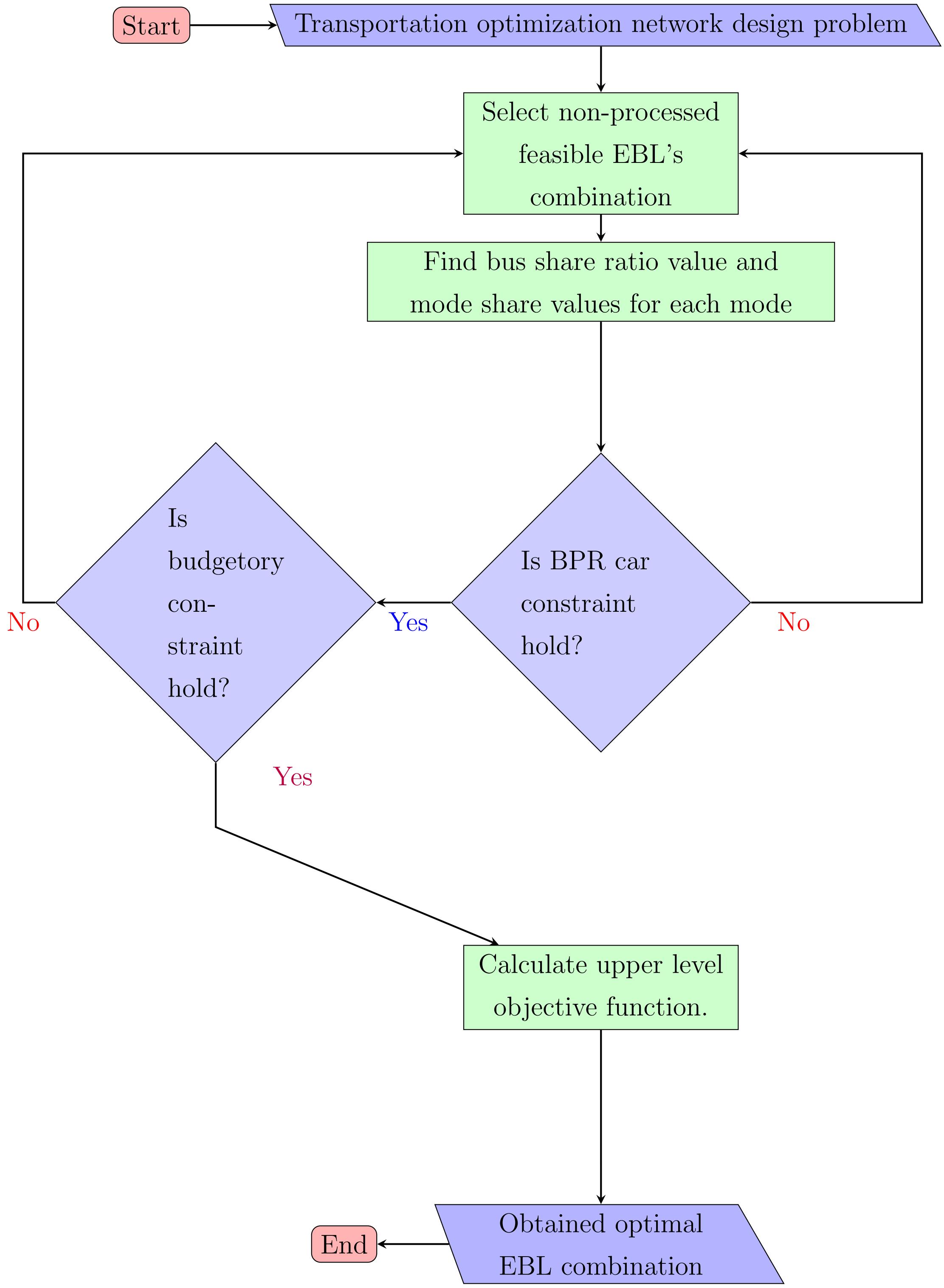

Figure 3.3: Outline of Purposed Methodology

Chapter 4

Solution Technique

4.1 Solution Strategy for Revised Bi-level EBL Optimization Planning Problem

For the large scale transportation optimization problems, single processor should not be suffice to find solution. For example, if we consider a medium sized network with 100 arcs, then worst case possible combination of EBL are 2^{100}. Let us assume that 99% EBL combinations is been reduced due to infeasible. Then, $\frac{2^{100}}{100}$ combinations are still remain to compute, which is still huge number of remaining feasible cases. That is why, parallel processors of powerful computers needed, despite they might take minutes instead of hours to evaluation optimal EBL combinations. In this section, we discussed parallel genetic algorithm from (Mesbah et al., 2011) due to applicability over reviewed bi-level network optimization planning problem.

Parallel genetic algorithm (PGA) is upgraded version of genetic algorithm, (Mesbah et al., 2011). In the PGA, feasible solutions have been produced, which satisfy budgetary constraint of upper level model. Bus share ratio of lower level model is determined by the technique of (Yang et al., 2020). In addition, mode share ratio is obtained during calculation of bus share ratio. The PGA processing is described as follows. Assume that $n^{'}$ processors run in parallel with unity core on multiple computers. In generation-0, $n^{'}$ feasible solutions (chromosomes) are randomly chosen, then optimization solution is calculated. This optimal solution acts as reference point to converge non-processing chromosomes for next generation-1 and so on. After overall processing, it either evaluates all feasible solutions or evaluate maximum possible feasible solutions. Hence, optimal solution comes through above process is optimal for given network.

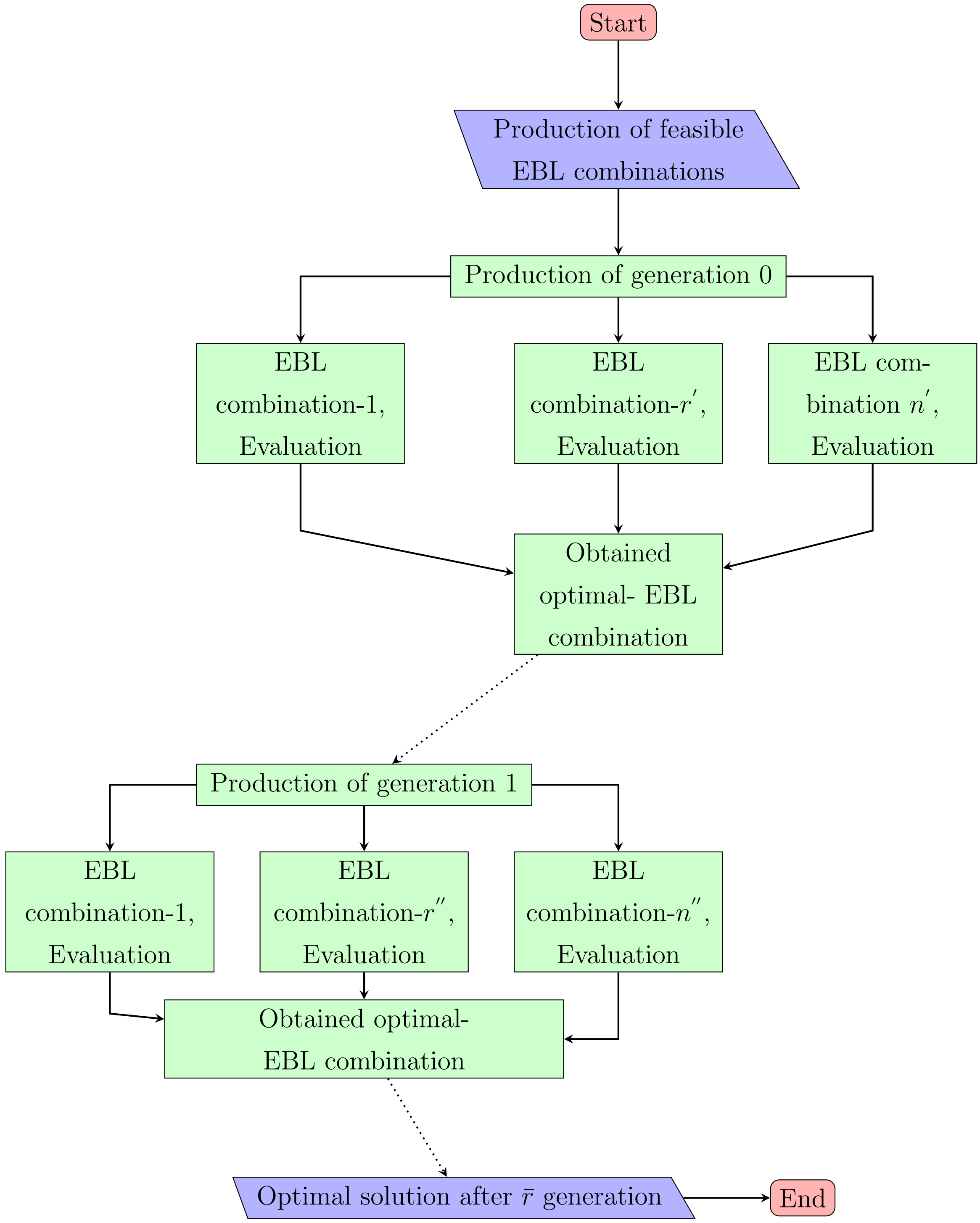

Figure 4.1: Procedure of Parallel Genetic Algorithm (PGA)

Remark 3. *Mode shift effect has different algorithm and simulation technique based on inference data.*

4.2 Complexity of Revised Bi-level EBL Optimization Planning Problem

Although our reviewed bi-level network optimization problem is not totally new problem, some of the constraint and a variable are apparently new like BPR constraint for lower level model and POC travel time variable. We show the NP hardness of our problem by the help of 0-1 Knapsack problem, (Wu et al., 2014).

Theorem 1. *Revised bi-level EBL optimization planning problem is NP-hard.*

Proof. Firstly, we have to construct proper subset of our purposed model. Suppose that only one route (i.e. $|K| = 1$) is selected for EBL and budget availability ϵ_{bdg} is enough to implement all EBL combinations on given single route. The upper level model is,

$MinZ^1 = \sum_{i=1}^{|R_1|} \left(x_{a_i}^b t_{a_i}^{b,POC} + x_{a_i}^c t_{a_i}^c + x_{a_i}^m t_{a_i}^m \right) \lambda_{a_i}, \forall a_i \in R_1$

s.t.

$\sum_{i=1}^{|R_1|} z_{a_i} \phi_{a_i} \leq \epsilon_{bdg}, \forall a_i \in R_1$

$\phi_{a_i} \leq \lambda_{a_i}, \forall a_i \in R_1.$

Relaxation state of above equation as follow:

$\sum_{i=1}^{|R_1|} z_{a_i} \lambda_{a_i} \leq \epsilon_{bdg}, \forall a_i \in R_1$

$\forall \lambda_{a_i} = \{0, 1\}, \forall a_i \in R_1.$

Without loss of generality with condition of lower level model, we assumed that this proper subset of model also satisfy the lower level mode share and assignments model, where $|R_1|$ is the total number of arcs in given route.

0-1 Knapsack Problem (KP) is defined as follows: For given n_1 commodities to pack in a knapsack, the objective is to maximize the profit for each commodity $i = 1, \ldots, n_1$, associated profit p_i, volume w_i and total capacity of knapsack is C_p. Mathematically, integer linear programming form of 0-1 Knapsack problem is:

$MaxKP = \sum_{i=1}^{n_1} p_i \gamma_i$

s.t.

$\sum_{i=1}^{n_1} w_i \gamma_i \leq C_p,$

$\gamma_i \in \{0, 1\}, i = 1, 2, 3, ..., n_1$

Now, comparing with above two models, n_1, w_i, C_p and p_i are corresponding to $|R_1|$, z_{a_i}, ϵ_{bdg} and $\left(x_{a_1}^b t_{a_1}^{b,POC} + x_{a_1}^c t_{a_1}^1 + x_{a_1}^m t_{a_1}^m \right)$, respectively. Parameter γ_i is replace by $1 - \lambda_{a_i}$ to achieve corresponding duality. The $0-1$ Knapsack Problem is NP-hard, (Wu et al., 2014), (Martello, Pisinger, & Toth, 2000) and (Gu, Nemhauser, & Savelsbergh, 1999). Hence revised bi-level EBL optimization planning problem is recognition problem and proved as polynomial reduced to $0 - 1$ Knapsack Problem. This refers as revised bi-level EBL optimization planning problem is NP-hard. □

4.3 A Numerical Example

Consider an example of transportation network as shown in Figure 4.2 that includes six zones/nodes, seven directed arcs and three bus routes (R1, R2 and R3). Origin zones are zone-1 and zone-5. Similarly, destination zones are zone-2 and zone-6. Flows from source zone-1 are sent to the sink zone-2 and from source zone-5 to the sink zone-6. Following terms are modified to the example of (Mesbah et al., 2008) with following reasons shown in Table 4.1:

Table 4.1: Review of Original Data and Reasons

Parameters	Reasons
1. $OD = \begin{bmatrix} q_{12} & q_{16} \\ q_{52} & q_{56} \end{bmatrix} = \begin{bmatrix} 6000 & 0 \\ 0 & 5500 \end{bmatrix}$	1. To incorporate motorcycle passengers. 2. To meet the condition of optimal solution in (Yang et al., 2020).
1. Free-flow speed of motorcycle is equal to free-flow speed of car in respective arcs. 2. Average motorcycle occupancy= 1 passenger per vehicle(PPV). 3. Motorcycle capacity =20 vehicle per hour(VPH) $\forall a$. 4. Numerical example satisfy mode share and bus share ratio. 5. BPR constraint for car and maximum number motorcycle constraint must be satisfy.	Without loss of generality with original network.
Average car occupancy= 2 passenger per vehicle(PPV).	To closure to real world observation.

Likewise, remaining parameters $\alpha = 0.2, \beta = 4$, car capacity for each arc $a = 1600$ VPH, average bus occupancy= 40 PPH and total capacity of each arc $a = 2400$ VPH are kept fixed with original example of (Mesbah et al., 2008). Further, consider time to prior origin count of bus for zone-1 is 1 minute and rest of zones (excluding sink zones) are zero minutes (detail of this concept is described in the chapter-3), which restricted bus passenger waiting time. For the sake of simplicity to numerical example, it is supposed that available arcs to build EBL's are (1,2), (3,4) and (4,2) for this example.

(Problem.) Calculate an optimal EBL combination for the Figure 4.2 with conditions: (a) Maximum budget available to construct at most two EBL's (lanes), (b) total BPR increment for car travel time Γ^{c}_{fix} (due to implemented EBL) are taken- 30%, 40%, 50%, respectively.

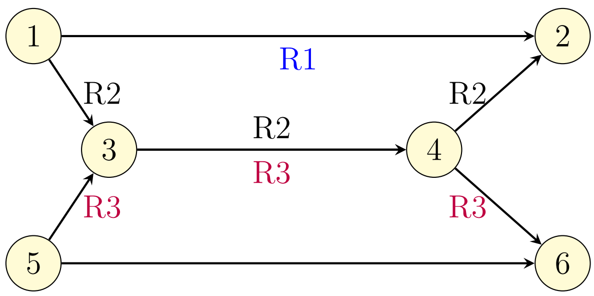

Figure 4.2: Example of Network Layout

Solution:

Case-1 Without EBL

Calculation of numerical example is done from data of (Mesbah et al., 2008) by using statistical tool of (Sukubhattu, 2013) with MS EXCEL-2010 software.

Background of motorcycle data(PPH/VPH) estimation:

To calculate, correlation coefficient between bus and car with data taken from (Mesbah et al., 2008), after using formula 4.4 is shown below. For this purpose, covarience is calculated using Table 4.2, standard deviation for car and bus are calculated using Tables 4.3 and 4.4, respectively is shown follows:

Table 4.2: Required Data for Covariance of Car and Bus (PPH):

a	x^c_a	x^b_a	$\bar{x^c}$	$\bar{x^b}$	$d1 = x^c_a - \bar{x^c}$	$d2 = x^b_a - \bar{x^b}$	$d1 * d2$
(1,2)	4582	869	3765	648	817.7	220.8	180552.576
(1,3)	3216	232	3765	648	-548.28	-416.2	228194.13
(3,4)	5768	1223	3765	648	2003.72	574.8	1151738.256
(4,2)	3216	232	3765	648	-548.2	-416.2	228194.13
(5,3)	2552	991	3765	648	-1212.28	342.8	-415569.58
(4,6)	2552	991	3765	648	-1212.28	342.8	-415569.58
(5,6)	4466	0	3765	648	701.7	-648	-454854.9

Formula for covariance of (Sukubhattu, 2013) for population data as follow:

$$COV(X^c_a, X^b_a) = \frac{\sum_{i=1}^{|A|}(x^c_{a,i} - \bar{x^c})(x^b_{a,i} - \bar{x^b})}{|A|} \quad (4.1)$$

Therefore, using Formula 4.1 yield $Covariance = 71812.14$ passenger square.

Table 4.3: Standard Deviation (SD) of Car (PPH)

a	x^c_a	$\bar{x^c_a}$	$d1 = x^c - \bar{x^c}$	$d1^2$	SD	SD% w.r.t. $\sum_{i=1}^{\vert A\vert} x^c_{a,1}$
(1,2)	4582	3765	817.5	668666	1115.8	4.234
(1,3)	3216	3765	-548	300610		
(3,4)	5768	3765	2003	4014893.8		
(4,2)	3216	3765	-548	300610		
(5,3)	2552	3765	-1212	1469622.8		
(4,6)	2552	3765	-1212	1469622.8		
(5,6)	4466	3765	701	492410.9		

Standard deviation(SD) from mean as follows:

$$SD(X^c) = \sqrt{\frac{\sum_{i=1}^{|A|}(x^c_a - \bar{x^c_a})^2}{|A|}} \quad (4.2)$$

Therefore, $SD(X^c) = 1115.8$ passengers.

Table 4.4: Standard Deviation (SD) of Bus (PPH)

a	x_a^b	$\bar{x_a^b}$	$d1 = x^b - \bar{x^b}$	$d1^2$
(1,2)	869	648	8220.8	48752.6
(1,3)	232	648	-416	173222.4
(3,4)	1223	648	574	330395
(4,2)	232	648	-416.2	173222
(5,3)	991	648	343	117511.8
(4,6)	991	648	343	117511.8
(5,6)	0	648	-648.2	420163

Similar way, standard deviation for bus passenger from mean as follow:

$$SD(X^b) = \sqrt{\frac{\sum_{i=1}^{|A|}(x_a^b - \bar{x_a^b})^2}{|A|}} \tag{4.3}$$

By solving, $SD(X^b) = 444$ passengers.

Now, correlation coefficient between car and bus (PPH) can be derive using the formula 4.4 below:

$$Correlation(X^c, X^b) = \frac{COV(X^c, X^b)}{(|A|)SD(X^c)SD(X^b)} \tag{4.4}$$

Hence, from this formula coefficient of correlation yield as 0.020.

Correlation between x_a^c(PPH) and x_a^b(PPH)) is positive **(i.e.** 0.020**)**. Supposed that correlation between x_a^c and x_a^m is positive and it's value equal to 0.020. Assumed if $\sum_{i=1}^{7} x_{a,i}^m = 98$, then $\bar{x_a^m} = 14$ is mean. Now, using SD of x_a^c (in percent with respect to $\sum_{i=1}^{|A|}$) as estimator for x_a^m. Standard deviation of x_a^c is $SD^c = 1116$ passenger (from Table 4.3), which is approximately 4% of total x_a^c and we assumed the same approximately 4% for motorcycle as well. Consequently, inference of x_a^m yield on Table 4.5 as shown below:

Table 4.5: Estimation Motorcycle data PPH $\forall a$

(Total motorcycles = 98 PPH on the Network)		
Ascending x_a^c	arc a	Estimated Motorcycle (PPH)
2552	(5,3)	6
2552	(4,6)	6
3216	(1,3)	10
3216	(4,2)	10
4466	(5,6)	18
4582	(1,2)	22
5768	(3,4)	26

As a whole, total data for base case network (without EBL network) is shown in Table 4.6 below, which used Tables of 4.2 and 4.5. After that objective function is calculated using Equation 3.1 is depicted in Table 4.7:

Table 4.6: Table for Base Case Network Data

a	u(VPH)	free-flow t_0(min)	x_a^c	x_a^b	x_a^m	$t_a^{b,POC} = t_a^c = t_a^m$(min)
(1,2)	2400	9	4582	869	22	10.617
(1,3)	2400	3	3216	232	10	3.123
(3,4)	2400	3	5768	1223	26	4.357
(4,2)	2400	3	3216	232	10	3.125
(5,3)	2400	3	2552	991	6	3.051
(4,6)	2400	3	2552	991	6	3.051
(5,6)	2400	9	4466	0	18	10.395

Table 4.7: Base Case Optimization Calculation

a	$x_a^c t_a^c$	$x_a^b t_a^{b,POC}$	$x_a^m t_a^m$
(1,2)	48624.27	10090.84	233.46
(1,3)	10052	957.530	31.25
(3,4)	25103.312	5322.058	113.373
(4,2)	10052.398	725.894	31.128
(5,3)	7790.171	3025.101	18
(4,6)	7792.238	3025.904	18
(5,6)	46414.570	0	187.512

Now, calculation of network is possible by using these estimated motorcycle parameters. Network optimization function 3.1 is calculated by the help of Table 4.6 and Table 4.7 through Microsoft Excel, which is obtained as 133006 passenger minute.

Case-2 With EBL

It is given that possible arcs for EBL project are $(1,2),(3,4),(4,2)$. Here, $2^3 = 8$ possible combination of EBL's in network are as follows.

1. ϕ
2. $(1,2)$
3. $(3,4)$
4. $(4,2)$
5. $(1,2)$ and $(3,4)$
6. $(1,2)$ and $(4,2)$
7. $(3,4)$ and$(4,2)$
8. $(1,2)$, $(3,4)$ and $(4,2)$

In this numerical example, we skip calculation of bus share ratio because of unavailability of required data for e.g. Q_p, Δ and P_b etc. Although, lower level model begins by calculating bus share ratio. In this situation, we assume that the bus share ratio is satisfied for this numerical example, calculation of which is possible by simulation from (Yang et al., 2020). According to the model, bus routes should be given. Otherwise, depth first search (DFS) algorithm can be applied to find the routes. An option (1) is discarded because it is same with base case (without EBL) or trivial. So rest of options (2) – (8) are survived.

Increment of BPR Car Travel Time (%)

Percent increment of average BPR car travel time is presented in Table 4.8. Calculation details is shown in Table 4.10.

Table 4.8: BPR Increment Percent of Car Over Base Case Travel Time

a	free-flow travel time	$t_a^{b,POC} = t_a^m = t_a^c$	BPR time for car	Increment (%)
(1,2)	9	10.61	16.56	35.94
av EBL(3,4)	3	4.35	9.33	53.37
av EBL(4,2)	3	3.12	3.61	13.46
av EBL's(1,2), (3,4)	6	7.48	10.81	44.65
av EBL's(1,2),(4,2)	6	6.87	8.56	24.68
av EBL's(3,4), (4,2)	3	3.735	4.97	33.41
av EBL's (1,2), (3,4), (4,2)	5	6.02	9.83	63.3

Transportation planner has right to select Γ_a^c in order to maximize lower level model value. Due to careful using of secondary data, we supposed that the average passenger per car is considered as 2 and that of motorcycle is 1 both of which are within the bound of the constraints in 3.19 and 3.20 of the model. Calculation of lower level model at different Γ_a^c as shown in Table 4.9 below: and

Table 4.9: Lower Level Objective Function Calculation

Γ_a^c	EBL implementation on	Value of $Z_a^{(c+m)}$ (VPH)	Maximum EBL on
$\Gamma_a^c = 30\%$	a. (4,2)	1618	
	b.(1,2) and (4,2)	1960	(1,2) and (4,2)
$\Gamma_a^c = 40\%$	a.(3,4) and (4,2)	2264	
	b. (1,2)	2313	(1,2)
$\Gamma_a^c = 50\%$	a.(1,2) and (3,4)	2635.5	(1,2) and (3,4)

Budget Constraint

Due to budget constraint, Option (8) is discarded, coincidentally this is already eliminated from BPR increment constraint. So, EBL on the arcs (1,2), (3,4), and (4,2) are not possible.

We calculate increased travel time of car due to BPR implementation for each arc of network using formula 1.2. After that, we convert car travel time in terms of base case travel time (in percent), which is depicted in Table 4.10. Similar procedure is done for motorcycle mode by using BPR formula 1.3, which is shown in Table 4.11. For this numerical example, number of passengers are kept fixed as mentioned above.

Table 4.10: BPR Increment Percent of Car Over Base Case Travel Time

a	free-flow travel time	$t_a^{b,POC} = t_a^m = t_a^c$	BPR time for car	Increment (%)	$df = t_a^c(1) - t_a^c$	$t_a^c - df$
(1,2)	9	10.61	16.56	35.94	5.95	4.65
(1,3)	3	3.12	3.61	13.46	0.48	2.63
(3,4)	3	4.35	9.33	53.37	4.98	-0.62
(4,2)	3	3.12	3.61	13.46	0.48	2.63
(5,3)	3	3.05	3.24	5.85	0.18	2.86
(4,6)	3	3.05	3.24	5.85	0.18	2.86
(5,6)	9	10.39	15.82	34.34	5.43	4.95

Table 4.11: BPR Travel Time Estimate for Motorcycle for Arc a

a	free-flow travel time	$t_a^{b,POC} = t_a^m = t_a^c$	$t_a^m(1)$	$df' = t_a^m(1) - t_a^m$	$t_a^m - df'$
(1,2)	9	10.61	11.63	1.023	9.58
(1,3)	3	3.12	3.03	-0.08	3.21
(3,4)	3	4.35	4.71	0.36	3.99
(4,2)	3	3.12	3.03	-0.088	3.21
(5,3)	3	3.05	3	-0.047	3.1
(4,6)	3	3.05	3.0	-0.047	3.1
(5,6)	9	10.39	10.18	-0.21	10.6

Value of Objective Function in Different EBL

Three EBL solutions are survived using lower level model calculation is shown in Table 4.9. They are listed below:

1. (1,2) and (4,2)
2. (1,2)
3. (1,2) and (3,4)

So, these possible solutions act as input for upper level model. Among them, minimum one will be an optimal solution using Formula 3.1. We calculate each of three possible solutions in Tables 4.12, 4.13 and 4.14, respectively. In each calculations, Tables 4.10 and 4.11 are used simultaneously for the corresponding arc. These, are shown accordingly as follows:

Table 4.12: Objective Function Calculation with EBL on (1,2) and (4,2)

a	$x_a^c t_a^c$	$x_a^b t_a^{b,POC}$	$x_a^m t_a^m$
(1,2)	**21341**	10090.84	**102.4**
(1,3)	10052	957.530	31.25
(3,4)	25103	5322.058	113
(4,2)	**8488.2**	725.894	**32.13**
(5,3)	7791	3025.101	18
(4,6)	7791	3025.904	18
(5,6)	-	-	-

Thus, using formula of upper level objective function yield 104137 passenger minutes with EBL implementation in (1,2) and (4,2).

Table 4.13: Objective Function Calculation with EBL on (1,2)

a	$x_a^c t_a^c$	$x_a^b t_a^{b,POC}$	$x_a^m t_a^m$
(1,2)	**21341**	10090.84	**102.4**
(1,3)	10052	957.530	31.25
(3,4)	25103	5322.058	113
(4,2)	10052.398	725.894	31
(5,3)	7791	3025.101	18
(4,6)	7791	3025.904	18
(5,6)	-	-	-

Therefore, upper level objective function for EBL $(1, 2)$ yield 105699.9 passenger minutes.

Table 4.14: Objective Function Calculation with EBL on (1,2) and (3,4)

a	$x_a^c t_a^c$	$x_a^b t_a^{b,POC}$	$x_a^m t_a^m$
(1,2)	**21341**	10090.84	**102.4**
(1,3)	10052	957.530	31.25
(4,2)	10052.398	725.894	31
(3,4)	**-2026**	5322.058	**40**
(4,6)	7791	3025.904	18
(5,6)	-	-	-

Thus, upper level objective function yield 92091 passenger minutes with EBL implementation in (1,2) and (3,4).

Thus, final optimal solutions with EBL's implemented is depicted in Table 4.15, which is used Tables 4.7, 4.13, 4.12 and 4.14 is shown below:

Table 4.15: Objective Function with EBL Cases

Cases	Objective Function	Difference from Base Case %
Base	133005	0
EBL's(1,2) and (4,2)	104136.6	-21.7%
EBL (1,2)	105699	-20.5%
EBL's (1,2) and (3,4)	92092	-30.7%

4.4 Result and Discussion of Numerical Example

Above numerical example refers to use EBL on arcs $(1, 2)$ and $(3, 4)$ because upper level objective function is minimum value 92092 passenger minutes. The value of objective function of purposed model occurs when the product of passengers and travel time is minimum. Value of Γ_a^c of lower level is selected with careful investigation. Furthermore, our key goal was calculating numerical example of (Mesbah et al., 2008) with additional parameter of motorcycle mode. On other hand, we took number of motorcycles is 98 which is less in compare to other modes of vehicle. Hence, it does not effect significantly on optimal solution of given example network.

Remark 4. *Value for arc* $(3, 4)$ *to column of* $x_a^c t_a^c$ *of Table 4.14 is negative, which indicates that the total time of car obtained by BPR is less than that of the given time, although value of passengers is positive.*

Chapter 5

CONCLUSION AND FUTURE WORK

Traffic congestion is becoming a hard issue in many cities because of the unplanned urbanization. One of the solutions for traffic congestion is implemention of EBL on some route within the city. Transportation plan with optimal EBL combination is outline of this thesis. We revised bi-level optimization planning model with objective of minimizing passenger travel time is based on models from (Mesbah et al., 2008, 2011) and (Yang et al., 2020). Purposed model is revised in terms of prior origin count of bus travel time, three modes of vehicles, maximization of number of cars and motorcycles at the lower level model, BPR constraint for car and the bus share ratio (instead of mode choice model in (Mesbah et al., 2008)). Revised bi-level EBL prolem is proved as NP-hard class problem. Parallel genetic algorithm is discussed of (Mesbah et al., 2011) because of applicability for this problem. By the help of numerical example, it yields an efficient solution for small scale problem. In the same way, extension of this numerical example is a nice remaining task for us.

Limitation in the numerical example is that only small number of motorcycle passenger should be assumed. Being of NP-hard type problem, small sized transportation network (let's say number of arcs = 15 and from which we choose 3 EBL combinations) is tedious task to show numerically. Simulation technique must be needed for real data or large data. In this numerical example, calculation of bus share ratio was not possible due to unknown value of some data (e.g. Δ and Q_p etc.). Many terms related with BPR function of transportation are left for future studies. To fit purposed model of EBL into simulation software (e.g. CPLEX and VISSIM) is fascinating future work also.

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