Chapter I

INTRODUCTION

This research wasfocused on conceptual and procedural difficulties in learning derivatives. Thus this chapter includes background of the study, statement of the problem, objective of the study, significance of the study, delimitation of the study and definition of the key words.

Background of the Study

Recognizing the procedure based curricula,National Council of Teachers of Mathematics(NCTM, 1989) recommended that the teaching of mathematics emphasizes the conceptual understanding, multiple represents and connection, mathematical modeling and problem solving (as cited in, 28th ICTM). Teaching by multiple representation of content, relational teaching may help to make mathematics as a popular subject(Hana shatila, 28th ICTCM). In the context of Nepal, if teaching mathematics emphasizes the conceptual understanding, connect with real life problem; multiple representation of content may help the mathematics teaching learning effective.

There are so many national and international organizations which have focused to deeper and conceptual understanding of mathematics and the popularization of mathematics is one of the key issues of today. The effective teaching and learning is the main slogan of today. Calculus is seen as difficult subject because of the weak presentation and understanding (Park, 2012).So, we must address the conceptual difficulties of our students to achieve the goals of the education. The derivative concept is one of the key concepts in calculus. Among

other things the derivatives measures the steepness of a function,slope of a tangent line to a curve at a given point, the rate of change of output relative to input, and helps in finding critical points of a graph. The derivative can also use as a tool to model the behavior of changing quantities such as population dynamics, finding velocity & acceleration of moving object & others. Therefore having a solid understanding of derivative is important.

Teaching and learning derivative is not a easy task. According to ORHUN(2012),students were not successful in analyzing derivative function .They have lack in conceptual understanding .This case may be the result of the traditional teaching methods .In the study the students haven't interpreted the graph of the derivative function. The students find it difficult to make connections between the graph of derived function and the original function(ORHUN, 2012). Most of the research showed that traditional teaching approach cannot give full conceptual understanding of the derivative. According to Sahin and et al (2015), traditional approach have very weak conceptual understanding. Also researches showed that there are so many teaching approaches such as computer based instruction, multiple representation approach, active learning approach etc. which are very effective in teaching and learning derivative.

Various researches show that derivative is very important topic.The students feel the calculus is one of the difficult subjects (Tall 1992). There are limited research in the topic of conceptual and procedural difficulties in learning derivative in the context of high school of Nepal although the maximum number of students feel it difficult .The derivative is field of my interest and I want to explore what the derivative is and what are the difficulties in learning it ? Also in my

experience every of my collegues feel it difficult too so I choose this topic for study.

In the context of Nepal the topic of derivative is introduced from the high school and it is widely introduced in the field of engineering, economics, science etc. But many students feel that the derivative is difficult concept and they always remember the formulae and the process of findingderivative. Even though the students can correctly solve the differentiation problems, they may not actually make the sense of what the concept of derivative truly mean. Thus in these contexts my studyis focused to explore the conceptual and the procedural difficulties in learning derivatives in high school students.

Statement of the Problem

The derivative is known as complex concepts for students to understand because it contains many other concepts - ratio, limit, and function, and it can be represented in various ways - the slopeof the tangent line, an instantaneous rate of change, and an expression using Leibniz's notation. Despite its complexity, sound understanding of the derivative is crucial to understanding advanced topics such as integral, Mean Value Theorem, and Fundamental Theorem of Calculus (Park 2012). From my informal talking of some calculus teacher and students the derivative is seen as a difficult subject in the high school of Nepal. Most of the student got a very few achievement in the case of derivative .The students just memorize the formulae and tried to solve the differential problems(Tarmizi, 2010). As the experience of former college tutor, I noticed that the students did not understand what the derivative of a function meant. Especially they felt difficult while asking the application problem like find the slopes of the tangent at a given

point of a function?So,teaching and learning derivativesbecome one of the rigor& difficult task in our context for both teacher and students. Because of lack of conceptual understanding of derivative the students feel it so much difficult in higher education too.Effective teaching in calculus course should be informal, intuitive and conceptually based on graphs and functions in order to improve the quality of learning and develop the understanding of calculus concepts (ORHUN, 2012). So, the today's need is the research in the different aspect of learning derivative should be conduct to improvise our teaching and learning culture. From that we can make the effective learning and can achieve the goals of education of our nation. Because of these problem the great deal of time, money, effort and manpower of the nation have been wasted .Thus the deeper understanding of the content is very important. The conceptual and procedural understanding of the students in learning derivative is become a great issues in our classroom and there are limited research in such issues in the context of Nepal. In such a context the conceptual and procedural difficulties in learning derivative is the problem of my study.

Objective of the Study

The main objectives of this study were given below:

- To explore the difficulties of students in conceptual understanding of derivative
- To explore the difficulties of students in procedural understanding of derivative

Research Questions

This study attempted to seek the answer of the following questions:

- How does student understand the meaning of derivative as a rate of change and a slope of a tangent at a given point?
- How does student experience in finding derivative using first principle?
- What are the difficulties of the students in finding derivative using formula?

Significance of the Study

Each and every research hasits own significance. This research wasinvestigating the both conceptual and procedural understanding of the students in learning difficulties in derivative. This study not only helps to learn about difficulty but also helps in improvising our teaching so that we can reduce the difficulties in learning derivative and make the effective learning. Now a days derivative becomes a difficult to learn and in such circumstances my research will be a fruitful and supportive document to teach math in productive way. My study helps to all mathematics teachers who taught in the high school calculus courses. From my study they could be aware about the student's difficulties of learning derivatives and they could improvise their teaching and learning. Also this study helps to those who are interested to research in the field of derivative. In short the following were the significance of my study to stakeholders:

- This research helps to do self- reflection for students so that they could improve their achievements.

- This research helps to teachers for making their class effective & productive.
- This research helps various stakeholders to guide in curriculum planning, textbookwriting, making teaching strategy etc.
- This research provides as a literature for those who want to study on the topic derivative.

Delimitation of the Study

Delimitation is the boundaries created by the researcher. The delimitations are those characteristics that limit the scope and define the boundaries of the study. They are created before any investigations are carried out in order to reduce the amount of time or effort spent in certain unnecessary, perhaps even unrelated, areas to the overall study. The main delimitations of this study were as follows:

- The study was limited to only one high school of Makwanpur District.
- The study was bounded on derivative at grade XI.
- The study was qualitative and descriptive in nature so the findings of the study are not generalized.
- The entire respondents were taken on the basis of researcher's purpose and convenience with non-probability sampling.
- This study was only limited with student's conceptual and procedural difficulties in learning derivative. It does not concern with other factors like home environment, teaching learning methodology etc.

Operational Definition of Terminologies

The operational definition and terminology clarify the meaning of the terms using in the study. Define terms is necessary if individuals outside the field of study may not understand and that go beyond common language (Locke, Spirduso& Silverman, 2007, as cited in Creswell,2007).It gives the operational meaning of variables of the study. The following were the operational definition of some terms which are used on this study:

Conceptual Understanding.conceptual understanding is when a learner is able to comprehend mathematical concepts, operations, and relations.It is the interrelationship between the basic elements within a larger structure that enables them to function together is the conceptual knowledge or understanding.Conceptual knowledge is a network of associations of mathematical procedures, integrated with the mathematical principles. It is the key of understanding the larger structure.Theconceptual understanding isthe meaning of derivative, as a rate of change and as a slope of a tangent line at given point in the curve and meaning of limit too.

Procedural Understanding.procedural understanding is knowledge of procedures: knowledge of how and when to use them appropriately, and the skills in performing them flexibly, accurately, and efficiently. Themethods of inquiry and criteria for using skills, algorithms, techniques and methodsis called the procedural knowledge. It is the understanding of the how the algorithms, formulae can be used in solving the problem.The understanding of process of finding derivative using formula is the procedural understanding.

Difficulties.the difficulties are thehinders which affect the learning clear picture of some concept. The causes and factors that hinders student to understand the concept of derivative and process of finding derivative is the difficulty.

Difficulties in Conceptual Understanding. The difficulties in the conceptual understandingare the factors that hinders in learning the basic and fundamental elements in the larger structure of the content. The difficulties in conceptual understanding means the factor and elements that always hinder in learning meaning of, derivative at a rate of change, derivative as a slope of the tangent and the meaning of limit.

Difficulties in Procedural Understanding. The difficulties in the procedural understanding is the factors and hinders that affect in learning the proper use of algorithms, formulas, rules of solving problem etc. So, the weaknesses or difficulties in finding derivative from first principle,using power rule and chain rule are the difficulties in procedural understanding.

Chapter II

REVIEW OF RELATED LITERATURE

Besides selecting a quantitative qualitative or mixed methods approach the proposal designer also needs to review the literature about a topic. This literature review helps to determine whether the topic is worth studying and it provides insight into ways in which the researcher can limit the scope to a needed area of inquiry (Cresswell, 2009). This chapter is devoted to the discussion of the previous research relevant to my study. The purpose of this literature review is to examine the existing research about how student perceive about derivative .This research also investigate the difficulties in the conceptual understanding about derivative. There have been limited studies about the main concept of the derivative as the slopes of a tangent at a point of a curve and derivatives as a rate of change.The literature consists four parts they are i) Derivative misconception and ii) conceptual and procedural knowledge iii) multiplerepresentation iv)learning difficulties. They are given below:

Derivative and Misconception

The misconception is the wrong concept of any subject whichhindrance the learning. The misconception plays a vital role in reduce the conceptual understanding. Misconception can be defined as the perception (conception) that is far from the consensus of the experts' perceptions for a specific subject (Zembat, 2010). Mistake can be considered as errors in learning while misconceptions can be considered as factors that block the learning (Keçeli, 2007; Ubuz, 1999, as cited in Kaplan, 2015). From the above definition the misconception is the wrong concept of any subject which hindrance the learning and misguided the learning.

According to Park(2012), the word "derivative" is colloquially used for both "derivative at a point" and "derivative of a function," which may confuse students about whether the word "derivative" refers to (a) a point-specific value or (b) a function. Student's misconception about derivative is also related to their thinking about the tangent line on graphical situation(Park, 2012). Another misconceptions is that a tangent line to the curve should intersect the curve only one at a tangency point and students tried to find the equation of tangent line of a curve .They did not make the connection between the equation and the graph of tangent line because it intersects the curve at other point also besides the tangency point when they extended the line (Kim,2005 ;Park 2007 ;as cited in Park 2012). In the context of Nepal the student have very weak geometrical concept of derivative. From my experience of teaching the high school students are memorize the derivative formula and they feel difficult while asking application problem like finding the slope of the tangent at a given point. So the graphical understanding is very important for conceptual understanding of derivative.

The study of Kaplan and et al. (2015), under the topic "Relieving misconceptions of derivative with derive" showed that students are unable to use the operation with the definition of the derivative .They did not consider the interval while finding the derivative .Students experience difficulty in doing the geometric representation of the derivative .The misconception rooted from the thinking derivative of a function as the derivative at a specific point .They have misconception rooted from not knowing the symbolic representation ,graphical representation and difficulty to construct a relation between the slopes ,tangent and normal. In their experiments the Derive software is more useful to overcome the misconceptions instead of traditional teaching method. Students feel the difficulty

in finding derivative from first principle. If students tried to understand the average rate of change and instantaneous rate of change and connect it to the slope of a tangent then they would become clear in the understanding of derivative concept. In the above study the causes of difficulty are not mentioned.

In the study of calculus has seen a difficult subject among the mathematics students and often they misunderstood the notion of function. A function y= f(x) is a process: input x carryout the procedure, output y. Students have some misconception in function so they need special treatment such as further tutorial session in correcting their misconceptions and they are fully confusing in Problem solving strategy according to George Polya(Tarmizi, 2010). Function is the one of the key concept for the derivative. So the understanding of function is the important in understanding derivative.This study also does not mention what are difficulties and how we can reduce them.

As similarly the study of Tyne (2016), explores that students have some wrong concept about slope. Students' misunderstandings of linear functions in general, and directly proportional relationships specifically (for which the slope is a ratio-of-totals), lead to an impoverished understanding of slope. Even with the slope interpretation questions, students performed poorly, and those who performed poorly on the slope interpretation question were likely to carry their misunderstandings on to their derivative interpretation (and later to the critiquing questions too) (Tyne, 2016). His/her research shows the students who used the ratio-of-totals approach for slope interpretation often went on to interpret the derivative similarly. A ratio-of-totals approach to slope interpretation was the dominant incorrect reasoning. Students do not have a full understanding of what slope and derivative mean as a rate of change in the context of modeling situations,

nor do they understand appropriate uses of slope and derivative to make predictions.

The research of Tyne (2016),also indicates that students struggle with knowing what the slope and derivative represent and how to use them appropriately to make predictions. The dominant incorrect reasoning by students (one-third of surveyed students and two-thirds of interviewed students) was to think of slope as the ratio-of-totals $\left(\frac{y}{x}\right)$ instead of the ratio-of-differences ($\Delta y/\Delta x$). Thinking of slope as a ratio-of-totals implies that all linear relationships are directly proportional (of the form $y=mx$, with a y-intercept of zero); students went on to interpret the slope as something that can be used to calculate the value of the dependent variable (by multiplying it by the value of the independent variable). This incorrect thinking about slope influences students' understanding of the derivative. As a result, they often interpreted the derivative as something that could be used to find the value of the dependent variable (by multiplying the derivative by the value of the independent variable). This led to the incorrect relationship, $f(x)=f'(x)*x$.

Furthermore, when students were asked to critique the reasoning of a hypothetical person's predictions, they showed little knowledge of how the derivative can be used to make valid predictions. Instead of demonstrating understanding that the derivative can because to estimate change only near the input value, 54% of interviewed students said once again that they could use the derivative to calculate the total value ($f(x)=f'(x)*x$). Hence the slope is the another key concept for understanding derivative. The ratio of change in y and change in x is the slope of the chord in the concept of derivative. Students'

impoverished views of slope are adversely impacting their ability to understand the more advanced related topic of derivative. In the study of Tyne the specific conceptual and procedural difficulties are not mentioned.

The derivative, once calculated, is the instantaneous rate of change at a given point on a curve. The derivative definition is giving students the idea that a derivative is a limit, or a form of a limit that is false. In learning calculus and doing calculations, most students just memorize the process or they do not understand on a conceptual level what they have found. In a study conducted by Tarmizi (2010), Tarmizi found that students did not fully understand function notation, memorized a procedure, and could not verbalize what they were doing in the problem. Even though this study was done with a simple function question, it still can be related to derivatives. Students memorize the tricks to findderivatives; however, students often cannot verbalize the meaning behind what they calculated. The students think sometime that x tends to 2 means exactly two which is a very wrong concept .The meaning of instantaneous is very important to understanding derivative. So the concept of limit may be one of the causes which affect the understanding of derivative. The study of Tarmizi explores the specific misconception which is not mention in other literature.

Similarly,In the study done by Constantinou (2014) showed that Students do not fully understand the concept of a derivate and have trouble correctly answering derivative application questions. Teachers with a strong understanding of their students' knowledge can tailor their lessons to accommodate students who do not fully understand the concept of a derivative in a first-year calculus course. Students will use the concept of a derivate in multiple classes through college depending on their major. It is crucial for them to easily understand the concept of

a derivative and be able to apply it. Teachers should review conceptual questions that are missed by students in order to help students understand derivatives at a higher reasoning level. In this study the specific misconception and difficulties are not mention. The teacher plays an important role to reduce the difficulty in learning derivative.

Conceptual and ProceduralKnowledge

Conceptual and procedural knowledge is the one of the key aspect of learning derivative. Various researches says that the procedure knowledge is the knowledge of doing a process or solving a problem while the conceptual knowledge is the deeper understanding of the content which makes help in procedural knowledge. Procedural knowledge is defined by Heibert and Carpenter (1992) as a "sequence of actions"or skills (p. 78). Solving routine problems is performed with minimal understanding of the mathematical principles involved (Heibert & Carpenter, 1992). Conceptual knowledge is a network of knowledge where the connections are meaningful (Heibert & Carpenter, 1992). Conceptual knowledge is a network of associations of mathematical procedures, integrated with the mathematical principles (Heibert & Carpenter, 1992,as cited in Abbey, 2008).Thus the procedural knowledge is the knowledge which helps to finding answer of the problems and the conceptual knowledge is the deeper understanding of the subject matter.

The study of ORHUN (2012), investigate that the students were not successful in analyzing derivative function .They have lack in conceptual understanding .This case could be the result of the traditional teaching methods .In the study the students haven't interpreted the graph of the derivative function. The students find it difficult to make connections between the graph of derived function

and the original function. Usually, they were to interpret the graph of derived function as the graph of function. The students did not use the mathematical language to describe the graph of derived function Effective teaching in calculus course should be informal, intuitive and conceptually based on graphs and functions in order to improve the quality of learning and develop the understanding of calculus concepts. Teaching derivative by using ICT tool helps the better conceptual understanding of the student than that of traditional method. So teaching method also played a vital role to reduce difficulties of the students. In this context the research of(FatihOcal, 2017) on the topic the effect of Geogebra on students conceptual and procedural knowledge find out that the instruction with Geogebra had positive effect on students' scores regarding conceptual knowledge and their overall scores. There is significant impact on student's achievement in regarding their conceptual knowledge. On the other hand, the result shows that there was no significant difference between experimental and control group students' scores regarding procedural knowledge. It could be concluded that students in both groups were focused on procedural knowledge to be successful in learning calculus subjects including applications of derivative in both groups. On the other hand, instruction with Geogebra supported student's learning these subjects meaningfully and conceptually. In the above study the difficulties of the students in conceptual and procedural understanding are not mentions specifically.

The concept of derivative should introduce and developed in relation to the rate of change, slope of tangent line and limit. Because unless the mathematical concept is understood relationally,student's compartmentalize the big ideas related to the concept in their conceptual system & cannot relate them each other (Zulal Sahin, 2015).This study also found that if any one of the big ideas (rate of change,

slope of tangent, limit) is ignored ,the concept of derivative may not be fully understood relationally. The three big ideas in derivative (rateof change, slope of tangent and limit) are connected to each other so learning by connecting all three makes derivative understanding very clear.

The case study done bySelloMakgakga(2012), in class 12 founded that the student performed better in the finding derivative of a function by using rules of differentiation than using first principles. Students have weak performance in finding derivative from first principles.Learners lack both conceptual and procedural knowledge of calculus, learners themselves claim that some of the difficulties are caused by teacher's approach of just wanting them to memorize formula's without explaining the meaning of the formula's(Sello Makgakga, 2012). In our classroom also the teacher do not makes the clear concept about the derivative because of this the students are memorize the formula and never tried to understand what the derivative is .Thus the student who is very good in procedural knowledge may not have aconceptual knowledge.

Recognizing the procedure based curricula , the 1989 standard issued by National Council of Teacher's of Mathematics(NCTM,1989) recommended that the teaching of mathematics emphasizes the conceptual understanding , multiple represents and connection ,mathematical modeling and problem solving (p.125) as cited in the paper of Shatila and et al in the 28th ICTCM.

The study of Dubinsky and et al (1997) explore the student's graphical understanding of the function and its derivative .The understanding analyzed through the APOS framework.The students whose course was based on the theoretical analysis of learning that they give there may have had more success in developing a graphical understanding of a function and its derivative, than students

from traditional courses. In our context because of various reasons the traditional teaching and learning approach is one of the causes of making derivative difficult. The teaching by ICT softwares makes the clear vision about the derivative. In the study of Dubinsky the specific place and concept where the student's get difficulties are not mentioned.

The study of Abbey(2008), shows students' preference for procedural knowledge, resulting from their prior success with those procedural skills, inhibits their conceptual learning. Their imbalance between procedural knowledge and conceptual knowledge is a key factor that contributes to their weakness in knowledge for graphing functions from information about the derivative. Breaking students free from learning procedurally is a critical component to expanding conceptual knowledge in a semester course in calculus. This study explores the procedural knowledge itself a cause of making low understanding of the conceptual understanding so to increase the conceptual understanding we have to reduce the procedural knowledge in our teaching and learning.

Multiple Representations of Derivative Concept

various study explores that the multiple representation approach is very fruitful for our teaching and learning. The representation of the concept from various way helps to the teacher and students to make a deeper understanding of the concept.in this context the study of Shatila and et al.(28th ICTCM), on the titled "Effects of Technology Aided Multiple Representation(numeric ,symbolic, graphical) Approach on students understanding of derivatives" resulted that the pedagogy used in the experimental group (multiple representation approach) is effective in helping students to develop a better understanding of concept of derivative.Students were able to explain the relationship between graphical

,numerical and symbolic representation of derivative than that of control group. Thus the representation of the content in a various way makes the deeper conceptual understanding of the derivative. It also helps to relational understanding of concept and blocks the compartmentalized knowledge.

Students expand their concept image when they include graphical and algebraic representations of the same concept (Aspinwall & Shaw, 2002a; Aspinwall & Miller, 2001). According to Aspinwall and Shaw (2002a), teachers enhance student learning by presenting graphical and algebraic methods to solve the same mathematical problem (as cited in Abbey, 2008). Students may primarily rely on geometric skills or analytic skills to solve calculus problems; while neither is incorrect, each provides a different understanding of the problem (Aspinwall & Shaw, 2002). Research has shown that students struggle with connecting graphical and algebraic representations and with converting symbolic information to graphical information (Ubuz, 2007, as cited in Abbey, 2008). Various studiesshow that the only one way of presenting the content makes difficult to understand the content conceptually. So multiple ways of representing the topic plays a vital role in reduce difficulties in learning derivative.

Difficulties in Learning Derivative

Difficulties are also a factor that affects the learning and deeper understanding of the derivative so various researches explores the difficulties of students in learning which helps to improve the learning. The paper related to difficulties in calculus by (Tall, 1992) indicated that the students have difficulties in , limit and infinite process, various terms and language such as limit, tends to , approaches, as small as we please etc., handling quantifiers ,symbolic

representation,consequent student preference for procedural method rather than conceptual understanding .To overcome such difficulties active learning method ,symbolic manipulation software ,computer programming are very effective. This paper explores the specific difficulties of the students in learning derivative which is the causes of the low achievement in learning calculus. This study also gives the methods of teaching which helps to reduce the difficulties.

Similarly the study of Acharya (2017),on the title "Factor Affecting Difficulties in Learning Mathematics by Mathematics Learner" explores the causes of learning difficulties in mathematics. The study concluded that students, teachers and parents have to play an important role as key and provider of sound environment for improvement of pass rate respectively. Teachers lack of linkage between new mathematical concept and previously learned mathematics structure, mathematics anxiety, negative felling of mathematics, economic condition and their educational backgrounds, school management system, lack of infrastructure of school and lack of regular assessment system of school are main causes of difficulties in learning mathematics. This study explores the specific causes of learning mathematics. Mathematics teachers, students, schooladministration, parents play a vital role in making effective and productive learning of mathematics.

Reflection on Literatures

The above literature is mainly focused on the misconceptions of the derivative and the student's conceptual understanding & procedural understanding of the derivatives. Many research focused on conceptual understanding and how to overcome from the misconception so in the above study the difficulties of the students and the causes of failure in finding derivatives are not mentioned. The

specific difficulties that are faced by the students while learning derivative are not mentioned in the above literature.Also there are the few study about the difficulties in understanding of derivative in the context of Nepal. My experience of teaching the students felt the derivative one of the difficult concept and thy always rote the process of finding derivative. so, I want to find out what are the difficulties of finding the derivative that the students of higher education courses of Nepal felt.

Theoretical Framework

The theoretical framework is the one of the important part of the research.The conceptual & procedural understanding of the students was analyzed according to the Action-Process-Object-Schema(APOS) theoretical framework according to (Dubnisky, 2001).

APOS Theory

APOS theory assumes the hypothesis that mathematical knowledge consists in an individual tendency, to deal with perceived mathematical problem situations by constructing mental action, process and objects and organizing them in schemas to make sense of the situations and solve the problem(Dubniskey& McDonald, 2001). It is the theory of how mathematical concepts can be learned. A brief summary of component of this theory were given below:

An *action* is a transformation of mathematical objects that is perceived by an individual as essentially external and as requiring, either explicitly or form memory, step by step instruction on how to perform the operation. Action questions require students to execute the steps to solve problems explicitly.

When an action is repeated and the individual reflects upon it, he or she can make an internalmental construction called a *process* which the individual can think of as performing the same kind ofaction, but no longer with the need of external stimuli. An individual can think of performing aprocess without actually doing it, and therefore can think about reversing it and composing it withother processes. The process part of this theory is for students to reflect on the action. The procedure of the process level is similar to the action one; however, the process level creates more of a thought process for a student. This means that, students would have to think about the question before applying the action technique.

An *object* is constructed from a process when the individual becomes aware of the process as a totality and realizes that transformations can act on it.Once a student is comfortable with an action and is proficient with the process level of this theory, then a student can move up to the object level. The object level is the third level of APOS theory. The object level is where a student can think more analytically about a problem.

Finally, a *schema* for a certain mathematical concept is an individual's collection of actions, process, objects and other schemas which are linked by some general principles to form a framework in the individual's mind that may be brought to bear upon a problem situation involving those concepts. This is the highest level of understanding for students. Schema links the action, process and object steps together. Schema questions show that students have a deeper understanding of the topic. Schema is a coherent collection of processes, objects and previously constructed schemas that is invoked to deal with a mathematical problem situation. As with encapsulated processes, an object is created when a

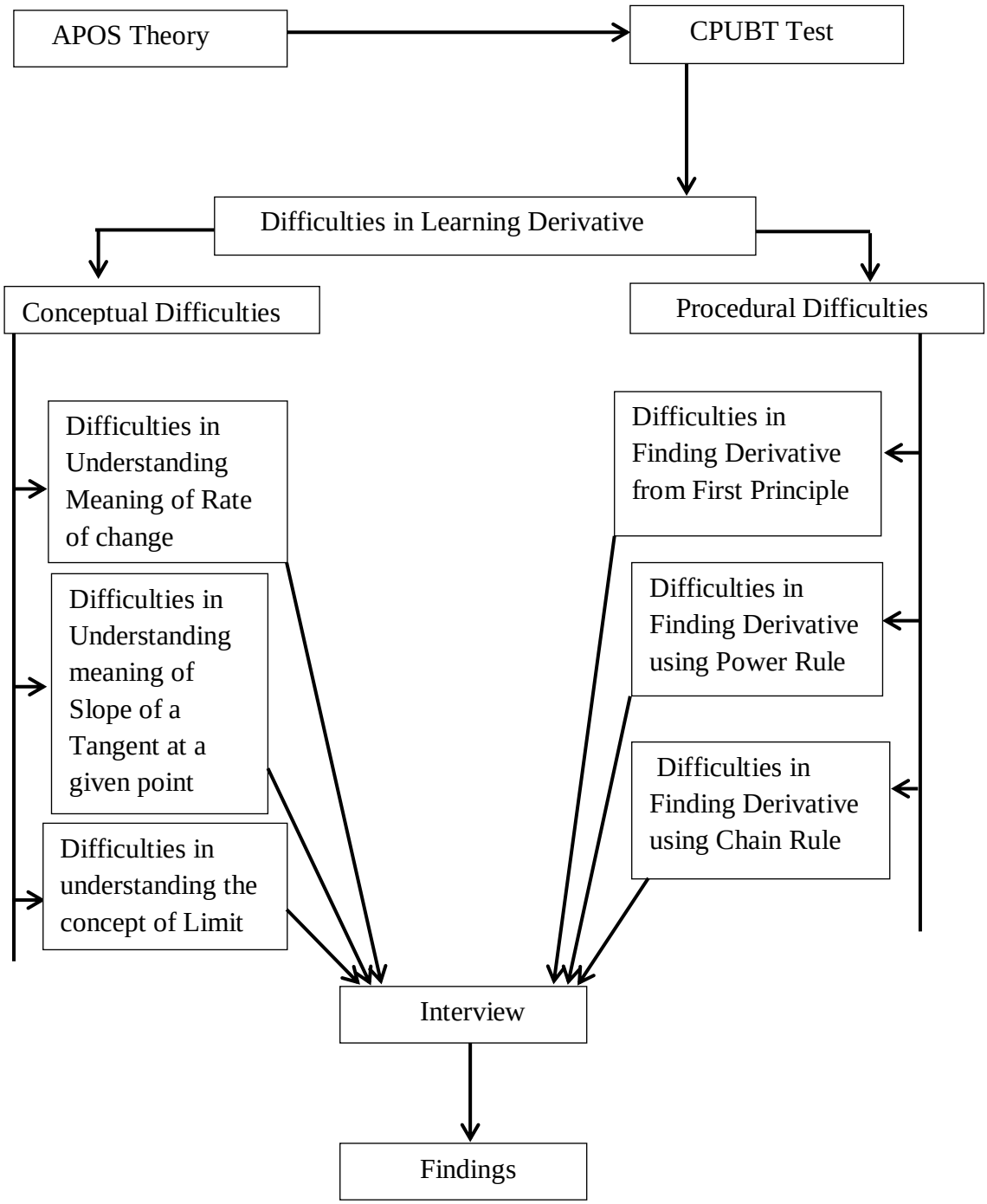
schema is thematized to become another kind of object which can also be de-thematized to obtain the original contents of the schema. The four components, action, process, object, and schema have been presented here in ahierarchical, ordered list. This is a useful way of talking about these constructions and, in some sense,each conception in the list must be constructed before the next step is possible. (Dubniskey, 2001)

Hence, APOS theory was used as a theoretical framework of my study. According to This theory I constructed the test consisting the conceptual and procedural understanding questions.The entire questions are selected on the CPUBT was on the basis of APOS framework which explores the conceptual and procedural difficulties in learning derivative.

Conceptual Framework

Conceptual framework helps to see clear picture of the thesis. It is also called the operational road map.The problem of my study is the conceptual and procedural difficulties in learning derivative. Before designing this conceptual framework I read several literatur on this topic. Thus, the conceptual framework for my study was constructed on the basis of review of the different literature.

On the basis of the study of Sahin and et al (2014), Ocal and et al (2015) and Tarmizi (2010), related to conceptual and procedural difficulties in learning derivative the following was the conceptual framework for my study



Ifound the result on the basis of above conceptual framework. How student feel difficulty in conceptual and procedural understanding and why they feel difficulty in learning derivative. That was found by using two tools i.e. CPUBT

test and interview guidelines. The researcher found the result with help of students performance in the written test. To find the difficulties in learning derivative researcher categorized it into two types of difficulty one is conceptual difficulty and the other is procedural difficulties. On the basis of APOS theory the researcher constructed the CPUBT test and on the basis of test result researcher selected the respondent and conducted the interview and examined the two types of difficulties.The study was found the difficulties in understanding of meaning, of rate of change, of slope of a tangent line at a given point of a curve and the concept of limit under the conceptual difficulties. Similarly under the procedural difficulties the study was found the difficulty, in finding derivative from first principle, in finding derivative using power rule and difficulty in finding derivative using chain rule.

Chapter III

RESEARCH METHODS AND PROCEDURES

Qualitative procedures demonstrate a different approach to scholarly inquiry than methods of quantitative research. Qualitative inquiry employs different philosophical assumptions; strategies of inquiry; and methods of data collection analysis and interpretation although the processes are similar(Creswell 2007).Research methodology is a science, which determines how to complete the research systematically. It is an important aspect of research work. Authenticity and reliability of any research depends upon the tools and methods which are used for data collection. Thus this chapter includes design of the study, population and sample, data collection tools, reliability and validity of tools, data collection procedure and data analysis procedure.

Research Paradigm

The research paradigms are general orientation about the world and the nature of research that a researcher holds. These world views are shaped by the discipline area of the student, the beliefs of advisers and faculty in a student's area and past research experiences(Cresswell, 2009).The research paradigm for the study was the interpretivism. The ontology of the interpretivism is there is multiple reality and epistemology is the subjective i.e. Knowledge is constructed through the underlying meanings of the individuals, events or activities and the methodology is the qualitative (Patel, 2015).

Upadhyay& et.al (2067) mentioned that interpretive research is also called qualitative research. The fundamental assumption of this type of research is that a

profound understanding of the world can be gained through correction and observation in natural setting rather than through experimental manipulation under artificial condition (pg.382). This paradigm assumes that the reality is relative terms based on the social term i.e. there is the multiple reality. Interpretive assumes relativist ontology, a subjective epistemology and a naturalistic methodological procedure (Upadhyay& et.al, 2067).

In this study the reality was multiple, there is no absolute reality. I tried to generate the knowledge through the views of respondents so, could not generalize the findings in different context. Also to find out the subjective reality I adopted the qualitative methodology under the study.Therefore, it was attempted to visualize the interpretation of their world of the research participants. Then I interpreted the meaning generated by my participants. Thus, there exists multiple interpretations of social realities from the perspectives of participants and me as a researcher. The data were collected through the in-depth interview and sometimes I tried to become critical.

Design of the Study

The case study design was adopted for study. Research designs are plans and the procedures for research that span the decisions from broad assumptions to detailed methods of data collection and analysis (Creswell, 2009). Research design is the conceptual structure, strategy of logical, systematic plan and direction of research. It is the way through which the researcher reaches to the goal of research. Case studies are a strategy or inquiry in which the researcher explores in depth a program, event, activity, process or one or more individuals (Creswell, 2009). A case study is a specific instance that is frequently designed to illustrate a more

general principle; it is 'the study of an instance in action. The single instance is of a bounded system, for example a child, a clique, a class, a school, a community. It provides a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply by presenting them with abstract theories or principles (Cohen, 2007). Cohen (2007) also mentioned that Case studies can establish cause and effect, indeed one of their strengths is that they observe effects in real contexts, recognizing that context is a powerful determinant of both causes and effects.

Case studies are set in temporal, geographical, organizational, institutional and other contexts that enable boundaries to be drawn around the case; they can be defined with reference to characteristics defined by individuals and groups involved; and they can be defined by participants' roles and functions in the case (Hitchcock and Hughes 1995: 319, cited in Cohen, 2007). In this study the case was the students of the grade XI.The study tried to find the difficulties in conceptual and procedural understanding of derivative. So the detail study was done.

The various researches show that the learning derivative is difficult for students. Such problem is also in the students of the Makwanpur district. Thus, the students in the public school of the Makwanpur district are the case for the study. The indepth study in the students conceptual and procedurals difficulties in learning derivative were done in this study.

Study Area and Selection of Respondents

The idea behind qualitative research is to purposefully select participants or sites (or documents or visual material) that best helps the researcher understand the

problem and the research question. This does not necessarily suggest random sampling or selection of a large number of participants and sites, as typically found in quantitative research. A discussion about participants and site might include four aspects identified by Miles and Huberman (1994): the setting (where the research will take place), the actors (who will be observed or interviewed), the events (what the actors will be observed or interviewed doing), and the process (the evolving nature of events undertaken by the actors within the setting) (Cresswell, 2009).

This research is the case study which tries to explore the conceptual and procedural difficulties in learning derivative. According to the purpose of the study I used purposive sampling to select the sample.Thus, the 40 students were selected as sample for the study.The study area of this studywasMakwanpur District. The case of this study was the students of the grade XI. In this study the students of the one of the public school of grade XIof Makwanpur District waschosen.The CPUBT (Conceptual and Procedural Understanding Based Test) was conduct to the each students of the class and for the purpose of the study researcher selected three students on the basis of result CPUBT for the interview. Also the interview was conducted to collect the data according to purpose.

Data Collection Tool

Research tool is the most important part for the data collection in the study. On the basis of data collection technique we can study and analyze every aspect of the study.In this study the researcher intended to find the conceptual and procedural difficulties in learning derivative. To fulfill the purpose of the study the CPUBT and interview guidelines are used as a tool for the data collection.

Conceptual and Procedural Understanding Based Test (CPUBT). The purpose of the study was to find the conceptual and procedural difficulties in learning derivative.To find out the conceptual and procedural difficulties in learning derivative the researcher made one test having some conceptual and procedural questions.The instrument that was administered was organized by the APOS (Action, Process, Object, Schema) framework (See Apendix-A). The first problems were action problems, then the process problems, then object and lastly a schema problem. The students had one hour to complete this assessment. Problems on the assessment were arranged from basic level of conceptual knowledge to the highest level of conceptual problems. Students were instructed to solve the problems to the best of their ability. Questions were made on the basis of the APOS model for observation of the student's conceptual & procedural understanding of derivative.

There was a total four problems. Problem no 1 is the action level problem. These problems are straight forward and are basic for students to do. Action questions required students to execute the steps to solve problems explicitly.On the CPUBT. The problem no 1 contain three problems related to power rule and chain rule.

Problem no 2 demonstrates the process component of APOS theory. The process part of this theory is for students to reflect on the action. The procedure of the process level is similar to the action; however, the process level creates more of thought process for a student. This means that, students would have to think about the questions before applying the action technique. On the CPUBT the problem no 2 contains four problems related to conceptual difficulties.

Problem no 3 represents an object questions for APOS theory. Once a student is comfortable with an action and is proficient with the process level of this theory, then a student can move up to the object level. The object level is where a student can think more analytically about a problem. There was only one questions mentioned in a CPUBT.

Problem number 4 was a schema questions. According to APOS theory this is a highest level of understanding for students. Schema links the action, process, and object steps together. Schema questions show that students have deeper understanding of the topic.

Interview Guidelines. In qualitative interviews, the researcher conducts face-to-face interviews with participants, interviews participants by telephone, or engages in focus group interviews, with six to eight interviewees in each group. These interviews involve unstructured and generally open-ended questions that are few in number and intended to elicit views and opinions from the participants (Cresswell, 2009).The interview is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard. The order of the interview maybe controlled while still giving space for spontaneity, and the interviewer can press not only for complete answers but also for responses about complex and deep issues. In short, the interview is a powerful implement for researchers. On the other hand, the researcher using interviews has to be aware that they are expensive in time, they are open to interviewer bias, they may be inconvenient for respondents, issues of interviewee fatigue may hamper the interview, and anonymity may be difficult (Cohen, 2007).

After conducting the CPUBT, the interview was conducted with the help of interview guidelines(See Apendix-B) to the participants. Semi-structure interview was used for finding the conceptual and procedural difficulty in learning derivatives.The necessary questions were asked to the participants although these are the main questions. Except these questions other situational questions were asked. I tried to explore the difficulty by conducting the depth interview. The interview was kept by recording and interview result was compared to the current literature.

Reliability and Validity of Test

Validity does not carry the same connotations in qualitative research as it does in quantitative research, nor is it a companion of reliability (examining stability or consistency of responses) or generalizability (the external validity of applying results to new settings, people, or samples). Qualitative validity means that the researcher checks for the accuracy of the findings by employing certain procedures, while qualitative reliability indicates that the researcher's approach is consistent across different researchers and different projects (Gibbs, 2007: as cited in Creswell, 2009,190).

Reliability and validity of tools is one of the most important aspects in any study. In this study the CPUBT was constructed on the basis of the study of Constantinou (2014). Also the pilot test was conducted to ensure the reliability of the CPUBT. I analyzed the reliability of test by using the split half method found reliability coefficient is 0.92(Appendix-C). It shows that the test questions were reliable. Also the test is constructed on the guidance of supervisor. Thus we can confirm that there is reliability and validity of CPUBT. Cross match or

triangulation method gives an accurate and reliable picture of situation. So the validity of tools was maintained by cross matching or triangulation the data collected from CPUBT and interview with students. Qualitative researchers use many methods for gathering information and interviewing is one of those methods with a research base .When conducting interviews, relationships and rapport must be established, and coupled with trust: “The purpose of interviewing is to find out what is in and on someone else’s mind. Active listening and nonjudgmental behavior are two of the common practices that should be prioritized when interviewing for case study research. Thus reliability and validity of interview schedule of my study will be calculated from consistency in response, multiple response, prolonged engagement, Triangulation etc.

Goodness and Trustworthiness

Because qualitative research entails the researcher taking an active role in the collection and interpretation of others’ meaning making, to be credible, qualitative researchers must be good and trustworthy. To ensure the goodness and trustworthinessGuba and Lincoln (1998), suggests the following criteria(as cited in McAninch, 2015). So, I followed these criteria to maintain the Goodness and Trustworthiness of my study. I used the triangulation of data, member check, prolonged engagement in the field,and peer review, rich thick description, Researcher role etc.

Credibility

This concept replaces the idea of internal validity, by which researchers seek to establish confidence in the 'truth' of their findings. Guba and Lincoln (1998) recommended several techniques inquirers may use to enhance the

credibility of the research: participant observation, triangulation, peer review, negative case analysis, progressive subjectivity checks and member checking. To maintain credibility of my study I performed triangulation, member check, peer review, prolonged engagement in the field.

Triangulation of data means the collection of data through the multiple sources to include interviews, observation and document analysis (Cresswell, 2009). In this study I used multiple sources of data to confirm emerging findings. Also I used multiple stand point to analyze the collected data using reviewed literature.

The member checking ensures the truth value of the data (Cresswell, 2009). In this study I performed the member checks by sending participants a copy of their interview transcript, Final report and asking them to verify the accuracy of their views. The next method that I used to increase the trustworthiness is the prolonged engagement in the field. I spent the long time in the study field. I spent the three months in the study area. Repeated observation of similar phenomenon was done by developing the good relationship with the respondents.

The peer review was performed to increase the trustworthiness. I requested to my collegue who had done similar study to examine my study. The researcher role is very important in qualitative research. I avoided my personal feelings, ideas, and judgments throughout the research period.

Transferability

Transferability replaces the concept of external validity. This criterion refers to the applicability of findings of one context (where the research is done) to

other context or settings (where the interpretation might be transferred). To maintain the transferability I had included the photos of the questions asked to them and answered of the student's practices in my research. I had tried to capture the most of scenario by using thick description of interviews and my meaning making.

Dependability

This concept replaces the idea of reliability i.e. the issue of dependability refers to the idea of another researcher being able to repeat the same work, in the same context, with the same methods and participants, and get similar results. For this study,Idocumented all processes in detail, and then shared with advisors to help evaluate the processes to confirm dependability.

Conformability

In qualitative research, conformability refers to the researchers concern with objectivity. This ensures that the ideas expressed in terms of findings result from the experiences and ideas of the participants rather than being biased by the researcher's (Shenton, 2004, as cited in McAninch, 2015). Triangulation was used as a strategy to reduce the effect of researcher bias. Also thee member checking process were performed to conformability.

Data Collection Procedure

The study was conducted in a sample selected from related population .The required data were collected from related school. Through the campus chief of the related campus I got in touch with my respondent. I had collect the data through

interview and CPUBT test by taking first consent from the campus chief and from participants.

The CPUBT test was made on the basis of APOS theory were given to the students and the result will be measured using rubric.After conducting the test I analyzed the test result and on the basis of test result I selected the three respondents who had more difficulty. The interview was conducted to the respondents and asking them the questions which are related to the difficulties in doing the problem.The data were collected through the in-depth interview with the students. The interview was recorded transcribed and coded.

Data Analysis and Interpretation Procedures

Result of the CPUBT was measured by using rubric score (Appendix-A).The data for this study was collected from the class XI students of education and science faculties' students of Makwanpur Multiple campus. Participants were administered the exam by using a CPUBT as mentioned above.Each student was given an assessment with various derivative questions. Each derivative question was categorized at a different level of the APOS model. Students' assessments were manually graded and scored out of eighteen points by the researcher. Each of the grades was recorded in a worksheet. Students showed strength with the action problems but had difficulty with process question. The students felt much difficult with the object &schema question, which conceptually is supposed to be the most difficult.

On the basis of CPUBT result, the three respondents were selected who had seen weak performance in test. Then the interview was conducted with these three respondents. The general inductive approach was used for analysis of the data

obtained from interview. According to Thomas (2006) the inductive approach is a systematic procedure for analyzing qualitative data in which the analysis is likely to be guided by specific evaluation objectives (p.8). Inductiveanalysisreferstoapproachesthatprimarilyusedetailedreadingsof rawdatatoderiveconcepts,themes,oramodelthroughinterpretationsmadefromtherawd ata by an evaluator or researcher.The inductive approach clarifies the data reduction process by describing a set of procedures for creating meaning in complex data throughthedevelopmentofsummarythemesorcategoriesfromtheraw data.

According to Thomas (2006), the following procedures are used for the inductive analysis of qualitative data:

1. Preparation of rawdata files (data cleaning)
2. Closereadingoftext
3. Creationofcategories
4. Overlappingcodinganduncodedtext
5. Continuing revision and refinement of category system

In this study I analyzed the interview result according as general inductive approach by Thomas (2006). At first interview was recorded and then transcripts of interview was made. After continuous reading of transcript so many times, the transcripts were coded. I categorized so many code into different category. Finally, the themes were generated such as understanding of derivative as a rate of change, understanding derivative as a slope of tangent, concept of limit, finding derivative using chain rule, finding derivative from first principle and finding derivative using chain rule.

Chapter IV

ANALYSIS AND INTERPRETATION

This chapter deals with the analysis and interpretation of the collected information derived from the case study during the research period. Analysis of data is a process of inspecting, cleaning, transforming, and modeling with the goal of highlighting useful information, suggestions, conclusions and supporting decision making(Best and Khan, 2009). According to Cohen (2007),Qualitative data analysis involves organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities.

The process of data analysis involves making sense out of text and image data. It involves preparing the data for analysis, conducting different analyses, moving deeper and deeper into understanding the data (some qualitative researchers like to think of this as peeling back the layers of an onion), representing the data, and making an interpretation of the larger meaning of the data. Data analysis involves collecting open-ended data, based on asking general questions and developing an analysis from the information supplied by participants(Creswell, 2009).

After the collection of data with the help of the relevant tools and techniques, the next step is to analyze and interpret them with the view to arrive at empirical solution to the problem. For the data collection, I used first the CPUBT. From the result of the test I selected those students who have difficulty in learning

derivatives and the interview was conducted. The data obtained from interview was analyzed analytically, descriptively and qualitatively. This is the case study about conceptual and procedural difficulties in learning derivatives. To meet the objective of the research I collected data from the only one campus of Makawanpur district, named as Makwanpur Multiple Campus.There were 40 students in a class XI. First I conducted the exam by using CPUBT. From the test result the researcher had selected three students whose results was very weak. The researcher asked the guideline questions (Appendix- B) for each student.

The interview is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard. The order of the interview maybe controlled while still giving space for spontaneity, and the interviewer can press not only for complete answers but also for responses about complex and deep issues. In short, the interview is a powerful implement for researchers. On the other hand, the researcher using interviews has to be aware that they are expensive in time, they are open to interviewer bias, they may be inconvenient for respondents, issues of interviewee fatigue may hamper the interview, and anonymity may be difficult(Cohen, 2007).

After the result of test I selected the three students on the basis of weak performance in the test. My cases were the three students read in the science faculty of class XI. They were permanent resident of out of makwanpur district. They have been taken a room for rent and they continue their reading. Researcher asked the questions on the basis ofguideline questions (Appendix-B) for selected participants.

Finally I categorized the collected data for analysis and interpretation in the following main two headings as conceptual difficulties (Meaning making of rate of change, meaning making on slope of tangent at given point of function, understanding and sense making on limit) and procedural difficulties (Finding derivative, using power rule, from first principle, using chain rule) in learning derivative which can be present as below on the basis of conceptual framework.

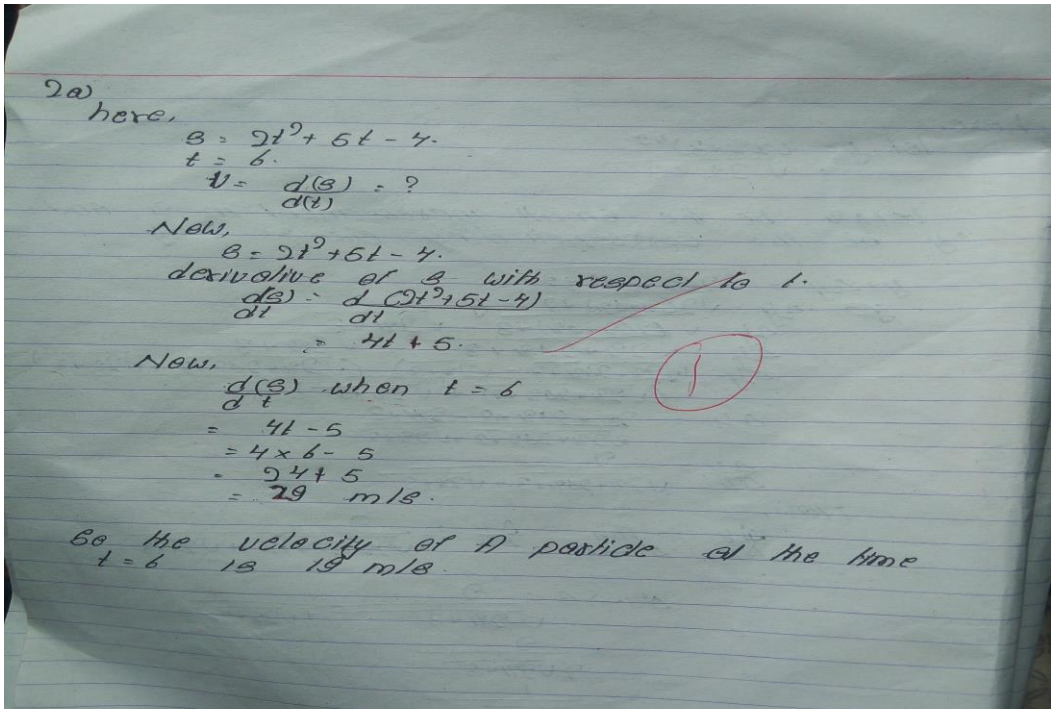
Conceptual Difficulties in Learning Derivative

The conceptual difficulties refers the difficulties in the meaning making of derivative as a rate of change, difficulties in the meaning making of limit and geometrical meaning of derivative and difficulties in connection between them.According to Heibert&Carpentar (1992)Conceptual knowledge is a network of knowledge where the connections are meaningful. Conceptual knowledge is a network of associations of mathematical procedures, integrated with the mathematical principles (as cited in Abbey, 2008).Difficulties in conceptual understanding are the factors that hinders in learning the basic and fundamental elements in the larger structure of the content. I used the three problems which are in the process, object and schema level questions based on APOS framework.On the basis of test result the answer and mistakes done by respondents were analyzed by taking an interview. Under the conceptual difficulties I categorized the three main difficulties called difficulties in meaning of rate of change & derivative, derivative as a slope of tangent line and meaning making the sense of limit.

Meaning Making of Rate of Change. This difficulty refers the difficulties related to explain how derivative refers the rate of change between two variables. The rate of change is the very broad and generalized concept of the

derivativeTarmizi (2010). Researcher analyzed the difficulties related to rate of change on the basis of function concept, meaning of average rate of change and instantaneous rate of change. If the respondents are able to explain the above concept then there is no any difficulties but if they cannot explain about the function, average rate of change, instantaneous rate of change then then such type of difficulty are categorized as a difficulty in the concept of rate of change.The derivative, once calculated, is the instantaneous rate of change at a given point on a curve (Tarmizi, 2010). According to ZulalSahin (2015)the concept of derivative should introduce and developed in relation to the rate of change, slope of tangent line and limit. Among the three big ideas the meaning of rate of change is also important. Because unless the mathematical concept is understood relationally,students compartmentalize the big ideas related to the concept in their conceptual system & cannot relate them each other (Zulal Sahin, 2015). Difficulties in the understanding of derivative as a rate of change means the elements that hinder to understand rate of change.

For finding difficulty in the meaning making of rate of change, the process level question was asked. The question was "explain the derivative as a rate of change. A particle moves in a straight line. The distance s covered by particle in time t is given by $s= 2t^2+5t -4$, where the distance is measured in meter and time in second. Find the velocity, v(t) of the particle at t=6 ".In this problem all students leave the first part and total number of student answered the second part. One respondent did the first part of the problem and other two did not do the first part. If the students were able to find the velocity at t = 6 with correct unit then, it is considered that students did not have any difficulty otherwise, it is considered they have difficulty. Example of one student's answersheet was presented below:



From the above solution, it can be seen that the student has difficulty in explaining the derivative as rate of change. There is no difficulty in solving the problem. Student easily solve but they did not able to explain the meaning of derivative as rate of change. Student A easily solves the questions about rate of change. Also, he wrote the unit of the velocity. This shows he had a well concept of velocity. Student solved the second part of the problem without understanding the meaning of rate of change. Students easily find out the ds/dt at t= 6 , this shows they had well concept of instantaneous velocity or in broad sense instantaneous rate of change. From their answers, it also can be seen that student has well understanding in the unit of velocity. Another thing is he could not sure why he differentiate with respect to t. So, it can be concluded that from the above answersheet student did not able to explain the meaning of rate of change but they

were able to find out the velocity. Some discussion with student A is presented below.

In this context, I conducted the interview to find out the difficulty in the concept of function, rate of change and connection with the derivatives with the respondents. To find the difficulty in the concept of function the interview with the students was as follows:

I: What is the meaning of function?

Student A: The function is the relation.

Student B: The function is given in x and y.

Student C: y= f(x) is called the function.

I: What is the meaning of y=f(x) and give me one example.

Student C: I think y is given in the variable x. So y is called the function of x and $y= 4x^2+5$ *is the example of function.*

I:If the function is given in s = p+5 then how you can write it?

Student C: Here, s is the function of p so it can be written as s=f(p).

Student A: y= f(x)

I: Do you know derivative is a function?

Student C: Derivative is a value,am I right?

This conversation shows that student A and student B has no clear concept of function. When I asked what is the function, the Student A said that the function is relation which was very good concept but from the answered given by the

student B & C it shows that student B & C had no cleat concept about function. The student has big problem in the notation of function which can be seen in above dialogue. They did not able to explain the meaning of y=f(x). Also when I asked to write the notation of function s= p+5 then respondent B&C did not give the right notation but A gives the right answer. So it can be said that the participants had difficulty in the functional notation. They did not able to understand how to represent the function in notation. Another big thing is the student could not understand derivative of a given function is again a function. In this regards, Park (2012) said that the student may confused in understanding of derivative is a function or a point specific. The study of Tarmizi (2010), explored that the students sometimes misunderstood the notion of function. According to Tarmizi (2010),A function y= f(x) is a process: input x carryout the procedure, output y. Students have some misconception in function so they need special treatment such as further tutorial session in correcting their misconceptions and they are fully confusing in Problem solving strategy according to George Polya. Thus, It can be concluded that students has difficulty in the concept of function and difficulty in understanding of derivative of a given function is again a function. Other discussion can be presented as below:

I: How could you explain about change in function y=f(x) i.e. the change in the value of x affect the value of y?

Student C: Of course, In the function y = f(x), when the value of x changes then the value of y will be changed this is called change.

I: If the function y is given in the variable t then in which respect we have to differentiate y?

Student C: We have to differentiate with respect to t.

Student A: We differentiate with respect to x? Am I right?

I: But you differentiate the given function s with respect to t in your paper why?

Student A: Ummm.. because we have to find ds/dt so.

All of the participants solved the second part of the problem related to rate of change. It can be seen from the above discussion that student didn't know why they are differentiating the function with respect to t in the solution of question 2a)(Appendix-A). They had lack of concept about the meaning of "with respect to". Student didn't care about in which variable the given function is given. It can be seen that they had lack understanding of meaning of "with respect to". When I asked when the function is given in the variable 't', then in which respect we should differentiate given function? Then student A replied with respect to x. But in the answer sheet of CPUBT the student A differentiate the given function with respect to t? When I asked why?, then he replied, we have to find out ds/dt. It can be concluded that student just memorize the process what their teacher taught in the classroom. They were not clear about the phrases like "with respect" "differentiate"etc. The same conclusion is made by the study of Tall (1992). The study explores that the students has difficulty in various terms and languages like tend to, approaches, with respect to etc. These are consequent to student preference for procedural understanding rather than conceptual understanding. Thus we can concluded that student have difficulty in the meaning making of word or phrases like "Differentiating" & "with respect to" and various terms and language.

The further discussion on the basis of answer of question number 2a) can be presented below:

I: Okay, so can you tell me how you able to solve the second part? you found ds/dt and put t=6 why you did it?

Student A: Such type of questions is in text book & I practiced it.

I: What is the meaning of ds/dt= 29 m/s can you explain it?

Student: ummm..No.

It can be seen from the above that, student A answered the question number 2a) he just memorized and practiced many times he don't know why he did so. Student did not understand the meaning of rate of change They could not connect the concept of rate of change in the application problems. Also they couldn't understand the meaning of ds/dt or notation of derivative. They have misconception that only the dy/dx is the derivative. From this, it can be concluded that they did not understand the notational representation of derivative. Student find out the velocity at t=6 i.e. ds/dt = 29 m/s but they could not explain what represent the ds/dt = 29 m/s. Also, they had a very weak concept in the notion of function. Student only depend on the textbook and just memorizing the process of doing the problems without understanding conceptual meaning. Similarly the study done by Constantinou (2014) showed that, Students do not fully understand the concept of a derivate and have trouble correctly answering derivative application questions. Teachers with a strong understanding of their students' knowledge can tailor their lessons to accommodate students who do not fully understand the concept of a derivative in a first-year calculus course.

All of participants leave the first part of the question 2a)(Appendix-A). They were unable to explain the concept of derivative as a rate of change. Student A and Student B left the question but student C answered about rate of change. In this context the interview can be presented as below:

I: Do you have any idea about rate of change?

Student A: Well I don't know any idea about this. I was absent when my teacher taught this chapter.

Student B: let y= f(x) be the continuous function then by the definition of function, y changes while x will change. If Δx *and* Δy *be the small change in x and y respectively then* $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$ *is known as the derivative as a rate of change.*

Again to make the confirmation about the students understanding of meaning of rate of change I asked the further questions. The example of face to face interview is mentioned as, first I asked"Are you sure this is a rate of change?" The respondent B replied *"I am not sure but I write it from the textbook. I am not very clear about the rate of change".* After that I asked "Do you have any idea about average rate of change and instantaneous rate of change?" Respondent replied *"No idea sir".* In the same manner I asked Okey, "you wrote $\frac{\Delta y}{\Delta x}$ is a derivative as a rate of change. Did you mean $\frac{\Delta y}{\Delta x}$ is a derivative? Is there difference between $\frac{\Delta y}{\Delta x}$ and dy/dx?", Then respondent replied *"umm…well I am confused but dy/dx also called derivative."*

After investigating the above interview, it can be seen that, respondents didn't know the average rate of change and instantaneous rate of change. Also, students feel very much difficult in recognizing the symbolic representation of average rate of change and instantaneous rate of change. They were unable to differentiate the symbolic representation of average rate of change and instantaneous rate of change. In this regards the study of Tyne (2016), explored that students were unable to explain the derivative as a rate of change. Thus, from the above facts we can conclude that the students had much difficulty to explain the derivative as a rate of change.

From the above discussion, it can be seen that the students had some concept about the rate of change but they couldn't explain it and they were not clear about what is rate of change. All of student didn't give the answer of the questions that is related to the rate of change. Student B wrote something about rate of change but he did not know what he wrote. From the answer given by B it can be concluded that B has some concept about rate of change but he did not able to clarify between average rate of change and instantaneous rate of change. Student A and C did not able to describe what is rate of change. Students are just depending on their textbooks and teachers and they just memorized the process without understanding the conceptual meaning. Thus we can conclude that all of the respondents feel difficult to explain the derivative means the rate of change. They didn't know the average rate of change and instantaneous rate of change. Also students feel very much difficult in recognizing the symbolic representation of average rate of change and instantaneous rate of change. They did not able to differentiate between the symbols $\frac{\Delta y}{\Delta x}$ and dy/dx. They were not able to explain

the how derivative is actually an instantaneous rate of change. The same result was found in the study of Tyne (2016). According to Carlson (1998) the most talented second-semester calculus students had trouble interpreting rate of change (As cited in Tyne, 2016). Thus the above interpretation justified the fact that the students had difficulty in understanding the derivative as instantaneous rate of change.

To clarify the derivative as a rate of change the further discussion with the students can be presented as below:

I: Do you have any idea about speed which you read in class ten?

Student A: Yes. Speed is a distance traveled by time taken.

I: Can we say speed is a ratio of change in distance by change in time i.e. $\frac{\Delta s}{\Delta t}$

where s is distance and t is time?

Student A: Of course.

I: So it is called average speed ok, in broad sense it is called average rate of change. Do you know about instantaneous speed?

Student A: No idea sir.

From the above discussion, it can be seen that students had concept of speed but they did not able to connect the concept of speed in the average rate of change. Student A said that speed is distance traveled by time taken but he did not able to understand the distance traveled is the change in distance and the time taken is the change in time. Another thing was he did not know about the instantaneous speed. He did not able to understand when we found the speed in the very small interval of time then it becomes the instantaneous speed at that time

interval. Thus, from the above discussion it can be concluded that, the students were unable to understand the average speed and instantaneous speed and did not able to connect these concept with the average rate of change and instantaneous rate of change. Thus, They had difficulty in meaning making of rate of change.

For the further investigation, I asked"let us check your answer sheet. For the question 2a),you answered the second part well, but you did not write the answer of first part where you had asked explain derivative as a rate of change." then, student A replied that *" I cannot explain it. I don't know anything about it. I think these things are not asked in our exam".* Again I asked "what is represented by the symbol $\frac{\Delta s}{\Delta t}$ and ds/dt?" then, student A replied that *"No".*

From the above discussion it can be concluded that students had a clear concept about speed but they could not relate the speed with average rate of change. When I asked to students A then he answered meaning of speed well but he did not relate the speed as average rate of change. Although Student A answered second part of the questions 2a), he did not relate instantaneous rate of change means. In the questions students were asked to find velocity at t= 6 sec. All of the participants answered the velocity at t=6 seconds is ds/dt = 29 m/s, which is correct but they did notconnect the concept of instantaneous speed. Students have no clear concept about average rates of change and instantaneous rates of change. They didn't know the difference between average speed and instantaneous speed. Another remarkable thing from the above interview is students were becomes very exam oriented. They didn't want to read those things which are not asked in the examination. Student didn't know the what represented by the symbol $\frac{\Delta s}{\Delta t}$ and

ds/dt. They had difficulty in the symbolic representation of average rate of change and instantaneous rates of change. Same kind of difficulty was presented in the study of Tyne (2016).So it can be concluded that student have difficulty in meaning making of rate of change concept.

Again to find out why they felt the derivative is difficult, I asked them some questions. In this regards I asked "*So tell me why do you feel so difficult?"* then the student B replied that

> *I feel it very difficult I don't know why but our teacher never goes to such detail and he just told something about chapter at the beginning of the chapter then he directly goes to the exercise and I think there was also our mistake because we did not attend the class regularly. So I don't know anything about the average rate of change and instantaneous rate of change.*

On the same question the student A replied that "*Sir I was absent in the class room and we always read for pass in the exam".* Again I asked"Did you repeat he lesson at your home when your teacher finished the lesson."Then students A and B replies *"No".*

The above discussion justified the fact that the derivative becomes difficulty cause by teacher and student both. When I asked why you do feel difficult then student B said that their teacher never goes to in detail of the subject. From this statement it can be concluded that teacher's interest of students procedural understanding is the main causes of difficulty which reduces conceptual understanding & students were memorized what they learn.The same conclusion is made by study of SelloMakgakga (2012), in which some of the difficulties are

caused by teacher's approach of just wanting them to memorize formula's without explaining the meaning of the formula's. Also the difficulties are caused by students.Student's exam oriented habits, not regularity in classroom and laziness were another causes of difficulties.

All of the above discussion justified the fact that the students had difficulty in understanding derivative as a rate of change. Students did not fully understand function notation, memorized a procedure, and could not verbalize what they were doing in the problem. Student did not able toexplain's' is the function of 't' so we differentiate 's' with respect to 't'. Student did not able to explain the meaning of ds/dt = 29 m/s. Students memorized the tricks to finding derivatives; however, students often cannot verbalize the meaning behind what they calculated which is same as the result of the study of Tarmizi (2010). Researcher also concluded that the role of teacher is very important in the class of derivative because it is very new concept for the XI students. Thus teacher should be very careful and he/she must explain the derivative as a rate of change in detail. Students just memorize the process of finding the velocity but they did not know why they are doing so. Thus there is a lack of multiple representation of same thing. The symbolic form and verbal form plays important role to understand the concept of derivative. Students could not tell the meaning of $\frac{\Delta y}{\Delta x}$ & the meaning of $\frac{dy}{dx}$. So researcher found that lack of conceptual explanation and multiple representation harms to students to understand derivative as a rate of change conceptually which is also mention in the study of Aspin wall and Shaw, (2002).

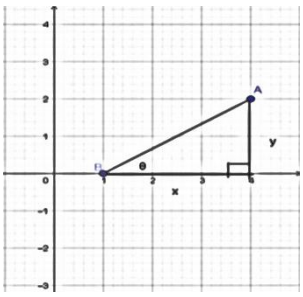
Meaning Making on Slope of Tangent at a Given Point. This difficulty refers the difficulty on the graphical understanding of derivative as a slope of

tangent, understanding meaning of slope, finding slope when the derivative function was given & choosing the correct derivative function of given function with reason. If the students are able to do all the above concept then there is no difficulty otherwise it should be considered that there was difficulty.Students have misconception rooted from not knowing the symbolic representation,graphical representation and difficulty to construct a relation between the slopes, tangent and normal (Kaplan & etal,2015).The three big ideas in derivative (rate of change, slope of tangent and limit) are connected to each other so learning by connecting all three makes derivative understanding very clear (ZulalShahin, 2015).To understand derivative students must be clear about the slope of tangent line at a given point of the function.

I asked the questions 2b) to measure difficulty related graphical understanding of derivative i.e. derivative as a slope of a tangent line which was "Define differential coefficient of a function f(x) at a point and interpret it geometrically. Compute the slope of a function $y = (x^2+3)^2$ at x=2". This problem was in a process level question. None of my respondents answered this problem. On the basis of the problem first I had taken an interview about slope which can be presented below:

I: What is the slope of straight line?

Student A: Slope of straight line is the $Tan\theta$



I: Good. Ok tell me can we say slope of given straight line is y/x in the figure alongside?

Student A: Sure sir $Tan\theta = \frac{p}{b}$ *so.*

I: When A(x_1,y_1) & B (x_2,y_2) given then what is the slope of line AB?

Sudent A: I forget sir.

Student B: $\text{Slope}(\text{Tan}\theta) = \frac{y_2 - y_1}{x_2 - x_1}$.

I: Can we say $\frac{\Delta y}{\Delta x}$ *is the slope of line AB?*

Student B:The slope is the y/x then how is it possible?

Student A &C: Agree with B.

I: Do you know what is Δy

Student A : No.

We can concluded from the above discussion that student had difficulty in the concept of slope. The student A couldn't give the right answer but student B answered some questions very well. But student B has incomplete understanding about the slope. He could not relate the slope of line as ratio of difference. Student had difficulty in symbolic representation of difference in variable x and y i.e. they were not able to say what the Δx and Δy represent. Student had incorrect reasoning about slope is the ratio of totals($\frac{y}{x}$). They couldn't connect the concept of slope as the ratio of difference ($\frac{\Delta y}{\Delta x}$) than ratio of totals($\frac{y}{x}$). The same conclusion was founded by the research of Tyne(2016). According to Tyne the ratio of totals approach to interpret the slope is the dominant incorrect reasoning. The above interpretation justified the fact that the students are struggling in understanding the concept of slope.

Majority of the student leave the questions 2b) which was "Define the differential coefficient of a function f(x) at a point and interpret it geometrically. Compute the slope of $y=(x^2+3)^2$ at x=2". If the students interpret correctly and able to find slope at x=2 then student has no difficult but if they were not able to interpret and find the slope then students had seen difficulty. Researcher and student's conversation about tangent and secant can be presented below:

I: Why do you leave the questions number 2b)?

Student A: I don't have any idea and I feel it so difficult.

I: Do you have any idea about tangent and secant?

Student A: Ummm.. tangent is line that touches the curve in one point and no idea about secant.

Student B&C: I think secant is line that touches the curve at two points.

I: Can we say tangent is limiting form of secant?

Student A: No idea.

Above interview justified the fact that student had incorrect concept about the tangent always touches only one point of the curve. Students had misconception about tangent that tangent touches in only one point of the curve. They had more difficult to interpret derivative as a slope of tangent geometrically.Student A could not differentiate the concept of secant and tangent. Student B&C has some understanding about secant line but they had also misconception about tangent line. Student didn't know the relationship between the tangent line and secant line. According to Park (2012) misconceptions in the

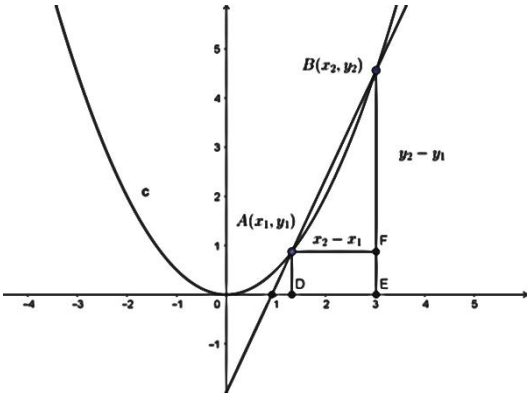
students is that a tangent line to the curve should intersect the curve only one at a tangency point and students tried to find the equation of tangent line of a curve. Hence we can easily concluded that they had very weak understanding about relationship about tangent line and secant line also they had misconception that tangent line touches the graph of function at only one point.Thus we can easily concluded that student had weak conceptual understanding about the tangent and secant.

Researcher showed the difference between tangent and secant by using graphical software and clearifies the tangent is limiting form of secant asked the questions given as below:

I: Also y_2-y_1 is called change in y which is denoted by Δy and x_2-x_1 is called change in x denoted by Δx then what is the slope of the secant line AB?

Student C: Confusing.

Student A: It is, $\frac{\Delta y}{\Delta x}$.



I: Good, When Point B tends to A then what happened can you see? (Researcher showed in a software)

Student A: Well the secant become tangent as we discussed above.

Interviewer: Good, can you find other thing?

Student A: No.

I: So, what about Δx *?*

Student A: No idea.

I: Ok, when B tends to A, Δx becomes very small that is tends to 0 in that time secant becomes the tangent and the slope of secant becomes slope of tangent. Are you clear?

Student A: Yes.

It can be seen that, students did not connect the concept of limit in the slope of tangent. Students had very weak concept about the tangent is limiting form of limit. Also they are struggle in the symbolic form of change in variables x and y. They didn't know the slope of tangent is the ratio of difference not a ratio of totals. Students were not able to connect the concept of limit in finding slope of tangent from the slope of secant. They had trouble in understanding how slope of secant becomes slope of tangent. When I showed them what happens when B tends to A in software then they were able to find out the main concept. They were confusing about what is meaning of tends to i.e. B tends to A. they were unable to understand tends to means not exactly but very close to. The same conclusion was found in the study of Ubuz (2007) that the students struggle with connecting graphical and algebraic representations and with converting symbolic information to graphical information (As cited in Abbey, 2008). So, the above information justified the fact that student had difficulty in the concept of tangent as limiting form of secant and they had difficult to interpret the derivative geometrically i.e. derivative as a slope of tangent line. Further discussion can be presented as below:

I: Now can you tell me the slope of tangent?

Student A: I think $\frac{\Delta y}{\Delta x}$.

I: But there is one condition we have to plus i.e. Δx tends to 0 right?

Students A: Ooo… Yes.

I: Now can we write slope of tangent = $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ *?*

Students A: Of course. Yes I understand, which is dy/dx also. Aaaa… it means derivative represents slope of tangent also right?

I: Excellent, Very good conclusion.

It can be concluded that student had lack of conceptual understanding of limit and they could not relate the limit with derivative. Students were not able to relate the concept that when we take limit in slope of secant then slope of secant becomes the slope of tangent. When I explained then now they were able to understand. To become a slope of secant equal the slope of tangent the one condition should be satisfied i.e. Δx should be tends to 0, then the students were able to connect the link between the slope of secant and slope of tangent. Also it can be seen that students had difficulty in symbolic representation. In this regards, study of Ubuz (2007), explored that students struggle with connecting graphical and algebraic representations and with converting symbolic information to graphical information (as cited in Abbey, 2008). On the same way the study of Aspinwall and Miller (2002), explored students expand their concept image when they include graphical and algebraic representations of the same concept. Teachers enhance student learning by presenting graphical and algebraic methods to solve the same mathematical problem. Hence, Student had so much difficulty in understanding geometrical meaning of derivative because of their teacher and themselves.Another reason is that they were not familiar the concept of slope of

straight line in their previous classes. Hence, lack of previous knowledge is the reason that makes derivative so difficult. Thus, we can concluded that students had more difficulty in the understanding of derivative as a slope of tangent.

For the further investigation to find out the causes of difficulty in understanding the geometrical meaning of derivative I asked "Tell me why you feel so difficult? Did your teacher explained like this?" then, Student A replied:

> *Sir we always focused on our examination and our teacher told us some little bit but not in detail like that. Also there is our mistakes too because I couldn't attempt class regularly. Every Friday we go to our home and we leave our class when we are returning.*

On the same question the student C replied that:

> *Sir firstly I don't know what is slope and what is tangent, the reason I think i did not take optional mathematics in class IX and X and also you know which type of teacher we have in our school. Our teacher never taught derivative using graph.*

The above conversation shows that the main reason of the students weak understandings were the not regularity in the class room, focusing the procedural knowledge by teacher, lack of pre knowledge etc. Students C said that he did not able to explain slope, tangent etc. He said that these difficulties were because of previous class of school level. Also it can be seen that the teacher should change their teaching style. They should focus on the conceptual understanding of derivative rather than the procedural understanding of derivative. Hence, Student

had so much difficulty in understanding geometrical meaning of derivative because of their teacher and themselves. Another reason is that they were not familiar the concept of slope of straight line in their previous classes. Hence, lack of previous knowledge is the reason that makes derivative so difficult. We can conclude that teacher's lack of explanations and focus on conceptual understanding about the derivatives makes the difficulty to the students. The same conclusion was found in the study of SelloMakgakga (2012). Thus, we can conclude that students had more difficulty in the understanding of derivative as a slope of tangent.

The entire student leaves the question number 3(Appendix-A). The students were asked to find the slope from the graph of derivative function. In the same context the interview with the students can be presented as below:

I: Do you know the derivative of given function is again the function?

Student A: Don't know. Derivative is value isn't it.

Student B &C: No idea.

I: Can we say the derivative function is function of slope of tangent?

Student A: No idea.

I: when we put the value of x in derivative function then the value of y gives the slope of the tangent at that point on original function?

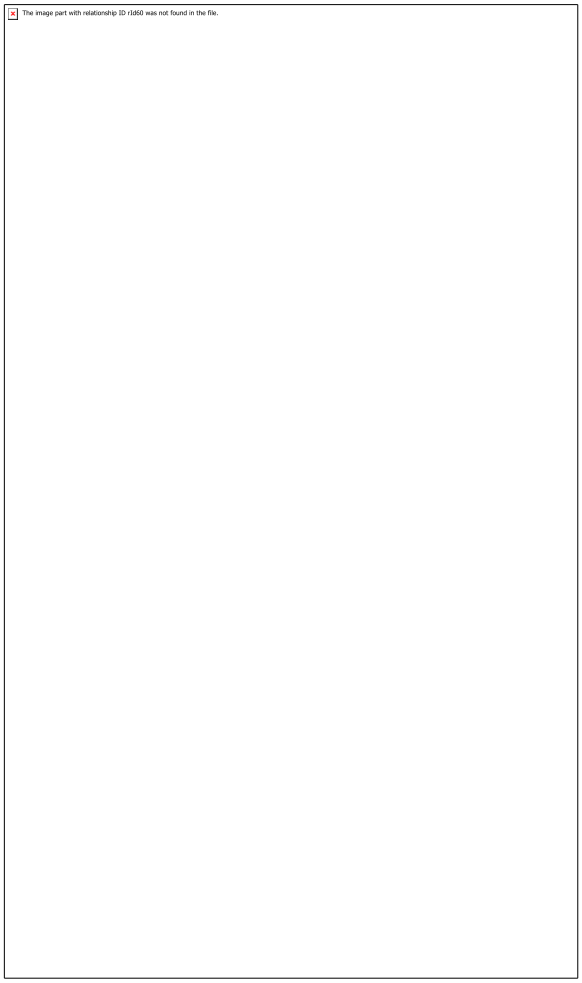
Student B: Oooo….we don't have any idea about this.

From the above discussion, researcher concluded that the student had no idea about the graphical understanding of derivative function. No one can explain the derivative function gives the slope of tangent of original function at given value

of x. In the given question (Appendix-A), when we put value of x=2 in the graph of derivative function then value of y gives the slope of tangent at x=2 in the original function. The student had no idea about the derivative of given function is again the function & which is the function of slope of tangent at a given point. The students were not able to find out the slope of the function at x=2.They were not able to understand that the graph of derivative function gives the slope of tangent at a given point in original function. They were become very surprised when I told them the derivative is a function. They had incorrect reasoning that derivative is value instead of function. The same conclusion was found by Park (2012), According to Park (2012) students may confused in two things first, derivative as a specific point value and the second is derivative as a function. Also the study of Kaplan (2015) concluded that students experience difficulty in doing the geometric representation of the derivative. Also in the study of ORHUN (2012) students were not able to interpret the graph of derivative function.The above interpretation justified the fact that student had more difficulty in the graphical understanding of the derivative function & finding the slope of original function when derivative function was given.

In the question number 4 (Appendix-A), it was asked to choose the correct graph of derivative function from given graph of original function with correct reason. It is the highest level questions based on APOS theory. If students were able to choose correct graph with correct reasoning then it can be considered as the students had no difficulty otherwise it can be considered they had difficult. From the test result it can be seen that the students had very much difficulty to choose the correct graph. They were unable to choose the correct graph of derivative of

given graph with correct reason. For example one of the answers of students can bepresented below:



hat Student students has difficulty en graph. Student A chosen the hows the maxima and minima. But in be seen that students had lack of They were not able to choose o choose the correct graph student should understand geometrical meaning of derivative i.e derivative is a slope of a tangent line. It can be seen that student were not able to understand the increasing (decreasing) function, stationary point and sign of slope of tangent at given interval. Hence from above it can be seen that students had very much difficulty in the graphical understanding of derivative function and original function.

The interview on the basis of questions 4 can be presented as below:

I: Do you know what is increasing or decreasing function?

Student C: *No.*

I: Is there any relationship in increasing (or decreasing) function and slope of tangent at a given point.

Student C: No idea.

Student A&B: Sorry.

I: Do you know interval and stationary point?

Student A: Not clear.

The above discussion shows that, student had very much difficulty in choosing the correct graph. My entire respondent felt much difficulty in graphical understanding of derivative function and original function. Students did not have clear concept about increasing function. They were not able to explain that the if the function is increasing on an interval then slope of tangent on that interval is always positive but when function is decreasing on an interval then the slope of tangent at every point of given interval negative. Also the students were not able to clear about the stationary point. Above interview justified the fact that students had no graphical understanding of derivative function. They were not able to identify derivative function of given function because of lack of graphical understanding of meaning of derivative. The same result was found in the study of Aspinwall& Shaw (2002), in which students expand their concept image when they include graphical and algebraic representations of the same concept. The study of Kaplan and et al. (2015) under the topic "Relieving misconceptions of derivative with derive" showed that students are unable to use the operation with the definition of the derivative .They did not consider the interval while finding the derivative.Students experience difficulty in doing the geometric representation of

the derivative. The study of ORHUN (2012) students hadn't interpreted the graph of the derivative function. They find it difficult to make connections between the graph of derived function and the original function. Thus from the above it can be concluded that the students have difficulty in understanding the derivative of given function.

All of the above discussion justified the fact that student had so much difficulty in understanding derivative as a slope of the tangent line. Students are struggling to understand the concept of tangent. Student has misconception about tangent which touches a curve at a point rather than it is a limiting form of secant which is the same result shown in the study of Park (2012). The students have difficulties in , limit , various terms and language such as limit, tends to , approaches, as small as we please etc., handling quantifiers ,symbolic representation,consequent student preference for procedural method rather than conceptual understanding as shown in the study of Tall (1992).Also student shave lack of graphical understanding of the derivative function. They were not able to find out the slope from the graph of derivative function. As a result of the study done by Acharya (2017) on the title "Factor Affecting Difficulties in Learning Mathematics by Mathematics Learner", Teachers lack of linkage between new mathematical concept and previously learned mathematics structure is also one of the reason that students felt difficult to understand the concept which is the same conclusion here. Teacher's approach of just wanting from students to memorize formula's without explaining the meaning of the formula's is again one of the another key things which affects to understanding students derivative conceptually which is also the result of the case study done by the SelloMakgakga (2012).

Understanding and Sense Making On Limit. This difficulty refers to the difficulty related to what exactly the limit is i.e. difficulty in define a limit of a function and meaning of tends to &other symbols and phrases and also difficulties to explain a limit of a function. If students are able to explain the all things which are mention above then it is considered that students has no difficulties in concept of limit. And if the students are unable to explain the things mention above then it is considered as students have difficulties in the concept of limit. The concept of limit is one of the three big ideas (rate of change, slope of tangent, limit) in understanding derivative (ZulalShahin 2015).In some literature, it is explained that the limit is the one of the fundamental concept to understand derivative. To find out the difficulties in concept of limit researcher has presented the question number 2d) in the CPUBT test and the difficulties were seen in the answer sheet and some interview questions.

The problem presented in the CPUBT was follows: "what is limit of a function and explain how the limit of a function y = x+3 as x tends to 0 is 3". In this problem student A and student C did not give any answer on their solution sheet but student B gave the definition of limit but he had not explain about how the limit of given unction is 3. The definition given by student B can be presents below as an example.

Analyzing the above definition of limit, it shows the students had more difficult to define a limit of a function and to explain the limit of a function. Students B defined it as a function f(x) is said to be a limit of function when it is in the form of lim x tends to a, (x-a)=0. From the definition given by student B we can easily understood that the students has no any clear concept about the limit. They were unable to define what the limit of a function is exactly. In the answer

sheet mentioned above he also gives the justification i.e. he answered the second part of the problem number 2d). From above we can easily conclude that students were unable to explain how the limit of a function x+3 is 3 when value of x tends to 0. It can be seen that students had no any conceptual understanding about right hand limit and left hand limit. They were struggling to understand the meaning of tends to. In order to confirm the difficulties related to the concept of limit researcher has conducted the interview with all respondents. The interview taken by researcher with student B can be mention as below:

I: Do you know what the limit of a function is?

Student B: Don't know clearly.

I: Is a limit of a function is again a function or particular value?

Student B: I don't know.

I: But you write limit of a function is again a function in your answer sheet?

Student B: I wrote it carelessly.

I: Can you find the limit of a function f(x) = x+3 as x tends to 0.

Student B: Sure. The limit of given function is 3.

I: But how?

Student B: Put the value of x=0 in given function.

From the above discussion, it can be seen that the student has very much difficulty in understanding the concept of limit. Students were unable to define limit of a function. Student B has the misconception that the limit is again a

function and it has a form. They were not able to understandthe limit of a function is always a value. Another thing is that Student B has no clear idea about the meaning of tends to. Students were understood that x tends to 2 means the exactly the value of x=2. When they calculate the limit of function they exercised in such a way that x tends to means exactly. Students were not able to justified the limit of a function f(x) = x+3 is 3 when x tends to 0. This shows students had procedural knowledge they found that the limit of a function is 3 but they were not able to explain how? Same as the study of Abbey(2008) shows students' preference for procedural knowledge, resulting from their prior success with those procedural skills, inhibits their conceptual learning.

Also,In a study conducted by Tarmizi (2010), found that students did not fully understand function notation, memorized a procedure, and could not verbalize what they were doing in the problem. The same conclusion was found by Tall(1992), indicated that the students have difficulties in , limit and infinite process, various terms and language such as limit, tends to , approaches, as small as we please with respect to etc., handling quantifiers ,symbolic representation,consequent student preference for procedural method rather than conceptual understanding. Hence from the above interpretation it can be easily concluded that students had very much conceptual difficulty in understanding limit. The further discussion can be presented below:

I: If then, could you tell me the limit of a function, $f(x) = \dfrac{x^2-4}{x-2}$ as $x \to 2$ *?*

Student B: It is 0/0 not determined.

I: What is the meaning of x tends to 2. Do you think x tends to 2 means, put the value of x exactly 2 on given function?

Student B: Yes we find the limit of a function in the same way but it is 0/0 I couldn't understand.

It can be seen that, Student B had misconception about the meaning of "tends to". Students thought that x tends to 2 means value of x is exactly 2. This shows they had lack of conceptual understanding of limit. Another remarkable thing is they had been focused on procedural understanding of limit. When I asked to find limit of function $f(x) = \frac{x^2-4}{x-2}$ as $x \to 2$ then they become very much surprising. These all facts justified that the students did not understands the symbols, phrases like "tends to" and other conceptual understanding. They memorized the process and practiced the exercise. The same conclusion was found by Tall(1992), indicated that the students have difficulties in , limit and infinite process, various terms and language such as limit, tends to , approaches, as small as we please with respect to etc., handling quantifiers,symbolic representation,consequent student preference for procedural method rather than conceptual understanding.

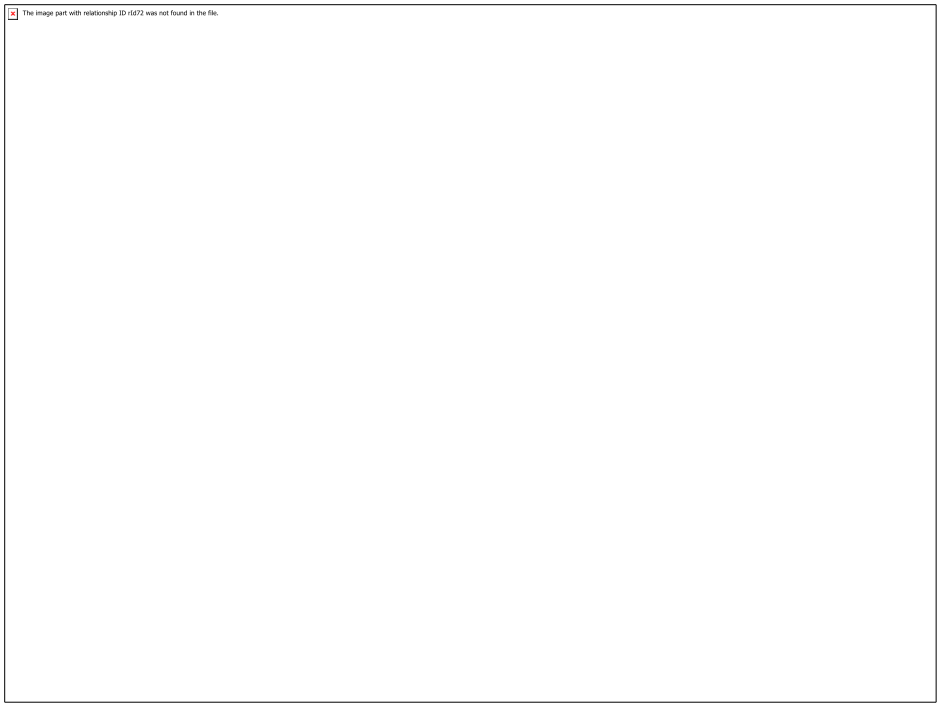
From the above interview I concluded that the students have very much difficulty about the concept of limit of a function. They have a misconception about limit of a function is a particular value or the limit of a function is again a function. Also students cannot explain what is the meaning of the symbol $x \to a$, They did not able to explain what the $x \to 2$ mean?. It can be concluded that student couldn't explain the how the function x+3 has the limit 3 as x tends to 0.

As a result of study It can be seen the students had difficulty to define a limit of a function. The students had difficulties in , limit and infinite process, various terms and language such as limit, tends to , approaches, as small as we please etc., handling quantifiers ,symbolic representation,consequent student preference for procedural method rather than conceptual understanding These all are because of lack of conceptual understanding.

Procedural Difficulties in Learning Derivative

The procedural difficulties in this research includes the difficulties in finding derivative by using power rule, difficulties in finding derivative by using chain rule and difficulties in finding derivative from the first principal.Procedural knowledge is defined by Heibert and Carpenter (1992) as a "sequence of actions"or skills (p. 78). Solving routine problems is performed with minimal understanding of the mathematical principles involved (Heibert& Carpenter, 1992) as cited in Abbey (2008). If the students are able, to find derivative from first principal, to find derivative using chain rule and to find derivative by using power rule then it is considered that they have no difficulty but if they could not find the derivative by using, chain rule, power rule and from first principle then it can be considered that student have procedural difficulty in learning derivative. To find out the procedural difficulty I used the three questions related to using power rule and chain rule and one question related to first principal on the CPUBT test. The procedural difficulties include the following:

Finding Derivative by Using Power Rule. There were two action level questions 1a) Find the derivative: $\frac{d}{dx}(3x^2+4x-2)$ & 1b) Find the derivative: $\frac{d}{dy}\left[\left(y^2+3\right)^2\right]$ in CPUBT test to find difficulties related to finding derivative by using power rule.The question number 1b) is the generalized power rule. If the students find out derivative correctly by using power rule and generalized power rule then student had seen no difficulty in using power rule otherwise it can be seen they had difficulty in using power rule to find derivative.The student B and student C had no difficulty in finding derivative. But student A had some difficulty. The answer of the student A can be shown below as an example:



Analyzing the above answer sheet, it can be seen that the student A has more difficulty to find the derivative by using power rule. He did first question

easily but in second question it can be seen that he had misconception about symbol of derivative. The questions asked to find derivative with respect to y but he did with respect to x. From this it can be easily concluded that the student A has difficulty in the concept of function and difficulties on understanding the meaning of finding derivative with respect to. To ensure the difficulty i conducted interview which can be presented as below:

I: In which variable the given function is given?

Student A: Well it is given in y.

I: The question is asked to you to find the derivative with respect to y but you find it with respect to x, do you know the meaning of with respect to?

Student A: This is a mistake. I realize it.

I: So in what respect you had to differentiate the given function?

Student A: I had to differentiate with respect y.

I: Why?

Student A: I am not sure about this, I have some confusion. Our teacher never goes in such detail.

The above discussion shows that even some student did not have any difficulty but some student had difficulty in finding derivative by using power rule. It can be seen that student A had no understanding about generalized power rule. And student did not know in which condition the power rule is used. Student A did not understand that the given function $(y^2+3)^2$ is the function of y^2+3 so first student should differentiate given function with respect to y^2+3 and again y^2+3 is

the function of y so student should differentiate with respect to y. This is called chain rule also. It can be seen that Students had lack understanding of when there is a function of function is given in such condition generalized power rule and chain rule is used. They had lack concept about the function and some terms used in differentiation.The same conclusions are also mentioned in the study of Tarmizi (2010). Also the difficulty arises because of teachers. Teacher did not clarify to the student about the situation, condition in which the power rule, chain rule is used which is also mentioned in the study of SelloMakgakga (2012). Hence we can conclude that the students had the difficulty in the using power rule to find the derivative.

Finding Derivative from First Principle. This difficulty refers the difficulty in finding derivative by the definition method of derivative. Only the process of finding derivative was seen to find difficulty in this category. If the student cannot able to find the correct derivative by using first principle then such difficulty is categorized as difficulty in using first principle. To find out the difficulty one process level question was to the student. Which is " Find the derivative of $(2x+3)^{1/2}$ by using first principle".

In this case, it was found that the respondents had correctly found the derivative by using the first principle. From this I came to conclude that there were no difficulties in the finding derivative using first principle. Because I had mentioned the difficulties as "Weather the students can or cannot find out correct derivative. In this way all the respondents find out the correct derivative. But this result is opposed to the result of the study of SelloMakgakga (2012) in which it is mentioned that Students have weak performance in finding derivative from first principles but the student performed better in the finding derivative of a function

by using rules of differentiation than using first principles. All of the fact mentioned above justified that there were no difficulty in finding derivative by first principle.

Understanding on Using Chain Rule. This difficulty refers the difficulty in finding derivative by using derivative. Only the process of finding derivative was seen to find difficulty in this category. If the student cannot able to find the correct derivative by using the chain then such difficulty is categorized as difficulty in using in finding derivative using chain rule. To find out the difficulty one action level question was asked to the student. Which is " Find dy/dx if $y = 4z^2-3z+5$ &$z = 2x^2-3$".

In this case it was found that the respondents had correctly find out the derivative by using the chain rule. From this I came to decide that there is no difficulties in the finding derivative using chain rule. Because I had mentioned the difficulties as "Weather the students can or cannot find out correct derivative. In this way all the respondents find out the correct derivative. But this result is same as the result of the study of SelloMakgakga (2012) in which the student performed better in the finding derivative of a function by using rules of differentiation than using first principles.All of the above mention fact depicts that in this research there were no difficulty in finding derivative using chain rule.

Chapter V

FINDINGS, CONCLUSIONS AND IMPLICATIONS

This chapter deals with the summary, findings, conclusion and recommendation concerning the student's conceptual and procedural difficulties in learning derivative. After rigorous analysis and interpretation of data, the findings of the study have been derived and conclusions have been made based on findings, the implications have been forwarded in different levels. This chapter is divided into three sections findings, conclusion and recommendation for educational implication.

This study was entitled "conceptual and procedural difficulties in learning derivative." The purpose of this study was to explore the difficulty related to the conceptual and procedural understanding of derivative. The design of the researchwas case study. The study was conducted the purposively chosen case students of grade XI of Makwanpur Multiple Campus. To achieve the objective of the study, data and information were collected through CPUBT test and interview was conducted with three students selected on the basis of test result. The researcher analyzed the interview data according to General inductive approach by Thomas (2006).Cross matching and triangulation was done to analyze the collected data. Hence, from the deep analysis and interpretation of the collected data the derived findings can be presented below:

Findings and Discussion

The findings of the study were drawn from the deep and systematic analysis and interpretation of the collected data. This study was case study about the conceptual and procedural difficulties in learning derivative. The main purpose of this study was to explore the conceptual difficulties and the procedural difficulties. For this purpose I included two types of difficulties conceptual(Derivative as a rate of change, Derivative as a slope of a tangent, Concept of limit) and procedural(Finding derivative using first principle, Finding derivative using chain rule, Finding derivative using power rule). From this case study the results of the data analysis showed that students felt difficulties in following areas:

Weak Concept to Understand the Derivative as a Rate of Change.

Students could not understand the concept of derivative as a rate of change. Students did not able to understand the meaning of average rate of change and instantaneous rate of change. The symbolic form and the verbal form is very important in the derivative concept. Students did not able to differentiate the symbolic form of average rate of change and instantaneous rate of change. Also students did not able to understand the concept of function and function notation. Also, they did not able to understand the derivative of a function is again a function not only the particular value. They just memorized the procedures of finding answer.Students did not able to explain their answer sheet. Students solved the problem which was asked in CPUBT test without understanding meaning of rate of change. The other difficulties are the students did not understand the various terms, languages, phrases like "with respect to", "differentiating", "tends to", "approaches" etc. All these things mentioned above justified the fact that the students had difficulties in understanding the meaning of derivative as a rate of

change. It's all because of lack of pre-knowledge, lack of multiple representations of concept, teacher's weak performance, focusing on procedural understandinglack of understanding of language and phrases etc.

Unable to Understand the Clear Geometrical Meaning of Derivative.

Students had difficulty to understand the geometrical meaning of derivative as a slope of tangent at a given point. Students did not able to understand the concept of slope. Students who have some concept about slope, failed to differentiate the slope is not the ratio of totals but is the ratio of difference. Students did not able to find the slope of the function $y= (x^2+3)^2$ at $x=2$. They did not able to understand the derivative of a given function is the slope function and if we put the value of x it gives the slope of tangent at that point in curve of function. Students had difficulty in connecting the geometrical meaning of average rate of change and instantaneous rate of change with the slope of secant line and slope of tangent line. Also they did not able to connect the symbolic representation of slope of secant and slope of tangent. The another things was they had weak understanding of concept of limit, various terms and languages.

None of the students had given the answer of question number three and four(See appendix -A). They did not able to find the slope when the curve of derived function of given function was given. This showed that they did not understand curve of derivative function gives the slope. Also all students did not able to answer question number four (Appendix- A). They did not able to choose the correct graph of the derivative of given graph with reason. All the fact justified that the students had very weak understanding of the geometrical meaning of derivative. The main causes of this difficulties are the misconception about the

slope as ratio totals instead the ratio of difference, misunderstanding of derivative as a particular value or derivative as a function, lack of pre knowledge, lack of understanding of graph, lack of understanding the multiple representation of same mathematical concept, weak performance of teacher, lack understanding of meaning of limit.

Unable to Make Exact Sense of Limit Necessary to Study Derivative.

Students did not able to understand the concept of limit of a function. They had misconception that limit means the directly putting the value of x in the function. Students did not have the concept of right hand limit and left hand limit. They did not able to explain the meaning of "x tends to 2". Students did not able to explain how the limit of function x+3 as x tends to 0 is 3? Students had weak understanding of phrases like tends to approaches to, limiting value, tending value, limit as infinite process etc.These all things justified the fact that the students had the difficulty in conceptual understanding of limit. The main causes of difficulties are the lack of understanding of quantifiers like tends to, limit, approaches etc, focusing on procedural understanding, teachers weak performance, students irregularities, becoming exam oriented etc.

Unable to Use Power Rule to Find Derivative.

Students had difficulty in using power rule. Even my two respondents use correctly but one respondent did not able to use it but they did not able to explain in which condition we use power rule. They could not understand the generalized power rule. They could not able to connect the link between generalized power rule and chain rule. They did not explain that when differentiate the function $(y^2+3)^2$ with respect to y, first we differentiate it with respect to y^2+3 because

$(y^2+3)^2$ is the function of y^2+3 and then we differentiate the y^2+3 with respect to y.Hence it can be seen that students have difficulty in the finding derivative using power rule.

Conclusion

From the deep analysis, interpretation and findings, I concluded that the students have more difficulty in conceptual understanding of derivative which may discourage the students to take the study of derivative easily. Based on above findings I concluded that the students had no any procedural difficulties in finding derivative although one respondent had weak understanding on using power rule. Also, I concluded the main causes of difficulties are the students more focusing on procedural understanding, lack of multiple representation of derivative, lack of graphical understanding of function, lack of pre knowledge, weak performance of teachers etc. Hence, it can be concluded that the teacher are focusing only procedural understanding of derivative and students becomes exam oriented which makes the derivative as one of the difficult topic. Thus, both teacher and students should focus the conceptual learning of derivative and pre knowledge of derivative such that they could easily understand the concept of derivative. Thus, teacher should change their teaching style so they can make their classroom very fruitful and learning derivative become a meaningful which avoids the rote learning.

Educational Implications

This study was the case study that provided the deeper understanding of the conceptual and procedural difficulties in learning derivative**.**The results which were drawn from the deep & systematic analysis and interpretation of the collected data have very strong educational implication.The results from this study can provide important consideration for teacher, students, curriculum developers and others policy makers as well. This study has a very strong educational implication. It helps to the teachers to focus more conceptual understanding of the derivative that makes easy in procedural understanding. Also this study helps to the curriculum developer and the textbook writer so that they include the sufficient content, methodology, evaluation technique in the related works. In addition, the policy maker also can make the suitable policy in the field of education. Furthermore, the educational implications of this study are given below:

- Students should not be exam oriented and process oriented rather they should focus on the conceptual understanding of the mathematical concept.
- Teacher should be more careful and should perform in classroom by focusing the conceptual knowledge and pre knowledge.
- Both teacher and students should think that conceptual understanding enhances the procedural understanding of derivative otherwise rote learning would increase.
- This study helps the school administration to enhance the conceptual understanding of their students.
- Teacher and students should be aware about rote learning rather they should try to seek the meaning of the concept.

Recommendations for Further Study

This is the case study of grade XI student's conceptual and procedural difficulties in learning derivative. This study consisted only the students of grade XI of one school. So, the findings and conclusion drawn from the study cannot be generalized. The researcher had tried to make some suggestion for further study in this field.

- This type of research can be conducted in the higher level.
- Further research can be done on the area of function and limit.
- Researcher only took the public school in this study. The researcher suggests conducting such type of research on the various private schools.

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