## Chapter I

## Introduction

## Background of the Study

Difficulties are the obstacles and problems of learning concept and process of knowledge. Therefore there are two types of difficulties that occur on mathematics learning; conceptual and procedural. Conceptual difficulties are the difficulties on perception of the concept of the subject. Conceptual understanding in mathematics means that students understand which ideas are key(by being helped to draw inferences about those ideas) and that they grasp the heuristic value of those ideas, knowledge and skills (Wiggins, 2015). Procedural difficulties are the difficulties on the appropriate use of already defined rule, structure and skills. Procedural fluency is the ability to apply procedures accurately, efficiently, flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply then other (NCTM, 2014). Curricular factors of difficulties occur due to the cause of abstractness of content, and organization of the content. Content related factor are the intrinsic factor of the subject which creates problem on learning geometry. There are some environmental factors those create the difficulties on learning geometry. Environmental factor creates artificial problems on learning geometry. If we can decrease the effect of environmental factors then it helps to decrease the effect of curricular content factor of difficulties.

Geometry comprised the various aspects of competencies such as understanding the geometrical objects and its characteristics, properties, logical proofs and its application. There are so many objects such as point, line, angle, plane, vertices, triangle, square, rectangle, rhombus, circle, etc. We used this concept widely
in our secondary school curriculum and these are equally important on daily life. When I was studying at secondary level, I was very interested to learn geometrical concept I was always confused about the theoretical proofs of the theorem. The text book gives the certain number of basic axiom and theorems. But question were asked from the aspect of combination of these theorems and applications on particular example. It becomes zig-zag for me about where I entered and how I completed it. But when I studied about geometry at upper level I felt very interesting and found that there were various number of manipulative, audio-visual as well as computer based software like as geo-gebra are influential for learning geometry. So, I am interested to research on the areas of difficulties on learning geometry in secondary level school students.

Geometry is the prime branch of mathematics which was developed initially during the development of mathematics. As further, we can easily say that Geometry is directly connected with the development of human civilization (Egyptian, Babylonian, Chinese, Hindu, etc.) where they used geometry as their needs differently. Geometry studies about physical objects. So, I am interested to say "Where there is no Geometry, there is no mathematics". Therefore, Geometry conceals a huge importance on various branch of knowledge like as arithmetic, algebra, Engineering etc. Geometry is an aspect of mathematics which deals with the study of different shapes. These shapes may be plane or solid. A plane shape is a geometrical form such that the straight line that joins any two points on it wholly lies on the surface. A Solid shape on the other hand is bonded by surfaces which may not wholly be represented on a plane surface (Emerg, 2011).

Mathematics is not itself a difficult subject. It has its own structure and language. David Hilbert said that 'mathematics is nothing more than a game played
according to certain rules with meaningless marks on paper' and Galileo said that 'mathematics is the language with which god wrote the universe '. These two definitions of great mathematician clearly justify the importance of mathematics. Mathematics education is not only concerned with what is learned? But concerned with what and how to teach and learn the mathematics? Mathematics education concealed the philosophical, psychological, cultural and technological foundations. Therefore teaching and learning mathematics is a complex procedure. There are various learning theory appears such as behavioral learning theory, cognitive learning theory, social learning theory, socio cultural learning theory etc. And there are various types of teaching and learning methods such as lector method, heuristic method, laboratory method, problem solving methods, problem based learning etc. Van Hiele's teaching and learning procedure is very significant and famous for geometry teaching and learning. Also, Polia's problem solving approach is very appropriate for geometrical problem solving. There are various materials such as manipulative, audio, audio- visual, visual, synchronized and unsynchronized as well as mathematical software such as geo-gebra are more influential for learning geometry.

Nepal gives the respected place for mathematics curriculum. School mathematics has eight fundamental sectors of mathematics among them, geometry is one. Secondary level school geometry contains chapters of triangles, similarity, parallelogram, circle, areas of triangle and quadrilateral, coordinate geometry, etc. (CDC, 2071 BS ). Output of the mathematics achievement is very low in SEE examination. And another great problem is outcome of the mathematical achievement on practical field. So, stakeholder, government of Nepal, parents and student itself dissatisfies among it and becomes subject of anxiety for all. Among the content of
mathematics, geometry becomes more tension for teachers and students and they speak that it is useless why? But, I think there is a wrong conception among them.

There are two types of school existed in school education one is public and other is institutional. Parents and students interests increase in institutional school where they pay more money. Government spent more money on public education. But student's achievement on mathematics has not satisfied. Where is the problem in mathematics achievement of students being going to low? Geometry is a one area of school mathematics. So, I studies about the difficulties faced by secondary level students in learning geometry.

## Statement of the Problem

Geometry studies about mathematical objects. It is fundamentally connected with the development of human civilization. It plays crucial role in the development of today's advance mathematics. When I studied primary level, teacher had given task to make straight line, triangle, square etc. At that time I was not able to make straight line, triangle, and square as their property but as topological structure. And when I studied secondary level, geometry became more rigorous because of the different terms, definition, axioms, theorems and its application. Teachers always have been given the priority for rote learning and teach some selected questions and theorems as examination point of view. I came very disappointed from it. But when I studied further about geometry, I see the interconnection between different branches of mathematics. Then, I decided to research about why geometry became difficult for students? Today there are various learning approach and resources invented such as mathematical software geo-gebra. If we teach and learn geometry appropriately, we expect it will help students understand geometry easily.

Nepalese school curriculum gives major emphasis for the mathematics. Geometry is a main area of school mathematics curriculum of secondary level. But teachers and students have taken geometry as abstract portion. They recognized geometry curriculum for gifted students and remembers geometric proofs are difficult to do for general student who have below average achievements (Upadhyay, Pardhan \& Dhakal, 2067 B.S.). Both the teacher and students faced several difficulties in teaching and learning geometry. So, students have negative believes which makes it more difficult to read and learn (Thakur, 2018). Geometry is a core difficult area where student performance has always been low (Emerge, 2011).

Geometry has been resolving more tension for students and teacher because of the low out-puts on SEE and outcomes on application. So I am interested on the curricular content factors and environmental factors that create the difficulties on learning geometry. Some research has been done on difficulties of the geometry learning. But I found these are more theoretical. But I see some gap between the existing research and the practical situation. So, my study has concerned with identify students learning difficulties in geometry who read on public school classroom. Both, teacher and student faced several difficulties in teaching and learning geometry. Many students have delicate negative believes which makes it more difficulties to read and learn (Thakur, 2018). So, my concern is here, why most student are becoming poor in learning geometry?

## Significance of the Study

Geometry is more influential and essential area of mathematics and it is also very realistic to know and useful in our daily life. This research had been done to find content related factors and environmental factors of difficulties on learning geometry in secondary school students. Teacher and students used classical approach and
material for teaching and learning geometrical content. After the research policy maker could be considered about quality education, Teacher and students can used recent developed methods and technologies such as geo-gebra for visualization of abstract concept of geometry, SMC can manage basic facilities for mathematics teaching and learning, researcher can use for further research etc.

The main importance of this research has been done to understand where are the difficulties on learning geometrical concept on secondary level school students? And what are the causes of difficulties? It provides appropriate information about the difficulties faced by the students in learning geometry. The findings could be supported to improve for mathematical achievement of student in geometry if research findings are appropriately adopted.

## Purpose of the Study

The main objectives of the study are;

- To explore the curricular content factors those produce the difficulty in learning of geometry.
- To identify the environmental factors those produce difficulties in learning geometry.


## Research Question

The study based on the following questions;

- What are the conceptual difficulties on visualization of geometrical objects?
- What are the conceptual and procedural difficulties on experimental verification of geometrical statement?
- What are the conceptual and procedural difficulties in construction of geometrical figures?
- What are the conceptual and procedural difficulties on theorem proving?
- What are the conceptual and procedural difficulties on application of the concept and theorems on application?
- What are the environmental factor that could be creates learning difficulties of geometry in secondary school students?


## Delimitation of the study

This study was conducted to find out the curricular content factor and environmental factors of difficulties of learning geometry in secondary level school students. This research is based only on remote area's public school. The sample of this research has taken from Arghakhanchi district. Only100 students were involved in this research. The finding of this research can't be generalized other than remote areas public school of Nepal.

## Operational Definition of Key Terms

Difficulties. The conceptual and procedural problems in understanding geometrical terms, definition, properties, theorems, and their application

Conceptual difficulties. Difficulties on perception of the concept of the subject such as understanding the definition, axioms and properties of geometrical objects. i.e. student can't find out the difference of complementary and supplementary angle.

Procedural difficulties. Difficulties on appropriate use of already defined rule such as appropriate use of formulas, logic development (convincing argument) on theorem proving, solution strategies used to solve the problem and calculation etc.

Definition. Mathematical statement defines based on the undefined and defined terms and justified about mathematical objects such as point, line, triangle etc. i.e. A point is that which has no part, a line is breathless length, square is a plane figure with four equal straight sides and right angles.

Axioms. A statement or proportion which is regarded as being established, accepted, or self-evidently true.

Experimental verification. Justification of mathematical statement by using meter scale and compass.

Theorems. A general proportion not self-evident but proved by a chain of reasoning; a truth established by means of accepted truths. i.e. The sum of opposite angle of cyclic quadrilateral is supplementary, the sum of interior angle of triangle is $180^{\circ}$.

Curricular content factors of difficulties in geometry. Difficulties appeared according to the content factor on visualization, experimental verification, construction, theorem proving and application.

## Chapter II

## Review of Related Literature

Geometry is the fundamental discipline of the mathematics teaching all over the world but how it is going to decline among other discipline of mathematics curriculum. And why teachers and students are feeling the difficulties to teach and learn the geometrical concepts which are extremely important in human life and the study and implementation of other discipline of knowledge like as algebra, arithmetic, economics, logical reasoning and others. So I am interested to diagnose the causes and problem where is lapses to learned the concept of geometry. I read and find out the following findings among my objectives of the study on the various literatures which are published in national and international scenarios.

## Thematic Review

Historical background of geometry curriculum. The word geometry comes from the Greek words geo+metron meaning "earth measure "which taken literately imply that geometry involves measuring earthly things. Ancient geometry, in past had its beginnings in the practical mensuration necessary for agriculture of the Babylonians and Egyptians (Wallace \& West, 1998). Webster dictionary defines Geometry as a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces and solids ,broadly studies of properties of given arguments that remain invariant under standard transformation. Mathematics is a compulsory subject from the initial stage of school education and informally subject is dealt in pre-primary level also. Therefore, learning of Mathematics is important for every person which help her/ him throughout her/his life. As Mathematics is a compulsory subject, receiving quality learning in Mathematics is a right to every child (NCERT, 2012). Geometry is one of the
curricular areas covered by the NCTM content standards and defined as a "branch of mathematics that deals with the measurement, properties and relationships of points ,lines, angles, surfaces, and solids; broadly : the studies of properties of given elements that remains invariant under specified transformations" (NCTM, 2000).

During the second half of twenty century geometry seems to have progressively lost its former central position of mathematics in most countries. The decrease has been both qualitative and qualitative .Geometry is one of the most important subjects of the school education. While teaching geometry, we not only give a pupil a lot of useful knowledge. But also develop him spiritually, intellectually, and bring a child up in a cultured way. In during so, the role of geometry is equally important for teaching both gifted and those fall behind with their development (Upadhyay, Pardhan \& Dhakal, 2067 B.S.). All over the world the mathematics curriculum recognizes geometry as a privileged field for the development of explanation, argumentation and proof (NCTM, 2000).

Nepal gives the respected place for mathematics curriculum. School mathematics has eight fundamental sector of mathematics among them geometry is one. Secondary level school geometry contends chapters of triangles, similarity, parallelogram, circle, areas of triangle and quadrilateral, coordinate geometry, etc. (CDC, 2071 BS).

Understanding difficulties of geometrical objects. Geometry is a part of mathematics that deals with forms, shapes and the sizes of objects. There are many different kinds of shapes. Some are two-dimensional like circles, squares or triangles; others are three-dimensional like cubes, balls or cones. Geometry shows us how we can construct or draw such forms and how to measure them (Rosmanitz, 2014). Plane geometry deals with objects that are flat, such as triangles and lines that can be drawn
on a flat piece of paper and three-dimensional, having width, depth and height, Solid geometry deals with objects in that space such as cubes and spheres (John, 2012).

Secondary level school mathematics contains abstract geometrical objects and notation like as similarity $(\approx)$, congruency, SAS, SSS, RHS, ASA etc. axioms, triangle( $\Delta$ ), quadrilateral( $\square$ ), parallel, perpendicular, tangent, chord, in-center, circumcircle, encircle, angle bisector, alternate angle, co-interior angle, corresponding angle, adjacent angle, etc. Whereas student must become confuse about how to use and where to use?

Geometry has abstract notations to be learned so that all those notations cannot be just transferred into students mind like a bunch of information that should be memorized. Students need to construct those concepts during their learning process. In learning mathematics, students interact with representations of mathematical objects and abstract mathematical ideas (Chazan \& Yerushalmy, 1998). Effective mathematics instruction calls for student engagement with mathematical objects to knowledge. In school mathematics, particularly the learning of geometry requires appropriate instructional experiences that can assert student to sequentially move their thinking from concrete to abstract.

Student difficulties in learning geometry. Geometry can be very fun for students, but if they start in a state of mind that they hate Geometry, it would be very unfortunate and can hinder their success ( Martins \& Melo, 2015). Many textbooks and many district pacing guides emphasize numeracy, arithmetic, and algebraic reasoning. Geometry (along with data and statistics) is often tucked into the final chapters of the book and the final weeks of the year after state testing (Fulton, 1994).

Although student with mathematics learning disabilities struggle, there exist a broader category of students with mathematics difficulties are in-defined as ranging
from low average performance to well below average performance. Most specially, identification of mathematics difficulties is commonly defined as a standard mathematical test score that falls below the $35^{\text {th }}$ percentile (Mazzocco, 2007). Maccini, Strickland, Gagnon, \& Malmgren, 2008), summarized characteristics common to student struggling in mathematics: a) memory problems that include remembering and using multiple steps to solve a mathematics problem, b) receptive (e.g. comprehending mathematics problems) and expressive (e.g. justifying an answer) mathematics vocabulary difficulties and: c) cognitive difficulties that may inhibit processing of mathematics concepts, procedural strategies, and rules. These characteristics may negatively affect secondary level student performance in geometry, particularly when computing the area of geometric figures.

Areas and causes of student difficulties of learning geometry. Every year achievement of student in SEE examination, the achievement of mathematics has seen as low achievement. Why it is? Why are not students been able to learn mathematics well? Where is the problem? Is the question always rising in my mind? There will be more causes and areas whereas the student does not give the interest to learn the mathematics and probably they read the mathematics just as to pass the examination and take it as a boring subject. Bhattrai (2005), made a study entitled "A study on problems face by the mathematics students in existing curriculum". He concluded that learning mathematics in secondary; level was distributed by so many factors such as: lack of teacher involvement in curriculum planning, deferential and instructional facilities and aids students with weak background in the subject matter, student defective promotion. Policy lacks of opportunity are given to upgrade their knowledge and huge number of personal problems faced by students and teachers. Poudel (2007), did a study on "problems faced by lower secondary mathematics teacher in teaching
geometry" with the aims to identify the problems faced by lower level mathematics teacher in teaching geometry related to curriculum. He conclude that the geometry teaching learning isn't effective because of curriculum, text book, physical facilities, teaching learning activities, materials, methods, and student evaluation techniques.

Moreover, both trained and untrained teachers are having similar problems like crowded numbers of students, lack of math's lab, poor evaluation process. Negative attitude towards geometry is also psychological problems. Sapkota (2017), did a study on "exploring learning difficulties in school level geometry ". she finds outs the main aspect of causes of difficulties in learning geometry are the poor teaching method, classroom environment, learning activities, evaluation technique, motivation and interest and lack of pre knowledge among content. Thakur (2018), did the study on "exploring student learning difficulties in geometry ". He concludes that causes of learning difficulties in geometry at secondary level are: lack of teaching materials, lack of time to use various methods, lack of locally available teaching materials, Lack of laboratory in school, lack of graph and bulletin board, lack of reference books and teachers guide, lack of training of teacher and lack of interacting between student and teachers. The research (Adolphus, 2011) on the problems of the mathematics curriculum and their implementation are as follows:

- Failure of the mathematics to relate the child's environment and thus the child can't see the importance and immediate application of mathematics, in particular geometry on day to day living.
- The Nepalese education system has been constituted in such a way that teacher training, curriculum development, and classroom practice are three separate activities.
- Non ability of instructional materials such as text books, work books, sliders, film strips, etc.to meets the pedagogical demands.

Instructional techniques of learning geometry. According to the NCTM standards, mathematics instruction should be active, social and interactive (NCTM, 2000). Effective mathematics instruction of the secondary level involves a)the use of small , interactive groups; b) using extended practice with feedback; all of which are components pre-mediated instruction( Swanson pre-mediated instruction( Swanson \& Hoskyn, 1998). Specially, peer-mediated instruction involves pairs of student working in a collaborative manner on structured, individualized activities (Kunsch, Jitendra, \& Sood, 2007). Many others have addressed geometrical thought based on Van Hiele's model. This model proposes a sequential progression in learning geometry through five discrete and qualitatively different levels of geometry thinking; visual, descriptive / analytic, abstract/ rational, formal deduction and rigor.

Teaching and learning of proofs in geometry. Secondary school geometry curriculum contains two types of proofs one is experimental proofs and other is theoretical proofs. Among them student feels more difficult in theoretical proof which comprises chain of reason to justifies the statement. A proof is an argument, a justification, a reason, that something is true. It's got to be a particular reasoning-logical-to be called a proof.

A number of reasons have been put forward for these student difficulties with proof. Amongst these regions are that learning to prove requires the co-ordination of a range of competencies each of which is, individually, far from trivial that teaching approaches tends to concentrate on verification and devalue or omit exploration and explanation, and that learning to prove students making the difficult transition from a computational view of mathematics to a view that conceives mathematics as a field of
intricately related structure (James , 2012). (Villiers, 2010), point out that, in addition to explanation, proof has a range of functions, including communication, discovery, intellectual challenge, verification, systemization, and so on. According to Polya (1957), mathematical proof should be taught because it helps in (1) acquiring the notation of intuitive proof and logical reasoning; (2) understanding a logical system; and (3) keeping what is learnt in one's memory.

## Review of Van Hiele's Theory of Geometry Teaching and Learning

There are so many learning theories that are widely used in the process of teaching and learning. Among them according to the nature of studies van Hieles model of teaching and learning is very influential and efficient for geometry teaching and learning. The van Hieles theory describes how young people learn geometry. It postulates five levels of geometric thinking which are labeled visualization, analysis, abstraction, formal deduction and rigor. Each level uses its own language and symbols. Students or pupils pass through the levels "step by step". This hierarchical order helps them to achieve better understanding and results. This article presents an overview of the model. It is focused on possibilities how to apply this theory on Czech mathematical education. First of all students needs to visualize the geometrical objects and clear the information about what is given and what is need to fiend and then student goes on the practical experimentation and then theoretical deduction and rigor.
van Hiele's also purposed the idea of teaching module for facilitating student to alleviate these levels of thinking based on van Hiele's model of teaching geometry, these are five phases that need to be accomplished by students: (1) inquiry or
information; (2) Direct orientation ; (3) Explication; (4) Free orientation; (5) Integration (Crowley, 1987). Inquiry or information phase is an internal stage when
teacher provides activities to identify student's prior knowledge and introduce the geometric term that will be learned during the lesion. In the directed orientation, teacher gives material which is designed gradually based on structure of the subjects. It can be stated that in these phase student are enriched by meaningful learning activities. After having experiences in two previous phases, students come to the explanation phase. In this phase teacher became facilitator to conduct classroom discussion when students are asked to express and exchange their emerging views about the structure that have been observed. One of the important roles the teacher in this phase is guiding student in order to use the accurate and appropriate terms or language. The next phase lays free orientation, in this stage student are given open problems to be solved through investigation. The last phase integration; student are given opportunity to build new level of thinking through summarizing what they have learned. Teacher plays a role as a facilitator in assisting the synthesis process. The new thinking domain will replace the old one in this phase, and then students are ready to replace the learning phase in the next level.

| Level | Stages | Characteristics |
| :--- | :--- | :--- |
| 0 | Visualization | Student recognize the figures on the basis of their <br> physical appearance |
| 1 | Analysis | Student analyze component part of figures <br> Deduction |
| 2 | Informal <br> properties both within figures |  |
| 3 | Rigor | Student able to construct proofs postulate or axioms <br> and deduction. |
| 4 | Student can work in a variety of axiomatic system. |  |

Level 0: Visualization
At this level pupils use visual perception and nonverbal thinking. They recognize geometric figures by their shape as "a whole" and compare the figures with their prototypes or everyday things ("it looks like door"), categorize them ("it is / it is not a..."). They use simple language. They do not identify the properties of geometric figures. At this stage student recognize the given problems as a total entities but does not able to identifies the other properties (triangle is thee sided closed figure but does not understand about the further properties like as right angle triangle, equilateral triangle, or isosceles triangle etc.)

Students recognize and name figures based on the global, visual characteristics of the figure a gestalt like approach to shape. Students operating at this level are able to make measurements and even talk about properties of shapes, but these properties are not thought about explicitly. It is the appearance of the shape that defines it for the student. A square is a square "because it looks like a square." Because appearance is dominant at this level, appearances can overpower properties of a shape. For example, a square that has been rotated so that all sides are at a $45^{\circ}$ angle to the vertical may not appear to be a square.

Level 1: Analysis
At this level pupils (students) start analyzing and naming properties of geometric figures. They do not see relationships between properties; they think all properties are important (there is no difference between necessary and sufficient properties). They do not see a need for proof of facts discovered empirically. At this level needs to analysis about the given problem of what is given? And what are the other properties on the given problem? Etc. But interrelationship between the properties cannot be explained.

Students at the analysis level are able to consider all shapes within a class rather than a single shape. Instead of talking about this rectangle, it is possible to talk about all rectangles. By focusing on a class of shapes, students are able to think about what makes a rectangle a rectangle (four sides, opposite sides parallel, opposite sides same length, four right angles, congruent diagonals, etc.). The irrelevant features (e.g., size or orientation) fade into the background. At this level, students begin to appreciate that a collection of shapes goes together because of properties. Ideas about an individual shape can now be generalized to all shapes that fit that class. If a shape belongs to a particular class such as cubes, it has the corresponding properties of that class. "All cubes have six congruent faces, and each of those faces is a square."

Level 2: Informal deduction
At this level pupils or students perceive relationships between properties and figures. They create meaningful definitions. They are able to give simple arguments to justify their reasoning. They can draw logical maps and diagrams. Student can establish interrelationship of properties within the problems (In triangle, all the properties of equilateral triangle are not on the isosceles triangle but some property are on both triangle, or a square is a rectangle because of all the properties of rectangle holds by square.). But at this stage student do not able to apply the properties on logical order to proofs the problem.

As students begin to be able to think about properties of geometric objects without the constraints of a particular object, they are able to develop relationships between and among these properties. "If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle." With greater ability to engage in "if-then" reasoning, shapes can be classified using only minimum characteristics. For example, four congruent sides and
at least one right angle can be sufficient to define a square. Rectangles are parallelograms with a right angle. Observations go beyond properties themselves and begin to focus on logical arguments about the properties. Students at level 2 will be able to follow and appreciate an informal deductive argument about shapes and their properties. "Proofs" may be more intuitive than rigorously deductive. However, there is an appreciation that a logical argument is compelling. An appreciation of the axiomatic structure of a formal deductive system, however, remains under the surface. Level 3: Deduction (Formal Deduction)

At this level students can give deductive geometric proofs. They are able to differentiate between necessary and sufficient conditions. They identify which properties are implied by others. They understand the role of definitions, theorems, axioms and proofs. At this level significance of deduction as way of establishing geometric theory with in axiomatic system is understood. The interrelationship and role of undefined terms, axioms, definitions; theorems and formal proof is seen.

At level 3, students are able to examine more than just the properties of shapes. Their earlier thinking has produced conjectures concerning relationships among properties. Are these conjectures correct? Are they "true"? As this analysis of the informal arguments takes place, the structure of a system complete with axioms, definitions, theorems, corollaries, and postulates begins to develop and can be appreciated as the necessary means of establishing geometric truth. The student at this level is able to work with abstract statements about geometric properties and make conclusions based more on logic than intuition

Level-4: Rigor
At this level students understand the way how mathematical systems are established. They are able to use all types of proofs. They comprehend Euclidean and
non-Euclidean geometry. They are able to describe the effect of adding or removing an axiom on a given geometric system. Students analyze various deductive systems with a high degree of rigor, while understanding such properties of a deductive system as consistency, independence and completeness of axioms.

## Conceptual Framework

I think there are two types of factor such as curricular content factor and environmental factors create difficulties on learning geometry in school education. Content factors are conceptual understanding of geometrical content such as visualization or experimental verification, construction or analysis, reasoning or abstraction and theorem proving or deduction. On the other hand environmental factors create difficulties from learning environment which are classroom environment, motivation and interest, evaluation techniques, physical facilities etc. By the help of literature review, expert consultation and peer discussion it is conceptualized systematically.


From the above conceptual framework there are mainly four factors of difficulties whereas various types of environmental factors affect and make more difficulties to learn the geometric concept. So here these factors discuss shortly.

Visualization or Experimental verification. There are some differences an inter-connection between visualization and experimental verification. When the student takes a problem from geometry, they need to visualize about what is given and what is the need to find? It is a basic level of understanding geometrical concept whereas student recognizes the fundamental concept of objects like as triangle has three side and three angles also is closed figure. Experimental verification is a task to verify the given statement through experimentation which is most physical and needs to proper use of geometrical tools like as meter scale, compass, etc. I think student feels difficulties on how to make a procedure and what relation is sufficient to conclude the given statement and appropriate use of geometric tools and accuracy of verification.

Construction or Analysis. Analysis comprise broad concept which is contextual along the content. And construction use in two types of sector among them one is figure construction according to the given direction and other is insufficient complication fulfill the given condition. In both aspect student feels the difficulties among them first creates a difficulties on appropriate and accurate use of geometrical tools through given direction and on the next creates a difficulties to distinguish what is appropriate construction to reach the conclusion?

Reasoning or Abstraction. Students establish the inter-relationship of properties both within figures. It is related to giving logic to find the measurement of given sides, angles, and distinguishing the relationship of figures. Theorem, postulate, definitions and basic rules etc. are the main basis for reasoning. Ideas and knowledge
of these terms also help on theorem proving which the causes of difficulties of learning geometry.

Theorem proving (Deduction). Theorem proving consist multiple tasks and requires high cognitive abilities. Therefore student feels more difficulties on theorem proving. It requires understanding about undefined and defined terms, basic definition, axioms and procedure of theorem proving. Also there shall be the problems in arranging the relation sequence.

From the above conceptual framework there are some causes of difficulties in learning geometry. The first cause of difficulties in learning geometry is pre knowledge of the student about geometry which includes base course and definition used in geometry. Similarly the other cause is physical facilities of learning geometry which includes geometrical lab (I.C.T. based), and other instrument used in learning geometry. Similarly teaching learning methods influence the achievement of the geometry which contains teacher perception, and activities in the classroom.

Various types of environment also create the difficulties in learning geometry. Home environment of reading, school environment and peer group environment and available of the sufficient learning materials and technological facilities etc. are the factors which provide the appropriate environment for learning. And motivation and interest, which includes the students interest, curiosity in learning geometry and giving motivation.

## Chapter III

## Methods and Procedures

The fulfillment of the research purpose depends on the reliable and valid methods and procedure. This chapter includes research design, population of the study, and sample of the study, data collection tools, reliability and validity of the tools, data collection procedure, and data analysis procedure.

## Research Types and Design

Research design provides a direction to achieve the desire, factual, reliable and valid conclusion from the research. I used mixed method approach to complete the research. Mixed method approach has more benefits which reduces the disabilities of qualitative and quantitate research approach and adopt the qualities of both quantitative and qualitative research. (Creswell, 2013), defines mixed methods as "Mixed methods research involves collecting and analyzing both quantitative and qualitative data. The quantitative data includes closedend information that undergoes statistical analysis and results in a numerical representation. Qualitative data, on the other hand, is more subjective and open-ended. It allows for the "voice" of the participants to be heard and interpretation of observations"

Sequential research design was adopted (QUAN+QUAL) procedure in my research.

## Population of the Study

The entire population of the study was all the remote areas public school of Nepal and specially Arghakhanchi district. The result of data can be generalized into whole students of public school. This study is used for data who were currently involved in learning programmed.

## Sample of the Study

The representative sample for the study takes a more significant role to make the reliable, factual, and valid research which is somehow large and somehow small that depends upon the research area, tools, researcher objectives and economy. This study used different tools. So, the sample depends upon it also. The participants of this study had been taken 100 students from the different community's schools of Arghakhanchi district. The sample was selected from random sampling for quantitative type and purposive sampling method for qualitative type. Purposive sampling was helped to take the required represented sample according to the purpose of research. So, I was taken one weak and one talent student for the interview.

## Data Collection Tools

Data collection tools are the instruments for the research which helps to collect the required and accurate data from the research area. There were actually two types of data that are obtained as qualitative and quantitative. (Petrson, Gibson, Tucci, Paratt, Lawton, \& Hokak, 2019), defines data collection as
"Data collection is the systematic approach to gathering and measuring information from a variety of sources to get a complete and accurate picture of an area of interest. Data collection enables a person or organization to answer relevant questions, evaluate outcomes and make predictions about future probabilities and trends"(pp. 14-15).

The major tools of data collection were math achievement test (includes question from different content of geometry with contains different level), questionnaire collect different information of difficulties on geometrical content and environmental causes such home and school environment) and interview (with the special disabilities students, special capabilities students and mathematics teacher).

Diagnostic test determined the area and nature of difficulties. The result of interview provides strong justification on difficulties, about which types of difficulties was suffered student.


Math achievement test. The test used in the research was math achievement test. Math achievement test is a form of pre-assessment that allows a researcher to determine student' individual strengths, weakness etc. This test used to determine the location, nature and areas of difficulties in solving geometries problems. This test provided the where is and how much difficulties faced by students on learning geometrical content. Fundamentally, Math achivement test was based on van Hiele's level of learning. So, it was determined that on which level students had more difficulties. Test was designed as simple to abstract approach. This math achivement test divided into four categories as visualization and experimental verification, construction, theoretical proofs and application as the conceptual framework and nature of content of geometry. There were 100 students involved on the examination from public school. There were objective and subject type questions sets on the test.

Questionnaire. A questionnaire is a data collection instrument consists of a series of questions and other prompts for the purpose of gathering information from respondents (Abawi, 2013).

The second research question or objectives of the research can't be fulfilled from math achievement test only. For this the researcher designed a questionnaire to find out the actual difficulties of learning geometry. The modified four point likerttype rating scale was adopted (Adolphus, 2011). Questionnaire was divided into four sections as strongly agree, agree, disagree, and strongly disagree. Students respond on only one section according to their opinion. Fundamentally, Questionnaire contends environmental factors such as physical facilities, pre- knowledge of students, teaching and learning procedure and activities and teaching learning environment and student motivation.

Interview schedule. Interviews were conducted in this study, are in depth interviews focused in finding out the respondent real situation and experience about geometry learning and explore the difficulties. (Best \& Khan, 2006) defines;
"Interviews are used to gather information regarding an individual's experiences and knowledge; his or her opinions, beliefs, and feelings; and demographic data. Interview questions can be asked so as to determine past or current information is tape record the interview if the respondent is willing. Otherwise, the interviewer must keep notes"(p.265).

This interview was used to explore in depth the difficulties of students in solving geometrical problem related to the concept and principal. Interviews were taken from the mathematics teacher and students. The interpreted data was based on the kept recodes. I used to open interview schedule to find out the experience and situation of the respondent because of open interview schedule help to collect the required data as the real situation.

Reliability and validity of tools. The reliability refers to the consistency of result and validity refers to the objective of the research that can measure. To establish the reliability and validity of research, Researcher consult with research guide, subject expert and school data book etc.

For reliability of the test, researcher would take a pilot study of the prepared test to 15 students of grade ten. Before advertising the test paper, investigator would instruct to students about the methods of responding the test paper. After collection the answer sheet of the student researcher scored of the answer. The reliability of the test should measure by test re-test method for which test conducted during the 15 days on the same group. By checking the reliability score was 0.77 (Appendix-E) which shows that the test paper has more reliable.

## Data Collection Procedure

The researcher selected three tools to collect data from the field. Researcher directly involved for data collection on this field. First of all I entered on the research area and report building with the teachers and students by justifying the objectives of research. Firstly, math achievement test conducted among the students. And then questionnaire filed from the students. I was taken diagnostic test to distinguish the area of difficulties of the student in secondary level geometry content. But, it was not sufficient to find out the special difficulties of weaker student. For this I used the interview schedule and took audio record the interview.

## Data Analysis Procedure

There were two types of data obtained from this research. Both types of data qualitative and quantitative obtained from the different tools. This research adopted the mixed method approach to collect the data and used the tools diagnostic test, questionnaire and interview. The analyze data were primary data taken from the
researcher participation. Both the qualitative and quantitative data has their specific nature and analysis procedure.

Quantitative data are found from the math achievement test and questionnaire. These data were filtered, tabulation, and interpreted from the result statically by using percentile and average. Whereas the qualitative data were obtained from the interview which were analyzed by them making and also it justified the result of quantitative data obtained from the questionnaire as sequential mixed method. To present the claim of objectives, collected data had presented in both types of data analysis procedure and inter-linked them to find out the intrinsic factor and extrinsic factor of learning difficulties on geometry of the secondary level school students. For the data analysis procedure, the detail procedure followed as;


Math achievement test score analyze into three categories (as appendix-A) excellent, Average and insufficient. Analyze the score of every question by dividing into different section such as visualization, experimental verification, construction, theoretical proofs and application. Obtained data interrupted by using percentage. Some figure of student answer included on data analysis. And the special cases of student difficulties.

Data obtained from questionnaire were tabulated in different section (as appendix-B). These analyzed by using percentage individually on every subject (as
appendix-B). Researcher analyzed interview data by theme development. There are some extract included on data analysis.

## Chapter IV

## Analysis and Interpretation

This chapter includes the analysis and interpretation of the collected data. Data analysis is very important task because it justifies the result of any research. I analyze the findings of my research through primary data which were collected from the respondent. These data were collected from Shree Sarsawati Bhawan Secondary School, Panini Rural Municipality-3 Khidem Harrabot, and Shree Sarswati Secondary School Panini-1 Panena of Arghakhanchi district. There are different tools like math achievement test, questionnaire, and interview to fulfill research work. The main task of this research is to explore the subjective factors that contribute to the difficulty in learning of geometry and identify the areas and causes of learning difficulties on secondary level school students. Math achievement test was conducted on 100 students and questionnaire was filled up from 100 students of grade nine and grade ten students. And interview was composed with the higher achiever student, lower achiever student and mathematics teacher which provide real situation and problems facing from them and what are the difficulties and how they are dealing with these problems?

Mathematics is very essential discipline of knowledge which is directly connected with the daily activities of human life. Almost all countries are given respective place for mathematics curriculum because it develops logical and mental ability on people and possible to make national unity through the mathematics. Geometry studies about physical objects which works the task of root on mathematics but it is going to decline on mathematics curriculum. Mathematics is not itself a difficulty subject but people perceive it as misconception.

This chapter includes two sub chapters according to the research objectives among them one is curricular content factors and environmental factors of learning difficulties of geometry. Content factor comprised the four fundamental topics; visualization, experimental verification, construction, theoretical proofs (formal deduction), application (rigor). Environmental factors concealed the student opinion about physical facilities, Pre-knowledge of students, teaching and learning procedure and activities, Teaching learning environment( school environment, home environment), evaluation techniques and student motivation.

## Curricular content Factors of Learning Difficulties on Geometry

Secondary level curriculum contains study about the two dimensional shapes and three dimensional shapes. Euclid (300BC) defines geometry on two and three dimension. The geometrical undefined terms such as point, line and plane etc. and defined terms triangle, square, straight angle, perpendicular etc., definition such as 'a point is that which has no part', Axioms such as if there are two point there exist exactly one line between them and theorems such as 'opposite angles of cyclic quadrilateral are supplementary' with the chapter triangles, area of rectangle and parallelogram, circle, construction, coordinate geometry and mensuration etc.

I sub-divided geometrical content as the five fundamental parts as visualization, experimental verification, construction, theoretical proofs and application. To find out students opinion from interview about the difficulties of geometry learning I asked with students what are the difficulties you are facing, student responded as;
"Geometry is more difficult parts of mathematics because it contains the abstract construction, theorem proving, application of the concept and theorem as well as it becomes more difficult due to the cause of
insufficient practice, difficult to understand about question, needs to rote the theorem, lack of sufficient material and reference books etc."

Also, student's responded mathematics as a formula based subject. Almost student feels geometry as problematic part of mathematics and also feels the great difficulties on proofs of the theorem and they used wrong procedure for numerical application of geometric content.

I composed the math achievement test to explore the difficulties faced by secondary level students in learning geometry. And the result of the test obtained as; Table 1: Student response about math achievement test in difficulties faced by secondary level students in learning geometry

| Sector of <br> geometry | Q.N. | Excellent <br> $(80 \%$ Above) | Average <br> $(40 \%-80 \%)$ | Insufficient <br> (Below 40\%) |
| :--- | :--- | :--- | :--- | :--- |
| Visualization | $1 . \mathrm{a}(\mathrm{I})$ | $80 \%$ | ---------------- | $20 \%$ |
|  | $1 . \mathrm{a}(\mathrm{II})$ | $10 \%$ | $40 \%$ | $50 \%$ |
|  | $1 . \mathrm{a}(\mathrm{IV})$ | $8 \%$ | ----------------- | $92 \%$ |
|  | $1 . \mathrm{c}$ | $10 \%$ | $50 \%$ | $40 \%$ |
| Construction | $3 . \mathrm{a}$ | $5 \%$ | $30 \%$ | $65 \%$ |
|  | $3 . \mathrm{b}$ | $2 \%$ | $18 \%$ | $80 \%$ |
| Theoretical | $4 . \mathrm{a}$ | $5 \%$ | $26 \%$ | $79 \%$ |
| proofs | $4 . \mathrm{b}$ | $4 \%$ | $22 \%$ | $84 \%$ |
| Application | 5 | $2 \%$ | $10 \%$ | $88 \%$ |
|  | Average | $13 \%$ | $21.6 \%$ | $65.4 \%$ |

From table 1 and appendix-A; students capabilities appeared according to the content and its abstraction on visualization of geometrical objects. $80 \%$ student
correctly responded excellently about diameter is twice the radius whereas only ten percent student correctly responded about radius is half the diameter. This concludes that student confused about the relationship of diameter and radius of the circle. They have conceptual difficulties about circle object because they couldn't distinguish the component of the circle. More than average students felt difficulties on visualization. van Hiele discussed it is the first step of learning. Student did not able to enter the next step without understanding the first step.

Experimental verification is harder than visualization. Only ten percent students excellently had done the visualization question. And also average number of students had done averagely. This concludes that they felt the difficulties on experimental verification.

Minimum number of students had done excellently whereas more then average number of students had done insufficient on this section. This concludes that they felt difficulties more than in visualization and experimental verification.

According to the nature of areas of geometry theoretical proofs requires the pre knowledge of visualization, experimental verification, and construction. Below ten percent students had done excellently in this section where more than $80 \%$ students had done insufficient and obtain below $40 \%$ marks. It concludes that they felt difficulties in theoretical proves.

Application is higher level of understanding the geometrical concept. It requires the knowledge of visualization, construction and theoretical proofs. Only two percent student had done excellently which is very minimum. $88 \%$ students were obtained below $40 \%$ marks concludes that they felt difficulties in this section. The detail analysis discussed on each section below.

Visualization. van Hiele's defines visualization is the first step of understanding the geometrical concept. At this level, students need to identify the geometrical figure as whole first and then find out the different parts of the figure and then its characteristics (Hiele-Geldof,1986). As the grade and mental age there is a different task of visualization like grade 1 student identified the shape of geometrical object as it is round and it is long and short only. But, secondary level students need to complete different task. Secondary level curriculum contains various two and three dimensional objects, their definition and properties such as circle, tangent, central angle, circumference angle, radius, diameter etc. of the circle, cyclic quadrilateral, area of triangle and quadrilateral, quadrant, reflection of the coordinate, cuboid, cube, cone, sphere, etc. I composed the math achievement test to find out the conceptual and procedural understanding of the students in visualization.

From the table1, students responded result appears most of the students have appropriate visualization skills whereas average student did not able to completed the given easy task. The questions were easier so it is not more important how much student gives only correct answer and wrong answer only. For this it is essential to look the first two questions has the same conclusion and relationship between the diameter and radius but why there is a gap appears between the two question because the lapses of correct understanding of the concept. Also question about tick the correct answer about straight angle have the measure $360^{\circ}$ derived the conclusion about how much student does not able to know the correct measure of straight angle. They have the lack of confidence about measure of straight angle haven't $360^{\circ}$ but it has $180^{\circ}$. I asked about write down the different parts of the circle on the alongside figure and the students difficulties appeared as;

Figure 1: Student response about difficulties in visualization


This question is also a very fundamental concept for the understanding the circle chapter. From figure-1, there was insufficient knowledge with the average number of students. I mentioned that example off two students answered correctly and other incorrectly which shows that some student have the difficulties on the visualization of the geometrical concept. Students have difficulties on correct mathematical vocabulary as well as understanding objects on respective place because students wrote special line for tangent line and misplacement of center of the circle and both insufficient fulfill the required answer.

I composed multiple choice question (Appendix-A) tick the wrong answer: a) all square are rectangle b) all angle of rectangle are equal c) all angles of quadrilateral are equal d) sum of interior angle of triangle is $180^{\circ}$ to find out the student concept about geometry; most of the students given wrong answer whereas distractors was very close and highly conceptual based. Students answer on this question showed that most of the student did not aware about all square become rectangle and all angles of the quadrilateral may not be equal whereas they also confused about all angles of rectangle are equal and all interior angle of triangle have the angle sum $180^{\circ}$. Only eight percent students respond the correct answer. This shows that most of the students have poor understanding about geometrical objects and their properties because minimum students responded correct answer on this concept based question.

Geometry can be very fun for students, but if they start in a state of mind that they hate geometry, it would be very unfortunate and hinder their success ( Martins \& Melo, 2015). I composed interview to find out the students difficulties about geometrical concept.

## Extract I

Researcher: hello A, Do you know about circle?
Student A: Yes sir. It is round.
Researcher: is ring and bicycle tire circle? Both are round.
Student A: Yes sir.
Researcher: hello sir, is student correct?
M.Teacher: I got confused about it. Circle is locus of point from fixed center and distance.

Researcher: hello B, can you say about relationship among radius and diameter.
Student B: Diameter is twice the radius.
Researcher: you mean radius is not half of the diameter.
Student B: Yes sir.
Researcher: hello C, is square also a rectangle?
Student C: No sir.

This interview showed that student pre- knowledge was poor about the geometrical object. Students confused about definition of circle and counter example. Student felt difficulties on understanding the actual meaning of definition. Student needed to understand ring and bicycle tire are not circle. Student C did not able to compare properties among square and rectangle. He understands square and rectangle separately. As Martins \& Melo(2015), Students faced the difficulties due to the lack of visualization of geometrical object. They were felt difficulties.

Experimental Verification. Experimental verification is also fundamental part of geometry which develops the practical and physical understanding of geometrical concept. According to van Hiele's it is a third step of understanding geometrical concept called the informal deduction. It is also a natural way to prove the theorem and also called as induction method of proof theorem. We take more than one case if these have same conclusion then we say that theorem is proved for other case. I composed some question on the math achievement test such as verified experimentally the statement "The sums of opposite angles of the cyclic quadrilateral are supplementary. Only ten percent students had done excellently. $50 \%$ students had done insufficient among them more students didn't attempt this question which appears as the average students had difficulties in learning experimental verification.

When I asked with the student which is more difficult among experimental verification or theoretical verification? They said that

Sir, "Experimental verification is more difficult than theoretical verification because experimental verification required correct use of geometrical tools and difficult to make the construction of the figure and develop the relationship and conclusion of the statement".
van Hiele's defines experimental verification is the previous level of learning geometrical concept then theoretical prove but students faced more difficulties on experimental verification which shows that something is the wrong with them. Students felt the difficulties to use the geometrical tools and construction of the geometrical figure. Also students faced the difficulties on develop conclusion from different case of relation. I mentioned here the students answer obtained from the math achievement test.

Figure 2: student response about difficulties of learning geometry in experimental verification


From figure 2, it appears that they have conceptual and procedural difficulties both. The figures shows that student don't know about actual meaning of cyclic quadrilateral present difficulties on the measure of the opposite angles and deduced the correct result. A cyclic quadrilateral or inscribed quadrilateral is a quadrilateral whose vertices all lie on a single circle. Students did not construct the appropriate cyclic quadrilateral by using compass tool and appears lacks of knowledge about which are the opposite of cyclic quadrilateral as well as student makes the procedural difficulties about correct measurement of the angles and present poor writing skills. They had lack of good skill of construct the cyclic quadrilateral. From the above data more than $50 \%$ students have difficulties on experimental verification.

Construction. Construction develops skills of making correct figure from the given information through geometrical tools. (John, 2015), defines geometrical construction as;
"The drawing of various shapes using only a pair of compasses and straightedge or ruler. No measurement of lengths or angles is allowed. The word construction in geometry has a very specific meaning: the drawing of geometric items such as lines and circles using only compasses and
straightedge or ruler. Very importantly, you are not allowed to measure angles with protractor, or measure lengths with a ruler".

So, it is very significant parts of the geometry. To find out the difficulties of students I asked some fundamental questions about construction of triangle and construction of parallelogram which has equal area with triangle.

Figure 3: Students response about difficulties of learning geometry in construction


Student seems to have the difficulties to make geometrical objects through use of compass and meter scale. They have unaware about the arc, joint point of arc, and side and the correct construction of the triangle which was easy task for grade ten students. Next shows that students are not aware about the procedure of parallelogram construction and create minor mistakes on procedural development of construction of the parallelogram which has the area equal to the triangle. So, students make error in construction of parallelogram with the same area of the given triangle. Student used wrong procedure and also appears the lack of appropriate knowledge for parallelogram construction.

Theoretical proofs (Formal deduction). Math achievement test was designed based on Van Hiele's model of teaching that consists of five phases among them
theoretical proofs is fourth steps of abstraction. Geometrical proofs are very important parts of geometrical content which develops the logical power of the students. It is more challenging than the other parts discussed previously for student without developing knowledge and skill on visualization, experimental verification and construction. It is based on approach of deduction. I prepared some questions on the math achievement test.

Figure-4: Students response about theoretical proofs


From figure-4 and appendix-A, question asked about prove that parallelograms on the same base and same parallel are equal and area an triangles on the same base and same parallel are equal in area. Fist figure shows that students don't knew about the given condition same base and same parallel. Student just tried to make the figure according to given condition. But student could not able to sketch the correct figure due to lack of conceptual understanding. And Student could not able to start the procedure of poofs the theorem. It showed that students who have not appropriate concept, they also couldn't able to develop the correct procedure. Lakshminarayan, P. (2018), supported that it's important to find out the given information first when solving proofs. It's also important to remember that the "given statements" are provided on purpose to help you decide the best way to prove what has already been established.

Also from figure second, student tried to write the given condition which was insufficient. Student wrote same base BC but did not wrote same parallels BC and EF. Also student formed the inappropriate figure. Student didn't write about what is given and what is needs to prove and didn't construct insufficient condition. But why student the correct prove? Whereas student didn't wrote correct notation on correct place. I composed interview to find out what is the real situation?

## Extract-II

Researcher: Do you know what is given in the question?
Student: Actually, I don't know sir.
Researcher: Why are you solving this question?
Student: Teacher solved this question in class. So it is on my note book.
Researcher: why don't you make the correct figure?
Student: I forget it but I remember the solution.
From the Extract-II, it is concluded that student attempt this question according to their rote learning. Rote learning everything without trying to develop an understanding of concepts in a math education is a bad idea, and this doesn't need an explanation. On the other hand rejecting rote learning completely is obviously highly inefficient, since there are things that cannot be understood, but merely memorized (Santos \& Murthy,2014). it concludes that rote learning also hamful to understanding the concept. But somehow it is essential for remember the terms and definition of geometry. When students does not understand the actual meaning of content then they give imhasis on rote learning because they need to pass the examination.Teaher and student needs to give imphasis to understand concept and process of learning because student could be frosted from rote earning. And so that student felt difficulties in learn geometry.

Figure-5: student response in theoretical proofs


From figure-5, it appeared that student find out what is given in the question?
And what is needed find? Also develop the correct procedure but student without construction used the perpendicular line DE. It concludes that student have conceptual difficulties.

From students answer and above analysis it is concluded that they have difficulties about what is given in the question? And what is needed to find? Means students have conceptual difficulties of understanding theorem. Students were confused about what is the insufficient construction to deduce the conclusion? Some question required to make construction to deduce the theorem. But students did not able to internalize the situation and they make the insufficient, extra and wrong constructions which shows that student have the difficulties on fulfill the insufficient given condition and neglect the extraneous condition on the question or statement. The major difficulties occurred on what are the statements needs to connect the theorem which provides the conclusion like as SAS, ASA, SSS, and relation between angles such as alternate angles, co-interior angles, etc. and how to connect required statement to make logic for finding out the conclusion? Theorem proving is major part of geometry. Almost chapter comprised some fundamental theorem.

Application of geometrical concepts. Application comprised the multi task on 1 question and needs to have more than one definition and theorems to obtain the required solution. Van Hiele's defined it as fifth level of learning geometrical content. It requires pre- knowledge and skills about conceptual understanding of definition, properties, theorem and formulas to compute the question of application level. I composed some question on the diagnostic test to find out what types of difficulties occurs for students. And students responded in this section were appeared as;

Figure-6: Students response in application of geometry content


The major areas in application section are distinguish between the definition and properties of the different geometrical figures like as square and parallelogram where students were unaware about every square is parallelogram but not converse and parallelogram on the same base are equal in area so square is also parallelogram that has same area. So student's feels the difficulties to use the appropriate theorem to justify the statement. From the above figure it can be easily said that student creates the difficulties to use appropriate strategies and formulas to find out the area of respective geometrical figure. Here students need to use the formula $\left\{\mathrm{A}=\mathrm{l}(\text { side })^{2}\right\}$ to find out the area of square where as they use $\mathrm{A}=21$ (side).

Many textbooks and many district pacing guides emphasize numeracy, arithmetic, and algebraic reasoning. Geometry (along with data and statistics) is often tucked into the final chapters of the book and the final weeks of the year after state testing (Fulton, 1994).Individuals who exhibit learning difficulties may not be intellectually impaired; rather, their learning problem may be the result of an inadequate design of instruction in curricular materials" (Carnine, Jitendra, \&Silbert, 1997). I composed interview to find out the difficulties of geometry learning.

## Extract III

Researcher: hello $A$, is geometry harder than arithmetic and algebra?
Student A: yes sir, it is difficult due to experimental verification, theorem and abstract application.

Researcher: hello sir, why student feels difficult in geometry learning?
M.Teacher: Lack of material, insufficient practice students and organization of curriculum makes difficulties.

Researcher: which time of academic year you teach geometry?
M.Teacher: Test book, it is the last chapter and I also teach at the end.

Students had difficulties on geometrical content because geometry teaching and learning content is at last of the section. As Fulton, curriculum of mathematics of Nepal gives the place for geometry at last. Carnine, Jitendra, \&Silbert (1997), gives emphasis on teaching learning process of curriculum content for better output. Due to this cause teacher teach the geometry content at last section of academic year. Student had very less time for learning geometrical content. They hadn't spent the more time for preparing the geometry content. Therefore they had left the question from geometry content on examination. Therefore, they felt difficulties. So, curriculum designers and teachers need to become serious on this context.

## Environmental Factors of Difficulties in Learning Geometry

Environmental factors comprise various sectors discussed as physical facilities of the school, pre-knowledge of the students, teaching and learning methods used in classroom, learning procedure and activities, evaluation techniques, teaching materials and access of the technologies, classroom environment and motivation, interest of the students (Thakur, 2018). Nepalese community's school faced the blame of low achievement in mathematics (Thakur, 2018). Is mathematics really difficult subject among other subject? And is geometry have difficult than other branch of mathematics? What are the environmental factors of difficulties on learning geometry? are fundamental question. I took an interview with the subject teacher and students and filled the questionnaire whose major outcomes are discussed below.

Physical facilities. The physical facilities contained land of school, building, classroom, furniture, teaching material, game material, I.C.T. lab, etc.

Figure-7: Students opinion about physical facilities


The physical facilities of the school show that land and furniture of schools had appropriate as number of students but teaching materials were insufficient for geometry teaching. There wasn't I.C.T. lab. From figure-7, students' response showed that all students had text book but only $40 \%$ students had reference material. Student's response concluded that there wasn't I.C.T. lab. $20 \%$ students didn't have the
geometrical box which is very essential during geometry learning. $30 \%$ students response was that mathematics teacher changed many times on one academic year. They did not attend the class regularly. Response of students showed that only two percent student strongly agreed about availability of learning material on the school.

I also composed the interview to find out what is the real situation of physical facilities for geometry learning. When I asked the question to the subject teacher about what is the condition of teaching material for the geometry teaching. He said that;
"We did not have appropriate teaching material, lack of essential material geo-board, graph-board, and I.C.T. approach to see the new approach of geometry teaching and learning. Government did not give the appropriate training about how to make and use the teaching material for the teacher.

This concludes that schools didn't have sufficient material for geometry learning. Geo-board and graph- board are essential for geometry teaching and learning. But schools couldn't provide. Some student hadn't basic material for geometry learning such as geometry box. Without geometrical box, it is very difficult to learn geometrical content. Students were depended only on text book. More than average students had not reference book. Students faced the lack of regular mathematics teacher because teacher hadn't interested to stay on remote area. Today's I.C.T. is also essential for geometry learning. Students hadn't opportunity to use it. It creates difficulties to learn geometry learning. (Bhattrai,2005), supported that lack of teaching material creates huge number of difficulties. (Thakur, 2018), concluded that causes of learning difficulties in geometry at secondary level are: lack of teaching materials, lack of locally available teaching materials, Lack of laboratory in school, lack of graph and
bulletin board, lack of reference books and teachers guide, lack of training of teacher and lack of interacting between student and teachers.

Pre-knowledge of the students. Secondary level course of mathematics is vertically organized on the basis of previous class learning capabilities. Therefore student is required to know about previous concept for learning the secondary level mathematics.

Table-2: Opinion about pre-knowledge of students

| S.N. | Subject | SA\% | A\% | D\% | SD\% |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | I have sufficient knowledge to read the <br> secondary level mathematics through the pre- <br> secondary level. | 2 | 40 | 50 | 8 |
| 2 | I agree that geometry is the easy area of <br> mathematics in your curriculum. | 0 | 50 | 42 | 8 |
| 3 | I agree that about the course of geometry is <br> vertically interconnected in your curriculum. | 44 | 46 | 10 | 0 |
| 4 | There is any new content on geometry in your <br> class course which was not read in previous <br> class. | 16 | 72 | 10 | 2 |

From students response on the table-2 about they have sufficient knowledge to read secondary level geometry viewed that only two percent students were respond on SA, $50 \%$ students respond on D , $40 \%$ only agreed where as eight percent respond on SD. It proved that more than average students were dissatisfied about their knowledge that gained from previous classes. More than average students disagreed that geometry is easy subject. $72 \%$ students agreed that there is a new concept in the class
course which is not connected with the previous course. It shows that student didn't understand geometry concept appropriately in their previous class. From the above findings it is clear that student's had poor pre-knowledge.

I composed interview to find student's pre -knowledge about geometry. When I asked question to both teacher and students "Is geometry is really difficult. Student answered that
"Geometry is not difficult. But I was not practice more time. Geometry requires more practice and time. I have no sufficient knowledge from previous years. So I feel that it is difficult. Geometry is more difficult than other content of mathematics".

And on the same question teacher said that,
"Mathematics is not difficult subject but artificial situation creates difficulties. Below $50 \%$ percent question asked on the base of determined text book where as necessary to ask more than $75 \%$ question".

When I asked question, is geometry is really different and difficult than other content of mathematics, why? And then student answered that
"Yes, geometry is harder than arithmetic, algebra etc. because it coincides with the difficult construction, experimental verification and a theoretical proof that makes difficulties to learn".

Whereas mathematics teacher said that,
"Geometry is foundation and not different with other branch of mathematics because it studies about the geometrical figures and properties. And this concept studies on arithmetic as a number and in algebra as symbol".

When I asked the question "there is insufficient organization of geometry content in mathematics curriculum" and then teacher said that,
"Vertical organization on curriculum creates the problem in teaching and learning mathematics because there are some new construction and theorem on secondary level"

This conclude that geometry is not itself a difficult which is widely used in our daily life. Student did not practice more and did not give more time. They have insufficient knowledge of previous class. Organization of curriculum became also causes of difficulties. Teacher and students depended on text book. They had not concern about curriculum. Students felt the difficulties due to the lack of preknowledge. Sapkota (2017), supported that pre- knowledge is essential for learning new concept and lack of pre-knowledge creates the difficulties.

Teaching and learning procedure and activities. Teaching learning procedure and classroom activities are the key part of the schooling because all other structure is developed for this. How much effective child makes depends upon teaching learning procedure and classroom activities. Teaching and learning procedure comprise the various aspects like as teaching learning methods, classroom management, fulfillment of the curricular activities, extra-curricular activities, creative activities, student enjoyment on the studies, teacher's skills and evaluation techniques etc.

Table-3: Student opinion about teaching and learning procedure and activities

| S.N | Subjects | SA \% A \% |  |  | D \% SD\% |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Geometry is a major prior area which I am reading <br> and learning before other areas. | 46 | 42 | 8 | 4 |
| 2 | The course of geometry is completed in our class. | 94 | 6 | 0 | 0 |
| 3 | Teacher use multiple ideas for solving geometry. | 56 | 42 | 2 | 0 |
| 4 | Teacher uses the sufficient teaching material to | 30 | 26 | 14 | 28 |


|  | teach the geometrical concept which becomes <br> easier to read. |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | I complete any extra- curricular activities like as <br> project work, group discussion, etc. in your <br> classroom. | 0 | 20 | 62 | 18 |
| 6 | I see some experiment of geometry through the <br> internet. | 4 | 8 | 14 | 74 |
| 7 | I am interested to see the theoretical concept into <br> the practical application through the mathematical <br> software like as geo-gebra. | 54 | 4 | 4 | 38 |

From table-3, students agreed that geometry is the fundamental part of mathematics. But they said on interview geometry is difficult part of mathematics and read at last of academic year. More than $90 \%$ students responded about course of geometry completed in the class. Only $30 \%$ students strongly agreed about teacher used the appropriate teaching material. No students respond about they complete extra-curricular activities. Only $20 \%$ students were agreed and more than average students had disagreed and strongly disagreed. Only four percent and eight percent students responded that they used the internet to learn the geometry content through the internet whereas $14 \%$ and $74 \%$ students respond on disagree and strongly disagree. $54 \%$ and four percent students responded that they want to use the software programmed such as geo-gebra whereas four percent and $38 \%$ students on disagreed and strongly disagreed.

This concludes that teacher and student used the classical approach for teaching and learning. They did not practice about extra- curricular activities such as
project work, group discussion etc. Teacher and student had given the less-emphasis for geometry then other branch. So, students felt difficulties.

Teaching learning environment and student motivation. Teaching learning environment contains home environment of learning and school environment which directly affects the motivation for learning mathematics. Firstly, students have needed to become positive to learn. When any student feels anxiety towards geometry about it is hard and difficult part of mathematics. It creates the difficulties to learn.

Table-4: Student opinion about teaching learning environment and student motivation

| S.N | Subjects | SA \% | A\% | D\% | SD\% |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | I am very interested in geometry learning | 26 | 62 | 10 | 2 |
| 2 | I have anxiety feeling about geometry. | 14 | 38 | 40 | 8 |
| 3 | I felt that I am not trying to hard work to learn <br> the geometry concept. | 10 | 50 | 22 | 18 |
| 4 | I have a problem even teacher gives the solution <br> of similar types in example. | 14 | 70 | 10 | 6 |
| 5 | I agree that geometry is directly related to your <br> daily life application and it gives a contribution <br> for your currier development. | 28 | 60 | 12 | 0 |
| 6 | I have interested to prove the theorem in <br> geometry | 14 | 48 | 24 | 14 |
| 7 | I am interest to make logic to solve the counter <br> example of the theorem. | 40 | 38 | 12 | 10 |
| 8 | I feel that experimental verification is more easy |  |  |  |  |
| then theoretical proofs of the theorem. | 72 | 18 | 10 | 0 |  |

From the response of students given on table- 4 , only $26 \%$ students were strongly agreed that they are very interested to learn geometry. Ten percent and two percent students disagreed and strongly disagreed showed that huge numbers of students were not interested in geometry. $14 \%$ and $38 \%$ students have anxiety about geometry. More than average student did not hard work to learn geometry. More than average students responded that they haven't able to compute similar problem when teacher solve the similar type of problem by teacher. It showed that students have the huge level of difficulties in learning geometry.

This concludes that students had not good interest on geometry. They had anxiety feeling about geometry. They had not interested in solving geometrical problem. Student had felt difficulties when teacher had given solution of similar type example. They had not strong believes about geometry develops their currier. More students hadn't interested to prove the theorem. Therefore they felt the difficulty in learning geometry.

## Chapter V

## Summary, Findings, Conclusion, Recommendation and Educational Implication

This chapter justified the main theme of the research. This research mainly conducted to find out the difficulties faced by secondary level students in learning geometry. Geometry is the prime branch of mathematics but why student feels difficulty in learning it? To find out the answer of this question there are two types of difficulties were emerged as curricular content factor of difficulties and environmental factors of difficulties. This chapter includes summary of the research, findings of the research, conclusion of the research and educational implication of the research.

## Summary of the Research

This study was designed to find out the difficulties faced by secondary level students in learning geometry with objective to find out curricular content factor of difficulties and environmental factors that could produce difficulty in learning geometry.

This study used mixed method design with sequential methods to collect and interpreted the data. Sample based on100 students of public school of Arghakhanchi district for questionnaire and math achievement text. The result of the research is analyzed on the basis of primary data.

According to research the difficulties in learning geometry occurred in students because of the lack of good understanding of conceptual knowledge on geometrical content and lack of better way to compute the geometrical problem. I analyze it as various sector of geometrical knowledge according to van Haile's theory and fundamental nature of mathematics. They are visualization, experimental verification, construction, theoretical proofs and Application (Rigor). There are
external factors which are also responsible to create the difficulties to learn the geometric content.

In curricular content, they felt conceptual difficulties in definition, Identifying and appropriate use of relevant properties of mathematical objects circles, triangles, parallelogram, square, different types of angle, understanding of statement on the theorem, and distinguish the appropriate formula for application and procedural difficulties such as use the geometrical tools for the construction and experimental verification, development of logic for theoretical proofs and appropriate computational skills of geometrical problem.

Otherwise curricular content factors environmental factors also create the difficulties in learning geometry. Infrastructure of the school, required teaching and learning material, I.C.T. lab, pre- knowledge of the students, teaching and learning procedure and activities, teaching learning environment and student motivation etc. has occur the problem on the research area. Students wanted to learn the geometrical content through the use of internet and different mathematical software but they did not have any opportunity of connectivity and computer lab. Teacher was also untrained about how to use the technologies and how to make teaching material in local level.

## Findings of the Research

According to this study curricular content factors of difficulties of learning geometry in secondary level school students are as follows;

- Students had difficulties in distinguish between the definition and properties of the different geometrical figures like as square and parallelogram where students were unaware about every square is parallelogram.
- Average number of students were not aware about basic objects of the circle: center, radius, diameter, center angle, tangent line and relationship of the properties of circle such as radius is half the diameter, central angle is twice the circumference angle, and opposite angle of cyclic quadrilateral, supplementary angle etc.
- More than average numbers of students feel difficulties about the experimental verification and appropriate use of geometrical tools and induct conclusion from finding data.
- Students faced the difficulties in both type of construction as construct the given geometrical figure according to the given condition and insufficient construction for the solution for theorem and question such as triangle construction and parallelogram construction with the same area as the given triangle.
- Students had following difficulties in the theorem proving;
i. Find out the given condition on the question and find out the required conclusion from the question.
ii. Find out the insufficient construction to deduce the conclusion.
iii. Connect the concept and theorems needed to prove the theorem which provides the required conclusion such as SAS, ASA, and SSS etc.
iv. Connect the required statement to make the logic for finding out the conclusion.
- Student had the difficulties to use the appropriate strategies and formulas to find out the area of geometrical figure.
- Vertical organization of the content on the curriculum creates the difficulties for teaching and learning the mathematics.

According to this research, environmental factors of difficulties in learning geometrical content were as follows;

- The teaching and learning environment were not conducive. There were lack of infrastructures and basic facilities for teaching and learning. 60\% student had not reference books, 20\% students had not geometrical box.
- Schools were not being able to provide the teaching material such as geo-bard, graph-board and I.C.T. lab as well as did not give emphasis on extra-curricular activities
- An attitude of students towards learning was poor. They have lack of willingness and readiness.
- Average students hadn’t appropriate pre-knowledge in geometry. As such they couldn't solve problem even when similar examples by the teacher.
- Students did not try to see any application of geometry in particular area and daily life. So, they consider geometry is difficult and feels anxiety towards mathematics.
- Students did not work hard and did not give sufficient time in learning geometry then other branch of mathematics such as arithmetic, algebra etc.
- Government couldn't give the appropriate training for teacher as their needs and was not provide appropriate infrastructure for the schooling.


## Conclusion

Fundamentally, the research is based on to find out the difficulties faced by secondary level students in learning geometry. The research was done to fill the heart of two fundamental objectives which were find the difficulties in the curricular content factor and which external factors produce the difficulties in learning geometry. I analyze the findings of research as fundamental parts of geometry 1)
visualization and experimental verification, 2) construction, 3) theoretical proofs 4) application on which every part appears two types of difficulties 1) conceptual, 2) procedural. Actually, geometry is not a difficult area. Other branches like arithmetic, algebra are based on it. But students were hardly convinced about it. van Hiele's mention that geometrical learning occurs on hierarchical structure but from the research most students felt difficulty in learning experimental verification then theoretical proofs made controversial.

Students had difficulties in conceptual understanding of the geometrical objects, definition, properties and theorems etc. and procedural difficulties such as appropriate use of the geometrical tool, develop the conclusion from the experimental verification, connect required condition to prove the theorem, use theorem to find the solution of the application and appropriate formula use in appropriate situation.

There were various environmental factors which were also responsible such as significant implementation of the government policies, inadequate infrastructure of the schools, lack of sufficient material for learning geometry, lack of I.C.T. lab, inappropriate and insufficient training for mathematics teacher, anxiety and low interest of student towards geometry, lack of reference books with the students, insufficient integration of geometry curriculum, insufficient pre-knowledge of students, insufficient use of multiple ideas of geometry learning like as project work, group discussion, work on software such as geo-gebra, mathematica etc.

## Recommendations

Based on the findings of this study, the following recommendations are made for government, students, parents, teachers and farther researcher.

- Students needs to learn the geometry and gives the priority to understand the meaning of geometrical knowledge but not on rote learning.
- The government should take as a matter of urgency sent mathematics teachers or training and seminars for effective teaching of mathematics and geometry in particular in secondary school.
- The government should endeavor to provide the necessary infrastructures and facilities that will motivate teaching and learning of mathematics
- Teachers should try as much as possible to relate their lesson to real life situation in order to reduce abstract nature of the subject
- Parent needs to organize the basic home environment and take the responsibility with the school.
- School needs to give the emphasis not only on development of physical structure only but maintain the quality of learning environment. For this needs to co-operate with teachers, students, society, government policies and government structure.
- The further researcher can take the deep and diverse research in different area and issues of mathematics and mathematics education differently by using the approach from my research


## Educational Implication

This research has great importance for those are who gives the interest towards mathematics education such as researcher, teacher, school member, students, and parents, curriculum designer etc. This research comprise the difficulties of geometry learning on which states that both the curricular factors of difficulties and environmental factor of difficulties. Also it provides the information of different branch of learning of geometry and specific difficulties on each field. Also the educational implications of this research are discussed as;

- The teacher should be encouraged to use the different teaching methods and materials to teach geometry.
- The administration of school should be engaged to develop the quality of learning and facilities such as I.C.T. lab and other materials for learning.
- The teacher should motive weak students and precise them to participate in teaching and learning.
- The administration of education of the government should be conscious about the quality of education and makes the structure and infrastructure for the better learning.
- Students itself should find out the difficulties in their learning and should get motivated towards geometry learning.
- Project works should be given to the students in different geometric topics. Workshops on various portion of geometry should be conducted effectively.
- Parents should make the good environment of learning for their children.


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## Appendix-A

## Math Achievement Test

Subject: Mathematics
Level: Secondary
Time: 1 Hrs
Creative answers with including the figures obtain the full marks. Full marks: $\mathbf{3 0}$
Experimental verification/visualization
1.) (a) Write T for true statement and F for false statement.
I. बृतको ब्यास अर्धब्यासको दुईगुणा हुन्छ। (.....)
II. बृतको ब्यास अर्धब्यासको आधा हुन्छ। (....)
III. दिइएको बृतका बिभिन्न $y$ भागको नाम लेख।
IV. सरल कोणको योगफल $360^{\circ}$ हुन्छ। (......)

(b) Tick the wrong answer.( तलका बिकल्पबाट गलत उतर छान्नुहोस ।)
I) सबै बर्ग आयात हुन्छन ।
II) आयातका सबै कोण बराबर हुन्छन ।
III) चर्तूभुजका सबै कोण बराबर हुन्छन । IV) सबै त्रिभुजमा भित्रि कोणको योगफल $180^{\circ}$ हुन्छ
(c) Verified experimentally the statement "The opposite angles of the cyclic quadrilateral are supplementary. चत्हय चर्तुभुजका विपरित कोणहरु बराबर हुन्छन भनी प्रयोगद्धारा प्रमाणिनत गर ।

## Construction

$4+4$
2.) (a) Construct the triangle $A B C$ in which $A B=6 \mathrm{~cm}, B C=4 \mathrm{~cm}$ and angle $A B C=60^{\circ}$. $A B=6$ $\mathrm{cm}, \mathrm{BC}=4 \mathrm{~cm}$. र कोण $\mathrm{ABC}=60^{\circ}$ भएको त्रिभुज ABC रचना गर ।
b) Construct a parallelogram having one side $7 \mathrm{c} . \mathrm{m}$ and equal in area of the triangle ABC . प्रस्न (a) मा बनेको त्रिभुज ABC को क्षेत्रफल सग बराबर हुने र कुनै एउटा भुजा 7 cm हुने समानान्तर चत्तुभुजको रचना गर।

## Theoretical Proofs (Formal Deduction)

3.) a) Theoretically proof the statement "Parallelogram on the same base and same parallels are equal in area. एउटै आधार र उहि समानान्तर रेखाबिच बनेका समानान्तर चर्तुभुजको क्षेत्रफल बराबर हुन्छ भनि सैद्धानतिक रुपमा प्रमाणित गर ।
b) Theoretically prove that "The triangle on the same base and same parallels are equal in area. एउटै आधार र उहि समानान्तर रेखाबिच बनेका त्रिभुजहरुको क्षेत्रफल बराबर हुन्छ भनि सैद्धानतक रुपमा प्रमाणित गर।

## Application (rigor)

4.) a) In a given figure, $A B C D$ is a square and $E B C F$ is a parallelogram. If $A B=4 \mathrm{c} . \mathrm{m}$. Calculate the area of the parallelogram" $\mathrm{EBCF} . \mathrm{AB}=4 \mathrm{~cm}$ भए समानान्तर चर्तुभुज EBCF को क्षेत्रफल निकाल।


## Appendix-B

## Questionnaire

## The personal information of the student will be secured.

Direction: Student needs to give a symbol $(\sqrt{ })$ on your opinion on the respective one option of room.

Here, SA means strongly agree
A means Agree

D means Disagree
SD means Strongly Disagree
Q.N. 1 Do you have a math text book?

Please fill your personal information
Name:

## School name:

## Class:

Q.N. 2 Do you have a personal geometrical box?
a. Yes b. No
Q.N. 3 is there mathematics and i.c.t. lab in your school? a. Yes b. No
Q.N. 4 Do you have additional reference books like as question sets, solution books etc.?
a. Yes
b. No
Q.N. 5 is there a regular mathematics teacher in your school?
a. Yes
b. No

| S. <br> N | Subjects | SA A D SD | Remarks |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Student opinion about physical facilities |  |  |  |  |  |
| 1 | There are basic facilities like as appropriate white board, desk, <br> bench, geo-board, graph- board etc. instrument in your <br> classroom. |  |  |  |  |
| Pre-knowledge of the students |  |  |  |  |  |
| 2 | I have a sufficient knowledge to read the secondary level <br> mathematics through the pre-secondary level. |  |  |  |  |
| 3 | I am agree about geometry is the easy area of mathematics in <br> your curriculum. |  |  |  |  |
| 4 | I am agree about the course of geometry is vertically <br> interconnect in your curriculum. |  |  |  |  |
| 5 | There is any new chapter in your class course which is not read <br> in previous class. |  |  |  |  |


Q.N. 1 Express your experience about the difficulties for learning Geometry?

## Q.N. 2 Do you have extreme interest an irritation about mathematics, why?

## Thank you

## Appendix-C

Open Interview Schedule

## Objective-1Curricular content Factor

Interview conducted in the following content,

- Difficulties on visualization
- difficulties in experimental verification
- Difficulties in construction
- Difficulties in theoretical proofs
- Difficulties in application of geometrical content


## Objective-2 Environmental Factors

Interview conducted in the following environmental factors,

- Physical facilities
- Pre-knowledge of the students
- Teaching and learning procedure and activities
- Teaching learning environment and student motivation


## Appendix-D



बिख्य:- तथ्याइड संकलन कार्य सम्पन्न गर्नु भएको वारे।
की
शीमान् प्रमुख इन्चार्ज ज्यू ,
श्री त्रिभुवन विश्वविद्यालय (किर्तिपुर काठमान्डौ)

प्रस्तुन विपयमा त्यस कार्यलयो र्मात र०जय19०नय गतेको प्राप्त पत्र अनुसार त्यस विद्यालयवाट आउनुमएका छात्र श्र्र कमल पराजुर्ना ले यस विद्यालयमा मर्मात २०७४|१9|०४ गतं दोरीख र्मात २०७थ19910 गतं संम्म यस विद्यालयका कक्षा १० र कक्षा $९$ का विद्यार्याहगलाई Difficulties Of Geometries Learing on Secondary Level School Student लाईं सोंध कार्य गर्नुका साथै तथ्याइड संकलन गर्ने कार्य संम्यन्न गनुं भएकों व्यहांग जानकारी गराइन्छ।

$$
\overline{\text { किरी व.के.सी. }}
$$

READRMASTER

## Appendix-E



बिषय (Subject):- सेधकार्यको तध्याड़ाक सड.कलन सम्बन्धमा

3i. (Shree):- शि जाशास्त्रक्द्रुय विभाग, गणित तथा i.c.t. शिजा विभाग कित्तिपुर, काठमाणड

उपथुक्क विवयमा रीद्यास्त केन्द्रय विभाग अर्त्रगत गणित तया İet. रिदा विभाग M.Ed. चौर्धो सेमेस्टर रोल न. 6 g का त्रती कमल पराजुलीले मिति $206 \times 199102$ देखि 206219910 g सम्म यस विधालयमा आई वध्याड्डक सैड्रकलनको लागी परिआ जानकारी गराइन्ध।

## Appendix-F

## Reliability by Test Re-Test Method

| Students | Test(X) | Re-Test(Y) | $\mathrm{X}^{2}$ | $\mathrm{Y}^{2}$ | XY |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A | 15 | 17 | 225 | 289 | 255 |
| B | 8 | 10 | 64 | 100 | 80 |
| C | 22 | 18 | 484 | 324 | 396 |
| D | 13 | 14 | 169 | 196 | 182 |
| E | 16.5 | 17 | 272.25 | 289 | 280.5 |
| F | 19 | 19.5 | 361 | 380.25 | 270.5 |
| G | 12 | 25 | 144 | 625 | 300 |
| H | 25 | 28 | 625 | 784 | 700 |
| I | 12.5 | 13 | 156.25 | 169 | 162.5 |
| J | 4 | 14 | 16 | 196 | 56 |
| K | 3.5 | 10 | 12.25 | 100 | 35 |
| L | 10 | 8 | 100 | 64 | 80 |
| M | 23 | 20 | 529 | 400 | 460 |
| N | 21.5 | 25 | 462.25 | 625 | 537.5 |
| O | 26 | 25.5 | 676 | 676 | 663 |
|  | $\sum \mathrm{X}=231$ | $\sum \mathrm{Y}=239.5$ | $\sum \mathrm{X}^{2}=4296$ | $\sum \mathrm{Y}^{2}=5021.25$ | $\sum \mathrm{XY}=4458$ |

$$
\mathrm{r}=\frac{\mathrm{N} \sum X Y-\sum X \sum Y}{\sqrt{N \sum X^{2}-\left(\sum X^{2}\right)} \sqrt{N \sum Y^{2}-\left(\sum Y^{2}\right)}}=0.77
$$

According to Garnet (Garnet, 2008), the interpretation of reliability coefficient as following table:

| Coefficient | Nature |
| :--- | :--- |
| 0.0 to $\pm 0.20$ | Indifferent or negligible |
| $\pm 0.20$ to $\pm 0.40$ | Present but slight |
| $\pm 0.40$ to $\pm 0.70$ | Substantial or marked |
| $\pm 0.70$ to $\pm 1$ | High to very high |

