CHAPTER I

INTRODUCTION

Chapter Overview

This chapter begins with its introductory parts, highlighting the background of study, statement of the problem, objective of the study, significance of the study, delimitation of the study and definition of related term.

Background of the Study

Mathematics has been taught as a compulsory subject at all levels in school system in Nepal. Besides compulsory mathematics optional mathematics is also offered to willing and worthy students. In both subject areas geometry is taught separately as an important part of the school mathematics curriculum. Thus, geometry is considered as an important component of school mathematics. There is a vital role of teacher to show all these importance of geometry to the students in their teaching. Moreover geometry has covered 30% area in mathematics curriculum according to N C F 2063. So the geometry is the most important part of mathematics to the school level curriculum.

According to Webster‘s Dictionary, geometry is a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solid; broadly: the study of properties of given elements that remain invariant under specified transformation. H. M. S. Coxeter described geometry as “the most elementary of the sciences that enable man, by purely intellectual processes, to make predictions (based on observation) about physical world. The power of geometry, in the sense of accuracy and utility of these deductions, is impressive, and has been a powerful motivation for the study of logic in geometry.” Geometry can also be seen as the mathematics of logic and reasoning. According to the average citizen, geometry is the mathematics of shapes and their characteristics.

According to historical evidence, geometry was developed faster than other mathematics. But now a day, it is regarded as one of the branch of mathematics. It is also oldest intellectual pursuits of man. The origin of geometry is the earliest Babylonia and Egypt. It is essential and empirical science cultivated solely for its utility with the art of practical measurement and logical subject, so the Plato advocated that every student of law must study geometry because law is logical subject.

As a two years teaching experience in school level mathematics I found that most of the student feel difficult to learning geometry. Among geometrical content, coordinate geometry is also the main branch of geometry, which deals about the distance between two points, the midpoint of a line joining two points, gradient of a line, parallel and perpendicular lines and the equation of a line, slope intercept form, perpendicular form, pair of straight line, homogeneous equations of second degree, point slope form, two point slope form, area of triangle and quadrilateral, by using coordinate, conic section, circle etc in secondary level. I feel that student feel many procedural and conceptual difficulties in learning coordinate geometry. In my literature review (Confrey 1990; Lim 2011; Fabiye 2017; Achraya 2017; Mahato 2018; Sapkota 2017; Thakur2018), I found that there are many research related to learning difficulties in geometry but not concrete about the specific topic coordinate geometry. So there was a research gap. I think this my research fulfill this gap. And also want to know what are the main conceptual and procedural difficulties that student felt in learning coordinate geometry. That’s why I selected the coordinate geometry as my research topic.

Statement of the Problem

There are so many researches done on the field of geometry. As to find out the learning difficulties, they are more general and only concerns on the geometry. coordinate geometry is new, very efficient and appropriate used field of geometry which deals about the distance between two points, the midpoint of a line joining two points, gradient of a line, parallel and perpendicular lines and the equation of a line, slope intercept form, perpendicular form, pair of straight line, homogeneous equations of second degree, point slope form, two point slope form, area of triangle and quadrilateral by using coordinate, conic section, circle etc. It requires various types of capability to learn whereas students feel the problem to work. According to my teaching experience, student’s difficulties are seen in problem solving in examination. Mainly students are facing the problem in the concept of the term used in coordinate and also they are facing in problem in solving process. So I feel that it’s a researchable area. So I intended that to find out the actual conceptual and procedural learning difficulties face by the student in learning coordinate geometry.

Objective of the Study

Objective is the main key of the research. The objectives of the study were:

- To explore the conceptual difficulties faced by students in learning co-ordinate geometry.
- To explore the procedural difficulties faced by students in learning co-ordinate geometry.

Research Questions

This study concerned with the study of conceptual and procedural difficulties face by the student in learning coordinate geometry. This study sought to answer the following research questions:

- What are the conceptual difficulties faces by student in learning coordinate geometry?
- What are the procedural difficulties faces by the student in learning coordinate geometry?

Significance of the Study

As my literature review I found that there are many difficulties in teaching and learning geometry, geometry as a broader concept. Geometry has different branches such as Euclidean geometry, coordinate geometry, transformation geometry, and analytic geometry. But there are not conceptual and procedural difficulties in learning coordinate geometry was mentioned. So there was a research gap. And I want to fulfill this research gap so my research will add one brick in coordinate geometry. That's why this research takes important role in learning and teaching coordinate geometry. I feel that this research will help to teacher, policy maker, administrator, curriculum maker and student for quality teaching and learning coordinate geometry.

Delimitation of the Study

Each study was no rigorously perfect and free from limitations. So this study had following delimitations;

- This study limited to one private school of Kaski district.

- This study was only concerned only the students of grade IX.
- This study focused on the conceptual and procedural difficulties in learning co-ordinate geometry.
- This study only based on the eighty students.
- Only mathematics achievement test and in-depth interview was used to collect data.

Definition of Related Term

Conceptual difficulties. A conceptual difficulty means such types of difficulties where students have poor in linking relationship, poor in thoughtful knowledge and reflective learning. For example problem in understanding definitions, understanding the objects, locating the point, knowing formula, transformation of knowledge, memorizing, visual spatial organization etc.

Procedural difficulties. Difficulties related to solving process of the problem of coordinate geometry. For example the procedural difficulties are the problem in appropriate use of formula, way of solving, numerical fluctuation, weak in verbal explanation etc.

School level Coordinate Geometry. School level coordinate geometry means the coordinate geometry which teaches in grade IX. For example the content of coordinate geometry be distance between two points; collinear; section formula; area of triangle and quadrilateral; slope of line etc.

Mathematics achievement Test. A mathematics achievement test is an examination to identify an individual's specific areas of learning difficulties. Mathematics achievement test measures an individual’s academic problem areas.

CHAPTER II

REVIEW OF RELATED LITERATURE

This chapter deals on the related literature review (on content of geometry, teacher’s instructional strategies, conceptual and procedural knowledge, misconception about geometry, learning errors in geometry, learning difficulties, and learning mathematics), theoretical literature review and conceptual framework.

Literature review

A literature review surveys books, scholarly articles, and any other sources relevant to a particular issue, area of research, or theory, and by so doing, provides a description, summary, and critical evaluation of these works in relation to the research problem being investigated. Literature reviews are designed to provide an overview of sources explored while researching a particular topic and to demonstrate to readers how this research fits within a larger field of study (Fink, 2014). This topic based on content of geometry, teacher’s instructional strategies, misconception about geometry, learning errors in geometry, difficulties factors, and learning mathematics.

Content of Geometry

School curricula worldwide cover four main learning outcomes in Geometry (Bassarear, 2012). By the time students complete school they should be able to:

- Analyze the characteristics, properties and relationships of two-dimensional and three-dimensional geometrical shapes (Euclidean Geometry).

- Specify locations and describe spatial relationships using coordinate geometry and other representation systems (Coordinate Geometry).
- Apply transformation and use symmetry to analyze mathematical situations (Transformation Geometry).
- Use visualization, spatial reasoning and geometric modeling to solve problems.

Atebe and Schafer (2011) assert that students' general mathematical competencies have been linked closely to their geometric understanding. Research has also noted that geometry is difficult to teach as well as to learn. Coordinate or analytical geometry, for instance, requires not only geometrical knowledge, but also a vast amount of knowledge in working with coordinates on a 2D (two-dimensional) or 3D (three-dimensional) set of axes. These additional concepts make geometry more complex and require an intricate manner of thinking. So co-ordinate geometry also play vitrol role to understand geometry. In geometrical content, there are procedure, conceptual, strategic, declarative knowledge included.. Analyzing transformation geometry involves many different types of knowledge as defined by Hiebert (1986) such as procedural, conceptual, strategic and declarative knowledge.

Teachers Instructional Strategies

Instructional strategies are techniques teachers use to help students become independent, strategic learners. These strategies become learning strategies when students independently select the appropriate ones and use them effectively to accomplish tasks or meet goals. Instructional Strategies is the most powerful aspect of teaching and learning process. The student learnt from the strategies adopted by the teacher. Research has delineated that errors occur mainly because students have difficulties in understanding the instructional strategies adopted by the teacher (Confrey, 1990). In geometry the communication of information at different levels of reasoning of the sender (the teachers) and the receiver (the student) become a major cause of misconception (Lim, 2011). This is especially true in the case of geometry. When the teacher operates and communicates at different levels of geometric thought to those of the students, concepts are not understood or acquired fully. It is necessary for teachers to know their students' level of geometrical thought and to operate at those levels. To teach geometry, teacher needs to know the child's spatial environment (Bishop, 1980) and they need to develop skills in geometry not only future use in teaching, but also as aids to their own study of mathematics. They particularly need to understand and to capitalize upon the interrelation among the several areas of mathematics (Meserve, 1973).

Elementary school teachers described mathematics learning difficulty as general problems in perception, learning and processing. Teachers also stated that the parents of these students are unconcerned, their communication with the teacher is weak and they ignore the problems with their children. Parents also stated that their children have difficulty in mathematics subjects such as operations, problem solving, mind calculation and multiplication table, the fact that they do not understand what they are told and they try to learn by memorizing. As a result of the research, it was seen that elementary school teachers are aware of mathematics learning difficulty but they struggle in detailed diagnosis of the difficulties of the students and need support in designing the learning process of these students. Moreover, it was understood that parents cannot describe their children's difficulties in detail and they are inadequate to support these students (Temur, Turgut, Ozdemir, .2018).

Conceptual and Procedural knowledge

Several theories of learning and cognition posit that our behaviour is shaped by at least two different kinds of knowledge: one providing an abstract understanding of the principles and relations between pieces of knowledge in a certain domain, and another one enabling us to quickly and efficiently solve problems. In recent empirical research on mathematics learning the former is frequently named conceptual knowledge, while the latter is labelled procedural knowledge (Baroody, 2003). Conceptual knowledge is a core knowlwdge. Conceptual knowledge is seen as the knowledge of the core concepts and principles and their interrelations in a certain domain. Accordingly, it is assumed to be stored in some form of relational representation, like schemas, semantic networks or hierarchies (Byrnes & Wasik, 1991). Because of its abstract nature and the fact that it can be consciously accessed, it can be largely verbalized and flexibly transformed through processes of inference and reflection. It is, therefore, not bound up with specific problems but can in principle be generalized for a variety of problem types in a domain (Baroody, 2003).

Procedural knowledge, in contrast, is seen as the knowledge of operators and the conditions under which these can be used to reach certain goals (Byrnes & Wasik, 1991). Further, it allows people to solve problems quickly and efficiently because it is to some degree automated. Automatization is accomplished through practice and allows for a quick activation and execution of procedural knowledge, since its application, as compared to the application of conceptual knowledge, involves minimal conscious attention and few cognitive resources (Johnson, 2003). Its automated nature, however, implies that procedural knowledge is not or only partly open to conscious inspection and can, thus, be hardly verbalized or transformed by higher mental processes. As a consequence, it is tied to specific problem types (Baroody, 2003)

Misconception about Geometry

Misconceptions have been determined as one of the most important barriers on learning Mathematics. (Michael, 2001) defines misconceptions as 'conceptual or reasoning difficulties that hinder students' mastery of any discipline'. According to (Drews 2005), a misconception could be the result of 'a misapplication of a rule, an over- or under-generalization, or an alternative conception of the situation'. (Swan 2001) views misconceptions as 'natural stage of conceptual development'. For students to be able to confront underlying conceptual difficulties, overcoming misconceptions is required (Van der Sandt & Nieuwoudt, 2008) as cited by Luneta, 2014).

The article written by Lee (2009), discuss nine common misconceptions about learning and teaching mathematics for young children that are widespread among prospective and practicing early childhood teachers in the United States. These misconceptions include: 1. Young children are not ready for mathematics education; 2. Mathematics is for some bright kids with mathematics genes; 3. Simple numbers and shapes are enough; 4. Language and literacy are more important than mathematics; 5. Teachers should provide an enriched physical environment, step back, and let the children play; 6. Mathematics should not be taught as a stand-alone subject matter; 7. Assessment in mathematics is irrelevant when it comes to young children; 8. Children learn mathematics only by interacting with concrete objects; 9. Computers are inappropriate for the teaching and learning of mathematics.

One of the most critical aspects of effective mathematics learning is developing a proper understanding of the nature of mathematics. The chairperson of the commission that writes in the National Council of Teachers of Mathematics (NCTM) Standards stated, "The single most compelling issue in improving school mathematics is to change the epistemology of mathematics in schools, the sense on the part of teachers and students of what the mathematical enterprise is all about" (Romberg, 1992).

Learning Errors in Geometry

Luneta (2008) defines errors as ‘simple symptoms of the difficulties a student is encountering during a learning experience’. According to Swan (2001), an error could be the result of ‘carelessness or misinterpretation of symbols or text’. Misconceptions manifest in students’ work as errors, which implies that errors are symptoms of misconceptions students possess. According to Confrey (1990) misconceptions emanate from ‘a line of thinking that causes a series of errors all resulting from incorrect underlying premises’. Knowledge of students’ errors is essential and teachers should provide opportunities for students to display their errors as these will be essential stepping stones for effective instruction. It can be argued that knowledge of students’ levels of geometric reasoning is essential for effective teaching.

Students did not understand most of the basic concepts in Euclidian transformation. Most of the errors were conceptual and suggested that students did not understand the questions and did not know what to do as a result. It is also noted that when students lack conceptual knowledge the consequences are so severe that they hardly respond to the questions in the examination (Luneta, 2013). The causes of difficulty in the learning of concepts in geometry could be traceable to teachers' method of instruction, unavailability of instructional materials, insufficient time allocation, students' gender, complexity, and misconception of concepts (Fabiyi, 2017).

Learning Difficulties

Learning difficulties as a problem that affects a person's ability to learn, get along with other a follow convention. Based on Sarama et.al. (2003) students tend reverse x and y, especially when they have to plot point that have 0 in its coordinate (e.g. (0,6)). In addition, some of students sometimes ignore the origin. They plot a new point started from the previous point, not the origin. Not only that, some students fail to relate coordinates and location of multiple points. For instance, they plot (20, 35) and (25, 35) each from the origin point. They cannot see that actually the second point is 5 point to the right from the first point. Moreover, based on Blades and Spencer 9 (2001), the children's struggle or mistake of determining the coordinate/location of an object is they often to see only one coordinate line rather than two coordinate lines.

According to Wongwanich and Sujiva (2014) There are several difficulties in problem solving, namely 1) Students have difficulties in understanding the keywords appearing in problems, thus cannot interpret them in mathematical sentences. 2) Students are unable to figure out what to assume and what information from the problem is necessary to solving it, 3) Whenever students do not understand the problem, they tend to guess the answer without any thinking process, 4) Students are impatient and do not like to read mathematical problems, and 5) Students do not like to read long problems.

Incomplete mastery of number facts, weakness in computational, inability to connect conceptual aspects of math, inefficiency to transfer knowledge, difficulty to make meaningful connection among information, incompetency to transform information mathematically, incomplete mastery of mathematical terms, incomplete understanding of mathematical language and difficulty in comprehend and visualizing mathematical concept might result in difficulties (Garnett, 1998).

Developing understanding in mathematics is an important but difficult goal. Being aware of student difficulties and the sources of the difficulties, and designing instruction to diminish them, are important steps in achieving this goal. Student difficulties in learning written symbols, concepts and procedures can be reduced by creating learning environments that help students build connections between their formal and informal mathematical knowledge; using appropriate representations depending on the given problem context; and helping them connect procedural and conceptual knowledge.(Elif, 2003).

Factor for Learning Mathematics

There are many factors difficulties in learning mathematics. They may responsible to make confusion in learning mathematics. Acharya(2017), argues that the followings are the factors affecting learning mathematics:

- Mathematical anxiety, lack of interest and negative felling towards mathematics, it makes difficult to learn mathematics.
- Lack of prior knowledge of the student creates problem to study mathematics.
- Lack of student’s labor in learning mathematics learning.

- Lack of parent's awareness, interest of the subject matters affects their children to study mathematics.
- Due to low economic condition of students, they have to engage their household work to fulfill their daily needs. So students are not interest to study mathematics.
- The students who did get support and inspiration from their parents students result is better.
- Lack of motivation and counseling crates misunderstanding to study mathematics.

Therefore, many difficulting factors are occur in learning mathematics, commonly mathematical anxiety, negative feelings towards mathematics, lack of pre knowledge, lack of motivation, economic status, parents awareness etc. are play vitrol role.

Similarly, Sapkota (2017) finds that the students feel difficulties in learning geometry because of school environment, home environment, lack of pre-knowledge, motivation and interest, learning activities, teaching method and technique. If the school provides sufficient teaching materials about geometry, extra classes for week students, teacher use appropriate teaching method and parents create good environment for practice then the difficulties in learning geometry would be minimize. Also Thakur(2018), agree that the learning environment of home and school, pre-knowledge of students, learning activity seems to be exam oriented rather than practical oriented, poor evaluation techniques, lack of appropriate teaching methods, communication problem, teaching without contextualization, lack of professional training, teaching overloaded irreverent teaching method, careless of school administration and no-effective learning management related exploring learning difficulties of students in geometry.

Similarly, Mahato(2018) says that learning geometry in secondary level is affected by so any factors such as lack of encouragement for the study, congested and uncomfortable classroom for students, unavailability of teaching learning materials, and lack of untrained teachers, lack of physical facilities and improperly arrangement, lack of good administration and negligence of students in learning geometry etc. The cause of difficulties in learning geometry due to lack of prerequisite knowledge, irregular in class attendance and method of teaching adopted by the teacher. Similarly, there is not proper interaction between teacher and students, not sufficiently practice at home and not getting appropriate problem solving ways and rules.

Theoretical Understanding of the Study

I take Van Hiele's theory as my theoretical literature review because he argues that, if we follow his five level of geometrical through we can learn and teach geometry properly. My research based on the conceptual and procedural learning difficulties, faced by student in learning coordinate geometry. The first three levels involve the development of conceptual understanding in geometry, whilst the last two display the development of procedural fluency (Kilpatrick, Swafford & Findell, 2001). Conceptual knowledge was a source of children's procedural knowledge, but not vice versa. In contrast to procedural knowledge, conceptual knowledge could be assessed with high internal consistency.For discussing five levels, van Hiele introduced the existence of five levels of geometrical thought, which are given below:

Level 0 (Basic Level): Visualization or Recognition (Visual Skills).

Students can identify a shape, but are not able to provide its properties. The shape is judged only by its appearance.

Level 1: Analysis (Drawing and Verbal Skills)

Analysis is predominantly descriptive i.e. students are able to identify particular properties of shapes, but not in a logical order.

Level 2: Ordering or Informal Deduction.

Students can combine shapes and their properties to provide a precise definition as well as relate the shape to other shapes. There is a logical order to the properties and they are deduced from one another.

Level 3: Deduction (Logical Skills)

Students apply formal deductive arguments such as in proofs. Theorems with an axiomatic system are established. The interrelationship and role of undefined terms, axioms, definitions, theorems and proof is seen.

Level 4: Rigors and Axiomatic (Applied Skills)

Rigors and axiomatic also known as the meta-mathematical level. (Van der Sandt,2007), is characterized by 'formal reasoning about mathematical systems by manipulating geometric statements such as axioms, definitions, and theorems' and at this juncture students can ‘compare systems based on different axioms and can study various geometries in the absence of concrete models.

All the above mentioned Van Hiele levels of geometrical thinking can be summarized in the following table:

Table 1: van Hiele’s Levels of Geometric Thinking

Levels	Stages	Characteristics
Level 0	Visualization	Students recognize the figure on the basis of their physical appearance
Level 1	Analysis	Students analyze the components parts of the figure
Level 2	Informal Deduction	Students establish the interrelationship of properties both within figure
Level 3	Deduction	Students able to construct proof using postulates or axioms and definitions
Level 4	Rigor	Students can work in a variety of axiomatic systems

Understanding these levels enables teachers to identify the general directions of students’ learning and the level at which they are operating. Pegg (1985) explains that van Hiele's theory is divided into two parts: the first part is the hierarchical sequence of the levels, which shows that each level must be fully developed by the student before proceeding to the next level. The second part is the development of intuition in students and the phases of learning that influence geometric learning. van Hiele's theory of geometry with its focus on geometrical reasoning has been linked to Piaget' five stages of child development and the role they play in learning geometry (Pusey, 2003).

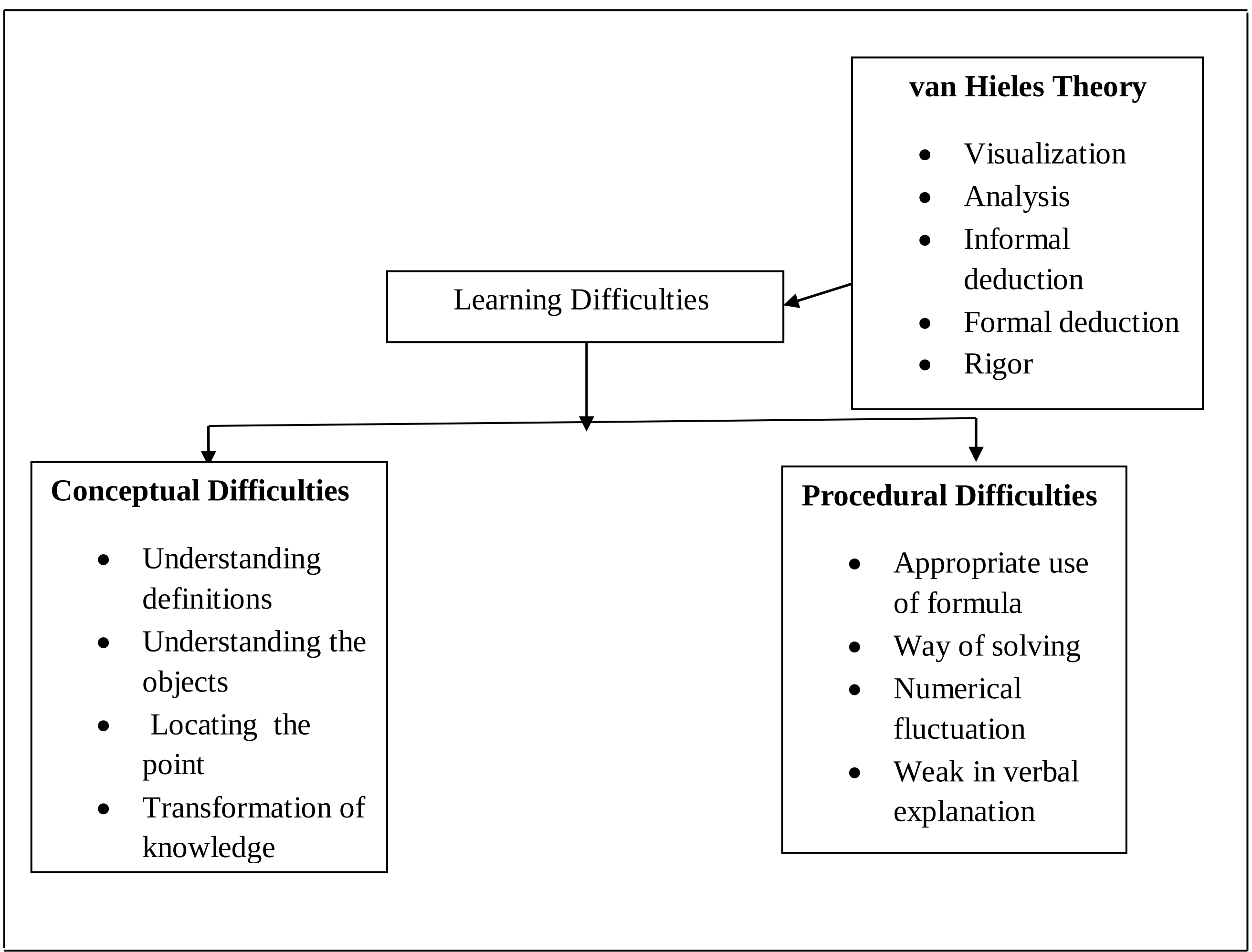
Implication of the review for the study

The above literature review indicate that misconception, lack of student's labor, teachers instructional strategies, errors in geometry, negative feelings of mathematics, difficulties in symbolization,visualizing the object, transformation of knowledge, difficulties in appropriate use of formula, irreverent teaching method, irregularity, lack of regular assessment system, lack of procedural of solving problem etc. are the main difficulties in learning geometry. Mainly learning difficulties can be classified two types: Conceptual and Procedural. According to Van Hiele's theory there are five level of geometrical thinking. They are visualization, analysis, informal deduction, formal deduction and rigor. From these five levels first three are concerned with the procedural understanding and the last two are concerned in conceptual fluency o geometrical thinking. Geometry has different braches such as Euclidean geometry, co-ordinate geometry, analytical geometry, Transformation geometry. From the above literature review I found many teaching and learning difficulties in geometry but there are no any exact learning difficulties in co-ordinate geometry. There is research gap. Therefore I select co-ordinate geometry as my research area.

Conceptual Framework

A conceptual framework represents the researcher's synthesis of literature on how to explain a phenomenon. It maps out the actions required in the course of the study given his previous knowledge of other researchers' point of view and his observations on the subject of research. The figure given below says that area of learning difficulties are based on Van Hiele's theory of geometrical thinking which gives area of procedural and conceptual difficulties. According to Kilpatrick, Swafford & Findell (2001), Van Hieles firsts three level are concerned with conceptual understanding in geometry and last two are procedural fluency. The conceptual difficulties may be problem in understanding definitions, understanding the objects, locating the point, knowing formula, transformation of knowledge etc. And the procedural difficulties may be problem in appropriate use of formula, way of solving, numerical fluctuation etc.

Figure 1: Conceptual Understanding



Above framework shows that there are two types of learning difficulties in coordinate geometry they are; conceptual difficulties and procedural difficulties (Hiebert, 1986). In conceptual difficulties, there are four subtopics. They are difficulties in understanding definition, understanding object, locating point, and transformation of knowledge. Also in procedural difficulties there are also four subtopics. They are difficulties in appropriate use of formula, way of solving, numerical fluctuation, and verbal explanation. In these conceptual and procedural difficulties I used van Hieles level of geometric through to analyze the data. van Hieles introduced the existence of five levels of geometrical thought, which are visualization, analysis, informal deduction, formal deduction, and rigor.

CHAPTER III

METHODS AND PROCEDURAL

This chapter deals with the methods and procedures of study. It determines how the research becomes complete, reliable, valid, and systematic. The major procedures and method applied in research was described in the following sections.

Design of the Study

The main concern of this study was to explorer the conceptual and procedural learning difficulties in school level Coordinate Geometry. For this purpose I used case study as my research design. Case study covers investigation within individual or small groups. Sagadin (1991) states that a "case study is used when we analyze and describe, for example each person individually (his or her activity, special needs, life situation, life history, etc.), a group of people (a school department, a group of students with special needs, teaching staff, etc.), individual institutions or a problem (or several problems), process, phenomenon or event in a particular institution, etc. Also Simons (2009) stated that "case study is an in-depth exploration from multiple perspective of the complexity and uniqueness of a particular project, policy, institution, program or system in a real life". The case study design is based upon the assumption that the case being studied is atypical of cases of a certain type and therefore a single case can provide insight into the events and situations prevalent in a group from where the case has been drawn. In my research I used this case study research design because it's give proper data. And the students whose performance was not satisfactory were my case.

Study Area and Participant

In this study all the student learning in school level became study area. In this area, I selected four schools from purposive sampling technique in Kaski district. Because in my two year teaching experience in secondary level in private school namely Marigold Secondary Boarding School Nayagoan and Children Academy, Miyapatan, I feel students face difficulties in learning coordinate geometry. So these two schools and another two schools namely Balbidhya Boarding school Miyapatan And Siddhibinayak Boarding School Bhandardhik fulfilled the objectives of the research. All the students, who are studied in grade IX of selected schools, are the participant of the study. So, I selected those schools and the participant.

Data Collection Tools

Data collection is the process of gathering and measuring information on targeted variables in an established system, which then enables one to answer relevant questions and evaluate outcomes. Data collection is a component of research in all fields of study. The main tools that I used in my research were mathematics achievement test and in-depth interview.

Mathematics achievement test. A mathematics achievement test is an examination to identify an individual's specific areas of learning difficulties. In my research first of all I find out the area of learning difficulties in school level coordinate geometry. So I select mathematics achievement test as first research tool. For this I made 27 questions which were in simple to complex order. First one to14 questions were very short questions which measures basic concepts and understanding formula of coordinate geometry. Middle eight questions (question number 15 to 22) were short questions related to the very short questions. Again remaining five questions (questions number 23 to 27) were long questions where two or more basic concepts are required to solve the question.

Reliability and validity of the test tool. To make reliable of the test paper, I did pilot test with ten students selected from the participant of the research. I used split half method to find reliability coefficient. Calculated reliability coefficient was 0.80 (Appendix-B), according to Garnet (2008) this coefficient was very high. For analyzing this results, the test question (Appendix-A) were reliable. For validation of the mathematics achievement test paper, I used curriculum of optional mathematics of grade IX developed by curriculum development center. Also I used scope and sequence table provided by curriculum development centre and I also consult with guidance teacher. This mathematics achievement test gave me the area of learning difficulties which helps to took in-depth interview.

In-depth interview. In-depth interview is a qualitative research technique that involves conducting intensive individual interviews with a small number of respondents to explore their perspectives on particular idea, program, or situation. The main concern of my research was to exploring conceptual and procedural learning difficulties in school level coordinate geometry. After doing mathematics achievement test for finding area of learning difficulties, in-depth interview gave me actual difficulties in related area so I used in-depth interview as my research tool. For In-depth interview I selected the seven students whose performance was not satisfactory in mathematics achievement test by purposive sampling method. And I took unstructured interview based on his learning difficulties. In in-depth interview basically I focused on the difficulties face by the students. I tried to find out the internal core facts where they feel difficulties and which contain or concepts make difficulties or which procedure makes difficulties by using in-depth interview. This in-depth interview gave me the primary data to analyze. So it was the core tool to take data from the field.

Data Collection Procedure

For data collection I selected the four school of Kaski district by purposive sampling method. Then I took mathematics achievement test for all the students of grade IX of the selected school. Mathematics achievement test gave me the area of learning difficulties. Among the students of grade IX, I selected seven students purposively. Then I took in-depth interview based on this area of learning difficulties and I recorded this interview. This in-depth interview was based on unstructured interview.

Data Analysis Procedure

For collecting data I used two tools they were mathematics achievement test and in-depth interview. I analyze the data (obtain from mathematics achievement test and in-depth interview) by inductive thematic approach. An inductive thematic approach, which is useful for identifying core meanings that were relevant to the research objects, was used for data analysis, in which quotes were coded and categorized into themes and subthemes. For analyzing data I will use some steps which is given by Thomos (2006) are (a) condense raw textual data into a brief, summary format; (b) establish clear links between the evaluation or research objectives and the summary findings derived from the raw data; and (c) develop a framework of the underlying structure of experiences or processes that are evident in the raw data. For analyze the data first of all I used mathematics achievement test to identify area of learning difficulties. That means mathematics achievement test gave me the area of learning difficulties. On the base of theses area of difficulties I took in-depth interview. Also the performance given by the students were scanned and analyze it on the basis of in-depth interview. The recorded interview data were translated, coding, and integrate similar coding and make a theme. Thus, in this study the data were analyzed by inductive thematic approach and using different theories.

CHAPTER IV

ANALYSIS AND INTERPRETATION OF DATA

This was the descriptive qualitative research related to the conceptual and procedural learning difficulties in school level coordinate geometry. Four schools of Kaski district were chosen for this study purposively, they are Marigold Secondary Boarding School, Nayagoan, Children Academy, Miyapatan, Balbidhya Boarding school Miyapatan And Siddhibinayak Boarding School Bhandardhik. Among these four schools, students of grade IX who studied optional mathematics were the samples of my research. The objectives of the study were to exploring conceptual and procedural difficulties faced by students in learning co-ordinate geometry. The collected data were tabulated and analyzed according to the objective of the study. Mathematics achievement test and In-depth interview were used for the data collection as a main tool.

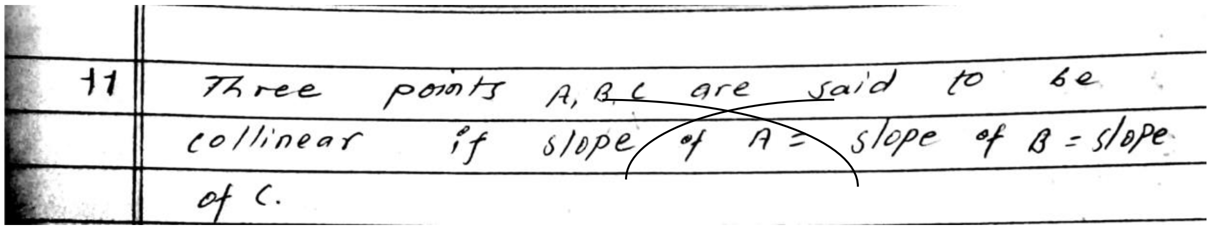
Then I took mathematics achievement test in 80 students to find the ground reality of the student's academic level which could help to find the difficulties in learning coordinate geometry. Among 80 students seven students who had greatest difficulties were selected for the in-depth interview.

This chapter includes the analysis and interpretation of the information obtained from the field of the study. The data collected from field are not in proper manner, so first they are coded and separated according to theme of information. The data were presented in terms of following difficulties (which was given in conceptual framework) conceptual difficulties and procedural difficulties. Conceptual difficulties face by the students in learning coordinate geometry is related to the objective first. In conceptual difficulties, it is based on the sub heading; difficulties in understanding definitions, understanding the objects, locating the point, transformation of knowledge, again procedural difficulties faced by the students in learning co-ordinate geometry are based on the second objective. In procedural difficulties, it is based on the subheading; difficulties in appropriate use of formula, way of solving, numerical fluctuation, weak in verbal explanation.

Conceptual Difficulties

Conceptual knowledge is a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete bits of information. A conceptual difficulty means such types of difficulties where students have poor in linking relationship, poor in thoughtful knowledge and reflective learning. Exploring conceptual difficulties in learning coordinate geometry is the first objective of my research. For this I divided four topics to explore difficulties. They are difficulties in understanding definition, Understanding objects, locating points, transformation of knowledge.

Difficulties related to understanding definition. Definition is the key term of the mathematics. Understanding definition is the main difficulties face by the students in learning coordinate geometry. I asked a one question related to the definition. This question was; Three points A, B, C are said to be collinear if a) slope of AB = slope of BC, b) slope of A= slope of B = slope of C, c) slope of A × slope of B = slope of C. This is in objective form. Among the 80 student only 34 students give correct answer. This quantitative data shows that there are vast difficulties in definition. The response given by the students are presented below:



11 Three points A, B, C are said to be collinear if slope of A = slope of B = slope of C.

The above answer shows that student wrote three points A, B, C are said to be collinear if slope of A= slope of B = slope of C. which is wrong. It should be three points A, B, C are said to be collinear if slope of AB= slope of BC. Then I asked the one student that why you select option (b) then student replied;

"*Sir, collinear means lies on same straight line. So that three points A, B, C should lies on same straight line. So slope of each point should be equal"*.

Then I asked another question that how to find out the slope by using one point then he thinks some time and he replied that "*sorry sir I don't know"*

Form the above response given by the student shows that student had difficulties in connecting slope in the definition of collinear. Because student know the meaning collinear term but due to not understanding the slope and how to calculate slope students feels difficulties. Student said that the slope of each point should be equal. This views shows that student have misconception about the slope. He thinks slope can be found in every point. This was wrong answer. Therefore misconception about the slope makes difficulties in knowing collinear. But Bishop (1890) says that misconception is necessary to know students geometrical through and level of students. So, misconception about the slope gives the level of students for the researcher and teacher.

Also in another discuss, I asked the another student why you did not know this definition then he replied that

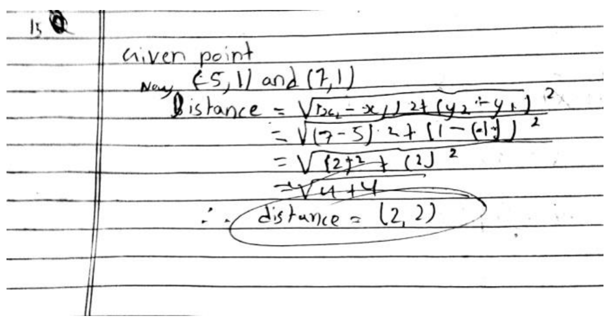
"We all are focus on exam. Definitions are not important for exam. So teacher and we (students) both do not focus on the definition. That's why we do not understand the definition."

Above students view shows that all students are not focus on definition. They only focus on the question and solution. In this matter Elif (2003) supported that students are not focused on the core concept of mathematics but they focused on the problem which are important for the examination. So evaluation system makes the difficulties in understanding definition. And also in another aspect, In question number 20, which is not directly based on the definition. This question was; show that A(0, 2), B(3, 0), C(6, -2) are collinear. Only 19 students give correct answer. In this aspect, I took two students for in-depth interview. They say that, *"We do not understand the term collinear*". When I clarify the meaning of the collinear then they replied that:

"*Sir, we do not understand the term collinear. If the term 'collinear 'is replaced by the 'lies on same straight line' then we should solve this problem."*

From the above response given by the student shows that student have difficulties in mathematical symbolization because students can solve the question if the term 'collinear' was replaced by the 'lies on same straight line. Therefore mathematical symbolization, representation of the term makes difficulties. Also Elif (2003) supported that students feels difficulties in written symbol and appropriate representation of the term used in mathematics. Wongwanich and Sujiva (2014) states that Students have difficulties in understanding the keywords appearing in problems, thus cannot interpret them in mathematical sentences. So, mathematical symbolization makes difficulties in learning coordinate geometry.

Another aspect, in a tool there is a one question related to distance formula, that is; find the distance between the points (5,-1) and (7, 1). The solution given by the student is given below:

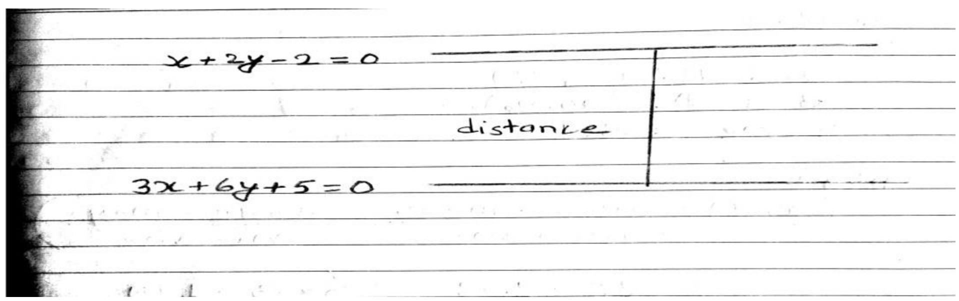


Above solution shows that student understands the solving way of this problem. But at last he mistake in opening root. He should wrote distance= $\sqrt{4+4}$ =$\sqrt{8}$ =2$\sqrt{2}$ but he wrote distance=$\sqrt{4+4}$ = (2, 2) which is wrong. Then I said the student that (2, 2) is distance? Then he replied:

"This question is from coordinate geometry. So I wrote distance is in coordinate form."

This answer given by the student shows that he did not know the definition of distance. He doesn't know the distance is scalar or vector. He said that in coordinate geometry all the concepts are in coordinate form. So distance also should be in coordinate form. Therefore he had difficulties in understanding what is scalar or vector.

Difficulties related to understanding object. Understanding object is the core thing of mathematics. Coordinate geometry is the concrete type. So student should have to know object to solve coordinate geometrical concept or problem. If the student can made figure of every problem then he easily can solve the problem. But I took one student who solves all problems except then question number 26. The representative performance is given below:



The above figure is related to the question number 26. This question was; find the distance between the parallel lines x+2y-2=0 and 3x+6y+5=0. The student was unable to solve this question. Then I took an interview with him. I asked him that why you did not solve this problem then he replied as "*I did not understand the problem so".* Then I asked him you made a figure but why you did not solve. Then he replied as follows:

"Our math's teacher said, in coordinate geometry all the problem can solve by making figure. We can make a figure if we understand a problem. If we make figure of every problem then we easily can solve the problem. I made a figure by taking help of you. But I did not understand this object (figure) so I did not solve this problem."

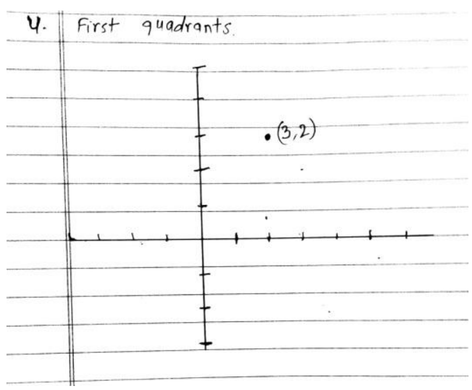
After justifying the figure student said:

"If I understand this object then I can solve it. One point and one line is required to solve this problem but I confused how to calculate one point. I also have knowledge if line is given then how to find out one point from this line. We already learn this in previous class. But due to not understanding this figure I unable to solve.

The response given by the student shows that student has difficulties in understanding the object. According to van hiele's level of geometrical thinking, students can recognize the figure on the basis of their physical appearance but difficulties in analyze the component parts of the figure. So student can visualize the object but difficulties in Analysis. Also Luneta (2014) states that when the students lack of conceptual knowledge the consequences are so serve that they hardly respond to the question in the examination. According to Garderen (2006) deficiency in visual-spatial skill might cause difficulty in differentiating, relating and organizing information. This could cause ineffectiveness in performing the problem-solving. Student had difficulties in visualizing the mathematical concept. According to Garnet (1998) visualizing mathematical concept might result of difficulties.

Difficulties related to locating points. The main characteristic of coordinate geometry is locating the points. All the concepts of coordinate geometry can be shown in graph by locating the points. All the objects of coordinate geometry can be located in the graph. So locating point is the basic concept of the graph. For locate the point student have to knowledge about the origin, abscissa, ordinate, quadrants. So I make four questions related to the locating the point. Where question number one, two and three was related to the writing coordinate point. And question number four is directly related to the classifying quadrants of given point and locating it in the graph. This question was;

which quadrants do the point (3, 2) lies. Locate it point in graph. But among 80 students only 48 student locate it properly. Also many students had not knowledge about the abscissa and ordinate. They can't able to write the point lies on x-axis, and y-axis. This shows that student have not proper concept about the abscissa and ordinate. So if they have not proper knowledge about the origin, abscissa and quadrants they can't locate the point. One student locates the point (2, 3) in spite of (3, 2). Whose response was presented below:

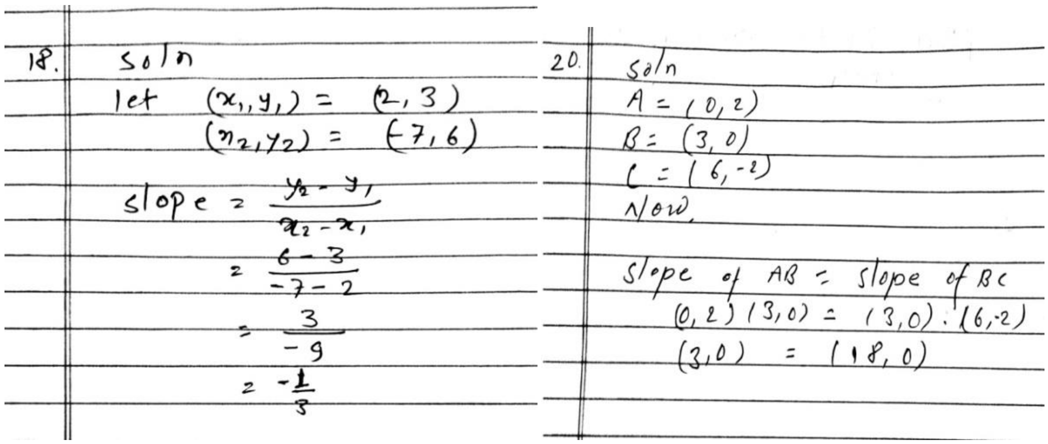


I asked him that why you locate it mistakenly then he shocked about it and said that he was right but later on clarifying this concept then he also said as follows:

"Sir I always confuse in locating the point. Sir just like we have a point (4, 5). Then 4 is the number of x-axis or y-axis I confuse."

The above response of the student shows that student have not proper knowledge about the abscissa, ordinate and its relationship. Based on Sarama et.al. (2003) students tend reverse x and y, especially when they have to plot point that have 0 in its coordinate (e.g. (0,6)). In addition, some of students sometimes ignore the origin. They plot a new point started from the previous point, not the origin. Not only that, some students fail to relate coordinates and location of multiple points. For instance, they plot (20, 35) and (25, 35) each from the origin point. They cannot see that actually the second point is 5 point to the right from the first point. Moreover, based on Blades and Spencer (2001), the children's struggle or mistake of determining the coordinate/location of an object is they often to see only one coordinate line rather than two coordinate lines. Relating to the Piaget's level of child's developmental thinking of locates a point (Piaget et al., 1960), the finding shows that the students' level of developmental thinking of location and object are different and most of the students are in the first or second level. Also According to Wongwanich and Sujiva (2014) Students are unable to figure out what to assume and what information from the problem is necessary to solving it.

Difficulties related to transformation of knowledge. Transformation of knowledge is the main key of understanding coordinate geometry. Mathematics is the sequence of knowledge. Student should transfer different mathematical knowledge in coordinate geometry. But if student can not able to transfer other mathematical knowledge into coordinate geometry there should be difficulties in learning coordinate geometry. The performance of the student who was not able to transfer one context into another context was presented below:



From the above figure, first figure is related to the question number 18. This question was; Find the slope of line passing through (2, 3) and (-7, 6). The solution given by the student was right. This shows that student understood the slope and he can calculate slope when two points given. But in another figure, which was question number 20. This question was; show that A(0,2),B(3,0) and C(6,-2) are collinear. The student try to solve the this problem by using slope i.e. to show the three points A, B and C are collinear, we have to show slope of AB = slope of BC. This second figure shows that student was clear that what to show but in second step the student wrote (0,2).(3,0)=(3,0).(6,-2) which was wrong. It should be slope of AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{0-2}{3-0} = -\frac{2}{3}$ and slope of BC = $\frac{y_2-y_1}{x_2-x_1} = \frac{-2-0}{6-3} = -\frac{2}{3}$. Hence slope of AB = Slope of BC = $-\frac{2}{3}$ So ABC are collinear. In the first figure, student calculate the slope but in second figure same student doing wrong. Then I asked a student that look at question number 18 you find out slope by using given two point which is right but in question number 20 why you can't apply like in question number 18.Then the response given by the student was presented below:

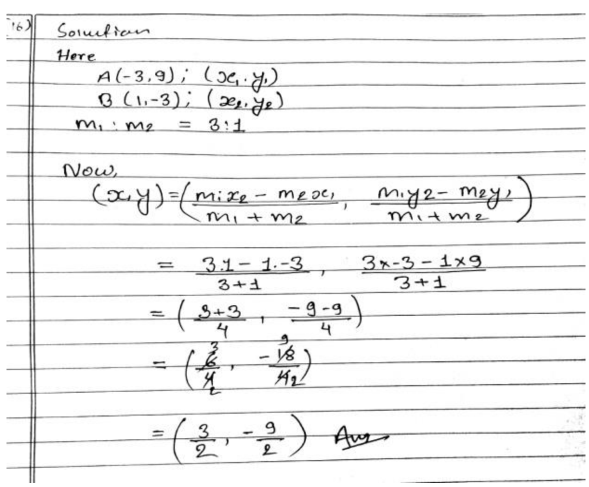
"Sir, question number 18 is a short question. It's easy to calculate slope. But question number 20 is little bit long. I know how to show collinear but confuse what to do in next step. I think it should be multiply so I multiply these two points".

It's showed that, there was a problem in transforming knowledge. Students are not able to transform the knowledge. Polya (1945) found that students had significance difficulties in transforming their problem solving skill from verbal representation to graphical representation. So it is concluded that transformation of knowledge from one problem to another problem seems to be difficulties. According to Garnet (1998) insufficient of transfer knowledge might results in difficulties.

Procedural Difficulties

A procedure is a series of steps, or actions, done to accomplish a goal. Knowledge of procedures is often termed procedural knowledge. For example, 'Procedural knowledge is 'knowing how', or the knowledge of the steps required to attain various goals. Procedures have been characterized using such constructs as skills, strategies, productions, and interiorized actions' (Byrnes & Wasik). The procedures can be (1) algorithms: a predetermined sequence of actions that will lead to the correct answer when executed correctly, or (2) possible actions that must be sequenced appropriately to solve a given problem (e.g. equation-solving steps). This knowledge develops through problem-solving practice, and thus is tied to particular problem types. Further, 'It is the clearly sequential nature of procedures that probably sets them most apart from other forms of knowledge' (Hiebert & LeFevre). Procedural difficulties mean the difficulties in solving process of any mathematical problem. This topic contains difficulties in appropriate use of formula, way of solving, numerical fluctuation, and weak in verbal explanation.

Difficulties related to appropriate use of formula. Using a Formula is a problem-solving strategy that students can use to find answers to math problems involving geometry, percents, measurement, or algebra. To solve these problems, students must choose the appropriate formula and substitute data in the correct places of a formula. A formula is a mathematical rule or relationship that uses letters to represent amounts which can be changed – these are called variables**.** Selecting appropriate formula seems difficulties for the students. Among 27 questions, 13 questions were related to the use of formula. I found that there were greatest problem in selecting appropriate formula. In question no. 16, I asked questions, which is related to the internal division but among 80 students, 9 students solve this question by using formula of external division which was wrong. The response given by the student was as follows:



I asked questions that why you used external division formula for solving this questions then student replied that:

"*Sir, there are so many formulas in coordinate geometry. One question can be solving by using various formula. In this case I confused which formula is suitable for this question".*

Another student said that;

"In exam period, searching suitable formula takes more time for me. And in this chapter we can't find exact questions on which we can apply direct formula."

Also on question number 19, which says that student have to find out the equation of straight line where one point and slope given, where it should use the formula $(y-y_1) = m(x-x_1)$. But 22 students used the formula $(x-x_1) = m(y-y_1)$ which is wrong.

It's showed that, using appropriate formula is the greatest difficulties in coordinate geometry. Student confuses in external division and internal division. Student did not understand the actual internal and external division. Also students have difficulties in searching suitable formula in exam period. Also it seems that students have connection difficulties because students feel difficulties in linking formula in related questions. According to Elif (2003) students have connection difficulties and it can be reduced by connecting procedural and conceptual knowledge.

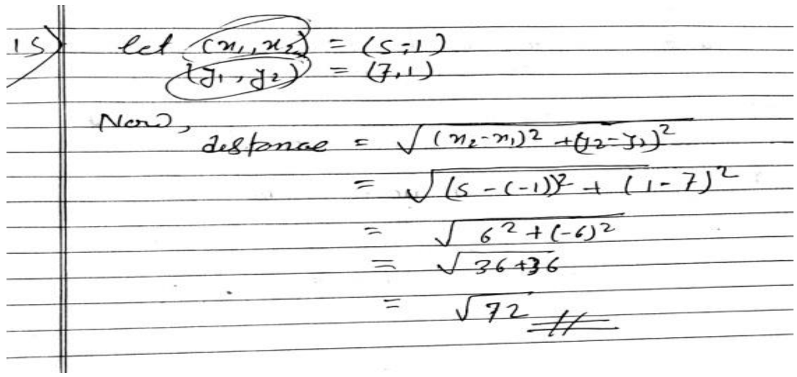
Difficulties related to way of solving. Way of solving means the solving procedure, knowledge of the steps required to attain various goals. The process of working through details of a problem to reach a solution is said to be way of solving. Problem solving may include mathematical or systematic operations and can be a gauge of an individual's critical thinking skills. In my research, I found that students had difficulties in way of solving. In relating to George Polya, difficulties in way of solving means students may understand the problem and may not be able to devise a plan or, may devise a plan but confuses in carry out the plan. Simply The difficulties in making plan or confuses in steps to accomplish a goal. We know that in the case of solving procedure, there are two type of problem in mathematics. First is the algorithm that means pre-determine sequences of solving steps and second is an approximate sequence of solving problem.

In this topic I divide into five sub topics which is based on the different topic of coordinate geometry. These sub topics are distance between two points, section formula, equation of straight line, collinear points, area of triangle and quadrilateral.

Difficulties related to way of solving in distance between two points: Distance between two points was the first topic on coordinate geometry in grade IX. There were three questions related to distance formula, one short and another long.

Q. No	Number of students who gave correct answer.	Number of students who gave partially correct answer	Number of students who gave wrong answer	Not touched
5	38	-	18	24
15	33	17	23	7
23	16	31	12	21

From the above table, in question number five, 38 students gave correct answer, 18 student gave wrong answer and 24 students didn’t touched. In question no. 15, 33 students gave correct answer, 17 students gave partially correct, 23 students gave wrong answer and 7 students were not touched. Also, in question number 23, 16 students gave correct answer, 31 students gave partially correct, 12 students gave wrong answer and 21 students didn't touch. In my research I found that there were many difficulties in solving way. For this I took question number 15. This question was "Find the distance between the points (5, -1) and (7, 1). The response given by the student presented below:



From the above response given by the student, shows that student had difficulties in way of solving. He assume $(x_1, x_2) = (5, -1)$ and $(y_1, y_2) = (7, 1)$ which was wrong. It should be $(x_1, y_1) = (5, -1)$ and $(x_2, y_2) = (7, 1)$. Then I asked the student that why you assume $(x_1, x_2) = (5, -1)$ and $(y_1, y_2) = (7, 1)$ in spite of $(x_1, y_1) = (5, -1)$ and $(x_2, y_2) = (7, 1)$? Then he replied that

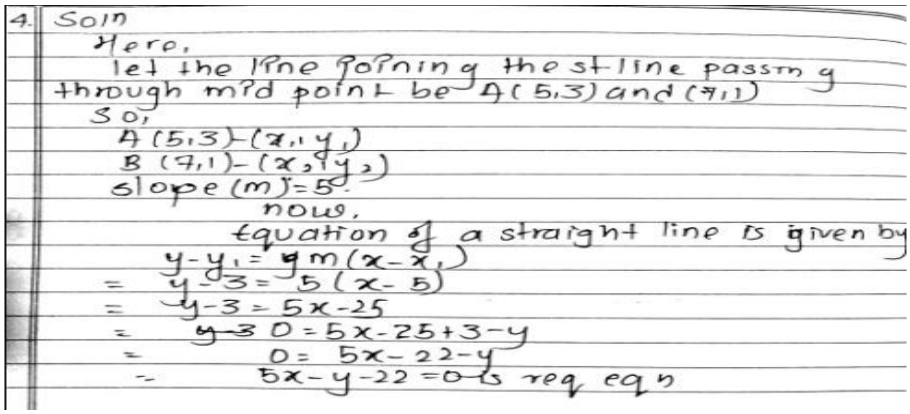
"Oh yes sir its mistake but I always confuse in supposition."

This shows that students have difficulties in assuming coordinate points. And also difficulties in recognizing x-value and y-value in coordinate. In next step he wrote right formula. Also in next step he substitutes the value accordingly to his supposition which was also mistake. Again in this step he also did mistake in substituting value. It's concluded that student had difficulties in substituting assuming value in formula.

Difficulties related to way of solving in Section Formula. Section formula is another content of coordinate geometry. There were five questions related to section formula. Two questions were very short questions which are directly related to the concept of section formula, another two questions were short questions and one is long question related to the two concepts: midpoint and equations of straight line.

Q.No	Number of students who gave correct answer.	Number of students who gave partially correct answer	Number of students who gave wrong answer	Not touched
7	36	-	25	19
8	38	-	18	24
16	23	6	17	34
17	30	4	27	19
24	22	13	20	25

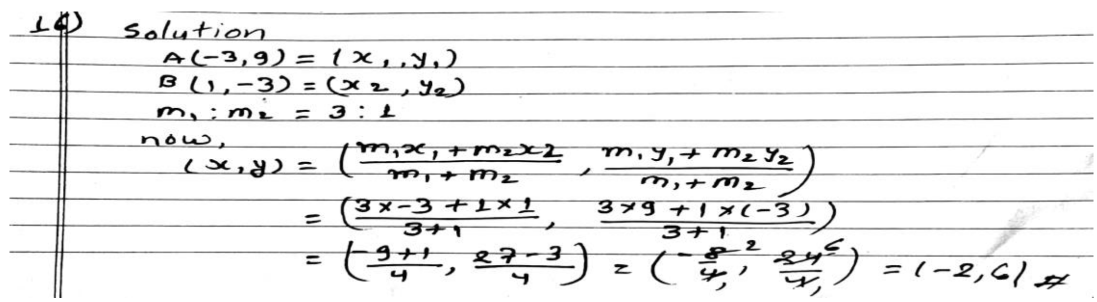
From the above table, in question number seven, 36 students gave correct answer, 25 students gave wrong answer and 19 students didn't touched. In question number eight, 38 students gave correct answer, 18 students gave wrong answer and 24 students didn't touch. In question number 16, 23 students gave correct answer, six students gave partially correct, 17 students gave wrong answer and 34 students were not touched. Also in question number 17, 30 students gave correct answer, four students gave partially correct, 27 students gave wrong answer and 9 students didn't touch. Also in question number 24, 22 students gave correct answer, 13 students gave partially correct, 20 students gave wrong answer and 25 students didn't touched. This table shows that there were vast difficulties in learning section formula. We can analyze the difficulties by using the following figure:


4. Soln
Here,
let the line joining the st line passing
through mid point be A(5,3) and (7,1)
So,
$A(5,3)-(x_1,y_1)$
$B(7,1)-(x_2,y_2)$
slope $(m)=5$
now,
equation of a straight line is given by
$y-y_1 = m(x-x_1)$
$= y-3 = 5(x-5)$
$= y-3 = 5x-25$
$= 0 = 5x-25+3-y$
$= 0 = 5x-22-y$
$\therefore 5x-y-22=0$ is req eqn

Above answer is related to the question number 24.This question was; find the equation of straight line passing through the midpoint of the line joining (5, 3) and (7, 1) and having slope 5. The solution given by the student indicates that student did not understand the nature of the problem. He did not calculate midpoint from the line joining the point's (5, 3) and (7, 1). Student used (5, 3) as a (x_1, y_1) and 5 as a slope which was wrong. Firstly student should have calculate midpoint using given two points (5, 3) and (7, 1) and then he should use the formula 'midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and then by calculation, required midpoint comes (6, 2). After that by supposing (6, 2) as a (x_1, y_1) and 5 as a slope and using formula $y-y_1=m(x-x_1)$, we will get required results. I took interview with the students. I asked the question that 'please describes your solution' then student describes in wrong way. Then I clarify the solution and asked why you did not calculate midpoint then student replied that as follows:

"I think midpoint is required here but I suppose (5, 1) = (x_1, y_1) and we have 5 as a slope so. I think there is a sufficient given value putting in formula".

Above response given by the student showed that student had difficulties in understanding the problem. According to Wongwanich and Sujiva (2014) students do not understand the problems; they tend to guess the answer without any thinking process. Also for analyzing another's student's response student had difficulties in understanding ratio, points and its relationship. Whose performance was presented below:



The above answer was related to question number 16. This question was; find the coordinate of the point which divides the line joining the point A(-3, 9) B(1, 3) internally in the ratio 3:1. For this question student wrote the formula (x, y) =

$\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$ this was wrong. It should be (x, y) =

$\left(\frac{m_2x_1+m_1x_2}{m_1+m_2}, \frac{m_2y_1+m_1y_2}{m_1+m_2}\right)$. Then I took interview. I asked the question for the student that your formula is right? Then student replied that "*yes sir*" then I said where is it? Then student search this formula on book and realized that he was wrong. Then I asked that why you wrote this formula then student replied as follows:

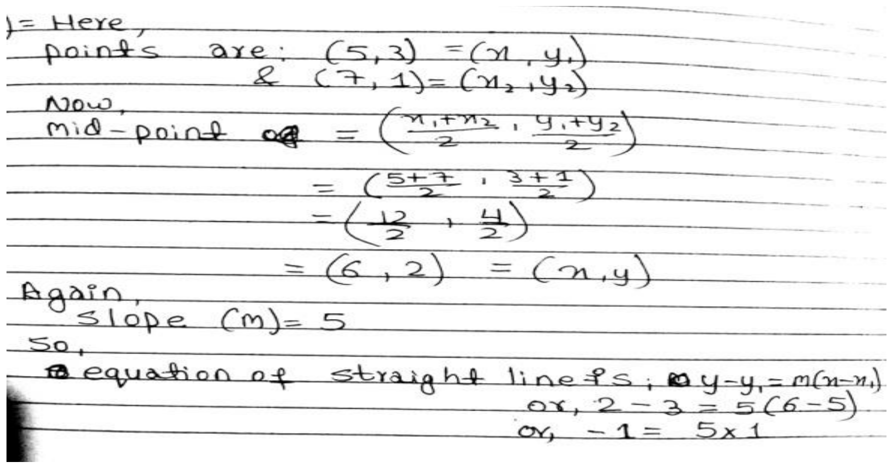
"Sir in exam time I think x_1 *&* m_1 *are like terms whose base is same so* m_1 *&* x_1*,* m_2 *&* x_2*,* m_1 *&* y_1*,* m_2 *&* y_2 *are related terms so."*

The response given by the student shows that student had difficulties in like terms. Student didn't know which terms were like terms. Student thinks ratio and points are like terms. So the student did not understand the ratios, point and its relationship in section formula. That's why student had difficulties in recognizing like terms, ratios and point and its relationship.

Difficulties related to way of solving in equation of straight line. Equation of straight line is the most interested contents of the coordinate geometry. I made three problems related to the finding equation of straight line. One problem was very short based on the formula, another one was short question and reaming one was long question. The following table shows that the number of students who give correct answer or the numbers of student who give partially correct answer or wrong answer or the numbers of students who didn't touched any questions.

Q.No	Number of students who gave correct answer.	Number of students who gave partially correct answer	Number of students who gave wrong Answer	Not touched
10	39	-	21	20
19	21	27	11	21
24	19	18	12	31

From the above table, in question number 10, 39 students gave correct answer, 21 students gave wrong answer and 20 students didn't touch. In question number 19, 21 students gave correct answer, 27 students gave partially correct, 11 students gave wrong answer and 21 students didn't touch. Similarly in question number 24, 19 students gave correct answer, 18 students gave partially correct, 12 students gave wrong answer and 31 students didn't touch. This table shows that there is a vast difficulty in Equation of straight line. The performance of the student is given as follows:



) = Here,
points are: $(5,3) = (x_1, y_1)$
& $(7, 1) = (x_2, y_2)$
Now,
mid-point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{5+7}{2}, \frac{3+1}{2}\right)$
$= \left(\frac{12}{2}, \frac{4}{2}\right)$
$= (6, 2) = (x, y)$
Again,
slope $(m) = 5$
So,
equation of straight line is; $y - y_1 = m(x - x_1)$
or, $2 - 3 = 5(6-5)$
or, $-1 = 5 \times 1$

The above answer was related to the question number 24. This question was; find the equation of straight line passing through the midpoint of the line joining (5, 3) and (7, 1) and having slope 5. For analyzing this figure, students made a plan for solving. And he finds out midpoint of given two points. Then in next step he use proper formula for equation of straight line. But substituting the value of variable was seems wrong. Students thinks that (5, 3) as a (x_1, y_1) and (6, 2) as a (x, y), which was wrong. It should be (6, 2) as a (x_1, y_1). I think that he had not a proper knowledge about the equation of straight line. Then I asked a question that why you assume (5,3) as a (x_1,y_1) and (6,2) as a (x, y). Then student replied that

"I had already assume (5, 3) as a (x_1, y_1) for finding midpoint. In formula $(y-y_1) = m(x-x_1)$, there was two variable like x, x_1 and y, y_1. I think that the calculated midpoint must be (x, y) so I assume this sir"

I asked the student that your last result was -1=5×1 this was like a equation of straight line then student replied that "*no sir, but I checked two times but I was not find my mistake.*"

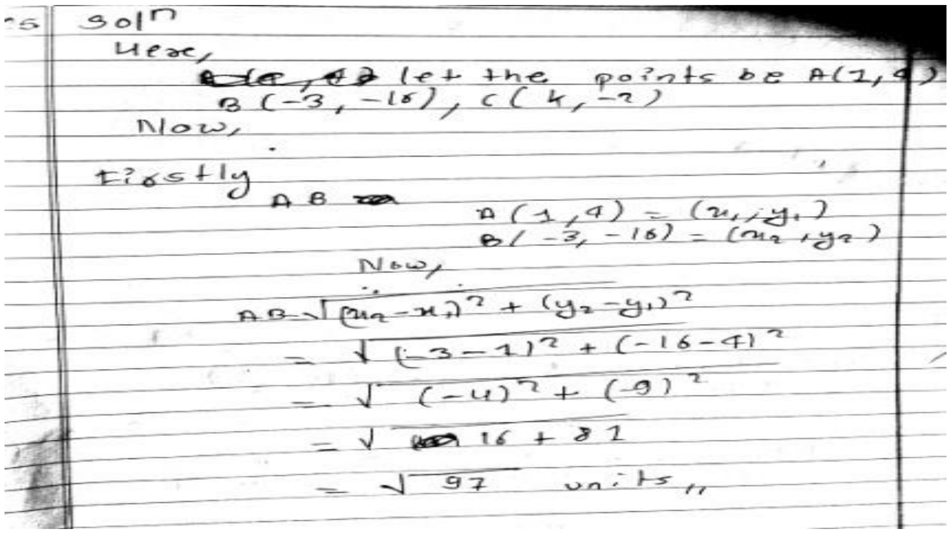
Above figure and interview shows that student had difficulties in solving equation. And also it seems that student can't recognize the variable used in the equation. So the students have difficulties in recognizing variable in the equation. According to poliya (1945) students face difficulties by not understanding the problem. And also in his theory poliya gave four steps of problem solving strategy. Among four steps understanding a problem is the first steps. So students had difficulties in polya's first stage. Students didn't understand what is given and what we have to find. So students face difficulties in understanding the problem, solving the equation and also recognizing the variable used in the equation.

Difficulties related to way of solving in collinear points. Three points are said to be collinear if all them lie on a same straight line. Three points A, B and C are said to be collinear if slope of AB = slope of BC. I made three questions related to the topic collinear. The following table shows that the number of students who give correct answer or the numbers of student who give partially correct answer or wrong answer or the numbers of students who didn't touched any questions:

Q.No	Number of students who gave correct answer.	Number of students who gave partially correct answer	Number of students who gave wrong Answer	Not touched
11	34	-	24	22
20	19	28	9	24
25	15	19	7	39

From the above table, in question number 11, 34students gave correct answer, 24 students gave wrong answer and 24 students didn't touched. In question number 20, 19 students gave correct answer, 28 students gave partially correct, 9 students gave wrong answer and 24 students didn't touch. Also in question number 25, 15 students gave correct answer, 19 students gave partially correct, 7 students gave wrong answer and 39 students didn't touch.

According to the response given by students shows that, the problem number 20 and 25 can be solved by three ways. The first way is by using slope (i.e. if three point be A, B, C then slope of AB = slope of BC), second way is by using area (i.e. area of triangle ABC = 0) and the third way is by using distance formula (i.e. distance of AB + distance of BC = distance of AC). The performance given by the students was presented below:



Soln
Here,
let the points be A(1, 4)
B(-3, -16), C(k, -2)
Now,
Firstly
AB
A(1, 4) = (x_1, y_1)
B(-3, -16) = (x_2, y_2)
Now,
$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(-3-1)^2 + (-16-4)^2}$
$= \sqrt{(-4)^2 + (-9)^2}$
$= \sqrt{16 + 81}$
$= \sqrt{97}$ units

Above answer shows that there was a difficulty. First figure is related to the question number 25. This questions was 'find value of k when the points (1, 4), (-3, 16) and (k, -2) lies on the same straight line'. This question can be solved in three ways. Student was trying to solve by using distance formula. He tried to find out distance between two points and he made a plan for solving by assuming A(1 ,4) = (x_1, y_1) and B(-3, -16)= (x_2, y_2). But in assuming B(-3, -16)= (x_2, y_2) was wrong. He wrote negative 16 but in question, there was positive 16. Also in next step he wrote -16-4 = -9 which was silly mistake. This leads wrong results. Therefore student had difficulties in adding two negative numbers. Also student should calculate distance of AB, distance of BC and also distance of AC. After that for finding value of k he should use the relation distance of AB + distance of BC = distance of AC but he didn't done. Then I asked a question for the student that how we can solve this question then student replied as follows

"To solve this question firstly we have to calculate distance of AB."

Then student stay silence and I said why? Then student replied "*actually sir I have not any idea for solving this problem*". Then I give hints for solving procedure and said him to solve now then student calculate only distance of AB, distance of BC and distance of AC. Also he stops in this step. Again I gave hints by please use the relation lies on same straight line. But he can't solve. Then I asked him that what's the meaning of lies on same straight line then he replied "*I don't know sir*". It's showed that, students have difficulties in understanding mathematical term.

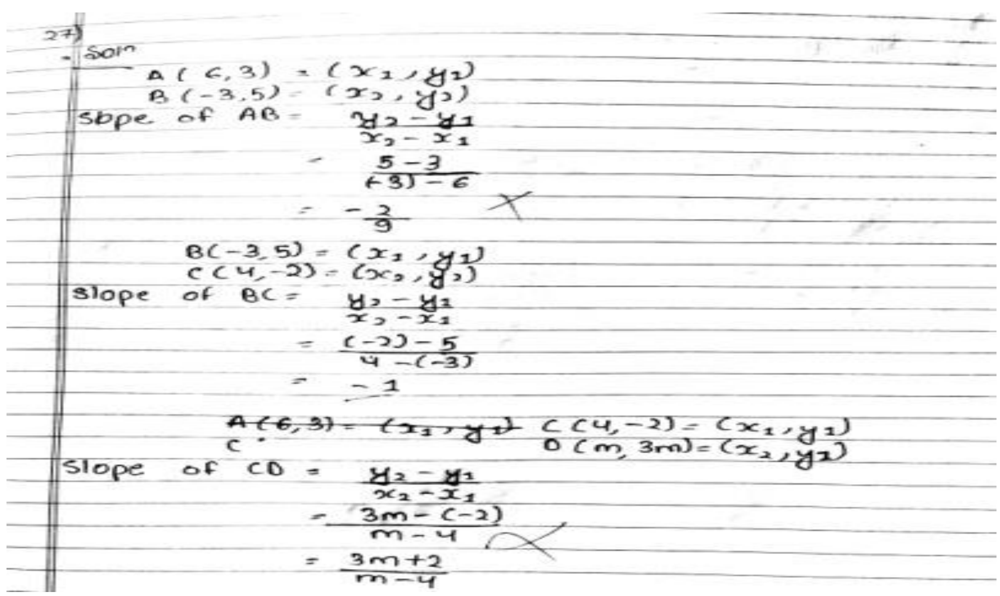
Also I asked him for showing straight line with three points then he made right figure. But he can't able to establish the interrelationship of properties. According to van Hile's levels of geometrical thinking level two is the stages informal deduction in this step student can established the interrelationship of the properties of the figures. So we can say that student had difficulties in informal deduction.

Also in continuing interview, I clarify this concept and gave him to solve the equation for finding value of k. But the student did not able to find value of k by solving equation. So we can say that student had also difficulties in solving equation.

Difficulties related to way of solving in area of triangle and quadrilateral. Calculating area of triangle and quadrilateral by using coordinate is the most interested contents of the coordinate geometry. I made three questions related to the area and quadrilateral. One question was very short based on the formula, another one was short question and reaming one was long question. The following table shows that the number of students who give correct answer or the numbers of student who give partially correct answer or wrong answer or the numbers of students who didn't touch any questions.

Q.No	Number of students who gave correct answer.	Number of students who gave partially correct answer	Number of students who gave wrong Answer	Not touched
14	31	-	18	31
22	19	28	9	24
27	15	19	7	39

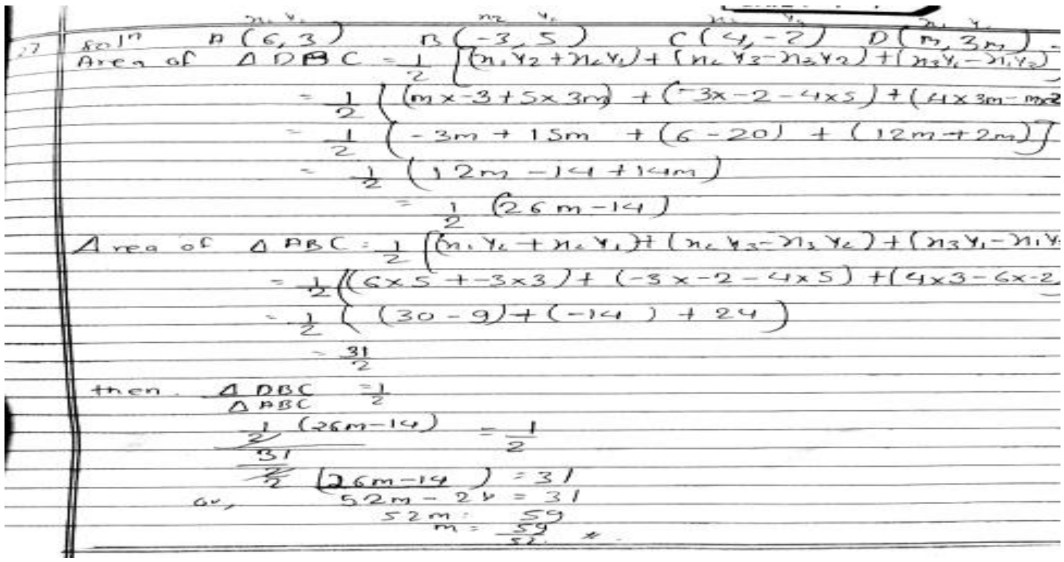
From the above table, in question number 14, 31 students gave correct answer, 18 students gave wrong answer and 31 students not touched. In question number 22, 19 students gave correct answer, 28 students gave partially correct, nine students gave wrong answer and 24 students didn't touch. Also in question number 27, 15 students gave correct answer, 19 students gave partially correct, 7 students gave wrong answer and 39 students didn't touch. The performance given by the student given as follows:



Above answer is related to the question number 27. The questions was "given a three points A(6,3), B(-3,5), C(4,-2) and D(m, 3m). Find the value of m if $\frac{\Delta DBC}{\Delta \mathrm{ABC}} = \frac{1}{2}$. For solving this question first of all we should find out the area of triangle DBC and ABC by using the formula Area of $\Delta = \frac{1}{2}\{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\}$. Then substitution of the value of triangle DBC and ΔABC in the given equation for finding the value of m is the solving way of this question. But the solution given by the student was wrong. He finds out slope of AB, slope of BC, slope of CD which was wrong process. This shows that there is a big problem in solving way of the student. He was in misconception. He didn't know the way of solving problem. I asked the student by why you find out slope in spite of area then student replied that

"I do not understand area of triangle and quadrilateral. I think that coordinate geometry is mostly related to slope and I can solve the problem related to the slope. If I calculated slope this should be little bit right and I will got half marks so I calculated slope."

According to the response given by the student indicate that he had misconception about coordinate geometry specially in finding area of triangle and quadrilateral. He did not understand the way of solving or process of solving. But in analyzing another student views it's showed that student know the solving way but difficulties in calculation. The representative views of student are presented below:

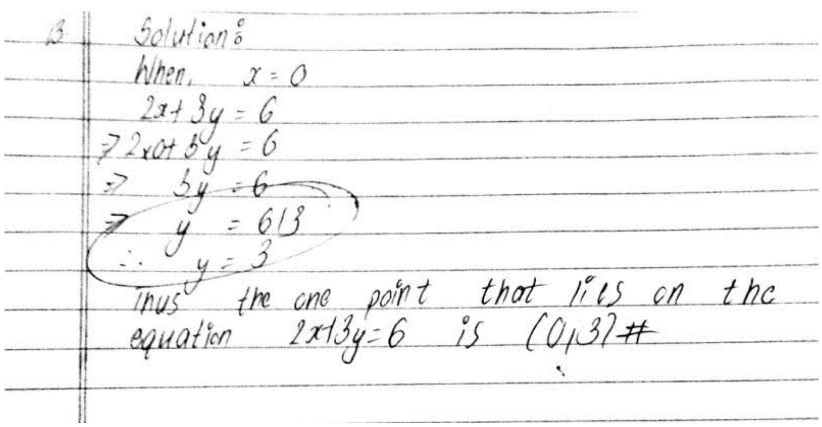


Also in this figure, student can make the plan for solving question by assuming points as a coordinate form used in formula. Student was not confusion in the solving way of this problem. But he wrote the formula Area of Δ= $\frac{1}{2}\{(x_1y_2+x_2y_1) + (x_2y_3-x_3y_2) + (x_3y_1-x_1y_3)\}$ which is wrong. It should be Area of Δ = $\frac{1}{2}\{(x_1y_2-x_2y_1) + (x_2y_3-x_3y_2) + (x_3y_1-x_1y_3)\}$. Also in the second steps he put wrong value in formula. i.e. he wrote $(x_1y_1+x_2y_2)$ this value in spite of $(x_1y_2 - x_2y_1)$. But in other place he put right value in formula. There were three questions related to area of triangle and quadrilateral. Among three questions he wrote right formula in two questions but in last question he wrote wrong formula. Then I asked a student that why you wrote wrong formula then he replied that

"Sir, this question is last question I had not proper time to recheck. And in exam period I felt hopeless so I think one but wrote another."

From the response given by the student shows that student feel output difficulties. Because student know all the procedure but he do mistake in calculation and solving which leads mistake output. It's concluded that students had misconception about the area of triangle and quadrilateral. Also he had difficulties in solving way, calculation, quantities and formula. Student, who could not bring meaning to the problems, did not know how to plan and perform the problem-solving strategies (Johan, 2002).

Difficulties related to numerical fluctuation. Numerical fluctuation is the error in numerical computations. In local language it is said to be silly mistake. Numerical fluctuation is the errors in adding subtracting, multiplying, dividing etc of numbers. In mathematics achievement test, most of the student knows problems and capable to solve problem but they do small mistake in adding, subtracting, multiplying or dividing numbers. That’s why we can say that students had feel difficulty.



From the above figure, this was related to question number 13. This questions was “find one point lies on equation 2x + 3y = 6. The starting of solving this question was very excellent. It showed that he was not difficulties in this question but at third last steps he wrote 3y = 6 which is right but in second last step he wrote y = (6/3) = 3 which was wrong. It should be y = 6/3 = 2. These steps showed that he had not a proper knowledge about solving equation. That’s why these types of errors seem numerical fluctuation buts leads mistake.

Weak in verbal explanation. Mathematics is a language. It should be in order. Some people think that mathematics means only a numerical explanation. But it is not a sufficient definition. There most should be a verbal explanation. Only numerical explanation cannot fulfill beauty of mathematics. But students did not care about the verbal explanation which leads mistake results. So verbal explanation was also seems as a difficulties in learning coordinate geometry. Among the 27 questions I take question number 23 as the types of verbal explanation. This question was not fully depends on verbal explanation but in last of the solution it's necessary to explain verbally.

The questions was "Prove that the points A(1, 1), B(2, 3) and C(5, -1) are the vertices of right angled triangle(By using distance formula)".This questions can be solve by the calculating distance of three side of triangle. i.e. we should had to calculate distance of AB, distance of BC and distance of AC. Then at last we have to use Pythagoras theorem like $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$, for checking this triangle was right angle triangle or not. Among 80 student 16 students give write answer with verbal explanation.31 students give partially correct answer. Among the 31 students 20 students did not wrote verbal explanation. They only calculate three sides in numerical figure like distance of AB, distance of BC and distance of AC. They didn't show this triangle was right angled triangle or not. I asked the students that why you did not show this triangle was right angle triangle?, You did not know Pythagoras rule? Then their answer is as follow.

"The teacher does not give us clear idea about coordinate terminology and they teach us without clarifying these terms, concepts, and procedure. So, that we have incomplete knowledge about coordinate rule. He only focused on the numerical quantity.

So we can't connect the axioms and postulates of mathematics to coordinate geometrical rules. As a result we can't deal with the questions of coordinate geometry and can't focus on the questions. These are the difficult point for us."

The above students view shows that the teacher does not use child centre method, he always uses lecture method and he did not focus about concepts and procedural which makes coordinate geometry learning difficulties. Teacher only focuses on the numerical quantity so doing verbal explanation was seems difficulties for the students. They are the cause but in fact student had difficulties in linking mathematical axioms and postulate of mathematics in coordinate geometry. That means student have connection difficulties. To teach geometry, teacher needs to know the child's spatial environment (Bishop, 1980) and they need to develop skills in geometry not only future use in teaching, but also as aids to their own study of mathematics. They particularly need to understand and to capitalize upon the interrelation among the several areas of mathematics (Meserve, 1973). Also Students cannot manage all the demands of a complex problem, such as a word problem, even thought he or she may know component facts and procedures.

CHAPTER V

SUMMARY, FINDINGS, CONCLUSION AND RECOMMENDATION

This chapter deals with the summary, major finding, conclusion and recommendations.

Summary and Findings

As a two years teaching experience in school level mathematics I found that most of the student feel difficult to learning coordinate geometry. Among geometrical content, coordinate geometry is also the main branch of geometry. I feel that student feel many procedural and conceptual difficulties in learning coordinate geometry. In my literature review, I found that there are many research related to learning difficulties in geometry but not concrete about the specific topic coordinate geometry. So there is a research gap. I think my research fulfill this gap. And also want to know what are the main difficulties that student feel in learning coordinate geometry. That's why I selected the coordinate geometry as my research topic. The main purposes of the study were to explore conceptual and procedural learning difficulties in coordinate geometry.

This is a qualitative research. The design of the research was case study in which meanings were derived from the total logical and reasoning of why and how it was difficult, by linking different theories. The research was conducted in four private schools of Kaski district. I selected four schools from purposive sampling technique. All the student's of grade IX were the participant of the study. Mathematics achievement test and the in-depth interview were used as a tool for the data collection procedure.

For the convenience of the study the difficulties were analyzes by categorizing into two types; conceptual difficulties and procedural difficulties. In conceptual difficulties I divided four topics to explore difficulties. They were difficulties in understanding definition, understanding objects, locating points, transformation of knowledge. Similarly In procedural difficulties, I divided four topics to explore difficulties. They were difficulties in appropriate use of formula, way of solving, numerical fluctuation, and weak in verbal explanation. On the basis of analysis and interpretation of the data the findings are stated below:

Conceptual difficulties

- Students have difficulties in defining the coordinate term, symbolization and mathematical representation of the term. They had difficulties in word problem also.
- Students face difficulties in visualizing and understanding the object.
- Students have difficulties in locating point. They have a problem in recognizing abscissa and ordinate.
- Student did not know that distance is scalar or vector. It's showed that student have difficulties in understanding scalar and vector.
- Transformation of knowledge is also the difficulties face by the student in learning coordinate geometry. Student can say or write the formula and can apply in short question but not apply in long question.
- Misconception about the coordinate geometry.

Procedural difficulties

- Using appropriate formula seems to be greatest difficulties in learning coordinate geometry. Most of the student used external division formula in spite of internal division. Students know the formulas but they feel difficulties in selecting which is proper formula.
- Solving procedure was also seems to be difficulties.
- Student's difficulties in assuming coordinate. This leads wrong results.
- Substituting the assuming value in formula was another difficulty.
- Student did not identifying like terms. Understanding ratios, points and its relationship also seems difficulties.
- Students have difficulties in solving equation and also recognizing variable used in equation makes difficulties.
- Students can visualize and analyze the component parts of the object. But they feel difficulties in establish the interrelationship of the properties of figure. So students had difficulties in informal deduction.
- Difficulties in calculation are also main problem face by the students. They did mistakes in addition, subtraction, multiplication of sign.
- Numerical fluctuation was also the difficulties in learning coordinate geometry.
- Student had organization difficulties. That means the students weak aspects of making sequential order of solution, steps mismatch for making final solution.
- Student had connection difficulties. Student knows mathematical axioms, postulates, formulas, definition but they had difficulties in linking it into coordinate geometrical concept.

Conclusion

The above findings of the study shows that students face difficulties in learning coordinate geometry. On the basis of finding, there are two types of difficulties face by student. They are conceptual difficulties and procedural difficulties. In conceptual difficulties, students feels difficulties in defining co-ordinate term, symbolization, mathematical representation of the term, visualizing and understanding the object, recognizing abscissa and ordinate, transformation of knowledge, and misconception about the coordinate geometry.

In procedural difficulties, students feel difficulties in using appropriate formulas, solving procedure, selecting proper formula, assuming coordinate, substituting assuming value in formula, understanding ratios, points and its relationship, solving equation, recognizing variable and constant in equation. Similarly students feel difficulties in establish the interrelationship of properties of figures, calculation, solving in sequential order, connecting other mathematical axioms and postulate in coordinate geometry.

Recommendations for Educational Implication

Recommendations have made to improve the learning situation happening the student's difficulties in learning coordinate geometry. I took in-depth interview with the student and also took suggestion of supervisor for the correct and behavioral solution of the above difficulties. I also read previous research documents which included practical effort and suggested to apply various techniques to achieve the goal despite the difficulties and problem stated above. The following recommendations are presented in the basis of above aspect.

- First, the results show that some students had difficulties in mathematical term or mathematical symbolization or mathematical representation. So teacher and stakeholder should focus on mathematical vocabulary.
- Teacher should visualize the mathematical object.
- Student can not able to transform the knowledge. So teacher and parents should help for transforming knowledge.
- Teacher should be focus on the concept of coordinate geometry.
- Student had difficulties in using and selecting proper formula due to less practice. So teacher and stakeholder should focus on their practice.
- Using teacher materials makes teaching and learning sustainable. So teacher should use teaching materials for teaching.
- Teacher should establish the interrelationship between the properties of figure or object.
- Teacher should focus on the problem solving way and sequential order for solving problem.
- Connecting other mathematical axioms and postulate in coordinate geometry seems difficulties. So stakeholder should focus on it.

Recommendation for the further Study

This present study may not be completed for all situations. Researcher can apply the different tools and methods related to the some problems. For this I presented the following recommendations for further studies:

- Similar study should be carried out with a large sample and various schools of different parts of Nepal.
- These types of study should be conduct different topics, levels, subjects in class.
- The further research should focus on the cause of conceptual and procedural difficulties in learning coordinate geometry.
- The further research should focused on “why student cannot able transfer other mathematical knowledge in coordinate geometrical content”

APPENDIX-A

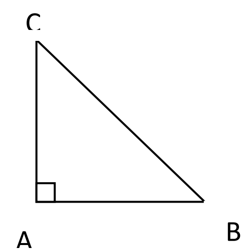
Mathematics Achievement Test

Attempt all questions.

Set A (14×1=14)

1. Write down the coordinate of origin.
2. Write down the coordinate point on x-axis whose abscissa is 7.
3. Write down the coordinate point on y-axis whose ordinate is 6.
4. Which quadrants do the point (3, 2) lie. Also locate it points on the graph.
5. Write down the distance formula between two points (x_1, y_1) and (x_2, y_2).
6. From the right angled triangle given in alongside,

   Which of the following is satisfied?

   a. $BC^2 = AB^2+AC^2$ b. $AC^2= BC^2+AB^2$

   c. $AB^2 = BC^2+ AC^2$ d. all of the above.
7. Write down the section formula for internal division where two points are $A(x_1, y_1$ and (x_2, y_2) and ratio $m_1:m_2$.
8. Write down the mid-point formula.
9. Which of the following is the formula for finding slope (m) when two points (x_1, y_1) and (x_2, y_2) are given?

   a. $m = \frac{y2-y1}{x2-x1}$ b. $y = mx+c$ c. $m = \frac{y2-x2}{y1-x1}$ d. all of the above.
10. Write down the equation of straight line passing through (x_1, y_1) having slope m.
11. Three point A,B,C are said to be collinear if

a. Slope of AB = Slope of BC

b. Slope of A = Slope of B = Slope of C

c. Slope of A × Slope of B = Slope of C

d. All of the above.

12. Length of perpendicular from a point (x_1, y_1) on the line Ax+By+C=0 is

a. $\frac{Ax+By+C}{\sqrt{A^2+B^2}}$ b. $\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}$ c. $\frac{Ax_1+By_1+C}{\sqrt{A+B}}$ d. $\frac{Ax_1+By_1+C=0}{\sqrt{A^2+B^2}}$

13. Find the one point lies on the equation 2x+3y=6.

14. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the three vertices of a triangle ABC.

Then Which is the formula for finding Area of ΔABC.

a. Area of $\Delta = \frac{1}{2}\{(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_1-x_1y_3)\}$ b. Area of $\Delta=\frac{1}{2}\times b\times h$

c. Area of $\Delta = \frac{1}{2}\{(x_1y_2+x_2y_1)+(x_2y_3+x_3y_2)+(x_3y_1+x_1y_3)\}$ d. all of the above.

Set B (8×2=16)

15. Find the distance between the points (5, -1) and (7, 1).

16. Find the co-ordinate of the point which divides the line joining the point A(-3, 9) B(1, -3) internally in the ratio 3:1.

17. Find the co-ordinate of the midpoint of the line joining (-3, -6) and (1,-2).

18. Find the slope of line passing through (2, 3) and (-7, 6).

19. Find the equation of straight line passing through (5,4) having slope 3.

20. Show that A(0, 2), B(3, 0), C(6, -2) are collinear.

21. Find the length of perpendicular line drawn from (2, 3) to the line 4x+3y+3=0.

22. Find the area of triangle whose vertices are A(2, -4), B(5, 3),C(-3, 6)

Set C (5×4=20)

23. Prove that the points A(1, 1), B(2, 3) and C(5, -1) are the vertices of right angled triangle(By using distance formula).
24. Find the equation of straight line passing through the midpoint of the line joining (5,3) and (7, 1) and having slope 5.
25. Find the value of k when the points (1, 4), (-3, 16) and (k, -2) lies on same straight line.
26. Find the distance between the parallel lines x+2y-2=0 and 3x+6y+5=0.
27. Given the points A(6,3), B(-3,5), C(4,-2) and D(m,3m). Find the value of m if $\frac{\Delta DBC}{\Delta \text{ABC}} = \frac{1}{2}$.

APPENDIX-B

Interpretation the results of Pilot test

The following table shows the students marks obtain from pretest.

Student	1	2	3	4	5	6	7	8	9	10
Marks obtain in odd questions (X)	15	12	20	18	21	5	10	13	11	10
Marks obtain in even questions (Y)	10	13	22	20	17	7	5	15	18	12

To calculate correlation coefficient of the above students marks, I used following formula:

$$r_{oe} = \frac{N\sum XY - \sum X \sum Y}{\sqrt{N\sum X^2 - (\sum X^2)}\sqrt{N\sum Y^2 - (\sum Y^2)}}$$

$r_{oe} = 0.67$

This correlation coefficient shows the half of the reliability of test item. So, for the reliability of whole item I used Spearman-Brown correlation coefficient formula as below:

$$r_t = \frac{2r_{oe}}{1+r_{oe}}$$

Where; r_t = Reliability coefficient of whole item

r_{oe} = Reliability between marks of odd and even items.

Now, $r_t = \frac{2\times0.67}{1+0.67} = 0.80$

According to Garnet (Garnet, 2008), the interpretation of reliability coefficient as following table:

Coefficient	Nature
0.0 to $\pm$ 0.20	Indifferent or negligible
$\pm$0.20 to $\pm$0.40	Present but slight
$\pm$0.40 to $\pm$0.70	Substantial or marked
$\pm$0.70 to $\pm$1	High to very high

APPENDIX-C

Name of the sample students taken for in-depth interview are as follows:

1. Luv Gurung
2. Sandeep Thakuri
3. Sanjeta Thakuri
4. Arpita Gurung
5. Diwash Jalari
6. Kanchan Sunar
7. Anisha Shrestha

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