

EFFECT OF COLLABORATIVE LEARNING ON MATHEMATICS

A

THESIS

BY

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LETTER OF APPROVAL

A

THESIS

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Entitled

"Effect Of Collaborating Learning On Mathematics" has been approved in partial fulfillment of requirements for the Degree of Master of Education.

Committee for The Viva-Voce

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LETTER OF CERTIFICATE

This is to certify that Mr. Ram Sagar Yadav, a student of academic year 2068/2069 with campus Roll No. 1684/068, Exam Roll No. 281766 (2069), T.U. Regd. No. 9-1-12-429-2006 and thesis No. 994 complete this thesis under my supervision, during the period prescribed by the rules and regulations of Tribhuvan University of Nepal. The thesis entitled "**Effect Of Collaborative Learning On Mathematics**", embodies the result of his investigation conducted during the period 2016 to 2017 under the Department of Mathematics Education, Tribhuvan University, Kirtipur. I recommend and forward that this thesis to be submitted for the evaluation and award the Degree of Master in Education.

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.....
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ABSTRACT

The major focus of this study was to explore "Effect of Collaborative Learning on Mathematics". The main objectives of the study were to compare the achievement of students taught through traditional and collaborative learning and to analyze the reflection of teachers and students towards collaborative teaching. The design of the study was experiment non equivalent. The sample of the study was 53 students collaborative method made students easy to understand the subject matter. The Laliguras higher secondary school at 30 students and Srijarajan higher secondary school at 23 were the sample of the study students. The experimental group was taught using collaborative learning approach and control group was taught using traditional approach. The duration of experiment was 12 days. After completing the experiment achievement test was administered on both groups and mean scores were calculated. The differences in mean achievement score was tested using t-test for determining statistical significance between them. Then it was found that the collaborative learning strategy was better than the traditional approach learning mathematics and the collaborative method made students easy to understand the subject matter.

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ABBREVIATIONS

CA	:	Collaborative Learning
CSCL	:	Computer Supported Collaborative Learning
EG	:	Experimental Group
CG	:	Control Group
HSS	:	Higher Secondary School
STAD	:	Student team achievement decision
LT	:	Learning together
AC	:	Academic Controversy
TGT	:	Team Game Tournament
SLC	:	School Leaving Certificate

Chapter -I

INTRODUCTION

Mathematics was dynamic in nature as discipline and essential part of human life is automatically derived from an ancient Greek word *mathema* taken from *mathancin* which means "to learn". "what one gets to know", hence also "study" and "science", and in modern Greek just "lesson". The word *mathema* is derived from *manthano*, while the modern Greek equivalent is *mathaino*, both of which mean "to learn". In Greece, the word for "mathematics" came to have the narrower and more technical meaning "mathematical study" even in Classical times. Its adjective is *mathematikos*, meaning "related to learning" or "studious", which likewise further came to mean "mathematical". In particular, *mathematike tekhne*, Latin: *arithmeticum*, *mathematica*, mean "the mathematical art". Similarly, one of the two main schools of thought in Pythagoreanism was known as the *mathematikoi* which at the time mean "teachers" rather than "mathematicians" in the modern sense. In Latin, and in English until around 1700 B.C., the term *mathematics* more commonly meant "astrology" rather than "mathematics"; the meaning gradually changed in to its present one from about 1500 to 1800 B.C. This has resulted in several mistranslations: a particularly notorious one is Saint Augustine's warning that Christians should beware of *mathematici* meaning astrologers, which is sometimes mistranslated as a condemnation of mathematicians. The apparent plural form in English, like the French plural form *les mathematiques*, goes back to the Latin neuter plural *mathematica* based on the Greek plural *mathematika*, used by Aristotle (384–322 BC), and meaning roughly "all things mathematical"; although it is plausible that English borrowed only the adjective *mathematic (al)* and formed the noun *mathematics* a new, after the pattern of *physics*

And metaphysics, which were inherited from the Greek. It is often shortened to maths or, in English-speaking North America, math. It has been development through the human endeavors in different area has come to the high of development and will still be in the process of development forever. (Wikipedia)

Mathematics can, broadly speaking, be subdivided into the study of quantity, structure, space, and change (i.e. arithmetic, algebra, geometry, and analysis). In addition to these main concerns, there are also subdivisions dedicated to exploring links from the heart of mathematics to other fields: to logic, to set theory (foundations), to the empirical mathematics of the various sciences (applied mathematics), and more recently to the rigorous study of uncertainty. While some areas might seem unrelated, the Langland's program has found connections between areas previously thought unconnected, such as Galois groups, Riemann surfaces and number theory.

It has come to hear that mathematics is very hard, complicated, irritating, time consuming and boring subject, but I am fully beyond of that assumption. In my opinion mathematics neither hard, nor complicated it is beauty subject. It is simplest and clearest way of thinking of calculation, means of increasing logical ability of learner. It is like as game, which is played by different players and all players fill very enjoy. There is need of mathematics learning strategy. Mathematics learning through many types learning then collaborative learning is most useful learning one of them.

Background of the Study

Historically, the importance of mathematics is related both for needs of everyday life and academic study. The nature of everyday mathematics and the mathematics for academicism study different. The modern science and technology

stands on the foundations of sophisticated mathematical structure and principals.

There are two branches of mathematics, In general, the pure and applied.

Applied mathematics concerns itself with mathematical methods that are typically used in science, engineering, business, and industry. Thus, "applied mathematics" is a mathematical science with specialized knowledge. The term applied mathematics also describes the professional specialty in which mathematicians work on practical problems; as a profession focused on practical problems, applied mathematics focuses on the "formulation, study, and use of mathematical models" in science , engineering and other areas of mathematical practice. In the past, practical applications have motivated the development of mathematical theories, which then became the subject of study in pure mathematics, where mathematics is developed primarily for its own sake. Thus, the activity of applied mathematics is vitally connected with research in pure mathematics.

(Wikipedia) The mathematics is the applications of mathematics in different field of study. "Understanding the different concept of the mathematics is important to the development and successful important programs in school mathematics as it is to conduct and interpretation of the research studies"(dossey, 1992, p36).

There are various methods of learning style in student's life. Some of the students enjoy and learning individually where as some enjoy on group. Here researcher wants to know which approaches are more appropriate between these two. In traditional method, instructor give lecture to make their foundation to strong and learner learn individually in their home also.

Collaborative Learning Leads to Student Success Even though the College Preparatory School is an elite school, schools with fewer resources can replicate College Prep's collaboration practices. The collaborative-learning style incorporated

into the fabric of the school helps students to be resilient by aiding them with identifying their resources (peers) and testing their theories to see if they are on the right track all while developing habits of mind that form the foundation of scholarship. To make this approach work, teachers must be willing to “cede the floor” to the students. Other things to consider are the need to create an effective classroom geography, focus on the process, build accountability, let students teach one another, and encourage students to be in tune with one another.

Building Strong Collaborative Learning

Teachers at the College Preparatory School encourage classroom collaboration by assigning students to groups to review their homework, do daily class worksheets, participate in moderated discussions, and complete hands-on projects. Often, teachers give students group tests, which, like the class worksheets, are designed to be harder than the individual assignments. Students quickly realize that they are able to solve problems as a group that they would not be able to solve as individuals. Some of the other ways teachers foster a collaborative-learning environment follow:

Creating an Effective Classroom Geography

In math classes, the students sit face-to-face in groups of four tables to collaborate. In English classes, students sit around a Harkness table (a large wooden table capable of seating the entire class), which allows every student to see the teacher and all the members of the class as they speak. The foundation is that students come prepared to discuss and collaborate.

Focusing on the Process, Not Right Answers

In math, four times a year, each student is given a set of values or codes to substitute in the equations so that even though the students are working together, they

have to focus on the mathematical process and not just the “right answer.” In English, the discussions are open-ended, allowing for multiple right answers.

Building Accountability

In math classes, students frequently take group tests and can consult with one another on the answers, but the teacher chooses only one test at random to grade for the group. Because the group work is intentionally more difficult, this process keeps individual students accountable for full participation in group work. To measure how well the groups work together, the teacher also gives out a group-collaboration grade for each unit, which is worth 10 percent of a student’s grade. While students participate in the group discussions around the Harkness table in English, the teacher selects one student to be the moderator and another to be the discussion tracker who records the flow of the conversations. The moderator can look at the discussion tracker’s notes and see which students he should invite to chime in.

Letting Students Teach One Another

In math teachers cede the floor to students so they can teach one another. In math class, students are given challenging class problems that encourage them to seek ideas and advice from their group members. In Math, juniors are grouped with seniors, which helps the younger students learn how the process works by watching and learning from the older students. Additionally, pairs of students are invited to lead the discussions. The Math discussions are also held online, and students are required to participate and comment on at least two other student comments. Encouraging students to be in tune with one another in math classes, students are grouped intentionally to provide a mix of skill levels, which helps them to be more sensitive to group members’ needs. The teacher provides feedback on helping the groups interact

well, which helps the more vocal students to step back and let the other students participate more.

Every English class starts with a moment of quiet after which students are asked to share their energy and stress levels. Often teachers assign different roles that allow students to focus on the discussion: the scribe takes notes for the entire class, the moderator ensures a smooth discussion, and the discussion tracker maps the flow of the discussion.

Myers(1975)points out that dictionary definition of "collaboration "drive from in Latin roots focus on the processes of working together, the root word for "cooperation" stresses the product of such work. Collaborative learning is British roots, based on the work of English teacher exploring ways to the help students responds to literature by taking a more active role in theorem learning. The collaborative learning traditional tends to use qualitative methods which look at achievement the product of learning.

Collaborative learning is grounded in constructivist theory. Knowledge is discovery by students and transformed into concept students can related to reconstructed and expanded trough new learning experiences. The learning consist of active participation by the student's verses passive acceptance of information representation by an expert lecture. Learning comes about through transaction and dialogue with their peers. collaboration is a philosophy of interaction and personal life style where individuals are responsible for their actions, including learning and respects the abilities and contributions of their peers; individuals is structure of internalization of knowledge designed to facilitate the accomplishment of a specific and product or goal through people working individually. Before we proceed with the

theoretical underpinning of each method. It would be helpful to describe the differences between the two paradigms in terms of an actual class.

The collaborative teacher would decide how many reasons they could identify. The teacher would be available for consultations and facilitate the process by asking for frequent process from the groups frequently, facilitate group discussion about dynamics, which help with conflict resolution etc.

Rookwood(1971)states" in the ideal collaborative environment, the authority for testing and determining the appropriateness of the group product rests with, first, the small group, second the plenary group and finally the requisite knowledge community.

Brafee (1995) holds "those shyer vocabularies values, convention and interests. The job of instructor is to help students learn to negotiate the boundaries between the communities they already belong to and the community represented by teachers academic discipline, which the students want to join every knowledge community has a core of foundation knowledge that its members consider as given to function indecently with in a knowledge community, the fielding scholar must master enough material to become conversant with the community".

Brody and David's (1998) look at the differences between the two paradigms epistemological. In the early 1970s some educators were formulating method based upon studies human social interaction and group learning these studies lead to learning strategies based upon social inter-dependence theory, cognitive developmental theory and the behavioral learning theory. Another group of educators based their framework for group working on theories derived from studies about the social nature of human knowledge. The different of constructivism formed the basis of collaborative learning.

The supportive environment helps us become more outgoing, encouraging and sociable helps us become more of who we want to be, we are all very different, with individual need, strengths and weakness. Yet, we have been able to find common ground, appreciate each other and find a place for all our voices". (sterahn 1991,p.108)

Lastly, according to the smith (1990) collaborating learning practitioners would say that all collaborative learning is about building learning communities. However we use the term learning community here in a broader and more specific sense in team of international reconfiguration of the curriculum. In the past 15 years the number of colleges have recognized that deep-seated structural factors weaken the quality of undergraduate learning and inhibit the development of community. These schools have attacked the problem directly by developing learning communities a "purposeful restructuring of the curriculum to link together courses so that students find greater chances in what they are learning and interaction with faculty and fellow students".

Statements of Problem

"Nepal is a multicultural, multi religious country with various language so, a teacher must know special instructional issues to accommodate students from different cultural background" (Acharya, 2014). As a professional, I have experienced a number of challenges for promoting collaborative learning in Nepali mathematics classrooms. This study was mainly concerned about the effect of the collaborative learning versus traditional learning. It needs focus on the answer of the following questions:

- How Does collaborative learning make students performance better than traditional teaching system? "

- What are the role of teacher and student in collaborative learning?

The question like this occur the researcher was motivated to carried out this research.

Research Questions

- Are there significances differences between achievement of the experimental and control group? Is there significance between the achievements of boys and girls of the experimental group?
- Is there significant difference between the achievement of boys and girls of the control groups?
- Is there a significant difference between the achievement of experimental boys group and control boys group?
- Is there significant difference between achievement of experimental girls and control girls groups?

Hypothesis Testing

The researcher develops the following statistical hypothesis to carry this study.

Null hypothesis (H_0): There is no significant difference in the mean achievements of collaborative group and traditional group.

Alternate hypothesis (H_1): There is significant difference in the mean achievements of collaborative group and traditional group.

Objectives of the Study

The objectives of this study were as follows:

- To compare the achievement of students taught through traditional learning and collaborative learning.
- To analyze the reflection of teachers and students towards collaborative teaching.

Significance of the study

The subject of mathematics is the useful tool for other relevant subject e.g. vocational teaching and highly specialized courses of learning. The main theme of something mathematics is not the teaching of knowledge but the acquisition of the acquiring knowledge because acquiring of knowledge is concern only with the product of knowledge is concern with process of knowledge. Which gives the system of mathematics? This study is important to the acquirement of the power of the acquiring knowledge.

This study was the following Significances in the teaching and learning strategy of mathematics:

- It, helps the teacher to organize his/her experience in appropriate and learning strategies.
- It helps to develop an effective instructional planning.
- It gives a model of innovative instructional planning and learning.

Delimitations of the Study

This study was limited to the certain area. If I saw aspects most decorous teacher and students that is problem for us. It was not included the students in group formation boys and girls or boys-boys and girls-girls. In our Nepalese context higher secondary mathematics and secondary level mathematics are high weight age courses. This study was not be generalized for all over our country or foreign country. But it will be a good explanation for the population with the same characteristics regarding of learning. It was not new learning system but arrangement systematic only. This study was be limited within a time period near about two weeks. This study was two higher secondary of science faculty of class eleven topics of matrix and determinants only.

Definition of the Related Terms

Achievements score -The achievements is the score obtained by the students in achievement test of higher secondary school mathematics constructed by the investigator.

Boy's group- The boy's mathematics students group of higher secondary level at grade eleven.

Collaborative-An environment where teachers are working together to problem solving around learning boys and girls of higher secondary level of grade eleven of mathematics students.

Collaborative learning- Collaborating learning is umbrella term encompassing many forms educational approach involving joint intellectual effort by students and teachers together. Teaching method in a group that requires positive interdependence, individual account ability and interpersonal skills, face to face primitive's interaction and processing.

Effect - The outcome of collaborative and traditional teaching and learning.

Experimental group-The collaborative is also experimental group.

Girls/Boys-The learner of grade eleven.

Girls group-The girl's mathematics students group of higher secondary level at grade eleven.

Traditional group-The learner who is participial in traditional or regular class.

Traditional learning-Method where the teacher is the authority of teaching learning activities and students are passively accept the facts exposed by the teacher. The interest and expectation of the students are highly underestimated by the teacher. The teacher explains and illustrates, questions but the students do not have approach to the explanation.

Chapter-II

REVIEW OF THE RELATED LITERATURES

The main purpose of review of the related literatures to find out what works have been done and what work has not been done, in the area of the study being the collaborative learning in mathematics, prior to beginning my project. I research and read many articles on the method of collaborative learning. Many of the research articles show that I have found which are related to my topic. Address achievement is not only mathematics but also all subject areas. This seems to be the main problem that many researchers are trying to address. Other underlying themes are student involvement within groups, over all group work and dynamic as well as teacher use understanding of collaborative learning. These are the main themes that I have focused on while finding research articles. I was addressing all of these topics at some point in my own research. During the past 1st century lots of and research about collaborative learning was in mathematics only. The collaborative learning is reviewed with some empirical, theoretical and framework literature. There are many literature of the field of study and around our world leave. Some researchers and their purposive findings are discussed which are given below:

Empirical Literature

Smith (1961) in American education encyclopedia states about mathematics as "mathematics, an inclusive term for the number of branches of learning that deals with magnitudes numbers, quantities and their relationships. Mathematics is a group of the experiences of all people regardless of how far they have gone in school. It is the very foundation of such a specific technological world as we live in today. The great advance which civilization has made ensconce and technology couldn't have been made without advantage of mathematics. It is the basis of physics, chemistry, astronomy and all branches of engineering. All our anything without use of

knowledge of mathematics. Fundamental process and the skill to them are the preliminary requirement of human being these days. "

Rout(2000) did an experiment research on "a study on the effectiveness of inductive to deductive teaching learning approach in secondary school." With the main to compare the achievement of the students taught by inductive-deductive approach and to determine effective approach of suitable classroom teaching learning in school. A present post-test equivalent group design was adopted to conduct the experiment group design was adopted to conduct the experiment in the content of equation and inequality for grade eleven for eight weeks. An achievement tests of 30 items was administered and t-test, t-test, correlation coefficient, analysis of variance used and concluded of the students were taught by inductive-deductive approach of teaching which improved significantly better achievement than the performance of students though by traditional approach.

Bhusal (2000) did a research on "a study on the effectiveness of teaching geometry using discovery module of teaching in secondary level". With the aim to find out whether the discovery is more useful than expository is to prove geometrical theorems as well as to compare the achievement between the group of the students though for three weeks. The t-test was applied to draw conclusion that the discovery method is better than expository method in teaching geometry.

Neupane (2001) did an experimental research "a study on the effectiveness of play way method in mathematics at primary level and to compare the achievement of students by play-ways method. A pre- test, post-test equipments group design was adopted. The research developed an achievement test. Two schools were sampled. The t-test was applied and concluded that the play way method resulted significantly better method over traditional method teaching at primary level.

Ghimire (2001) did a research on "a study on the effectiveness on experimental verification in teaching the deductive proofs of geometric theories at secondary level" with the aim to study the effect of Procedure of experimental verification in proving geometric theories and enhancements of understanding the fact principals and concept of geometric ideas. In post test only equipment group was adopted, though both the groups on the same selected units of grade nine geometry and made achievement test. The t-test and t-test were applied and concluded that experimental verification had significant effect on teaching geometry than the next.

Pokhrel (2006) studied in teacher education program and in the students of grade nine was an action research and published an article as journey of mathematics educator from his student life in school to the life of educator. How teaching and learning of mathematics was practiced in the past and how practiced in the Post and how this is done at present was highlighted in the history particularly concerned in the learning culture development in school in Nepalese context in the course of mathematics teaching and educating teacher discussed on the basis of an action research of the writer. The research, "is people generally, understand a high sounding meaning. But it is the result of implementers" research intervening something in a situation. The study of writer showed that cooperative learning is possible to introduce in Nepalese concept with slight modification in the approach. This creates positive and hope in the improvement of mathematics education in Nepal.

Kattel (2009) did research on "Effectiveness of Collaborating learning in mathematics at secondary level" with the aims to find out the effectiveness of collaborative learning method in teaching mathematics is more useful than traditional methods of secondary level. The researcher selected 31 students of grade IX from two school in Okhaldhunga district. The students of the selected two schools were divided

into experimental and control group. Experimental group was taught using collaborative learning method and control group was taught using collaborative learning method and control group was taught using traditional method. The researcher experiment was 12 days. The difference in mean achievement scores was tested in t-test for determining statistical significance between them.

The researcher used t-test at the 0.05 level of significance showed that the traditional method in teaching at secondary level mathematics.

Theoretical Literature

The "collaborative learning "is an umbrella term for a variety of educational approaches involving joint intellectual effort by students and teachers together. Usually, students are working in groups if two or more mutually searching for understanding, solutions or meaning or creating a product, collaborative learning has activities vary widely, but most centers on students exploration or application of the course materials, not simply the teachers exploration or application of the course materials not simply the teachers presentation or application of it. In collaborative classrooms, lecturing/listening/note talking process may not disappear entirely, but it lives alongside other process that is based on student discussion and active work with the course of material. Teachers who use collaborative learning approaches tend to think themselves less as expert transmitters of knowledge to students and more and expert designers. If intellectual experiences for students as coaches or mid wives of a more emergent learning process(Barbara,1992)

According to David D-Ohrislip (2002), describe that the following phases of collaboration:

Phases	focus
• Getting started.	process

- Setting up for success Process
- Working together. Content
- Moving to action. Implication

According to Dhoffner, Dias and Thomas are likely to benefit programs in a similarly nearly phase of collaboration. In this article the author proposes that there are three phrase of collaboration. In this article the author proposes that there are three phrases of collaboration. Different programs are a different stage of collaboration. Some are struggling to established collaboration. Some are struggling to establish collaborating relationship, other are collaboration and trying to incorporate strategies such as those noted previously while others programs have not only implemented these strategies but are revising longitudinal studies to assess their ultimate impact. This might be visualized as successive phase of collaborating:

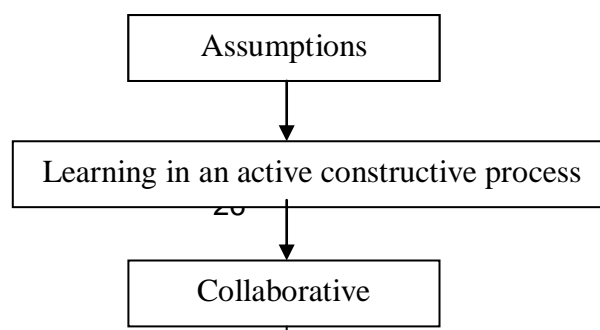
Phase 1:Developing a collaborative relationship.

Phase 2:Addressing content specific needs.

Phase 3:Assessing the long-term effects of collaboration.

While, we use collaborative learning because we believe it helps students learn more effectively, many of us also place a high premium on teaching strategies that go beyond mere mastery of content and ideas. We believe collaborative learning promotes a larger educational agenda, one that encompasses several intertwined rationales.

The address the research statements mentioned above topic the research will set an appropriate framework.



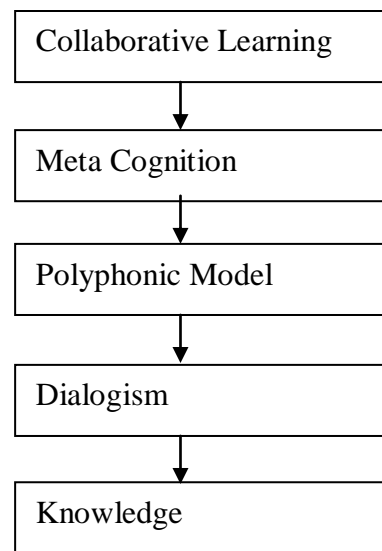
Source: David D-Ohrislip (2002)

One issue is whether the collaborative work was done "in class" or "out of class " for me, "out of class "works best in upper level and graduate course. In my department we already know how to do this give some hard problem sets (which might be called projects in other field) and give the students a week or more to work on them. Our graduate students have been good at working together (they work out of several large offices) and in general this has worked without requiring very much organization. On several occasions, I've tried to organize work in class, but it hasn't been very effective. I think that perhaps the steps in the work are too big. Individuals need to do some background work before they can collaborate effectively. At that leave, a mathematics course is generating genuinely new material all the time. So the collaborative learning is interested in focuses on classes at the first and second year level, algebra through calculus. The work can be done effectively in class because it can be given in relatively small increments.

According to M.Dascolu, S Trauson-matu, P Desius (2007) are conclude that computer supported collaborative learning (CSCL) technologies play and increasing role simultaneously with the appearance of the social web. The polyphonic analysis method based on Bakhtin's dialogical model reflects the multi voiced nature of a CSCL Conversation and the related learning process. We propose the extension of the

model and the previous applications of the polyphonic method to both collaborative CSCL chats and individual meta cognitive essays performed between collaboration and textual complexity, all integrated in all implemented system, which uses natural language processing techniques.

In diagram,



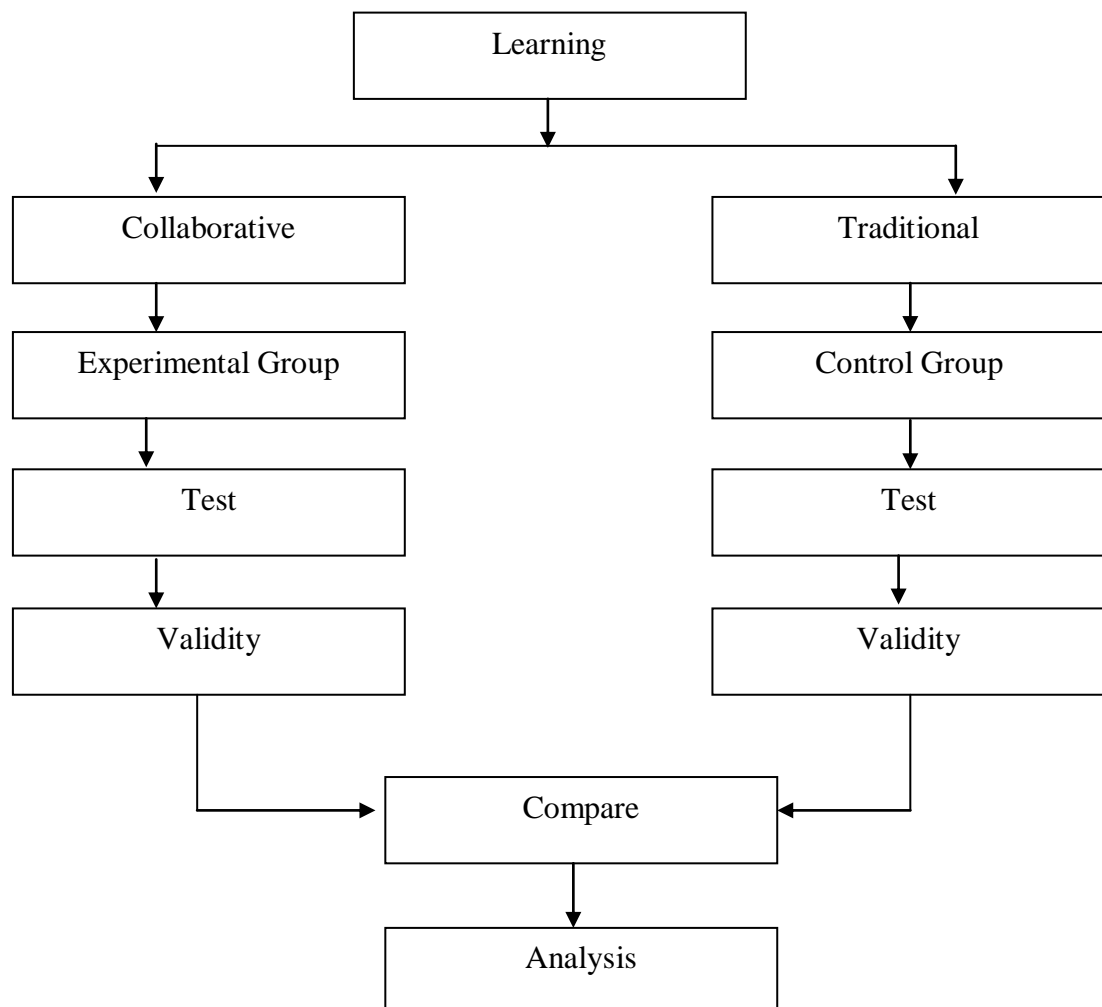
Source: CSCL, P. Desius (2007)

According to Stefan Trausan Mates (2009) are introduces a theoretical framework for analyzing collaborative problem solving in chats, based on the concept of polyphong and Bakhtish theory a dialog polyphony, a notion taken from music theory, may be considered as a general model for interaction and creativity by a group of people following patterns of counter point' As Bakhtin emphasized, polyphony may occur in texts, we will show that it can occur in problem solving tents. One of the features of polyphonic music is its potential development of complex architecture starting from a given them. Polyphonic structuring of dialogs may transform the international into a thinking device. Different voices jointly construct a method, something adopting different positions and then generating, denying or solving dissonances polyphony consist of several. "Horizontal" longitudinal method lines that are "Vertically" transversally integrated.

Similarly in chats, the contributions of utterances are tied together over time providing a methodic lin. Simultaneously, they are coordinated with the utterances of others, maintaining the integration toward. Unity across various themes and variations that sometimes can introduces differences. The collaborative process software tools for the visualization.

Conceptual Framework About Learning

To address the researcher statements mentioned the above topic research was set an appropriate framework. The main conceptual basis of the collaborative learning.



As the above table compare between two learning approaches. which are traditional and collaborative. In other word researcher is considered to control and

experimental groups. In first linkage of experimental group was taken achievement test after the class, determined the validity of the learning. And same procedure of control group also. Lastly, Two groups were compared to analyze the result.

According to Stefan Trausan-matu (2009) introduces a theoretical framework for analyzing collaborative problem solving in chats, based on the concept of polyphony and Bakhtin's theory of dialog. Polyphony, a notion taken from music theory, may be considered as a general model for interaction and creativity by a group of people ("voices," in an extended sense) following patterns of counterpoint. As Bakhtin emphasized, polyphony may occur in texts; we will show that it can occur in problem-solving chat texts. One of the features of polyphonic music is its potential development of complex architectures starting from a given theme. Polyphonic structuring of dialogs may transform the interaction into a "thinking device":

Different voices jointly construct a melody (story or solution), sometimes adopting different positions and then generating, idenepting or solving dissonances (unsound, rickety stories or solutions). Polyphony consists of several "horizontal," longitudinal melody lines that are "vertically," transversally integrated. Similarly, in chats, the continuations of utterances are tied

Together over time providing a melodic line. Simultaneously, they are coordinated with the utterances of others, maintaining the integration toward unity across various themes and variations that sometimes can introduce differences. This chapter also proposes software tools for the visualization of the polyphonic weaving in chats. These tools idenept and visualize the explicit and implicit links among utterances, and may determine or visualize the contributions of each participant in a chat.

Chapter III

METHODS AND PROCEDURES

This chapter includes the procedure adopted in the study which was carried out to achieve the objective of the study and to get answer to the research questions. It was described the design of the study site selection, selection of the case respondent tools used to collect the information and data analysis procedure.

Design of the Study

The design of the study is framework of the research. This is a class interaction research approach. In this study the researcher has performed pre-test, post-test equivalent control group experimental research in which researcher was made two groups. Experimental group was though by traditional method and for the process, the researcher used stratifies random sampling from experimental and control group in to equivalent classes. Then the period of research time the researcher was observe students groups or individually involving the participation or exam participation. The research was based on analysis and quantative in nature. Randomization (K.C.2006) assures statically equivalence of two groups prior to introduction of the experimental treatment of the instructional strategy. Though researcher research was quantative in nature, researcher has used some quantative data. To analyze the data. These data were classified to reflect the activities happened in the classroom.

This study was conducted by pre-test, post-test non-equivalent group research design as given by table.

Table 3.1**Experimental Design**

Pre-test	Group	Treatment	Post-test
First test(T_1)	Experimental	Collaborative	Second test(T_2)
First test(T_1)	Control	Traditional	Second test(T_2)

Population of the Study

Researcher considered students of class eleven studying mathematics science faculty of Srijana Higher Secondary School (Misson campus) and Laliguras Higher Secondary School (paradieze campus) as the population of the study. These schools are located in Saptary district of Rajbiraj.

Variable Studied

This study was experimental in nature every experimental study deal with independent and dependent variable so it is also deal with the dependent and independent variable which can be describe as follows.

Independent Variable

In this study, the independent variable was method of teaching mathematics in the class room instruction in accordance with the characteristics of social constructivism.

Dependent Variable

In this study, achievement score in the test of mathematic was dependent variables.

Experimental Variables and Control Exercised in the Experiment

The following non experimental variables were tried to control or equate to each go the following ways.

- Same subject matter were taught for both group
- The same test was used for both group.
- Experimental and control groups were divided by randomization.
- The same teacher taught both groups.

Sample of the Study

Researcher selects the two higher secondary schools which have also running the higher education programs. The researcher selected the control group, Srijana Higher Secondary School (Misson campus), there are 23 students out of the 14 boys and 9 girls. Whereas experimental group, Laliguras Higher Secondary School (paradieze campus), there were 30 students out of the 26 boys and 4 girls.

Table 3.2

Students Demography in the Selected Schools

Laliguras Higher Secondary School (paradieze campus) (EG)			Srijana Higher Secondary School(Misson campus) (CG)		
Total	Boys	Girls	Total	Boys	Girls
30	26	4	23	14	9

Out of the total number of the students, so as to make of sample unbiased and control different variables that could positively affect student's achievement, some students were identified and excluded from the participation in the experiment.

Criteria for exclusion from the participation in the experiment.

Table 3.3

Criteria for Exclusion from the Participation are Shown on the Following Table

School/criteria	Laligurash Higher Secondary School (Paradise Campus) (EG)	Srijana Higher Secondary School (Mission Campus) (CG)

	Boys	Girls	Total	Boys	Girls	Total
Under16 years	1	0	1	0	0	0
Above 18years	1	0	1	0	0	0
Irregular in class	1	1	2	0	0	0
Not taking extra class	1	0	1	0	0	0
Total	4	1	5	0	0	0

There was 1 boys of under 16 years and 1 boys above 18 years in Laliguras Higher Secondary School (paradieze campus). So these two boys were excluded to control the effect of extraneous variables related to maturation. It was found that 1 boys and 1 girls in Laliguras Higher Secondary School (paradieze campus) were irregular in class with the help of the attendance register of grade eleven faculty of science. They were present in the class less than sixty present out of the total working days of school. They were excluded to control the extraneous effect of the variable related to be experimental mortality. Other one student (paradieze campus) hadn't taken extra class so it was also excluded. After this exclusion the remaining students are shown the following table.

Table 3.4

Sample Student For Experimental Research

Laliguras Higher Secondary School(paradieze campus) (EG)	Srijana Higher Secondary School(Misson campus) (CG)
---	--

Boys	Girls	Total	Boys	Girls	Total
22	3	25	14	9	23

Remaining students were also sub divided four sub-groups according to marks obtained by them in school leaving certificate (SLC) 2071 as sampled by stratified random sampling because the groups of the students were homogeneous regarding equal number of boys and girls.

Table 3.5

Selected Students For Sample Study

Marks obtained	Laliguras Higher Secondary School(EG)			Srijana Higher Secondary School. (CG)		
	Boys	Girls	Total	Boys	Girls	Total
Distraction	1	0	1	0	1	1
First Division	1	1	2	1	1	2
Second Division	1	1	2	1	1	2
Third Division	0	1	1	1	1	2
Failed	1	1	2	1	1	2
Total	4	4	8	4	4	8

Note: According to SLC Examination Distinction (80% -100%)

First Division (60% - 79%)

Second Division (45%-59%)

Third Division (32%-44%)

Fail Division (Below 32%)

Finally two groups of students were made ready for collaborative and traditional group. There were of Student in each group consisting 4 boys and 4 girls in

both group. At last, a fair coin was tossed to choose the collaborative and traditional group. The students of Laliguras Higher Secondary School were choose to be a collaborative group and the students of Srijana Higher Secondary School (Misson campus) were chosen be traditional group. Teaching was done in the whole grade and the scores were collected from the sample group only.

Data Collection Tool

The researcher used daily lesson plan assessment of class as tools of teaching and the achievement test consisted of 35 questions, in which were 15 objective, 10 short and long 5 categories of questions related to the HSEB new curriculum. The test consisted of 16 knowledge, 10 skill and 3 problem solving questions based on chapter matrix and determinant. Before the test was developed, researcher discussed with subject teacher of three difference schools and export to minimize the complexity and language error of the items. The specification chart of the questions is given in the appendix A₁ for the refined achievement test (pre test, post-test). Researcher did pilot test among 30 students of grade XI in Shree Bhawani H.S.S Manbu, Gorkha when students started to submit their answer sheet researcher started to note the time taken by them to complete the test. Researcher collected the achievement score of piloting to analyzed the item for this research arrange the answer sheet from higher score to lower score and took 27 percent of the answer sheet from high score and 27 percent of the answer sheet from low score. In this way researcher found that 8 answer sheet on top most marks and 8 answer sheets on low marks so all together 16 were indentified for the purpose of calculating reliability coefficient and item analysis.

Item Analysis

The process of evaluating test item to maintain the optimum difficult level and discriminating index it is known as item analysis. Item analysis indicates which items

are very easy and which are difficult. The prepared achievement test was administered in Laligurash higher secondary school. To students of grade eleven for item analysis.

To analysis the item answer sheets were ordered from highest score to lowest score then researcher took from, 27 %upper and 6 %lower score students answer sheet. 9% .upper scores and 3%. Lower scores answer sheet were sheet where taken out of 30 answer sheets. The correct response of each item was marked with and the incorrect.

Difficulty Level (p)

The difficulty level of an item is the index of difficulty, which is the percentage of examinees who correctly answered the item. It is the percentage of students able to pass each item. It takes the values ranging from 0 to 100. The index of difficulty (p-value) of each item will be calculated by using the formula of using in Appendix -C. It is the percentage of students able to success achievement each item which scores is from 0 to 100.

Table No. 3.6

The Criteria of Analysis of Item Difficulty Level are as Given by Table.

Criteria	Item evaluating	No of items	Remarks
Above 80%	Easy	16	Need improvement or reject
80%-20%	Good	19	Accepted
Below 20%	Difficult	0	Need improvement or rejected

Discrimination Index

The index is a number which differentiates the strong and poor students. It takes the value ranging from -1 to +1. The index of discrimination or D-value of each item is calculated by using formula (Appendix C). The P-level and D -index of two steps subjective item is the average of p-level and D-index of each steps respectively. The item analysis table is given in the Appendix B₁ to B_g. The items having P-level between 20 to 80 percentage and D-index above 0-19 were accepted otherwise the items were rejected. The item numbered 1, 3, 5, 6, 7, 8, 9, 11, 17, 18, 20, 22, 24, 26, 27, 29, 32, 33, 34 and 35 were rejected after item analysis in Appendix D. But Q.No. 1 was improved. In subjective items the students who answer correctively full question for the item analysis. The final test contained 8 objective 5 subjective short question and 3 subjective long questions. The final achievement test paper is given in the appendix E and F respectively.

The Criteria of Analysis of Item, Discriminating Index are Given by Table.

Criteria	Item evaluating	No of item	Remarks
0.4and above	Very good	14	Accepted
0.3-0.39	Good	1	Accepted
0.20-0.29	Marginal	2	Accepted
0.19and below	Poor	22	Need improvement or rejected

The P-level and D-level of two steps subjective items is the average of P-level and D-level index of each steps respectively. The item analysis table is given in the appendix B₁ to B₉. The items having P-level between 20 to 80 percentage and D-index above 0.19 were accepted otherwise the items were rejected. The item numbered 1,3,5,6,7,8,9,11,12,13,15,17,18,20,22,24,26,27,29,32,33,34 and 35 were rejected after item analysis in appendix D. But question 1 was improved. In subjective items, the students who answered correctively full question for the item analysis. The final test contained 8 objective 5 subjective short questions and 3 subjective long questions. The achievement test paper is given in the appendix E and F respectively.

The Reliability of the Study

The reliability is the degree in which a test consistently measures whatever it measures. The more reliable test is the more confidently itself presentation, we can have that the score obtained from the repeated administration of the test are essentially the same. Thus reliability refers to the consistency of the result. Applying rational equivalence reliability was adopted to determine internal consistency by determining how each item on the test related to all other items and to the total test. The use of application of point blistered correlation coefficient, which is calculated by using the formula given in the appendix E. Applying the Blisterel correlation

coefficient the mean calculated value of the reliability was meet to be 0.35. It indicated that the mathematical achievement test was reliable. The calculation of the reliability table is presented in appendix E.

The Validation of the Study

The test is valid for a particular purpose and for a particular group, therefore the mathematics achievement test would developed for measuring the effectiveness of collecting learning as well as the achievement of eleven grade boys and girls. For the estimation of validity of this test, it had already been maintained earlier that school. Teacher and mathematics educator was also consult to judge however the test measure what it was intended to measure. To maintain validity, researcher prepared specification chart a part from this; research was also get the feedback from experts and thesis supervisor guide. Therefore, the test is valid.

The control of valid minimizes using enjoyable environment of class room, study topics, time manage and teaching exam oriented. Then control of external valid threats starting the research on students. Before taken permission by subject teacher school administration and do the agreement of clear time table view.

Data Collection Procedures

The researcher taught both the experimental and control groups for two weeks. Experimental is Laliguras Higher secondary school by collaborative learning and another in Srijana Higher secondary school by traditional method from 2071-07-10 to 2071-07-30. The experimental group was taught matrix and determinant by collaborative method according to lesson module in appendix I. The content validity and reliability of the teaching module was conducted with subject experts and senior mathematics educator. The control group was taught the same topic by traditional method. No difference was in class work and home work.

- In experimental research teachers personality is also of the factors so to maintain the personality researcher himself taught in both of the classes.
- Researcher spoke same types of words in both of the classes, so that the verbal problem words in both of the classes so that the verbal would not affect the achievement of the students.
- At the end of the classes, researcher had given same types of problem to solve at home and next class student of both groups in the same day and returned them.
- Researcher created the friendly environment on both the groups.
- To give the equal chance for all students in a group researcher allowed them to change their sheet everyday in both of the groups.
- Any types of question positive in the sense of topic or negative were respectfully accepted and the solutions of their problem were given
- Group were formed maintain ing the ethnicity maturity, sex, locality, knowledge ability in both of the groups, so that it could be sure that the achievement test score were not affected by any other independent variable other than learning method.
- During the post test period researcher was announced the test schedule class ahead in both of the group and the examination halls of both the groups were similar in size.
- Researcher himself invigilated for both groups same question paper were given.
- Though the time of test was different the duration of the time period was same.
- Researcher himself invigilated for both groups researcher had conducted an achievement test at the end of the class.
- The test paper were collected by using the making scheme and the score obtained by those student in the test were collected to analyze.

- At last, a mini seminar was conducted for the opinion of teachers and students towards the collaborative learning.

Experimental Process

There are several process to teach students by cooperative learning method. To control the effect due to other variables such as personality, emotion and behavior of the teacher, both experimental and control group were by me. Control group were taught by using collaborative learning method by preparing teaching module. Experimental were selected the students, STAD, because it is simplest of all cooperative learning methods.

STAD is one of the simplest and Most flexible of the collaborative learning methods. There are several procedures to teach students by collaborative such as the group investigation, LT, AC, TGT, STAD, Jig-Saw and Group Interaction method etc. It is the good model for the new educator who is going to follow collaborative method.

At Experimental group, to use STAD (Revert E. Slavin, 1995) researcher made six teams. Each team contains five students of different level of ability, sex and ethnicity. In this way there were 30 students engaged all together. Researcher followed STAD major components grade presentation, Team Study, Quizzes (test), individual improvement, Team recognition and play way learning.

Data Analysis and Interpretation Procedure

To analyze the obtained data researcher used the following statistical procedure for quantitative data.

1. Researcher has calculated mean, standard deviation and variance for both the group with their secured marks in the test.
2. Researcher used t-test at 0.05 level of significant to find whether the difference of mean is statistically significant by using the method pooled variance formula.

For achieving the second objective the researcher was taken the interview with three teachers and three students who involving collaborative teaching and learning the analyzed thorough different theories.

Chapter-IV

DATA ANALYSIS AND INTERPRETATION

In this chapter researcher has analyzed and interpreted the collected data. Researcher did six sets of comparison of the achievements of collaborative and traditional groups.

- i. Comparison of the achievement score in pre-test of the collaborative and traditional groups.
- ii. Comparison of the achievement of students in the collaborative and traditional groups in post-test.
- iii. Comparison of the achievement between boys and girls of the collaborative groups in post test.
- iv. Comparison of the achievement between boys and girls of the traditional groups in post-test.
- v. Comparison of the achievement between collaborative boys group and traditional boys groups in post-test.
- vi. Comparison of the achievement between collaborative girls and traditional girls groups in post-test.

Researcher collected the achievement score and organized, tabulated and subjected to statically test and interpreted. The marks obtained by the students are tabulated in Appendix J.

Comparison of the Achievement of Score in Pre-Test of Collaborative and Traditional Group

This study was to compare the achievement between two groups. In this research, the researcher found that the mean marks obtained by collaborative and the traditional group in pre-test were 3.1 and 3.5 respectively. Over the full marks 30

which were different. So researcher calculate the statistical tool t-value to check whether the difference is statistically significant or not then the statistical data are tabulated in the following table.

Table 4.1

Mean, Standard Deviation and t-Scores on Pre-Test of Collaborative and Traditional Groups

Group	Sample	Mean	Standard deviation	Variance	Tabulated t-value	Calculated t -value	Remarks
Collaborate	8	3.1	1.81	3.29	2.98	1.96	Null hypothesis accepted
Traditional	8	3.5	1.96	3.85			

Table 4.1 shows that, the calculated (t-value) (1.96) is smaller than the critical or tabulated t-value (2.98) at 0.05 level of significance with degree of freedom 18 in two tail test, which shows that the null hypothesis, there is no significance difference between the mean score of the collaborative group and traditional group on the pre test achievement of the students' was accepted. This shows that there is no significant difference between two groups. This means initially both of the groups were similar.

Comparison of the Achievement of the Collaborative and Traditional Groups in Post-Test

This study was to compare the achievement between group of students taught by collaborative learning method and traditional method. In this research, the researcher found that the mean mark obtained by collaborative and traditional groups were 25.2 and 16.1 respectively. There is no significant different between the achievement of the student taught by collaborative and traditional method. So

researcher calculated the t-value to check whether the different is statistically significant or not then the statistical data are tabulated in the following table.

Table 4.2

Mean and Standard Deviation of Scores on Post -Test of Collaborative and Traditional Groups

Group	Sample	Mean	Standard Deviation	Variance	Tabulated t-value	Calculated t-value	Remarks
Collaborate	8	25.2	3.75	13.76	1.96	2.78	Null hypothesis is rejected
Traditional	8	16.1	5.4	29.09			

Table 4.2 shows that, the calculated t-value (2.78) is greater than the tabulated t-value (1.96) at 0.05 level of significance with degree of freedom 18 in two tail test. Therefore the null hypothesis, there is no significant difference between the mean score of the collaborative group and traditional group on the achievement of the students was rejected i.e. the alternative hypothesis there is significant difference between the mean score of the collaborative group and traditional group on the achievement of the students was accepted. Thus the difference of the mean is found significant at 0.05 level of significant for two tails test. This shows that the students in the collaborative group were benefited in the achievement of matrix and determinate than the students in traditional group. It means that collaborative learning method is better than the traditional method in matrix.

Comparison of the Achievement Between Boys and Girls of the Collaborative Group

The achievement score of boys and girls are compute by using formula and is shown in the following table.

Table 4.3

**Comparison of Achievement Between Boys and Girls of the Collaborative
Groups**

Group	Sample	Mean	Standard deviation	Variance	Tabulated t-value	Calculated t-value	Remarks
Collaborative							
Boys	4	25.4	4.41	19.44	2.306	1.79	Null hypothesis is accepted
Girls	4	25	1.89	5.4			

The table 4.3 shows that the mean achievement scores of boys and the girls group in the collaborative group were equally benefited by the collaborative learning method in teaching matrices and determines in class eleven. Also the test of significance indicates that the calculated t-value in the test is 1.79 which is less than the critical or tabulate t-value 2.306 at the 0.05 level of significance with the degree of freedom 8 in two-tailed tests. Therefore the null hypothesis there is no significant different between the achievement of boys and girls group taught by the collaborative learning method in topic matrices and determinate in class eleven was accept. Thus the different of the mean at 0.05 level of significant. Thus boys and girls are equally benefited by this method.

**Comparison of the Achievement Between Boys and Girls of the Traditional
Group**

The achievement score of boys and girls are computed by using formula and is shown in the following table.

Table 4.4**Achievement Between Boys and Girls of the Traditional Groups**

Group traditional	Sample	Mean	Standard deviation	Variance	Tabulated t-value	Calculated t-value	Remarks
Boys	4	16.4	5.54	38.8	2.306	1.65	Null hypothesis is accepted
Girls	4	15.8	5.93	27.36			

Table 4.4 shows that the mean achievement scores boys and girls groups in the traditional groups were 16.4 and 15.8. This implies traditional boys read more adequately the unit matrices than girls group in class nine. Also the significant test indicates that the calculated t-value in 1.65 is less than the critical or tabulated t-value 2.306 at the 0.05 level of significance with the degree of freedom 8 in two-tails test. Therefore the null hypothesis there is no significant different between the achievement of boys and girls group taught by the traditional learning method in topic matrices and determinate in class eleven was accepted. Thus, the difference of the mean at 0.05 level of significant is found insignificant. Thus boys and girls are equally benefited in this method.

Comparison of the Achievement Between Collaborative Boys and Traditional Boys Group

The achievement score of boys and girls are computed by using formula and is shown in the following table.

Table 4.5**Achievement Between Collaborative Boys Group and Traditional Boys Group**

Boys Group	Sample	Mean	Standard deviation	Variance	Tabulated t-value	Calculated t-value	Remarks
Collaborative	4	25.4	4.41	19.44	2.101	2.639	Null hypothesis is rejected
Traditional	4	16.4	5.54	38.8			

Table 4.5 show that the mean achievement scores of collaborative boys group and traditional boys group were 25.4 and 14.6. The significant test shows that the calculated t-value 2.836 is greater than the tabulated-value at the 0.05 level of significance with the degree of freedom 8 in two-tails test. Therefore the null hypothesis rejected i.e. there is significant different between the achievement of boys in collaborative group taught by collaborative method and boys in traditional group taught by the traditional learning method in the topic matrices and determinate in class eleven. Thus the difference of the mean at 0.05 level of significant is found significant. Thus the collaborative boys group learnt more effectively than traditional boys group.

Comparison of the Achievement Between Collaborative Girls and Traditional Girls Group

The achievement score of boys and girls are computed by using formula and is shown in the following table.

Table 4.6**Achievement Between Collaborative Girls and Traditional Girls Group**

Girls Group	Sample	Mean	Standard deviation	Variance	tabulated t-value	Calculated t-value	Remarks
Collaborative	4	25	1.89	5.4	2.306	2.59	Null Hypothesis is rejected
Traditional	4	15.8	5.93	27.36			

Table 4.6 shows that the mean achievement scores of collaborative girls group and traditional girls group were 25 and 15.8. This implies that the collaborative girls group learnt matrices more adequately than the traditional girls group in grade nine. Also the significant test indicates that the calculated t-value 2.59 is greater than tabulate t value 2.306 at the 0.05 level of significance with the degree of freedom 8 in two-tailed test. Therefore the null hypothesis is rejected. There is no significant difference between the achievement of girls in collaborative group taught by collaborative method and girls in traditional group taught by traditional learning method in topic matrix and determinants in class eleven. Thus the difference of the mean at 0.05 level of significant. Thus there is significance difference between the achievement on the unit of matrix and determinant by collaborative girls group more benefited than traditional girls group. Finally, researcher concludes that from this research, collaborative learning method is more practicable as compare to traditional learning method according to the quantitative data.

Researcher took instructed direct personal interview to understand the opinion of teachers and student about which method is better in learning matrix? Why? There were 5 mathematics, +2 teachers and 20 participant students. All of the mathematics, teachers and students were focus on collaborative method is better than traditional

method because of collaborative method motivates students to learn, helps students to understand and cooperate the every problem of students 80 the collaborative learning method is more effective than tradition method in teaching matrix and determinant at grade XI.

During the period of data collection researcher spent 12 days with students and became a friend of them. They could ask to researcher the question without any hesitation so both of the group benefited on their learning. In traditional groups researcher used to give lecture to make their concept clear and researcher used to do some problems to make their concept clear. In collaborative group in most of the classes researcher gave the materials to study and they used to read the materials to find the facts things and to make their foundation clear. In most of the classes researcher felt tried at the end of the class of traditional group but in collaborative group most of the class time finished in group work and their presentation so researcher fell easy. In collaborative group, collaboration in mathematics learning is the focus groups on this study so the focus group were conducted to explore the hypothesis that the collaborative learning groups were serving their purpose in helping students to learn. These focus groups were held for 12 days. Several recurring themes emerged through these focus groups including gender issues, attendance, and group learning verses lecture learning.

To fulfill second objective of my research I have done the following interview with the three teachers and three who involving applied collaborative teaching and learning activities.

Teacher 'A' said

"Students learn both passively and actively passive learning takes place when students take on the role of receptacles of knowledge. We they do not directly

participate in the learning process. Collaborative learning is more likely to take place when students are doing something besides listening."

I found that the strategies promoting and motivation collaborative learning it is even more helpful to envision a continuum of possible classroom actions that increase in student's activity.

Teacher 'B' said

"In a collaborative setting, the success of one person is dependent on the positive interdependence. All members should rely on one another to achieve the goal and need to believe that they are linked together to successes. Teachers is the belief of anyone in the group that there is value in working together and the results of both individuals learning and working products would be better when they are done in collaboration."

In the above information, I concluded that teacher's motivation positive interdependence. Positive interdependence is the learnt of collaborative tasks. According to collaborative learning setting is the belief that the group setting is the belief that the group members.

In this regards, Teacher C said

"I use research findings on the positive inter dependence academic unpreparedness, group dynamics, poor listening skills. In a collaborative setting it refers to group members relying on one another in order to succeed"

In the above information, I conclude that each individual has only a part of the information resources, materials needs for his or her task, positive interdependence is met when particular roles are assigned to group members.

So from above information, I concluded that collaborative communications can pay an important role in learning mathematics. Teaching-learning method is very

important to achieved the knowledge of mathematics and helps to engaged the students in learning mathematics. It also helps to motivated to learn mathematics meaningfully.

Similarly, when I took interview of three students toward collaborative learning.

Student 'A' said

"In collaborative classroom the teacher as a facilitator who facilitating involves creating rich environments and activities for linking new information to prior knowledge, providing opportunities for collaborative work and problem solving and offering students a multiplicity of authentic learning task".

In the above information I concluded that teacher facilitate collaborative learning is to establish classrooms with diverse and flexible social structures that promote the sort of classroom behaviour they deem appropriate for communication and collaboration. In this regarded (Acharya, 2017).

Student 'B' said

"In collaborative classroom the students especially important is good setting, a critical process that helps to guide many other before during and after learning activities"

In the above information I concluded that the students become more actively involved in the unit after their discussion about goals, and at the end of the unit, could better evaluate whether they had attained them. In this regard (Acharaya, 2017). In this suggested (Cole, 2008).

Student 'C' said

"The critical role of dialogue in collaborative classrooms has been stressed throughout these modules. The collaborative classroom is alive with two-way

communication. A major mode of communication is dialogue, which in a collaborative classroom is thinking made public a major goal for teachers in to maintain this dialogue among student."

In the above information I concluded that students engaged in interaction often exceed what they can accomplish by working independently. (In this regard Acharya, 2017).

Chapter-V

SUMMARY, FINDINGS, CONCLUSION AND RECOMMENDATIONS

This chapter includes the Summary finding, conclusion and recommendation. The purpose of this study was to compare the achievement of student taught through traditional learning and collaborative learning to determine the effective approach in learning mathematics.

Summary

Researcher did the experimental research which was carried out in mixed method that is quantitative and qualitative. The main objective of this study was to find the effectiveness of collaborative learning method in teaching matrix in class eleven. In order to conduct the experiment. Researcher developed a teaching module and taught the students of experimental group by collaborative learning method and control group by traditional method for 12 days. At the end of the experiment, researcher performed a standardized achievement test. This test was administered on both groups. The scores obtained by the students in the test were analyzed using mean, variance, standard deviation and t-test. Initially researcher took a pre test and the so formed result indicated that there is no significant difference between the achievement between the control and experimental group. But in post test the result indicated that there is significant difference between the control and experimental group.

Findings

On the basis of the analysis and interpretation of the data obtained from the achievement test, the findings of the study are as follows:

- i. The average score of the students of experimental group is higher than the average score of the students of control group. Statistically the mean

difference was significant. Thus it was concluded that the collaborative learning method was effective method in teaching matrix in class eleven.

- ii. The mean score of both boys group and girls group of experimental group was similar. Also it was not statistically significant. Thus it was concluded that collaborative learning method could reduce the difference in achievement between boys and girls. It means collaborative learning method was good for both boys and girls.
- iii. The mean score of boys group was slightly more than girls of control group but it was not significant. Thus it was concluded that traditional method was also helpful to minimize the gender difference in learning mathematics as collaborative learning method.
- iv. The mean score of the experimental boys group was higher than that of control boys group. Also, statistically, it was significant. Thus, it was concluded that the boys who were taught matrix with the use of collaborative learning method could achieve significant difference in achievement than boys who were taught with the use of traditional method. So the traditional learning method is not favorable for boys group.
- v. The mean score of experimental girls group was greater than the mean score of control girls group. Statically, the mean difference was significant. Thus it was concluded that the collaborative learning method of teaching was effective for girls then traditional method of teaching in teaching matrix at grade XI.
- vi. According to the interview, students and teacher's expressed that collaborative methods made them easy to understand the subject matter.

Conclusions

On the basis of statically analysis the following conclusion are derived:

- i. The mean achievement score of experimental group is higher than control group. Also statistically it is significant. Since there is no significant difference between the mean score of boys group and girls in experimental group, collaborative learning method minimize the gender difference in learning mathematics.
- ii. The mean score of experimental boys group and control boys group similarly there is significant difference between the mean score of experimental girls group and control girls group. The difference of the mean mark obtained by experimental boys and control boys group is more than the difference of the mean marks obtained by the experimental girls and control girls groups. It means boys were more benefited than girls by collaborative learning method.
- iii. In opinions, the teachers and students were collaborative learning method is effective than traditional method of teaching matrix and determinant.
- iv. The collaborative learning strategy was better than the traditional approach learning mathematics.

Finally it is conclude that collaborative learning method is more effective than traditional method in teaching matrix and determinate at class eleven. Collaborative learning method better motivates students to learn, helps students to understand and perform better in achievement test over traditional method in teaching learning 'Matrix and determinates class XI.

Recommendations

Based on finding and conclusion, review and related theory the following recommendations are presented. Collaborative learning method is suggested to adopt in teaching Matrix and determinant at class XI.

1. The mathematics teacher should be encourage to use collaborative learning method in teaching 'Matrix and determinate at class eleven.
2. The teacher training institutes should focus their attention on collaborative learning method.
3. Curriculum designer, textbook writer and subject experts should emphasize on collaborative learning method.
4. Mathematics teacher should emphasize on collaborative learning method.
5. Study on designing different teaching and learning modules should be used on classroom teaching.
6. Different collaboration learning strategies should be carried out in all breaches of school mathematics.
7. A larger sample would help to validate the conclusions made in this research; it will be generalized if it is under taking nationally.
8. It is helpful for every teacher to understand collaborative classroom and to apply collaborative relevant teaching learning activities.
9. It is also helps myself to develop my professional career.

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Appendix A₁ : The schedule and Lesson Plan for Experimental Group : Specification Girls

Topics	Time	Knowledge				Skill				Problem Solving				Total
		Objective	short	long	interview	objective	short	long	interview	objective	short	long	interview	
Matrix and its notation	1 periods	1 (1)			1(1)		1 (2)							3 (4)
Types of Matrix	1 periods	1 (1)				1 (1)					1 (2)			3(4)
Addition and subtraction of Matrix	1 periods	1 (1)			1 (1)		1 (2)							3 (4)
Algebra of Matrix	1 periods	1 (1)							1 (1)					2 (2)
Multiplication of Matrix	2 periods	1(1)					1 (2)			1(1)		1 (4)		4 (8)
Adjoin and inverse of Matrix	1 periods	1(1)	1(2)				1 (2)						1 (1)	4 (6)
Determinant of square matrix	2 periods	1(1)			1(1)	1(1)				1(1)				
Determinant	1 periods	1(1)		1 (4)				1 (4)			1(2)			4 (11)

of square matrix (up to order up to order 3×3)														
Minors and cofactors	1 periods	1 (1)	1(2)							1(1)	1(2)			4 (6)
Properties of Determinants	1 periods		1(2)		1(1)	1 (1)		1 (4)				1 (4)		4(11)
Total	12 period	9(9)	3 (6)	1 (4)	3 (3)	3(3)	4 (8)	2 (8)	1 (1)	3 (3)	3 (1)	2 (8)	1 (1)	35 (60)

Appendix A₂ : Number of Questions of Different Level for Item Analysis

Level Question	Knowledge	Skill	Problems Solving
	1,2, 4, 5, 6, 7, 8, 12, 13, 16, 18,19, 26, 31, 32, 35	3,4, 10,15, 17, 20, 22, 25, 27, 29, 33	9, 10, 11, 21, 23, 24, 28, 30, 34
	16	10	9

Appendix A₃ : Specification Grids

Topics	Time	Knowledge			Skill			Problem Solving			Total
		objective	short	long	objective	short	long	objective	short	long	
Matrix and its notation	1 periods	1 (1)	1 (2)								2 (3)
Types of Matrix	1 periods			1 (4)							2 (4)
Addition and subtraction of Matrix	1 periods	1 (1)									1 (1)
Algebra of Matrix	1 periods	1 (1)									1 (1)
Multiplication of Matrix	2 periods						1 (4)		1 (2)		2 (6)
Adjoin and inverse of Matrix	1 periods	1 (1)			1 (1)	1(2)			1 (2)		4 (6)
Determinant of square matrix	2 periods							1 (1)			1 (1)
Determinant of square matrix (up to order up to order 3×3)	1 periods	1 (1)								1(4)	2 (5)
Minors and cofactors	1 periods				1(1)						2 (1)
Properties of Determinants	1 periods					1 (2)					1 (2)
Total	12 period	5 (5)	1 (2)	1 (4)	2 (2)	2 (4)	1 (4)	1 (1)	2 (4)	1 (4)	16 (30)

Appendix B₁ : Item Analysis Table Upper Topmost Mark of the Objective

Questions

Students Questions	U ₁	U ₂	U ₃	U ₄	U ₅	U ₆	U ₇	U ₈
Q ₁	1	1	1	1	1	1	1	1
Q ₂	1	1	1	1	1	1	1	1
Q ₃	1	1	1	1	1	1	1	1
Q ₄	1	1	1	1	1	0	1	1
Q ₅	1	1	1	1	1	0	1	1
Q ₆	1	1	1	1	1	1	1	0
Q ₇	1	1	1	1	1	1	1	1
Q ₈	1	1	1	1	1	0	1	1
Q ₉	1	1	1	1	0	0	1	1
Q ₁₀	1	1	0	1	1	1	1	1
Q ₁₁	1	1	1	1	0	1	0	0
Q ₁₂	1	1	1	1	1	1	1	1
Q ₁₃	1	1	1	1	1	1	1	1
Q ₁₄	1	1	1	1	0	1	1	0
Q ₁₅	1	1	1	1	1	1	1	1

Appendix B₂ : Item Analysis Table Lower Mark For Objective Questions

Students Questions	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈
Q ₁	1	1	1	1	0	1	0	0
Q ₂	0	1	0	1	0	1	0	0
Q ₃	1	1	0	1	0	1	1	1
Q ₄	1	1	0	1	1	1	1	0
Q ₅	1	1	1	1	1	1	1	1
Q ₆	1	1	1	1	1	1	1	1
Q ₇	1	1	1	1	1	1	0	1
Q ₈	1	1	1	0	1	0	1	0
Q ₉	1	1	1	1	1	1	1	1
Q ₁₀	0	1	1	0	1	0	0	1
Q ₁₁	1	1	1	0	0	0	0	1
Q ₁₂	1	1	1	1	1	1	1	1
Q ₁₃	1	1	1	1	1	1	1	0
Q ₁₄	0	1	0	0	0	0	1	0
Q ₁₅	1	1	0	0	0	0	0	0

Appendix B₃ : Interpretation of P-Value and D-Level of Objectives Questions

Q.N.	U _R	L _R		Analysis		D= $\frac{U_R}{U_N} - \frac{L_R}{L_N}$	Analysis
1	8	5	5	81	Easier	0.4	V-Good
2	8	3	3	68	Good	0.6	Good
3	8	7	7	97	Easier	0.12	Poor
4	7	5	5	75	Good	0.25	Marginal
5	6	8	8	87	Easier	-0.3	Poor
6	8	7	7	97	Easier	0.12	Poor
7	8	7	7	97	Easier	0.12	Poor
8	7	7	7	87	Easier	0	Poor
9	7	6	6	81	Easier	0.12	Poor
10	7	3	3	62	Good	0.5	Poor
11	5	4	4	56	Good	0.1	Poor
12	8	8	8	100	Easier	0	Poor
13	8	7	7	97	Easier	0.12	Poor
14	6	3	3	56	Good	0.4	Poor
15	6	6	6	100	Good	0	V-Good

Appendix B₄ : Item Analysis Table Top Most Mark For Short Questions

Students Questions	U ₁	U ₂	U ₃	U ₄	U ₅	U ₆	U ₇	U ₈
Q ₁₆	2	2	1	2	2	1	2	2
Q ₁₇	2	2	2	2	1	2	2	2
Q ₁₈	2	2	2	1	2	2	1	2
Q ₁₉	2	1	2	2	2	2	2	2
Q ₂₀	2	2	2	2	2	2	2	2
Q ₂₁	2	2	2	2	2	2	1	2
Q ₂₂	2	2	2	2	2	2	2	2
Q ₂₃	2	2	2	2	2	2	2	2
Q ₂₄	1	2	2	2	2	2	2	1
Q ₂₅	2	2	2	2	2	2	2	2

Appendix B₄: Item Analysis Table Lower Most Mark For Short Questions

Students Questions	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈
Q ₁₆	1	1	0	1	1	0	0	0
Q ₁₇	2	2	2	1	2	2	2	1
Q ₁₈	1	1	0	0	0	1	0	0
Q ₁₉	2	2	2	2	1	2	1	1
Q ₂₀	2	2	2	2	2	1	2	2
Q ₂₁	1	2	1	1	1	1	0	1
Q ₂₂	0	0	0	0	0	0	0	0
Q ₂₃	1	0	1	0	1	0	1	0

Q ₂₄	1	2	1	0	0	1	0	0
Q ₂₅	2	2	2	2	1	1	1	1

Appendix B₅ : Interpretation of P-Value and D-Level for Short Questions

Q.N.	U _R	L _R	P= (U _R + L _R)/U _N + L _N	Analysis	D = $\frac{U_R}{U_N} - \frac{L_R}{L_n}$	Analysis
16	8	4	75	Good	0.5	V-Good
17	8	7	93	Easier	0.12	Poor
18	8	7	93	Easier	0.12	Poor
19	8	3	68	Good	0.6	V-Good
20	7	7	87	Easier	0	Poor
21	8	7	93	Easier	0.12	Poor
22	7	4	68	Good	0.37	Good
23	8	4	75	Good	0.5	V-Good
24	8	4	75	Medium	0.5	V-Good
25	6	6	100	Easier	0	Poor

Appendix B₆ : Item Analysis Table Top Most Mark For Long Questions

Students Questions	U ₁	U ₂	U ₃	U ₄	U ₅	U ₆	U ₇	U ₈
Q ₂₆	4	4	4	4	4	4	4	4
Q ₂₇	4	4	4	4	4	4	4	4
Q ₂₈	4	4	3	4	4	4	4	4
Q ₂₉	4	4	4	4	4	4	4	4
Q ₃₀	4	4	4	2	2	2	4	2

Appendix B₈ Item Analysis Table Lower Mark For Long Questions

Students Questions	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈
Q ₂₆	2	2	4	1	1	1	0	0
Q ₂₇	0	0	0	0	0	0	0	0
Q ₂₈	1	1	2	2	2	1	1	1
Q ₂₉	2	2	4	3	3	2	2	2
Q ₃₀	2	2	2	0	2	0	0	0

Appendix B₉ : Interpretation of P-Value of D-Level for Short Questions

Q.N.	U _R	L _R	$P = (U_R + L_R) / U_N + L_N$	Analysis	$D = \frac{U_R}{U_N} - \frac{L_R}{L_n}$	Analysis
26	8	0	50	Easier	1	V-Good
27	8	0	50	Good	1	V-Good
28	8	4	75	Good	0.5	V-Good
29	8	8	100	Easier	0	Poor
30	8	4	75	Good	0.5	V-Good

Appendix B₁₀ : Item Analysis Table Upper Topmost Marks of the Interview Questions

Students Questions	U ₁	U ₂	U ₃	U ₄	U ₅	U ₆	U ₇	U ₈
Q ₃₀	0	1	1	0	1	0	0	1
Q ₃₁	1	1	1	0	0	0	0	1
Q ₃₂	1	1	1	1	1	1	1	1
Q ₃₃	1	1	1	1	1	1	1	0
Q ₃₄	0	1	0	0	0	0	1	0
Q ₃₅	1	1	0	0	0	0	0	0

Appendix B₁₁ : Item Analysis Table Lower Marks of the Interview Questions

Students Questions	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈
Q ₃₀	0	1	1	0	1	0	0	1
Q ₃₁	1	1	1	0	0	0	0	1
Q ₃₂	1	1	1	1	1	1	1	1
Q ₃₃	1	1	1	1	1	1	1	0
Q ₃₄	0	1	0	0	0	0	1	0
Q ₃₅	1	1	0	0	0	0	0	0

Appendix B₁₂ : Interpretation of P-Value of D-Level for Interview Questions

Q.N.	U _R	L _R	P = (U _R + L _R) / (U _N + L _N)	Analysis		D = $\frac{U_R}{U_N} - \frac{L_R}{L_n}$	Analysis
30	7	3	3	62	Good	0.5	Poor
31	5	4	4	56	Good	0.1	Poor
32	8	8	8	100	Easier	0	Poor
33	8	7	7	97	Easier	0.12	Poor
34	6	3	3	56	Good	0.4	Poor
35	6	6	6	100	Good	0	V-Good

Appendix C

Formula used to Analyze Data in Study

1. Mean (\bar{x}) = $\frac{\sum x}{N}$

2. Standard Deviation (σ) = $\frac{\sum d^2}{N} \left(\frac{\sum d}{N} \right)$ where X-A

3. Variance (σ^2) = $\frac{\sum (X - \bar{X})^2}{N}$

4. Calculated value of t = $\frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_p^2 \left(\frac{1}{N_1} + \frac{1}{N_2} \right)}}$ Tabulated Value of t = $t_{\alpha, (n_1 + n_2 - 2)}$

- 2)

$$S_p^2 = \frac{\sum X_1^2 + \sum X_2^2}{(N_2 - 1) + (N_2 - 1)} = \frac{(N_1 - 1)\sigma_1^2 + (N_2 - 1)\sigma_2^2}{N_1 + N_2 - 2}$$

Where $x_1 = X_1 - \bar{X}_1$ and $x_2 = X_2 - \bar{X}_2$

Point Biserial Correlation Coefficient : $r_{pbis} = \frac{Y_1 - Y}{S_Y} \sqrt{\frac{P_x}{1 - P_x}}$

Where,

Y = Mean achievement score of total students

Y_1 = Mean achievement score of the students who gave the true answer on each question

S_Y = Standard deviation of total students

P_x = Ratio of Students giving true answer in each questions.

5. $P = \frac{U_R + L_R}{U_N + L_N} \times 100\%$ where

P = Item difficulty level

U_R = No of student in upper 27% who answer correctly

L_R = No. of student in lower 27% who answer correctly

U_N = Total number of student in upper 27%

L_N = Total number of student in lower 27%

$$6. \quad D = \frac{U_R}{U_N} - \frac{L_R}{L_N} \text{ where,}$$

D = Item discrimination index

U_R = No. of student in upper 27% who answer correctly

L_R = No. of student in Lower 27% who answer correctly

U_N = Total number of student in upper 27%

L_N = Total number of student in lower 27%

Appendix G: Lesson Models

Subject: Basic Mathematics

Time Duration: 12 Period of 45 minutes each

Unit: Matrix and Determinants

Grade: XI

Theme/Concept Meaning and Concept of Cont. Contextualized teaching method in the area of matrix and determinants

Long Term Objectives

1. To develop the positive attitude of teachers toward their students and mathematics.
2. To provide foundation for the students in matrix and Determinant
3. To provide the sufficient background in matrix and its determinants for the statements.
4. To develop the skill of representing matrix and determinants, finding sum, differences and products of matrices.

Short-term objectives

1. To discuss the meaning of matrix and its determinants.
2. To demonstrate the concept of matrix and its determine as for as practically.
3. To contextualize the meaning of matrix and determinant
4. Generalize the problems of matrices on the corresponding field.
5. To make able to notation of matrix and its elements in their corresponding field.
6. To make familiar on matrix notation, order and its types.

7. To find the sum, difference, multiplication of a matrix by a scalar and transpose of matrix and algebraic properties of matrix addition.
8. To investigate the Inverse of matrix adjoints of matrix and Determinants of matrix.

Materials

Flash Cards, Pencils, paper, Scale, Compass, text book, Cracker, Calendar, Graph Chart, Camera, Chit paper etc.

Major Activities

1. Brain Strumming
2. Graphing activities
3. Bridging activities
4. Questioning activities
5. Engaging activities
6. Reflection activities

Activities in Detail

First Day's Activities

Date: 2070/07/10

Brain Storming Activities

I will enter the class with a smiley face and call one of the interested students to count the number of the students in the class and classify them in a two-way table with headings boys, girls, wearing tie or not wearing tie. Probably they will prepare the table in horizontal or vertical arrangement.

	Number of Boys	Number of
Girls		
Wearing tie	15	17
Not wearing tie	7	5

I will ask some questions related to this table

- i. How many students in your class?
- ii. How many wear tie? Or not wear tie?
- iii. What does the first and second horizontal line represent?
- iv. What does the first and second vertical line represent?
- v. How many boys and girls wearing tie separately?

Groping

I will divide the students into six groups based on their abilities in mathematics to form heterogeneous groups. They will sit together group wise A, B, C, D, E and F each group contains 5 students.

Bridging

I will try to bridge then show a calendar asked some questions like Saturday in a month the Sunday and Saturday in the calendar in brain storming activities.

- i. how many Sunday and Saturday in this month?
- ii. how do you find it.

Questioning

I will provide chance to ask question for each group among themselves so that they will be independent on their group work and they can ask the question to me to clarify their subject matter.

Engaging

I will give the topics, matrix its order and notation to all groups to discuss and to find the more convenient method to instruct and present. They will get chance to read text book to get the idea about notations, rows, columns order of matrix. I will give them necessary and available teaching material for the contextual understanding. After the discussion group will presents.

Group A: What is matrix?

Group B: How matrixes and its members are symbolized?

Group C: Row of matrix

Group D: Column of matrix

Group E: On order of matrices.

Group F: Members of matrix

Each group will be present on the basis of their finding on text book. I will help them if necessary

Reflection

Student will be reflecting the class activities in a short note so that they will get the strong foundation for conceptual understanding they will reflect their understanding in home work also.

Assessment

I will watch their participation in group work. I will make like question for group wise or individually also

2nd Day activities:

Date: 2070/07/11

Discussion with in Group and Discussion

Connecting the last day activities give a work out

Example in class

If $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 4 & -1 \\ -2 & 5 & 6 \end{pmatrix}$ Find i. the order of A

ii. The elements $a_{12}, a_{21}, a_{23}, a_{33}, a_{31}$

iii. i, j for $a_{ij} = 5$

Some special types of matrixes

(Rectangular, column, square, null, Diagonal, scale, Identify, Triangular)

Major Activities

To storm the brain of students towards the lesson I will ask the question about the types of vehicles like how many types of vehicles do you see and what are they?

Grouping Activities

I will divide the students in six groups. A, B, C, D, E, F each group contains 2 students.

Bridging Activities

I will summarize the yesterday's class within 25 minutes and ask them to differentiate between the following matrices.

- a. A and B b. A and C c. A and D d. B and D
e. C and D f. A and E

Can present their findings towards the class

Questioning Activities

I will ask the following question to the student

- a. How many rows and columns in the matrices A, B, C and D?
- b. What are their geometrical shapes?

I will let them to ask questions

Engaging Activities

I will post chart writing different types of matrices with their definition and examples and let them time to discuss about the different types of matrices and ask to prepare similar chart paper to present by kipping different examples of matrices. To save the time and makes the class more effective I will give the topic rectangular matrices and row matrices. For group and sub matrix they will take the concept by reading the chart paper pasted in front of the class and I also will be there to help them. I will give 15 minutes time to discuss and to prepare chart paper. After preparing the chart each group will get 2 minutes time to present their work towards the class.

Reflection Activities

They will reflect the class activities by doing the following homework.

Homework

Study the give matrices and answer the following questions with reason.

$$A = \begin{bmatrix} 3 & 4 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 & 5 \\ -1 & 2 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad G = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad I = \begin{bmatrix} a & b & c \\ o & d & e \\ o & o & f \end{bmatrix} \quad J = [1 \ 3 \ 4]$$

$$K = \begin{bmatrix} p & o & o \\ o & q & o \\ o & o & r \end{bmatrix} \quad L = \begin{bmatrix} p & 0 & 0 \\ q & r & 0 \\ s & t & u \end{bmatrix}$$

- a. Which of the above matrices are rectangular matrices?
- b. Which of the above matrices are row matrices?
- c. Which of the above matrices are column matrices?
- d. Which of the above matrices are diagonal matrices?
- e. Which of the above matrices is square matrix?
- f. Which of the above matrices is scalar matrix?
- g. Which of the above matrices is null matrix?
- h. Which of the above matrices is identity matrix?
- i. Which of the above matrices is triangular matrix?

Third Day Activities**Date: 2073-07-12****(Contextual Application)****(Addition and Difference of****Matrix)**

To teach about the additions and difference of the matrices I will do the following activities. I will make writing the following information.

A man has two telephone sets matrix A summaries the total number of telephone calls for one week. Row 1 represents trunk-calls, Row 2 represents local calls, column 2 represents the first telephone set and the column 1 represents the second telephone set.

Similarly the matrix B represents the telephone calls for the second week.

Telephone calls for the first week are as follows:

	Set I	Set II
Trunk Calls	20	24
Local Calls	200	198

Telephone calls the second week drew as follows:

	Set I	Set II
Trunk Calls	30	36
Local Calls	200	198

$$\therefore A = \begin{bmatrix} 20 & 24 \\ 200 & 198 \end{bmatrix}, \quad B = \begin{bmatrix} 30 & 36 \\ 200 & 198 \end{bmatrix}$$

How many local and trunk calls where phoned in each set during the two weeks period?

To find the answer we have to add the corresponding elements of the two matrices as follow:

$$A + B = \begin{bmatrix} 20 & 24 \\ 200 & 198 \end{bmatrix} + \begin{bmatrix} 30 & 36 \\ 200 & 198 \end{bmatrix}$$

$$= \begin{bmatrix} 20+30 & 24+36 \\ 200+200 & 198+198 \end{bmatrix}$$

$$= \begin{bmatrix} 50 & 60 \\ 400 & 396 \end{bmatrix}$$

Similarly, other example for the difference of matrix

To introduce subtraction I will raise issue of News paper available and sold in a particular day in two shops in the following matrices matrix A represent the news paper available in the shop A and B represents the sold item and C represents the remaining newspaper at the end of the day.

		Shop I	Shop II
A =	Gorkhapatra	40	60
	Anpurna Post	25	51
		Shop I	Shop II
B =	Gorkhapatra	32	62
	Annapurna Post	23	492

Write the above information in matrix from A and B. Find the matrix C as mention above

$$A = \begin{bmatrix} 90 & 75 \\ 60 & 51 \end{bmatrix}, B = \begin{bmatrix} 32 & 62 \\ 23 & 59 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 40 & 75 \\ 60 & 51 \end{bmatrix} - \begin{bmatrix} 32 & 62 \\ 23 & 59 \end{bmatrix}$$

$$= \begin{bmatrix} 90-32 & 75-62 \\ 60-23 & 51-59 \end{bmatrix} = \begin{bmatrix} 58 & 13 \\ 37 & -8 \end{bmatrix} = C$$

I hope that above two examples will be sufficient to the edition and difference of two matrixes.

I will give them some work for some matrices under addition and difference or subtraction of them.

4th Day Activities: Problem Solving**Date: 2070/07/13****Brain Storming Activities:**

It is continuous from 3rd day activities. On that class I will make the students engage on solving the following problem they solve these problems using algebraic properties

- i. Commutative ($A+B = B+A$)
- ii. Associative [$(A+B)+C = A +(B+C)$]
- iii. Existence of identity element ($A+0 = 0 +A = A$)
- iv. Existence of additive inverse [$A+ (-A) = 0$]
- v. Scalar multiplication $C (A+B) = CA +CB$
- vi. Associative law of scalar [$C (KA) = (CK)A$]

The above properties are verified by following examples:

Bridging Activities

$$\text{Let } A = \begin{pmatrix} 2 & 3 & -4 \\ 3 & 6 & 5 \end{pmatrix}, B = \begin{pmatrix} 0 & -1 & 2 \\ 4 & 2 & 3 \end{pmatrix}, C = \begin{pmatrix} 3 & 6 & 5 \\ -2 & 3 & -4 \end{pmatrix}$$

Then, Participial group wise:

For Group (A) Prove it $A + B = B+A$

Group (B) (F), Prove it $(A+B) + C = A + (B+C)$

Group (C), Prove it $(A+ 0) = 0+A = A$

$$\text{Where } O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Group (D) Prove it: $C (A+B) = C (A) + C (B)$

Where $C = 5$

After the birding activities after discussion in group then after collaborated in whole class group wise

Engaging Activities:

I will make the students engage on solving the following problem they solve these problems in groups they could take help with me,

I. If $A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix}$ & $B = \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$ find the matrix $AB-3I$, where I is the unit

matrix of order 2.

II. If $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, shows that $A^3 - 3A - 2I = 0$

III. Find the sum $\begin{bmatrix} -2x & y \\ 2y & 4 \end{bmatrix} + \begin{bmatrix} 2x & -y \\ -2y & -y \end{bmatrix}$

IV. Find the difference $\begin{bmatrix} -2x & y \\ 2y & 4 \end{bmatrix} - \begin{bmatrix} 2x & -y \\ -2y & -4 \end{bmatrix}$

V. Find the value of x, y, z and if $\begin{bmatrix} 2x & y \\ 5 & -6 \end{bmatrix} + \begin{bmatrix} 8 & 4 \\ 6 & 3z \end{bmatrix} = \begin{bmatrix} 16 & -2y \\ t & 9 \end{bmatrix}$

Fifth day Activities:**Date:****2070/07/14**

To start today's activity I will prepare a chart of symmetric and skew - symmetric matrix.

Brain Storming Activities

Compare the two matrices.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 3 & 7 \\ 3 & 0 & -6 \\ 7 & -6 & 8 \end{pmatrix}$$

Here, $a_{12} = a_{21} = 3$, $a_{13} = a_{31} = 7$, $a_{23} = a_{32} = -6$

There is no restriction on elements on the principal diagonal.

Then after student will be convince the definition of symmetric matrix.

Similarly, A square matrix $A = a_{ij}$ is called a skew symmetric matrix, if its $(i, j)^{\text{th}}$ element is the negative of its $(j, i)^{\text{th}}$ element, i.e. if $a_{ij} = -a_{ji}$ for all i, j
(Note that $i = j$, then $a_{ij} = -a_{ij}$ or $2 a_{ij} = 0$ or $a_{ij} = 0$, This shows that in a skew - symmetric matrix, each element is the principal diagonal is zero.)

For example: $A = \begin{pmatrix} 0 & 5 & -7 \\ -5 & 0 & 3 \\ 7 & -3 & 0 \end{pmatrix}$

is a skew symmetric matrix.

Here, $a_{12} = -a_{21} = 5$, $a_{13} = -a_{31} = 7$, $a_{23} = -a_{32} = 3$

$a_{11} = 0$, $a_{22} = 0$ and $a_{33} = 0$

Questioning Activities

I will ask the following question to the student

- Which values are equal in symmetric matrix
- What is difference between symmetric and skew symmetric matrix.

Engaging Activities:

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 4 & 2 & 3 \end{pmatrix}$, find all possible answer to each of the followings:

- i. $a_{23} = ?$ ii. $a_{3j} = ?$ if $1 \leq j \leq 3$
- iii. If $a_{ij} = 1$ then $i = ?$ iv. if $a_{ij} = 0$ then $i = ?$
- v. if $a_{ij} = 1$ then $i = ?$, $j = ?$ and
- vi. Proved it $a_{ij} = 2a_{ji}$

Student will be solving continuously in group A for (i) group for (ii) respectively.

Home work

1. Construct a 3×3 matrix whose elements a_{ij} are given by (i) $a_{ij} = i + 2j$ (ii) $a_{ij} = 3j - 2j$ (iii) $a_{ij} = 2ij$ (iv) $a_{ij} = i$
2. **Write down**
 - i. A row matrix containing 9 elements
 - ii. A column matrix containing 3 elements
 - iii. A scalar matrix of order 3
 - iv. An upper triangle matrix of order 3
 - v. An lower triangle matrix of order 2.
 - vi. A skew -symmetric matrix of order 2.

Sixth day Activities**Determinant****Date: 2070/07/15****Definition Activities:**

I will divide the students in to five divide one maker pen and one post card for each group. Write the definition of determinant.

Determinant of a 1×1 matrix $A = (a_{21})$ is defined to be the number a_{11} .

For example : $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

The numbers a_{11} , a_{12} , a_{21} , a_{22} are the elements of the determinant. Since it has two rows and the columns, it is called the determinant of order.

Brain Storming Activities

I will summarize the yesterday's class within 3 minutes and ask them to find the following matrices determinants.

$$\begin{array}{lll} \text{i.} & A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} & \text{(ii) } B = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix} & \text{(iii) } C = \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix} \\ \text{iv.} & D = \begin{bmatrix} 8 & 7 \\ 3 & 2 \end{bmatrix} & \text{(iv) } E = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} & \text{(vi) } F = \begin{bmatrix} 3 & 0 \\ 4 & 9 \end{bmatrix} \text{ each} \end{array}$$

group A, B, C, D, E and F respectively.

Discussion Activities

I will discuss about order 3×3 matrix determinant of 2×2 matrix formed by omitting the i^{th} row and j column of A is called minor of the element a_{ij} and it

is denoted by number m_{ij} , where is $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Such that : $M_{11} = \text{Minor of } a_{11} = \text{Minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$$M_{12} = \text{Minor of } a_{12} = \begin{vmatrix} a_{22} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$M_{13} = \text{Minor of } a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Hence : The co factor A_{ij} i^{th} row j^{th} column element a_{ij} of the 3×3 matrix. A is the number $A_{ij} = (-1)^{i+j} M_{ij}$, where $i = 1, 2, 3$ and $j = 1, 2, 3$

Engaging Activities

$$\text{Let } A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

So, $a_{11} = 1$, $a_{12} = 2$, $a_{13} = -3$

$a_{21} = 2$, $a_{22} = 0$, $a_{23} = 4$

$a_{31} = 3$, $a_{32} = 2$, $a_{33} = 1$

Then I will give the 15 minutes time to discussion and find the minor's of matrix of M_{12} , M_{23} , M_{21} , M_{22} , M_{23} , M_{31} , M_{32} , M_{33} are respectively given the group wise after left also solve which one group are first second and third respectively.

After that I will prepare the matrix $|A| = [\quad]$.

Homework

- Without expanding the determinants show that that value of each of the following determinants is zero.

$$(i) \begin{vmatrix} 6 & 1 & 9 \\ 2 & 4 & 7 \\ 18 & 3 & 24 \end{vmatrix}$$

$$(ii) \begin{vmatrix} x & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -2 & 0 \end{vmatrix}$$

$$(iii) \begin{vmatrix} x & 3 & 3 \\ 2 & 3 & x \\ 2 & 3 & 3 \end{vmatrix} = 0$$

7th day Activities: Problem Solving Activities Date: 2070/07/18

I will give the question for discussion after hints such that,

Without expanding the determinant show that,

$$\begin{vmatrix} b-c & b+c & b \\ c-a & c+a & c \\ a-b & a+b & a \end{vmatrix} = 0$$

Hints

$$\begin{vmatrix} 2b & b+c & b \\ 2c & c+a & c \\ 2a & a+b & a \end{vmatrix} \text{ by adding the column 2 to the column 1. i.e. by applying } c_1 + c_2$$

$$= 2 \begin{vmatrix} b & b+c & b \\ c & c+a & c \\ a & a+b & a \end{vmatrix} \text{ by taking 2 common from } C_1$$

$$= 2 \times 0 \text{ as } c_1 = c_3$$

$$= 0$$

Questioning activities

$$\text{Show that } \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-z)(z-x)(y-z)$$

Bridging Activities

I will discuss about the value of third order determinant can also be evaluated by the rule of Sarrus. Such that

$$\text{Let } |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{vmatrix}$$

$$= (a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}) - (a_{31} a_{22} a_{13} + a_{32} a_{23} a_{11} + a_{33} a_{21} a_{12})$$

Then after give the exercise such that: find the value of $\det. A =$

$$\begin{vmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & 2 & 1 \end{vmatrix} \text{ by the rule of Sarrus.}$$

Home work

1. Without expanding show that $\begin{vmatrix} 30 & 2 & 5 \\ 18 & 4 & 3 \\ 6 & 8 & 1 \end{vmatrix} = 0$

2. Find the value of $\det. A = \begin{vmatrix} 3 & 4 & 5 \\ 15 & 21 & 26 \\ 21 & 29 & 34 \end{vmatrix}$ by the rule of sarrus.

8th day's Activities (Inverse of a Matrix)**Date:****2070/07/19**

To teach about the Inverse of Matrix I will do the following activities:

$$\text{Consider the matrixes } AB = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\text{and } BA = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Hence, If A and B are square matrices such that $AB = BA = I$ then B is called the inverse of A and is written as $A^{-1} C = B$. Similarly A is called the inverse of B and written as $B^{-1} = A$.

For Example:

$$\text{let } A = \begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix} \text{ then } |A| = \begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} = 20$$

$$A_{11} = \text{co factor of } a_{11} = 6$$

$$A_{12} = \text{cofactor of } a_{12} = 1$$

$$A_{21} = \text{cofactor of } a_{21} = -2$$

$$A_{22} = \text{Cofactor of } a_{22} = 3$$

$$\text{Matrix of cofactors} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}$$

$$\text{Adj } A = \text{transpose of } \begin{pmatrix} 6 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix} \text{ and}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{20} \begin{pmatrix} 6 & -2 \\ 1 & 3 \end{pmatrix}$$

Engaging Activities

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then find the possible answers of transpose of matrix of A.

Similarly $B = \begin{bmatrix} 4 & -5 \\ 3 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 8 & 9 \\ 3 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$ $E = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$ and $F = \begin{bmatrix} 3 & 3 \\ 2 & 1 \end{bmatrix}$

find transpose matrix of B,C, D, E and F respectively group wise.

Question Activities

I will give following question there is need the help I will give the hints.

1. Prove that the two matrices $\begin{pmatrix} -3 & -2 \\ 5 & 3 \end{pmatrix}$ and $\begin{pmatrix} 3 & 2 \\ -5 & -3 \end{pmatrix}$ are the inverse of each other.
2. If $A = \begin{pmatrix} 7 & -3 \\ 6 & 2 \end{pmatrix}$ prove that $A^{-1} = \frac{1}{32} \begin{pmatrix} 2 & 3 \\ -6 & 7 \end{pmatrix}$ then after discuss in group.

Homework

- i. If $A = \begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}$ Verify that $A \cdot (\text{adj}_A) = (\text{adj}_A) \cdot A$.
- ii. If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 0 \\ -5 & 2 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1} A^{-1}$.

9th Day Activities**Date:****2070/07/20****Inverse of Matrix (3×3 matrix)**

Connecting the last day activities I will respect the teams and terms and terminology which was each and distribute work question number respectively A,B, C, D, E and F.

I. What is called inverse matrix?

II. If $AB = BA = I$ is called inverse matrix.

III. If matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ then find adjoin of matrix A.

IV. If matrix $M = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$ then find the transpose of matrix M.

V. Find the co-factor of matrix $M = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

VI. Can you find the inverse of $\begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$

They discuss and solve these problems in group. I will help to omit their difficulty. After the compulsion of their group work each student at a same place.

Problems Section

I will give the deft concept of matrix

i.e. $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ engaging the all group and help the conceptual teaching

such that,

$$\text{Let, } A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ - & -2 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

then we have, $a_{11} = 1, a_{12} = 2, a_{13} = 2$

$$a_{21} = -1, a_{22} = 3, a_{23} = 0$$

$$\text{and } a_{31} = 0, a_{32} = -2, a_{33} = 1$$

So their co factors are

$$A_{11} = \text{cofactor of } a_{11} = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix} = 3$$

$$A_{12} = \text{cofactor of } a_{12} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = 1$$

Similarly give the assignment group wise $a_{12}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$ respectively.

Then after by the definition

$$\text{Adj. } A = \text{transpose of } \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

Hence student group find the 3×3 matrix

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} \text{ inverse matrix } A^{-1} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$

Homework

a. Find inverse of matrix

$$\text{i. } C = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\text{ii. } B = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 0 & 3 & -1 \end{pmatrix} \quad \text{iv. } M = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & 4 \end{bmatrix}$$

$$\text{iii. } A = \begin{pmatrix} -3 & 2 & 6 \\ -1 & -1 & -3 \\ 2 & -3 & -4 \end{pmatrix} \quad \text{v. } N = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & -4 \\ -1 & 3 & -2 \end{pmatrix}$$

10th day activities**Date:****2070/07/21****(Matrix Multiplication****Brain Storming Activities**

I will enter the class with a smiley face and call one of the interested students to count the number of the students in the class after linking the summaries of the yesterday class activities through an interested student then after I will give the two matrices with some questions.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Find the order of matrix A and B, how many members of matrix A and B. which types of matrix A and B and so on.

Bridging

I will try to bridge the student with an example of matrix multiplication

$$\text{Let } A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \text{ and } B = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \text{ then } A \cdot B = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 0 & aq+bs \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ cp+dr & 0 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & cq+ds \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ ep+fr & 0 \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & eq+ts \end{pmatrix}$$

The entire product AB is written as

$$AB = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & oq+bs \\ cp+dr & cq+ds \\ ep+fr & eq+fs \end{pmatrix}$$

So, the product AB is a matrix of order 3×2 , if the orders of A and B are 3×2 and 2×2 respectively

Questioning Activities

If $A = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$, $B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \end{pmatrix}$ and $c = \begin{pmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 0 \end{pmatrix}$ find if possible AB, BC, AC,

AA, BA, BC respectively group A,B,C,D,E and F.

Home work

Let $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ find the multiply of

1. AB 2. BC 3. AC
2. Let A, B and C be the matrix given in above then proved that: $A(B+C) = AB + AC$.

Eleventh Day Activities**Date:****2070/07/22****Group-wise problem solving properties of transport**

In this class I will follow the problem solving strategy to conduct the class for the problem solving strategies. I will divide the students in four homogenous groups of heterogeneous students each group containing students. I will give them Roll No. as 1, 2, 3, 4, 5, 6 will provide the work sheet for each group as mentioned below.

Learning Activities

The new matrix obtained from a given matrix A by interchanging its rows and columns is called the transpose of A. It is denoted by A' or A^T .

Properties of Transpose

- i. $(A')' = A$ ii. $(A+B)' = A'+B'$ iii. $(cA)' = cA'$, c is constant
 iv. $(AB)' = B'A'$

The following examples verify the properties:

$$\text{Let } A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 3 \\ -1 & 2 \\ 0 & -3 \end{pmatrix} \text{ and } c = 3.$$

$$\text{Then } A' = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} B' = \begin{pmatrix} 0 & -1 & 0 \\ 3 & 2 & -3 \end{pmatrix}$$

$$\text{i. } (A')' = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{pmatrix} = A$$

$$\text{ii. } A+B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ -1 & 2 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ -1 & 1 \\ 3 & -2 \end{pmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} 1 & -1 & 3 \\ 5 & 1 & -2 \end{bmatrix} \text{ and } A' + B' = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 \\ 3 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 3 \\ 6 & -3 & 3 \end{pmatrix}$$

$$\text{iii. } CA = 3 \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 0 & -3 \\ 9 & 3 \end{pmatrix} \therefore (CA)' = \begin{pmatrix} 3 & 0 & 9 \\ 6 & -3 & 3 \end{pmatrix}$$

$$\text{iv. } B'A' = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 0 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 6 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 2 & 6 \end{pmatrix}$$

$$\text{So that } (AB)' = \begin{pmatrix} 1 & 2 \\ -2 & 6 \end{pmatrix} = B'A'$$

I will give to verify the above properties of the matrixes by taking $A =$

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} \text{ and } B = \begin{pmatrix} -1 & -4 & 4 \\ 7 & 6 & 3 \end{pmatrix} \text{ } K = 2 \text{ and } r = 3 \text{ for group A.}$$

Group B will verify the same property by taking

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \text{ and } r = -3, K = 2.$$

$$\text{Group C will verify this property } A = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ -3 \\ 5 \end{bmatrix} \text{ } k = \frac{1}{2} \text{ } r = \frac{-2}{3}$$

$$\text{Group D will verify the same property by taking } A = \begin{bmatrix} -1 & 6 & 6 \\ -2 & 3 & -2 \\ 3 & 2 & 1 \end{bmatrix}, B =$$

$$\begin{bmatrix} -1 & 6 & 9 \\ -2 & 3 & -2 \\ 3 & 2 & 1 \end{bmatrix} \text{ } K = 1 \text{ and } r = -1$$

After the completion of their group work I will rearrange the students roll number wise like roll number of each group will set together and so on.

Then they will share their knowing to each other.

12th Day's Activities

Date:

2070/07/24

In this class I will follows the problem solving strategy to conduct the cloths for the problem solving strategies. I will divide the students is four homogenous group of heterogeneous students each group contain 5 students.

Learning Test

Properties of matrix multiplication

- i. $AB \neq BA$ ii. $A(BC) = (AB)C$
 iii. $AI = IA = A$ iv. $A(B+C) = AB + AC$ v. $A.A = A^2$

Engagement Activities

The above properties are verified by the following examples.

Let $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$. I will give to verify the above properties of

the matrixes by taking above value of A and B for group A. That is $AB = BA$.

Group B will verify the before properties $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. i.e. $AI =$

$IA = I$

Group C will verify the third properties $A (BC) = (AB)C$ where $A =$

$\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

Group D' will verify the fourth properties ($A(B+C) = AB + AC$) distribute properties using the value of A,B, C of above.

Group E will verify the fifth properties of square matrix. Where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then

$$A \times A = A^2 = ?$$

and lastly Group F will prove that $A \times B = B \times A$ of along where $A =$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 2 \\ 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Homework

If $A = \begin{pmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -5 \\ 0 & 4 \\ -5 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 5 & -3 \\ 0 & 2 & 4 \end{pmatrix}$ compute the following

if possible

- i. AB ii. BC iii. CA iv. BA v. CB vi. AC

Appendix H: Time Schedule for Traditional Group

S.N.	Topic	Period
1.	Matrix and its notation	1
2.	Types of Matrix	1
3.	Addition and subtraction of Matrix	1
4.	Algebra of Matrix	1
5.	Multiplication Matrix	2
6.	Adjoin and inverse of Matrix	1
7.	Determinant of square matrix	2
8	Determinant of square matrix (up to order up to order 3×3)	1
9.	Minors and cofactors	1
10	Properties of Determinants	1
Total Period		12

Appendix Interview Questions

- How many elements of matrix order of 6×9 ?
- Say the example of Identity matrix of 3×3 order.
- What is determinant of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- Matrix of $(A)^T$ of Transpose = ? Where $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$
- What is adjacent of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $A^{-1} = ?$ Can you say the formula please.

Appendix I: Making Scheme of Achievement Test 2070

Objectives Questions:

Q.N.	a	b	c	d
1	√			
2			√	
3	√			
4			√	
5	√			
6		√		
7				√
8				√
9				√
10			√	
11				√
12			√	
13	√			
14				√
15			√	

Appendix J: Achievement score of the post -test

Student	Experiment	Control
1	30	26
2	28	24
3	24	17
4	26	14
5	27	16
6	29	18
7	27	14
8	20	8

Achievement score of the pre-test of the participate student

Student	Experiment	Control
1	6	7
2	6	6
3	3	6
4	4	3
5	4	2
6	3	3
8	2	2
8	1	1

Achievement score of Boys and Girls in collaborative groups

Students	1	2	3	4	5
Boys	30	29	23	27	18
Girls	28	24	26	20	27

Achievement Score of Traditional Boys and Girls Groups

Students	1	2	3	4	5
Boys	26	17	16	9	14
Girls	24	18	14	15	8

Achievement Score of Collaborate and Traditional Boys Groups

Students	1	2	3	4	5
Experimental	30	29	23	27	18
Control	26	17	16	9	14

Achievement Score of Collaborative Girls and Traditional Girls Groups

Students	1	2	3	4	5
Experimental	28	24	26	20	27
Control	24	18	14	15	8

Appendix F

Post-Test

Objective Questions

(8×1=8)

1. Which of the following is not a matrix?

- a. $\begin{bmatrix} 1 & 4 & 2 \\ 3 & 6 & 5 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ c. [-5] d. $\begin{bmatrix} 1 \\ \frac{2}{3} & 2 \end{bmatrix}$

2. What types of matrix is $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

- a. Scalar b. Diagonal c. Square d. All of these

3. Which is not the example of Identify matrix.

- a. $I = (1)$ b. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ d.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Which of the following two matrices are compatible under addition?

- a. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & -8 \end{bmatrix}$ b. $\begin{bmatrix} x & y \\ z & a \end{bmatrix}$ and $\begin{bmatrix} x \\ z \end{bmatrix}$

- c. $\begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$ and [1, 4] d. $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$

5. $A + B = B + A$ is called

- a. associative b. identify c. Commutative
d. additive is algebraic properties.

6. If $A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ then $5A$ is equal to

- a. $\begin{bmatrix} 10 & 15 \\ 25 & 25 \end{bmatrix}$ b. $\begin{bmatrix} 15 & 10 \\ 25 & 20 \end{bmatrix}$ c. $\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}$ d.

None of them

7. $[30, 55, 28]$ of $\begin{bmatrix} 20 \\ 2 \\ 3 \end{bmatrix}$ is equal to

- a. 794 b. 974 c. 497 d. 749

8. If matrix $A = [-5]$ then def. of A is equal to

- a. -5 b. 5 c. 25 d. -25

Short Questions

(5×2=10)

9. Find the order of all possible matrices which has 6 elements.

10. If $A = \begin{bmatrix} x & 5 \\ 3 & y \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 5 & -2 \end{bmatrix}$ such that $A^T = B$ then find the value of x

and y .

11. If $A = \begin{bmatrix} 3 & 5 & 4 \\ 5 & 6 & -1 \end{bmatrix}$ verify that $(A')' = A$

12. If $D = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$ prove that $BB' = B'B = I$

13. Find the inverse of $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix}$

Long Questions

(3×4 = 12)

14. Find the Value of x , y , a and b . If $\begin{bmatrix} x+y & a+b \\ a-b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & -5 \end{bmatrix}$

15. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 5x + 7$ find the value of $F(A)$.

$$16. \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b-c)$$

Appendix E

Pre-Test

Objective Questions (8×1=8)

1. What is called order of Matrix?
 - a. Number of row \times column
 - b. number of column row
 - c. total number of elements
 - d. a and b

2. What is the value of a_{21} in the matrix

$$A = \begin{bmatrix} 2 & 4 & 5 \\ -2 & 3 & -4 \end{bmatrix}$$
 ? Which a_{ij} is element of i^{th} rows and j^{th} column in the matrix A.
 - a. 4
 - b. 3
 - c. 5
 - d. -4

3. From Matrix $\begin{bmatrix} 1 & 6 \\ 2 & 4 \end{bmatrix}$ $a_{ij} = 6$ what is the sum of i and j?
 - a. 2
 - b. 3
 - c. 4
 - d. 1

4. What is the value of 'a' if $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & x \\ 4 & 2 \end{bmatrix}$
 - a. 7
 - b. 4
 - c. 1
 - d. 3

5. If $\begin{bmatrix} 2x+3 & 0 \\ y & 8 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ x+1 & 8 \end{bmatrix}$ then what is the value of x?
 - a. -1
 - b. 1
 - c. 4
 - d. -4

6. Which one of the following is null matrix.
 - a. $A = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$
 - b. $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 - c. $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 - d. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. Which is the example of Unit matrix?

a. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ d. $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

8. For the matrix equation $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$ the values of x and y are respectively.

a. 3 and 3 b. 2 and 4 c. 3 and 0 d. 11 and 1B

Short Questions (5×2 =10)

9. Write down one example of 2×3 matrix.

10. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ verify that $A + B = B + A$

11. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ and $K = 2$ verify that $K(A+B) = KA + KB$

12. Find the Sum of matrices $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ -8 & 2 \end{bmatrix}$.

13. Simplify $\begin{bmatrix} 2 & -4 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 5 & -5 \\ 7 & 6 \end{bmatrix}$

Long Questions (3×4=12)

14. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 4 & 2 & 3 \end{bmatrix}$ find all possible answers to each of the followings.

a. $a_{23} = ?$

b. $a_{3j} = ?$ if $1 \leq j \leq 3$

c. if $a_{i2} = 1$ then $i = ?$

d. If $a_{ij} = 1$ then $i = 2j = ?$

15. Find the values of x, y a and b if $\begin{bmatrix} x+y & a+b \\ a-b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & -5 \end{bmatrix}$

16. Find the value of matrix $X = \begin{pmatrix} x \\ y \end{pmatrix}$ where $\begin{bmatrix} 4 & -3 \\ 3 & 7 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 11 \\ -1 \end{bmatrix}$

Appendix D: Item Analysis

Objective Questions

(8×1=8)

1. Which of the following is not a matrix?

- a. $\begin{bmatrix} 1 & 4 & 2 \\ 3 & 6 & 5 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ c. [-5] d. $\begin{bmatrix} 1 \\ 2 \\ 3 & 2 \end{bmatrix}$

2. What types of matrix is $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

- a. Scalar b. Diagonal c. Square d. All of these

3. Which is not the example of Identify matrix.

- a. $I = (1)$ b. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ d.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Which of the following two matrices are compatible under addition?

- a. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & -8 \end{bmatrix}$ b. $\begin{bmatrix} x & y \\ z & a \end{bmatrix}$ and $\begin{bmatrix} x \\ z \end{bmatrix}$

- c. $\begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$ and [1, 4] d. $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$

5. $A + B = B + A$ is called

- a. associative b. identify c. Commutative
d. additive is algebraic properties.

6. If $A = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$ then $5A$ is equal to

a. $\begin{bmatrix} 10 & 15 \\ 25 & 25 \end{bmatrix}$ b. $\begin{bmatrix} 15 & 10 \\ 25 & 20 \end{bmatrix}$ c. $\begin{bmatrix} 20 & 25 \\ 10 & 15 \end{bmatrix}$ d.

None of them

7. $[30, 55, 28]$ of $\begin{bmatrix} 20 \\ 2 \\ 3 \end{bmatrix}$ is equal to
 a. 794 b. 974 c. 497 d. 749
8. If matrix $A = [-5]$ then def. of A is equal to
 a. -5 b. 5 c. 25 d. -25
9. Which one is determinant is order 2
 a. $a_{11}a_{22}-a_{12}a_{21}$ b. a_{11} c. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ d. a and c
10. $|-a_{11}| = a_{11}$ is denoted oforder of determinant.
 a. 1×1 b. 2×2 c. 1×2 d. 2×1
11. If two rows (or column) of a determinant are identical then value of determinant is
 a. 0 b. 1 c. -1 d. 0 or 1
12. If $A = \begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$ then which one is the cofactor of a_{11}
 a. 6 b. 3 c. 2 d. -1
13. A square matrix A is called a non-singular matrix, If
 a. $A' = 0$ b. $|A| \neq 0$ c. a and b d.

None of them

14. The inverse of matrix A is

- a. $\frac{adjA}{|A|}$ b. $A = \frac{adjA}{|A|}$ c. $\frac{adjA}{|A|}A$ d. All

of above

15. If matrix $M = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ the transpose of matrix M^T is

- a. $\begin{pmatrix} -3 & 4 \\ 1 & -2 \end{pmatrix}$ b. $\begin{pmatrix} 3 & -4 \\ 2 & -1 \end{pmatrix}$ c. $\begin{pmatrix} -1 & 2 \\ -4 & 3 \end{pmatrix}$ d.

$$\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$$

Short Questions

(10×2 = 20)

16. Find the order of all possible matrices which has 6 elements.
17. If $A = \begin{bmatrix} x & 5 \\ 3 & y \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 5 & -2 \end{bmatrix}$ such that $A^T = B$ then find the value of x

and y.

18. If $A = \begin{bmatrix} 3 & 5 & 4 \\ 5 & 6 & -1 \end{bmatrix}$ verify that $(A')' = A$

19. If $D = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$ prove that $BB' = B'B = I$

20. Find the inverse of $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix}$

21. Define singular matrix. Text when the matrix if $A = \begin{pmatrix} 3 & 1 & 0 \\ -2 & 1 & -1 \\ -1 & 3 & 2 \end{pmatrix}$ is

singular or not

22. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ then determine the matrix A.

23. Find the determinants of A. If

a. $A = (-3)$ b. $A = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}$

24. Find the value of $\det. A = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & 2 & 1 \end{vmatrix}$ by the rule of Sarrus.

25. Prove that the following two matrices are inverse of each other.

$$A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

Long Questions

(5×4 = 20)

26. Find the Value of x, y, -a and b. If $\begin{bmatrix} x+y & a+b \\ a-b & 2x-3y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & -5 \end{bmatrix}$

27. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 5x + 7$ find the value of $F(A)$.

28. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b-c)$

29. Use Properties of determinant of show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = \begin{vmatrix} 1 & x & xz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

30. If a, b, c are non-zero and $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$ then show that $abc = 1$

Interview Questions (5×1 =5)

31. How many elements of matrix order 6×9 ?
32. If A and B are matrices then how to denoted A difference B matrix?
33. $A + (-A)$ is existence of additive inverse equal to?
34. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then say the value of adj. of A?
35. What is formula's of determinants of 2×2 order matrix where $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ a matrix is.