ALGEBRA WITH GEOGEBRA: CONNECTING ABSTRACTION TO VISUALIZATION

A

THESIS

BY

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LETTER OF CERTIFICATE

This is to certify that Mr Vijay Kumar Thapa a student of academic year 2070/1 with Campus Roll Number 539, Thesis Number 1362, Exam Roll Number 280552 (2073) and TU registration number 9-2-632-122-2009 has completed this thesis for the period prescribed by the rules and regulations of Tribhuvan University, Nepal. This thesis entitled **'Algebra with GeoGebra: Connecting Abstraction to Visualization'** has been prepared based on the results of his investigation. I, hereby recommend and forward that his thesis be submitted for the evaluation as the partial requirements to award the degree of Master of Education.

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(Head)

December, 2019

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LETTER OF APPROVAL

This thesis entitled "**Algebra with GeoGebra: Connecting Abstraction to Visualization**" submitted by Mr Vijay Kumar Thapa in partial fulfilment of the requirements for the Master's Degree in Education has been approved.

Viva-Voce Committee	**Signature**
Assoc. Prof. Laxmi Narayan Yadav (Chairman)	……………..
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RECOMMENDATION FOR ACCEPTANCE

This is to certify that Mr Vijay Kumar Thapa has completed his M. Ed. thesis entitled "**Algebra with GeoGebra: Connecting Abstraction to Visualization**" under my supervision during the period prescribed the rules and regulations of Tribhuvan University, Kirtipur, Kathmandu, Nepal. I recommended and forward his thesis to the Department of Mathematics Education to organize final viva-voce.

…………………………….

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DECLARATION

This dissertation contains no material which has been accepted for the award of another degree in any institutions. To the best of my knowledge and belief, this thesis contains no material previously published by any authors except due acknowledgement has been made.

…………………………………

Vijay Kumar Thapa

December, 2019

Poush, 2076

Defense Date: December, 2019

DEDICATION

To My Parents:

Mr. Rohit Thapa Magar and Mrs. Lila Thapa Magar

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December, 2019

Poush, 2076

ABSTRACT

The title of this research was **“Algebra with GeoGebra: Connecting Abstraction to Visualization”**. The use of technology in mathematics teaching and learning has become a popular strategy in Nepal. The primary purpose of this research was to measure the effectiveness of GeoGebra in teaching algebra making the connection between algebraic and geometric concepts. For achieving this purpose, the researcher used an experimental research method in which the researcher adopted the pre-test post-test quasi nonequivalent design. Two private secondary schools were selected from Barahathawa, Sarlahi to observe the impact of the use of GeoGebra in algebra teaching regarding students’ achievement, and students’ algebraic thinking. The researcher used achievement test in algebra, algebraic thinking test and memo writing as the main data collection tools. The internal consistency of each test was ensured by calculating Cronbach’s Alpha model with SPSS 21.0 setting 0.05 level of significance. After one month of regular treatment to the experimental group, achievement of students from both groups was recorded in terms of post-test. From the result, the study has succinctly shown that there was a significant difference in students’ achievement of experimental and control groups in post-test. More importantly, the result indicated that the use of GeoGebra in algebra teaching improve the algebraic thinking of students by connecting algebra to its geometric interpretation. Furthermore, errors of choosing correct variables in verbal problems of algebra had been reduced and students did not confuse about the using variables in algebraic problems. Thus, it is affirmed that the use of GeoGebra can be valuable to improve students algebraic thinking skills.

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ACRONYMS

ATT	=	Algebraic Thinking Test
ICT	=	Information and Communication Technology
NASA	=	National Assessment of Students Achievement
NCTM	=	National Council of Teachers of Mathematics
SD	=	Standard Deviation
SPSS	=	Statistical Package for Social Science

CHAPTER I

INTRODUCTION

Background of the Study

Algebra teaching is the all-important part in mathematics teaching at the school level because of its vertical and horizontal relationship with the higher level of education and other contents of mathematics. However, in many schools, algebra is taught at the periphery of the traditional approach in which there is no connection between algebra and geometry. In order to improve students' score in mathematics, many schools and hence mathematics teachers have introduced Technology in Geometry but not in algebra. Even though, many teachers lack the knowledge of how to properly incorporate technology in the classroom (Doering, Huffman, & Hughes, 2003) to connect various branches of mathematics. So, there has not a fruitful connection between Algebra and Geometry that hinged on the overall performance of students in mathematics.

Use of Information and Communication Technology, ICT, in mathematics is fundamental pedagogy (Clark, 1983) that helps to directly change trends of teaching and learning (Dede, 1996). There are a number of affordable ICT tools, for example, GeoGebra, MatLab, Mathematica, etc. possible to introduce in algebra classroom making the connection between Algebra and Geometry. Among these, GeoGebra is best one to facilitate explorations that promote the conjecturing process (Baki, 2005).

GeoGebra is dynamic geometry software having algebraic and geometry view of the object together. Moreover, GeoGebra is a relatively new software system that integrates possibilities of both dynamic geometry and computer algebra in one tool for mathematics education. GeoGebra has the unique ability to illuminate both algebraic and geometric concepts. It allows a closer connection between the symbolic manipulation and visualization capabilities and dynamic changeability (Hohenwarter and Fuchs, 2004). It means that GeoGebra provides a dynamic platform to teach algebra and geometry together. GeoGebra constructs geometrical figures and demonstrates the relationship between geometry and algebra (Raju, 2007). That is, GeoGebra makes possible teaching algebra regarding geometric interpretation. Mathematics teachers can use GeoGebra in the Algebra classroom to introduce the "big idea" of Linear and exponential growth patterns (Laynch-Davis and Goodson-Espy, 2010). It seems that teaching algebra through GeoGebra is a worldwide trend in mathematics education.

In the context of Nepal, national achievement of grade VIII students in algebra (28%) is impoverished, which is 7 per cent less than the national achievement in mathematics and, similarly in geometry 1 per cent less than from national achievement (NASA, 2015). However, the government of Nepal has supposed that proper use of ICT tools mathematics may be remedial for this situation by stating the ICT as the principle for curriculum development (NEF, 2007, as cited in Bist, 2017).

As a consequent, the SLC result of 2073 in mathematics has improved in those schools where ICT has introduced in mathematics teaching (Bist, 2017). It means that the use of ICT provides a mechanism for improving mathematics achievement. With its unique ability to illuminate both algebraic and geometric concepts, GeoGebra provides a mechanism to improve how mathematics is taught and learned in middle grades/secondary classrooms (Hall and Chamblee, 2013). However, GeoGebra yet not widely used in the Nepalese education system (Bist, 2017) and use of GeoGebra may not be effective pedagogy every content of mathematics, particularly Algebra. So, this study is intended to examine the effectiveness of GeoGebra in teaching Algebra connecting it to geometry in the context of Nepal.

Statement of the problem

In teaching and learning Algebra, a solution of the equation is a central theme in school level, which is directly related to geometric interpretation. Moreover, to be an effective teaching, regarding algebra at the school level, geometric interpretation of system of equations and geometric interpretation of solution of the equations should be clearly illustrated because NCTM (2000) stated that algebra and algebraic thinking are closely linked to the communication of all areas of mathematics such as geometry and statistics. However, it is known that students face the challenge regarding distinguish linear equations from nonlinear equations. Hall and Chamblee (2013) stated that GeoGebra has an outstanding mechanism to improve mathematics teaching connecting algebra and geometry.

Mathematics teachers, educators and mathematics researchers agree that use of technology is necessary at teaching and learning linear algebra. But it is a question that when, and how linear algebra teachers use technology (Herrero, 2000) because of the incredible role of technology in algebra instruction such as: provide a dynamic platform for actively exploring algebraic concepts and structures. Clark (1983) avowed that the use of ICT is the fundamental pedagogy that provides a mechanism for teaching and learning. Moreover, mathematics software provides students with a means of instantly and effortlessly performing in linear algebra, and thus free them to concentrate on what the computations mean, and when and why to perform them (Aydin, 2009).

Despite the considerable advances in the field of mathematics teaching, it is still realizing that achieving the goal for developing algebraic thinking in early grades is challenging. So that it is necessary to be something new in teaching and learning algebraic concepts that makes a clear connection between linear algebra and geometry and that refines students' algebraic ability. Therefore, the main concern of this study was to examine the following statements:

- How does achievement of students taught by GeoGebra differ from traditional approach in algebra?
- What is the students' conceptual understanding of algebraic thinking?

Objectives of the study

The primary objective of the study was to identify the effectiveness of GeoGebra in teaching linear algebra making the connection between algebraic and geometric concepts. This objective is specifically stated in the following way:

- To compare the achievement of students taught by GeoGebra and traditional approach.
- To analyse the conceptual understanding regarding algebraic thinking.

The significance of the study

This study would be advantageous to improve the mathematics achievement of students. More specifically this study has the following significance:

Mathematics Teachers. This study may provide a clear framework to the mathematics teachers to teach algebra in school level using GeoGebra that connects algebraic concepts to the geometry.

Mathematics Educators. This study may also be beneficial to mathematics educators to amend the contents of school algebra in terms of technology, particularly GeoGebra.

Mathematics Researchers. This study might make some hidden aspects as the recommendation for further researcher which is a cue to conduct new research in the field of mathematics. Finally, this study could be valuable to the curriculum developers, students and independent learners of Algebra.

The Hypothesis of the Study

The following null hypotheses were investigated based on the research questions.

- H_0: There is no significant difference between the average achievement of students taught through GeoGebra and traditional approach.
- H_1: There is a significance difference between the average achievement of students taught with GeoGebra and traditional approach.

Delimitation of the Study

Delimitation refers to the boundary of the research work. It specifies the research topic which is going to be studied. This study has the following delimitations:

- This was experimental research conducted at grade VIII regarding teaching linear algebra.
- The study was conducted on two groups of students, namely control group and experimental group, taken from two private schools of Sarlahi district of Nepal.
- The researcher used purposive sampling techniques to select the schools due to lack of availability of ICT tools in all school, however, the researcher selected the group in terms of randomness.
- Effectiveness only defines the increase in the students' test scores and improvement of algebraic thinking.

Definition of Key Words

Experiment group. The group of students who were taught algebra through GeoGebra in their classroom.

Control group. The group of students who were taught through a traditional approach.

Geometric Interpretation. Geometric interpretation refers to a graphic model of algebraic equations and their solutions.

Algebraic Thinking. Algebraic thinking refers to students' ability to recognise and analyse pattern, represent the relationships, and analyse how things change that starts with mathematical learning.

Achievement Test. An achievement test refers to a test including test items from algebra lesson only.

CHAPTER II

REVIEW OF RELATED LITERATURE

This chapter conveys the review of relevant literature, which is divided into three sections. The first section presents the review of Empirical Literature, the second section includes the review of Theoretical Literature and final section contains the Conceptual Framework.

Review of Empirical Literature

Throughout this study four key terms are in focused: Teaching Algebra, Use of GeoGebra, Geometric interpretations, and algebraic thinking, which are discussed below:

Teaching algebra is one of the all-important parts of mathematics, in which teachers need to explain the system of solutions and the way for findings its solutions. A solution of the system of equations and problems of algebra, which are abstract, should be geometrically interpreted (Narasimhan, 2013). Because geometry and algebra are central to mathematics and have been called its “two formal pillars” (Atiyah, 2001). This may be possible through introducing advanced technology, particularly GeoGebra, in Algebra classroom. GeoGebra enables teachers and students to make strong connections between geometry and algebra (Hohenwarter & Jones, 2007). Research based on introducing ICT in mathematics education has observed that positive effect of implementing GeoGebra in Algebra in the classroom on students' achievement (Isikal & Askar, 2005). Research shows that students' achievement in Algebra and Algebraic thinking are related. Narasimhan (2013) stated that GeoGebra, mainly, spreadsheet can be a powerful ally in facilitating algebraic thinking.

Algebraic thinking includes recognizing and analysing patterns, studying and representing relationships, making generalizations, and analysing how things change (Seeley, 2004). Kaput describes five forms of algebraic thinking (p. 254)

- Making generalizations from arithmetic and patterns
- Meaningful use of symbols
- Study structure in the number system
- Study patterns and functions
- Mathematical modelling integrating the first four list items

To solve problems performing algebra, students need to know about the algebraic symbols and properties. However, trying to understand abstract symbolism without foundation in thinking algebraically is likely to lead to frustration and failure because algebraic thinking can begin when students begin their study of mathematics (Seeley, 2004). Therefore, there is a still gap the how GeoGebra improves students achievement in Algebra and makes the connection between algebra for fostering students' algebraic thinking.

Despite the above mentioned vital points, the related empirical literature is reviewed as below:

Bist (2017), conducted a study entitled on "Use of GeoGebra in geometric construction" to determines the effectiveness of GeoGebra in Geometric Construction This was a quasi-experimental study conducted in Kathmandu district of Nepal. The experiment tenure was a month along. The researcher had used test items, students attitudes scale and interview as the data collection tools. The reliability of the tools was determined by calculating Cronbach's Alpha and validity was assured by expert judgement. The result of this study showed that the use of GeoGebra improved visual thinking of students in mathematics, particularly in geometry.

Hohenwarter and Jones (2007), conducted a study entitled on "Ways of linking geometry and algebra: the case of GeoGebra" to find out ways of enhancing the teaching of mathematics through enabling learners to gain stronger links between geometry and algebra. The researcher concluded a freely-available open-source software package that combines both geometry and algebra, GeoGebra has much to offer. As the same way, Edwards and Jones (2006), that utilising software like GeoGebra could inspire a change to forms of classroom problems “that need high-level thinking, and things that students may find themselves wanting to follow-up outside of regular school lessons” (p30).

Aydin (2009), conducted a study entitled "factors affecting teaching linear Algebra" to find out the factors affecting teaching and learning Linear Algebra. The result of this study underscored that the use of technology was important factors that effects are teaching and learning Linear Algebra. Similarly, Herero (2000) used computer projects in linear algebra aiming at introducing students to a new subject in linear algebra through a hands-on approach. Using linear algebra software, the projects were intended to provide motivation for new definitions, show the need for the new theorems, make conjectures, and realize the usefulness of the new theorems by applying them to solve various problems. In these projects, each student was allowed to choose an applied problem in the student’s area of interest.

Diković (2009), carried out the study entitled on "Applications GeoGebra into Teaching Some Topics of Mathematics at the College Level" to identify the opportunities and examples how GeoGebra can be used in Linear Algebra and Calculus. The result showed that GeoGebra has many possibilities to help students to get an intuitive feeling and to visualize adequate math process. The use of this software’s tools allows students to explore a wider range of function types and provides students to make the connections between symbolic and visual representations.

From, the above review of the related literature, it seems clear that mathematics education studies could not give a certain solution to overcome all the difficulties of learning and teaching algebra. Many of the work have been considered to address difficulties in teaching while others focusing on improving students' achievement. However, there are a few numbers of research that emphasis uses of GeoGebra on teaching algebra connection it with geometry for refining algebraic thinking. There is still research gap that how use of GeoGebra helps students to increase their performance in achievement test, especially in algebra and to improve their algebraic thinking together.

Theoretical Review

This study was based on constructivism approach to concept learning of mathematics. Constructivism is a learning theory found in psychology which explains how a learner might acquire knowledge and experience based on past and present state of knowledge and experience of an individual. Constructivism is not a specific pedagogy. There are two types of constructivism: cognitive constructivism and social constructivism. Cohen, Manion and Morrison (2013), stated that both constructivisms shares some common characteristics such as the view that the knowledge is constructed through reflective abstraction, through learners' cognitive structures and processing, through active and participative learning, and through the recognition that learning is not fixed and inert, but is continually developing.

Piaget's theory of constructivist learning has wide-ranging impact on learning theories and teaching methods in education and is an underlying theme of many education reform movements. Research support for constructivist teaching techniques has been mixed, with some research supporting these techniques and other research contradicting those results. Jerome Bruner is a psychologist who focused much of his research on the cognitive development of children and how it relates to education. While he has made many contributions to the field of psychology, his greatest contributions have been in the educational field.

Initially, Bruner was interested in how the mind organized and categorized information. Because his early career focused on cognitive psychology, Piaget's theories played a large role in his initial studies. Over time, however, as he began to specialize more on learning, Vygotsky and his ideas on the Zone of Proximal Development and scaffolding came to be increasingly influential to Bruner's research (Smith, 2002). Each of Bruner's stages of representation builds off of the knowledge and information learned in the previous stage, or in other words, the stage before acts as scaffolding for the next stage. The theory has come to play a huge role in mathematics education, particularly with the encouraged use of manipulatives.

Eventually, Bruner's stages of representation came to play a role in the development of the constructivist theory of learning as well (Culatta, 2012).
While Bruner has influenced education greatly, it has been most noticeable in mathematical education. The theory is useful in teaching mathematics which is primarily conceptual, as it begins with a concrete representation and progresses to a more abstract one. Initially, the use of manipulatives in the enactive stage is a great way to "hook" students, who may not be particularly interested in the topic.

Furthermore, Bruner's theory allows teachers to be able to engage all students in the learning process regardless of their cognitive level of the concept at the moment. While more advanced students may have a more well-developed symbolic system and can successfully be taught at the symbolic level, other students may need other representations of problems to grasp the material (Brahier, 2009, p. 54). In addition, by having all students go through each of the stages, it builds a foundation for which the student can fall back on if they forget or as they encounter increasingly difficult problems. For these reasons, it is essential that the teacher goes through each of the stages with the whole class; however, the time spent on each stage can and would vary depending on the student, topic.

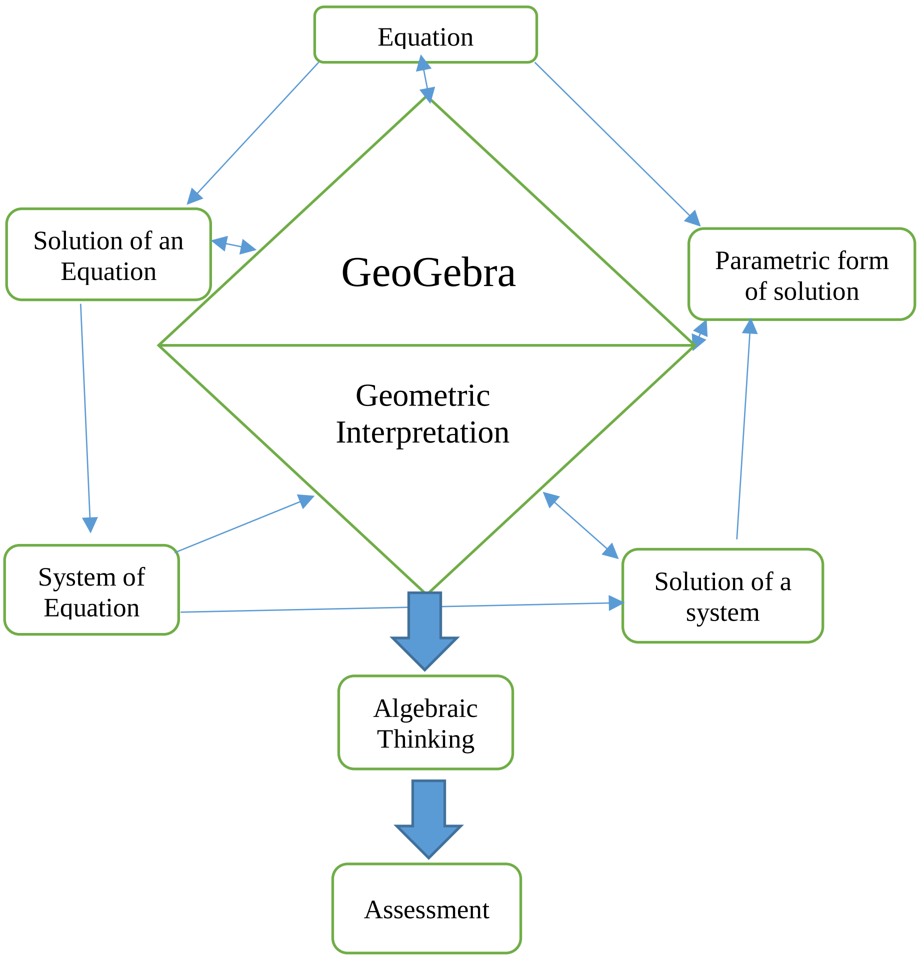
Another important part of the theory's application is the academic language. The development and use of an academic language are crucial for successfully learning the concept. This primarily takes place in transitioning from the iconic stage to the abstract, language-based, symbolic stage. "Since language is our primary means of symbolizing the world, [Bruner] attaches great importance to language in determining cognitive development"(Mcleod, 2002). The correct academic language needs to be taught and used in the symbolic stage in order for the student to demonstrate that they can not only come up with the correct answer but that they understand the problem and process for getting it. In this context, the academic language involves not only vocabulary and mathematical terms but also mathematical symbols.

From this, it is concluded that learner constructs the meaning of mathematics based on individual cognitive development and symbolic language. In this study, teaching and learning Algebra with GeoGebra based instruction is defined in terms of Action, Concepts, Perceptions and Reflection. The researcher has selected constructivism as theoretical grounds for this study because, in GeoGebra based instruction, students refine the algebraic thinking through geometric interpretation of each algebraic problems and solutions.

Conceptual Framework

This study was based on the constructivism learning theory regarding action, construction and reflection as the main concepts. In Algebra teaching and learning, students were taken as the central part of the learning, however, GeoGebra and Geometric interpretation would also be placed in the central part of the conceptual framework. The periphery of discussion is the algebraic contents, for example, equation, the system of equations, a solution of equations, a solution of a system, and parameter form of a solution. This shows that GeoGebra makes the connection between Algebra and Geometry. After making a strong connection between these both areas of mathematics, students refine their algebraic thinking and finally, students were assessed in terms of an achievement test.

Figure I: Conceptual Framework



In the above framework, students were encouraged to do at least one doable action to learn algebra better, especially focusing on geometric interpretation. Then the students were encouraged to reconstruct their knowledge and finally reflect their own task. It was assumed that GeoGebra was used in every phase of learning such as action, construction and reflection.

From these empirical and theoretical review of literature, it is more worthy to conduct research on this particular topic. In other words, the statement of the problem is an exigent problem to be studied in the context of Nepal, where use of technology in mathematics has been provisioned as the principle of curriculum development. Furthermore, contemporary literature insinuates that it is necessary to carry out the study to assess the effectiveness of GeoGebra in algebra lesson. Moreover, use of GeoGebra in algebra to improve students algebraic thinking has also become most significant part of this study.

CHAPTER III

METHODS AND PROCEDURES

This chapter expresses about the Research Design, Sample and Population, Variables of the Study, Data collection tools, Sources of Data collection, Determination of Reliability and Validity of Data Collection Tools, Data Collection Procedures and the way of Data Analysis. Apart from this Data section, this chapter includes ethical consideration throughout the research process.

Research design

In this study, the researcher used quasi-experimental design which is a quantitative research method. More precisely, the researcher used pre-test post-test non equivalent design which is presented in the following table:

Table I: Research Design

Groups	Test	Treatment	Test
Experimental Group	Pre-test	Use of GeoGebra in Algebra Teaching	Post-test
Control Group	Pre-test	No Treatment	Post-test

The researcher adopted this design because there is a lack of randomness in the selection of schools and the non-equivalency in the students in experimental and control groups. However, the researcher needed to measure the relationship and impact of the independent variable on dependent variables and over a specified period.

Population and Sample

The population of the study was all the students who have been studying at grade VIII in Sarlahi District. The study was conducted with the two schools of Barahathawa Municipality, based on purposive sampling. However, the randomness was used for selecting them as control and experimental groups. One group is supposed as control which comprises 35 students and next as experimental which comprise 40 students.

Variables of the Study

In this study, GeoGebra is an independent variable whereas students' achievement in algebra and algebraic thinking are the dependent variables of the study. Also, extraneous variables are selection of school, researcher's perceptions, subject matter and time stamp in teaching module for this experiment.

Control mechanism for extraneous variables. The extraneous variables are special types of variables that are not the primary concern of the researcher. The extraneous variables were controlled throughout the experiment. The extraneous variables were controlled in the following way:

- The same subject matter was taught in both group
- The researcher himself taught in both groups
- The same test was used in both groups
- Point scoring system was used for scoring the answer sheet
- The interaction between the groups was controlled by maintaining a distance of two schools

Sources of Data

The researcher used both primary and secondary sources of data to collect more realistic and relevant data to the research. The primary sources of data were based on achievement test in algebra, memo writing, and algebraic thinking test. The researcher arranged the secondary sources of the data by searching and reviewing the computer database, Journal articles and authentic books.

Data Collection Tools

To collect the primary data, the researcher used achievement test in algebra, algebraic thinking test and memo writing. These tools are described briefly in the following way:

Achievement Test. An Achievement Test comprised a set of test items developed from algebra having direct or indirect relationships with geometry. The researcher developed two types of test items objective and subjective from grade VIII algebra lessons with the help of a supervisor. These test items (subjective and objective) were designed purposefully to identify students' factual knowledge and to explore the relationship between algebra and geometry. There were 15 objectives and 15 subjective types of test items for pilot test. After the pilot test, the researcher retained 10 subjective and 10 objective items.

Algebraic Thinking Test. The researcher used the modified form of Algebraic Thinking Test (ATT) developed by Chimoni and Pitta-Pantazi (2016) to find out the algebraic thinking of the students. ATT contained four categories: Generalized Arithmetic, functional thinking, modelling as a domain of expressing and formalizing generalizations, and Algebraic proof. These four categories are described below:

Generalized Arithmetic. The generalized arithmetic involves expressing and formalizing generalizations and in this section solving equations and inequalities types questions are asked, without any reference to the meaning of the symbols.

Functional Thinking. In this category, questions refer to generalizing numerical patterns to describe functional relationships (functional thinking). These items required finding the nth term in patterns and functional relationships and expressing them in a verbal, symbolic or any other form.

Modelling as a domain of expressing and formalizing generalizations. These items required the expression and formalization of generalizations by analysing information that is presented verbally, symbolically or in a table.

Algebraic Proof. These items reflected different activities and associated abilities of algebraic proof. For example, one of these items required the use of a generalization that was previously established (what is the sum of two odd numbers) for building a new generalization (what is the sum of three odd numbers).

The test included 10 items representing each category to pilot the test. The researcher retained only 8 items in this study, after ensuring reliability and validity.

Memo Writing. Through-out the experiment, the activities of experimental group students were observed including their participation in classroom, and completion of the assigned task in time. The memo writing was based on three dimensions, namely Using Symbols, Including Letters, as Variables, Exploring Properties and Relationships and Using Inverse Operations, and that contains only seven statements which are related to activities of students reflecting the algebraic thinking activities.

Validity and Reliability of the Tools

Achievement Test. The researcher conducted a pilot test of achievement test among the students who represented the population of the study but not included in the sample of the study. The achievement test comprised 10 subjective and 10 objective test items. The reliability coefficient of the test was calculated in terms of Cronbach's Alpha model by performing SPSS 21.0 setting 0.05 level of significance. The reliability coefficient (Cronbach's Alph) was 0.88. This reliability coefficient was very good with reference to the interpretation criteria provided by George and Mallery

(2003, 231). It means that there was greater internal consistency of the items in the scale. The validity of the achievement test was assured by expert judgement.

Algebraic Thinking Test. The researcher used ATT developed by Chimoni and Pitta-Pantazi (2016) with modified test items. Then the researcher conducted a pilot test of ATT among the students who represented the population of the study and who had some experience of the use of GeoGebra in mathematics but not included in the sample of the study. The ATT comprised 10 test items in the pilot test. The reliability coefficient of the test was calculated in terms of Cronbach's Alpha model by performing SPSS 21.0 setting 0.05 level of significance. The reliability coefficient (Cronbach's Alph) was 0.90. This reliability coefficient was excellent with reference to the interpretation criteria provided by George and Mallery (2003, 231). It means that there was greater internal consistency of the items in the scale. The validity of the achievement test was assured by expert judgement. The researcher retained only 8 items in this study, after ensuring the validity and reliability of the test.

Memo Writing. The validity and reliability of the observation were ensured by more than two mathematics subject experts.

Experimental Validity Threats

Any uncontrolled extraneous variables affecting performance on the dependent variable are threats to the validity of an experiment (Gay, Mills and Airasion, 2012). Thus, it is necessary to control these threats to ensure the validity of the experimental results. In this study, the internal and external validity threats were addressed in the following ways:

Threats to Internal Validity. Internal validity is the degree to which observed differences on the dependent variable are a direct result of manipulation of the independent variable, not some other variable" (Gay el.al; 2012, 253). The following internal threats were addressed in this study:

History. History refers to any event such as the endemic, epidemic of measles, a bomb scares occurring during the study that is not part of the experimental treatment but may affect the dependent variable. But during the experiment, such historical events did not happen.

Maturation**.** Maturation refers to physical, intellectual, and emotional changes that naturally occur within individuals over a period of time (Gay et.al; 2012). In a research study, these changes may affect participants' performance on a measure of the dependent variable. To remedy this problem, individuals of the same grade level were selected as participants and treatment did not last a long time.

*Testing***.** A potential threat to internal validity is that participants may become familiar with pre-test post-test and remember facts for later testing. This study was based on pre-test post-test and there might be such a situation. To remedy this situation, experimenter-administered two tests less frequently and some different items were used on the post-test than those used in earlier testing.

*Instrumentation***.** The measuring instrument is changed between pre- and post-testing or a single measuring instrument is unreliable (Gay et.al; 2012). The instrumentation threat refers to a lack of consistency in measuring instruments that may result in an invalid assessment of performance. To correct this potential problem, the researcher had used standardize a procedure that is the same measuring instrument throughout the experiment.

*Statistical Regression***.** Statistical regression refers to the tendency of participants who score highest on a pretest to score lower on a posttest and the tendency of those who score lowest on a pretest to score higher on a posttest (Gay et.al; 2012). In this study, the naturally intact groups were taken as experimental and control groups. Both the groups were of mixed ability.

*Differential Selection***.** Differential selection of participants in the selection of subjects who have differences before the start of a study that may at least partially account for differences found in a posttest (Gay et.al; 2012). To correct this potential problem, two groups were selected within the same locality and pretest was used to check initial equivalence.

*Mortality***.** Mortality, or attrition, refers to a reduction in the number of research participants during the study (Creswell, 2014). Mortality creates problems with validity particularly when different groups drop out for different reasons and with different frequency. But in this study, no participants dropped out during the experiment. Thus, same number and same respondents were involved in pre-test and post-test.

*Selection Bias***.** The participants selected into treatment groups have different maturation rates. Selection interactions also occur with history and instrumentation. To address this problem, experimental and control groups were divided based on randomization.

Threats to External Validity Threats. External validity is the degree to which study results are generalizable to groups and environments outside the experimental setting. "Threats to external validity are problems that threaten our ability to draw correct inferences from the sample data to other persons, settings, treatment variables, and measures" (Creswell, 2014, 306). The following external threats were addressed in this study:

Pre-test Treatment Interaction. Pre-test treatment interaction indicates that the pretest sensitizes participants to aspects of the treatment and thus influences posttest scores. This would be minimal when the study is conducted with very young pupils (Gay et.al; 2012). In this study, the effects of the pretest were being greatly diminished by maintaining a one-month gap between pre-test and post-test and students' age.

*Multiple Treatment Interaction***.** When participants receive more than one treatment, the effect of prior treatment can affect or interact with later treatment, limiting generalizability. In this study, a single treatment was used in the experimental group.

*Specificity of Variables***.** Poorly operationalized variables make it difficult to identify the setting and procedures to which the variables can be generalized. In this study, the generalizing variable was clearly specified.

Treatment Diffusion. Treatment diffusion occurs when the treatment group communicate with and learn from each other, altering the initial status of the treatment's comparison (Gay et. al; 2012). To reduce treatment diffusion, the researcher requested to the teachers and his colleagues to use the same treatment until the study was completed and thus, it has reduced following only one treatment per school.

Experimenter Effects. Experimenter effects refer to conscious or unconscious actions of the researchers that affect participants' performance and responses. It is difficult to identify experimenter bias in the study. To counter this problem, the researcher had become as a striver to avoid communicating emotions and expectations to participants in the study. It also reduced by blind scoring, in which the researcher doesn't know whose performance is being evaluated.

*Reactive rearrangement***.** In experimental design, the experimenter may create a highly artificial environment but not easily generalizable to the non-experimental setting for his/ her experiment which is known as reactive rearrangement. To reduce reactive rearrangement, affect participants were formed in naturally assembled class.

Phases of the Experiment

This experiment had been completed in the following three phases:

***Pre-experimental Phase*.** This was the first phase of the experiment which had ranged from 10 August 2018 to 11 September 2018. In this phase, the preparation of episodes, slides of teaching, preparation of materials, validations of these episodes with the help of the subject expert were completed. Furthermore, the researcher had completed planning, preparation, and piloting the achievement test, ATT for administration of pre-test, analysis of pre-test result in this phase.

Experimental Phase. The tenure of the experimental phase was ranged from 12 September 2018 to 28 November 2018. During this phase, the researcher taught the experimental group by using GeoGebra and control group with the traditional method.

Post-experimental Phase. In the final phase of the experiment the post achievement test was administered in both groups. The post-test of ATT was administered in experimental group only.

Ethical Consideration

In this research, some ethical issues were considered to make standardisation in the data collection process, tools and accuracy in report writing. The researcher considered the following ethical issues:

Approval. The researcher had granted permission to research the institution before planning and launching the experiment providing to the accurate information of the intention of the research.

Consent. Before experimenting, the researcher made informed consent between participants appraising of the tenure of the experiment and possible risk factors in this experiment.

Language. The researcher used appropriate language throughout his experiment which is reasonably understandable to all the students and supervisor.

Reporting. The researcher did neither fabricate the data nor falsify the results for thesis writing or publication.

Data Collection Procedures

After the consent between the researcher and the head of the schools, mathematics teachers and the students, the data collection process was started. In the first phase of data collection, the pre-test of achievement test was administered to both groups for the purpose of identifying the homogeneity in terms of students' achievement in algebra. After collecting the pre-test related data, the researcher performed SPSS 21.0 setting at 0.05 level of significance to test the hypothesis for homogeneity. The researcher decided to name these two groups one as experiment and another as control randomly. Then, the researcher conducted the pre-test of ATT in the experimental group only to identify their level of algebraic thinking. During the experiment, the experimental group was taught through GeoGebra and the control group was taught through traditional method.

Furthermore, the researcher observed experimental classroom including their task completion, active participation and identifying the relationships and made a memo writing for the purpose of analyzing the students' algebraic thinking. After the one month of the experiment, students of both groups were assessed in terms of the same post-test for measuring the effectiveness of GeoGebra and Traditional approach in students' achievement in Algebra. Then researcher collected the students answer sheet. After this, the researcher administered the post-test of ATT to analyze students' progress in algebraic thinking after the use of GeoGebra in algebra lessons. The post-test of ATT comprised of same test items as in the pre-test. The researcher arranged the secondary data through the literature review and computer database.

Data Analysis Procedures

The researcher started the data analysis with the coding of the data in the personal computer and protecting the data with security key. This was the experimental study. So, preliminary data analysis was based on descriptive analysis, such as mean and standard deviation. Then the researcher used inferential statistics, particularly t-test and paired t-test to test the stated hypothesis. In other words, the researcher used an independent t-test in pre-and post-test data and paired t-test, used after the completion of the experiment, for analyzing algebraic thinking of the students. More importantly, the researcher used SPSS 21.0 statistical package setting 0.05 level of significance for the purpose of analyzing the data.

The data obtained from the memo writing was analyzed based on thematic approach. The researcher generated the primary theme based on the frequency of the activities mentioned on memo writing. The statements of memo writing were supposed as secondary themes in this study. The results of each piece of data were presented in the pictorial graphs.

CHAPTER IV

ANALYSIS AND INTERPRETATION OF DATA

This chapter conveys the analysis and interpretation of the data collected. The chapter is organized in order of the hypothesis and objectives of the study stated in chapter I. Each hypothesis discussion entails a discussion of the test conducted and findings from data. For the purpose of analyzing all types of statistical data, SPSS 21.0 statistical analysis software was used setting at the 0.05 confidence level. The researcher analyzed and interpreted the data under the following headings:

Achievement in Algebra

Comparison of achievement score of control to experimental groups in the pre-test. Algebra achievement test comprising ten subjective and twenty objective items was administered in both experimental and control groups to determine the homogenity of two intact groups. The Individual scores of the students from the control group and experimental group in pre-test are presented in Appendix A. The summary of independent t-test is presented in the following table II:

Table II: Results of Independent t-test on the pre-test

Groups	Mean	SD	t	Significance (2-tailed)
Control (n = 35)	17.0	4.6	0.379	0.705
Experimental (n = 40)	16.6	4.2		

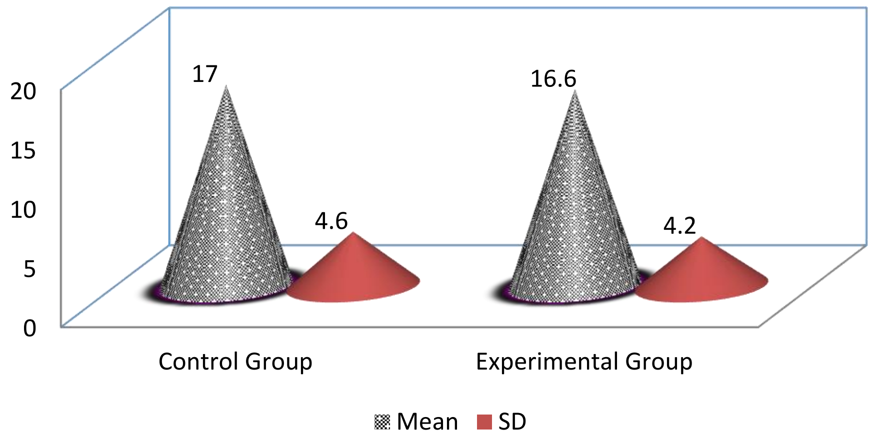
t-value significant at $p < 0.05$.

An independent-sample t-test was used to compare the students' mean achievement scores for control and experimental groups. The above table reveals that, there was no significant difference in scores for control group students (mean = 17.0, SD = 4.6) and experimental students (mean = 16.6, SD = 4.2) in pre-test.

Thus, this result demonstrates that students in the control and experimental group were homogenous in terms of their achievement or rather their similar abilities.

Figure II provides a graphical illustration of the mean and standard deviation of the students of the control group and experimental group, diagrammatically. It also shows that two groups were homogeneous in terms of their average achievement in the pre-test.

Figure II: Comparison of Students' Achievement in pre-test



Comparison of achievement score of control to experimental groups in post-test. After providing the certain treatment to the experimental group, control and experimental groups were again assessed in terms of same test namely post-test. A set of individual scores of students on post-test is presented in Appendix A. The summary of independent t-test is presented in table III:

Table III: Results of independent t-test on post-test

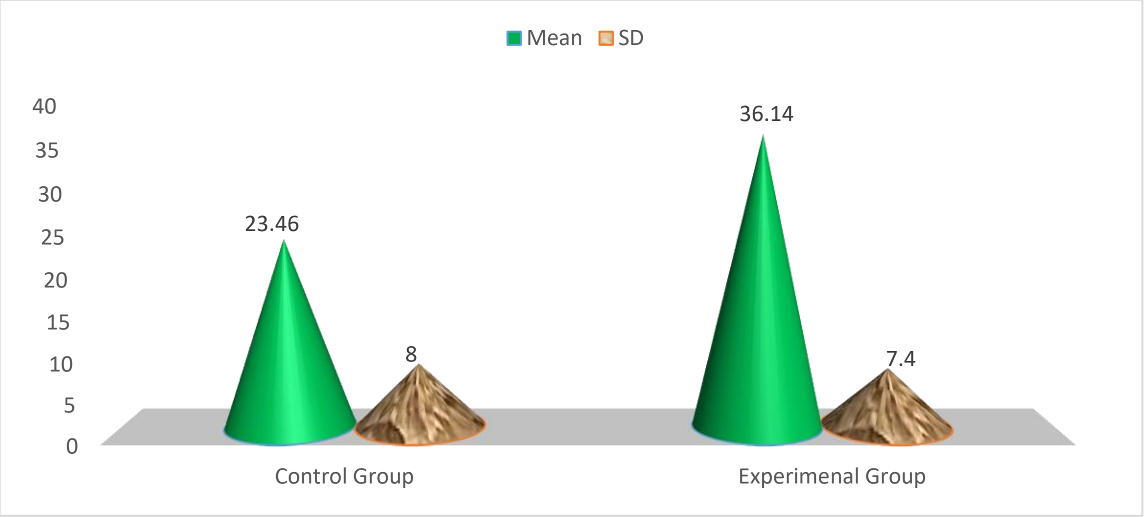
Groups	Mean	SD	t	Significance (2-tailed)
Control (n = 35)	23.46	8.0	-7.084	0.000
Experimental (n = 40)	36.14	7.4		

t-value significant at $p < 0.05$.

An independent-samples t-test was carried out to compare students' achievement scores for control and experimental groups. The above table exhibits that there is a statistically significant difference in scores for control group students (mean = 23.46, SD = 8.0) and experimental students (mean = 36.14, SD = 7.4) with $|t| = 7.084 > 1.96\ at\ 0.05$ level of significant, which indicates that the null hypothesis is rejected, and concluded that there is significant difference between the achievement of students in experiment and control group. Moreover, the mean achievement of experimental group was higher than control group. Thus, results of this study succinctly demonstrate that use of GeoGebra has positive impact on students' achievement in the algebra test.

Figure III provides the graphical illustrations that mean achievement of experimental students significantly higher in comparison with control group.

Figure III: Comparison of Students' Achievement in post-test.



Algebraic Thinking

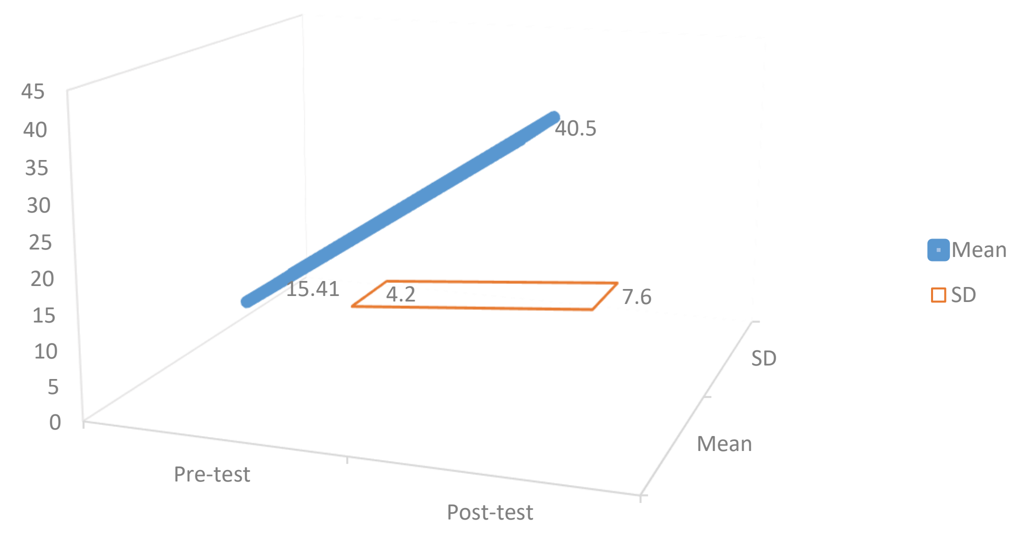
The researcher conducted the ATT to the students of the experimental group. The ATT was conducted as pre-test and post-test in the experimental group. However, the test items were the same in both tests. A paired t-test was used to measure the level of algebraic thinking of students of experimental group from pre-test to post-test. The individual score of students is presented in appendix B. The summary of paired t-test is presented in the following Table IV:

Table IV: Results of the paired sample t-test of Experimental group

Experimental Group	Mean	SD	t	Sig (2-tailed)
Posttest score – Pretest score	25.1	9.53	16.64	0.000

A paired-samples t-test was conducted to evaluate the impact of the intervention on students' scores on the Algebraic Thinking Test (ATT). There was a

Figure IV: Students' progress in Algebraic Thinking



statistically significant increase in ATT scores from Pretest (Mean = 15.41, SD = 4.2 to Posttest (Mean = 40.50, SD = 7.6), t (29) = 5.39, p <. 005 (two-tailed). The mean increase in ATT scores was 25.1 with a 95% confidence interval ranging from 22.04 to 28.14. It seems that the use of GeoGebra in the algebra lesson helped the students to improve their algebraic thinking skills. The following figure IV shows that how much students algebraic thinking increased, numerically:

Memo Writing

The researcher observed the students of the experimental group in order to identify whether students engaged in different kinds of algebraic thinking activities. In this observation process, the researcher acted as an observer and prepared the memo writing. The researcher analyzed this memo using the thematic approach. Thus, the researcher found the following themes:

Relationship. The relationship refers to how two or more variables are related to each other. It was seen that students easily used letters as variables in the algebraic equations. Furthermore, they were able to transcribe the verbal problems in the algebraic equations. The researcher observed that students did not confuse about the symbols or letters that their peer used and shared with them. It was observed that students did get to identify existing relationships in algebraic problems.

Inverse operation. Inverse operation is important aspect to understand the algebraic thinking. The researcher observed inverse operations such as addition and multiplication. Students used succinctly inverse operations such as Subtraction is the inverse operation of addition and vice versa. Similarly, the division is the inverse operation of multiplication and vice versa.

Using Symbols. The correct use of symbols including letters, as variables ensures the students to have good algebraic thinking (Manly & Ginsburg, 2010). The researcher observed that students used letters as variables in the algebraic equations. Furthermore, it was observed that students did not confuse about the variables in learning algebra with GeoGebra in the classroom.

Moreover, the researcher recorded frequency of how much students used relationships and inverse operations in their daily activities. The complete table of frequency is presented in the appendix C. It is seen that the majority of the students are enabled to use different sorts of activities that are useful to improve algebraic thinking. Nearly all, about 91 % of students' activities, are related to identifying the relationships in the verbal problems and stating them in algebraic form. Moreover, students aptly used algebraic properties for their problem-solving. Furthermore, a student could understand the existing relationships of variables in the problems.

It also shows that nearly three of fourth students activities were observed as concise use of letters and symbols as the variables in the algebra learning. Students feel free to choose variables to state the problem in mathematical form and to solve such problems. More importantly, they did not confuse while using different sort of variables by their peers in the same problem-solving. Using inverse properties seems to be a little bit complicated in comparison with the other two activities of algebraic thinking. The observed frequency for using inverse properties is only 62 %.

CHAPTER V

SUMMARY, FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

This chapter synthesizes the analysis of the data gathered from primary and secondary sources as findings of the study and draws the conclusion of the study. And finally, the implication of the research and recommendation for future areas of research are presented.

Summary and Findings

This is experimental research related to the effectiveness of GeoGebra in teaching algebra at grade VIII. Apart from this, the aim of this research was to identify the students Algebraic thinking and help to improve it. The study was conducted based on quasi-experimental design selecting two private schools from Sarlahi District, Province 2 of Nepal. Achievement test containing Algebra items, ATT scale with four dimensions, and observation were used as data collection tools. The reliability of these tools was determined performing Cronbach's alpha by using SPSS 21.0 and validity of the tools was ensured by expert judgment and secondary school mathematics curriculum.

Extraneous variables were controlled as much as practicable. The data collection process started with the conducting with pre-test of achievement test on both groups of students and ATT which was conducted only on Experimental group. During the process of experiment, the researcher used observation protocol to observed activities of students related to algebraic thinking. After certain treatment to the experimental group, the students of both groups were assessed in terms of the same post-test. Then, the experimental group was also assessed in terms of post-test of ATT. The obtained data were analyzed by using SPSS 21.0 and then the results were interpreted.

For the analysis of qualitative data, the researcher calculated the frequency of the observed statements. Moreover, students' achievement in pre-test and post-test of algebra achievement and ATT were analyzed by using both descriptive and inferential statistics but observation data were analyzed in terms of frequency. Based on the analysis of the data, utilization of GeoGebra has established as an efficacious tool for teaching algebra in order to connect it to geometry at the very early age of education. It has helped students to connect algebra and geometry. Thus, the following were the main findings of the study:

- There was no significant difference between the mean achievement of control group students and experimental group students in the pre-test.
- The mean achievement of experimental group students was higher than the control group students.
- The algebraic thinking of the students was improved by the use of GeoGebra in teaching algebra.
- GeoGebra has provided an opportunity to each individual for improving their algebraic thinking. GeoGebra helped students to reconstruct their existing state of knowledge and enforces them to engage in enquiry based activities such as searching application of construction.
- The students were able to use different sorts of activities that are useful to improve algebraic thinking.
- The students aptly used algebraic properties for their problem solving by understanding the existing relationships in the questions.
- The students felt difficulty to apply inverse properties to solve problems.

Conclusion

The use of GeoGebra in Algebra has proven that the best strategy in algebra teaching for improving the algebraic thinking of students in the context of Nepal. The use of GeoGebra not only increases students' scores in mathematics but also helps students to become more creative, independence as well as to improve their visual thinking. Thus, the use of GeoGebra in algebra provides an ample opportunity to each individual for unlocking algebraic thinking step by step through active participation. More importantly, it up-rises students' algebraic thinking because GeoGebra helps students to reconstruct their existing state of knowledge and enforces them to engage in enquiry based activities such as searching application of construction. Furthermore, the use of GeoGebra in algebra lesson enables students to use existing cognitive and visual skills to develop efficiency, experiences, autonomy and hence confidence in their verbal problem-solving. Thus, it is necessary to encourage the use of GeoGebra gradually into school algebra for the purpose of improving higher order thinking skills of students.

It is affirmed that GeoGebra helps students to construct geometric figures and concepts systematically owing to its visualization of subsequent steps. This is also true in this research student were able to draw an apt graphical representation of algebraic equations. In addition, the use of GeoGebra exhorts productive interaction between students- students and teacher- students, during the learning scenarios. Moreover, this study also underscores that GeoGebra provides a cognitive and procedural understanding of the geometric construction through visualizing corresponding steps. Therefore, the use of GeoGebra in algebra teaching is necessary for teaching algebra for better algebraic thinking and better understanding.

The implication of the Research

The results of research might lead valuable insight into improving students' achievement and placing worth of GeoGebra in teaching algebra for connecting it with geometry. Based on the findings, there is the following potential implication:

- Teachers should be urged in on using GeoGebra software in every branch of mathematics in accordance with policy provisions of penetrating ICT tools in the secondary level curriculum.
- GeoGebra has a highly positive impact on algebra lessons so that school level mathematics teachers are first encouraged for participating in training packages allied to the application of mathematics software.
- The traditional approach has a lot of difficulties in algebra teaching and it might be encouraged to make meaningful through the use of technology.
- Policy makers are suggested to intensively articulate the use of GeoGebra in all branches of mathematics.

Recommendations for further research

This study has focused on the only improvement of students' achievement and algebraic thinking through the use of GeoGebra in algebra lessons based on an experimental design with a middle size of the sample. Other researchers may conduct a study within large sample in order to increase the effectiveness of GeoGebra in mathematics lessons. Based on the research, the research has made following recommendation for the further study:

- It is recommended that further study may be carried out in order to identify the articulation of mathematics curriculum in terms of the flow of ICT tool in schools and effectiveness of policy provisions of integrating ICT into school mathematics especially geometry teaching of Government of Nepal.

- It is recommended that further research may be conducted to explore the effectiveness of ongoing of GeoGebra based instruction in other algebra lessons.
- It is recommended to compare other ICT tools to GeoGebra with reference to mathematics teaching at school level in the context of Nepal for the purpose of increasing the effectiveness of GeoGebra in secondary level mathematics.
- It is recommended to survey the use of GeoGebra in mathematics classroom covering all schools namely public and private and identify the relation between them and its effective use in mathematics learning.

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Appendix A

Score of students in Achievement Test in Pre-test and Post-Test

SN	Group	Pre-Test	Post-Test
1	Experimental Group	17.0	38.0
2	Experimental Group	17.5	33.0
3	Experimental Group	19.0	49.0
4	Experimental Group	17.0	44.0
5	Experimental Group	18.0	41.0
6	Experimental Group	17.0	42.0
7	Experimental Group	20.0	24.0
8	Experimental Group	15.5	26.0
9	Experimental Group	16.0	25.0
10	Experimental Group	17.0	30.0
11	Experimental Group	20.0	33.0
12	Experimental Group	16.0	29.0
13	Experimental Group	20.0	34.0
14	Experimental Group	16.0	29.0
15	Experimental Group	10.0	30.0
16	Experimental Group	22.0	30.0
17	Experimental Group	20.0	43.0
18	Experimental Group	20.0	31.0
19	Experimental Group	18.0	45.0
20	Experimental Group	19.0	31.5
21	Experimental Group	18.0	25.0

22	Experimental Group	22.0	43.0
23	Experimental Group	12.0	41.0
24	Experimental Group	18.0	48.0
25	Experimental Group	18.0	43.0
26	Experimental Group	8.0	39.0
27	Experimental Group	16.0	49.0
28	Experimental Group	15.0	35.0
29	Experimental Group	12.0	49.0
30	Experimental Group	24.0	47.0
31	Experimental Group	9.0	40.0
32	Experimental Group	8.0	35.0
33	Experimental Group	5.0	34.0
34	Experimental Group	24.0	41.0
35	Experimental Group	15.0	28.0
36	Experimental Group	15.0	32.0
37	Experimental Group	16.0	28.0
38	Experimental Group	18.0	30.0
39	Experimental Group	19.0	35.0
40	Experimental Group	18.0	36.0
41	Control Group	30.0	33.0
42	Control Group	26.0	29.0
43	Control Group	17.0	22.0
44	Control Group	14.5	19.0
45	Control Group	19.0	19.0
46	Control Group	16.5	17.5

47	Control Group	17.5	19.5
48	Control Group	21.0	22.5
49	Control Group	18.0	16.0
50	Control Group	12.0	18.0
51	Control Group	22.0	29.5
52	Control Group	12.0	16.0
53	Control Group	15.0	19.0
54	Control Group	15.0	18.5
55	Control Group	15.0	12.0
56	Control Group	10.0	15.0
57	Control Group	12.0	17.5
58	Control Group	13.0	16.0
59	Control Group	23.0	44.5
60	Control Group	19.0	32.0
61	Control Group	12.0	21.0
62	Control Group	15.0	24.0
63	Control Group	20.0	28.0
64	Control Group	22.0	36.0
65	Control Group	23.0	30.0
66	Control Group	20.0	44.0
67	Control Group	10.0	22.0
68	Control Group	19.0	35.0
69	Control Group	16.0	24.0
70	Control Group	17.0	21.0
71	Control Group	15.0	24.5

72	Control Group	12.0	17.0
73	Control Group	10.0	12.0
74	Control Group	19.0	25.0
75	Control Group	18.0	22.0

Appendix B

Score of students of Experimental Group in ATT

SN	Pre-Test	Post-Test
1	17.0	45.0
2	20.0	40.0
3	15.5	42.0
4	16.0	36.0
5	17.0	29.0
6	20.0	22.0
7	16.0	29.0
8	20.0	35.0
9	16.0	49.0
10	12.0	45.0
11	20.0	41.0
12	20.0	40.0
13	20.0	40.0
14	15.0	45.0
15	19.0	48.0
16	18.0	43.0
17	16.0	49.0
18	15.0	48.0
19	18.0	45.0
20	18.0	45.0
21	8.0	49.0
22	16.0	49.0

23	15.0	50.0
24	12.0	50.0
25	24.0	34.0
26	9.0	50.0
27	8.0	32.0
28	5.0	50.0
29	21.0	30.0
30	15.0	35.0
31	15.0	40.0
32	16.0	45.0
33	8.0	40.0
34	10.0	42.0
35	12.0	41.0
36	15.0	29.0
37	16.0	40.0
38	18.0	41.0
39	15.0	31.0
40	10.0	26.0

Appendix C

Result of observed activities in Experimental Group

SN	Algebraic thinking related Activities	Observed Frequency (%)
1.	Using Symbols, Including Letters, as Variables a. Students easily used letters, as variables in the algebraic equations. b. Students were able to transcribe the verbal problems in the algebraic equations c. Students did not confuse about the symbols or letters that their peer used and shared with them.	72
2.	Exploring Properties and Relationships a. Students did get to identify existing relationships in algebraic problems. b. Students were able to explore a relationship easily because they used different sorts of properties in solving and stating the problems. c. Students identified useful algebraic properties.	91
3	Using Inverse Operations a. Students used inverse operations such as Subtraction is the inverse operation of addition and vice versa. Similarly, the division is the	62

SN	Algebraic thinking related Activities	Observed Frequency (%)
	inverse operation of multiplication and vice versa.	

Appendix D

Algebraic Thinking Test.

1. The sum 245676 + 535731 is odd or even number? Explain your answer.
2. Bill is arranging squares in the following way. How many squares there will be in the 16th figure?

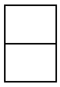


Figure 1

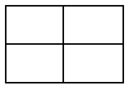


Figure 2

Figure 3

3. Joanna will take computers lesson twice a week. Which is the best offer?

   OFFER A: €8 for each lesson

   OFFER B: €50 for the first 5 lessons of the month and then €4 for every additional lesson.
4. Solve the equation 3(x - 2) = 36
5. The sum of ages of Tom and Mary is 55 years. If Mary is 4 times older than Tom how old is Tom? Show at least two different methods to solve this problem
6. Of 8 students, there are more girls than boys. How many girl students could there be?
7. Ram and Shyam play in the same football team. Last Saturday Ram scored 3 more goals than Shyam, but together they scored less than 9 goals. What are the possible number of goals Ram scored?
8. A rectangular room fits at least 7 tables that each have 1 square meter of surface area. The perimeter of the room is 16 m. What could the width and length of the room be?

Appendix E

Algebra Achievement Test

Pre-/Post-test Question set

Full Marks: 50

Name________________________

Group 'A'

Instruction: *Read the question carefully and circle the best answer*

1. Which of the following is the common factor of 21 x^2y and 35 xy^2?
    a) 7
    b) xy
    c) 7 xy
    d) none of these.
2. Which of the following is quotient obtained on dividing –18 xyz^2 by –3 xz?
    a) 6 yz
    b) –6 yz
    c) 6 xy^2
    d) 6 xy
3. If 2x/5 = 4, the value of x is-
    a) 10
    b) -10
    c) -8/5
    d) 8/5
4. If the sum of two consecutive numbers is 71 and one number is x, then the other number is-
    a) x + (x+1) = 71
    b) x + (x+2) = 71
    c) x + x = 71
    d) none of these
5. The graph of which of the following equations passes through the origin?
    a) y = 2x + c
    b) y = 2x – c

c) y = 2x

6. Through which of the following points, the graph of the linear equation 3x – 2y = 0, passes?

(i) $\left(\frac{2}{3}, -\frac{2}{3}\right)$ (ii) $\left(\frac{2}{3}, \frac{3}{2}\right)$ (iii) $\left(\frac{1}{3}, \frac{1}{2}\right)$

7. How old will I be after 10 years, if my age before 10 years was ‘x’ years?

a) X + 20

b) X - 20

c) X + 10

d) X - 10

8. Which of the following is the numerical coefficient of -5xy?

a) 5

b) -x

c) -5

d) -y

9. Which of the following is the value of $(x + 1/x)^2$?

a) $x^2 + 1/x^2$

b) $x^2 - 1/x^2$

c) $x^2 + 1/x^2 + 2$

d) $x^2 + 1/x^2 + 2x$

10. Which of the following is correct?

a) Cube of a negative number is always positive.

b) Cube of a negative number is always negative.

c) Cube of a negative number may be positive or negative.

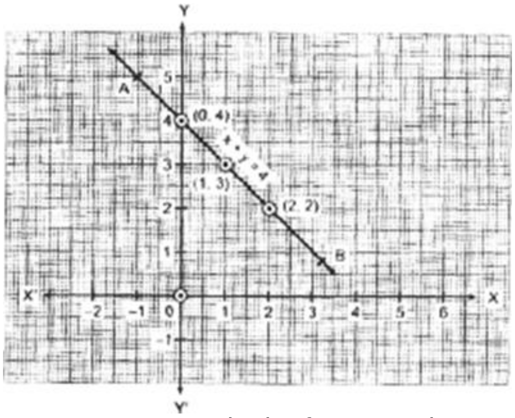
d) All of the above

Group ‘B’

Instruction:

- *Attempt all questions and Each contains equal marks*

1. Is (3, 2) a solution of x + y = 6?



2. Express 2x = 5 in the form ax + by + c = 0 and find the value of a, b and c.
3. Write two solutions of 3x + y = 8.
4. Evaluate $[(1/2)^{-1} - (1/3)^{-1}]^{-1}$.
5. Look at the following graphical representation of an equation. Which of the points (0, 0) (0, 4) or (– 1, 4) is a solution of the equation?
6. Show that x = 4 is a solution of the equation x + 7 – 8x/3 = 17/6 – 5x/8
7. A is twice old as B. Five years ago A was 3 times as old as B. Find their present ages.
8. Solve: 5x – 3 = 3x + 7
9. Find out the HCF of x^2 +6 x +8, x^2 -4 and x^2 +4 x +4
10. The digits of a 2-digit number differ by 5. If the digits are interchanged and the resulting number is added to the original number, we get99. Find the original number.

Episode 1

Grade: 8 **Time: 45 Min.**

Topic: Inequalities in One Variable

Objective

- To solve the following inequalities and present the solution in the number line
  - $3x < 27$,
  - $4x + 3 \geq 23$
  - $5x - 7 \geq 3x - 15$

Introductory Activity (10 minutes)

Divide the class into three groups. Each group can split into pairs for this activity.

Students of each group will be asked the following question:

Question: Given that x is an integer. State the possible integer values of x in the following inequalities.

- $3x < 27$
- $x \leq -3$

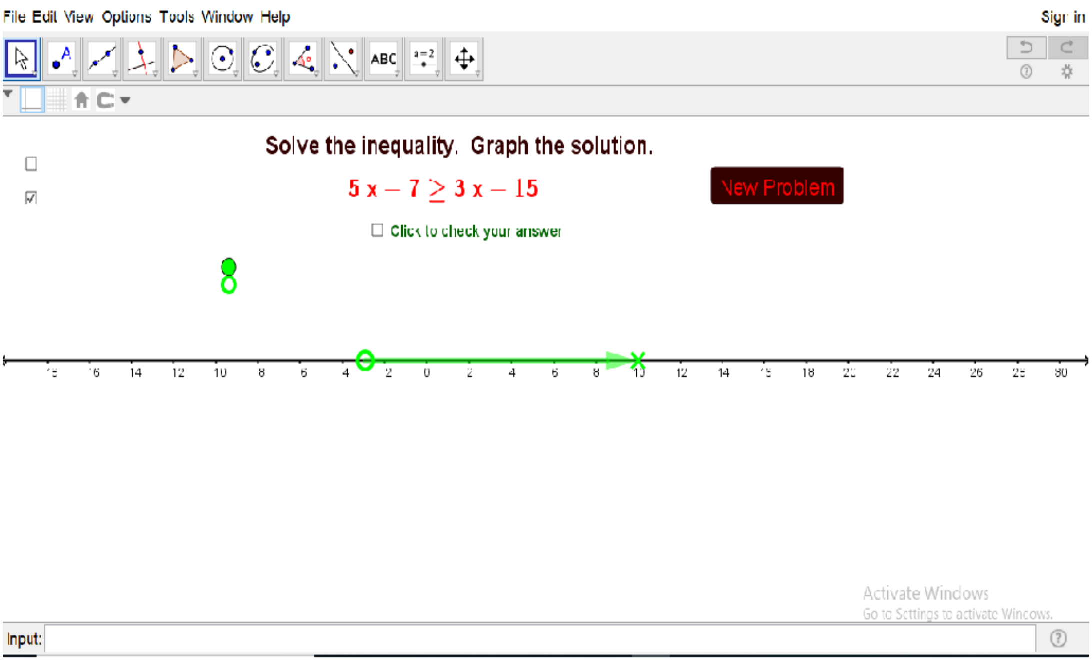
Then they will be encouraged to work on their pair. After pairs of students have completed solving their example problem, have them share the process with their group and see that all reach a consensus about the answer. Students already learned such types of examples in grade VI. So that, students will be encouraged to draw its number line based on their past knowledge and experiences. In these examples, GeoGebra could be used to demystify the concepts.

Class Discussion **(20 minutes)**

Each group of students will be presented different types of questions to work on their group. Then the teacher will encourage students to work on the following:

- $3x < 27$,
- $4x + 3 \geq 23$,
- $5x - 7 \geq 3x - 15$.

Then, students will be engaged to perform at least one actionable activity such as solving the inequalities. In this phase, students will have opportunity to check their answers with GeoGebra platform, which is like playing game. This platform is presented in the following way:



Wrap-Up—Mathematical Terms Review **(7 minutes)**

Then students will be encouraged to define the meaning of terms related to this lesson. Moreover, they will be encouraged to discuss each one of the following and keep a running list in a notebook:

Variable

Inequality Symbols

Linear Inequality

Direction of Inequality

Assessment (7 minutes)

Solve and present its solution in the line graph

$$-11 \leq 3x - 2 \leq -5$$

Episode 2

Grade: 8 **Time: 45 Min.**

Topic: Inequalities in One Variable

Objective

- To solve the following inequalities
  - $2x \leq 9$, Graphically
  - $^{x}/_{4} > ½$, Graphically
  - $-2x < 5$, Graphically
  - $^{(2x-3)}/_{4} \leq 2$, Graphically

Introductory Activity (10 minutes)

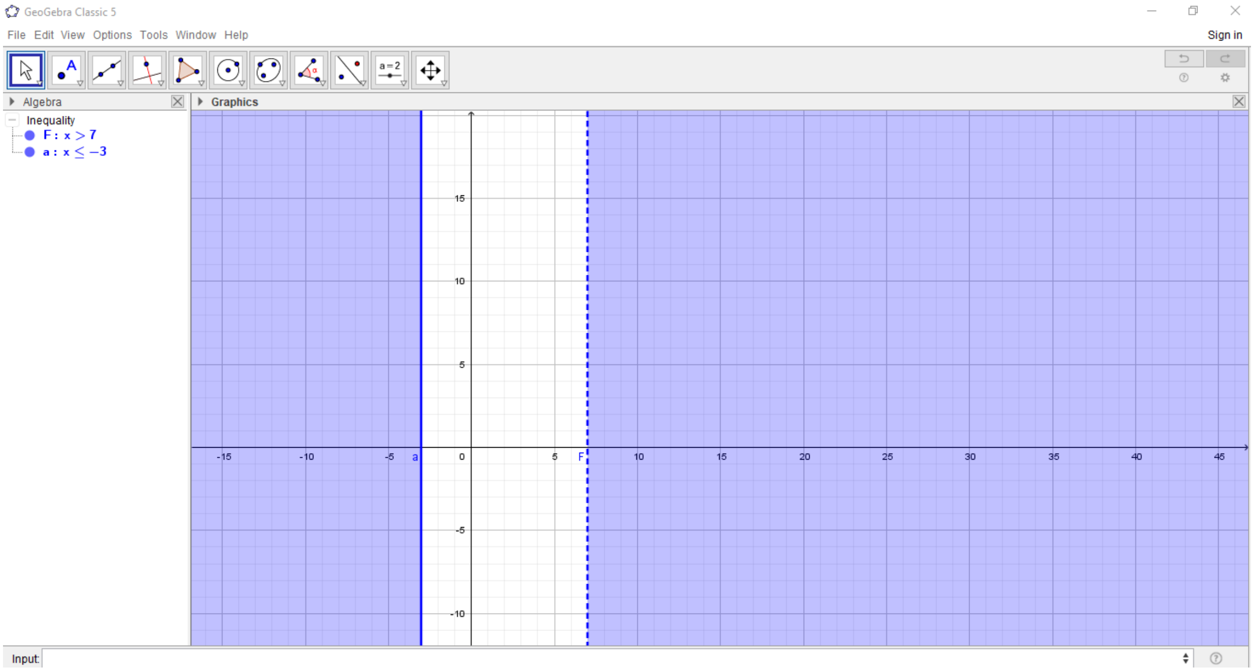
Divide the class into four groups. Each group can split into pairs for this activity.

Students of each group will be asked the following question:

Question: Given that x is an integer. State the possible integer values of x in the following inequalities.

- $x > 7$
- $x \leq -3$

Then they will be encouraged to work on their pair. After pairs of students have completed solving their example problem, have them share the process with their group and see that all reach a consensus about the answer. Then students will be encouraged to draw its graph. The GeoGebra will be used to show the graphic solution of these inequalities. For Instance,



Then students will be asked how to know in which direction the shadow will appear.

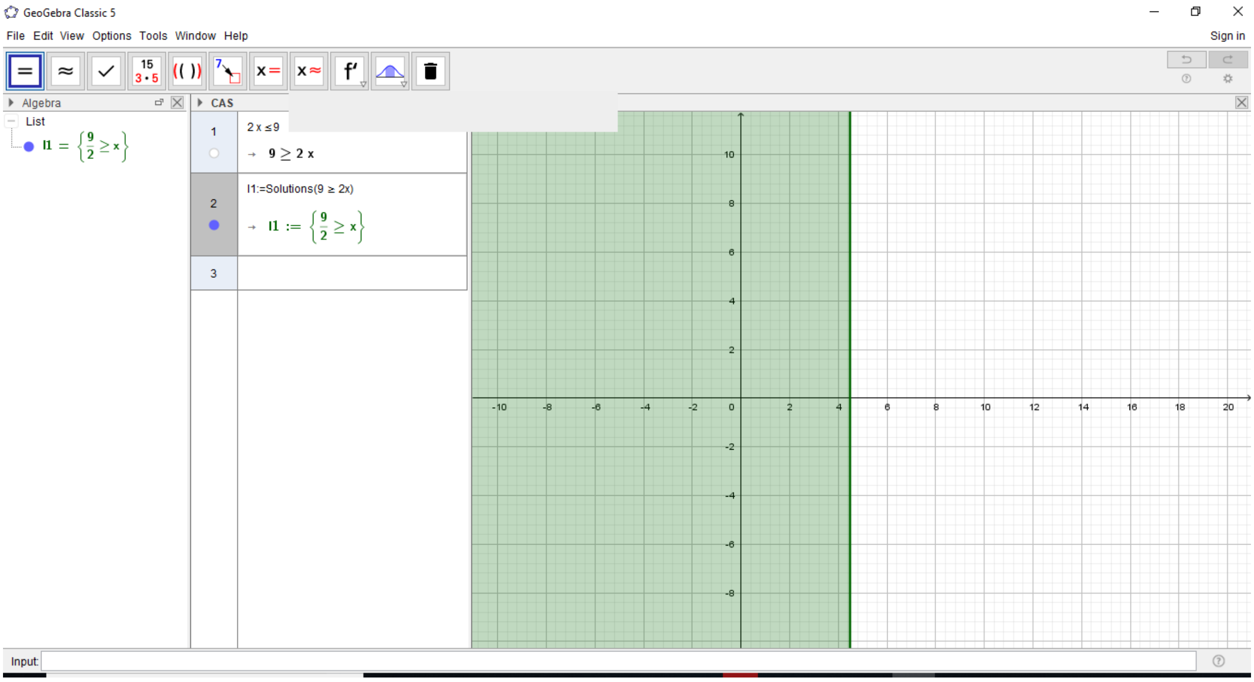
Then further classroom discussion will be based on testing points.

Class Discussion (20 minutes)

Each group of students will be presented different types of questions to work on their group. Then the teacher will encourage students to work on the following:

Group C: Solve $2x \leq 9$, Graphically
Group B: Solve $^{x}/_{4} > \frac{1}{2}$, Graphically
Group D: Solve $-2x < 5$, Graphically
Group A: Solve $^{(2x-3)}/_{4} \leq 2$, Graphically

Then, students will be engaged to perform at least one actionable activity such as drawing graphs. In this phase, GeoGebra will be used in order to solve questions presented to the students and this can be visualized as in the following:



Wrap-Up—Mathematical Terms Review (7 minutes)

Then students will be encouraged to define the meaning of terms related to this lesson. Moreover, they will be encouraged to discuss each one of the following and keep a running list in a notebook:

Variable

Inequality Symbols

Linear Inequality

Direction of Inequality

Assessment (7 minutes)

Solve 10 ≤ 3*x* + 4 ≤ 19, Graphically

Home Assignments

What is Linear Inequality?

How many possible solutions does an inequality in one variable have?

Final Note

- It should be noted that GeoGebra will be used each phases if the students feel awkward.

Episode 3

Grade: 8 **Time: 45 Min.**

Topic: Inequalities in Two Variable

Objective

To solve the system of inequalities

$$y \geq 2\,x - 3$$

$$y \geq -3$$

$$y \leq 0.8\,x + 2.5$$

Graphically.

Introductory Activity (10 minutes)

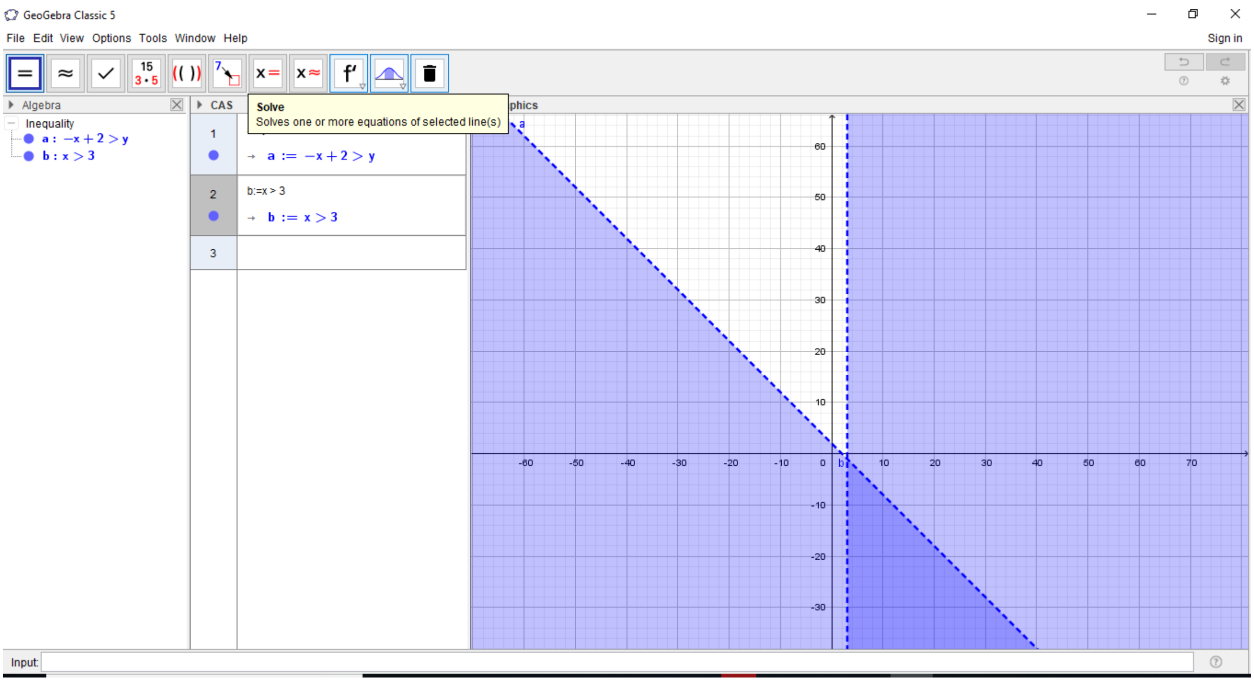
Divide the class into three groups. Each group can split into pairs for this activity. Students of each group will be asked the following question:

Question: Decide the variables of the following inequalities

$x > 3$

$y \leq -x + 2$

After this, students will be encouraged to draw the system of inequalities. Then they will be encouraged to work on their pair. After pairs of students have completed solving their example problem, have them share the process with their group and see that all reach a consensus about the answer. The GeoGebra will be used to show the graphic solution of these inequalities. For Instance,



Class Discussion (20 minutes)

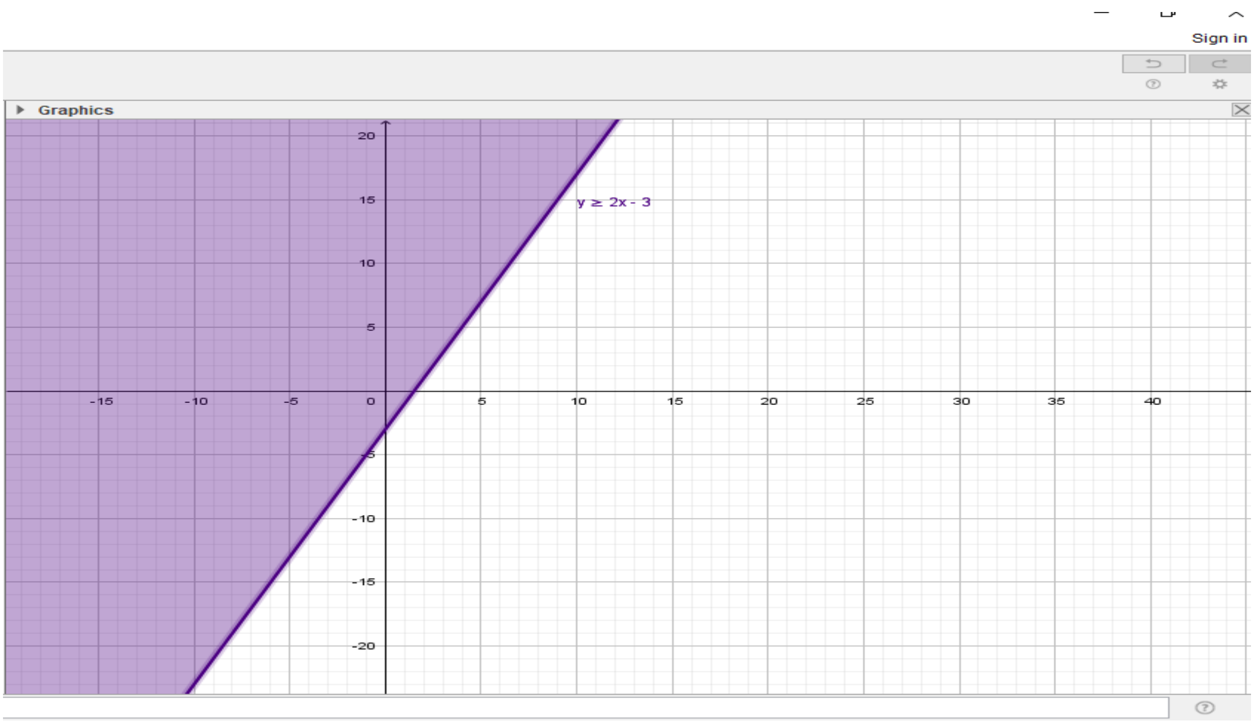
Students will be encouraged to work on each of the following inequality in their group.

$$y \leq 2\,x - 3$$

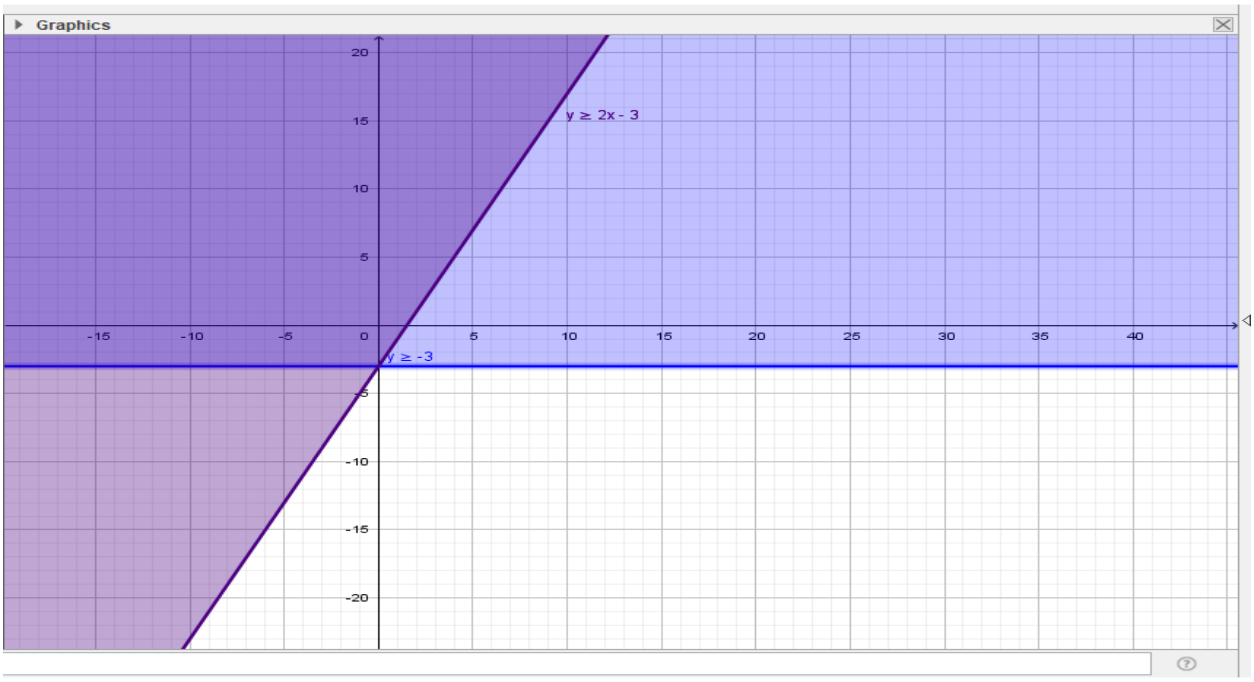
$$y \geq -3$$

$$y \leq 0.8\,x + 2.5$$

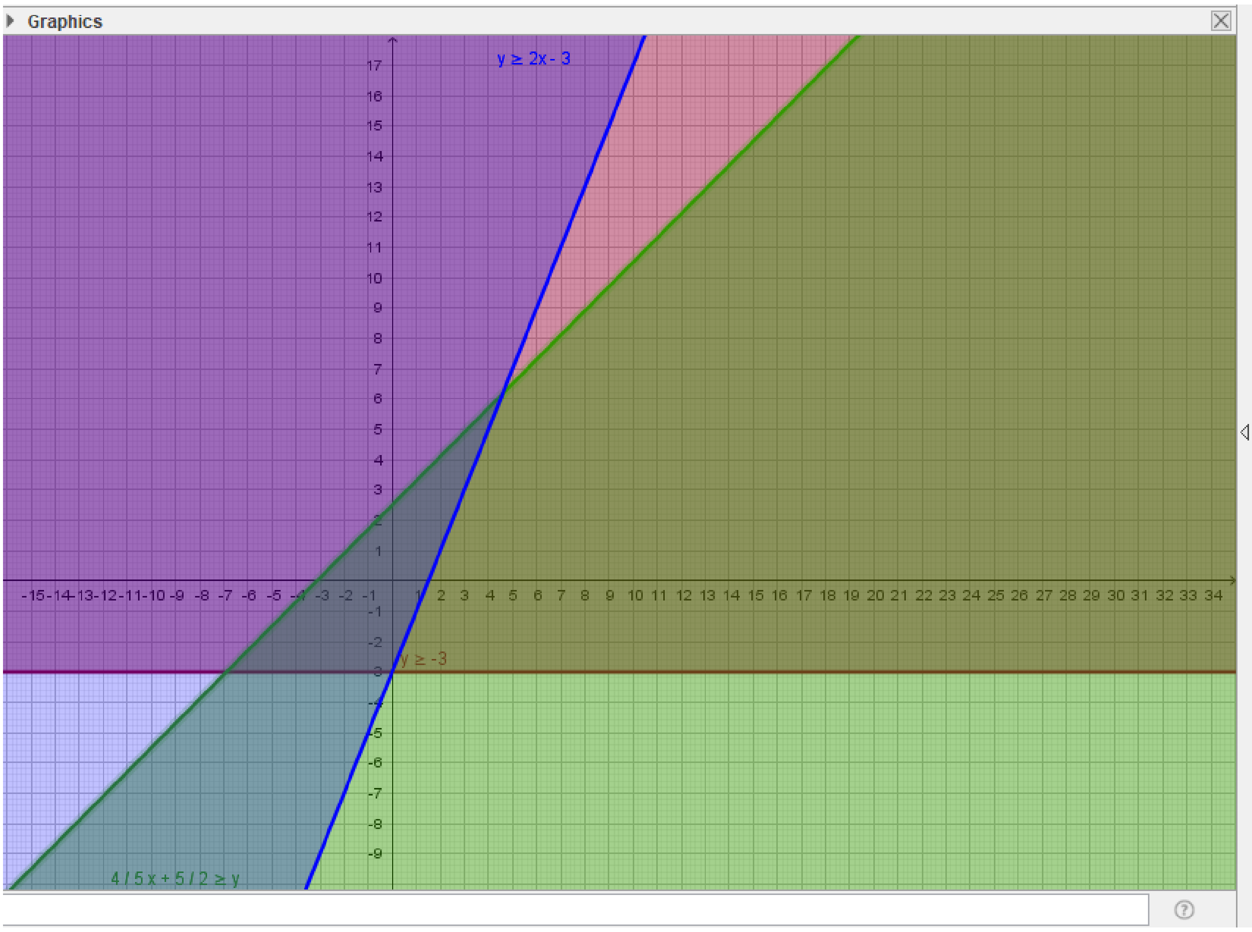
Then, students will be engaged to perform at least one actionable activity such as drawing graphs. In this phase, GeoGebra will be used in order to solve questions presented to the students visualized in the following step by step:



In Next Step: The solution region which is the intersection of the half-planes is shown in a darker shade.



In third Step: Usually only the solution region is shaded which makes it easier to see which region is the solution region.



Wrap-Up—Mathematical Terms Review (5 minutes)

Then students will be encouraged to define the meaning of terms related to this lesson. Moreover, they will be encouraged to discuss each one of the following and keep a running list in a notebook:

Two Variables

Direction of Inequality

Assessment (10 minutes)

Solve the following system, Graphically:

$2x - 3y \leq 12$

$x + 5y \leq 20$

$x > 0$

Home Assignments

Does the number of variable change the inequality? How?

How many possible solutions does an inequality in Two variable have?

Final Note

- It should be noted that GeoGebra will be used each phases if the students feel awkward.

Episode 4

Grade: 8 **Time: 45 Min.**

Topic: factorization

Objective

- To generate the formula of a^2-b^2 using graphic method

Introductory Activity (10 minutes)

Divide the class into eight groups. Each group can split into pairs for this activity.

Students of each group will be asked the following question:

Question: What is factor of a^2-b^2?

The teacher will encourage more and more responses from students. After that students will be engaged in a pair for solving one particular case of this formula. For example: 5^2-3^2

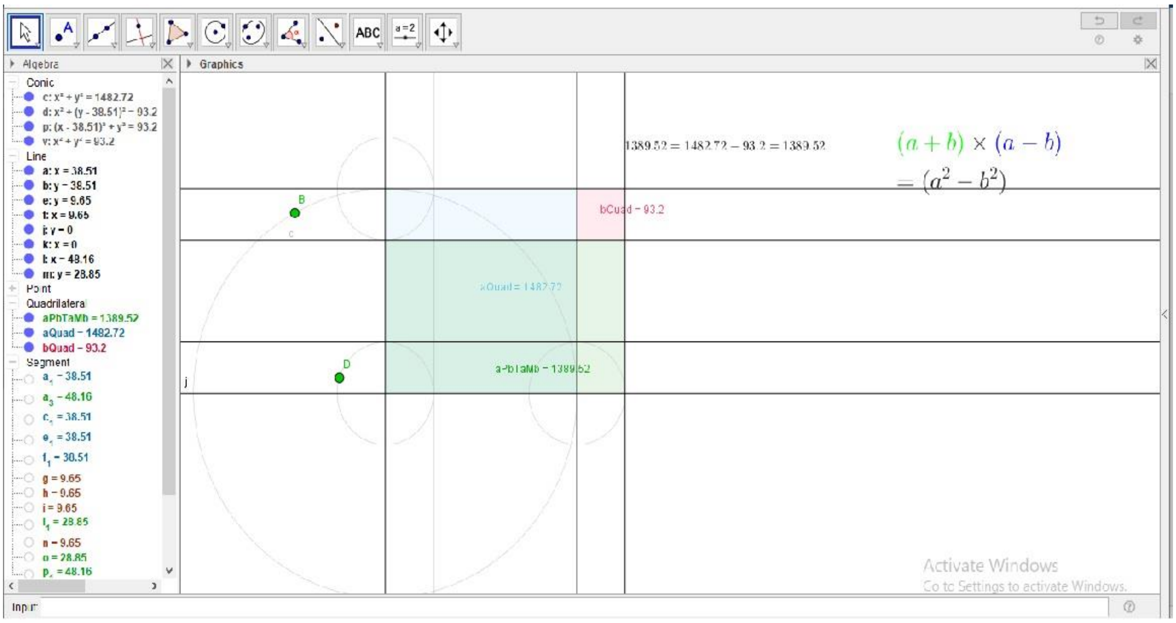
Then students will be encouraged to cut the paper based on their results. For above example: 16 and students will be encouraged to cut paper, whose area is 16.

Class Discussion (20 minutes)

Based on the results of introductory session. Each group of students will have opportunity to share their results to another groups and teacher will ask following questions:

- Does your shape similar to each other?
- How to generalize your results?

Then teacher will use GeoGebra based on the results to generalize the results. This platform would become as the following:



Wrap-Up—Mathematical Terms Review (7 minutes)

Then students will be encouraged to define the meaning of terms related to this lesson. Moreover, they will be encouraged to discuss each one of the following and keep a running list in a notebook:

Square

area

Assessment (7 minutes)

Solve

$4x^2 - 25$

Objective Item Analysis

Kalpavriksha Secondary School, Sarlahi

SN	Upper 27%											Middle 46%																		Lower 27%														
QN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	C	P (%)	D	Remarks
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1	0	1	1	0	0	1	35	87.5	0.36	Delete
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1	0	1	1	1	1	1	1	0	0	1	0	0	1	0	1	0	30	75	0.54	Retain
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	0	0	1	1	1	0	0	0	1	1	1	0	0	0	30	75	0.54	Retain
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	28	70	0.9	Retain
5	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	33	82.5	0.54	Delete
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	0	0	0	0	32	80	0.72	Delete
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	0	0	1	1	0	33	82.5	0.36	Delete
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	31	77.5	0.72	Retain
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	1	33	82.5	0.63	Delete
10	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1	0	1	0	1	1	1	0	1	0	1	0	0	1	1	0	0	0	0	0	27	67.5	0.72	Retain
11	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	28	70	0.72	Retain
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	0	0	28	70	0.72	Retain
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	1	0	1	0	0	0	1	33	82.5	0.54	Retain
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	24	60	1	Retain
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	0	1	0	0	1	0	0	32	80	0.63	Retain
Total	15	15	15	15	15	15	15	15	15	15	14	14	14	14	14	14	14	13	13	13	13	13	13	12	12	12	12	12	12	11	9	5	5	5	5	5	5	3	3	3				

Note:

P= Difficualty level of Items

D=Diiscrimination Index

C= Correct Answer

Interpretation Criteria for P	
Indicator	Meaning
0-39	Very Difficult
40-60	General
61-75	Sustantial
76-90	Easy
91-100	Very Easy

Reference: Khanal, 2012 p.245

Interpretation Criteria for D	
Degree	Meaning
-1 - 0.19	Negligible
0.20 - 0.	General
0.30 - 0.	Good
0.40 - 1.	Very Good

Source: Ebel and Frisbie, 1991 p.132,(as cited in Khanal, 2012)

Subjective Item Analysis

Kalpavriksha Secondary School, Sarlahi

SN	Upper 27%												Lower 27%														
QN	1	2	3	4	5	6	7	8	9	10	11	Total	1	2	3	4	5	6	7	8	9	10	11	Total	D	P (%)	Remarks
16	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	1	0	1	0	0	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	1	0	1	0	0	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	1	0	1	0	0	0	4	0.63	68	
	1	1	1	1	1	1	0	1	0	1	0	8	0	1	0	0	0	1	0	0	0	0	0	2	0.54	90	
Total																									0.6	73	Delete
17	1	1	1	1	1	1	1	1	1	0	1	10	1	1	1	1	1	1	0	0	0	0	0	6	0.36	72	
	1	1	1	1	1	1	1	1	1	0	1	10	0	1	1	0	0	0	0	0	0	0	0	2	0.72	54	
	1	1	1	1	1	1	1	1	1	0	1	10	0	1	1	0	0	0	0	0	0	0	0	2	0.72	54	
	1	1	1	1	0	1	1	0	1	0	1	8	0	1	1	0	0	0	0	0	0	0	0	2	0.54	45	
Total																									0.61	56.25	Retain
18	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	1	0	0	0	0	0	1	4	0.63	68	
	1	1	1	1	1	1	1	1	1	0	1	10	0	1	1	0	1	0	0	0	0	0	1	4	0.54	63	
	1	1	1	1	1	1	0	1	1	0	1	9	0	0	0.5	0	0.5	0	0	0	0	0	0	1	0.72	45	
	1	1	1	1	1	1	0	1	1	0	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0.81	40	
Total																									0.67	54	Retain
19	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	1	0	0	0	1	0.9	54	
	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	1	0	0	0	1	0.9	54	
	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	0.5	0	0	0	0.5	0.95	47	
	1	1	1	1	1	1	0	1	1	1	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0.9	45	
Total																									0.91	50	Retain
20	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	1	1	0	0	0	0	1	1	5	0.54	72	
	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	1	1	0	0	0	0	1	1	5	0.54	72	
	1	1	1	1	1	1	1	1	1	1	0	10	1	0	0	1	0.5	0	0	0	0	0.5	0.5	3.5	0.59	61	
	1	1	1	1	1	0	1	0	1	1	0	8	0.5	0	0	1	0	0	0	0	0	0	0	1.5	0.59	29	
Total																									0.56	58	Retain

21	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	1	0	1	0	1	0	0	5	0.54	72	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	1	0	1	0	1	0	0	5	0.54	72	
	1	1	1	1	1	1	1	0.5	1	1	1	10.5	0	1	1	0	0.5	0	0.5	0	1	0	0	3.5	0.63	63	
	1	1	1	1	1	1	1	0	1	1	1	10	0	0	1	0	0	0	0	0	1	0	0	2	0.72	54	
Total																									0.6	65	Delete
22	1	1	1	1	1	1	1	1	1	1	1	11	1	0	1	1	1	1	1	0	0	1	0	7	0.45	81	
	1	1	1	1	1	1	1	1	1	1	1	11	1	0	1	1	1	1	1	0	0	1	0	7	0.45	81	
	1	1	1	1	1	1	0	0.5	1	1	1	9.5	1	0	0.5	1	1	1	0.5	0	0	0	0	5	0.31	65	
	1	1	1	0	1	1	0	0	1	1	1	8	1	0	0	0.5	1	1	0	0	0	0	0	3.5	0.4	20	
Total																									0.4	61	Delete
23	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	1	0	1	1	0	1	0	0	5	0.54	72	
	1	1	1	1	1	0	1	1	1	1	1	10	1	0	0	1	0	1	1	0	1	0	0	5	0.45	68	
	1	1	1	1	1	0	1	0.5	0	1	1	8	1	0	0	0.5	0	0.5	0.5	0	1	0	0	3	0.45	50	
	1	1	1	1	1	0	1	0	0	1	0	6.5	0	0	0	0	0	0	0	0	0	0	0	0	0.59	29	
Total																									0.5	54	Retain
24	1	1	1	1	1	1	1	1	1	1	1	11	1	1	1	0	1	0	0	0	0	0	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	1	1	1	0	1	0	0	0	0	0	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	10.5	1	1	0.5	0	0.5	0	0	0	0	0	0	3	0.68	61	
	1	1	1	1	1	1	1	1	1	0	0	9	0.5	1	0	0	0	0	0	0	0	0	0	1.5	0.68	47	
Total																									0.65	61	Delete
25	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	0	1	1	1	0	0	5	0.54	72	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	0	1	1	1	0	0	5	0.54	72	
	1	1	1	1	1	1	1	0.5	1	1	1	10.5	0	1	1	0	0	0	0	0.5	1	0	0	3	0.68	61	
	1	1	1	1	1	1	1	0	1	1	0	9	0	0	1	0	0	0	0	0	1	0	0	2	0.63	50	
Total																									0.59	63	Retain
26	1	1	1	1	1	1	1	1	1	1	1	11	1	1	0	1	0	0	0	0	0	1	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	1	1	0	1	0	0	0	0	0	1	0	4	0.63	68	
	1	1	1	0	1	0.5	1	1	1	0	1	8	1	1	0	1	0	0	0	0	0	1	0	4	0.36	54	
	1	0	1	0	0	0	1	0	1	0	0	4	0	1	0	1	0	0	0	0	0	0	0	1.5	0.22	25	
Total																									0.46	53	Retain
27	1	1	1	1	1	1	1	1	1	1	1	11	1	0	1	0	0	1	0	0	0	0	0	3	0.72	63	
	1	1	1	1	1	1	1	1	1	1	1	11	1	0	1	0	0	1	0	0	0	0	0	3	0.72	63	
	1	1	1	1	1	1	1	1	0.5	1	1	10.5	1	0	0.5	0	0	1	0	0	0	0	0	2.5	0.72	59	
	1	1	1	1	1	0	1	1	0	1	1	9	0.5	0	0	0	0	1	0	0	0	0	0	1.5	0.68	34	
Total																									0.71	54	Retain

28	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	1	1	0	1	1	0	0	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	1	1	0	1	1	0	0	0	4	0.63	68	
	1	1	1	0	1	1	1	1	1	0	1	9	0	0	0	1	0.5	0	1	0.5	0	0	0	3	0.54	54	
	1	1	1	0	1	1	1	0.5	0	0	1	7.5	0	0	0	0.5	0	0	0	0	0	0	0	0.5	0.63	36	
Total																									0.6	56	Retain
29	1	1	1	1	1	1	1	1	1	1	1	11	1	1	0	1	1	1	0	0	0	0	0	5	0.54	72	
	1	1	0	1	1	0	1	1	1	1	1	9	1	1	0	1	0	1	0	0	0	0	0	4	0.45	59	
	1	1	0	1	1	0	1	1	1	1	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0.63	40	
	1	1	0	1	1	0	1	0	0	1	0	6	0	0	0	1	0	0	0	0	0	0	0	1	0.45	31	
Total																									0.51	50.5	Retain
30	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	0	0	0	0	0	1	50	
	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	0	0	0	0	0	1	50	
	1	1	1	1	1	1	1	1	0.5	1	0	9.5	0	0	0	0	0	0	0	0	0	0	0	0	0.86	43	
	1	1	0	1	1	1	1	1	0	1	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0.72	36	
Total																									0.89	44	Delete
Total	60	59	56	55	58	51	53	50	52	48	49		24	25	25	23	19	18	13	11	11	7.5	4.5				

Note:

P= Difficualty level of Items

D=Diiscrimination Index

C= Correct Answer

Interpretation Criteria for P

Indicator	Meaning
0-39	Very Difficult
40-60	General
61-75	Sustantial
76-90	Easy
91-100	Very Easy

Reference: Khanal, 2012 p.245

Interpretation Criteria for D

Degree of D	Meaning
−1 - 0.19	Negligible
0.20 - 0.29	General
0.30 - 0.39	Good
0.40 - 1.00	Very Good

Source: Ebel and Frisbie, 1991 p.132,(as cited in Khanal, 2012)

ATT Item Analysis

Kalpavriksha Secondary School, Sarlahi

SN	Upper 27%												Lower 27%														
QN	1	2	3	4	5	6	7	8	9	10	11	Total	1	2	3	4	5	6	7	8	9	10	11	Total	D	P (%)	Remarks
1	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	1	0	0	0	0	0	1	4	0.63	68	
	1	1	1	1	1	1	1	1	1	0	1	10	0	1	1	0	1	0	0	0	0	0	1	4	0.54	63	
	1	1	1	1	1	1	0	1	1	0	1	9	0	0	1	0	1	0	0	0	0	0	0	1	0.72	45	
	1	1	1	1	1	1	0	1	1	0	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0.81	40	
Total																									0.67	54	Accept
2	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	0	0	1	0	0	0	0	0	2	0.81	59	
	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	0	0	1	0	0	0	0	0	2	0.81	59	
	1	1	1	1	1	1	1	1	1	0	1	10	1	0	0	0	0	0	0	0	0	0	0	1	0.81	50	
	1	1	1	1	1	1	0	0	0	0	1	7	0	0	0	0	0	0	0	0	0	0	0	0	0.63	31	
Total																									0.76	50	Reject
3	1	1	1	1	1	1	1	1	1	1	1	11	0	1	0	0	1	0	1	0	0	0	0	3	0.72	63	
	1	1	1	1	1	1	1	1	0	1	1	10	0	1	0	0	0	0	1	0	0	0	0	2	0.72	59	
	1	1	1	0	1	1	1	1	0	1	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0.81	40	
	1	1	1	0	1	1	1	1	0	1	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0.81	40	
Total																									0.76	51	Accept
4	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	1	1	0	0	0	0	0	0	3	0.72	63	
	1	1	1	1	1	1	1	1	1	1	0	10	1	0	0	1	1	0	0	0	0	0	0	3	0.63	59	
	1	1	1	1	1	1	1	1	1	1	0	10	1	0	0	1	0	0	0	0	0	0	0	2	0.72	54	
	1	1	1	0	1	1	1	1	0	1	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0.72	36	
Total																									0.69	53	Accept
5	1	1	1	1	1	1	1	1	1	1	1	11	1	1	0	0	0	0	1	0	1	0	1	5	0.54	72	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	0	0	0	0	1	0	1	0	0	3	0.72	63	
	1	1	1	1	1	1	1	0	1	1	1	10	0	0	0	0	0	0	1	0	1	0	0	2	0.72	54	
	1	1	1	1	1	1	0	0	1	1	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0.81	40	
Total																									0.69	57	Reject
6	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	0	1	0	0	1	0	1	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	1	0	0	0	1	0	0	1	0	0	0	3	0.72	63	
	1	1	1	1	1	1	1	1	1	1	0	10	1	0	0	0	0	0	0	0	0	0	0	1	0.81	50	
	1	1	1	1	1	1	1	1	0	1	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0.72	40	
Total																									0.72	55	Accept

7	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	1	0	0	1	0	0	4	0.63	68	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	0	0	0	1	0	0	3	0.72	63	
	1	1	1	1	1	1	1	1	1	1	1	11	0	1	1	0	0	0	0	0	0	0	0	2	0.81	59	
	1	1	1	1	1	1	1	1	1	1	0	10	0	0	1	0	0	0	0	0	0	0	0	1	0.81	50	
Total																									0.74	60	Reject
8	1	1	1	1	1	1	1	1	1	0	1	10	1	0	1	0	0	0	0	1	0	1	1	5	0.45	68	
	1	1	1	1	1	1	0	1	1	0	1	9	0	0	1	0	0	0	0	1	0	0	0	2	0.63	50	
	1	1	1	1	0	1	0	1	1	0	0	7	0	0	1	0	0	0	0	1	0	0	0	2	0.45	40	
	1	1	1	1	0	1	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0.45	22	
Total																									0.56	45	Reject
9	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	0	0	0	0	0	1	50	
	1	1	1	1	1	1	1	1	1	1	1	11	0	0	0	0	0	0	0	0	0	0	0	0	1	50	
	1	1	1	1	1	1	1	1	0.5	1	0	9.5	0	0	0	0	0	0	0	0	0	0	0	0	0.86	43	
	1	1	0	1	1	1	1	1	0	1	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0.72	36	
Total																									0.89	44	Reject
10	1	1	1	1	1	1	1	1	1	1	1	11	1	1	0	1	1	1	0	0	0	0	0	5	0.54	72	
	1	1	0	1	1	0	1	1	1	1	1	9	1	1	0	1	0	1	0	0	0	0	0	4	0.45	59	
	1	1	0	1	1	0	1	1	1	1	0	8	0	0	0	1	0	0	0	0	0	0	0	1	0.63	40	
	1	1	0	1	1	0	1	0	0	1	0	6	0	0	0	1	0	0	0	0	0	0	0	1	0.45	31	
Total																									0.51	51	Accept
Total	40	40	36	37	38	37	33	35	31	31	28		13	11	10	7	9	5	5	5	5	2	4				

Note:

P= Difficualty level of Items

D=Diiscrimination Index

C= Correct Answer

Interpretation Criteria for P	
Indicator	Meaning
0-39	Very Difficult
40-60	General
61-75	Sustantial
76-90	Easy
91-100	Very Easy

Reference: Khanal, 2012 p.245

Interpretation Criteria for D	
Degree of D	Meaning
−1 - 0.19	Negligible
0.20 - 0.29	General
0.30 - 0.39	Good
0.40 - 1.00	Very Good

Source: Ebel and Frisbie, 1991 p.132,(as cited in Khanal, 2012)