MEANINGFUL LEARNING OF LIMIT AND CONTINUITY IN SCHOOL LEVEL: A CASE STUDY

A

THESIS

BY

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IN THE PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF MASTER OF EDUCATION

SUBMITTED

TO

DEPARTMENT OF MATHEMATICS EDUCATION

CENTRAL DEPARTMENT OF EDUCATION

UNIVERSITY CAMPUS

TRIBHUVAN UNIVERSITY

KIRTIPUR NEPAL

2022

LETTER OF CERTIFICATE

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Prof. Dr. Bed Raj Acharya

Date : ………………………

LETTER OF APPROVAL

This thesis entitled **"Meaningful Learning of Limit and Continuity in School Level: A Case Study"** Submitted by **Mr. Ram Prasad Ghimire** for the partial fulfilment for the requirement of Master's Degree in Education has been approved.

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RECOMMENDATION FOR ACCEPTANCE

This is to certify that **Mr. Ram Prasad Ghimire** has completed his M.Ed. thesis entitled **"Meaningful Learning of Limit and Continuity in School Level: A CaseStudy"** under my supervision during the period prescribed by the rules and regulations of Tribhuvan University, Kirtipur, Kathmandu, Nepal. The study embodies the results of investigation conducting during the period under the Department of Mathematics Education, University Campus, Tribhuvan University, Kirtipur, Kathmandu. I recommend and forward his thesis to the Department of Mathematics Education for final Viva-Voce.

…………………………..

Mr. Krishna Prashad Bhatt

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Date: ……………………

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DEDICATION

This work is heartily dedicated to my respected parents father Bhojraj Ghimire and mother Dhanikala Ghimire, their love, care, support and sacrifices made me a person who I am now.

DECLARATION

This thesis does not contain any other work which is offensive and beyond the copy right norms. To the best of my knowledge and beliefs, this research is truly based on my effort and it does not match with any researches that were published earlier in the university as well as others. I take all the ethical and legal responsibility for submitting this thesis.

…………………..

Ram Prasad Ghimire

ACKNOWLEDGEMENT

I am heavily indebted to my respected supervisor Mr. Krishna Prasad Bhatt, Department of Mathematics Education, Central Department of Education, Kirtipur, Kathmandu. His valuable and constructive suggestions, instructions and scholarly guidance have become the greatest property of the thesis. It was impossible without his constant supervision and intellectual guidance.

At the same time, I am very grateful to my respectful teachers of Central Department of Mathematics Educations for the valuable comments, suggestions and invaluable knowledge. I am also very much indebted to all the teachers, administrations and students studying optional mathematics in class 9 and 10 at Prabhasecondary school for their kind co-operations and help.

I must extend my heartily thanks to the department head of mathematics education Prof. Dr. Bed Raj Acharya for his constant inspirations and suggestions to bring this thesis in this form. I am also thankful with the member of office assistant and Liberians for their support during my study.

Last but not least, I wish to acknowledge my parents and all my friend who always support me to go forward.

……………………..

Ram Prasad Ghimire

Abstract

This study entitled "Meaningful learning of limit and continuity in school level: a case study" is a case study. This study aimed to seek the answer of the question as "are the students able to learn limit and continuity in meaningful way avoiding possible misconceptions?" and suggest some ways to teach limit and continuity so that it would be meaningful for students.

This study was based on social constructivism and expository teaching/learning model in some case. The mixed method study was adopted in this research. For this study 30 students studying optional mathematics at Prabha Secondary School were selected as the sample using conventional sampling method. The data were collected using tools as analytical memo, achievement test and interview. The data obtained from the achievement test were analyzed in two ways: quantitatively and descriptively, and the data obtained from other tools were analyzed descriptively.

The result of the study showed that most of the content added in the school level curriculum can be learned in meaningful ways by the students avoiding possible misconceptions. To teach it in meaningful ways this study has suggested some steps such as: (i) tell the objective (ii) begin with concrete and real life examples (ii) start to discuss about the concept of limit and continuity using graphical representations, for this GeoGebra Applet is more appropriate than traditional ways (iv) discuss about symbol with the help of graphical representations (v) discuss about the solutions procedures of algebraic expressions (vi) use verbal representations only to describe other representations (vii) involve students in all activities and provide time for group

discussions (vii) be aware about the student's misconceptions, use figures, examples and non-examples to avoid misconceptions. This study also concluded that some content such as ∞, $\frac{0}{0}$ could not be learned in meaningful way by the students studying in class 9 and 10, although we can convince the students through the way of discussion using various figures, examples and non-examples.

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ABBREVIATIONS

ZPD: Zone of Proximal Development

CDC: Curriculum Development Center

TU: Tribhuvan University

C.S. & C. R.: Correct solution and Correct reasons

C. S. &P.C.R.: Correct Solution and Partial correct reasons

C.S.: Correct Solution

LHL: Left Hand Limit

RHL: Right Hand Limit

P.C.S: Partial Correct Solutions

S.: Solution

MKO: More Knowledgeable other

CSA: Continuous Assessment System

SEE: Secondary Education Examination

Chapter I

INTRODUCTION

This research will focus on exploring the level of meaningful learning of the chapter limit and continuity in optional mathematics in class 9 and 10. This chapter includes background of the study, statement of the problem, objective of the study, research questions of the study, significance of the study, delimitation of the study and operational definition of the key terms.

Background of the Study

Generally, mathematics has been taken as abstract subject. Many students feel difficulty in learning mathematics. Although twentieth-century mathematics encompasses hundreds of academic specialties, theses specialties all begin with two human activities such as counting and measuring (Cooke, 1997). It is said that mathematics was originated with human civilization. Its origins lie shrouded in the mists of antiquity (Burton. 2011). Some archaeological evidences, such as Ishango bone, show that some sorts of counting procedure was being carried on at very early date. Not only in human activities some psychologists, such as, Koehler, have established that birds and animals also have counting sense. So in this sense learning mathematics is a natural process.

History of mathematics reveals that all the human cultures create and use mathematics but the degree of development may be different. It is commonly accepted that mathematics originated with the practical problems of counting and recording numbers (Burton, 2011). It is said that mathematics came into practice along with the origin of human civilization. In the beginning, different human civilizations created and used mathematics with their practical needs. When people began to live in houses they had to build themselves and manufacture the objects they used. The practical geometry, for instance, was discovered by Egyptians to measure their lands. It was necessary for them because the Nile River obliterates the boundaries between their properties. Similarly, other human civilizations created and used mathematics as per their practical needs, although Greek, since the time of Euclid, developed axiomatic mathematics.

According to Hermann Hankel "In most sciences one generation tears down what another has built and what one has established another undoes. In Mathematics alone each generation builds a new story to an old structure"(Burton, 2011). So in the case of mathematics we have to transmit the accumulated discoveries of previous generations to each succeeding generations. So we must take account previous development in mathematics to learn today mathematics. In short cut, as mentioned by Roger Cooke forming a sort of pyramid there three levels of mathematical sophistication as: (i) Mathematics as use of numbers, shapes and topological considerations to create tools and art and to engage in commerce, this level of mathematical awareness was exhibited by all known societies, (ii) Mathematics as specialized subject of study and (iii) Mathematics as logically organized deductive system. Nowadays students have to learn mathematics as logically organized deductive system. Modern children are not creating arithmetic and geometry; they are learning it as did their teachers (Cooke, 1997). So in this sense students may feel difficulty to learn mathematics which is not matched with their daily activities.

All of our activities ranging from simple daily life activities to science and technology, music and arts, engineering etc. has been affected by the development of mathematics. Although the initial emphasis was on utilitarian mathematics, the subject began eventually to be studied for its own sake (Burton, 2011). Most of the content of optional mathematics are pure mathematical in nature and beyond the simple daily life activities. By this reason most the secondary level teachers are obliged to use "Pi in the sky" model in teaching. Because of the abstractness of the subject only the few number of students choose optional mathematics as their optional first subject. The changing trend in the content of optional mathematics shows that contents has been lowering down from higher mathematics to school mathematics. It may have both beneficial and harmful. Recently (2075/76 B.S.), the curriculum of secondary level of optional mathematics has changed and some contents such as limits and continuity, Cramer's rule, conic section, inversion transformation has added in this curriculum which contents we used to study in higher mathematics curriculum. Among them limit and continuity, which is considered as basic concept of calculus, is the concern of my study.

The watershed in the history of mathematics is the invention of the calculus (Cooke, 1997). The invention of calculus is considered as one of the great intellectual achievement of the 17th century, although the certain pre-figurations of the calculus can be seen in the early work which are, as mentioned by Roger Cooke, can be mentioned as: (i) in the work of Archimedes (ii) in the method used by ZuChongzhi and ZuGeng to find the volume of sphere (iii) in the recursive methods of approximating certain geometric quantities used by the Japanese mathematicians (iv) the infinite series expansions of Hindus, and other places. But the formal development of calculus began with the work of Newton and Leibniz in the seventeenth century. Nowadays, it has been occupying more space in higher mathematics and playing important role in the development of various field. So the meaningful understanding of calculus plays the very important role in the development of various field. In higher mathematics calculus is considered as the central concept and limit and continuity is taken as preliminary knowledge of calculus. The understanding of subsequent concepts is hardly possible if pre-requisite concepts are not clearly established (Areaya&Sidelil, 2012). For instance, the understanding of derivative is not possible without clear understanding of the concept limit. So much attention has been paid to limit and continuity in the field of mathematics education research. In the present curriculum limit and continuity has added in class 9 and 10. As a mathematics teacher, my concern is about the meaningful learning of limit and continuity in class 9 and 10.

Problem Statement

Generally, mathematics takes important position in school level as a compulsory subject. Optional mathematics is chosen by some students as their interest and ability in mathematics in secondary level that is in class 9 and 10 although some private school taught this subject from lower secondary level just like compulsory subject. But in public school some interested students choose this subject as an optional I subject specially in class 9 and 10. There are 24 different subjects in the list of optional first but if we look at practice most of the students choose economic and optional mathematics as optional I subject. From my experience I have found that mainly the following type of students choose optional mathematics as their optional I (i) The student who wants to choose mathematics and science as major subject in his/her higher study (ii) The student who wants to secure higher grade (iii) The student who wants to show that s/he is also talent in mathematics (iv) The students who choose optional mathematics in the request of their guardians. So the students choose optional mathematics with various propose. Whatever intentions the students may have, the meaningful understanding is the concern of my study.

The changing trends in the content of optional mathematics of school level show that some topics have been lowering down from higher mathematics to school mathematics. There are eight chapters altogether in optional mathematics. Among them some contents, such as, limit and continuity which is main chapter among the eight chapter, Cramer's rule which has included in the chapter matrix as a sub-topic, conic section which has included in the chapter coordinate geometry as a sub-topic and inversion transformation which has included in the chapter transformation as a sub-topic, are new in the curriculum changed in 2075/76 B.S.

Among these new content the limit and continuity is the area of my study because "the concepts limit and continuity are central for further mathematics learning" (Amatangelo, 2013) and previous studies show that there are various misconceptions and difficulties even in higher level regarding the conceptual understanding of limit and continuity. But in the present curriculum the chapter limit and continuity has added in school level. Obviously this change can work as a bridge between school mathematics and higher mathematics but my concern is that "can the students learn the concepts limit and continuity in meaningful way in class 9 and 10"?

Objectives of the Study

The general objectives of this study is to explore students' meaningful understanding of limit and continuity in class 9 and 10. To complete this general objective the following specific objectives are considered:

1. To explore the students' meaningful understanding of verbal representation of limit and continuity.
2. To explore the students' meaningful understanding of algebraic representation of limit and continuity.
3. To explore the students' meaningful understanding of graphical representation of limit and continuity.

Justification of the Study

Every research has its own significance. This research was concerned with the meaningful understanding of limit and continuity in class 9 and 10. This research has also suggested some way to teach and learn limit and continuity in more meaningful way. The chapter limit and continuity is new for school level which is considered as basic concepts for calculus and analysis. Moreover, there are many misconceptions and difficulties in learning limit and continuity even in higher level. In such conditions this research has the following significance:

- This research is helpful for every mathematics teachers to teach limit and continuity in meaningful way.
- This research is helpful for every students to learn limit and continuity in meaningful way.
- This research is helpful to avoid possible misconceptions and difficulties.
- This study is beneficial for curriculum designer.

Delimitation of the Study

The delimitations are those factors that limit the scope and define the boundaries of the study. These boundaries are created by the researcher. The delimitations of my study are as follow:

- This study was limited in the secondary school (9 and 10) of Nisikhola municipality of Baglung district, especially at Prabha secondary school.
- This study was focused in optional mathematics of grade 9 and 10.
- This study was focused only in the course content of limit and continuity added in class 9 & 10.
- The meaningful understanding of function and sequence & series are the pre-requisite of this research.
- During this study the researcher worked as a mathematics teacher at Prabha secondary school of the Nisikhola municipality and other respondents was selected as per researcher's purpose and convenience with non-probability sampling from the Nisikhola municipality.
- This study is qualitative and descriptive in nature so the finding of the research are not so generalized.

Operational Definition of Key Terms

Meaningful learning. The learning in which learner can explain a particular concept with examples and non-examples, can solve the problem with appropriate reasons, can demonstrate with suitable graphical representation and appropriate reasons is called meaningful learning.

Limit and continuity. During this research, the learning of limit and continuity means the learning only that content of limit and continuity added in the present (2075/76 B. S.) curriculum of optional mathematics of class 9 and 10.

Verbal representation. In this study, the verbal representations means the expressions of the notion and problems of limit and continuity in words.

Algebraic representation. In this study, the algebraic representations means the expressions of the notion and problems of limit and continuity in algebraic forms.

Graphical representation. In this study, the graphical representations means the expressions of the notions and problem of limit and continuity in graph and in geometric figures.

Misconceptions. Those incorrect conceptions used by students while reasoning about any mathematical concepts (or skill or principle) as correct reasoning.

School level. In this research school level means the study up to class 10

Higher level. In research higher level means the study after class 10.

Chapter II

REVIEW OF RELATED LITERATURE

Literature reviews are critical evaluations of material that has been already published (APA, 2010). Literature review is basis for research in every academic field. Literature review is secondary source and do not report new or original experimental work but it includes the current knowledge including substantive finding as well as theoretical and methodological contributions to a particular topic. As a researcher I have reviewed some related literatures as follows:

Review of the Empirical Literature

Areaya and Sidelil (2012) made an article on the topic "Students' Difficulties and Misconceptions in learning concept of Limit, Continuity and Derivative". This study aimed at examining students' difficulties and misconceptions in learning concepts of calculus at preparatory secondary school of Dire Dawa city. This study was done using descriptive survey approach involving one-hundred thirty-five study. This study has concluded that students can perform procedural aspects of the mathematics better than the conceptual aspects, significant number of students were able to compute derivative of a function but failed to relate with its application, students also have difficulties in algebraic manipulation. This study has also concluded that students have some misconception such as the limit of a function at a point is always equal to the functional value at the point, some students considered limit values are always computed just by substitution. In addition, this study has asserted that teaching-learning process in mathematics classroom as well as the practice of assessment were dominantly procedural. Because of inadequate and poor level of understanding of pre-calculus concepts, lack of commitment on the part of students, traditional beliefs that calculus is difficult and teachers' pedagogical limitation students' conceptual understanding of concepts of calculus has been being influenced. To overcome these problem, the researchers have advised that teachers while teaching calculus concepts must take pedagogical-content care, provide highly vivid illustrations by the way of maximizing students' understanding and avoiding possibilities of misconception on the part of their students, use visual demonstration in teaching calculus.

Amatangelo (2013) conducted a thesis entitled "Students understanding of Limit and Continuity at a point: A Look into Four potentially Problematic Conceptions" at Brigham Young University. Questionnaires were administrated to 861 students in various mathematics courses and interviews were conducted with nine first calculus to get an idea of how students reason about continuity and limit and how that influences whether they use the PPCs where the PPCs means the four potentially problematic conceptions which are as: 1) if the function is defined at a point, it is continuous at that point 2) the limit value and function value are the same at a point 3) if the limit exists at a point, the function is continuous at that point 4) if we take the limit at a point the function must be discontinuous at that point. The result showed evidence of holding the four PPCs with decrease in these conceptions typically after they took a course in analysis. The result also showed that students also did not understand the formal definition of a limit until they took a course in analysis. This study has concluded that students were able to reason appropriately using many different conceptions of continuity and while considering the limit conceptions students using a dynamic conception of limit tended to be better able to reason about continuity and limit at a point and students who did not use a dynamic conception of limit tended to use PPCs in general and incorrectly more often.

Karates, Guven and Cekmez (2011) did a study on the topic "A Cross Age study of students' Understanding of Limit and Continuity concepts". Achievement test was performed with sample of 268 respondents with 61 grade 12 students, 73 first year pre-service mathematics teachers, 60 second year mathematics teachers and 74 third year pre-service mathematics teachers which were selected randomly from two cohort secondary schools and department of elementary mathematics education at Karadeniz Technical University in the city of Trabzon in Turkey. The aim of this study was to reveal concept development and the way limit and continuity concepts are understanding by students from different level of education. For this purpose a test comprising open-ended question was administrated to students from the different level of education in order to evaluate the students understanding of limit of continuity concepts about verbal, algebraic and graphical representations of concepts. In this test the respondents had to choose correct answer with reasons. Data was analyzed using traditional statistical means and standard deviation and the data were analyzed categorically. The result showed that the pre-service teachers in their 3rd year of study were found much less successful than other students in algebraic, verbal and graphical representations of limit and continuity concepts. This study has suggested that while designing the instructional activities verbal, graphical and algebraic representations should be prioritized to enhance the development of students' interpretation skills of different representation of functions.

Shrestha (2017) conducted a case study entitled "Problems faced students in learning mathematics: An interpretive inquiry. This case study was conducted on

Learning Zone Academic School of Gangabu, Kathmandu choosing 6 students of grade 10 and two mathematics teachers and one head teacher as respondents of this study using purposive sampling. This study was focused on exploring the problem faced by students in learning mathematics and suggesting remedial way to overcome these problem. The data were collected using classroom observation form and interview guideline. The data were collected using qualitative approach. This study had concluded that because of lack of proper interaction between students and teacher. Discrimination of teachers, socio-economic conditions of students, less motivation students have been feeling mathematical anxiety and she has suggested that we should create child friendly environment, apply equality and equity pedagogy, use contextual pedagogy, involve parent and create group discussion.

Kafle (2019) conducted a case study entitled "Conceptual and Procedural difficulties in learning Derivative". This case study was conducted at Makwanpur Multiple Campus of Makwanpur district focusing on 40 students of grade of XI. The data were collected by using CPUBT and in-depth interview on the basis of APOS theory. This study was focused on finding the conceptual and procedural difficulties in learning derivative and had concluded that students had weak conceptual understanding about derivative and both students and teacher have been focusing on procedural understanding because of their exam orientation. At last his suggestion was that teacher should change their teaching style so that it will make classroom more fruitful and avoids rote learning.

Reflection on Literatures

From the above literature review we can see that various studies have been doing regarding the topic limit and continuity but most of the studies start from class 11 and 12.

Almost all studies showed that there are various type of students' misconceptions and teacher's weakness in teaching strategies. Such misconceptions and various difficulties about the learning of limit and continuity have found in the learning of students studying higher level of mathematics but my concern is that how the students learn limit and continuity in class 9 and 10. Can we teach these concepts avoiding misconceptions? Can the students learn limit and continuity meaningfully? From the suggestions of various literature I saw that there is a problem in teaching strategies also, most of the teachers insist in procedural understanding rather than conceptual understanding and promoting rote learning. The previous studies suggest that to allow the students' better understanding of the concept limit and continuity concepts students' interpretation skills of different representation of function should be emphasized.

The different between the previous studies and my study is that my study is focused on the students of class 9 and 10 studying optional mathematics as their optional first subject and in this research I tried to find out the answer of the question "is meaningful learning of limit and continuity possible in class 9 and 10?" following the advices provided by the previous studies.

Review of Theoretical Literature

Social Constructivism

This study was conducted through the social constructivist lens. Social constructivism is a learning theory developed by Russian psychologist L. V. Vygoksky (1896-1934). According to the social constructivism knowledge is constructed through interaction with others and problem solvers must see a problem from different perspectives. Social constructivism views mathematics as a social construction (Ernest, 1991). According the Paul Ernest, social constructivism is a descriptive as opposed to a prescriptive philosophy of mathematics. Knowledge is socially constructed by learners who convey their meaning making to others (Upadhyay, 2067 B. S.). Interaction and collaboration is needed for meaningful learning. In this sense, the concept of MKO (More knowledge others), ZDP (Zone of proximal development), and scaffolding are the main contributions of social constructivism.

ZDP means the difference between capacity of solving problem by learners themselves and solving problem from the support of other (Acharya, 2072). It is said that mathematics is also social product so that the concept ZPD can play significant role in teaching mathematics. If there is zone of proximal development, the learner can get full learned. This development can be obtained from the social interaction, So that mathematics should be taught by method of collaboration with the group work under the guidance of capable peer which is the main concept of ZDP.

Expository Teaching/Learning Model

Meaningful verbal learning is a learning theory developed by David Paul Ausubel in 1960s. David Paul Ausubel was an American psychologist who advanced a theory which contrasted meaningful verbal learning from rote learning. The learning theorist David P. Ausubel argued that expository teaching was the only efficient way to transmit the accumulated discoveries of countless generations to each succeeding generations (Bell, 1978). Ausubel identified that the reception learning using expository teaching method is the most effective and useful for helping students to construct new knowledge. So in this sense, Ausubel meaning verbal learning has given idea that how can we organized lecture type method to teach a discipline such as mathematics to make learning more meaning for students. According to the expository teaching model the model for teaching any mathematics topic which is new for students should include seven to nine teacher directed activities, which can be shown in the following table.

S.N.	Activity	Mathematical objects for which the activity is appropriate
1	Discuss objectives with students.	skill, concepts, principle
2	Name the skill, concept, principle.	skill, concept, principle
3	Identify and discuss prerequisite skills, concepts, and principles through a preassessment strategy.	skill, concept, principle
4	Develop the skill through an example. Define the concept. Deduce or demonstrate the principle.	Skill concept principle
5	Demonstrate the skill, concept, or principle through several more examples.	skill, concept, principle
6	Have students develop the algorithm for the skill. Compare examples and non-examples of the concept. Apply the principle in several cases.	Skill concept principle
7	Have the students practice the skill on several exercises. Have students identify irrelevant dimensions of the concept. Evaluate student mastery of the principle through a post assessment strategy.	Skill concept principle

8	Evaluate student mastery of the skill. Have students practice using the concept.	Skill Concept
9	Evaluate student mastery of the concept.	Concept

(Source: Bell, 1978, p.224)

Generally, this research has seen through constructivist lens but the chapter limit and continuity is totally new for students studying in class 9 and 10, so while giving the basic information and practical examples as a teacher the researcher should take initiation. So in such case the expository teaching model organized in meaningful way, as argued by Ausubel, is effective and efficient method for meaningful learning. In this phase the teacher as the researcher, because of some fundamental reasons, in the present context, we have to adopt lecture method of teaching but we should organize it in more meaningful way. Because of the some fundamental reasons such as existing physical infrastructure, course amount and time duration it may be problematic to use methods such as discovery, laboratories etc. So the teacher should take initiation and inform students by motivating them. So in this conditional expository model is the appropriate model for teaching the chapter limit and continuity. While using the expository teaching model modern computer based technology, such as GeoGebra and mathematica, were also used for different representations of limit and continuity that were helpful to avoid possible misconceptions.

In the second phase the researcher as teacher used the concept of Social constructivism, namely the concept of ZDP. For this, 30 students were divided into 4 group, the teacher playedthe role of facilitator and created the environment of interaction and motivate students to help each other. Some questions were raised and it left for students for discussion.

Conceptual Framework

The conceptual framework is the general road map of any study. It is the researcher's explanation of how the research problem would be solved. The conceptual framework of this study is as follows:

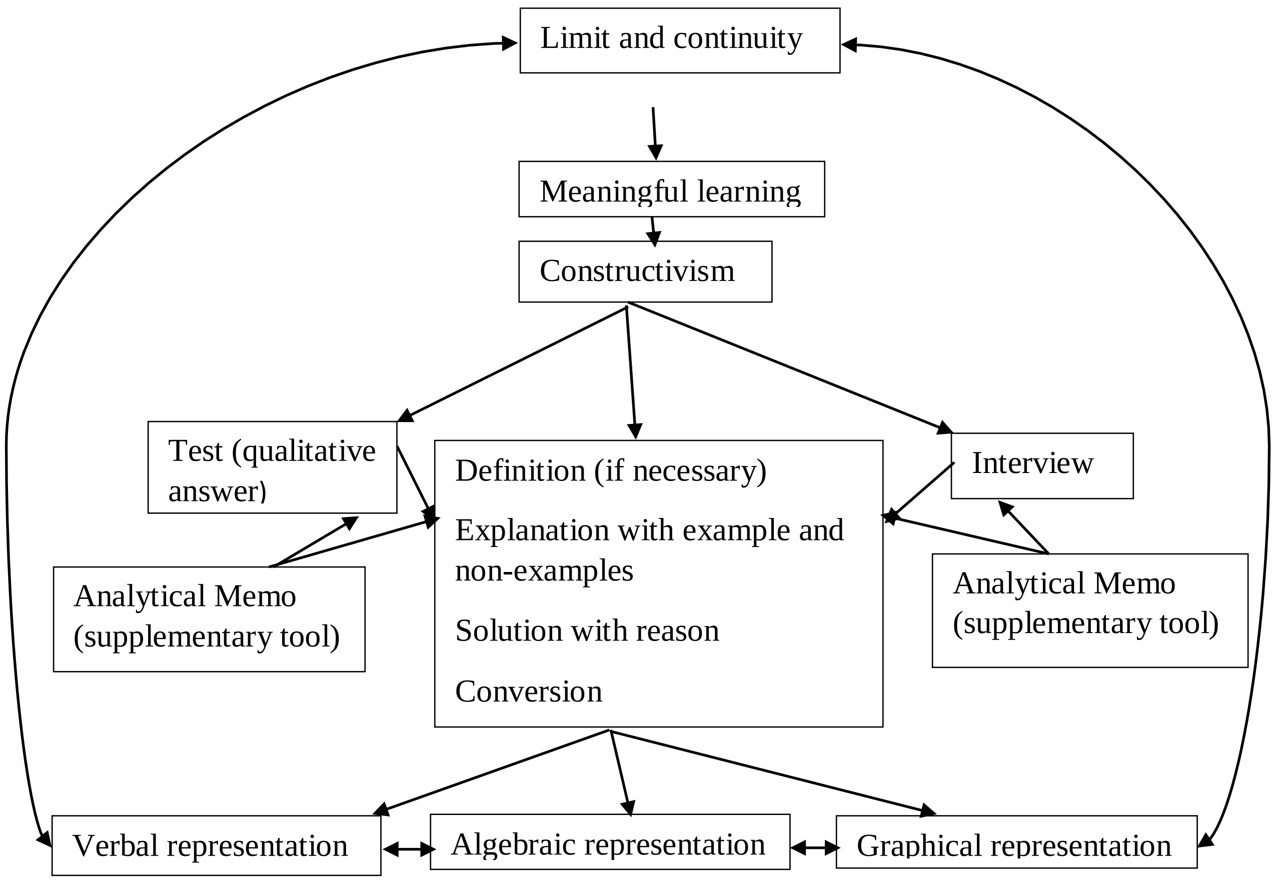


Figure 1: Conceptual Framework

The research was based on the above conceptual framework. The answer of the question "Can the students learn limit and continuity meaningfully in class 9 and 10?" was found using the tools such as achievement test, interview and analytical memo as supplementary tool. Analytical memos writing were taken as the continuous process and were written by analyzing students' responses in daily class activities during teaching and learning period of limit and continuity. The achievement tests were conducted after completing the learning activities. In the test questions were involved being based on the specific objectives expecting both quantitative and qualitative answers. The interviews were administrated to the students being based on the students' responses in the test in order to conform that students' responses in the test was not because of rote learning. The whole process was based on the social constructivism and expository teaching learning model in some case and for the learning to be meaningful students should be able to give appropriate reasons, explanation with examples and non-examples and convert one representation into another with appropriate demonstrations. The results were found being based on the data obtained from these three tools.

Chapter III

METHODS AND PROCEDURES

This chapter deals with the procedure of the study. It determines how the research becomes complete and systematic. The method applied in this study as discussed in the following sections: research design, population of the study, sample of the study, source of data, tools/instruments, data collection procedure, scoring procedure and data analysis procedure.

Design of the Study

This is a mixed method study. Mainly this study is qualitative in nature. Some quantitative method were used to help qualitative method. At first quantitative method was adopted and this became helpful for qualitative method. Case study was adopted as design of this study. Case studies are analyses of persons, events, decisions, periods, projects, polices, institutions or other systems that are studied holistically by one or more methods (Thomas, 2011). As a research design, the case study claims to recommend a wealth and depth of information which is not usually offered by other methods (Astalin, 2013). Case studied is considered as multipurpose research method and may be descriptive or explanatory. Like survey, it may be both qualitative and quantitative and it may employ any and all tools of data collections from testing to interviewing. This case study was conducted at Prabha secondary school of Nisikhola municipality as the researcher is the mathematics teacher of this school.

Research site/ Respondents of the Study

All the students studying optional mathematics (about 200) in Nisikhola municipality are the population of this study. For the study, all students studying optional mathematics (30) in Prabha secondary school were selected as the sample of the study using convenience sampling method where the researcher himself is a teacher of the school.

Data Collection Tools

Data collection tools refers to the instruments used to collect data such as interview guidelines, questionnaires, achievement test etc. Data collection tools are taken as the most important aspect of any research field. It is important to decide the tools for data collections because research is carried out in different ways for different purposes. For this study achievement test, analytical memo, interview guidelines were used as main data collection tools during this study.

Achievement Test

Achievement test is a test of knowledge and skills developed in students after teaching a particular subject matter. It is mostly used in educational area to determine the level of instruction for which a student is prepared. Achievement test is an important tool to collect the data for the study related to educational field. By the nature of the study the achievement test playedan important role. Researcher developed the achievement test with the help of other experience teacher, available subject expert and specification grid of optional mathematics of grade IX and X developed by CDC Nepal 2076BS. All type of questions i.e. questions to measure knowledge, understanding, ability and higher ability was involved in the test.

For the pilot test, achievement test was developed involving the same level of questions asked in SEE with the help of specific objectives, specification grid and subject expert. But while administrating the test for the research with the help of subject experts, experience teachers qualitative part, such as how, why etc., were added in every questions being based on specific objectives in order to gather richer information about the students' meaningful understanding of limit and continuity. The marking scheme was as (a) For correct response with correct reason 4 points, (b) For correct response with partially correct reason 3 points, (c) For correct response 2 points, (d) For partially correct response 1 points and (e) For no response 0 points.

Item Analysis of the Test

Researcher conducted pilot test to measure the quality of questions. For standardized the questions, mainly difficulty index (P- value) and discrimination index (D- value) were calculated. After analyzing the items, researcher prepared a set of questions for pre- test and post- test.

Interview Guidelines

Interview is another data collection tool of this research. Interview was conducted to in order to collect richer information about the students' meaningful understanding of limit and continuity. An open-ended interview was held, so it was not be in pre-determined format. But generally it was based on student's performance in achievement test. Being based on the achievement test, more questions such as why, how, what, were asked in interview in order to gather richer data about the students' understanding of limit and continuity. Both group and personal interviews were conducted after completing the achievement test.

Analytical Memo

Analytical memo is another supplementary data collection tool of this research. Learning is ongoing process and CAS is considered as the most important aspect in educational field. The aforementioned tools test and interview were used after the teaching learning activities, but in such situation various important aspects of learning process may be missed. So, to involve students' feeling during learning period of limit and continuity in this research analytical memos was used as supplementary data collection tool of this research. For this, the researcher wrote memo after analyzing teaching learning activities and students' reactions during the teaching learning process of limit and continuity.

Reliability and Validity of the Tool

In this study, researcher conducted a pilot test to measure the P- value and D-value. Using the same result of pilot test reliability was estimated. For this, split- half method was used. To estimate the reliability of the tool, the formula developed by Spearman- Brown was used. The interview also administrated to the students being based on achievement test. But it is an in-depth interview so it was not in pre-determined format.

To establish the validity of achievement test, it was prepared with the help of supervisor, subject expert being based on every specific objectives presented in the curriculum. And "what type of qualitative aspect should be added in the questions while conducting test and interview?" was decided with the help of supervisor and subject experts.

Data Collection Procedures

This is a case study, so the sample of the study may be small in size. All the students of class 9 and 10 ofPrabha secondary school were selected as the sample of the study. The related data were collected from the sample. For this, as a mathematics teacher of this school, the researcher made close contact with the students and administration. After this the related data were collected with the help of in-depth interview, mathematical achievement test and memo writing.

During this study the teaching learning activities were run being based on social constructivism and expository teaching model was also used, and achievement test was be made as it is. The researcher wrote memo writing from the beginning up to the end of the data collection. Generally the interview were conducted at the end after administrating the test. It was not conducted in pre-specified format but theoretically it was based on the social constructivism

Data Analysis and Interpretation Procedures

After collecting the data, the next step is to analyze and interpret the collected data. In this phase, the researcher use the collected data to understand what actually happened in the studied case. The basic objective of this step is, as in other analysis, to derive the conclusions from the data keeping a clear chain of evidences.

This is a case study and as case study research is flexible research method, both qualitative and quantitativedata analysis procedures were adopted in this research. Mainly the qualitative technique was used and the quantitative were also used while analyzing the achievement test. For this the traditional statistical technique such as weighted mean was calculated. This result became helpful to administrate the interview. As in others qualitative research, general inductive approach was adopted for this study. The general inductive approach begins with detailed observations of the world and theories or generalizations are proposed towards the end of the research process as a result of observations i.e. inductive research involves the search for pattern from the observations and development of explanations theories. As mentioned by Thomas in the article entitled "A General inductive approach for analyzing qualitative evaluation data" published in Americans Journal of Evaluation the procedures for inductive analysis of qualitative data are as follows:

1. Preparation of raw data files (data cleaning)
2. Close reading text
3. Creation of categories
4. Overlapping coding and un coded text
5. Continuing revision and refinement of category system

The researcher adopted inductive approach for this study. For this the recorded interviews were transcribed properly and the rough memo writings were also written fairly. After continuous reading of the transcriptions time and again, the transcriptions were coded and these code were categorized into different categories. Finally, themes were generalized from these categories.

The data obtained from mathematical achievement test were also analyzed meaningfully and logically.

Quality standard

This is a mixed method study and mainly it is qualitative in nature, it is not possible to use statistical technique to maintain quality standard of qualitative data and result. To maintain the quality standard of the qualitative data and result, the following facets of quality were adopted:

Credibility

This concepts replaces the idea of internal validity. It tries to seek the answer of the questions such as "Do participants feel that the finding represent their experience?" and for this I followed some techniques as triangulations, prolonged engagement with participants and negative case analysis.

Transferability

Transferability replaces the idea of external validity. It tries to seek the answer of the questions such as "Are the findings applicable in other context?" and to maintain this I provided rich descriptions of participants responses and researchers responses.

Dependability

Dependability refers to the idea of reliability i.e. it tries to seek the answer of the questions as "Would similar finding be produced if someone else also undertook the research?" and to maintain this I followed some techniques as triangulations and auditing.

Conformability

In qualitative research, conformability refers to the researcher's concern with objectivity i.e. it tries to seek the answer of the questions such as "Are the findings a product of participants' responses and not the researcher's biases, motivations interests and perspectives?" and to maintain this I followed the techniques such as triangulations, auditing.

In sum up, it is a case study and is both qualitative and quantitative in nature. To maintain the quality standard of the quantitative data, pilot test was conducted. For the standardization of the test, mainly the difficulty index (P-value) and discrimination index (D-value) was calculated. And to maintain the quality standard of qualitative data and result, some techniques such as triangulations, thick descriptions, auditing obtained data, prolonged engagement in the field, member checking, negative cases analysis, avoiding individual bias were followed.

Ethical Considerations

As a researcher I am a permanent mathematics teacher teaching at Prabha secondary school. I took the permission with school administrations and students for the research. During the research period for data collections procedures extra time may be needed in such condition the permission was taken from school administrations, guardians and students. Students was informed about all activities during the research. All the responses of participants were respected. The name of particular participants has not mentioned without their permission. Any individual information have not published without permission. After completing the research also the researcher provided debriefing to the participants. The conclusion of the research were seen from the participant's point of view also.

Chapter IV

ANALYSIS AND INTERPRETATION OF DATA

This chapter deals with the analysis and interpretation of collected data. In this study, achievement test, interview and analytical memo were used to collect data. The achievement test and interview were used as the main data collection tools and the analytical memo was used as supplementary tool. The data were collected from the students studying optional mathematics in class 9 and 10 at Prabha secondary school. This is a case study and mixed method has adopted but quantitative data were collected to help qualitative data. So descriptive method was adopted to analyze the data.

After collecting the data, the next step is to analyze and interpret data as per the nature of collected data and purpose of the study to come up with the solution of the problem. To collect the data, I used achievement test, interview and analytical memo-writing as the data collection tool. This is a case study aimed to find out the level of meaningful understanding of the chapter limit and continuity in class 9 & 10 and both qualitative and quantitative in nature. Mainly it is qualitative in nature but quantitative data was used as the helping point for qualitative data. So, at first, the data obtained from the achievement test were analyzed quantitatively in which simple technique such as weighted mean was calculated. And the data obtained from the achievement test and analytical memo-writing were analyzed analytically and descriptively.

This is a case study about students' meaningful learning of limit and continuity in class 9 and 10. To meet the objectives of the research I collected the data from the Prabha secondary school situated at Nisikhola municipality 3, Baglung. There were 30 students in class 9 and 10 studying optional mathematics. Firstly I conducted achievement test among the students expecting qualitative answers. Being based on the students' performance interview was conducted with 10 students selected being based on the student's performance with the purpose of finding the deeper information.

The collected data were analyzed by dividing into the mainly three heading as verbal expression, algebraic expression and graphical or geometrical expression.

Verbal Representation:

Here the verbal representations means the expressions of the notion and problems of the limit and continuity in words. In this section, the notion and problems of limit and continuity were expressed verbally, that is, in this section questions were asked without using the algebraic expressions and geometrical figures. The students' responses in the questions added in Appendix-A are as follows:

Class 9

Table 4.1: *Result of student's response in verbal representation of limit*

Q. No.	No. of student's response with…					Mean
	C. S. & C. R.	C. S. & P. C. R.	C. S.	P. C. S.	No. S.	
1(i)	1	0	10	0	6	1.4
1(ii)	1	2	11	0	3	1.88
1(iii)	0	3	9	0	5	1.47
2(i)	5	6	4	2	0	2.82
2(ii)	2	3	10	1	1	2.29
2(iii)	3	4	8	1	1	2.23

2(iv)	1	2	11	1	2	1.94
2(v)	1	3	10	1	2	2
2(vi)	1	2	10	3	1	1.94

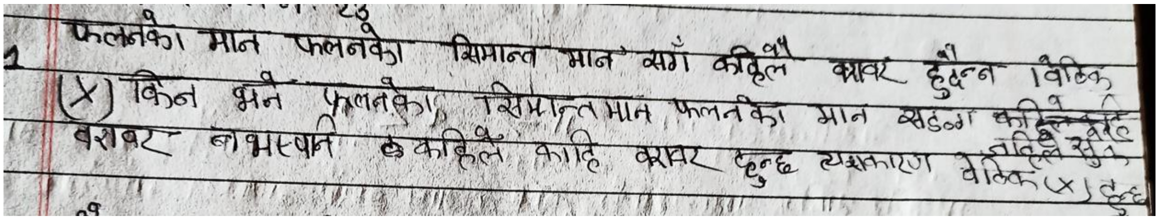
The information from the table show that mean marks in the questions representing verbal representation of limit is between 1.4 and 2.8. In 5 questions the mean marks is less than 2. Generally, this shows students' poor performance in verbal representation. In some questions, the mean marks is 2 or more than two. This shows that some students can give the answer with reasons. But this ratio differs questions wise. In questions 1(i), 1(ii) and 1(iii) many students have not given any response. In the questions 2(i), 2(ii), 2(iii), 2(iv), 2(v), 2(vi) all most students have given correct response but failed to give correct reasons. This also differs questions-wise. Generally, in the questions involving the concept of infinity students have not given the correct response with correct reasons but in the other questions some students have given partially correct response.

In the first three questions, where the students were to write true or false for the given statements and then the students were asked to give appropriate reasons for their answers, the students did poor performance. About 30 to 40 percent students became fail to differentiate true or false for the given statements. About 60 to 70 percent students have chosen the correct response but none of the students has given the reason for their response. It may have two reasons as (i) students do not know the appropriate reasons (ii) students do not understand questions and can't explain even knowing the answers.

In the first three questions the students had to differentiate as true or false and give to reasons for their choice. Lets' see some students' responses:

Question: The limiting value of a function never equals to functional value. ….

Because…..



I: Can you give me an example in which the functional value and limiting value are equal to each other?

Students: Sir I know that in some case they may be equal but I can't give example.

I: Can you show me by drawing figures?

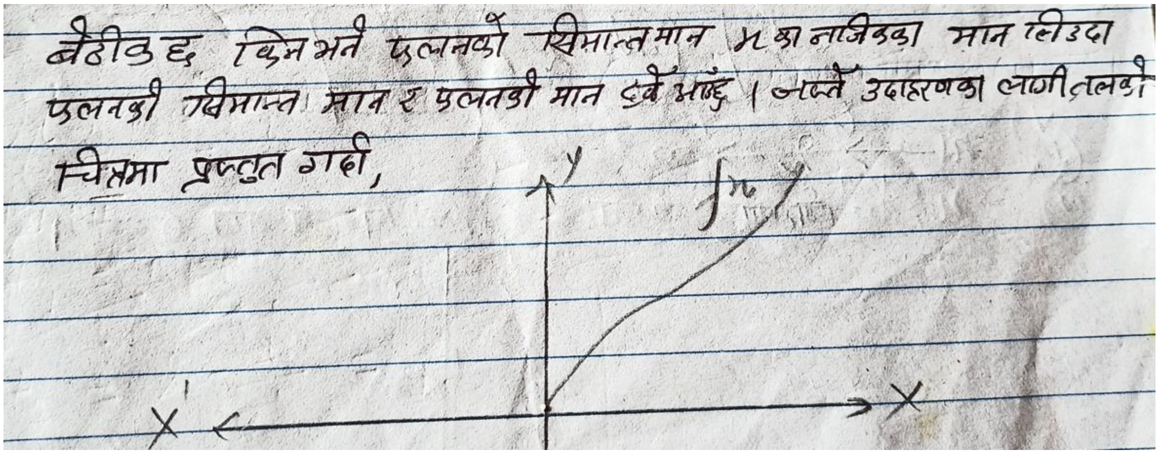
Student: Sir I have written some examples and drawn figures in my note copy but now I forgot sir.

I: Did you copy without understanding?

Student: No sir, at that time I had understood but now I forgot sir. Sir if you give me a chance, tomorrow I can explain it with example.

I: it’s ok.

This conversation shows that this student somehow has written correct response. To clarify this I asked him an example, he failed to give example. He told that "I forgot sir, but I have written on my note copy." This shows that the students have remembered something told by teacher but could not make his own conceptual understanding.



This students' response is correct but the reason seems insufficient. The figures shows that in such type of function the functional and limiting value are equal to each other's, but the students can't express in this way.

I: What is the meaning of the figures drawn by you?

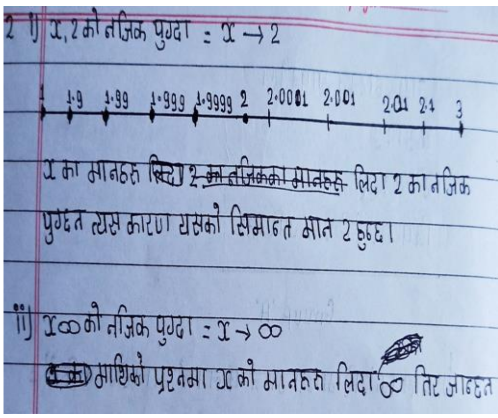
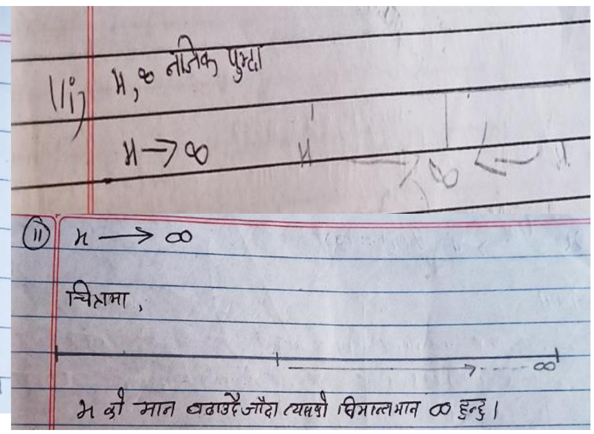
Student: I meant that in such type of function the limiting value and functional value are equal to each other at a point.

I: It's ok, but are you think that is the reason given by you correct?

Student: No sir, I have written the reason on my note copy. I had understood at that time but now I could not remember sir.

This conversation repeats the same problem that students only attempt to remember note copy and class discussion but could not make conceptual understanding. Most of the students have not written reasons on these questions and who attempted to write could not explain clearly. This shows that either the students did not understand meaningfully or the students could not understand questions and reply it verbally.

In the first three questions almost all students could not write appropriate reasons but in the remaining six questions, where students were to write the given sentence in symbol and explain it using figures, some students became able to write correct response with appropriate reasons. In the case of infinity, students could not draw the appropriate figure but almost all students have written correct answer in symbolic form. Lets' see some models:

The above figures show that almost all students have given correct response, but we can see some problem while drawing figures. Most of the students could not draw the figure of questions regarding infinity. Most of the students could not express the concept of ∞ in figures. Some students thinks it as other numbers and draw the figures as others. In the individual or group discussions the students have given three type of answers as (i) I could not understand about infinity (ii) I know but I forgot how to draw figures (iii) infinity means larger than any number (only one students). This shows that many students could not understand ∞ meaningfully but we can convince them through the way of individual or group discussion using various figures. But the students have done better in these questions than in the first three questions.

Class 10

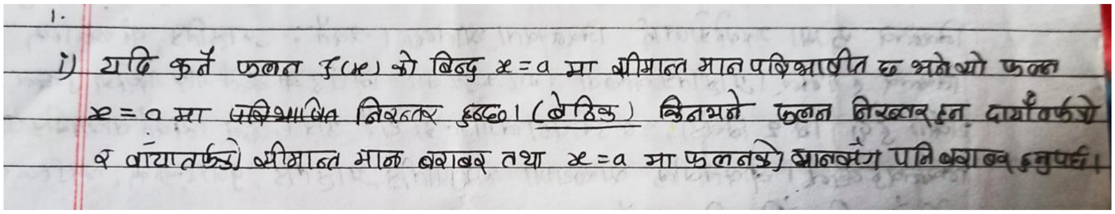
Now lets' see class 10 students' responses in the questions added in the Appendix-D

Table 4.2: *Result of student's response in verbal representation of limit and continuity*

Q. No.	No. of student's response with…					Mean
	C. S. & C. R.	C. S. & P. C. R.	C. S.	P. C. S.	No. S.	
1(i)	3	1	8	0	0	2.58
1(ii)	2	3	2	5	0	2.16
1(iii)	2	2	8	0	0	2.5
2	3	2	5	2	0	2.5
3	2	4	4	2	0	2.33
4	1	4	7	0	0	2.5

The above table shows that the mean marks in the questions representing verbal representation of continuity is between 2.16 and 2.58. In all questions, the mean marks is more than 2. This shows that the students have done better than in class 9. Most of the students have given correct responses but failed to give correct reasons. Only few number of students have given correct reasons. At least 1 students has given appropriate reasons in every questions. This gives us the significant meaning. It means that there is at least one students who can understand the notion of continuity meaningfully.

Let's see the student's responses in some questions as model:


1.
i) यदि कुनै फलन f(x) को बिन्दु x = a मा सीमान्त मान परिभाषित छ भने त्यो फलन x = a मा परिभाषित निरन्तर हुन्छ। (बेठिक) किनभने फलन निरन्तर हुन दायाँतर्फको र बायाँतर्फको सीमान्त मान बराबर तथा x = a मा फलनको मानसँग पनि बराबर हुनुपर्छ।

This students which is name as A has responded perfectly. Similar type of questions were asked in interview as follows:

Teacher: Do you think that your reason is correct?

Student: Yes. LHL, RHL and functional value at a point must be equal for a function to be continuous at that point.

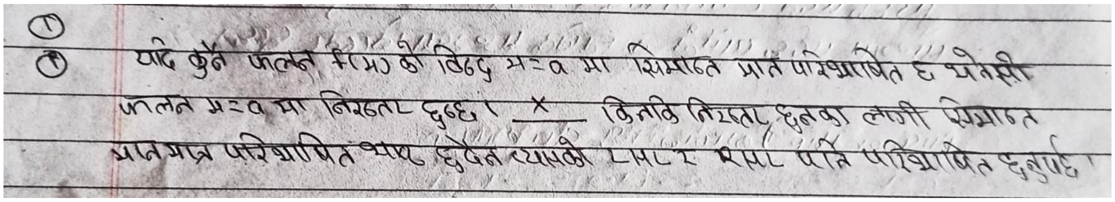
Teacher: Can you give example in which $\lim_{x \to a} f(x)$exists but the function is discontinuous that $x = a$.

Student: Umm, I can't give example in algebraic form, I have no idea about it but I can solve problem if you asked me. But I can make figure in which$\lim_{x \to a} f(x)$ exists but the function is discontinuous that $x = a$.

Teacher: Then show me

Students: Sir, we need not go far. In questions number 3 from group C, $\lim_{x \to 4} f(x)$ exits but $f(4)$ is not defined. So the function is discontinuous that point, you can see hole in the graph of the function at $x = 4$.

The student A has given similar type of reasons in most of the questions. This shows that the student A is able to understand limit and continuity in meaningful way. He can explain, give examples and non-examples. This shows that this student has learned limit and continuity in meaningful way from his level. Similarly other students have also given correct response but failed to give appropriate reason, although they have done better than in class 9. Let's see one as example.


यदि कुनै फलन f(x) को बिन्दु x=a मा सिमान्त मान परिभाषित छ भने सो फलन x=a मा निरन्तर हुन्छ। ___×___ किनकि निरन्तर हुनका लागी सिमान्त मान मात्र परिभाषित भएर हुँदैन त्यसको LHL र RHL पनि परिभाषित हुनुपर्छ।

This students has given the correct response but the reason is insufficient. In the individual discussion this student told that "I became confusion while writing reason". Then I showed some related graphs and this students found his insufficiency in the reasons and told the correct reasons.

These students could not write correct reasons. In the interview also they could not clarify their answers. Their expressions showed that they could not understand questions clearly and could make a clear figures of answers if we asked them questions in verbal representations only. Not only this, according as my teaching experience, students could not understand the concept of limit and continuity if we explain it only by verbal representations. The group discussion showed that the graphical representation of limit and continuity is more appropriate for avoiding possible misunderstanding.

One of the students from class 10 have given the correct reasons in almost all questions. This students secures more than 90 marks in most of the exam of mathematics. This also shows that the meaningful understanding depends on the level of students. In mathematics, some students try to understand the solutions steps but not meaning. This habit also effects in meaningful leaning. So every mathematics teacher should motivate students to learn mathematics in meaningful way from earlier class.

Algebraic Representations:

In this section, the students were asked to response to the problem of limit and continuity expressed in algebraic form. In the section, some questions were asked using symbol of limit and continuity. Sometimes students were asked to write the meaning of symbol.

Class 9:

The following tables shows the students' response in the questions added in Appendix-B.

Table 4.3: Result of student's response in algebraic representation of limit

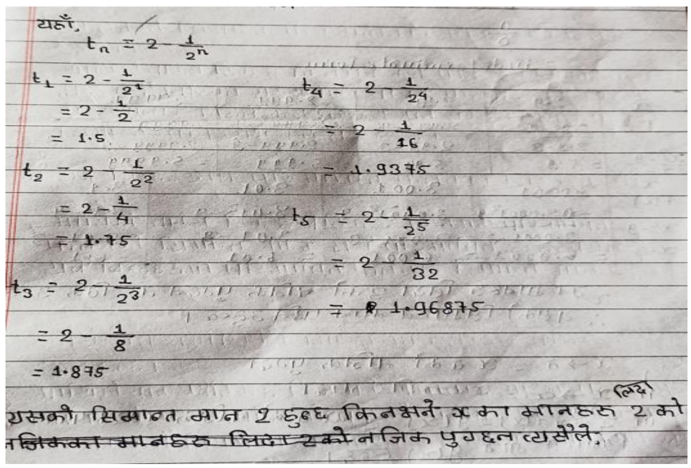
Q. No.	No. of student's response with…					Mean
	C. S. & C. R.	C. S. & P. C. R.	C. S.	P. C. S.	No. S.	
1	6	4	4	3	0	2.76
2	9	2	3	3	0	3
3	2	7	5	3	0	2.47
4(i)	7	5	5	0	0	3.11
4(ii)	4	3	10	0	0	2.64
4(iii)	2	4	11	0	0	2.47
4(iv)	1	4	12	0	0	2.35
4(v)	4	3	10	0	0	2.64
4(vi)	1	2	14	0	0	2.23

The above table shows that the students' mean marks in the questions expressed in algebraic form is between 2.23 to 3. In the every questions the mean marks is more than

2. In some questions the students have given response with appropriate reasons but in some questions the students' failed to give reasons to their response.

Lets' see the students' response in some questions:

Question: Find the limiting value of the sequence $t_n = 2 - \frac{1}{2^n}$with suitable reason. If possible, also demonstrate using figure.



Being based on the student's solutions I asked some questions as below:

I: In the given questions, you have found only first five term. What will happen more terms, that is, $t_6, t_7,\ t_8, t_9, \ldots ....$

Students: The value of these termswill approach to 2.

I: Not equal with 2?

Student: No

I: Why?

Student: Sir, I do not know the reason.

Then, I said if we subtracted something from 2, it would be equal to or less than or more than 2?" and after then I asked the student to discuss with his friend. After sometimes I asked the same questions, now do you know reason? Then the student replied that "yes sir, now I know" and he explained giving other examples also.

While teaching such type of questions I had also taught giving various examples, but the students could not remember and make conceptual understanding. So the above discussion shows that we should give chance to interact each other between the students. While teaching this type of questions, we need to explain them expressing in graphical form, this representation is more appropriate to avoid possible misunderstanding.

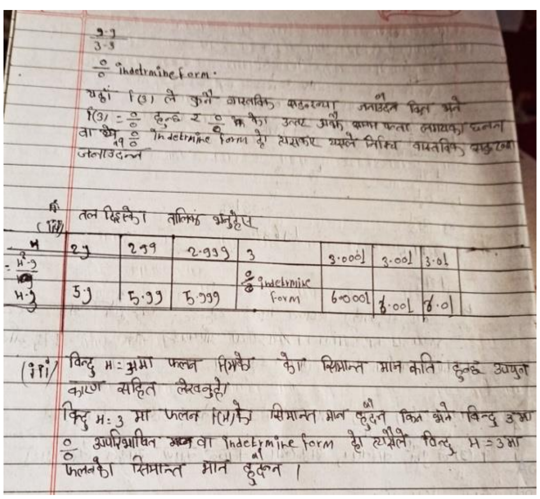
Question: On the basis of the function defined by $f(x) = \frac{x^2-9}{x-3}$ give the answer of the following questions:

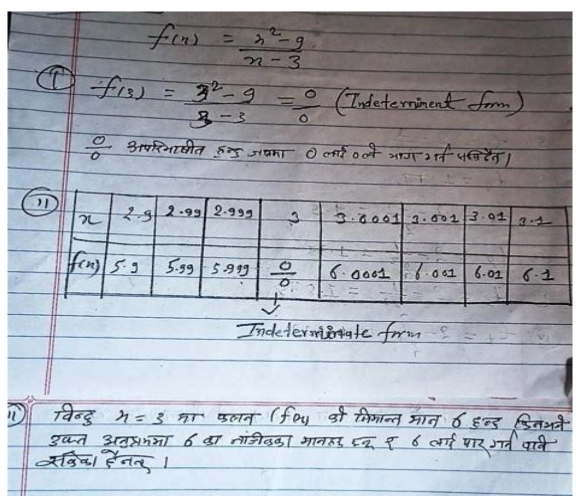
(i) Does $f(3)$ represent any real number? Explain.

(ii) Fill the following table

x	2.9	2.99	2.999	2.999	3	3.0001	3.001	3.01	3.1
$f(x)$									

(iii) What is the limiting value of the function $f(x)$ at $x = 3$? Give the answer with suitable reason.





In this type of questions, most of the students have written correct response except some students. Here the first students have written that the function do not have limiting value at $x = 3$ because $f(3) = \frac{0}{0}$(indeterminate form). This shows that the students have some misconceptions about limit.

In this type of questions, most of the students have written correct response except some students. In the above figures, the second student have given the correct response. In the similar way other students also have given the correct response but failed to give the correct reasons. To clarify this I presented the example and students convinced with my logic but failed to explain in their own words. In the group discussion, students repeat the similar type of confusion. They write correct answers as they have understood but they failed to give appropriate. While teaching also I felt difficult to connect with reasons. But if we use graphical representation students feel easy and can give appropriate reasons. The students' response, result of achievement test and individual and group discussions all show that the graphical representation is more appropriate for school level students.

So we can teach and they can learn the most of the content of limit presented algebraically in meaningful way. The students can solve the problem and give reasons for their solutions. The above discussion also shows that the average students studying optional mathematics cannot understand the meaning of $\frac{0}{0}$, but through the way of discussion, we can convince the students that $\frac{0}{0}$ is an indeterminate form, it do not represent certain real numbers. But again if we asked the students about it, students would not explain about it.

Class 10:

Now let's see the student's response in the questions added in the Appendix-E

Table 4.5: *Result of student's response in algebraic representation of limit and continuity*

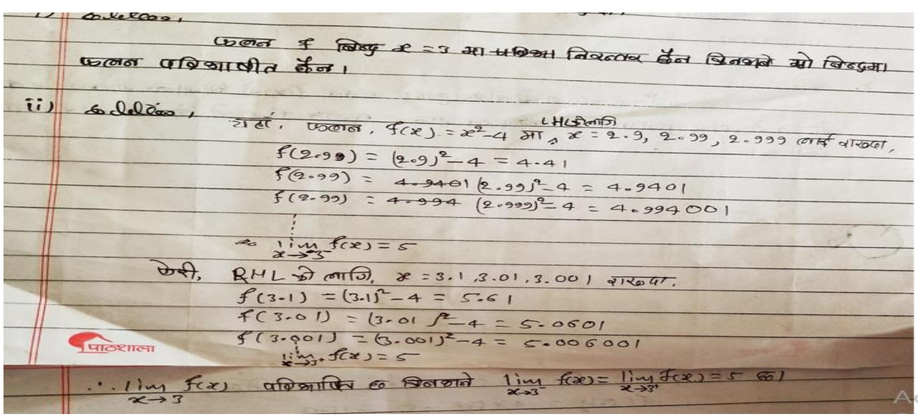
Q. No.	No. of student's response with…	Mean

	C. S. & C. R.	C. S. & P. C. R.	C. S.	P. C. S.	No S.	
1	8	2	0	2	0	3.8
2	9	3	0	0	0	3.75
3	10	2	0	0	0	3.8
4	10	2	0	0	0	3.8
5	3	7	2	0	0	3.08

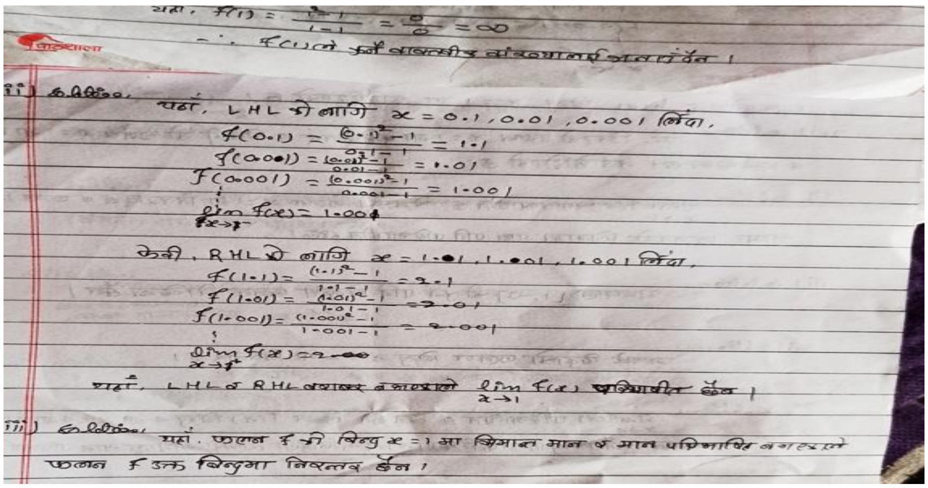
The information from the above table show that the mean marks in the questions representing the algebraic representation of continuity is between 3.08 and 3.8. In every questions the mean marks is more than 3. Most of the students have given correct response with appropriate reasons and few number of students have given partial correct reasons.

Let's see the students' responses in some questions as model:

At first, lets' see the students' 10A response in the following question
Given that, $f(x) = x^2 - 4 \quad if\ x \neq 3$,
(i) Is the function f given above continuous at $x = 3$?Explain?
(ii) Does $\lim_{x\to 3} f(x)$ exist? If so, write your answer with reason.



For the first question, that is the question of continuity, one of the students from class 10 has responded very shortly. His logic is that given function has no definition at $x = 3$, so that the function is not continuous at that point. The student 10A has given the similar type of answer in most of the questions. He has responded with suitable reasons. In the interview also he have given good response in most of the concept. Now let's see his response in the next question.



This students from class 10 has responded with reasons in most of the questions but in this questions, he wrote that $\frac{0}{0} = \infty$. He did another mistake while taking nearest value of 1 from the left side. I gave him his paper and asked some questions as:

I: In this questions, you have done some mistake. Can you find out your mistake?

Student: (After few minute later) yes sir, I did mistake while taking the value of x for LHL.

I: Then what should we take?

Student: $x = 0.9, 0.99, 0.999, \ldots\ldots$

I: It's ok, and do we know the value of $\frac{0}{0}$?

Student: No sir, it is indeterminate form.

I: Then why did you determine that $\frac{0}{0} = \infty$?

Student: Sir I thought that they are same. In $\frac{0}{0}$, 0 is in denominator, so in this sense I wrote that $\frac{0}{0} = \infty$.

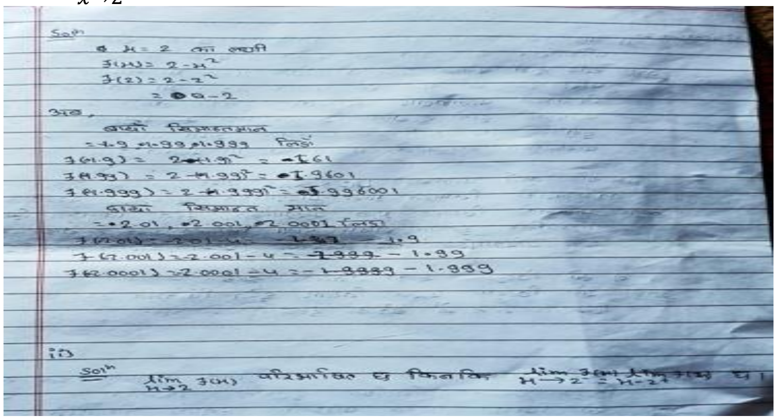
The above discussion shows that sometimes students adopt wrong procedure by mistake and did mistake in algebraic representations. This student has responded with correct reasons in similar type of questions. But in this questions, he did mistake while taking the value of x for LHL. This shows that students should do more practice to do better in algebraic representations. The conceptual understanding is not enough for the algebraic representations. The above discussion also shows that the school level students could not understand $\frac{0}{0}$ in meaningful way, although we can convince the school level students that $\frac{0}{0}$ is an indeterminate, do not represent a certain real number through the way of discussion using various examples.

Now lets' see the student 10B response in the question given as, a function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2 - x^2, & if\ x \leq 2 \\ x - 4, & if\ x > 2 \end{cases}$$

(i) Is the function f given above continuous at $x = 2$?Explain.

(ii) Does $\lim_{x \to 2} f(x)$ exist? If so, write your answer with reason.



Almost all students have responded in the similar way in such type of questions. The above figure shows that the school level students can choose appropriate option in step function.

I: Why did you choose $2 - x^2$ to calculate f(2), f(1.9), f(1.99), f(1.999) … … and $x - 4$ to calculate f(2.1), (2.01), f(2.001) … …

Student: Sir, the functions has been defined in this way.

I: What does it mean? Clarify.

Student: Sir, in its definition the function has two expressions, namely $2 - x^2$ and $x - 4$, in which first one holds for all value of x less than or equal to 2 and the second one is true for all values of x greater than 2. So I did in this way.

From the above discussion and student's solution, we can say that the school level students can choose appropriate option in step functions. They can find the limiting value of the step function at a point. They can draw graph of step functions.

Graphical Representations:

Here the graphical representation means the expressions of the notion and problem of the limit and continuity in the form of figures. In this section, questions were asked showing the figures. Some figures were drawn in graph showing relation of x and y, and some figures were drawn in general plane copy representing the sequence.

Class: 9

Let's see the result of students response in the questions added in Appendix-C:

Table 4.5: *Result of student's response in graphical representation of limit*

Q. No.	No. of student's response with…	Mean

	C. S. & C. R.	C. S. & P. C. R.	C. S.	P. C. S.	No. S.	
1	5	6	3	3	0	2.76
2	6	6	2	3	0	2.88
3	8	4	4	1		3.11
4	5	4	5	3	0	2.64
5	9	5	3	0	0	3.55

The above table shows that the mean marks in the questions expressed in figures and graph is between 2.64 and 3.55. The table also shows that many students are able to give complete or partial reasons for their responses. The above information also shows that students' have done better in the questions expressed geometrically than in other representations.

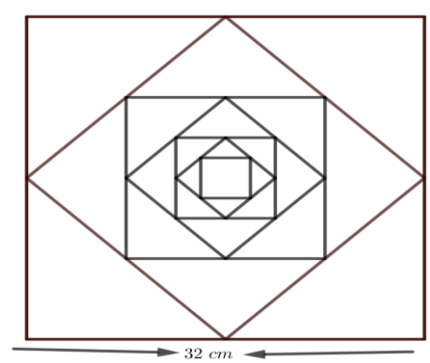
Lets' see students' response in some questions.

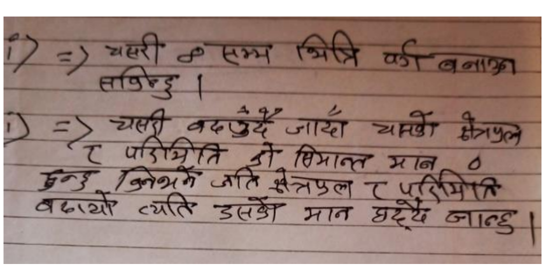
Question: In the figure given alongside, it is shown the sequence of squares formed by joining the midpoints of sides.

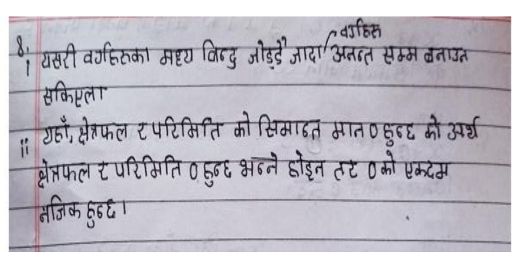
Answer the following questions:

(a) How many polygons can be formed?

(b) What are the limiting values of perimeters and areas of the squares? Explain your answer with reason.




i) => यसरी ∞ सम्म त्रिभुज वर्ग बनाउन सकिन्छ ।
ii) => यसरी बढाउँदै जाँदा यसको क्षेत्रफल र परिमिति को सिमान्त मान 0 हुन्छ किनभने जति क्षेत्रफल र परिमिति बढायो त्यति उसको मान घट्दै जान्छ ।


8. i यसरी वर्गहरुका मध्य बिन्दु जोड्दै जादा वर्गहरु अनन्त सम्म बनाउन सकिएला
ii यहाँ, क्षेत्रफल र परिमिति को सिमान्त मान 0 हुन्छ को अर्थ क्षेत्रफल र परिमिति 0 हुन्छ भन्ने होइन तर 0 को एकदम नजिक हुन्छ ।

Being based on the student's response have asked some questions as:

I: Here you have written that we can make squares up to ∞. What does it mean?

Students: Sir, in such condition we can make a new square continuously in every step.

I: How is it possible?

Student: Sir, here we are making new square by joining the mid-point of previous square. Here the mid-point means the half part remains in every steps, so we can make a new square by joining the mid-point of the remaining parts.

I: In question, it was asked to write a number. But you have written ∞, is it a number?

Students: Sir, one man may say I can make 10 squares, then another man can make 11th square by joining the mid-point of the 10th square and so on. So in this sense we wrote∞.

I: (For the first Student)here you have written increasing the area and perimeter its value decreases, how is it possible?

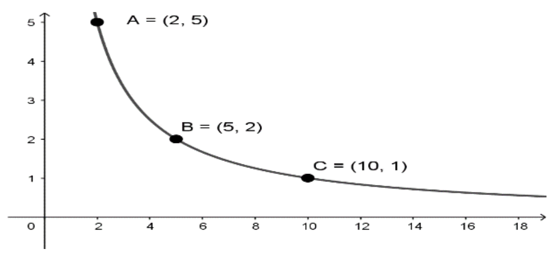
Student: (After looking paper) No sir, I wrote it by mistake. I mean to write increasing the number of squares.

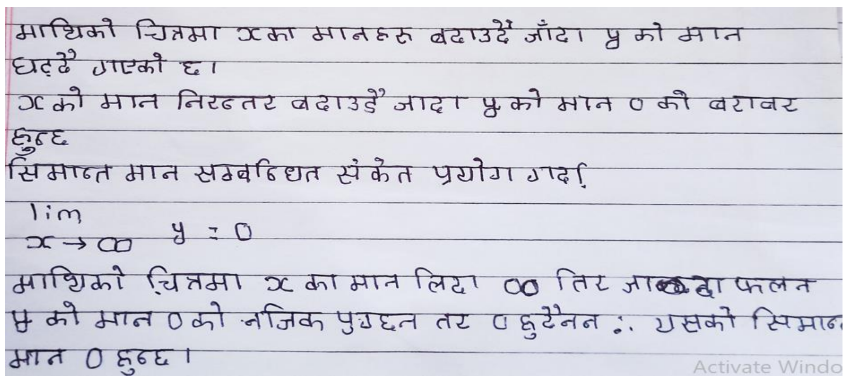
I: Then, how is the limiting value of the area and perimeter 0?

Students: Sir, we are making new square by joining the mid-point of the previous square, so every succeeding square becomes smaller than the previous. Then the area and perimeter of the smaller square is less than the previous one. So if we increase the numbers of square, the perimeter and area of the square tend to zero.

Similarly let's see another student's response in the following question which is also related with infinity.

Question: In the graph given alongside, the values of y have been decreasing as x increases. If we continuously increase the value of x, what would be the value of y? Explain it using limit notion.




माथिको चित्रमा x का मानहरु बढाउँदै जाँदा y को मान घट्दै गएको छ।
x को मान निरन्तर बढाउँदै जाँदा y को मान 0 को बराबर हुन्छ
सिमान्त मान सम्बन्धित संकेत प्रयोग गर्दा

$$\lim_{x \to \infty} y = 0$$

माथिको चित्रमा x का मान लिँदा ∞ तिर जाँदा फलन y को मान 0 को नजिक पुग्छन तर 0 हुँदैनन ∴ यसको सिमान्त मान 0 हुन्छ।

The above figures and the discussions shows that students studying in class 9 can response questions of limit related to infinity if we asked the questions using geometrical representations. The students could not response properly while asking the questions using verbal and algebraic expressions in the similar type of questions.

Class 10:

Now let's see how do class 10 students response in graphical representations. The table given below shows the student's response in the questions added in the Appendix- F

Table 4.6: *Result of student's response in graphical representation of limit and continuity*

Q. No.	No. of student's response with…					Mean
	C. S. & C. R.	C. S. & P. C. R.	C. S.	P. C. S.	No S.	

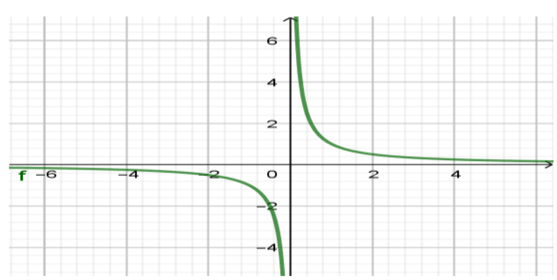
1.	5	4	2	0	1	3
2.	6	3	2	1	0	3.16
3.	4	5	3	0	0	3.08
4.	6	4	2	0	0	3.33
5.	5	4	2	0	1	3
6.	6	4	2	0	0	3.16
7	4	4	2	2	0	2.83
8	4	3	4	1	0	2.83
9	6	2	4	0	0	3.16

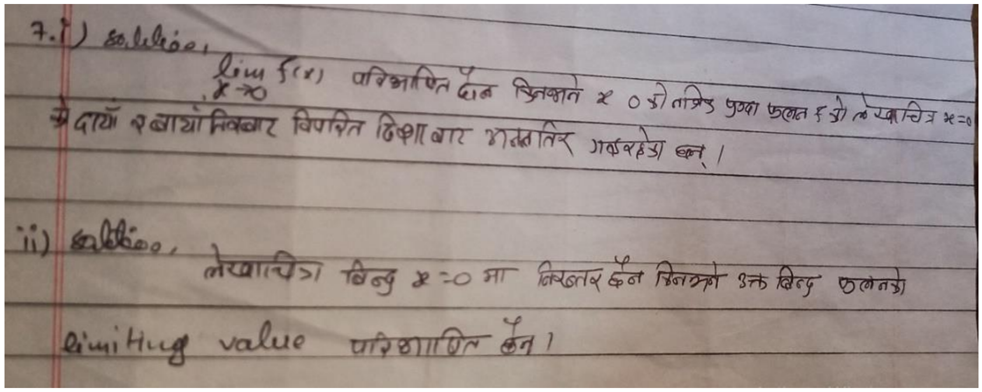
The above table shows that the students' mean marks in the questions expressed geometrically is between 2.83 and 3.16. In most of the questions students obtained marks near to 3. The most of the students have given the correct reasons or partial correct response, although some students could not give reasons but they have given the correct responses.In graphical representation, the number of students giving correct response with correct reasons is more than in that of other representations. This also shows that students have better understanding in geometrical representation than in other representations.

Lets' see some students' response as a model:

Lets' see the student 10A response in the question given by

(i) Does $\lim_{x \to 0} f(x)$ exist? Justify your answer.
(ii) Is the function continuous at x=0?




7. i) Solution,
lim f(x) परिभाषित छैन किनभने x 0 को नजिक पुग्दा फलन f को लेखाचित्र x=0
x→0
को दायाँ र बायाँ निकट विपरित दिशातर्फ अनन्ततिर गइरहेको छ ।

ii) Solution,
लेखाचित्र बिन्दु x=0 मा निरन्तर छैन किनभने उक्त बिन्दु फलनको
limiting value परिभाषित छैन ।

From the above answer, it can be seen that the student has better understanding in geometrical representations. The mean marks obtained by students is more in geometrical representation than in other representations. Most of the students have given response with reasons. About this I discussed with students in group which as follows:

I: I were asked you questions dividing whole paper into three group, namely group A, group B and group C. Which group do you feel easy to give answer?

Students: Group C sir (Almost all students)

I: Why?

Students: Sir, in group C, we were to answer by looking figures. We can clearly see in figures, so we can give answer and reasons to answer.

I: Then what happened in Group A and Group B?

Students: In group A, questions were asked only using sentence, we could not understand the questions. In group B, at first we need to solve problem. After solving the problem, we felt difficult to write suitable reasons because it is not clearly seen just like in figures. And one of the student also told that "sir in the classroom also we understood clearly in which you used figures and examples and non-examples".

The above discussion shows that students can understand the concept of limit and continuity if we present it graphically. Moreover, students understand easily and reply properly if we ask question using geometrical representations. So the graphical representations of limit and continuity is more appropriate for the school level students. This will be more helpful for conceptual understanding and will help the students to learn higher level content in higher level. To avoid the possible misconceptions, the graphical representation is more appropriate because the students can visualize directly what we describe. For example, while inscribing polygons inside a polygons, students can see the areas of the polygons approaching the area of the circle. So in this way, the graphical of limit and continuity seems more appropriate for school level students to develop intuitive idea about it.

Chapter V

FINDING, CONCLUSION AND IMPLICATIONS

This chapter deals with the summary, findings, conclusions, implementations and recommendations of the study. All these are based on the analysis and interpretation of the study. This section is divided into three sub-sections namely, summary of the study, finding of the study, conclusion of the study and recommendation for educational implication.

Finding and Discussion

Verbal Representation

Students felt difficult with verbal representation of limit and continuity. While teaching if we explain the concept of limit and continuity verbally, students do not understand meaningfully. They may write correct response by rote learning but if we check the level of their understanding, it would be in low level. They can't give reasons, example and explain about their answer.

While asking the problem also the verbal representation is not good for school level students. From the discussion it seems that students do not understand the questions asked in this way. If we asked the same question in other way, they would reply properly. We saw it in various questions and discussions. So as a teacher, we should only use the verbal presentations while giving concrete examples, explaining graphical presentations and explaining the solution of algebraic representations.

Algebraic Representation

In the algebraic representation we see that most of the students only try to remember procedural steps, they escape the meaning. Almost it became their habit. So a teacher should be aware and motivate them for conceptual understanding. If we took pedagogical content care, most of students would understand notion of limit and continuity in school level.This study also shows that most of students do not understand the meaning of the indeterminate form$\frac{0}{0}$. But their level of understanding in school level will be helpful to understand the meaning of the indeterminate form $\frac{0}{0}$ in class 11. But sometimes the misconceptions may lead to wrong direction. So a teacher should be aware about it. So while teaching the concept of limit and continuity in school level, we should not begin with algebraic expression. If the students became clear with the concept of limit and continuity, we should begin to solve the problem given in algebraic form.

Graphical Representation

From the analysis and interpretationof the collected data, it can be seen that students have done better in graphical representations. In this sections questions were asked using figures. In most of the questions, students have given correct responses and also have given correct reasons except in some students. This shows that students can understand the concept of limit and continuity but the concept should be presented through figures and graphs. In the same questions asked via verbally and algebraically students could not give reasons but students have responded with reasons when we asked the questions graphically. So for the school level students, most of the concept of limit and continuity should be presented graphically. After then we should begin to solve problem given algebraically. To avoid possible misconceptions and for more conceptual understanding algebraic representation should present into graphical representations while solving algebraic problem. And for the school level students verbal expression should use only for explaining the concrete examples, graphical representations and the solution of algebraic representations.

But while teaching limit and continuity we should begin with concrete and real life examples. Then students somehow will feel the concept of infinitesimal, limit and continuity. After then we should go through graphical representations. For this Geogebra applet is must appropriate because we can make dynamic applet which is more useful to clarify the concept.

Conclusion

From the analysis of the data and findings, I concluded that the meaningful understanding of limit and continuity is possible in school level. Except some concept, students can learn the basic concept of limit and continuity in meaningful way avoiding possible misconceptions. But the school level students could not express in various ways. The above discussion shows that the school level students feel easily with graphical representations. The conclusion of above discussion is that a teacher should take pedagogical content care to teach it in meaningful way. This chapter is new for school level students and it is taught in higher level. So to teach it in school level in meaningful way a teacher should be aware and follow some procedures. As a researcher I would like to suggest the following steps to teach limit and continuity in school level.

- At first, tell something about the chapter and inform the students what we will learn in today class.
- Begin the chapter with concrete and real life example. For example, ask the student to divide an apple (or stick or others) continuously into half and discuss about the result.

- In the third step, we should begin to discuss about limit and continuity through graphically. For more clarificationwe should use various figures and we should ask the students to explain the figures and as a tutorial we should help the students in their difficulty. For this discussion, the GeoGebra applet is more effective tool. If we draw the figures on whiteboard, the figures may not be accurate and if we go through the GeoGebra we can make dynamic figures which is more helpful to clarify the concept and manage the time in classroom.
- Then after, if the students became clear with the concept of limit (class 9) and continuity (class 10), we should begin to discuss about the symbol and to solve the problem given algebraically. We should take help of graphical representation while solving symbolic and algebraic problem also.
- The verbal expression should use to describe the graphical representation and to explain the solutions of the algebraic expressions. If the students became clear with the concept, we can ask the some short questions verbally as "is the set of natural numbers continuous?(Why or why not?)"
- Through all these activities, for more clarification we should give chance for group discussion and some problem should be given to solve in group. Through observations of the group discussion, a teacher can find out the student's real problem.
- As mentioned in literature review section, students may have some misconceptions. So a teacher should aware about it and check it and

should present example and non-example to remove the misconceptions. The graphical representations is more appropriate to avoid possible misconceptions.

If we go through this way we can teach the chapter limit and continuity in meaningful way avoiding possible misconceptions and students can learn it meaningful way. But this study also shows that some content added the chapter limit and continuity such as ∞ and $\frac{0}{0}$, could not learned meaningfully by the students studying in class 9 and 10. But my experience during this study also shows that we can convince them about ∞ and $\frac{0}{0}$ using various figures, examples and non-examples through the way of discussion. But again if we asked to explain, show in figures, students would feel difficult. So their level of understanding obviously will help in higher study. The students feel more difficult with $\frac{0}{0}$. So a teacher should administrate enough discussion using various examples to clarify that $\frac{0}{0}$ is an indeterminate form.

Educational Implications

The meaningful understanding of limit and continuity in school level will be helpful to learn calculus in higher level. This study suggests some ways to teach limit and continuity meaningfully in school level avoiding possible misconceptions. This study may be important reading materials in the field of mathematics teaching. In sum up, the educational implications of the study are as follows:

- This study is helpful for a students because it suggests some ways that how a students can learn it in meaningful way.

- This study is helpful for a teacher because it suggests some useful idea to teach limit and continuity in meaningful way.
- This study helps the curriculum designer to add suitable content.
- This study is helpful for teaching and learning of limit and continuity avoiding possible misconceptions.
- This study promotes the use of technology in the area of teaching.

Recommendations for Further Study

This is a case study entitled "meaningful understanding of limit and continuity in school level". Here the school level means this study was focused on students studying optional mathematics in class 9 and 10. This study was done at Prabha secondary school in which about only 30 students were studying optional mathematics in class 9 and 10. So the conclusion of this study is not generalized. So as researcher I would like suggest some points for the further study in this field.

- This type of study also should be conducted in higher level.
- This study was limited to Prabha secondary school suited at Nisikhola municipality. So similar type of study should be done in other schools.
- To draw the proper conclusion, a research should be conducted in higher level with the aim of finding students' feeling while learning limit and continuity in class 11 and 12 after learning it in class 9 and 10.

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Appendix-A

1. Mark the following questions as true or false and justify your answer.

   (i) The function which is defined at all points has no limiting value.

   ……….. Because ….

   (ii) The limiting value of a function never equals to functional value.

   ………. Because………

   (iii) The limiting values of infinite sequence is always infinity.

   ……Because….

2. Write the following sentence in notational representation and demonstrate their meaning.

   (i) x approaches to a

   (ii) x approaches to ∞

   (iii) The limiting value of a function $f(x)$ is l as x tends to a.

   (iv) The limiting value of a function $f(x)$ is ∞ as x tends to a.

   (v) The limiting value of a function $f(x)$ is 0 as x tends to ∞.

   (vi) The limiting value of a function $f(x)$ is ∞ as x tends to ∞.

Appendix-B

1. What will be the limiting values of the sum$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \cdots$? Find the limiting value with suitable reason and figure.

2. Find the limiting value of the sequence $t_n = 2 - \frac{1}{2^n}$with suitable reason. If possible, also demonstrate using figure.

3. On the basis of the function defined by $f(x) = \frac{x^2-9}{x-3}$ give the answer of the following questions:

(iv) Does $f(3)$ represent any real number? Explain.

(v) Fill the following table

x	2.9	2.99	2.999	2.999	3	3.0001	3.001	3.01	3.1
$f(x)$									

(vi) What is the limiting value of the function $f(x)$ at $x = 3$? Give the answer with suitable reason.

4. Write the following notation in sentence and demonstrate their meaning.

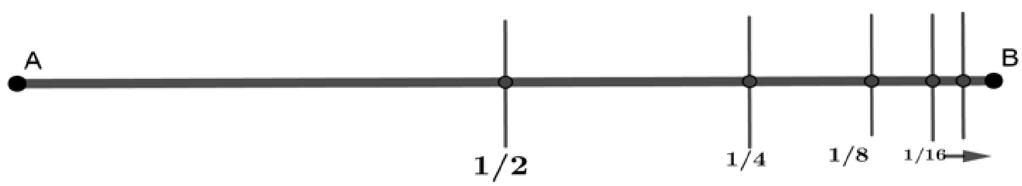
(i) $x \to a$ (ii) $x \to \infty$ (iii) $\lim_{x \to a} f(x) = g(x)$

(vii) $\lim_{x \to a} f(x) = \infty$ (v) $\lim_{x \to \infty} f(x) = 0$ (vi) $\lim_{x \to \infty} f(x) = \infty$

Appendix-C

1. Suppose you have a line segment AB = 1 m. If you start to bisect the segment continuously into half as shown in the below figure, how many time can you do that? What is limit of the length? Explain your answer with reasons

2.



3. Observe the figure given alongside, where the different regular polygon have been inscribed inside the circle of radius 1.5 cm. Fill the following table and answer the questions.

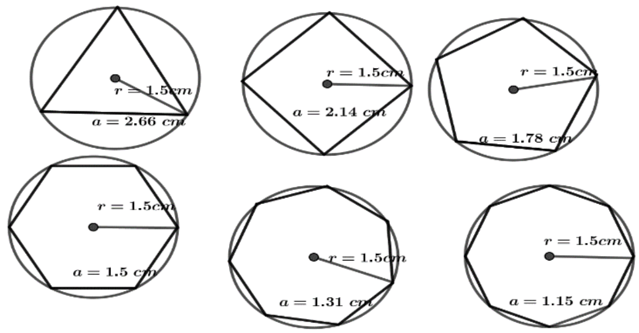
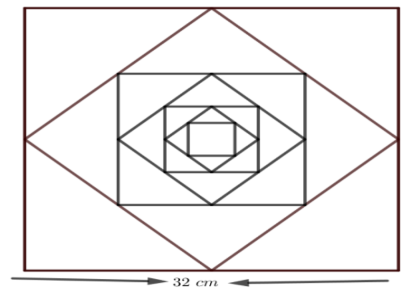


Figure no.	No. of sides in Polygon(n)	Perimeter of polygon(P_n)	Circumference of Circle
1	3		
2	4		
3	5		
4	6		
5	7		
6	8		
……..	……………….	…………………………..	
	$n \rightarrow$	$P_n \rightarrow$	

(a) How many such regular polygon can be constructed inside the circle?

(b) What is the limiting value of the perimeter of the polygon? Explain with reason.

4. In the figure given alongside, it is shown the sequence of squares formed by joining the midpoints of sides.

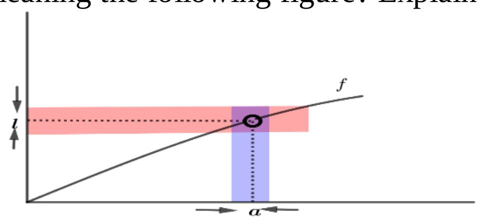


Answer the following questions:

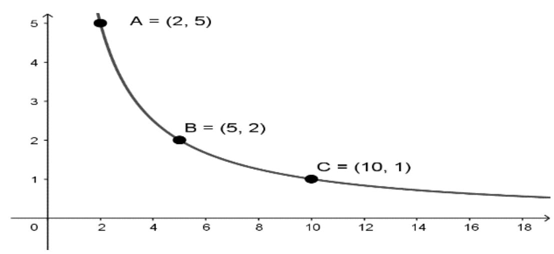
(c) How many polygons can be formed?

(d) What are the limiting values of perimeters and areas of the squares? Explain your answer with reason.

5. What may the meaning the following figure? Explain in your own words.



6. In the graph given alongside, the values of y have been decreasing as x increases. If we continuously increase the value of x, what would be the value of y? Explain it using limit notion.



Appendix-D

1. Answer the following question as true or false and give reason about your selection.

(i) If the limit of a function exists at a point $x = a$then the function is continuous at that point. …. Because ……

(ii) If the RHL and LHL of a function exist at a point $x = a$ then

(a) The limit of the functions exists at that point. ….. Because……..

(b) The function is continuous at that point. …… Because…………

(iii) The function must be defined at a point to exist the limit of the function at that point. …. Because. ….

(iv) The set of natural numbers is continuous ….. Because………

(v) The set of rational numbers is discontinuous…… Because……

(vi) The set of natural numbers is continuous…… Because ……

2. Write in a sentence: $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = l$ and explain its meaning with the help of figure.

3. What do you mean by continuity? Explain in your own words and give some examples and non-examples.

Appendix-E

Answer the following questions:

1. $f(x) = x^2 - 4 \quad if\ x \neq 3,$

(iii) Is the function f given above continuous at $x = 3$?Explain with suitable reason.

(iv) Does $\lim_{x \to 3} f(x)$ exist? If so, write your answer with reason.

2. A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2 - x^2, & if\ x \leq 2 \\ x - 4, & if\ x > 2 \end{cases}$$

(iii) Is the function f given above continuous at $x = 2$?Explain.

(iv) Does $\lim_{x \to 2} f(x)$ exist? If so, write your answer with reason.

3. A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 4x - 5, & if\ x \neq 5 \\ 15, & if\ x = 5 \end{cases}$$

(i) Is the function f given above continuous at $x = 5$?Explain.

(ii) Does $\lim_{x \to 5} f(x)$ exist? If so, write your answer with reason.

4. A function $g(x)$ is defined as follows:

$$g(x) = \begin{cases} 2x + 1, & if\ x < 3 \\ 5, & if\ x = 3 \\ 6x + 1, & if\ x > 3 \end{cases}$$

(i) Is the function g defined above continuous at $x = 3$? If $g(x)$ is not continuous, how can it be made continuous?

(ii) Does $\lim_{x \to 3} g(x)$ exist? If so, write your answer with reasons.

5. A function is given by $f(x) = \frac{x^2-1}{x-1}$.

(i) Is the function defined at x=1?

(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, find your answer with reason.

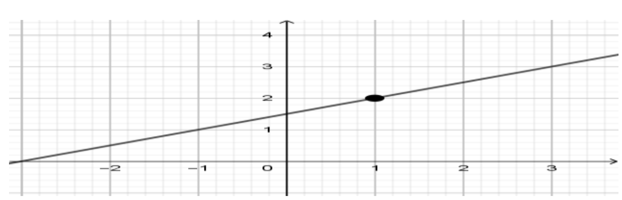
(iii) Is the function given above continuous or discontinuous at $x = 1$? Explain with reason.

Appendix-F

1. What type of graph is called continuous graph? Explain with examples and non-examples.

2.

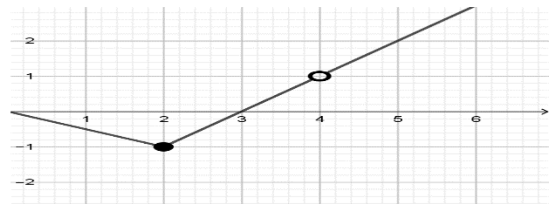
(i) Is the function given alongside continuous at x= 1?



(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, write your answer with reason.

(iii) Is there any points of discontinuity in the interval [-1, 4]? If yes, explain

3.

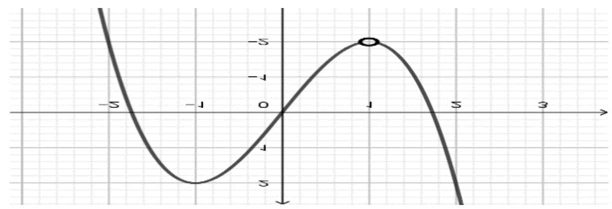


(i) Is the function given in the above figure continuous at x= 2?

(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, write your answer with reason.

(iii) Is there any points of discontinuity in the interval [-1, 4]? If yes, explain.

4.

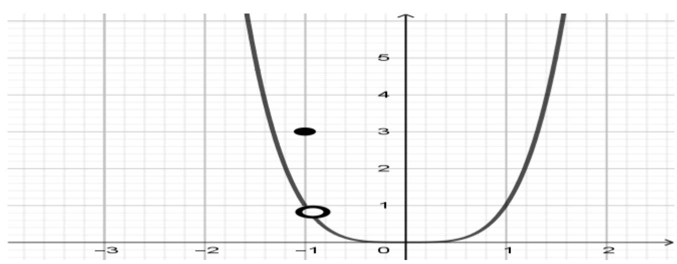


(i) Is the function given in the above figure continuous at x= 2?

(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, write your answer with reason.

(iii)Is there any points of discontinuity in the interval [-2, 2]? If yes, explain.

5.

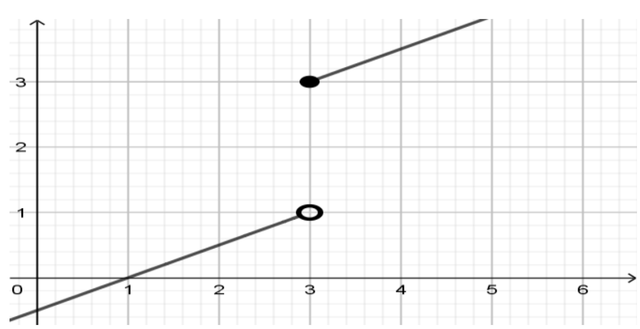


(i) Is the function given in the above figure continuous at x= -1?

(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, write your answer with reason.

(iii)Is there any points of discontinuity in the interval [-2, 2]? If yes, explain.

6.

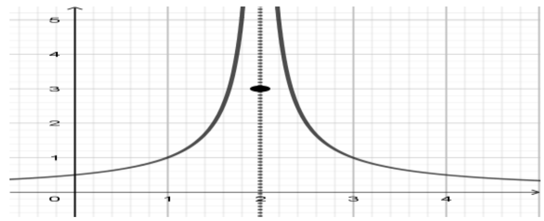


(i) Is the function given in the above figure continuous at x= 3?

(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, write your answer with reason.

(iii) Is there any points of discontinuity in the interval [-2, 2]? If yes, explain.

7.

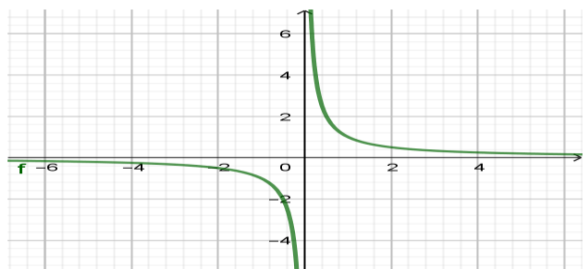


(i) Is the function given in the above figure continuous at x= 2?

(ii) Does $\lim_{x \to 1} f(x)$ exist? If so, write your answer with reason.

(iii) Is there any points of discontinuity in the interval [-2, 2]? If yes, explain.

8.

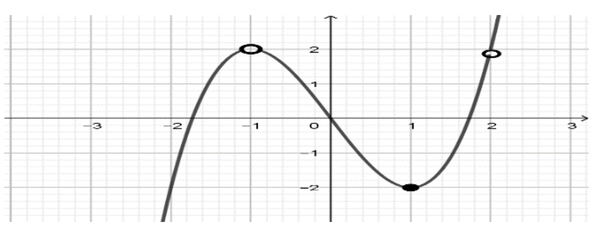


(iii)Is the function continuous at x=0?

(iv)Does $\lim_{x \to 0} f(x)$ exist? Justify your answer.

(v) Are there any point of discontinuity in the interval [-4, 4]? Justify your answer.

9.



(i) Is the function continuous at x= 1? Justify your answer.

(ii) Does $\lim_{x \to 1} f(x)$ exist? Justify your answer.

(iii) Are there any points of discontinuity in the interval [-2, 3]? Justify your answer.

Appendix- G

Item Analysis of test (class 9)

S. N.	Upper 27% 1	2	3	4	Total	Average 46% 5	6	7	8	9	10	11	Total	Lower 27% 12	13	14	15	Total	Grand Total	P% value	D Value	Remarks
1	1	1	0	1	3	1	1	0	0	1	0	1	4	1	0	0	1	2	9	60.00	0.25	Accepted
2	0	1	1	1	3	1	1	1	0	0	1	0	4	0	1	1	0	2	9	60.00	0.25	Accepted
3	1	0	1	1	3	0		0	1	1	1	0	3	1	1	0	0	2	8	53.33	0.25	Accepted
4	1	1	1	1	4	1	1	1	1	0	0	1	5	0	0	1	0	1	10	66.67	0.75	Accepted
5	1	1	1	1	4	1	0	1	0	1	1	1	5	1	0	1	0	2	11	73.33	0.50	Accepted
6	1	1	1	1	4	1	1	1	1	1	1	0	6	1	0	0	1	2	12	80.00	0.50	Accepted
7	1	1	0	1	3	0	1	0	1	1	1	1	5	0	1	0	1	2	10	66.67	0.25	Accepted
8	1	1	1	1	4	1	1	1	0	1	0	0	4	0	0	1	1	2	10	66.67	0.50	Accepted
9	1	1	0	1	3	1	0	1	1	0	1	1	5	1	1	0	0	2	10	66.67	0.25	Accepted
10	1	1	0	1	3	1	1	0	1	0	1	1	5	0	1	1	0	2	10	66.67	0.25	Accepted
11	0	0	0	1	1	0	0	1	1	0	0	0	2	0	0	0	0	0	3	20.00	0.25	Rejected
Group B 1	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	1	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	0	2				
	1	0	1	1	3	1	0	1	1	1	0	1	5	0	1	0	0	1				
	1	0	1	1	3	0	0	0	1	0	0	0	1	0	0	0	0	0				
Total	4	2	4	4	14	3	2	3	4	3	2	3	20	2	3	0	1	6	40	66.67	0.50	Accepted
Group B 2	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	0	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	0	1	0	2				
	1	0	1	1	3	0	1	1	1	1	0	0	4	0	0	1	0	1	40	66.67	0.50	Accepted

	1	0	1	1	3	0	0	1	0	1	0	0	2	0	0	0	0	0				
Total	4	2	4	4	14	2	3	4	3	4	2	2	20	2	1	3	0	6				
Group B 3	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	1	3				
	1	0	1	1	3	1	1	1	1	1	1	1	7	0	1	0	0	1				
	1	0	1	1	3	0	1	0	1	1	0	1	4	0	0	0	0	0				
Total	4	2	4	4	14	3	4	3	4	4	3	4	25	2	3	1	2	8	47	78.33	0.38	Accepted
Group B 4	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	0	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	0	0	0	1				
	0	1	1	0	2	1	0	0	1	1	0	1	4	0	0	0	0	0				
Total	3	4	4	3	14	4	3	3	4	4	3	4	25	3	2	2	1	8	47	78.33	0.38	Accepted
Group B 5	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	0	1	3				
	1	1	1	1	4	1	0	1	1	0	1	0	4	0	0	0	0	0				
	1	1	0	1	3	1	0	1	0	0	1	0	3	0	0	0	0	0				
Total	4	4	3	4	15	4	2	4	3	2	4	1	20	2	2	1	2	7	42	70.00	0.50	Accepted
Group B 6	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	0	0	1	1	2				
	1	1	1	0	3	1	1	1	1	1	1	1	7	0	0	0	1	1				
	1	1	1	0	3	0	1	0	0	0	1	1	3	0	0	0	1	1				
Total	4	4	4	2	14	3	4	3	3	3	4	4	24	1	1	2	4	8	46	76.67	0.38	Accepted
Group B 7	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	0	0	1	1	1	5	0	0	0	0	0				
	0	1	1	0	2	1	0	1	0	0	0	0	2	0	0	0	0	0				
Total	3	4	`	3	14	4	3	3	2	3	3	3	21	2	2	2	2	8	43	71.67	0.38	Accepted
Group B 8	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	0	1	6	1	1	1	0	3				
	1	1	1	1	4	1	1	1	1	1	0	0	5	1	0	0	0	1				
	1	0	1	0	2	1	0	0	1	1	0	0	3	0	0	0	0	0				
Total	4	3	4	3	14	4	3	3	4	4	1	2	21	3	2	2	1	8	43	71.67	0.38	Accepted
Group B 9	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	1	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	0	1	0	1	2	48	80.00	0.31	Accepted

	1	0	1	0	2	0	1	1	0	0	1	1	4	0	0	0	0	0				
Total	4	3	4	3	14	3	4	4	3	3	4	4	25	2	3	1	3	9				
Group B 10	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	0	1	6	1	1	1	1	4				
	1	1	1	1	4	1	0	1	1	1	0	0	4	0	1	1	0	2				
	0	1	1	1	3	1	0	0	1	0	0	0	2	0	1	1	0	2				
Total	3	4	4	4	15	4	2	3	4	3	1	2	19	2	4	4	2	12	46	76.67	0.19	Rejected
Group C 1	1	1	1	1	4	1	1	1	0	1	0	1	5	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	0	0	0	1				
	1	1	1	0	3	0	0	0	0	1	1	1	3	0	0	0	0	0				
Total	4	4	4	3	15	3	3	3	2	4	3	3	21	3	2	2	2	9	45	75.00	0.38	Accepted
Group C 2	1	1	1	1	4	1	1	1	0	1	1	1	6	0	0	1	1	2				
	1	1	1	1	4	1	1	1	1	1	0	1	6	0	1	1	0	2				
	1	1	1	1	4	1	1	0	1	1	1	1	6	1	1	1	0	3				
	1	1	1	1	4	0	0	0	0	0	0	0	0	1	0	1	0	2				
Total	4	4	4	4	16	3	3	2	2	3	2	3	18	2	2	4	1	9	43	71.67	0.44	Accepted
Group C 3	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	0	0	1	2				
	1	1	1	1	4	0	0	1	1	1	1	1	5	0	0	0	1	1				
	1	1	0	1	3	0	1	1	0	0	0	0	2	1	0	0	1	2				
Total	4	4	3	4	15	2	3	4	3	3	3	2	20	3	1	1	4	9	44	73.33	0.38	Accepted
Group C 4	1	1	1	1	4	1	1	1	1	1	1	1	7	0	1	1	1	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	0	1	1	1	3				
	1	1	1	1	4	1	1	1	1	1	1	0	6	0	1	0	0	1				
	0	1	1	1	3	0	0	0	0	0	0	0	0	1	0	1	0	2				
Total	3	4	4	4	15	3	3	3	3	3	3	2	20	1	3	3	2	9	44	73.33	0.38	Accepted
Group C 5	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	0	2				
	1	1	1	1	4	1	0	0	1	0	1	0	3	0	1	1	0	2				
	1	1	1	1	4	0	0	0	1	0	0	1	2	0	1	0	1	2				
Total	4	4	4	4	16	3	2	2	4	2	3	2	18	2	4	2	2	10	44	73.33	0.38	Accepted
Group C 6	1	1	1	1	4	1	1	0	1	1	0	1	5	1	1	1	0	3				
	1	1	1	1	4	1	1	1	0	1	1	1	6	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	1	0	3	47	78.33	0.19	Rejected

	1	1	1	0	3	0	1	0	1	0	1	0	3	1	1	0	0	2				
Total	4	4	4	3	15	3	4	2	3	3	3	2	20	4	4	3	1	12				

Appendix-H

Item Analysis of test (class 10)																						
	Number of Students																	Total	Grand Total	P% value	D Value	Remarks
	Upper 27%				Total	Average 46%							Total	Lower 27%								
S. N.	1	2	3	4		5	6	7	8	9	10	11		12	13	14	15					
1	1	1	1	1	4	1	1	0	0	0	0	1	3	1	0	0	0	1	8	53.33	0.75	Accepted
2	1	1	1	1	4	1	1	1	0	0	0	0	3	0	0	0	0	0	7	46.67	1.00	Accepted
3	1	0	1	1	3	0		0	1	1	1	0	3	1	0	0	0	1	7	46.67	0.50	Accepted
4	1	1	1	1	4	1	1	0	1	0	0	1	4	0	0	0	0	1	9	60.00	0.75	Accepted
5	1	1	1	1	4	1	0	1	0	1	1	1	5	1	0	1	0	2	11	73.33	0.50	Accepted
6	1	1	1	1	4	1	1	1	1	1	1	0	6	1	0	0	1	2	12	80.00	0.50	Accepted
7	1	1	0	1	3	0	1	0	1	1	1	1	5	0	1	0	1	2	10	66.67	0.25	Accepted
8	1	1	1	1	4	1	1	1	0	1	0	0	4	0	0	1	1	2	10	66.67	0.50	Accepted
9	1	1	0	1	3	1	0	1	1	0	1	1	5	1	1	0	0	2	10	66.67	0.25	Accepted
10	1	1	0	1	3	1	1	0	1	0	1	1	5	0	1	1	0	2	10	66.67	0.25	Accepted
11	0	0	0	1	1	0	0	1	1	0	0	0	2	0	0	0	0	0	3	20.00	0.25	Rejected
Group B 1	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	1	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	0	2				
	1	0	1	1	3	1	0	1	1	1	0	1	5	0	1	0	0	1				
	1	0	1	1	3	0	0	0	1	0	0	0	1	0	0	0	0	0				
Total	4	2	4	4	14	3	2	3	4	3	2	3	20	2	3	0	1	6	40	66.67	0.50	Accepted
Group B 2	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	0	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	0	1	0	2				
	1	0	1	1	3	0	1	1	1	1	0	0	4	0	0	1	0	1	40	66.67	0.50	Accepted

	1	0	1	1	3	0	0	1	0	1	0	0	2	0	0	0	0	0				
Total	4	2	4	4	14	2	3	4	3	4	2	2	20	2	1	3	0	6				
Group B 3	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	1	3				
	1	0	1	1	3	1	1	1	1	1	1	1	7	0	1	0	0	1				
	1	0	1	1	3	0	1	0	1	1	0	1	4	0	0	0	0	0				
Total	4	2	4	4	14	3	4	3	4	4	3	4	25	2	3	1	2	8	47	78.33	0.38	Accepted
Group B 4	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	0	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	0	0	0	1				
	0	1	1	0	2	1	0	0	1	1	0	1	4	0	0	0	0	0				
Total	3	4	4	3	14	4	3	3	4	4	3	4	25	3	2	2	1	8	47	78.33	0.38	Rejected
Group B 5	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	0	1	3				
	1	1	1	1	4	1	0	1	1	0	1	0	4	0	0	0	0	0				
	1	1	0	1	3	1	0	1	0	0	1	0	3	0	0	0	0	0				
Total	4	4	3	4	15	4	2	4	3	2	4	1	20	2	2	1	2	7	42	70.00	0.50	Accepted
Group B 6	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	0	0	1	1	2				
	1	1	1	0	3	1	1	1	1	1	1	1	7	0	0	0	1	1				
	1	1	1	0	3	0	1	0	0	0	1	1	3	0	0	0	1	1				
Total	4	4	4	2	14	3	4	3	3	3	4	4	24	1	1	2	4	8	46	76.67	0.38	Accepted
Group C 1	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	0	0	1	1	1	5	0	0	0	0	0				
	1	0	0	1	2	1	0	1	0	0	0	0	2	0	0	0	0	0				
Total	4	3	3	4	14	4	3	3	2	3	3	3	21	2	2	2	2	8	43	71.67	0.38	Accepted
Group C 2	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	0	1	6	1	1	1	0	3				
	1	1	1	1	4	1	1	1	1	1	0	0	5	0	0	1	0	1				
	1	0	1	0	2	1	0	0	0	1	0	0	2	0	0	0	0	0				
Total	4	3	4	3	14	4	3	3	3	4	1	2	20	2	2	3	1	8	42	70.00	0.38	Accepted
Group C 3	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	1	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	0	0	0	0	0	43	71.67	0.44	Accepted

	1	0	1	0	2	0	0	0	0	0	0	1	1	0	0	0	0	0				
Total	4	3	4	3	14	3	3	3	3	3	3	4	22	2	2	1	2	7				
Group C 4	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	0	1	6	1	1	1	1	4				
	1	1	1	1	4	1	0	1	1	1	0	0	4	0	1	1	0	2				
	0	1	1	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0				
Total	3	4	4	4	15	3	2	3	3	3	1	2	17	2	3	3	2	10	42	70.00	0.31	Accepted
Group C 5	1	1	1	1	4	1	1	1	0	1	0	1	5	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	0	1	0	2				
	1	1	1	0	3	0	0	0	0	1	0	0	1	0	0	0	0	0				
Total	4	4	4	3	15	3	3	3	2	4	2	2	19	3	2	3	2	10	44	73.33	0.31	Accepted
Group C 6	1	1	1	1	4	1	1	1	0	1	1	1	6	0	0	1	1	2				
	1	1	1	1	4	1	1	1	1	1	0	1	6	0	1	1	0	2				
	1	1	1	1	4	1	1	0	1	1	1	1	6	1	1	1	0	3				
	1	1	1	1	4	0	0	1	0	0	0	0	1	1	0	1	0	2				
Total	4	4	4	4	16	3	3	3	2	3	2	3	19	2	2	4	1	9	44	73.33	0.44	Accepted
Group C 6	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	0	0	1	2				
	1	1	1	1	4	0	0	1	1	1	1	1	5	0	0	0	1	1				
	1	1	0	1	3	0	0	1	0	0	0	0	1	1	0	1	1	3				
Total	4	4	3	4	15	2	2	4	3	3	3	2	19	3	1	2	4	10	44	73.33	0.31	Accepted
Group C 7	1	1	1	1	4	1	1	1	1	1	1	1	7	0	1	1	1	3				
	1	1	1	1	4	1	1	1	1	1	1	1	7	0	1	1	1	3				
	1	1	1	1	4	1	1	1	1	1	1	0	6	0	1	0	0	1				
	0	1	1	1	3	0	1	0	0	0	0	0	1	1	0	0	0	1				
Total	3	4	4	4	15	3	4	3	3	3	3	2	21	1	3	2	2	8	44	73.33	0.44	Accepted
Group C 8	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	1	7	1	1	0	0	2				
	1	1	1	1	4	1	0	0	1	0	1	0	3	0	1	1	0	2				
	1	1	1	1	4	1	1	0	1	0	0	1	4	0	1	0	1	2				
Total	4	4	4	4	16	4	3	2	4	2	3	2	20	2	4	2	2	10	46	76.67	0.38	Accepted
Group C 9	1	1	1	1	4	1	1	0	1	1	0	1	5	1	1	1	0	3				
	1	1	1	1	4	1	1	1	0	1	1	1	6	1	1	1	1	4				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	1	0	3	44	73.33	0.19	Rejected

	1	1	1	0	3	0	0	0	0	0	0	0	0	1	1	0	0	2				
Total	4	4	4	3	15	3	3	2	2	3	2	2	17	4	4	3	1	12				
Group C 10	1	1	1	1	4	1	1	0	1	1	0	1	5	1	1	1	0	3				
	1	1	1	1	4	1	1	1	0	1	1	1	6	1	0	1	1	3				
	1	1	1	1	4	1	1	1	1	1	1	0	6	1	1	1	0	3				
	1	1	1	0	3	0	0	0	0	0	1	0	1	0	1	0	0	1				
Total	4	4	4	3	15	3	3	2	2	3	3	2	18	3	3	3	1	10	43	71.67	0.31	Accepted

Appendix-I

Reliability test using spearman-Brown formula (class 9)

Students	X (Odd item)	Y (Even item)	r (PEARSON)	r_c(Spearman-Brown)
1	34	32	**0.978**	**0.989**
2	33	31		
3	30	31		
4	32	35		
5	26	24		
6	27	26		
7	24	20		
8	25	26		
9	23	22		
10	22	20		
11	19	18		
12	13	10		
13	12	11		
14	10	9		
15	12	13		

Appendix-J

Reliability test using spearman-Brown formula (class 10)

Students	X (Odd item)	Y (Even item)	r (PEARSON)	r_c(Spearman-Brown)
1	30	36	**0.870**	**0.931**
2	32	28		
3	31	27		
4	27	35		
5	24	21		
6	23	26		
7	24	20		
8	20	26		
9	18	22		
10	20	21		
11	15	18		

12	14	11
13	11	10
14	10	10
15	11	14