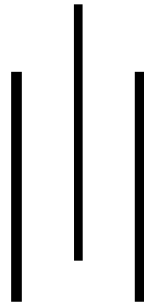




Tribhuvan University
Institute of Science and Technology

Optimal Coloring of a Plane Using Unit Distance Graphs



Dissertation
Submitted to

Central Department of Computer Science and Information Technology
Kirtipur, Kathmandu, Nepal

In partial fulfillment of the requirements
for the Master's Degree in Computer Science and Information Technology

by

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21st June, 2012 A.D.

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Student's Declaration

I hereby declare that I am the only author of this work and that no sources other than the listed here have been used in this work.

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Supervisor's Recommendation

I hereby recommend that this dissertation prepared under my supervision by **Mr. Harendra Raj Bist** entitled "Optimal Coloring of a Plane using Unit Distance Graphs" in partial fulfillment of the requirements for the degree of M.Sc. in Computer Science and Information Technology be processed for the evaluation.

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LETTER OF APPROVAL

We certify that we have read this dissertation and in our opinion it is satisfactory in the scope and quality as a dissertation in the partial fulfillment for the requirement of Masters Degree in Computer Science and Information Technology.

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ACKNOWLEDGEMENTS

I would never have been able to finish my dissertation without the guidance of my supervisor, help from friends, CDCSIT members, and support from my family.

I would like to express my deepest gratitude to my supervisor, **Mr. Jagdish Bhatta**, for his excellent guidance, caring, patience, and providing me with excellent ideas and encouragement for doing research.

I am also thankful to all the CDCSIT members for creating suitable environment in the completion of my dissertation. I would also like to thank current acting head of department, **Asst. Prof. Nawaraj Paudel** for advising me in several steps during dissertation. I am also thankful to the all the faculty members of CDCSIT, who have provided me the knowledge during the academic period.

Especially, I would like to thank **Mr. Ashok Kumar Pant** who helped me by giving ideas about implementation of the problem and patiently checked my code.

I would also like to thank my parents and elder brother and all my family members. They were always supporting and encouraging me with their best wishes.

ABSTRACT

Generally, plane coloring is the assignment of different colors to different points of a plane. Plane can be represented via graphs and hence can be colored using graph coloring. So the chromatic number of a plane is equivalent to finding the chromatic number of the equivalent graph of the plane. The unit distance graph can be used to find the chromatic number of a plane. The main application of unit distance graphs is the unit distance wireless networks (UDW), in cellular networks; the geometric regions are partitioned into hexagonal cells. Unit distance graph can represent the hexagonal cells, and coloring the hexagonal regions implies distribution of frequencies among the hexagonal cells. Different heuristic based algorithms viz. contraction based RLF, DSATUR and IDO based graph coloring algorithms are used in this study in order to color the unit distance graphs. And, different simple unit distance graphs used during the study include wheel graph, cycle graph, grid graph, etc. as test cases for these coloring algorithms.

The optimality of plane coloring, nowadays, is no more just confined to the minimal number of colors rather the coloring time makes importance. Since, coloring time is important in the field of cellular networks as the assignments of the frequency in a geometric regions and assignment of data to registers in program execution in minimum time. Since, these problems can be resolved through graph coloring. In this context, this study analyzes on optimal coloring of planes using heuristic based approaches so as to suggest an effective coloring paradigm.

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List of Abbreviations

UDG	Unit Distance Graph
UDW	Unit Distance Wireless Networks
RLF	Recursive Largest First
SC	Sequential Coloring
DSATUR	Degree of Saturation
LDO	Largest Degree Ordering
SDO	Saturated Degree Ordering
IDO	Incidence Degree Ordering
MATLAB	MATrix LABoratory