



**Evaluating Heuristic Solutions  
for  
*NP*-Hard Single Machine Scheduling Problems**

**Dissertation**

**Submitted To**

**Central Department of Computer Science and Information Technology**

**Institute of Science and Technology**

**Tribhuvan University**

**In Partial Fulfillment of the Requirements for the Degree of**

**Master of Science**

**in**

**Computer Science and Information Technology**

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**January 2008**

**Kirtipur, Nepal**



# **Tribhuvan University**

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We certify that we have read this dissertation work and in our opinion it is satisfactory in the scope and quality as a dissertation as the partial fulfillment of the requirement of Master in Computer Science and Information Technology from Tribhuvan University, Nepal.

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### **Recommendation**

I hereby recommend that the dissertation prepared under my supervision by **Mr. Nischal Regmi** entitled “**Evaluating Heuristic Solutions for *NP*-Hard Single Machine Scheduling Problems**” be accepted as a partial fulfillment of the requirement for the degree of Master in Computer Science and Information Technology, from Tribhuvan University, Nepal. To my best knowledge this is an original work in computer science.

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# Acknowledgements

Master's level dissertation is for acquisition of profound knowledge in a certain field. I suspect whether I could attain such a depth during this study, but I have a keen interest in algorithms and the theory of computational complexity. I was willing to do something related to this core area of computer science, nevertheless could not pick some topic for the dissertation. Thanks to Dr. Tanka Nath Dhamala, who suggested me to write something on scheduling algorithms, and kindly accepted to supervise me. Dr. Dhamala, apart from guiding me, also provided plenty of relevant research papers - Every Nepali student knows the difficulty of collecting research materials in Nepal, and thus can appreciate this immense help. Dr. Dhamala, albeit strictly, regularly speculated in my progress and provided me guidelines till the completion of this dissertation.

I must also thank Mr. Bishnu Gautam, my co-supervisor for this dissertation. He instructed me at critical moments, provided a great help in theoretical matters, and even generously edited this document. Similarly, I would like to express my gratitude to the respected teachers of CDCSIT - Prof. Dr. Shashidhar R. Joshi, Prof. Dr. Onkar Sharma, Mr. Sudarshan Karanjit, Dr. Subarna Shakya, Mr. Arun Timilsina, Mr. Min Bahadur Khati, Mr. Samujjwal Bhandari, Mr. Hemanta G.C., Mr. Dinesh Bajracharya and others, who donated their precious knowledge during my two years' study.

I would also like to thank my friend Madhav Dhakal, who helped me during the first phase of this work. Similarly, I thank Mr. Niraj Manandhar, Mr. Sanat Dahal, and all friends of CDCSIT, whose cheerful gossips frequently wiped away the fatigue of this tiresome study. Lastly, in our culture, one seldom thanks the parents and the almighty. Their existence is only for giving, they never take anything from their children, even avowals of appraisals.

## Abstract

*Scheduling, though being a classical problem of computer science, is still an evolving area of research. Unfortunately, many scheduling problems having high practical significance belong to the class NP-hard, or in simple words, they are not solved exactly by any efficient algorithm on any computer. In this dissertation, scheduling problems for the case of single machine problem is studied. The schemes of evaluating near-to-exact solutions for NP-hard problems are examined, and an algorithm based on tabu search is devised for the single machine scheduling problem  $1 \mid r_j \mid \sum C_j$ , where jobs arrive over time, preemption is not allowed, and the objective is to minimize the total completion time.*

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