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Determination of Optimal Number and Placement of Phasor Measurement Units (PMU) in Transmission Networks

by
Deepa Upadhyaya Subedi

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The undersigned certify that they have read, and recommended to the Institute of Engineering for acceptance, a thesis entitled **" Determination of Optimal Number and Placement of Phasor Measurement Units (PMU) in Transmission Networks "** submitted by Deepa Upadhyaya Subedi in partial fulfillment of the requirements for the degree of Master of Science in Renewable Energy Engineering.

Supervisor, Dr. Ajay Kumar Jha
Associate Professor
Department of Mechanical Engineering

External Examiner, Er. Dhupendra Jha
Assistant Manager
Nepal Electricity Authority

Committee Chairperson, Dr. Nawraj Bhattarai
Head of Department
Department of Mechanical Engineering

November 15, 2019
Date

ABSTRACT

With power demand in the world escalating day by day, interconnected power system networks are becoming progressively complex. Therefore, it is necessary to monitor and control operation of the system more accurately for smooth operation of the system. The classical state estimation of power system is based upon measurements collected from Supervisory Control and Data Acquisition System (SCADA), which is inadequate to measure fast and dynamic phenomenon. Phasor Measurement Unit (PMU) provides real time and synchronized phasor measurements with the use of Global Positioning System (GPS) with great measurement accuracy. However, placement of PMU at each and every node of the system is economically and technically not viable. Hence, PMUs are optimally placed in the network using Recursive Security N (RSN) algorithm, ensuring system's full observability and maximum reliability. Three step RSN algorithm is coded in MATLAB and utilized here to optimally place PMUs in Integrated Nepal Power System (INPS), considering presence of ZI buses. Then, System Observability Redundancy Index (SORI) was calculated for the obtained sets of optimal solution to ensure that the solution is most reliable among obtained sets. Further, to ensure validity of the method, it is implemented in IEEE 14-bus, 30-bus and 57-bus system.

In INPS, 2 sets of optimal solution with 21 number of PMUs was determined. Using the same algorithm, a set of optimal solution with 3 PMUs, 4 sets of optimal solution with 7 PMUs and 2 sets of optimal solution with 12 PMUs was determined for IEEE 14-bus, 30-bus and 57-bus system. Finally, SORI is calculated for each case and the solution set that offers maximum SORI was selected. Maximum SORI for 30-bus, 57-bus and INPS was found to be 33, 54 and 82 respectively.

The results are compared with solutions obtained using Simulated Annealing (SA) method from literature and is valid. Further, the study of effect of change in number of ZI buses in optimal solution showed that with the increase in number of ZI buses in the system, required number of PMUs for system's full observability would decrease.

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LIST OF ABBREVIATIONS

BOI	Bus Observability Index
CT	Current Transformer
DE	Differential Evolution
DFS	Depth First Search
DFT	Discrete Fourier Transform
GA	Genetic Algorithm
GPS	Global Positioning System
HMI	Human Machine Interface
IA	Immune Algorithm
ICT	Information and Communication Technology
IED	Intelligent Electronic Devices
IEEE	Institute of Electrical and Electronics Engineers
ILP	Integer Linear Programming
ILS	Iterated Local Search
INLP	Integer Non Linear Programming
INPS	Integrated Nepal Power System
IQP	Integer Quadratic Programming
LDC	Load Dispatch Centre
MATLAB	Matrix Laboratory
NEA	Nepal Electricity Authority
OPP	Optimal PMU Placement
PMU	Phasor Measurement Unit
PPS	Pulse Per Second
PSO	Particle Swarm Optimization
PT	Potential Transformer
RSN	Recursive Security N
RTU	Remote Terminal Unit
SA	Simulated Annealing
SCADA	Supervisory Control and Data Acquisition
SCDR	Symmetrical Component Distance Relay
SE	State Estimation
SORI	System Observability Redundancy Index
TS	Tabu Search
WAMS	Wide Area Monitoring System
ZI	Zero Injection

CHAPTER ONE: INTRODUCTION

1.1. Background

With enormous demand of power in the world, the interconnected power system networks are becoming progressively complex. An interconnected power system can be divided into three main systems: generation, transmission and sub transmission and distribution. For smooth operation of such a system, it is necessary to monitor and control the operation of the system most accurately. Further, power grids around the world are witnessing a remarkable development in recent years due to the challenges faced with the integration of renewables. This leads to the development of smarter grid by integrating information and communication (ICT) layers above the conventional power layer allowing faster and accurate control of grid. One of the factors that caused the major blackout in North America in 2003 was lack of real time data gathering and monitoring. This prevented the necessary steps from being taken before the incident happened, leading to the catastrophic blackout. Fifty million people in eight US states and two Canadian provinces were affected (Anderson, et al., 2005). In order to cope with such challenges the power system faces, the system should be continuously monitored by providing measured data and using computer applications such as state estimation to find other variables to help in controlling the network (Bandak, 2013).

The classical state estimation of a power system is based upon measurements collected from Supervisory Control and Data Acquisition System (SCADA) where measurements might be unsynchronized and have low sampling rates i.e. such a system cannot capture measurements of fast and dynamic phenomenon (Mabaning, et al., 2017). Another drawback of SCADA measurements is that they do not include the phase angle of bus voltages and line currents. To address these issues, Phasor Measurement Unit (PMU) was developed that provides real time and synchronized phasor measurements with the use of global positioning system (GPS) only in microseconds with great measurement accuracy (Morais, et al., 2014) (Wang, et al., 2014) which is faster than the speed of existing SCADA technologies. This data obtained with the help of PMU allows better monitoring of power system because it allows one to detect, anticipate and correct problems during abnormal system conditions. Whenever a power system is monitored using PMU, it enhances the system in many aspects ranging from optimal power dispatch, monitoring tie line power, detecting unacceptable voltage profile to prevent faults or minimize their effects. These features may already seem to appear in conventional power networks that use SCADA. However, the efficiency and correctness of SCADA regarding giving real time data is relatively small due to many estimations, since there are many missing variables that SCADA needs to estimate. Another drawback of SCADA is its inability to properly react to major faults that might eventually lead to blackouts (Mahgoub, 2017).

Over time, greater number of PMUs have become necessary to improve monitoring, control and protection of power system. For proper monitoring and control, a power system needs to be observable. A power system is said to be observable if the measurements made on it allow determination of bus voltage magnitude and angle at every bus of the network. The observability of a system is significantly related to the location of data measuring devices and the way they are distributed. It could be possible to make a power system observable by placing a PMU at each bus of the system. However, due to redundancy and high price of PMUs and associated communication equipment, it is not an option to place PMU at every node in the network. Rather, a certain number of PMUs are required to be placed at certain location which make the system fully observable i.e. optimal placement of PMU.

The average overall cost per PMU for procurement, installation and commissioning ranges from $ 40,000 to $180,000. The major factors that are responsible for increased cost of PMU installation include communication requirements and installations, cybersecurity requirements, labor and the PMU hardware (U.S. Department of Energy, 2014).

A wide range of applications of PMU have been developed after its emergence, when it was recognized that best estimate of power system's state is required to prevent the system from failures and improve its performance. These applications include model validation, state estimation (SE), protection and closed loop control, fault detection and more (Phadke & BI, 2018). Wide Area Monitoring System (WAMS) is a system that combines the data provided by the synchrophasor and conventional measurements with capability of new communication systems in order to monitor, operate and control power systems in wide geographical area (Junce & Zexiang, 2005). As such, PMU is the key element of WAMS. Its application can be found in the power systems networks all around the globe including North America, Mexico, nearly all countries in Europe, China, India and Russia (Phadke & BI, 2018).

1.2. Problem Statement

Modern power system has evolved into a huge network as the demand for power goes on increasing due to advancement in technology and improvements in living standard worldwide. It requires constant monitoring in order to operate with high efficiency while minimizing the chances of failure. Further, penetration of renewable energy sources into the system has led to the operation of the system near to the stability limit elevating the requirement of accurate measurement devices with monitoring capabilities. Conventional monitoring system using SCADA has become insufficient to maintain an acceptable control and protection of power system leading to system collapse and blackouts.

PMU can facilitate the real time computing and synchronized measurement of voltage and current phasors. These devices can achieve accuracy and precision by using GPS thus remarkably improving system monitoring and control.

Traditionally state estimation is formulated as a weighted least squares problem due to absence of measurements which can measure phasors. The introduction of PMUs make state estimation able to be achieved via linear estimators which significantly increases the efficiency of solving state estimation problems for identifying the system observability. It is possible to directly measure all the systems states simply by placing the PMUs at all the nodes without running state estimation problem. However due to high cost of PMU, it is not economically viable to place PMUs at each and every nodes of the system. Therefore, optimizing the number of PMUs and the locations of these units for maximum observability of the system is important.

1.3. Research objectives

1.3.1. Main objective

The main objective of this research is to determine the optimal number and location of Phasor Measurement Unit (PMU) to be installed in transmission network while ensuring system's full observability and maximum reliability.

1.3.2. Specific objectives

1. To formulate mathematical problem for optimal placement of PMU by recursive security N algorithm.
2. To calculate BOI and SORI to determine the most reliable solution.

3. To analyze system observability after optimal PMU placement.
4. To evaluate the impact of change in the number of ZI bus in optimal number of PMU.

1.4. Limitations

1. The study has considered implementation of RSN for optimal placement up to 74-bus system only. It does not guarantee optimal solution for much larger system, as it may take a lot of time to run the algorithm and determine solution for larger systems.
2. In INPS, transmission lines only up to 66 kV are considered for study and loads below 66 kV voltage level are considered lumped.
3. Power from all generators connected to a bus are added and represented as a single generator in INPS.

CHAPTER TWO: LITERATURE REVIEW

2.1. Power System Monitoring and Control

Being dynamic in nature, power system is prone to many contingencies and disturbances. In recent years, major blackouts have been recorded through the world. Disturbances such as loss of line or generators, changing of load, surges, faults etc. can cause the system's operating point to deviate from the limits. In order to account on these problems and find out the reason of their occurring, one has to keep a check on parameters related all these phenomenon (Chouhan & Jaiswal, 2016).

Since the power system is a huge system due to interconnections in order to improve reliability and economic efficiency, there are always some difficulties in evaluating and maintain the stability of whole system in case of disturbances. Recently a new issue in power system has come out, which is the penetration of renewable energy resources, bringing more uncertainty that requires more severe operation. For energy security, the introduction of renewable energy sources is indispensable; therefore, to maintain system reliability and make efficient use of sustainable energy, power system monitoring should be a key technology to achieve flexible operation in the system. Further, in recent years, the development of information and communication technology (ICT) has enabled more flexibility in wide area monitoring of power system with fast and large data transmission. In order to monitor the system, many measuring instruments and apparatuses are installed. Typically, active power, reactive power, node voltage and frequency must be measured all the times. So far SCADA system is most widely adopted monitoring system (Bevrani, et al., 2014).

SCADA system is as a collection of equipment that will provide an operator at a remote location with sufficient information to determine the status of particular equipment and cause action to take place regarding that equipment without being physically present. This system is composed of four major components viz. Remote Terminal Unit (RTU), Communication System, Master Station and Human Machine Interface (HMI) (Thomas & McDonald, 2015). Conventional data such as real/reactive power flow-injections, magnitudes of bus voltages and branch currents are obtained from SCADA and RTU. A typical SCADA system is represented in Figure 2-1.

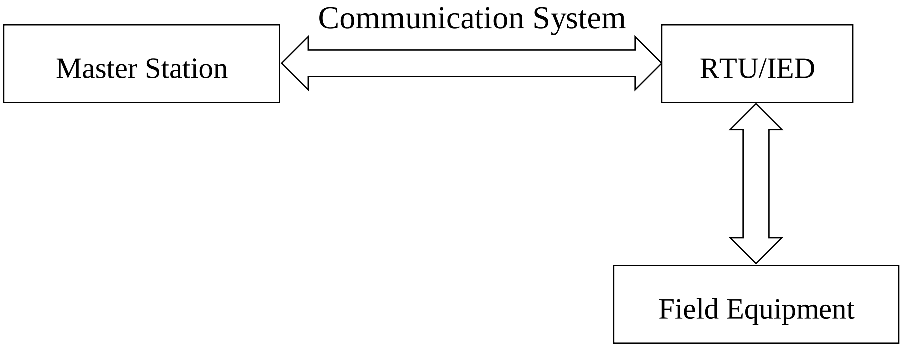

Figure 2-1: A typical SCADA system

The most common task of SCADA, is the state estimation of grid, which should be done most accurately and fast for smooth operation of the power system. However, the data collected from conventional SCADA system are not enough as they are prone to errors such as delays in measurement because of its slow duty cycle and telecommunication bias (Sefid & Rihan, 2018).

With the help of synchronized measurement of voltage and current, the limitations and problems associated with SCADA can be eliminated. This is facilitated by PMU by providing real time computed and synchronized phasor measurement of voltage and current in the power grid (Wache, 2013). The algorithm used for state estimation for ensuring system's observability using phasor data are computationally simpler, systematic and efficient than conventional methods (Shahraeini & Javidi, 2011).

A comparison between SCADA and PMU is presented in Table 2-1.

Table 2-1: Comparison between SCADA and PMU

	SCADA	PMU
Resolution	1 sample evert 2-4 seconds (steady state observability)	10-60 samples per second (Dynamic observability)
Measured Quantities	Magnitude only	Magnitude and phase angle
Time Synchronization	No	Yes
Total input/output channels	100+ analog and digital	~10 phasors, 16+ digital. 16+ analog

	SCADA	PMU
Focus	Local monitoring and control	Wide area monitoring and control

2.2. Historical Development of Phasor Measurement Unit (PMU)

Phase angle between the voltage phase and current phase as the basic measuring function of PMU has been utilized to monitor the conditions of power networks. In early 1980s, modern equipment for direct measurement of phase angle difference was introduced (Phadke & Thorp, 2008). For maintaining reference signal for synchronization, communication channels based on LORAN-C, GOES satellite transmissions and the HBG radio transmissions in Europe was used. Researchers used local phase angle with respect to time reference for resolving zero crossing of the phase voltage. The phase angle difference between voltages at two buses was established utilizing the difference of measured angles to common reference, for both locations. However, measurement accuracy in order of only 40 microseconds was obtained by using the best achieved time reference from the communication channels mentioned above. These were still insufficient to capture the harmonics in voltage waveform. Later when GPS satellite were deployed in significant number, it was realized that GPS time signal can be utilized as an input to sampling clock. The GPS provides high precision timing, ranging from 1 nanoseconds to 10 nanoseconds (Ashby, et al., 1997). Also, GPS receiver can supply a unique pulse signal in 1 second interval, which is known as 1 pulse per second (PPS). Hence, by embedding GPS system in measuring device, this system offered the most effective way of synchronizing power system measurements over great distance. Eventually, utilization of PMU using GPS system started worldwide.

PMUs were introduced in 1988 by Arun G. Phadke and James S. Throp at Virginia Tech as a developed version of symmetrical component distance relay (SCDR). SCDR was developed in order to overcome the problem of solving six equations to cover the protection of all possible faults in three phase transmission line by solving a single complex equation. SCDR used symmetrical component voltages and currents instead of traditional phase quantities. Later, when SCDR was no longer needed, it was felt that there is a great advantage in salvaging the portion of SCDR that computes the sequence components of voltage and currents. Hence, the portion of SCDR up to the computation of positive sequence voltages and currents was pulled out as standalone measurement unit which could measure with great accuracy positive sequence voltages and currents in a period of fundamental frequency, which finally with the help of GPS to synchronize measurements across power system, evolved as PMU (Phadke & BI, 2018).

2.3. Phasor Measurement Unit (PMU)

PMU can be defined as a device that produces synchronized phasor, frequency and rate of change of frequency (ROCOF) estimates from voltage and/or current signals and a time synchronized signal (IEEE Std. , 2011).

Phasor is a complex representation of pure sinusoidal signal. A pure sinusoidal quantity can be represented by

$$x(t) = X_m cos(\omega t + \phi) \qquad \text{Equation 2.1}$$

Where, ω is the frequency of the signal in radians per second

ϕ is phase angle in radians

X_m is the peak amplitude of the signal

Equation 2.1 can also be represented as

$$x(t) = \frac{X_m}{\sqrt{2}} e^{j\phi} = \frac{X_m}{\sqrt{2}} (cos\phi + jsin\phi) \qquad \text{Equation 2.2}$$

Equation 2.2 is the phasor representation of a pure sinusoidal quantity given by Equation 2.1.

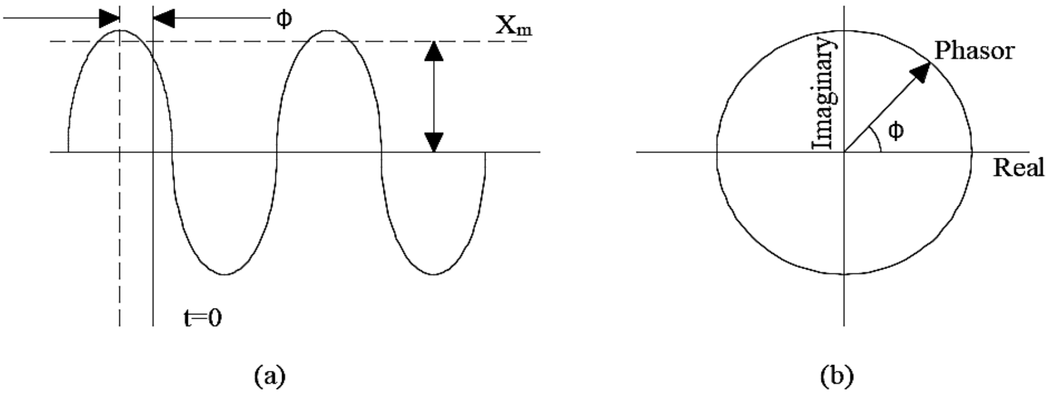

Figure 2-2: A sinusoid and its phasor representation

In practice, a waveform is not a pure sinusoid but is often corrupted with other signals of different frequencies. In such case, whenever there is a necessity to extract a single frequency component, Fourier transform is used.

A simplified block diagram of PMU is given in Figure 2-3. The main components of PMUs are: analog inputs, anti-aliasing filters, GPS receiver, phase locked oscillator, A/D converter, micro-controller unit and modem. Function of each block is explained below:

Analog inputs: The analog inputs to PMU are voltages and currents obtained from the secondary of current and potential transformers.

Anti-aliasing filters: Anti-aliasing filter is an analog low pass filter which is used to filter out those components from the actual signal whose frequencies are greater than or equal to half of Nyquist rate to get the sampled waveform. Nyquist rate is equal to twice the highest frequency component of input analog signal. If anti-aliasing filters are not used, error will be introduced in the estimated phasor.

Phase locked oscillator: Phase locked oscillator along with GPS reference source provides the needed high speed synchronized sampling.

A/D Converter: It converts analog signal to digital signal. The output of ADC is a sequence of digital values that convert continuous time and amplitude analog signal to a discrete time and discrete amplitude signal.

GPS: The synchronized time given by GPS is obtained by using high accuracy clock from satellite technology. The satellite transmit one pulse per second signal. This pulse as received by any receiver on earth is coincident with all other received pulse within 1 microsecond. The GPS satellite provides a very accurate time synchronization signal, available via antenna input, throughout the power system. Thus, the voltage and current recordings from different substations can be directly displayed on the same time axis and in the same phasor diagram.

Micro-controller Unit: The micro controller calculates positive sequence estimates of all current and voltage signals using DFT. Other parameters such as frequency and rate of change of frequency are also obtained as PMU output. The time tag is created from two of the signals derived from GPS receiver.

Modem: Modem produces a signal that can be transmitted and decoded to make a replica of original digital data and transfers those data to required location for future reference or initiating necessary actions.

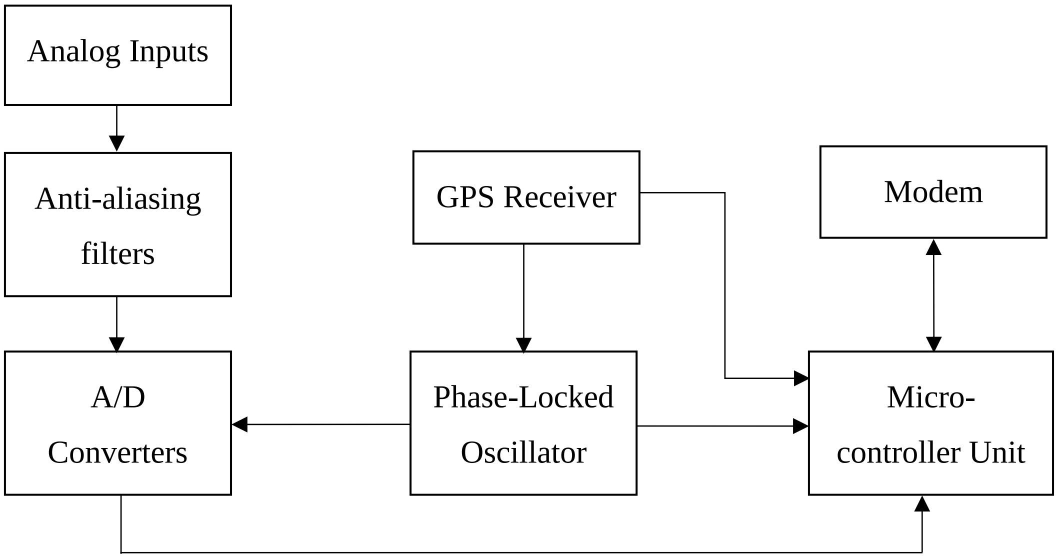

Figure 2-3: Simplified functional block diagram of PMU

2.4. State Estimation

As power system network is becoming more and more complex due to increase in energy demand, need for real time model is crucial for monitoring and controlling of power system networks. In order to build realistic models, state estimation (SE) is required, especially for non-linear systems similar to power grids (Bandak, 2013). SE is a technique used to find the values of the state variables based on some imperfect measurements. This technique uses statistical criteria to estimate the actual value of those unknown variables to minimize or maximize the selected criterion (Wood, et al., 2014). Typically, a state estimator consist of following functions: Topology processor, observability analysis, state estimation solution, bad data processing and parameter and structural error processing (Abur & Exposito, 2004).

State estimators, based on the types of measurement devices can be classified into conventional, PMU based and hybrid state estimators. Traditional SEs, using conventional measurements i.e. power flow injections and voltage magnitudes, are known as conventional SEs. When state estimations utilize phasor data, they are referred as PMU based SEs. Hybrid SEs use data obtained from both conventional and phasor measurements (Shahraeini, et al., 2011).

Traditionally state estimation is formulated as a weighted least squares problem due to absence of measurements which can measure phasors. This problem can only be solved via non-linear iterations. The introduction of PMUs make state estimation able to be achieved via linear estimators which significantly increases the efficiency of solving state estimation problems. It has been observed in (Saha Roy, et al., 2012) that if a power system's observability is achieved through placing PMUs in the system, the system state can be obtained by running linear state estimator in a single iteration.

State estimation can be classified according to the methods or criteria used for estimating best values of state variables. Out of various criteria that have been used in various applications, following three are most widely used.

a. **The maximum likelihood criterion:** Here, the objective is to maximize the probability that the estimate of the state variable, $\hat{x}$, is the true value of the state variable vector, x (i.e. $P(\hat{x}) = x$).
b. **The weighted least squared criterion:** Here, the objective is to minimize the sum of the squares of the weighted deviations of the estimated measurements, $\hat{z}$, from the actual measurements, z.
c. **The minimum variance criterion:** Here, the objective is to minimize the expected value of sum of squares of the deviations of the estimated components of the state variable vector from the corresponding components of the true state variable vector.

2.5. Power System Observability

A power system is said to be observable if the measurements available in the system are sufficient in number and location to allow the state vector of whole system to be estimated. Power system observability is an essential concept for state estimation. Only when a power system is guaranteed to be observable, state estimation of the system is accurate and comprehensive (Krumpholz, et al., 1980). Observability analysis is done off-line during initial phase of a state estimator installation, in order to check the adequacy of the existing measurement configuration. Observability analysis can also be done on-line, prior to running the state estimator, in order to ensure that a state estimate can be obtained using the set of measurements received at the last measurement scan (Abur & Exposito, 2004).

Observability of a given network is determined by the type and location of available measurements as well as by the topology of the networks. Network observability analysis can be performed using either numerical or topological approaches.

2.5.1. Numerical Observability Analysis

In state estimation, the measurement model can be described as

$$\bar{z} = h(\bar{x}) + \bar{e} \qquad \text{Equation 2.3}$$

Where $\bar{z}$ is the measurement vector, $\bar{x}$ is the system state vector that contains all buses' voltage phasor, $\bar{e}$ is the measurement error or noise vector and $h(\bar{x})$ is vector function relating measurement vector and state vector. In conventional and hybrid SE cases, Equation 2.3 may include non-linear equations, while in PMU based SE case, Equation 2.3 is a vector of linear equations (Baldwin, et al., 1993).

Considering the accuracy of the data provided by PMUs, $\bar{e}$ is very small and thus is neglected in general. If PMU is used as measurements in power system, then the state estimator for this system is linear, which can be described as

$$\bar{z} = \mathbf{H}\bar{x} \qquad \text{Equation 2.4}$$

Where, **H** is the measurement function matrix and also a coefficient matrix related to the system state vector $\bar{x}$.

For a power system with n buses and m PMUs, $\bar{x}$ is a vector with $2n - 1$ dimensions and **H** is a matrix with $m * (2n - 1)$ dimensions. If the system is fully observable, $\bar{z}$ provided by Equation 2.4 should have $(2n - 1)$ valid elements. This means that a power system with n buses is observable if

$$Rank\ (\boldsymbol{H}) = 2n - 1 \qquad \text{Equation 2.5}$$

2.5.2. Topological Observability Analysis

Topological algorithms for system observability are not investigated separately based on their measurement types (Monticelli, 2000). (Shahraeini & Javidi, 2011), however, have classified topological observability algorithms into conventional and PMU based observability algorithms.

Conventional observability algorithm: Conventional observability algorithms use real/reactive power flows, power injections, and magnitudes of bus voltage and branch currents for system observability. When topology is used to represent a power system, the system can be taken into account as a graph with n apexes, which represent n buses, and b edges, which represent b branches each of which connects two buses. This graph can be described as

$$G = (V, E) \qquad \text{Equation 2.6}$$

Where V is the set of the apexes in the graph and E is the set of edges in this graph.

A subgraph can be described as

$$G' = (V', E') \qquad \text{Equation 2.7}$$

Where $V' \subseteq V$ and $E' \subseteq E$.

The subgraph G' is defined as a full ranking spanning tree of graph G if G' contains all of the graph apexes and exactly $n - 1$ edges. Hence, a power system is topologically observable if its measurements are placed in the way that at least one full rank spanning tree of measurements is existed. This method is known as spanning tree method.

PMU based observability algorithm: According to Kirchhoff and Ohm's law, three kind of virtual measures can be applied in power system as follows (Peng & Xu, 2008).

- When the voltage of one node of a branch and the branch current are directly measured by measurements, then the voltage of the other node is virtually measured.
- If the voltages of two nodes of a branch are directly measured by measurements, the current of this branch can be virtually measured.
- If all branch currents of a node are known except one, the unknown branch current is virtually measured.

According to above rules for virtual measurement, if all system PMUs have enough voltage and current channels, then three main topological observability rules can be concluded as follows (Peng & Xu, 2008).

- If a PMU is placed at a bus, this bus and all of its neighbor buses can be observed (Figure 2-4(a)).
- For a zero injection node, which is observed, if all of its connected nodes are observable except one, then the unobserved node can be observed (Figure 2-4(b)).
- If all the nodes connected to a zero injection node are observable, then the zero injection node can be observed too (Figure 2-4 (c)).

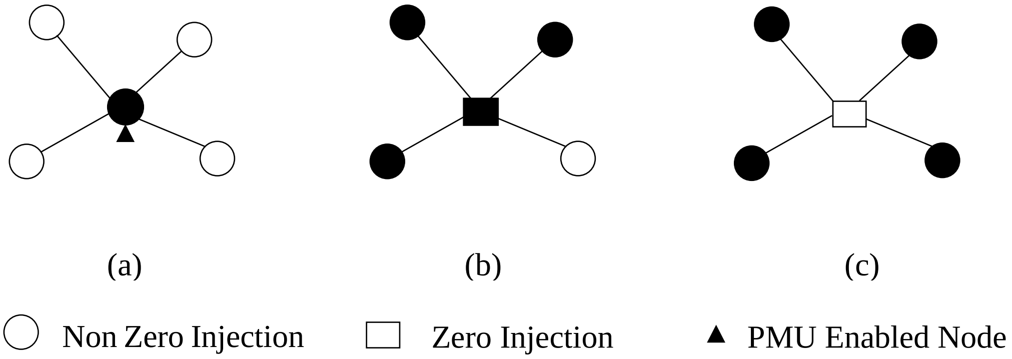
(a) (b) (c)

Figure 2-4: Topological Observability Rules based on PMU

For systems whose phasors are directly measured by PMUs, above rules together with some topological characteristics of network introduce some topological observability algorithms such as spanning tree, topology transformation and augmented incidence matrix.

2.6. Optimization Techniques

In order to create a fully observable power system, various optimization techniques have been proposed that can be classified mainly into two categories: mathematical and heuristic algorithms (Manousakis, et al., May 2012).

2.6.1. Mathematical Algorithms

a. **Integer Programming:** Integer programming is a mathematical programming method of solving an optimization problem having integer design variables, while the objective function and the constraints are linear, non-linear or quadratic thus leading to integer linear programming (ILP), integer nonlinear programming (INLP) and integer quadratic programming (IQP), respectively. The ILP method is a deterministic optimization method used to solve OPP problem. In ILP, in order to achieve best solution, the constraints to solve OPP problem needs to be defined accordingly. Therefore, the constraints formed are very crucial when using ILP method to solve OPP problem. In (Abbasy & Ismail, 2009), OPP problem has been formulated as binary integer linear programming (BILP). It has integrated the impact of both existing conventional power flow and the possibility of single or multiple PMU loss in decision strategy of the optimal PMU allocation. (Chakrabarti, et al., January 2009) has used integer quadratic programming (IQP) to minimize total number of PMUs required and to maximize measurement redundancy at the power system buses also considering existing conventional measurements.

b. **Exhaustive search (ES):** Exhaustive search is a technique that lists all possible candidates for the solution and selects the candidate that satisfies the constraints at optimum value of objective function. Its main advantage is that it guarantees finding of the global optimum. However, it is not suitable for large scale systems with huge search space. An exhaustive search method has been adopted in (Chakrabarti & Kyriakides, August 2008) to solve OPP problem, considering single branch outage with and without the presence of ZI buses. In cases of multiple solutions, the algorithm selects the one with most preferred pattern of measurement redundancy. (Mabaning & Orillaza, 2016) has presented an optimum version of exhaustive search technique that reduces search space and number of computations of traditional ES. Conditions such as presence of ZI bus and n-number of PMU outage are also considered.

2.6.2. Heuristic Algorithms

In contrast to mathematical methods, heuristic algorithms rely on parameters which require fine tuning to ensure that the algorithms are able to find the best possible solutions, instead of a set of constraints.

a. **Genetic algorithm (GA):** A genetic algorithm is an optimization technique based on natural selection and genetics. GA operates on a population of individuals, known as "chromosomes", which are potential solutions to a given problem and are combined to breed new individuals. In (Milosevic & Begovic, 2003), a combination of graph theoretical procedure and a simple GA is used to determine best tradeoffs between competing objectives such as minimization of PMU's number and maximization of measurement redundancy. Binary genetic algorithm (BGA) has been used to solve the OPP in (Shahriar, et al., 2018). It found out that BGA performs better than heuristic approach in reaching global solution.

b. **Tabu search (TS):** Tabu search is a neighborhood search decent method which avoids "local minimum traps" by accepting worse solutions and constraining the current solution neighborhood by the solutions' "search history". The search history is stored in the form of a tabu (forbidden) list. In (Peng, et al., 2006), OPP is solved by TS algorithm and fast observability analysis method based on augmented incidence matrix that only manipulates integer numbers and can conveniently and quantitatively assess network observability.

c. **Simulated Annealing (SA):** The SA algorithm is an optimization method which mimics the slow cooling of metals, which is characterized by a progressive reduction in the atomic movements that reduce the density of lattice defects until a lowest energy state is reached. In (Akhlaghi, 2016), a two-step optimization method is proposed. In first step, minimization model is applied to convex programming to achieve minimum number of PMU. In second step, SA is applied to maximize measurement redundancy. Similar, a multistage SA method is utilized in (Gopakumar, et al., 2013) to provide optimal joint placement of PMUs with current regular measurement units. This method is faster than other conventional SA algorithms as it finds the solution on the basis of uphill movements throughout different steps (Gopakumar, et al., 2013).

d. **Differential Evolution (DE):** The DE algorithm is a population based algorithm like genetic algorithm using the similar operators: crossover, mutation and selection. The main difference in constructing better solutions is that genetic algorithms rely on crossover while DE relies on mutation operator as a search mechanism and selection operation to direct the search towards the prospective regions in the search space. The paper (Rajasekhar & Chandel, 2013) has solved joint optimal PMU and conventional measurements placement problem with the objective to minimize number of PMUs and maximize PMU measurements redundancy. The NLIP problem is solved by DE method considering different cases of power system viz. IEEE 7-bus and 14-bus system.

e. **Particle Swarm Optimization (PSO):** PSO is a stochastic optimization method simulating the foraging behaviors of birds. In PSO, every solution is entitled as particle, and the combination of particles constitutes the whole swarm. PSO uses velocity and position concepts in order to find the optimal point(s) in working space. (Peppanen, et al., 2012), gives local optimum solution using binary particle swarm optimization (BPSO) for PMU placement trying to attain two objectives of minimizing PMU's number and maximizing measurement redundancy. In (Wang, et al., 2012), an improved PSO (IPSO) algorithm is used to optimize PMU placement where GA and SA is incorporated into PSO to overcome limitations of PSO. Exponential binary PSO (EBPSO) technique has been suggested in (Maji & Acharjee, 2017) with the goal of overcoming the optimization problem and achieving maximum observability in a power system considering different practical possibilities such as zero injection, single-PMU outage along with the usual operating situation.

f. **Immune Algorithm (IA):** The immune algorithm (IA) is a search strategy based on genetic algorithm principles and inspired by protection mechanisms of living organisms against bacteria and viruses. (Aminifar, et al., 2009), has incorporated immune operator in the canonical genetic algorithm (GA), on the condition of preserving GA’s advantages and has utilized some characteristics and knowledge of the problems for restraining the degenerative phenomena during evolution, so as to improve the algorithm efficiency.

g. **Iterated Local Search (ILS):** Iterated local search (ILS) is a global optimization technique that explores a sequence of solutions created as perturbations of the current best solution, the result of which is refined using an embedded heuristics. In (Hurtgen & Maun, 2010), the OPP problem is solved assuming that a PMU placed at one node is capable of measuring all current phasor leaving the node. The proposed method suggest an initial PMU distribution which makes the network observable and then ILS is used to minimize size of PMU configuration needed to observe the network.

h. **Spanning Tree Search:** This algorithm dynamically determines the best path from source to destination avoiding bridge loops that can cause misinterpret results. In (Mahadeva, et al., 2015), the concept of depth of observability is used and its impact on the number of PMU placements is explained. The spanning tree approach is used for the power system graphs and a tree search technique is used for finding the optimal location of PMUs.

i. **Greedy Algorithm:** The greedy algorithm is an optimization methodology that follows the problem solving heuristic of making the locally optimal choice at each stage with the hope of finding the global optimum. A system reconfiguration approach for OPP in the distribution network to achieve system reconfiguration, and the ant colony algorithm has been applied as an optimization tool to minimize energy losses in (Abdelsalam, et al., 2014). Further, a greedy algorithm is utilized to find the PMU placement location by minimizing the number of PMUs with consideration to the maximum observability of the system. (Yang, et al., 2016) enhanced a method to develop an effective greedy algorithm for OPP to defend against data integrity attacks. The least-effort attack model computes the minimum number of sensors that must be compromised to manipulate a given number of states. Regarding the least-effort attack model, it proved the existence of the smallest set of sensors. Concerning the defense strategy, an effective PMU-based greedy algorithm was applied, and this algorithm not only defends against data integrity attacks but also ensures system observability with low overhead.

j. **Recursive security N algorithm:** The recursive security N algorithm is a spanning tree search of multiple solutions, with a different starting point. In (Denegri, et al., 2002), recursive and single shot security N approaches of PMU placement, with the aim of linear static estimation, are presented ensuring network observability. Recursive and single shot security N-1 algorithms considering both line losses and PMU outages are also presented.

Above mentioned study has focused only on determining minimum number of PMUs to minimize the cost. Further, very few literatures are available in PMU optimization using recursive security N algorithm. In this study, apart from calculation of optimal sets of solutions using RSN, further calculation of two indices, viz. Bus Observability Index (BOI) and System Observability Redundancy Index (SORI) has been done in order to ensure that the final solution obtained is most reliable as well as most economic and ensures system's full observability among multiple sets. Also, the effect of change in number of ZI bus in optimum solution is studied.

2.7. Recursive Security N (RSN) Algorithm

Recursive security N (RSN) is a spanning tree search of multiple solutions for minimal PMU positioning issue. It is a modified depth first approach, where the search is repeated as many times as the number of nodes, with a different starting PMU location. Depth first search (DFS) approach is one of the tree search methods of PMU placement.

This method uses only rules from 1 to 3 i.e. it does not consider ZI buses. The first PMU is placed at the bus with the largest number of connected branches. If there is more than one bus with this characteristics, one is randomly chosen. Then the following PMUs are placed with the same criterion, until the network becomes completely observable. This method, however, increases unwanted redundancy. Therefore, this method is modified in RSN algorithm by repeating the search as many times as the number of nodes, for e.g. if there are N buses in the system, the search is repeated N times, every time using a different bus as the initial bus at which the first PMU is installed. There are three major steps in implementing this algorithm:

a. Generation of N minimum spanning trees
b. Search of alternative patterns
c. Reducing PMU number in case of pure transit nodes

a. Generation of N minimum spanning trees

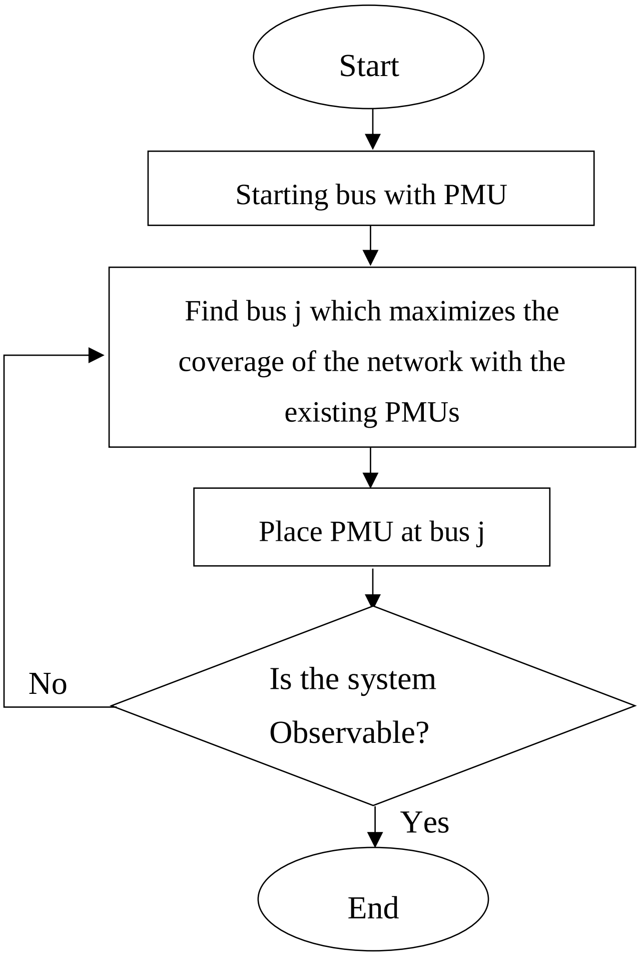

Figure 2-5: Flowchart for minimum spanning trees generation

Figure 2-5 shows the flowchart for generation of minimum spanning tree. If N is the number of buses if the network, the algorithm is performed N times, thus using all the nodes as starting position for PMU placement. After choosing the first PMU position, the remaining PMUs are recursively set in those which are found both to be closer to the observability region and to provide higher number of observed buses.

PMU's location end when the entire network is observable, and thus a minimum spanning tree is built. This modified DFS does not always guarantee an efficient positioning of PMUs, because the growing of the spanning tree is strongly conditioned by the first PMU choice. It has been found that a pre-ordering of the bus numbers can improve the results, leading to a higher number of sets with a minimum of PMU number. At this aim, the symmetric reverse Cuthill-McKee permutation of the admittance matrix and of the network nodes, seems to lead to the best results (Denegri, et al., 2002).

b. Search of alternative patterns

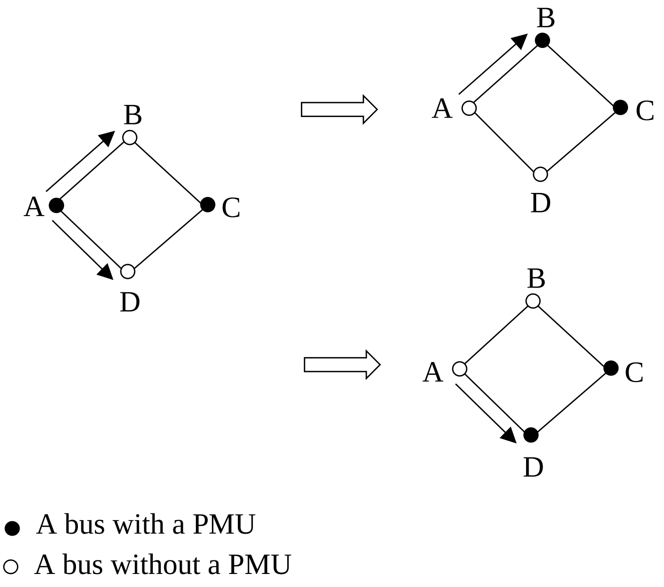

Figure 2-6: Search for alternative pattern in RSN method

The PMU sets obtained by the preliminary spanning tree generation are reprocessed for further improvement. One at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set, as shown in Figure 2-6. PMU placement which lead to a complete observability are retained. This kind of 'small signal' variation prove to provide some other equivalent minimum sets that may present practical advantages for the physical allocation of PMU.

c. Reducing PMU number in case of pure transit nodes (ZI bus)

If no pure transit nodes are present in the network, the procedure ends at previous step. Otherwise a last filtering of the actual sets is performed, by means of eliminating one PMU at a time and verifying if the network remains observable. In order to save simulation time, this procedure proved to be effective when applied only to the sets which present minimum number of PMUs.

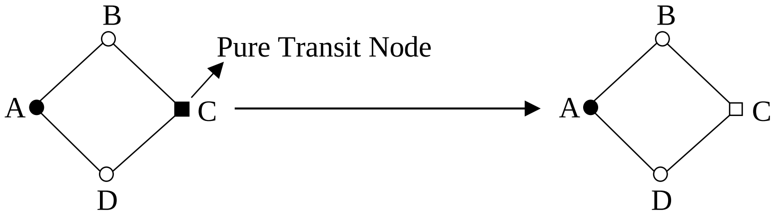

Figure 2-7: Pure transit node filtering

2.8. BOI and SORI

Bus Observability Index (BOI) is defines as the times the bus *i* is observed, and is equal to the number of PMUs observing bus i. The maximum bus observability index is limited to maximum connectivity of a bus plus one, which happens only when all adjacent buses and the bus itself is equipped with PMUs.

System Observability Redundancy Index (SORI) can be obtained by adding up BOI of all buses in the system. Higher SORI value indicates that the PMU based monitoring system is more reliable, where reliability is the probability that the system will perform its designated function for the given period of time under the conditions in which it was designed to operate.

These indices are used to determine the most reliable solution among multiple sets of optimal solution. In this study, first multiple optimum solution sets are determined using RSN and further BOI and SORI are calculated to ensure maximum reliability of PMU based monitoring system.

CHAPTER THREE: RESEARCH METHODOLOGY

3.1. Methodology Approach

The methodological approaches that has been adopted for the study is shown in Figure 3-1.

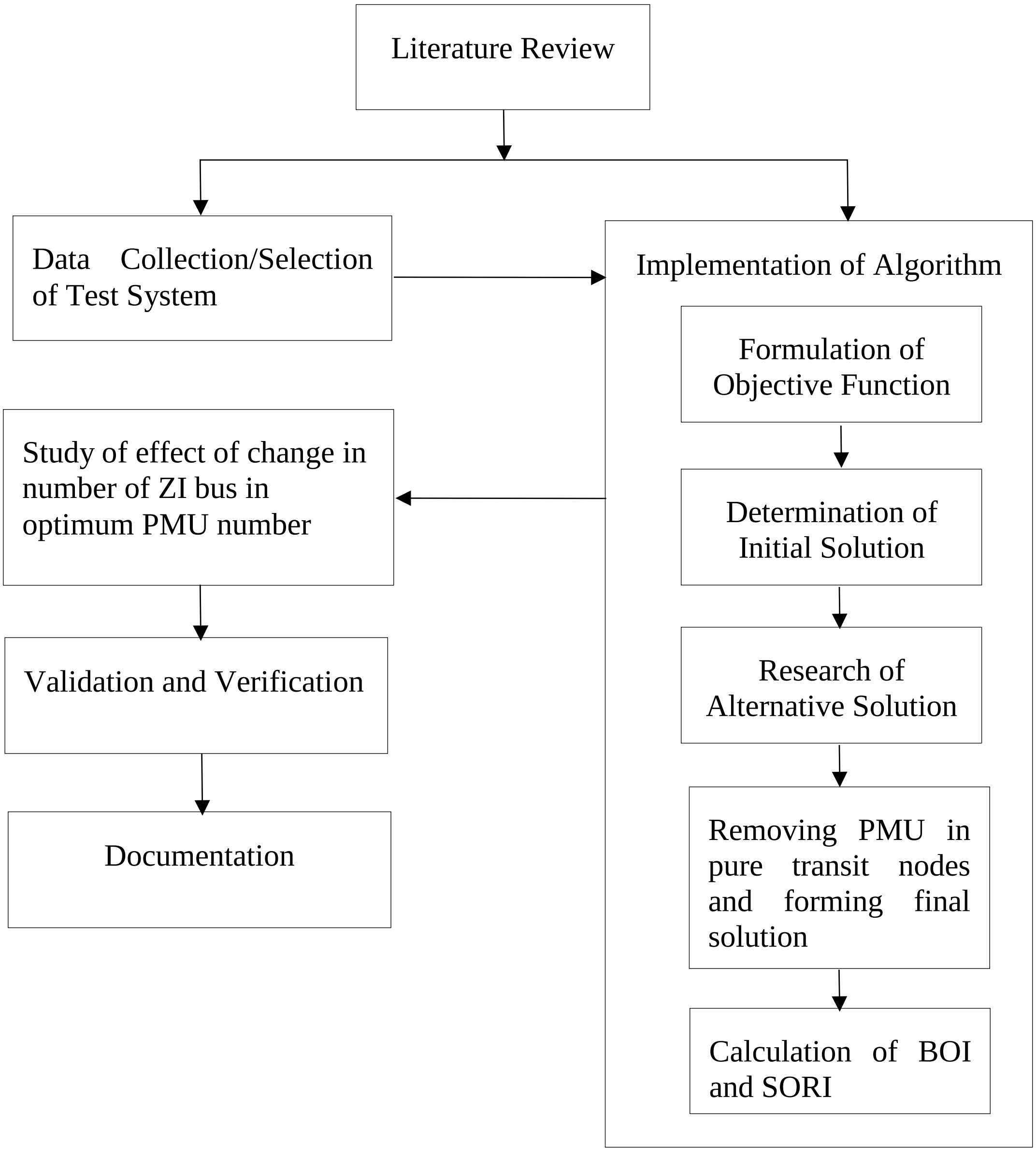

Figure 3-1: Methodology approach

3.2. Required Data

Data related to INPS were collected from System Planning Department and LDC, NEA. Bus data, line data, generator data and single line diagram related to 14-bus, 30-bus and 57-bus were collected through different websites. These data are attached in appendices. The major aspects of the data used are discussed in section below.

14-Bus System: In 14-bus system, there is only one zero injection (ZI) bus, bus number 7. There are 5 nos. of generator buses viz. bus number 1, 2, 3, 6 and 8 and the system has 20 branches.

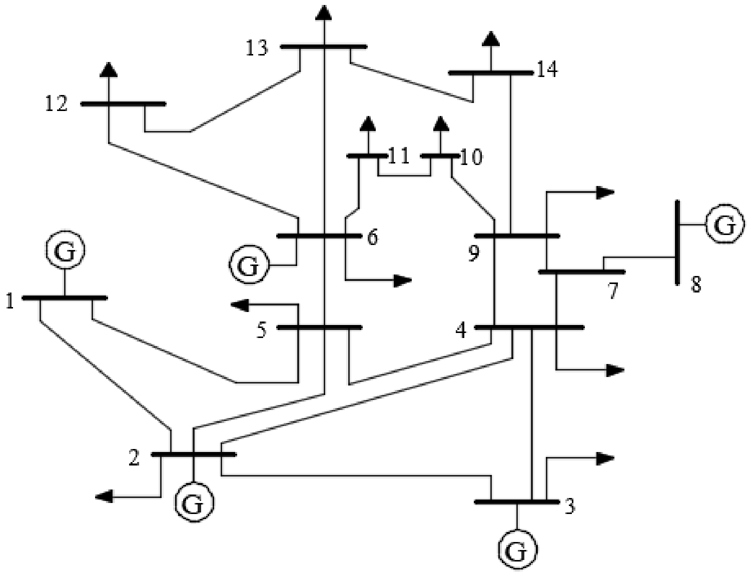

Figure 3-2 : Single Line Diagram of IEEE 14-bus Test System

30-Bus System: In IEEE 30-bus test system, there are six (6) nos. of ZI bus, viz. bus no. 6, 9, 22, 25, 27 and 28, six (6) nos. of generator bus, viz. 1, 2, 5, 8, 11 and 13 and the system has 41 branches.

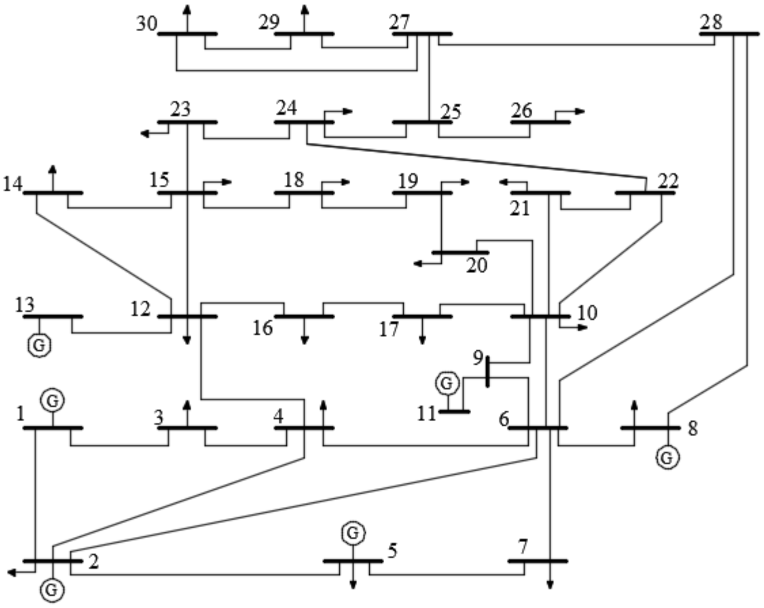

Figure 3-3: Single Line Diagram for IEEE 30-Bus Test System

57-Bus System

In IEEE 57-bus test system, there are fifteen (15) nos. of ZI bus, viz. bus no. 4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46 and 48. Moreover, there are six (7) nos. of generator bus, viz. 1, 2, 3, 6, 8, 9 and 12 and the system has 80 branches.

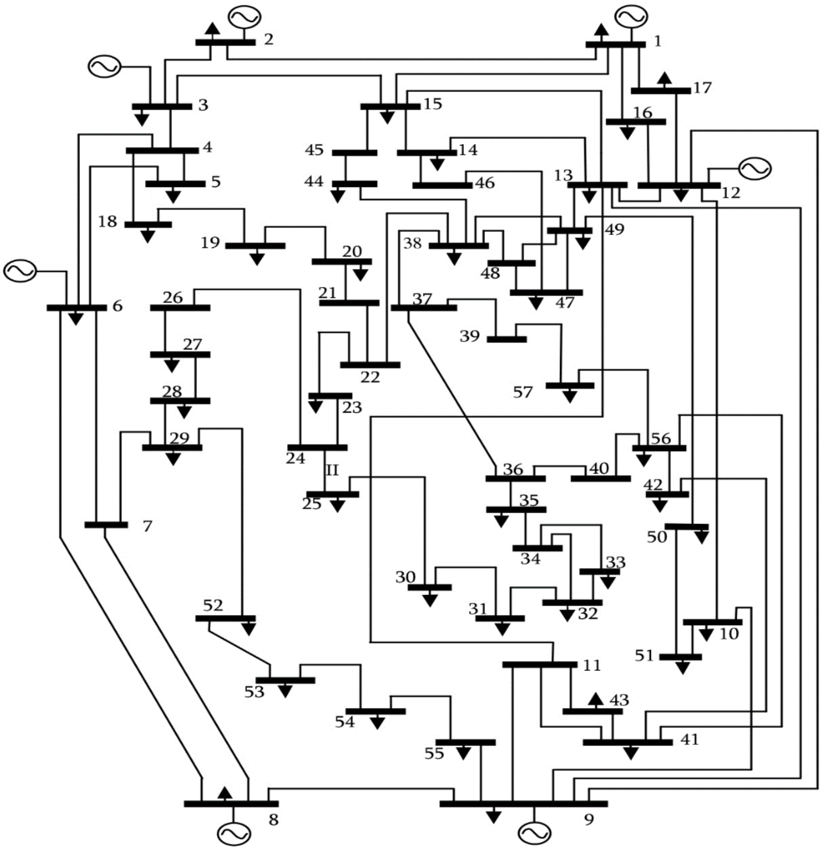

Figure 3-4: Single Line Diagram for IEEE 57-Bus System

Integrated Nepal Power System (INPS)

Every country has its own interconnected power system known as the Integrated National Power System. The Integrated National Power System ensures integrated operation of the power system. It consists of power generation, transmission and distribution network and its primary function is smooth evacuation of power from generating stations to consumers. INPS is responsible for overall reliability, security, economy and efficiency of power system (Shrestha, 2015). The INPS system considered here consists of 74 buses, 84 branches, 16 number of ZI buses viz. 5, 6, 10, 11, 12, 14, 25, 26, 33, 39, 43, 44, 51, 67, 69 and 71 and 24 number of generator buses viz. 3, 4, 7, 23, 24, 35, 41, 42, 46, 47, 48, 49, 50, 52, 53, 56, 57, 58, 59, 62, 63, 66, 70 and 73. The INPS system's single line diagram is attached in **Appendix R**.

3.3. Formulation of Objective Function

It is possible to make a system observable by simply placing PMUs at all buses without running any state estimator. However, it is obviously not economical. Besides the consideration of economy, the increasing scale of power system also makes it tough to control and manage PMUs installed at all buses. Hence, it is inadvisable to simply install a PMU at every bus. The goal here is to maintain power system's observability through as few PMUs as possible. This is the optimal PMU placement (OPP) problem. The objective of the OPP problem is to minimize the installation cost of PMUs. For an n-bus system, the OPP problem can be formulated as

$$\text{Minimize, } \sum_{i=1}^{n} w_i x_i \qquad \text{Equation 3.1}$$

$$\text{subject to} \qquad F(\bar{X}) = \bar{I}$$

where, w_i is the cost of installing a PMU at bus i.

x_i = 1 if a PMU is installed at Bus i otherwise x_i = 0,

$\bar{X}$ is a vector composed of x_i,

$F(\bar{X})$ is a vector function that represents the observability constraint functions,

$\bar{I}$ is a vector which has n entries and whose all elements are 1 when the system is fully observable.

According to first three rules for system observability mentioned in (Peng & Xu, 2008), the constraint vector function is obtained as

$$F(\bar{X}) = \mathbf{A}\bar{X} \qquad \text{Equation 3.2}$$

Where, **A** is a binary adjacency matrix. The element of matrix A is defined as

$$a_{mn} = \begin{cases} 1 \; if \; m = n \\ 1 \; if \; bus \; m \; and \; bus \; n \; are \; connected \\ 0 \; otherwise \end{cases} \qquad \text{Equation 3.3}$$

Where a_{mn} represents the element located at the m^{th} row and the n^{th} column in **A**.

For the bus i in an n-bus system, the corresponding constraint function f_i can be represented as

$$f_i(\bar{X}) = \sum_{j=1}^{n} a_{ij}x_j \qquad \text{Equation 3.4}$$

If $f_i(\bar{X}) \neq 0$, namely, if any $a_{ij} = 1$ and $x_j = 1$ $(j = 1,2,\ldots.,n)$, the bus i is observable. If all buses are observable, i.e. if all $f_i(\bar{X})$ in $F(\bar{X})$ are non-zero, the power system is fully observable.

Above equations are for a system without consideration of zero injection (ZI) buses. ZI buses are the buses with no generation and no load. When ZI buses are considered, the constraint function vector $F(\bar{X})$ is modified into a nonlinear constraint function vector.

According to the last three rules for system observability mentioned in (Peng & Xu, 2008), for each bus in the group composed of ZI bus and its adjacent buses, a bus is observable if other buses in that group are observable. Therefore, we introduce an auxiliary binary variable y_{ij} into Equation 3.4 to represent that bus i is observable supported by other buses but not bus i in the group of ZI bus j and all of its adjancent buses. Hence the Equation 3.4 can be modified as

$$f_i = \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n} a_{ij}z_j y_{ij} \qquad \text{Equation 3.5}$$

Where, z_j is a binary parameter whose value is 1 if the bus j is a ZI bus or zero otherwise and y_{ij} is an auxiliary binary variable whose value is 1 if all of bus j and its adjacent buses but not including bus i are observable or zero otherwise.

In the $a_{ij}z_j y_{ij}$ part of Equation 3.5, a_{ij} ensures bus i and j must be connected and z_j ensures that the bus j must be a ZI bus.

In practice, y_{ij} has to be calculated via logical operation. If, for example, there is a ZI bus m to which k normal buses composed of bus 1 to k where $m \notin [1, k]$, are connected, then the constraint function for bus 1 to k and for bus m can be obtained as

$$\begin{aligned} f_1 &= \sum_{j=1}^{n} a_{1j}x_j + (f_2 \wedge f_3 \wedge \ldots \wedge f_k \wedge f_m) \\ f_2 &= \sum_{j=1}^{n} a_{2j}x_j + (f_1 \wedge f_3 \wedge \ldots \wedge f_k \wedge f_m) \end{aligned} \qquad \text{Equation 3.6}$$

…

$$f_k = \sum_{j=1}^{n} a_{kj}x_j + (f_1 \wedge f_2 \wedge \ldots \wedge f_{k-1} \wedge f_m)$$

$$f_m = \sum_{j=1}^{n} a_{mj}x_j + (f_1 \wedge f_2 \wedge f_3 \ldots \wedge f_k)$$

Where, $\wedge$ is the logical AND operator.

Consider a situation where four PMUs are respectively installed at Bus 1, 2, 6 and 7. There are two sources to support the observability of bus 3 as well as bus 5.

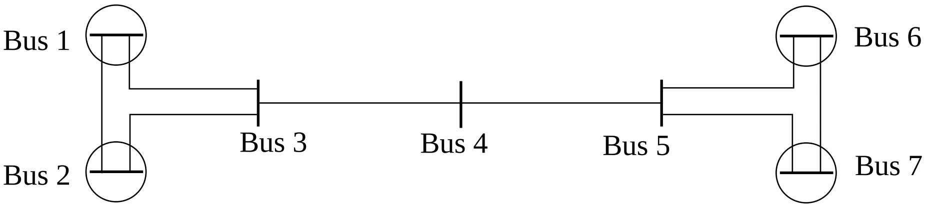

Figure 3-5: Observability supported via multiple sources applying ZI buses' properties

Using Equation 3.6, we have

$$\begin{aligned}
&x_1 = x_2 = x_6 = x_7 = 1\\
&x_3 = x_4 = x_5 = 0\\
&f_1 = x_1 + x_2 + x_3 = 2\\
&f_2 = x_1 + x_2 + x_3 = 2\\
&f_3 = x_1 + x_2 + x_3 + x_4 + (f_4 \wedge f_5) = 3\\
&f_4 = x_3 + x_4 + x_5 + (f_3 \wedge f_5) = 1\\
&f_5 = x_4 + x_5 + x_6 + x_7 + (f_3 \wedge f_4) = 3\\
&f_6 = x_5 + x_6 + x_7 = 2\\
&f_7 = x_5 + x_6 + x_7 = 2
\end{aligned} \qquad \text{Equation 3.7}$$

Since all f here are greater than 1, the system is considered observable.

From above equations, we can observe that the constraints equations, that compose the constraint function, are coupled. If we calculate the value f_1 to f_n, (where n is the total number of buses in a given system) only once in a certain order, the final result may be inaccurate. Therefore, we need to calculate f_1 to f_n repeatedly and take into account their values obtained in the last iteration, until the values of f_1 to f_n does not change after an iteration.

3.4. Determination of Initial Solution

The N number of minimum spanning trees created are the initial solutions. As mentioned in earlier in section 2.7, if N is the number of buses if the network, the algorithm is performed N times, thus using all the nodes as starting position for PMU placement.

An algorithm for determination of initial solution

Step 1: Start

Step 2: Read system data (Bus data, branch data, generator data)

Step 3: Determine the adjacency matrix and identify ZI buses

Step 4: Perform symmetric reverse Cuthill-McKee permutation of adjacency matrix A.

Step 5: Construct topological observability formula using Equation 3.5 and Equation 3.6

Step 6: Initialize variables (X_best, xb_len, xb_used, cand_X, cx_len)

Step 7: Create a diagonal matrix tmp with PMU placed at each bus

Step 8: Take transpose of first row of tmp and place it in a variable c_X and determine the product of c_X and A to know how many buses are observable when PMU/s is/are placed at current starting position.

Step 9: Make first row of tmp empty (for eg. If tmp is [14x14] matrix, now it will become [13x14] matrix)

Step 10: Determine which bus has maximum number of connected adjacent buses.

Step 11: Place PMU at the bus which makes maximum number of buses observable.

Step 12: If all buses are observable in step 11, store the solution in variable cand_X. If not, go to step 13.

Step 13: Check if solution obtained in step 11 is already in tmp, If yes, go to step 14, else, add the solution as last row of tmp.

Step 14: Check if tmp is empty or not. If tmp is empty, go to step 15, else, go to step 8 and repeat until tmp is empty.

Step 15: Enumerate all the solutions obtained from steps mentioned above in variable cand_X.

Step 16: End

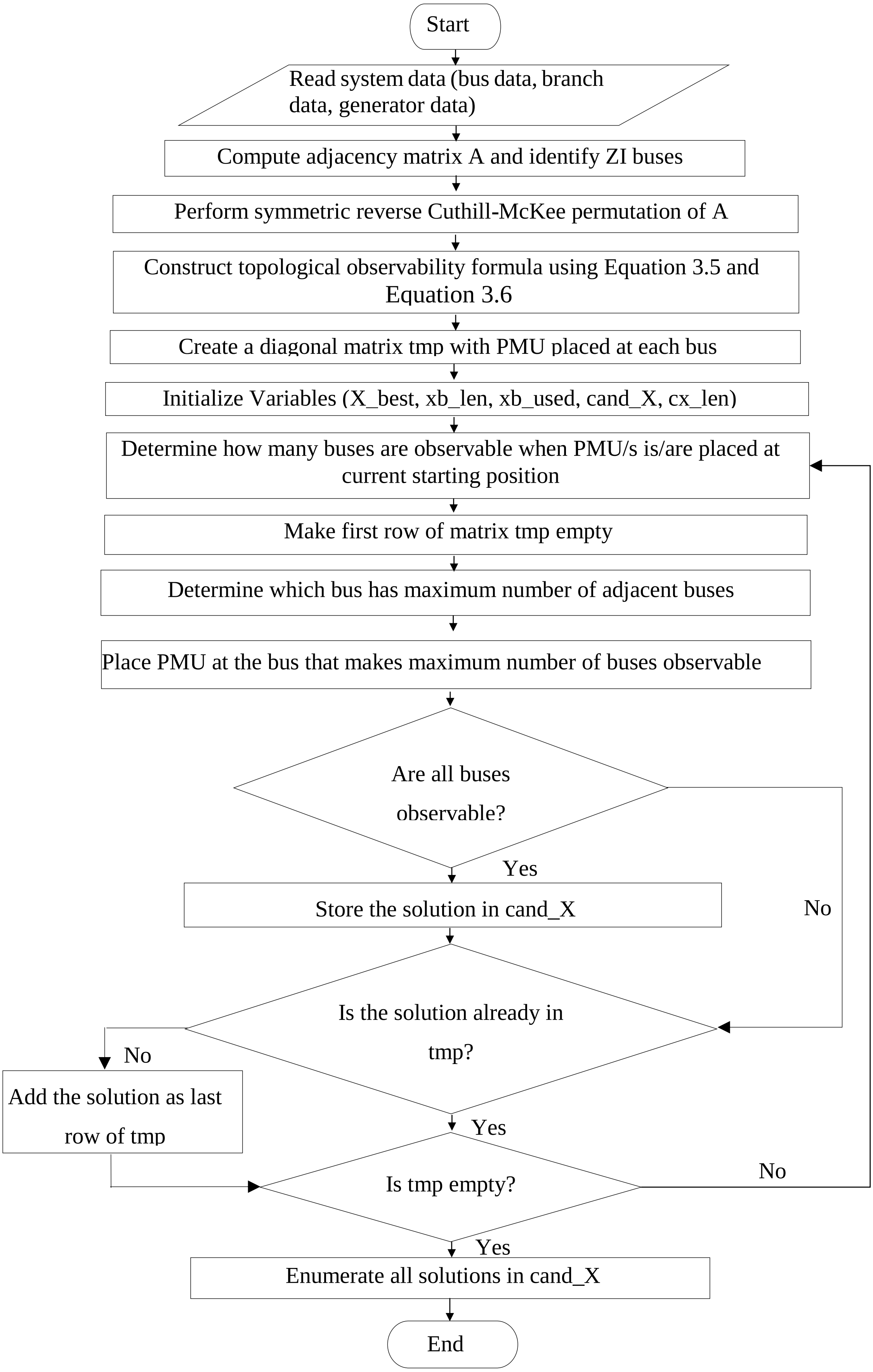

Figure 3-6: Flowchart for the determination of initial solution

3.5. Search of Alternative Patterns

Once the sets of initial solution is determined, they are reprocessed for further improvement. One at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set. PMU placement which lead to a complete observability are retained.

An Algorithm to Search Alternative Patterns

Step 1: Create an empty matrix Xs of dimension [0 x num_buses].

Step 2: Take first row of cand_X and place it in a variable c_X.

Step 3: Make first row of cand_X empty.

Step 4: Determine the number and position of PMU in the first initial solution set.

Step 5: For each PMU in the solution set, remove the PMU from its original position, and place it in each adjacent position, and check observability.

Step 6: However, if there is already a PMU in adjacent position, skip that position and check for another adjacent position. If all buses are observable when placing PMU in adjacent positions, store the alternative solution thus obtained in variable Xs.

Step 7: Repeat the process from step 2 to 6 until cand_X is empty.

Step 8: Enumerate all unique alternative solutions obtained from above steps in Xs.

Step 9: End

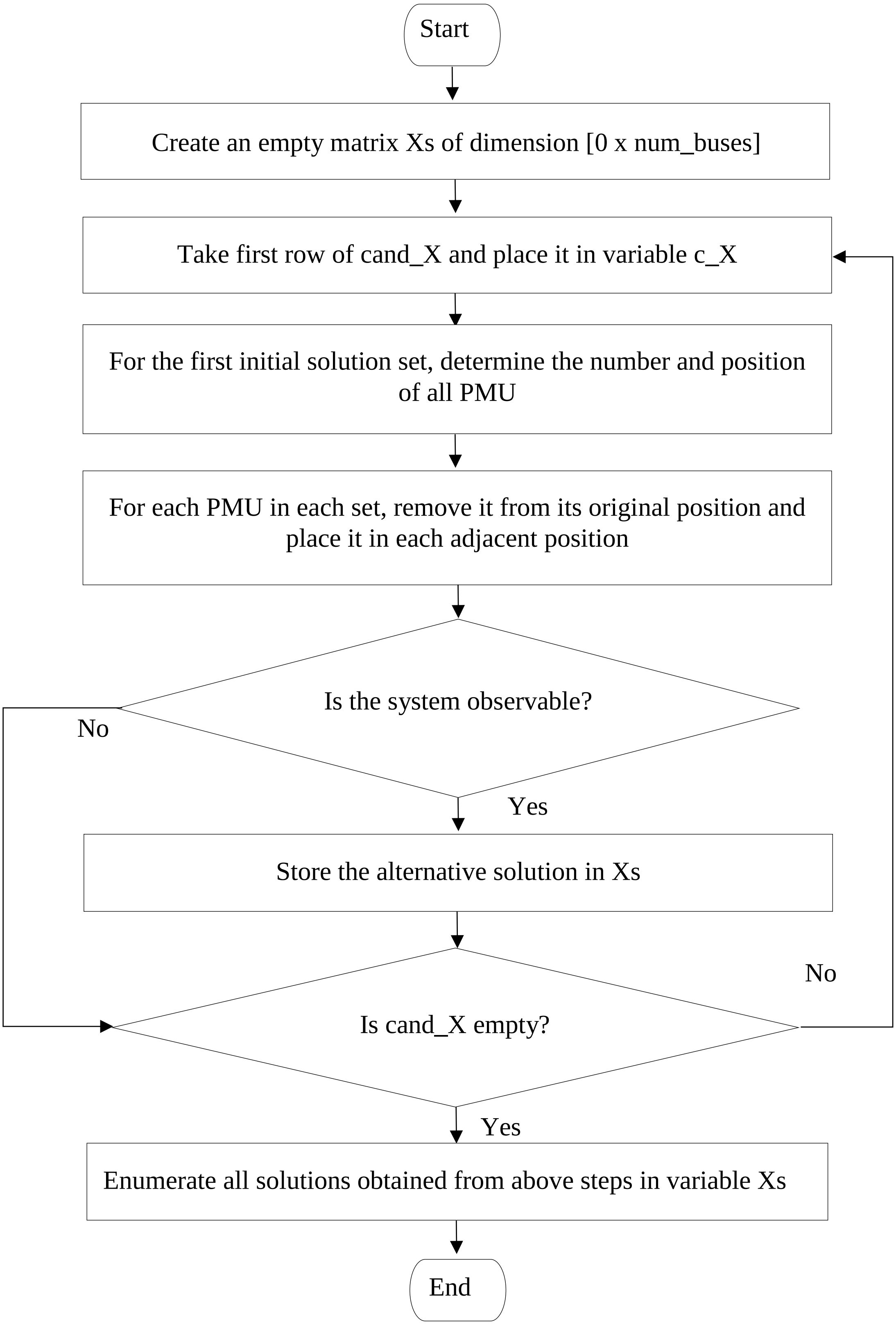

Figure 3-7: Flowchart for the determination of alternative solution

3.6. Removing PMU in Pure Transit Nodes and Determination of Final Solution

If no pure transit nodes are present in the network, the procedure ends at previous step. However, a final filtering of the obtained solution set is done, by eliminating PMU present in ZI buses and verifying if the network still remains observable. In order to save simulation time, this procedure is carried out only in the solutions that have minimum number of PMUs.

An Algorithm to Remove PMUs from Pure Transit Nodes

Step 1: In the solution Xs obtained before, determine the set with minimum number of PMU placement.

Step 2: Place the sets with minimum number of PMUs obtained in variable cand_X.

Step 3: For each set of solutions, change every non zero element to zero one at a time and check observability.

Step 4: If it is observable, store the solution to check if any other solution will be better than the current solution. If not, go to step 3.

Step 5: If the current solution is better than the solution previously stored in X_best, replace it in a variable X_best.

Step 6: Obtain the arranged actual solution by performing symmetric reverse McKee-Cuthill permutation of solution placed in X_best.

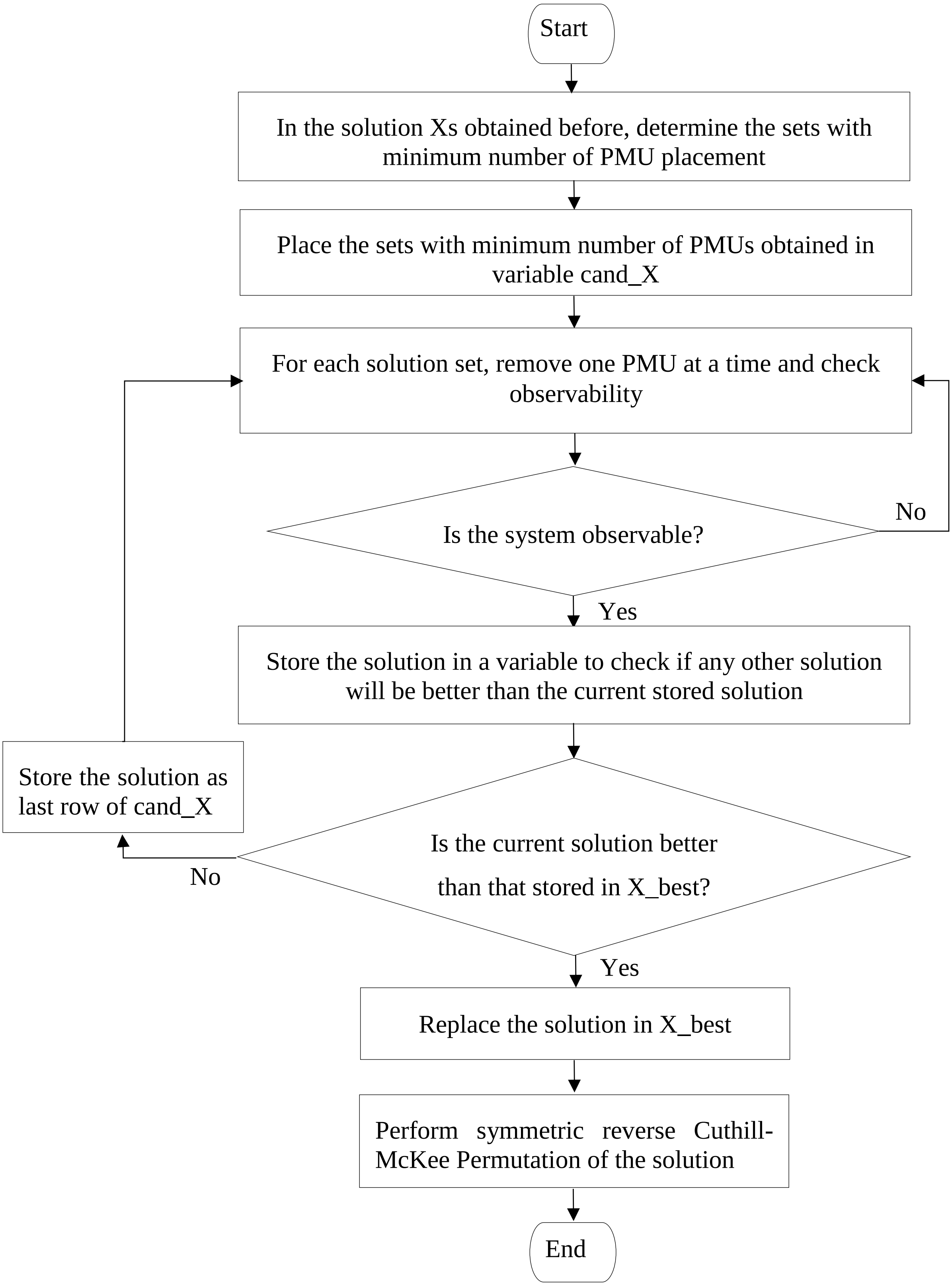

Figure 3-8 : Flowchart for the determination of final solution

CHAPTER FOUR: RESULTS AND DISCUSSION

With a selected subset of buses with PMUs, entire system can be made observable. This will be possible only by proper placement of PMUs among the system buses. The problem is formulated and solved using recursive security N method. When the collected data were processed and fed as input data to the developed algorithm, it searched the optimal solution and gave the following outputs.

a. Minimum number of PMUs set to be installed in the system for full system's observability.
b. Location at which each of the PMU has to be installed.

4.1. Study Case I: IEEE 14-Bus Test System

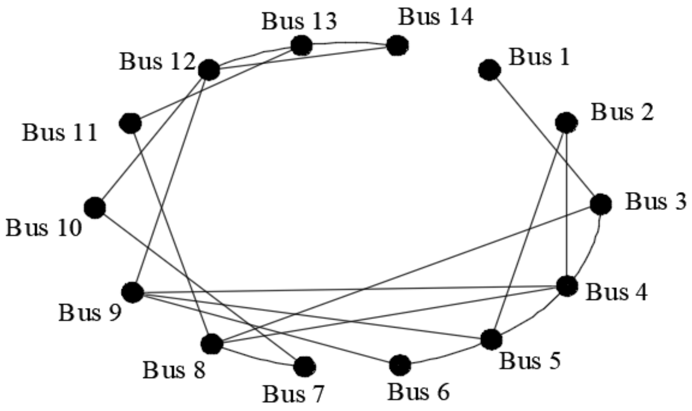

Figure 4-1: IEEE 14-Bus Test System after Cuthill-McKee Permutation

The first step for the determination of optimal solution is the determination of adjacency matrix and identification of ZI buses. Adjacency matrix is determined using the algorithm and Cuthill-McKee permutation of the matrix is done for accuracy of the solution.

Table 4-1: Original adjacency matrix for IEEE 14-bus system in vector form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	[2,5]	8	7
2	[1,3,4,5]	9	[4,7,10,14]
3	[2,4]	10	[9,11]
4	[2,3,5,7,9]	11	[6,10]
5	[1,2,4,6]	12	[6,13]
6	[5,11,12,13]	13	[6,12,14]
7	[4,8,9]	14	[9,13]

There is only one ZI bus in this system which is bus no. 7. After Cuthill-McKee permutation, bus 3 becomes ZI bus.

Table 4-2: Adjacency Matrix for 14-Bus System after Cuthill-McKee Permutation in Vector Form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	3	8	[3,4,7,11]
2	[4,5]	9	[4,5,6,12]
3	[1,4,8]	10	[7,12]
4	[2,3,5,8,9]	11	[8,13]
5	[2,4,6,9]	12	[9,10,13,14]
6	[5,9]	13	[11,12,14]
7	[8,10]	14	[12,13]

Topological observability formula is constructed using Equation 3.5 and Equation 3.6. The topological observability equation is shown in Table 4-3.

Table 4-3: Topological Observability Formulae for IEEE 14-Bus Test System

f(1)=X(1)+X(3)+~any([f(4) f(8) f(3)]==0)
f(2)=X(2)+X(4)+X(5)
f(3)=X(3)+X(1)+X(4)+X(8)+~any([f(1) f(4) f(8)]==0)
f(4)=X(4)+X(2)+X(3)+X(5)+X(8)+X(9)+~any([f(1) f(8) f(3)]==0)
f(5)=X(5)+X(2)+X(4)+X(6)+X(9)
f(6)=X(6)+X(5)+X(9)
f(7)=X(7)+X(8)+X(10)
f(8)=X(8)+X(3)+X(4)+X(7)+X(11)+~any([f(1) f(4) f(3)]==0)
f(9)=X(9)+X(4)+X(5)+X(6)+X(12)
f(10)=X(10)+X(7)+X(12)
f(11)=X(11)+X(8)+X(13)
f(12)=X(12)+X(9)+X(10)+X(13)+X(14)
f(13)=X(13)+X(11)+X(12)+X(14)
f(14)=X(14)+X(12)+X(13)

In the topological observability formula, it can be observed that there are four equations, equation for bus no. 1, 3, 4 and 8 whose observability formula is affected by the presence of ZI buses. These buses are connected to the ZI bus. Hence, such equations are obtained using Equation 3.6 for those buses which are connected to ZI bus.

a. Determination of Initial Solution

After following the steps for the generation of minimum spanning tree as mentioned in Section 3.4, the initial solution obtained is given in Table 4-4.

Table 4-4: Initial Solution for IEEE 14-Bus Test System

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Solution Set														
1	0	0	1	0	1	0	1	0	0	0	1	1	0	0
2	1	0	0	0	1	0	0	1	0	0	0	1	0	0
3	1	0	0	1	0	1	0	0	0	1	0	0	1	0
4	1	0	0	1	0	1	1	0	0	0	0	0	1	0
5	1	0	0	1	0	1	1	0	0	0	1	0	0	1
6	1	0	0	1	0	1	1	0	0	0	1	1	0	0
7	1	1	0	0	0	0	0	1	1	1	0	0	1	0

All 7 solution sets provide complete observability of the system. The maximum number of PMU required for complete system observability according to the initial solution is six and minimum number of PMU required for complete system observability is four.

b. Determination of Alternative Solution

Alternative solution is determined using the steps mentioned in Section 3.5. One at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set. PMU placement which lead to a complete observability are retained. The retained solution for IEEE 14-bus test system are listed in **Appendix A**.

c. Determination of Final Solution

A final filtering of the obtained solution set is done, by eliminating PMU present in ZI buses and verifying if the network still remains observable. In IEEE 14-bus system, alternative solution set number 4 and solution set number 16 have 4 number of PMUs. However, pure transit node is only present in solution set 4. There are four PMUs, one each in bus number 3, 5, 8 and 12. The PMU in pure transit node 3 is removed and the system is checked for full observability. The system was found to be observable. Hence the final optimal placement of PMU for IEEE 14-bus system was found to be 5, 8 and 12. However, since this solution is obtained for the system after Cuthill-McKee permutation, the permutation is again performed to obtain the original placement. The original placement was found to be bus number 2, 6 and 9. Table 4-5 shows that the 14-bus system is completely observable after PMU placement in bus number 2, 6 and 9.

Table 4-5: Value of Constraint Function for 14-Bus System

Constraint Function	Value	Constraint Function	Value
f(1)	1	f(8)	2
f(2)	1	f(9)	2
f(3)	2	f(10)	1

Constraint Function	Value	Constraint Function	Value
f(4)	3	f(11)	1
f(5)	1	f(12)	1
f(6)	1	f(13)	1
f(7)	1	f(14)	1

4.2. Study Case II: IEEE 30-Bus Test System

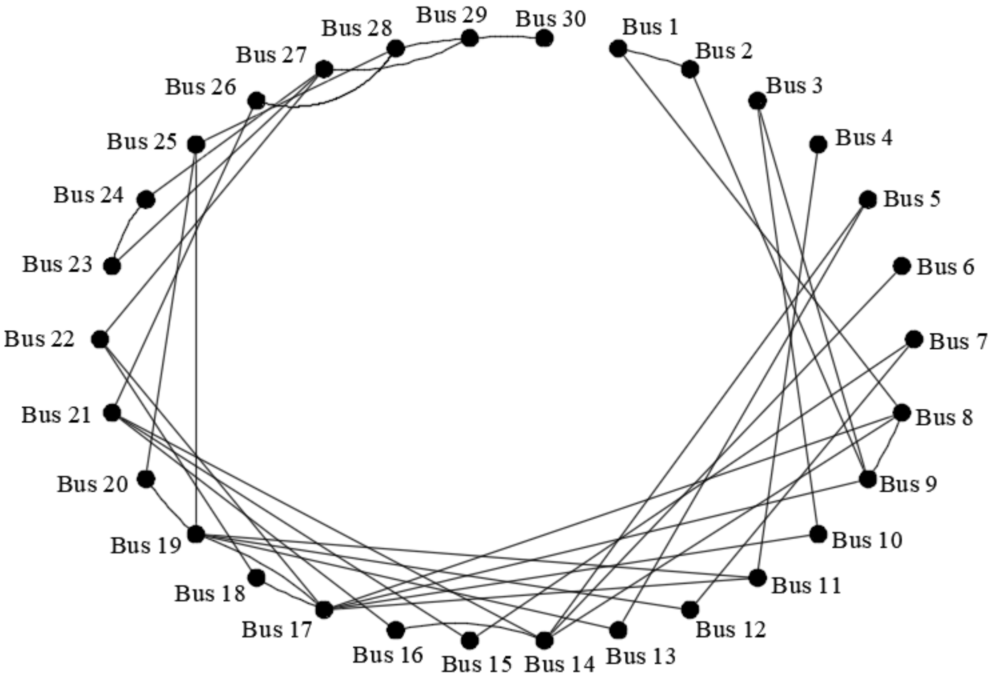

Figure 4-2: IEEE 30-Bus Test System after Cuthill-McKee Permutation

The same steps that were used for IEEE 14-bus system are utilized in this system. Original adjacency matrix and adjacency matrix after Cuthill-McKee permutation in vector form is given in **Appendix B** and **Appendix C** respectively. There are six ZI buses in this system which are bus no. 6, 9, 12, 25, 27 and 28. After Cuthill-McKee permutation, ZI buses are changed to 11, 17, 22, 25, 27 and 29. Topological observability formula is constructed using Equation 3.5 and Equation 3.6. The topological observability equation for 30-bus system is shown in Table 4-6.

Table 4-6: Topological Observability Formulae for IEEE 30-Bus Test System

f(1)=X(1)+X(2)+X(8)
f(2)=X(2)+X(1)+X(9)
f(3)=X(3)+X(9)+X(10)
f(4)=X(4)+X(11)+~any([f(17) f(19) f(11)]==0)
f(5)=X(5)+X(13)+X(14)
f(6)=X(6)+X(14)

f(7)=X(7)+X(12)+X(15)
f(8)=X(8)+X(1)+X(9)+X(14)+X(17)+~any([f(9) f(10) f(11) f(18) f(19) f(22) f(17)]==0)
f(9)=X(9)+X(2)+X(3)+X(8)+X(17)+~any([f(8) f(10) f(11) f(18) f(19) f(22) f(17)]==0)
f(10)=X(10)+X(3)+X(17)+~any([f(8) f(9) f(11) f(18) f(19) f(22) f(17)]==0)
f(11)=X(11)+X(4)+X(17)+X(19)+~any([f(8) f(9) f(10) f(18) f(19) f(22) f(17)]==0)+~any([f(4) f(17) f(19)]==0)
f(12)=X(12)+X(7)+X(19)
f(13)=X(13)+X(5)+X(19)
f(14)=X(14)+X(5)+X(6)+X(8)+X(16)+X(21)
f(15)=X(15)+X(7)+X(21)
f(16)=X(16)+X(14)+X(21)
f(17)=X(17)+X(8)+X(9)+X(10)+X(11)+X(18)+X(19)+X(22)+~any([f(8) f(9) f(10) f(11) f(18) f(19) f(22)]==0)+~any([f(4) f(19) f(11)]==0)+~any([f(18) f(27) f(22)]==0)
f(18)=X(18)+X(17)+X(22)+~any([f(8) f(9) f(10) f(11) f(19) f(22) f(17)]==0)+~any([f(17) f(27) f(22)]==0)
f(19)=X(19)+X(11)+X(12)+X(13)+X(17)+X(20)+X(25)+~any([f(8) f(9) f(10) f(11) f(18) f(22) f(17)]==0)+~any([f(4) f(17) f(11)]==0)+~any([f(20) f(28) f(25)]==0)
f(20)=X(20)+X(19)+X(25)+~any([f(19) f(28) f(25)]==0)
f(21)=X(21)+X(14)+X(15)+X(16)+X(26)
f(22)=X(22)+X(17)+X(18)+X(27)+~any([f(8) f(9) f(10) f(11) f(18) f(19) f(17)]==0)+~any([f(23) f(24) f(29) f(27)]==0)+~any([f(17) f(18) f(27)]==0)
f(23)=X(23)+X(24)+X(27)+~any([f(22) f(24) f(29) f(27)]==0)
f(24)=X(24)+X(23)+X(27)+~any([f(22) f(23) f(29) f(27)]==0)
f(25)=X(25)+X(19)+X(20)+X(28)+~any([f(19) f(20) f(28)]==0)
f(26)=X(26)+X(21)+X(28)
f(27)=X(27)+X(22)+X(23)+X(24)+X(29)+~any([f(28) f(30) f(29)]==0)+~any([f(22) f(23) f(24) f(29)]==0)+~any([f(17) f(18) f(22)]==0)
f(28)=X(28)+X(25)+X(26)+X(29)+~any([f(19) f(20) f(25)]==0)+~any([f(27) f(30) f(29)]==0)
f(29)=X(29)+X(27)+X(28)+X(30)+~any([f(27) f(28) f(30)]==0)+~any([f(22) f(23) f(24) f(27)]==0)
f(30)=X(30)+X(29)+~any([f(27) f(28) f(29)]==0)

In the topological observability formula, it can be seen that observability equation for bus no. 4, 8, 9, 10, 23, 24, 25 and 30 are similar to that obtained for the 14-bus system, where these buses are connected with only one ZI bus. However, in this system there are buses that are connected with more than one ZI bus. The observability equation for these buses are obtained using Equation 3.6. Bus 11, 17, 18, 19, 22, 27, 28 and 29 are connected with more than one ZI bus.

a. Determination of Initial Solution

After following the steps for the generation of minimum spanning tree as mentioned in Section 3.4, the initial solution obtained is given in Table 4-7.

Table 4-7: Initial Solution for IEEE 30-Bus Test System

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Solution Set																														
1	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
2	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
3	1	0	0	0	0	0	1	0	1	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1
4	1	0	0	1	0	0	1	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	1
5	1	0	0	1	0	0	1	0	1	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1
6	1	0	1	1	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
7	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
8	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1	0
9	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	1	1	0	0	1	0
10	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	1	1	0	1
11	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
12	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	1

All 12 solution sets provide complete observability of the system. The maximum number of PMUs required for complete system observability according to the initial solution is 13 and minimum number of PMUs required is 11.

b. Determination of Alternative Solution

Alternative solution is determined using the steps mentioned in Section 3.5. One at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set. PMU placement which leads to complete observability are retained. The retained solutions for IEEE 30-bus test system are listed in **Appendix H**.

Altogether, there are 263 number of solutions including the initial solutions. All of these solutions ensure complete observability.

c. Determination of Final Solution

Table 4-8: Final Solution for IEEE 30-Bus Test System

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Solution Set																														
1	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
2	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0
3	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0
4	1	0	0	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0

A final filtering of the obtained solution set is done, by eliminating PMU present in ZI buses and verifying if the network still remains observable. To save simulation time, this filtering is done only in the solutions that consist of minimum number of PMU. Pure transit nodes are removed and checked for observability. For 30-bus system, 4 sets of optimal solutions were obtained each consisting of 7 PMUs. However, since this solution is obtained for the system after Cuthill-McKee permutation, the permutation is again performed to obtain the original placement. The original placement found is given in Table 4-9.

Table 4-9: Final Placement for IEEE 30-Bus Test System

Solution Set	Position of PMU						
1	3	19	7	12	10	27	24
2	3	19	7	12	10	30	24
3	3	19	2	12	10	27	24
4	3	19	2	12	10	30	24

All four sets of optimal solution were checked for observability. Table 4-10 shows that 30-bus system is completely observable with all four sets of solutions.

Table 4-10: Value of Constraint Function for 30-Bus System

Constraint Function	Values				Constraint Function	Values			
	Set 1	Set 2	Set 3	Set 4		Set 1	Set 2	Set 3	Set 4
f(1)	1	1	1	1	f(16)	1	1	1	1
f(2)	1	1	2	2	f(17)	5	5	5	5
f(3)	1	1	1	1	f(18)	2	2	2	2
f(4)	1	1	1	1	f(19)	4	4	4	4
f(5)	1	1	1	1	f(20)	2	2	2	2
f(6)	1	1	1	1	f(21)	1	1	1	1
f(7)	1	1	1	1	f(22)	4	3	4	3
f(8)	3	3	4	4	f(23)	2	2	2	2
f(9)	1	1	2	2	f(24)	2	2	2	2
f(10)	2	2	1	1	f(25)	3	3	3	3
f(11)	3	3	3	3	f(26)	1	1	1	1
f(12)	2	2	2	2	f(27)	4	4	4	4
f(13)	1	1	1	1	f(28)	3	3	3	3
f(14)	1	1	1	1	f(29)	4	3	4	3
f(15)	1	1	1	1	f(30)	1	1	1	1

4.3. Study Case III: IEEE 57-Bus Test System

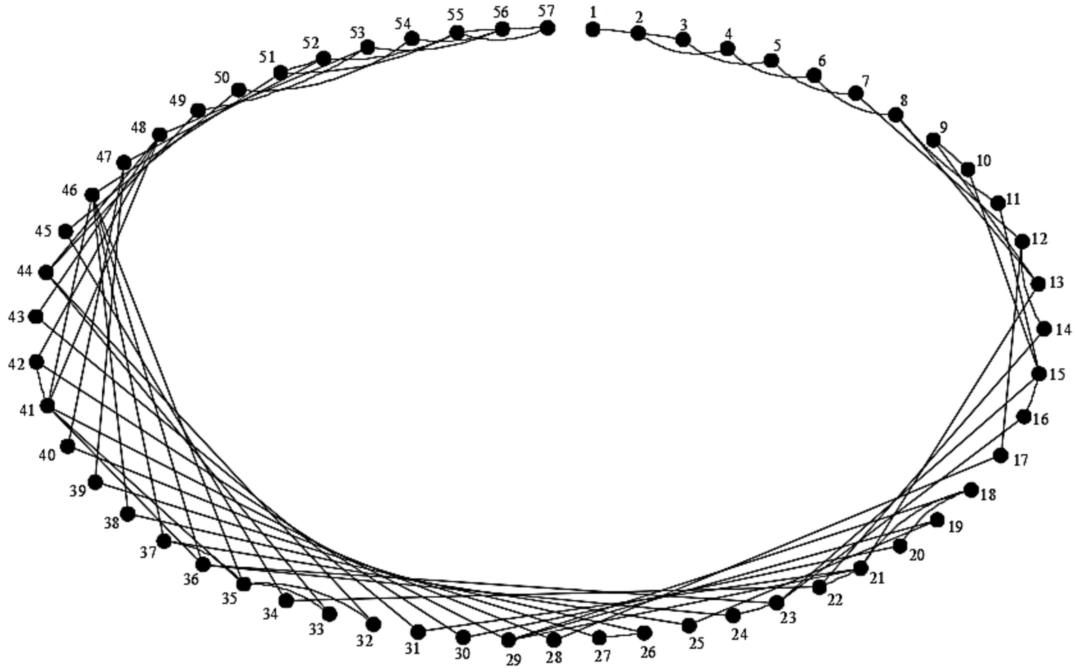

Figure 4-3: IEEE 57-Bus Test System after Cuthill-McKee Permutation

The same steps that were utilized for 30-bus system are used here. The original adjacency matrix and the adjacency matrix after Cuthill-McKee permutation are shown in vector form in **Appendix D** and **Appendix E** respectively. ZI buses after Cuthill-McKee permutation are bus no. 56, 52, 36, 34, 22, 12, 17, 4, 8, 13, 9, 11, 43, 30 and 18. Topological observability formulae for 57-bus system is obtained as shown in Table 4-11.

Table 4-11: Topological Observability Formulae for IEEE 57-Bus Test System

f(1)=X(1)+X(2)
f(2)=X(2)+X(1)+X(3)+X(4)+~any([f(6) f(4)]==0)
f(3)=X(3)+X(2)+X(5)
f(4)=X(4)+X(2)+X(6)+~any([f(2) f(6)]==0)
f(5)=X(5)+X(3)+X(7)
f(6)=X(6)+X(4)+X(8)+~any([f(2) f(4)]==0)+~any([f(11) f(13) f(8)]==0)
f(7)=X(7)+X(5)+X(12)+~any([f(14) f(17) f(12)]==0)
f(8)=X(8)+X(6)+X(11)+X(13)+~any([f(6) f(11) f(13)]==0)+~any([f(9) f(21) f(13)]==0)+~any([f(15) f(11)]==0)
f(9)=X(9)+X(10)+X(13)+~any([f(8) f(21) f(13)]==0)+~any([f(10) f(13)]==0)
f(10)=X(10)+X(9)+X(15)+~any([f(13) f(9)]==0)
f(11)=X(11)+X(8)+X(15)+~any([f(6) f(13) f(8)]==0)+~any([f(8) f(15)]==0)
f(12)=X(12)+X(7)+X(14)+X(17)+~any([f(7) f(14) f(17)]==0)+~any([f(28) f(17)]==0)

f(13)=X(13)+X(8)+X(9)+X(21)+~any([f(6) f(11) f(8)]==0)+~any([f(8) f(9) f(21)]==0)+~any([f(10) f(9)]==0)
f(14)=X(14)+X(12)+X(22)+~any([f(21) f(34) f(22)]==0)+~any([f(7) f(17) f(12)]==0)
f(15)=X(15)+X(10)+X(11)+X(16)+X(23)+~any([f(8) f(11)]==0)
f(16)=X(16)+X(15)+X(23)
f(17)=X(17)+X(12)+X(28)+~any([f(7) f(14) f(12)]==0)+~any([f(12) f(28)]==0)
f(18)=X(18)+X(20)+X(21)+X(29)+~any([f(20) f(21) f(29)]==0)
f(19)=X(19)+X(25)+X(29)
f(20)=X(20)+X(18)+X(30)+~any([f(42) f(30)]==0)+~any([f(21) f(29) f(18)]==0)
f(21)=X(21)+X(13)+X(18)+X(22)+X(29)+X(31)+~any([f(14) f(34) f(22)]==0)+~any([f(8) f(9) f(13)]==0)+~any([f(20) f(29) f(18)]==0)
f(22)=X(22)+X(14)+X(21)+X(34)+~any([f(45) f(34)]==0)+~any([f(14) f(21) f(34)]==0)
f(23)=X(23)+X(15)+X(16)+X(24)+X(36)+~any([f(24) f(41) f(46) f(36)]==0)
f(24)=X(24)+X(23)+X(36)+~any([f(23) f(41) f(46) f(36)]==0)
f(25)=X(25)+X(19)+X(37)
f(26)=X(26)+X(27)+X(38)
f(27)=X(27)+X(26)+X(39)
f(28)=X(28)+X(17)+X(40)+~any([f(12) f(17)]==0)
f(29)=X(29)+X(18)+X(19)+X(21)+X(41)+~any([f(20) f(21) f(18)]==0)
f(30)=X(30)+X(20)+X(42)+~any([f(20) f(42)]==0)
f(31)=X(31)+X(21)+X(43)+~any([f(48) f(43)]==0)
f(32)=X(32)+X(35)+X(44)
f(33)=X(33)+X(35)+X(44)
f(34)=X(34)+X(22)+X(45)+~any([f(22) f(45)]==0)+~any([f(14) f(21) f(22)]==0)
f(35)=X(35)+X(32)+X(33)+X(37)+X(41)+X(46)
f(36)=X(36)+X(23)+X(24)+X(41)+X(46)+~any([f(23) f(24) f(41) f(46)]==0)
f(37)=X(37)+X(25)+X(35)+X(46)
f(38)=X(38)+X(26)+X(46)
f(39)=X(39)+X(27)+X(47)
f(40)=X(40)+X(28)+X(47)
f(41)=X(41)+X(29)+X(35)+X(36)+X(42)+X(46)+X(48)+~any([f(23) f(24) f(46) f(36)]==0)
f(42)=X(42)+X(30)+X(41)+X(48)+~any([f(20) f(30)]==0)
f(43)=X(43)+X(31)+X(48)+~any([f(31) f(48)]==0)
f(44)=X(44)+X(32)+X(33)+X(48)+X(49)
f(45)=X(45)+X(34)+X(50)+~any([f(22) f(34)]==0)
f(46)=X(46)+X(35)+X(36)+X(37)+X(38)+X(41)+X(51)+~any([f(23) f(24) f(41) f(36)]==0)
f(47)=X(47)+X(39)+X(40)+X(52)+~any([f(51) f(55) f(52)]==0)
f(48)=X(48)+X(41)+X(42)+X(43)+X(44)+X(53)+~any([f(31) f(43)]==0)
f(49)=X(49)+X(44)+X(53)
f(50)=X(50)+X(45)+X(54)
f(51)=X(51)+X(46)+X(52)+X(55)+~any([f(47) f(55) f(52)]==0)
f(52)=X(52)+X(47)+X(51)+X(55)+~any([f(47) f(51) f(55)]==0)
f(53)=X(53)+X(48)+X(49)+X(56)+~any([f(54) f(55) f(57) f(56)]==0)

f(54)=X(54)+X(50)+X(56)+~any([f(53) f(55) f(57) f(56)]==0)
f(55)=X(55)+X(51)+X(52)+X(56)+X(57)+~any([f(53) f(54) f(57) f(56)]==0)+~any([f(47) f(51) f(52)]==0)
f(56)=X(56)+X(53)+X(54)+X(55)+X(57)+~any([f(53) f(54) f(55) f(57)]==0)
f(57)=X(57)+X(55)+X(56)+~any([f(53) f(54) f(55) f(56)]==0)

The IEEE 57-bus test system has fifteen ZI buses. More than 50% of buses are connected with ZI bus. Initial solution set is attached in **Appendix I**. There are 24 sets of solution that ensures system's full observability. The minimum number of PMUs required for full system observability according to initial solution is 20 and maximum number of PMUs required is 23. 1156 sets of alternative solutions were determined from these 24 sets of initial solution. Out of 1156 solutions, when final filtering was done by removing PMU present in pure transit nodes, only 2 sets of solution were optimal. The optimal solution is given in Table 4-12. The total number of PMU required for full system's observability for 57-bus system is 12.

Table 4-12: Final Optimal PMU Placement for IEEE-57 Bus Test System

Solution Set	PMU Position (Bus No.)											
1	32	30	56	38	51	54	27	13	1	29	19	4
2	32	30	24	56	38	51	54	13	1	29	19	4

Both sets of optimal solution were checked for observability. Table 4-13 shows that IEEE 57-bus system is completely observable with both sets of solutions.

Table 4-13: Value of Constraint Function for 57-Bus System

Constraint Function	Values		Constraint Function	Values	
	Set 1	Set 2		Set 1	Set 2
f(1)	1	1	f(29)	3	3
f(2)	2	2	f(30)	1	1
f(3)	2	2	f(31)	2	2
f(4)	2	2	f(32)	1	1
f(5)	1	1	f(33)	1	1
f(6)	2	2	f(34)	2	2
f(7)	2	3	f(35)	1	1
f(8)	3	3	f(36)	2	2
f(9)	2	2	f(37)	1	1
f(10)	2	2	f(38)	1	1
f(11)	3	3	f(39)	1	1
f(12)	2	3	f(40)	2	1
f(13)	4	4	f(41)	2	2
f(14)	2	3	f(42)	2	2
f(15)	2	2	f(43)	1	1

Constraint Function	Values		Constraint Function	Values	
	Set 1	Set 2		Set 1	Set 2
f(16)	1	1	f(44)	1	1
f(17)	3	3	f(45)	2	2
f(18)	2	2	f(46)	2	2
f(19)	1	1	f(47)	2	2
f(20)	2	2	f(48)	3	3
f(21)	4	4	f(49)	1	1
f(22)	3	3	f(50)	1	1
f(23)	2	2	f(51)	1	1
f(24)	1	1	f(52)	2	2
f(25)	1	1	f(53)	2	2
f(26)	1	1	f(54)	3	3
f(27)	1	1	f(55)	3	3
f(28)	2	1	f(56)	2	2
f(29)	3	3	f(57)	2	2

4.4. Study Case III: INPS 74-Bus Test System

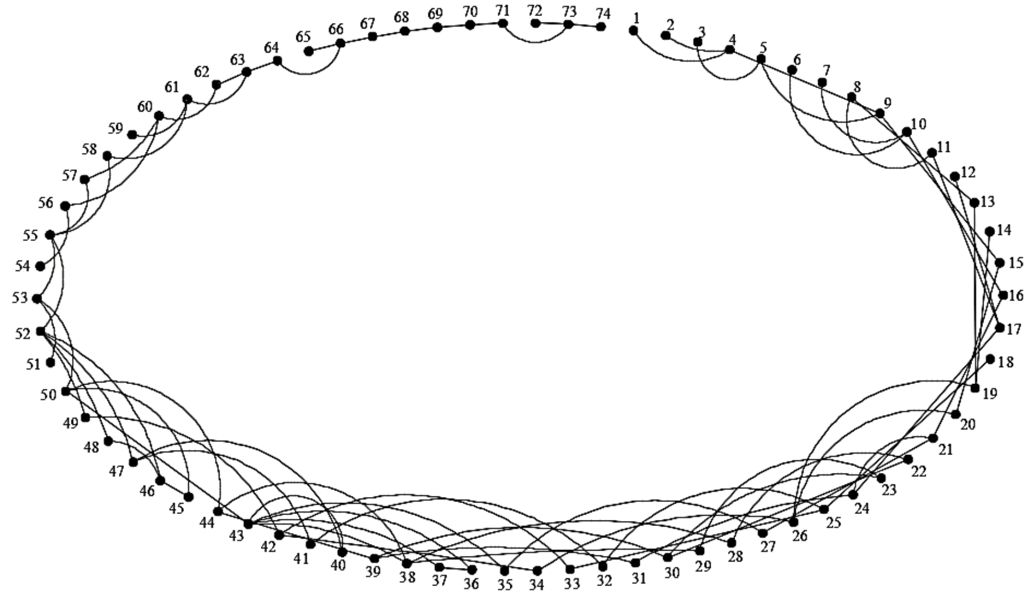

Figure 4-4: INPS 74-Bus Test System after Cuthill-McKee Permutation

The INPS consists of 16 number of ZI buses viz. 5, 6, 10, 11, 12, 14, 25, 26, 33, 39, 43, 44, 51, 67, 69 and 71 and 24 number of generator buses viz. 3, 4, 7, 23, 24, 35, 41, 42, 46, 47, 48, 49, 50, 52, 53, 56, 57, 58, 59, 62, 63, 66, 70 and 73. The adjacency matrix before and after Cuthill-McKee permutation can be written in vector form as shown in **Appendix F** and **Appendix G** and respectively. After Cuthill-McKee permutation, ZI buses are changed to bus number 5, 3, 19, 13, 14, 42, 48, 45, 21, 44, 39, 31, 8, 67, 69 and 71. Bus which is connected to maximum number of buses is bus number 43 and has six adjacent buses.

Topological observability formulae is obtained using Equation 3.6. This formulae is used for observability analysis of the system, every time the system's observability is to be determined. Topological observability equations for INPS is given in Table 4-14.

Table 4-14: Observability Equations for INPS 74-bus System

f(1)=X(1)+X(4)
f(2)=X(2)+X(4)
f(3)=X(3)+X(5)+~any([f(9) f(5)]==0)+~any([f(5)]==0)
f(4)=X(4)+X(1)+X(2)+X(9)
f(5)=X(5)+X(3)+X(9)+~any([f(3) f(9)]==0)+~any([f(3)]==0)
f(6)=X(6)+X(10)
f(7)=X(7)+X(10)
f(8)=X(8)+X(11)+X(13)+~any([f(19) f(13)]==0)+~any([f(11) f(13)]==0)
f(9)=X(9)+X(4)+X(5)+X(15)+~any([f(3) f(5)]==0)
f(10)=X(10)+X(6)+X(7)+X(16)
f(11)=X(11)+X(8)+X(17)+~any([f(13) f(8)]==0)
f(12)=X(12)+X(17)
f(13)=X(13)+X(8)+X(19)+~any([f(14) f(26) f(19)]==0)+~any([f(8) f(19)]==0)+~any([f(11) f(8)]==0)
f(14)=X(14)+X(19)+~any([f(13) f(26) f(19)]==0)+~any([f(19)]==0)
f(15)=X(15)+X(9)+X(20)
f(16)=X(16)+X(10)+X(21)+~any([f(24) f(27) f(21)]==0)
f(17)=X(17)+X(11)+X(12)+X(24)
f(18)=X(18)+X(25)
f(19)=X(19)+X(13)+X(14)+X(26)+~any([f(13) f(14) f(26)]==0)+~any([f(8) f(13)]==0)+~any([f(14)]==0)
f(20)=X(20)+X(15)+X(26)
f(21)=X(21)+X(16)+X(24)+X(27)+~any([f(16) f(24) f(27)]==0)
f(22)=X(22)+X(28)+X(30)
f(23)=X(23)+X(29)+X(30)
f(24)=X(24)+X(17)+X(21)+X(31)+~any([f(16) f(27) f(21)]==0)+~any([f(39) f(31)]==0)
f(25)=X(25)+X(18)+X(32)+X(33)
f(26)=X(26)+X(19)+X(20)+X(34)+~any([f(13) f(14) f(19)]==0)
f(27)=X(27)+X(21)+X(35)+~any([f(16) f(24) f(21)]==0)
f(28)=X(28)+X(22)+X(38)
f(29)=X(29)+X(23)+X(38)
f(30)=X(30)+X(22)+X(23)+X(39)+~any([f(31) f(44) f(39)]==0)
f(31)=X(31)+X(24)+X(39)+~any([f(30) f(44) f(39)]==0)+~any([f(24) f(39)]==0)
f(32)=X(32)+X(25)+X(41)
f(33)=X(33)+X(25)+X(42)+~any([f(34) f(49) f(42)]==0)

f(34)=X(34)+X(26)+X(42)+~any([f(33) f(49) f(42)]==0)
f(35)=X(35)+X(27)+X(43)
f(36)=X(36)+X(37)+X(43)
f(37)=X(37)+X(36)+X(43)
f(38)=X(38)+X(28)+X(29)+X(43)+X(44)+~any([f(39) f(50) f(44)]==0)
f(39)=X(39)+X(30)+X(31)+X(44)+~any([f(38) f(50) f(44)]==0)+~any([f(30) f(31) f(44)]==0)+~any([f(24) f(31)]==0)
f(40)=X(40)+X(43)+X(47)
f(41)=X(41)+X(32)+X(47)
f(42)=X(42)+X(33)+X(34)+X(49)+~any([f(33) f(34) f(49)]==0)
f(43)=X(43)+X(35)+X(36)+X(37)+X(38)+X(40)+X(50)
f(44)=X(44)+X(38)+X(39)+X(50)+~any([f(38) f(39) f(50)]==0)+~any([f(30) f(31) f(39)]==0)
f(45)=X(45)+X(46)+X(50)+~any([f(46) f(50)]==0)
f(46)=X(46)+X(45)+X(48)+X(52)+~any([f(52) f(48)]==0)+~any([f(50) f(45)]==0)
f(47)=X(47)+X(40)+X(41)+X(52)
f(48)=X(48)+X(46)+X(52)+~any([f(46) f(52)]==0)
f(49)=X(49)+X(42)+X(52)+~any([f(33) f(34) f(42)]==0)
f(50)=X(50)+X(43)+X(44)+X(45)+X(53)+~any([f(46) f(45)]==0)+~any([f(38) f(39) f(44)]==0)
f(51)=X(51)+X(53)
f(52)=X(52)+X(46)+X(47)+X(48)+X(49)+X(55)+~any([f(46) f(48)]==0)
f(53)=X(53)+X(50)+X(51)+X(55)
f(54)=X(54)+X(56)
f(55)=X(55)+X(52)+X(53)+X(57)+X(58)
f(56)=X(56)+X(54)+X(60)
f(57)=X(57)+X(55)+X(60)
f(58)=X(58)+X(55)+X(61)
f(59)=X(59)+X(61)
f(60)=X(60)+X(56)+X(57)+X(62)
f(61)=X(61)+X(58)+X(59)+X(63)
f(62)=X(62)+X(60)+X(63)
f(63)=X(63)+X(61)+X(62)+X(64)
f(64)=X(64)+X(63)+X(66)
f(65)=X(65)+X(66)
f(66)=X(66)+X(64)+X(65)+X(67)+~any([f(68) f(67)]==0)
f(67)=X(67)+X(66)+X(68)+~any([f(66) f(68)]==0)
f(68)=X(68)+X(67)+X(69)+~any([f(66) f(67)]==0)+~any([f(70) f(69)]==0)
f(69)=X(69)+X(68)+X(70)+~any([f(68) f(70)]==0)
f(70)=X(70)+X(69)+X(71)+~any([f(68) f(69)]==0)+~any([f(73) f(71)]==0)
f(71)=X(71)+X(70)+X(73)+~any([f(70) f(73)]==0)
f(72)=X(72)+X(73)
f(73)=X(73)+X(71)+X(72)+X(74)+~any([f(70) f(71)]==0)
f(74)=X(74)+X(73)

Initial solution set is attached in **Appendix J**. There are 28 sets of solution that ensures system's full observability. The minimum number of PMUs required for full system observability according to initial solution is 26 and maximum number of PMUs required is 29. 1181 sets of alternative solutions were determined from these 28 sets of initial solution. Out of 1181 solutions, when final filtering was done by removing PMU present in pure transit nodes, only 2 sets of solution were optimal. The optimal solution is given in Table 4-15. The total number of PMU required for full system's observability for 57-bus system is 21.

Table 4-15: Optimal PMU positions for INPS 74-bus System

Solution Set	PMU Position (Bus No.)																				
1	2	5	45	7	48	10	33	42	37	17	43	20	14	28	22	52	58	56	61	65	72
2	2	5	51	45	7	48	33	42	37	17	43	20	14	28	22	52	58	56	61	65	72

Both sets of optimal solution were checked for observability. Table 4-16 shows that INPS is completely observable with both sets of solutions.

Table 4-16: Value of Constraint Function for INPS

Constraint Function	Values		Constraint Function	Values		Constraint Function	Values	
	Set 1	Set 2		Set 1	Set 2		Set 1	Set 2
f(1)	1	1	f(26)	2	1	f(51)	1	1
f(2)	1	1	f(27)	2	2	f(52)	2	2
f(3)	3	3	f(28)	1	1	f(53)	1	1
f(4)	1	1	f(29)	1	1	f(54)	1	1
f(5)	3	3	f(30)	4	4	f(55)	2	2
f(6)	1	1	f(31)	3	3	f(56)	2	2
f(7)	1	1	f(32)	2	2	f(57)	1	1
f(8)	2	3	f(33)	3	3	f(58)	1	1
f(9)	4	4	f(34)	2	2	f(59)	1	1
f(10)	1	1	f(35)	1	1	f(60)	1	1
f(11)	2	3	f(36)	1	1	f(61)	1	1
f(12)	1	1	f(37)	1	1	f(62)	1	1
f(13)	4	4	f(38)	2	2	f(63)	1	1
f(14)	3	2	f(39)	4	4	f(64)	1	1
f(15)	1	1	f(40)	1	1	f(65)	1	1
f(16)	3	3	f(41)	1	1	f(66)	2	2
f(17)	1	1	f(42)	2	2	f(67)	2	2
f(18)	1	1	f(43)	1	1	f(68)	2	2
f(19)	4	3	f(44)	3	3	f(69)	1	1

Constraint Function	Values		Constraint Function	Values		Constraint Function	Values	
	Set 1	Set 2		Set 1	Set 2		Set 1	Set 2
f(20)	1	1	f(45)	1	1	f(70)	2	2
f(21)	2	2	f(46)	3	3	f(71)	2	2
f(22)	1	1	f(47)	2	2	f(72)	1	1
f(23)	1	1	f(48)	2	2	f(73)	2	2
f(24)	4	4	f(49)	3	3	f(74)	1	1
f(25)	1	1	f(50)	4	4			

4.5. Calculation of BOI and SORI

In order to determine most reliable solution out of calculated sets of optimal solutions, SORI was determined for each case. Table 4-17 shows SORI values for all sets of optimal solution. Optimal solution with maximum SORI is most reliable PMUs arrangement. BOI for IEEE 30-bus system, 57-bus system and INPS is attached in **Appendix P**.

Table 4-17: SORI Calculation Table

Test System	Solution Set	SORI
INPS	Set 1 (2, 5, 45, 7, 48, 10, 33, 42, 37, 17, 43, 20, 14, 28, 22, 52, 58, 56, 61, 65, 72)	82
	Set 2 (2, 5, 51, 45, 7, 48, 33, 42, 37, 17, 43, 20, 14, 28, 22, 52, 58, 56, 61, 65, 72)	81
IEEE 30-Bus	Set 1 (3, 19, 7, 12, 10, 27, 24)	31
	Set 2 (3, 19, 7, 12, 10, 30, 24)	29
	Set 3 (3, 19, 7, 12, 10, 27, 24)	33
	Set 4 (3, 19, 2, 12, 10, 30, 24)	31
IEEE 57-Bus	Set 1 (32, 30, 56, 38, 51, 54, 27, 13, 1, 29, 19, 4)	52
	Set 2 (32, 30, 24, 56, 38, 51, 54, 13, 1, 29, 19, 4)	54

4.6. Impact of Change in Number of ZI Bus in Optimum Solution

Zero injection bus, known as ZI bus or pure transit nodes are the buses that neither have generator nor load. To analyse what effect number of ZI bus has in optimum number of PMU required, number of ZI buses were varied from zero to double the number in original data. Optimum number of PMUs required for each case was calculated. Figure 4-5 shows the effect of change in number of ZI buses on optimum number of PMU. Further, correlation coefficient was calculated by feeding optimal PMU number and number of ZI buses into excel. The data fed is attached in **Appendix O**. Table 4-18 shows values of correlation coefficient between number of ZI buses and optimum number of PMUs in INPS, IEEE 57-bus, 30-bus and 14-bus system.

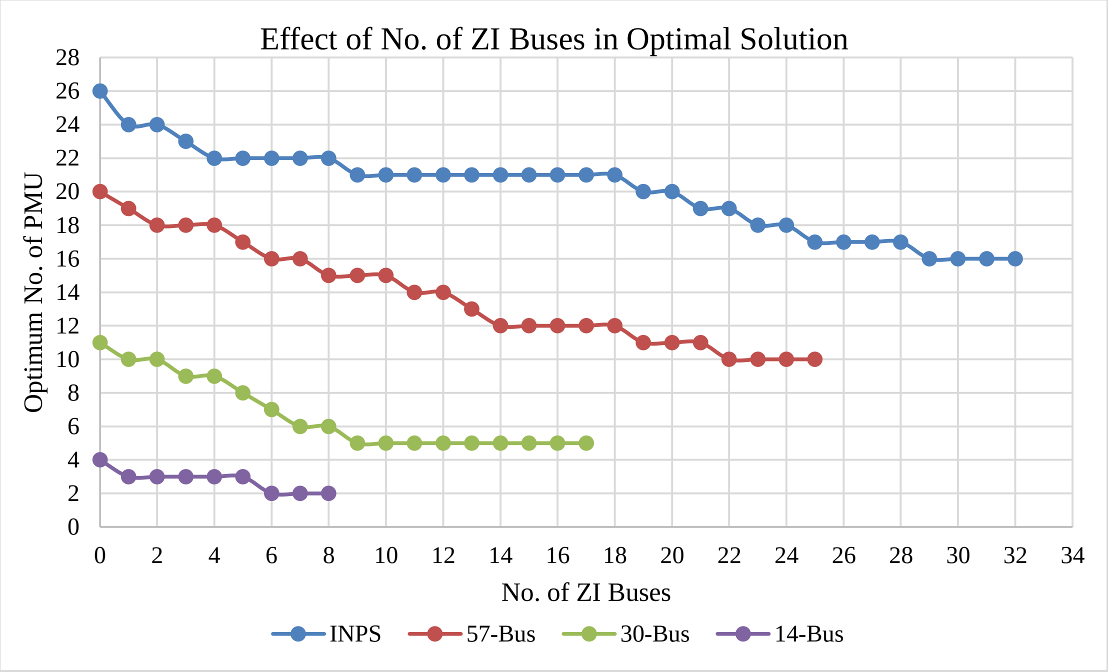

Figure 4-5: Effect of number of ZI buses in optimal solution

The data shows that there is strong negative correlation between number of ZI bus and optimum number of PMU. This indicates that for increase in number of ZI bus, there is a decrease in the optimum number of PMUs required for complete observability. This is because one unobservable bus among a ZI bus set can be made observable according Kirchhoff's Current Law (KCL) as suggested by observability rules mentioned in earlier section.

Table 4-18: Correlation Coefficient between Optimum Number of PMUs and Number of ZI Buses

Correlation Coefficient	
No. of PMUs in	**ZI Bus**
INPS	-0.96
57-Bus	-0.98
30-Bus	-0.91
14-Bus	-0.89

4.7. Validation of Results

Recursive security algorithm developed here for INPS has also been implemented in IEEE 14-bus, 30-bus and 57-bus system to ensure validity of the method. Results obtained is compared with that obtained from Simulated Annealing (SA) suggested in literature. Comparison shows that results obtained are valid.

Table 4-19: Comparison of Solution with SA

Test System	RSN		SA	
	Without ZI	With ZI	Without ZI	With ZI
14-Bus	4	3	4	3
30-Bus	11	7	10	7
57-Bus	20	12	20	13

CHAPTER FIVE: CONCLUSION AND RECOMMENDATION

5.1. Conclusion

1. Optimal number and location of PMUs to be installed in INPS and IEEE 14-bus, 30-bus, 57-bus system while ensuring system's full observability was determined. Optimum number of PMUs required for INPS, 57-bus, 30-bus and 14-bus system considering ZI buses are 21, 12, 7 and 3 respectively. Optimum positions of PMU after calculation of SORI are 2, 5, 45, 7, 48, 10, 33, 42, 37, 17, 43, 20, 14, 28, 22, 52, 58, 56, 61, 65, 72 for INPS, 2, 6, 9 for IEEE 14-bus system, 3, 19, 7, 12, 10, 27, 24 for IEEE 30-bus system and 32, 30, 24, 56, 38, 51, 54, 13, 1, 29, 19, 4 for IEEE 57-bus system.
2. Mathematical problem was formulated for optimal placement of PMU using recursive security N algorithm, where the objective function was to minimize total cost of PMU by minimizing total number of PMU, constrained by observability and maximum reliability of the system.
3. BOI and SORI were calculated to determine the most reliable solution. Maximum SORI among the solution sets were found to be 82, 54 and 33 respectively for INPS, IEEE 57-bus and IEEE 30-bus system.
4. The effect of change in number of ZI bus in optimal number of PMU required for system's full observability was analysed. It showed that for decrease in number of ZI buses, optimum number of PMUs required would increase and vice versa.

5.2. Recommendation

1. Recursive security N algorithm has been used here for the purpose for optimization of PMUs in transmission networks. This algorithm can also be used for solving other optimization problems in transmission and distribution system.
2. Further study can be done in optimization of PMUs considering different cases such as presence of existing measurement systems, loss of one or multiple PMUs, loss of one or multiple transmission lines, expansion of transmission networks, etc.
3. Since presence of ZI bus offers a practical advantage in installation of PMU by reducing required number of PMU and improve observability redundancy, further study exploiting this advantage of ZI bus can be done.

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APPENDICES

Appendix A Alternative Solutions for IEEE 14-Bus Test System

Bus No. Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	1	0	1	1	0	0	1	1	0	0
2	0	0	0	1	1	0	1	0	0	0	1	1	0	0
3	0	0	1	0	1	0	0	0	0	1	1	1	0	0
4	0	0	1	0	1	0	0	1	0	0	0	1	0	0
5	0	0	1	0	1	0	0	1	0	0	1	1	0	0
6	0	0	1	0	1	0	1	0	0	0	0	1	1	0
7	0	0	1	0	1	0	1	0	0	0	1	0	0	1
8	0	0	1	0	1	0	1	0	0	0	1	0	1	0
9	0	0	1	0	1	0	1	0	0	0	1	1	0	0
10	0	0	1	0	1	0	1	1	0	0	0	1	0	0
11	0	0	1	1	0	1	0	0	0	1	0	0	1	0
12	0	0	1	1	0	1	1	0	0	0	0	0	1	0
13	0	0	1	1	0	1	1	0	0	0	1	0	0	1
14	0	0	1	1	0	1	1	0	0	0	1	1	0	0
15	0	1	1	0	0	0	0	1	1	1	0	0	1	0
16	1	0	0	0	1	0	0	1	0	0	0	1	0	0
17	1	0	0	0	1	0	0	1	1	1	0	0	1	0
18	1	0	0	0	1	0	1	0	0	0	1	1	0	0
19	1	0	0	0	1	1	0	0	0	1	0	0	1	0
20	1	0	0	0	1	1	1	0	0	0	0	0	1	0
21	1	0	0	0	1	1	1	0	0	0	1	0	0	1
22	1	0	0	0	1	1	1	0	0	0	1	1	0	0
23	1	0	0	1	0	0	0	0	1	1	0	0	1	0
24	1	0	0	1	0	0	0	1	1	1	0	0	1	0
25	1	0	0	1	0	0	1	0	1	0	0	0	1	0
26	1	0	0	1	0	0	1	0	1	0	1	0	0	1
27	1	0	0	1	0	0	1	0	1	0	1	1	0	0
28	1	0	0	1	0	1	0	0	0	1	0	0	1	0
29	1	0	0	1	0	1	0	0	0	1	1	0	0	1
30	1	0	0	1	0	1	0	0	0	1	1	1	0	0
31	1	0	0	1	0	1	0	1	0	0	1	1	0	0
32	1	0	0	1	0	1	1	0	0	0	0	0	1	0
33	1	0	0	1	0	1	1	0	0	0	0	0	1	1
34	1	0	0	1	0	1	1	0	0	0	0	1	1	0
35	1	0	0	1	0	1	1	0	0	0	1	0	0	1
36	1	0	0	1	0	1	1	0	0	0	1	0	1	0
37	1	0	0	1	0	1	1	0	0	0	1	1	0	0
38	1	0	0	1	0	1	1	1	0	0	0	0	0	1
39	1	0	0	1	0	1	1	1	0	0	0	1	0	0

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Solution Set														
40	1	0	0	1	1	0	0	0	0	1	0	0	1	0
41	1	0	0	1	1	0	1	0	0	0	0	0	1	0
42	1	0	0	1	1	0	1	0	0	0	1	0	0	1
43	1	0	0	1	1	0	1	0	0	0	1	1	0	0
44	1	1	0	0	0	0	0	0	1	1	1	0	1	0
45	1	1	0	0	0	0	0	1	1	0	0	1	1	0
46	1	1	0	0	0	0	0	1	1	1	0	0	0	1
47	1	1	0	0	0	0	0	1	1	1	0	0	1	0
48	1	1	0	0	0	0	0	1	1	1	0	1	0	0
49	1	1	0	0	0	0	1	0	1	1	0	0	1	0
50	1	1	0	0	0	0	1	1	1	0	0	0	1	0
51	1	1	0	0	0	1	0	0	0	1	0	0	1	0
52	1	1	0	0	0	1	0	1	0	1	0	0	1	0
53	1	1	0	0	0	1	1	0	0	0	0	0	1	0
54	1	1	0	0	0	1	1	0	0	0	1	0	0	1
55	1	1	0	0	0	1	1	0	0	0	1	1	0	0
56	1	1	0	0	1	0	0	1	0	1	0	0	1	0
57	1	1	0	1	0	0	0	0	1	1	0	0	1	0
58	1	1	1	0	0	0	0	0	1	1	0	0	1	0

Appendix B Original adjacency matrix for IEEE 30-bus system in vector form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	[2,3]	16	[12,17]
2	[1,4,5,6]	17	[10,16]
3	[1,4]	18	[15,19]
4	[2,3,6,12]	19	[18,20]
5	[2,7]	20	[10,19]
6	[2,4,7,8,9,10,28]	21	[10,22]
7	[5,6]	22	[10,21,24]
8	[6,28]	23	[15,24]
9	[6,10,11]	24	[22,23,25]
10	[6,9,17,20,21,22]	25	[24,26,27]
11	9	26	25
12	[4,13,14,15,16]	27	[25,28,29,30]
13	12	28	[6,8,27]
14	[12,15]	29	[27,30]
15	[12,14,18,23]	30	[27,29]

Appendix C Adjacency Matrix for IEEE 30-Bus System after Cuthill-McKee Permutation in Vector Form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	[2,8]	16	[14,21]
2	[1,9]	17	[8,9,10,11,18,19,22]
3	[9,10]	18	[17,22]
4	11	19	[11,12,13,17,20,25]
5	[13,14]	20	[19,25]
6	14	21	[14,15,16,26]
7	[12,15]	22	[17,18,27]
8	[1,9,14,17]	23	[24,27]
9	[2,3,8,17]	24	[23,27]
10	[3,17]	25	[19,20,28]
11	[4,17,19]	26	[21,28]
12	[7,19]	27	[22,23,24,29]
13	[5,19]	28	[25,26,29]
14	[5,6,8,16,21]	29	[27,28,30]
15	[7,21]	30	29

Appendix D Original Adjacency Matrix For IEEE 57-Bus Test System in Vector Form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	[2,15,16,17]	29	[7,28,52]
2	[1,3]	30	[25,31]
3	[2,4,15]	31	[30,32]
4	[3,5,6,18]	32	[31,33,34]
5	[4,6]	33	32
6	[4,5,7,8]	34	[32,35]
7	[6,8,29]	35	[34,36]
8	[6,7,9]	36	[35,37,40]
9	[8,10,11,12,13,55]	37	[36,38,39]
10	[9,12,51]	38	[22,37,44,48,49]
11	[9,13,41,43]	39	[37,57]
12	[9,10,13,16,17]	40	[36,56]
13	[9,11,12,14,15,49]	41	[11,42,43,56]
14	[13,15,46]	42	[41,56]
15	[1,3,13,14,45]	43	[11,41]
16	[1,12]	44	[38,45]
17	[1,12]	45	[15,44]
18	[4,19]	46	[14,47]
19	[18,20]	47	[46,48]
20	[19,21]	48	[38,47,49]
21	[20,22]	49	[13,38,48,50]
22	[21,23,38]	50	[49,51]
23	[22,24]	51	[10,50]
24	[23,25,26]	52	[29,53]
25	[24,30]	53	[52,54]
26	[24,27]	54	[53,55]
27	[26,28]	55	[9,54]
28	[27,29]	56	[40,41,42,57]
29	[7,28,52]	57	[39,56]

Appendix E Adjacency matrix for IEEE 57-Bus System after Cuthill-McKee Permutation in Vector Form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	2	29	[18,19,21,41]
2	[1,3,4]	30	[20,42]
3	[2,5]	31	[21,43]
4	[2,6]	32	[35,44]
5	[3,7]	33	[35,44]
6	[4,8]	34	[22,45]
7	[5,12]	35	[32,33,37,41,46]
8	[6,11,13]	36	[23,24,41,46]
9	[10,13]	37	[25,35,46]
10	[9,15]	38	[26,46]
11	[8,15]	39	[27,47]
12	[7,14,17]	40	[28,47]
13	[8,9,21]	41	[29,35,36,42,46,48]
14	[12,22]	42	[30,41,48]
15	[10,11,16,23]	43	[31,48]
16	[15,23]	44	[32,33,48,49]
17	[12,28]	45	[34,50]
18	[20,21,29]	46	[35,36,37,38,41,51]
19	[25,29]	47	[39,40,52]
20	[18,30]	48	[41,42,43,44,53]
21	[13,18,22,29,31]	49	[44,53]
22	[14,21,34]	50	[45,54]
23	[15,16,24,36]	51	[46,52,55]
24	[23,36]	52	[47,51,55]
25	[19,37]	53	[48,49,56]
26	[27,38]	54	[50,56]
27	[26,39]	55	[51,52,56,57]
28	[17,40]	56	[53,54,55,57]
29	[18,19,21,41]	57	[55,56]

Appendix F   Original Adjacency Matrix for INPS in Vector Form

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	2	38	[37,42,43]
2	[1,3,4]	39	[27,40,43]
3	2	40	[28,36,39,41]
4	[2,5,7]	41	[40,42]
5	[4,6]	42	[38,41]
6	5	43	[38,39,44]
7	[4,8]	44	[35,43]
8	[7,9]	45	[34,46,47]
9	[8,10,13]	46	45
10	[9,11,12]	47	45
11	[10,51]	48	[35,49,50]
12	10	49	48
13	[9,14]	50	[48,51]
14	[13,15,16]	51	[11,50]
15	[14,22]	52	[27,53,54]
16	[14,17]	53	52
17	[16,18,19]	54	[22,52,55,60]
18	17	55	[54,56]
19	[17,20]	56	[55,57,59]
20	[19,21]	57	[56,58]
21	[20,22,23]	58	57
22	[15,21,24,25,54]	59	[56,63]
23	[21,28]	60	[54,61]
24	[22,25,26]	61	[60,62,63]
25	[22,24]	62	61
26	[24,27]	63	[59,61,64]
27	[26,28,39,52]	64	[63,65]
28	[23,27,29,30,31,40]	65	[64,66,67]
29	[28,30]	66	65
30	[28,29]	67	[65,68]
31	[28,32]	68	[67,69]
32	[31,33]	69	[68,70]
33	[32,34,35]	70	[69,71]
34	[33,45]	71	[70,72]
35	[33,44,48]	72	[71,73,74]
36	[37,40]	73	72
37	[36,38]	74	72

Appendix G Adjacency Matrix for INPS after Cuthill McKee Permutation

Bus No.	Adjacent Buses	Bus No.	Adjacent Buses
1	4	38	[28,29,43,44]
2	4	39	[30,31,44]
3	5	40	[43,47]
4	[1,2,9]	41	[32,47]
5	[3,9]	42	[33,34,49]
6	10	43	[35,36,37,38,40,50]
7	10	44	[38,39,50]
8	[11,13]	45	[46,50]
9	[4,5,15]	46	[45,48,52]
10	[6,7,16]	47	[40,41,52]
11	[8,17]	48	[46,52]
12	17	49	[42,52]
13	[8,19]	50	[43,44,45,53]
14	19	51	53
15	[9,20]	52	[46,47,48,49,55]
16	[10,21]	53	[50,51,55]
17	[11,12,24]	54	56
18	25	55	[52,53,57,58]
19	[13,14,26]	56	[54,60]
20	[15,26]	57	[55,60]
21	[16,24,27]	58	[55,61]
22	[28,30]	59	61
23	[29,30]	60	[56,57,62]
24	[17,21,31]	61	[58,59,63]
25	[18,32,33]	62	[60,63]
26	[19,20,34]	63	[61,62,64]
27	[21,35]	64	[63,66]
28	[22,38]	65	66
29	[23,38]	66	[64,65,67]
30	[22,23,39]	67	[66,68]
31	[24,39]	68	[67,69]
32	[25,41]	69	[68,70]
33	[25,42]	70	[69,71]
34	[26,42]	71	[70,73]
35	[27,43]	72	73
36	[37,43]	73	[71,72,74]
37	[36,43]	74	73

Appendix H Alternative Solutions for IEEE 30-Bus Test System

Bus No. Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	0	0	0	0	0	0	1	1	1	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1
2	0	0	0	1	0	0	1	1	1	1	0	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	1
3	0	0	0	1	0	0	1	1	1	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1
4	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
5	0	1	0	0	0	0	1	0	1	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1
6	0	1	0	1	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
7	0	1	0	1	0	0	1	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
8	0	1	0	1	0	0	1	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	1
9	0	1	0	1	0	0	1	0	1	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1
10	0	1	0	1	1	1	1	1	0	1	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
11	0	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
12	0	1	1	0	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
13	0	1	1	0	1	1	1	1	0	0	1	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
14	0	1	1	1	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
15	0	1	1	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	1	1	1	0	0	1
16	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	1
17	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	1
18	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1
19	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	1
20	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1
21	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
22	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	1	0
23	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1
24	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1	0
25	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	1	1	0	0	0	1
26	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	1	1	0	0	1	0
27	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
28	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	1	1	0	1
29	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	1	1	0	0	1
30	0	1	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	1
31	0	1	1	1	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
32	0	1	1	1	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
33	0	1	1	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
34	0	1	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
35	0	1	1	1	0	1	1	1	0	0	0	0	0	1	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
36	0	1	1	1	0	1	1	1	0	0	0	0	1	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
37	0	1	1	1	1	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
38	0	1	1	1	1	0	1	1	0	0	0	0	0	1	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
39	0	1	1	1	1	1	0	1	0	0	0	0	0	0	1	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
40	0	1	1	1	1	1	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
41	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
42	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	1	0	1
43	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	1
44	0	1	1	1	1	1	1	0	0	0	0	0	0	1	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
45	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1

Bus No. Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
46	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1
47	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	1
48	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	1	0	1
49	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	0	1	1
50	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
51	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	1	0
52	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	1	1	0	0	1
53	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	1	0	1	0	0	1
54	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	1	0	0	0	1	0	1
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Bus No. Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
96	1	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	1	1	1	0	0	1	0
97	1	0	0	1	0	0	1	0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	1	1	0	0	1	0
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Bus No. Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
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180	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	1	1	1	0	0	0
181	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
182	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	1	1	0	0	0	1	0
183	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0	1	0	1	0	0	0	1	0
184	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	1	1	0	1
185	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	0	1	1	1	0
186	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0	1	1	0	0	1
187	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	1
188	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	0	1	0	1	0	0	1	0
189	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	0	0	1	0	1
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193	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	0	1	0
194	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	1	0	0	1	0	0	1	0
195	1	0	1	1	0	0	1	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
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197	1	0	1	1	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
198	1	0	1	1	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0

Bus No. Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
199	1	0	1	1	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0
200	1	0	1	1	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	1
201	1	0	1	1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	1
202	1	0	1	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1
203	1	0	1	1	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
204	1	0	1	1	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0
205	1	0	1	1	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0
206	1	0	1	1	0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	1
207	1	0	1	1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	1
208	1	0	1	1	0	0	1	0	1	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1
209	1	0	1	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
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211	1	0	1	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0
212	1	0	1	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	1
213	1	0	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1
214	1	0	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0
215	1	0	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0	1	0
216	1	0	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	1
217	1	0	1	1	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
218	1	0	1	1	0	1	1	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	1
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223	1	0	1	1	1	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
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226	1	0	1	1	1	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
227	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	1
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229	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	0	0	1
230	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1
231	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
232	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	1	0
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234	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	1
235	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	1	0	0	0	0	1
236	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	1	0	0	1
237	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1
238	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	1	1	0	0	1
239	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	1	1	0	0	1
240	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	1	1	0	0	1
241	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	1
242	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	1	1	0	1
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248	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	1

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Solution Set																														
249	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	1	1	0	0	1
250	1	0	1	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	1	1	0	0	1
251	1	0	1	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1
252	1	0	1	1	1	1	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	1
253	1	0	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1
254	1	0	1	1	1	1	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	1
255	1	0	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1
256	1	0	1	1	1	1	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	1
257	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1
258	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	1
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Appendix I Results for IEEE 54-Bus Test System

Initial Solution

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
1	0	1	0	0	0	0	1	1	1	0	0	0	0	1	1	0	1	0	1	1	1	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	0
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14	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	0	0	0	1	1	1	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	0	1	0	1	1	0	0	1
15	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	0	0	0	1	1	1	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	0
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19	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	0	0	0	1	1	1	0	0	1	0	1	0	1	0	0	0	1	1	1	0	0	1	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0	0	0	0	1	0
20	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	0	0	1	1	0	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0	1	0	0	0	0	1	0
21	0	1	0	0	1	1	0	0	0	0	0	1	1	0	1	0	0	1	1	0	0	1	0	1	0	1	0	1	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	1	1	0	1	0	0	0	1	0	0	0	0	1	0
22	0	1	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0	0	1	1	1	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	0
23	0	1	0	0	1	1	0	0	1	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	0
24	1	0	1	1	0	0	0	1	1	0	0	1	0	0	1	0	0	0	1	1	1	0	0	1	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	0

Final Solution before Second Cuthill-McKee Permutation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0
2	0	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0

Appendix J Results for Integrated Nepal Power System (INPS)

Initial Solution Set

Bus No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	0	0	0	1	1	0	0	0	0	1	0	0	1	1	1	0	1	0	0	0	1	0	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0
2	0	0	0	1	1	0	0	0	0	1	1	1	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
3	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	1	1	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
4	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0
5	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	1	1
6	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0	1	1	0	1
7	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
8	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
9	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
10	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
11	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
12	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
13	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
14	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
15	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
16	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
17	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	0
18	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	0	0	0
19	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
21	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0
22	0	0	0	1	1	0	0	1	0	1	0	0	0	1	1	0	1	0	0	0	1	0	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0
23	0	0	0	1	1	0	0	1	0	1	0	1	0	0	1	0	0	0	1	0	0	0	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0

24	0	0	0	1	1	0	0	1	0	1	0	1	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
25	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
26	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	0	1	1	1	1	1	0	0	0	0	0	0	0
27	0	0	1	1	0	0	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0
28	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1	1	1	0	0	0	1	0	0	1	1	1	1	0	0	0	0	0	0	0

Bus No.	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
1	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
2	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
3	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
4	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
5	0	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
6	0	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
7	0	0	0	1	1	1	0	1	1	0	0	0	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
8	0	0	0	1	1	1	1	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
9	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	1	1	0	0	0	1	0	1	0	1	0	0	1	0
10	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	0	1	0	0	0	1	0
11	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	0	1	1	0	1
12	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
13	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	1	0	1
14	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	1	1	0	1	0	1	0	0	0	1	0
15	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1	0	0	1	0	0	1	0	1	0	1	0	0	1	0
16	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
17	0	0	0	0	1	1	0	1	1	0	1	0	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
18	0	0	1	0	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
19	1	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
20	0	0	0	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0

21	0	0	0	0	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
22	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
23	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
24	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
25	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
26	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
27	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0
28	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	0	1	0	0	1	0

Final Solution before Second Cuthill-McKee Permutation

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1	0	1	0	1	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
1	1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	1	0
2	1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	1	0

Appendix K Data for IEEE 14-Bus Test System

Bus Data

Bus No.	Pd	Qd	Bus No.	Pd	Qd
1	0	0	8	0	0
2	21.7	12.7	9	29.5	16.6
3	94.2	19	10	9	5.8
4	47.8	-3.9	11	3.5	1.8
5	7.6	1.6	12	6.1	1.6
6	11.2	7.5	13	13.5	5.8
7	0	0	14	14.9	5

Generator Data

Bus No.	Active Power Generation (Pg)	Reactive Power Generation (Qg)
1	232.4	-16.9
2	40	42.4
3	0	23.4
6	0	12.2
8	0	17.4

Branch Data

Branch No.	From Bus	To Bus	Branch No.	From Bus	To Bus
1	1	2	11	6	11
2	1	5	12	6	12
3	2	3	13	6	13
4	2	4	14	7	8
5	2	5	15	7	9
6	3	4	16	9	10
7	4	5	17	9	14
8	4	7	18	10	11
9	4	9	19	12	13
10	5	6	20	13	14

Appendix L Data for IEEE 30-Bus Test System

Bus Data

Bus No.	Pd	Qd	Bus No.	Pd	Qd	Bus No.	Pd	Qd
1	0	0	11	0	0	21	17.5	11.2
2	21.7	12.7	12	11.2	7.5	22	0	0
3	2.4	1.2	13	0	0	23	3.2	1.6
4	7.6	1.6	14	6.2	1.6	24	8.7	6.7
5	94.2	19	15	8.2	2.5	25	0	0
6	0	0	16	3.5	1.8	26	3.5	2.3
7	22.8	10.9	17	9	5.8	27	0	0
8	30	30	18	3.2	0.9	28	0	0
9	0	0	19	9.5	3.4	29	2.4	0.9
10	5.8	2	20	2.2	0.7	30	10.6	1.9

Generator Data

Bus No.	Active power Generation (Pg)	Reactive Power Generation (Qg)
1	260.2	-16.1
2	40	50
5	0	37
8	0	37.3
11	0	16.2
13	0	10.6

Branch Data

Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus
1	1	2	15	4	12	29	21	22
2	1	3	16	12	13	30	15	23
3	2	4	17	12	14	31	22	24
4	3	4	18	12	15	32	23	24
5	2	5	19	12	16	33	24	25
6	2	6	20	14	15	34	25	26
7	4	6	21	16	17	35	25	27
8	5	7	22	15	18	36	28	27
9	6	7	23	18	19	37	27	29
10	6	8	24	19	20	38	27	30
11	6	9	25	10	20	39	29	30
12	6	10	26	10	17	40	8	28
13	9	11	27	10	21	41	6	28
14	9	10	28	10	22	42	6	28

Appendix M   Data for IEEE 57-Bus Test System

Bus Data

Bus Number	Pd	Qd	Bus Number	Pd	Qd	Bus Number	Pd	Qd
1	55	17	20	2.3	1	39	0	0
2	3	88	21	0	0	40	0	0
3	41	21	22	0	0	41	6.3	3
4	0	0	23	6.3	2.1	42	7.1	4.4
5	13	4	24	0	0	43	2	1
6	75	2	25	6.3	3.2	44	12	1.8
7	0	0	26	0	0	45	0	0
8	150	22	27	9.3	0.5	46	0	0
9	121	26	28	4.6	2.3	47	29.7	11.6
10	5	2	29	17	2.6	48	0	0
11	0	0	30	3.6	1.8	49	18	8.5
12	377	24	31	5.8	2.9	50	21	10.5
13	18	2.3	32	1.6	0.8	51	18	5.3
14	10.5	5.3	33	3.8	1.9	52	4.9	2.2
15	22	5	34	0	0	53	20	10
16	43	3	35	6	3	54	4.1	1.4
17	42	8	36	0	0	55	6.8	3.4
18	27.2	9.8	37	0	0	56	7.6	2.2
19	3.3	0.6	38	14	7	57	6.7	2

Generator Data

Bus Number	Active Power Generation (Pg)	Reactive Power Generation (Qg)
1	128.9	-16.1
2	0	-0.8
3	40	-1
6	0	0.8
8	450	62.1
9	0	2.2
12	310	128.5

Branch Data

Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus
1	1	2	6	6	7
2	2	3	7	6	8
3	3	4	8	8	9
4	4	5	9	9	10

Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus
5	4	6	10	9	11
11	9	12	46	34	32
12	9	13	47	34	35
13	13	14	48	35	36
14	13	15	49	36	37
15	1	15	50	37	38
16	1	16	51	37	39
17	1	17	52	36	40
18	3	15	53	22	38
19	4	18	54	11	41
20	4	18	55	41	42
21	5	6	56	41	43
22	7	8	57	38	44
23	10	12	58	15	45
24	11	13	59	14	46
25	12	13	60	46	47
26	12	16	61	47	48
27	12	17	62	48	49
28	14	15	63	49	50
29	18	19	64	50	51
30	19	20	65	10	51
31	21	20	66	13	49
32	21	22	67	29	52
33	22	23	68	52	53
34	23	24	69	53	54
35	24	25	70	54	55
36	24	25	71	11	43
37	24	26	72	44	45
38	26	27	73	40	56
39	27	28	74	56	41
40	28	29	75	56	42
41	7	29	76	39	57
42	25	30	77	57	56
43	30	31	78	38	49
44	31	32	79	38	48
45	32	33	80	9	55

Appendix N Data for INPS 74-Bus System

Bus Data

Bus No.	Pd	Qd	Bus No.	Pd	Qd	Bus No.	Pd	Qd
1 (Anarmani)	39	-0.5	26 (Matatirtha)	0	0	51 (NewKhimti220)	0	0
2 (Damak)	21.2	10.6	27 (Siuchatar132)	0	-25	52 (Marsyangdi)	2.2	1.1
3 (Ilam)	0	0	28 (Siuchatar66)	35.1	17.6	53 (M-Marsyangdi)	0	0
4 (Duhabi)	173.6	19.8	29 (K-3)	35.7	17.79	54 (Bharatpur)	89.4	17.2
5 (Kushaha)	0	0	30 (Teku)	40.7	20.4	55 (Damauli)	26	13.3
6 (Katiya)	0	0	31 (Patan)	46.4	-1.7	56 (Lekhnath)	9.4	4.7
7 (Lahan)	64.4	2.2	32 (Baneshwor)	14.05	-17.5	57 (Pokhara)	29.9	14.6
8 (Mirchaiya)	7.4	3.7	33 (Bhaktapur)	0	0	58 (Modi)	7.6	3.8
9 (Dhalkebar)	80.3	15.1	34 (Banepa)	13.7	6.9	59 (KGA)	7.6	3.8
10 (Dhalkebar 400)	0	0	35 (Bhaktapur132)	34.4	-32.8	60 (Kawasoti)	15.3	7.7
11 (Dhalkebar 200)	0	0	36 (Lainchaur)	45.9	23	61 (Bardaghat)	7.4	3.7
12 (Muzzafarpur)	0	0	37 (NewChabahil)	53.6	1.8	62 (Gandak)	0	0
13 (Chapur)	44.9	22.5	38 (Chapali)	22.7	11.4	63 (Butwal)	111.5	15.7
14 (Patlaiya)	0	0	39 (Balaju)	0	0	64 (Shivapur)	23.6	1.8
15 (Kamane)	13.5	6.8	40 (Balaju66)	15.6	-17.2	65 (Lamahi)	47.8	26.4
16 (N-Parwanipur)	52.3	-22	41 (Trishuli)	13.3	6.7	66 (Jhimruk)	4.6	2.3
17 (Parwanipur)	5.2	2.6	42 (Devighat)	6.5	3.3	67 (Kusum)	0	0
18 (Birgunj)	61.6	-5.3	43 (Chapali132)	0	0	68 (Kohalpur)	26.2	13.1
19 (Simara)	81.2	25.6	44 (Changunarayan)	0	0	69 (Bhurigaun)	0	0
20 (Amlekhgunj)	2.1	1.1	45 (Panchkhal)	8.2	4.1	70 (Lumki)	5.7	2.8
21 (Hetauda66)	26.8	13.5	46 (Sunkoshi)	6.5	3.3	71 (Pahalmanpur)	0	0

Bus No.	Pd	Qd	Bus No.	Pd	Qd	Bus No.	Pd	Qd
22 (Hetauda132)	0	-10	47 (Indrawati)	0	0	72 (Attariya)	28	14
23 (Kulekhani1)	7	-17	48 (Lamosangu)	0	0	73 (Balanch)	0	0
24 (Kulekhani2)	0	0	49 (Bhotekoshi)	0	0	74 (Mahendranagar)	7.7	3.8
25 (Kulekhani3)	0	0	50 (Khimti)	0	0			

Generator Bus

Bus No.	Pg	Qg	Bus No.	Pg	Qg	Bus No.	Pg	Qg
3 (Ilam)	104.3	50.51	46 (Sunkoshi)	10.05	4.87	57 (Pokhara)	7.3	3.54
4 (Duhabi)	10.8	5.23	47 (Indrawati)	12.3	5.96	58 (Modi)	59.9	29.01
7 (Lahan)	7	3.39	48 (Lamosangu)	9.53	4.62	59 (KGA)	160.8	77.88
23 (Kulekhani1)	66.36	32.14	49 (Bhotekoshi)	48	23.25	62 (Gandak)	15	7.26
24 (Kulekhani2)	32	15.49	50 (Khimti)	72.4	35.06	63 (Butwal)	1.024	0.5
35 (Kulekhani3)	2.4	1.16	52 (Marsyangdi)	85	41.17	66 (Jhimruk)	12	5.81
41 (Trishuli)	60.1	29.11	53 (M-Marsyangdi)	138.4	67.03	70 (Lumki)	3.7	1.79
42 (Devighat)	14.1	6.83	56 (Lekhnath)	42.5	20.58	73 (Balanch)	38.5	18.65

Branch Data

Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus
1	1	2	29	24	25	57	45	46
2	2	3	30	24	26	58	45	47
3	2	4	31	26	27	59	48	49
4	4	5	32	27	28	60	48	50
5	4	7	33	27	39	61	50	51
6	5	6	34	27	52	62	52	53
7	7	8	35	28	29	63	52	54
8	8	9	36	28	30	64	54	55
9	9	10	37	28	40	65	54	60
10	9	13	38	28	31	66	55	56
11	10	11	39	29	30	67	56	57
12	10	12	40	31	32	68	56	59
13	11	51	41	32	33	69	57	58
14	13	14	42	33	34	70	59	63

Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus	Branch Number	From Bus	To Bus
15	14	15	43	33	35	71	60	61
16	14	16	44	34	45	72	61	62
17	15	22	45	35	44	73	61	63
18	16	17	46	35	48	74	63	64
19	17	18	47	36	37	75	64	65
20	17	19	48	36	40	76	65	66
21	19	20	49	37	38	77	65	67
22	20	21	50	38	42	78	67	68
23	21	22	51	38	43	79	68	69
24	21	23	52	39	40	80	69	70
25	22	24	53	39	43	81	70	71
26	22	25	54	40	41	82	71	72
27	22	54	55	41	42	83	72	73
28	23	28	56	43	44	84	72	74

Appendix O Effect of change in number of ZI bus in optimum solution

No. of ZI Bus	**Optimum PMU Number**			
	INPS	**57-Bus**	**30-Bus**	**14-Bus**
0	26	20	11	4
1	24	19	10	3
2	24	18	10	3
3	23	18	9	3
4	22	18	9	3
5	22	17	8	3
6	22	16	7	2
7	22	16	6	2
8	22	15	6	2
9	21	15	5	
10	21	15	5	
11	21	14	5	
12	21	14	5	
13	21	13	5	
14	21	12	5	
15	21	12	5	
16	21	12	5	
17	21	12	5	
18	21	12		
19	20	11		
20	20	11		
21	19	11		
22	19	10		
23	18	10		
24	18	10		
25	17	10		
26	17			
27	17			
28	17			
29	16			
30	16			
31	16			
32	16			

Appendix P Bus Observability Index

30 Bus system's Bus Observability Index

Bus No./ Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1	1	1	0	1	1	1	2	0	1	1	2	1	1	1	1	2	0	1	1	1	1	1	1	2	1	1	1	2	0
2	1	1	1	0	1	1	1	2	0	1	1	2	1	1	1	1	2	0	1	1	1	0	1	1	2	1	1	1	1	0
3	1	2	1	0	1	1	1	3	1	0	1	2	1	1	1	1	2	0	1	1	1	1	1	1	2	1	1	1	2	0
4	1	2	1	0	1	1	1	3	1	0	1	2	1	1	1	1	2	0	1	1	1	0	1	1	2	1	1	1	1	0

57 Bus system's Bus Observability Index

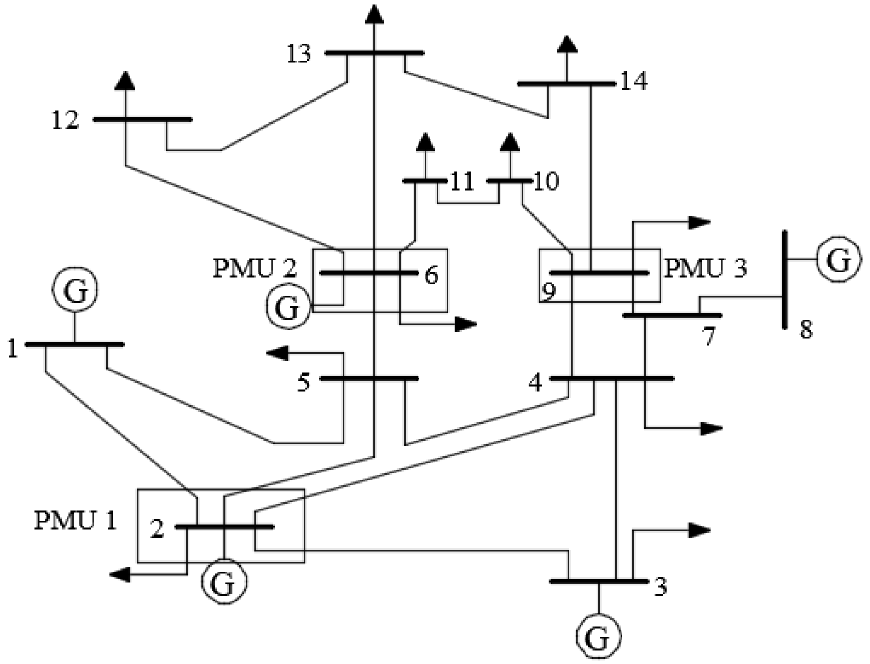
Bus No./ Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	1	1	2	1	1	0	1	0	0	1	1	0	1	0	1	1	1	1	1	0	1	1	1	0	1	1	1	1	2
2	1	1	2	1	1	0	3	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	2
Bus No./ Solution Set	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	
1	0	1	1	1	0	1	1	1	1	1	2	1	1	0	1	1	1	1	2	1	1	0	1	1	3	1	1	1	
2	0	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	2	1	1	0	1	1	3	1	1	1	

INPS 74-Bus system's Bus Observability Index

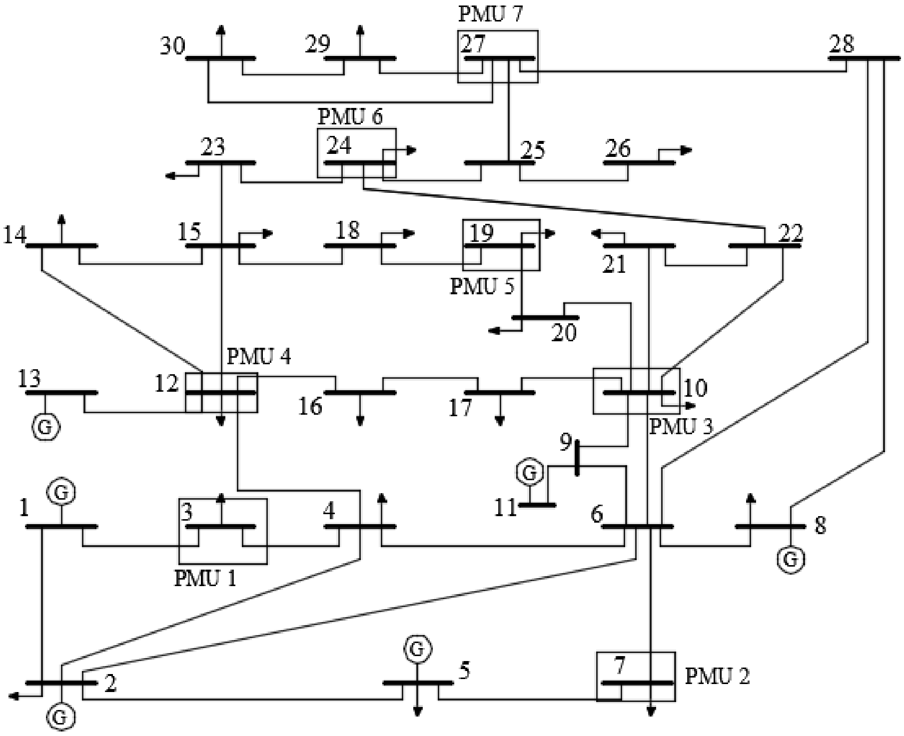
Bus No./ Solution Set	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	1	1	1	1	1	1	0	3	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1
2	1	1	1	1	1	1	1	1	3	1	2	1	1	0	1	2	1	1	0	1	1	1	1	2	1
Bus No./ Solution Set	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	1	1	1	1	3	1	2	2	1	1	1	1	1	1	1	1	1	1	1	0	1	2	1	2	2
2	0	1	1	1	3	1	2	2	1	1	1	1	1	1	1	1	1	1	1	0	1	2	1	2	2
Bus No./ Solution Set	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	
1	1	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	
2	1	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	

Appendix Q Optimum Location of PMU

Optimum PMU Placement in IEEE 14-Bus System



Optimum PMU Placement in IEEE 30-Bus System



Optimum PMU Placement in IEEE 57-Bus System