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Analysis and Control of Vehicle Anti-lock Braking System using Fuzzy Logic

by

Prashant Shah

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Approval page

The undersigned certify that they have read, and recommended to the Institute of Engineering for acceptance, a thesis entitled "Analysis and Control of Vehicle Anti-lock Braking System using Fuzzy Logic" submitted by Prashant Shah in partial fulfilment of the requirements for the degree of Master of Science in Mechanical Systems Design and Engineering.

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Abstract

Antilock Braking System (ABS) prevents a vehicle from ceasing to rotate when brakes are applied. The non-linear behaviour of different road conditions makes it difficult to predict the optimal brake forces to be applied to minimize the stopping distance and maintain steerability. A quarter car model has been used and a mathematical and MATLAB/Simulink model of components of an ABS has been developed. Two Fuzzy Logic Controllers (FLC) have been used to control different parameters like optimal slip and brake pedal force on the basis of input parameters which are slip, road condition, coefficient of friction and wheel acceleration. The nature of slip curve during whole braking period with and without FLCs has been analyzed. The nature of curve involving wheel velocity and vehicle velocity, and the time to reach the stopping distance with and without FLCs have been analyzed. Fuzzy logic control mechanism resembles human brain in the sense of decision making and hence provides a better real time control over parameters in comparison to a simple bang-bang controller. Fuzzy logic controllers provide better steerability, slip control and braking distance in comparison to a simple bang-bang controller.

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List of Acronyms and Abbreviations

ABS	: Antilock braking system
CPU	: Control processing unit
ESP	: Electronic stability program
EV	: Electric Vehicle
FIS	: Fuzzy interference system
FLC	: Fuzzy logic controller
FOSMC	: Functional order sliding mode controller
GPS	: Global positioning unit
GUI	: Graphical user interface
IOE	: Institute of Engineering
ISMS	: Intelligent sliding mode scheme
MATLAB	: Matrix Laboratory
MF	: Membership function
PID	: Proportional integral derivative
RBFNN	: Radial basis function neural networks
Simulink	: Simulation and Link
SM	: Sliding mode
SMC	: Sliding mode controller

CHAPTER ONE: INTRODUCTION

The different type of situations that threaten the safety of a moving vehicle occur when the driver tries to decelerate or stop the vehicle while braking or cornering or both on slippery surfaces and on the surface with asymmetric coefficients of friction.

In most of the accidents, an obstacle appears in front of the vehicle and the driver has to take action after recognising the danger (Unlusoy, 2008). This action depends on many parameters such as the distance between the vehicle and the obstacle, the state of the other lanes (being occupied or not), the road surface conditions, etc. A vehicle without an Anti-lock Braking System (ABS) is safe only when there is sufficient clearance before the obstacle, the road is straight, and the friction coefficient is same for the both vehicle sides. If any of these conditions do not apply, single or multiple vehicle crashes may occur. Even with an ABS with only longitudinal motion control capability, single vehicle crashes are not a far risk.

1.1 Background

The Anti-lock Braking idea emerges far back since 1930s. In around 1950s, the early examples appear in aerospace applications. During initial 1970s, ABS was experimented on passenger cars. Automobile drivers were assisted with enhanced stability and capability of braking also increased. This made antilock braking a standard system most of the vehicles (Douglas & Schafer, 1971).

In initial 1900s, the systems were developed for trains from which the present hydraulic ABS are adapted. Anti-lock brakes were then developed to help aircrafts for stopping quickly and straight on the runways that are slippery. B-47 bomber aeroplanes used antilock brakes in 1947 to avoid blowout of tire on dry concrete and icy runways spinouts. In 1954, the early use of ABS was done in automotive sector. From a French aircraft, Lincolns, in a limited number were fitted with an ABS. Vacuum actuated modulators and analogue computers were used in earlier ABS that were offered in few models of Cadillac, Ford and Chrysler in the late 1960's. Actual stopping distance of the vehicle increased due to very slowly cycling of vacuum actuated modulators. This brought attention of the legal authorities which then put the development of ABS on hold in the US, which made the Europe based companies take the lead in the next two decades. BMW and Mercedes introduced ABS that were electronically controlled in the late 1970's. Bosch ABS was introduced by Audi, BMW and Mercedes by 1985, and first Teves system was introduced by Ford. Many high-priced sports and sports cars includes ABS by the late 1980's. Most light duty vehicles and passenger cars today comprise computer controlled, complex braking systems. ABS systems in dozens were introduced by vehicle manufacturing companies from middle of 1980's. Control strategy and hardware configurations of these ABS vary (Hattwig, 1993).

The ability of providing vehicle's performance improvements under braking when compared to conventional braking system is the soul motivation for an ABS. Stopping distance, steerability and stability require performance improvement. Each wheel's slip is controlled by an ABS to save it from getting locked up to achieve a high friction and maintain steerability. Robust adaptive behaviour characterizes ABS controllers with respect to highly uncertain characteristics of tire and properties of road surface which change quickly (Petersen, 2003).

Fuzzy set theory gives rise to a logic which is then called fuzzy logic. This logic is multi-valued which deals with reasoning that is not precise but approximate. Binary sets have binary logic also called crisp logic whereas a truth value that ranges between zero and one is taken by fuzzy logic variables, not bounded with two truth values of classic logic. These degrees can be managed with the use of linguistic variables by specific functions (Klir & Folger, 1988).

1.2 Problem Statement

In today's motorcycles, cars and trucks, the applications of automotive safety have become very common. ABS and electronic stability control types of vehicle stabilization systems are becoming standard in almost all passenger cars (More et al., 2017). Production ABS in today's time is a control system that is rule-based and has exhaustive tables for varying braking scenarios. Using exhaustive field testing and simulations in a trial and error manner, the controllers are tuned. The further development and analysis of the current production ABS are seriously limited due to the level of complexity of these systems.

The control of wheel slip is a problem that is very challenging (Cirovic & Aleksendric, 2013). This is because of the model uncertainties, nonlinear dynamics of braking process and a complex behaviour of tire-road interaction (Aleksendric & Barton, 2009). Behaviour of tire force saturation results in a high degree of nonlinearity. Changing of the vehicle parameters, un-modelled dynamics and coefficient of tire-road friction are additional main sources of uncertainties which exist in vehicle dynamics. Degradation of the control performance is significant due to these uncertainties. While designing the controller for an ABS, key issue is the achievement of robustness. From the above discussion, it is found that these problems can be solved by designing a nonlinear robust control law for the ABS.

The uncertainties and high nonlinearities that exist in mathematical model make it difficult to design an ABS. In nonlinear systems control framework, ABS therefore, is becoming an attractive area to research due to these difficulties.

In complex, nonlinear and systems that is not mathematically describable, an intelligent, relatively new and knowledge based control technique which is Fuzzy Control performs exceptionally well. Thus fuzzy logic can be used for an ABS promisingly. In absence of fuzzy ABS, the brake pressure reaches extreme level and wheels are locked up very fast. Due to these, vehicle's behaviour become unstable, the vehicle cannot be further steered as desired and the stopping distance increases (Clair et al., 1997). With the activation of ABS controller that uses fuzzy logic, steerability is retained during whole duration of braking, and the slowing down length is noticeably shortened (Ross, 1995).

1.3 Research Gap

After studying different types of the literatures published in different years, the research gap identified is summarized in the table below.

Table 1.1: Research gap identifications

S.N.	**Researchers**	**Findings**	**Research Gap**
1.	Kaufmann & Gupta, (1991)	To ratain steerability under hard braking and to minimize vehicle's stopping distance is the aim of an ABS.	Road conditions are not defined.
2.	Mergenthaler et al., (1993)	For the functionability of an ABS, the corresponding switching logic is of high importance.	Road conditions are not defined.

3.	Maier & Miller, (1995)	A combination of wheel acceleration control and slip control is the basic philosophy of control for conventional ABS.	No use of Fuzzy logic controller.
4.	Clair et al., (1997)	In absence of fuzzy ABS, the brake pressure reaches extreme level and wheels are locked up very fast. Due to these, vehicle's behaviour become unstable, the vehicle cannot be further steered as desired and the stopping distance increases.	Different kinds of road conditions are not defined for optimal slip.
5.	Harifi et al., (2007)	To achieve maximum retardation with prevention of wheel locking, an ABS was designed. Sliding mode (SM) controller for controlling the slip of wheel was designed and usage of improved integral switching surface was carried for lowering the chattering effects.	FLC is not used and the time for vehicle to reach the stopping distance is not mentioned.
6.	Sharkawy, (2010)	The use of proportional integral derivative (PID) controllers in ABS minimizes vehicle's stopping distance and keeps the slip ratio of the tires within set-points.	Comparison is not done with FLCs.
7.	Vazquez et al., (2010)	Sliding mode (SM) was applied for an ABS and found that the use of this type of controller robust against unmatched and matched perturbations. Also, the capability to lower the sliding friction increases.	Time to reach stopping distance is not mentioned and FLCs are not used.
8.	Bera et al., (2011)	In extreme driving conditions, ABS actuation rate needs to be very quick. During mechanically designing the brake system	Different road conditions and their

		actuator, consideration is given to peak brake force, actuation rate, etc., for the achievement of desired actuator response time, maximum force, etc., while taking proper care to extend the fatigue life of components of brake system and lower the wear in brake pads.	respective optimal slip are not mentioned.
9.	Aly et al., (2011)	Different control techniques for ABS systems were developed for different road conditions and suggested that intillegent control systems like fuzzy logic controller should be employed to the ABS system for its smooth functioning.	No controller is used to determine the optimal slip.
10.	Sanchez-Torres et al., (2011)	For an ABS control problem, development of a sliding mode (SM) regulator was done and found that the performance of ABS was increased and the closed loop system become more efficient.	Results are not compared with a model using FLCs.
11.	Tang et al., (2013)	Combining Functional Order Sliding Mode Controller (FOSMC) with Sliding mode controller (SMC) along with fractional order dynamics shows that FOSMC can deal with the ABS system’s uncertainties and can track the required slip of wheel faster than conventional integer order SMC with sliding surface of proportional-derivative or proportional.	No use of FLCs.
12.	Dousti et al., (2014)	To estimate the optimal reference slip value, development of a an algorithm for multiple model switching observer was done.	Comparison with other controllers is not done.

13.	Aksjonov et. al, (2016)	Automatic driving safety system such as an ABS and an electronic stability program (ESP) help vehicle driver in better controlling of the vehicle for avoiding road accidents.	Optimal wheel slip is not taken using any controller.
14.	Xiao et al., (2016)	Based on fuzzy control, ABS can effectively prevent locking of the wheels, braking more effectively and slip rate is also more close to the optimal slip ratio of near 0.2.	Time for vehicle to reach stopping distance is not mentioned.
15.	Gowda & Ramachandra (2017)	Due to the control of vehicle speed and wheel speed at the same time, braking performance with bang-bang controller is better.	Only three types of road conditions are considered.
16.	Eze et al., (2018)	Linear slip control is done with four types of road conditions using PID controller.	Slip control is not done using FLCs.
17.	Mirzaeinejad, (2018)	The use of radial basis function neural networks (RBFNN) in ABS gives better performance than that of sliding controller.	Performance is not compared with FLCs.

1.4 Objectives

General Objective:

i. To analyse and control a vehicle ABS using fuzzy logic.

Specific Objectives:

i. To develop mathematical model for the vehicle ABS.
ii. To develop MATLAB/Simulink model of the mathematical model.
iii. To apply fuzzy logic controllers in the ABS model and analyse the system's performance in comparison to an ABS model with bang-bang controller.

CHAPTER TWO: LITERATURE REVIEW

An extensive review of the current literature is required to gain familiarity with an antilock braking system and fuzzy logic, before entering into the methodology process of the research problem. Since fuzzy logic, being relatively new technique, is hardly applied to improve the efficiency of an ABS, the methods of applications are still in progress and developments are going on to reach a saturation point. So, here we will review the works that were done earlier to enhance the braking system of vehicles and how the application of comparatively new term, fuzzy logic and fuzzy sets play the important role in such enhancements.

2.1 Antilock Braking System

An anti-lock braking system (ABS) is a safety system that prohibits the wheels on a motor vehicle to get locked up (or ceased to rotate) during braking. To ratain steerability under hard braking and to minimize vehicle’s stopping distance is main aim of an ABS (Kaufmann & Gupta, 1991).

A number of subsystems are incorporated in any production ABS. There exists a subsystem called slip controller most ABS systems whose objective is to prevent wheel locking a braking process by making sure that the wheel slip stays within a desired range or at a predefined set-point. All ABS systems do not estimate and explicitly control the slip of wheel. They work on speed and acceleration. Particularly important is the logic which is responsible for coordinating all the four controllers of wheel slip among these subsystems. These controllers for each wheel slip are only active in adverse situations acting as safety devices. Therefore, when the wheel is out of danger of being locked, these controllers are switched off and the manual operation of the brake is set. At the same time, the controller for slip has to be switched on early enough for avoiding the wheel from being locked. Thus, for the functionability of an ABS, the corresponding switching logic is of high importance (Mergenthaler et al., 1993). A combination of wheel acceleration control and slip control is the basic philosophy of conventional ABS control (Maier & Miller, 1995).

The measured angular velocities of wheel are used for the control of wheel acceleration to indirectly control the slip by regulating the wheel’s acceleration or retardation. A hydraulic solenoid valve is used as the actuator used in conventional ABS that has three modes of brake pressure, viz., reduce, hold and increase.

When the wheel retardation drops below a specified value for a given time period, the controller is switched on. During whole ABS active duration, the switching between reduce, hold or increase of the actuator modes is controlled either by using several acceleration and slip thresholds or by mentioning a switching surface that uses a weighted sum of acceleration and slip. The slip will lie around the critical slip by selecting these thresholds appropriately. Thus, the force of friction between the road surface and tires will be near its maximum value and the distance of braking is reduced. Vibration as a side effect will be occurring in this type of algorithm which are noticeable while braking (Petersen, 2003).

For non-decreasing tire force characteristics, slip control works satisfactorily. For tire characteristics having pronounced maximum, wheel acceleration control works better. The reason behind this is the reality that a larger wheel retardation or acceleration can be obtained in the case of pronounced maximum. Since ABS controllers can tolerate a large amount of uncertainties in the coefficient of friction and tire force characteristics, they have been shown to be highly adaptive. There exist some limitations in performance and control of conventional ABS. A remarkable disadvantage of these systems is their dysfunctional nature in controlling wheel slip and tracking of a specified desired slip in an allowable span.

Production ABS in today's time is a control system that is rule-based and has exhaustive tables for varying braking scenarios. Using exhaustive field testing and simulations in a trial and error manner, the controllers are tuned. The further development and analysis of the current production ABS are seriously limited due to the level of complexity of these systems.

A large number of active safety devices have been developed for helping the driver to enhance the safety of vehicle during adverse driving conditions. ABS is a well-known common active technology for safety among them for controlling the braking systems of automobiles. The wheels can get locked up in a situation where hard braking is applied. Due to this, the braking forces get reduced to their values of sliding and the lateral forces are also decreased to almost zero (Wong, 2001). Such condition gives rise to increment of the stopping distance and loss of vehicle steerability in turning manoeuvres. Therefore, the longitudinal wheel slip plays an important role in affecting the performance of traction, braking and steerability systems in ground vehicles. ABS is recognized as a good measure for maintaining the wheel slip at specified value and avoiding the wheel from getting locked up. Hence, the steerability and the vehicle safety are improved by obtaining the shortened stopping distance. Considering other applications of ABS, electronic stability control (ESC) systems apply ABS to give the necessary yaw moment for vehicle lateral dynamics stabilization by differential braking techniques (Lee et al., 2014).

A desired vehicle motion is imposed in the ABS control problem and as a result, adequate vehicle stability is provided. Due to the presence of uncertainties and high nonlinearities in the mathematical model, difficulty arises in the ABS design. Thus, in the control framework of nonlinear systems, ABS is becoming an attractive area of research. Several works are reported there in the literature using the sliding mode technique (Cueva et al., 2010).

2.2 Components of ABS

An ABS is comprised of many subsystems. These subsystems are individual complex systems in themselves. However, these are arranged together, as shown in the Figure 2.1 below, to form a complete antilock braking system. The individual subsystem or say components of antilock braking system are mainly controller, actuator, tire slip and sensor.

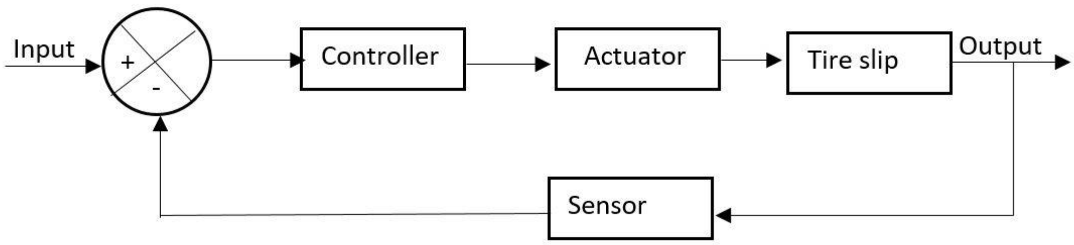

Figure 2.1: ABS Control Block diagram

2.2.1 Controller

Controllers are the basic component of any antilock braking system. Different types of controllers such as functional order sliding mode controller (FOSMC), radial basis function neural networks (RBFNN), sliding mode controller (SMC), fuzzy logic controller (FLC), etc. are used to send the command signal that is received from the input to the actuator.

2.2.2 Brake Actuator

Brake actuator is a subsystem that permits the hydraulic pressure to pass through the circuit of ABS according to a command signal which is received from the brake controller. The change rate of brake pressure is proportional to the brake fluid flow rate. Also, the flow rate depends on the control valve opening. Values of initial condition cannot be chosen randomly because of the sensitivity of actuator at optimization. Hydraulic pressure reaching the wheel cylinder can be controlled up to 5 bars for some applications. For regenerative braking, an electric actuator is used which executes quicker than the friction brake. At declaration, disability of the regenerative brake will arise when the actuators are switched off. As a consequence, there arises ABS reliability on the friction brake. An actuator with combined electrical and hydraulic properties should have fast response time as well as performance tracking. With the usage of brake actuators the slip of each wheel can be controlled separately. Hydraulic accumulator contains gas inside which experiences an adiabatic process when braking force is applied. So, the application of gas state equation is valid.

2.2.3 Wheel Slip

Wheel slip is nothing but the defined as the normalized difference between vehicle velocity and wheel velocity.

2.2.4 Sensor

An inductive coil sensor which is contoured to a reflector ring measures the wheel speed. Recently, accelerometer and rate sensors are implemented for control of wheel slip. In order to detect conditions of road surface, sensors work on different principles of light, sound or microwave. Noises developed by actuator and sensor are not considered in ABS research. The transfer function of a sensor for wheel speed should be linearized in the frequency domain to obtain an ideal performance.

2.3 Fuzzy Logic and Fuzzy Sets

Zadeh (1965) stated that a class of objects with a continuum grades of membership can be termed as a fuzzy set. Fuzzy set is distinguished by a membership function that allocates to each object a group of membership ranging from 0 to 1. The concepts of union, complement relation, intersection, convexity, inclusion, etc. are extended to fuzzy sets. In the context of these sets, various properties of these concepts are established.

In the real physical world, the grades of objects experienced do not have exactly defined benchmarks of membership. Taking example, a class of animals does not include objects like plants, rocks, fluids, etc., but clearly include cows, buffaloes, horses, etc. However, there exists an unclassifiable status for some members like bacteria, starfish, etc., with respect to the animal class. Similar type of vagueness rises up in the case of a number like 5 in relation to the "class" of all integers which are greater than 1. Some categories do not constitute sets or classes in the general mathematical sense such as "the class of short men" or "the class of intelligent women" or "the class of integers which are greater than 1". Still the truth exists that such vaguely defined "classes" have significant role in human thinking while recognizing patterns, abstraction and communicating information. The concept of a fuzzy set gives a suitable point of withdraw for the establishment of a conceptual framework. This framework has similarities with framework used in ordinary sets but it can prove to have a vague scope of applications in the fields of information processing and pattern classification. The problems in which the cause of imprecision is the non-appearance of clearly defined benchmarks of class membership and not the appearance of random variables, this kind of framework gives an easy way to cope with them (Zadeh, 1965).

The fundamental concept in classical set theory has to be understood for explaining the idea of fuzzy sets. The idea of classical set is very simple in mathematics. A collection of well-defined objects is called a set. These objects either belong to the set or not and cover almost.

Ragin (2000) indicated a very simple clarification for fuzzy sets. Permitting the scaling of membership scores is the fundamental concept behind as illustrated by Ragin. The scaling allows fuzzy membership. If 1 is the membership score, it shows full membership in a set. If 0.8 or 0.9 is the score, it shows strong and partial membership in a set. If less than 0.5 but 0.2 or 0.3 is the score, it shows that objects are weak membership in a set. If 0 is the score, it shows full non membership of the set. So, fuzzy sets are the combination of quantitative and qualitative assessment. The categorization of memberships in fuzzy sets give extensive opportunities to this theory to be flawlessly invaded in various sectors of knowledge that also include multi criteria decision making.

Chowdhary et al. (2010) stated that the fuzzy set theory concept, introduced by Lotfi A. Zadeh, is generalization of classical set theory. An element in fuzzy set represents a class of its membership. Let U be universe of discussion, x be collective element of U such that U = {x, x _ U}. A fuzzy set S in U is then distinguished by membership function $\mu_S(x)$, that relates x a real number in the interval [0,1], where $\mu_S(x)$ represents class of membership of collective element x in set S. As the value of $\mu_S(x)$ reaches near to unity, class of membership of collective element x in set S goes higher. Membership function consists any of two values either 0 or 1 in classical set theory.

When S is a fuzzy set and x is an appropriate element, the proposition "x is a member of S" can be either true or false, as required by the crisp logic. It can be true only to some level, to which element x is actually a member of S. Element x is a real number in the interval [0, 1] (Chowdhary et al., 2010).

Theoretically, if X is a group of elements indicated collectively by x, then a fuzzy set F in X is a set of ordered pairs, such that F = {(x, $\mu_S(x)$) |x _ X},

$\mu_S(x)$is termed as the membership function of x in F. A subset of non-negative real numbers whose least upper bound is finite constitutes the range of membership function.

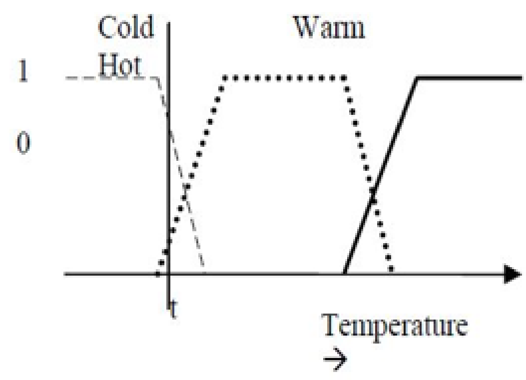

Figure 2.2: Membership Graph

Each input variable have a membership function in fuzzy system that can be plotted over membership graph as shown by Figure 2.2. The terms cold, warm, and hot are represented by membership function values over temperature scale. In Figure 2.2, the vertical line indicates that temperature t is "not hot" as the line of hot is located at zero. The line of cold indicates the temperature as "very cold" and line of warm as "slightly warm".

Fuzzy logic imitates human logical way of thinking. Fuzzy logic gives a simple way to reach at a definite conclusion based on noisy, ambiguous, vague, imprecise or missing input information. This logic gives powerful and meaningful portrayal of measurement of uncertainties. This logic also gives powerful and meaningful portrayal of indefinite concepts which are practically expressed in natural language. It enables to use linguistic variables such as good, bad, slow, fast, cold, hot, etc. It also allows the use of hedges, adjective of variables, for further refining of elements like very good, slightly slow, too hot, etc.

Human awareness of approximation has become highly significant for answer deduction capability and recovery of information like never before. Due to this reason, principality of model of crisp relational database is shifting to the principality of model of fuzzy real world database.

2.4 Fuzzy Logic and Fuzziness

In our daily life, arbitrarily many words are used that are normally fuzzy in terms of usual meanings. Words such as slow, fast, young, old, cold, hot, short, tall, etc., are used to express or describe an event or a system. Depending on his/her age, human calls a person young or old. According to the road condition, human presses the brake pedal less or more. These examples indicate the nature of acting of human brain and taking decisions during the situations which are not clear and fuzzy. This way of human brain like decision making logic is called fuzzy logic. Fuzzy logic is largely used in many fields these days as in human machine interactions, signal aliasing, image processing, operational behaviour of machine, commercial products, motion control, flow control, temperature control, tracking systems, automation, robotics and many more. One of the first application areas of fuzzy logic is control systems.

Fuzzy logic is nothing but the computation of words in practice. There is possibility of computation with words. Because of which, computerized systems can be generated by implanting human competence attached in daily language. Such a fuzzy rule base or fuzzy inference kind of system can perform estimated reasoning somewhat alike to but more primal than that of the human brain (Sivanandan et. al., 2007).

Fuzzy logic's core elements are the fuzzy sets that are distinguished by some short of fuzzy numbers called as membership functions. Uncertainty gives rise to the idea of fuzziness. If data are hard to categorize from one to another and are not crisp, they are represented by involvement degrees in related grades. Suppose if we mix two colours, say, black and white and try to draw a spectrum on a paper, we will get many shades of either black or white. Those transition shades are neither fully white nor fully black and hence the fuzziness between white and black occurs.

2.5 Fuzzy Membership Functions

There has to be a bridge between fuzzy world and uncertain data. Fuzzy membership functions are considered for that job of being a bridge. Fuzzy membership functions represent the partitioned subsections of muddy data of the crisp universe. Fuzzy sets represent and use the fuzzy membership functions as a tool. Fuzzy sets are represented by different pictorial and geometric shapes. Membership functions use known mathematical functions due to their simplicity (Atlas, 2017).

Certain kinds of mathematical formulas define membership functions which represent fuzzy sets having different shapes. Triangular, Gaussian, trapezoid, Cauchy, sinusoid, bell and sigmoid are the membership functions that are mostly used. The membership functions are formulized in terms of their parameters, which consist of location range in the universe of discussion and information about fuzziness so that fuzzy sets operations become easier. Membership functions are made adjustable due to the flexibility of adjusting parameters. Membership functions like triangular and trapezoid-type are largely used than other membership functions because of the linearity in their structure. In learning algorithms that need derivations such as in artificial neural networks, the membership functions are adjusted and determined accordingly.

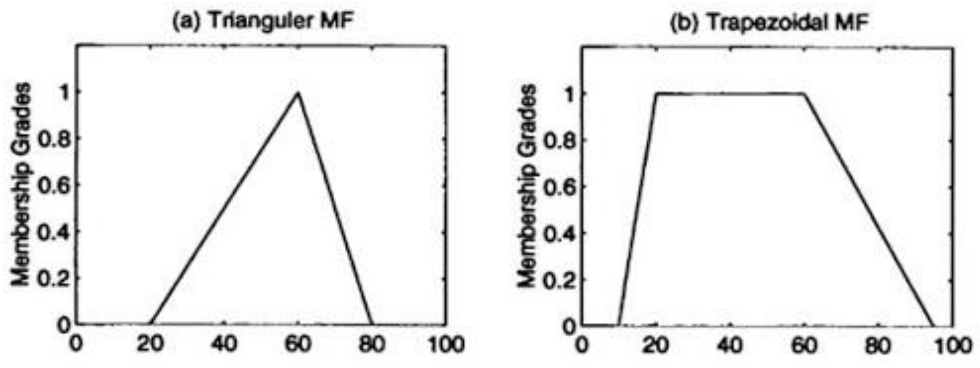

Figure 2.3: Triangular and Trapezoidal membership function

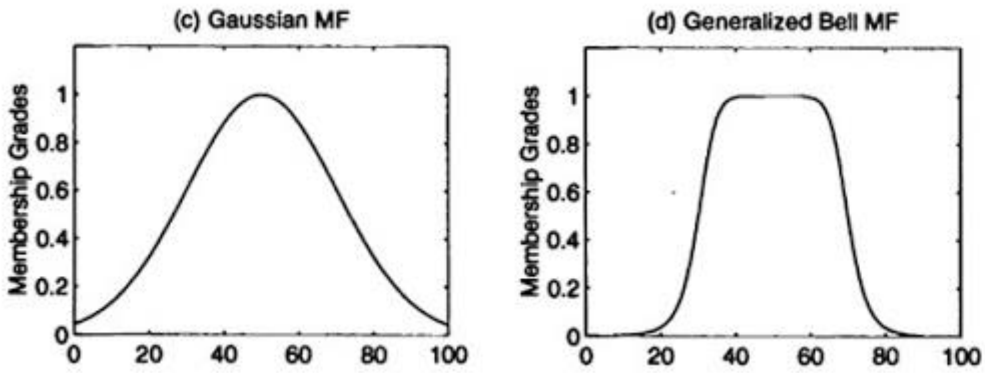

Figure 2.4: Gaussian and Generalized Bell membership function

2.6 Classic Sets and Fuzzy Sets

Suppose in a classical set, X denotes the universal sets and x denotes the individual elements in the universe X. The properties of the elements in X can be countable integers, discrete, or continuous valued quantities on the real line. Some examples of elements of different universal sets are the integers from 1 to 100, the operating temperature of an air conditioner, the operating currents of an electronic motor, the clock speeds of computers CPUs, etc.

Selecting a universe which is continuous and infinite or one which is discrete and finite is a choice of modelling. The characterization of sets defined on the universe is not altered by the choice. The corresponding set defined on the universe will be continuous if the universe owns continuous elements. The total number of elements in X is said to be its cardinal number. Finite collection of elements which are countable compose discrete universe that has a finite cardinal number. Similarly, infinite collection of elements which are uncountable compose continuous universe that has an infinite cardinal number.

The subclasses of uncertainty are classified by fuzzy sets. The inclusion of elements in a classified group is represented by fuzzy sets similarly to crisp sets. In crisp sets, the boundaries are sharp. This sharpness is absent in fuzzy sets. The membership degree becomes 0 if an element does not belong to a crisp set. Similarly, the membership degree becomes 1 if an elements belongs to a crisp set. No other choice exists for the element from 0 or 1, which is the main difference between fuzzy and crisp sets. Showing the degree of belonging of an element to the set, the boundaries of fuzzy sets change moderately from 1 to 0 or from 0 to 1. So, the membership degrees are varied between 1 and 0, and are not just 1 or 0 in fuzzy sets. Each fuzzy set which is defined in the interval [0,1], is distinguished fairly by a membership function which then allocates values of membership between 0 to 1 to each element. If an element is said to be fully included in that set, 1 is the membership value that is allocated as a full membership degree. A membership value is defined as 0 if an element is not a member of that set. A membership value between 1 and 0 is assigned by the interval [0,1] for an element, that is more or less incorporated in the set with an inclusion degree which can be any number between 1 and 0. The crisp sets do not incorporate any uncertain element.

2.7 Fuzzy Set Operations

Fuzzy numbers and fuzzy membership functions signalize the fuzzy sets. The basis of fuzzy sets and fuzzy logic are the operations which are done using fuzzy numbers or fuzzy membership functions. Fuzzy sets themselves do the fuzzy set operations or the operations are done by membership functions which carry their characteristic properties. Definitions like complement of a set, union set, intersection set, empty set, subset, etc., do exist in fuzzy sets similarly as in the crisp sets.

Intersection, union and complement are the three main fuzzy set operations.

CHAPTER THREE: RESEARCH METHODOLOGY

This chapter includes the research methodology adopted for the analysis and control of vehicle ABS using fuzzy logic. The steps adopted for the research methodology are as follows:

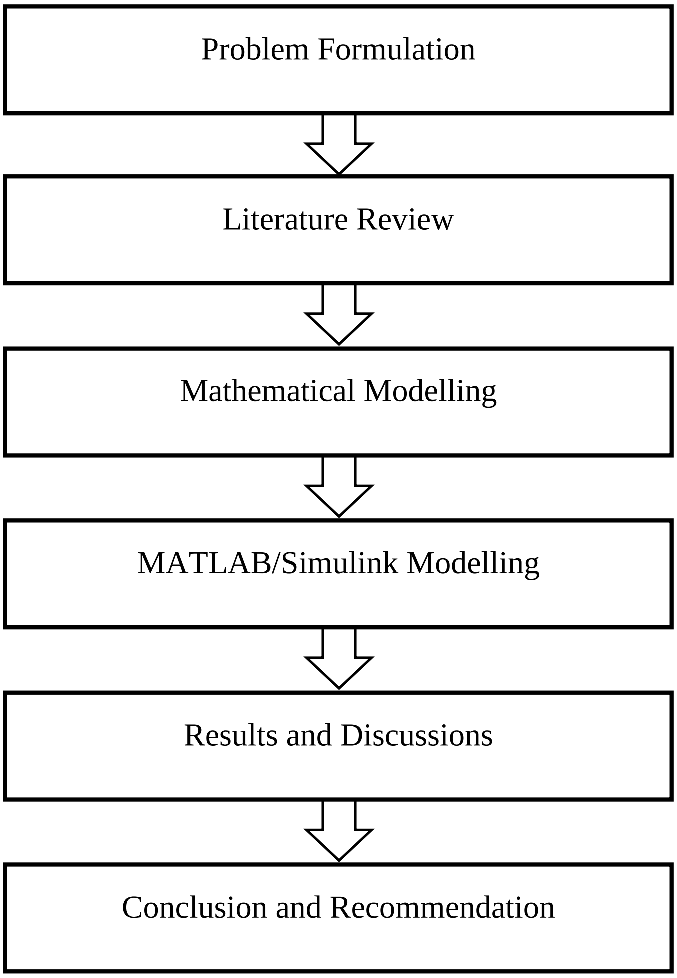

Figure 3.1: Research methodology

3.1 Problem Formulation

Fuzzy set theory gives rise to a logic which is then called fuzzy logic. This logic is multi-valued which deals with reasoning that is not precise but approximate. Binary sets have binary logic also called crisp logic whereas a truth value that ranges between zero and one is taken by fuzzy logic variables, not bounded with two truth values of classic logic. These degrees can be managed with the use of linguistic variables by specific functions (Klir & Folger, 1988).

Applications of fuzzy logic generally uses the non-numeric linguistic variables to facilitate the expression of facts and rules while numerical values are usually taken by variables in mathematics (Yuan & Klir, 1995).

In absence of fuzzy ABS, the brake pressure reaches extreme level and wheels are locked up very fast. Due to these, vehicle's behaviour become unstable, the vehicle cannot be further steered as desired and the stopping distance increases (Clair et al., 1997). With the activation of ABS controller that uses fuzzy logic, steerability is retained during whole duration of braking, and the slowing down length is noticeably shortened (Ross, 1995).

Since many researches have been done in the field of ABS, the use of fuzzy logic in the controller of ABS is found to be very less. Thus, in this thesis, we try to control the stopping distance of a vehicle by shortening the slowing down time with smooth slip on different types of road conditions, as the braking force is applied, with the use of fuzzy logic controllers in the ABS.

3.2 Literature Review

The extensive review of literature was conducted with the help of internets, books, journals and research papers available. The literatures related to MATLAB/Simulink and how it is used to analyse data were studied. The literatures related to application of different types of modern type antilock braking system were studied. Also the study about special features of MATLAB/Simulink modelling process was done. Current trends of the antilock braking system used in modern vehicles were studied as this provided an extra aid to this research activity. Various aspects of control logics were studied. The effects of the road conditions and other factors on the vehicle dynamics and mathematical modelling as well as non-linear equations were studied. Previous research on the similar topic were also studied.

3.2.1 Analysis of Antilock Braking System

The aim of an ABS is to keep the braking distance to minimum while retaining the steerability even when the braking is hard. Figure 3.2 shows the wheel model for a quarter car vehicle and the curve that shows the relation between wheel slip and coefficient of friction. Coefficient of friction depends upon the wheel slip. The terminologies related to the figure are explained below:

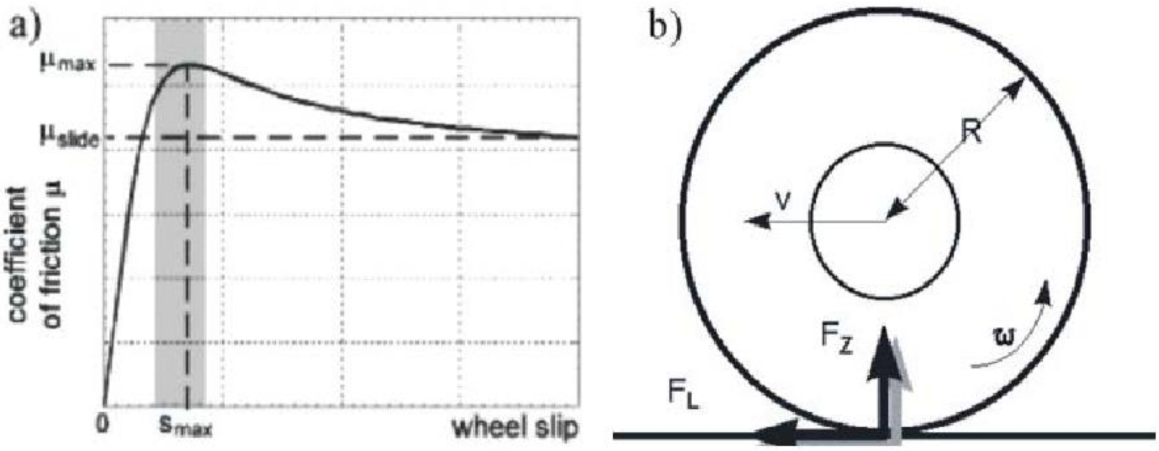

Figure 3.2: Friction characteristics and wheel model

Fz: Wheel load

R: Radius of the wheel

ω: Angular velocity of the wheel

v: Wheel centre velocity

F_L: Force (in longitudinal direction)

When an uncontrolled full brake is applied, wheel slip is zero i.e. S =0 when the operation starts and then the slip rises and reaches the maximum peak level i.e. Smax as shown in Figure 3.2. After that, the coefficient of friction starts declining rapidly and the wheel lock occurs after few milliseconds. Steerability of the vehicle is lost at this point. Therefore, to avoid such happening a stable and robust controller is required to maintain the slip of the vehicle inside the shaded are as shown in Figure 3.2.

For an ABS control of longitudinal slip towards maximum braking performance is not the only problem. Cornering force, which is the main factor for cornering ability for the vehicle, depends on the longitudinal slip ratio besides the lateral slip angle. A small amount of increase in longitudinal slip results in a very steep decrease in lateral force generating ability. Thus, the cost of a small improvement in braking distance may be a large reduction in the directional control and also lateral stability of a vehicle. Usually, the ability of changing direction provides a better advantage in emergency situations, than ability to stop in a shorter distance. The braking and steering simulation is important because it shows the results of the ability of the vehicle to gain lateral deflection during maximum deceleration. Just clearing a road obstacle may not be safe because other encounters may occur after the first obstacle is cleared. Both of the steering and braking actions must take place in a stable manner for safest results. Without an ABS this manoeuvre yields strong deceleration with no lateral deflection capability if the braking and steering commands are simultaneously applied. A driver may even lose control and spin out, as it is successfully observed in the simulations here with various magnitudes of braking and steering performed in the name of controlled panic braking in an obstacle avoiding manoeuvre (Unlusoy, 2008).

3.3 Mathematical Modelling

A set of equations that represents dynamics of the system at least fairly well or accurately is called the mathematical model of a dynamic system. A mathematical model is not distinctive to a given mechanical system. Depending on one's point of view, a system can be expressed in various ways and hence, it can have various mathematical models. Differential equations describes the dynamics of a mechanical system. Such differential equations can be obtained by using laws of physics which govern specific system like in case of a mechanical system, Newton's laws are used. Different forms can be assumed by mathematical models. One mathematical model can be more suitable than other models depending on the specific circumstance and the specific system. Different computer and analytical tools can be used for modelling and analysis purpose when a system's mathematical is obtained.

Formulating the basic equations of motion for the dynamic system is the first step towards achieving a simulation for an ABS. The function of the mathematical model of car is identifying the problems that takes place in the braking system of the cars. The precise physical characteristics of the car can be acquired by simulating the mathematical model with the help of data obtained from the vehicle. MATLAB software is mainly used to simulate a nonphysical model of car for this purpose.

3.4 Quarter Car Model

The study of an ABS can be performed by using different ABS models. The major antilock braking system models used for the analysis of braking system are full car model, half car model and quarter car model. Among these, quarter car model of a vehicle ABS is generally employed for the analysis of vehicle braking system. Due to the facts that quarter car model is simple and can capture essential characteristics of full model, it is frequently used for braking system analysis. Quarter car model which is referred as a dynamic model, describes the relationship between the input and output for understanding the system's behaviour.

Quarter car (driver seat) model is used in this research work for further analysis due to the fact that it provides the qualitatively correct information, especially for handling and ride studies. This model provides more comprehensive, accurate and exhaustive studies with more involved dynamical car model. Quarter car models can be generated in MATLAB/Simulink.

3.5 MATLAB/Simulink Modelling

For technical computing used by industries, research institutes, scientists, engineers and students in universities all over the world, MATLAB is a very popular language. Due to the reason that MATLAB is powerful and easy to use, this software is very popular. MATLAB can be used as the next tool for graphic calculators.

Simulink (**Simu**lation and **Link**) is an extension of MATLAB by Math Works Inc. Simulink works with MATLAB to offer analysis of mechanical system, modelling and simulation under a graphical user interface (GUI) environment. It supports nonlinear and linear systems, which are modelled in sampled time, continuous time, or a hybrid of the two. Systems may also have different parts termed as multi rate that are updated or sampled at different rates. By using block notation, Simulink allows engineers to accurately and rapidly generate the computer models of mechanical systems. A comprehensive block library of sources, sinks, nonlinear and linear components and connectors, is included in Simulink. Users are allowed to create and customize their own blocks by Simulink. For modelling and simulating mechanical systems, Simulink has become the most frequently used software package in industry and academia in the last few years. It has a heavy industrial usage, and is credited with reducing the development of most projects of control system.

Finally after collecting all the data inputs required to model the ABS using fuzzy logic, the MATLAB/Simulink models of different components based upon their mathematical equations/models were developed.

3.5.1 Components used in Simulink modelling

The components of the Simulink that are used in modelling of the antilock braking system are as follows:

a. Add Block

This block performs subtraction or addition on its inputs. It can subtract or add matrix, scalar or vector inputs. Signal elements can also be collapsed by add block.

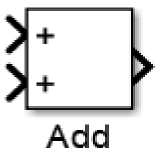

Figure 3.3: Simulink add block

b. Bus Creator Block

A set of signals is combined into a bus by this block. The block parameter number of inputs are set to the number of signals in the group to bundle a group of signals with this block. It displays the number of ports that are specified.

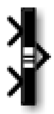

Figure 3.4: Simulink bus creator model

c. Constant Block

This block generate a complex or real value. It generates matrix, vector or scalar output depending on the setting of the parameters of interpret vector as one dimensional parameter and the dimensionality of the parameter of constant value. The block output has the same dimensions and elements as the constant value parameter.

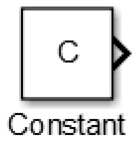

Figure 3.5: Simulink constant block

d. Gain Block

The input is multiplied by a gain (constant value) in this block. The constant value and the input can each be a matrix, scalar or vector. The constant value in the gain parameter is specified. The parameter allows to specify the order of the multiplicands for matrix multiplication.

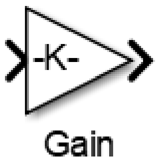

Figure 3.6: Simulink gain block

e. Inport Block

This block is the link from outside a system into the system. According to certain rules, Simulink assigns inport block port numbers.

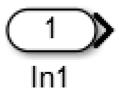

Figure 3.7: Simulink inport block

f. Integrator Block

This block outputs the value of the integral of its input signal with respect to time. The only difference between integrator limit block and integrator block is that the output of the integrator limit block is limited based on the upper and lower saturation limits, otherwise they are similar. This block is treated as a dynamic system with one state by Simulink.

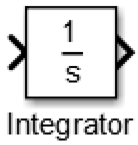

Figure 3.8: Simulink integrator block

g. Outport Block

This block is the link from a system to a station outside the system. According to certain rules, Simulink assigns outport block port numbers.

Figure 3.9: Simulink outport block

h. Scope Block

With respect to simulation time, this block displays input signals.

- **Scope Window-** Scope data is written to the connected Scope even if a scope window is closed when simulation starts. Due to this, if a Scope is opened after a simulation, window displays the input signals or signals.
- **Plotting signals-** The Scope draws a point-to-point plot if the input signal is continuous whereas, it draws a stair-step plot if the signal is discrete.
- **Time steps values-** Major time-step values are only displayed by this block.
- **Multiple graphs (y-axes)-** Multiple graphs (y-axes) can be displayed by this block with one graph per input port. It allows to alter the range of input values and the amount of time displayed.
- **Data type support-** Scope accepts real signals of any type of data which is supported by Simulink. This includes enumerated and fixed-point types of data.

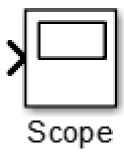

Figure 3.10: Simulink scope block

i. **Signum Function Block**

An odd mathematical function which can extract the sign of a real number is termed as signum function.

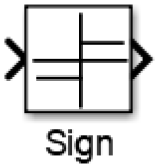

Figure 3.11: Simulink signum function block

j. **Step Input Block**

At a specified time, step input gives a step between two definable levels. Output of this block is the initial parameter value if the time of simulation is less than the value of step time parameter. Output is the final parameter value for time of simulation equal to or greater than the step time.

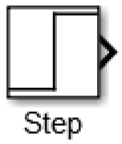

Figure 3.12: Simulink step input block

k. **Sum Block**

This block carries out subtraction or addition on its inputs. It can subtract or add matrix, scalar or vector inputs. Signal elements can also be collapsed by this block.

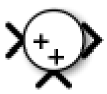

Figure 3.13: Simulink sum block

3.6 Modelling of antilock braking system (ABS)

The mathematical modelling of different components, starting from vehicle dynamics to wheel slip, tire, brake actuator and controller for an ABS, are done in this section using the mathematical equation and expressions.

3.6.1 Vehicle dynamics

While taking the simplified design into account, and the higher requirements of real time control, we choose a quarter car model to create the simulation system. For simplifying the system model, the secondary factors are neglected and some assumptions are made such as the system ignores the influence of the lateral wind, the tires are rigid and the aerodynamic drag is ignored.

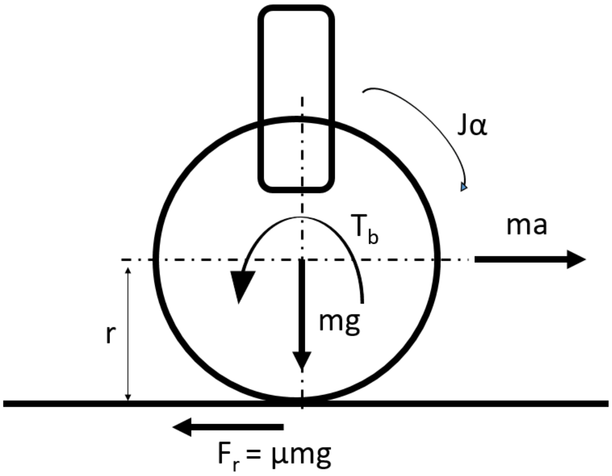

Figure 3.14: Vehicle dynamics

A quarter car model which carries the essential characteristics of the whole vehicle was selected. Figure 3.14 shows the free body diagram of the quarter car model. In longitudinal direction, the force balance equation is expressed as

$$ma = -\mu(\lambda)mg \qquad (3.1)$$

$$J\alpha = r\mu(\lambda)mg - T_b \qquad (3.2)$$

Where,

m = 1/4 × vehicle mass,

a = vehicle's linear acceleration,

μ(λ) = friction coefficient between tire and road, which is nonlinear function of road dynamics and slip ratio,

J = wheel's moment of inertia,

ω = wheel's angular velocity,

α = wheel's angular acceleration,

r = wheel's radius,

T_b = brake torque which acts on the wheel, and

λ = wheel slip ratio

The stability of the closed-loop control system is affected because of the existence of the measured noise and plant uncertainties in the ABS controller design. Hence, the dynamics and plant uncertainties which is not modelled are considered to evaluate the robustness of the controller.

3.6.2 Wheel Slip

In most friction models, basically it is assumed that the quotient μ fundamentally is a function of the slip λ. Slip is defined as the normalized difference of vehicle velocity and the velocity of tire circumference. When v > ωr, slip is expressed as:

$$Slip\ (\lambda) = \frac{v - \omega r}{v} \qquad (3.3)$$

The value of the slip is ensured to lie between 0 and 1 by the slip equation. When the wheel has full contact with the road and it rolls freely, a slip value of λ = 0 is obtained. When the wheel is completely locked up and slides on the road, a slip value of λ = 1 is obtained.

The wheel slip is modelled in MATLAB/Simulink as shown in APPENDIX D.

3.6.3 Tire modelling

The modelling of the tire friction force, F_x has an important role in deciding the vehicle behaviour. It has shown to be infamous difficult to model. There exist various models with different properties which are in use. The friction force is dependent on friction and other physical processes such as deformation. A friction-like definition is used to name the effective quotient of force and the word friction is used.

$$\mu = Fx/Fz \qquad (3.4)$$

The normal force F_z is considered known in all models, and when μ has been modelled, the friction force can be calculated. μ is a function of the normal force F_z, vehicle speed v and wheel rim velocity ωr, where the normal force is of minor significance, road and tire materials and their interaction in most of the models.

Modelling of a tire in the most basic models is done neglecting the deformation of tire during braking. In these kind of static models, the effective quotient of force μ is causal. By term casual here means the dependence on present values of v, ω and F_z only.

The tire deformation cannot be neglected while modelling the wheel more carefully. The force which acts on the tire from the road surface deforms it, which then apparently acts a spring because the tire is made of soft material i.e., rubber. An extra state of tire deformation is added physically by this to the process, and then μ cannot be determined from measurement of present values only. By modelling or estimating the state of tire deformation The actual friction force *Fx* can be calculated by estimating or modelling the state of deformation of tire from the values of v, ω and F_z.

Based on the key factors such as anistropic stiffness properties, dynamic friction coefficient, translational, bending and twisting compliance of the carcass and arbitrary pressure distribution, the analytical tire model is developed. Some of the trending tire models available are Burckhardt model, Pacejka model, Brush model, Magic formula, etc. Among all the different tire models available, Burckhardt model has been used for tire modelling in this thesis.

The equation governing this tire model is given as (Hi, et. Al., 2019)

$$\mu(\lambda) = C_1\left(1 - e^{-C_2\lambda}\right) - C_3\lambda \qquad (3.5)$$

where,

μ(λ) = friction coefficient, λ = slip and C_1, C_2, C_3 are constants which depend upon road condition (Kant et. al, 2013) as shown in Table 3.1.

Table 3.1: Coefficients of Burckhardt equation

Road surface condition	***C1***	***C2***	***C3***
Dry Asphalt	1.2801	23.990	0.52
Dry Concrete	1.1973	25.186	0.5373
Wet Asphalt	0.86	33.82	0.35
Cobblestone	1.37	6.46	0.67
Snow	0.1946	94.129	0.0646
Ice	0.05	306.39	0

By applying the given expression (3.5), the graphs for coefficient of friction v/s slip ratio is obtained which are shown in Figure 3.15 and Figure 3.16. From the graphs we can conclude that there is a range of slip around 0.2 approx. in which the coefficient of friction reaches to its highest range for every road condition and hence we term that value as an optimal slip.

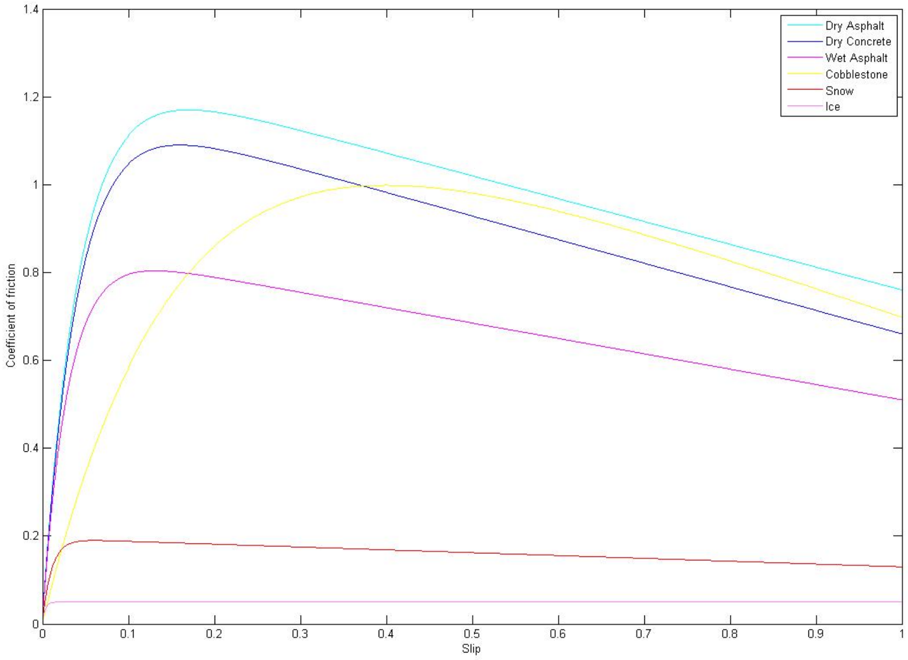

Figure 3.15: Coefficient of friction v/s slip

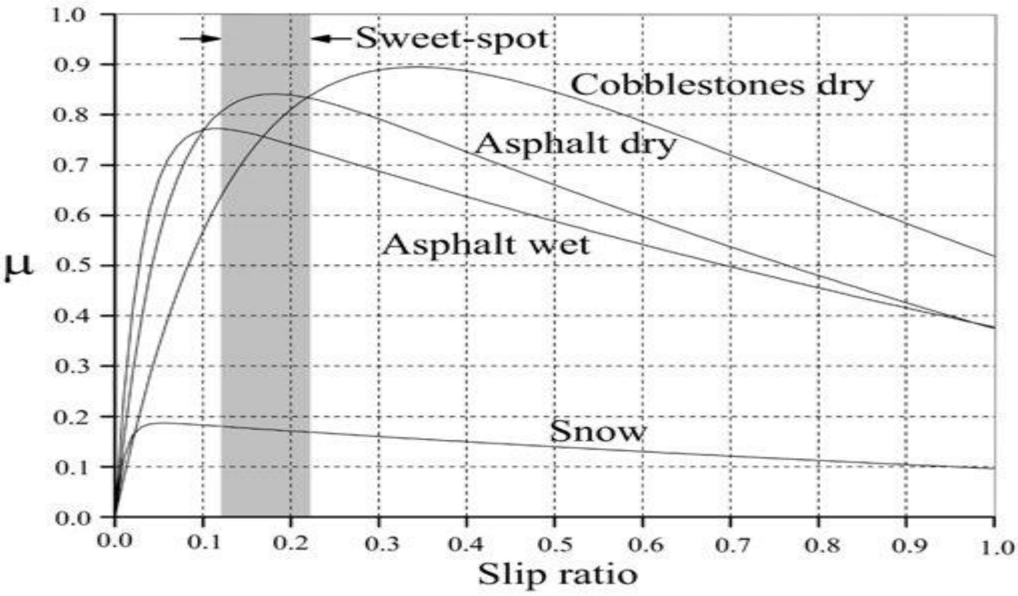

Figure 3.16: Coefficient of friction v/s slip with sweet spot

3.6.4 Brake Actuator model

Brake actuator is a subsystem that permits the hydraulic pressure to pass through the circuit of ABS according to a command signal which is received from the brake controller. The change rate of brake pressure is proportional to the brake fluid flow rate. Also, the flow rate depends on the control valve opening. Values of initial condition cannot be chosen randomly because of the sensitivity of actuator at optimization. Hydraulic pressure reaching the wheel cylinder can be controlled up to 5 bars for some applications. For regenerative braking, an electric actuator is used which executes quicker than the friction brake. At declaration, disability of the regenerative brake will arise when the actuators are switched off. As a consequence, there arises ABS reliability on the friction brake. An actuator with combined electrical and hydraulic properties should have fast response time as well as performance tracking. With the usage of brake actuators the slip of each wheel can be controlled separately. Hydraulic accumulator contains gas inside which experiences an adiabatic process when braking force is applied. So, the application of gas state equation is valid.

A MATLAB/Simulink model of brake actuator is shown in APPENDIX E.

3.6.5 Sensor

An inductive coil sensor which is contoured to a reflector ring measures the wheel speed. Recently, accelerometer and rate sensors are implemented for control of wheel slip. In order to detect conditions of road surface, sensors work on different principles of light, sound or microwave. Noises developed by actuator and sensor are not considered in ABS research. The transfer function of a sensor for wheel speed should be linearized in the frequency domain to obtain an ideal performance.

3.6.6 Bang-bang controller

A bang-bang controller, also known as on-off or two step controller in control theory, is a feedback controller which shifts abruptly between two states. It is considered with regard to any element that provides hysteresis. To control a plant which accepts a binary input, this controller can be used.

In minimum-time problems, the use of bang-bang controllers frequently arises. In some cases, bang-bang controls are actually optimal controls, also they are usually applied because of convenience or simplicity. This controller electronically or mechanically turns something off or on when a predefined target has been reached. This two-step controller or say hysteresis controller is used in many industrial and home control systems.

When a bang-bang controller is used in an ABS, the control is based upon the error between actual slip and desired slip. Desired slip is the value of slip at which slip-friction coefficient curve reaches a peak value. This slip value is called optimum value for minimum braking distance. The brake torque as the control input is shifted between the maximum value and minimum value, for keeping the slip operating in the desired region (Gowda & Ramchandra, 2017).

3.6.7 Fuzzy logic controller

Fuzzy logic controllers (FLC) are based upon the theory of fuzzy sets and fuzziness. They are theoretically very simple. An input stage, a processing stage and an output stage are consisted in these controllers. The input stage does the mapping of sensor or other inputs to the suitable truth values and membership functions. The processing stage calls on each suitable rule and creates an outcome for each, after that combines the outcomes of the rules. At last, the output stage transforms and merges outcome back into a specific value of control output. A collection of logic rules which are in the form of if-then statements forms the platform for the processing stage. These rules use the truth value of input to create an outcome in the fuzzy set for the output. The outcome is used with the outcomes of other rules to ultimately create the crisp composite output.

In MATLAB/Simulink, the fuzzy logic controller block executes a fuzzy inference system (FIS). The FIS is specified to evaluate using the FIS name parameter.

3.7 Controller for Antilock Braking System

The method applicable for a non-linear continuous time is adaptive fuzzy logic, which however, is not suitable for discrete time. For the coefficient of friction μ, an ideal value is chosen.

For development, the behaviour of logging the driving and applying it for the control system is functional. The wheel speed sensor transmits the signal which derives the tire velocity and tire angular acceleration. Mamdani model is used for the selection of parameters of a membership function. To measure the lateral and longitudinal velocities as well as the vehicle tilt angle, vehicles are provided with systems like Global Positioning System (GPS) navigation and RT2500 inertial system. The tire speed becomes less than the vehicle speed while braking. Maximum braking force depends on the coefficient of tire friction.

In this model, two Fuzzy Logic Controllers are used. One controller is used to determine the optimal slip based on different road conditions and their coefficient of friction based on slip. The other controller is used to determine the brake force required based on the slip error and wheel acceleration. The complete MATLAB/Simulink model is shown in APPENDIX C.

3.8 ABS Simulink model with fuzzy logic controllers

APPENDIX C shows the Simulink model for control of anti-lock braking system using fuzzy logic controllers. Two fuzzy logic controllers (FLC) have been used for control of ABS. Fuzzy logic controller 1 uses vehicle speed and wheel slip as input to determine optimal slip based on the present road condition. Fuzzy Logic Controller 2 uses slip error and wheel acceleration as input to determine the required brake force that needs to be applied to minimize excessive slip and avoid locking of the wheels. The brake actuator subsystem is set up according to its mathematical model which gives angular acceleration of the wheels at the output node. Similarly, vehicle speed and stopping distance is calculated using blocks that serve the purpose to apply the mathematical equations discussed in the earlier sections. The vehicle speed and wheel speed is then used to determine the relative slip of the vehicle which is then fed as feedback to the summation block where and error signal is generated by comparing the existing slip with the optimal slip. Further decision for brake force is calculated based on the positive or negative value of the error slip signal. The feedback slip signal is also fed to the mu-slip conversion block where the slip is converted into coefficient of friction using Burchardt's mathematical model. Different signals are logged before running the simulation to obtain required curves for wheel velocity and slip.

3.9 ABS Simulink model with bang-bang controller

APPENDIX F shows the Simulink model for control of anti-lock braking system using bang-bang controllers. A bang-bang controller that takes slip error signal as input is used to determine the required brake force that needs to be applied to minimize excessive slip and avoid locking of the wheels. These controllers switch between minimum and maximum value when a certain set-point is reached. The brake actuator subsystem is set up according to its mathematical model which gives angular acceleration of the wheels at the output node. Similarly, vehicle speed and stopping distance is calculated using blocks that serve the purpose to apply the mathematical equations discussed in the earlier sections. The vehicle speed and wheel speed is then used to determine the relative slip of the vehicle which is then fed as feedback to the summation block where and error signal is generated by comparing the existing slip with the optimal slip. Further decision for brake force is calculated based on the positive or negative value of the error slip signal. The feedback slip signal is also fed to the mu-slip conversion block where the slip is converted into coefficient of friction using Burchardt's mathematical model. Different signals are logged before running the simulation to obtain required curves for wheel velocity and slip.

3.10 Rule formation and simulation

The use of two controllers provides better control over slip and steerability of the vehicle. The fuzzy logic controller 1 which determines the optimal slip has two inputs: vehicle speed and wheel slip as shown in Figure 3.17. Velocity and wheel slip are divided into five ranges of values:

- Very Low (VL)
- Low (L)
- Medium (M)
- High (H)
- Very High (VH)

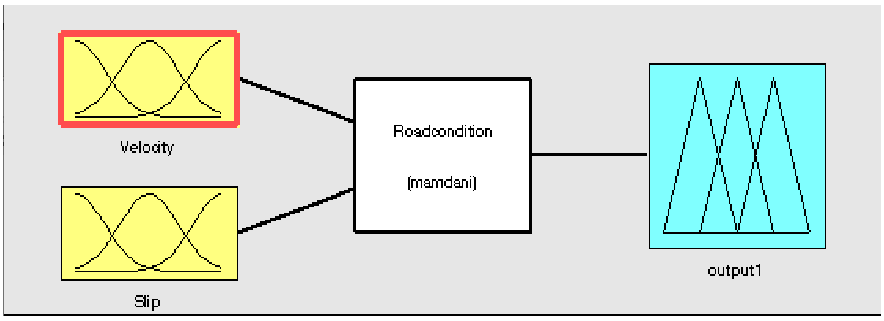

Figure 3.17: Fuzzy inputs for fuzzy controller

Fuzzy logic controller 2 which is used to determine required brake force has two inputs: slip error and wheel acceleration as shown in Figure 3.18. Slip error and wheel acceleration are divided into five range of values:

- Negative Large (NL)
- Negative Small (NS)
- Zero (Z)
- Positive Small (PS)
- Positive Large (PL)

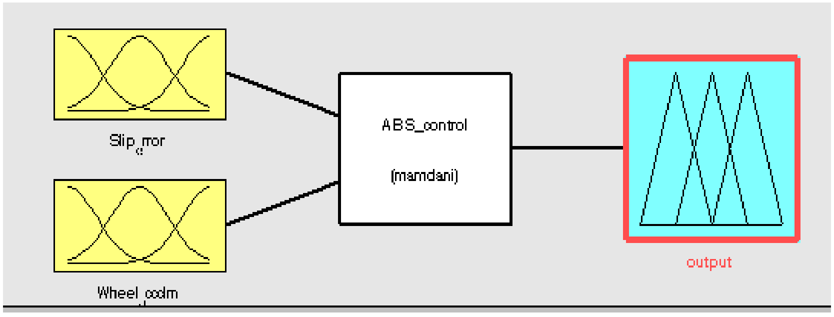

Figure 3.18: Fuzzy inputs for fuzzy controller 2

For fuzzy logic controller 1, the membership function for different input parameters such as wheel velocity and slip are defined as shown in Figure 3.19 and Figure 3.20 respectively.

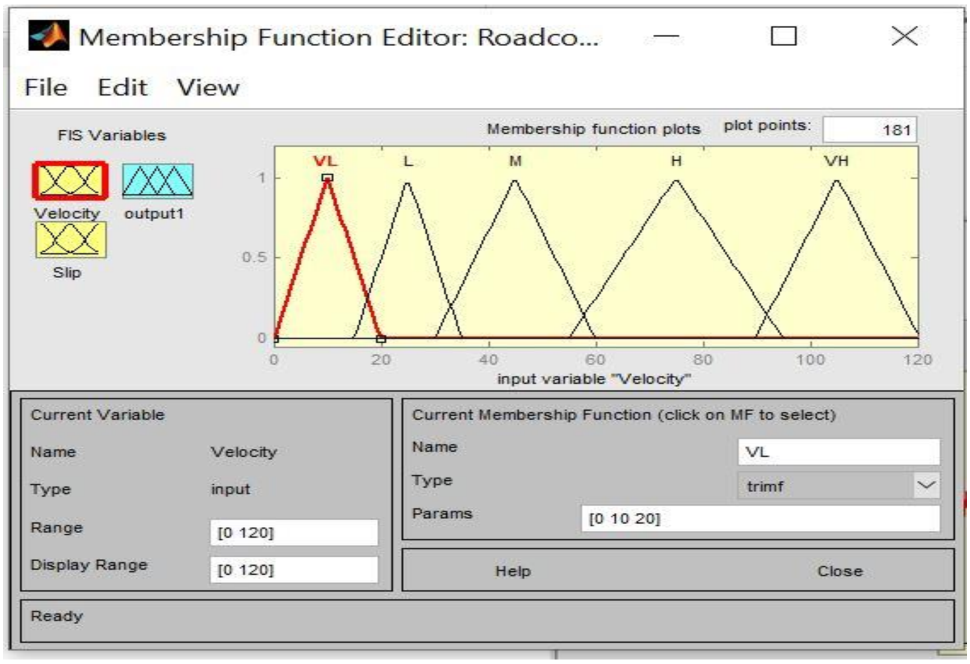

Figure 3.19: Membership function for wheel velocity

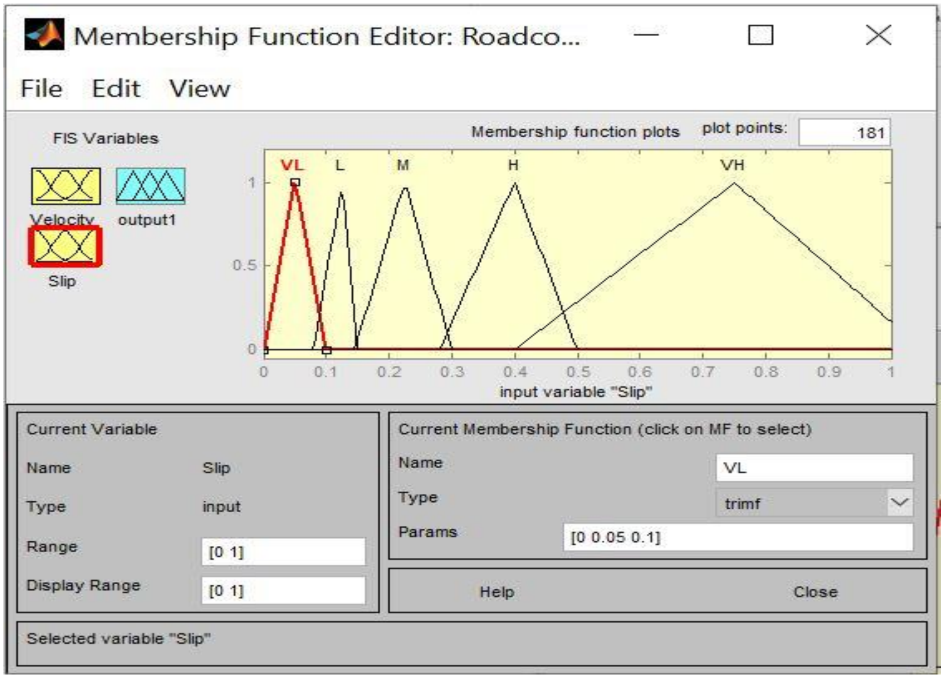

Figure 3.20: Membership function for slip

Once the membership functions for input and output are created, fuzzy rule set has to be formed. Fuzzy rules are formed to define the dependence between input and output parameters. It defines the output for different combinations for input. Rule viewer for fuzzy rules set is shown in Figure 3.22 and the surface viewer is shown in Figure 3.23.

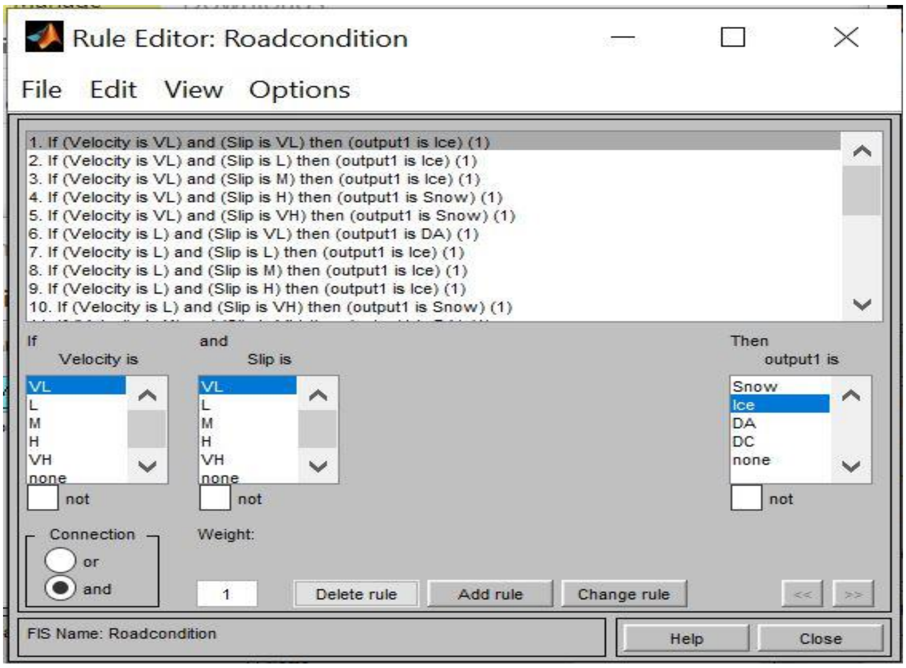

Figure 3.21: Rule editor for fuzzy controller 1

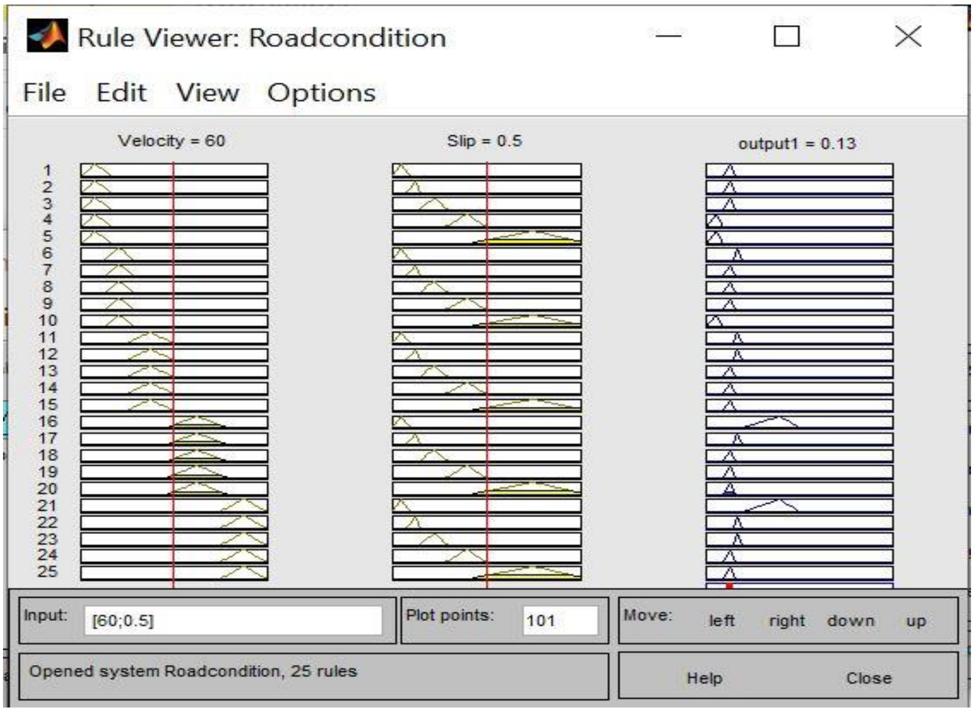

Figure 3.22: Rule viewer for fuzzy controller 1

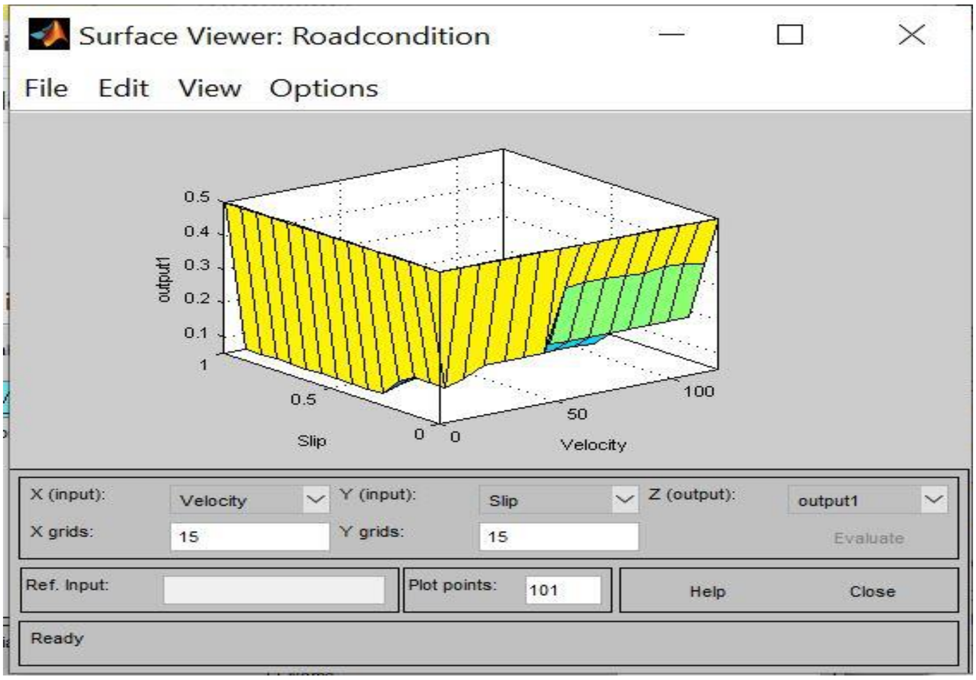

Figure 3.23: Surface viewer for fuzzy controller 1

Table 3.2: Fuzzy Rule Matrix 1

Slip	**Wheel Velocity**				
	VL	**L**	**M**	**H**	**VH**
Very Low (VL)	Ice	Dry Asphalt	Dry Asphalt	Dry Concrete	Dry Concrete
Low (L)	Ice	Ice	Dry Asphalt	Dry Asphalt	Dry Asphalt
Medium (M)	Ice	Ice	Ice	Ice	Dry Asphalt
High (H)	Snow	Ice	Ice	Ice	Ice
Very High (VH)	Snow	Snow	Ice	Ice	Ice

Table 3.2 shows the fuzzy rule matrix to determine optimal slip from different available combinations. These rules are fed to the fuzzy rule editor as shown in Figure 3.21.

Similarly, for fuzzy logic controller 2, the membership function for different parameters such as slip error and wheel acceleration are defined as shown in Figure 3.24 and Figure 3.25 respectively and fuzzy rules are set for the same as shown in Figure 3.26. Rule viewer and surface viewer of the rules are shown in Figure 3.27 and Figure 3.28 respectively.

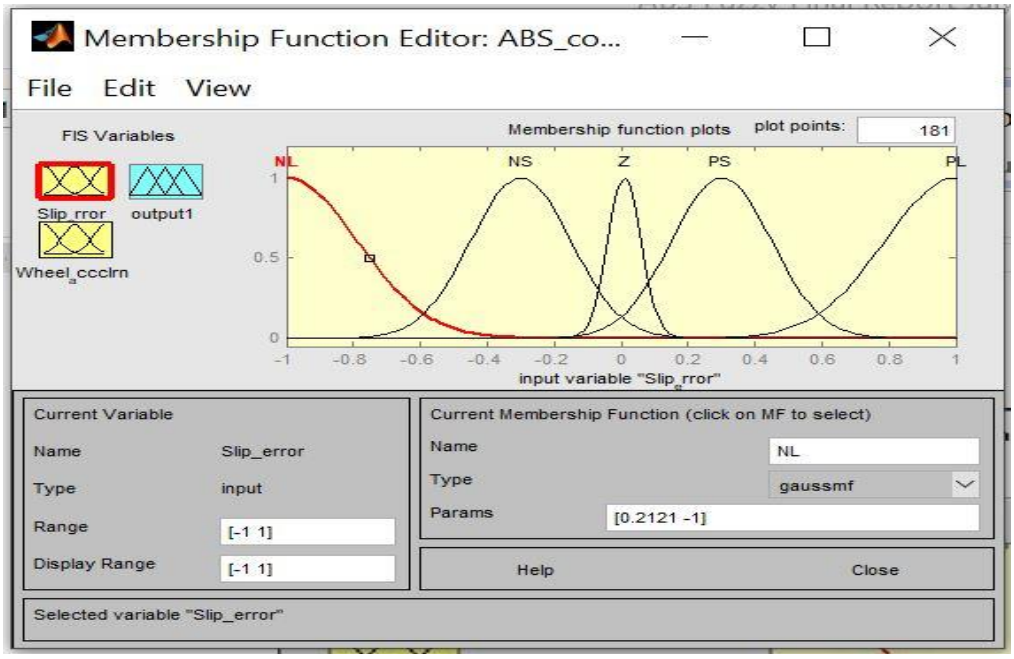

Figure 3.24: Membership function for slip error

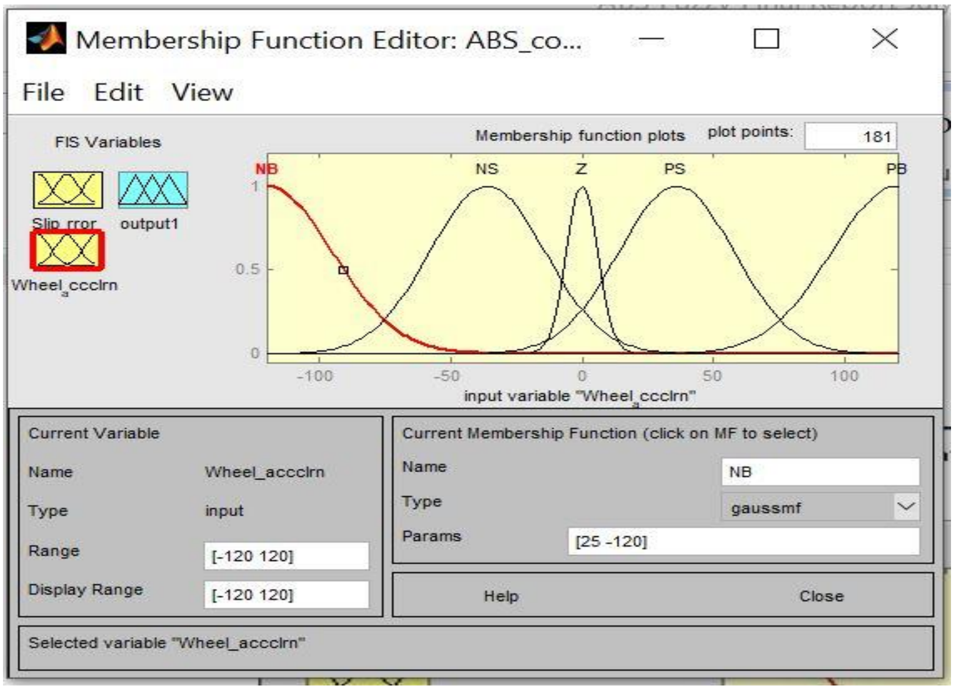

Figure 3.25: Membership function for wheel acceleration

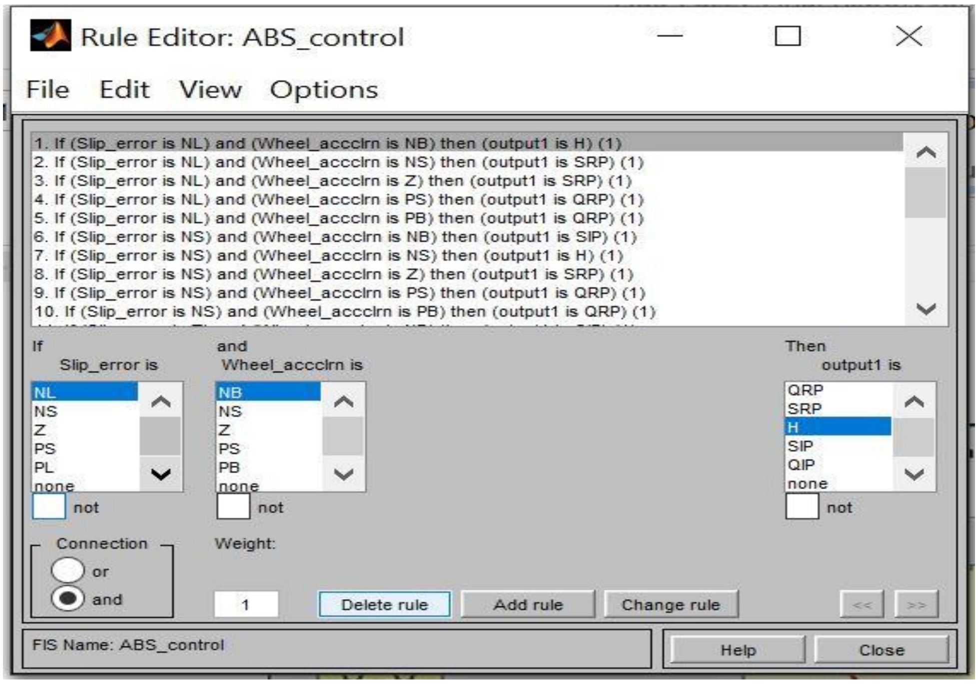

Figure 3.26: Rule editor for fuzzy controller 2

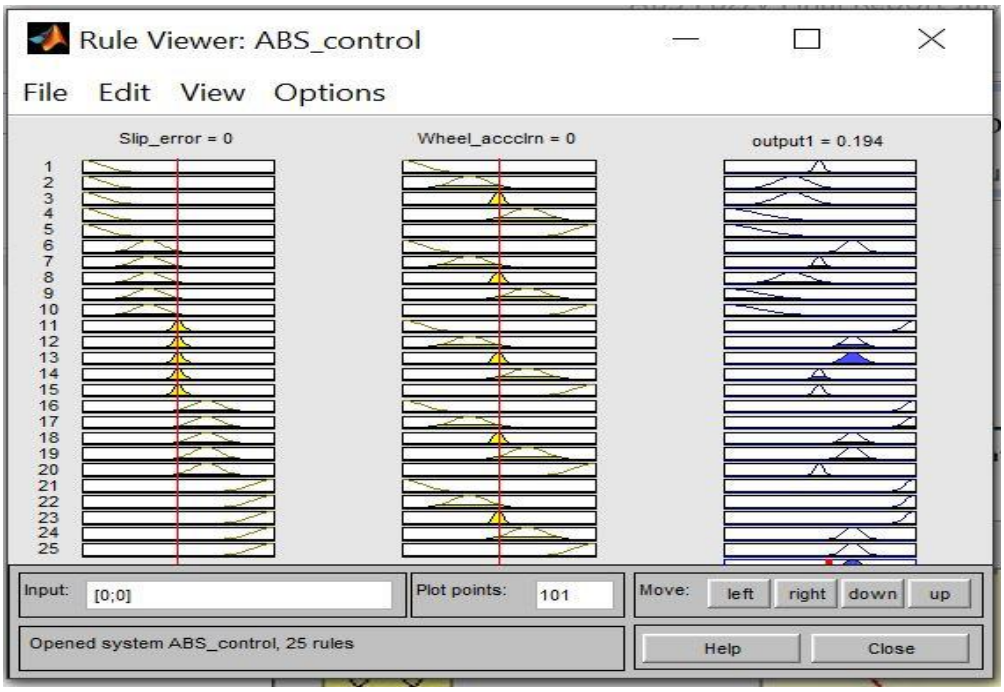

Figure 3.27: Rule viewer for fuzzy controller 2

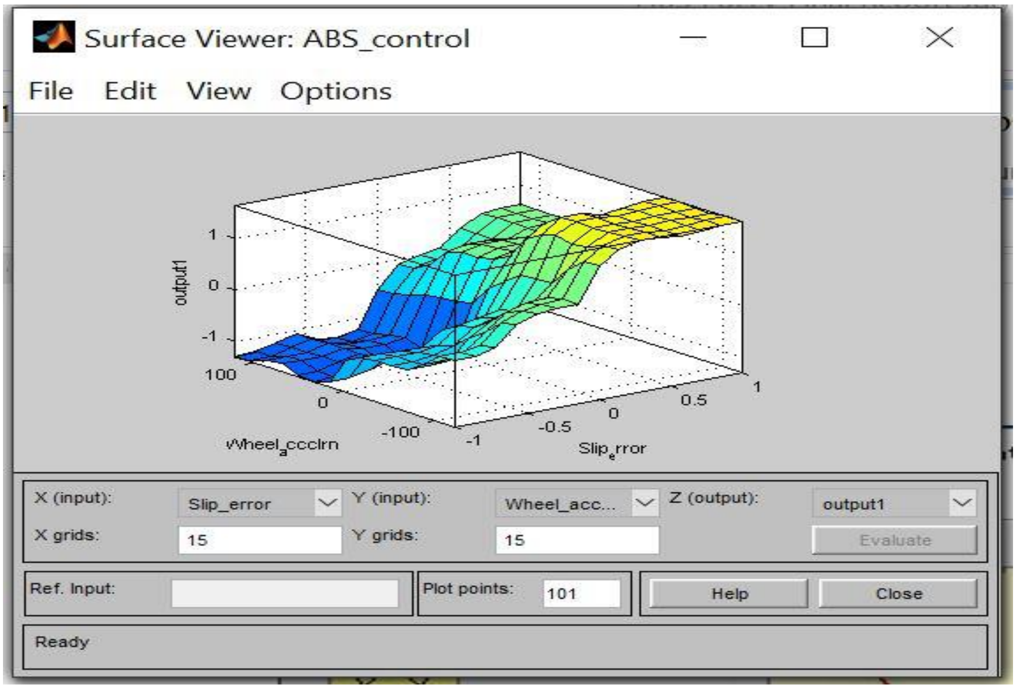

Figure 3.28: Surface viewer for fuzzy controller 2

Table 3.3 shows the fuzzy rule matrix to determine nature of brake force from different available combinations. These rules are fed to the fuzzy rule editor as shown in Figure 3.26. Following terms are used for ABS control in the Table 3.3:

- QRP (Quickly Release Pressure)
- SRP (Slowly Release Pressure)
- H (Hold)
- SIP (Slowly Increase Pressure)
- QIP (Quickly Increase Pressure)

Table 3.3: Fuzzy Rule Matrix 2

Slip Error	**Wheel Acceleration**				
	NL	**NS**	**Z**	**PS**	**PL**
Negative Large (NL)	H	SRP	SRP	QRP	QRP
Negative Small (NS)	SIP	H	SRP	QRP	QRP
Zero (Z)	QIP	SIP	SIP	H	H
Positive Small (PS)	QIP	QIP	SIP	SIP	H
Positive Large (PL)	QIP	QIP	QIP	SIP	SIP

CHAPTER FOUR: RESULTS AND DISCUSSION

4.1 Input and output of fuzzy logic controllers

The mathematical models of components of an antilock braking system were developed and then the MATLAB/Simulink models of the same were modelled. Different inputs for two fuzzy logic controllers used in this model were given with different ranges. The Table 4.1, Table 4.2 and Table 4.3 show the range, display range and parameters of the given inputs: wheel velocity and slip and the output road condition of fuzzy logic controller 1 respectively.

Table 4.1: Velocity categories

Wheel velocity	Range	Display Range	Params
Very Low (VL)	[0 120]	[0 120]	[0 10 20]
Low (L)	[0 120]	[0 120]	[15 25 35]
Medium (M)	[0 120]	[0 120]	[30 45 60]
High (H)	[0 120]	[0 120]	[55 75 95]
Very High (VH)	[0 120]	[0 120]	[89.7 105 120]

Table 4.2: Slip categories

Slip	Range	Display Range	Params
Very Low (VL)	[0 1]	[0 1]	[0 0.05 0.1]
Low (L)	[0 1]	[0 1]	[0.08 0.125 0.15]
Medium (M)	[0 1]	[0 1]	[0.145 0.225 0.3]
High (H)	[0 1]	[0 1]	[0.28 0.4 0.5]
Very High (VH)	[0 1]	[0 1]	[0.4 0.75 1]

Table 4.3: Road condition categories

Road condition	Range	Display Range	Params
Snow	[0 1]	[0 1]	[0 0.061 0.1]
Ice	[0 1]	[0 1]	[0.1 0.13 0.16]
Dry Asphalt	[0 1]	[0 1]	[0.15 0.17 0.2]
Dry Concrete	[0 1]	[0 1]	[0.2 0.4 0.5]

Similarly, Table 4.4, Table 4.5 and Table 4.6 show the range, display range and parameters of the given inputs: slip error, wheel acceleration and the output ABS control of fuzzy logic controller 2 respectively.

Table 4.4: Slip error categories

Slip error	Range	Display Range	Params
Negative Large (NL)	[-1 1]	[-1 1]	[0.2121 -1]
Negative Small (NS)	[-1 1]	[-1 1]	[-0.15 -0.3]
Zero (Z)	[-1 1]	[-1 1]	[-0.05 0.01]
Positive Small (PS)	[-1 1]	[-1 1]	[0.15 0.3]
Positive Large (PL)	[-1 1]	[-1 1]	[0.2121 1]

Table 4.5: Wheel acceleration categories

Wheel acceleration	Range	Display Range	Params
Negative Large (NL)	[-120 120]	[-120 120]	[25 -120]
Negative Small (NS)	[-120 120]	[-120 120]	[22 -36]
Zero (Z)	[-120 120]	[-120 120]	[6 0]
Positive Small (PS)	[-120 120]	[-120 120]	[22 36]
Positive Large (PL)	[-120 120]	[-120 120]	[25 120]

Table 4.6: ABS control categories

ABS control	Range	Display Range	Params
QRP	[-1.8 1.8]	[-1.8 1.8]	[0.63 -1.797]
SRP	[-1.8 1.8]	[-1.8 1.8]	[0.3 -0.5]
H	[-1.8 1.8]	[-1.8 1.8]	[0.09 0]
SIP	[-1.8 1.8]	[-1.8 1.8]	[0.18 0.63]
QIP	[-1.8 1.8]	[-1.8 1.8]	[0.18 1.8]

4.2 Optimal slip v/s time

The simulation results from the model with fuzzy logic controllers were obtained and then compared with the simulation result of a model that uses a simple bang-bang controller.

The curve of optimal slip with same parameters but using another logic controller which is bang-bang controller came very non-linear as shown in Figure 4.1. We can see that the nature of curve remains somehow linear when brake is applied till 6 seconds but then changes vigorously till 12 seconds and gain a final linearity at 12 seconds. The steerability during this rapid changing of optimal slip is very hard to achieve. To attain an optimal slip with this kind of curve is not recommended for the safe and effective performance of the antilock braking system.

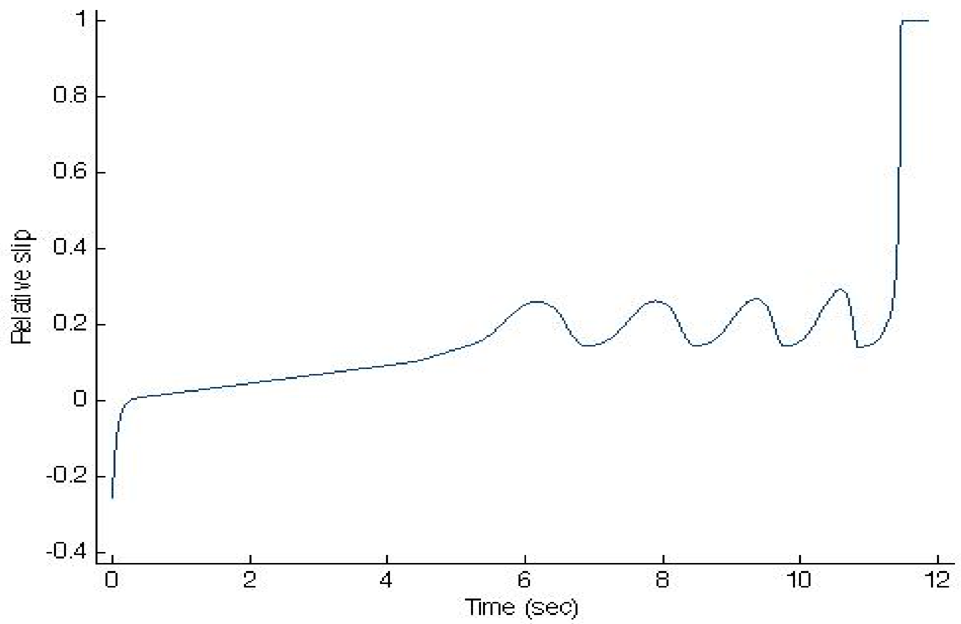

Figure 4.1: Slip v/s Time curve with bang-bang controller

The curve of optimal slip with parameters using fuzzy logic controller came quite linear than that with bang-bang controller as shown in Figure 4.2. We can see that the nature of curve remains somehow linear when brake is applied till 10 seconds and gains its optimal value of slip just after 10 seconds and remains constant till the vehicle stops. The steerability during this period is easy to achieve and hence it is recommended for the safe and effective performance of the antilock braking system.

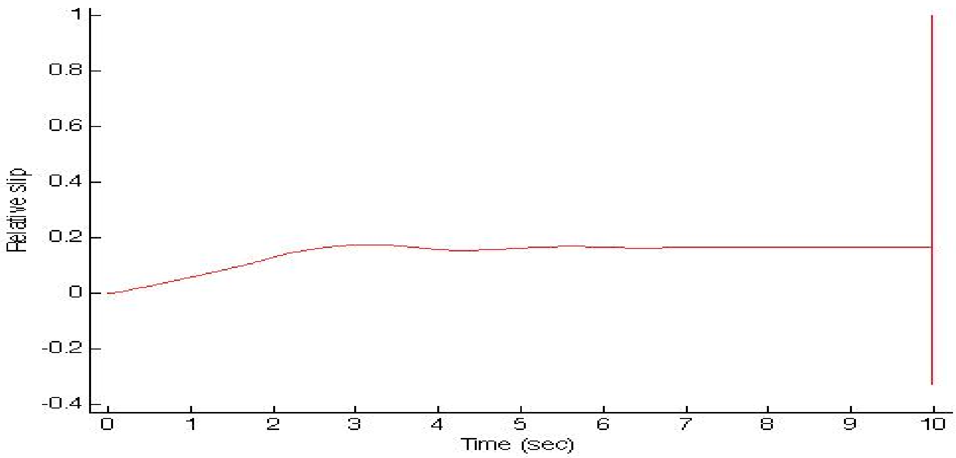

Figure 4.2: Slip v/s Time curve with fuzzy logic controller

We now compare both the results of optimal slip and the time required to attain it with and without fuzzy logic controller as shown in Figure 4.3. It is clearly seen that the model using fuzzy controllers provides much better control over slip. The curve smoothly rises to the optimal value and stays there until the vehicle comes to stop. Better slip brings better control and steerability to the vehicle.

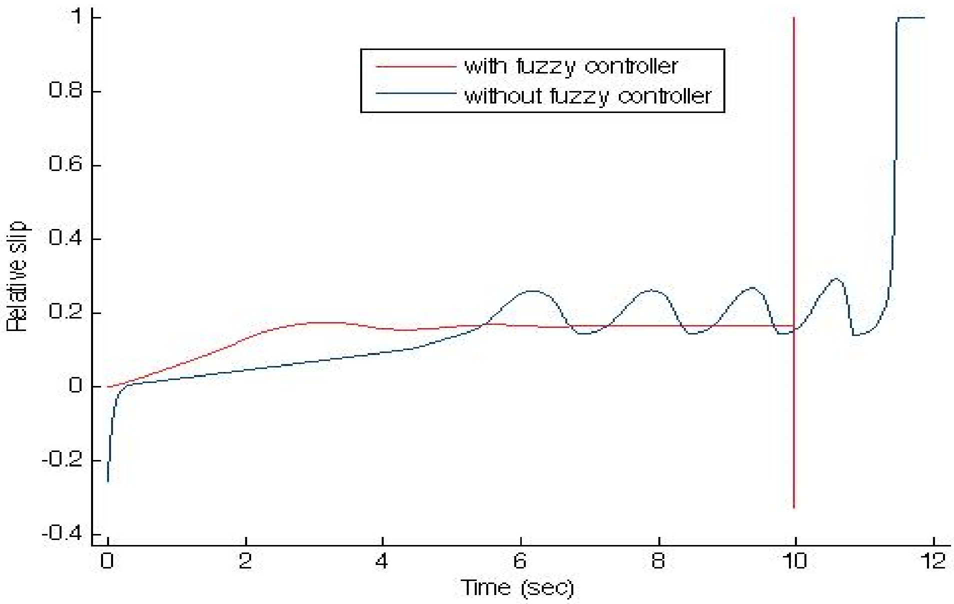

Figure 4.3: Comparison of slip with and without fuzzy controller

4.3 Velocity v/s Time

Figure 4.4 shows the velocity v/s time curve for the model that uses a bang-bang controller. The curve for wheel velocity and vehicle velocity converge to zero after 12 seconds.

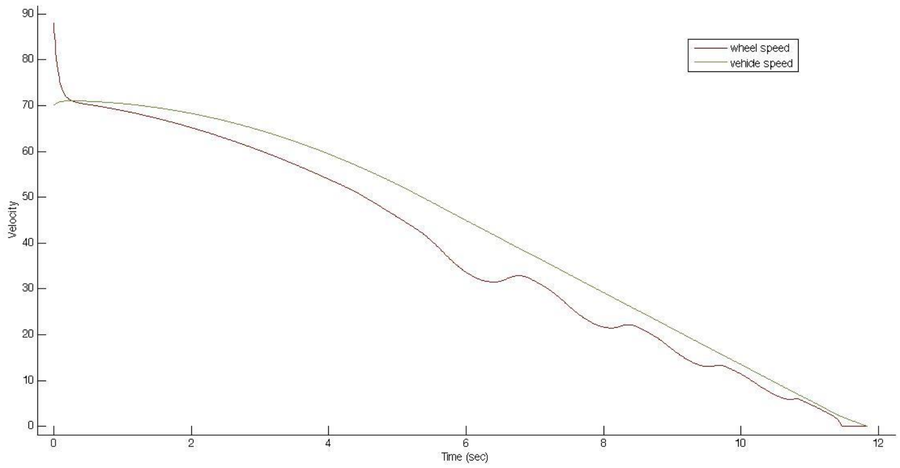

Figure 4.4: Speed v/s time curve for bang-bang controller

Figure 4.5 shows the velocity v/s time curve for the model that uses fuzzy controller. The model that used fuzzy controllers shows improved results in bringing the vehicle to stop with better steerability and control.

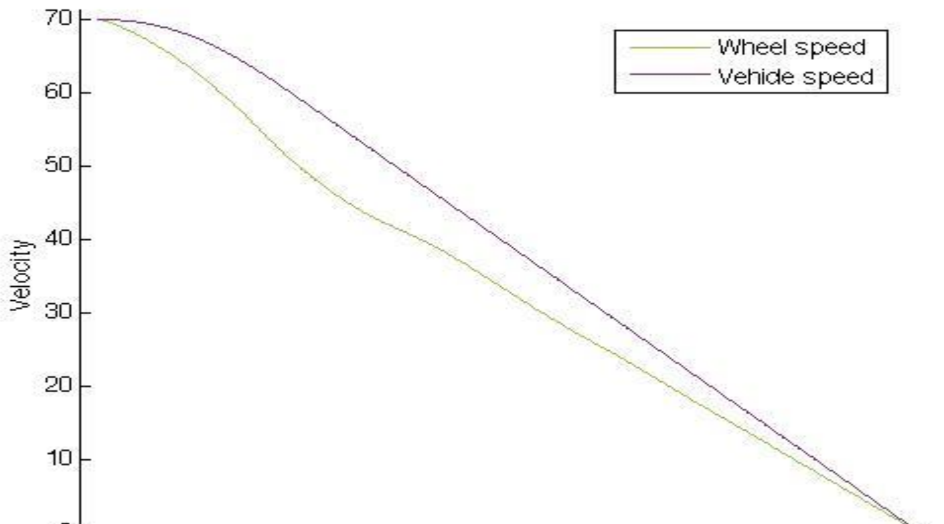

Figure 4.5: Speed v/s Time curve for fuzzy controllers

With the comparison of Figure 4.4 and Figure 4.5, we can clearly see that the model without fuzzy controller takes more time which is near 12 seconds and the model with fuzzy controllers takes less time which is near 10 seconds for the vehicle to stop. Hence, the valuable 2 seconds is achieved for stopping the vehicle when fuzzy controllers are used in the antilock braking system of a vehicle.

CHAPTER FIVE: CONCLUSION AND RECOMMENDATION

5.1 Conclusion

A mathematical model of different components of Antilock braking system such as vehicle dynamics, tire, wheel slip and brake actuator has been developed. MATLAB/Simulink models of tire, wheel slip, brake actuator and two fuzzy logic controllers have also been developed. Input parameters have been given to the first fuzzy controller to obtain optimum slip of the road condition as the output. Input parameters have been given to the second fuzzy controller to get the ABS control as the output. Finally, the concept of how the braking force should be applied in different road conditions has been clarified.

An Anti-lock Braking System (ABS) with fuzzy logic controllers has been created in MATLAB/Simulink and the simulation results has been compared with an ABS system that uses a simple bang-bang controller. The use of fuzzy controller provides better results in the slip control, steerability and stopping distance of the vehicle. The system without fuzzy controllers brings the vehicle to stop after 12 seconds with fluctuations in slip whereas the system with fuzzy controllers brings the vehicle to stop after 10 seconds with stable wheel slip and better steerability. Hence, fuzzy controllers provide better wheel slip control, better steerability and better stopping distance in comparison to a bang-bang controller.

5.2 Recommendations

This research has its own limitations which provide scopes for future studies in the same field. Following are some recommendations that can be incorporated to widen the scope of this thesis work:

- The model can be further modified by adding more inputs to the fuzzy controller. Slope of the road can be added as an input parameter while controlling the brake force applied to the vehicle.
- The comparisons of ABS model using fuzzy logic controller with the ABS models using other controllers such as PID controller, Sliding mode (SM) controller, FOSMC controller, etc. can be done and the results with the difference in time for stopping the vehicle can be analysed.

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APPENDIX A: Program codes for optimum slip

```
[System]
Name='Roadcondition'
Type='mamdani'
Version=2.0
NumInputs=2
NumOutputs=1
NumRules=25
AndMethod='min'
OrMethod='max'
ImpMethod='min'
AggMethod='max'
DefuzzMethod='centroid'

[Input1]
Name='Velocity'
Range=[0 120]
NumMFs=5
MF1='VL':'trimf',[0 10 20]
MF2='L':'trimf',[15 25 35]
MF3='M':'trimf',[30 45 60]
MF4='H':'trimf',[55 75 95]
MF5='VH':'trimf',[89.7 105 120.5]

[Input2]
Name='Slip'
Range=[0 1]
NumMFs=5
MF1='VL':'trimf',[0 0.05 0.1]
MF2='L':'trimf',[0.08 0.125 0.15]
MF3='M':'trimf',[0.145 0.225 0.3]
MF4='H':'trimf',[0.28 0.4 0.5]
MF5='VH':'trimf',[0.4 0.75 1.05]

[Output1]
Name='output1'
Range=[0 1]
NumMFs=4
MF1='Snow':'trimf',[0 0.061 0.1]
MF2='Ice':'trimf',[0.1 0.13 0.16]
MF3='DA':'trimf',[0.15 0.17 0.2]
MF4='DC':'trimf',[0.2 0.4 0.5]

[Rules]
1 1, 2 (1) : 1
1 2, 2 (1) : 1
1 3, 2 (1) : 1
1 4, 1 (1) : 1
1 5, 1 (1) : 1
2 1, 3 (1) : 1
2 2, 2 (1) : 1
2 3, 2 (1) : 1
2 4, 2 (1) : 1
2 5, 1 (1) : 1
3 1, 3 (1) : 1
3 2, 3 (1) : 1
3 3, 2 (1) : 1
3 4, 2 (1) : 1
3 5, 2 (1) : 1
```

```
4 1, 4 (1) : 1
4 2, 3 (1) : 1
4 3, 2 (1) : 1
4 4, 2 (1) : 1
4 5, 2 (1) : 1
5 1, 4 (1) : 1
5 2, 3 (1) : 1
5 3, 3 (1) : 1
5 4, 2 (1) : 1
5 5, 2 (1) : 1
```

APPENDIX B: Program codes for ABS control for braking decision

```
[System]
Name='ABS_control'
Type='mamdani'
Version=2.0
NumInputs=2
NumOutputs=1
NumRules=25
AndMethod='min'
OrMethod='max'
ImpMethod='min'
AggMethod='max'
DefuzzMethod='centroid'

[Input1]
Name='Slip_error'
Range=[-1 1]
NumMFs=5
MF1='NL':'gaussmf',[0.2121 -1]
MF2='NS':'gaussmf',[-0.15 -0.3]
MF3='Z':'gaussmf',[-0.05 0.01]
MF4='PS':'gaussmf',[0.15 0.3]
MF5='PL':'gaussmf',[0.2121 1]

[Input2]
Name='Wheel_accclrn'
Range=[-120 120]
NumMFs=5
MF1='NB':'gaussmf',[25 -120]
MF2='NS':'gaussmf',[22 -36]
```

```
MF3='Z':'gaussmf',[6 0]
MF4='PS':'gaussmf',[22 36]
MF5='PB':'gaussmf',[25 120]

[Output1]
Name='output1'
Range=[-1.8 1.8]
NumMFs=5
MF1='QRP':'gaussmf',[0.63 -1.797]
MF2='SRP':'gaussmf',[0.3 -0.5]
MF3='H':'gaussmf',[0.09 0]
MF4='SIP':'gaussmf',[0.18 0.63]
MF5='QIP':'gaussmf',[0.18 1.8]

[Rules]
1 1, 3 (1) : 1
1 2, 2 (1) : 1
1 3, 2 (1) : 1
1 4, 1 (1) : 1
1 5, 1 (1) : 1
2 1, 4 (1) : 1
2 2, 3 (1) : 1
2 3, 2 (1) : 1
2 4, 1 (1) : 1
2 5, 1 (1) : 1
3 1, 5 (1) : 1
3 2, 4 (1) : 1
3 3, 4 (1) : 1
3 4, 3 (1) : 1
3 5, 3 (1) : 1
4 1, 5 (1) : 1
```

4 2, 5 (1) : 1

4 3, 4 (1) : 1

4 4, 4 (1) : 1

4 5, 3 (1) : 1

5 1, 5 (1) : 1

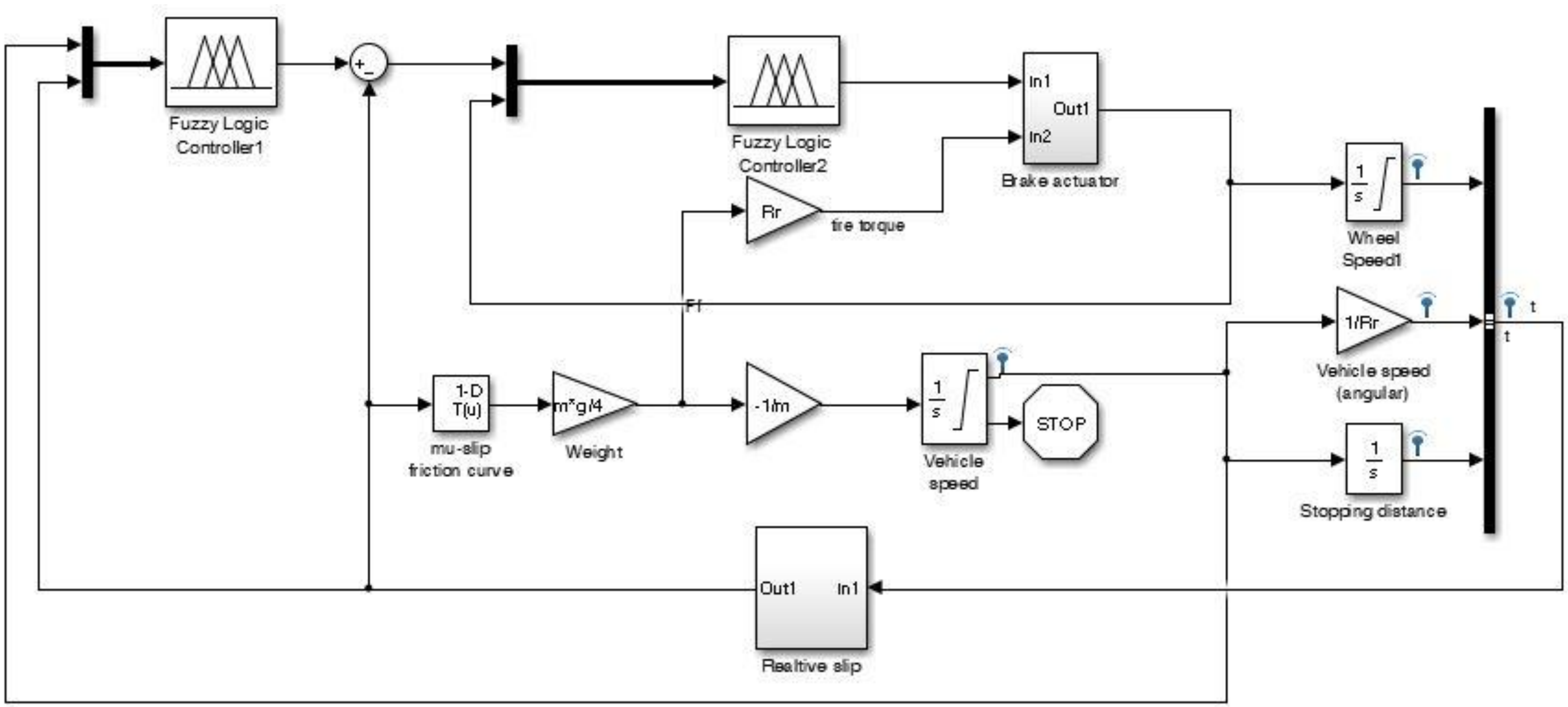
5 2, 5 (1) : 1

5 3, 5 (1) : 1

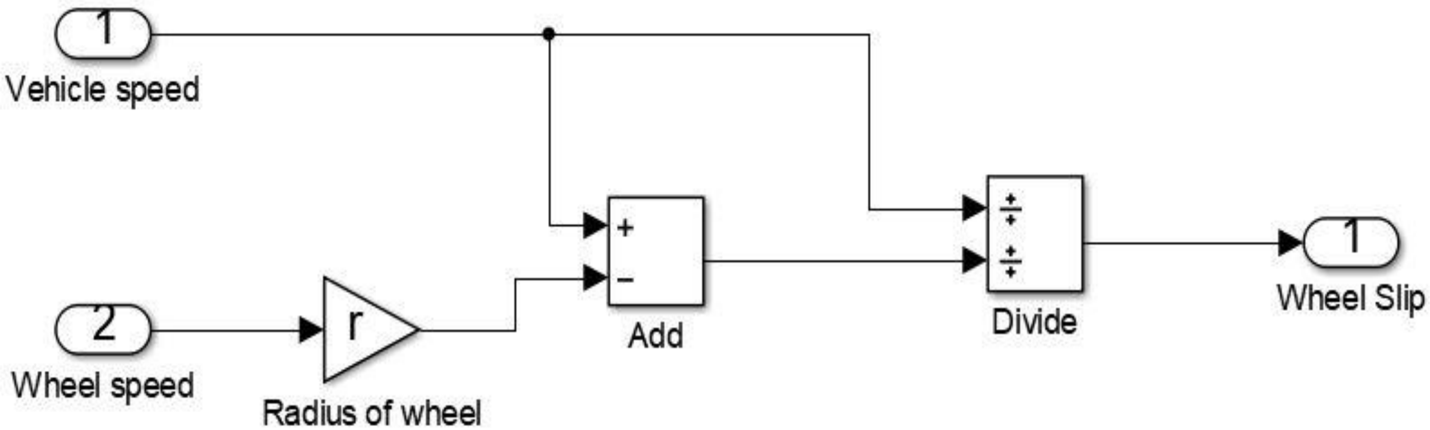
5 4, 4 (1) : 1

5 5, 4 (1) : 1

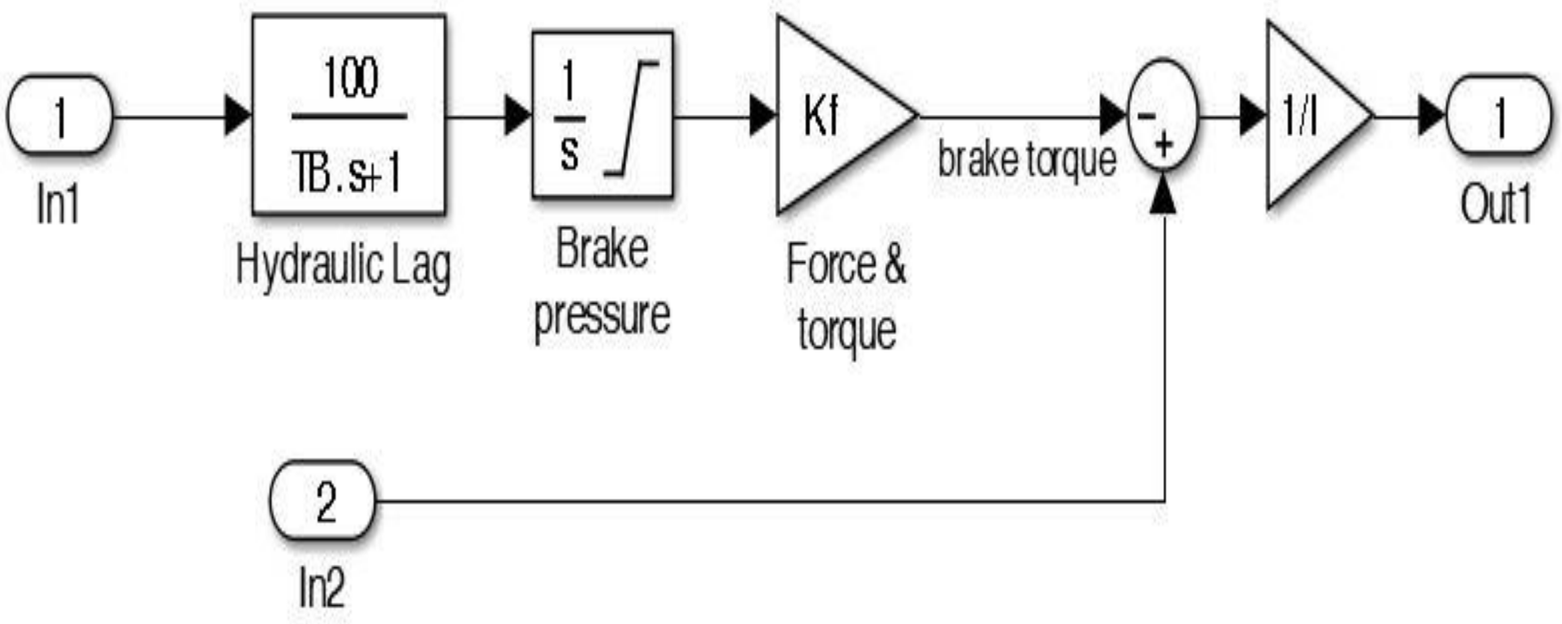
APPENDIX C: ABS Simulink model with fuzzy controllers



APPENDIX D: Simulink model for wheel slip



APPENDIX E: Simulink model for brake actuator



APPENDIX F: ABS Simulink model with bang-bang controller