

Tribhuvan University Institute of Engineering Pulchowk Campus

Thesis No: IC065609

Factors Affecting the Particle Tracking Velocimetry & Approach to the New Particle Tracking Algorithm

Thesis

Submitted to

Masters of Science in Information and Communication Engineering

(MSICE) Committee

(Department of Electronics and Computer Engineering)

Submitted by Pramod Ghimire (065/MSI/609)

Thesis Supervisor Professor. Dr. Shashidhar Ram Joshi

Thesis No: IC065609

Factors Affecting the Particle Tracking Velocimetry & Approach to the new particle Tracking Algorithm

By Pramod Ghimire (065/MSI/609)

Thesis Supervisor Professor. Dr. Shashidhar Ram Joshi

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Information and Communication Engineering.

Department of Electronics and Computer Engineering Institute of Engineering, Pulchowk Campus Lalitpur, Nepal

January 2011

COPYRIGHT ©

The author has agreed that the library, Department of Electronics and Computer Engineering, Institute of Engineering, Pulchowk Campus, may make this thesis freely available for inspection. Moreover the author has agreed that the permission for extensive copying of this thesis for scholarly purpose may be granted by the professor, who supervised the thesis work or, in his absence, by the head of the institute in which the thesis work was done. It is understood that the recognition will be given to the author of thesis and to the Department of Electronics and Computer Engineering, Pulchowk Campus in any use of the material of this thesis. Copying of publication or other use of this thesis for financial gain without approval of the Department of Electronics and Computer Engineering, Institute of Engineering, Pulchowk Campus and author's written permission is prohibited.

Request for permission to copy or to make any use of the material in this thesis in whole or part should be addressed to:

Head of the Department Department of Electronics and Computer Engineering Institute of Engineering Pulchowk Campus Lalitpur Nepal

Tribhuvan University

Institute of Engineering

Pulchowk Campus

DEPARTMENT OF ELECTRONICS AND COMPUTER ENGINEERING RECOMMENDATION

The undersigned certify that the thesis that they had read and recommended to the Department of Electronics and Computer Engineering for the acceptance of the thesis entitled **"Factors Affecting the Particle Tracking Velocimetry & Approach to the new particle Tracking Algorithm"**, submitted by **Pramod Ghimire** in partial fulfillment of the requirement for the award of the degree of "Master of Science in Information and Communication Engineering" as a bonafide work carried out by him.

.....

Dr. Shashidhar Ram Joshi Professor Department of Electronics and Computer Engineering, Institute of Engineering (IOE), Pulchowk, Nepal (Thesis Supervisor)

Mr. Santosh Bhandari Telecom Engineer Nepal Telecom Lalitpur, Nepal (External Examiner)

DEPARTMENTAL ACCEPTANCE

The thesis entitled **"Factors Affecting the Particle Tracking Velocimetry & Approach to the new particle Tracking Algorithm**", submitted by Mr. Pramod Ghimire in partial fulfillment of the requirement for the award of the degree of "Master of Science in Information and Communication Engineering" has been accepted as a bonafide record of work carried out by him in our department.

.....

Dr. Shashidhar Ram Joshi Professor and Head of Department Department of Electronics and Computer Engineering, Institute of Engineering, Pulchowk, Nepal (Head of Department)

TO MY PARENTS

Acknowledgement

I am grateful to Kazuo Ohmi, Christian Cierpka, Pascal Henry Biwole, and M Naito for providing their paper published by IOP publishing Ltd for my thesis work. I am thankful to Sarad Shrestha, MNIT, India, Asshish Sarada, NIT-T, Tamil Nadu, India for providing me with the reference paper published by IEEE, Springer-Verlag and Science Direct. I am very much grateful to the suggestions provided by our M.Sc co-ordinator Sharad Kumar Ghimire regarding the thesis work to be performed. I am thankful to my supervisor Professor Shashidhar Ram Joshi for encouraging me to work on the topic consistently. Feedback from Mr.Saroj Shakya was useful for me to come up with the proper working code. So I would like to thank him. I am thankful to Sanjeeb Prasad Panday and Basanta Joshi for providing different paper used in this work and the useful website link and suggestions for proceeding with my thesis work. I am thankful to Assistant Professor Rajendra Ral Rajbhandari for giving useful suggestions and providing access to the e-mail addresses of some of the alumni of IOE. I am thankful to Sundar Kunwar for his help regarding the flow of the program in C language. I am thankful to Mr. Santosh Bhandari, External Examineer for checking and correcting this report. I sincerely thank all the staff of the department of the electronics and computer engineering who have provided the suitable environment for the conduct of this thesis work.

Pramod Ghimire 065-MSI-609

Abstract

The Particle Tracking Velocimetry (PTV) can be splitted into three sub-processes; the recording of the experiment, the image processing and determination of the velocity components. It is essential that each of the three steps be accurate to determine the actual velocity components. Many factors can affect the end result in the particle tracking velocimetry. These factors are scattered in many papers. This thesis collects all the factors that can affect the velocity components in each three steps. The thesis then deals with the modified relaxation based algorithm for tracking the tracer particles between the two image frames. The effect of the maximum threshold value used in the relaxation method on the matching percent and the time for computation is deducted.

List of Figures

Figure 1-1: PIV system Layout	2
Figure 1-2: Captured Image	2
Figure 1-3: Velocity Map Schematic	2
Figure 2-1: Basic Setup Schematic of PIV system	7
Figure 2-2: The fundamental principle of PIV	
Figure 2-3:Three types of particle tracking velocimetry. (a) low density	PTV (b)
Medium density PIV and (c) High density (LSV)	9
Figure 4-1:Heuristics used (a) Maximum velocity; (b) small velocity cha	nge; (c)
common motion; (d) consistent match	16
Figure 4-2: Maximum threshold Tm and neighborhood threshold Tn	16
Figure 4-3: Quasi-rigidity threshold tq	17
Figure 4-4: Particle tracking routine	
Figure 6-1: Image frame 0	24
Figure 6-2: Image frame 1	24
Figure 6-3: Image frame 0-1	24
Figure 6-4: Neighborhood threshold plotted against the tracer particle	25
Figure 6-5: Velocity Map	25
Figure 6-6: Velocity map with 30 particles being considered	
Figure 6-7: Velocity map with 100 particles being considered	
Figure 6-8: Matching Percent Vs Maximum Threshold Value	
Figure 6-9: CPU time Vs Maximum Threshold Value	

List of Tables

Table 6-1 Particle tracking result	27
Table 9-1: List of standard images	36

List of Abbreviations

CCD PIV	Charge Coupled Device Particle Image Velocimetry
PTV	Particle Tracking Velocimetry
LSV	Laser Speckle Velocimetry
SOM	Self Organizing Map
VSJ	Visualization Society of Japan
3-D	Three dimensional
2-D	Two dimensional
GA	Genetic Algorithm
EPTV	Enhanced Particle Tracking Velocimetry
NRX	New Relaxation Method
ICCRM	Integrated Cross-Correlation Relaxation Method
CFD	Computational Fluid Dynamic

Table of Contents

Ack	nowledgement	vii
Abs	tractv	iii
List	of Figures	ix
List	of Tables	. x
List	of Abbreviations	xi
Tab	le of Contents	xii
1.	Introduction	. 1
1.1	Background	. 1
1.2	Objective	3
1.3	Scope of the work and the applications	3
1.4	Problem Identification	. 5
1.5	Organization of the report	6
2.	Particle Image Velocimetry	7
2.1	Introduction	7
2.2	Principle of particle image velocimetry (PIV)	8
2.3	PTV Versus PIV	8
3.	Factors affecting the particle tracking velocimetry	10
3.1	The Recording of the experiment	10
3.2	The image-processing	12
3.3	The evaluation of velocities	14
4.	Relaxation Method	15
4.1	Introduction	15
4.2	Baek and Lee's Algorithm	15
4.3	Derivation of Formula	16
4.3	Ohmi, Hang and Joshi's Algorithm	19
4.4	New Relaxation Algorithm	19
4.5	Basanta Joshi's Thesis work Review	21
4.6	Recent Approaches	21
4.7	New Approach to Relaxation Algorithm	22
5.	Research Methodology	23
5.1	Image/Data Collection	23

5.2	Algorithm Implementation	. 23
6.	Result and Observations	. 24
6.1	Results	. 24
6.2	Observations	. 28
7.	Conclusions and Future Enhancements	. 31
7.1	Conclusions	. 31
7.2	Future Enhancements	. 31
8.	References	. 32
9.	Appendix	. 36
A.1	PIV Standard Images	. 36
A.2	Standard Images	. 37

1. Introduction

1.1 Background

The Particle Tracking Velocimetry (PTV) is a multi-disciplinary research field. The scope includes the fluid mechanics, image processing, computer vision, nuclear engineering, particle physics, civil engineering, space engineering, bio-medical and much more. The PTV quantifies the displacement of individual dynamic particles between two or more image frames [1]. In the PTV experiments the particles are first extracted from the recorded images and the particles are tracked from one frame to the other frame. The particles can be tracked among the three or four or more frames. But the particle tracking velocimetry where the particles are tracked between only the two image frame has been recently adopted due to the process of simplicity.

The image frames are obtained by recording the PTV experiment. The inaccuracies involved with the seeding of the tracer particles, illumination by the pulsed laser, and the resolution of the camera plays an important deciding factor for the final accuracy of the PTV result.

The recorded image frames are then analyzed and used to extract the particle size and the centroid of the particles in those image frames. The methods used to calculate the size and the position of the particle also plays an important role in the final result. Not only that the time-step involved in considering the two frame images also plays a very important role in deciding the right result at the end.

The next step after calculating the position and number of the particles in the two image frame is to track the path of the particles between the two image frames. The tracer particles may be overlapping in any one of the image. The tracer particle which is not present in the first frame image may appear in the second image frame. Alternatively a tracer particle may be present in the first image frame but it may not be present in the considered second image frame. That is the tracer particle may be lost between the image frames. This is the case of the loss of pair particles. All these types of the considerations have to be considered in the tracking algorithm being used and a lot of paper has been published focusing on this tracking algorithm. The Figure 1-1 [29] shows the basic schematic of the PIV system.



Figure 1-1: PIV system Layout



Figure 1-2: Captured Image



Figure 1-3: Velocity Map Schematic

The flow field seeded with tracer particles is illuminated by the laser sheet as shown in the Figure 1-1. The CCD camera is used to record the two or more no of the image frames for the further processing to determine the velocity information of the fluid flow. The computer hardware associated with the PIV system is not shown in the Figure 1-1. The recorded image is of the nature as shown in Figure 1-2 [1]. This image is processed by the PTV algorithm to

obtain the velocity map as shown in the Figure 1-3 [1]. So the goal of the PIV/PTV system is to observe the velocity map as depicted in Figure 1-3 from the image obtained from the PIV/PTV experimental setup.

1.2 Objective

The thesis has mainly two objectives as follows.

- Collect the different factors that can affect the PIV/PTV result.
- Investigate the approach to the new modified particle tracking algorithm based on relaxation method.

1.3 Scope of the work and the applications

The PTV algorithm for non-uniform and non-spherical particles has been developed in [2]. The application involves the calculation of the sediments that are below the fluid surface and the tracking algorithm can be used to determine the motion of the fish in the ocean water.

The detection of the C-Reactive Protein(CRP) based on the measurement of Brownian motion by micro- particle tracking velocimetry is developed in [3]. The paper is able to use the bio-sensing technique to measure the antigen-antibody interactions. This is the application of the particle tracking velocimetry in the bio-medical field. Another application related to bio-medicine can be referred in [4]. The paper gives the measurement of the individual red blood cell motions under high hematocrit conditions using a confocal micro-PTV system.

The deep water wave acceleration measurement is performed in [5] by the application of the particle tracking technique. The paper has used the Monte Carlo simulations in the measurement. This is the application of the PTV in the field of the oceanography.

[6] gives us some light on the use of the particle tracking velocimetry in the field of the nuclear engineering. The thesis details the three-dimensional transient, experimental study of the bubbly dynamics in the restricted medium.

The use of particle tracking velocimetry in the optic is being done by the paper [7]. This PhD thesis deals on the optical sorting and the manipulating of the microscopic particles with the help of the particle tracking velocimetry technique and the software developed by the author.

The first demonstration of the PIV/PTV method for ice movement through a two river confluence is demonstrated in [8]. The PIV/PTV method is used in a case study of ice discharge in the confluence of the Missouri and Mississippi rivers is done in this paper. The paper says this is the ice jam prone area. So this is the very much diverse field where the particle tracking velocimetry is being used for the first time in the history of the particle tracking velocimetry.

The most interesting application of the particle tracking velocimetry is the one mentioned in [9]. The paper finds the trail of the fire ant (Solenopsisgeminata) with the application of the PTV after the ants are being activated artificially. This shows the application of the PTV algorithms being used in the field of the Zoology.

The particle tracking velocimetry as a convenient and reliable technique for non-invasive measurement of granular flows is demonstrated by the authors in [10]. The granular flows are generally encountered in the materials, metallurgical, cement, fertilizer, chemical, coal, food, and pharmaceuticals companies. And the paper uses the PTV methods in the study of these granular flows. This application is entirely different than the one considered everywhere else.

A simple single camera 3C3D velocity measurement is possible [11] with the particle tracking velocimetry. Normally more than two cameras are used to find the third component of the velocity in the PTV experiment. This paper simplifies this and has a great importance in the PIV/PTV field. The three-dimensional fluid transport study in an electrothermal microvortex (EMV) using the wavefront deformation particle tracking velocimetry is shown in [12] where again a single camera is used to calculate the third component of the velocity vector. [13] illustrate the comparative study of the three types of the PIV/PTV: stereoscopic micro-PIV, wavefront-deformation, or astigmatism, micro-particle tracking velocimetry and the standard micro-PIV. These three paper [11][12][13] try to give the advances in the micro PIV/PTV and the use of the single camera even to measure the third component of the velocity component.

1.4 Problem Identification

The preprocessing algorithm is used for extracting individual particles and identifying their respective centers from the densely distributed particle images and the particle-identifying algorithm enables to quantify the displacement of moving particles between two or more image frame [1].

The Genetic algorithm is used by [14] for the particle matching of the particles between the image frames. The algorithm uses a new fitness function as well as unique genetic operations. The algorithm takes a bit longer time to converge than other algorithm but the algorithm gives the perfect matching result and it also detects the loss of the pair particles.

[15] makes the use of the Self-Organizing Map(SOM) algorithm to organize the vast document collection according to the textual similarities. Laborte in [16] has modified SOM based algorithm given in [15] by Kohonen to implement the correspondence problem encountered in the particle tracking velocimetry experiment. Ohmi in [17] has modified the concept initiated by the Kohonen and implemented by Laborte to accommodate the large density of the tracer particles used in the PTV experiments. And Joshi has further modified the Ohmi's modification in [1] to further level by the use of delta-bar-delta rule. And [18] makes the use of the recurrent neural network for the flow field computation with the help of the PTV algorithm.

The particle tracking algorithm using the velocity gradient tensor(VGT) is dealt in [19]. The algorithm can deal with strong deformations and is applicable to rigidly rotating flow, Couette flow, and expansion flow and compression flow.

The new relaxation method is used by [20] for the particle tracking velocimetry. It uses the two types of probabilities for the matching or the correspondence problem. The match probability and the loss of pair probability. Ohmi and Li has improved the relaxation method implemented by Baek & Lee [21] formulated by Barnard and Thompson[22]. Sang-Joon Lee *et al* [23]has improved the performance of the tracking algorithm by the use of hybrid adaptive scheme which makes the combined use of the PIV technique and the relaxation based tracking algorithm. Mikheev and Zubstov[23] have made the use of particle-size-based

tracking along with the relaxation based algorithm for the PTV. And they have named the new algorithm as EPTV. They have shown it better than the NRX used by Ohmi[20]. Jirka et al [24] have integrated cross-correlation and relaxation algorithms for particle tracking velocimetry. Thesis has been submitted to the department of Electronics and Computer Engineering at IOE by Sanjeeb Prasad Panday [27] and Basanta Joshi [28]. Thesis by Sanjeeb Prasad Panday is based on Genetic Algorithm and thesis by Basanta Joshi is related to the comparison of the SOM based algorithm with the Relaxation based algorithm. One of the references detailed in [28] is the paper by K ohmi, Li Hang Yu and Shashidhar Ram Joshi [30]. It deals with the modification to some parameters involved in [21].

The current thesis work deals with the algorithm based on Relaxation method. The author has found some parameters that can be improved with reference to [30] [28][20][21]. So the main focus of the thesis is to implement the modified algorithm with the standard images provided by [26]. In addition to it, the thesis collects the factors that have effect on the final result. The main paper for this will be [25].

1.5 Organization of the report

The thesis is divided into seven chapters. The first chapter is the introduction about the PIV/PTV. It deals with the background, objective and the scope of the work. The chapter two deals with the basics of the PIV and PTV and the difference between them. The chapter three deals with the different factors that can affect the end result of the PIV/PTV. The fourth chapter deals with the different approaches that have been used in different paper regarding the relaxation based algorithm. Chapter five describes the methodology used in this thesis. It includes the implementation phase of the thesis. Chapter six and chapter seven indicate the result of the thesis and the conclusion. And at the end of the thesis is provided with the references and appendix.

2. Particle Image Velocimetry

2.1 Introduction

The experimental set-up of a typical Particle Image Velocimetry (PIV) consists of three subsystems: Image recording, Image Processing, Post-Image processing. To record the flow of the fluid, the fluid is seeded with the tracer particles. These particles are illuminated by the double pulsed laser. The setup is recorded with the CCD camera. The recorded image or the video is fed to the computer for the image processing. Here the different types of the tracking algorithm can be applied to the captured image sequences and finally the velocity field is determined at the end of the post-image processing. The simple diagrammatic schematic of PIV setup system is depicted in the Figure 2-1 [31].



Figure 2-1: Basic Setup Schematic of PIV system

2.2 Principle of particle image velocimetry (PIV)

The fundamental principle underlying the PIV is quite simple. It measures the displacements X_2 - X_1 and the displacements Y_2 - Y_1 as shown in the Figure 2-2. The displacement should be small if we assume that the tracer particle follows the velocity flow of the fluid and we want to calculate the velocity U and V corresponding to the x-axis and the y-axis.



 $U = \lim_{t2 \to t1} \frac{X2 - X1}{t2 - t1}, V = \lim_{t2 \to t1} \frac{Y2 - Y1}{t2 - t1}$ Figure 2-2: The fundamental principle of PIV

2.3 PTV Versus PIV

Depending on the number of the tracer particles used in the seeding of the flow, PIV is categorized into three types: Particle Tracking Velocimetry (PTV), Particle Image Velocimetry (PIV) and Laser Speckle Velocimetry (LSV). The number of the tracer particles in the PTV is so less (<2000) that it is possible to track each particle in both the frames. In PIV the density of the tracer particles is large (>2000 & <4000) so it is difficult to track each and every particle but it is not impossible to track each of them. In the LSV there is overlapping of the tracer particles (>4000) so it is impossible to track the individual particle. The figure 2-3 shows the difference between them. In short it can be said that PTV can be used in the experiments with low tracer density, PIV can be used in the experiment with medium tracer density whereas the LSV can be used in the case of the high tracer density. Historically the PIV and the LSV techniques have been evolved from the conventional PTV

technique. In PIV and LSV, the velocity information in the interrogation region is obtained from the number of the particles. In LSV the number of the particles is so large that the scattered light looks like a speckle. And PTV can be considered as the low density PIV in one way [31].



Figure 2-3:Three types of particle tracking velocimetry. (a) low density PTV (b) Medium density PIV and (c) High density (LSV)

Tracer particle density refers to the number of the tracer particles used in the PIV experiment divided by the area of the image. And from the Figure 2-3, it is very much easy in distinguishing among the low density PTV, medium density PIV and the high density LSV. And in the current thesis makes the use of the low density image and to some extent medium density image corresponding to PIV.

3. Factors affecting the particle tracking velocimetry

The PTV system can be subdivided into three categories: the recording of the experiment, the image processing and the evaluation of velocities. The different factors that affect the each step of these sub-categorized systems affect the end result of the PTV system. And in PTV, the velocity map of the flow fluid considered is the main end result in any type of the PTV experiment.

3.1 The Recording of the experiment

A typical particle tracking velocimetry (PTV) setup consists of a digital camera with fast access memory in it (CCD camera), a high power laser, optical lens, the tracer particles, fluid under consideration and the computer hardware. The tracer particles are seeded in the fluid under consideration. The laser light is used to illuminate the field under investigation. The laser is pulsed at least twice to capture the reflected light from the tracer particles to record at least two images of the two dimensional flow fields of the tracer particles in the fluid flow. The recorded images are transferred to the computer hardware for the image processing purpose to obtain the velocity field diagram. It is assumed that the tracer particles smoothly follow the motion of the fluid flow without disturbing the velocity field. The affect of all these factors is described in the sub-section to follow.

3.1.1 Tracer Particles:

The tracer particles are also called as the seeding particles. The tracer particles are an important component of the PIV/PTV system. The tracer particles should be able to match the fluid properties reasonably well. If not, the tracer particles will not follow the fluid flow correctly. And if this happens, whatever we get as the result of the PIV/PTV, which will not be able to show the true or actual behaviour of the fluid flow motion. The nature of the tracer particles used by the investigator depends mostly on the nature of the fluid being investigated. Generally for the macro PIV/PTV investigations or experiments, glass beads, polystyrene, oil drops has been used. The most important factor of the tracer particles is that they should be inherently reflective in nature. And it is ideally assumed that the laser light

incident on the fluid flow will reflect off of the tracer particles and that will be scattered towards the CCD camera.

The next important thing related to the tracer particles is the number of the tracer particles being used to seed the flow field. If the number of the tracer particle is more but still the tracer particles can be identified, then we will be dealing with the particle imaging velocimetry (PIV). If the number of the tracer particle is so high that individual tracer particles cannot be distinguished, then we will be dealing with Laser Speckle Velocimetry (LSV). And if the tracer density is low enough that each tracer particles can be identified and tracked separately then in that case we will be dealing with the particle tracing velocimetry (PTV).

Thus it can be concluded that the number of the tracer particles affect the output velocity map. If it is PTV, the velocity map is easy to obtain. If the number of the tracer particles is increased beyond 1000, it is difficult to track each and every particle but with improved algorithm it is still feasible.

3.1.2 Digital Camera

Two or more exposures of laser light are required to accomplish the target objective of the PIV/PTV system. A decade back, it was not possible to record the multiple frames of the images with the analog camera available at that time. All the multiple exposures were recorded in the single frame of the image. That single frame was used to track the particle between the frames to determine the velocity map of the field flow. Auto-correlation had been used for the analysis at that time. And from this result it is not clear about the direction of the flow field. With the advent of the 21st century and the introduction of the high speed CCD camera, now it is possible to record two images or more images in the different frames and that too very fast, it has been easy to work with the multiple frames to observe or derive the velocity map. And cross-correlation technique analysis and other important techniques have been introduced which is better than the auto-correlation technique.

Thus the modern camera does affect in the eventual evaluation of the PTV velocity map.

3.2.3 Laser and Lens

Lasers are dominant in the PIV/PTV experimental setup. The reason behind it is their capability to produce high-power light beams with a very short pulse durations. This gives short exposure times for the each frame recorded in the PIV/PTV experiment. Normally many of the PIV/PTV experiments make the use of Nd:YAG lasers. They emit at 1064 nm wavelengths and at its harmonics. A bandpass filtered output is used to get the green laser light which is visible to human eye at 532nm wavelength. It is better to use the laser source which has the ability to take the images at the very high speed. And it depends on the pulse of the laser light illumination being used.

The spherical and cylindrical lens combination is used to focus the emitted light in the experimental setup. The lens is important for the reason that they are only the device to collect and focus the laser light in the first step. So the final PIV/PTV result is dependent on this factor. Hence the good quality lens should be used for the cameras so that proper focusing and visualization of the tracer particles within the region of interest can be done perfectly.

3.2 The image-processing

The two image frame recorded is then processed by the computer. The correspondence between the tracer particles between the two frames has to be found out. Application of the algorithm to deduce the correspondence requires the initial information about the maximum velocity and the time-step between the two frames being considered. High maximum velocity increases the number of incorrect identifications and small maximum velocity reduces the number of correct identifications [25]. The images obtained are processed to calculate the centroid of the tracer particles in each frames. The particle image centroid error is a function of parameters linked to the particle image, such as the image particle radius, the pixel size, and Gaussian peak location on the CCD array, and to other factors increasing the error, such as the CCD read noise and variation in the pixel intensity[25].

The recorded images are binarized before applying the algorithm to match the tracer particles in the two image frames. Two types of binarization are there; Single threshold binarization and the dynamic threshold binarization. It has been shown by Ohmi in [20] that the dynamic threshold binarization yields the better PTV result. This paper also gives the relationship between the correct matching and the time-step. The paper shows that the correct matching is 89% for the frame 0-1, 95% for the frame 0-2, 92% for the frame 0-3 and 96% for the frame 1-3. So it can be concluded that the less is the time-step, better is the corresponding result and better is the PTV result.

3.3 The evaluation of velocities

The evaluation of the velocity field mostly depends on the type of the particle tracking algorithm being used. The particle tracking algorithm enables to quantify the displacement of the moving tracer particles between two image frames[1]. Different algorithm has been proposed for this purpose like, Genetic Algorithm based PTV[27], Velocity Gradient based PTV[19], Neural Network based PTV[15], Self Organizing Map (SOM) based PTV[1], Swarm Intelligence based PTV based on ant colony optimization [34]. Match probability based PTV is used by Baek and Lee [21], Improved Relaxation method is proposed by Ohmi et al in [20]. Recently many hybrid algorithm integrating the cross-correlation based PTV has been published in many paper. Among them few of them are by Lee's [32], Zubstov [23] and Jirka[24]. Zubstov, Jirka have integrated the cross-correlation with the Relaxation based algorithm and they have implemented the algorithm in Matlab. And in addition to this, these two paper suggest that the best tracking algorithm up to date is the Relaxation based tracking algorithm as implemented by [20]. So in short it can be said that the evaluation of the velocity map does depends on the algorithm used. The details about each algorithm can be referred from the references provided. And in the next section, the thesis deals in depth about the relaxation based particle tracking algorithm.

4. Relaxation Method

4.1 Introduction

The basic concept of the particle tracking based on relaxation method is to search for the most probable link of a reference particle while assuming similar movements of its neighboring tracer particles [30]. The method for tracking particle paths from only two image frames is based on the iterative estimation of the match probability and no-match probability as a measure of the matching degree. The particle tracking routine depends on a statistical calculation of match probabilities and an updating iteration process to determine the most probable particle path [21]. During the iteration process, the correct link probabilities gradually tend to unity whereas the other link probabilities tend to zero.

4.2 Baek and Lee's Algorithm

There are few heuristics used for matching the tracer particles coordinate of the sequential images separated by a small time interval Δt [21]:

4.2.1 Maximum Velocity

If a particle is known to have maximum velocity U_m in the flow field, then it can move at most $U_m\Delta t$ between two images within the time interval Δt . So for a given particle in the first image it is only necessary to search the corresponding particle in the second image within the radius given by this value.

4.2.2 Small Velocity change

The tracer particles possess the finite mass. So the very small velocity is the consequence of this phenomenon. So any abrupt velocity cannot take place in the velocity field.

4.2.3 Common motion

Small number of the tracer particles shows a similar pattern of motion.

4.2.4 Consistent match

Two points cannot match to a single point in the another frame. The small velocity change and common motion heuristics may be combined into a quasi-rigidity condition [21]. Figure 4-1 [21]explains the maximum velocity, small velocity change and quasi-rigidity condition.



Figure 4-1:Heuristics used (a) Maximum velocity; (b) small velocity change; (c) common motion; (d) consistent match

4.3 Derivation of Formula

Let x_i and y_j in Figure 4-2 [21]denote the particles centers on the first frame F^1 and the second frame F^2 . Then according to the first heuristics of the maximum velocity, candidate particles y_j with respect to x_i must be within the maximum displacement threshold T_m .



Figure 4-2: Maximum threshold Tm and neighborhood threshold Tn

 $|\mathrm{d}ij| = |\mathrm{x}i - \mathrm{y}j| < \mathrm{Tm} \ (1)$

 $T_m = U_m \Delta t$ where U_m is the maximum velocity in the flow field and Δt is the time interval between two consecutive image frames. From figure above it can be visualized that only the three points(y_1 , y_2 , y_3) inside the circle of radius of T_m are considered the match point possible.

The Baek and Lee's algorithm for tracking discrete particles from only two image frames is based on the iterative estimation of match probability p_{ij} and no-match probability p_i^* as a measure of the matching degree. p_{ij} is the probability of match between the points y_j and the point x_i . p_i^* represents the no match of the point x_i to any of the point y_j . The following condition has to be satisfied.

 $\sum_{i} pij + p_i^* = 1.....(2)$

The initial initialization of these two types of the match and no-match probabilities is done as follows in the Baek and Lee's algorithm.

 $p_{ij}^{(0)} = p_i^{*(0)} = \frac{1}{N+1}.$ (3)

where N is the number of the possible match of the individual particle in the first frame to the particle in the second frame as defined by the equation (1).

If T_n is the neighborhood threshold, then the particle x_k within this radius shows the similar motion with respect to the individual particle x_i . These neighborhood point satisfy the following equation.

 $|xi - xk| < T_n$(4) From the figure 4-2, {x₂, x₃, x₄, x₅} are the eligible candidate to become the neighbor of the point x₁.



Figure 4-3: Quasi-rigidity threshold tq

The iterative updating procedure adjusts match probabilities for the neighboring points. Match probabilities $p_{kl}^{(n-1)}$ of the neighboring points x_k satisfying the quasi-rigidity criterion are used to iteratively update the match probability $p_{ij}^{(n)}$ of x_i . Match probability is proportional to the neighboring match probabilities where the neighboring match is consistent. The iterative update followed by Baek and Lee is as follows:

 $\tilde{P}_{ij}^{(n)} = A \cdot P_{ij}^{(n-1)} + B \cdot Q_{ij}^{(n-1)}$(6) where (A< 1) and (B> 1) are the constants which affect the speed of the convergence. In their algorithm A=0.3 and B=4.0. The tilde symbol over P_{ij} represents the non-normalized value and the superscript n represents the iteration step number.

The summation ranges to all the values of k and corresponding value l which satisfy the quasi-rigidity condition.



Figure 4-4: Particle tracking routine

Figure 4-4 [21] shows the general example of particle tracking using the concept of the neighboring match probability. This figure also shows the tentative relationship between the different types of the threshold used in the algorithm by the authors.

$$P_{ij}^{(n)} = \frac{\tilde{P}_{ij}^{(n)}}{\sum_{j} \tilde{P}_{ij}^{(n)} + P_{ij}^{*(n-1)}}.$$
(8)

$$P_i^{*(n)} = \frac{P_i^{*(n-1)}}{\sum_j \tilde{P}_{ij}^{(n)} + P_{ij}^{*(n-1)}}.$$
(9)

After the successful termination of the particle tracking routine, correct matches have high probabilities and incorrect matches have very low probabilities. The most probable match point y_j for the object point x_i is selected as one which has the highest match probability value. The velocity vector for the point x_i is calculated using the tracked displacement vector d_{ij} , the time interval Δt .

4.3 Ohmi, Hang and Joshi's Algorithm

[30] uses a modified version of the algorithm then used by Baek and Lee [21]. The following equations are the modified one used in this case of the algorithm.

The no-match probability is defined as follows:

$$\tilde{P}_{i}^{*(n)} = \sum_{m < C} \frac{D \cdot P_{i}^{*(n-1)}}{N}.$$
(11)

And the normalized values are defined as follows:

$$P_{ij}^{(n)} = \frac{\bar{P}_{ij}^{(n)}}{\sum_{j} \bar{P}_{ij}^{(n)} + \bar{P}_{i}^{(n)}}.$$
(12)

$$P_i^{*(n)} = \frac{P_i^{(n)}}{\sum_j \tilde{P}_{ij}^{(n)} + \tilde{P}_i^{(n)}}.$$
(13)

C=0.1 and D=5.0 are the new constants introduced in this modified algorithm. Here m denotes the ratio of the particles satisfying the quasi-rigidity condition to the no of the particles in the neighborhood threshold. T_q is taken as 10-15% of T_m in this algorithm. The paper investigates the performance of the relaxation method of this modified algorithm on the basis of the standard PIV images [26].

4.4 New Relaxation Algorithm

The New Relaxation Method (NRX) implemented by [20] is the upgrade version of the algorithm used by [30] as described in the previous section 4.3. In the new algorithm, the

authors have made the quasi-rigidity constant T_q a variable parameter to incorporate the more turbulent fluid flow. The following equation gives the modified equation:

 $|dij - dkl| < E + F|xi - yj| \dots (14)$

E and F are the new constants introduced in this algorithm and E=1.0-4.0 and F=0.05. Apart from this modification, the algorithm used the dynamic threshold binarization in the binarization process of the recorded images.

4.5 Basanta Joshi's Thesis work Review

In the master's thesis submitted to the Department of Electronics and Communication, Institute of Engineering [28], the author brings about the comparison of the Relaxation method used in [30] and the Modified Self Organizing Map (SOM) used in [1].

4.6 Recent Approaches

Hyoung-Bum Kim and S.J Lee(2002) in [32] have improved the performance of the match probability based algorithm by integrating the cross-correlation method with the original match probability based technique used in [21]. They have introduced a new variable T_i and from the data obtained by the cross-correlation based PIV, the input thresholds in the original algorithm are varied to achieve the better performance.

Pereira and et al (2006) in [33] compares the three different types of the algorithm: the nearest neighbour, the neural network and the relaxation method implemented Baek and Lee [21]. They have demonstrated that the relaxation based algorithm is the robust algorithm among them. And they have shown that by combining the PIV result in PTV, the algorithm can be the fastest of all.

Enhanced-particle tracking velocimetry (EPTV) introduced by Mikheev and Zubtsov (2008) [23] is the particle-size-based tracking along with the relaxation method. EPTV involves tracer particle size as a direct matching indicator to search particle pairs in PTV measurement. In EPTV, initial match probabilities are calculated on the basis of the diameter/total energy reflected to the CCD camera lens and the match probability based on the relaxation method is used to update the initialized probabilities.

Jirka et al (2010) [24] have integrated the cross-correlation and relaxation algorithm for the particle tracking velocimetry. Their integration is different from the previous types of integration in the sense that their proposed algorithm is the two step process. And they have justified their work by saying that their algorithm uses the best of both the techniques. The cross-correlation is used in the low density region and their match probability is initialized to unity. In the high density region, relaxation method is used. And in using relaxation method,

the match probability of the determined tracer particles from the first step is used to update the match probability of the particle in the high density region. So the authors justify the performance of the relaxation based matching algorithm.

4.7 New Approach to Relaxation Algorithm

The recent approaches followed in the article published in the reputed journal on relaxation method are integrating the cross-correlation based PIV result for the simplification of the original relaxation method proposed by Baek and Lee [21]. It has been observed that few papers have tried to make $_{Tq}$ a variable so as to decrease the quasi-rigidity threshold and decrease the convergence of the relaxation algorithm. And the author has felt that no work has been done to check the variability of the neighborhood threshold T_n used in the original algorithm. In Baek and Lee [21], this neighborhood threshold is estimated on the basis of the average distance between the particle centroids in the image frame. The current thesis estimates this parameter on the basis of the average distance between the particle centroids in the image frame within the radius of maximum movement threshold T_m within the first frame of the recorded image. It should be noted that T_m is the search radius in the second frame. The thesis looks into the effect of T_m on the overall pairing of the particles and the final velocity map with respect to the standard output available on the VSJ website link. The main objective of the thesis work is still the correspondence problem.

5. Research Methodology

5.1 Image/Data Collection

The algorithm is tested on the synthetic test images available from the Visualization Society of Japan (VSJ) via internet <u>http://www.piv.jp/image3d/image-e.html</u>. In the present work, the algorithm is applied to the datasets available from the website mentioned above. The images are implemented within the 'PIV-STD project' to provide a common standardized means of evaluating any PIV image-analysis technique [26]. The series #301 derived from a CFD simulation of a turbulent jet impinging on a wall was selected for the synthetic test of the algorithm described in this thesis. The detail about the image is given in the appendix A. This particular series of image has been widely used by [1][20][23][27][28][30]. So this series of image was selected to work on this thesis.

5.2 Algorithm Implementation

1] Datasets for image frame 1 and 2 is taken from a text file obtained from VSJ website.

2] Number of the particles within the maximum threshold in the second frame is calculated.

3] Mean of the distance among these particles is assigned as the value for the neighborhood threshold value.

4] The number of the particles within the radius of this sub search region is found out.

5] These points are used for the update.

6] The velocity map diagram is derived.

6. Result and Observations

6.1 Results

The standard image #301 is used as a test image in this thesis work. The datasets available from the VSJ website is taken as an input to the program. This image has been extensively used by many authors whole over the world. So it is used in this thesis work.



Figure 6-1: Image frame 0



Figure 6-2: Image frame 1



Figure 6-3: Image frame 0-1

Figure 6-1 shows the tracer particles in the first image. Figure 6-2 shows the tracer particles in the second image frame. And the Figure 6-3 gives the image frame 0 and image frame 1

being overlapped. These two particle positions were plotted within the program. The datasets were obtained from the VSJ website.





Figure 6-4: Neighborhood threshold plotted against the tracer particle



Figure 6-5: Velocity Map

Figure 6-4 shows the plot of the neighborhood threshold value obtained as an average of the distance among the tracer particles within the maximum threshold value in the second image frame. And it can be observed from the above graph that the value stabilizes to around 6.5. The maximum value is about 7.9 about the considered tracer particle. Since the maximum threshold velocity value is 10 for the standard image # 301, it is concluded that it is very much suitable that neighborhood threshold value be taken approximately 65% of the maximum threshold velocity value. And the Figure 6-5 indicates the velocity map obtained for the standard image # 301.

Particles	Paired	Paired%	seconds
20	20	100	0.85
30	29	96.66	0.09
40	33	82.5	0.17
50	45	90	0.11
60	52	86.6	0.29
70	64	91.42	0.39
80	69	86.25	0.59
90	78	86.66	0.81
100	83	83	0.78
200	199	99.5	6.57
300	298	99.33	9.97
400	396	99	16.34
500	487	97.4	28.21
600	585	97.5	49.49
700	689	98.42	58.5
800	792	99	70.39
900	884	98.22	95.52
1000	983	98.3	113.08

Table 6-1 Particle tracking result

6.2 Observations



Figure 6-6: Velocity map with 30 particles being considered



Figure 6-7: Velocity map with 100 particles being considered

The Table 6-1 shows the observed result. It shows the relationship among the number of the particles used, maximum threshold value, and the time taken for computation.

The two velocity plot given above in Figure 6-6 and Figure 6-7 show the plot with the number of tracer particles being used equal to 30 and 100. The red arrow indicates the loss of pair particle obtained with the modified algorithm. The green arrow shows the actual plot `obtained from the standard data sets. So wherever there is red arrow, it is assumed that particle is being lost by the run algorithm. In the first case with 30 tracer particles, 29 particles are paired. So the loss of pair particle corresponds to 1 particle. In the second case with 100 particles, 83 particles are matched, so the number of the loss of pair particles is 17 in this case.



Figure 6-8: Matching Percent Vs Maximum Threshold Value



Figure 6-9: CPU time Vs Maximum Threshold Value

Figure 6-8 shows the effect of the maximum threshold value on the matching percent. It is observed that, with the increase of the maximum threshold value, the matching percent increases. The Figure 6-9 shows the effect of the maximum threshold value on the cpu time computation. And thus it was observed that the time computation increases for the increase of the maximum threshold value whereas on the other hand the matching percent increases in parallel.

7. Conclusions and Future Enhancements

7.1 Conclusions

The different factors that affect the end result of the particle tracking velocimetry (PTV) were collected in this thesis work. A modified algorithm based on the relaxation method for tracking the particles was implemented. It was observed that when the value of the maximum threshold was maximum (256), the neighborhood value was equal to the one suggested by Lee. The modified algorithm converges to the original algorithm in this case. The effect of the maximum threshold value on the matching percent and the time computation was deducted in this thesis work. It was observed that the matching percentage increases with the increase in the maximum threshold value and on the other hand the time for computation was increased. So in this thesis work, factors affecting PTV were collected and the effect of maximum threshold value was deducted and its effect was observed.

7.2 Future Enhancements

The thesis works on the coordinates data sets obtained from the VSJ library available from the internet. So the work is of indirect in nature. So the enhancement can be that the image may be considered directly for the computation. The code was run under Matlab platform. So the computation time was a bit more. Future work can incorporate the faster platform like C language.

8. References

- [1] Joshi, Shashidhar Ram, "Improvement of Algorithm in the particle tracking velocimetry using self-organizing maps", Journal of the Institute of Engineering, Vol 7, No 1, pp 1-18.
- [2] H. Salinas Tapia, J. A. Gracia Aragon, D. Moreno Hernandez, B. Barrientos Gracia, "Particle Tracking Velocimetry (PTV) algorithm for non-uniform and non-spherical particles", *IEEE, Proceeding of the Electronics, Robotics and Automotive Mechanics Conference* (CERMA'06).
- [3] Y. J, Fan, C.J, Hsu, C. P. Liu, T.H. Wu, H.J. Sheen, S. Lin, "Detection of C-Reactive Protein Based on measurement of Bownian motion by micro-particle tracking velocimetry," *IEEE, MEMS* 2008, *Tuscon, AZ, USA, January* 13-17,2008.
- [4] Rui Lima, Takuji Ishikawa, Yohsuke Imai, Motohiro Takeda, Shigeo Wada, and Takami Yamaguchi"Measurement of the Individual Red Blood Cell Motions Under High Hematocrit Conditions Using a Confocal Micro-PTV system", Anals of Biomedical Engineering, Vol. 37, No. 8, August 2009 pp, 1546-1559.
- [5] Kuang-An Chang, Edwin A. Cowen and Philip L.-F. Liu," Wave Acceleration Measurement Using PTV Technique", Department of Civil Engineering, Texas A & M University, College Station, TX77843-3136, U.S.A.
- [6] Javier Ortiz-Villafuerte," Three dimensional Experimental Investigation of the Shape and Dynamics of a Rising bubble in Stagnant water with Particle Tracking Velocimetry", PhD Thesis, December 1999.
- [7] Graham Milne," Optical Sorting and Manipulation of Microscopic Particles", *PhD Thesis, March* 2007.
- [8] Robert Etteme, Marian Muste, Anton Kruger, and Joe Zufelt,"Factors Influencing Ice Conveyance at River Confluences," Special report, December 1997.
- [9] Lichuan Gui and John M. Seiner," Application of Image Tracking Algorithm in Fire Ant Motion Experiment," 30th April 2009, Algorithms 2009; doi: 10.3390/a2020735.

- [10] N.Jesuthasan, B. R. Baliga and S. B. Savage," Use of Particle Tracking Velocimetry for Measurements of Granular Flows: Review and Application-Particle Tracking Velocimetry for Granular Flow Measurements," KONA No 24 (2006)
- [11] C Cierpka, R Hain and C J Kahler, "A simple camera 3C3D velocity measurement technique without errors due to depth of correlation and spatial averaging for micro fluidics", *Measurement of Science and Technology* 21(2010) 045401 (13pp), *IOP publishing*.
- [12] Aloke Kumar, Christian Cierpka, Stuart J. Williams, Christian J. Kahler, Steven T. Wereley," 3D3C velocimetry measurement of an electrothermal microvortex using wavefront deformation PTV and a single camera", *Microfluid Nanofluid, Springer Verlag* 2010.
- [13] Christian Cierpka, Massimiliano Rossi, Rodrigo Segura, Francesco Mastrangelo, Christian. J. Kahler," Comparitive study of the uncertainty of stereoscopic micro-PIV, wavefront-deformation micro-PTV, and standard micro-PIV", 15th Int Symp on Applications of Laser Techniques to Fluid Mechanics, Lisbon, Portugal, 05-08 July, 2010.
- [14] Kazuo Ohmi and Sanjeeb Prasad Panday,"Particle Tracking Velocimetry Using the Genetic Algorithm", Journal of Visualization, Vol. 11, No. 1 (2008)1-16, The Visualization Society of Japan.
- [15] Teuvo Kohonen, Samuel Kaski, Krista Lagus, Jarkko Salojarvi, Jukka Honkela, Vesa Paatero, and Antti Saarela," Self Organization of a massive document collection", *IEEE Transaction on neural networks*, Vol. 11, No. 3, May 2000.
- [16] G. Labonte," A new neural network for particle-tracking velocimetry", Experiments in Fluids 26 (1999) 340-346, Springer-Verlag 1999.
- [17] K. Ohmi," Neural network PIV using a self-organizing maps method", Proceedings of PSFVIP-4, June 3-5, 2003, Chamonix, France, F4006.
- [18] M. Knaak, C, Rothlubbers, R. Orglmeister," A Hopfield Neural Network for Flow Field Computation Based on Particle Image Velocimetry/Particle Tracking Velocimetry Image Sequences", *IEEE* 1997.

- [19] M. Ishikawa, Y. Murai, A. Wada, M. Iguchi, K. Okamoto, F. Yamamoto," A novel algorithm for particle tracking velocimetry using the velocity gradient tensor", *Experiments in Fluids* 29 (2000) 519-531, *Springer-Verlag* 2000.
- [20] Kazuo Ohmi and Hang-Yu Li," Particle-tracking Velocimetry with new algorithms," *Meas. Sci. Technol.* 11 (2000) 603-616, *IOP publishing Ltd.*
- [21] S. J. Beak, S. J. Lee"A new two-frame particle tracking algorithm using match probability", Experiments in Fluids 22(1996) 23-32, Springer-Verlag
- [22] Stephen T. Barnard, William B. Thompson,"Disparity Analysis of Images" IEEE Transactions on pattern analysis and machine intelligence, Vol. Pami-2, No. 4, July 1980.
- [23] A V Mikheev and V M Zubstov, "Enhanced particle-tracking velocimetry(EPTV) with a combined two-component pair-matching algorithm", Meas. Sci. Technol. 19 (2008) 085401 (16pp), IOP publishing.
- [24] W. Brevis, Y. Nino, G.H. Jirka "Integrating cross-correlation and relaxation algorithms for particle tracking velocimetry", Exp Fluids, Springer-Verlag 2010.
- [25] P. Veber, J. Dahl, R. Hermansson, "Study of the phenomena affecting the accuracy of a video-based Particle Tracking Velocimetry technique", *Experiments in Fluids* 22 (1997) 482-488, Springer-Verlag.
- [26] K Okamoto, S Nishio, T Saga and T Kobayashi," Standard images for particle-image velocimetry", Meas. Sci. Technol. 11 (2000) 685-691,IOP publishing.
- [27] Panday Sanjeeb Prasad, 2006,"Particle tracking velocimetry using Genetic Algorithm", Master's thesis.
- [28] Joshi Basanta, 2008"Performance of Self Organizing Map PTV on the basis of the PIV Standard Images", Master's thesis.
- [29] <u>http://www.tut.fi/units/me/ener/laitteistot/EFD/PIV.html</u>
- [30] Ohmi, K, Li Hang Yu, Joshi Shashidhar Ram, 1998, "Performance of relaxation method PTV on the basis of the PIV standard images", CD-ROM Proceedings of the VSJ/SPIE (Optical Technology and Image Processing in Fluid, Thermal and combustion Flow) Symposium, AB-122.
- [31] Edward L. Paul, Victor A, Atiemo-Obeng, Suzanne M. Kresta, "Handbook of Industrial mixing: science and practice, Volume 1, pp 207.

- [32] Hyoung-Bun Kim and Sang-Joon Lee," Performance improvement of twoframe particle tracking velocimetry using a hybrid adaptive scheme", Meas. Sci. Technol. 13 (2002) 573-582, Institute of Physics Publishing.
- [33] Francisco Pereira, Heinrich Stuer, Emilio C Graff and Morteza Gharib, " Two-frame 3D particle tracking", Meas. Sci. Technol. 17 (2006) 1680-1692, Institute of Physics Publishing.
- [34] Kazuo Ohmi, Sanjeeb Prasad Panday, Achyut Sapkota," Particle tracking velocimetry with an ant colony optimization algorithm", Exp Fluids (2010) 48:589-605, Springer-Verlag.

9. Appendix

A.1 PIV Standard Images

The PIV Standard Images are provided for the evaluation of PIV techniques.

The PIV technique is roughly classified into two steps, i.e., Visualization and Image Processing. The Image Processing includes image enhancement, particle detection, particle tracking, image cross correlation, velocity averaging and erroneous evaluation. To evaluate the Image Processing technique for PIV, The following standard images are generated.

No	Title
01	2D Wall shear flow/Reference
02	2D Wall shear flow/Large displacement
03	2D Wall shear flow/Small displacement
04	2D Wall shear flow/Dense Particle
05	2D Wall shear flow/Sparse Particle
06	2D Wall shear flow/Constant particle size
07	2D Wall shear flow/Large particle size
08	2D Wall shear flow/Large out-of-plane velocity
21	3D jet impingement/Reference
22	3D jet impingement/Large out-of-plane velocity

Table 9-1: List of standard images

A.2 Standard Images

Transient 3D flow field with slit light sheet(300 series)

- #301 (Jet shear flow)
- Number of particles = 4000
- Maximum particle displacement = 10 pixel/interval
- Maximum out-of-plane displacement = 15% of laser thickness/interval
- Particle diameter = about 5 pixel
- Number of images = 10(A), 51(B) and 145(C)

More about the PIV Standard Image can be found at <u>http://www.piv.jp/image3d/image-e.html</u>.