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# Wavelet Transform Method for the Analysis of ECG Signals for Ambulatory Cases

by

Rita Thapa

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# Wavelet Transform Method for the Analysis of ECG Signals for Ambulatory Cases

by

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Thesis Supervisor

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A thesis submitted in partial fulfillment of the requirements for the Degree of Master of Science in Information and Communication Engineering

Department of Electronics and Computer Engineering

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Lalitpur, Nepal

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# RECOMMENDATION

The undersigned certify that they have read and recommended to the Department of Electronics and Computer Engineering for acceptance, a thesis entitled **"Wavelet Transform Method for the Analysis of ECG Signals for Ambulatory Cases**", submitted by **Rita Thapa** in partial fulfillment of the requirement for the award of the degree of "**Master of Science in Information and Communication Engineering**".

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# DEPARTMENTAL ACCEPTANCE

The thesis entitled **"Wavelet Transform Method for the Analysis of ECG Signals for Ambulatory Cases",** submitted by **Rita Thapa** in partial fulfillment of the requirement for the award of the degree of "**Master of Science in Information and Communication Engineering**" has been accepted as a bonafide record of work independently carried out by her in the department.

-----------------------------------------------------------------------

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# TO MY PARENTS

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# ABSTRACT

The electrocardiograph (ECG) is a common clinical and biomedical tool used for diagnosis of heart patients. The thesis is aimed towards the development of beat detection algorithm with high level of accuracy for ambulatory monitoring of arrhythmia patients. The thesis has been inspired by the need to find an efficient method for ECG signal analysis which is simple and has good accuracy. The initial task for efficient analysis is the removal of effect of noise. It actually involves the use of wavelet filters which extract the required cardiac components by rejecting the background noise and the second task is that of R peak detection. Efficiency of the method is measured in terms of sensitivity and positive predictivity. The development, simulation and the evaluation of the methodology is done in MATLAB environment and the database of MIT-BIH is used for the purpose of the evaluation. The accuracy of the algorithm is evaluated against the MIT-BIH arrhythmia database, giving an average sensitivity of 99.71% and positive predictivity of 99.64% respectively.

**Keywords:** Electrocardiogram, Motion artifacts, Mexican hat wavelet, First derivative of Gaussian, Continuous Wavelet Transform.

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# ABBREVIATIONS

AHA: American Heart Association

AV: Atrio-ventricular

BIH: Beth Israel Hospital

BW: Baseline Wander

CWT: Continuous Wavelet Transform

DC: Direct current

DWT: Discrete Wavelet Transform

ECG: Electrocardiogram

EEG: Electroencephalogram

EMG: Electromyogram

IIR: Infinite Impulse Response

MIT: Massachusetts Institute of Technology

PP: Positive Predictivity

SA: Sino-atrial

Se: Sensitivity

WT: Wavelet Transform

# CHAPTER 1: INTRODUCTION

## 1.1 Background

An electrocardiogram (ECG) is a Cartesian representation of the electrical potential generated by the heart. Since its invention in 1887, it has been an invaluable diagnostic tool for the clinician. Traditionally, the ECG is recorded in a hospital setting, or by an ambulatory device and the analysis is done offline by trained clinical personnel. This restricts the ease in the mobility and the comfort of the patient. Moreover various methods have been developed for this purpose [13]. Continuous monitoring of the ECG signal using a body-worn wireless system for people at increased risk potentially allows detecting anomalies earlier and reducing the hospitalization needs [14].

The development of new advanced techniques of signal processing together with the fast improvement of computational systems have led to the design of various devices and algorithms for the analysis of ECGs. Many methodologies have been developed for this purpose and different techniques have been used and the researches are going on for this problem. Among these techniques are Time Domain methods such as Zero-Crossing [15] and Signal Derivatives [16], Digital Filters [17], Filter Banks [18], Neural Networks [19] and Wavelet Analysis [7][12].

The challenge of beat detection increases in case of ambulatory monitoring of ECG signal as the motion artifacts is considerably higher than in hospital monitoring. Thus, such an algorithm is required which is robust against noise induced by daily life activities and also gives a high level of accuracy. Therefore, the thesis is aimed towards development of the algorithm which is focussed on removal of effect of noise and beat detection with high level of accuracy.

## 1.2 Problem Statements

The major challenge with the ambulatory cases is the occurrence of high level of noises that corrupt the ECG signal and development of the algorithm that removes the effects of noise and detects the beats with high level of accuracy. The major concerns with the method are as follows:

1. Removal of effect of noise.
2. Detection of the QRS complex of the ECG signals, also known as the beat detection.

An algorithm using continuous wavelet transform has been designed that works under reasonable levels of noise due to movement of electrodes. For the performance evaluation of the methodology, MIT-BIH database has been used and compared with the methodologies that already have high level of accuracy (above 99%).

## 1.3 Objectives

There are mainly two objectives:

1. To develop an algorithm to remove effects of noise that occurs in ambulatory monitoring.
2. To develop an algorithm to detect R peaks in an ECG signal with high accuracy using wavelet transform for ambulatory cases.

## 1.4 Applications

The results of this thesis will help develop a system that will be beneficial with the good health and healthy heart of human being. Moreover its applications can be summarized as follows:

1. Continuous monitoring of the ECG signal without hospitalization of heart patients.
2. Development of the system for cardiac monitoring with high level of accuracy.

## 1.5 Organization of the report

The thesis is divided into eight chapters. The first chapter deals with the background, problem statement, objectives and applications. Chapter two gives brief introduction about ECG; it deals with the overview of ECG, it's physiological interpretation, features, noise in ECG and a brief introduction to arrhythmia. Chapter three gives a brief introduction about wavelet transform, its types and its bio-medical applications. This chapter also gives a short introduction about the mother wavelets used in the thesis. Chapter four presents literature review; it deals with cardiac signal analysis, various existing methodologies with their drawbacks and need for new algorithm. The chapter five deals with methodology; this chapter gives description about the new algorithm divided into three sections. The details of evaluation procedure have been discussed in chapter six. Chapter seven shows results and analytical discussion of the result and Chapter eight presents conclusion of the thesis work and future enhancements. The end of thesis is provided with references.

# CHAPTER 2: ELECTROCARDIOGRAM

## 2.1 Overview of ECG

ECG is a trans-thoracic interpretation of the electrical activity of the heart over time captured and externally recorded by skin electrodes. Normal electric impulse is derived from the SA node, located in the upper wall of right atrium. SA node is a group of special muscle cells in heart, capable of generating impulses and contraction of the cardiomyocytes. Cardiomyocytes are the major structure of a heart which builds two atriums and two ventricles.

Each cardiac cell is surrounded by and filled with solutions of Sodium (Na+), Potassium (K+), and Calcium (Ca++). The interior of the cell membrane is considered to be negative with respect to outside during resting conditions. When an electric impulse is generated in the SA node, the interior part becomes positive with respect to the exterior. This change of polarity is called depolarization. After depolarization, the cell comes back to its original state. This phenomenon is called repolarization. The ECG records the electrical signal of the heart as the muscle cells depolarize (contract) and repolarize. The effects of depolarization and repolarization are shown in Table 2.1.

Table 2.1 Electrophysiology

| Action | Effect |
|---|---|
| Depolarization | Shifting of electrolytes across the cell membrane causes change in electric charge of the cell. It results in contraction. |
| Repolarization | Internal negative charge is restored and the cells return to their resting state. |

SA node is also called pacemaker since it is the leader of the heart beat rhythm. It produces 60-100 heart beats (cycles) per minute which is considered a normal heart rate. In case of the Human physiology: primary impulse from the SA node is taken by inter-nodal ways that connect the SA node and AV node, which is placed between ventricles and atriums of a heart. Further, signal goes through His's bundle into ventricle muscle causing ventricular depolarization. The generation of ECG signal is described in detail in section 2.2. The three elements: SA node, AV node and His's bundle are components of the conduction system of a heart as shown in Fig. 2.1.

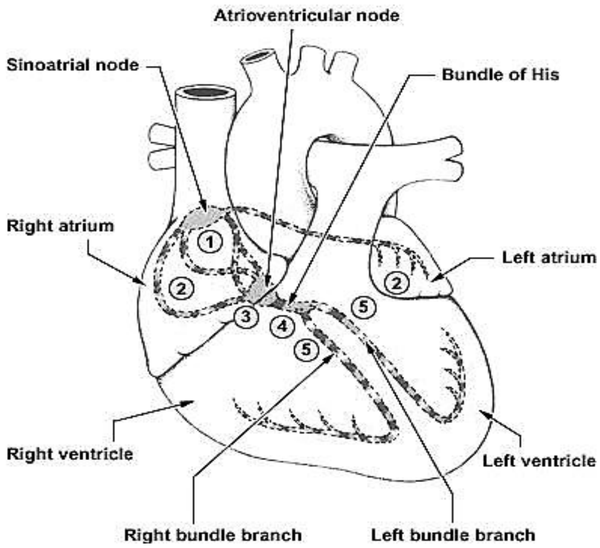



Fig. 2.1 Heart and its conduction system

ECG signal mainly consists of P wave, QRS complex and T wave as shown in Fig. 2.2. P wave shows us the activation of atriums i.e. depolarization. QRS complex shows the ventricles action (depolarization). T wave is a representation of repolarisation of ventricle muscle cells, coming back into stable stage for another contraction.

## 2.2 Physiological Interpretation

An ECG recorded for offline analysis is conventionally written to graph paper, with a horizontal scale of 40ms per division and a vertical scale of 0.1mV per division, with divisions occurring at 1mm intervals. As a depolarized region moves towards an electrode, a positive deflection will be recorded on the ECG, and a negative deflection will be recorded as the regions closest to the electrode become repolarized. For historical reasons, the turning points of a normal ECG are conventionally labeled P, Q, R, S and T (see Fig. 2.2). Some texts also include a U wave, but U is often of very low amplitude or absent altogether. The P wave occurs as the atria are depolarizing, and hence contracting. This typically takes approximately 120ms. Following the P wave, comes the QRS complex. This represents the ventricles depolarizing, and completes in about 100ms. Since the ventricles are much larger cavities than the atria, a larger electrical potential results from their depolarization and the QRS complex is of larger magnitude than the P wave. During the T wave, the ventricles are repolarizing. In the human heart, the repolarization takes place in the direction of the endocardium to epicardium (i.e. in the opposite direction to polarization). Hence, the T wave extends in the same direction as the R peak.

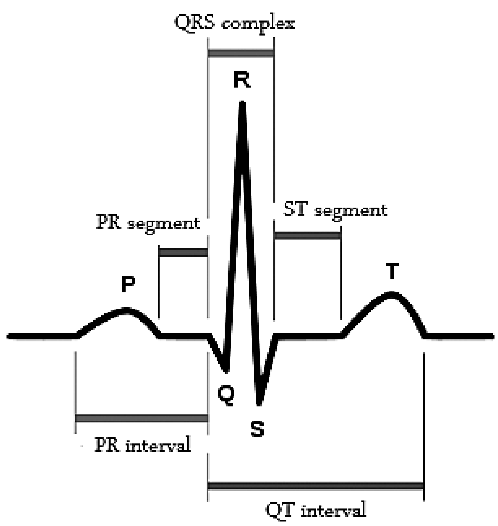



Fig. 2.2 ECG signal

## 2.3 Noise in ECG

Like any other physical signals, ECGs suffer from various forms of noise. Noise can arise from various sources. Some of the important sources of the noise have been discussed herein. The major categories of noise are: low frequency base line wander (BW) caused by respiration, high frequency random noise caused by power line interference (50 or 60Hz) and random shifts of the ECG signal amplitude caused by motion artifacts.

### 2.3.1 Power line interference and other electrical environmental noise

A carefully prepared ECG can significantly reduce the magnitude of this kind of noise. However ECGs recorded during emergency situations may not have the benefit of such careful preparation. Fortunately, this type of noise is generally of a higher frequency ($\geq$ 50Hz) than the components normally of interest in ECG analysis.

### 2.3.2 Motion Artifacts

Motion artifact is the noise introduced to the ECG signal that results from motion of the electrode. More specifically, movement of the electrode or lead wire produces deformations of the skin around the electrode site. The deformations of the skin change the impedance and capacitance of the skin around the sensing electrode. The impedance and capacitance changes are sensed by the ECG electrode and result in artifacts that are manifest as large amplitude signals on the ECG.

The presence of motion artifact may result in misdiagnosis, can prolong procedure duration, and may lead to delayed or inappropriate treatment decisions. This type of noise manifests itself as near or complete saturation lasting for up to one second. This type of noise becomes worse in ambulatory cases. In some applications where multi-channel ECG is used; alternative channels maybe used during periods where this noise is present.

### 2.3.3 Respiration noise

This is caused by the patient's normal respiratory function giving rise to electrical activity in the intercostal muscles. It manifests itself as low frequency "baseline shift" with a frequency of less than 0.4 Hz.

### 2.3.4 Muscle noise

The ECG from any conscious patient will exhibit noise due to muscle contractions. These can be particularly troublesome for ECG analysis since their spectrum and waveform can closely match that of the wanted signal.

## 2.4 Important Features of ECG

A simple example of ECG analysis is the measurement of heart rate. This involves detecting the R peaks and measuring the RR intervals (the time between each ventricular contraction). Normally, this will be identical to the PP intervals, however under pathological conditions, the two may become independent. Hence detection of the P wave and R wave provides useful data. Advanced ECG analysis typically calls for segmentation of each beat into its component waves. The relative durations and amplitudes of each wave are often indicative of certain clinical conditions. For example, an abnormally wide P wave can be a predictor of atrial fibrillation [4].

Similarly, an elevated voltage in the ST segment is commonly associated with acute transmural myocardial ischemia (i.e. loss of blood supply to the cardiac muscles) [6]. Other studies have used the relative positions and magnitudes of the segmented components in stochastic learning tools to perform beat classification [5]. Con-temporary research, thus frequently requires detection not only of the peaks, but also an accurate location of the onset and offset of each “complex”, and the points delimiting them. These are generally known as the fiducial points. There is no consensus as to which points are the most useful in ECG analysis, but many studies have been concerned with the peaks of P, Q, R, S and T as well as the onset and offset of P and T [7][8][9].

## 2.5 Arrhythmia

For a normal healthy person the ECG comes off as a nearly periodic signal with depolarization followed by repolarization at equal intervals. However, sometimes this rhythm becomes irregular.

Cardiac arrhythmia (also dysrhythmia) is a term for any of a large and heterogeneous group of conditions in which there is abnormal electrical activity in the heart. The heart beat may be too fast or too slow, and may be regular or irregular. Arrhythmia comes in varieties. It may be described as a flutter in chest or sometimes “racing heart”. The diagnosis of arrhythmia requires electrocardiogram. By studying ECG, doctors can diagnose the disease and prescribe the required medications.

# CHAPTER 3: WAVELET TRANSFORM

## 3.1 Introduction

A wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components. Usually one can assign a frequency range to each scale component. Each scale component can then be studied with a resolution that matches its scale. A wavelet transform is the representation of a function by wavelets.

Wavelet is a wave-like oscillation with an amplitude that starts out at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one might see recorded by a seismograph or heart monitor. Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. Wavelets can be combined, using a "reverse, shift, multiply and sum" technique called convolution, with portions of an unknown signal to extract information from the unknown signal. Such a process is known as wavelet transform.

## 3.2 Classification

Wavelet transforms are classified into discrete wavelet transforms (DWTs) and continuous wavelet transforms (CWTs). They can be used to represent continuous-time (analog) signals. CWTs operate over every possible scale and translation whereas DWTs use a specific subset of scale and translation values or representation grid.

### 3.2.1 Continuous Wavelet Transform

The continuous wavelet transform (CWT) is a time–frequency analysis method which differs from the more traditional short time Fourier transform (STFT) by allowing arbitrarily high localization in time of high frequency signal features. The CWT does this by having a variable window width, which is related to the scale of observation—a flexibility that allows for the isolation of the high frequency features. Another important distinction from the STFT is that the CWT is not limited to using sinusoidal analyzing functions. Rather, a large selection of localized waveforms can be employed as long as they satisfy predefined mathematical criteria (described below). The wavelet transform of a continuous time signal, $x(t)$, is defined as:

$$T(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} x(t)\, \psi * \left(\frac{t-b}{a}\right) dt \tag{3.1}$$

Where, $\psi*(t)$ is the complex conjugate of the analyzing wavelet function $\psi(t)$, $a$ is the dilation parameter of the wavelet and $b$ is the location parameter of the wavelet.

### 3.2.2 Discrete Wavelet Transform

In its most common form, the DWT employs a dyadic grid (integer power of two scaling in $a$ and $b$) and orthonormal wavelet basis functions and exhibits zero redundancy. (Actually, the transform integral remains continuous for the DWT but is determined only on a discretized grid of $a$ scales and $b$ locations. In practice, the input signal is treated as an initial wavelet approximation to the underlying continuous signal from which, using a multiresolution algorithm, the wavelet transform and inverse transform can be computed discretely, quickly and without loss of signal information.) A natural way to sample the parameters $a$ and $b$ is to use a logarithmic discretization of the $a$ scale and link this, in turn, to the size of steps taken between $b$ locations. To link $b$ to $a$ we move in discrete steps to each location $b$, which are proportional to the $a$ scale. This kind of discretization of the wavelet has the form

$$\psi m,n(t) \;=\; \frac{1}{\sqrt{a_0^m}}\varphi\left(\frac{t-nb_0a_0^m}{a_0^m}\right) \tag{3.2}$$

where the integers $m$ and $n$ control the wavelet dilation and translation respectively; $a_0$ is a specified fixed dilation step parameter set at a value greater than 1, and $b_0$ is the location parameter which must be greater than zero. A common choice for discrete wavelet parameters $a_0$ and $b_0$ are 2 and 1 respectively [21].

## 3.3 Mother Wavelet

The wavelet analysis is thus performed using a prototype function called the wavelet base, $\varphi(t)$ ($\varphi(t) \in$ L2, i.e. finite energy functions). The main characteristic of the wavelet base is given by

$$\int_{-\infty}^{\infty} \varphi(t)dt = 0 \tag{3.3}$$

This means that the wavelet base is oscillatory and has zero mean value. Also, this function needs to satisfy the admissibility condition so that the original signal can be reconstructed by the inverse Wavelet Transform.

$$\int_{-\infty}^{\infty} \frac{|\varphi(\omega)|^2}{|\omega|} d\omega = C_\varphi < \infty \tag{3.4}$$

The admissible condition implies that the Fourier transform of the wavelet must have a zero component at zero frequency. Hence, the wavelet transforms are inherently band-pass filters in the Fourier domain, defined as Wavelet filters. Any function that has finite energy is square integrable and satisfies the wavelet admissibility condition can be a wavelet [20].

The mother wavelets used in the algorithm are first derivative of Gaussian and Mexican hat wavelet which have been described in detail below:

### 3.3.1 First Derivative of Gaussian

Gaussian function (named after Carl Friedrich Gauss) is a function of the form:

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{3.5}$$

Where, μ is the mean, $\sigma$ is the standard deviation and $\sigma^2$ is known as variance. Equation 3.5 can be written as:

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}} \tag{3.6}$$

For some real constants *a*, *b*, *c* > 0, and e ≈ 2.71828... (Euler's number).

The graph of a Gaussian is a characteristic symmetric "bell curve" shape that quickly falls off towards plus/minus infinity. The parameter '*a*' is the height of the curve's peak, '*b*' is the position of the centre of the peak, and '*c*' controls the width of the "bell".

Suppose, $t = \frac{x-\mu}{\sigma}$ (3.7)

The first-derivative of the Gaussian is shown in Fig. 3.1 which is given by,

$$\psi(t) = te^{-t^2} \tag{3.8}$$

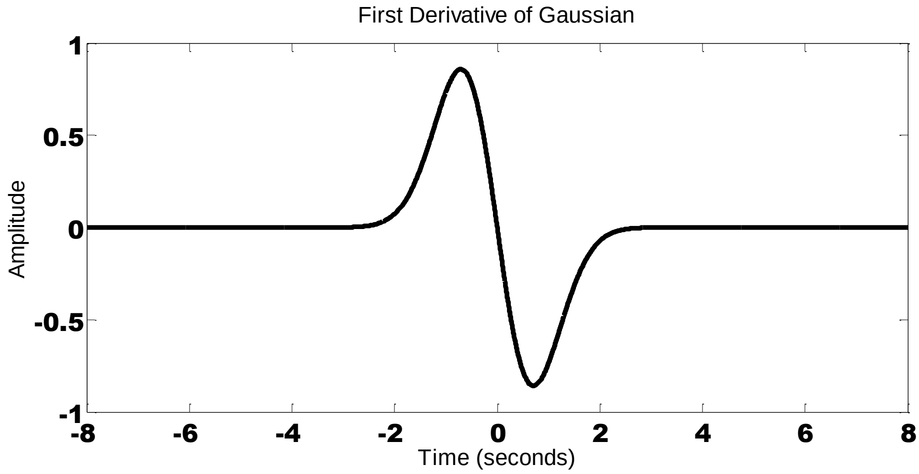



Fig. 3.1 First derivative of Gaussian

### 3.3.2 Second Derivative of Gaussian/ Mexican Hat Wavelet

The Mexican hat wavelet is the second derivative of a Gaussian function given by,

$$\psi(t) = (1 - t^2)\, e^{-t^2} \tag{3.9}$$

This wavelet, shown in Fig. 3.2 , has been used in practice for a number of data analysis tasks in science and engineering including: the morphological characterization of engineering surfaces, the interrogation of laser-induced ultrasonic signals used to measure stiffness coefficients in a viscoelastic composite material and the analysis of turbulent flows. In addition, the Mexican hat is used extensively in studies requiring the use of modulus maxima methods as its maxima lines (and those of all other derivatives of Gaussian functions) are guaranteed continuous across scales for singularities in the signal.

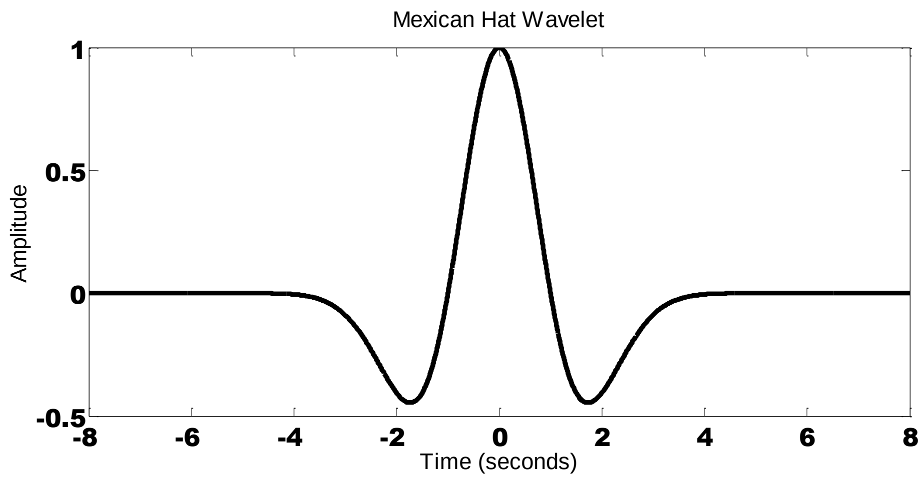



Fig. 3.2 Mexican hat wavelet

## 3.4 Biomedical applications of the wavelet transform

Physiological signals are mostly non-stationary, such as the electrocardiogram (ECG), the electroencephalogram (EEG) and the electromyogram (EMG). Those signals represent the electrical activity of the heart, the brain and the muscles, respectively. The main difficulty in dealing with biomedical signal processing is the extreme variability of the signals and the necessity to operate on a case by case basis [22]. The Wavelet transform (WT) has been extensively used in biomedical signal processing, mainly due to the versatility of the wavelet tools. The WT has been shown to be a very efficient tool for local analysis of non-stationary and fast transient signals due to its good estimation of time and frequency (scale) localizations [23].

# CHAPTER 4: LITERATURE REVIEW

## 4.1 Cardiac Signal Analysis

Many new approaches to cardiac signal analysis can be found in the literature; such as algorithms based on filter banks [18], neural networks [19], non-linear transformations [17] and the wavelet transform [7][10]. In Fig. 4.1, the number of publications in the IEEE online database related to electrocardiogram (ECG) signal detection for three different types of algorithms, being filter-based, wavelet transform and neural networks are shown. Besides the fact that wavelet analysis is still relatively new, the wavelet-based signal processing methods have been evolving very rapidly and the rate of publication keeps increasing steadily.

There are a number of methods that can be found in the literature which are being used so far for the analysis of the ECG signals.

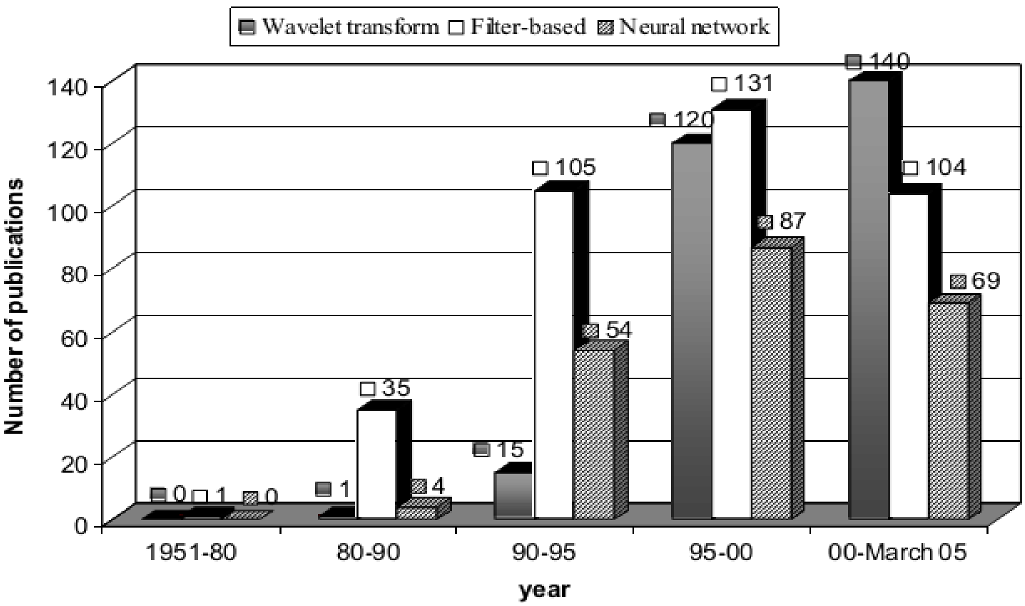



Fig. 4.1 IEEE online database publications of algorithms for cardiac signal detection

## 4.2 Existing Methodologies and their drawbacks

### 4.2.1 Pan – Tompkins Algorithm

In the eighties, J. Pan and J.W. Tompkins have developed an algorithm that found a well-balanced trade-off between detection performance and computational complexity, which was a very important design parameter at that time [10]. The Pan-Tompkins Algorithm is represented in terms of block diagram which is shown in Fig. 4.2.

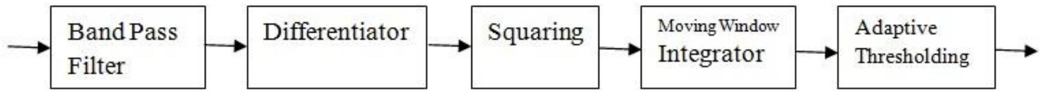



Fig. 4.2 Block Diagram of Pan-Tompkins Algorithm

The components in the block diagram of Pan-Tompkins algorithm has been described below:

#### 4.2.1.1 Band pass Filter

The band pass filter for the QRS detection algorithm reduces noise in the ECG signal by matching the spectrum of the average QRS complex. This attenuates noise due to muscle noise, power line interference, baseline wander, T wave interference. The pass band that maximizes the QRS energy is in the 5Hz-35Hz range. The filter implemented in this algorithm is composed of cascaded high pass and low pass Butterworth IIR filters.

#### 4.2.1.2 Differentiator

Differentiation is a standard technique for finding the high slopes that normally distinguish the QRS complexes from other ECG waves. The derivative procedure suppresses the low frequency components of P and T waves, and provides a large gain to the high-frequency components arising from the high slopes of the QRS Complex.

#### 4.2.1.3 Squaring

The squaring operation makes the result positive and emphasizes large differences resulting from QRS complexes; the small differences arising from P and T waves are suppressed. The high frequency components in the signal related to the QRS complex are further enhanced. This is a non-linear transformation that consists of point by point squaring of the signal samples.

#### 4.2.1.4 Moving Window Integrator

The squared waveform passes through a moving window integrator with window of 150 ms duration. This integrator sums the area under the squared waveform over a suitable interval, advances one sample interval, and integrates the new predefined interval window. This helps to select features which have both large slope and width, reducing false detections caused by spikes.

#### 4.2.1.5 Adaptive Thresholding

Adaptive thresholding is the process of smartly updating thresholds and estimating the time where the coming beat is about to occur. This makes this algorithm robust against noise and even against data loss. An adaptive dual threshold is applied both to the output of filter and integrator. A QRS complex is considered to be present, if and only if the signal exceeds the upper threshold in both cases. If a QRS is not detected within 1.66 times the running average of the RR period, then the thresholding stage is applied again, but using the lower of the two thresholds.

The Pan-Tompkins detector suffers a number of problems which are listed below:

1. The algorithm tends to misidentify T waves as QRS complexes. So it is necessary to compare the slope of the detected feature with that of the previously detected QRS complex. If the slope is less than one half, then it is declared to be a T wave and discarded.

2. Any QRS occurring within 200ms of the previous one is not detected. Further, the dual thresholds are lowered, depending upon the time from the previous detection.

3. The complex nature of the algorithm involves many parameters, e.g. the ratio between upper and lower threshold, the period over which the first threshold stage applies, the size of the moving window etc. The authors present results based on empirical optimizations of these parameters. Whilst the results seem impressive, it cannot be determined to what extent the parameters have been over-optimized to suit the test data.

These problems mean that abrupt changes in rhythm or ectopic beats can be missed or delayed. It also means that the output lags the input by 200ms. In general, noise, a very irregular heart rhythm and sudden changes in amplitude of the peak might all lead to this problematic behavior. More details on this can be found in [11].

### 4.2.2 Discrete Wavelet Transform method

Algorithms based on DWT use the discrete wavelet transform to analyze band-pass filtered versions of the signal on a dyadic scale. When a suitable mother wavelet is chosen, the filtered signals will have sharp peaks at points where there is occurrence of beat. Nimmala proposes such an algorithm in [10], where the redundant algorithm is proposed to yield time-invariant behavior and the use of this for the beat detection. The first derivative of Gaussian is used as mother wavelet and a multi-scale analysis is used for robust peak detection in which scales $2^1 – 2^4$ are used for peak detection (at a sampling frequency of 250 Hz). The

DWT algorithm is quite sensitive to noise, which is relatively often detected as a peak. DWT misses a beat too because the actual beat is not present or has a very odd shape. This can be caused by packet losses or distortion in the signal due to movement. The algorithm is good with its sensitivity but is weak in terms of positive predictivity. This method gives large number of false positives.

### 4.2.3 Band Power Method

The use of the Band Power Method along with the defined system can give good result with the use of very less power but the algorithm is considered to be weak in terms of its accuracy. The flowchart for the band power based beat detection algorithm is shown in Fig. 4.3. It shows a detailed threshold calculation used in the method.

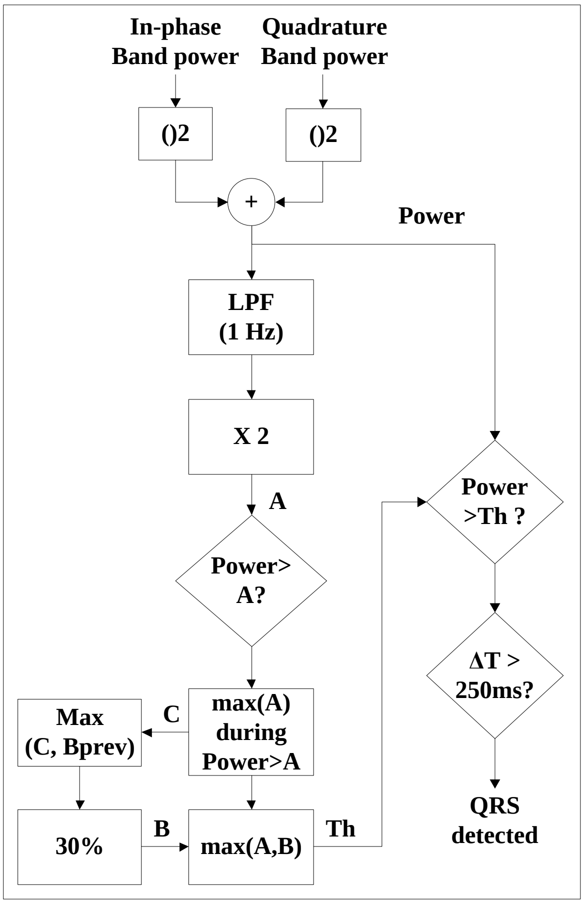



Fig. 4.3 Flowchart for Band power based beat detection algorithm

The band power based beat detection algorithm is described below:

1. The analog preprocessor ASIC provides two band power channels in-phase band power and quadrature band power which gives both magnitude and phase information. The algorithm takes only magnitude component and takes the sum of squares of the in-phase and quadrature components after removing DC component from the signals.
2. The result is then compared with an adaptive threshold to detect the presence of a QRS complex. Two peaks detected within a 250 ms interval are considered to be a part of a single QRS complex.
3. The exact R peak location is found using a time domain peak search on the absolute value of the ECG signal after subtraction of its DC component.
4. A search window of size 200 ms is used around the band power threshold crossing.
5. The adaptive threshold is calculated based on a 1Hz low pass filtered version of the band power magnitude limited to a minimum value of 30 % of the peak value.

More Details on this algorithm can be found in [1] and [2].

### 4.2.4 Romero Algorithm

Romero et al. [12] developed an algorithm based on the Modulus Maxima calculated from the CWT. The algorithm gives good accuracy but consumes more power as compared to the band power based beat detection algorithm.

The algorithm is described below:

1. Perform CWT (using as mother wavelet the second derivative of the Gaussian function or Mexican hat) on the input data, in selected scales. The scales considered were those corresponding to the frequency band of 15 to 18 Hz because those were found to be optimal for QRS detection.

2. Extract the Modulus Maxima of the CWT computed.
3. Square the values obtained.
4. Search the maximum value and determine the threshold as a percentage of the maximum.
5. Each value above the threshold within the scales considered (regardless in which scale) is regarded as a peak. Organize them so that only unique peaks are found. All modulus maxima found within an interval of 0.25 seconds of each other are then inspected in turn and the point with the maximum value is selected as the fiducial point.
6. For each detected beat, search in time domain for the actual maximum of the absolute value in a small area around the detection point (200 milliseconds).

## 4.3 Need for new algorithm

Pan-Tompkins algorithm implements adaptive thresholding method in which thresholds are smartly updated and the time where the coming beat is about to occur is estimated. This process makes this algorithm robust against noise and even against data loss. However, if the estimated temporal heart rate is false due to a burst of noise (the estimated heart rate will then become too high), it can get stuck, because it does not find a beat within the expected time window, hence it will search back, and might possibly detect a T-wave resulting in lowering the threshold for the R-peak and shortening the expected beat time.

DWT algorithm is quite sensitive to noise. This algorithm detects noise signal as peak. Therefore, this method has relatively large number of false positives. DWT misses a beat too because the actual beat is not present or has a very odd shape. This can be caused by packet losses or distortion in the signal due to movement. The algorithm is good with its sensitivity but is weak in terms of positive predictivity. The use of the Band Power Method along with the defined system can give good result with the use of very less power but the algorithm is considered to be weak in terms of its accuracy.

Romero algorithm uses CWT for R peak detection. This algorithm gives good accuracy but consumes more power as compared to the band power based algorithm. In the study described by the authors they concluded that the frequency band that best matches with the QRS complex energy is the range of 15 - 18 Hz. This range has been concluded to be the best frequency band after the analysis of different types of QRS morphology. Tests performed in real signals also proved this frequency band to be successful in differentiating QRS complex from other components within the ECG. This implementation ignores all beats that occur within a time window of 200ms of the detected beat [12]. Several tests performed by the authors of the paper also concluded that the optimal threshold is of 30% from the maximum. This algorithm performs continuous wavelet transform on ECG signal and Mexican hat wavelet is used as mother wavelet.

The thesis is focussed on the development of the algorithm that removes the effect of noise and detects location of R peaks with high level of accuracy. The algorithm performs continuous wavelet transform separately on the input ECG signal using mother wavelets as first derivative of Gaussian and Mexican hat wavelet and computes sum of the resulting CWT coefficients. The new algorithm gives better accuracy and robustness to noise than using Mexican hat alone. Certain modifications have been done in the Romero algorithm to give new improved algorithm which is described in detail in chapter 5.

# CHAPTER 5: METHODOLOGY

The thesis is focussed on the development of the algorithm which is robust to noise and detects R peaks with high level of accuracy**.** The algorithm performs continuous wavelet transform on input ECG signal. First derivative of Gaussian and Mexican hat wavelet are used as mother wavelets to perform CWT. The algorithm was first implemented using first derivative of Gaussian only. Large number of false peaks was detected. The algorithm was then implemented using Mexican hat wavelet. Number of false peaks reduced; however increasing the number of missed detections. Thus, the algorithm named as Combinatory CWT algorithm was derived which combines useful features of both first derivative of Gaussian and Mexican hat wavelet to give best results.

The Combinatory CWT algorithm can be broadly classified into following three sections:

## 5.1 CWT computation for R peak detection

The detailed process for computation of continuous wavelet transform of the input ECG signal for R peak detection is described by the following steps:

1. Perform continuous wavelet transform separately on the input ECG signal using mother wavelets as first derivative of Gaussian function and second derivative of Gaussian function (Mexican hat wavelet).
2. Compute absolute values of the CWT coefficients obtained.
3. Add the coefficients.
4. Calculate maximum value of the result obtained from step 3. Calculate threshold recursively. The threshold calculation method is described in detail in section 5.2.
5. Compare signal obtained from step 3 with the threshold obtained from step 4 to detect the presence of R peak.

6. The time duration at which the peak is detected after the last detection of peak is calculated.
7. If the duration is greater than 250 ms, the peak detected is considered to be a new peak and stored. Otherwise, it is compared with the previous maximum value of peak and stored along with its location value if it is greater among the two.

The flowchart for the CWT computation for R peak detection is shown in Fig. 5.1

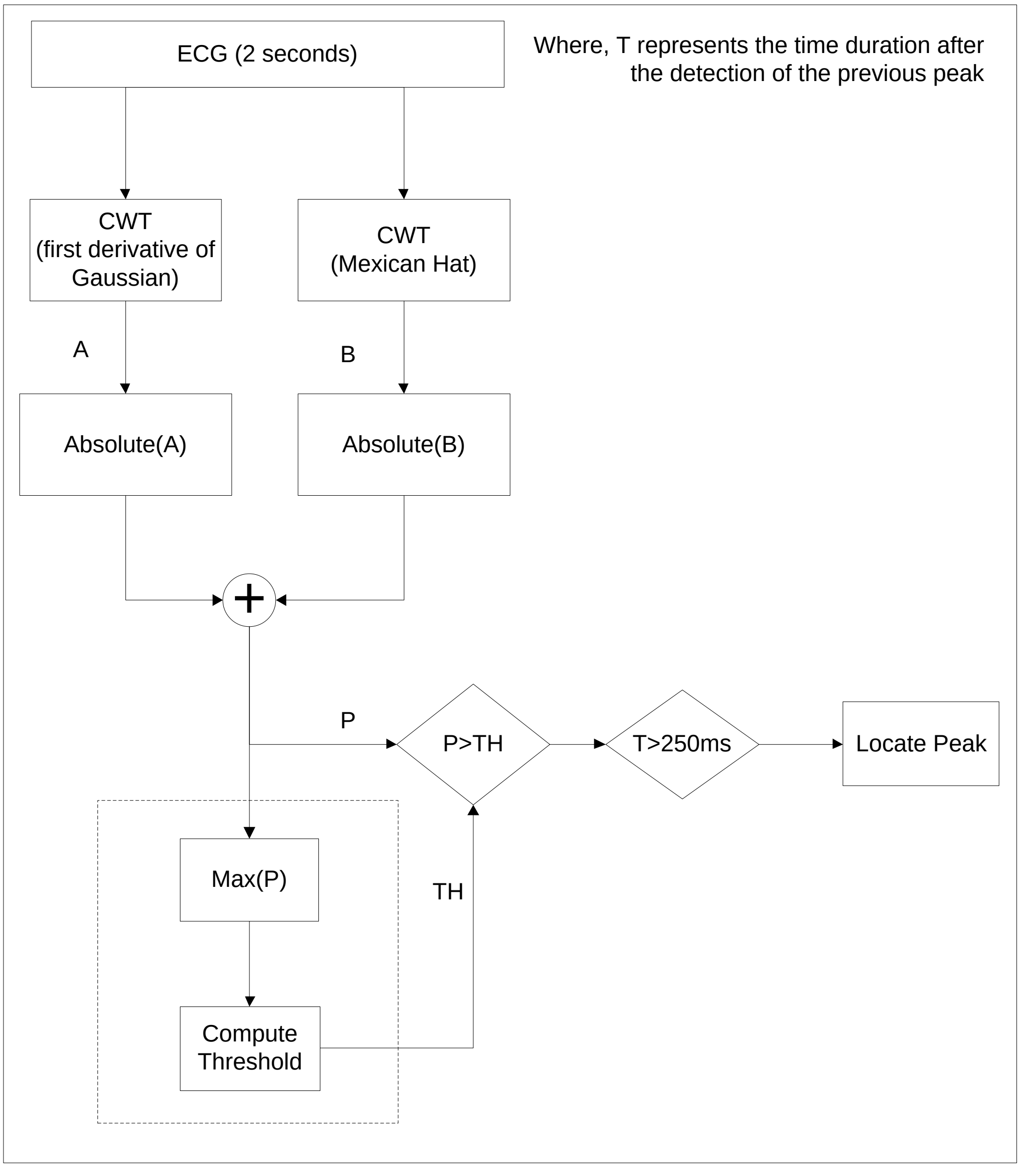



Fig. 5.1 Flowchart showing CWT computation for R peak detection

## 5.2 Threshold Calculation

The threshold value is computed recursively – by using a weighted sum of previous thresholds and the newly calculated threshold. Several weights have been investigated to find the optimal values. In this implementation, the threshold is changed in case of abrupt changes in rhythm, in order to verify whether the rhythm change is due to a change in cardiac activity or to false detections as a result of noise.

The threshold calculation method can be enumerated by the following steps:

1. Calculate maximum value of P where, P is the sum of absolute values of the CWT coefficients obtained by performing continuous wavelet transforms on the input ECG signal separately using first derivative of Gaussian and Mexican hat wavelet.
2. Compute 40 % of the value obtained from step 1.
3. Calculate the threshold value recursively using a weighted sum of eight previous thresholds and newly calculated threshold.

The flowchart for the detailed threshold calculation is shown in Fig. 5.2.

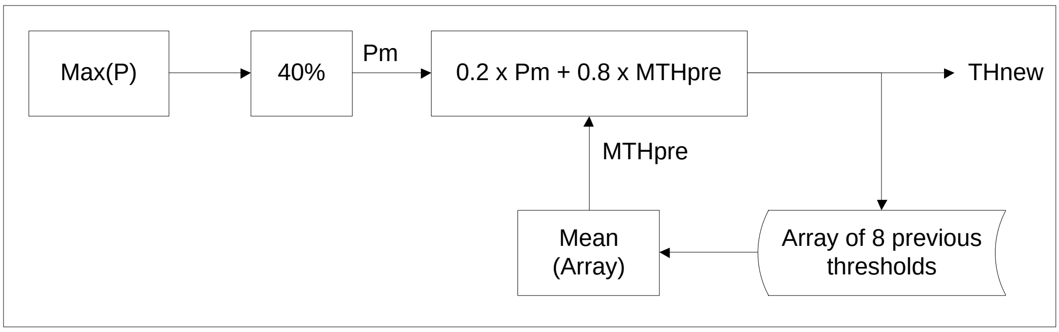



Fig. 5.2 Flowchart showing threshold calculation

## 5.3 Overlapping of ECG segments

A segment (time window) of 2 seconds of data is extracted from an ECG signal giving an overlap of 0.3 seconds with the previous window and again 0.3 seconds with the next one. To avoid mismatches in the overlap sections, the detections obtained in the first 0.3 seconds and the last 0.3 seconds within the window are ignored as seen in Fig. 5.3. This was considered necessary in order to avoid boundary effects in the CWT computation and beat detection.

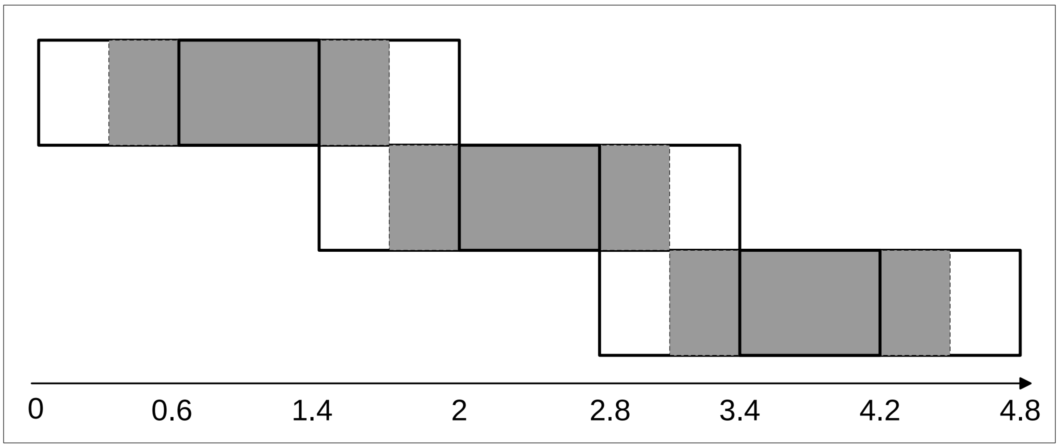



Fig. 5.3 Three ECG segments overlapped 0.3 seconds with window immediately before and after

# CHAPTER 6: EVALUATION PROCEDURE

To determine the efficacy of a system intended to detect or classify features in an ECG, the system will need to be tested and its performance reported. For the evaluation of the algorithm being developed, an evaluation protocol is defined that permits the evaluation of the performance as a function of a wide range of conditions. This test protocol has been used for benchmarking the different methods and to study the improvement made with the methodology defined within this thesis work.

Efficiency of the method is measured in terms of sensitivity and positive predictivity. The development, simulation and the evaluation of the methodology is done in MATLAB environment and the database of the MIT-BIH arrhythmia is used for the purpose of the evaluation.

## 6.1 MIT-BIH database

For the development and testing of beat detection algorithms there are available several standard databases that are accepted as reference benchmarks. They are well annotated and validated and contain a high number of representative ECGs and some not so common examples of cardiomyopathies that are important in the clinical practice. The most common ones are the MIT-BIH and the AHA (American Heart Association) databases. The MIT-BIH database is recorded by the Boston's MIT and Beth Israel Hospital and consists in several datasets for different test purposes, as the Arrhythmia database. This dataset was started to be collected in the 70's and it has been distributed for almost 30 years until today. Nowadays it is probably the most popular ECG database that is widely used in the scientific community for the development and test of algorithms.

For testing and improving the algorithm described in this thesis, the MIT-BIH Arrhythmia database was considered. This database is publicly and freely available on the website of physionet (www.physionet.org). It has become a standard and is a good tool to compare the performance of different algorithms with the same dataset. The database contains half-hour recordings of 23 patients, randomly selected from a large database with Holter recordings, partly recorded in the hospital (60%) and partly out of the hospital (40%). 25 other signals were selected to include a variety of clinically relevant phenomena. All signals are numbered – signals numbered below 125 are from the first category whereas signals 200 and higher are from the later one.

In most recordings, two electrode configurations are available: modified limb lead II and modified lead V1. The analysis in this thesis work has been restricted to the first lead configuration. All signals have been recorded analogue, and are digitized in a later stage at 360 Hz. Annotation has been done by two independent cardiologists, with a simple beat detection algorithm output as starting point. Discrepancies were solved by consensus. In the course of years, some mistakes in the annotations have been corrected. This study has used the most recent annotations (as of 2011).

## 6.2 Algorithm validation

After running the algorithm using a testing ECG, the output was compared with the annotations and the sensitivity and positive predictivity were calculated. These parameters are the ones that are usually used to compare beat detection algorithms and most of the publications offer them to show the performance of the algorithms. The sensitivity is defined as:

$$Se = \frac{TP}{TP+FN} \tag{6.1}$$

And the positive predictivity:

$$PP = \frac{TP}{TP+FP} \tag{6.2}$$

Where, *TP* is true positives (the number of correctly detected beats), *FN* is false negatives (the number of missed beats) and *FP* is false positives (the number of wrongly detected beats).

A beat is considered to be correctly detected if there is an annotated beat within a certain time window around the detected beat (typically +/- 100 ms). If two or more beats are detected for one annotated beat, there is one true positive and one false positive. The value of these parameters is measured in terms of percentage and therefore they range from 0 to 100. An optimal algorithm should try to maximize both parameters, so the sum of both parameters (Se + PP) can also be used as a unique parameter to maximize within a range from 0 to 200.

## 6.3 Evaluation on the signal with varying level of noise

To evaluate the algorithm, ECG signals with varying signal to noise ratio ranging from -10dB to +10dB were generated. For this purpose, a clean ECG signal (first channel of recording 100 in MIT-BIH arrhythmia database) was superimposed with the noise available from the MIT-BIH database containing physiological and electrode motion artifact noise mixed at an SNR varying from +10dB down to -10dB. The good performance of the algorithm on these signals could assure the good performance for the ambulatory cases.

# CHAPTER 7: RESULTS AND DISCUSSION

The evaluation procedure as described in previous section has been followed. The results obtained so far are described in detail in this section.

## 7.1 Evaluation results on MIT-BIH database

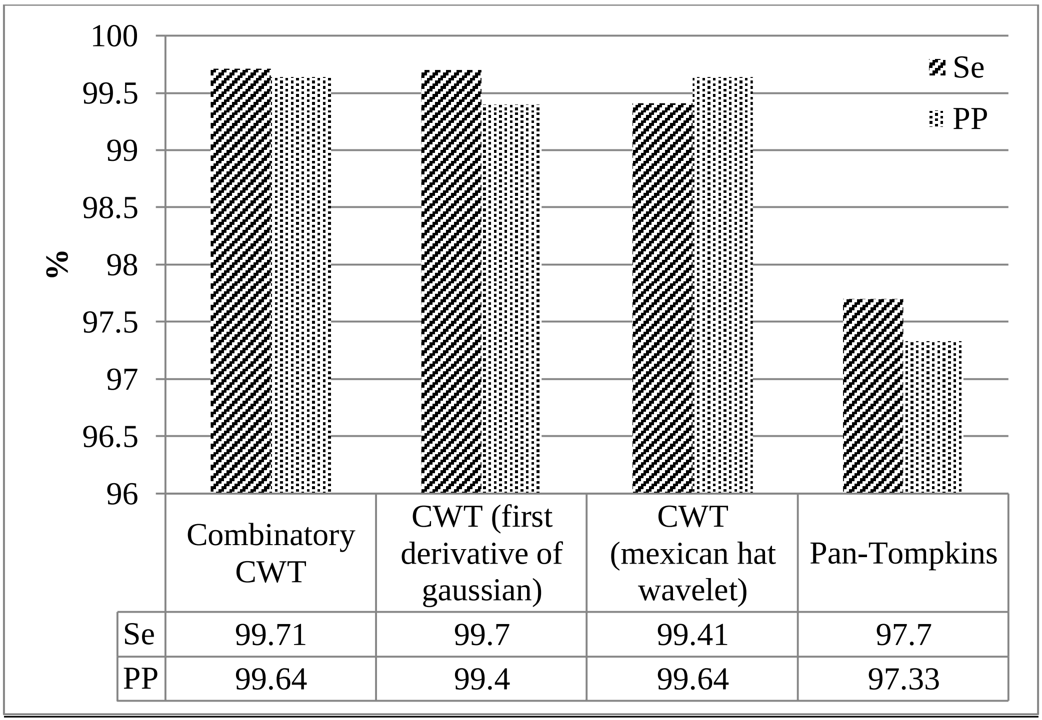
The MIT-BIH arrhythmia database has been chosen for the evaluation procedure. Table 7.1 shows the sensitivity and positive predictivity values obtained from the Combinatory CWT algorithm for 35 different files of MIT-BIH arrhythmia database.

Table 7.1 Sensitivity and positive predictivity of MIT-BIH arrhythmia database using Combinatory CWT algorithm

| S. N. | MIT-BIH File | Combinatory CWT | | S. N. | MIT-BIH File | Combinatory CWT | |
|---|---|---|---|---|---|---|---|
| | | Se | PP | | | Se | PP |
| 1 | 100.mat | 100.00 | 99.91 | 19 | 208.mat | 99.15 | 99.73 |
| 2 | 101.mat | 99.89 | 99.62 | 20 | 209.mat | 100.00 | 99.87 |
| 3 | 102.mat | 99.91 | 99.77 | 21 | 210.mat | 98.90 | 99.66 |
| 4 | 103.mat | 99.95 | 99.81 | 22 | 212.mat | 100.00 | 99.82 |
| 5 | 104.mat | 99.82 | 99.06 | 23 | 213.mat | 99.81 | 99.75 |
| 6 | 105.mat | 99.30 | 97.92 | 24 | 214.mat | 99.73 | 99.87 |
| 7 | 107.mat | 99.91 | 99.91 | 25 | 215.mat | 99.94 | 99.85 |
| 8 | 108.mat | 99.49 | 98.48 | 26 | 217.mat | 99.86 | 99.82 |
| 9 | 109.mat | 99.80 | 99.96 | 27 | 219.mat | 99.95 | 99.72 |
| 10 | 111.mat | 99.95 | 99.67 | 28 | 220.mat | 100.00 | 99.76 |
| 11 | 112.mat | 100.00 | 99.80 | 29 | 221.mat | 99.67 | 99.92 |
| 12 | 116.mat | 99.38 | 99.33 | 30 | 222.mat | 99.84 | 99.68 |
| 13 | 118.mat | 100.00 | 99.82 | 31 | 223.mat | 99.81 | 99.88 |
| 14 | 122.mat | 100.00 | 99.92 | 32 | 228.mat | 99.51 | 99.37 |
| 15 | 200.mat | 99.92 | 99.73 | 33 | 230.mat | 100.00 | 99.73 |
| 16 | 201.mat | 98.36 | 99.53 | 34 | 233.mat | 99.84 | 99.84 |
| 17 | 203.mat | 98.42 | 99.02 | 35 | 234.mat | 99.89 | 99.75 |
| 18 | 205.mat | 99.81 | 99.96 | | **Average** | **99.71** | **99.64** |

The average sensitivity and positive predictivity for the Combinatory CWT algorithm are obtained as 99.71 and 99.64 respectively.

The Combinatory CWT algorithm has been compared with the Pan-Tompkins algorithm, CWT computation for R peak detection using first derivative of Gaussian and CWT computation for R peak detection using Mexican hat wavelet in terms of sensitivity and positive predictivity. The comparison chart in Fig.7.1 shows average sensitivity and positive predictivity for the different algorithms. These values are calculated for all the 35 files of MIT-BIH arrhythmia database.




| | Combinatory CWT | CWT (first derivative of gaussian) | CWT (mexican hat wavelet) | Pan-Tompkins |
|---|---|---|---|---|
| Se | 99.71 | 99.7 | 99.41 | 97.7 |
| PP | 99.64 | 99.4 | 99.64 | 97.33 |

Fig.7.1 Comparison chart showing average Se and PP for different algorithms

The comparison chart in Fig.7.1 shows that the average sensitivity has improved while performing CWT using first derivative of Gaussian for R peak detection whereas average positive predictivity is only 99.4. Similarly, there was improvement in positive predictivity with the trade-off in average sensitivity while performing CWT using Mexican hat wavelet. In Combinatory CWT algorithm, useful features of both first derivative of Gaussian and Mexican hat wavelet have been combined to optimize the performance of the new algorithm giving better result which is clearly shown in Fig.7.1. Pan-Tompkins Algorithm gives poor result in terms of both average sensitivity and positive predictivity i.e., Se=97.7% and PP=97.33% whereas, Combinatory CWT algorithm gives average sensitivity of 99.71% and average positive predictivity of 99.64% which is the best result among the four algorithms shown in the comparison chart.

## 7.2 R peak detection

The algorithm has been tested against the 2 seconds of ECG segments (time window) with overlap time of 300 ms with the previous window and again 300 ms with the next one. To avoid mismatches in the overlap sections, the detections obtained in the first 0.3 seconds and the last 0.3 seconds within the window are ignored. This was considered necessary in order to avoid boundary effects in the CWT computation and beat detection. The R peaks of the ECG have been detected properly by the algorithm with a very slight variation in the time. The time difference is in the range of 1-2 samples, however, the accuracy is well maintained.

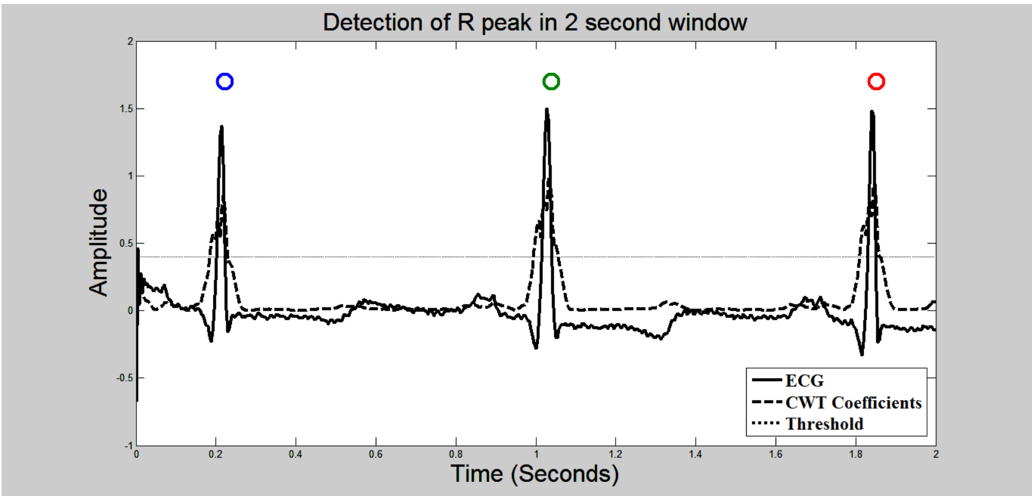



Fig. 7.2 R peak detection in a 2 second window using Combinatory CWT algorithm

Fig. 7.2 shows the CWT computation performed on the ECG signal based on the methodology defined in this thesis work. Based on the threshold being calculated and finding the region of the crossing of the CWT coefficients by the threshold, the R peak of the ECG is detected.

## 7.3 Varying SNR values

Motion Artifacts is added to the ECG signal (100.mat) varying values of SNR ranging from -10dB to +10dB and the algorithm is also tested against the noisy signal to detect the presence of beats. Sensitivity and positive predictivity are computed for each value of SNR. The graph showing the variation of Se and PP with the variation in the values of SNR for Combinatory CWT algorithm and Pan-Tompkins algorithm is shown in Fig. 7.3.

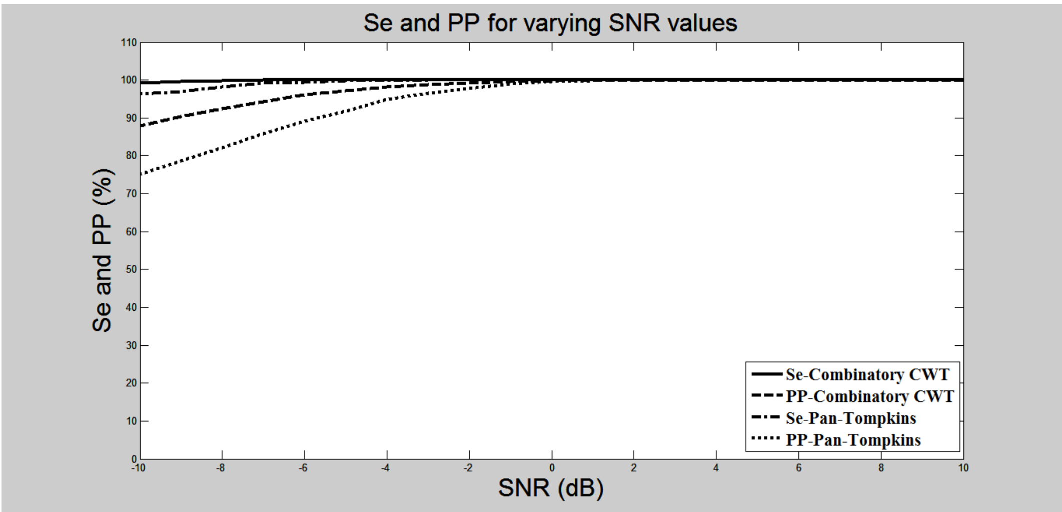



Fig. 7.3 Se and PP versus SNR (dB) for Combinatory CWT algorithm and Pan-Tompkins algorithm

It can be seen that as the SNR value decreases, there is consequent decrease in the performance of the algorithm and basically with the positive predictivity i.e. number of false detections has been increased. The values for Se and PP for lower values of SNR are even worse in case of Pan-Tompkins Algorithm.

# CHAPTER 8: CONCLUSION AND FUTURE ENHANCEMENTS

## 8.1 Conclusion

With the results obtained so far, it can be concluded that the Combinatory CWT algorithm can be used for the removal of effect of noise such as motion artifacts and detection of R peaks in an ECG signal with high level of accuracy. The Combinatory CWT algorithm combines useful features of both first derivative of Gaussian and Mexican hat wavelet to give best results. Average sensitivity and positive predictivity computed against MIT-BIH arrhythmia database were found to be 99.71% and 99.64% respectively. The algorithm was also compared with Pan-Tompkins algorithm. It performed far better in comparison to the Pan-Tompkins algorithm. The algorithm was also tested against the noisy ECG signal by adding motion artifacts to the clean ECG signal varying values of SNR from -10 dB to +10 dB. Despite slight decrement of the performance with the decrement in the SNR values, the results can be considered to be good based on the evaluation procedure that has been followed.

## 8.2 Future Enhancements

The algorithm has been tested against the 2 seconds of ECG segments (time window) with overlap time of 300 ms with the previous window and again 300 ms with the next one so that the algorithm can be used for low power real time implementation in future. This segmentation of the ECG section and the overlapping method confirms its applicability in the real time system with low power usage since, the memory requirement for such system will also be low. Despite the evaluation being performed in the MATLAB environment, for the confirmation of its real time applicability in the embedded system, its evaluation should again be done with the real time inputs and in some hardware systems.

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