TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Pulchowk Campus

Thesis No: 068/MSI/613

A THESIS REPORT ON

OPTIMIZATION OF ENERGY DETECTION APPROACH OF SPECTRUM SENSING IN COGNITIVE RADIO NETWORK

By

NIRANJAN BARAL

068/MSI/613

Thesis Report

Submitted to

Masters of Science in Information and Communication Engineering

Department Of Electronics and Computer Engineering

November 2013

OPTIMIZATION OF ENERGY DETECTION APPROACH OF SPECTRUM SENSING IN COGNITIVE RADIO NETWORK

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A thesis submitted in partial fulfillment of the requirements for the Degree of Master of Science in Information and Communication Engineering

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Lalitpur, Nepal

November 2013

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RECOMMENDATION

The undersigned certify that it has been read and recommended to the Department of Electronics and Computer Engineering for acceptance, a thesis entitled **“Optimization of Energy Detection Approach of Spectrum Sensing in Cognitive Radio Network”**, submitted by **Mr. Niranjan Baral** in partial fulfillment of the requirement for the award of the degree of **“Master of Science in Information and Communication Engineering”**.

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DEPARTMENTAL ACCEPTANCE

The thesis entitled **"Optimization of Energy Detection Approach of Spectrum Sensing in Cognitive Radio Network"**, submitted by **Mr. Niranjan Baral** in partial fulfillment of the requirement for the award of the degree of **"Master of Science in Information and Communication Engineering"** has been accepted as a bonafide record of work independently carried out by him in the department.

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ACKNOWLEDGEMENT

Firstly, I would like to thank my supervisor, Dr. Dibakar Raj Pant for providing continual valuable encouragement, support and direction throughout thesis work. His advice, guidance and support are gratefully acknowledged. I would also like to thank him for reviewing the versions of this thesis.

I would like to show my greatest appreciation to Dr. Arun Kumar Timalsina, Sharad Kumar Ghimire, Daya Sagar Baral and Department of Electronics and Computer Engineering for providing the platform to me for showing venture in this thesis work.

I would also like to acknowledge the assistance of Prof. Dr. Shasidhar Ram Joshi, Prof. Dr. Subarna Shakya, Dr. Sanjeeb Prasad Pandayand Dr. Nanda Bikram Adhikari, for supporting me.

Many thanks to my friend Pawan Dhakal for his help and for many insightful and useful discussions at different point of time during this period.

My sincere and heartfelt appreciation goes to Shraddha Manandhar and Om Bikram Thapa for their valuable suggestions during mid-term and final presentation respectively.

And finally, I would like to express my gratitude for the support of my parents and family members during these studies.

Niranjan Baral

068/MSI/613

ABSTRACT

Cognitive Radio is the emerging technology that allows dynamic access of radio spectrum. Spectrum Sensing is the first task needed to be done to check the presence of licensed users in the spectrum. This thesis is focused on understanding the underlying principles of “Energy detection for Spectrum Sensing” in Cognitive Radio technology which does not need any prior information about the type of signal and optimizing its performance. In this research, spectrum sensing algorithms basically Energy Detection (ED) is considered under a typical fading unknown channel and White Gaussian Noise scenario. Knowledge of the noise power is imperative for the optimum performance of ED. Unfortunately the variation and unpredictability of noise power is unavoidable. Introducing an idea of auxiliary noise variance estimation for combating the absence of prior knowledge of noise power, Hybrid Energy Detection 1 (HED1)/Hybrid-2 (HED2) approach of signal detection was set forth. For HED noise variance is estimated in S auxiliary noise only slots and for HED2 noise variance is estimated in S auxiliary slots which are declared only noise signal slots by ED. The detection performance of the considered methods are derived and expressed by a closed form analytical formulas. The impact of noise estimation accuracy on the performance of ED is compared based on Receiver Operating Characteristic curves and Performance Curves. Accordingly, this study shows that even if the performance gap may be significant under some circumstances (few sensors, low signal-to-noise ratio, small number of slots used for noise power estimation), the performance gap can be decreased in terms of ROC performance by increasing the number of slots used for noise variance estimation.

Keywords: Cognitive Radio, Spectrum Sensing, Energy Detection, Receiver Operating Characteristics

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS

AWGN	Additive White Gaussian Noise
CDF	Cumulative Distribution Function
CDR	Constant Detection Rate
CFAR	Constant False Alarm Rate
CR	Cognitive Radio
CRN	Cognitive Radio Network
DSA	Dynamic Spectrum Access
ED	Energy Detection
FCC	Federal Communication Commission
GSM	Global System for Mobile
HED	Hybrid Energy Detection
HED2	Hybrid ED-2
MAC	Medium Access Control
MSE	Mean Square Error
NP	Neyman Pearson
NTA	Nepal Telecommunications Authority
OFCOM	Office of Communications
OSA	Opportunistic Spectrum Access
OSI	Open System Interconnection
PDF	Probability Density Function
PU	Primary User
ROC	Receiver Operating Characteristic
SNR	Signal-to-Noise Ratio
SU	Secondary User
UHF	Ultra High Frequency
WiMAX	Worldwide Interoperability for Microwave Access
WLAN	Wireless Local Area Network

CHAPTER 1: INTRODUCTION

1.1 Background

Proliferation of wireless communication technologies is leading to the increased demand for high speed wireless application thus increasing demand for Radio frequency. While consumer devices such as cell phones, PDAs and laptops receive a lot of attention, the impact of wireless technology is much broader e.g. through sensor networks for safety applications and home automation system, smart grid control system, medical wearable and embedded wireless devices and entertainment systems. Wireless Channels are characterized by a fixed Spectrum Assignment Policy. Electromagnetic spectrum is strictly regulated and licensed by governmental entities, for instance, Federal Communication Commission (FCC) in US, Office of Communications (OFCOM) in UK and Nepal Telecommunications Authority (NTA) in Nepal.

Table: 1.1 Spectrum assigned by NTA [18]

S. No.	*Frequency Band*	*Bandwidth*	*Technology*
1	800 MHz	824-841.25 MHz paired with 869-886.25 MHz (2x17.25 MHz)	CDMA
2	900 MHz	887.6-915 MHz paired with 932.6-960 MHz (2x27.4 MHz)	GSM
3	1800 MHz	1710-1755 MHz paired with 1805-1850 MHz (2x45 MHz)	GSM
4	2100 MHz	1960-1980 MHz paired with 2150-2170 MHz (2x20 MHz)	IMT-2000
5	2300 MHz	2300 MHz- 2400 MHz (100 MHz)	IMT
6	2600 MHz	2500 MHz- 2690 MHz (190 MHz)	IMT
7	1900 MHz	1850-1880 MHz paired with 1930-1960 MHz (2x30 MHz)	CDMA

It has been a big challenge for every licensing organization to accommodate all the new applications and services noted above with the limited electromagnetic spectrum. Thus that day is not too far, when the increasing demands of spectrum access will lead to spectrum scarcity and will be one of the main limitations in next-generation wireless systems. The demand for spectrum is increasing but report shows that most portion of the spectrum is assigned already.

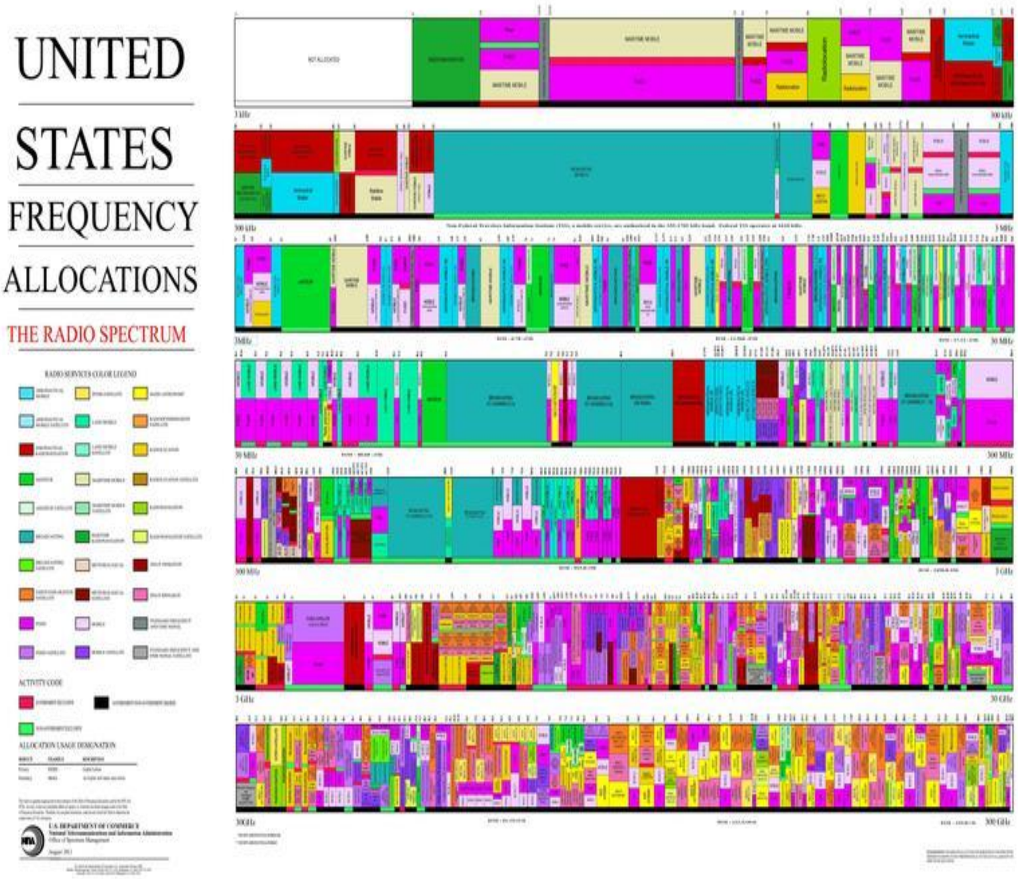


Fig 1.1: US Spectrum allocation Chart [16]

The frequency allocation chart (e.g. US (fig 1.1)) shows that large portion of the radio spectrum is already assigned to traditional services (Mobile, Television, Fixed Satellite Services and so on). New demand for spectrum for latest technologies may create problem for spectrum regulating organizations. Several surveys performed on different parts of world on “spectrum usage” shows that the allocated spectrum is underutilized. Table 1.2 showing spectrum efficiency in GSM and UMTS/ISM bands and figure 1.2 showing spectrum efficiency of different bands in different regions of Europe shows that the spectrum is underutilized even in developed countries where communication technologies are developing rapidly. This limited available spectrum, increasing demand and inefficiency in Spectrum usage necessitate to the development of new communication paradigm to exploit the existing wireless spectrum dynamically and "Cognitive Radio is one of them".

Tables 1.2: Spectrum utilizations in GSM and UMTS/ISM bands in different parts of Europe [17]

Region	System	Utilization [%]
No.1	(E-) GSM900 / GSM1800	38.0/22.0
No.2		47.9/29.3
No.3		44.4/15.6

Region	System	Utilization [%]
No.1	UMTS/ISM	2.1/0.2
No.2		10.8/4.5
No.3		11.1/7.63

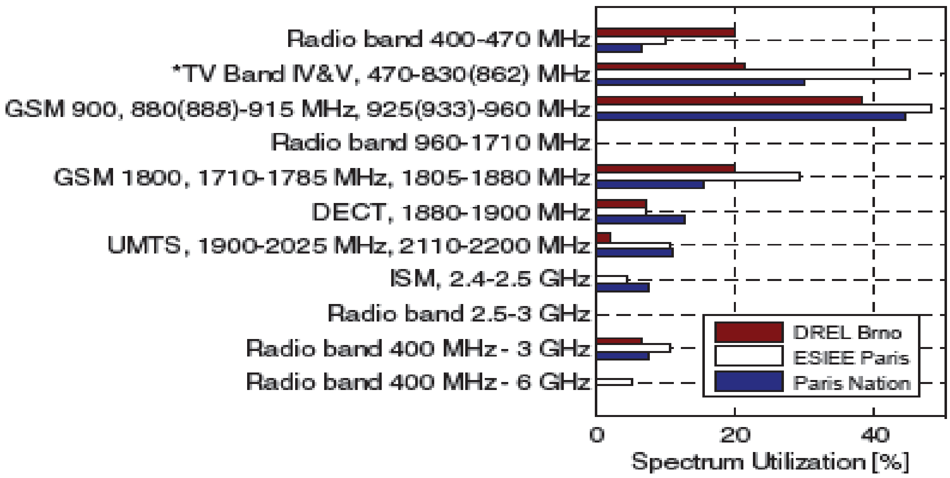


Fig 1.2: Spectrum utilization in different regions of Europe [17]

Cognitive Radio reduces the problem of Spectrum Scarcity by using the unused frequency slots of licensed users (Primary Users) without interfering them. A cognitive radio network consists of secondary radio users (unlicensed users) who can operate at the appropriate pieces of unused spectrum dynamically without any interference to Primary Users (PU). Spectrum Sensing is the first task one need to perform for dynamic Spectrum access. In the aspect of spectrum sensing, the sensing device located at secondary user section measures certain characteristics of the radio waveform and then decides if a primary user is actively using the spectrum. Among the different techniques of spectrum sensing such as matched filter detection (coherent detection through maximization of the signal to noise ratio) and cyclostationary detection (exploitation of the inherent periodicity of primary signals), “Energy detection is the most popular method addressed in literature” [3].

Measuring only the received signal power, the Energy detector is a non coherent detection device with low implementation complexity. It doesn’t need any prior information about the signal characteristics. The energy detector measures the energy of the input wave over a specific period and compares it with the threshold for deciding the presence/absence of spectrum usage by primary user. In spectrum sensing, our goal is to meet a given ROC constraint at very low SNR. Small modeling uncertainties are unavoidable in any practical system and so robustness to them is a fundamental performance metric. The impact of modeling uncertainties can be quantified by the position of SNR Wall below which a detector will fail to be robust no matter how long it can observe the channel [13]. Threshold can be calculated based on two principles, Constant False Alarm Rate (CFAR) and Constant Detection Rate (CDR). In both CDR and CFAR cases the Noise Power is needed to determine the threshold. In practice however the information is rarely available. This work deals with the study of the detection performance of the energy detection using threshold calculated from Noise Variance.

1.2 Problem Statement

The explosion or rapid development of wireless technologies creates an ever-increasing demand for more radio spectrum as explained by figure 1.3. However, most easily usable spectrum bands have been allocated and licensed to a particular organization. Several surveys performed on Spectrum usage as explained already in section 1.1 have shown that these bands are significantly underutilized and also have stated that the main problem behind shortage of spectrum is inefficient utilization of Spectrum. These considerations have motivated the search for breakthrough radio technologies that can scale to meet future demands both in terms of spectrum efficiency and application and allow for dynamic spectrum access. As a result of several researches done for efficient bandwidth utilization gave birth to the term “Cognitive Radio”.

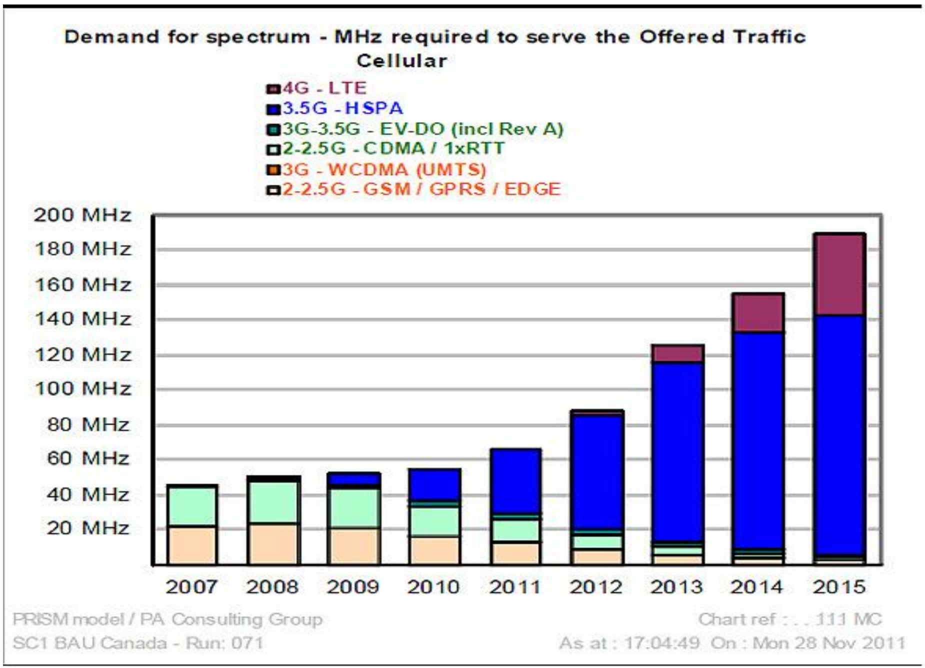


Fig 1.3: Cellular demand for spectrum by technology [19]

“Cognitive radio” is a fully programmable wireless device that can sense their environment and dynamically adapt their transmission waveform, channel access method, spectrum use, and networking protocols as needed for good network and application performance. Cognitive radio network is a combination of Spectrum sensing, Spectrum management, Spectrum mobility and Spectrum sharing. This network will perform efficiently if all the functions of the Cognitive radio network are attained at a desirable rate. Cognitive radio gives the unlicensed users access to the licensed spectrum whenever the primary users are not active within the given spectrum and thus contribute to the effective bandwidth utilization. The first step in cognitive radio is Spectrum Sensing and several methods are stated for this step. This thesis deals with Energy detection approach of Spectrum sensing is the most popular method addressed in literature [3]. But the question definitely arises, “How can it be attained”? Several papers published have given several approaches to energy detection in different communication scenario with different parameters. Some state about matched filter detection and some talked about cyclostationary approaches. These two approaches need prior information about the type of signals sent. But what happens when we can’t predict the type of signal to be received .The one we are dealing in this work is the Energy detection approach of spectrum sensing which does not need prior knowledge about the type of signals. Considering only the Energy detection has many modeling uncertainties in the system which can be quantified by the position of SNR wall below which a detector will fail to be robust. The accuracy of threshold and its robustness is important in Energy detection. Proper threshold calculation by accurately estimating the noise power is important task for spectrum sensing which should meet a given ROC constraint at very low SNR.

1.3 Objectives

The main objectives of this thesis are:

1. To study and analyze Energy detection, Energy detection with noise power estimation (Hybrid Energy Detection 1 and Hybrid Energy Detection 2) for Spectrum Sensing.

2. Compare Energy Detection with Hybrid Energy Detection techniques of noise power estimations by ROC curves.

1.4 Scope and Applications

This thesis aims to introduce the concepts of cognitive radio and techniques for spectrum sensing in a cognitive radio network. It mainly aims to provide clear concepts about the “Energy Detection” method of spectrum sensing. This energy detection scheme for Spectrum Sensing can be used for the detection of presence of primary users by the secondary users in Cognitive Radio Network. The project simulates the Energy detection process and explains several ways to optimize the process along with the comparison of those different processes with the general Energy detection process.

Cognitive radio has become a hot topic these days because of its application in effective bandwidth utilization. Effective bandwidth management will help to get rid of shortage of bandwidth to provide highly reliable services. Reliable spectrum sensing is the very task upon which the entire operation of the cognitive radio rests. Energy detection is one of the solutions that have been proposed for enabling opportunistic spectrum access to determine the presence of vacant band in Cognitive radio network. Energy detector is useful in detecting target signals about which prior information is not known in advance. It allows systems to use their spectrum sensing capabilities to optimize their access to and use of the spectrum efficiently.

1.5 Organization of Thesis Report

The rest of thesis report is divided into following chapters:

Chapter 2 gives insight into the literature review of Energy detection technique of spectrum sensing.

Chapter 3 deals with theoretical background of Cognitive Radio, Spectrum Sensing and different techniques of Spectrum sensing and the System model in detail.

Chapter 4 deals with mathematical system model of Energy detection, HED1 and HED2.

Chapter 5 explains the methodology that is used in thesis.

Chapter 6 shows the Simulation result and gives an analysis of the results obtained during the experiment

Chapter 7 concludes the thesis work.

CHAPTER 2: THEORITICAL BACKGROUND

2.1 Cognitive Radio

Cognitive radios offer the promise of being a disruptive technology innovation that will enable the future wireless world. Cognitive radios are fully programmable wireless devices that can sense their environment and dynamically adapt their transmission waveform, channel access method, spectrum use and networking protocols as needed for good network and application performance. It is an emerging solution to the problem of spectrum scarcity caused by under-utilized frequency slots of licensed or unlicensed frequency bands.

Some definitions of Cognitive Radio:

The term Cognitive Radio was first coined by Joseph Mitola."Cognitive Radio is a radio that includes a transmitter in which operating parameters such as frequency range, modulation type or maximum output power can be altered by software [2]".

According to **FCC**: "A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets[1]."

"Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment and uses the methodology of understanding by building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind: highly reliable communication whenever and wherever needed: efficient utilization of the radio spectrum[11]."

The ultimate objective of the cognitive radio is to obtain the best available spectrum through cognitive capability and reconfigurability as described before. Since most of the spectrum is already assigned, the most important challenge is to share the licensed spectrum without interfering with the transmission of other licensed users as illustrated in following figure.

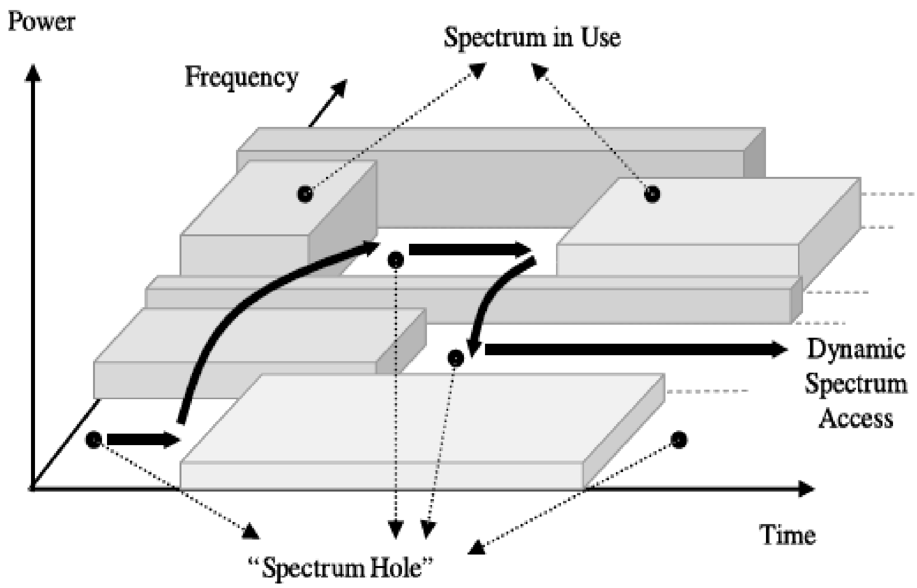


Fig 2.1 Spectrum hole Concept [1]

The cognitive radio enables the usage of temporally unused spectrum, which is referred to as spectrum hole or white space. If this band is further used by a licensed user, the cognitive radio moves to another spectrum hole or stays in the same band, altering its transmission power level or modulation scheme to avoid interference.

Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel. Once the cognitive radio supports the best available channel, the next challenge is to make the protocols adaptive to the available spectrum. To support this adaptivity, Cognitive radio require following functionalities [1].

- Spectrum Sensing: Detecting unused spectrum and sharing the spectrum without harmful interference with other users.

- Spectrum Decision: Capturing the best available spectrum to meet user communication requirements.

- Spectrum Mobility: Maintaining seamless communication requirements during the transition to better spectrum as a SU changes its frequency of operation when a PU appears in the same band.
- Spectrum Sharing: Providing the fair spectrum scheduling method among coexisting next generation users.

Thus we see that Spectrum Sensing is the first task one need to perform for Dynamic Spectrum access in a Cognitive radio network which enables the cognitive radio to adapt to its environment by detecting spectrum holes.

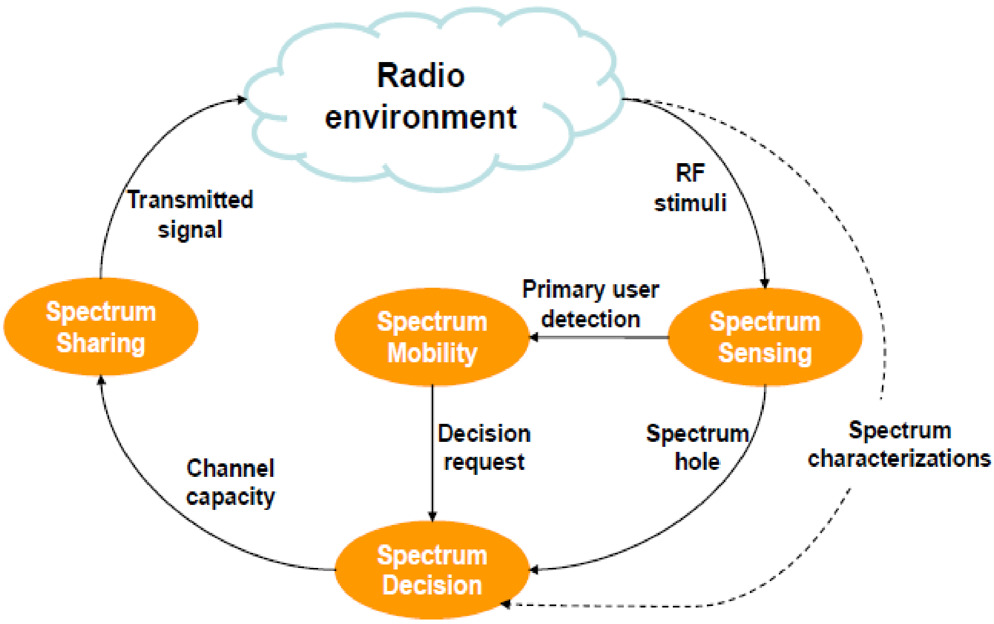


Fig 2.2 Spectrum Management in Cognitive Radio Network

2.2 Spectrum Sensing

Spectrum sensing is an important element of cognitive radio. The most efficient way to detect spectrum holes is to detect the primary users that are receiving data within the communication range of a secondary user. In reality, however, it is difficult for a secondary user to have accurate channel information between a primary receiver and a transmitter due to the inherent property of cognitive radio.

Thus, the most recent work engages in primary transmitter detection based on local observations of secondary users. The current spectrum sensing methods can be classified as three categories: Non-cooperative spectrum sensing, cooperative spectrum sensing and interference temperature spectrum sensing [3].

In the aspect of spectrum sensing, the sensing device located at secondary user section measures certain characteristics of the radio waveform and then decides if a primary user is actively using the spectrum. Among different spectrum sensing techniques such as the matched filter detection (coherent detection through maximization of the signal to noise ratio) and the cyclostationary (exploitation of the inherent periodicity of primary signals) “Energy detection” is the most popular method addressed in literature [3].

Measuring only the received signal energy, the energy detector is a non coherent detection device with low implementation complexity. It doesn’t need any prior information about the signal characteristics. The energy detector measures the energy of the input wave over a specific period and compares it with the threshold for deciding the presence/absence of spectrum usage by primary user. Absence of primary user provides opportunity for secondary users to access the spectrum until the former use that spectrum.

2.2.1 Non-cooperative Spectrum Sensing

Non-cooperative spectrum sensing, also known as transmitter detection, is a sensing mechanism where a CR user distinguishes used and unused spectrum bands. A secondary user should be able to decide whether a signal from a primary transmitter is present or not within a certain time and spectrum band. This method is based on the detection of the relatively weak signal from a primary emitter through the local observations of individual secondary users. Basic hypothesis model for non-cooperative spectrum sensing can be defined as follows,

a. $H_{0:}$ (Absence of Signal)

   The input signal x(t) consists of noise only i.e. r(t)=n(t).

b. H_1

The input signal x(t) consists of signal and noise i.e. r(t)=s(t)+n(t)

Where x(t) is the signal received by CR user, s(t) is the transmitted signal from primary user, n(t) is the additive white Gaussian noise and h is the amplitude gain of the channel. H_0 and H_1 are a null and a non-null hypothesis respectively, indicating the presence or absence of the primary user's signal. Typically, there are three main schemes for non-cooperative spectrum sensing which are matched filter detection, energy detection and cyclostationary feature detection.

A. Matched Filter

In Match Filter Sensing Technique, match filter maximizes the received signal-to-noise ratio. But a matched filter requires demodulation of primary user signal this means cognitive radio should have a priori knowledge of primary user signal at both Physical and MAC layers. Some information like modulation type and order, pulse shaping, packet format might be pre-stored in Cognitive Radio memory, but for demodulation it needs timing and carrier synchronization with primary user, even channel equalization [1]. This is still possible since most primary users have pilots, preambles, synchronization words or spreading codes that can be used for coherent detection. The main advantage of matched filter is that due to coherency it requires less time to achieve high processing gain. However, a significant drawback of a matched filter is that a cognitive radio would need a dedicated receiver for every primary user class to share the signal parameters priory.

B. Cyclostationary Feature Detection

Modulated signals are in general coupled with sinusoidal wave carriers, pulse trains, or cyclic prefixes which has certain periodicity. Even though the data is a stationary random process, these modulated signals are characterized as *cyclostationary*, since their statistics, mean and autocorrelation, exhibit periodicity. This periodicity of the signals is used for: parameter estimation such as carrier phase, pulse timing, or direction of arrival. This can then be used for detection of a random signal with a particular modulation type in a background of noise and other modulated signals. These features are detected by analyzing a spectral correlation function. The main advantage of the spectral correlation function is that it differentiates the noise energy from modulated signal energy, which is a result of the fact that the noise is a wide-sense stationary signal with no correlation, while modulated signals are cyclostationary with spectral correlation due to the embedded redundancy of signal periodicity. Therefore, a cyclostationary feature detector can perform better than the energy detector in discriminating against noise due to its robustness to the uncertainty in noise power [1].However; it is computationally complex and requires significantly long observation time.

C. Energy Detection

Energy detection is one of the spectrum sensing techniques that evaluate the signal energy over a certain time interval and compare it with the threshold defined or calculated from noise power estimations to decide whether the spectrum is in use or not. The effect of noise in the signal may affect the decision of energy detector thus causing false alarm or even miss detection.

2.2.2 Cooperative Spectrum Sensing

Cooperative sensing is a method in which multiple cognitive radios collaborate by sending their decision statistics where the final decision is done by the base station. This method is more powerful than other methods in a sense that it achieves multiuser diversity and mitigates the Hidden Node Problem for proper detection and low interference. Since it is usually impossible for secondary users to detect the location of primary receiver, the interference cannot be avoided. Moreover, a CR transmitter may not be able to detect the primary transmitter due to the channel fading or shadowing. Consequently, the sensing information from other users is required for more accurate detection and cooperative spectrum sensing arises. Compared to the non-cooperative sensing which is based on the local observation from each CR user independently, cooperative sensing method collects and incorporates sensing information from multiple CR users in order to improve the performance of spectrum sensing.

CHAPTER 3: LITERATURE REVIEW

Energy detection is one of the spectrum sensing techniques that evaluate the signal energy over a certain time interval and compare it with the threshold to decide whether the spectrum is in use or not. The effect of noise in the signal may affect the decision of energy detector thus causing false alarm or even miss detection. Energy detection was first discussed in [3] which evaluates the closed form expressions of the performance parameters P_d(probability of Detection) and P_f(Probability of false alarm rate) based on the scenario of unknown signal of known amplitude transmitted over a flat-band limited Gaussian noise channel. The result of this work was extended in [5] which derived the closed form expression of P_d and P_f of the signal with random amplitude for different types of distributions. [6] studied about the problem of Unknown signals over different fading channels. Starting with no diversity case, he presented the closed form of expression of system performance when different diversity techniques are employed. Based on the assumption that we cannot estimate the actual noise variance of the channel, which has a direct effect in the estimation of P_f, [7] analyze the performance of spectrum sensing based on energy detection using an estimated noise variance to calculate the threshold where noise variance was estimated using a spare channel dedicated for analyzing noise characteristics. Performance of ED in AWGN and different fading channels has been studied in many works including [12, 13, 14]. These works assumed a perfect knowledge of the noise power at the receiver, which allows for the perfect threshold design. In that case ED can work with arbitrarily small value of False Alarm Probability and Miss-detection Probability ($P_{Md)}$, by using sufficiently large observation time, even in low SNR environment [15]. However, in real systems the detector does not have a prior knowledge of the noise level. Variation and unpredictability of the precise noise level at the sensing device came as a critical issue, which is also known as noise uncertainty. With the motive of reducing the impact of noise uncertainty on the signal detection performance of ED, several researches have been proposed. Hybrid Spectrum Sensing Algorithms based on the combination of ED and Feature Detection techniques are put forwarded for the reduction of the effect of noise variance uncertainty [20, 21]. In [15] the fundamental bounds of signal detection in presence of noise uncertainty are analyzed. This study showed that there are some SNR thresholds under noise uncertainty known as SNR Wall that prevents achieving the desired performance even if the detection interval is made infinitely large. It concludes that the robustness of any detector can be quantified in terms of the SNR Wall giving the threshold below which weak signals cannot be detected reliably no matter how many samples are taken. In [17] author performed the asymptotic analysis of the estimated noise power (ENP) to derive the condition of SNR Wall phenomenon which suggested that the SNR Wall can be avoided if the variance of the noise power estimator can be reduced while the observation time increases. [21] proposed an uniform noise power distribution model for the noise uncertainty study of ED in low SNR regime.

This thesis formulates the detection performance of Hybrid approach of ED which is named as Hybrid Energy Detection (HED) and Hybrid Energy Detection 2 (HED2) to optimize the performance of Energy detection techniques with high probability of detection.

CHAPTER 4: MATHEMATICAL MODEL

In our system model, we consider a single sensor Energy Detector which senses and decides the presence or absence of the primary signal within a defined spectrum band W. In a given sensing time interval T, the Energy Detector calculates its detection statistic $\boldsymbol{T_{ED}}$ taking N samples of received signal $y(n)$. Let

$$\boldsymbol{Y} = [\, y(1) \dots \dots \dots \dots y(n) \dots \dots \dots y(N)]^T \qquad (4.1)$$

be the $N \times 1$ received vector at an arbitrary sensing interval T, where the element $y(n)$ is the discrete baseband complex sample at the receiver at time n. Now the spectrum sensing problem using Energy Detection can be viewed as a binary detection problem. The decision of the Energy Detector is the test of the following hypothesis.

$$\boldsymbol{Y} = \begin{cases} \boldsymbol{V}, & H_o(Noise\ alone) \\ \boldsymbol{hS} + \boldsymbol{V}, & H_1(Signal\ and\ Noise) \end{cases} \qquad (4.2)$$

Where,

- n = 1, 2, 3…N, represents the samples (detection period).
- $\boldsymbol{Y}$ is the received signal vector at the sensor and $\boldsymbol{h}$ is a channel vector which is assumed to be constant and memory less within the sampling interval,
- $\boldsymbol{S}$ is the primary signal vector, $\boldsymbol{S} = [s(1) \dots \dots s(n) \dots \dots s(N)]^T$, which is assumed to be Complex Gaussian Distributed signal with zero mean and variance σ_s^2 having flat band limited power spectrum density PSD: $s(n) \sim N_{¢}(0, \sigma_s^2)$
- $\boldsymbol{V}$ is noise vector, $\boldsymbol{V} = [v(1) \dots \dots v(n) \dots \dots v(N)]^T$also assumed to be Complex Gaussian Distributed Noise signal with zero mean and variance σ_v^2: $v(n) \sim N_{¢}(0, \sigma_v^2)$.

[Note: $N_{¢}$ is the notation for Complex Normal Distribution Random Variable and N_R is the Real Normal Distribution Random Variable]

Using the information of the received signal vector $\boldsymbol{Y}$ to develop a test statistic $\boldsymbol{T_{ED}}$, which is the measure of the average energy of the received signal over a sensing interval N, the detector compares $\boldsymbol{T_{ED}}$ against a predefined threshold $\boldsymbol{t}$. If $\boldsymbol{T_{ED}} < t$ then it decides in favor of Null Hypothesis $\boldsymbol{H_0}$ otherwise in favor of Alternate Hypothesis $\boldsymbol{H_1}$. The average energy of the received signal vector $\boldsymbol{Y}$ normalized by the noise variance can be represented as,

$$\boldsymbol{T_{ED}} = \left(\frac{1}{N\sigma_v^2}\right)\sum_{n=1}^{N}|y(n)|^2 \tag{4.3}$$

Now the important part is that how we calculate the noise variance to be compared with threshold to detect the presence/absence of signal and derive the relation for Probability of detection and Probability of False Alarm

4.1 Energy Detection

In energy detection we compare the calculated decision statistic of the sample with the predefined threshold.

For Null Hypothesis rearranging the above equation (4.3) using $y(n) = v(n)$, we get,

$$\boldsymbol{T_{ED}}|_{H_0} = \left(\frac{1}{N\sigma_v^2}\right)\sum_{n=1}^{N}|v(n)|^2 \tag{4.1.1}$$

$$\boldsymbol{T_{ED}}|_{H_0} = \left(\frac{1}{2N}\right)\sum_{n=1}^{N}\left|\frac{v_R(n)}{\sigma_v/\sqrt{2}} + j\,\frac{v_C(n)}{\sigma_v/\sqrt{2}}\right|^2 \tag{4.1.2}$$

$$\boldsymbol{T_{ED}}|_{H_0} = \left(\frac{1}{2N}\right)\sum_{n=1}^{N}|\beta_R + j\beta_C|^2 \tag{4.1.3}$$

$$\boldsymbol{T_{ED}}|_{H_0} = \left(\frac{1}{2N}\right)\sum_{n=1}^{N}\left({\beta_R}^2 + {\beta_C}^2\right) \tag{4.1.4}$$

Where $v_R(n)$ and $v_C(n)$ are real and imaginary part of the noise signal $v(n)$ respectively. $\beta_R = \sqrt{2}\, v_R(n)/\sigma_v$ and $\beta_C = \sqrt{2} v_C(n)/\sigma_v$. As $v(n)$ is a zero mean and σ_v^2 variance Complex Valued Gaussian Random Variable, we can easily infer β_R and β_C as Standard Normal Random Variables with mean zero and unity variance. The numerator of $\boldsymbol{T_{ED}}$ in equation (4.1.4) is sum of square of 2N standard normal random variable with mean zero and variance 1, thus we can say that the decision statistic $\boldsymbol{T_{ED}}|_{H_0}$ follows the Chi Square Distribution with 2N degrees of freedom scaled by the factor$\left(\frac{1}{2N}\right)$.

$$\boldsymbol{T_{ED}}|_{H_0} = \left(\frac{1}{2N}\right) \chi^2_{2N} \tag{4.1.5}$$

Similarly for Alternate Hypothesis, considering the channel coefficient as a constant value rather than a vector and rearranging the equation (4.3) using $y(n) = hs(n) + v(n)$, we get,

$$\boldsymbol{T_{ED}}|_{H_1} = \left(\frac{1}{N\sigma_v^2}\right)\sum_{n=1}^{N}|hs(n) + v(n)|^2 \tag{4.1.6}$$

$$\boldsymbol{T_{ED}}|_{H_1} = \left(\frac{\sigma_t^2}{2N\sigma_v^2}\right)\sum_{n=1}^{N}\left|\frac{hs(n) + v(n)}{\sigma_t/\sqrt{2}}\right|^2 \tag{4.1.7}$$

$$\boldsymbol{T_{ED}}|_{H_1} = \left(\frac{\sigma_t^2}{2N\sigma_v^2}\right)\sum_{n=1}^{N}|\alpha|^2 \tag{4.1.8}$$

Where $\alpha = \frac{hs(n)+v(n)}{\sigma_t/\sqrt{2}}$. As h is assumed to be constant for the sensing interval and both the signal and noise are Complex Valued Gaussian Signals with variances σ_v^2 and σ_s^2 respectively and both are independent signals, $hs(n) + v(n)$ is also Complex Valued Gaussian Signal with mean zero and variance $\sigma_t^2 = h^2\sigma_s^2 + \sigma_v^2$. It is clear that α is also a Complex Valued Standard Normal Random Variable with mean zero and unity variance. So the numerator of $\boldsymbol{T_{ED}}|_{H_1}$ is the sum of square of N Complex Valued Normal Random variable, from which we can say that the decision statistic $\boldsymbol{T_{ED}}|_{H_1}$ also follows the Chi Square Distribution with 2N degrees of freedom scaled by the parameter $\left(\frac{\sigma_t^2}{2N\sigma_v^2}\right)$. If we define Signal to Noise Ratio as $\rho = \left(\frac{\sigma_s^2}{\sigma_v^2}\right)$. Equation (4.1.7) can be re-written as,

$$\boldsymbol{T_{ED}}|_{H_1} = \left(\frac{h^2\rho + 1}{2N}\right)\chi_{2N}^2 \tag{4.1.9}$$

Noting the result we get,

$$\boldsymbol{T_{ED}} = \begin{cases} \left(\frac{1}{2N}\right)\chi_{2N}^2 & H_o \\ \left(\frac{\omega}{2N}\right)\chi_{2N}^2 & H_1 \end{cases} \tag{4.1.10}$$

where $\omega = h^2\rho + 1$. It is clear from equation (4.1.9) that the decision statistic $\boldsymbol{T_{ED}}$ follows the Chi Square Distribution for both the hypothesis with different scaling factor. Including the scaling factor and using the property of Chi Square Distribution and its relation to Gamma Distribution Function, we get,

$$p\chi_N^2 \cong \Gamma(k = N/2\,, \theta = 2p) \tag{4.1.11}$$

Where p is a constant ($p > 0$), $\Gamma(\,)$ is Gamma Distributed Random variable. Thus, above equation (4.1.9) can be re-written in the form of Gamma Distribution as shown below.

$$\boldsymbol{T_{ED}} = \begin{cases} \Gamma(k = N, \theta = 1/N) & H_o \\ \Gamma(k = N, \theta = \omega/N) & H_1 \end{cases} \tag{4.1.12}$$

For Large N

According to the Central Limit Theorem, when N is made sufficiently large, the Chi Squared Distributed Random Variable in equation (4.1.10) converse to a Gaussian distribution. Figure 4.1.1 shows the PDF plot of the Chi Squared Distribution for 3, 5, 10, and 20 degrees of freedom superimposing $N_{¢}(10{,}20)$ to illustrate a good approximation of χ_N^2 by a Gaussian $N_{¢}(N/2{,}2N)) \cong N_R(N, 2N)$ for large N. In the figure we found that the Chi Square Distribution Variable with

Degrees of Freedom 20 (N = 20) shows a similarity with the Normal Distribution with mean and variance 20 and 40 (N and 2N).

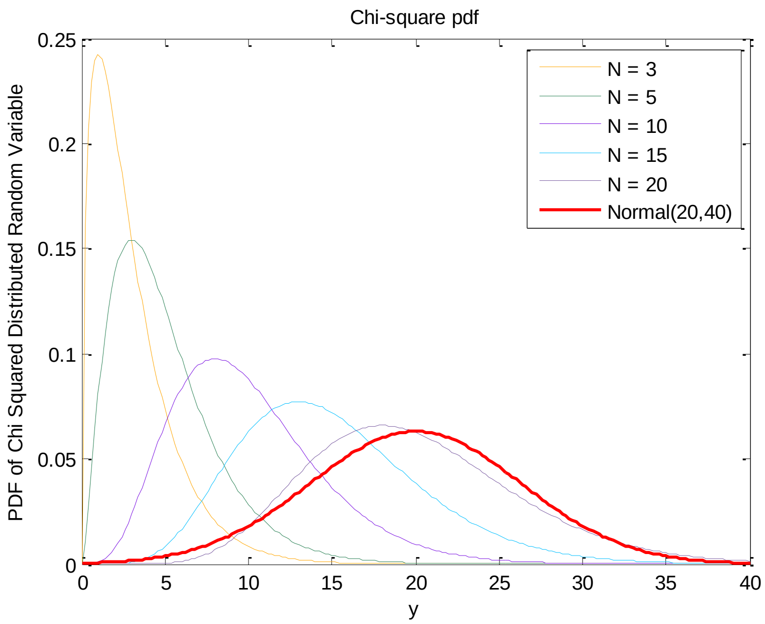


Figure 4.1 χ_N^2 distribution with 5, 10, and 20 degrees of freedom. A normal distribution is superimposed to illustrate a good approximation to χ2n by $N_{\phi}(10, 20)$ for N large

For good approximation, different models have been developed such as Berkeley Model which analyzed the accuracy of different Energy Detection models in approximating the exact solution of the $\boldsymbol{T_{ED}}$ and concluded that these models almost have the same performance for such scenario.

For the result in equation (4.1.10), using Berkeley Model [22] for approximating Chi Squared Distribution Function to a Normal Distribution Function, we get,

$$\boldsymbol{T_{ED}} = \begin{cases} \left(\frac{1}{2N}\right) N_{\phi}(N, 2N) & H_o \\ \left(\frac{\omega}{2N}\right) N_{\phi}(N, 2N) & H_1 \end{cases} \tag{4.1.13}$$

$$\boldsymbol{T_{ED}} = \begin{cases} N_{\phi}(1/2, 1/2N) & H_o \\ N_{\phi}(\omega/2, \omega^2/2N) & H_1 \end{cases} \tag{4.1.14}$$

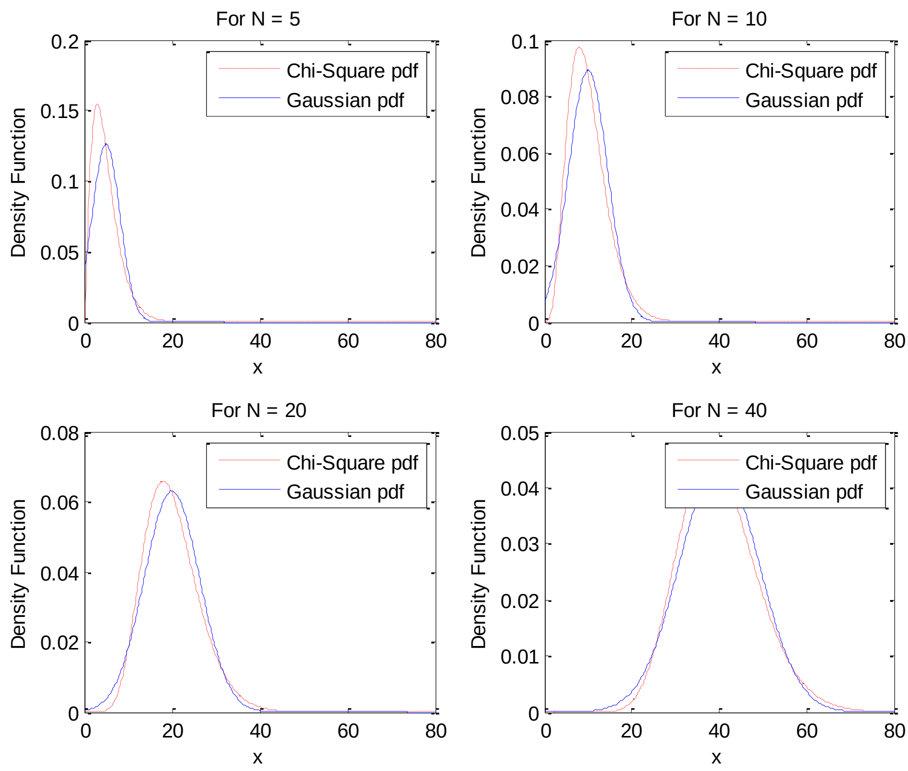


Figure 4.2 Comparison of Chi Square Distribution with Gaussian approximation for different N

To get an insight of accuracy of the approximation, we simulated the Gaussian approximation of Chi Square Distribution for different N. Figure 4.2 shows the pdf of Chi Square Random Variable and its Gaussian Approximation counterpart for N taking values 5, 10, 20 and 40. Also in figure 4.3, we have plotted the Mean Square Error (MSE) of the approximation considering pdf of distributions as a comparison criteria for varying N and found that the approximation shows perfect result for N greater than 40. From figure 4.1.3, it is clear that the MSE is nearly zero, i.e. the Gaussian approximation is perfectly true for N > 40, which indicated that the degree of freedom greater than 40 is sufficient for approximating Chi Squared Distribution of the detection statistic to a Gaussian distribution.

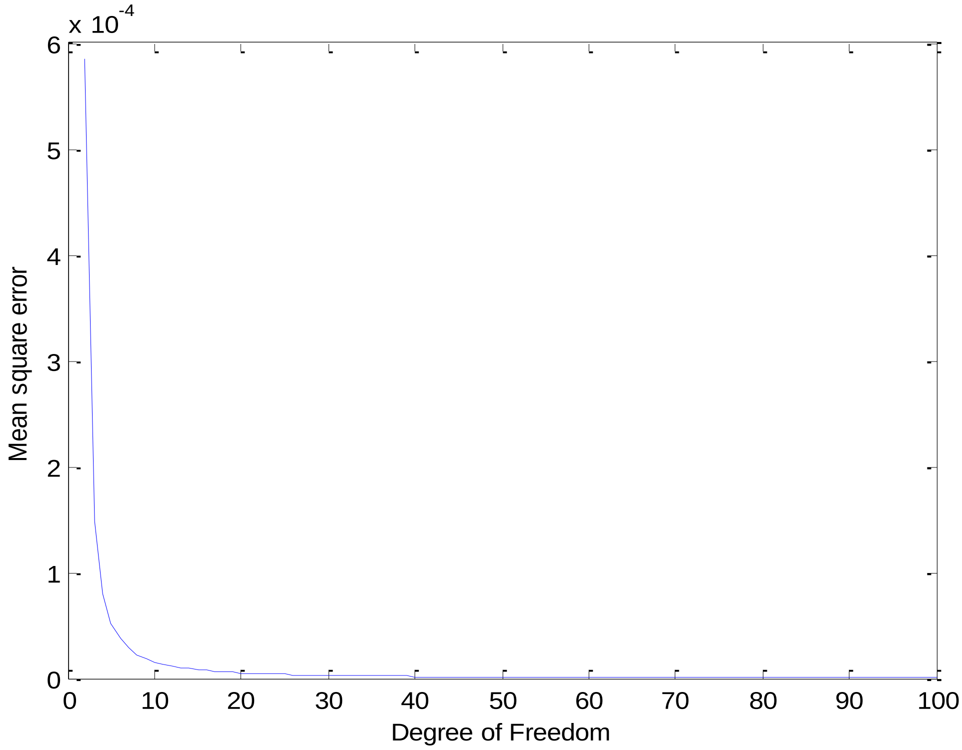


Figure 4.4 Mean Square Error of the approximation considering pdf as a comparison criteria

Expression for detection and false alarm Probabilities

A numerical study shows that Energy Detection ROC curve based on chi-squared distributions may be accurately represented by binomial receiver operating characteristics (ROC) curves. This allows the detector accuracy and the ROC shape (Assymetry) to be expressed simply in terms of distribution parameters[5]. For any description model, it would be useful to have a concise description of the ROC curves that is meaningful in terms of underlying signal and noise distributions. Such a description would facilitate the comparison of model with experiment, and help in the assessment of effect of changing model parameter. The ROC curve is obtained by plotting the probability of correct detection versus the probability of false alarm.

Hypothesis test is a procedure which divides the space of observations into 2 regions, Rejection Region (R) and Acceptance Region (A). The two important characteristics of a test are called signifcance and power referring to errors of type

I and II in hypothesis testing which relates to Probability of False alarm and Probability of Detection respectively. The probabilities of false alarm P_f and probability of detection for a given threshold energy $\boldsymbol{T_{ED}}$ is given by,

$$P_f = Prob\{\boldsymbol{T_{ED}} > \boldsymbol{t}/H_o\} \quad (4.1.15)$$

$$P_d = Prob\{\boldsymbol{T_{ED}} > \boldsymbol{t}/H_1\} \quad (4.1.16)$$

<u>For Chi Square Distributed/ Gamma Distributed $\boldsymbol{T_{ED}}$</u>

Based on the statistics of $\boldsymbol{T_{ED}}$, P_f can be evaluated as,

$$P_f = 1 - Prob\{\boldsymbol{T_{ED}} < \boldsymbol{t}/H_o\} \quad (4.1.17)$$

$$P_f = 1 - \int_{\boldsymbol{0}}^{\boldsymbol{t}} F(t, k = N, \theta = 1/N)dt \quad (4.1.18)$$

$$P_f = 1 - \frac{\gamma(N, N\boldsymbol{t})}{\Gamma(N)} \quad (4.1.19)$$

$$P_f = \frac{\Gamma(N, N\boldsymbol{t})}{\Gamma(N)} \quad (4.1.20)$$

Where $\gamma(,)$ is the lower incomplete Gamma function given by $\gamma(s, x) = \int_0^x t^{s-1}e^{-t}dt$, $\Gamma(s, x)$ is the upper incomplete gamma function and $\Gamma(\ )$ is the Gamma function given by $\Gamma(n) = (\mathrm{n} - 1)!$.

Following the same line of reasoning, we get the expression of P_d as shown below.

$$P_d = \frac{\Gamma\left(N, \frac{N\boldsymbol{t}}{\omega}\right)}{\Gamma(N)} \quad (4.1.21)$$

For Gaussian distributed $\boldsymbol{T_{ED}}$

Based on the statistics of $\boldsymbol{T_{ED}}$ shown in equation , P_f can be evaluated as,

$$P_f = \int_{\boldsymbol{t}}^{\infty} \boldsymbol{T_{ED}}/H_o dt \tag{4.1.22}$$

$$P_f = 1 - \phi(t) \equiv 1 - \frac{1}{2}\left[1 + erf\left[\frac{\boldsymbol{t} - \mu)}{\sqrt{2\sigma^2}}\right]\right] \tag{4.1.23}$$

$$P_f = \frac{1}{2}\left(1 - erf\left[\frac{(\boldsymbol{t} - \mu)}{\sqrt{2\sigma^2}}\right]\right) \tag{4.1.24}$$

$$P_f = \frac{1}{2}\left(erfc\left[\frac{(\boldsymbol{t} - \mu)}{\sqrt{2\sigma^2}}\right]\right) \tag{4.1.25}$$

$$P_f = Q\left(\frac{(\boldsymbol{t} - \mu)}{\sqrt{\sigma^2}}\right) \tag{4.1.26}$$

Where $\phi(t)$ is the Cumulative Distributive Function CDF of Normal Distribution, $erf()$ is the error function given by $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ and $erfc()$ is the complementary error function. Now putting the value of mean and variance for H_0 from equation (4.1.13), we get.

$$P_f = Q\left[(\boldsymbol{t} - 1)\sqrt{N}\right] \tag{4.1.27}$$

Similarly, for the same threshold level $\boldsymbol{T_{ED}}$ the probability of detection is given by

$$P_d = Prob\{\boldsymbol{T_{ED}} > \boldsymbol{t}/H_1\}$$

Following the same line of reasoning, we get the expression of P_d as shown below.

$$P_d = Q\left[\frac{(\boldsymbol{t} - w)\sqrt{N}}{w}\right] \tag{4.1.28}$$

4.2 Hybrid Energy Detection

In this work Noise Variance is estimated on L auxiliary noise only slots and supposing the noise variance is constant over the adjacent slots, we perform independent noise estimation in auxiliary "noise only" slots where we are sure that the primary signal is absent. Since we are sure that there is only noise in each slots, noise variance can be estimated by using all the samples. Consider a sampling window of length M before and adjacent to the detection window which is containing only the noise samples for sure. Then the estimated noise variance from the noise only samples using a Maximum Likelihood noise power estimate can be written as,

$$\hat{\sigma}_v^2 = \frac{1}{M}\sum_{k=1}^{M}|v(k)|^2 \tag{4.2.1}$$

k denotes that the noise only samples are adjacent to the detection window. If the estimated variance is constant, the estimation can be averaged over S successive noise-only slots. Thus, above equation (4.2.1) can be modified by averaging over S successive noise-only slots as,

$$\hat{\sigma}_v^2(S) = \frac{1}{MS}\sum_{j=1}^{S}\sum_{k=1}^{M}|v(k)|^2 \tag{4.2.2}$$

Now the Energy Detection Test Statistic using equation (4.2.1) becomes,

$$\boldsymbol{T_{ED}^{H}} = \left(\frac{1}{N\hat{\sigma}_v^2}\right)\sum_{n=1}^{N}|y(n)|^2 \tag{4.2.3}$$

Here $\boldsymbol{T_{ED}^{H}}$ denotes the detection statistics for the Hybrid Energy Detection scheme and the statistical distribution of $\boldsymbol{T_{ED}^{H}}$ depends upon the particular estimation technique considered. Moreover equation (4.2.3) can be considered as the parametric likelihood ratio test when the signal to be detected is assumed to be Gaussian with zero mean and variance σ_s^2. Now for Null Hypothesis rearranging the above equation (4.2.3) using $y(n) = v(n)$ and equation (4.2.1), we get,

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_0} = \left(\frac{1}{N\hat{\sigma}_v^2}\right)\sum_{n=1}^{N}|v(n)|^2 \qquad (4.2.4)$$

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_0} = \left(\frac{SM}{N}\right)\frac{\sum_{n=1}^{N}\left|\frac{v(n)}{\sigma_v/\sqrt{2}}\right|^2}{\sum_{j=1}^{S}\sum_{k=1}^{M}\left|\frac{v(k)}{\sigma_v/\sqrt{2}}\right|^2} \qquad (4.2.5)$$

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_0} = \left(\frac{MS}{N}\right)\frac{\sum_{n=1}^{N}|\beta_1|^2}{\sum_{j=1}^{S}\sum_{k=1}^{M}|\beta_2|^2} \qquad (4.2.6)$$

Where $\beta_1 = \sqrt{2}v(n)/\sigma_v$, $\beta_2 = \sqrt{2}v(k)/\sigma_v$ and $\sigma_v^2$is the actual noise variance. As $v(n)$ is a zero mean and σ_v^2 variance Gaussian Random Variable, we can easily infer β_1 and β_1 as Standard Normal Random Variables with mean zero and unity variance. The numerator and the denominator of $\boldsymbol{T_{ED}^{H}}\Big|_{H_0}$ in equation (4.2.6) are sum of square of standard normal random variables, thus we can say that the decision statistic $\boldsymbol{T_{ED}^{H}}\Big|_{H_0}$ is the ratio of Chi Square Distributed random variables with 2N and 2MS degrees of freedom respectively, scaled by the factor$\left(\frac{MS}{N}\right)$ noted as,

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_0} = \left(\frac{MS}{N}\right)\frac{\chi_{2N}^2}{\chi_{2MS}^2} \qquad (4.2.7)$$

Finally $\boldsymbol{T_{ED}^{H}}\Big|_{H_0}$ in equation (4.2.7) can be represented in the form of F-Distribution as,

$$\boldsymbol{T_{ED}}|_{H_0} = F(2N, 2MS) \qquad (4.2.8)$$

Similarly for Alternate Hypothesis, considering the channel coefficient as a constant value rather than a vector and rearranging the equation (4.2.3) using $y(n) = hs(n) + v(n)$ and equation (4.2.1), we get,

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_1} = \left(\frac{1}{N\hat{\sigma}_v^2}\right)\sum_{n=1}^{N}|hs(n) + v(n)|^2 \tag{4.2.9}$$

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_1} = \left(\frac{SM\sigma_t^2}{N\sigma_v^2}\right)\frac{\sum_{n=1}^{N}\left|\frac{hs(n)+v(n)}{\sigma_t/\sqrt{2}}\right|^2}{\sum_{j=1}^{S}\sum_{k=1}^{M}\left|\frac{v(-k)}{\sigma_v/\sqrt{2}}\right|^2} \tag{4.2.10}$$

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_1} = \left(\frac{MS\sigma_t^2}{N\sigma_v^2}\right)\frac{\sum_{n=1}^{N}|\alpha|^2}{\sum_{j=1}^{S}\sum_{k=1}^{M}|\beta_2|^2} \tag{4.2.11}$$

Where $\alpha = \sqrt{2}(hs(n) + v(n))/\sigma_t$. As h is assumed to be constant for the sensing interval and both the signal and noise are Gaussian with variances σ_v^2 and σ_s^2 respectively, $hs(n) + v(n)$ is also Gaussian with mean zero and variance $\sigma_t^2 = h^2\sigma_s^2 + \sigma_v^2$. It is clear that α is also a Standard Normal Random Variable with mean zero and unity variance. So both the numerator and the denominator of $\boldsymbol{T_{ED}^{H}}\Big|_{H_1}$ excluding $\left(\frac{\sigma_t^2}{\sigma_v^2}\right)$ are the sum of square of N and MS Complex Normal Random variables respectively, from which we can say that the decision statistic $\boldsymbol{T_{ED}^{H}}\Big|_{H_1}$ is the ratio of Chi Square Distribution with 2N and 2MS degrees of freedom respectively, scaled by the parameter $\left(\frac{SM\sigma_t^2}{N\sigma_v^2}\right)$. If we define Signal to Noise Ratio as $\rho = \left(\frac{\sigma_s^2}{\sigma_v^2}\right)$. Equation (4.2.11) can be re-written as,

$$\boldsymbol{T_{ED}^{H}}\Big|_{H_1} = \left(\frac{SM(h^2\rho + 1)}{N}\right)\frac{\chi_{2N}^2}{\chi_{2MS}^2} \tag{4.2.12}$$

If $\omega = h^2\rho + 1$, then equation (17) can be simplified using F-distribution function as,

$$\boldsymbol{T_{ED}^{H}}\big|_{H_1} = wF(2N, 2MS) \tag{4.2.13}$$

$$\frac{\boldsymbol{T_{ED}^{H}}\big|_{H_1}}{w} = F(2N, 2MS) \tag{4.2.14}$$

Noting the result we get,

$$\begin{cases} \boldsymbol{T_{ED}^{H}}\big|_{H_0} = F(2N, 2MS) \\ \dfrac{\boldsymbol{T_{ED}^{H}}\big|_{H_1}}{w} = F(2N, 2MS) \end{cases} \tag{4.2.15}$$

It is clear from equation (4.2.15) that the decision statistic $\boldsymbol{T_{ED}^{H}}$ follows the F-Distribution for both the hypothesis with different scaling factor.

Normal Approximation of decision statistic

According to the Central Limit Theorem, when N and M are made sufficiently large, the F-Distributed Random Variable in equation (4.2.15) converges to a Gaussian distribution [23]. It gave two approximation models where both transform the C-CDF of F-Distributed Random Variable to a Q-function with different parameter which can be noted as,

Approximation 1

$$C - CDF\left(F|_{N,M}\right) \cong Q(x) \qquad \textit{For all N and M} \tag{4.2.16}$$

Where,

$$x = \frac{\left(\sqrt{(2M-1)\frac{N}{M}F} - \sqrt{2N-1}\right)}{\sqrt{1 + \frac{N}{M}F}}$$

Approximation 2

For Large N and M,

$$C - CDF\left(F|_{N,M}\right) \cong Q(x) \tag{4.2.17}$$

Where,

$$x = \frac{F - \frac{M}{(M-2)}}{\frac{M}{(M-2)}\sqrt{\frac{2(N+M-2)}{N(M-4)}}}$$

Figure 4.4 shows the $Complementary\ CDF$ (C-CDF) plot of **Approximation 1** of F-Distribution using equation (4.2.15), $Complementary\ CDF$ (C-CDF) plot of the original F-Distribution using equation (4.2.16) and $Complementary\ CDF$ (C-CDF) plot of the **Approximation 2** given by equation (4.2.17) for different values of N viz. N = M = 5, 10, 20 and 50 in each subplot. For all values of N and M in each subplot, the **Approximation 1** of F-Distribution closely matches with its original F-distribution counterpart. But in case of **Approximation 2**, if we analyze each subplot in the figure, we can find that $Complementary\ CDF$ (C-CDF) plot with no overlapping with the original F-Distribution C-CDF counterpart for small value of N and M starts to overlap for increasing N and finally overlaps more accurately when N = M = 50.

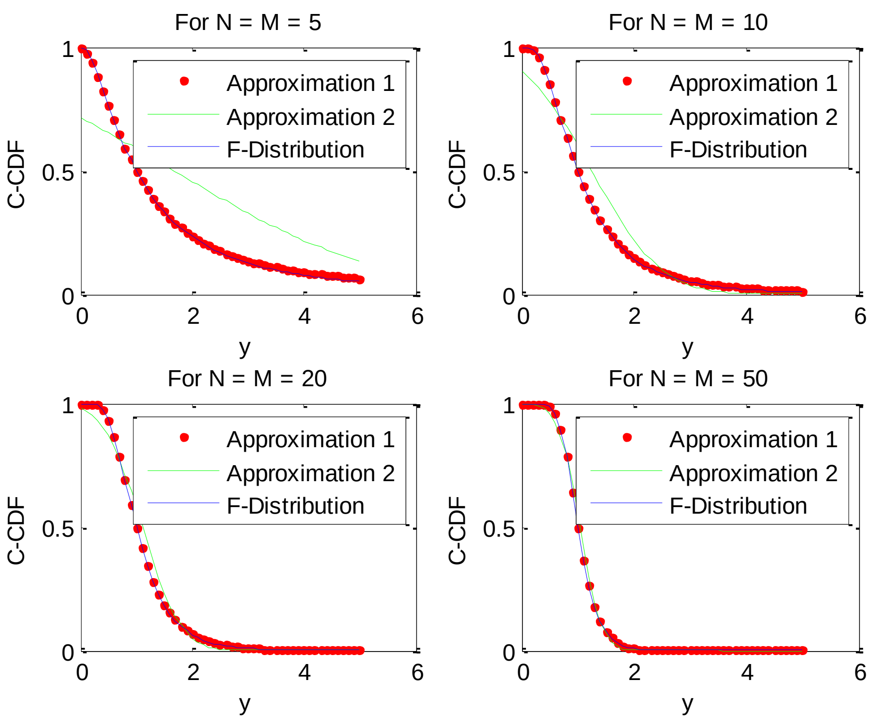


Figure 4.4 Comparison of Gaussian approximation models of F-Distribution for different values of N and M

For the result in equation (4.2.15), using **Approximation 2** for approximating F-Distribution Function to a Normal Distribution Function, we get,

$$\begin{cases} \boldsymbol{T_{ED}^{H}}\big|_{H_0} = N_{¢}\left(\alpha, \dfrac{\alpha^2(2N+2MS-1)}{4N(MS-1)}\right) \\ \dfrac{\boldsymbol{T_{ED}^{H}}\big|_{H_1}}{w} = N_{¢}\left(\alpha, \dfrac{\alpha^2(2N+2MS-1)}{4N(MS-1)}\right) \end{cases} \tag{4.2.18}$$

Where,

$$\alpha = \frac{2MS}{(2MS-1)}$$

Expression for detection and false alarm Probabilities

The probabilities of false alarm P_f^H for a given threshold energy $\boldsymbol{T_{ED}^{H}}$ is given by,

$$P_f^H = Prob\{\boldsymbol{T_{ED}^{H}} > \boldsymbol{t}/H_o\} \tag{4.2.19}$$

Similarly, for the same threshold level $\boldsymbol{T_{ED}^{H}}$ the probability of detection is given by,

$$P_d^H = Prob\{\boldsymbol{T_{ED}^{H}} > \boldsymbol{t}/H_1\} \tag{4.2.20}$$

<u>For F-Distributed $\boldsymbol{T_{ED}^{H}}$</u>

Based on the statistics of $\boldsymbol{T_{ED}^{H}}$ as in equation (4.2.15), P_f^H can be evaluated as,

$$P_f^H = 1 - Prob\{\boldsymbol{T_{ED}^{H}} < \boldsymbol{t}/H_o\} \tag{4.2.21}$$

$$P_f^H = 1 - \int_{\boldsymbol{0}}^{\boldsymbol{t}} f_{\boldsymbol{T_{ED}^{H}}}(t, 2N, 2MS)dt \tag{4.2.22}$$

$$P_f^H = 1 - I\left(2N, 2MS, \frac{N\boldsymbol{t}}{N\boldsymbol{t}+MS}\right) \tag{4.2.23}$$

$$P_f^H = I\left(2N, 2MS, \frac{MS}{N\boldsymbol{t} + MS}\right) \tag{4.2.24}$$

Where $I(,)$ is the incomplete Beta function which can be expressed with its integral form $I(a,b,z) = \left(\frac{1}{B(a,b)}\right)\int_0^z t^{a-1}(1-t)^{b-1}dt$, and $B(a,b)$ is the complete Beta function.

Following the same line of reasoning, we get the expression of P_d as shown below.

$$P_d^H = I\left(2N, 2MS, \frac{MSw}{N\boldsymbol{t} + MSw}\right) \tag{4.2.25}$$

<u>For Gaussian distributed T_{ED}^H following Approximation 2</u>

Based on the statistics of $\boldsymbol{T_{ED}^H}$ in equation (4.2.18), P_f^H can be evaluated as,

$$P_f^H = Q\left[\frac{(\boldsymbol{t} - \alpha)}{\alpha\sqrt{\frac{(N + MS - 1)}{N(MS - 2)}}}\right] \tag{4.2.26}$$

Where $Q(x)$ is the Q-function which is also known as the tail probability of the Standard Normal Distribution. Similarly, for the same threshold level $\boldsymbol{T_{ED}}$ the probability of detection is given by,

$$P_d^H = Prob\{\boldsymbol{T_{ED}} > \boldsymbol{t}/H_1\} \tag{4.2.27}$$

Following the same line of reasoning, we get the expression of P_d as shown below.

$$P_d^H = Q\left[\frac{(\boldsymbol{t} - \alpha w)}{\alpha w\sqrt{\frac{(N + MS - 1)}{N(MS - 2)}}}\right] \tag{4.2.28}$$

4.3 Hybrid Energy Detection 2

In this work Noise Variance is estimated on S auxiliary noise only slots and supposing the noise variance is constant over the adjacent slots, we perform independent noise estimation in auxiliary "noise only" slots which are declared H_0 by ED. Consider a sampling window of length M before and adjacent to the detection window which is containing only the noise samples, the estimated noise variance from the noise only samples using a Maximum Likelihood noise power estimate can be written as,

$$\hat{\sigma}_v^2 = \frac{1}{M}\sum_{k=1}^{M}|y(k)|^2 \tag{4.3.1}$$

k is used to denote that the received signal slot with noise only samples is adjacent to the detection window.

If the estimated variance is constant, the estimation can be averaged over S successive noise-only slots. Noting that since each auxiliary slot are declared Noise only slot by ED, there is a chance with probability $(1 - P_d^{ED})$ that ED decides in favor of H_0 even if the received signal is the sum of noise and the primary signal. If the estimated variance is the mean of S successive noise variances calculated from the noise only slots decided by ED, then we can easily say that out of S noise variance estimates from S noise only slots decided by ED, $P_s \times (1 - P_d^{ED})S$ noise variance estimates are calculated from the received signal containing noise and the primary signal. P_d^{ED} is the probability of detection of ED and P_s is the probability of received signal containing primary signal. Thus the average noise variance or estimated noise variance from the noise only samples declared by ED in equation (4.3.1) can now be modified as,

$$\hat{\sigma}_v^2(S) = \frac{1}{MS}\left[\sum_{j=1}^{S_S}\sum_{k=1}^{M}|h(k)s(k)+v(k)|^2 + \sum_{j=1}^{S_N}\sum_{k=1}^{M}|v(k)|^2\right] \quad (4.3.2)$$

Where $S_s = P_s \times (1 - P_d^{ED})S$ and $S_N = S - S_S$

Now the Energy Detection Test Statistic in equation (3) using equation (7) becomes,

$$\boldsymbol{T_{ED}^{H2}} = \left(\frac{1}{N\hat{\sigma}_v^2}\right)\sum_{n=1}^{N}|y(n)|^2 \quad (4.3.3)$$

Here $\boldsymbol{T_{ED}^{H2}}$ denotes the detection statistics for the Hybrid Energy Detection-2 scheme and the statistical distribution of $\boldsymbol{T_{ED}^{H2}}$ depends upon the particular estimation technique considered. Moreover equation (4.3.3) can be considered as the parametric likelihood ratio test when the signal to be detected is assumed to be Gaussian with zero mean and variance σ_s^2. Now for Null Hypothesis, considering the channel coefficient as a constant value rather than a vector and rearranging the above equation (8) using $y(n) = v(n)$ and equation (4.3.2), we get,

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_0} = \left(\frac{1}{N\hat{\sigma}_v^2}\right)\sum_{n=1}^{N}|v(n)|^2 \quad (4.3.4)$$

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_0} = \left(\frac{MS}{N}\right)\left[\frac{\sum_{j=1}^{S_S}\sum_{k=1}^{M}|hs(k)+v(k)|^2}{\sum_{n=1}^{N}|v(n)|^2} + \frac{\sum_{j=1}^{S_N}\sum_{k=1}^{M}|v(k)|^2}{\sum_{n=1}^{N}|v(n)|^2}\right]^{-1} \quad (4.3.5)$$

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_0} = \left(\frac{MS}{N}\right)\left[\frac{\sigma_t^2}{\sigma_v^2}\frac{\sum_{j=1}^{S_S}\sum_{k=1}^{M}|\alpha_2|^2}{\sum_{n=1}^{N}|\beta_1|^2} + \frac{\sum_{j=1}^{S_N}\sum_{k=1}^{M}|\beta_2|^2}{\sum_{n=1}^{N}|\beta_1|^2}\right]^{-1} \quad (4.3.6)$$

Where $\beta_1 = \sqrt{2}v(n)/\sigma_v$, $\beta_2 = \sqrt{2}v(k)/\sigma_v$, $\alpha_2 = \sqrt{2}(hs(k) + v(k))/\sigma_t$and σ_v^2& σ_t^2 are actual noise and signal variances respectively. As $v(n)$ is a zero mean and σ_v^2 variance Complex Gaussian Random Variable, we can easily infer β_1 and β_1 as Standard Normal Random Variables with mean zero and unity variance. Similarly, as h is assumed to be constant for the sensing interval and both the signal and noise are Gaussian with variances σ_v^2 and σ_s^2 respectively, $hs(n) + v(n)$ is also Gaussian with mean zero and variance $\sigma_t^2 = h^2\sigma_s^2 + \sigma_v^2$. So it is clear that α_2 is also a Standard Normal Random Variable with mean zero and unity variance. Thus each summation is the sum of square of Standard normal random variable thus each follows a Chi Square Distribution with degree of freedom equal to the summation order as shown in equation (4.3.7) below,

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_0} = \left(\frac{MS}{N}\right)\left[\frac{\sigma_t^2}{\sigma_v^2}\frac{\chi^2_{2MS_S}}{\chi^2_{2N}} + \frac{\chi^2_{2MS_N}}{\chi^2_{2N}}\right]^{-1} \tag{4.3.7}$$

If we define Signal to Noise Ratio as $\rho = \left(\frac{\sigma_s^2}{\sigma_v^2}\right)$ and $\omega = h^2\rho + 1$, $\boldsymbol{T_{ED}^{H2}}\Big|_{H_0}$ in equation (4.3.7) can be represented in the form as shown below,

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_0} = \left(\frac{MS}{N}\right)\left[w\frac{\chi^2_{2MS_S}}{\chi^2_{2N}} + \frac{\chi^2_{2MS_N}}{\chi^2_{2N}}\right]^{-1} \tag{4.3.8}$$

Similarly for Alternate Hypothesis, rearranging the equation (4.3.3) using $y(n) = hs(n) + v(n)$ and equation (4.3.2), we get,

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_1} = \left(\frac{1}{N\hat{\sigma}_v^2}\right)\sum_{n=1}^{N}|hs(n) + v(n)|^2 \tag{4.3.9}$$

Following the same line of reasoning as in case of Null Hypothesis, $\boldsymbol{T_{ED}^{H2}}\Big|_{H_1}$ can be written as shown below,

$$\boldsymbol{T_{ED}^{H2}}\Big|_{H_1} = \left(\frac{MS}{N}\right)\left[\frac{\chi^2_{2MS_S}}{\chi^2_{2N}} + \frac{1}{w}\frac{\chi^2_{2MS_N}}{\chi^2_{2N}}\right]^{-1} \tag{4.3.10}$$

Noting the result we get,

$$\begin{cases} \boldsymbol{T_{ED}^{H2}}\big|_{H_0} = \left(\frac{MS}{N}\right)\left[w\frac{\chi^2_{2MS_s}}{\chi^2_{2N}} + \frac{\chi^2_{2MS_N}}{\chi^2_{2N}}\right]^{-1} \\ \boldsymbol{T_{ED}^{H2}}\big|_{H_1} = \left(\frac{MS}{N}\right)\left[\frac{\chi^2_{2MS_s}}{\chi^2_{2N}} + \frac{1}{w}\frac{\chi^2_{2MS_N}}{\chi^2_{2N}}\right]^{-1} \end{cases} \tag{4.3.11}$$

<u>For Large N and M</u>

For large N and M, equation (4.3.11) with Chi Square Distributions in numerator and denominators can be approximated with their Normal approximates given by the approximation formula $\chi^2_N \cong N_{¢}(N/2, N))$. Thus, simplifying the expression of $\boldsymbol{T_{ED}^{H2}}$ for Null hypothesis in equation (4.3.11) using the normal approximates, we get,

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_0} = \frac{MS}{N}\chi^2_{2N}\left[w\chi^2_{2MS_s} + \chi^2_{2MS_N}\right]^{-1} \tag{4.3.12}$$

$$T_{ED}^{H2}\big|_{H_0} = \frac{MS}{N}\chi^2_{2N}\left[w\chi^2_{2MS_s} + \chi^2_{2MS_N}\right]^{-1} \tag{4.3.13}$$

$$\begin{aligned} \boldsymbol{T_{ED}^{H2}}\big|_{H_0} = \frac{MS}{N} N_R(2N, 4N)[wN_R(2MS_s, 4MS_s) \\ + N_R(2MS_N, 4MS_N)]^{-1} \end{aligned} \tag{4.3.14}$$

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_0} = S\,N_R\left(2, \frac{4}{N}\right)\left[N_R\left(2(wS_s + S_N), \frac{4}{M}(S_s w^2 + S_N)\right)\right]^{-1} \tag{4.3.15}$$

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_0} = \frac{N_R\left(S, \frac{S^2}{N}\right)}{N_R\left((wS_s + S_N), \frac{1}{M}(S_s w^2 + S_N)\right)} \tag{4.3.16}$$

Now, the test statistic in equation (4.3.16) which is a ratio of two Normal Random variables can be represented in a single Normally Distributed Random Variable as,

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_0} = N_R\left(\frac{S}{(S_s w + S_N)}, \qquad \frac{\boldsymbol{t}^2 N(S_s w^2 + S_N) + MS^2}{MN(S_s w + S_N)^2}\right) \tag{4.3.17}$$

For the result in equation (4.3.11) for alternate hypothesis, using the same line of reasoning we can evaluate the expression of the decision statistics as shown below,

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_1} = N_R\left(\frac{S}{\left(S_s + \frac{S_N}{w}\right)}, \qquad \frac{\boldsymbol{t}^2 N\left(S_s + \frac{S_N}{w^2}\right) + MS^2}{MN\left(S_s + \frac{S_N}{w}\right)^2}\right) \tag{4.3.18}$$

Thus, noting the final result, we get,

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_0} = N_R\left(\frac{S}{(S_s w + S_N)}, \qquad \frac{\boldsymbol{t}^2 N(S_s w^2 + S_N) + MS^2}{MN(S_s w + S_N)^2}\right) \tag{4.3.19}$$

$$T_{ED}^{H2}|_{H_1} = N_R\left(\frac{S}{\left(S_s + \frac{S_N}{w}\right)}, \qquad \frac{\boldsymbol{t}^2 N\left(S_s + \frac{S_N}{w^2}\right) + MS^2}{MN\left(S_s + \frac{S_N}{w}\right)^2}\right) \tag{4.3.20}$$

$$\equiv$$

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_0} = N_R(\,\mu_1, \sigma_1^2) \tag{4.3.21}$$

$$\boldsymbol{T_{ED}^{H2}}\big|_{H_1} = N_R(\mu_2, \sigma_1^2) \tag{4.3.22}$$

Where,

$$\mu_1 = \frac{S}{(S_s w + S_N)}$$

$$\sigma_1^2 = \frac{\boldsymbol{t}^2 N(S_s w^2 + S_N) + MS^2}{MN(S_s w + S_N)^2}$$

$$\mu_2 = \frac{S}{\left(S_s + \frac{S_N}{w}\right)}$$

$$\sigma_1^2 = \frac{\boldsymbol{t}^2 N\left(S_s + \frac{S_N}{w^2}\right) + MS^2}{MN\left(S_s + \frac{S_N}{w}\right)^2}$$

Expression for detection and false alarm Probabilities

The probabilities of false alarm P_f^{H2} for a given threshold energy $\boldsymbol{T_{ED}^{H2}}$ is given by,

$$P_f^{H2} = Prob\{\boldsymbol{T_{ED}^{H2}} > \boldsymbol{t}/H_o\} \tag{4.3.23}$$

Similarly, for the same threshold level $\boldsymbol{T_{ED}^{H2}}$ the probability of detection is given by,

$$P_d^{H} = Prob\{\boldsymbol{T_{ED}^{H2}} > \boldsymbol{t}/H_1\} \tag{4.3.24}$$

Based on the statistics of $\boldsymbol{T_{ED}^{H2}}$ shown in equation(4.3.28) , P_f^{H2} can be evaluated as,

$$P_f^{H2} = \int_{\boldsymbol{t}}^{\infty} \boldsymbol{T_{ED}^{H2}}/H_o dt \tag{4.3.25}$$

$$P_f^{H} = 1 - \phi(t) \equiv 1 - \frac{1}{2}\left[1 + erf\left[\frac{\boldsymbol{t} - \mu)}{\sqrt{2\sigma^2}}\right]\right] \tag{4.3.26}$$

$$P_f^{H} = \frac{1}{2}\left(erfc\left[\frac{(\boldsymbol{t} - \mu)}{\sqrt{2\sigma^2}}\right]\right) \tag{4.3.27}$$

$$P_f^{H} = Q\left(\frac{(\boldsymbol{t} - \mu)}{\sqrt{\sigma^2}}\right) \tag{4.3.28}$$

Where $\phi(t)$ is the Cumulative Distributive Function CDF of Normal Distribution, $erf()$ is the error function given by $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt$ and $erfc()$ is the complementary error function. Now putting the value of mean and variance for H_0 from equation (31), we get,

$$P_f^{H2} = Q\left[\frac{(\boldsymbol{t} - \mu_1)}{\sqrt{\sigma_1^2}}\right] \tag{4.3.29}$$

Similarly, for the same threshold level $\boldsymbol{T_{ED}^{H2}}$ the probability of detection is given by,

$$P_d^{H2} = Prob\{\boldsymbol{T_{ED}^{H2}} > \boldsymbol{t}/H_1\} \tag{4.3.30}$$

Following the same line of reasoning and using equation(4.3.21), we get the expression of P_d as shown below.

$$P_d^{H2} = Q\left[\frac{(\boldsymbol{t} - \mu_2)}{\sqrt{\sigma_2^2}}\right] \tag{4.3.31}$$

CHAPTER 5: METHODOLOGY

Objective of every spectrum sensing scheme is to find out the detection statistic which can be used in the decision making by comparing the detection statistic with the threshold value. In context to Energy Detector, the detection statistic can be obtained by integrating the energy of the signal over certain time interval T.

The purpose of this Energy Detection technique is to find the presence or absence of primary network signal in a certain frequency band licensed to primary user's network by using "Energy Detection and Hybrid Energy Detection method. The input for energy detector is x(t) which is a unknown primary signal with known amplitude distorted by band unlimited white Gaussian Noise. Signal is present only within a certain bandwidth (W).Thus we can filter the signal x(t) using a band pass filter which removes the noise present in other than the signal pass band. Now for the calculation of energy over a time period 'T' we can simply integrate the squared signal of x(t) i.e. $x^2(t)$ over the defined time period.

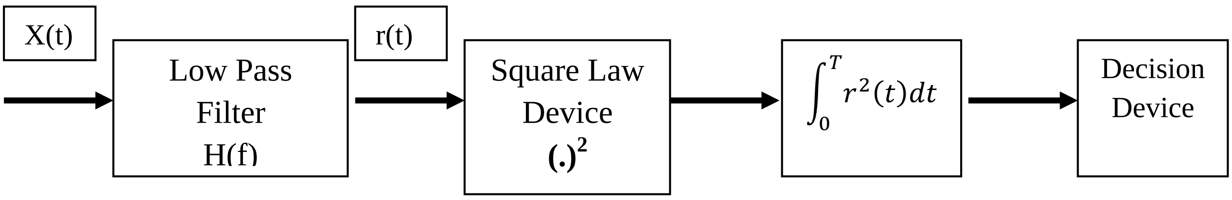


Fig 5.1 Energy based detector

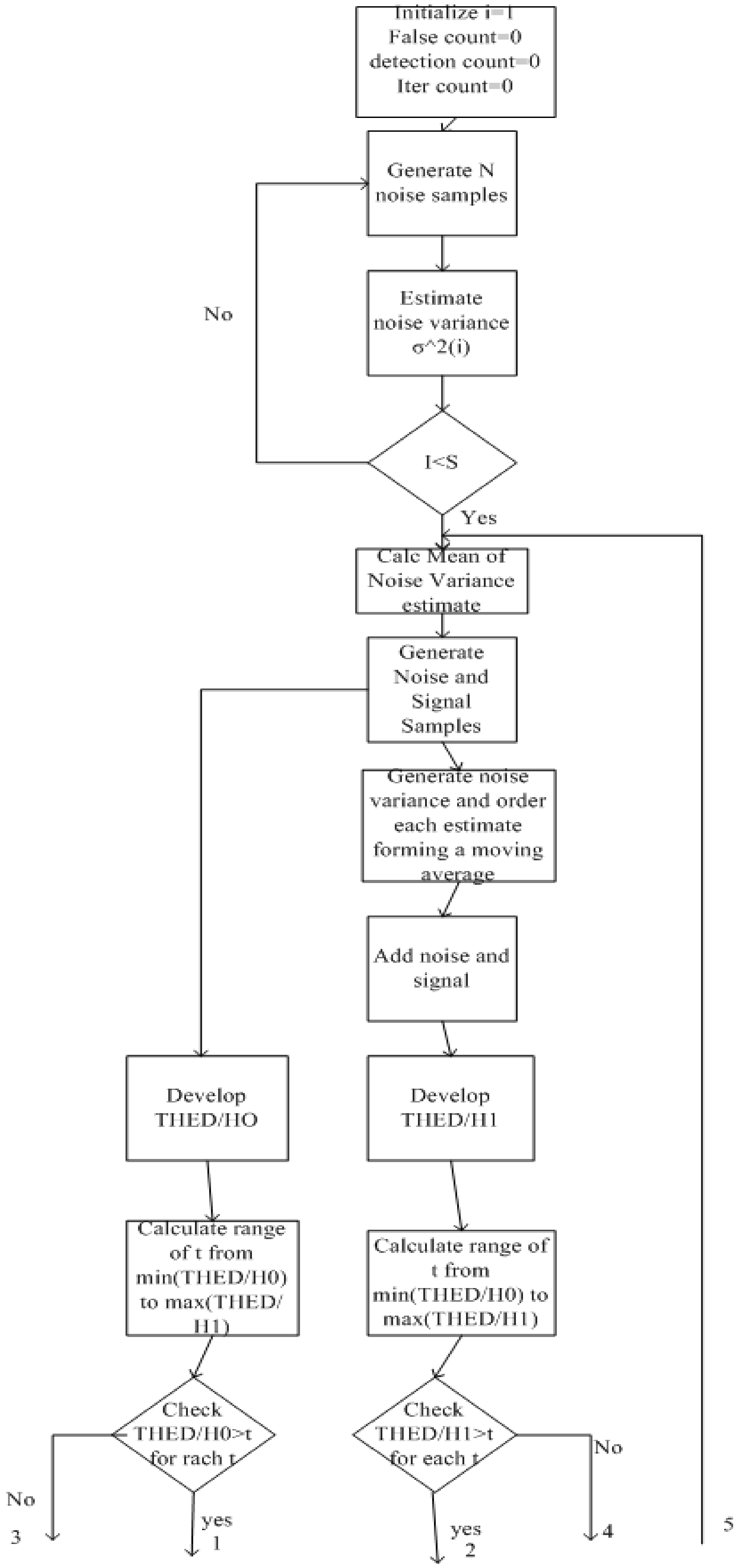
Using the information of the received signal matrix Y to develop a test statistic T_{ED}, which is the measure of the average energy of the received signal over a sensing interval N, the detector compares T_{ED} against a predefined threshold t. If $T_{ED} < t$ then it decides in favor of Null Hypothesis H_0 indicating absence of primary users otherwise in favor of Alternate Hypothesis H_1 indicating presence of primary user i.e. The obtained energy is then compared with the threshold value and determines whether the primary user is active or not. The threshold can be calculated based on two principles: constant false alarm rate (CFAR) and constant detection rate (CDR). In both CFAR and CDR cases, the noise power is needed .

The knowledge of the noise power is one of the critical limitations of ED for its operation in low SNR. The only option is to estimate the noise power. A straight forward and even reasonable way is to treat the estimate of the noise power as the true noise power and use in Energy Detection accordingly. For any description model, it would be useful to have a concise description of the ROC curves that is meaningful in terms of underlying signal and noise distributions. Such a description would facilitate the comparison of model with experiment, and help in the assessment of effect of changing model parameter.

The ROC curve is obtained by plotting the probability of correct detection versus the probability of false alarm. In order to compare the performances for different threshold values ROC curves can be used. ROC curves allows us to explore the relationship between the sensitivity (Probability of detection) and specificity (Probability of false alarm) of a sensing method for a variety of different threshold, thus allowing the determination of a optimal threshold. Probability of detection provides us an estimate of the detectability of the receiver and tells us about the capacity of the receiver to detect the signal during the transmission of the signal by the primary user. Better the probability of detection, better would be the utilization of the frequency band. In contrast, probability of false alarm provides us an estimate of misdetection capability of the receiver in presence of channel noise. Larger the probability of false alarm, higher would be the interference from the secondary users to the primary users.

5.1 HED1

S (assumed number) auxiliary noise only slots where we are sure that the primary signal is absent is considered and Noise variance is estimated from those slots. For Energy Detection with noise power estimation from noise only slots, which we call Hybrid Energy Detection (HED) model, the analytical expression of performance parameters P_D and P_{Fa} is derived and simulated as expressed in above chapter. We consider a sampling window of length M before and adjacent to the detection window which is containing noise only samples for sure.


Initialize i=1
False count=0
detection count=0
Iter count=0
Generate N noise samples
Estimate noise variance σ^2(i)
No
I<S
Yes
Calc Mean of Noise Variance estimate
Generate Noise and Signal Samples
Generate noise variance and order each estimate forming a moving average
Add noise and signal
Develop THED/HO
Develop THED/H1
Calculate range of t from min(THED/H0) to max(THED/H1)
Calculate range of t from min(THED/H0) to max(THED/H1)
Check THED/H0>t for rach t
Check THED/H1>t for each t
No
3
yes
1
yes
2
No
4
5

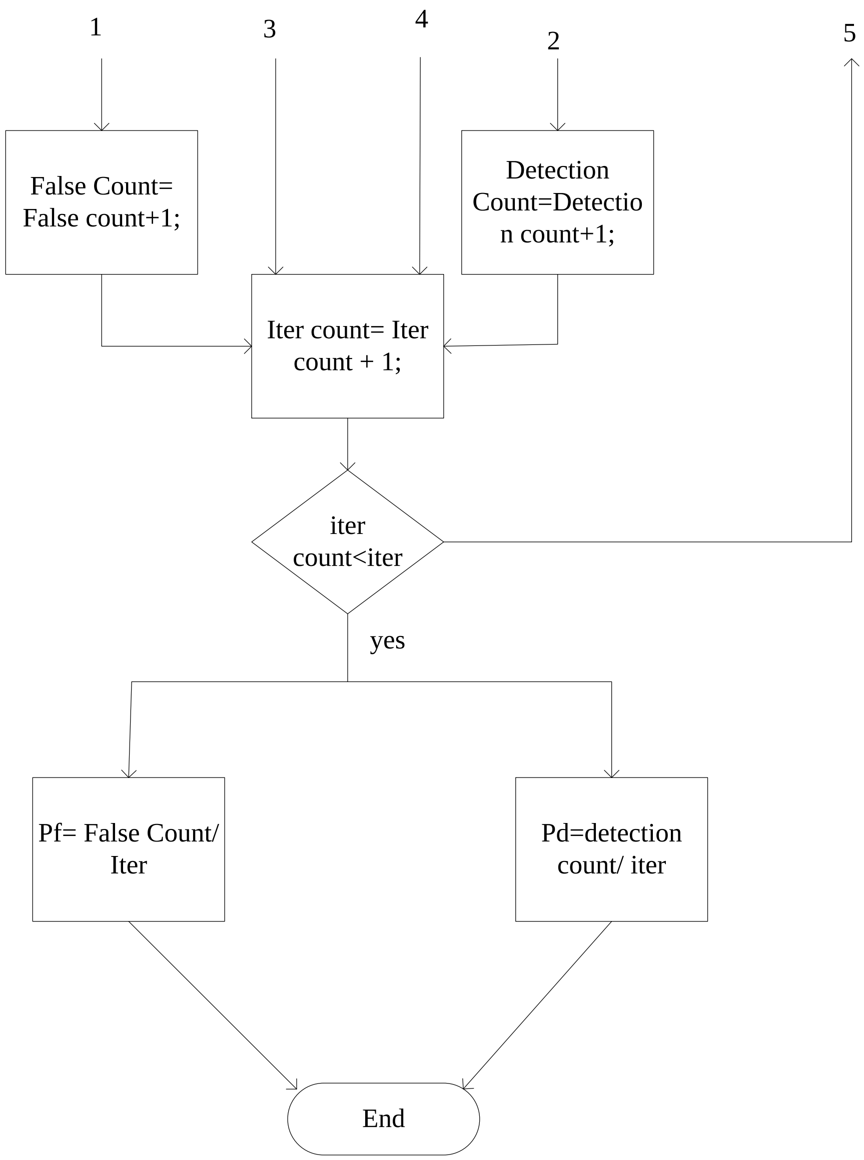
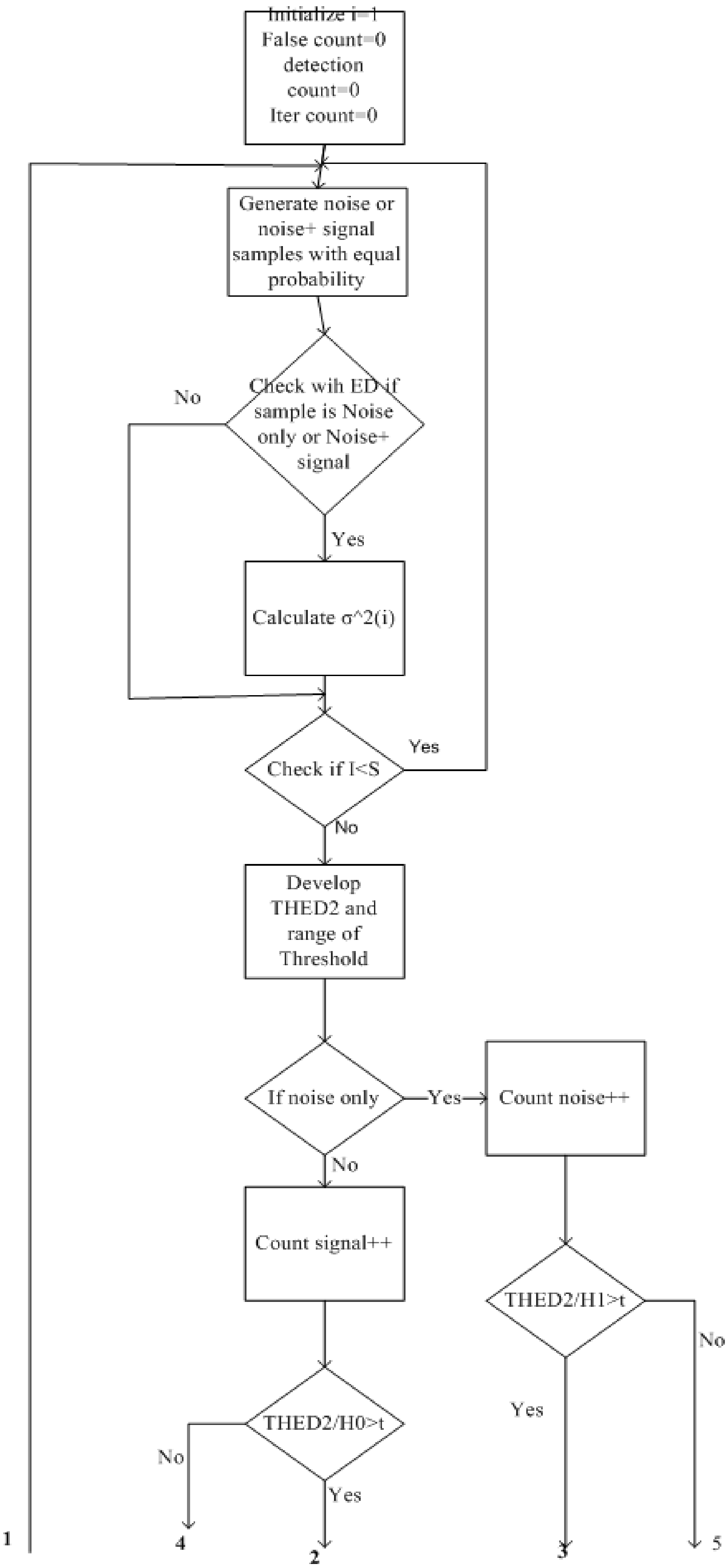


Figure 5.1 Flowchart for HED1

5.2 HED2

In HED2, Noise Variance is estimated on S auxiliary noise only slots and supposing the noise variance is constant over the adjacent slots, we perform independent noise estimation in auxiliary "noise only" slots which are declared H_0 by ED by considering a sampling window of length M before and adjacent to the detection window which is containing only the noise samples.


Initialize i=1
False count=0
detection
count=0
Iter count=0
Generate noise or
noise+ signal
samples with equal
probability
Check wih ED if
sample is Noise
only or Noise+
signal
No
Yes
Calculate σ^2(i)
Check if I<S
Yes
No
Develop
THED2 and
range of
Threshold
If noise only
Yes
Count noise++
No
Count signal++
THED2/H1>t
No
Yes
THED2/H0>t
No
Yes
1
4
2
3
5

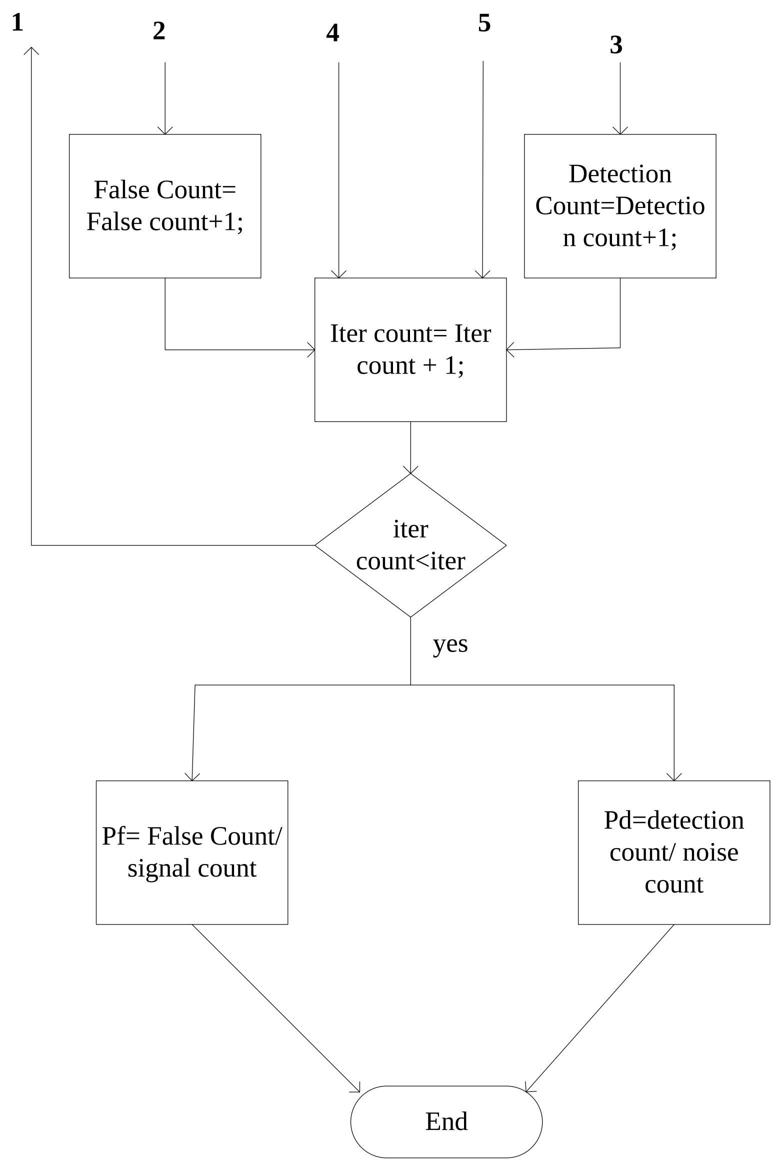


Fig: 5.2 Flowchart for HED2

CHAPTER 6 :RESULTS AND DISCUSSION

This section implements the simulation of Energy Detection in single sensor environment. Signal, channel and the noise environment is set in such a way it matches the scenario explained in previous chapters. Figure 6.1 illustrates the ROC plot of single sensor Energy Detector with its detection statistic following Normal distribution. Analytical result of ROC for Energy Detection computed for SNR = -10dB, number of sensors K = 1 and N=20, 50, 100 and 200 respectively where N represents the number of samples taken to calculate the decision statistics. This figure shows high detection probability with higher number of samples (i.e. N=200) taken to calculate decision statistics and the proper detection rate decreases simultaneously with lower number of samples considered.

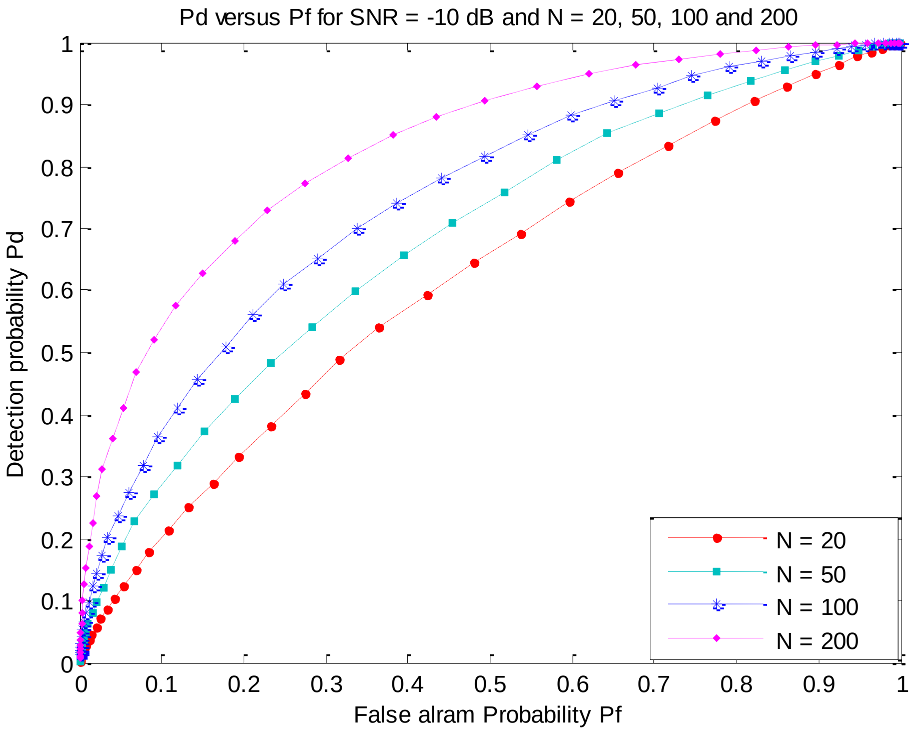


Figure 6.1: ROC plot of Energy Detection with SNR = -10dB for varying number of samples

Fig 6.2 shows the performance of the energy detector in AWGN channel. It illustrates ROC curve for the AWGN channel condition for varying SNR (dB). For N = 5 and SNR varying from 10 dB to 0 dB, this curve shows the energy detector capabilities degrade rapidly when the average SNR of the channel decreasing from 10 dB to 0 dB. Thus, we see from these two figures SNR and number of samples are important parameters to be considered for higher performance.

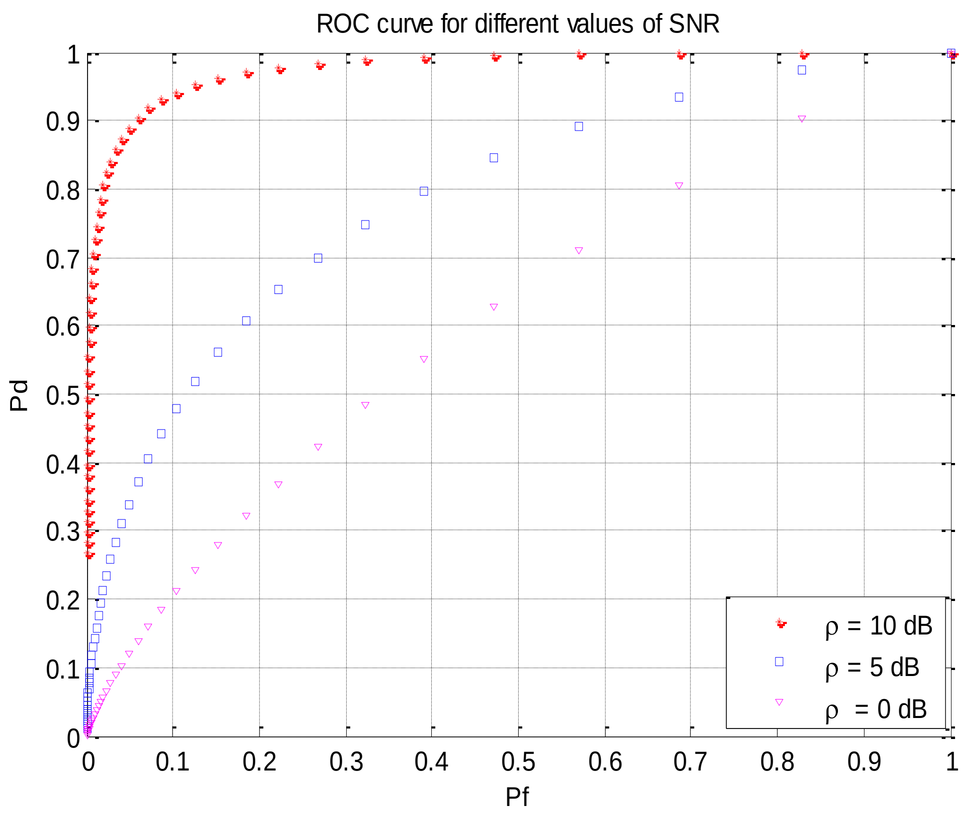


Figure 6.2 Receiver Operating Characteristic curves of an Energy Detector over AWGN channel at different SNR and N = 5

Figure 6.3 illustrates the performance analysis plot of single sensor Energy Detector for SNR varying from -16db to 4db with P_{fa}= -10dB, number of sensors K = 1 and N=20, 50, 100 and 200 respectively where N represents the number of samples taken to calculate the decision statistics. This figure also validates our result in figure 6.1 which shows the detection performance increases with higher number of samples to calculate the decision statistics.

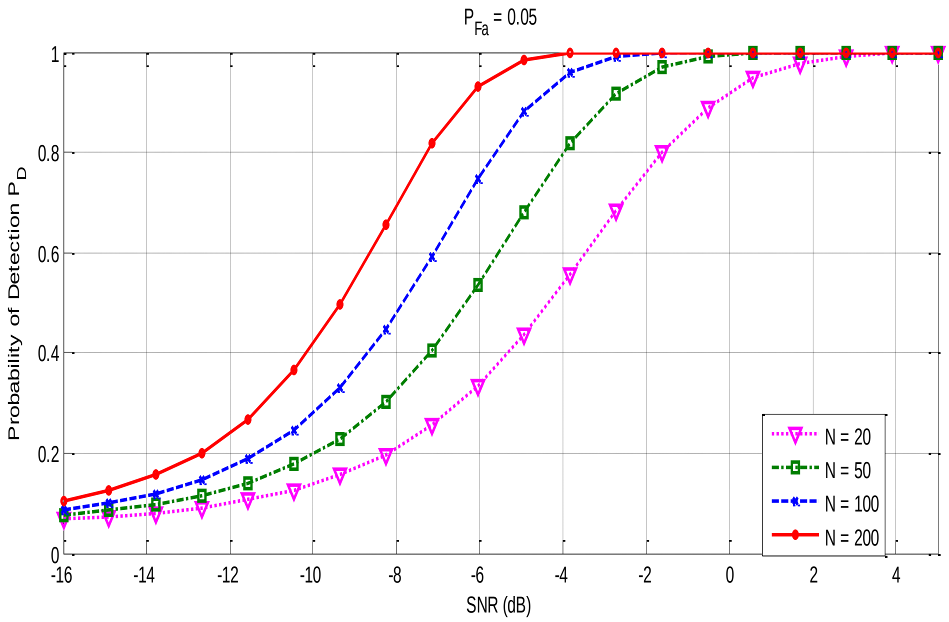


Figure 6.3. Probability of Detection vs. SNR for P_{fa}=0.05

The accuracy of the closed-form expression of the theoretical formula is compared against simulated detection performance over S auxiliary noise only slots (S ranges from 1 to 10). It can be realized from Figure 6.4 that the analytical and the numerical curves are perfectly matching which validates the analytical expressions. Also, it can be noted that, the increase in number of slots for noise variance estimation correspondingly increases the performance of HED and approaches closer to the optimal one (ED with known noise variance). Under the considered scenario with M = N = 400 which is the number of noise samples in each slot, just S = 10(i.e., 1000 samples) provides very near convergence to ideal performance. For each numerical curve, its analytical counterpart is superimposed to evaluate the accuracy of the model for different values of S.

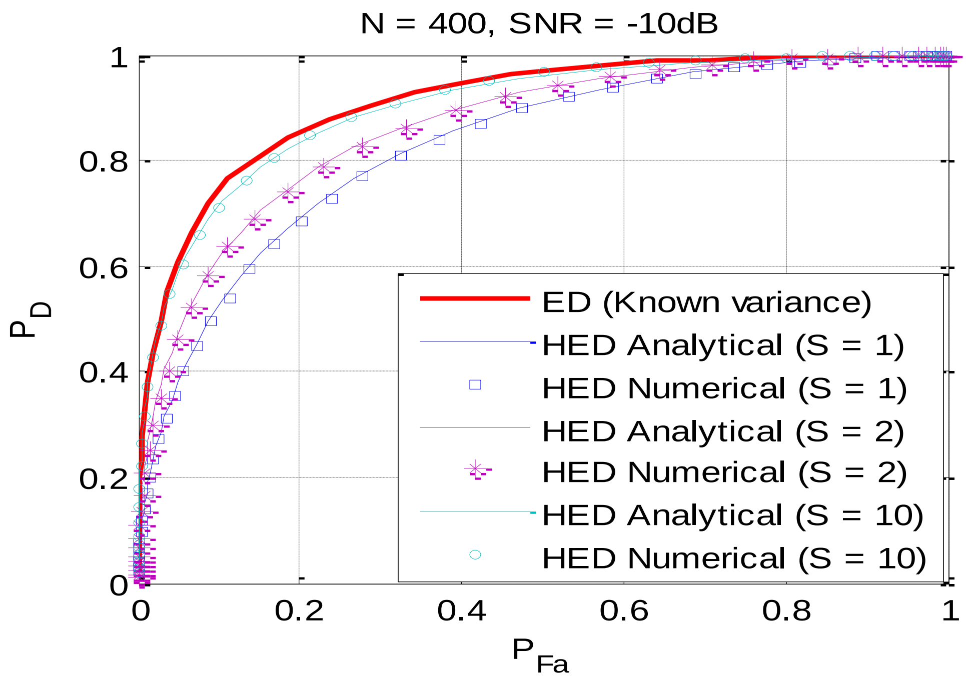


Figure 6.4 ROC curve of Single Sensor Hybrid Energy Detection for N = 100, M = 100, S = {1, 3, 5 and 10}, $\sigma_v^2 = 400$, $\sigma_s^2 = 40$, SNR = -10dB

For given probability of false alarm $P_{Fa} = 0.05$ and considered parameter (K = 1;N = 50;M = 50), the performance of Hybrid Energy Detection is also evaluated in terms of probability of detection against different values of SNR as shown in figure 6.5. This validates our result in figure 6.1 as well showing that the increase in number of slots for noise variance estimation correspondingly increases the performance of HED. The probability of detection is increased with increase in number of slots and plot for S=10 almost coincide with the optimal one (known noise variance)

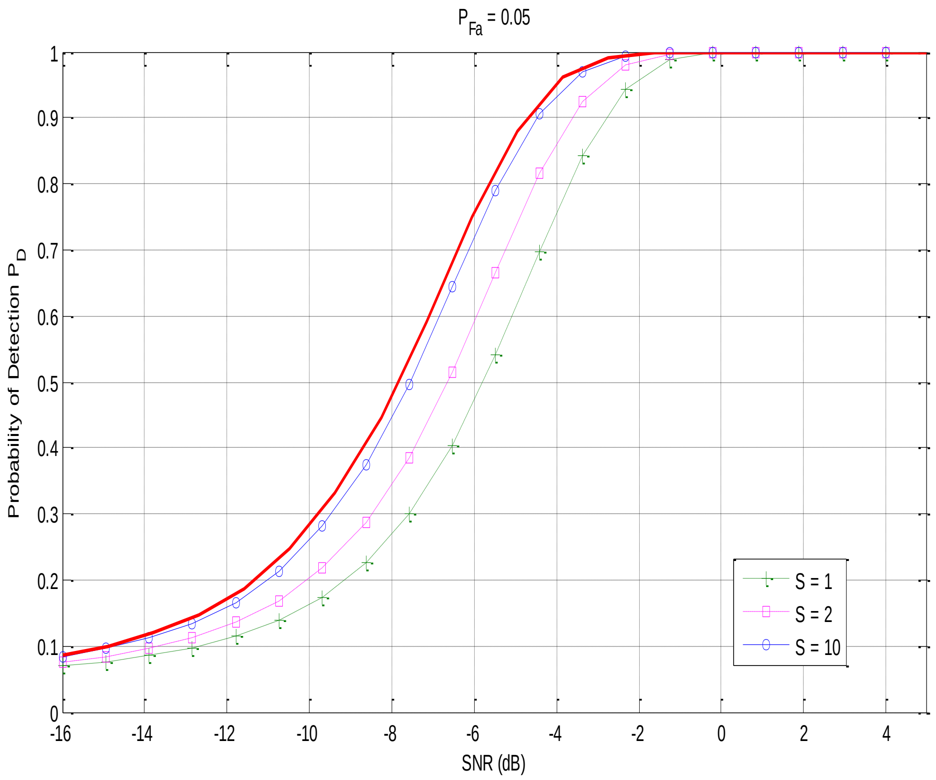


Figure 6.5 Probability of detection vs SNR for Hybrid Energy Detection

Figure 6.6 illustrates the simulation of Single Sensor HED2 ROC curves when the noise variance is independently estimated by applying the obtained equation over S auxillary only noise slots determined by ED and the estimate $\hat{\sigma}_v^2$ is used in equation (3) for all other slots recursively. For generating the HED2 performance curve, ED parameters within HED2 are, N = 10, pd_ed = 0.5, assuming signal probability 0.5 and noise probability 0.5. Since, there is a chance of misdetection in case of HED2, performance of HED2 is slightly lower than HED but still no visible difference can be noted in extreme high or low SNR values. With the increase of the number of slots used for the estimation of the noise variance, HED and HED2 curves approximate the ED with known variance.

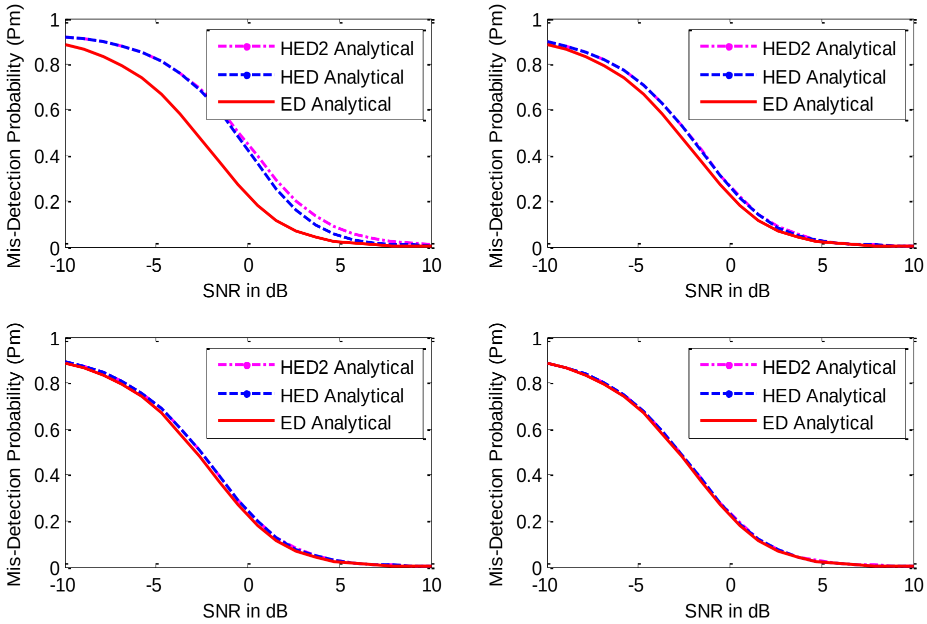


Figure 6.6 P_m vs SNR plot of Single Sensor Hybrid Energy Detection-2 for Gaussian Approximation of the Decision Statistics for N = 10, S = (2, 10, 20, 50), M = 10, SNR = -10dB to 10 dB, $h = 1$ and false alarm probability pf = 0.1.

CHAPTER 7: CONCLUSION

As the demand of radio spectrum increases in past few years and licensed bands are used inefficiently, improvement in the existing spectrum access policy is expected. Dynamic spectrum access is imagine to resolve the spectrum shortage by allowing unlicensed users to dynamically utilize spectrum holes across the licensed spectrum on no interfering basis. In this study, different methods of existing spectrum sensing were studied and the performance of different channels is analyzed terms of Receiver Operating Characteristic (ROC) curves. Objective of every spectrum sensing scheme is to find out the detection statistic which can be used in the decision making by comparing the detection statistic with the threshold value. In context to Energy Detector, the detection statistic can be obtained by integrating the energy of the signal over certain time interval T. In this thesis, the analysis of semi-blind spectrum sensing algorithms, especially, ED is carried out in context to CRN. The analysis is then extended to hybrid approaches of ED. Analytical expressions for the performance parameters, P_D and P_{Fa} is derived for each algorithms. Analytical results are verified by simulation and by numerical methods. Impact of noise variance estimation on ED was carried out based on ROC curves and Probability of detection vs SNR curves. The results showed that the effect of fluctuation of noise variance estimate from nominal value is severe in case of small number of auxiliary slots used for the estimation of noise variance. High detection probability is attained with higher number of samples taken to calculate decision statistics and the proper detection rate decreases simultaneously with lower number of samples considered to calculate the decision statistics. Even at low SNR, the performance of ED, HED and HED2 is better in regards to probability of detection if higher number of samples is considered to calculate the decision statistics.

This thesis only considers presence of single sensor and can be extended considering the presence of multiple sensors to detect the presence of primary users for higher detection probability even with lower number of samples considered to calculate the decision statistics. It also can be extended to develop simple and computationally efficient local spectrum sensing algorithms to detect Orthogonal Frequency Division Multiplexing (OFDM) based PU transmissions as OFDM is a key technology for the present and future broadband wireless communication systems and is used in various applications. System can be extended to develop fast, energy efficient, and practical collaborative sensing algorithms. Sensing policy, which resolves different issues related to central fusion of data such as user selection and sensing scheduling is also a sector that can be studied.

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