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NON LINEAR DISTORTION AND COMPENSATION TECHNIQUE IN WIMAX BROADBAND

BY:
PRADIP ADHIKAREE

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A

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The undersigned certify that it has been read and recommended to the Department of Electronics and Computer Engineering for acceptance, a report of thesis entitled **"Non Linear Distortion and Compensation Technique in WIMAX Broadband"**, submitted by **Mr. Pradip Adhikaree** in partial fulfillment of the requirement for the award of the degree of **"Master of Science in Information and Communication Engineering"**.

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DEPARTMENTAL ACCEPTANCE

The thesis entitled **"Non Linear Distortion and Compensation Technique in WIMAX Broadband"**, submitted by **Mr. Pradip Adhikaree** in partial fulfillment of the requirement for the award of the degree of **"Master of Science in Information and Communication Engineering"** has been accepted as a bona fide record of work independently carried out by him in the department.

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ABSTRACT

WiMAX (Worldwide Interoperability for Microwave Access) is a wireless communications standard designed to provide 30 to 40 Mbps data rates. The physical layer of WiMAX is based on Orthogonal Frequency Division Multiplexing (OFDM) technique. OFDM is a Frequency Division Multiplexing (FDM) technique used as a digital multi-carrier modulation method. The major advantages of OFDM are robustness in multipath fading and high spectral utilization efficiency. However, it has some serious problem like highly sensitive to timing and frequency offset, and more specifically Peak-to-Average Power Ratio (PAPR) problem. Since OFDM is constructive superposition of the subcarriers, it has significant numbers of large peak resulting in high PAPR causing large fluctuation in input signal which requires the use of highly linear amplifier. Due to the amplifier imperfection the peaks are distorted non-linearly which generates inter modulation product causing both in-band distortion and Out-of-Band (OOB) radiation. The design of a compensation technique to improve the linearity of the power amplifier is performed in this thesis work. The adaptive pre-distortion method to compensate the non-linearity of power amplifier includes the LSE estimation of non-linearity and pre-distortion techniques. The analysis of Phase Realignment (PR) and Modified Phase Realignment (MPR) techniques for PAPR reduction for the performance improvement of the system showed that the combination of pre-distortion and PR/MPR technique has superior capability in mitigation of the non-linear distortion as compared to the implementation of individual techniques.

Keywords: OFDM, WiMAX, PAPR, non-linear distortion, HPA

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF ABBREVIATIONS

ACI	Adjacent Channel Interference
ADSL	Asymmetric Digital Subscriber Line
CCDF	Complementary Cumulative Distribution Function
CDMA	Code Division Multiple Access
CI	Carrier Informatory
CP	Cyclic Prefix
DAB	Digital Audio Broadcasting
DSL	Digital Subscriber Line
DVB	Digital Video Broadcasting
DVB-H	Digital Video Broadcasting – Handheld
FDM	Frequency Division Multiplexing
FFT	Fast Fourier Transform
HPA	High Power Amplifier
IBO	Input Back Off
IEEE	Institute of Electrical and Electronics Engineers
IFFT	Inverse Fast Fourier Transform
ISI	Inter-Symbolic Interference
LAN	Local Area Network
LSE	Least Square Error
LTE	Long Term Evolution
LTE-A	Long Term Evolution – Advanced
LUT	Look Up Table
MATLAB	MATrix LABoratory
MPR	Modified Phase Realignment
OFDM	Orthogonal Frequency Division Multiplexing
OOB	Out-of Band
PA	Power Amplifier
PAPR	Peak to Average Power Ratio

PC	Peak Clipping
PR	Phase Realignment
RF	Radio Frequency
SER	Symbol Error Rate
SSM	Spread Spectrum Multiplexing
TDM	Time Division Multiplexing
TWTA	Travelling Wave Tube Amplifier
VDSL	Very High Bit rate Digital Subscriber Loop
Wi-Fi	Wireless Fidelity
WiMAX	Worldwide Interoperability for Microwave Access
WLAN	Wireless Local Area Network

CHAPTER 1

INTRODUCTION

1. INTRODUCTION

1.1. Background

Worldwide Interoperability for Microwave Access (WiMAX) is a wireless communications standard designed to provide 30 to 40 Mbps data rates. The name "WiMAX" was created by the WiMAX Forum, which promotes conformity and interoperability of the standard. The forum describes WiMAX as "a standards-based technology enabling the delivery of last mile wireless broadband access as an alternative to cable and Digital Subscriber Line (DSL). WiMAX was developed to overcome the data rate limitation of mobile phones and to add mobility to broadband Internet access [1].

WiMAX is based on IEEE 802.16 standardization. It is similar to the Wi-Fi standard IEEE 802.11, but supports a far greater range of coverage. While Wi-Fi is a good wireless Internet solution for home networks and coffee shops, it is impractical for larger areas. In order to cover a large area, multiple Wi-Fi repeaters must be set up at consistent intervals. For areas that span several miles, this is a rather inefficient method to provide Wi-Fi as a wireless access and typically requires lots of maintenance. WiMAX, on the other hand, can cover several miles using a single station. This makes it much easier to maintain and offers more reliable coverage. A Wi-Fi signal can cover a radius of several hundred feet whereas a fixed WiMAX station can cover a range of up to 30 miles.

The physical layer of WiMAX is based on Orthogonal Frequency Division Multiplexing (OFDM) technique. OFDM is a multicarrier transmission technique which is conveniently implemented using Inverse Fast Fourier Transform (IFFT) operation and Fast Fourier Transform (FFT) operation. OFDM is the transmission scheme of choice in order to enable high-speed data, video, and multimedia communications. OFDM is highly expected to provide efficient implementation of wireless system with high signal dynamics. It is robust to frequency selective fading

[2]. OFDM is used by a variety of commercial broadband systems, including DSL, Wi-Fi, Digital Video Broadcast-Handheld (DVB-H) etc. besides WiMAX.

1.2. Orthogonal Frequency Division Multiplexing

Orthogonal Frequency Division Multiplexing (OFDM) is a frequency division multiplexing (FDM) technique used as a digital multi-carrier modulation method. In OFDM, the available spectrum is utilized by a large number of closely spaced orthogonal sub-carrier signals to carry data on several parallel data streams or channels as shown in Figure 1.1.

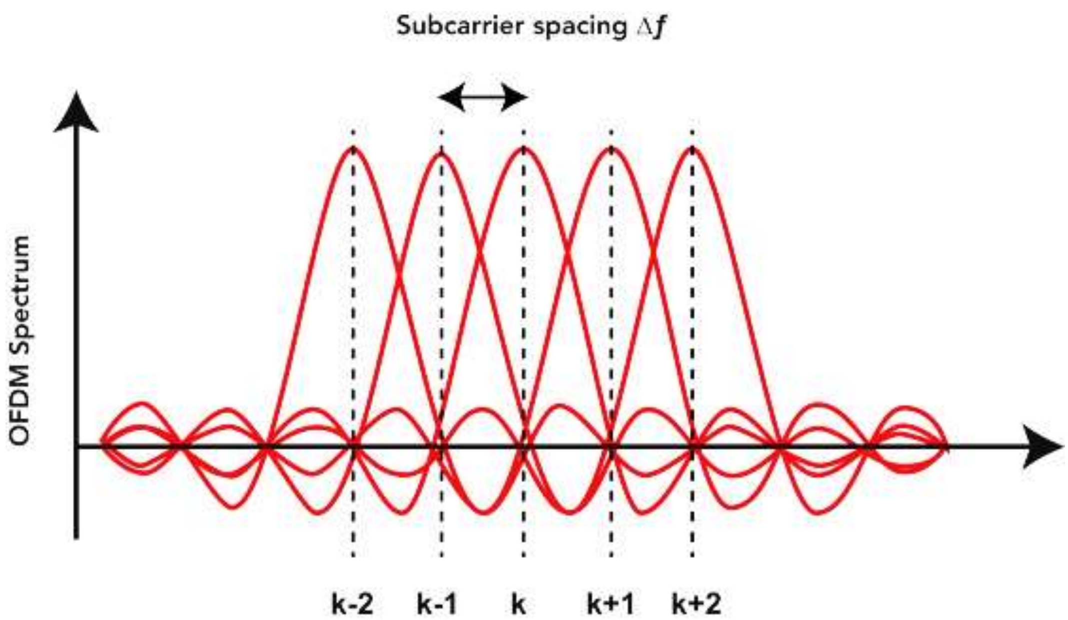

Figure 1.1 OFDM spectrum with five sub-carriers

(Courtesy: http://www.electronics-eetimes.com)

In OFDM, a subcarrier carrying a portion of the user information is transmitted in each band. Each subcarrier spacing is selected so as they are mathematically orthogonal with every other subcarrier [3]. Due to the orthogonal property of the sub-carriers, the signals transmitted in parallel over a common channel can still be detected individually. The possibility of overlapping of the sub-carriers due to their orthogonal property results in less spectrum or bandwidth wastage. OFDM technique offers high spectral efficiency, improved ISI effect and is resistant to frequency selective fading. As a result, OFDM provides higher data rates, superior performance and other benefits over traditional multiplexing techniques such as Frequency Division Multiplexing (FDM), Time Division Multiplexing (TDM) and Spread Spectrum Multiplexing (SSM).

OFDM is conveniently implemented using IFFT and FFT operations. It is being considered for many other high rate wireless systems like Digital Subscriber Line (ADSL, VDSL), Digital Audio Broadcasting (DAB), Digital Videos Broadcasting (DVB), Wireless LAN (WLAN), WiMAX etc. OFDM is also an ultimate requirement for 4^{th} generation cellular telecommunications standards LTE/LTE-A.

1.3. OFDM Advantages

OFDM has been used in many high data rate wireless systems because of the many advantages it provides. One of the main advantages of OFDM is that is more resistant to frequency selective fading than single carrier systems because it divides the overall channel into multiple narrowband signals that are affected individually as flat fading sub-channels. Using adequate channel coding and interleaving it is possible to recover symbols lost due to the frequency selectivity of the channel and narrow band interference. Thus, not all the data is lost [2].

Another major advantage of OFDM is the spectrum efficiency. Using close-spaced overlapping sub-carriers, a significant OFDM advantage is that it makes efficient use of the available spectrum. Another advantage of OFDM is that it is very resilient to inter-symbol and inter-frame interference. This results from the low data rate on each of the sub-channels.

One of the issues with CDMA systems was the complexity of the channel equalization which had to be applied across the whole channel. An advantage of OFDM is that using multiple sub-channels, the channel equalization becomes much simpler.

1.4. Statement of the Problem

However, OFDM has some serious problems/demerits; mainly prone to frequency and phase offset errors and peak to average power ratio (PAPR) problem.

The OFDM signal is prone to frequency and phase offset due to frequency inaccuracy and instability (phase noise). This can be mitigated using the Cyclic Prefix (CP) in which the copy of certain percent of the signal at the end is glued at the beginning of the symbol [4].

The PAPR problem is the most serious problem in OFDM. Since OFDM is constructive superposition of the subcarriers, it has significant numbers of large peak resulting in high PAPR. High PAPR causes large fluctuation resulting in higher crest factor in input signal which requires the use of highly linear amplifier, thus creating a problem in linear amplification. Due to the amplifier imperfection, the peaks are distorted nonlinearly which generates inter modulation product causing both in-band distortion and OOB radiation [5]. The in-band distortion causes ISI, which results in high SER (Symbol-Error Rate) while Out Of Band (OOB) radiation causes Adjacent Channel Interference (ACI) due to spectral re-growth. The imperfection in amplifier causes spectral spreading of the OFDM signals, besides it also causes wrapping (amplitude and phase distortion) and clustering of the signal constellation in each sub-channel at the output of amplifier [6].

1.5. Objectives

The overall objective of this thesis work is to design and analyze compensation techniques to overcome the non-linear distortions in high power amplifier of WiMAX broadband systems.

The specific objectives of the thesis work are listed as follows:

- To design a compensation technique to improve the linearity of the power amplifier
- To analyze PAPR reduction techniques

1.6. Organization

The report is organized as follows. Chapter 1 describes the background of the thesis with the research problems and objectives. Chapter 2 highlights the major works carried out in similar kind of research as literature review. The system model for simulation including the mathematical model is discussed in Chapter 3. The theoretical background for the compensation technique and PAPR reduction techniques is presented in Chapter 4. Chapter 5 shows the comparative results of the simulation. Chapter 6 is the final chapter that presents the conclusions of the thesis work.

CHAPTER 2

LITERATURE REVIEW

2. LITERATURE REVIEW

OFDM technique is widely adopted due to its numerous advantages like immunity to selective fading, spectrum efficiency, resilient to ISI effects, etc. Although OFDM has wide ranges of advantages, there are still a few disadvantages to its use which need to be addressed when considering its use.

The first problem in OFDM systems is that it is sensitive to carrier frequency offset and drift. It can be removed by the use of Cyclic Prefix. Another problem in using OFDM technique is its high Peak to Average Power Ratio (PAPR). High PAPR of the input signal requires the use of highly linear amplifier. This impacts the RF amplifier efficiency as the amplifiers need to be linear and accommodate the large amplitude variations and these factors mean the amplifier cannot operate with a high efficiency level.

There are many techniques proposed for the remedy of the PAPR problem which basically can be divided into four categories. First, there are signal distortion techniques, which reduce the peak simply by nonlinearly distorting the OFDM signal like clipping, peak windowing, and peak cancellation [3].

Clipping is a nonlinear process and may cause significant in-band distortion, which degrades the SER performance and out-of-band noise, which reduces the spectral efficiency, filtering after clipping can reduce the spectral splatter but causes some peak re-growth [7]. Further PAPR reduction can be achieved by repeated clipping and filtering operations [8] but the SER performance will further degrade with longer processing time.

Windowing allows multiplying large signal peaks with a certain window function but there is always a trade-off between the OOB radiation and SER performance. To maintain the OOB radiation within a certain level, the window length needs to be increased. But on the other hand, long window length affects many signal samples degrading the SER performance [9].

In peak cancellation a time shifted and scaled reference function is subtracted from the signal to reduce the peak power of at least one signal sample. It is described in detail in [10]. Peak cancellation can achieve smaller OOB radiation compared to clipping which leads to a better PAPR reduction without the loss of SER Performance [11]. But there is an overhead of reference signal.

The literature in [12] presents the PAPR reduction method using clipping and peak-windowing in Carrier Informatory (CI)/OFDM system but this may add more complication and computation time for the processor. The second category is coding techniques that use special-forward error correction code set. By adding the error correction code, the error probability depends on the power of a number of consecutive symbols rather than on the power of individual symbol [3]. Third category is based on scrambling and using the peak power reducing codes. Each signal is scrambled with a different scrambling sequence and selecting the sequence that gives smallest PAPR. Some of the technique for scrambling sequence is previously mentioned in [13]. Peak power reduction code uses a code which only produces OFDM symbol for which the PAPR is below some desirable level. Some of the peak power reduction codes are mentioned in [14].This category of PAPR reduction introduces complexity to the system with longer processing time.

Lastly, the fourth category includes the use of adaptive filters and pre-distorter prior to high power amplification. Pre-distortion of the signal is one of the most popular techniques to combat the nonlinearity of the power amplifier. It is often necessary to suppress spectral re-growth, contain ACI, and reduce SER. Many models of pre-distortion are presented in [15]. The concept of pre-distortion is the insertion of a nonlinear module between the input signal and the power amplifier, such that the nonlinear module generates inter-modulation product that are in anti-phase to that of the power amplifier thereby canceling out the undesirable inter-modulation products [16]. Adaptive pre-distortion algorithms prove a feasible alternative where the non-constant amplitude modulation like OFDM is concerned.

In [17] the pre-distortion algorithm is formulated valid for the whole dynamic range of the input to the amplifier using the activation function and its associated coefficient. The pre-distortion coefficients estimation can be done by the application of the LS algorithm or by gradient based equation, but the result is shown only for the former approach of pre-distortion coefficient estimation, which seems to be very complex to be practically implemented. In [18] the polynomial based non-linear and inverse non-linear function is derived using Least Square Error (LSE) estimation. The inverse non-linear polynomial function is used to generate the polynomial pre-distortion function which is so far found to have enhanced the power efficiency of mobile OFDM, however there is divergence from the linearity for pre-distortion when the input increases to a critically high level, thus the output becomes deeply saturated. The pre-distortion works only in a limited input range without saturation. Also there is already pre-distorter presented with a Look Up Table (LUT) based implementation [19] which has disadvantage with quantization noise caused by the limited size and a high implementation time required. There are other techniques which use more than one category to provide effective compensation technique for high PAPR problem [20]. Besides, there are also techniques other than these mentioned categories. In [21], a technique to reduce the PAPR of OFDM signal by signal set expansion is presented. Each point in the original signal set is associated with two or more points in the expanded signal set by mapping the symbol in an OFDM data block into points in the expanded signal set with more signal points. The result shows the PAPR reduction only for the low number of sub-carriers which might not be equivalent to most of the application used today. The drawback of the proposed technique is the increased transmission power to achieve a certain performance. To alleviate this problem the modified version of the proposed technique is used in which, the extended set is used only as a part of the sub-carrier. The technique is more effective for small number of sub-carriers. Also there is a trade-off between the PAPR reduction and the overhead and computational complexity by using modified version.

CHAPTER 3

THE SYSTEM MODEL

3. THE SYSTEM MODEL

The WiMAX system is the special case of the OFDM system since it is one of the application fields of the OFDM. In OFDM a high data rate is transmitted over parallel sub-carriers with a lower rate on each sub-carrier. It is a FDM technique for transmitting large amount of digital data over a radio wave. The system mode for WiMAX is presented in this chapter. The parameters relevant to the WiMAX system are also discussed to model the system.

3.1. System Components

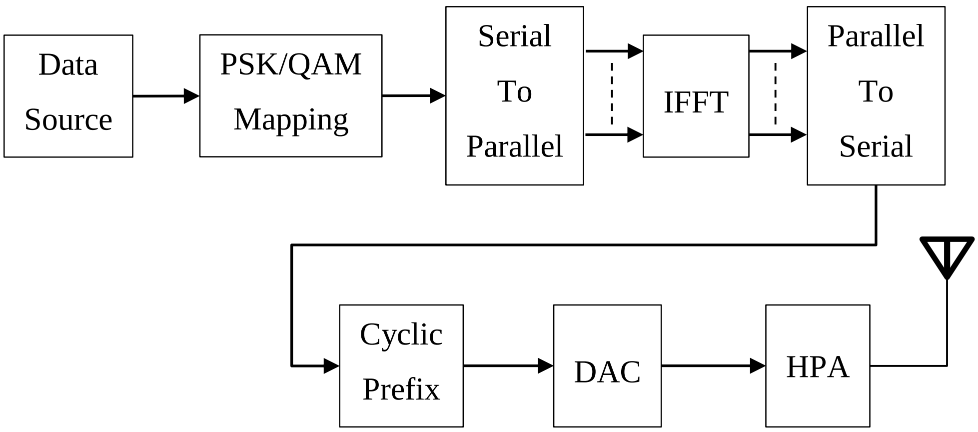

Figure 3.1 WiMAX Transmitter Block Diagram

The system components of a WiMAX can be represented by three blocks namely, transmitter, channel medium and the receiver block. Figure 3.1 depicts the transmitter block diagram being considered for WiMAX system.

A data generator is used to generate the data on OFDM subcarriers, which is mapped (modulated) using common digital modulation scheme Quadrature Phase Shift Keying (QPSK) or Quadrature Amplitude Modulation (QAM) forming the baseband symbol. The symbols are packed together and converted to serial which is followed by the IFFT operation. In IFFT operation, each subcarrier is frequency modulated to different carrier frequencies which are orthogonal with each other, hence the name OFDM. The conceptual IFFT operation in OFDM is depicted in Figure 3.2, where the input signals are allocated to the different sub-carrier with orthogonal carrier frequency.

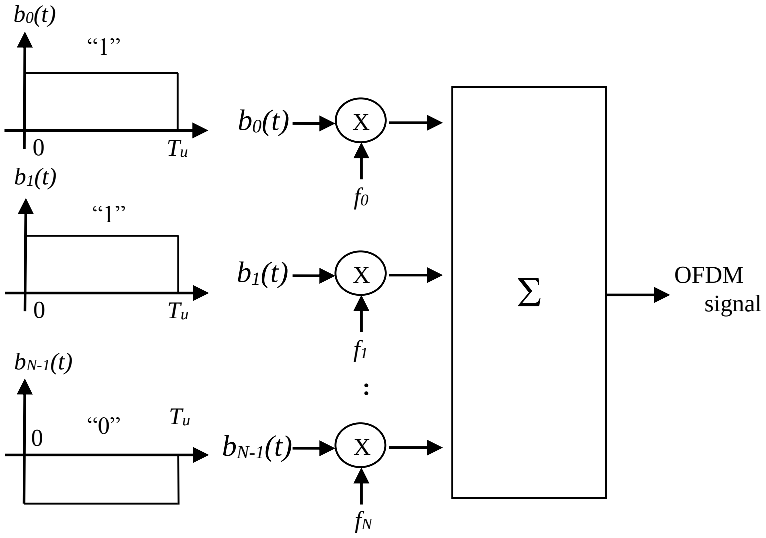

Figure 3.2 Conceptual IFFT operation in OFDM

The guard interval, considered as CP, anywhere from 10% to 25% of the symbol time is then added to compensate for the frequency and phase offset errors. Finally, the signal is up-converted, amplified and transmitted.

The channel environment can be considered as the Additive White Gaussian Noise (AWGN) Channel. And the receiver is in reverse order to the transmitter.

3.2. Mathematical Model

The OFDM in WiMAX system takes a set of input data bits [b_1 b_2 b_3 … b_M] which is modeled using the QAM or PSK modulation technique to the I/O channel baseband symbol. $X_n^{r,i}$ N such modulated symbols i.e., $\left(X_1^{r,i},\ X_2^{r,i}, X_3^{r,i}, \ldots, X_N^{r,i}\right)$ are packed together at the input of the IFFT block using a serial to parallel converter.

The OFDM signal in the time domain is generated by IFFT operation which is represented by the equation below,

$$x(n) = x_r(n) + jx_i(n) = \frac{1}{N}\sum_{k=1}^{N} X_k e^{\frac{j2\pi(k-1)(n-1)}{N}} \qquad (1)$$
$$for\ n = [1,2,\dots,N]$$

Normally, N is very large, so according to the Central Limit Theorem, the real $x_r(n)$ and imaginary $x_i(n)$ values of $x(n)$ become Gaussian distribution, each with mean zero and variance of *0.5*. The amplitude of an OFDM signal, therefore, has Rayleigh distribution. Thus the signal has a very high PAPR. The PAPR which is generally used to describe the fluctuant, can be obtained by the equation,

$$PAPR = \frac{\max|x(n)|^2}{\varepsilon[|x(n)|^2]}, n \in [1, N] \qquad (2)$$

Where $\varepsilon\{\cdot\}$ denotes the expectation and $\varepsilon[|x(n)|^2]$ gives the average power.

The analog signal after DAC can be obtained as,

$$\tilde{x}(t) = Re\left[x(t)e^{j(\omega_c t + \phi(t))}\right] \qquad (3)$$

The analog signal in equation (3) contains the amplitude and phase of the input signal which is fed to the HPA for RF transmission.

Finally, the complex RF output from the HPA with non-linear distortion can be expressed by the following equation,

$$\tilde{X}(t) = A\{\rho(t)\}e^{j\{\varphi(t) + \phi(\rho(t))\}} \qquad (4)$$

Where $\rho\ (t)$ and $\varphi\ (t)$ are amplitude and phase of the input signal. $A(\rho)$ and $\phi(\rho)$ are AM/AM and AM/PM conversion of the nonlinear distortion respectively.

3.3. Power Amplifier

The high PAPR of OFDM signal requires a very good linear transmission of the signal. Power amplifiers are typically the most power hungry components of RF transceivers. The design of PAs, especially for linear, low-voltage operations, remains a difficult problem defying an elegant solution.

There are two types of PAs for communication system namely, Travelling Wave Tube Amplifier (TWTA) and Solid State Power Amplifier (SSPA). TWTA is generally used for high power wireless communications and SSPA is generally used for several communication systems, especially for mobile communication systems.

The power amplifier is mathematically modeled for analysis and simulation. The common model for TWTA amplifier is given by Saleh model by using two simple parameter functions to model the AM/AM and AM/PM characteristics of nonlinear amplifiers, which are as shown below respectively in equation (5) and (6),

$$A(\rho) = \frac{\alpha_a \rho}{1 + \beta_a \rho^2} \quad (5)$$

$$\phi(\rho) = \frac{\alpha_\phi \rho^2}{1+\beta_\phi \rho^2} \quad (6)$$

The set of parameters that closely matches TWTA data is $\alpha_A = 2.1587$, $\beta_A = 1.1517$ and $\alpha_P = 4.033$, $\beta_P = 9.1040$.

Another common approach to model the power amplifier is using a polynomial to represent the amplitude transfer characteristics,

$$y(t) = \sum_{i=0}^{P} \alpha_i x^i(t) = \alpha_0 + \alpha_1 x(t) + \sum_{i=2}^{P} \alpha_i x^i(t) \qquad (7)$$

Where $x(t)$ and $y(t)$ are input and output of the non-linear amplifier respectively. α_0 represents the DC offset, α_1 is the linear scalar, $\{\alpha_2, \ldots \alpha_p\}$ contribute to non-linearity in the system and P is the highest order of the system.

CHAPTER 4

PROPOSED COMPENSATION TECHNIQUE

4. PROPOSED COMPENSATION TECHNIQUE

There are several compensation methods to reduce the effect of non-linearity of WiMAX systems. Most of them have some form of expenses, either in the form of bandwidth, system complexity, loss of critical signal or requirement of reference signal.

An elegant compensation technique for WiMAX system is presented in this chapter. Since power amplifiers are the most power hungry and highly nonlinear in nature, the large fluctuation in the signal amplitude causes non-linear distortion in the RF transmission. To mitigate this problem, an adaptive pre-distortion technique is used. With the use of pre-distortion overall system becomes linear but does not ensure that the power amplifier is not driven in the saturation region. To overcome this problem a new technique of PAPR reduction called Phase Realignment is proposed. In addition, Modified-PR which has high PAPR reduction is also proposed. The result shows that the performance of both the techniques is much better. Here in this thesis work, the PR and MPR process in conjunction with adaptive pre-distortion for TWTA non-linear distortion of WiMAX is used.

Equation (4) gives a simple model for analysis but further a memory less polynomial given in equation (7) is used to capture the non-linear characteristics of the transmitter, regardless of the PA type as the inverse of (4) is not easy to solve. For the sake of simulation, the seventh order odd polynomial is used for representing the non-linear TWTA, i.e., the coefficients $\alpha_0, \alpha_2, \alpha_4$ and α_6 are set to zero and remaining $\alpha_1, \alpha_3, \alpha_5$ and α_7 are set carefully having positive and negative values alternately. Here only the AM/AM conversion is considered ignoring the AM/PM conversion of the PA.

4.1. Adaptive Pre-distortion

The design of adaptive pre-distortion technique consists of two steps. The first step is the least square estimation of non-linearity of the system (mainly power amplifier of the system). The second step is the determination of the pre-distortion function with the help of the non-linear estimation obtained in first step.

4.1.1. LS Estimation of Non-linearity

A training sequence with sufficient dynamic range is used to probe the non-linearity. The training sequence is assumed to be available. A feedback of the RF output to the base-band with this sequence transmitted in ascending order is sampled,

$$y_n = f(x_n) = \sum_{i=0}^{P} \alpha_i x_n^i \quad (8)$$

The signal vectors of size $N \times 1$ are defined as $Y_t = [y_1\ y_2\ ...\ y_N]^T$, is the feedback vector of the RF output. $X_t = [x_1\ x_2\ ...\ x_N]^T$ is the input training sequence vector in ascending order and the non-linear polynomial coefficient vector are $\alpha = [\alpha_0\ \alpha_1\ \alpha_2\ ...\ \alpha_P]^T$. Now, equation (8) can be represented in matrix formulation as,

$$Y_t = \overline{X_t} * \alpha \quad (9)$$

where $\overline{X_t}$ is a Vandermonde matrix of $N \times P$, each element of which is calculated as,

$$\bar{X}_{t_{i,j}} = x_i^{j-1}, i\epsilon[1, N], j\epsilon[1, P] \quad (10)$$

Finally, the Least Square estimation of the non-linear PA can be represented by the polynomial coefficients which can be computed as,

$$\underline{\dot{\alpha}} = [\bar{X}_t^H\ \bar{X}_t]^{-1}\bar{X}_t^H * Y_t \quad (11)$$

The term $\bar{X}_t^H$ in the above equation denotes the Hermitian of the Vandermonde matrix $\bar{X}_t$. Hermitian is simply the conjugate and transpose of a matrix. Thus the optimal estimation of the non-linearity can be achieved by using the estimated coefficients $\underline{\dot{\alpha}}$.

4.1.2. Pre-distortion

Sample-by-sample pre-distortion is performed by feeding the base band digital signal to the processer. The main objective of the pre-distortion is to make the overall effect of RF output linear. Let us suppose a function *g(x)* to be the another polynomial of order *Q* such that it would pre-distort the input signal as,

$$\breve{x}(n) = g\big(x(n)\big) = \sum_{i=0}^{Q} \beta_i x^i(n) \qquad (12)$$

So if $\breve{x}(n)$ is passed through the PA the output should be linearly scaled. That is,

$$\tilde{y}(n) = \sum_{i=0}^{P} \alpha_i \left(\sum_{j=0}^{Q} \beta_j x^j(n) \right)^i = \sum_{k=0}^{P*Q} \gamma_k x^\wedge k(n) \approx \alpha_1 x(n) \qquad (13)$$

With γ_k representing both α_i and β_i.

For determining the solution $f^{-1}(y)$ is fitted by another *Q* order polynomial from training data,

$$x_n = f^{-1}(y_n) = l(y_n) = \sum_{i=0}^{Q} \lambda_i y_n^i \qquad (14)$$

Similarly, the solution can be obtained as

$$\underline{\dot{\Lambda}} = [\bar{Y}_t^H \bar{Y}_t]^{-1} \bar{Y}_t^H * X_t \qquad (15)$$

with Y_t as Vandermonde matrix of the output and $\bar{Y}_t^H$ as the Hermitian of $\bar{Y}_t$. $\underline{\lambda} = [\lambda_0\ \lambda_1 \ldots \lambda_Q]$ is the estimated inverse non-linearity coefficient vector.

Now, from equations (12), (13) and (14) the solution of *g(x)* can be obtained as,

$$g\big(x(n)\big) = f^{-1}\big(\tilde{y}(n)\big) = \sum_{i=0}^{Q} \lambda_i \alpha_1^i x^i(n) = \sum_{i=0}^{Q} \beta_i x^i(n) \qquad (16)$$

Therefore, the coefficient vector $\beta_i = \lambda_i \alpha_1^i$ creates the pre-distortion function *g(x)*.

4.2. PAPR Reduction Technique

It has been discussed that the pre-distortion alone is not enough for the overall compensation of the non-linear problem. Here a simple new non-bijective constellation technique of PAPR reduction called Phase Realignment (PR) is presented along with its modified version Modified Phase Realignment (MPR) with practical algorithm that shows promising results for the commercial use in WiMAX broadband.

4.2.1. Phase Realignment

PR uses non-bijective constellation to reduce the PAPR by appropriately encoding the data symbols. The idea can be easily explained in the case of flat-power lying in each quadrant of the complex plane equidistance from the real and the imaginary axis. Error occurs when the noise causes the received sample to fall into one of the other three quadrants. The error rate can be decreased by increasing the distance of the constellation point from the decision border. The effect of this might cause to increase the transmitted power for data block. So an idea is introduced to increase the Euclidean distance from the decision border without increasing the transmitted power of data block. It will be shown later that with the PR processing, there is improvement in SER performance. A 1705-point QPSK OFDM time-domain signal is considered for illustration of PAPR reduction with PR.

Before the PR process, the OFDM signal is first clipped in order to reduce the peak with the appropriate clipping level determined by using the Clipping Ratio (*CR*), the ratio of the clipping level power and the average power.

$$Clipping\ level\ (l_{clp}) = \sqrt{CR * P_{avg}} \quad (17)$$

Clipping of the peak is followed by the filtering process to remove possible OOB radiation. The clipping of the OFDM signal highly reduces PAPR but introduces the extra distortion which leads to the degradation in SER performance. Thus, the role of the PR comes into play. After the peak clipping, the phase of the clipped signal is changed back to its original state and the magnitude is restricted along certain amplitude threshold i.e., magnitudes lower than the threshold are scaled up to the level of threshold and the others are left untouched. The PR process is carried out in frequency domain.

The discrete frequency domain peak clipped OFDM signal can be represented as,

$$X_{clp}(n) = \rho_{clp}(n)e^{j\varphi_{clp}(n)} \quad (18)$$

where $\rho_{clp}(n)$ and $\varphi_{clp}(n)$ are the magnitude and phase of the clipped signal respectively.

Now, the phase $\varphi_{clp}(n)$ is realigned to its original position as it was before peak clipping, for this, first the possible phase is set to the array of phases. A 4QAM/QPSK is assumed for this case,

$$\psi[.] = \left[-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right] \quad (19)$$

$$\varphi_{pr}(k) = \psi\left[\left|\varphi_{clp}(k) - \psi[.]\right| = \min\left|\varphi_{clp}(k) - \psi[.]\right|\right] \quad (20)$$

And the amplitude lower than ρ_{th} are scaled up to it,

$$\rho_{pr}(n) = \begin{cases} \rho_{th}, if\, \rho_{clp}(n) < \rho\_th \\ \rho_{clp}(n), \quad else \end{cases} \quad (21)$$

Thus forming the Phase Realigned signal as,

$$X_{pr}(n) = \rho_{pr}(n) e^{j\varphi_{pr}(n)} \quad (22)$$

The procedural concept of the PR technique can be explained as below.

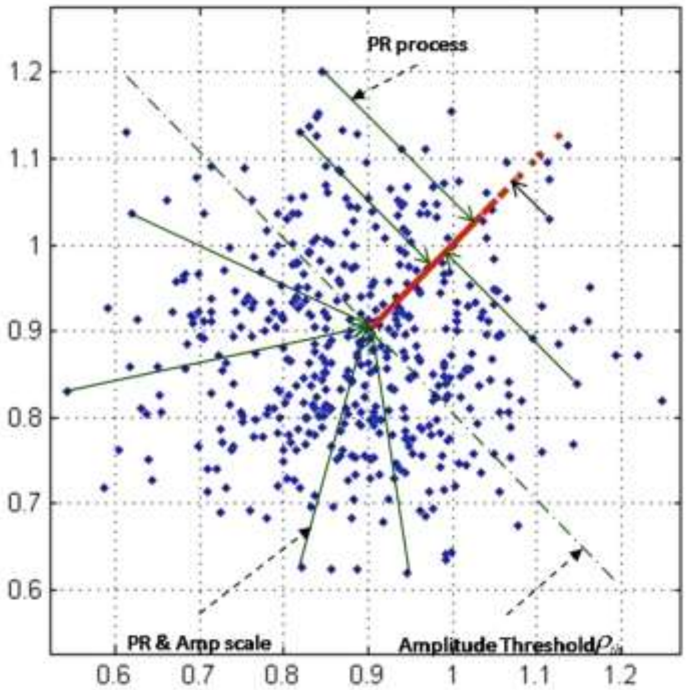

Figure 4.1 PR process concept

The conceptual process of PR is depicted in Figure 4.1. It shows the constellation of QPSK signal around the point (1+j1). All constellations below the amplitude threshold ρ_{th} are scaled and phase realigned, whereas all other constellation are only phase aligned. The scattered dots are the constellation of the peak clipped signal. The line approximately having negative unity slope (alternately dashed and dotted line) represents the amplitude threshold. The arrow indicates the shift of the constellation as the PR process is carried on.

All the constellation goes through the change in the phase to its original phase which is shown in Figure 4.1. The constellations having the amplitude lower than the amplitude threshold, undergoes amplitude scaling to the amplitude threshold level. The resulting constellation in the figure is the PR constellation.

4.2.2. Modified PR

The PR processing has considerable PAPR reduction. This can be further reduced by applying phase threshold in addition. Since the PR is carried out after peak clipping, introducing the optimal phase threshold would help in retaining the low PAPR resulted by peak clipping.

The modified version of PR, MPR has the phase threshold in addition to the amplitude threshold. It is also performed in frequency domain after the clipping of the peak. The phase of the peak-clipped constellation residing inside the allowable phase from the center phase i.e., $\pm\varphi_{th}$ is left untouched and others are changed to respective phase threshold. This can be represented by the equation below,

$$\varphi_{mpr}(n) = \begin{cases} \varphi_{pr}(n) + \varphi_{th}, & if\ \varphi_{clp}(n) > \varphi_{pr}(n) + \varphi_{th} \\ \varphi_{pr}(n) - \varphi_{th}, & if\ \varphi_{clp}(n) > \varphi_{pr}(n) - \varphi_{th} \\ \varphi_{clp}(n), & else \end{cases} \quad (23)$$

where, φ_{th} is the additional allowable phase on each side of the phase of the original constellation.

The procedural concept of the MPR technique is explained below.

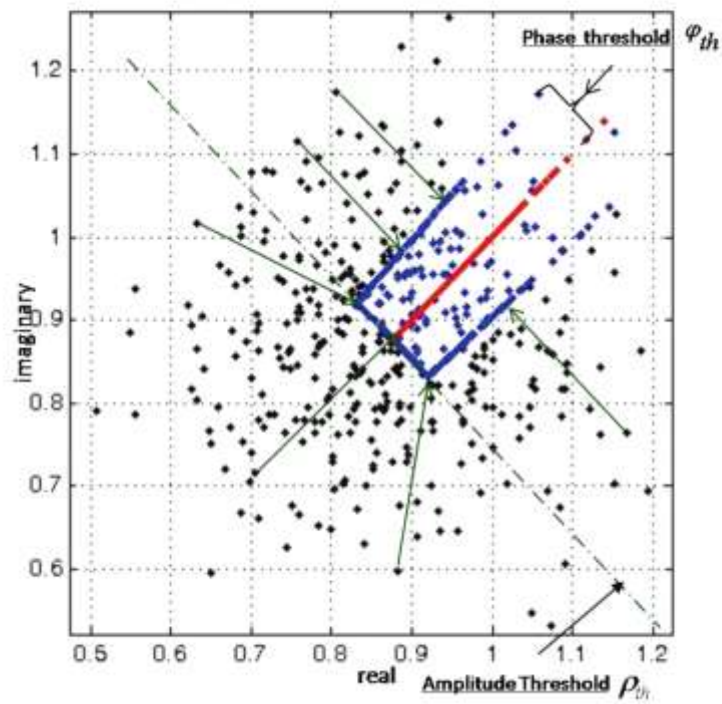

Figure 4.2 MPR Process concept

The process involved in MPR technique is shown in Figure 4.2. The MPR process is similar to the PR process with amplitude scaling. But there is a phase threshold $\pm\varphi_{th}$ instead of changing the phase to the original phase. This can be seen in the Figure 4.2 as the phase of the constellation inside the allowable phase are not changed whereas the constellation outside the phase threshold are aligned to the threshold.

CHAPTER 5

SIMULATION RESULTS

5. SIMULATION RESULTS

The simulation for the WiMAX system is carried out using simulation tool MATLAB. The simulation parameters are chosen to closely represent the WiMAX system. The simulation results for the proposed compensation technique are discussed in this chapter.

5.1. Simulation Parameters

Different parameters are used to closely match the WiMAX system. The number of subcarrier used is *N=1705*, the modulation technique is QPSK for the OFDM symbol and the channel environment is chosen as AWGN channel. The analysis uses the normalized PA model given by the 7th order odd polynomial (α_1 = 1.9817, α_3 = -1.3638, α_5 = 0.4507, α_7 = -0.0524), with the gain of 20dB. This specification is typically for the WiMAX system with TWTA as a PA.

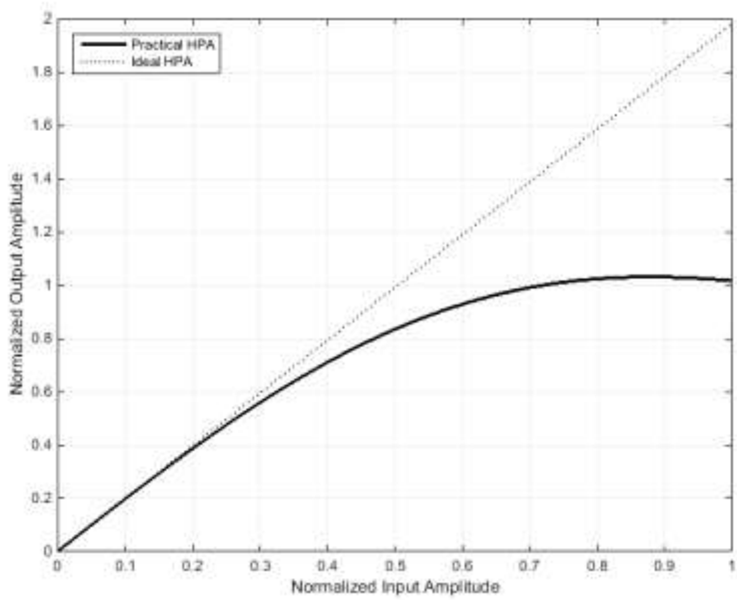

Figure 5.1 Normalized Input-Output Curve for Ideal vs. Practical Power Amplifier

The comparison of the normalized input to output characteristics for the ideal and practical high power amplifier can be seen in Figure 5.1. In comparison to the linear characteristics of the ideal HPA, the practical HPA shows non-linear characteristics. To linearize the characteristics of the practical HPA, some compensation technique has to be implemented. The use of a compensation technique helps to modify the non-linear curve to match the ideal linear characteristics curve.

5.2. Simulation Results for Adaptive Pre-distortion

An adaptive pre-distortion technique discussed in section 4.1 can be implemented to linearize the non-linear characteristics of practical HPA. This technique is composed to two steps: LS Estimation of Non-linearity and Pre-distortion as explained in sections 4.1.1 and 4.1.2.

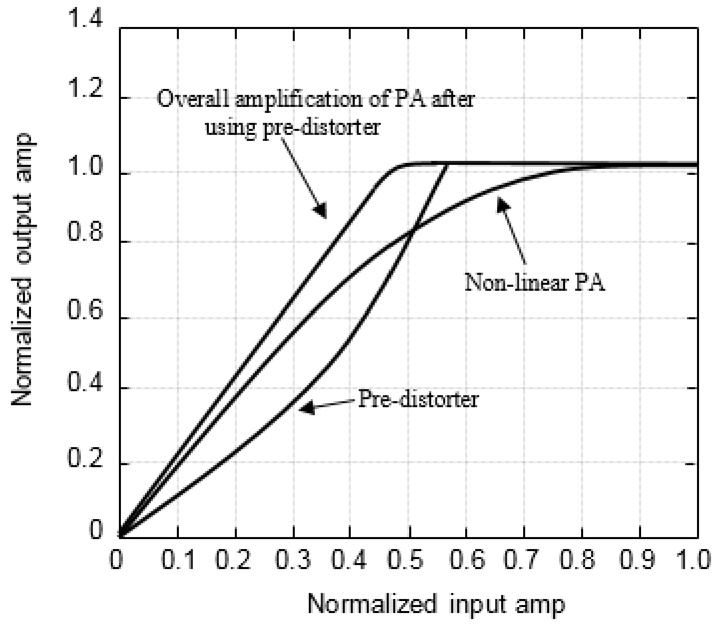

Figure 5.2 Normalized Input-Output Curve for Compensated Power Amplifier

The response of the system with and without the pre-distortion is shown in Figure 5.2. The input signal is first fed to the pre-distorter and the output of the pre-distorter is fed to the non-linear power amplifier. This results in the linear amplification of the overall system as shown in Figure 5.2.

However, according to Figure 5.2, it can be seen that the amplification is still not completely linear. For high input values, the output is constant for the overall system of pre-distorter and non-linear power amplifier. The system is linear only for lower values of input signal. Hence, by using the pre-distorter, only the lower input values can be used. The higher input values or peak values have to be clipped and removed.

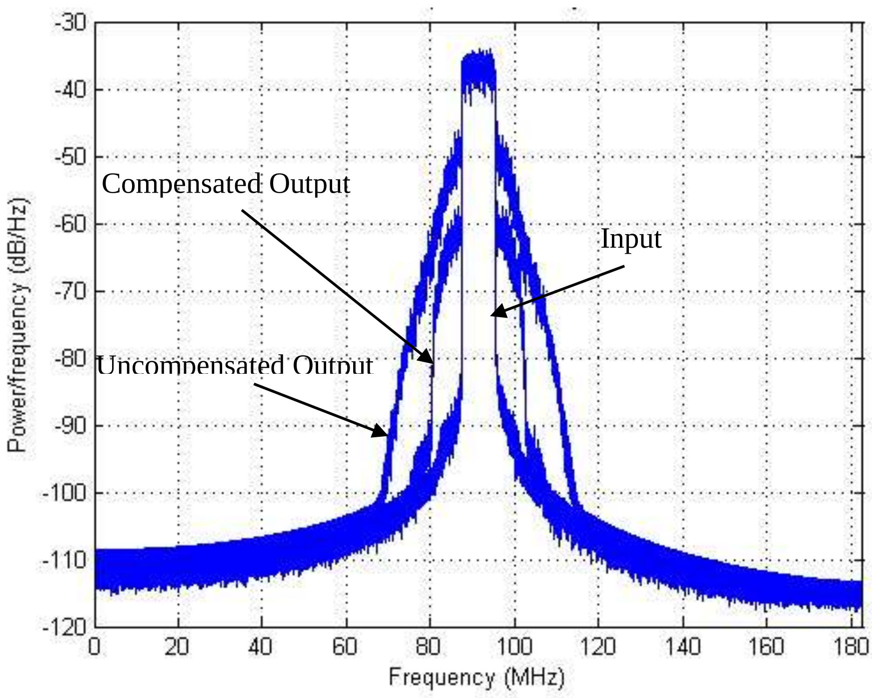

Figure 5.3 Power Spectrum Density

The Power Spectrum Density (PSD) of the input signal, uncompensated output signal and compensated (with the application of adaptive pre-distortion) output signal is shown in Figure 5.3. Figure 5.3 clearly shows that the out-band radiation is suppressed effectively by using adaptive pre-distortion for compensation.

5.3. Simulation Results for Phase Realignment

A 1705-point QPSK OFDM time-domain signal is considered for the simulation of PAPR reduction technique with Phase Realignment. The OFDM signal is first generated without any PAPR reduction technique. Second, the OFDM signal with Phase Realignment PAPR reduction technique is generated.

5.3.1. Effect on Time Domain Signal

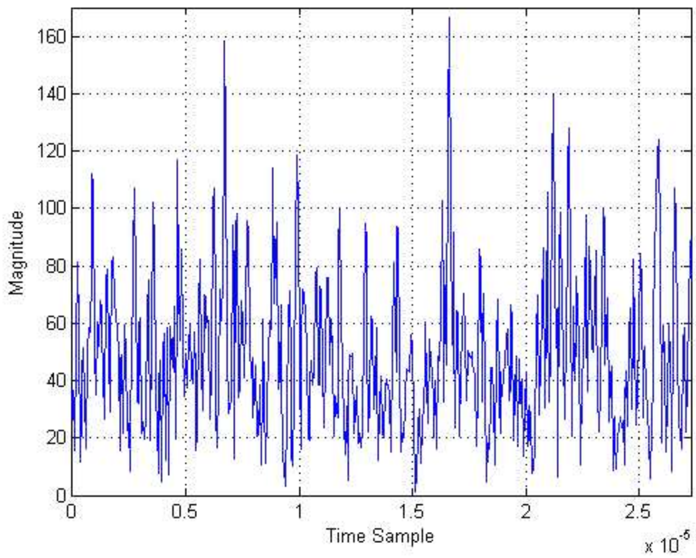

Figure 5.4 Time domain magnitude of normal signal

The time domain plot of the magnitude of the portion of the OFDM signal without any PAPR reduction technique applied is shown in Figure 5.4. The normal OFDM signal has the PAPR of 9.12 dB.

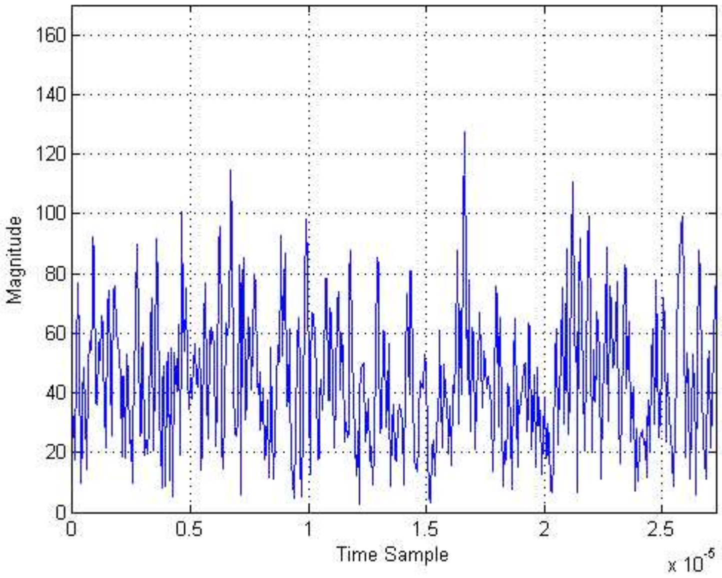

Figure 5.5 Time domain magnitude of signal after PR processing

The magnitude of the portion of the signal after the PR processing is shown in Figure 5.5. On comparison with Figure 5.4, it can be seen that the peak amplitudes have been reduced. The signal shown in Figure 5.5 has the PAPR of 8.52 dB. Thus, it can be illustrated that the PR processing results in the reduction of average power by 0.79 dB and also the reduction in PAPR by 0.6dB.

5.3.2. Effect on Signal Constellation

The normal QPSK signal without peak clipping and phase realignment has the constellation as shown in Figure 5.6. The QPSK constellations lie at points 1+j1, 1-j1, -1-j1 and -1+j1. According to Figure 5.6, the original signal without peak clipping and phase realignment has its constellation lie exactly at one of the above points.

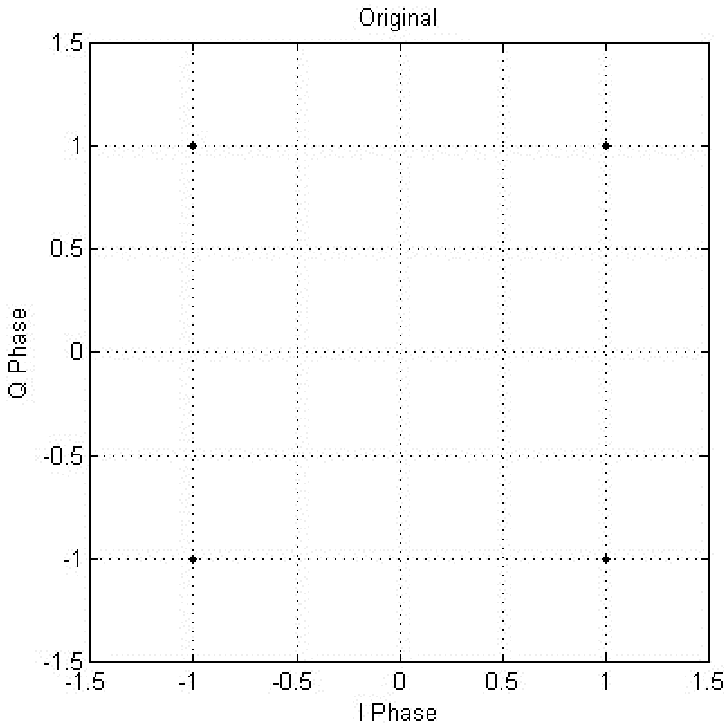

Figure 5.6 Original QPSK constellation

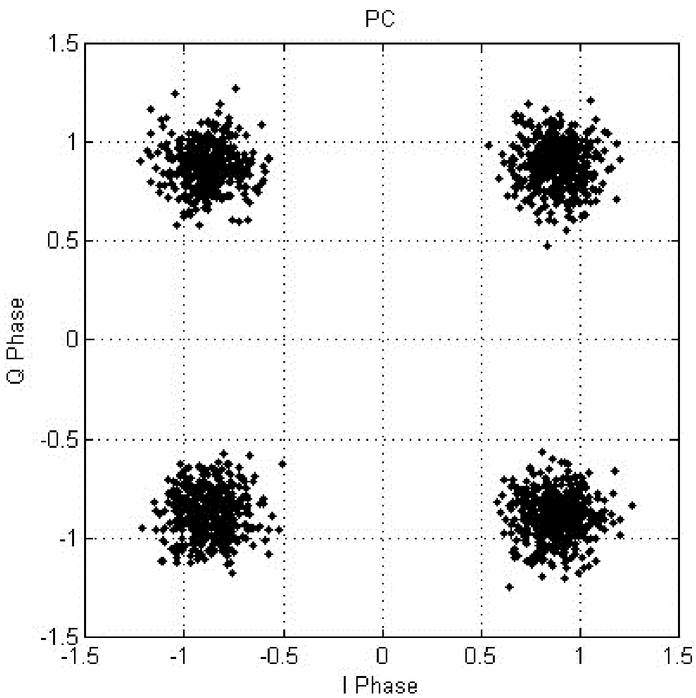

Figure 5.7 PC constellation

The QPSK constellation of peak clipped signal is shown in Figure 5.7. Comparing Figures 5.6 and 5.7, the effect of peak clipping is that the phase of resulting signal is distorted. The PAPR of the signal can be further reduced by realigning the distorted phase.

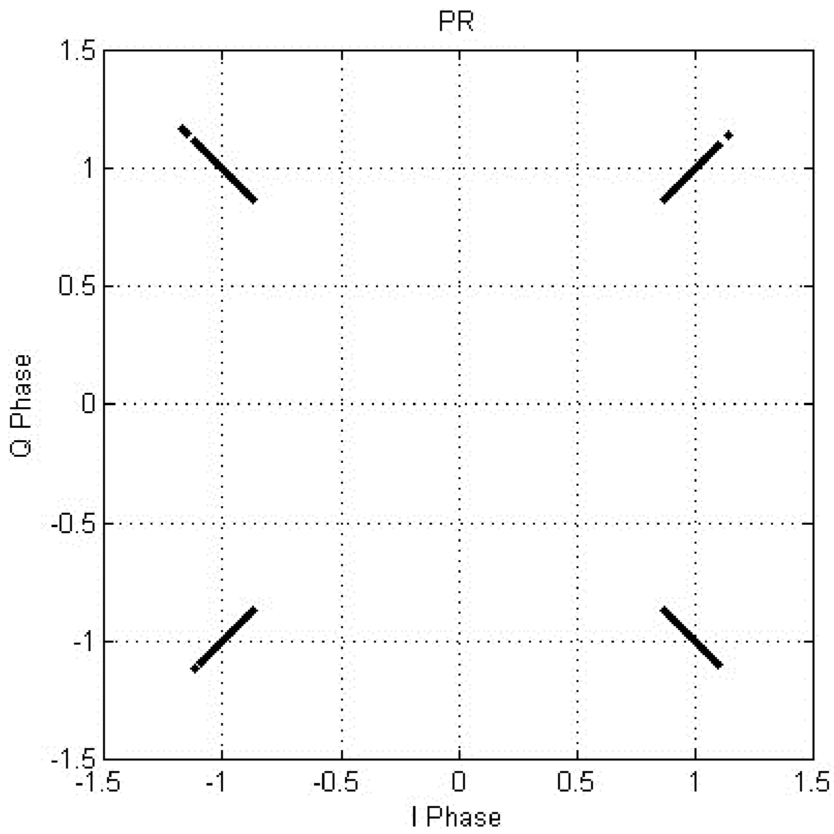

Figure 5.8 PR constellation

The constellation of PR signal is depicted in Figure 5.8. The scattered constellation shown in Figure 5.7 is realigned according to PR process concept discussed in section 4.2.1.

5.4. Simulation Results for Modified Phase Realignment

Similar to the simulation of Phase Realignment technique, a 1705-point QPSK OFDM time-domain signal is considered for the simulation of PAPR reduction technique with Modified Phase Realignment technique as well. The OFDM signal is first generated without any PAPR reduction technique. Then, the OFDM signal with Modified Phase Realignment PAPR reduction technique is generated.

5.4.1. Effect on Time Domain Signal

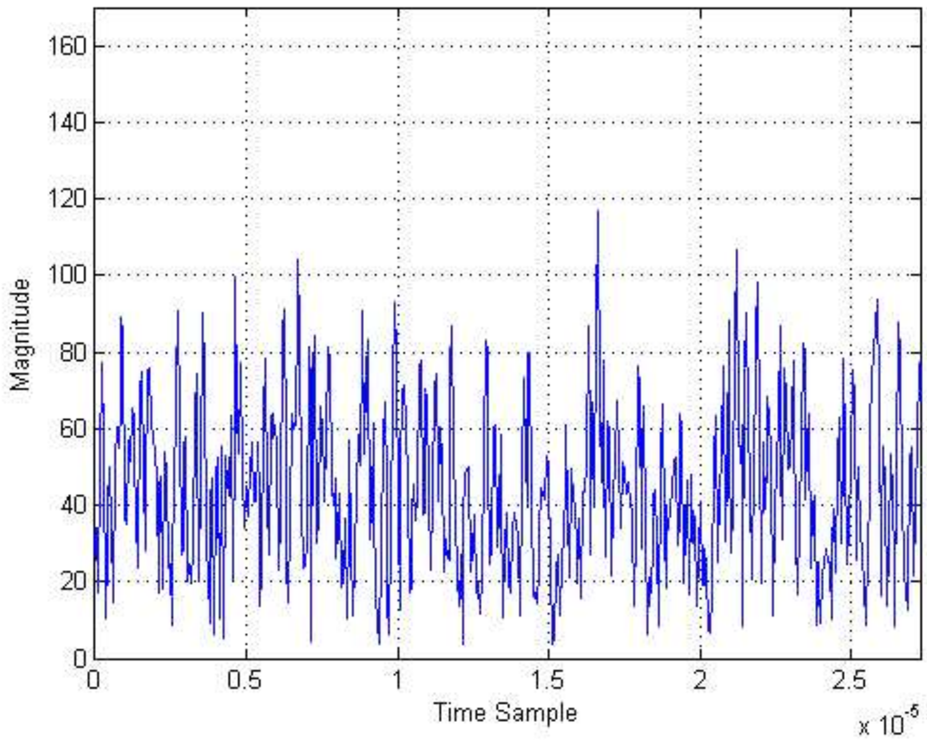

Figure 5.9 Time domain magnitude of signal after MPR processing

Figure 5.9 shows the magnitude of the portion of the signal after MPR processing to the signal shown in Figure 5.4. This signal has the PAPR of 7.652dB. The MPR processing results in 1.47dB of PAPR reduction and the average power is reduced by 0.79dB. Although the reduction in average power is similar to that of PR processed signal, the reduction of PAPR is higher in case of MPR technique as compared to the PR technique.

5.4.2. Effect on Signal Constellation

The constellation diagram of MPR signal is shown in Figure 5.10. The scattered constellation shown in Figure 5.7 is realigned according to MPR process concept discussed in section 4.2.2. Unlike in PR technique, the phase is realigned only if it crosses the phase threshold.

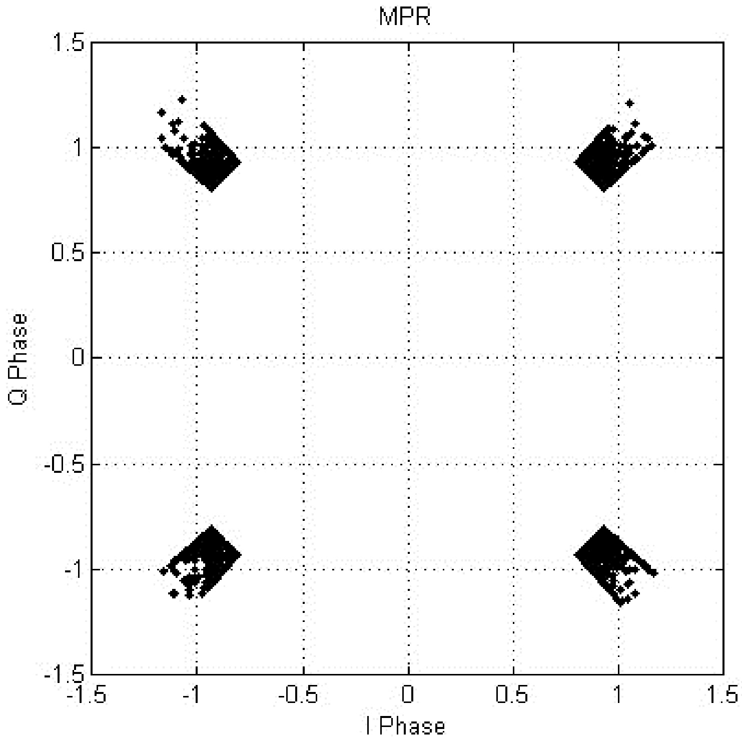

Figure 5.10 MPR constellation

5.5. Comparison of PAPR Reduction Techniques

5.5.1. Comparison using CCDF

Figure 5.11 shows the Complementary Cumulative Distribution Function (CCDF) of PAPR for PR and MPR techniques after Peak Clipping.

The CCDF is given by,

$$CCDF\big(PAPR(x)\big) = \Pr(PAPR(x) > PAPR_{th}) \qquad (24)$$

It can be interpreted as the probability that the PAPR of a symbol exceed some threshold level $PAPR_{th}$. From Figure 5.11, it can be observed that for 0.1% of the CCDF the PAPR improvement of PR process is around 1.3dB and for MPR process, it is around 2.5dB.

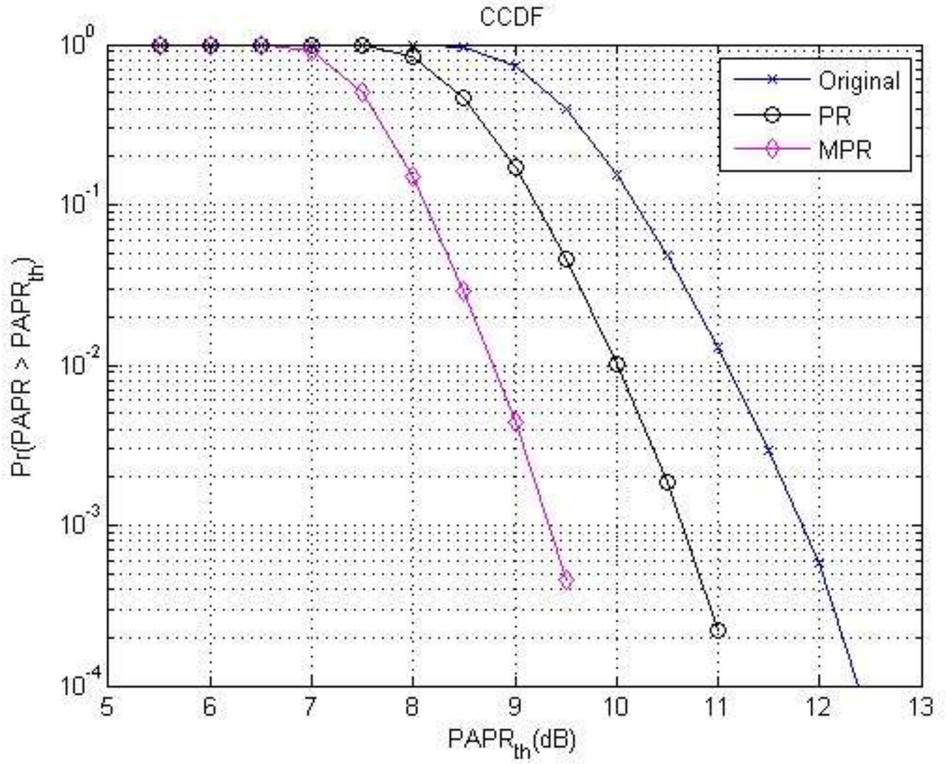

Figure 5.11 CCDF of PAPR for PR and MPR

5.5.2. Comparison using SER

The SER performance for pre-distortion with PR and MPR techniques were computed simultaneously, along with the uncompensated signal using the non-linear PA and the ideal PA. The result of the simulation for SER performance is discussed below.

The SER performance of the signal using ideal PA and non-linear PA is shown in Figure 5.12. It shows the change in SER for non-linear PA with and without pre-distortion. Also, it shows the variation in SER for PR as well as MPR techniques. The SER performance of pre-distorted signal is much better than the normal non-linear amplification without pre-distortion is shown in Figure 5.12. For 0.1% (10^{-3}) SER, the normal OFDM signal (marked as 'Normal') requires an SNR of around 13 dB whereas the pre-distorted OFDM signal requires an SNR of only about nearly 7.9 dB. With the application of pre-distortion, the SER curve approaches towards the ideal curve.

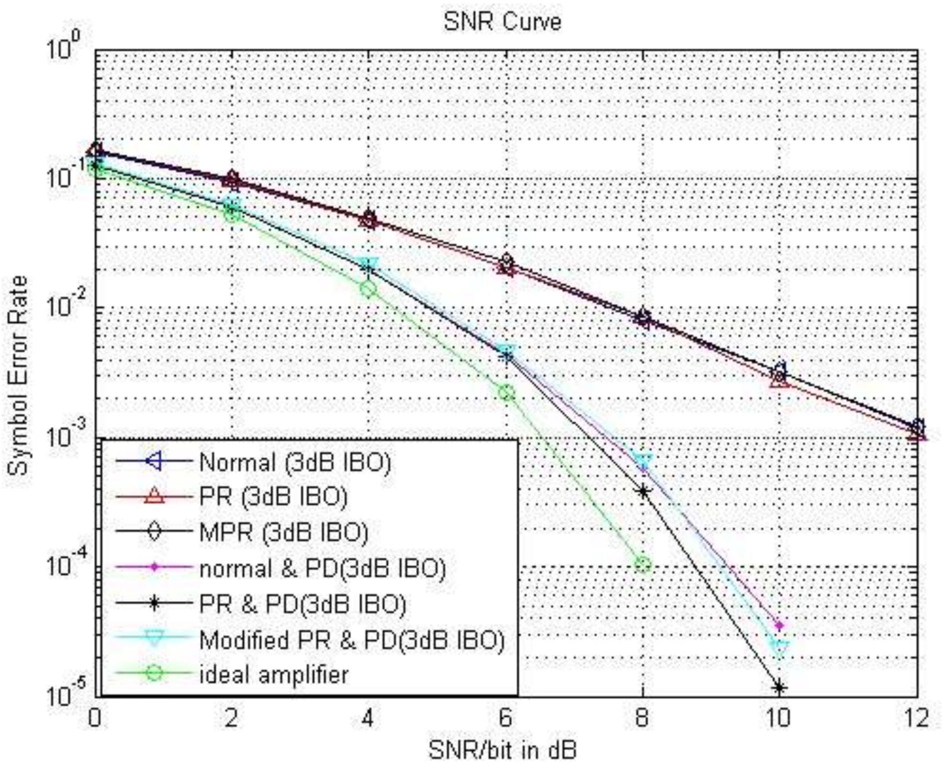

Figure 5.12 SER curve over SNR for PR, MPR and pre-distortion

The SER curve further approaches to the ideal curve as the PR and MPR are applied along with the pre-distortion of the OFDM signal individually as shown in Figure 5.12. Again, for 0.001% (10^{-5}) SER, the SNR is improved by 1.1 dB for the PR process, whereas it improves by around 1 dB for the MPR process respectively than the normal pre-distorted signal. We chose the phase threshold (φ_{th}) as ($\pi/46$) radian for MPR process.

The SER curve of PC and MPR, with and without pre-distortion is shown in Figure 5.13. As seen from the Figure 5.13, the SER curve of the MPR process is much better than that of PC in both cases (with and without the application of pre-distortion). For 1% of SER, MPR has the SNR improvement by around 2.5 dB to that of PC. Similarly, for 0.1% SER, MPR & pre-distortion has around 1.2 dB improvement in SNR to that of PC & pre-distortion.

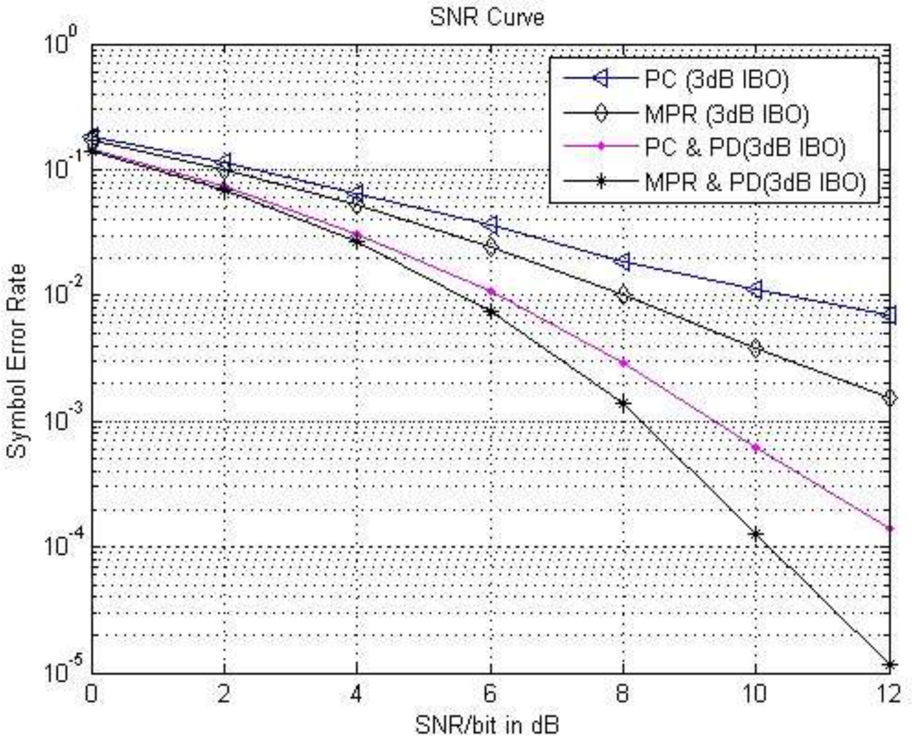

Figure 5.13 SER curve comparing PC and MPR

The curve of MPR & PD is close to the PR & PD in Figure 5.13. Also, there is high PAPR reduction in MPR process. So in MPR process, the HPA power can be sufficiently increased with almost the same SER performance as PR process.

CHAPTER 6

CONCLUSIONS

6. Conclusions

The compensation techniques to overcome the flaws in the WiMAX system due to non-linearity property of HPA and high PAPR problem of OFDM technique has been discussed and compared in this thesis work. The effect of non-linearity of HPA in WiMAX systems has been mitigated by using an adaptive pre-distortion technique in conjunction with PR and MPR techniques for PAPR reduction. The Polynomial model is used for estimation of both the non-linearity and inverse non-linearity of PA in a WiMAX communication system using the LSE estimation. Seventh order odd polynomial is used to represent the non-linear PA of the WiMAX system.

The Phase Realignment scheme offers relatively high PAPR reduction. It also offers enhanced SER performance and its modified version Modified-PR offers even more PAPR reduction. At specifically chosen amplitude threshold and phase threshold (ρ_{th} = 1.24 and $\varphi_{th} = \pi/46$), the PAPR reduces by *1.47 dB* in MPR, whereas it reduces by 0.6 dB in PR process. In addition the average power of the OFDM signal is reduced by *0.79 dB* after the processing in both PR and MPR. The SER curve of MPR is found to be close to that of the PR. Furthermore, PR and MPR are preformed after the peak clipping to mitigate the unwanted distortion introduced by PC, leading to degradation in SER performance. Among these MPR is more effective to enhance the SER performance retaining possibly low PAPR. PR scheme provides best SER performance, whereas MPR provides large PAPR reduction with the better SER performance close to that of PR scheme.

Pre-distorter is used along with the PR and MPR scheme. The normal (non-compensated) OFDM signal from the PA is compared with the compensated signal (i.e., PR and MPR with pre-distortion). The result showed the superiority of compensated technique in implementation to mitigate the non-linear distortion.

REFERENCES

[1] M. Tran, G. Zaggoulos, A. Nix and A. Doufexi, “Mobile WiMAX: Performance Analysis and Comparison with Experimental Results”, Vehicular Technology Conference, 2008.VTC 2008-Fall. IEEE, Sept. 2008.

[2] B. R. Ballal, A. Chadha and N. Satam, “Orthogonal Frequency Division Multiplexing and its Applications,” IJSR, vol. 2, pp. 325-328, Jan. 2013.

[3] R. Van Nee and R. Prashad, “OFDM Wireless Multimedia Communication,” Artech House, Boston - London, pp. 119-151, 2000.

[4] R.K. Bhatial, G. Joshi and S. R. Anjum, "Comparison analysis of OFDM signal performance with Cyclic prefix and LDPC codes over AWGN channel," IEEE Int. Conf. on Advances in Engineering, Science and Management (ICAESM) 2012, pp. 202-206, March 2012.

[5] S. P. Yadav and S. C. Bera, “Nonlinearity effect of Power Amplifiers in Wireless Communication Systems,” IEEE Int. Conf. on Electronics, Comm. and Computational Engg. (ICECCE) 2014, pp. 12-17, 2014.

[6] E. Costa, M. Midrio and S. Pupolin, “Impact of Amplifier Nonlinearities of OFDM Transmission System Performance,” IEEE Communication Letters, vol. 3, no. 2, pp. 37-39, Feb. 1999.

[7] W.F. Al-Azzo; B.M. Ali.; S. Khatun; N.K. Noordin; S.M. Bilfagih, “Peak-to-average power ratio reduction in OFDM systems using smoothing technique,” Communications (MICC), 2009 IEEE 9th Malaysia International Conference on, pp. 11 – 14, Dec. 2009.

[8] J. Armstrong, “Peak-to-Average Power Reduction for OFDM by Repeated Clipping and Frequency Domain Filtering,” IEEE Electronics Letters, vol. 38, no. 5, pp. 246-247, Feb. 2002.

[9] R. Van Nee and A. De Wild, "Reducing the peak-to-average power ratio of OFDM," in proc. IEEE Vehicular Technology Conference, vol. 3, pp. 2072-2076, May 1998.

[10] T. May, and H. Rohling, "Reducing the Peak-to-Average Power Ratio in OFDM Radio Transmission System," in proc. IEEE Vehicular Technology Conference '98, Ottawa, Canada, pp. 2774-2778, May 1998.

[11] L. Dan, T. Li, Y. Xiao and S. Li, "Performance of Peak Cancellation for PAPR Reduction in OFDM system," in proc. Comm., Circuits and Systems, ICCCAS 2008, pp. 283-287, May 2008.

[12] M. Ojima and T. Hattori, "PAPR Reduction Method Using Clipping and PeakWindowing in CI/OFDM System," in proc. IEEE Vehicular Technology Conference 2007 Fall, pp. 1356-1360, Sep. 2007.

[13] C. Y. Hsu and H. C. Chao, "Peak Power Reduction Scheme Based on Transform Kernel Scrambling for OFDM Systems," Springer Wireless Pers Commun (2007), pp. 1499-1509, June 2007.

[14] K. Kasiri and M. J. Dehghani, "A Blind SLM Scheme for Reduction of PAPR in OFDM Systems," in proc. World Academy of Science, Engineering and Technology, vol. 38, pp. 403-406, Feb. 2009.

[15] C. H. Lin, H. H. Chen, Y. Y. Wang and J. T. Chen, "Dynamically Optimum Lookup-Table Spacing for Power Amplifier Predistortion Linearization," IEEE Transaction on Microwave Theory and Techniques, vol. 54, no. 5, pp. 2118-2127, May 2006.

[16] S. P. Stapleton, G. S. Kandola and J. K. Cavers, "Simulation and analysis of an adaptive predistorter utilizing a complex spectral convolution," IEEE Trans. Comm., vol. 41, no. 4, Nov. 1992.

[17] J. Sala and H. Durney, “Unconditionally Convergent Pre-Distortion of nonLinear High Power Amplifiers,” in proc. of IEEE Vehicular Technology Conference 2001-Spring, vol. 3, pp. 1649-1653, 2002.

[18] Y. Guo and J. R. Cavallaro, ”Enhanced Power Efficiency of Mobile OFDM Radio using Pre-distortion and Post-compensation,” in proc. of IEEE Vehicular Technology Conference 2002-Fall, Vol. 1, pp. 215-218, 2002.

[19] C. H. Lin, H. H. Chen, Y. Y. Wang and J. T. Chen, “Dynamically Optimum Lookup-Table Spacing for Power Amplifier Predistortion Linearization,” IEEE Transaction on Microwave Theory and Techniques, vol. 54, no. 5, pp. 2118-2127, May 2006.

[20] R. Shrestha, M. H. Seo, G. W. Go and B. S. Lee, “PAPR reduction and Predistortion techniques against Non-linear Distortion of Satellite WiBro,” in proc. JCSAT 2008 Conference, Nov. 2008.

[21] S. H. Han, J. H. Lee, “Peak-to-Average Power Ratio Reduction of an OFDM Signal by Signal Set Expansion,” in proc. Comm., 2004 IEEE Int. Conference, vol. 2, pp. 867-871, June 2004.