AN ERROR ANALYSIS ON SOLVING VERBAL PROBLEMS OF ALGEBRA

A

THESIS

BY

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FOR THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF MATHEMATICS EDUCATION

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LETTER OF CERTIFICATE

This is to certify that Mr. Krishna Prasad Aryal, a student of academic year 2070/071 with Campus Roll No: 69, Thesis No: 1352, Exam Roll No: 280439 and TU registration No: 9-2-394-72-2008 has completed his thesis under the supervision and guidance of Mr. Lok Nath Bhattarai during the period prescribed by the rules and regulation of Tribhuvan University, Nepal. The thesis entitled on **"An Error Analysis on Solving Verbal Problems of Algebra"** has been prepared based on the result of his investigation conducted during the prescribed period under the Department of Mathematics Education, Central Department of Education, University Campus, Tribhuvan University, Kirtipur. I recommend and forward that this thesis submission for the evaluation to award the degree of Masters of Education.

.....

Prof. Dr. Bedraj Acharya

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LETTER OF APPROVAL

This thesis entitled **"An Error Analysis on Solving Verbal Problems of Algebra"** submitted by Mr. Krishna Prasad Aryal in partial fulfillment of the requirements for the Master Degree in Mathematics education has been approved.

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Recommendation for Acceptance

This is to recommend that the thesis entitled "**An Error Analysis on Solving Verbal Problems of Algebra"** has been carried out by **Krishna Prasad Aryal** for the partial fulfillment of Master Degree in Mathematics Education. This original work was conducted under my supervision. I would like to recommend for the final evaluation of this thesis.

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ABSTRACT

Algebra is the one of the most challenging topics to learn and teach in many countries. In this context, this study wants to analyze the errors made by the students in solving verbal problems of algebra. It was also intended to compare the error made by the students of community and institutional schools on solving verbal problems of algebra.

The sample in the study consists of 59 students from Glowshine Academy and 63 students, from Janjivan secondary school of Chitwan District. The schools were selected randomly. Newmann technique was adopted as theoretical based of the study. A test consisting of eight problems from grade VIII mathematics was administrated to the sample students to collect the required data. The identified errors were classified into five categories as described by Newmann. The frequency of each type of error was tabulated and converted into percentages to make comparison easy. The identified errors according to Newmann were: Reading Errors, Comprehension Errors, Transformation Errors, Process Skills Errors and Encoding Errors.

The result of the study indicated that students had committed numbers of errors on solving problems in algebra. The most prominent error was seen on the phase of encoding level, whereas, least was found in reading level. In comparison, government schools students' committed more errors than institutional schools student. The result shows that the maximum number of students they solved the problem correctly but forgot to write the answer in acceptable written form. This shows that those students who have understood the problem are failed to make appropriate mathematical form or expression.

DEDICATED TO

This work is affectionately dedicated to

My father Mr. Hom Nath Aryal,

My mother Mrs. Shanta Devi Aryal

and my wife Mrs. Ruchila Sapkota Aryal,

Who even in a very difficult situations gave me a great span of their life for what I am now.

DECLARATION

I hereby declare that this thesis entitled "**An Error Analysis on Solving Verbal Problems of Algebra"** has been prepared by me under the close guidance and supervision of Mr. Lok Nath Bhattarai, Lecturer, Department of Mathematics Education T.U., Kirtipur as a guide in the partial fulfillment of the requirements for the degree of Master of Mathematics Education T.U.is entirely my original work. I have made due acknowledgements to all ideas and information borrowed from different sources in the course of preparing this thesis. The result of this thesis has not been presented or submitted anywhere else for the award of any degree or for any other purposes. I assure that no part of the content of this thesis has been published in any form before.

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Krishna Prasad Aryal

Date: ……

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Chapter I

INTRODUCTION

Background of the Study

Mathematics is considered as the study of quantity, structure, space, and change. Philosophically mathematics has no universally accepted definition. With the increasing philosophical thought, its meaning and definition gradually become more sophisticated. Different school of thoughts perceives mathematics differently. According to Benjamin Peirce (1870) mathematics is the science that draws necessary conclusion. This definition of mathematics is very near to absolute-logical-deductive thought. Later on Intuitionist defines mathematics as the mental activity which consists in carrying out, one after another, and those mental constructions which are inductive and effective. Similarly, formalist philosophy defines mathematics as the manipulation of the meaningless symbols of the first order language according to explicit, syntactical rules (Ernest, 1991).In this way the meaning and definition of mathematics becomes a unified body of knowledge which is social culture, phenomena and social process. Thus, mathematics deals with the social phenomena of learning.

Etymologically, the word “mathematics” has been derived from an ancient Greek words “Mathematica” which mean “to learn” .Mathematics has grown with the development of humanity from its ancient civilization up to the modern civilization Mesopotamian, Babylonian, Egyptian, Roman and Greek civilization played central role in the development of mathematics as it is learned and used today.

Now a day mathematics is defined as science of measurement, quantity and magnitude. According to Oxford Advanced dictionary mathematics is a strict sense is the abstract science which investigates deductively the conclusion implicit in the elementary concept of spatial and numerical relations. The term mathematics has been explained in various ways such as; it is the numerical and calculation part of man's life and knowledge. Mathematics as we know today, is the science of numbers and operation, interrelation and collection of skills and method and to solve equation and to find unknown i.e. Algebra. The word 'Algebra' is Arabic in origin. Etymologically it came from the word 'Aljabr wal Muqubulah', where 'al' means 'the' 'jabr' refers to the 'operation of transferring quantities just from one side of an equation to another'; while 'Muqabulah' means 'the process of subtracting similar quantities from both side of an equation.

Many people believe that algebra is the generalization of arithmetic. It also thought to be most difficult and abstract among all branches of mathematics. But it is primarily thought for manipulative skills, solution of problem through the equation, a power of generalization and use of formulation and idea of functionality. As it has been called generalized arithmetic so it may be related to geometry by saying that algebra is only written geometry and geometry is merely pictured algebra. Algebra is useful in other branches of mathematics. It is especially simplified for learner for many problems of arithmetic. It gives compact formulae or generalization to be used in all causes. The solution of problems by equation and factorization are the example of this. It has a practical value in many of the trades and industries. It provides an effective way for expressing complicated relations. It gives a new, good approach, to the study of abstract

mathematical relationships through the use of a new language and new symbolism. It includes the power of analytical verification of result in simpler and more satisfactory in algebra than in other branch of mathematics. It develops confidence among the subject. It helps in the generalization of the scientific truth in to simple and compact formulae. It is a good instrument for mental training.

In an effective mathematics program, students learn the reasons to communicate mathematically and become confident in their own mathematical abilities to solve problems in situations that may not be familiar to them. Students need to be equipped as confident, competent, engaged and persistent problem solvers. The key to mathematical competence is learning with understanding so that students are able to give reason for solving problems, and applying their learning to new situations. Students need to be computationally fluency to carry out mathematical procedures flexibly accurately, efficiently, and appropriately. Students need to be able to understand and use mathematics in everyday life. This understanding must be continually strengthened and expanded throughout their educational experiences.

Teachers often had negative feeling about the mistake that children would make, regarding them as “unfortunate” events that needed to be eliminated and possibly avoided at all times. However, the strategy of more drill and practice has been replaced by regarding errors as valuable sources of students thinking. As a teacher, it is difficult to escape from children’s mistakes. So it is worthwhile finding out why children make mistakes in the first place. So error analysis is the first step toward doing something relevant which helps to remove the cause of the mistake.

Error pattern analysis provides an effective and efficient method for diagnosis specific problems students are having with competition. By determining that student is consistently using an inaccurate procedure for solving computation problems, teacher can provide specific instruction and monitoring to assist the student to use an effective procedure for solving specific types of computations. Additionally, teacher may discover through error pattern analysis that a student does not have an accurate working knowledge of a major mathematical concept. In other words, specific types of error patterns can use that a student not only uses an ineffective procedure to compute a problem, but that the student also does not accurately understand an important mathematical concept. More of error pattern analysis is much more than a diagnostic tool for determining a student's procedural effectiveness; often, it provides you a window for determining that a student lacks basic conceptual understanding. This situation is due to various student learning characteristics as well as instructional factors including: a student's slow rate of processing information relative to instructional pace, lack of sufficient opportunities to respond (practice), lack of specific feedback regarding Misunderstanding or non-understanding, anxiety about mathematics, and visual as well as auditory processing difficulties.

Lower secondary level is the basic level of acquiring prerequisites knowledge for further study. Piaget’s cognitive development stage states that it is initial time for abstractness of mathematical concepts. In practical life student of mathematics are typically evaluated by counting the number of correct and incorrect response. Those students committing a few errors learn mathematics and student committing more error do not learn mathematics. If any entire school is taking a standardized test, the result may

be used to decide grade placement, promotion, scholarship or perhaps the success of the school program. Occurrence of errors study differs from quantitative evaluation by attending to patterns of missed problems. Its purpose is not to evaluate how much the students know but rather to describe what and where the students could not get the subject matter or knowledge.

Statement of the Problem

We know that the algebra is one of the important branches of mathematics at Basic level. This level is the foundation for the upper levels. Verbal as well as calculation problems are the basis for the understanding of mathematics. It is necessary that every students should enjoy mathematics i.e. students should be interested to learn mathematics. So this study mainly concerned with the identification of error and comparison of errors committed by grade eight students in solving algebraic problem. There is low achievement score and increased failure rate in mathematics. Now, new curriculum has created a debate over a curriculum that it is not relevant to Nepalese context. Why most of students fail in mathematics? What are the causes behind the decline of achievement score? What are difficulties faced by the students in learning mathematics? The issues raised above are the major issues among the subject expert, math educator, mathematics teacher, and parents and policy makers? On the other hand, many people believe that algebra is easy portion of mathematics but most of the students used to commit mistakes in algebra while solving verbal problem of algebra.

As a researcher, I wanted to investigate why the students committed error while solving verbal problem of algebra. How could we overcome these problems, so that we

could make mathematics more interesting and popular among students, trying to answer these questions, I have selected the topic, "An error analysis on solving verbal problem of algebra". Newman's hierarchy of error analysis was applied to investigate the errors that the students of grade eight committed in solving mathematical problems. This study is mainly concerned with analysis of mathematics performance; enroll in public primary schools students in rural area of Nepal. In this regards, this research was conducted to find the answers of following questions:

- What is the pattern of errors made by the students in learning algebra?
- Does the error made by students on solving algebraic problems differ significantly?
- Do the public school students have similar error with institutional school students?

Objective of the Study

To analyze the error committed by the students in solving verbal problem of algebra, the following objectives were stated:

- To analyze the error made by students in solving verbalproblem of algebra.
- To compare the error with respect to school type (community and institutional).
- To suggest the way of minimizing errors committed by students in solving verbal problems.

Significance of the Study

The purpose of this research was to identify the errors and classify the relative frequency of the most common errors made by the students in their attempts to solve the problems of algebra. Algebra is a useful and important topic from where no one can escape, but it was necessary to find out what errors committed by the learners. So the mathematicians in the field of teaching learning should conscious on the questions viz. Why they feeling difficult? What are the ways to make it simple while teaching? Remembering these facts as central point, the researcher analyzed the error committed by grade VIII students in solving verbal problems of algebra. Therefore, this study would be helpful for teachers, school administrators, policy makers, curriculum designers, researchers and concern students as well. It would help teacher to organize his experience and teaching strategies and adopt suitable method to teach. It would help the curriculum planners and text books writers to organize the learning experience in the appropriate manner. This study would be significant for the students of mathematics to minimize their errors in solving algebraic problems.

Delimitation of the Study

Each study is not rigorous, perfect and free of limitation. So this study had the following delimitation because of time and resources constraints.

- The study was limited to grade VIII students of Chitwan district.
- Only two schools were selected as the sample schools randomly by the researcher, so the result may not be generalized to all other context.

- The study was limited only to the error made by students of grade VIII on solving verbal problems of algebra.
- Test was made by the researcher himself with the help of subject teacher, expert, and supervisor.

Definition of the Key Terms

Verbal/Word Problem. A verbal problem in this study is a mathematical exercise where significant background information on the problem is presented as text rather than in mathematical notation

Errors. The mistake done by the students during the solution of the problems were considered as an error in this study.

Error Analysis. The systematic study and analysis of error committed by the students.

Reading Error. Students have not been able to read all the words in the question such that s/he can't grasp all the information or key words given in the question.

Comprehension Error. The error related to the act of understanding the meaning of word problem. If the student can't grasp the overall meaning of the given problem, this type of error is classified as comprehension error.

Transformation Error. The error related to translate the word problem in to mathematical words. If the students understand the question but cannot transform it in to mathematical expression, this type of error is classified as transformation error.

Process skill Error. The error related to given sequence of work and the nature of the events in solving word problem .If the student make the mathematical expression but cannot identify the correct and do not know the procedure to carry out the solution.

Encoding Error. The error related to formal written form of the solution of word problem, where student makes correct solution to the question but, they cannot express solution into acceptable written form.

Chapter - II

REVIEW OF RELATED LITERATURE

A literature review is a written summary of journal article, books, and other document that describes the past and current state of information related to the topic of research study. With so much information available, searching and locating good literature on the topic of research study can be challenging (Creswell, 2012). The previous studies cannot be ignored because they provide the information to the present study. The following empirical and theoretical researches were reviewed in this study:

Review of Empirical Literature

Newman(1977, 1983) maintained that when a person attempted to answer a standard ,written ,mathematics word problems than that person had to able pass over a number of successive hurdles level: Reading, Comprehension, Transformation ,Process Skills and Encoding. Along the way, it was always possible to make a carelessness error and there some who gave incorrect answers because they were not motivated to the answer to their level of ability. Clements (2004) believes that at the upper primary level most errors on mathematics tests and examinations are caused by Reading, Comprehension or Transformation errors, or by Carelessness. Often, pupils are able to carry out one or more of the four operations (+, -, x, ÷) needed to answer a question, but they do not know which operations is to use. Not surprisingly, there are those who disagree and who argue for the centrality of language in the teaching and learning of mathematics. Others would also argue that a deeper level of mathematics is needed beyond procedural proficiency, and that a conceptual knowledge of mathematics is the

goal. Some would maintain that language provides a vehicle for rich classroom discussions and assists teachers and students to appreciate the power of mathematics in making sense of their world.

On the other hand, Aremu (1998) explained that; when pupils express lack of interest in the subject, it affects the way they react or listen to the teacher. And when many of the pupils believe that they cannot pass, the teacher is affected. This is because of this negative response from the pupils; he/she as well is already being confronted by many other factors (e.g., low income, low status in society, large teacher-pupils ratio) and so on. These may cause him or her to resorts to the easiest way of disseminating knowledge that is 'chalk and talk' without the use of instructional materials. He may not also bother to vary his teaching styles to suit individuals; therefore, the cycle goes on.

One unfortunate outcome of this is that, the negative attitude towards the subject is passed down from one generation of pupils to another and therefore the cycle keeps enlarging. What then could be done to break such a cycle of failure? This has been the question by many mathematics educators and researchers. A lot of new and modified old methodologies have been proposed to improve performance in the subject. Instructional materials have also been designed and developed to aid mathematics teaching and learning. The issue as relating to mathematics education would then be, is it possible to motivate pupils to learn mathematics? And how could it be done? One needs to therefore look at the effect of motivation on learning. He suggested that the place and importance of mathematical word problems in the school curriculum have attracted diverse opinions. "Teachers seem not to like word problems. Many have asked me why these are used to

'trick' children in assessments" It is well recognized that students appear to struggle with both the literacy and mathematical demands of typical mathematical word problems.

He explained that if the essence of mathematics is the setting up of working with mathematical models, and if we treat word problems in such a way, then they may have a role to play in helping children better understand the process of mathematizing. And with the increasing mathematizing of the world (from national test scores to pension prospects), informed and critical citizens need to be aware that mathematizing is not something that arises from the world, but something that is done to the world. In a small way, working on word problems might help begin to develop this awareness.

By supporting above views, Marahatta (2002) studied on “A study on computational errors on fraction by grade VI student in Chitwan district”. The descriptive survey method was adopted to find the error of students on solving mathematical problems of fraction. After analyzing the data he conclude that the mean errors occurring the addition of fraction and subtraction of fraction were the same. It can be concluded that the grade VI students had the same difficulty in addition of fraction and subtraction of fraction. His conclusion also shows that the location of school i.e. rural or urban didn’t play significant role in committing error. And there is no effect of sex to commit the error in areas of operation of fraction considered in the study.

Likewise, Bhatt (2003), conducted research on the topic “An error analysis in quadratic equation at grade X”, with objective to investigate for identification and comparison of errors committed by grade X students in quadratic equations. He concluded that there is no effect of location committing equal number of errors in

understanding knowledge of solving and application of quadratic equation. And the role of gender is less important to commit errors in understanding knowledge of solving and application of quadratic equation.

Similarly, Pokhrel (2013) studied on "Error analysis on solving word problems in mathematics." The main objective of this study was to find out the error of grade V students on solving word problems of mathematics. The samples of the study were four public schools from rural area of Syangja districts. Descriptive survey design was used for this research. He concluded that Primary Level's students feel difficult to solve word problem. Most of the students are unable to give the appropriate meaning of mathematical term properly and they are unable to choose appropriate operation to solve the problem. In this study, the researcher have found maximum errors in transformation level because maximum number of students is unable to choose appropriate operation, so that conceptual teaching for minimize the transformation errors. On the other hand comprehension errors appear lack of pre-requisite knowledge and concept of mathematical terms.

In addition, Ningrum *et al*. (2019) stated that the error factor in solving contextual problems in algebraic material is that students cannot gain and understand the information well; students do not understand the transformation of problems, weaknesses in the concept of prerequisites possessed by students, lack of student experience in working on contextual problems. Most students are not able to transform contextual problems into algebraic forms. Mathematics teachers are expected to provide much practice in contextual problems for students. Therefore students are accustomed to

solving problems related to daily life. Teachers are also expected to conduct learning that focuses on the process, so students are trained to do the best planning in solving various problems faced.

Theoretical Framework

Error analysis can also be conducted for mathematics, for computation as well as for concepts. There are a number of benefits to conducting an error analysis in mathematics. The teacher can:

- Identify the steps a student can perform correctly
- Identify patterns of errors
- Determine whether the error is a one-time miscalculation or whether it is a persistent error indicating an important misunderstanding of a math concept

To conduct an error analysis for mathematics, the teacher can analyze the student's errors on a worksheet, test, or progress monitoring measure. The teacher should score each problem, marking each incorrect digit in the student's answer from right to left for addition, subtraction, and multiplication problems. By scoring from right to left, the teacher can be sure to note incorrect digits in the place value columns. Division problems should be scored left to right. Evaluating each numeral in the answer allows the teacher to gain more information. This study adopted Newman Procedure for error analysis as a theoretical framework. Newman proposes to analyze and minimize the error we have to proceed in the following hierarchy:

- Read the problem
- Comprehend what is read
- Carry out the mental transformation from the words of the question to the selection of an appropriate mathematical strategy.
- Apply the process skills demanded by the selected strategy

- Encode the answer in the acceptable written form.

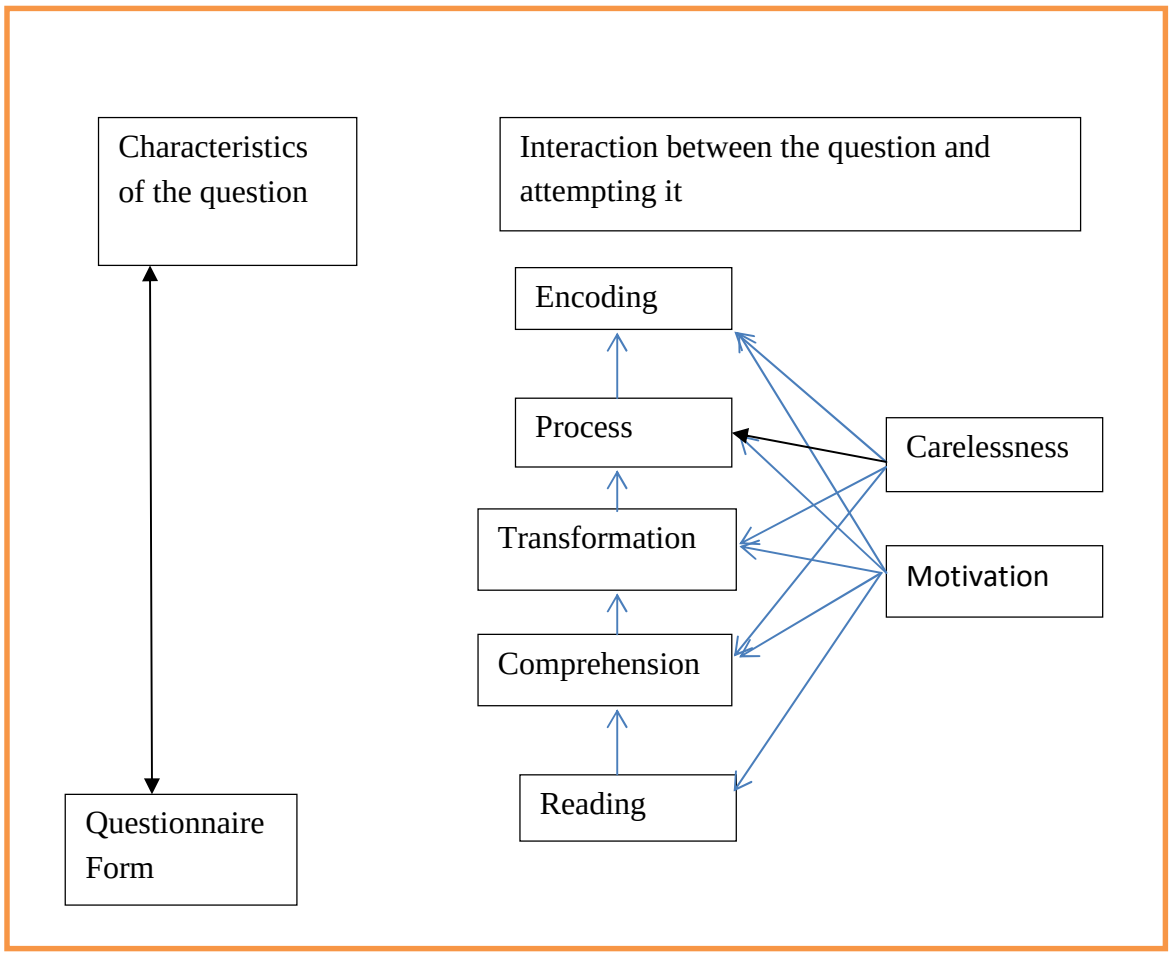


Fig 1: The Newman hierarchy of errors causes

Conceptual Framework

A conceptual framework is representation, either graphically or in narrative form, of the main concepts or variables, and their presumed relationship- with other. A conceptual framework covers the main features (aspects, dimensions, factors, variables) of a research and their presumed relationship. To conduct the research on error analysis in solving verbal problems in mathematics at grade VIII ; the researcher has read several other thesis, research articles and journals and was performed the study on the basis of following framework:

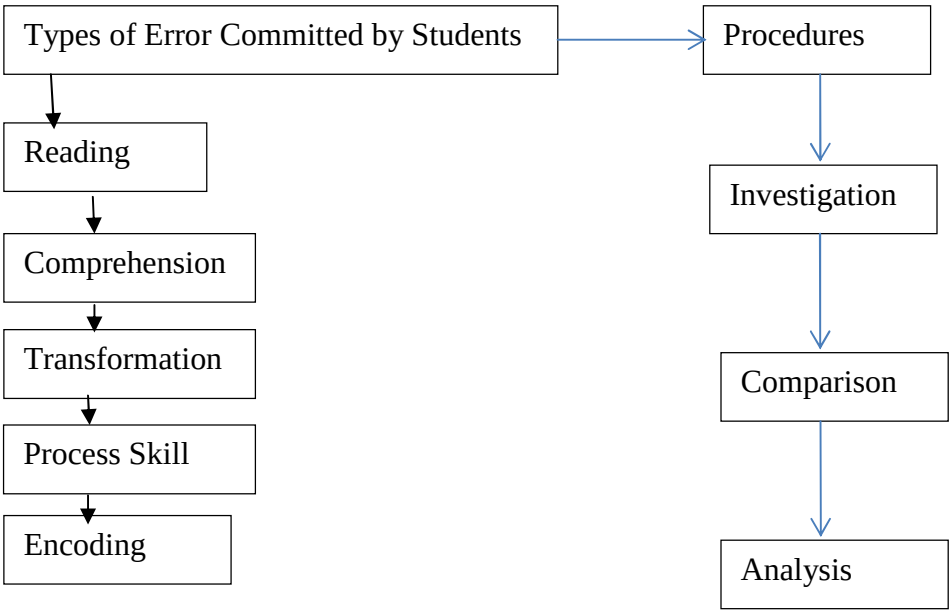


Fig 2: ***Method of Analysis Error Committed by Students***

The above conceptual framework depicts that the types of error committed by students in solving verbal problems. Among them reading error refers whether students read the question or not. Comprehension refers can students understand the meaning of question? Transformation refers how students use their prior knowledge in the context. Process skill is the skill in which students are able to perform correct mathematical operation accurately. Encoding level is the highest level where students cannot represents the verbal problem into correct mathematical output.

As shown in the conceptual framework, the researcher investigates the errors as in the framework then the variables are compared to each other and finally the information was analyzed.

Chapter-III

METHODS AND PROCEDURES

Research methodology is the most important aspect of research work. It is a bridge to achieve the objectives of the study in systematic way. Authenticity and reliability of any research depends upon the tools and methods used for data collection. Hence, the primary purpose of this chapter is to discuss and design the framework for the research.

Research Design

The research design of this study is descriptive survey. Both the quantitative and qualitative techniques were used to analyze the data of the study. It is descriptive because Its aim is to describe the events or situation addressing the present activities of the students. The researcher used the Newman's procedure of error analysis as theoretical background and on the basis of them, result was analyzed.

Population of the Study

A population is all the individuals or units of interest related to the study. The population of this study consisted of all the students of Chitwan district of grade VIII enrolled in the academic year 2074/2075.

Sample of the Study

Using simple random sampling method one institutional and one public school were selected. Then all the students of grade VIII were selected as sample students. Hence the total sample students selected in this study is 122. Also four mathematics

teachers were selected for the interview purpose. Similarly four mathematics students who commit many errors were selected purposively for interview purpose.

Data Collection Tools

To analyze the errors committed by the students, the researcher has developed an achievement test. The test items included ten questions based on the curriculum of grade VIII Mathematics. The entire test items were selected from algebra and were in the form of word (verbal) problem. After finding the error, I developed open ended interview guidelines for teacher as well as students for analyzing the error and to find the idea to minimize error. So achievement test and interview guidelines for students and teachers were main tools developed by the researcher in this study.

Data Collection Procedure

Data is the foundation of any research. Therefore data collection is the essential part of research. For this study, I was visited one institutional and one community school to collect the data and meet with the concern students.

The paper- pencil test was administered to sample students of grade VIII. After paper pencil test, four sample students were interviewed by using Newman's question for interview. And as soon as possible I took interview with four sample students and four mathematics teachers about the error and idea to minimize it. For this purpose, the researcher went to the class with class teacher and made clear to the students for the purpose of this study and the test. I tried to make confident that the result of the test would not affect in their final examination and their position. Further, they were

informed to give answer freely and fairly. The researcher gave the necessary instruction to the students how to give answer the question with examples. Then the test was administered to the students select for the sample. I was selected those types of students who committed errors in solving word problem. Similarly four sample students and four teachers were interviewed using the prepared interview guidelines.

Data analysis Procedure

Data analysis procedures were based upon the data collection tool. The collected data were analyzed by using descriptive method. The errors were identified as reading error if the student was not being able to read all the questions. Similarly the comprehension error was identified if the student is able to read all the word problems but not able to comprehends the overall meaning of the problem. An error was classified as comprehension error, if the student couldn't grasp the overall meaning of the verbal problem. An error was classified as transformation error, if the student couldn't transform the sentences into mathematical forms. An error was classified as process skill error, if the student could choose the appropriate operation but couldn't complete accurately. An error was classified as encoding error, if the student couldn't write down the answer in acceptable written form. An error was classified as motivation error, if the student did not want to respond the problem at any level of hierarchy. An error was classified as carelessness error, if the student obtained correct answer in second attempt during interview i.e. if the student could spot his /her own mistakes.

The Newman procedure for analyzing errors on written mathematical task was adopted to analyze and interpret the data obtained from written test and interview. The

numbers of errors were converted into percentage so that the errors can be compared more easily. Furthermore, the error committed by community and institutional school students were counted separately to determine whether there were any differences in committing errors while solving verbal problem in algebra.

Chapter IV

ANALYSIS AND INTERPRETATION OF DATA

This is a survey research which is related to find the errors committed by students in solving verbal problems of algebra, to analyze the errors in solving algebraic problems, and to compare the error with respect to school types(community and institutional) and to suggest the ways of minimizing errors committed by students. This study was based on descriptive survey research design because the aim of this study is to describe the events or situation addressing the present activities of students. The researcher had used Newman technique of error analysis as the theoretical base of the study and on the basis of them result was analyzed.

` After making the tools ready, the researcher visited both of the sample secondary schools. All the students of class VIII were taken for written test. All the test items included in the test paper was prepared by researcher himself as per objective of the research and the items were selected from different topics of Algebra. After taking paper-pencil test, researcher checked the test paper in which the first mistake was taken as error and the errors following were neglected. To analyze the collected data, Newman's technique of error analysis was used as theoretical base. The data analyzing procedures is based on the following headings:

- Distribution of errors on the basis of test items
- Error committed by the students as school type
- Reading Error

- Comprehension Error
- Transformation Error
- Process skill Error
- Motivational Error
- Encoding Error

Distribution of Errors on Community School Students

The following table shows the distribution of errors on the basis of test items of community school students

Table 1. ***Distribution of Errors on the Basis of Test Items for Community Schools***

S.N.	**Test Items**	**No. of Errors**			**Percent**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Total**
1	The length of the rectangle is 6m less than three times its breadth. Find the length and breadth of the rectangle if its perimeter is 148m.	34	48	82	13.5	13.2	13.4
2	The sum of the three consecutive numbers is 30, find the numbers.	32	48	80	12.7	13.2	13
3	A man is twice as old as his son. If their total age is 60.fin their ages.	27	42	69	10.7	11.6	11.3

4	Divide 184 into two parts such that one-third of one part exceeds one –seventh of another part by 8.	51	50	101	20.3	13.8	16.5
5	The difference of two numbers is 15. The smaller number is 13. Find the greater number.	23	52	75	9.2	14.4	12.2
6	The product of 150 and a number is 300. Find the number.	25	46	71	10	12.7	11.6
7	The sum of two numbers is 58 and their difference is 12. Find the numbers.	41	41	82	16.3	11.3	13.4
8	A number consists of two digits whose sum is 5. If 9 is subtracted from the number, its digits are interchanged. Find the number.	18	35	53	7.2	9.7	8.6
Total		251	362	613	100	100	100

Table 1 show that the total errors committed by boys and girls students of community schools. This table also shows the percentage of errors committed by boys and girls students of community schools. It shows that only 13.4 % of the total errors were committed in the first problem which was about finding length and breadth of rectangle by making algebraic equation. In first test item girls committed 13.2% error and

boys committed 13.5% error. This shows that the girls committed less error than boys. In this problem, students committed less error in comparison of other test items that indicate students were more familiar on the concept of rectangle and its area. Second test item was about the consecutive numbers in which 13% of the errors committed by the students. Boys committed 12.7% errors whereas girls committed 13.2% errors in this problem. Test item third was taken from father and son's age. In this problem, 11.3% of the total errors were occurred. Boys and girls committed 10.7% and 11.6% errors respectively. This shows that girls committed fewer errors than boys in solving mathematical problems related to the age of father and their son. Forth test item was about to find the numbers if one is exceed by other. In this problem, boys committed 20.3% and girls committed 13.8% of the total errors in aggregate of 16.5% error.

Similarly, the fifth test item was also related to finding numbers if the sum and difference of two numbers are given. In this problem boys committed 9.2% error where girls committed 14.4% error. The average error was 12.2%. Sixth test item was related to finding number when product of numbers is given. Here boys and girls committed 10% and 12.7% errors respectively and average error was 11.6%. Seventh test item was related find numbers if the sum and difference of two numbers are given.In this problem, students committed 13.4% error on which boys committed 16.3% errors and girls committed 11.3% errors. Test item eight was related to two digits numbers. Comparatively less error errors were committed in this problem. Boys committed 7.2% and girls 9.7% errors in this problem. 8.6% of the total errors were committed in this problem. This shows that the community school students were more practiced on these type of problems in previous classes as well.

Distribution of Errors on Institutional School Students

The following table shows the distribution of errors committed by the students of institutional schools to solve verbal problems of algebra**.**

Table 2. ***Distribution of Errors on the Basis of Test Items for Institutional School***

S.N.	**Test Items**	**No. of Errors**			**Percent**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Total**
1	The length of the rectangle is 6m less than three times its breadth. Find the length and breadth of the rectangle if its perimeter is 148m.	22	35	75	12.94	13.15	13.07
2	The sum of the three consecutive numbers is 30, find the numbers.	14	25	39	8.23	9.39	8.94
3	A man is twice as old as his son. If their total age is 60.find their ages.	27	35	62	15.88	13.15	14.22
4	Divide 184 into two parts such that one-third of one part exceeds one –seventh of another part by 8.	18	32	50	10.58	12.03	11.46
5	The difference of two numbers is 15. The smaller number is 13. Find the greater	22	43	65	12.94	16.16	14.90

	number.						
6	The product of 150 and a number is 300. Find the number.	24	36	60	14.11	13.53	13.76
7	The sum of two numbers is 58 and their difference is 12. Find the numbers.	18	22	40	10.58	8.27	9.17
8	A number consists of two digits whose sum is 5. If 9 is subtracted from the number, its digits are interchanged. Find the number.	25	38	63	14.70	14.28	14.44
Total		170	266	436	100	100	100

The above table 2 shows that the numbers and percentage of errors committed by boys and girls students of institutional school. This shows that total of 170 errors were committed by boys and 266 errors were committed by girls. It indicates that girls committee more errors than boys on solving verbal problems of algebra. It also shows that only 13.07% of the total errors were committed in the first problem. In this test item girls committed more error (13.15%) than boys (12.94%). In this problem, students committed less error in comparison of other test items. In the second test item,8.94% errors were committed among all the items. Boys committed 8.23% errors whereas girls committed 9.39% errors in this problem. In test item third,14.22% of the total errors were occurred. On which boys and girls committed 15.88% and 13.15% errors respectively. In this problem there is a high percentage of committing errors than other problems. In the

fourth, boys committed 10.58% and girls committed 12.03% of the total errors in aggregate of 11.46% error. Fifth test item was also related to father and son's ages. In this problem boys committed 12.94% error and girls committed 16.16% errors. The average error was 14.90%. In the sixth test items boys and girls committed 14.11% and 13.53% errors respectively and average error was 13.76%. In the seventh problem, students committed 9.17% error in which boys committed 10.58% errors and girls committed 8.27% errors. Test item eight was related to fraction. Comparatively less error errors were committed by the students in this problem. This shows that students were more competent in fraction then other contents of algebra. Boys committed 14.70% and girls 14.28% errors in this problem 14.44% of the total errors were committed in this problem.

From the comparison of table 1 and table 2, it can be seen that total 613 errors were committed by community school students and total 436 errors were committed by institutional school students. So, we can say that community school students committed more errors than institutional school students. In the content of algebra, community school students seems weak than the institutional school students. The community school students have little confidence level to solve algebraic problems than the students of institutional school. This also gives the glimpse of the total error investigation and their distribution in an institutional and community schools in solving the verbal problems of algebra.

Item Wise Analysis of the Errors Committed by Students

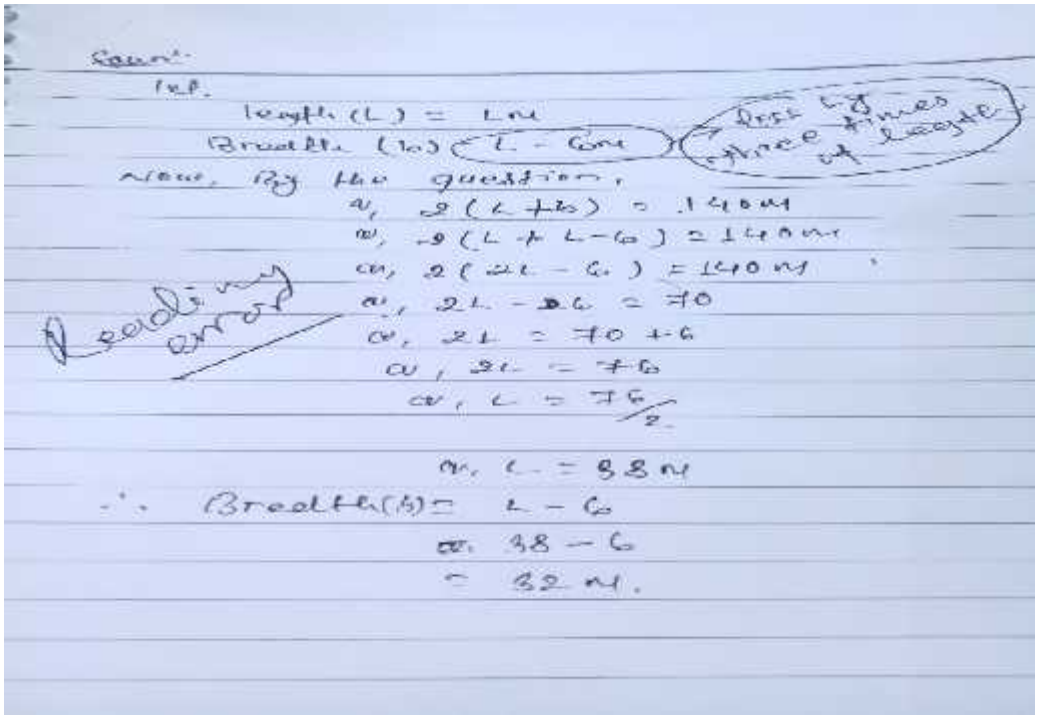
The following section presents the distribution of errors and the sample of students answer sheet with their errors in solving the verbal problems of algebra. The error committed by the students was analyzed item wise as follows:

Analysis of First Test Item

The first test item was "A two digit number is 3 times the sum of its digits. The sum of the number formed by reversing its digits and 9 is equal to 3 times the original number. Find that number". At first, I analyzed the items wise errors and expressed total conclusion based on error wise.

There were altogether eighty two errors in this test item. Out of these errors, thirty four errors were committed by boys and forty eight errors were committed by girls. When this question was asked to read, three boys and one girl couldn’t read the problem. The students were more confused only on the word “reversing”.

These students did not make the appropriate mathematical expression to solve the problem and one of the sample students solve the problem as:



The researcher took interview with four sample students and their respective teacher by asking why they committed such kind of mistakes. On my question the students and teachers respond as:

"I tried as what I have understood." *Student*

"The question was little bit confusing" *Student*

"Most of the students are able to read the question but they are unable to write the problem mathematically." *Teacher*

"In general, these kinds of errors are due to the lack of inner understanding and drill."

Teacher

From the above views of students and teacher, and analysis of the test item I came to the conclusion that there should be adequate opportunity to practice verbal problems of mathematics. For the solution of first question the sample student understood the question but could not express the verbal problems into mathematical expression. So, it is taken as transformation error. According to the Newman framework this student is less motivated to the problem and he also had reading error. To minimize these type of errors teacher should focus on the verbal problems and the process of translating the verbal problem in to mathematical form. Similarly, more practice of students is also necessary for the minimization of such type of errors.

Distribution of Errors of First Test Item

The following table shows the distribution of errors of first test item.

Table 3. ***Distribution of Errors of First Test Item***

S.N.	**Types of Error**	**No. of Errors**			**Percentage**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	3	1	4	8.8	2.1	4.9
2	Comprehension Errors	4	6	10	11.8	12.5	12.2
3	Transformation Errors	12	15	27	35.3	31.2	32.9
4	Process Skill Errors	3	7	10	8.8	14.6	12.2
5	Encoding Errors	2	12	14	5.9	25	17.1

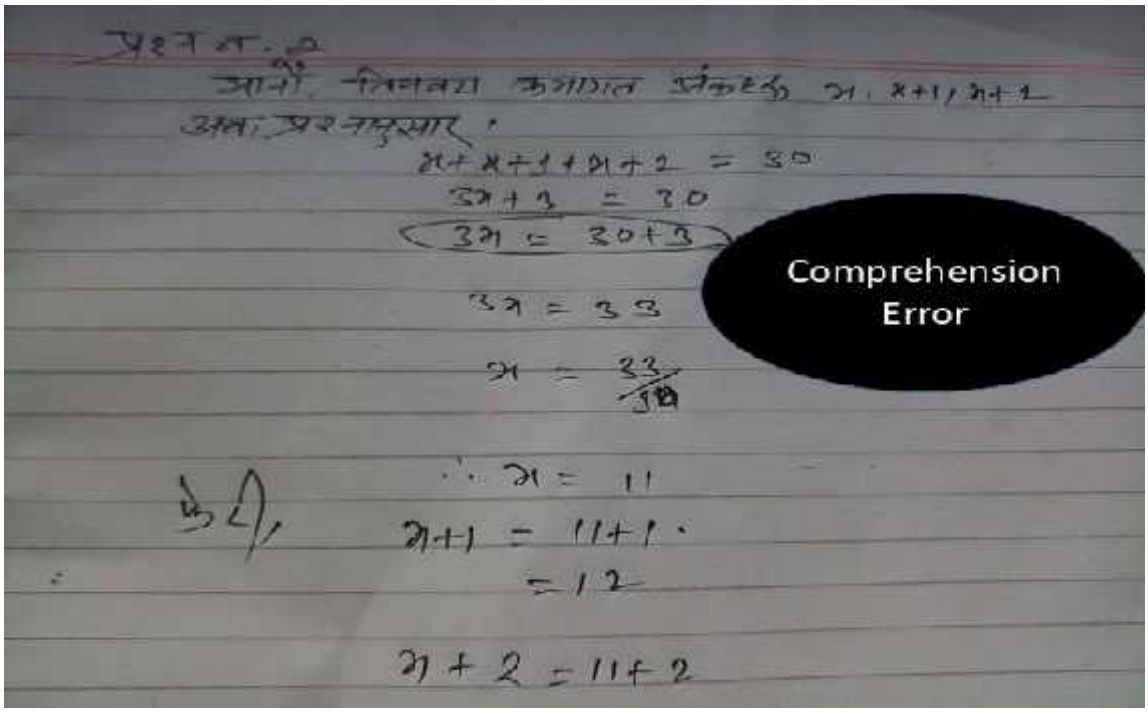
6	Motivation Errors	3	4	7	8.8	8.3	8.5
7	Carelessness Errors	7	3	10	20.6	6.3	12.2
Total		34	48	82	100	100	100

The above table shows that, there was 4.9% reading error. Ten students read the problem, but couldn't understand what is given and what is to find from this problem. There were fourteen (17.1%) encoding errors which were committed by boys and girls. Similarly, there were twenty seven students (32.9%) who could not make the appropriate mathematical expression. Out of these twenty seven errors, twelve errors were committed by boys and fifteen errors were committed by girls. The maximum number of errors was seen in the process of transformation. This shows that there is some problem of language as well as carelessness of students on mathematics. The data also shows that the girls commit more errors than boys in solving algebraic problems.

Analysis of Second Test Item

When the researcher asked the question as, the sum of the three consecutive numbers is 30, find the numbers. There were eighty one errors in this test item. Thirty three errors were committed by boys and forty eight errors were committed by girls. Most of the errors (30.8%) were committed in encoding step. When asked to read the problem, six girls and five boys couldn't read the question properly. But they corrected themselves

when interviewed. When asked to solve the problem, one of the sample student solved like below:



For the solution of the above question, the sample student could not grasp over all information from the question. So, it is taken as comprehension error. In this solution, the student was able to identify the operation but did not know the procedure to carry out this operation correctly. After the test the researcher shows the answer sheet of the concern student and asks him to find your mistake. In this time she found her mistake and realizes her mistake. This type of carelessness of students can be removed by realizing them in their mistakes. Similarly from the observation, it is seen that students were not well motivated and curious on mathematics. This is the great challenge for mathematicians and mathematics teachers.

Distribution of Errors of Second Test Item

The following table shows the distribution of errors committed by the students in solving second question

Table 4. ***Distribution of Errors of Second Test Item***

S.N.	**Types of Error**	**No. of Errors**			**Percentage**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	5	6	11	15.2	12.5	13.5
2	Comprehension Errors	5	3	8	15.2	6.3	9.9
3	Transformation Errors	8	7	15	24.2	14.6	18.5
4	Process Skill Errors	4	4	8	12.1	8.3	9.9
5	Encoding Errors	7	18	25	21.2	37.5	30.8
6	Motivation Errors	0	4	4	0	8.3	4.9
7	Carelessness Errors	4	6	10	12.1	12.5	12.3
Total		33	48	81	100	100	100

The above table 4 shows that, there was eleven (13.5%) reading error. (9.9%) comprehension errors of which five were committed by boys and three errors were

committed by girls. In this question they could not understand what is to find and were confused on the process as well. Similarly, there were fifteen (18.5%) transformation errors, eight (9.9%) process skill errors committed by the sample students. Similarly, four motivation errors were committed by girls. There are ten carelessness errors on this problem on which four errors committed by boys and six by girls.

From this problem the researcher identified different types of errors are in Newman framework. Maximum number of errors was on the Encoding category. This indicates that students made error related to formal written form of the solution of word problem. The student makes correct solution to the question but, they cannot express solution into acceptable written form. In this regards, the researcher took interview with sample students and the teacher. The edited versions of them were as follows:

I had understood the problem but I am unable to follow the appropriate solution procedure. (Student)

In my concern that what type of role did you play to minimize such kind of error? One of the participant teachers replied as:

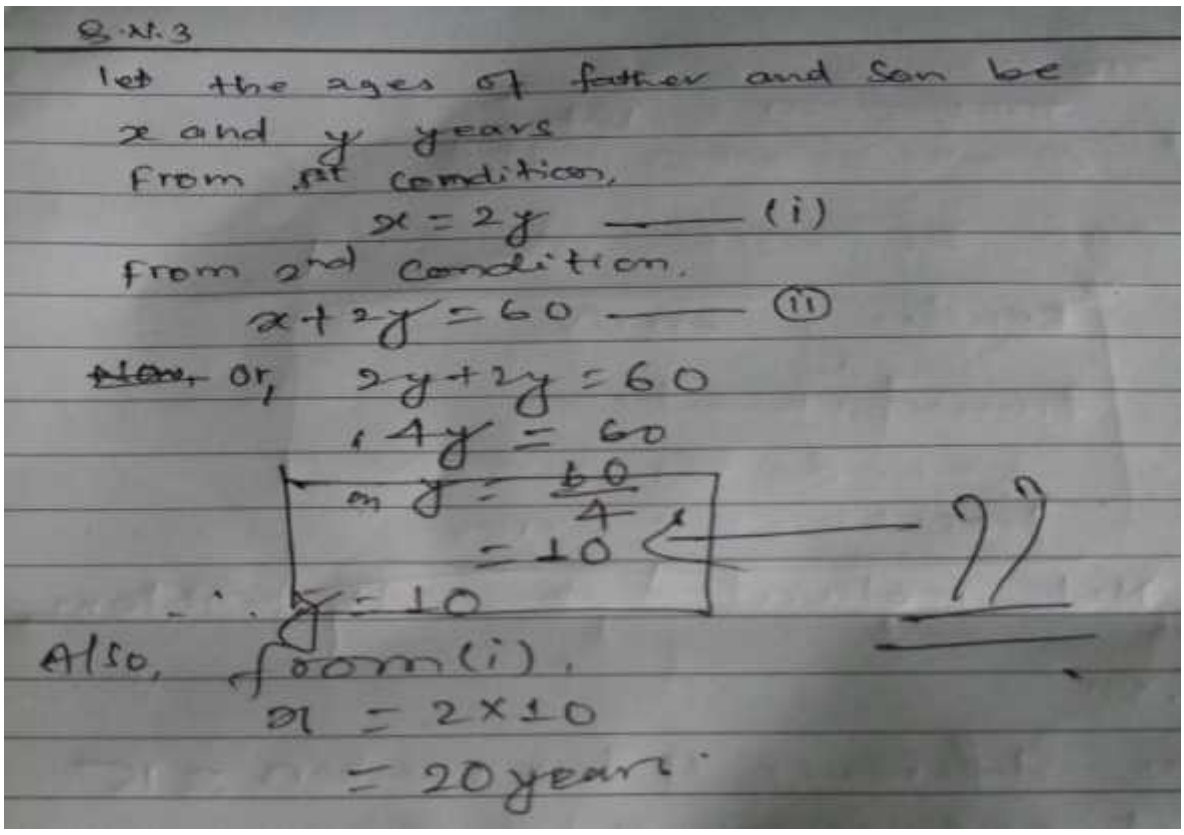
In my opinion, these students can read the problems and they understood that problem. But they committed error in the process. So the student must be careful to minimize such errors (Teacher A)

Students should actively participate in the classroom and they need to ask question to the teacher if they have any misconception (Teacher B)

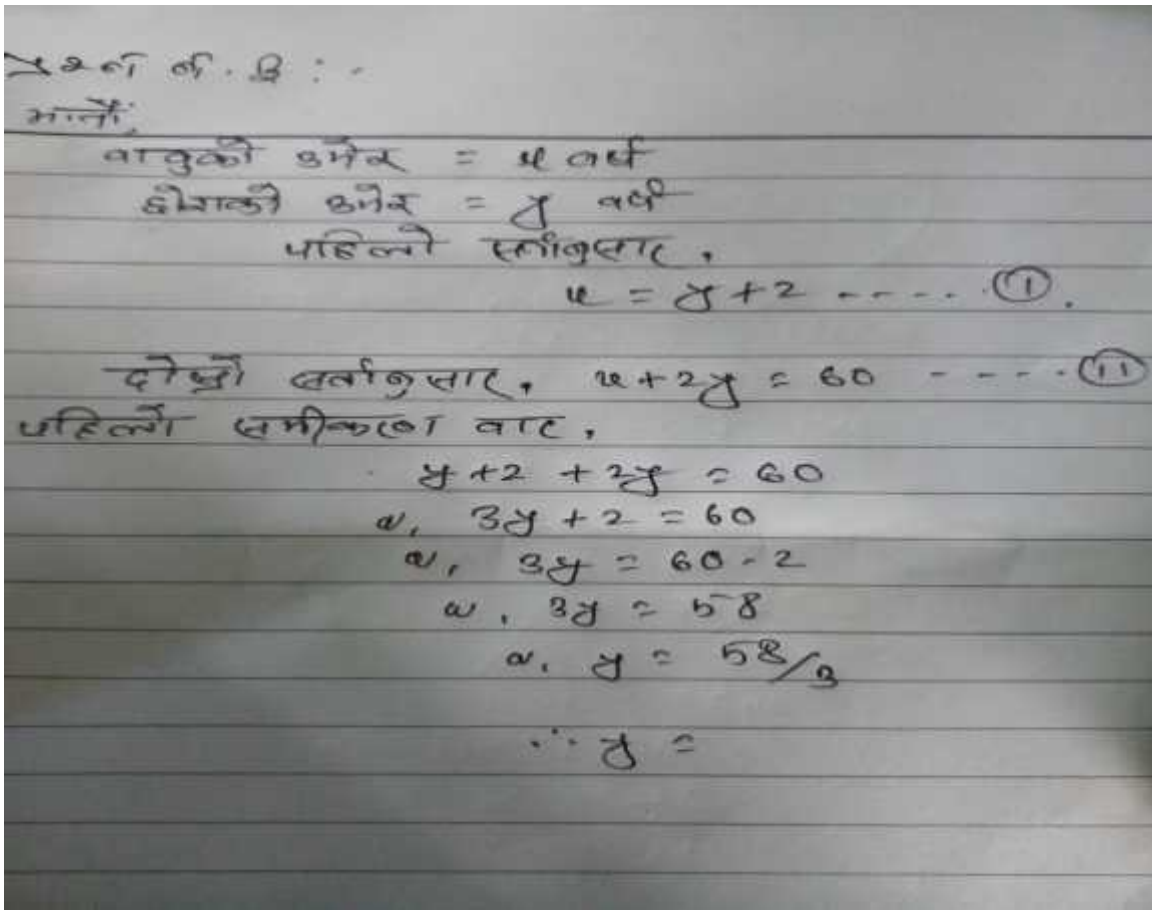
The conclusion drawn from above table and interview of teacher and students, we can conclude that the student should read the question very carefully, they should generate the ideas and they should verify their ideas to the teacher. They should be assured whether they have made correct mathematical statement. To minimize the process skill error they have to be sensitive while preceding the solution. There is lack of student teacher interaction in the mathematics classes.

Analysis of Third Test Item

The third question in the test paper was, a man is twice as old as his son, if their total age is 60, find their ages. The answer given by one of the student is given below:



For the solution of third question, she makes the solution to the problems but did not express the answer acceptable written form. The process is correct but the minor mistake on the division was seen in this solution. This type of errors was committed by the students due to the less practice and carelessness on the subject matter. The error committed by the students is considered as encoding error. By the version of mathematics teacher and Newman's theory these type of errors should be minimized through the process of learning by doing approach. Hence teacher should apply the practical approach in mathematics teaching. The sample solution of one of the sample student of community school is presented below:



In this solution the sample student does not understand the question. He understands double means more than two, so he add 2 on the age of son to equate with his father's age. So he worked out the solution to the problems but did not express the solution in acceptable written form. This type of error is considered as comprehension error.

Distribution of Errors of Third Test Item

The following table shows the distribution of error of third test item. The numbers of errors committed in different headings by boys and girls and their percentage are presented below.

Table 5. ***Distribution of Errors on Third Test Item***

S.N.	**Types of Error**	**No. of Errors**			**Percentage**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	0	1	1	0	2.4	1.4
2	Comprehension Errors	0	0	0	0	0	0
3	Transformation Errors	6	3	9	22.2	7.1	13
4	Process Skill Errors	4	17	21	14.8	40.5	30.4
5	Encoding Errors	9	16	25	33.3	38	36.2
6	Motivation Errors	3	3	6	11.1	7.1	8.7
7	Carelessness Errors	5	2	7	18.5	4.8	10.1
Total		27	42	69	100	100	100

Above table shows that, a total of sixty nine errors were committed in this test item. Twenty seven errors were committed by boys and forty two errors were committed by girls. Out of them only one error (1.4%) was reading error, whereas maximum errors were committed on encoding process. There were twenty five errors (36.2%) in the encoding step of error analysis hierarchy. Among the 69 students committed error in this problem, nine (13%) students could not make the word problem in mathematical expression properly. Twenty five errors (36.2%) were encoding error. Similarly six (8.7%) were committed in motivation errors and seven (10.1%) carelessness errors. Maximum numbers of errors were encoding error. Twenty five (36.2%) errors were committed in this step. Students could not complete operation accurately. From this table it can be concluded that; on solving verbal problems in Simultaneous Equation most of the errors occurred in encoding step. Therefore to solve such problems this step must be considered as vital step and students and teachers must focus on this step.

In this regards on my question how can we minimize encoding error? The sample teachers say,

"Encoding is the high mathematical ability. Indeed this ability is not developed in 8th grade students." *(Teacher C)*

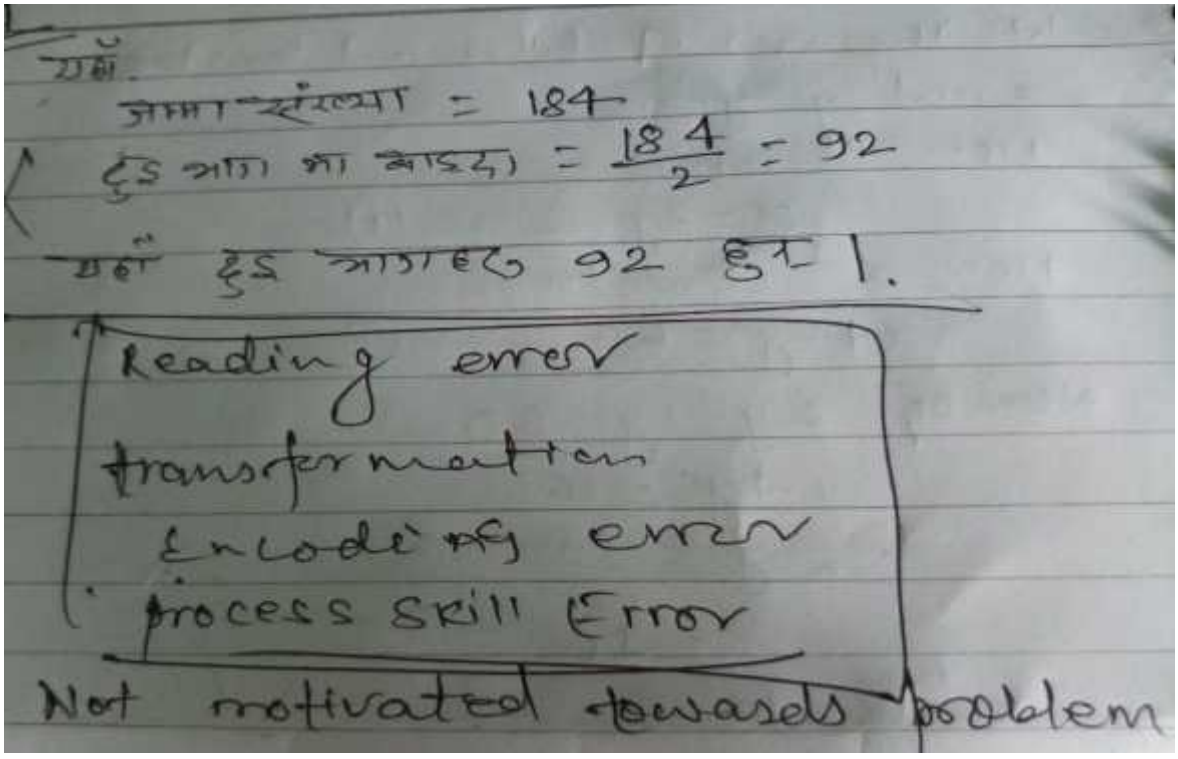
"I think this error can be minimized by developing the creative and cognitive thinking skills" *(Teacher D)*

From the above views of teachers it can be clear that encoding means to internalize the learned mathematical concepts in their own ways. The students fail to

internalize the concept properly. So the duty of teacher should be develop the creative and cognitive thinking on their students. This process helps to become the students' independent and capable. These type errors sometime occur due to age factor.

Analysis of Fourth Test Item

When researcher asked the question as, Divide 184 into two parts such that one-third of one part exceeds one –seventh of another part by 8. One hundred one errors were committed in this test item. Out of these errors fifty one errors were committed by boys and fifty errors were committed by girls. The answer given the student is shown below:



For the solution of fourth problem, the sample student couldn't grasp the overall meaning of the words and therefore could not proceed correctly. So, it is taken as comprehension error. In this problem, they understood what the question was asking

about but could not transform sentences into mathematical form. So, it is taken as transformation error as well.

Distribution of Errors of Forth Test Item

The following table shows the distribution of error committed by the students on forth test item.

Table 6. ***Distribution of Errors of Forth Test Item***

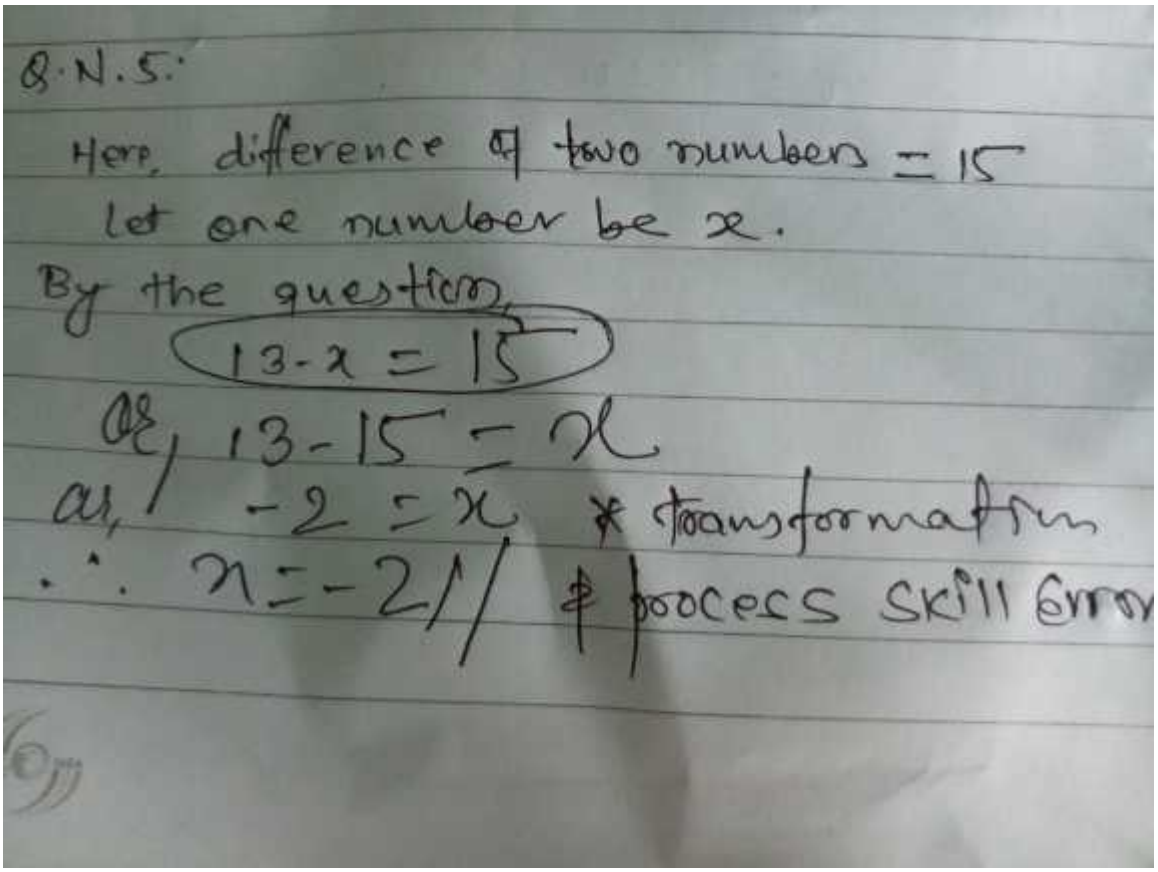
S.N.	**Types of Error**	**No. of Errors**			**Percentage**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	3	2	5	5.9	4	4.9
2	Comprehension Errors	3	2	5	5.9	4	4.9
3	Transformation Errors	26	12	38	50.9	24	37.6
4	Process Skill Errors	12	15	27	23.5	30	26.7
5	Encoding Errors	3	12	15	5.9	24	14.8
6	Motivation Errors	1	1	2	1.9	2	1.9
7	Carelessness Errors	3	6	9	5.9	12	8.9
Total		51	50	101	100	100	100

From the above table it is found that only five errors (4.9%) was reading error. Similarly, five (4.9%) errors were comprehension errors, thirty eight (37.6%) were transformation errors, twenty seven (26.7%) process skill errors and nine (8.9%) carelessness errors. Hence maximum error was found in transformation level. This also

reveals that 26.7% errors are committed in process skill errors. Therefore, these steps must be kept in mind by the students and teachers on solving verbal problems of simultaneous equation.

Analysis of Fifth Test Item

When the researcher asked the question as, the difference of two numbers is 15. The smaller number is 13. Find the greater number. One of the sample student solved this problem as follows:



For the solution of fifth problem, the student correctly worked out the solution of the problems but did not express the answer in acceptable written form. So, it is taken as

encoding error. The student is unable to express the verbal problem in mathematical form correctly. For the minimization of encoding error, the views of the teacher are presented below:

Transformation skill is the prominent skill in mathematics. Every student must be able to use pre-learning in their current learning. (Teacher A)

To minimize the transformation error teachers should link the previous mathematical concepts, ideas with the new concepts and ideas. (Teacher B)

From the above views of teachers it can be clear that the transformation skill is most important skill for effective mathematics learning. The transformation errors can be minimized by relating the pre-learned concepts with the new concept of mathematics.

Distribution of Errors of Fifth Test Item

The following table shows the distribution of errors committed by the students in fifth test item.

Table7. ***Distribution of Errors of Fifth Test Item***

S.N.	Types of Error	No. of Errors			Percentage		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	1	1	2	4.3	1.9	2.7
2	Comprehension Errors	4	3	7	17.4	5.7	9.3
3	Transformation Errors	5	19	24	21.7	36.5	32
4	Process Skill Errors	3	6	9	13	11.5	12
5	Encoding Errors	8	19	27	34.8	36.5	36
6	Motivation Errors	1	1	2	4.3	1.9	2.7
7	Carelessness Errors	1	3	4	4.3	5.7	5.3
Total		23	52	75	100	100	100

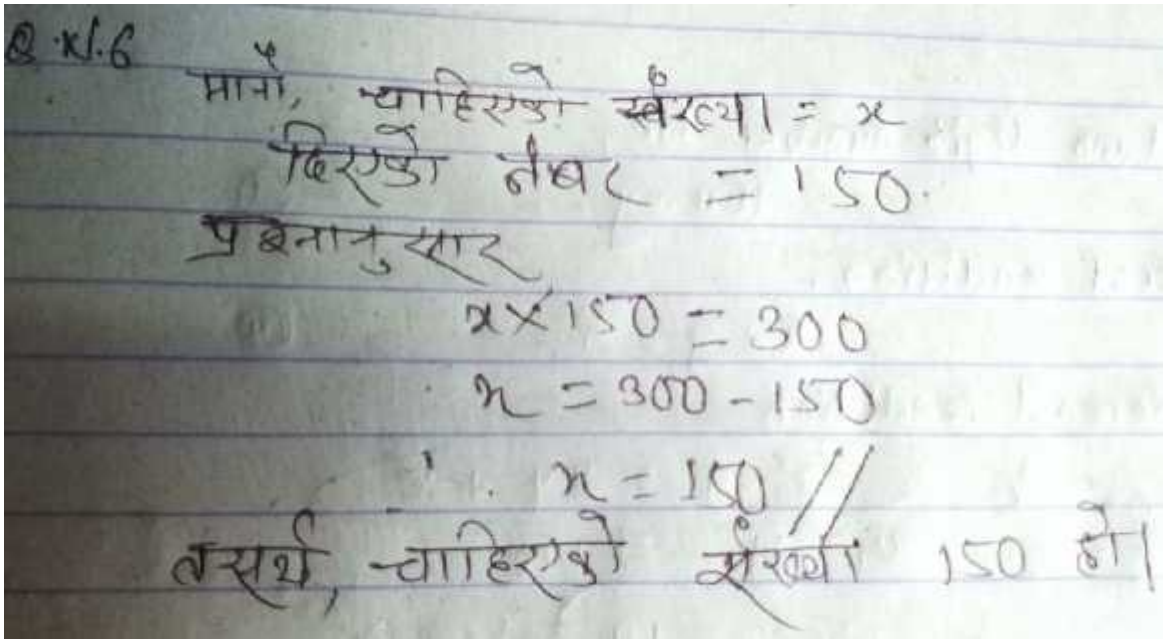
The above table shows that, seventy five errors were committed in this test item. In these item boys students committed twenty three errors whereas girls' students committed fifty two errors. Almost all students are able to read the question properly, but maximum number of students felt difficulty in understanding the concept of time that should be increased or decreased from present time when the terms before and after are given.

In this question, there were 2.7% reading errors and 9.3%comprehension Errors. Twenty four (32%) students did transformation error. Nine students (12%) committed errors in process skill which were committed by three boys and six girls. The maximum errors committed by students are encoding error. Twenty seven (36%) students committed encoding errors which were committed by eight boys and nineteen girls. Only two students committed motivation errors and four (5.3%) students committed carelessness errors.

This table shows that there are about 68% transformation and encoding errors. While solving verbal problems of algebra, students tried to transform the problem into mathematical form but were unable to transform properly. So the teachers must focus on this step in teaching verbal problems and they guide students about the way of transforming verbal problems into mathematical form.

Analysis of Sixth Test Item

When researcher asked the question "the product of 150 and a number is 300. Find the number". This problem was related to two digits number. They were quite interested to solve this problem. Total eighty eight errors were counted in this test item. Maximum number of errors was encoding errors. Five students with two boys and three girls, feel difficulty in reading the problem correctly. The word which caused the problem was "consists". They did not understand the meaning of this word and couldn't pronounce this word accurately. Five (5.7%) students knew what's to find but they did not know what is given. Therefore, these errors were counted as comprehension error. Some sample copy of the students is presented here:



For the solution of sixth problem, the student use subtraction operation instead of division. He is unable to express the answer in acceptable written form and come to the correct solution. This type of error committed by the students is considered as encoding error.

Distribution of Errors of Sixth Test Item

The following table shows the distribution of errors committed by the students in sixth test item.

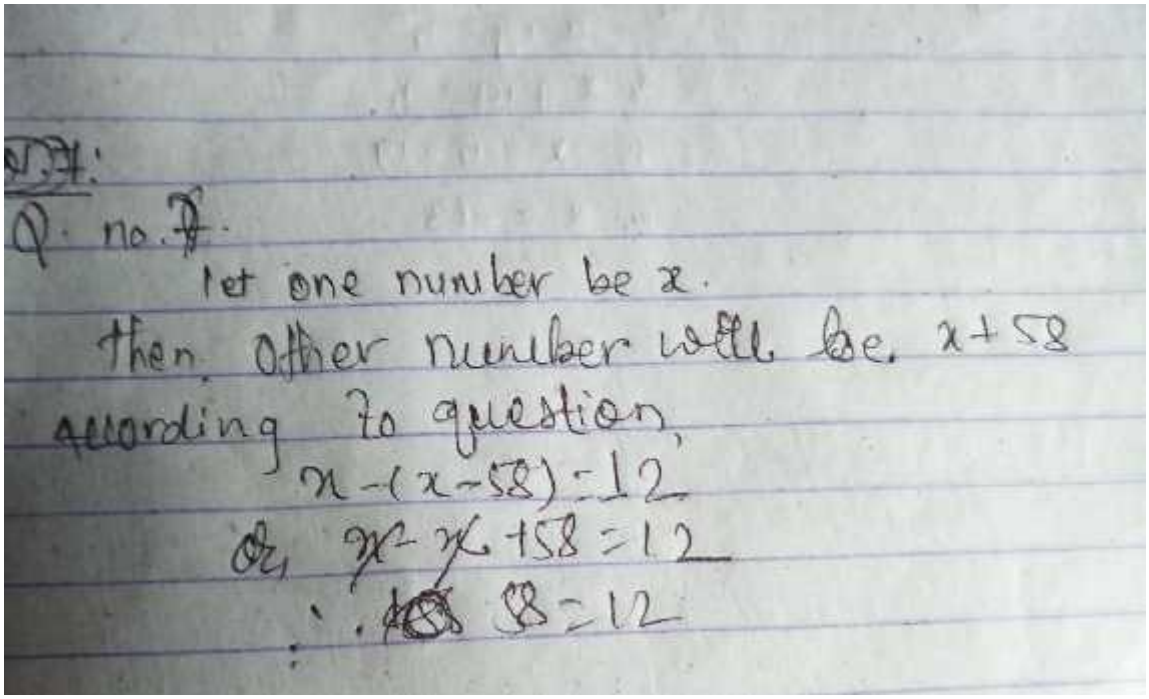
Table 8. ***Distribution of Errors on Sixth Test Item***

S. No.	Types of Error	No. of Errors			Percentage		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	2	3	5	5.7	5.7	5.7
2	Comprehension Errors	2	3	5	5.7	5.7	5.7
3	Transformation Errors	7	18	25	20	33.9	28.4
4	Process Skill Errors	2	6	8	5.7	11.3	9
5	Encoding Errors	14	17	31	40	32	35.2
6	Motivation Errors	0	2	2	0	3.8	2.3
7	Carelessness Errors	8	4	12	22.8	7.5	13.6
Total		35	53	88	100	100	100

The above table shows that, twenty five students (28.4%) who read and comprehended the problem were failed to make accurate expression and eight students (9%) could not precede the solution accurately. There are 31 encoding errors committed by the students which are the highest error and only two errors seen in motivation level which is lowest errors. This table again shows most troublesome steps on solving verbal problem are transformation and encoding type. Students who are willing to solve such problems must be careful in the key words. They must try to transform the problem in their own language but never try to attach their own fuzzy thinking.

Analysis of Seventh Test Item

The seventh question asked by the researcher was, the sum of two numbers is 58 and their difference is 12, find the numbers. Here nearly equal numbers of errors were committed by boys and girls. Eighty one errors were counted in this test item. Boys committed forty one errors whereas girls committed forty errors. Only three boys couldn't read the question correctly. There was no reading error for girls in this question. Some of the students, without making appropriate expression, tried to solve the problem and committed errors as follows:



For the solution of seventh problem she identified the operation but did not proceed to carry out this operation correctly. So, it is taken as process skill error. In this problem, he understood the problem what the question was asking but could not

transform the sentence into mathematical form. So, it is taken as transformation error. The following table shows the distribution of errors of seventh test item

Distribution of Errors of Seventh Test Item

The following table present the error committed by the students in solving the given problems related to simultaneous equation:

Table 9. ***Distribution of Errors on Seventh Test Item***

S.No.	**Types of Error**	**No. of Errors**			**Percentage**		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	3	0	3	7.3	0	3.7
2	Comprehension Errors	3	3	6	7.3	7.5	7.4
3	Transformation Errors	9	4	13	21.9	10	16
4	Process Skill Errors	18	17	35	43.9	42.5	43.2
5	Encoding Errors	4	11	15	9.7	27.5	18.5
6	Motivation Errors	0	0	0	0	0	0
7	Carelessness Errors	4	5	9	9.7	12.5	11.1
Total		41	40	81	100	100	100

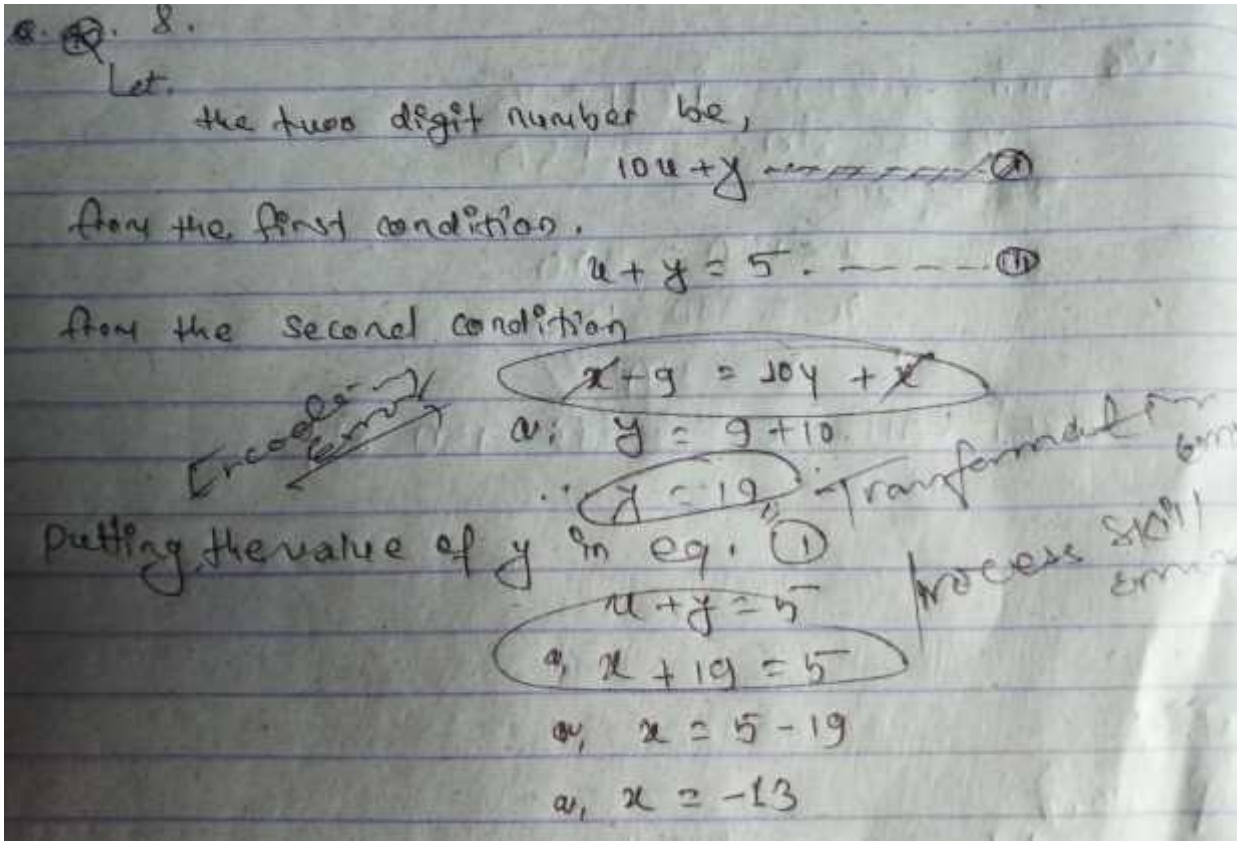
The above table 9 shows that, thirteen students (16%) couldn't transform the question into mathematical form. But nine (11.1%) students made mistake due to carelessness where four boys and five girls were found unwilling to solve the problem. This table also indicate that students were trying to solve verbal problem of simultaneous equation understand the problem easily but could not find out an appropriate mental solution and become failed to proceed further. Motivation errors and carelessness errors were found in this problem. This type of problems was found due to anxiety of students toward the problem. Due to the lack of practical activities in solving the mathematical problems the errors were committed. So teacher should focus on the learning by doing approach for students in their teaching.

Analysis of Eighth Test Item

When the researcher asked the problem as a number consists of two digits whose sum is 5. If 9 is subtracted from the number, its digits are interchanged. Find the number

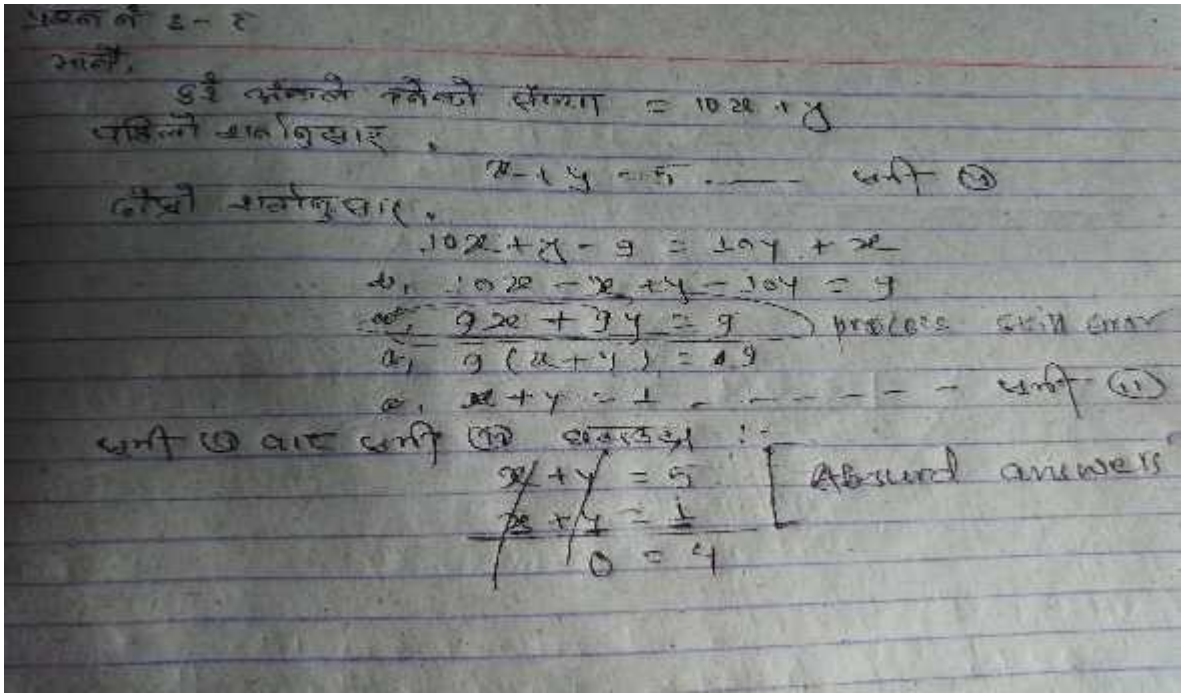
There were fifty three errors committed by the students in this test item which were mostly related to process skill and encoding error type. Some comprehension and encoding errors were also committed by the students. In this regards one of the sample

students of institutional school have solved this as follows:



For the solution of eighth question, the student was unable to grasp overleaping of question. So, could not select appropriate procedure to carry out correct solution. So, it is taken as comprehension error. The error of the students reflected that the students were fail to translate the essence of the question in appropriate manner. In this situation they are unable to solve the given problem properly.

Similarly, one of the community school students solved the same problem and commit the following errors as follows:



In this solution the sample student correctly carried out the solution to the problem but he failed to follow the steps correctly. He did very minor mistake of sign rule in his solution that create the wrong solution. There is some carelessness of students in this solution as well. This type of errors should be removed by instructing the students to focus on the problem.

Distribution of Errors of Eighth Test Item

The following table shows the distribution of errors made by the students on solving eighth test item.

Table 10. ***Distribution of Errors of Eighth Test Item***

S.N.	Types of Error	No. of Errors			Percentage		
		Boys	**Girls**	**Total**	**Boys**	**Girls**	**Average**
1	Reading Errors	2	2	4	11.1	5.7	7.5
2	Comprehension Errors	1	4	5	5.5	11.4	9.4
3	Transformation Errors	5	1	6	27.8	2.8	11.3
4	Process Skill Errors	3	13	16	16.7	37.1	30.2
5	Encoding Errors	1	11	12	5.5	31.4	22.6
6	Motivation Errors	3	1	4	16.7	2.8	7.5
7	Carelessness Errors	3	3	6	16.7	8.6	11.3
Total		18	35	53	100	100	100

The information obtained from above table shows that, there were sixteen (30.2%) process skill errors and twelve (22.6%) encoding errors. Those students who read the problem couldn't grasp the aggregate meaning of the question. Six (11.3%) transformation errors, four (7.5%) motivation and six (11.3%) carelessness errors were also found in this test item. The maximum error was found on process skill and the least error on motivation. This indicates that the students were not following the appropriate

process to solve this problem. If the students become failed in these steps, they obviously failed the further steps of solution.

Distribution of Errors on the Basis on Newman's Five (Plus Two) Error Hierarchy

The following table shows the distributions of errors committed by the students in solving verbal problems of algebra and its distributions according to the hierarchy of Newman:

Table11. ***Distribution of Errors on the Basis on Newman's Five (Plus Two) Error Hierarchy***

S. No.	Types of Error	No. of Errors			Percentage		
		Boys	Girls	Total	Boys	Girls	Total
1	Reading Errors	16	16	32	6.4	4.4	5.2
2	Comprehension Errors	20	24	44	7.9	6.6	7.2
3	Transformation Errors	76	79	155	30.3	21.8	25.3
4	Process Skill Errors	49	82	131	19.5	22.6	21.4
5	Encoding Errors	48	113	161	19.1	31.2	26.3
6	Motivation Errors	9	16	25	3.6	4.4	4.1
7	Carelessness Errors	33	32	65	13.1	8.8	10.6
Total		251	362	613	100	100	100

From the above table what I found that there are altogether 613 errors of which 251 errors were committed by boys and 362 errors were committed by girls. It is concluded from this table that111more errors were committed by the girls in comparison of boys. This table also indicates that lowest percentage of errors was committed in motivation level whereas highest percentage of errors was committed on encoding level. This indicates that students of grade VIII commit less error due to motivation. They always try to solve the problem but they didn't encode answer in acceptable written form.

The students committed 5.2% reading errors due to their poor background in language. Similarly, 7.2% errors were comprehension error and 25.3% errors were transformation error. This indicated that most of the students can't transform the verbal problems into mathematical expression. These two levels of error were found most troublesome steps in solving verbal problem. There was 21.4% process skill error, which is the second highest error. This shows that large number of error was committed in process skill level in solving verbal problem of simultaneous equation. Similarly, 26.3% encoding errors, 4.1% motivation errors, 10.6% carelessness errors were found in the study. This table also shows the comparison of number of errors and their percentage between in errors committed by boys and girls.

Causes of Error

Through the interview with teachers and students including achievement test the researcher also found some learning problems of students due to which they commit errors in solving the verbal problems of algebra. In this regards one sample teacher say:

> *Our students were confused on the meaning of the words used in verbal problems by attaching their own meaning. The students pay only partial attention to the*

teacher's explanation as a result of boredom, tiredness or monotonous tone of teacher and they just listen the explanation and do not participate in the discussion. Consequently, they can recollect only part of explanation and then try to patch it up with their own logic, which may be faulty.

From the view of teacher, the major causes of errors can be listed as follows:

Attach own Meaning

Some students were confused on the meaning of the words used in verbal problems by attaching their own meaning on the students previously. For example: In the solution of problem four students put their own meaning to solve the question in question no. 2, 3, 7 and 8.

Incomplete of Fuzzy Thinking

Sometimes the students pay only partial attention to the teacher's explanation as a result of boredom, tiredness or monotonous tone of teacher. Consequently, they can recollect only part of explanation and then try to patch it up with their own logic, which may be faulty.

Mix-up the Rules

Students often mix-up rules because they do not really have relational understanding of what they are doing.

Salient Teachers

In attempt to make things easy for the students some teachers give incomplete explanation by focusing on salient feature that illustrate only some of the features of the concept.

A Conformist Attitude

Since students are often trained to follow instruction meticulously, seldom supported by conceptual justification, they do not think of alternatives and uncomfortable with them.

Teacher Talks and Students Listen

When teacher teaches, the students just listen the explanation and do not participate in the discussion. The teachers must involve the students in the teaching and learning process to do better in solving verbal problems. The teachers should give the orientation that provide sufficient guidance to solve the problem and should give the opportunity to solve the problem for students themselves.

The Ways of Minimizing Error in Different Stage

The third objective of this study was to find the way of minimizing errors in solving word problem of grade VIII student. Errors were identified through the test. And to find the error in different level, researcher took interview by Newman's procedure. Next, researcher had to find the ways to minimize such type of error. To find the way of minimizing the error first we need to find the cause of error. Then the researcher found the conclusions which are bellow in different topic. Errors/mistakes are seemed in simple in solving the world problems. In this research, the researcher discussed to their respected class teacher in related on regarding that have to minimize the error in different stage/level to solve the word problem of mathematics.

Way to Minimize Reading Errors

In this research, researcher found very little reading errors. If this kind of errors seemed, make the habit of studying to the students. In some cases, the lack of students careless and motivation, this kind of error seems where the students may give rapid answer. This error may reduce by concentrating students while reading the questions. In this regards the teacher say;

Teacher should give the knowledge of mathematical symbols properly in his class and he should focus for reading question on classroom. (Teacher B)

From the view of teacher students with Newman's theory, the following ways to minimize the reading errors were found:

- Give the knowledge of mathematical symbols in the mathematics.
- Teacher should focus for reading question on classroom..
- Some of the cases it may bring the problem because of sound, so help them to clear pronunciation.

In conclusion, poor background of language, misconception of mathematical terms, symbols, mother tongue of students, voice of respondent were the responsible causes of reading errors. To minimize this error, teacher should try to minimize these causes. In addition, to minimize the error, teacher needs to encourage the students to read question correctly. Teacher should pay special attention to the language.

Way to Minimize Comprehension Error

This error was kept under the comprehension error when students were unable to receive what the question asked or student were unable to give the meaning of typical word. This type of error was committed by all the students. There is lack of pre requisite knowledge and environment of the classroom where no student friendly. in this regards the teacher B says:

> *We should give the knowledge before starting the lesson and teach the students by dividing their ability, interest and wishing. We must involve them actively in class by creating reading environment. Also we try to improve their poor level by checking homework and class work regularly and improve the error in comprehension individually.*

From the discussion with teachers the comprehension error may be reduced with following ways:

- Give the knowledge before starting the lesson
- Teach them by dividing their ability, interest and wishing.
- Involve them actively in class not only listening.
- Create reading environment at home.
- Try to improve his level in which level he/ she is poor.
- Check homework and class work regularly and improve the error in comprehension individually.
- Make attention to teacher's explanations.
- Try to improve classroom management.

- Involve them to mathematical quiz, which helps to remember to pre request knowledge.

In conclusion, comprehension error occurred due to monotonous environment, own meaning, lack of knowledge of technical terms and symbols, lack of attention to the teachers' explanation. To minimize the comprehension error teacher should try to minimize these causes. To minimize the comprehension error the mathematics teacher need to give fundamental knowledge about the topic before starting the new chapter. Opportunity should be given to the students to express the question in their own words after reading. Also these errors can minimize by applying mathematical games and quiz in teaching learning process.

Way to Minimize Transformation Errors

An error was classified on transformation error if the students had understood what the questions ask but were unable to transform sentence into mathematical form and unable to choose appropriate operations. The interview with teacher and students it is found that:

While teaching in the classroom, the teacher should careful on how and where to use operation of mathematics. (Student A)

We must check and find about students' pre knowledge and knowledge of pre class, and give suitable feedback and motivation. Also we must focus on meaning of question and which operation is used for this, then solving the problem. (Teacher C)

From the view of teacher and students following are the ways to reduce the transformation errors.

- Clarify the students about the mathematical words' concept and meaning.
- Practice them as many as about choosing appropriate operation.
- While teaching in the classroom teacher should careful how to use operation and where.
- Check and find about students' pre knowledge and knowledge of pre class, and give suitable feedback and motivation.
- Focus meaning of question and which operation is used for this, then solving the problem.
- Make students to discuss actively on classroom.

In conclusion, transformation errors are occurred due to incomplete explanation, unclear statement of problem, lack of mathematical concept, cannot choose appropriate operation, skill and more emphasis on calculation part on mathematics. So minimize the error teacher should focus above point. To minimize the transformation error the teacher should give clear concept about mathematical terms which are used in respective topic.

Way to Minimize Process Skill Errors

An error was classified as process skill error when students were unable to identify the correct operation and did not know the procedures to carry out these operations accurately. The opinion of students and teachers in the interview suggest following ways to minimize process skill errors.

- To practice the students more and more simplified form of word problem.
- To learn the problems having mathematical operation according to rule.
- Teacher should focus the practice than explanation.

- To keep students out of bore / tired, it is better to use different techniques.
- To construct the good environment for students actively practice to their interested subject.
- Teachers should avoid lack of teaching material and should use properly teaching material which helps students to construct the conceptual structure vary easily.
- To care about process skill error while checking homework and class work and give suitable feedback.

In conclusion, process skill is occurred due to mix-up of rules, lack of thinking alternatively, not concentrate teachers' explanation properly, less practicing mathematical word problem, less careens in solving word Problem. So to minimize this type of error teacher should care these point. To minimize the errors teachers must force the students to solve the problem time and again.

Way to Minimize Encoding Errors

An error was classified as encoding if the students correctly choose operation and correct process but could not express the solution or answer in acceptable written form. At the time of discussion the teacher A says:

Teachers and students only think that how to find out answer but this errors seems without thinking the causes that how answer has come and to solve it , make the habit of attention. Teachers should focus in giving answer to join part answer of the question and should focus the class work and homework.

From the view of teacher if we follow the following way, encoding errors may be reduced.

- To discourage the students habit to write fast answer.
- To knowledge the students while solving the word problem, the answer should also write in word.
- Teachers should focus in giving answer to join part answer of the question.
- To focus the proper utilization of time.
- To focus the class work and homework.

In conclusion, encoding error is occurred due to carelessness, importance is given to finding the answer and less important to arranging the procedure. Write some sentences.

Chapter –v

SUMMARY, FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This chapter deals with the result of the study an error analysis on solving verbal problems of algebra by grade viii student**.** This chapter is divided into four sections which consists summary, findings, conclusions and the recommendations.

Summary of the Study

Words problems in algebra play a vital role in an individual to solve daily life problems. Realizing its importance, Simultaneous Equation is included as an important component of Mathematics up to secondary level. But in the teaching experience of researcher, student feel more difficulty in doing Simultaneous Equation, especially in solving verbal problems, than any other topics of Mathematics. Keeping this in mind, the researcher decided to find out the types of error committed by the students on solving verbal problems on Simultaneous Equation so that some remedial solutions can be drawn to minimize the errors.

The sample for the study consisted of one hundred twenty grade VIII students from two different schools (community and institutional) of Chitwan district. These schools were selected by stratified random sampling technique. A test of eight verbal problems from grade VIII was administrated to all the 120 sample students. All the test items were related to verbal problems of algebra. The collection of data or the information was done in two phases. In the first phase, data were collected from answer sheets and errors were identified. In second phase in-depth interviews with each students

and teachers was conducted to conform the errors which were seen in the answer sheets. Newman technique of error analysis was adopted as the theoretical base of study.

The identified errors were classified into seven categories as described by Newman and Clements frequency of each type of error were tabulated, the seven categories of errors were reading error, comprehension error, transformation error, process skill error, encoding error, motivation error and carelessness error. An error was classified as reading error, if the student couldn't recognize the key word of the written problem. An error was classified as comprehension error, if the student couldn't grasp the overall meaning of the verbal problem. An error was classified as transformation error, if the student couldn't transform the sentences into mathematical forms. An error was classified as process skill error, if the student could choose the appropriate operation but couldn't complete accurately. An error was classified as encoding error, if the student couldn't write down the answer in acceptable written form. An error was classified as motivation error, if the student did not want to respond the problem at any level of hierarchy. An error was classified as carelessness error, if the student obtained correct answer in second attempt during interview i.e. if the student could spot his /her own mistakes.

Findings of the Study

Verbal problems in algebra take a major part in Algebra of Mathematics in school level. Most of the exercises in Simultaneous equations are found in the verbal problems. The main objective of the study was to identify the types of error committed by students of grade VIII. According to the objectives of the study, the data and information were

collected and analyzed. After classifying the errors into seven categories, the errors committed by the students in solving different eight test items were listed as findings of the study. The following are the major findings of the study:

- Total number of errors committed by the students of community school was 613.Out of these errors 251 were committed by boys and 362 errors by girls. Similarly, 436 errors were committed by institutional school students among them 170 errors were committed by boys and 266 by girls.
- A total of 5.2% of the errors were committed in reading level followed by 7.2% error in comprehension level,25.3% error were committed in transformation level,21.4% errors were committed in process skill level, 26.3% errors were committed in encoding level, 4.1% errors were committed in motivation level, and 10.6% errors were committed in carelessness level.
- Students attach their own meanings to the confusing words of the problem.
- Incomplete thinking is one of the major problems in understanding of verbal problem.
- A conformist attitude restricts the students from alternative ways of solving problems.
- Students committed 25.3% errors in transformation level because they adopted wrong operation to transform the words in mathematical forms. Next concentration of errors was on process skill level where students committed 21.41% errors because they could not find out an appropriate mental solution and became failed to proceed further.

- Similarly, next concentration of error was on carelessness level where students committed 10.6% errors because of their overconfidence, nervousness and lack of practice for written test.
- Students committed 7.2% errors in comprehension level because they attached their own meaning to key words rather than word's mathematical meaning.
- The next concentration of error was on reading level where students committed 5.2% errors due to poor background in language, grammatical structure and lack of practices. Students committed 4.1% errors in motivation level due to lack of motivation, health problem, tiredness and lack of preparation for the test.

Conclusion

From the analysis and interpretation of data it is concluded that; the students commit errors from the beginning (reading level) to the deduction of the result (encoding level), the concentration of errors were seen on encoding level where the students committed 26.3% errors. The result shows that the maximum number of students they solved the problem correctly but forgot to write the answer in acceptable written form. Next concentration of errors was on transformation level where the students committed about one fourth percent of the errors. This shows that those students who have understood the problem are failed to make appropriate mathematical form or expression. Similarly, next concentration of was on process skill level where the students committed just above one fifth of the errors. This shows that students were able to choose appropriate mathematical expression but did not know the procedure to carry out the solution.

Recommendations for Educational Implications

On the basis of above result and conclusion the following recommendations are made for educational implications:

- Most of the errors are committed by students because of poor background in language. Teachers should pay special attention to the language and techniques to understand it. Mathematical concepts should be explained in detail using simple language with illustration.
- Teachers should adopt appropriate strategies to minimize the errors committed by the students.
- Teachers should encourage students to solve the verbal problems themselves.
- Teachers should be committed on how to make the verbal problem understandable.
- Teachers can use Newman error analysis hierarchy to teach verbal problems efficiently.

Recommendation For Further Study

The present study generates some questions, which need to be verified. Following are the some areas recommended for future course for investigator.

- What are the measures to minimizing errors committed by the students in solving verbal problem?
- How and why the students commit errors in solving verbal problems of algebra?
- The similar type of researches can be conducted on other areas of mathematics as well as other classes.

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Appendix A

Interview Guidelines for Students

- Please read the question for me. (Reading)
- Tell me what is asked you to do?(Comprehension)
- Tell me method you can use to find the answer.(Transformation)
- Show me how you worked out the answer to the question.(Process Skill)
- Now tell me your answer to the question.(Encoding)

Appendix B

Test Questions

1. The length of the rectangle is 6m less than three times its breadth. Find the length and breadth of the rectangle if its perimeter is 148m.
2. The sum of the three consecutive numbers is 30, find the numbers.
3. A man is twice as old as his son. If their total age is 60.fin their ages.
4. Divide 184 into two parts such that one- third of one part exceeds one –seventh of another part by 8.
5. The difference of two numbers is 15. The smaller number is 13. Find the greater number.
6. The product of 150 and a number is 300. Find the number.
7. The sum of two numbers is 58 and their difference is 12. Find the numbers.
8. A number consists of two digits whose sum is 5. If 9 is subtracted from the number, its digits are interchanged. Find the number.