

# CHAPTER I

## INTRODUCTION

### 1.1 Background of the Study

The concept of number and the process on counting developed so long before the time of recorded history that the manner of this development is largely conjectural. It seems fair to argue that humans, even in most primitive times, had some number sense, at least to the extent of recognizing more and less when some objects were added to or taken from a small group with a gradual evolution of society, simple counting became imperative. At the past human used mathematics what they need in their daily life. But now mathematics has been playing key role for the development of several field like science, economics, commerce, engineering, astronomy, information technology and so on. The need and importance of mathematics has been increasing by leaps and bounds in everyday human life. Supporting the importance of mathematics, the School Mathematics Study Group (SMSG) published a progressive report in 1959 stating the role of mathematics as:

*“The world of today demands more mathematics knowledge on the part of more people than the world of yesterday, and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens, skilled in mathematics must be greatly increased and understanding of the role of mathematic in our society is a prerequisite for intelligent citizenship. Since no one can predict with certainty*

*his future profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematics skill which the future will surely demand of many of them.”*

The word ‘mathematics’ has been derived from an ancient Greek word *μάθημα* (máthēma) which means ‘to learn’. It seems to indicate that mathematics was considered as a process of learning and interpreting the natural phenomena or surroundings of an individual. Historically, mathematics as a discipline is the outgrowth of different human civilization in developing the rules, formula and mathematical systems based on solving their social problems for the continuation of the society.

Mathematics occupies a well established position in the school of all times. Supporting this view Traverse and others (1977) write:

*“Even since the school of ancient Greek over 2000 years ago, mathematics has been a key subject in the curriculum. The four liberal arts, the aquarium consisting of arithmetic, geometry, astronomy and music were basically mathematical studies.”*

The liberal arts in the ancient Greek were studied basically for the aesthetic values, but when the society also changed and this resulted in a change in the aim of education. Mathematics education, is not limited to aesthetic values but as a subject to be taught to meet the needs and demands of the rapidly growing society due to the change in science and technology. The opinion is clearly supported by Brissenden (1980) in the following words:

*“During the last twenty years a succession of challenges has arisen within the teaching of mathematics. More than in any other subject, new methods and materials have been created, argued over, developed or discarded.”*

The necessity of mathematical understanding for better living in modern scientific and technologically changing society is inevitable. In order to keep pace with the change in the needs and demands of the society, mathematics education needs to be changed supporting this view (1963) reported by Aichele and others (1977) reported that:

*“Mathematics education is to fulfill the need continual scrutiny and it is the responsible of all mathematics, working in the university, school or industry to concern themselves, with the problem of keeping mathematics education up-to-date.”*

To keep mathematics up-to-date changes are taking place in the schools and the universities throughout the world. Worldwide reformation in mathematics education has been taken place during the recent decades to meet the need and demands of the society. As a result of this old courses are being replaced by the newer ones and mathematics courses, especially are changing rapidly. The developments have resulted from a reappraisal of the contents and purpose of mathematics taking place and they lead to further reappraisals.

The above statement is clearly supported on a report published by SMSG (1959) quoted in Stephen K. and Ingrid B.W. (1977) in the following words:

*“The world of today demands more mathematics knowledge on the part of more people than the world of yesterday, and the world of tomorrow will make still greater demands.”*

In Nepal remarkable step was undertaken to introduce new topics in mathematics education after the adoption of NESP (1971-1975 AD) with the stress of vernacular language. According to the growing needs and demands of the society and the nation as a whole, new topics are tried out in mathematics curriculum as Wilder (1970) states:

*“Change in mathematics and mathematics curriculum have been taking place and as mathematics itself grows ... We can expect what most topics which were considered as university subject in the recent past (probability and statistics are examples) will be provided at high school level.”*

In a seminar paper on “Problems and Issues of Mathematics Education in Nepal” supporting the above argument, Shrestha (1990) reported that with the modern reform in the teaching of mathematics, both content and teaching methods have undergone a drastic change. Many contents includes in our school curriculum are no longer taught as a part of mathematics in recent programmes. Our contents in school mathematics are often remarked to be century old. They are not relevant to the needs of science and technology of the modern world (Cited Upadhyaya, 1992).

By the recommendation of NEC (1992 AD) and higher level NEC (1998 AD), the school curriculum is being revised according to international trends, recent political change and the need of the

people to meet the demand of present situation. To revise the curriculum of each level, organized commission has recommended a number of new topics to be incorporated in the school curriculum.

In American mathematics education, precalculus is an advanced form of secondary school; algebra is a fundamental mathematical discipline. Precalculus explores the topics that will be applied in calculus. In details precalculus deals with sets, real numbers, complex numbers, solving equations and inequation, properties of functions, composite functions, polynomial functions, trigonometry, trigonometric functions and their inverse, trigonometric identities, cubic sections, exponential functions, logarithmic functions, sequence and series, Binomial theorem, vectors, mathematical inductions, limits and continuity.

Some of the above topics have already been kept in the secondary level mathematics in Nepal. But the topic limits and continuity has not been included yet. So the research of appropriateness of limits and continuity is quite important. Limit and continuity are quite fundamental for the development of calculus. These two concepts are closely linked together with the involvement of the concept of limit in the definition of continuity.

Archimeds and Syrause first developed the idea of limits to measure curved figure and the volume of a sphere in the concept of limit formally express the notion of arbitrary closeness. That is, a limit is a value that a variable quantity approaches as closely as one desire. The operation of differentiation and integration from calculus are both based on the theory of limits. The theory of limit

is based on a particular property of the real number between any two real numbers, no matter how close together they are, there is always another one. Nearness is a key to understanding limits. Only after nearness is defined does a limit acquire an exact meaning relatively a neighbourhood. Neighbourhoods are definite components of infinite limits of a sequence.

The ancient Greek philosopher Zeno may have been one of the first mathematicians to ponder the limit of a sequence and wonder how it related to the world around him. Zeno argued that all motion was impossible because in order to move a distance it is first necessary to travel half the distance, then half the remaining distance then half of that remaining distance and so on. Thus he argued that the distance can never be traversed.

The limit and continuity concept is essential to understand the real number system and its distinguishing characteristics. In one sense real numbers can be defined as the numbers that are the limits of convergent sequence of rational numbers. One application of the concept of limit is on the derivative. The derivative is rate of flow or change, and can be computed based on some limits concepts. Limits are also key to calculate integrals. The integral calculates the entire area of a region by summing of an infinite numbers of small pieces of it. Limits are also part of the integrative process. An iteration repeatedly interacts, can get as close as desired to a theoretical exact value.

Secondary school education is the gateway of higher education to different fields. So the teachers, educationist, educational planners and administrators have been engaged in

providing as much as knowledge of mathematics at all level of school. The consequence of this practice is to raise the achievement of students in mathematics, but there are several affecting factors such as students' age, gender, environment. About the factors which affect or relate to students' school achievement in mathematics, Richard said:

The important factors related to students school achievement in mathematics are classroom behaviour (time spent in learning, student's attention, method of teaching and so on), teacher's background (teacher training, experience, ability), school characteristics (boarding or general distance to school, location of school), students attitude, daily attendance, maturation and health) and family characteristics (socio-economic status, parental education, educational environment of home and family size). Ausubel says:

*“If I had to reduce all of educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.”*

According to Jean Piaget and other learning theorists and psychologists the age of students of secondary level is formal operational state. So in this state students are able to find out the cause and effect relation and they can think about the abstract too. So to keep this course of precalculus learning theorist and psychologists will not be obstacles.

In Nepal transformation, matrix, set, probability topics were introduced gradually and different research had shown that these

topics are very much suitable and useful in this stage. But limits and continuity has been introduced only from eleven class. There is not made any research whether it is appropriate in secondary level or not. Thus researcher made a purposed course for secondary level. The course was designed to present basic and elementary concepts of limits and continuity in simple and practical ways.

## **1.2 Statements of the Problem**

Modern mathematics had been developed in 17<sup>th</sup> century and after. Precalculus, also a main part of modern mathematics, was developed in the time of 17<sup>th</sup> century by Kepler's Mosaics, Descartes, Fermat, Newton, Leibniz, Chauchy etc. Precalculus has been introduced in the university level of Nepal in different fields and school level in other countries, though it has not been introduced in school level of Nepal. In Nepal, study has not been conducted to asset whether the topic is appropriate or not for secondary grade students. For this reason, the researcher was interested to research whether it was appropriate or not to introduce precalculus in secondary level. This is why the statement of the problem emerged as "Appropriateness of Precalculus in Grade IX of Nepal".

## **1.3 Significance of the Study**

Mathematics has been a key subject in school curriculum throughout the world. In our country, it has been taught from primary to secondary level as different subject. Among the many areas of mathematics, precalculus is still an important subject in university level because of its many practices and technical uses



and applications in other areas of mathematics. If the general concept of precalculus is given in secondary level then students performance will be increased in the university level. There is the vast difference between the course of higher secondary and secondary level. So to fulfill this gap try should be made for the appropriateness of precalculus in secondary level grade IX. Today the problem of teaching mathematics is to cover as much content of mathematics to improve students' achievement in mathematics. To overcome such problem mathematics educators and experts have been engaging in organizing curriculum and adopting meaningful teaching mathematics (Nath, 2002).

Most students feel mathematics as a subject having no application. Quaiyum and Shah (1982) reported that students should apply mathematical concepts and skills in daily life situation as well as higher studies. But generally it is known that students are weak in problem solving skills and day to day application of mathematics. To elaborate further; CDC (1977), reported that students have poor background of mathematics to study it at higher level, specially mathematics as a main area of study.

The difficulty of the subject might be due to its inclusion at the upper grades without preliminary foundation in the lower grades of the school. The curriculum should be designed in such a way that the students should feel the familiarity of the subject in earlier grades thorough practical and play way method which would help in future to learn the subject in more formal ways. Since the study is an attempt to introduce precalculus in grade IX in practical way to provide base for higher level calculus, its

importance lies on organization of concept of precalculus suitable to beginners.

This study further seeks to enrich elementary school mathematics curriculum by adding the concept of precalculus in optional mathematics of grade IX. Also researcher believes that by this research work, mainly the educationist who are involved in curriculum construction of secondary level and all the students of that level will be benefited.

#### **1.4 Objectives of the Study**

- i) To develop a unit of precalculus (together with instructional activities).
- ii) To determine the appropriateness of introductory course of precalculus through learning achievement of students.
- iii) To compare the achievement scores of students in precalculus unit with that of achievement scores in mathematics.

#### **1.5 Statement of the Hypothesis**

This research work aims at the development of an introductory course in precalculus for the secondary level of Nepal and the intention was to explore the practicability of the course content at the specified grades. For this purpose, the following null hypotheses were formulated.

- i) There is no significant difference between grade IX students' achievement in the precalculus and their achievement in the course of mathematics.
- ii) There is no significant relationship between the achievement score in precalculus and their achievement score in course of mathematics.

### **1.6 Limitation of the Study**

None of the study addresses all the variables at a time. This study has some limitations they are as follows:

- i) The study was confined to only Kaski District.
- ii) Study is confined only to the private schools.
- iii) The study was based on the sample selected from grade IX students of Shantideep English School of Majheri Patan-14, Pokhara and Harvard Boarding School of Falepatan-15, Pokhara.
- iv) Students' socio-economic, cultural and family background and relation that effect to some extent were not controlled.

### **1.7 Meaning of Terms**

- i) **Precalculus:** It means those topics of mathematics which are prerequisites for calculus. Among these limits and continuity are taken as topics of precalculus.
- ii) **Secondary Level:** Secondary level means grade 9 and 10 but here the study is done in class 9 only.

- iii) **Mathematics:** Mathematics refers a subject taught in secondary level. Here study was done in optional mathematics.
- iv) **Achievement:** Achievement in this study is defined in terms of the score obtained by the students on the achievement test that was conducted by researcher.
- v) **Students:** Students refer to those boys and girls of class IX.
- vi) **Class-Test:** Achievement test conducted by researcher at the end of whole course teaching.
- vii) **Group 'A':** Sample students of Shantideep English School.
- viii) **Group 'B':** Sample students of Harvard Boarding School.

## CHAPTER II

### REVIEW OF THE RELATED LITERATURE

This chapter reviews the different feature of articles and finding of different researches in order to explain the basis and explore the trends of research in this field; provide insight and direction; find area which are less explored and finally formulated a set of research questions.

There are so many researches about mathematics but there are not exactly same as “The Study on the Appropriateness of the Topic ‘Precalculus’ at Secondary Level School Mathematics” in the context of Nepal. Some of the related researches are as follows:

Das (2001) studied the appropriateness of the mathematics textbook of grade ten and concluded that the present textbook of grade ten is appropriate and most of the sampled teachers agreed and satisfied with the textbook in its academic, technical and psychological aspects. They disagree that textbook is inappropriate only in terms of its price; printing, references, index, suggestive hints and few chapters are difficult to handle due to some small problems. There is no significant difference between the opinion of teachers of both rural schools and urban schools. Above all, the researcher comes to conclusion that the textbook is appropriate.

Dhakal (2001) did a research on the topic “Appropriateness of the Compulsory Mathematics” with the aims to study the appropriateness of mathematics textbook on the basis of its academic, physical and psychological aspects and to investigate

the different problems faced by the teachers if they existed in teaching the new textbook. He took twenty teachers of Nuwakot district by stratified random sampling technique. He prepared questionnaires to gathered data. By analyzing the data the investigator concluded that the organization, presentation, examples, figures, graphs were found to be appropriate and the teachers remained undecided on the question whether the quality of the paper used was also found to be appropriate. He also concluded that teachers were undecided whether it could help students participate in self-learning. He found that teachers felt difficulties to teach newly included topics without any training.

Ghimire (2001) did his thesis for Masters' Degree on "A Study on the Relationship between Students' Attitudes and Achievement in the Topic Set" with the objective to gather the basic information about the attitudes of the students towards the unit set across the variables schools types, gender and to investigate the relationship between attitudes and achievement. Two sets of research tools were developed for collection data for the study. One set related to opinionnaire based on David Krathwohl and another the achievement test. Two hundred twenty seven students were selected from Chitwan District. Mann-Whitney U-Test and Z-Test were applied to conclude that the students studying in both types of schools (public and private) had similar attitudes and there was no gender difference in attitudes towards the unit of the set. Study also revealed that no significant correlation existed between students' attitude scores and achievement scores of ninth graders on the test.

KC (2001) did a research on “A Study on the Appropriateness of the Mathematics Textbooks of Group ‘B’ with aims to study the appropriateness of the mathematics textbook of Group ‘B’ about academic, technical and psychological aspects and to compare the opinion of teachers teaching mathematics of Group ‘B’ in urban and rural school. A descriptive study was undertaken with stratified random sample. A questionnaire was developed by the researcher and administered on seventy teachers and t-test was applied. H concluded that presentation of the content was found to be clear but examples were not practical according to standard of students. No significant difference was noticed in the opinions of teachers teaching mathematics in urban and rural schools about appropriateness in each of the three aspects of the textbook of Group ‘B’.

Kharel (2002) did a study on “A Correlational Study of Attitude of Specification Grid of Secondary Level Mathematics and Its Practices”. He concluded that the secondary level mathematics teachers’ attitude towards specification grid was positive but it was negatively related to their practices in it. This implies that the teachers seem to be unable to care less in using the specification grid.

Pandit (1999) studied the attitude of secondary level students and teachers towards geometry and he found that the students’ attitude towards geometry was positive but teachers’ negative. The boys’ attitude is greater than that of girls and students’ attitude score was significantly greater than that of their teachers.

Pokhrel (2000) did his thesis on the topic “A Study of Present States and Current Problems in New Curriculum of Group ‘B’ Mathematics in Gorkha District” with two major objectives. They were to find out the present status of the implementation of the existing Group ‘B’ mathematics curriculum (2049 BS) and to find out the existing problems of the implemented curriculum of mathematics Group ‘B’. The sample size of sixty secondary schools in Gorkha district in which are hundred twenty students, twelve teachers, fifteen guardians and three supervisors were taken. Simple percentage analysis was used to conclude that some lesson were difficult to understand and some lessons being longer than necessary, no proper use of teaching materials for new topics course could not be finished with assigned time. Teacher guide was not found available in sample school. Most of the school were lacking in physical facilities, most of the sample schools’ teachers were untrained.

Quaiyum and Shah (1982) did a research on “Preference in Secondary school Mathematics Curriculum Contexts and Resources” with the aims to find preferences in mathematical contexts and resources to be included in the curriculum for secondary school level and to identify contents and resources desired to introduce in the secondary school mathematics curriculum. A questionnaire regarding curriculum was prepared and administered on three groups of teachers: one hundred six school teachers, twenty teacher-educators and twenty eight campus teachers, f-test was applied to conclude that their opinion of school teachers, teacher-educators and campus-teachers were found the same for the contents and resources, except the



computation devices were highly preferred by three groups to be used in teaching mathematics. Out of one hundred and thirty one content items of secondary school mathematics, one hundred and sixteen items were accepted by the sample groups for inclusions in the secondary level curriculum.

Sharma (2000) did a research on “A Comparative Study of the Achievements of the Students of Grade Nine in the Topic ‘Vectors’ of the Secondary School Mathematics Curriculum” and she found that boys’ and girls’ performance were better and nearly equal and also concluded that teachers and students were agreed not to remove the topic vector from the curriculum of additional mathematics.

Timilsina (2002) did a research work on “Analysis of Contents of Vector of Optional Mathematics of Grade IX” with the aims to study the appropriateness of vector contents of optional mathematics of grade IX and to compare the opinions of teacher of urban and rural area. Forty teachers were sampled and t-test was applied. He concluded that the respondents agreed that the content vector was found to be good for the further study and training was visualize as essential for the also trained and untrained teachers to teach vectors.

Tiwari (1984) did a research on “A Comparative Study of Boys and Girls Attitude towards Mathematics” and found that the students has positive attitude towards mathematics and he also concluded that the mean attitude scores of boys was greater than that of girls.

Tiwari (2002) did a research on “A Study of Attitude of Farmer and Non-Farmer Parents towards the School Mathematics” with the objectives to investigate the attitudes of farmers and non-farmer parents towards the school mathematics towards male and female child. Researcher developed two types of opinionnaire. Ch-square-test, t-test were adopted to conclude that the farmer and non-farmer parents had positive attitude towards the school mathematics but scores of attitude on non-farmer parents was higher than the score on attitude scale of farmer parents. It means non-farmer parents have emphasized their children towards the school mathematics than the farmer parents. Both types of parents had given higher priority towards their male child than their female child about the mathematics.

NEC (1992) has introduced new topic in school level of Nepal in mathematics: like as probability, set, transformation, matrix, vectors etc. Though research had been done about the related topics there is not any topic like “Appropriateness of Precalculus in Grade IX of Nepal”. Therefore, this present study deals to explore the possibility of precalculus in secondary level of school mathematics of Nepal.

## **CHAPTER III**

### **RESEARCH METHODOLOGY**

Research methodology is a useful bridge to solve the research problems in a systematic way. It describes the methods and process applied in the entire aspect of the study. In this chapter procedure for the selection of the sample preparation, validation and administration of the instrument, data collection procedure and procedure of analysis and interpretation of data and use of statistical tools are presented.

#### **3.1 Design of the study**

This study is quantitative type. It depends up on the score obtained by students in achievement test.

#### **3.2 Sample**

Many limitations such as time, energy and money didn't allow the researcher to widen and extend the area and scope of this study. Therefore, the researcher selected only two secondary schools in Kaski District, Shantideep English School and Harvard Boarding School situated in Majheri Patan-14 and Falepatan-15, Pokhara respectively. These two schools were selected by purposive sampling method for the purpose of the field test of the study. From grade IX of each school 15 students were selected randomly by lottery method. All together 30 students were taken for the study it was done to conduct teaching on small groups.

### **3.3 Development of Instruments**

#### **3.3.1 Development and Validation of Introductory Course in Precalculus.**

Prior to the development of the instruments for this study, a survey of the available books, magazines was done and introductory course on precalculus for the purpose of field test was designed. The introductory course was validated with the help of thesis advisor.

While developing the introductory course on precalculus the following criteria was adopted:

##### **a) Criteria Related to Skills and Application**

Teaching of mathematics as suggested by Traverse and Others (1977) is that the students through the learning mathematics need to know how mathematics fits into the 'big picture', of daily life. Daily life certainly requires application of the concepts learned today to other similar situations. Therefore the course was so designed as to encourage application of arithmetical, algebraical concepts in the learning of precalculus. This course was also designed as to encourage the development of students' skill power.

##### **b) Criteria Related to Difficulty Level**

The course was designed in such a way that it uses the past experience and so that their past experiences were used to get the concept of limit and continuity that is why the students got the

objectives in a simple way. Millington (1972) quoted in mathematics through school in the following words:

“We start with experience (assuming there is motivation for the moments), then we play, think, generalize and test the generalization in order.” The organization of the content, activities and evaluation schemes has been developed in such a way that students get the concept in easy way and in practical way. Teaching episodes were organized and implemented in scientific and practical way.

### **3.3.2 Construction, Validation and Reliability of the Test Items**

After the development of the introductory course in precalculus the test items were constructed with a wide coverage of the content. The test items have reflected the objectives of contents. There were two class tests and one comprehensive post test constructed by researcher. The first class test had 5 items of two marks each, all of which were subjective. Second class test was conducted on 6<sup>th</sup> day on which 3 subjective questions all of which were 2 marks each and 4 objective questions of 1 mark each were asked. On the 9<sup>th</sup> day the comprehensive post test was conducted which was of twenty full marks out of which 8 were objectives of 1 mark each and six were subjective all of which were of two marks each.

The validation of the test items was done by consulting the reference books and selecting the items from them.

The reliability of test items was calculated by test-retest method and used formula Pearson's correlation. The correlation

between two tests was found 0.94 it was highly positive correlation. It indicated that the test was reliable.

**Table No: 1**

**Test Retest reliability of the post test**

S.N.	Test 1 (out of 20 marks) $X_1$	Test 2 (out of 20 marks) $Y_1$	$X_1^2$	$Y_1^2$	$X_1Y_1$
1	16	14	256	196	224
2	15	18	225	324	270
3	17	18	189	324	306
4	12	13	144	169	156
5	13	14	169	196	182
6	10	10	100	100	100
7	9	10	81	100	90
8	7	8	49	64	56
9	7	6	49	36	42
10	6	6	36	36	36
11	6	7	36	49	42
12	5	6	25	36	30
13	15	14	225	196	210
14	7	8	49	64	56
15	9	7	81	49	63
$N_1 = 15$	$\Sigma X_1 = 154$	$\Sigma Y_1 = 159$	$\Sigma X_1Y_1=1863$	$\Sigma X_1^2 = 1814$	$\Sigma Y_1^2 = 1939$

$$r = \frac{N_1 \Sigma x_1 y_1 - \Sigma x_1 \Sigma y_1}{\sqrt{N_1 \Sigma x_1^2 - (\Sigma x_1)^2} \cdot \sqrt{N_1 \Sigma y_1^2 - (\Sigma y_1)^2}}$$

$$= 0.94$$

### **3.3.3 Conduction of the Purposed Course**

The researcher himself had taught the introductory course of precalculus in both the schools on a same day by using the same method with the same lesson plan. At the end of 3 days, 6 days and 9 days class test I, class test II and post test of 10, 10 and 20 full marks were taken respectively. Teaching style, evaluation, techniques, classroom activities all were same in both the schools.

### **3.4 Data Collection Procedures**

Test items were administered to find out the achievement level of the students. Average of class test I, class test II and post test was calculated. The scores of two class tests and the post tests generated the required amount of data for both analysis and interpretation of the study. The procedure stated above produced the primary data. Students' scores in the whole course mathematics in grade IX of both schools were also recorded with the help of subject teacher.

### **3.5 Data Analysis Procedures**

The mean, standard deviation and correlation of the two samples were calculated and t-test and correlation coefficient test were carried out. All the hypotheses were tested for their significance at 0.05 level.

## CHAPTER IV

### ANALYSIS AND INTERPRETATION OF DATA

In the preceding chapters, introduction of the study, review of related literature and method of study were discussed. Its chapter has been designed for analysis of the data and interpretation of results.

The analysis was done to test the following hypotheses.

1.  $H_0$ : There is no significant difference between mean achievement of Group 'A' in precalculus and their mean achievement in the course of mathematics.  
 $H_1$ : There is significant difference between mean achievement of Group 'A' in precalculus and their mean achievement in the course of mathematics.
2.  $H_0$ : There is no significant difference between mean achievement of Group 'B' in precalculus and their mean achievement in the course of mathematics.  
 $H_1$ : There is significant difference between mean achievement of Group 'B' in precalculus and their mean achievement in the course of mathematics.
3.  $H_0$ : There is no significance relationship between Group 'A' students' achievement score in precalculus and their achievement score in course of mathematics.



$H_1$  : There is significance relationship between Group 'A' students' achievement score in precalculus and their achievement score in course of mathematics.

4.  $H_0$  : There is no significant relationship between Group 'B' students' achievement score in precalculus and their achievement score in course of mathematics.

$H_1$  : There is significant relationship between Group 'B' students' achievement score in precalculus and their achievement score in course of mathematics.

To test the appropriateness of purposed course of precalculus for grade IX null hypothesis 1 and 2 were formulated so as to compare the average achievement of each of Group 'A' and Group 'B' on precalculus with their average achievement on the whole course of mathematics.

To test the relationship between precalculus course with course of mathematics for Group 'A' and Group 'B', null hypothesis 3 and 4 were formulated so as to compare correlation coefficient between achievement score of each of Group 'A' and 'B' on precalculus with their achievement score of mathematics.

## Part I

### 4.1 Comparison: Achievement in precalculus vs. achievement in whole course of mathematics.

This comparison was made to test the hypotheses:

Ho 1: There is no significant difference between mean achievement of Group 'A' in precalculus and their mean achievement in the course of mathematics.

Ho 2: There is no significant difference between mean achievement of Group 'B' in precalculus and their mean achievement in the course of mathematics.

To test these hypotheses by using t-test for dependent samples mean, standard deviation and t-value have been calculated. There have been recorded in table 2 for Group 'A' and in table 3 for Group 'B'.

**Table No. 2**

#### **Comparison of Group 'A' Achievement in Precalculus vs. Mathematics.**

Subject	N	Mean	SD	d.f.	Critical Region	t-Value	Conclusion
Precalculus	15	10.60	4.08	28	t $\geq$ 2.145	8.97	Reject H <sub>0</sub>
Mathematics	15	7.33	4.14				

In above table no. 2 the observed mean differences between the achievement in precalculus and mathematics for Group 'A' found to be significant at 0.05 level. The mean difference was too big enough to explain in terms of chance factor alone. This difference in mean could be explained in terms of the treatment given to teach precalculus.

**Table No. 3**

**Comparison of Group 'B' Achievement in Precalculus vs. Mathematics.**

Subject	N	Mean	SD	d.f.	Critical region	t-value	Conclusion
Precalculus	15	12.00	4.17	14	$t \geq 2.145$	3.84	Reject $H_0$
Mathematics	15	10.40	4.14				

In above table no. 3, the observed mean difference between the achievement in precalculus and mathematics for Group 'B' is found to be significant at 0.05 level. The mean difference was too big enough to explain in terms of chance factor alone. This difference in mean could be explained in terms of the treatment given to teach precalculus.

**Part II**

**4.2 Relation between Group 'A' and Group 'B' Students' Achievement Score on Precalculus with Their Score in Mathematics**

Under this heading analysis was done to test the following hypotheses:

Ho 3: There is no significance relationship between Group 'A' students' achievement score on precalculus and their achievement score in course of mathematics.

Ho 4: There is no significance relationship between Group 'B' students' achievement score in precalculus, and their achievement score in course of mathematics.

To test the hypothesis, correlation coefficient between scores on precalculus and scores on the whole course of mathematics for Group 'A' and Group 'B' were calculated. Those correlations have been given in table 4.

**Table No. 4**

**Correlation between Precalculus and Mathematics**

	N	Correlation coefficient	Value of $t_r$	d.f. (N-2)	Tabulated value ( $t_{\alpha, N-2}$ )	Conclusion
i) Group 'A' students' achievement score on precalculus and mathematics	15	0.94	9.93	13	2.160	Significant at 0.05
ii) Group 'B' students' achievement score on precalculus and mathematics	15	0.91	8.46	13	2.160	Significant at 0.05

In the above table no 4, the correlation coefficient between Group 'A' and Group 'B' students' achievement score on precalculus was found to be 0.94 and 0.92 respectively. This

shows that the relationship between precalculus and mathematics is very high. It indicates that students who have done good in mathematics also have done good in precalculus and students who have lesser achievements in mathematics also have lesser achievement in precalculus.

To test for significance of correlation coefficient researcher calculated t-value which is given in appendix 5(i) and appendix 5 (ii). As above table no 4, the calculated t-value of both Group 'A' and Group 'B' were greater than the tabulated value at level of significance  $\alpha = 0.05$  and degree of freedom  $N-2$ . Therefore correlation coefficient was significant at 0.05. So we conclude that there is a significant relationship between each of Group 'A' and Group 'B' students' achievement score on precalculus and score on mathematics.

## CHAPTER V

### SUMMARY, FINDING, CONCLUSION AND RECOMMENDATION

After the analysis and interpretation of the collected data an attempt has been made to summarize, to enlist the finding and some recommendations for further study. The first section presents the summary of the study and the second section presents its findings, conclusion and the last section presents recommendations based on the findings of the study.

#### 5.1 Summary

This study was concerned with the study of exploration of the possibility of precalculus in secondary level in Grade IX of Nepal. The objectives of this study were:

- i) To develop a unit of precalculus.
- ii) To determine the appropriateness of introductory course of precalculus through learning achievement of students.
- iii) To compare the achievement score of students in precalculus unit with that of achievement scores in mathematics.

For this researcher introduced the new curriculum of precalculus for grade IX. The new course of precalculus developed with criteria related to skills and application, criteria related to difficulty level and basic form of higher calculus which was validated with thesis advisor.

This study was experimental type so the purposed unit was taught by researcher at grade IX of Shantideep English School and Harvard Boarding School of Kaski District. Those schools were selected by purposive method for the convenience of researcher.

From grade IX, 15/15 students from each school were selected by lottery method. By the same method using same lesson plan the course was taught upto 8<sup>th</sup> day. During this on 3<sup>rd</sup>, 6<sup>th</sup> and 9<sup>th</sup> day class test I, class test II and one post test was conducted for the collection of primary data. Researcher used secondary data for students' achievement score in mathematics which was average score of 1<sup>st</sup> and 2<sup>nd</sup> terminal as well as of two unit test. Both of scores in precalculus and in mathematics were converted to 20 marks to facilitate comparison.

After collecting data the scores of 30 students were analyzed by using mean, standard deviation, correlation coefficient,, t-test for independent sample and one tail test for dependent sample at 0.05 alpha level of significance to compare following headings:

- i) Comparison in achievement on precalculus and achievement in whole course of mathematics of both Group 'A' and 'B'.
- ii) Relation between achievement score in precalculus and achievement score in mathematics of both Group 'A' and Group 'B'.

## **5.2 Major Findings of the study**

Statistical analysis of the collected data yielded the following result as findings of the study.

- i) Mean achievement of each Group 'A' and 'B' in precalculus was found higher than the mean achievement in mathematics the difference was significant at 0.05 level.
- ii) The correlation coefficient between students' score on precalculus and mathematics of both Group 'A' and 'B' are highly positive at 0.05 level of significant.

### **5.3 Conclusion**

On the basis of findings of the study the conclusion can be drawn about the possibility of precalculus in grade IX of secondary level of Nepal i.e. precalculus can be introduced from grade IX in optional mathematics subject. Since the achievement score of both Group of students in precalculus is higher than that in mathematics it can be concluded. Furthermore the students whose performance was good in mathematics their performance was good in precalculus too. Hence precalculus is appropriate to introduce in secondary level in Grade IX of school of Nepal

### **5.4 Recommendation**

From the findings and conclusion of this study the researcher suggested the following recommendations:

- i) To get a more valid and generalize solution, this type of study should be carried out on an extensive scale.
- ii) Design of curriculum should be integrated through school level to upper level.
- iii) Curriculum should be made up to date according to the need and ability of children.
- iv) There is vast difference in course of mathematics in secondary level and in higher secondary level, this gap should be made narrower for this further study should be carried out.
- v) Similar studies including the opinions and attitudes of parents', teachers and students' should be carried out.



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## Appendix 1

### Proposed Unit Curriculum for Grade Nine

#### 1. Introduction

The proposed unit of precalculus in Grade IX of optional mathematics curriculum was designed by the researcher himself with the advice of thesis advisor and consulting different related books and subject experts. In this chapter two topics limit and continuity are included and in these topics also it is tried to provide the basic concept of limit and continuity.

It is important topic for mathematics. If it feels to be appropriate it will help the students to read mathematics of higher level very easily. Many researches have proved that previous knowledge makes positive effect in the achievement of the students. Thus it helps to play the role of bridge between secondary and higher secondary school.

This proposed curriculum has only two units. In the first unit there is about limit in which concept of indeterminate form, meaning of  $x \rightarrow a$ , limit at infinity, left hand and right hand limit. Similarly the second unit is about the continuity in which the intuitive idea of continuous function, types of discontinuity, test of continuity and discontinuity are included.

## 2. Objectives

The objectives of this unit are following:

- i) To identify the basic terms limit, indeterminate form, tends to, infinity and finding the limiting value of algebraic functions.
- ii) To introduce the term continuity and discontinuity and to determine whether any function is continuous or discontinuous at a given point.

## 3. Content

### Unit I: Limit

- Indeterminate form
- Meaning of  $x \rightarrow a$
- Intuitive idea of limit
- Meaning of infinity
- Limit at infinity
- Limit theorems
- Limit of algebraic function
- Right hand and left hand limit

### Unit II: Continuity

- Intuitive idea of continuous function
- Types of discontinuity

- Test of continuity and discontinuity
- Continuity theorem
- Properties of continuous function

#### **4. Teaching Materials**

Daily used materials, chart, graph board, projector.

#### **5. Method of Teaching**

- Discussion Method
- Question-Answer method
- Problem Solving Method
- Demonstrative Method

#### **6. Evaluation Techniques:**

- Observation
- Class work, homework
- Unit test
- Comprehensive Test

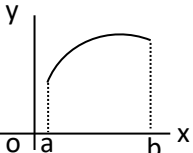
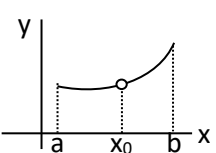
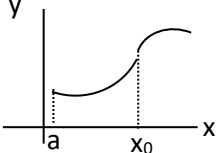


## Appendix 2 Content Specification Grid

Unit	Objectives	Content Specification	Materials	Activities	Evaluation	Period
I Limit	<ul style="list-style-type: none"> <li>To identify indeterminate form.</li> <li>To describe the meaning of <math>x \rightarrow a</math></li> <li>To Explain the intuitive idea of limit</li> </ul>	<ul style="list-style-type: none"> <li>Indeterminate form</li> <li>Meaning of <math>x \rightarrow a</math></li> <li>Intuitive idea of limit</li> </ul>	Graph Projector	<ul style="list-style-type: none"> <li>Consider a function <math>y = f(x) = \frac{x^2-1}{x-1}</math>. Let the students to find the value of <math>f(1) = \frac{0}{0}</math> which does not give any number. Such form is said to be indeterminate form. Give other example of indeterminate form <math>\frac{\infty}{\infty}</math>, <math>\infty-\infty</math>, <math>1^\infty</math>, <math>0^\infty</math> and <math>0^\infty</math>.</li> <li>Take a variable x and its value be 1.9, 1.99, 1.999, ... 1.9999... as the number of 9's increases the value of x will be nearer to 2 but will never be 2. In such case the numerical difference between x and z will be very small.</li> <li>Similarly, take the values of x as 2.1, 2.01, 2.001, ... as the number of zero increases the value of x comes nearer to 2 but it is never 2. In such case we say x approached 2 or x tends to 2 and we write <math>x \rightarrow 2</math>.</li> <li>Consider a regular polygon inscribed in a circle. As the number of sides increases the area and perimeter goes nearer to the area and perimeter of circle as the value of n taken sufficiently large in this case the different between the area and perimeter of polygon is very closely near to that of circle but never be equal. <math>\lim_{n \rightarrow \infty} A_n = \text{Area of circle.}</math> <math>n \rightarrow \infty</math> Similarly discuss the other examples as <math display="block">1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots</math> <math>\lim_{n \rightarrow \infty} S_n = 2</math></li> </ul>	<ul style="list-style-type: none"> <li>Students' participation can be observed during class activities.</li> <li>What are the examples of indeterminate form?</li> <li>Define indeterminate form?</li> <li>What is the difference between <math>x \rightarrow 2</math> and <math>x=2</math>?</li> <li>What is the value of f(3) when <math>f(x) = \frac{2x-6}{x-3}</math></li> <li>If <math>f(1)=0.9</math>, <math>f(2) = 0.99</math> <math>f(3) = 0.999</math> ... so on, find value of <math>\lim_{n \rightarrow \infty} f(n) = ?</math></li> <li>Distinguish between the limit and value of the function at a point.</li> </ul>	1

<ul style="list-style-type: none"> <li>• To use the meaning of infinity</li> <li>• To find the limit of infinity.</li> <li>• To state and use the limit theorem.</li> </ul>	<ul style="list-style-type: none"> <li>• Meaning of infinity.</li> <li>• Limit at infinity.</li> <li>• Limit theorems.</li> </ul>	Projector	<ul style="list-style-type: none"> <li>• Consider the sequence <math>x=1, 0.5, 0.1, 0.001, 0.001, \dots</math> where limit is 0. If <math>f(x)=\frac{1}{x}</math> then corresponding values of <math>f(x)=1, 2, 10, 100, 1000, \dots</math> which go on increasing. If we take value of <math>x</math> very near to 0 the value of <math>f(x)</math> is greater than any positive number, however large. In such case we say:  <math>\lim_{x \rightarrow 0} f(x) = \infty</math></li> <li>• Consider the function <math>f(x)=\frac{1}{x^2}</math>. When the value of <math>x</math> increases the corresponding value of <math>f(x)</math> decrease. When the value of value of <math>f(x)</math> becomes large enough the corresponding value of <math>f(x)</math> becomes small enough. In such a situation, we say that as <math>x</math> tends to infinity, <math>f(x)</math> tends to zero and is indicated by the symbol, <math>f(x) \rightarrow 0, \text{ when } x \rightarrow \infty</math>  Or, <math>\lim_{x \rightarrow 0} \frac{1}{x^2} = 0.</math></li> <li>• (Only Statement)  Let <math>f(x)</math> and <math>g(x)</math> be two functions of <math>x</math> such that <math>\lim_{x \rightarrow a} f(x) = l</math> and <math>\lim_{x \rightarrow a} g(x) = m</math>, then we have the following theorems on limits:  i) The limit of the sum (or difference) of the functions <math>f(x)</math> and <math>g(x)</math> is the sum (or difference) of the limits of the function.  i.e. <math>\lim_{x \rightarrow a} f(x) \pm g(x) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x) = l \pm m</math>  ii) <math>\lim_{x \rightarrow a} [f(x) \cdot g(x)] = (\lim_{x \rightarrow a} f(x)) \cdot (\lim_{x \rightarrow a} g(x)) = l \cdot m</math>  iii) <math>\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{l}{m}</math>  iv) The limit of <math>n^{\text{th}}</math> root of a function <math>f(x)</math> is the <math>n^{\text{th}}</math> root of the limit of the function i.e. <math>\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{l}</math></li> </ul>	<ul style="list-style-type: none"> <li>• Give another example which clarifies the meaning of infinity.</li> <li>• Give an example which illustrates the limit at infinity.</li> <li>• State limit theorems, verify limit theorem with a particular example. Use limit theorem to solve the problems.</li> </ul>	1
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	<ul style="list-style-type: none"> <li>Students will be able to find the limit of algebraic function of:               <ol style="list-style-type: none"> <li>Simple Polynomial</li> <li><math>\frac{0}{0}</math> form</li> <li><math>\frac{\infty}{\infty}</math> form</li> <li><math>\infty - \infty</math> form</li> </ol> </li> </ul>	<ul style="list-style-type: none"> <li>Limit of algebraic function</li> </ul>	Projector	<ul style="list-style-type: none"> <li>Teacher, with discussing students, will solve the problems of these types: Find limits:               <ol style="list-style-type: none"> <li><math>\lim_{x \rightarrow 4} (x^2 + 2x - 9)</math></li> <li><math>\lim_{x \rightarrow 4} \frac{x^3 + 64}{x^2 + 16}</math></li> <li><math>\lim_{x \rightarrow 4} \frac{3x^2 - 4}{4x^2}</math></li> <li><math>\lim_{x \rightarrow 4} (\sqrt{x - a} - \sqrt{x - b})</math></li> </ol> <p>Many other similar type of problems can be discussed in class.</p> </li> </ul>	<ul style="list-style-type: none"> <li>Find the limit of:               <ol style="list-style-type: none"> <li><math>\lim_{x \rightarrow 1} \frac{3x^2 + 2x - 4}{x^2 + 5x - 4}</math></li> <li><math>\lim_{x \rightarrow a} \frac{x^{2/3} - a^{2/3}}{x - a}</math></li> <li><math>\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - x - 2}</math></li> <li><math>\lim_{x \rightarrow \infty} \frac{2x^2}{3x^2 + 2}</math></li> <li><math>\lim_{x \rightarrow \infty} \frac{5x^2 + 2x - 7}{3x^2 + 5x + 2}</math></li> <li><math>\lim_{x \rightarrow \infty} (\sqrt{x} - \sqrt{x - 3})</math></li> <li><math>\lim_{x \rightarrow 1} (\sqrt{x - a} - \sqrt{bx})</math></li> </ol> </li> </ul>	1
	<ul style="list-style-type: none"> <li>Students will be able to find the limit if exist by using the concept of right hand and left hand limit</li> </ul>	<ul style="list-style-type: none"> <li>Interval, neighbourhood, right hand and left hand limit</li> </ul>		<ul style="list-style-type: none"> <li>Teacher will discuss on following:               <p>A set of points between any two points a and b is said to be an interval. Closed interval [a, b] and open interval (a, b). A neighbourhood of a point a is an open interval containing the point a denoted by (a - <math>\delta</math>, a + <math>\delta</math>) also denoted by <math> x - a  &lt; \delta</math>. For any function f, l, is said to be the right hand limit of f at x = a if corresponding to any positive number <math>\epsilon</math>, there exist a positive number <math>\delta</math> such that <math> f(x) - l_1  &lt; \epsilon</math>, whenever <math>x \in (a, a + \delta)</math>. We denote it by <math>\lim_{x \rightarrow a^+} f(x)</math>.</p> <ul style="list-style-type: none"> <li><math>\lim_{x \rightarrow a} f(x)</math> exist if <math>\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)</math></li> </ul> <p>Do in class with discussion.</p> <p>Find the limit of the function f at x = 0.</p> <ol style="list-style-type: none"> <li><math>f(x) = x + 2</math>, for <math>x \geq 0</math> = <math>4x + 2</math> for <math>x &lt; 0</math></li> <li><math>f(x) = 3x + 2</math> for <math>x \geq 1</math> = <math>2x</math> for <math>x &lt; 1</math></li> </ol> </li> </ul>	<ul style="list-style-type: none"> <li>Define the following interval, closed interval, open interval, neighbourhood, limit of a function at a point, left hand limit, right hand limit, existence of limit of a function at a point.</li> <li>Find the limit at the points specified:               <ul style="list-style-type: none"> <li><math>f(x) = 3x + 1</math> for <math>x \geq 2</math> = <math>2x^2 - 1</math> for <math>x &lt; 2</math> } at <math>x = 2</math></li> <li><math>f(x) = 3x - 2</math> for <math>x \geq 2</math> = <math>2x^2 + 1</math> for <math>x &lt; 2</math> } at <math>x = 2</math></li> <li><math>\lim_{x \rightarrow 0} \frac{ x }{x}</math></li> </ul> </li> </ul>	1

<p>II Conti nuity</p>	<ul style="list-style-type: none"> <li>• To explain the intuitive idea of continuity.</li> <li>• To define continuity of a function at a point.</li> <li>• To state the types of discontinuity</li> </ul>	<ul style="list-style-type: none"> <li>• Intuitive idea of continuity.</li> <li>• Definition of continuity.</li> <li>• Types of discontinuity.</li> </ul>	<ul style="list-style-type: none"> <li>• The intuitive idea of a continuity functions <math>f</math> in the interval <math>[a,b]</math> gives the impression that the graph of the function <math>f</math> in this interval is a smooth curve without any break in it. Actually this curve is such that it can be drawn by the continuous motion of pencil without lifting it in a sheet of paper. Similarly, a discontinuous function gives the picture consisting of disconnected curves.</li> </ul> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p>Continuous</p> </div> <div style="text-align: center;">  <p>Discontinuous</p> </div> <div style="text-align: center;">  <p>Discontinuous</p> </div> </div> <p>Definition of continuous: The function <math>f(x)</math> is said to be continuous at the point <math>x = x_0</math> if <math>\lim_{x \rightarrow x_0} f(x) = f(x_0)</math> which implies that <math>f(x)</math> will be continuous at <math>x = x_0</math> if</p> $\lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow x_0^+} f(x) = f(x_0).$ <p><u>Types of Discontinuities:</u></p> <ol style="list-style-type: none"> <li>If <math>\lim_{x \rightarrow x_0} f(x)</math> does not exist i.e. <math>\lim_{x \rightarrow x_0^+} f(x)</math> then it is called ordinary discontinuity.</li> <li>If <math>\lim_{x \rightarrow x_0} f(x) \neq f(x_0)</math> then <math>f(x)</math> is said to have removable discontinuity.</li> <li>If <math>\lim_{x \rightarrow x_0} f(x) \rightarrow 0 \text{ or } -\infty</math> then <math>f(x)</math> is said to have infinite discontinuity.</li> </ol>	<ul style="list-style-type: none"> <li>• Define continuity.</li> <li>• State the types of discontinuity.</li> <li>• Draw the graph of <math>y = 2x-3</math> and state whether the function is continuous or not.</li> <li>• Draw the graph of <math>y = \frac{1}{x-2}</math> and state what type of discontinuity do you find at <math>x=2</math>.</li> <li>• What condition is necessary for a function <math>f(x)</math> to be continuous at the point <math>x = a</math>?</li> <li>• In what conditions will <math>f(x)</math> be discontinuous at <math>x=a</math>?</li> </ul>	
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<ul style="list-style-type: none"> <li>• Students will be able to test the continuity or discontinuity of the given function at specified point.</li> </ul>	<ul style="list-style-type: none"> <li>• Test of continuity and discontinuity.</li> </ul>		<p>With discussion at class some problems will be solved:</p> <ul style="list-style-type: none"> <li>• Test the continuity or discontinuity of the given function by calculating left hand limits, right hand limits and the value of function at specified point. <ul style="list-style-type: none"> <li>i) <math>f(x) = \frac{1}{2x}</math> at <math>x = 0</math>.</li> <li>ii) <math>f(x) = \frac{x^2-16}{x-4}</math> at <math>x = 4</math>.</li> <li>iii) <math>f(x) = \frac{ x-2 }{x-2}</math> at <math>x = 2</math>.</li> </ul> </li> <li>• Discuss the continuity of the following function: <ul style="list-style-type: none"> <li>i) <math>f(x) = \begin{cases} 2 - x^2 &amp; \text{for } x \leq 2 \\ x - 4 &amp; \text{for } x &gt; 2 \end{cases}</math> at <math>x = 2</math></li> <li>ii) <math>f(x) = \begin{cases} 2x + 1 &amp; \text{for } x &lt; 1 \\ 2 &amp; \text{for } x = 1 \\ 3x &amp; \text{for } x &gt; 1 \end{cases}</math> at <math>x = 1</math>.</li> </ul> </li> </ul>	<p>Discuss the continuity of following function.</p> <ul style="list-style-type: none"> <li>i) <math>f(x) = \begin{cases} 2x^2+1 &amp; \text{for } x \leq 2 \\ 4x+1 &amp; \text{for } x &gt; 2 \end{cases}</math> at <math>x = 2</math></li> <li>ii) <math>f(x) = \begin{cases} x^2+2 &amp; \text{for } x \leq 5 \\ 3x+12 &amp; \text{for } x &gt; 5 \end{cases}</math> at <math>x = 5</math></li> <li>iii) <math>f(x) = \begin{cases} 2x - 3 &amp; \text{for } x &lt; 2 \\ 2 &amp; \text{for } x = 2 \\ 3x - 5 &amp; \text{for } x &gt; 2 \end{cases}</math></li> </ul> <p>How can the function <math>f(x)</math> be made continuous at <math>x = 2</math>?</p>	
<ul style="list-style-type: none"> <li>• To state continuity theorem and use in problem solving.</li> <li>• To state properties of continuous function.</li> </ul>	<ul style="list-style-type: none"> <li>• Continuity Theorem</li> <li>• Properties of continuous function.</li> </ul>		<ul style="list-style-type: none"> <li>• Continuity theorems: <p>Let the function <math>f</math> and <math>g</math> be continuous at <math>x = a</math> then</p> <ul style="list-style-type: none"> <li>i) <math>f(x) \pm g(x)</math> is continuous at <math>x = a</math>.</li> <li>ii) <math>f(x) \cdot g(x)</math> is continuous at <math>x = a</math>.</li> <li>iii) <math>\frac{f(x)}{g(x)}</math> is the continuous at <math>x = a</math>, provided <math>g(a) \neq 0</math></li> <li>iv) <math>\sqrt[n]{f(x)}</math> is continuous at <math>x=a</math> provided <math>f(a) &gt; 0</math> when <math>n</math> is even.</li> </ul> </li> <li>• Properties of continuous function. <ul style="list-style-type: none"> <li>i) Let <math>f</math> be continuous on a closed interval <math>[a, b]</math> suppose <math>f(a)</math> and <math>f(b)</math> have opposite signs. Then there exists a point <math>c</math> such that <math>a &lt; c &lt; b</math> and <math>f(c) = 0</math>.</li> <li>ii) A continuous function on a closed interval is bounded.</li> <li>iii) A continuous function on a closed interval has a maximum and a minimum values on the interval.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• State continuity theorem.</li> <li>• State properties of continuous function.</li> <li>• <math>f(x) = 2x+3</math> is continuous at <math>x=2</math> similarly <math>g(x) = x+5</math> is continuous at <math>x=2</math>. Prove that: <ul style="list-style-type: none"> <li>i) <math>f(x) \pm g(x)</math> is continuous at <math>x=2</math>.</li> <li>ii) <math>f(x) \cdot g(x)</math> is continuous at <math>x=2</math>.</li> <li>iii) <math>\frac{f(x)}{g(x)}</math> is continuous at <math>x=2</math>.</li> </ul> </li> </ul>	

## Appendix 3 (I)

### Class Test I

Full Marks: 10

Pass Marks: 4

Time: 20 Minutes

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Attempt all the questions. Figure at the right indicate the full marks.

1. Define function. [2]
2. For what value of  $x$  the function  $f(x) = \frac{1}{x+2}$  is undefined? [2]
3. What is the value of  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ ? [2]
4. Find limiting value of  $\lim_{x \rightarrow 3} f(x)$  where  $f(x) = 3x - 2$ . [2]
5. The limit of the product of the function  $f(x)$  and  $g(x)$  is the product of the limits of the function i.e.  
 $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \dots$  [2]

## Appendix 3 (II)

### Class Test II

Full Marks: 10

Pass Marks: 4

Time: 20 Minutes

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Attempt all the questions.

#### Group 'A' [1x4 =4]

1. What the value of  $\lim_{x \rightarrow 0} \frac{1}{x}$  ?  
a) 0            b)  $\infty$             c) 1            d) x
2. Let f be continuous on a close interval [a, b]. Suppose f(a) and f(b) have opposite signs, then there exists a point c such that  $a < c < b$  and ...  
a)  $f(c) < 0$    b)  $f(c) > 0$    c)  $f(c) = 0$    d)  $f(c) = a$ .
3. Let f(x) be defined in a neighbourhood of a, then f(x) is said to tend to the limit l, as x approaches a or symbolically  $\lim_{x \rightarrow a} f(x) = l$ . If, to every positive number  $\epsilon$ , however small, there corresponds a positive number  $\delta$ , such that  $|f(x) - l| < \epsilon$  whenever ...  
a)  $|x - a| < \epsilon$    b)  $|x - a| < \delta$    c)  $|x - a| > \epsilon$    d)  $|x - a| > \delta$
4. What is the value of  $\lim_{x \rightarrow 3} \frac{4x-5}{2x+3}$  ?  
a)  $\frac{9}{7}$             b)  $\frac{7}{9}$             c)  $\frac{4}{3}$             d)  $\frac{3}{4}$

#### Group 'B' [2x3 =6]

5. Write the necessary and sufficient condition for a curve or the function f to be continuous at  $x = x_0$
6. Give one example of ordinary discontinuity or a jump discontinuity.
7. Prove that  $\lim_{x \rightarrow -\frac{2}{3}} \frac{2}{2+3x}$  doesn't exist.

## Appendix 3 (III)

### Post Test

Full Marks: 20

Pass Marks: 8

Time: 40 Minutes

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Attempt all the questions.

#### Group 'A' [1x8 = 8]

- Which of the following is not indeterminate form?  
a)  $\frac{\infty}{\infty}$       b)  $\infty - \infty$       c)  $\frac{5}{0}$       d)  $1^\infty$
- Which of the following function tends to infinity when x tends to 2.  
a)  $\frac{x^2-4}{x-2}$       b)  $\frac{3}{x-x-3}$       c)  $x-2$       d)  $x^2-4$ .
- Which of the following is true?  
a)  $\lim_{x \rightarrow 2} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$   
b)  $\lim_{x \rightarrow 2} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$   
c)  $\lim_{x \rightarrow 2} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$   
d)  $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = (\lim_{x \rightarrow a} f(x))^n$
- Which value is correct when  $\lim_{x \rightarrow a} \frac{x^5 - a^5}{x^4 - a^4}$ .  
a)  $\frac{a}{4}$       b)  $\frac{5a}{4}$       c)  $\frac{5}{4}$       d)  $\frac{4a}{5}$
- Which of the following is the necessary and sufficient condition for a function to have a limit at  $x=a$ ;  
a) Left hand limit and right hand limit of f at  $x=a$  should exist and coincide.  
b)  $|f(x) - l| < \epsilon$  whenever  $|x - f(x)| < \delta$   
c)  $\lim_{x \rightarrow a} f(x) = f(a)$   
d)  $\lim_{x \rightarrow a-0} f(x) \neq \lim_{x \rightarrow a+0} f(x)$
- Which of the following represent neighbourhood of a ?  
a)  $\{a - \delta, a + \delta\}$       b)  $[a - \delta, a + \delta]$   
c)  $(a - \delta, a + \delta)$       d)  $(a - \delta, a + \delta]$



7. Which of the following is incorrect?
- a)  $f(x)$  is said to be ordinary discontinuity if  $\lim_{x \rightarrow x_0} f(x)$  does not exist.
- b) Discontinuity of  $f(x)$  can be removed by redefining if  $\lim_{x \rightarrow x_0} f(x) \neq f(x_0)$
- c) If  $\lim_{x \rightarrow x_0} f(x) \rightarrow \infty$  or  $-\infty$ , then  $f(x)$  is said to have infinite discontinuity at  $x = x_0$ .
- d)  $f(x)$  is said to be removable discontinuity if  $\lim_{x \rightarrow x_0} f(x) = \infty$
8. Let the function  $f$  and  $g$  be continuous at  $x = a$  then which is false. ]
- a)  $f(x) \pm g(x)$  is continuous at  $x = a$ .
- b)  $f(x) \cdot g(x)$  is continuous at  $x = a$ .
- c)  $\frac{f(x)}{g(x)}$  is continuous at  $x = a$  for all value of  $g(a)$ .
- d)  $\sqrt[n]{f(x)}$  is continuous at  $x = a$  provided  $f(a) > 0$  when  $n$  is even.

**Group 'B' [1x8 = 8]**

- 1) Compute  $\lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16}$
- 2)  $\lim_{x \rightarrow 64} \frac{\sqrt[6]{x} - 2}{\sqrt[3]{x} - 4}$
- 3)  $\lim_{x \rightarrow \infty} (\sqrt{x} - \sqrt{x - 3})$
- 4) Find the limit of  $f(x)$  at  $x = 2$  when  
 $f(x) = 3x + 1$  for  $x \geq 2$   
 $= 2x^2 - 1$  for  $x < 2$
- 5) Discuss the continuity a function  $f$  such that  
 $f(x) = 2 - x^2$  fro  $x \leq 2$   
 $= x - 4$  for  $x > 2$  } at  $x = 2$
- 6) Define continuity of the function  $f(x)$  at  $x = x_0$ .

### Appendix 4 (i)

#### Achievement Score of Group 'A' Students in Mathematics and in Precalculus Unit

S.N.	Average Scores in Mathematics(out of 20 marks)	Average Scores in Precalculus (out of 20 marks)
1	15	17
2	13	16
3	14	17
4	9	13
5	8	14
6	6	10
7	5	9
8	5	8
9	7	7
10	4	7
11	2	6
12	3	5
13	10	14
14	1	6
15	8	10
N = 15	$\bar{x} = 7.33$ $\sigma = 4.14$ $\Sigma x = 110$ $\Sigma x^2 = 1064$	$\bar{x} = 10.60$ $\sigma = 4.08$ $\Sigma x = 159$ $\Sigma x^2 = 1935$

### Appendix 4 (ii)

#### Achievement Score of Group 'B' Students in Mathematics and in Precalculus Unit

S.N.	Average Scores in Mathematics(out of 20 marks)	Average Scores in Precalculus (out of 20 marks)
1	12	14
2	15	14
3	14	17
4	17	15
5	7	6
6	12	15
7	8	10
8	7	9
9	11	14
10	4	6
11	15	17
12	13	16
13	12	15
14	3	6
15	6	6
N = 15	$\bar{x}=10.4$ $\sigma = 4.14$ $\Sigma x = 156$ $\Sigma x^2 = 1880$	$\bar{x}=12$ $\sigma = 4.17$ $\Sigma x = 180$ $\Sigma x^2 = 2422$

### Appendix 5 (i)

#### Correlation Coefficient between Group 'A' Students' Scores in Precalculus and total scores in Mathematics

S.N.	Score in Precalculus(out of 20 marks) $X_1$	Score in Mathematics (out of 20 marks) $Y_1$	$X_1^2$	$Y_1^2$	$X_1Y_1$
1	17	15	289	225	255
2	16	13	256	169	208
3	17	14	289	196	238
4	13	9	169	81	117
5	14	8	196	64	112
6	10	6	100	36	60
7	9	5	81	25	45
8	8	5	64	25	40
9	7	7	49	49	49
10	7	4	49	16	28
11	6	2	36	4	12
12	5	3	25	9	15
13	14	10	196	100	140
14	6	1	36	1	6
15	10	8	100	64	80
$N_1 = 15$	$\Sigma X_1 = 159$	$\Sigma Y_1 = 110$	$\Sigma X_1^2 = 1935$	$\Sigma Y_1^2 = 1064$	$\Sigma X_1Y_1 = 1405$

$$r_1 = \frac{N_1 \Sigma x_1 y_1 - \Sigma x_1 \Sigma y_1}{\sqrt{N_1 \Sigma x_1^2 - (\Sigma x_1)^2} \sqrt{N_1 \Sigma y_1^2 - (\Sigma y_1)^2}}$$

$$= \frac{15 \times 1405 - 159 \times 110}{\sqrt{15 \times 1935 - (159)^2} \sqrt{15 \times 1064 - (110)^2}} = 0.94$$

$$\text{Now, } t = \frac{r_1 \sqrt{N_1 - 2}}{\sqrt{1 - r_1^2}} = \frac{0.94 \sqrt{15 - 2}}{\sqrt{1 - (0.94)^2}} = 9.93$$

### Appendix 5 (ii)

#### Correlation Coefficient between Group 'B' Students' Scores on Precalculus and Mathematics

S.N.	Scores in Precalculus(out of 20 marks) $X_2$	Scores in Mathematics (out of 20 marks) $Y_2$	X <sub>2</sub> <sup>2</sup>	Y <sub>2</sub> <sup>2</sup>	X <sub>2</sub> Y <sub>2</sub>
1	14	12	196	144	168
2	14	15	196	225	210
3	17	14	289	196	238
4	15	17	225	289	255
5	6	7	36	49	42
6	15	12	225	144	180
7	10	8	100	64	80
8	9	7	81	49	63
9	14	11	196	121	154
10	6	4	36	16	24
11	17	15	289	225	255
12	16	13	256	169	208
13	15	12	225	144	180
14	6	3	36	9	18
15	6	6	36	36	36
N <sub>2</sub> = 15	$\Sigma X_2 = 180$	$\Sigma Y_2 = 156$	$\Sigma X_2^2 =$ 2422	$\Sigma Y_2^2 =$ 1880	$\Sigma X_2 Y_2 = 2111$

$$\begin{aligned}
 r_2 &= \frac{N_2 \Sigma x_2 y_2 - \Sigma x_2 \cdot \Sigma y_2}{\sqrt{N_2 \Sigma x_2^2 - (\Sigma x_2)^2} \sqrt{N_2 \Sigma y_2^2 - (\Sigma y_2)^2}} \\
 &= \frac{15 \times 2111 - 180 \times 156}{\sqrt{15 \times 2422 - (180)^2} \sqrt{15 \times 1880 - (156)^2}} \\
 &= 0.92
 \end{aligned}$$

$$\text{Now, } t = \frac{r_2 \sqrt{N_2 - 2}}{\sqrt{1 - r_2^2}} = \frac{0.92 \sqrt{15 - 2}}{\sqrt{1 - (0.92)^2}} = 8.46$$

## Appendix 6

### List of Formulae

#### 1. t-test for dependent samples

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sigma_D}$$

Where,

$\bar{X}_1$  = Mean score of first treatment

$\bar{X}_2$  = Mean score of second treatment

$$\& \sigma_D = \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 - 2.r.\sigma_{x_1}.\sigma_{x_2}}$$

N-1 = degree of freedom

Here,

$\sigma_{\bar{x}_1}$  = Standard error of the mean of I treatment

$$\therefore \sigma_{\bar{x}_1} = \frac{\sigma}{\sqrt{N_1}}$$

$\sigma_{\bar{x}_2}$  = Standard error of the mean of II treatment

$$\therefore \sigma_{\bar{x}_2} = \frac{\sigma_2}{\sqrt{N_2}}$$

$\sigma_1$  = are standard deviation of I

$\sigma_2$  = standard deviation of II treatment.

r = correlation coefficient

#### 2. Correlation coefficient:

$$r = \frac{N\Sigma xy - \Sigma x.\Sigma y}{\sqrt{\{N\Sigma x^2 - (\Sigma x)^2\}\{N\Sigma y^2 - (\Sigma y)^2\}}}$$

3. Significance test for the correlation

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}}$$

$$\text{d.f.} = N - 2$$

4.  $\sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

5.  $\bar{x} = \frac{\Sigma x}{N}$