

## Chapter- I

### INRODUCTION

#### Background of the Study

Mathematics directly deals with human life. It is believed that the development of mathematics and development of human civilization was together. Mathematics was created to fulfill the human needs. Though Mathematics was introduced later in the formal education system, it had been developed simultaneously with the development of society. Mathematics is not only taught and practical through the formal institution but also the contemporary society has been practicing it with its own and belief systems.

Mathematics is a body of knowledge in the area of science with its symbolism, terminology, contents, theorem and techniques. Mathematics is in nature highly symbolic subject. Mathematics is numerical and calculative part of human life and knowledge which helps us to give exact interpretation.

Mathematics deals with idea, not pencil marks or chalk mark, not physical triangles or physical sets, but ideas (which may be represented or suggested by physical objects). What are the main properties of mathematical activity of mathematics knowledge as known as to all of us from daily experience? (i) Mathematical objects are invented or created by humans. (ii) They are created, not arbitrarily, but arise from activity with already existing mathematical objects from the needs of science and daily life. (iii) once created mathematical objects have properties which are well determined difficulty discovering, but which are passed identity

of our knowledge of them. Mathematics is at the heart of many successful careers and successful lives (National Council of Teachers of Mathematics (NCTM, 1998)).

(Greek *γεωμετρία*; geo = earth, metria = measure) arose as the field of knowledge dealing with spatial relationships. Geometry was one of the two fields of pre-modern mathematics, the other being the study of numbers (arithmetic).

Geometry is one of the most useful and important branches of mathematics. Geometry includes an enormous range of ideas and can be viewed in many different ways.

It has been interlocked with many other subjects and different views of human activity. The basic ideas of mathematical systems originated in geometry some twenty-two to twenty-three hundred years ago (Kelly and Ladd, 1986). Furthermore, Kelly and Ladd write, it is not certain who first had the idea of trying to prove the mathematical rule by reasoning rather than by testing it in different cases. Although both Thales (640 -546 B.C.) and Pythagoras (born 572 B.C) have been given credit for the idea, originated in Greece around the sixth century B.C. Once the idea of this mathematical method had been discovered or invented, the mathematics of geometry grew with astonishing speed. By 300 B.C, a large body of geometric knowledge was in existence. At this time, the mathematician Euclid brought together and unified this knowledge by constructing the first definite, formal system of mathematics in the treatise 'The Elements'. It is probable that Euclid's 'Elements' is a highly successful compilation and systematic argument of works of earlier writers... Euclid's Elements is devoted to geometry along

but contains much number theory and geometric algebra. The work composed of 13 books with 455 propositions (Eves, 1986).

Geometry concerned with the properties of configurations of geometric objects, points, lines and circles.

Classic geometry was focused in compass and straightedge constructions. Geometry was revolutionized by Euclid, who introduced mathematical rigor and the axiomatic method still in use today. His book, *The Elements* is widely considered the most influential textbook of all time, and was known to all educated people in the West until the middle of the 20th century.

In modern times, geometric concepts have been generalized to a high level of abstraction and complexity, and have been subjected to the methods of calculus and abstract algebra, so that many modern branches of the field are barely recognizable as the descendants of early geometry.

Furthermore, Butler and Weren have suggested that students of junior high school had to learn geometrical concept. Therefore, junior high school has to systematize geometric information and extend it to some of the broader and more general aspect of the geometry of everyday life. To aid the pupil in becoming familiar with the basic geometrical concepts and understanding the fundamental techniques, such as the use of the straightedge, protractor, compasses and the techniques of measurement and construction; to acquaint the pupil with the characteristics of good geometrical notion; to bridge the gap from the largely manipulative types of geometric experiences to the more formal logical processes of demonstrative geometry. Such geometry has been

called "intuitive" but it is rather a geometry sui-generis which is characterized by intuition, experiment and an informal approach to the more formal logical processes of demonstrative geometry. To omit any one of these three aspects (intuition, experiment and in informal of the geometry of the junior high school "(The Teaching of Secondary Mathematics. P....363).

Mathematics is introduced as a compulsory subject at all levels of school education in Nepal. Of the total school hours 30 percent is allocated to mathematics at the primary level, 20 percent at the secondary level and 12 percent at the secondary level.

It is clear indication that understanding of mathematics is essential for everyday life as well as for higher studies in the field of science and technology, useful and practically life related mathematics in human life. National education system plan (NESP 1971) had realized the meaningful approach of mathematics teaching.

Mathematics is hierarchical in nature and learning is growth process that involves children to learn. If mathematics is not learned with understanding in elementary level, then it would be very difficult for later learning. The teaching procedure or method of teaching that can motivate the learners in the learning process through active participation. It is also believed that how to teach is really a difficult problem of the teacher, equally mathematics teaching is also a challenging and exciting profession. To become an effective and efficient teacher, it is necessary to understand the relationship among the mathematics contents and various teaching strategies.

In Nepal, studies have shown that mathematics education was largely managed and imparted through traditional lecture approach, rote memorization, and cramming. Many students have the habit of only memorizing factual information from their textbook without thinking "why?" worst of all, the students merely copy what the teachers have written on the blackboard and then memorize only that information while even neglecting their textbooks. It is not only painful for the students to engage in such rote memory; it also takes the long period of time to do this work.

Constructivists and other cognitive theorists believe that the meaningful learning depends upon the teaching procedure of method that can motivate the learners in the learning process through active participation doing things themselves. It is also believed that how to teach is really a difficult problem for the teacher. Various methods and techniques of teaching have been developed by the pedagogists and utilized to develop various fact of the cognitive and affective domain. The old methods of teaching are criticized severely and have been replaced by newer methods of teaching. In the traditional method of teaching, the main actor is the teacher and the teacher is the exposure of all activities so this traditional method gives much focus on lecture and recitation from text books, the teaching and learning process is based upon the assumption that the teacher explains and students memorize, which has never become the meaningful learning.

Mathematics has been taught as a compulsory subject at all levels of school system in Nepal besides compulsory mathematics optional mathematics is also offered to willing and worthy students. In both

subject areas, geometry is taught separately as important area and has an integral part of the whole school mathematics curriculum. Thus, geometry is considered as an important component of school mathematics programme. There is a vital role of teacher to show all these importance of geometry to the student in their teaching.

Though, the importance has been given to the geometry in the school curriculum, there is no considerable attainment that has been expected. There are major problems and issue in the teaching, Shrestha (1991) claimed in his Master's thesis that school level students did better performance in algebra and arithmetic rather than in geometry. This implies that geometry has become a difficult subject for the students.

Many of the problems of different profession and discipline are related to geometry. Therefore, it plays an important role in our society. Geometry is taught as an important component of school mathematics programme. It should be taught in such a way that it may stress its relationship to the daily life of the students and help to study the other subjects. Some researchers indicate that out of number of students who like mathematics, only least number of students like geometry. This attitude towards geometry may be due to the lack of appropriate teaching methods. The history of mathematics indicates that the development and presentation of geometrical concepts were different in different period.

In the context of Nepal there are curriculum, instructional materials, textbooks, teacher's guide to conduct the regular teaching activities in the classroom and teacher training package for improving the achievement of the students. In spite of these efforts. Significant achievement is not found. It implies that there is need suggest new

method of learning management and teaching for geometry based on research and experiment. The meaningful learning depends upon the teaching procedure or method that can motivate the learners in the learning process through active participation doing things themselves. It is also believed that how to teach is really a difficult problem to the teacher. Mathematics teaching is a difficult task for the teacher. In order to become as efficient and effective teacher, it is necessary to understand the relationship among the mathematics contents and various teaching strategies for presenting mathematics lessons.

Various methods and techniques of teaching have been developed by the pedagogics. Now, traditional method of teaching are criticized and replaced by newer methods through researches. Teachers need to have exposure of the developed method to bring into use in classroom teaching. Most of the teachers use traditional method in teaching geometry due to the lack of justification of the suitability of other new methods. Traditional method is taken as expository method. This methods assumes that learning is possible through the activities of teachers explanation and students' memorization. At present, this process of learning is claimed not as the meaningful learning.

Geometry is difficult part is mathematics due to the lack of suitable approaches of teaching. Even the excessive use of expository approach of geometry teaching, students of secondary level are found unable to understand the properties of geometrical figures. Worst of all, the students memorize/copy what the teachers write on the board, it is not only painful for the students to engage in such rote memory but also no an efficient method of teaching and learning of geometry. In spite of

excessive time spent on study, they felt geometry is a difficult subject to study. With the aim of overcoming the learning problem of geometry, different cognitive theories of learning have been developed. Van-Hiele's level of understanding geometrical ideas and the theory of instruction could be an alternative pedagogy for teaching geometry. Therefore, the researcher wanted to introduce and examine the effectiveness of Van-Hiele's approach in teaching geometry in secondary level of geometry teaching.

### **Statement of the Problem**

In the issue of ineffectiveness of usual method of geometry teaching and claiming of suitability of Van-Hiele's developmental approach in teaching strategies, the researcher intended to study the effectiveness of van-Hiile approach in teaching geometry (triangles and quadrilateral) at secondary level. This study was an experimental study. The only one parameter of the effectiveness used in this study is the level of learning achievement of the student. In order to ascertain the effectiveness of the van-Hiele's approach toward geometry teaching, the study intended to answer the following question.

'Is van-Hiele's approach of teaching more appropriate in classroom than usual approach of teaching geometry (triangles and quadrilateral) at secondary level?'

### **Significance of the Study**

Geometry is the most important part of the mathematics as the human's heart without geometry. One cannot imagine the existence of mathematics. There are different techniques by using the geometry figure



in the environment. In Nepal, the geometrical concepts introduced in school education have some aspects without necessary conceptualization. There are still alive won traditional geometrical concepts.

Method of teaching geometry in schools at present is not really appreciable. Most of the students are found negative to learning geometry. They feel that learning of geometry is learning to prove geometric propositions and proving propositions in difficult tasks. Even, teachers do feel difficult to teach geometry and scare the students that they are under qualified for learning geometry. Equally, most of the teachers in school do not like to bother to adopt new methods of teaching, because it becomes challenging to them. This experimental study has given a sort of justification for the effectiveness of Van-Hiele's approach for meaningful teaching of geometry and conducting this study, the research had given a detail of the methods and approaches for classroom teaching. Therefore, this study is significant in this matter. Fundamentally, the implementers get one of the tested instructional designs for geometry in the Nepalese context.

Furthermore, the results of the study is useful to the concerned personnel and agencies to chose the appropriate technique of teaching unit now no research has given the mental developmental level of secondary students in geometry. This study helps to the curriculum designer, teachers, planners, trainers to their respective field.

This study has significance in the followings.

- i. This study experimentally verified and justified the effectiveness of Van-Hiele's approach in geometry teaching. So, teachers can use it following the strategies as used in the experimental phase. It may

provide one more instance to establish a new method of teaching in the Nepalese contexts.

- ii. The result of the study has given the mental development level of the secondary students. This could be a great information to curriculum dsinger and even the textbook writers, so that they could organize and equence their contents according to the mental development level in geometry.
- iii. The teacher training institution gets an opening of a possibility of introducing new approach with a small instance of justification in geometry teaching in Nepalese. This approach has not yet been introduced in the practices of training of the teachers.

### **Objectives of the Study**

The objectives of the study were:

- i. To explore the effectiveness of Van-Hiele's approach in teaching geometry (triangles and quadrilateral) at secondary level;
- ii. To explore the effectiveness of van –Hiele's approaches in reducing the gender difference in achievements in geometry (triangles and quadrilateral) at secondary level.

### **Hypothesis of the Study**

In order to verify collected information statistically, the following hypothesis were stated:

## **Research Hypothesis**

1. There is no significant difference in the achievements of students exposed to the Van-Hiele's model and usual method of teaching, while teaching toptic triangle and quadrilateral.
2. There is no significant difference in the achievement in triangles and quadrilateral at secondary level of students by gender when exposed to the method in experimental.
3. There is no significant difference in the achievements in triangles and quadrilateral of secondary level students by gender when exposed to the usual method of teaching.

## **Delimitation and Limitation of the Study**

The study was delimited methodologically consisting in the experiment.

- i. The result of the study can be generalized in the context of Jhapa district and in the other part of the country having the same context. Consequently it cannot be claimed that the results of the study is applicable throughout the country.
- ii. The effectiveness of this approach can be justifiable for the unit of (triangles and quadrilateral) of grade ten.
- iii. The experimental duration is a short period, as well as the coverage of the variables influencing in the learning are not adequately controlled in such a small study. Thus there is a danger to generalize the statement that van-Hiele's method is the most suitable ones for all situations and context.

## **Definition of the Related Terms of the Study**

*Van-Hiele's approach:* It is a sequential learning program for the classroom to examine the students' level of thinking at different developmental levels with skills visual, verbal, logic, drawings and application and proceed classroom instruction according to the phases given in the Van-Hiele's theory of geometry learning. It is also recognized as an example of a scientific approach of learning, which mentions the rational competition of the students in a group or individual. It is student-centered approach since the students gain knowledge according to their level of understanding in geometry. Both the researcher and students are good participants on the learning and the researcher as teacher become a facilitator much than exposure of content to the students.

*Method:* It refers to a planned way to teaching and learning management in the class. It consists of the activities done in the class by both teacher and students.

*Achievement:* The achievement on thesis study is defined in terms of scores obtained by the learners in the mathematics achievement test constructed by the researcher.

*Effectiveness:* The effectiveness in this study is defined in terms of the magnitude of the score obtained by experimental and control group in the mathematics achievement.

*Usual Method :* In this method the teacher is the authority of teaching learning activity and the students passively accept the facts exposed by the teacher. The interest and expectation of the students are highly underestimated by the teacher. The teacher explains, illustrate, question

but nothing done by the student in the process of reform. This model illustrates, question to the students but nothing done by themselves for learning. This modality of teaching–learning is considered as traditional/usual method of teaching.

## Chapter- II

### REVIEW OF THE RELATED LITERATURE

Shrestha (2005), in his master thesis entitled "Attitudes of primary level teachers of Kathmandu district towards Geometry" concluded that:

- (i) There was a positive attitude of male and female teachers towards geometry.
- (ii) There was no gender wise difference in attitudes among teachers towards geometry.
- (iii) There was no difference in attitude among urban teachers and rural teachers towards geometry.

In the study entitled "A study of effectiveness of co-operative teaching method in teaching mathematics in secondary level" Budhathoki, Tara Bahadur (2004) found that the co-operative learning method is more effective than the traditional method in teaching. From the result of the study, it can be concluded that the co-operative learning method helps students to understand probability and consequently perform better in achievement test over traditional method, motivates students to learn and apply the non concepts of probability in unfamiliar condition.

Chamlagain (2005) conducted the research on "A study of Problem faced by Secondary School Mathematics Teacher in Teaching Geometry". Students evaluation techniques, geometry instruction, teachers' professional development constructing and using instructional materials, school's, administration, students' various background,

characteristics and problem related to curriculum and test were found as major problems.

Gautam, (2006) cites on his thesis entitled "A Study on the Assessment Actionable Learning Strategy at Mathematics classroom in Higher Secondary Education" that H. Furth (NY Oxford, 1978) in his volume Piaget in the classroom adds to emphasize of Dewey's thought to piaget "Dewey writes continuity and interaction were two critical dimensions in the development of experiences as educative engagements. By continuity, Dewey meant that the meaning derived from experience should be built progressively from prior learning forming on even more complex scaffold of knowledge and understanding". The second attributes noted by Dewey, was interaction, active engagement with human and physical environment, through interaction with the environment. Jean Piaget has emphasized that dual development process of accommodation and assimilation are energized. Assimilation involves incorporating knowledge within pre-existing conceptual categories of the mind.

According to van Hiele's Theory (Clements and Batlista, 1992), students' Geometric thoughts develops in five discontinuous level called van Hiele's level (VHLS) namely: Visual (VHL-0), Analysis (VHL-1), Non-formal Deduction (VHL-2), Formal Deduction (VHL-3) and Rigor (VHL-4). These five levels from four shifts to complete geometry thought. Transition (a shift) from one to another is very difficult as the levels belong to different paradigms, similarly, to create effective teaching-learning situation in geometry teaching.

Upadhyay, (2010) "Method of Teaching Mathematics in Secondary Level" has emphasized five Van- Hiele's levels of geometric thoughts such as level-I: Visualization, level-II: Analysis, level-III: Informal Deduction, Level-IV: Deduction and level-V: Rigor. Furthermore, he has illustrated various teaching-learning activities for each van Hiele's levels of geometric thought.

Pandit, (1999) on his master's thesis entitled "A study of Attitudes of Secondary Level students and teachers towards Geometry". There were four major objectives and selected 15 teachers and 224 students from the Tanahu district concluded that:

- (i) The students studying in secondary level had positive attitudes towards geometry.
- (ii) The teachers had negative attitudes of secondary level geometry.
- (iii) Secondary level boys had better attitudes than these of girls attitudes towards geometry.
- (iv) The mean attitude scores of students towards geometry was significantly greater than that of their teachers.

There are several researches carried out in the in the department of mathematics education related to learning approach, it is seen that teaching mathematics by using different approaches is useful than usual method. The researchers try to present the effectiveness of van Hiele's approach on teaching geometry on the area of triangle and quadrilateral in the eastern part of Nepal.



## **Theoretical Framework of the Study**

Although this is an experimental research but Van-Hiele's theory of geometrical reasoning and understanding was used to analyse the problem.

### **Van-Hiele's Levels of Geometric Thinking**

In the later 1950's, Pierre and Dina Van-Hiele's (a Dutch husband and wife team) introduced their developmental model of geometric thinking. Their belief was that students progressed through various levels of cognition as they gained experience with the properties and relationships of geometric concepts. The five levels of geometric thought (numbered Levels 0-4 or 1-5) do not correspond with student age. As students develop the cognitive skills necessary to master one level, they progress to the next. For the classroom teacher, it is important that to assess what level students are functioning on when they arrive in his/her class. Too often, elementary school mathematics lessons stagnate on Level 0. By middle school, students should be able to function at Level 2.

Van-Hiele's sample model of how teachers can help students' progress through the cognitive levels of thinking is given below:

**Level 0: (Recognition/ Visualization)** The student identifies names, compares and operates on geometric figures (e.g., triangle, angles, intersecting or parallel lines) according to their appearance.

**Level 1: (Analysis)** The student analyzes figures in terms of their components and relationship among components and discovers properties rules of a class of shapes empirically (e.g. folding, measuring and using a grid or diagram).

Level 2: (Informal Deduction/Relationship) The students logically interrelates previously discovered properties rules by giving or following informal argument.

Level 3: (Deduction/ Formal Proof) The student proves theorems deductively and establishes interrelationships among network of theorems.

Level 4: (Axiomatic / Rigor) The students establish theorems in different postulation systems and analyzes/ compares these systems.

## **Chapter- III**

### **METHODS AND PROCEDURE**

The general goal of this study is to investigate the effectiveness of the use of Van-Hiele's hierarchy of understanding geometrical idea in the approach instruction, particularly in the topic quadrilateral and triangle at secondary level mathematics course. This study is an experimental in nature. This chapter is focused on the methodology used in this study. In this chapter, the researcher describes the methods and procedures used in the study. It gives the description of the research design, population, the method used in sample and sampling technique, data collection procedure and analysis of the collected data. This chapter further explains the principle and method used in preparation of test items, reliability and validity of the test and administration of the test and procedure of analyzing the data.

#### **Design of the Study**

The research plan developed before starting the research work is a research design. Therefore, research design is a baseline of each research. Research design is needed to conduct the research in proper way. Main target of the research design is to collect data systematically with minimum cost and effort and give much validity to the findings of the study. This study is an experimental type, having two groups: experimental group and control group. The experimental and control groups were established by random. Experimental group was taught by using Van- Hiele's developmental stage in understanding geometrical ideas and phases in teaching geometry, and control group was taught by usual method of teaching. The design is properly known as pre-test, post-test non- equivalent group design.

The design represented graphically as follows:

Table: 1

Equivalent Group	Treatment	Pre-test	Post- test
E	use of Van-Hiele's approach	O <sub>1</sub>	O <sub>2</sub>
C	use of usual methods	O <sub>3</sub>	O <sub>4</sub>

Whereas,

E = Experimental group

C = Control group

O<sub>1</sub> = Pre –test given to the experimental group

O<sub>2</sub>= Post –test given to the experimental group

O<sub>3</sub> = Pre-test given to the control group

O<sub>4</sub>= Post -test given to the control group

At the end of the predetermined period as described later on, the groups were tested using a test and the difference between means was subjected to a test of statistical significance. Z- test and T-test will be used to compare the achievement.

### **Population and the Sample of the Study**

The tenth grade students enrolled in the public schools of Jhapa district of Nepal considered as the population of the study. In order to carry out the experiment, the study was carried out in Moti Higher

Secondary School of Jhapa district. The school was selected purposively by the researcher because of the access and expectation of help and cooperation needed from the school to accomplish the study. There are 38 students in class X. Among them 13 are boys and 25 were girls. Which were divided into two groups: control group and experimental group each having 19 students. In experimental group, 7 were boys and 12 were girls. In control group, 8 were boys and 11 were girls. After dividing into groups, experimental group was taught by (Van-Hiele's approach as stated in the theoretical frame of the study) on the basis of test score of pre-requisite test, students were divided into two equivalent groups. Experimental group was taught by researcher in the first period (10:30 - 11.10) using Van-Hiele's approach and control group was taught by himself in the fourth period (12:50-1:30) in usual method from 2068-01-08 to 2068-01-19.

## **Tools**

Module for teaching according to the designed treatment in the experiment and a test for measurement of achievement were the main tools of the study. The researcher developed the teaching module on the basis of Van-Hiele's approach was one of the tools for the study. The researcher developed teaching module consisting 8 lessons for the experiment (Appendix-III) as the involvement in the experimental group. Achievement test paper consists of subjective (short and long answer) type questions. The researcher developed the tools in consultation with the senior subject teacher for the study, based on the content covered in the curriculum by this experiment. The test items were taken according to

the skill of geometry as stated in the Van-Hiele's theory-visual, verbal, drawing, logical and applied.

### **External Threats:**

The researcher has visited the parents of each student and requested them not to guide their children at the research period.

### ***Description of Test Items and Scoring Criteria***

Scoring Criteria: After having the experiment students by dividing students into two equivalent group i.e. experimental and control group. The post test exam was taken with full marks 30 with two types of questions short and long. Five short questions each question has weight age two marks and five long questions each weight age four marks including each Van-Hiele's level. Two marks has given for correct answer of short question and 0 for wrong, four marks was given for correct answer of long question and 0 for wrong answer also marking scheme of S. L.C. exam was followed in order to check the paper.

### ***Reliability***

Reliability is the degree to which a test consistently measures whatever it measure. All the test questions of pre-requisite test and post-test papers are selected from the government book and previous S. L. C. questions. That is why, it is considered that the reliability of question is already assumed.

### ***Validity***

Validity is the degree to which a test measure what is supposed to measure, since a test is valid for a particular purpose and a particular

group. All the test questions of pre-requisite test and post test papers are selected from the government book and previous S. L. C. questions and being S. L. C. questions standardized, it is considered that the validity of question is already maintained.

### **Data Collection Procedure**

The experimental group and control group were taught by the researcher himself from 2068-01-08 to 2068-01-19 B. S. The researcher taught the control group in the fourth period of each school day (12:50 to 1:30) by usual method of teaching while experimental group was taught by using Van-Hiele's approach with developed teaching module in the first period of each school day (10:30 to 11:10). The 8 day's instructional activities were made as a part of regular school activities. The strategy used in the experimental teaching was group work co-operative learning approach. There were 19 students in experimental group having 7 boys and 12 girls. Whereas in the control group of 19 students, there were 8 boys and 11 girls. These groups were formed on the basis of their obtained marks in pre-requisite tests which was held in 2068-02-05 B.S. The two equivalent groups were divided on the basis of marks, score and the pre-requisite test paper.

At the end of completing the teaching, the standardized achievement test was administrated on both group of the sample students. They were inspired to answer freely and without any discussion among themselves. The time allotted to the test was 1 hour. After the time duration of examination, the answer sheets were collected and answer were scored by researcher and then the scores were tabulated for the analysis.

## **Data Analysis Procedure**

The collected data were analysed using statistical devices by giving critical evaluation using the following procedure. The researcher used the following the statistical tools for analyzing and interpreting data.

- i. Mean standard deviation and variance were calculated for the both groups with their obtained marks in the test.
- ii. The t-test and z-test were used at 0.05 level of significance to find whether the difference of mean is significant.



## **Chapter- IV**

### **ANALYSIS AND INTERPRETATION**

This is an experimental study in nature. The main focus of study was to explore the effectiveness of Van-Hiele's approach in teaching Geometry. On the topic "Triangle and Quadrilateral" at secondary level school mathematics curriculum. The main parameter to explore the effectiveness of teaching approach is the achievement of the students' performance. For this purpose, achievement test of secondary school students who were taken as sample of the study were taken.

This chapter focuses on analyzing the effectiveness of strategy for teaching geometry developed by researcher based on Van-Hiele's learning phases. Class activities, students' concept and response at class works are analysed according to the strategies. More over the development of level of thinking towards geometry are discussed critically, and statistical analysis of the obtained achievement scores between experimental and control group was done.

#### **Analysis of Achievement of Experimental and Control Group**

Statistical analysis of the achievement scores are presented in three sections.

- a) Achievement of experimental and control group of students
- b) Achievement of experimental group students by gender
- c) Achievement of control group students by gender

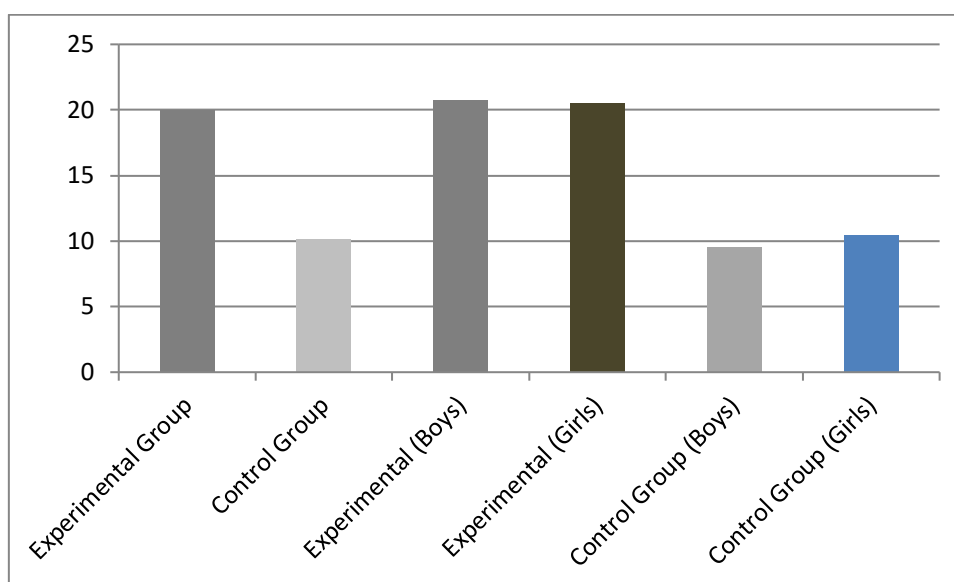
Achievement of students in post-test exam are given under the table:

Table. 2

	Experimental Group	Control Group	Experimental Group (Boys)	Experimental Group (Girls)	Control Group (Boys)	Control Group (Girls)
mean	19.95	10.16	20.71	20.5	9.5	10.45
$\sigma$	5.07	6.66	5.22	4.83	7.38	6.35

Achievement of students in post-test exam is represented on bar diagram

Figure -1



(a) Achievement of Experimental and Control Group Students

Table: 3

Achievements of experimental group and control group of post-test exam.

	Mean $\bar{X}$	Standard Deviation	$Z_{0.05}$	Calculated Value
Experimental Group	19.95	5.07	1.196	5.05
Control group	10.16	6.66		

(1) Null hypothesis ( $H_0$ ) = There is no significant difference between the achievement score of student teaching by Van-Hiele's approach and usual method. ( $\mu_1 = \mu_2$ )

(2) Alternate hypothesis ( $H_1$ ) : There is significant difference between the achievement score of students, teaching by Van-Hiele's approach and usual method. ( $\mu_1 \neq \mu_2$ )

(3) Level of significance is 0.05

$z_{0.05} = 1.96$  it is two tailed test. Null hypothesis is rejected if calculated value of  $z \geq 1.96$  or,  $z \leq -1.96$  where  $z$  is calculated value using formula:

- Calculated value of  $z = 5.05$  is greater than table value ( $z_{0.05} = 1.96$ ) i.e. the null hypothesis is rejected i.e. ( $\mu_1 \neq \mu_2$ ) there is significant difference between the achievement score obtained by teaching Van-Hiele's approach and usual method and the reasons behind that are considered as follows:
  - Preparation of teacher towards Van-Heile model of teaching
  - Active participation of students
  - Active creativity to show best performance
  - Curiosity of students over the newly adopted method

**(b) Achievement of Experimental Group students by gender**

Table: 4

	Mean $\bar{X}$	Standard Deviation	$Z_{0.05}$	Calculated Value
Boy	20.71	5.22	1.740	0.028
Girl	20.5	4.83		

(1) Null Hypothesis (H<sub>0</sub>) There is no significant difference between the achievement of boys and girls of the experimental group. ( $\mu_2 = \mu_3$ )

(2) Alternate hypothesis (H<sub>1</sub>): There is significant difference between the achievement of boys and girls of the experimental group. ( $\mu_2 \neq \mu_3$ )

Level of Significance :  $\alpha = 0.05$

Degree of Freedom =  $n_1 + n_2 - 2 = 7 + 12 - 2 = 17$

$$t_{0.05, 17} = 1.740$$

Null hypothesis will be rejected if  $t \geq 1.740$  where t is the calculated value according to formula and 1.740 is the value  $t_{0.05, 17} = 1.740$

$t = 0.028$ , calculated value of t is less than 1.740 (table value) hence, the null hypothesis is accepted i.e. there is no significant difference between the achievement of boys and girls of experimental group. Although the mean of boys group is slightly greater than the group of girls, but it is not significantly different. The reason behind this is the students have mixed strategy so, they adopt co-operative learning to each other activity.

### (c) Achievement of Control Group Students by Gender

Table: 5

	Mean $\bar{X}$	Standard Deviation	$Z_{0.05}$	Calculated Value
Boy	9.5	7.38	1.740	0.282
Girl	10.45	6.35		

(1) Null Hypothesis ( $H_0$ ) There is no significant difference between the achievement of boys and girls of the Control group. ( $\mu_4 = \mu_5$ )

(2) Alternate hypothesis ( $H_1$ ): There is significant difference between the achievement of boys and girls of the Control group. ( $\mu_4 = \mu_5$ )

Level of Significance :  $\alpha = 0.05$

Degree of Freedom =  $n_1 + n_2 - 2 = 8 + 11 - 2 = 17$

$$t_{0.05, 17} = 1.740$$

Null hypothesis will be rejected if  $t \geq 1.740$  where  $t$  is the calculated value according to formula and 1.740 is the value  $t_{0.05, 17} = 1.740$ .

$t = 0.282$ , calculated value of  $t$  is less than 1.740 (table value) hence, the null hypothesis is accepted i.e. there is no significant difference between the achievement of boys and girls of Control group.

## **Chapter-V**

### **SUMMARY, FINDINGS, CONCLUSION AND RECOMMENDATIONS**

After analysing and interpretation the researcher has tried to summarize, draw findings, derived conclusions and recommendations for the further study. The first section of this chapter reveals the summary, second section findings of study and the third section about conclusions on the basis of research analysis and last section presents recommendation based on findings of the study.

#### **Summary**

The study was an experimental in nature. In order to conduct the experiment on the effectiveness of Van-Hiele's approach in teaching 'triangle and quadrilateral' at secondary level, the researcher develop teaching module and taught himself in both experimental and control group of the students. The experimental group was taught using module on Van-Hiele's teaching approach and the control group was taught using usual teaching method. At the end of teaching experiment, an achievement test was administered on both groups. The scores obtained in the test by the students in each group was analyzed and obtained the findings.

#### **Finding of the Study**

On the basis of analysis of data obtained from achievement test which has been described in the Chapter IV, the following findings were drawn:

1. The mean score of students of experimental group was greater than the mean score of the students of control group. This difference was statistically significant. Thus, the researcher concluded that the Van-Hiele's approach of teaching was found to be an effective method of teaching geometry at secondary level
2. The mean score of boys was greater than girls mean score achievement test. But the difference was not statistically significant. This result implies that Van-Hiele's approach of instruction, to some extent, can reduce the difference in achievement between boys and girls.
3. Similarly, mean score of boys was less than the girls in the achievement test given at the end of the experiment in the control group. But the difference was not statistically significant. This result implies that usual teaching method can contribute in reducing the difference in students' achievement by gender. It cannot be authentically claimed, there needs further exploration, because pre-requisite test scores of control group students by gender was equal. It can be the replication of the previous status in the usual method of teaching geometry.

## **Conclusion**

In this study the researcher found that the mean achievement score of the student taught by Van-Hiele's approach was higher than the students taught by using usual teaching method. In conclusion, this study reveals that the Van-Hiele's approach can be more effective than the usual method in teaching geometry at secondary school level. From the result of this study it can be concluded that Van-Hiele's approach helps students

to understand geometry and consequently perform better in achievement test over usual teaching method. Additionally the Van-Hiele's approach helps students to motivate and apply the known geometrical concepts in unfamiliar conditions.

### **Recommendations and Suggestions for Further Study**

On the basis of the findings of this study, some measures have been recommended for the improvement of the teaching learning situation at secondary level classes and for further study are given below:

- 1) Van-Hiele's approach is suggested to adopt in teaching the topic "triangle and quadrilateral" at secondary level curriculum.
- 2) In addition, the following recommendations are made on the basis of the process adopted during experiment.
  - a) The mathematics teacher should be encouraged to use Van-Hiele's approach in teaching geometry.
  - b) The writers of teacher's guide should emphasize the use Van-Hiele's approach in giving sample activities in particular area's in teaching.
  - c) The teacher training institutes should focus their attention on Van-Hiele's approach of teaching mathematics in pedagogy course which is not yet introduced.
  - d) Curriculum designer, textbook writer should emphasize on the Van-Hiele's approach. In the preparation of mathematics textbook, emphasis should be given on Van-Hiele's approach and strategies throughout school mathematics.



## **Suggestions for the Further Study**

The following suggestions are made for the further study:

- a) To make teaching and learning effective and make easy, different kinds of teaching modules should be used.
- b) This study was confined only on Jhapa district. Therefore, further studies should be done in different parts of Nepal and the results of the study can be generalized all over Nepal.
- c) It would be worthwhile to study the opinions and attitudes of teachers and pupils towards the use of Van-Hiele's approach with teaching module.
- d) Similar study can be carried out at each grade level of school in order to have a wider view of effectiveness of Van-Hiele's approach in school level mathematics.

## BIBLIOGRAPHY

- Budhathoki, T. B. (2004). *A study of effectiveness of co-operative teaching method in teaching mathematics in secondary level*. Kathmandu: An unpublished dissertation in math edu., TU.
- Chamlagain, R. K.(2005). *A study of Problem faced by Secondary School Mathematics Teacher in Teaching Geometry*. Kathmandu: An unpublished dissertation in math edu., TU.
- Freund, J. E. (2004) *Mathematical Statistics with application*. Delhi: Pearson Education Pvt.ltd.
- Gautam, D. P. (2000) *Study on the assignment actionable learning strategy*. Kathmandu: An unpublished dissertation in math edu., TU.
- Gautam, T. R. (2005). *Teachers believe on learning geometry and its manifestation in teaching at Secondary level*. Kathmandu: An unpublished dissertation in math edu., TU.
- Janak Shiksha Samagri Kendra: Ganit for 9 and 10.
- Killy, P. J. and Lad, N. E. (1986). *Fundamental Geometry*, Delhi: Eurasia Publishing House (P) Ltd.
- Lamsal, S. (2005). *A study on effectiveness of van Hiele's approach in teaching geometry at lower secondary level*. Kathmandu: An unpublished dissertation in math edu., TU.
- Pandit, E. R. (1999). *A study of Attitudes of Secondary Level students and teachers towards Geometry*. Kathmandu: An unpublished dissertation in math edu., TU.
- Pandit, R. P. ed. al. (2006). *An elementary approach to Mathematical Statistics*. Kathmandu: Mrs, Indira Pandit Publisher 265/32 Santinagar Marga.
- Pangeni, B. R. (2006). *An ethno-mathematics study of Chitwan district*. Kathmandu: An unpublished dissertation in math edu., TU.

Shrestha, L. (2005), *Attitudes of primary level teachers of Kathmandu district towards Geometry*. cited in Chamlagain (2005). Kathmandu: An unpublished dissertation in math edu., TU.

Upadhyay, H. P. (2010) *Method of teaching mathematics*.

Van Hiele's Theory (Clements and Batlista, 1992), *Students' Geometric thoughts develops in five discontinuous level called van Hiele's level (VHLS) namely: Visual (VHL-0), Analysis (VHL-1), Non-formal Deduction (VHL-2), Formal Deduction (VHL-3) and Rigor (VHL-4)*.

[http://en.wikipedia.org/wiki/History\\_of\\_geometry](http://en.wikipedia.org/wiki/History_of_geometry)

[http://en.wikipedia.org/wiki/History\\_of\\_mathematics](http://en.wikipedia.org/wiki/History_of_mathematics)

## APPENDIX- I

### Pre-test Test paper

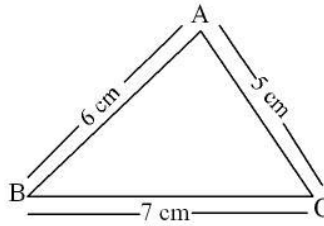
;a} k|Zgx? clgjfo{ 5g\ .

$$10 \times 3 = 30$$

1. bfofFsf] lrqdf

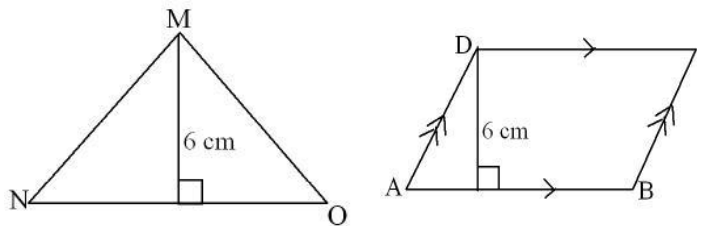
lbOPsf] lqe'hsf]

c4{ kl/lolt lgsfn .



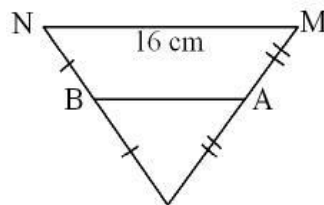
2. tn lbOPsf] lqe'h / ;dfgfGt/ rt'e'{hdf s'gsf] lf]qkmn a9L

x'G5, lgsfn



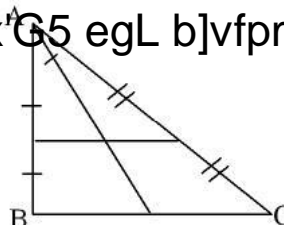
3. lrqdf A / B qmdzM LM / LN e'hfx?sf dWoljGb'x? x'g\

eg] HofldtLo sf/Of;lxt AB sf] nDafO lgsfn .



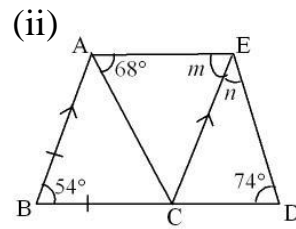
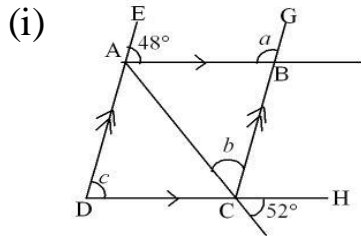
4. lrqdf M / N qmdzM AB / AC sf dWoljGb'x? x'g\ . P, MN sf]

s'g} ljGb' xf] eg]  $AP = PB$  x'G5 egL b]vfpm .

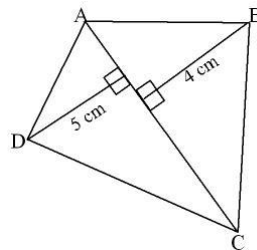




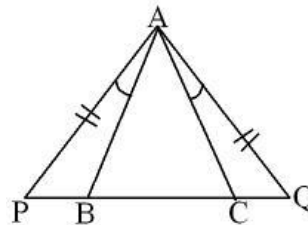
5. IbOPsf] lrqdf a, b, c, m, n cflbn] hgfOPsf sf]0fx?sf] dfg kQf nufpm .



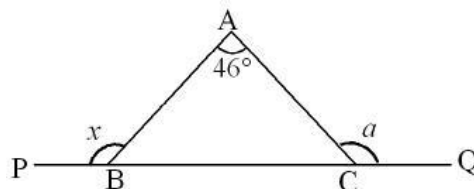
6. lrqdf lbOPsf] rt'e' {hsf] lf]qkmn lgsfn h;df  $AC = 14$  cm 5 .



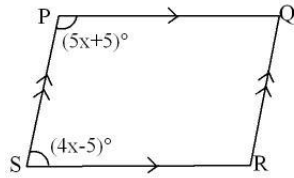
7. lbOPsf] lrqdf  $\angle PAB = \angle QAC$   $AP = AQ$  /  $PB = QC$  5g\, k|df]0ft ug' {xf];\  $\Delta ABC$  ;dlåjfx' lqe'h xf] .



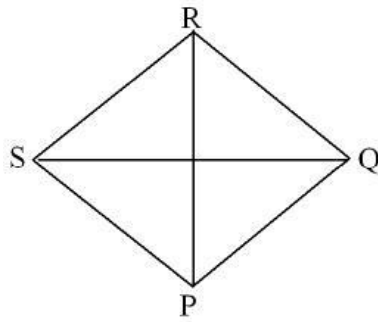
8. lbOPsf] lrqdf sf]0f a nfO { x df JoQm u/ .



9. lbOPsf] ;dfgfGt/ rt'e' {h  $\square PQRS$  df x sf] dfg lgsfn .



10. In  $\square PQRS$  if  $SP = RQ$  /  $RP = SQ$  5 eg]  $RT = ST$   
 x'G5 egL k|df10ft u/ .



## APPENDIX –II

### Post -test Paper

**Full Marks: 30**

Attempt all the questions.

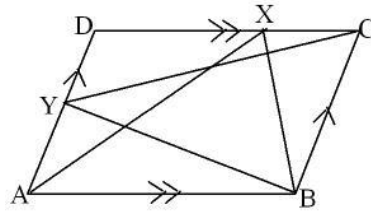
Group-A

5 × 2 = 10

1. In a parallelogram ABCD, X is a point on DC such that AX is drawn. Y is a point on AB such that AY is drawn. Prove that  $\Delta AXB = \Delta BYC$ .

Y is a point on AB such that AY is drawn.

Prove that  $\Delta AXB = \Delta BYC$ .



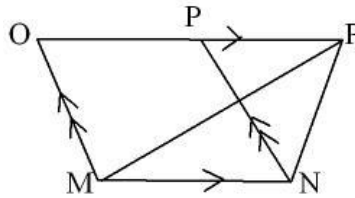
$$\Delta AXB = \Delta BYC$$

u/ .

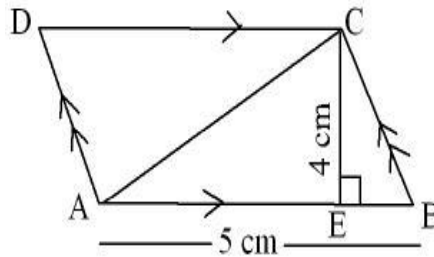
2. In a parallelogram ABCD, P is a point on AC such that BP is drawn. Q is a point on BD such that AQ is drawn. Prove that  $\Delta APQ = \Delta BQP$ .

Q is a point on BD such that AQ is drawn.

Prove that  $\Delta APQ = \Delta BQP$ .



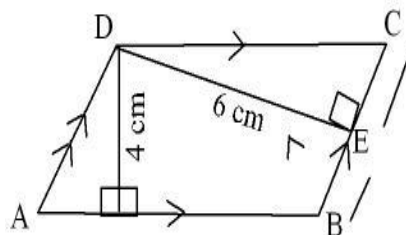
3. In a parallelogram ABCD, E is a point on AB such that CE is drawn. F is a point on CD such that CF is drawn. Prove that  $\Delta CEF = \Delta BFC$ .



4. In a parallelogram ABCD, E is a point on AB such that DE is drawn. F is a point on BC such that DF is drawn. Prove that  $\Delta DEF = \Delta BFD$ .

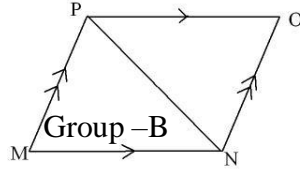
E is a point on AB such that DE is drawn.

F is a point on BC such that DF is drawn. Prove that  $\Delta DEF = \Delta BFD$ .





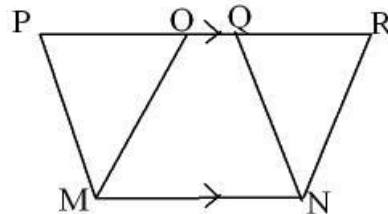
5. Irqdf  $\triangle MNP$  sf] If]qkmn  $12\text{cm}^2$  eP ;dfgfGt/ rt'e' {h MNOP sf] If]qkmn lgsfn .



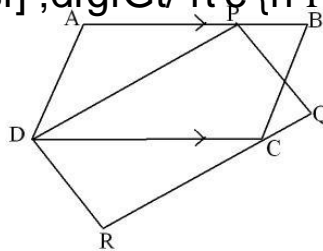
$$5 \times 4 = 20$$

6. Pp6} cfwf/ / pxL ;dfgfGt/ /]vfx? aLr /x]sf lqe'hx?sf] If]qkmn a/fa/ x'G5 egL b]vfpm .

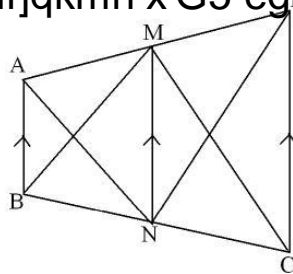
7. Irqdf ;dfgfGt/ rt'e' {h MNRO / ;dfgfGt/ rt'e' {h MNPQ sf] If]qkmn a/fa/ x'G5 egL b]vfpm .



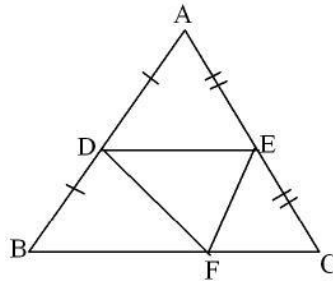
8. ;Fu}sf] Irqdf ABCD / PQRD ;dfgfGt/ rt'e' {hx? x'g\ egL l;4 u/ . ;dfgfGt/ rt'e' {h ABCD sf] ;dfgfGt/ rt'e' {h PQRD sf] If]qkmn .



9. IbOPsf] ;dnDa rt'e' {h  $AB \parallel CD \parallel MN$  eP lqe'h BCM sf] If]qkmn =  $\triangle ADN$  sf] If]qkmn x'G5 egL l;4 u/ .



10.  $\triangle ABC$  में  $D$  /  $E$  क्रमशः  $AB$  /  $AC$  के मध्यबिंदु हैं।  $DE$  को  $BC$  के मध्यबिंदु  $F$  से जोड़ें।  $\triangle DEF$  का क्षेत्रफल  $\triangle ABC$  के क्षेत्रफल का  $\frac{1}{4}$  भाग है।



### APPENDIX -III

#### Teaching Episode-1

#### Unit: Area of triangles and quadrilateral

Main teaching points

Diagonal of parallelogram, divides it into two halves.

Objectives:

At the end of this lesson, the students will be able to prove the theorem 'Diagonal divides parallelogram into two halves theoretically.'

1. Introductory Task (12 min.)

Technique: Group Work

Task:

With the help of Geo board, the researcher presents the plane figure of parallelogram drawing a diagonal and ask students about the properties of parallelogram, e.g. opposite sides of parallelogram are equal, opposite

angles are equal. This completes Van Hiele's level 0 (recognition /visualization) of thinking.

Ask them the five axioms (i.e. SSS, SAS, AAS, ASA and RHS) of making triangle congruent and will ask the step, of proving the theorem 'theoretically'. He will ask the students whether area of triangle are equal or not by folding on diagonal on paper, which is related to Van Hiele's level 1 (Analysis) of thinking.

## 2. Discussion:

What are the step of proving the theorem theoretically, how many diagonals can be drawn on a parallelogram?

Draw any one diagonal and, search the equality between two triangles also investigate by which these two triangle can be made congruent. Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

## 3. Extended task: (proving by theorems)

Let them prove the theorem theoretically in suitable step.

Diagonal divides parallelogram into two halves.

Given: MNOP be a parallelogram in which diagonal PN is joined.

To prove: Area of  $\triangle MNP$  = area of  $\triangle PNO$

Proofs

Statements	Reasons
1. In $\triangle MNP$ and $\triangle PNO$ (i) $MN = PO$ (ii) $MP = NO$ (iii) $PN = PN$ 2. $\triangle MNP \cong \triangle PNO$ 3. Area of $\triangle MNP =$ area of $\triangle PNO$	1. (i) Opposite sides of parallelogram are equal. (ii) Opposite sides of parallelogram are equal. (iii) Common sides 2. By SSS axiom 3. From 2.

Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

Summarizing (5 min.): Diagonal divides the parallelogram into two halves. Since the students established theorem in different postulation system and analysis/comparates these system so they completes Van Hiele's level 4 (Axiomatic / Rigor) of thinking.

## Teaching Episode -2

### Area of triangles and quadrilateral

Main teaching points:

Parallelograms on the same base and between the same parallel lines are equal in area.

Objectives:

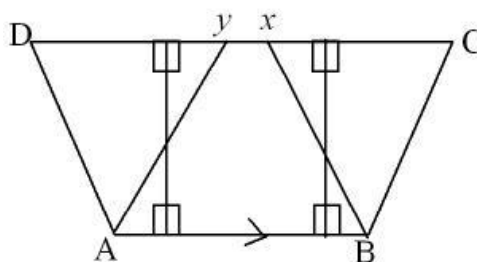
At the end of this lesson the students will be able to prove that theorem 'parallelogram on the same base and between the same parallel lines are equal in area theoretically'.

1. Introductory Task: (10 min.)

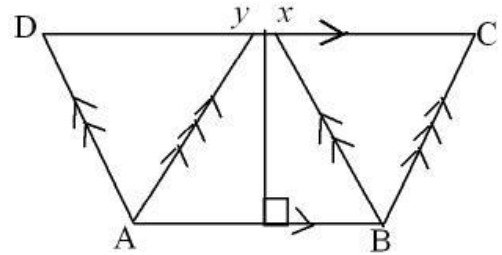
Technique (Group Work)

Researcher draws plane figure of parallelogram with its height and asks the students which one is the base of the parallelogram and which one is the height. This completes Van Hiele's level 0 (recognition /visualization) of thinking.

Ask the students to draw the parallelograms on the same base and between the same parallel lines and also ask them to draw their height and measure their base and height, he would also ask them to find their area, which is related to Van Hiele's level 1 (Analysis) of thinking.



Discussion questions (10 mins.):



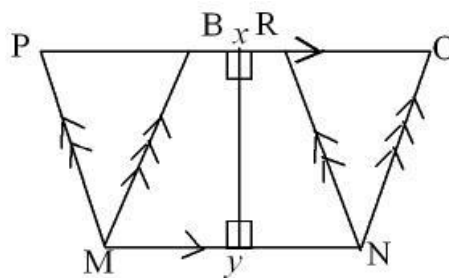
The researcher asks the students different questions e.g. what are name of parallelogram on the given figure? What is their base and between which two lines do the lie? What is their height is it common? Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

How can you find the area of them? In this way the researcher collects the necessary information and draws the conclusion.

3. Extended task: Proving theorem (20 min.):

Let them ask to prove the theorem theoretically.

Statement: Parallelogram on the same base and between the same parallel lines are equal in area.



Given: Parallelogram MNOB and MNRP are on the same base MN and between the same parallel lines MN and OP.

To prove: Area of parallelogram MNOB = Area of parallelogram MNOP.

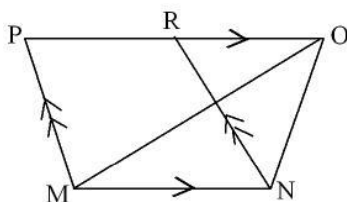
Plan: Draw common height xy.

Proof:

Statements	Reasons
1. Area of parallelogram MNOB = $MN \times xy$	1. Area of parallelogram is base $\times$ height
2. Area of parallelogram MNRP = $MN \times xy$	2. Area of parallelogram is base $\times$ height 3. from (i) and (ii)
3. Area of parallelogram MNOB = area of parallelogram MNRP	

Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

Summary (5 min.): Parallelogram on the same base and between the same parallel lines are equal in area.



Since the students established theorem in different postulation system and analysis/comparisons these systems so they complete Van Hiele's level 4 (Axiomatic / Rigor) of thinking.

## Teaching Episode -3

### Unit: Area of triangles and quadrilateral

Topic: Area of triangle is one half area of parallelogram on the same base and between the same parallel lines.

Objectives:

At the end of this lesson, the students will be able to prove the theorem 'Area of Triangle is one half area of parallelogram on the same base and between the same parallel lines.'

1. Introductory Task (10 min.)

Technique Group Work

The researcher represents the cut put plane figure of parallelogram and triangle on the same base and between the same parallel lines and ask the student to measure the base and the height of triangle and parallelogram. This completes Van Hiele's level 0 (recognition /visualization) of thinking.

He asks students to find the area of parallelogram and triangle, which is related to Van Hiele's level 1 (Analysis) of thinking.

2. Discussion question (10 min.):

The researcher draws the plane figure of parallelogram and triangle on the same base between the same parallel lines on the blackboard and asks the following questions:

What is the base and height of parallelogram as well as triangle?

How can we find the area of parallelogram?



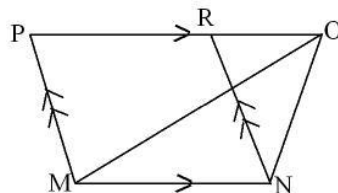
How can we find the area of triangle?

What relation did you see in their area?

In this way with the help of their answer the researcher reach at the conclusion. Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

Extended Task for proving the theorem (20 min.):

The researcher ask the students to prove the area of  $\triangle MNO =$  area of parallelogram MNRP



Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

Summarizing Task (5 min.):

The area of triangle is one half area of parallelogram on the same base and between the same parallel lines. Since the students established theorem in different postulation system and analysis/compares these system so they completes Van Hiele's level 4 (Axiomatic / Rigor) of thinking.

## Teaching Episode-4

### Unit: Area of triangle and quadrilateral

Main teaching point: Triangles on the base and between the same parallel lines equal in area.

**Objectives:** At the end of this lesson the students will be able to prove the theorem 'triangles on the same base and between the same parallel lines are the equal in area.'

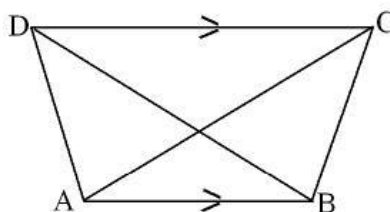
1. Introduction Task (10 min.):

Technique: (Group Work)

The researcher asks the students to draw the triangles on the same base and between the same parallel lines and also asks them their base and let them draw height and asks them to find area and compare also the teacher shows the same thing on Geo board, which is related to Van Hiele's level 1 (Analysis) of thinking.

Discussion: (15 min.)

The researcher asks the students from the discussed figure to write given to prove, plan and proof of the theorem and the students will give their answer and the researcher draw the conclusion with the help of their response.



Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

Extended task (15 min.)

The researcher asks the students to prove  $\text{area of } \triangle ABC = \text{area of } \triangle ABD$  from the given figure alongside. Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

Summarizing (5 min.):

From the above discussion the students will draw the conclusion that the area of triangles on the same base and between the same parallel lines are equal. Since the students established theorem in different postulation system and analysis/compares these system so they completes Van Hiele's level 4 (Axiomatic / Rigor) of thinking.

## Teaching: Episode-5

### Unit: Area of triangle and quadrilateral

Main teaching points:

Area of triangle and quadrilateral (problem) (from exercise 12)

Objectives:

At the end of this lesson the students will be able to solve the problem no. 1 and 2 from exercise 12. (Class 10)

Introductory Task (10 min.):

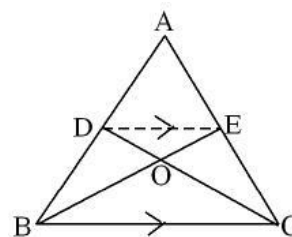
The researcher asks the students to write question no. 2 on their copy and draw the figure as shown in figure alongside.

Question:

In  $\triangle ABC$ , medians  $BE$  and  $CD$  intersect at  $O$  prove that-

(i)  $\triangle CAD = \triangle BAE$

(ii) Quadrilateral  $ADOE = \triangle BOC$



This completes Van Hiele's level 0 (recognition /visualization) of thinking.

After copying question the researcher asks the students:

What is a median? and how many medians can be drawn in triangle? Does a median divides a triangle into 2 halves and draw the conclusion that the line segment drawn from the vertex to the mid point of the opposite side is called median, three medians can be drawn in a triangle

and each diagonal divides triangle onto 2 halves. which is related to Van Hiele's level 1 (Analysis) of thinking.

Discussion Questions (15 min.):

The researcher asks the students following questions:

What are DC and BE? What is the relation between  $\triangle CAD$  and  $\triangle ABC$ ?

What is the relation between  $\triangle BAE$  and  $\triangle ABC$ ?

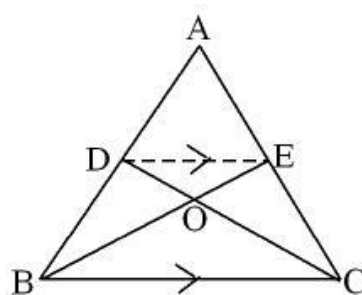
What would be the relation between  $\triangle CAD$  and  $\triangle BAE$ ?

What conclusion do we get if we subtract it  $\triangle COE$  from  $\triangle BAE = \triangle CAD$ ?

After asking such questions and collecting different response the researcher draws the conclusion. Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

3. Extended Task (15 min.):

In the figure alongside,  $DE \parallel BC$ , prove that  $\triangle BOD = \triangle COE$ .



Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

4. Summarizing (5 min.):

The line segment joining the vertex to the opposite side is called median.

Three medians can be drawn in a triangle. Median divides triangles into 2 halves. Since the students established theorem in different postulation system and analysis/compares these system so they completes Van Hiele's level 4 (Axiomatic / Rigor) of thinking.

## Teaching Episode-6

### Unit: Area of triangle and quadrilateral

Topic: Problem related to area of triangle and quadrilateral.

Objectives:

At the end of this lesson the students will be able to solve the problem no. 3 and 4 from exercise -4.

Introductory Task (5 min.):

The researcher tells the summary of previous learned theorem related to area of triangle and quadrilateral as follows:

Parallelogram on the same base and between the same parallel lines are equal in area.

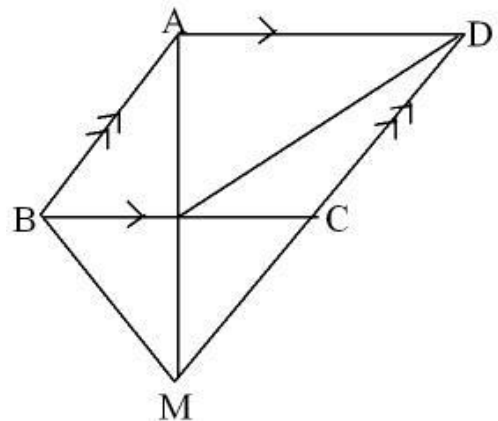
Area of triangle on the same base and between the same parallel lines are equal.

Diagonal divides parallelogram into 2 halves, which is related to Van Hiele's level 1 (Analysis) of thinking.

Discussion Question (15 min.):

The researcher asks the students to write question number 3 and the researcher writes question and draw the figure on blackboard as shown below and have discussion.

In figure ABCD is parallelogram. E is any point on BC. DC and AE is extended to meet at M, then prove  $\triangle DEC = \triangle BEM$ . Here, student logically interrelates previously discovered properties by giving informal argument



and completes Van Hiele's level 2 (informal deduction) of thinking.

After drawing figure the researcher has the discussion as: what is base  $\triangle ABM$  and  $\square ABCD$ ?

What is the base of  $\triangle ABCD$ , between which parallel lies do  $\triangle AED$  and  $\square ABCD$ ?

If  $\triangle AED$  is  $\frac{1}{2} \square ABCD$ , what will be the  $\triangle ABE + \triangle CED$ ?

Is  $\triangle ABM = \triangle ABE + \triangle CED$ ? Can we divide  $\triangle ABM$  into  $\triangle ABE + \triangle BEM$ ?

Is  $\triangle BEM = \triangle CDE$ ?

In this way collecting different response the researcher reaches the conclusion.

Extended Task:

The researcher asks the students to copy question number 4 and solve also the researcher asks the students if they get confusion they will consult the teacher. Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

Reflection: Whole part axiom can be applied in geometry.

The remaining part of half of the parallelogram is also half of it. Since the students established theorem in different postulation system and analysis/comparates these system so they completes Van Hiele's level 4 (Axiomatic / Rigor) of thinking.



## Teaching Episode- 7

### Unit: Area of triangle and quadrilateral.

Topic: Problem related to area of triangle and quadrilateral.

Objectives: At the end of this lesson the students will be able to solve the problem of question no. 5 and 6 of exercise-12.

#### 1. Introductory Task (5 min.):

The researcher revises the previous lesson and also tells them the use of whole part axiom solving problem especially question no. 5.

#### 2. Discussion Question (20 min.):

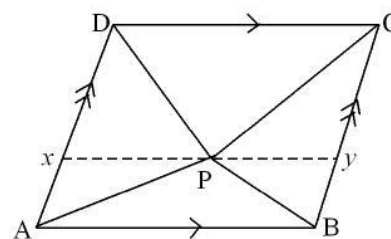
The researcher asks the students to write questions and draws the figure of question no. 5. The researcher also writes the question and draw the figure on blackboard as shown below:

Questions:

In parallelogram ABCD,  $\triangle ABCD$ ,  $\triangle PAD$  and  $\triangle PBC$  are drawn by taking P inside it. Prove that-

$$(i) \triangle PAB + \triangle PDC = \frac{1}{2} \square ABCD$$

$$(ii) \triangle PAD + \triangle PBC = \frac{1}{2} \square ABCD$$



This completes Van Hiele's level 0 (recognition /visualization) of thinking.

After drawing the figure on the blackboard the researcher asks students to draw  $xy \parallel AB$  or  $CD$  from P and have discussion as follows:

$$\square AB_{yx} + \square xyCD = \square ABCD?$$

$$\Delta PAB = \frac{1}{2} \square AB_{yx} = ?$$

$$\Delta PCD = \frac{1}{2} \square xyCD?$$

which is related to Van Hiele's level 1 (Analysis) of thinking.

What relation do you get from three statements?

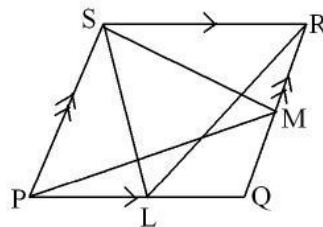
The researcher will ask and collect the response of the students and reaches to the conclusion. Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

Extended Task (20 min.):

The researcher asks the students to copy question 6 and uses the relation of triangle and parallelogram on the same base and find the solution, which is copied alongside.

Question: PQRS is a parallelogram, L and M are the point on PQ and QR respectively, prove that:

$$\Delta RLS = \Delta PQM + \Delta RSM$$



Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

## Teaching Episode-8

### Unit: Area of triangle and quadrilateral

Topic: Problem related to area of triangle and quadrilateral

Objectives:

At the end this lesson the students will be able to solve question no. 8 and 9 of exercise -12.

Introductory Task (5 min.):

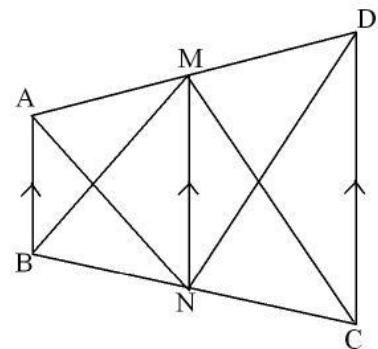
The researcher revises the previous lesson and encourages students to apply those theorems to solve the problem.

Discussion Question (15 min.):

The researcher asks the students to write question no. 9 in copy also to draw its give figure. The researcher will also writes question and draws figure on blackboard and have discussion.

In the given rhombus alongside  
 $AB \parallel CD \parallel MN$

Prove that:  $\triangle BCM = \triangle AND$



This completes Van Hiele's level 0  
(recognition /visualization) of thinking.

Are  $\triangle BMN$  and  $\triangle AMN$  in B same base? What will be their relation?

Are  $\triangle MNC$  and  $\triangle MND$  in same base? What will be their relation?

If equal is added to the equal quantity, what will be the result? which is related to Van Hiele's level 1 (Analysis) of thinking.

Can you use addition axiom between  $\triangle BMN$  and  $\triangle MNC$  also  $\triangle AMN$  and  $\triangle MND$ ? If so, what result do you get?

Do you get  $\triangle AND = \triangle BMC$ ?

Here, student logically interrelates previously discovered properties by giving informal argument and completes Van Hiele's level 2 (informal deduction) of thinking.

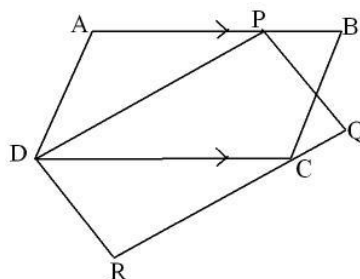
In this way, asking such several questions and collecting response from the students the researcher reaches at the conclusion. Since the students prove the theorem deductively and establishes interrelationship among network of theorem so they complete Van Hiele's level 3 (deduction/formal proof) of thinking.

Extended Task (20 min.):

The researcher asks the students to copy the question no. 8 in their copy as shown alongside figure and suggests them to join PC and compare the relationship of  $\triangle PCD$  and  $\square ABCD$  also  $\triangle PCD$  and  $\square PQRD$  and reached the conclusion.

Question: In the figure alongside, ABCD and PQRD are parallelogram, prove that:

$$\square ABCD = \square PQRD$$



Since the students established theorem in different postulation system and analysis/compares these system so they completes Van Hiele's level 4 (Axiomatic / Rigor) of thinking.

## APPENDIX –IV

### Achievement Score of Students in Experimental and Control Group

Experimental Group			Control Group	
S. N.	Name the students	Obtained Marks	Name the students	Obtained Marks
1	Anil Subba	28	Kishan Mahara	26
2	Yubaraj Rai	27	Narbala Sapkota	24
3	Apsara Adhikari	27	Babita Rai	20
4	Sushma Bhattarai	25	Man Kr. Shrestha	15
5	Sharmila Kalikote	25	Rubi Devi Karki	14
6	Anil Baral	23	Prabita Bajgain	14
7	Nira Rai	22	Pradip Nepal	12
8	Shanta Khajum	22	Kamala Limbu	8
9	Mima Devi Budhathoki	21	Srijana Agrawal	8
10	Udaya Murmu	20	Ritu Bastola	7
11	Chandrakala Dahal	20	Saraswati Tudu	6
12	Man Br. Khulal	19	Bir Br. Budhathoki	6
13	Januka Adhikari	18	Saraswati Adhikari	6
14	Mina Thakur	16	Bishal Kirku	6
15	Ganga Lamichhane	14	Aisa Rai	5
16	Bir Br. Tamang	14	Bhawana Dahal	5
17	Bhakta Tajpuriya	14	Prakash Sapkota	4
18	Kamala Chemjong	13	Umesh Hembram	4
19	Sita Paudel	11	Pratap Rai	3

**(a) Achievement of Experimental and Control Group Students**

Table No. 3

Experimental Group				Control Group			
S.N.	Obtained Score (X <sub>1</sub> )	x <sub>1</sub> =X <sub>1</sub> - $\bar{X}_1$	x <sup>2</sup>	S.N.	Obtained Score (X <sub>2</sub> )	x <sub>2</sub> =X <sub>2</sub> - $\bar{X}_2$	x <sup>2</sup>
1	28	8.05	64.8025	1	26	15.84	250.9056
2	27	7.05	49.7025	2	24	13.84	191.5456
3	27	7.05	49.7025	3	20	9.84	96.8256
4	25	5.05	25.5025	4	15	4.84	23.4256
5	25	5.05	25.5025	5	14	3.84	14.7456
6	23	3.05	9.3025	6	14	3.84	14.7456
7	22	2.05	4.2025	7	12	1.84	3.3856
8	22	2.05	4.2025	8	8	-2.16	4.6656
9	21	1.05	1.1025	9	8	-2.16	4.6656
10	20	0.05	0.0025	10	7	-3.16	9.9856
11	20	0.05	0.0025	11	6	-4.16	17.3056
12	19	-0.95	0.9025	12	6	-4.16	17.3056
13	18	-1.95	3.8025	13	6	-4.16	17.3056
14	16	-3.95	15.6025	14	6	-4.16	17.3056
15	14	-5.95	35.4025	15	5	-5.16	26.6256
16	14	-5.95	35.4025	16	5	-5.16	26.6256
17	14	-5.95	35.4025	17	4	-6.16	26.6256
18	13	-6.95	48.3025	18	4	-6.16	26.6256
19	11	-8.95	80.1025	19	3	-7.16	51.2656
			488.9475				841.8856

$$\text{Mean } \bar{X}_1 = \frac{\sum X_1}{N} = \frac{379}{19} = 19.95 \quad \sigma_1 = \sqrt{\frac{\sum x_1^2}{N}} = \sqrt{\frac{488.9475}{19}} = \sqrt{25.734} = 5.07$$

**(b) Achievement of Experimental Group Students by Gender**

Table No. 4

Boy				Girl			
S.N.	Obtained Score (X <sub>1</sub> )	x <sub>1</sub> =X <sub>1</sub> - $\bar{X}_1$	x <sup>2</sup>	S.N.	Obtained Score (X <sub>2</sub> )	x <sub>2</sub> =X <sub>2</sub> - $\bar{X}_2$	x <sup>2</sup>
1	28	7.29	53.14	1	27	6.5	42.25
2	27	6.29	39.56	2	25	4.5	20.25
3	23	2.29	5.24	3	25	4.5	20.25
4	20	-0.71	0.50	4	22	1.5	2.25
5	19	-1.71	2.92	5	22	1.5	2.25
6	14	-6.71	45.02	6	21	0.5	0.25
7	14	-6.71	45.02	7	20	-0.5	0.25
				8	18	-2.5	6.25
				9	16	-4.5	20.25
				10	16	-4.5	20.25
				11	13	-7.5	56.25
				12	11	-9.5	90.25
			145				281

$$\text{Mean } \bar{X}_1 = \frac{\sum X_1}{N_1} = \frac{145}{7} = 20.71 \quad \sigma_1 = \sqrt{\frac{\sum x_1^2}{N}} = \sqrt{\frac{191.4}{7}} = \sqrt{27.342} = 5.22$$

$$\text{Mean } \bar{X}_2 = \frac{\sum X_2}{N_2} = \frac{246}{12} = 20.5 \quad \sigma_2 = \sqrt{\frac{\sum x_2^2}{N}} = \sqrt{\frac{281}{12}} = \sqrt{23.416} = 4.83$$

**(c) Achievement of Control Group Students by Gender**

Table No. 5

Girl				Boy			
S.N.	Obtained Score (X <sub>1</sub> )	x <sub>1</sub> =X <sub>1</sub> - $\bar{X}_1$	x <sup>2</sup>	S.N.	Obtained Score (X <sub>2</sub> )	x <sub>2</sub> =X <sub>2</sub> - $\bar{X}_2$	x <sup>2</sup>
1	24	13.55	183.36	1	26	16.5	272.25
2	20	9.55	91.20	2	15	5.5	30.25
3	14	3.55	12.60	3	12	2.5	6.25
4	14	3.55	12.60	4	6	-3.5	12.25
5	8	-2.45	6	5	6	-3.5	12.25
6	8	-2.45	6	6	4	-5.5	30.25
7	7	-3.45	11.9	7	4	-5.5	30.25
8	6	-4.45	19.8	8	3	-6.5	42.25
9	5	-5.45	29.7				
10	5	-5.45	29.7				
11	4	-6.45	41.6				

$$\sum X_1 = 97$$

$$\sum X_2 = 76$$

$$\text{Mean } \bar{X}_1 = \frac{\sum X_1}{N_1} = \frac{115}{11} = 10.45 \quad \sigma_1 = \sqrt{\frac{\sum x_1^2}{N}} = \sqrt{\frac{444.4625}{11}} = \sqrt{40.411} = 6.35$$

$$\text{Mean } \bar{X}_2 = \frac{\sum X_2}{N_2} = \frac{76}{8} = 9.5 \quad \sigma_2 = \sqrt{\frac{\sum x_2^2}{N}} = \sqrt{\frac{436}{8}} = \sqrt{54.5} = 7.38$$



